Outline

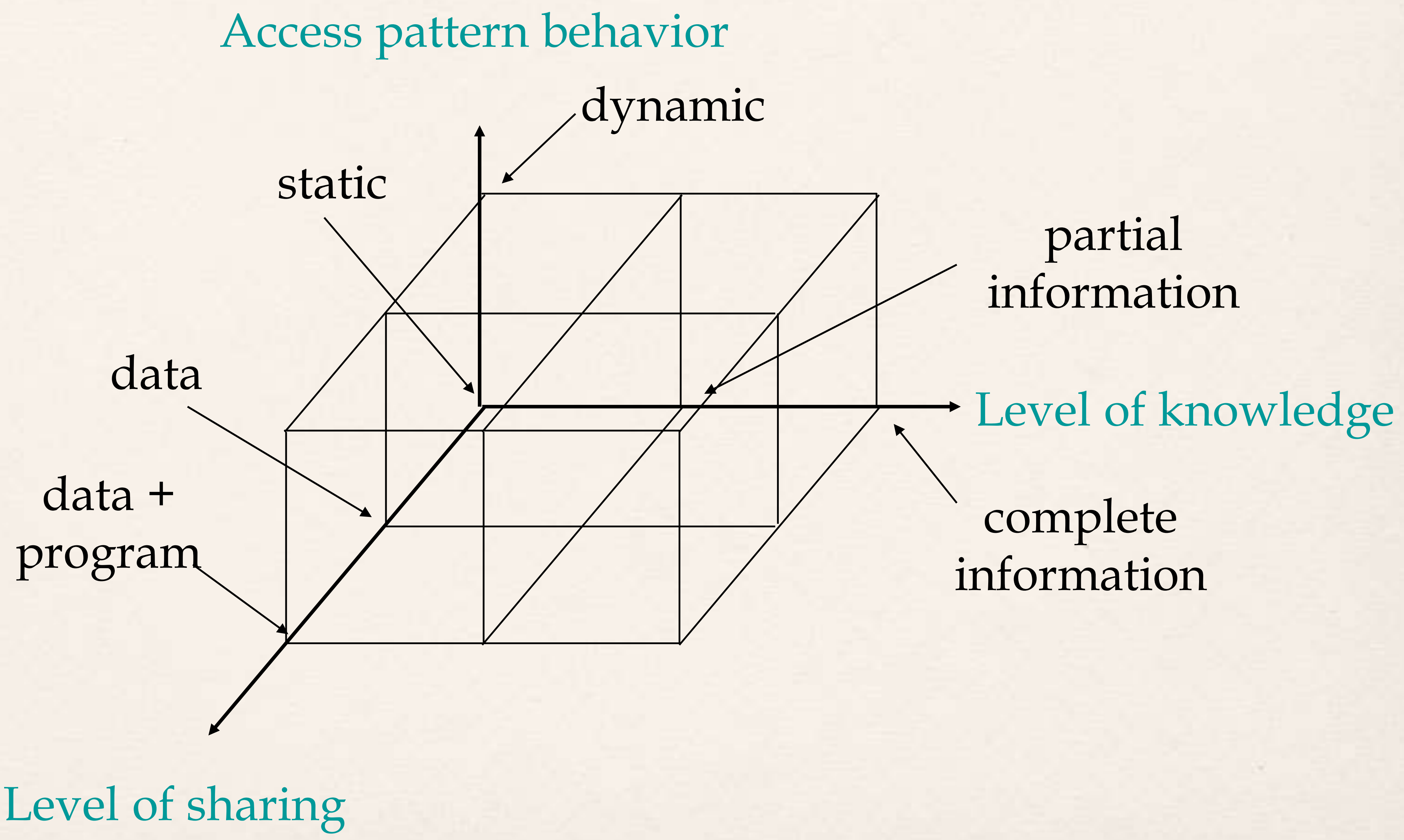
Design Problem

- In the general setting :

  Making decisions about the placement of data and programs across the sites of a computer network as well as possibly designing the network itself.

- In Distributed DBMS, the placement of applications entails
  - ➡ placement of the distributed DBMS software; and
  - ➡ placement of the applications that run on the database

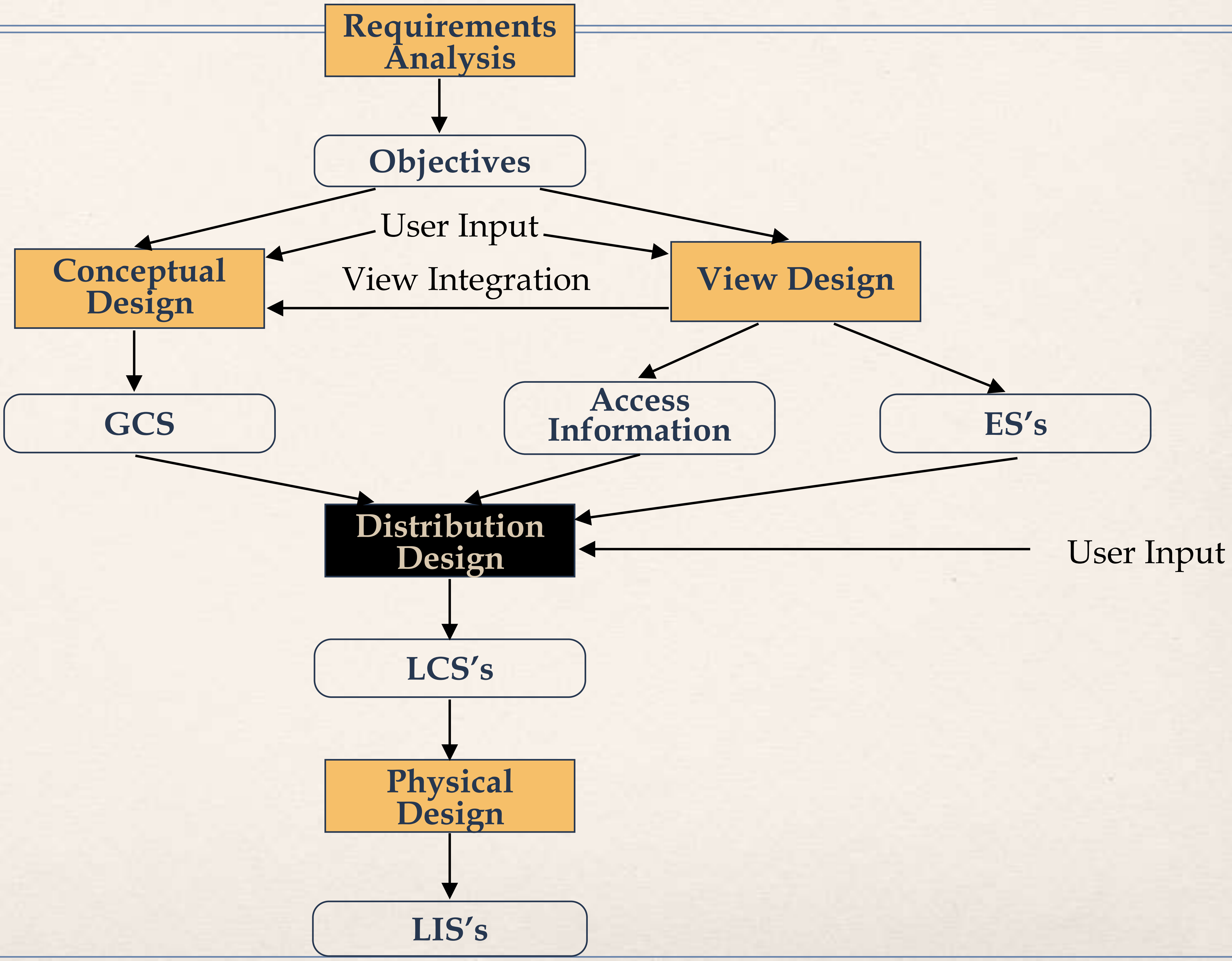
Dimensions of the Problem



Distribution Design

- Top-down
  - ➞ mostly in designing systems from scratch
  - ➞ mostly in homogeneous systems
- Bottom-up
  - ➞ when the databases already exist at a number of sites

Top-Down Design



Distribution Design Issues

1. Why fragment at all?
2. How to fragment?
3. How much to fragment?
4. How to test correctness?
5. How to allocate?
6. Information requirements?

Fragmentation

- Can't we just distribute relations?
- What is a reasonable unit of distribution?
  - relation
    - views are subsets of relations ➔locality
    - extra communication
  - fragments of relations (sub-relations)
    - concurrent execution of a number of transactions that access different portions of a relation
    - views that cannot be defined on a single fragment will require extra processing
    - semantic data control (especially integrity enforcement) more difficult

Fragmentation Alternatives – Horizontal

$PROJ_1$: projects with budgets less than $200,000

$PROJ_2$: projects with budgets greater than or equal to $200,000

PROJ

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal
P2	Database Develop.	135000	New York
P3	CAD/CAM	250000	New York
P4	Maintenance	310000	Paris
P5	CAD/CAM	500000	Boston

$PROJ_1$

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal
P2	Database Develop.	135000	New York

$PROJ_2$

PNO	PNAME	BUDGET	LOC
P3	CAD/CAM	250000	New York
P4	Maintenance	310000	Paris
P5	CAD/CAM	500000	Boston

Fragmentation Alternatives – Vertical

PROJ$_1$: information about project budgets

PROJ$_2$: information about project names and locations

PROJ

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal
P2	Database Develop.	135000	New York
P3	CAD/CAM	250000	New York
P4	Maintenance	310000	Paris
P5	CAD/CAM	500000	Boston

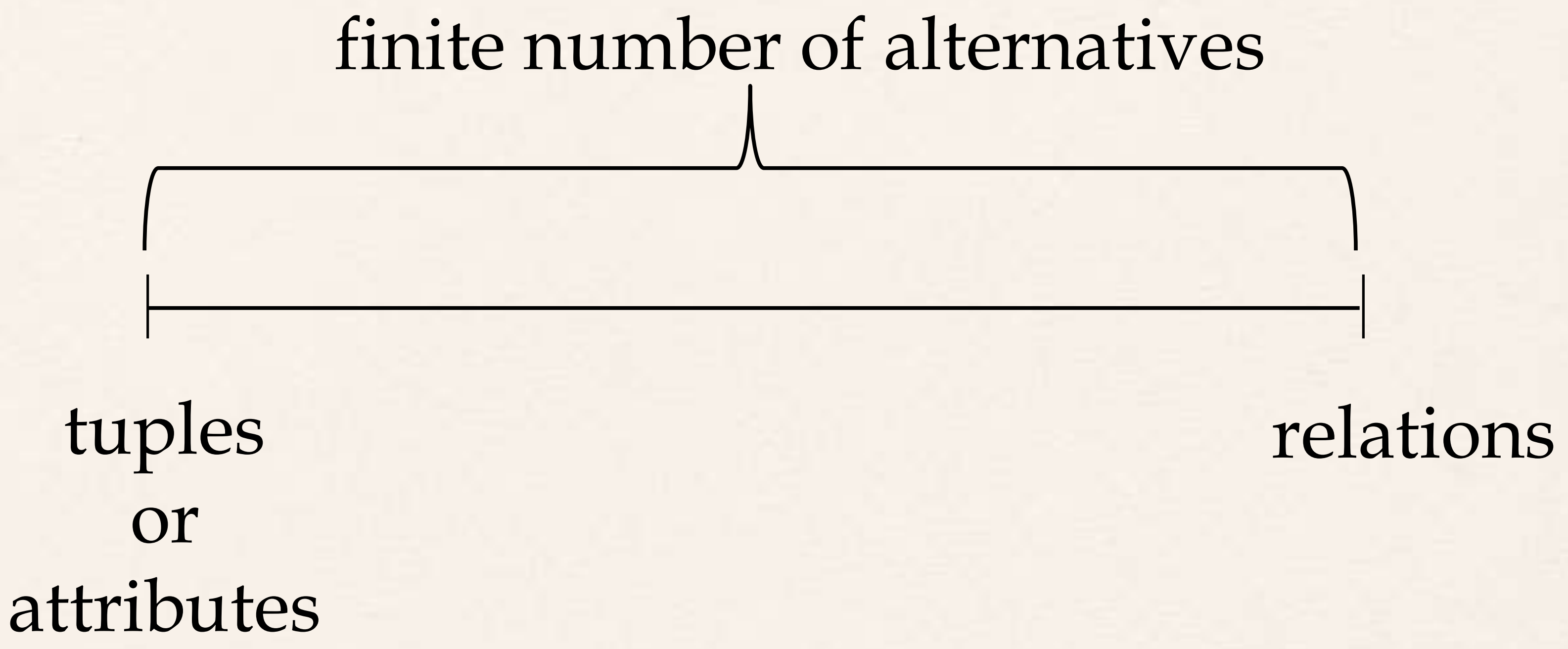
PROJ$_1$

PNO	BUDGET
P1	150000
P2	135000
P3	250000
P4	310000
P5	500000

PROJ$_2$

PNO	PNAME	LOC
P1	Instrumentation	Montreal
P2	Database Develop.	New York
P3	CAD/CAM	New York
P4	Maintenance	Paris
P5	CAD/CAM	Boston

Degree of Fragmentation



Finding the suitable level of partitioning within this range

Correctness of Fragmentation

- Completeness
  - ➡ Decomposition of relation R into fragments $R_1, R_2, ..., R_n$ is complete if and only if each data item in R can also be found in some R_i
- Reconstruction
  - ➡ If relation R is decomposed into fragments $R_1, R_2, ..., R_n$, then there should exist some relational operator ∇ such that

    $$R = \nabla_{1 \le i \le n} R_i$$
- Disjointness
  - ➡ If relation R is decomposed into fragments $R_1, R_2, ..., R_n$, and data item d_i is in R_j, then d_i should not be in any other fragment R_k ($k \neq j$).

Allocation Alternatives

- Non-replicated
  - ➡ partitioned : each fragment resides at only one site
- Replicated
  - ➡ fully replicated : each fragment at each site
  - ➡ partially replicated : each fragment at some of the sites
- Rule of thumb:

  If $\frac{\text{read-only queries}}{\text{update queries}} << 1$, replication is advantageous,

  otherwise replication may cause problems

Comparison of Replication Alternatives

	Full-replication	Partial-replication	Partitioning
QUERY PROCESSING	Easy	← Same Difficulty →	
DIRECTORY MANAGEMENT	Easy or Non-existant	← Same Difficulty →	
CONCURRENCY CONTROL	Moderate	Difficult	Easy
RELIABILITY	Very high	High	Low
REALITY	Possible application	Realistic	Possible application

Information Requirements

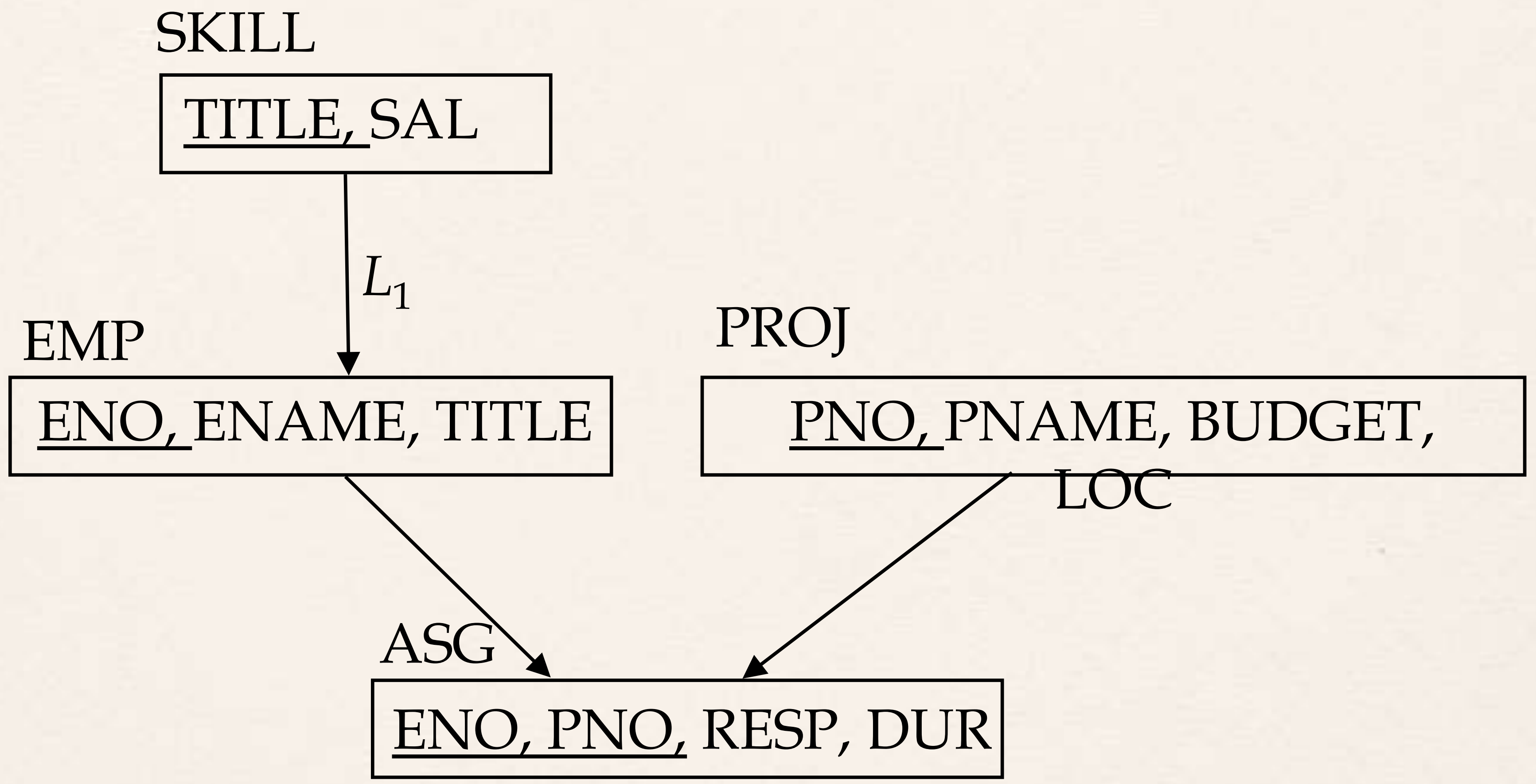
- Four categories:
  - ➡ Database information
  - ➡ Application information
  - ➡ Communication network information
  - ➡ Computer system information

Fragmentation

- Horizontal Fragmentation (HF)
  - → Primary Horizontal Fragmentation (PHF)
  - → Derived Horizontal Fragmentation (DHF)
- Vertical Fragmentation (VF)
- Hybrid Fragmentation (HF)

PHF – Information Requirements

- Database Information
  - relationship



  - cardinality of each relation: *card*(*R*)

PHF - Information Requirements

- Application Information
  - **simple predicates** : Given $R[A_1, A_2, \ldots, A_n]$, a simple predicate p_j is

    $$p_j : A_i \, \theta \, Value$$

    where $\theta \in \{=,<,\leq,>,\geq,\neq\}$, $Value \in D_i$ and D_i is the domain of A_i.

    For relation R we define $Pr = \{p_1, p_2, \ldots, p_m\}$

    Example :

    PNAME = "Maintenance"

    BUDGET ≤ 200000
  - **minterm predicates** : Given R and $Pr = \{p_1, p_2, \ldots, p_m\}$

    define $M = \{m_1, m_2, \ldots, m_r\}$ as

    $$M = \{ m_i \mid m_i = \bigwedge_{p_j \in Pr} p_j^* \}, 1 \leq j \leq m, 1 \leq i \leq z$$

    where $p_j^* = p_j$ or $p_j^* = \neg(p_j)$.

PHF – Information Requirements

Example

m_1: PNAME="Maintenance" $\wedge$ BUDGET≤200000

m_2: **NOT**(PNAME="Maintenance") $\wedge$ BUDGET≤200000

m_3: PNAME= "Maintenance" $\wedge$ **NOT**(BUDGET≤200000)

m_4: **NOT**(PNAME="Maintenance") $\wedge$ **NOT**(BUDGET≤200000)

PHF – Information Requirements

- Application Information
  - **minterm selectivitie**s: *sel*(m_i)
    - The number of tuples of the relation that would be accessed by a user query which is specified according to a given minterm predicate m_i.
  - **access frequencies**: *acc*(q_i)
    - The frequency with which a user application *qi* accesses data.
    - Access frequency for a minterm predicate can also be defined.

Primary Horizontal Fragmentation

Definition :

$$R_j = \sigma_{F_j}(R),\ 1 \le j \le w$$

where F_j is a selection formula, which is (preferably) a minterm predicate.

Therefore,

A horizontal fragment R_i of relation R consists of all the tuples of R which satisfy a minterm predicate m_i.

$$\Downarrow$$

Given a set of minterm predicates M, there are as many horizontal fragments of relation R as there are minterm predicates.

Set of horizontal fragments also referred to as minterm fragments.

PHF – Algorithm

Given: A relation *R*, the set of simple predicates *Pr*

Output: The set of fragments of *R* = $\{R_1, R_2, \ldots, R_w\}$ which obey the fragmentation rules.

Preliminaries :

- *Pr* should be *complete*
- *Pr* should be *minimal*

Completeness of Simple Predicates

- A set of simple predicates *Pr* is said to be *complete* if and only if the accesses to the tuples of the minterm fragments defined on *Pr* requires that two tuples of the same minterm fragment have the same probability of being accessed by any application.

- Example :
  - ➡ Assume PROJ[PNO,PNAME,BUDGET,LOC] has two applications defined on it.
  - ➡ Find the budgets of projects at each location. (1)
  - ➡ Find projects with budgets less than $200000. (2)

Completeness of Simple Predicates

According to (1),

Pr={LOC="Montreal",LOC="New York",LOC="Paris"}

which is not complete with respect to (2).

Modify

Pr ={LOC="Montreal",LOC="New York",LOC="Paris",
BUDGET≤200000,BUDGET>200000}

which is complete.

Minimality of Simple Predicates

- If a predicate influences how fragmentation is performed, (i.e., causes a fragment f to be further fragmented into, say, f_i and f_j) then there should be at least one application that accesses f_i and f_j differently.
- In other words, the simple predicate should be *relevant* in determining a fragmentation.
- If all the predicates of a set *Pr* are relevant, then *Pr* is *minimal*.

$$\frac{acc(m_i)}{card(f_i)} \neq \frac{acc(m_j)}{card(f_j)}$$

Minimality of Simple Predicates

Example :

Pr ={LOC="Montreal",LOC="New York", LOC="Paris", BUDGET≤200000,BUDGET>200000}

is minimal (in addition to being complete). However, if we add

PNAME = "Instrumentation"

then *Pr* is not minimal.

COM_MIN Algorithm

Given:	a relation *R* and a set of simple predicates *Pr*
Output:	a *complete* and *minimal* set of simple predicates *Pr'* for *Pr*

Rule 1: a relation or fragment is partitioned into at least two parts which are accessed differently by at least one application.

COM_MIN Algorithm

❶ Initialization :

- find a $p_i \in Pr$ such that p_i partitions R according to *Rule 1*
- set $Pr' = p_i$; $Pr \leftarrow Pr - \{p_i\}$; $F \leftarrow \{f_i\}$

❷ Iteratively add predicates to Pr' until it is complete

- find a $p_j \in Pr$ such that p_j partitions some f_k defined according to minterm predicate over Pr' according to *Rule 1*
- set $Pr' = Pr' \cup \{p_i\}$; $Pr \leftarrow Pr - \{p_i\}$; $F \leftarrow F \cup \{f_i\}$
- if $\exists p_k \in Pr'$ which is nonrelevant then

  $Pr' \leftarrow Pr - \{p_i\}$

  $F \leftarrow F - \{f_i\}$

PHORIZONTAL Algorithm

Makes use of COM_MIN to perform fragmentation.

Input: a relation R and a set of simple predicates Pr

Output: a set of minterm predicates M according to which relation R is to be fragmented

❶ $Pr' \leftarrow$ COM_MIN (R,Pr)

❷ determine the set M of minterm predicates

❸ determine the set I of implications among $p_i \in Pr$

❹ eliminate the contradictory minterms from M

PHF – Example

- Two candidate relations : PAY and PROJ.
- Fragmentation of relation PAY
  - Application: Check the salary info and determine raise.
  - Employee records kept at two sites application run at two sites
  - Simple predicates

    p_1 : SAL $\leq$ 30000

    p_2 : SAL $>$ 30000

    $Pr = \{p_1, p_2\}$ which is complete and minimal $Pr' = Pr$
  - Minterm predicates

    m_1 : (SAL $\leq$ 30000)

    m_2 : **NOT**(SAL $\leq$ 30000) = (SAL $>$ 30000)

PHF – Example

PAY_1

TITLE	SAL
Mech. Eng.	27000
Programmer	24000

PAY_2

TITLE	SAL
Elect. Eng.	40000
Syst. Anal.	34000

PHF – Example

- Fragmentation of relation PROJ
  - Applications:
    - Find the name and budget of projects given their no.
      - ✓ Issued at three sites
    - Access project information according to budget
      - ✓ one site accesses ≤200000 other accesses >200000
  - Simple predicates
  - For application (1)

    p_1 : LOC = “Montreal”

    p_2 : LOC = “New York”

    p_3 : LOC = “Paris”
  - For application (2)

    p_4 : BUDGET $\leq$ 200000

    p_5 : BUDGET $>$ 200000
  - $Pr = Pr' = \{p_1, p_2, p_3, p_4, p_5\}$

PHF – Example

- Fragmentation of relation PROJ continued
  - Minterm fragments left after elimination

m_1 : (LOC = “Montreal”) $\wedge$ (BUDGET $\leq$ 200000)

m_2 : (LOC = “Montreal”) $\wedge$ (BUDGET $>$ 200000)

m_3 : (LOC = “New York”) $\wedge$ (BUDGET $\leq$ 200000)

m_4 : (LOC = “New York”) $\wedge$ (BUDGET $>$ 200000)

m_5 : (LOC = “Paris”) $\wedge$ (BUDGET $\leq$ 200000)

m_6 : (LOC = “Paris”) $\wedge$ (BUDGET $>$ 200000)

PHF – Example

$PROJ_1$

PNO	PNAME	BUDGET	LOC
P1	Instrumentation	150000	Montreal

$PROJ_2$

PNO	PNAME	BUDGET	LOC
P2	Database Develop.	135000	New York

$PROJ_4$

PNO	PNAME	BUDGET	LOC
P3	CAD/CAM	250000	New York

$PROJ_6$

PNO	PNAME	BUDGET	LOC
P4	Maintenance	310000	Paris

PHF – Correctness

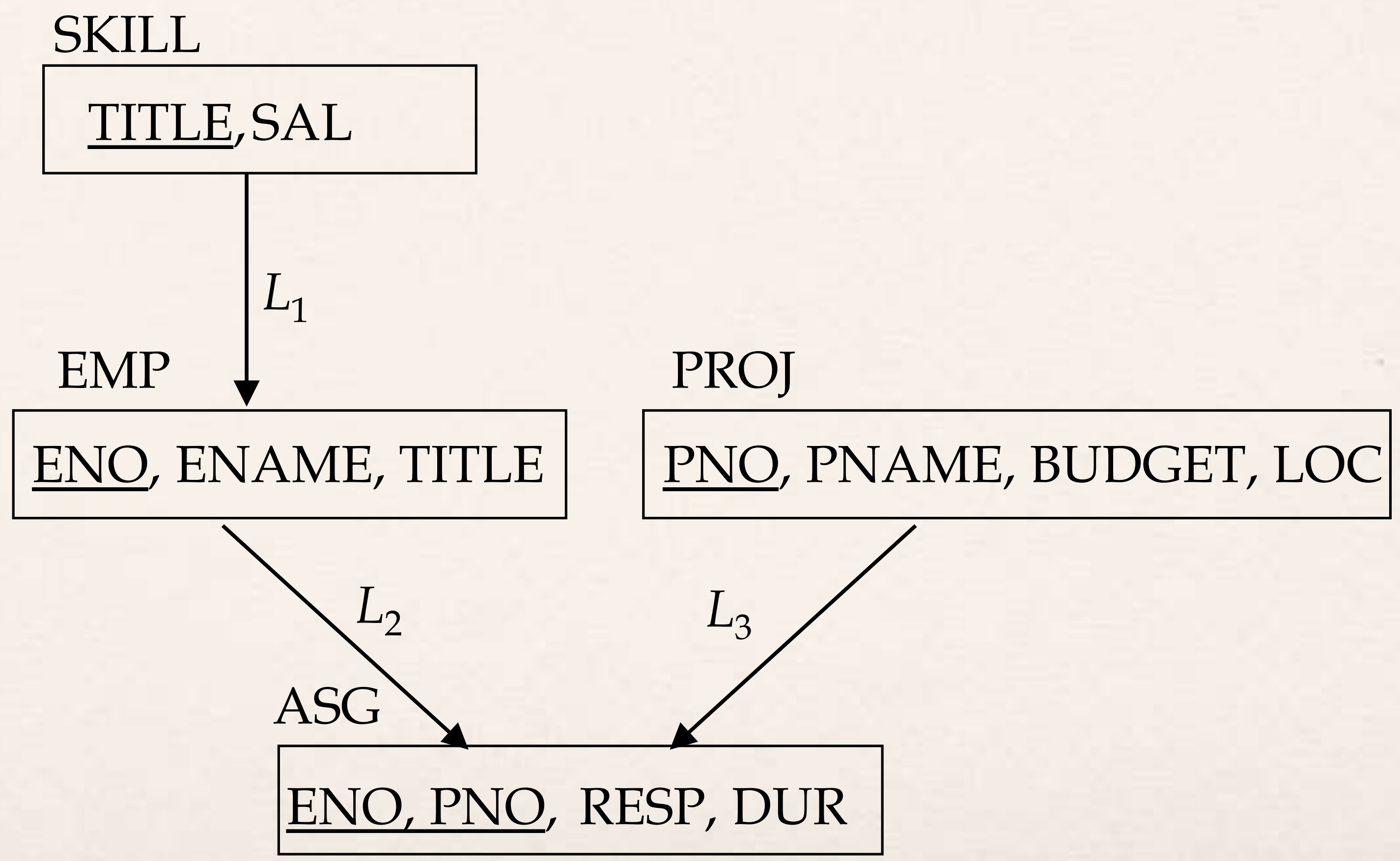
- Completeness
  - → Since *Pr*' is complete and minimal, the selection predicates are complete
- Reconstruction
  - → If relation *R* is fragmented into $F_R = \{R_1, R_2, \ldots, R_r\}$

$$R = \bigcup_{\forall R_i \in F_R} R_i$$

- Disjointness
  - → Minterm predicates that form the basis of fragmentation should be mutually exclusive.

Derived Horizontal Fragmentation

- Defined on a member relation of a link according to a selection operation specified on its owner.
  - Each link is an equijoin.
  - Equijoin can be implemented by means of semijoins.



DHF – Definition

Given a link L where $owner(L)=S$ and $member(L)=R$, the derived horizontal fragments of R are defined as

$$R_i = R \ltimes_F S_i,\ 1 \leq i \leq w$$

where w is the maximum number of fragments that will be defined on R and

$$S_i = \sigma_{F_i}(S)$$

where F_i is the formula according to which the primary horizontal fragment S_i is defined.

DHF – Example

Given link L_1 where owner(L_1)=SKILL and member(L_1)=EMP

$EMP_1 = EMP \ltimes SKILL_1$

$EMP_2 = EMP \ltimes SKILL_2$

where

$SKILL_1 = \sigma_{SAL \leq 30000}(SKILL)$

$SKILL_2 = \sigma_{SAL > 30000}(SKILL)$

EMP_1

ENO	ENAME	TITLE
E3	A. Lee	Mech. Eng.
E4	J. Miller	Programmer
E7	R. Davis	Mech. Eng.

EMP_2

ENO	ENAME	TITLE
E1	J. Doe	Elect. Eng.
E2	M. Smith	Syst. Anal.
E5	B. Casey	Syst. Anal.
E6	L. Chu	Elect. Eng.
E8	J. Jones	Syst. Anal.

DHF – Correctness

- Completeness
  - ➡ Referential integrity
  - ➡ Let R be the member relation of a link whose owner is relation S which is fragmented as $F_S = \{S_1, S_2, ..., S_n\}$. Furthermore, let A be the join attribute between R and S. Then, for each tuple t of R, there should be a tuple t' of S such that

$$t[A] = t'[A]$$

- Reconstruction
  - ➡ Same as primary horizontal fragmentation.
- Disjointness
  - ➡ Simple join graphs between the owner and the member fragments.

Vertical Fragmentation

- Has been studied within the centralized context
  - ➡ design methodology
  - ➡ physical clustering
- More difficult than horizontal, because more alternatives exist.

  Two approaches :
  - ➡ grouping
    - ✦ attributes to fragments
  - ➡ splitting
    - ✦ relation to fragments

Vertical Fragmentation

- Overlapping fragments
  - ➡ grouping
- Non-overlapping fragments
  - ➡ splitting

We do not consider the replicated key attributes to be overlapping.

Advantage:

Easier to enforce functional dependencies

(for integrity checking etc.)

VF – Information Requirements

- Application Information
  - Attribute affinities
    - a measure that indicates how closely related the attributes are
    - This is obtained from more primitive usage data
  - Attribute usage values
    - Given a set of queries $Q = \{q_1, q_2,\ldots, q_q\}$ that will run on the relation $R[A_1, A_2,\ldots, A_n]$,

$$use(q_i,A_j) = \begin{cases} 1 \text{ if attribute } A_j \text{ is referenced by query } q_i \\ 0 \text{ otherwise} \end{cases}$$

$use(q_i,\bullet)$ can be defined accordingly

VF – Definition of *use*(q_i,A_j)

Consider the following 4 queries for relation PROJ

q_1: **SELECT** BUDGET
FROM PROJ
WHERE PNO=Value

q_2: **SELECT** PNAME,BUDGET
FROM PROJ

q_3: **SELECT** PNAME
FROM PROJ
WHERE LOC=Value

q_4: **SELECT** **SUM**(BUDGET)
FROM PROJ
WHERE LOC=Value

Let A_1= PNO, A_2= PNAME, A_3= BUDGET, A_4= LOC

$$\begin{array}{c|cccc|} & A_1 & A_2 & A_3 & A_4 \\ q_1 & 1 & 0 & 1 & 0 \\ q_2 & 0 & 1 & 1 & 0 \\ q_3 & 0 & 1 & 0 & 1 \\ q_4 & 0 & 0 & 1 & 1 \end{array}$$

VF – Affinity Measure $aff(A_i,A_j)$

The attribute affinity measure between two attributes A_i and A_j of a relation $R[A_1, A_2, \ldots, A_n]$ with respect to the set of applications $Q = (q_1, q_2, \ldots, q_q)$ is defined as follows :

$$aff(A_i, A_j) = \sum_{\text{all queries that access } A_i \text{ and } A_j} (\text{query access})$$

$$\text{query access} = \sum_{\text{all sites}} \text{access frequency of a query} * \frac{\text{access}}{\text{execution}}$$

VF – Calculation of *aff*(A_i, A_j)

Assume each query in the previous example accesses the attributes once during each execution.

Also assume the access frequencies

$$\begin{array}{c|ccc} & S_1 & S_2 & S_3 \\ \hline q_1 & 15 & 20 & 10 \\ q_2 & 5 & 0 & 0 \\ q_3 & 25 & 25 & 25 \\ q_4 & 3 & 0 & 0 \end{array}$$

Then

$aff(A_1, A_3) = 15*1 + 20*1+10*1$

$= 45$

and the attribute affinity matrix *AA* is

$$\begin{array}{c|cccc} & A_1 & A_2 & A_3 & A_4 \\ \hline A_1 & 45 & 0 & 45 & 0 \\ A_2 & 0 & 80 & 5 & 75 \\ A_3 & 45 & 5 & 53 & 3 \\ A_4 & 0 & 75 & 3 & 78 \end{array}$$

VF – Clustering Algorithm

- Take the attribute affinity matrix *AA* and reorganize the attribute orders to form clusters where the attributes in each cluster demonstrate high affinity to one another.
- Bond Energy Algorithm (BEA) has been used for clustering of entities. BEA finds an ordering of entities (in our case attributes) such that the global affinity measure is maximized.

$$AM = \sum_{i} \sum_{j} \text{(affinity of } A_i \text{ and } A_j \text{ with their neighbors)}$$

Bond Energy Algorithm

Input:	The *AA* matrix
Output:	The clustered affinity matrix *CA* which is a perturbation of *AA*

❶ *Initialization*: Place and fix one of the columns of *AA* in *CA*.

❷ *Iteration*: Place the remaining *n-i* columns in the remaining *i*+1 positions in the *CA* matrix. For each column, choose the placement that makes the most contribution to the global affinity measure.

❸ *Row order*: Order the rows according to the column ordering.

Bond Energy Algorithm

"Best" placement? Define contribution of a placement:

$$cont(A_i, A_k, A_j) = 2bond(A_i, A_k)+2bond(A_k, A_l) -2bond(A_i, A_j)$$

where

$$bond(A_x,A_y) = \sum_{z=1}^{n} aff(A_z,A_x)aff(A_z,A_y)$$

BEA – Example

Consider the following *AA* matrix and the corresponding *CA* matrix where A_1 and A_2 have been placed. Place A_3:

$$AA = \begin{array}{c|cccc|} & A_1 & A_2 & A_3 & A_4 \\ A_1 & 45 & 0 & 5 & 0 \\ A_2 & 0 & 80 & 5 & 75 \\ A_3 & 45 & 5 & 53 & 3 \\ A_4 & 0 & 75 & 3 & 78 \end{array} \qquad CA = \begin{array}{c|cccc|} & A_1 & A_2 & & \\ & 45 & 0 & & \\ & 0 & 80 & & \\ & 45 & 5 & & \\ & 0 & 75 & & \end{array}$$

Ordering (0-3-1) :

$cont(A_0,A_3,A_1) = 2bond(A_0 , A_3)+2bond(A_3 , A_1)–2bond(A_0 , A_1)$

$= 2* 0 + 2* 4410 – 2*0 = 8820$

Ordering (1-3-2) :

$cont(A_1,A_3,A_2) = 2bond(A_1 , A_3)+2bond(A_3 , A_2)–2bond(A_1,A_2)$

$= 2* 4410 + 2* 890 – 2*225 = 10150$

Ordering (2-3-4) :

$cont (A_2,A_3,A_4) = 1780$

BEA – Example

- Therefore, the CA matrix has the form

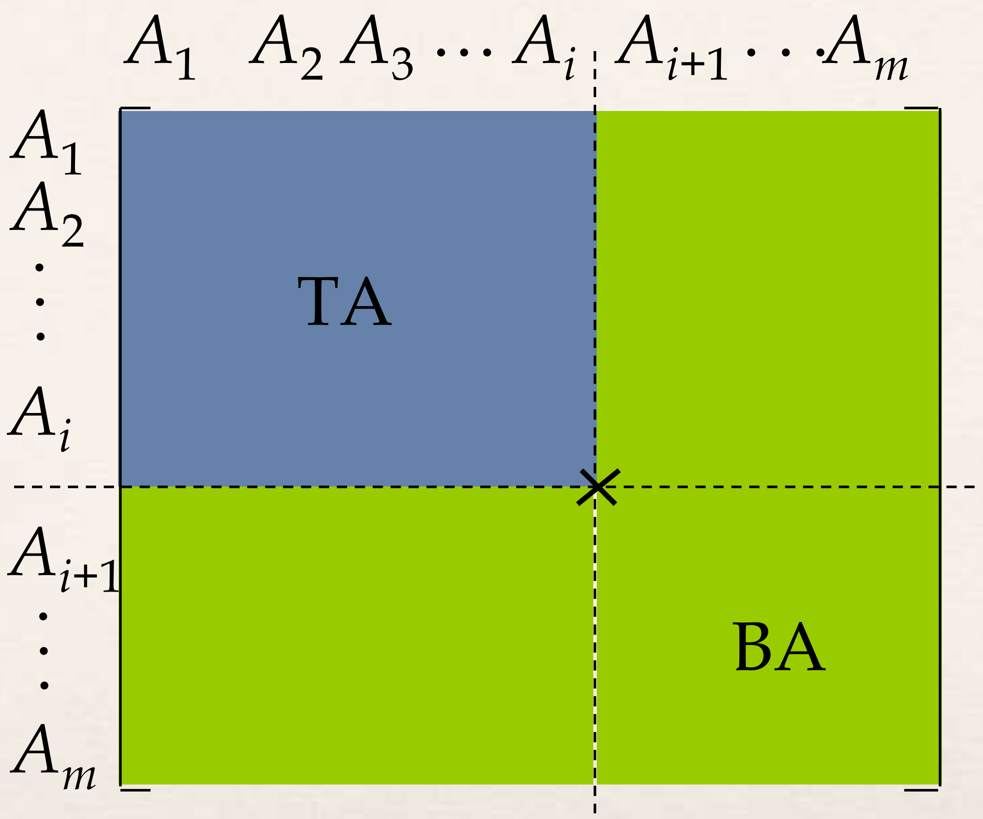
$$\begin{array}{ccc} A_1 & A_3 & A_2 \\ \end{array}$$
$$\begin{bmatrix} 45 & 45 & 0 \\ 0 & 5 & 80 \\ 45 & 53 & 5 \\ 0 & 3 & 75 \end{bmatrix}$$

- When A_4 is placed, the final form of the *CA* matrix (after row organization) is

$$\begin{array}{c|cccc} & A_1 & A_3 & A_2 & A_4 \\ \hline A_1 & 45 & 45 & 0 & 0 \\ A_3 & 45 & 53 & 5 & 3 \\ A_2 & 0 & 5 & 80 & 75 \\ A_4 & 0 & 3 & 75 & 78 \end{array}$$

VF – Algorithm

How can you divide a set of clustered attributes $\{A_1, A_2, \ldots, A_n\}$ into two (or more) sets $\{A_1, A_2, \ldots, A_i\}$ and $\{A_i, \ldots, A_n\}$ such that there are no (or minimal) applications that access both (or more than one) of the sets.



$A_1 \quad A_2\ A_3 \cdots A_i \ \vdots\ A_{i+1} \cdots A_m$
A_1
A_2
A_i
A_{i+1}
A_m
TA
BA

VF – ALgorithm

Define

TQ	=	set of applications that access only TA
BQ	=	set of applications that access only BA
OQ	=	set of applications that access both TA and BA

and

CTQ =	total number of accesses to attributes by applications that access only TA
CBQ =	total number of accesses to attributes by applications that access only BA
COQ =	total number of accesses to attributes by applications that access both TA and BA

Then find the point along the diagonal that maximizes

$$CTQ*CBQ-COQ^2$$

VF – Algorithm

Two problems :

- Cluster forming in the middle of the *CA* matrix
  - ➡ Shift a row up and a column left and apply the algorithm to find the "best" partitioning point
  - ➡ Do this for all possible shifts
  - ➡ Cost $O(m^2)$
- More than two clusters
  - ➡ m-way partitioning
  - ➡ try 1, 2, …, m–1 split points along diagonal and try to find the best point for each of these
  - ➡ Cost $O(2^m)$

VF – Correctness

A relation R, defined over attribute set A and key K, generates the vertical partitioning $F_R = \{R_1, R_2, \ldots, R_r\}$.

- Completeness
  - ➡ The following should be true for A:

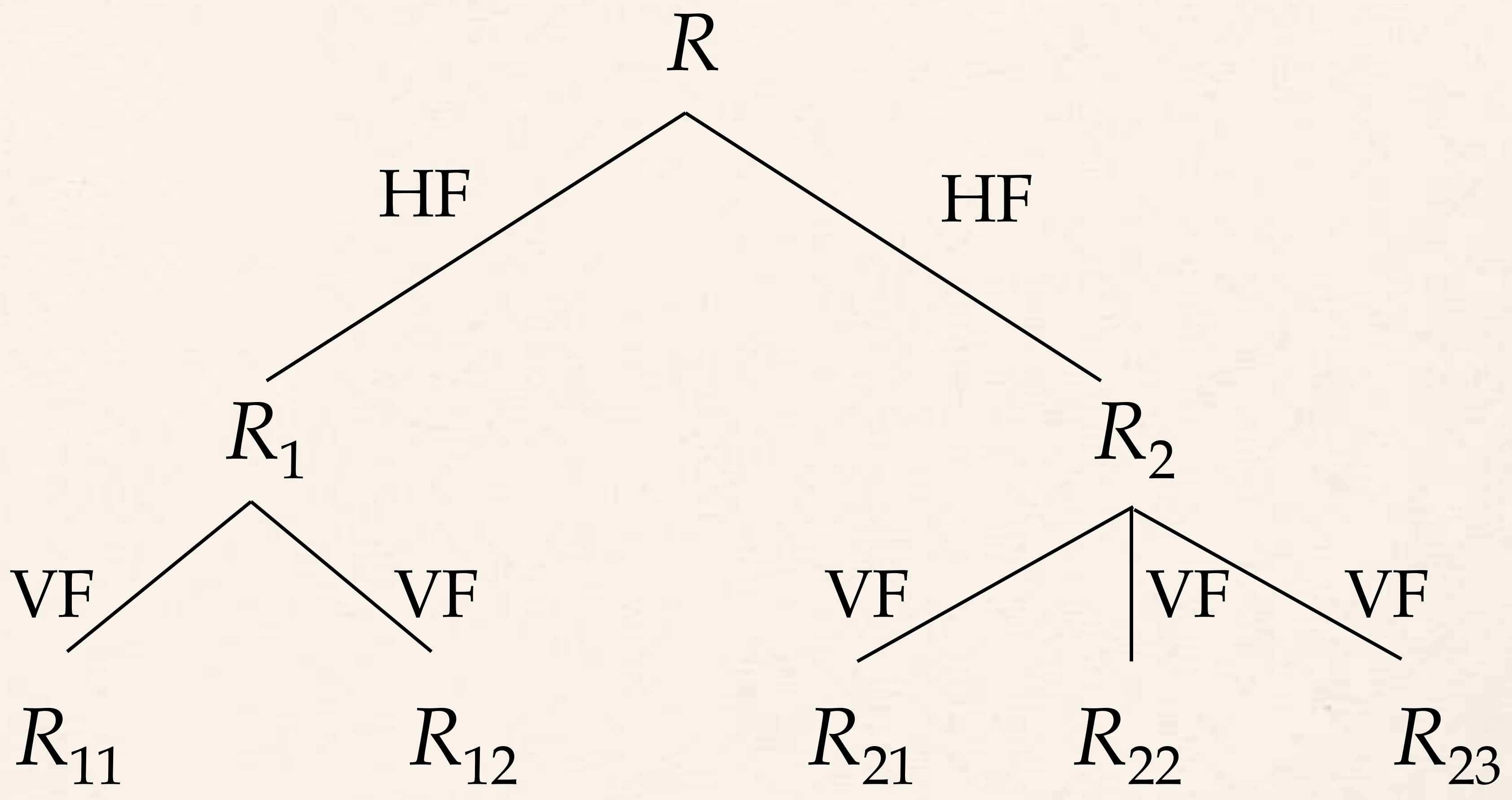
$$A = \cup A_{R_i}$$

- Reconstruction
  - ➡ Reconstruction can be achieved by

$$R = \bowtie_K R_i, \forall R_i \in F_R$$

- Disjointness
  - ➡ TID's are not considered to be overlapping since they are maintained by the system
  - ➡ Duplicated keys are not considered to be overlapping

Hybrid Fragmentation



Fragment Allocation

- Problem Statement

  Given

$F = \{F_1, F_2, \ldots, F_n\}$	fragments
$S = \{S_1, S_2, \ldots, S_m\}$	network sites
$Q = \{q_1, q_2, \ldots, q_q\}$	applications

  Find the "optimal" distribution of F to S.

- Optimality
  - ➡ Minimal cost
    - ✦ Communication + storage + processing (read & update)
    - ✦ Cost in terms of time (usually)
  - ➡ Performance

    Response time and/or throughput
  - ➡ Constraints
    - ✦ Per site constraints (storage & processing)

Information Requirements

- Database information
  - ➞ selectivity of fragments
  - ➞ size of a fragment
- Application information
  - ➞ access types and numbers
  - ➞ access localities
- Communication network information
  - ➞ unit cost of storing data at a site
  - ➞ unit cost of processing at a site
- Computer system information
  - ➞ bandwidth
  - ➞ latency
  - ➞ communication overhead

Allocation

File Allocation (FAP) vs Database Allocation (DAP):

- Fragments are not individual files
  - relationships have to be maintained
- Access to databases is more complicated
  - remote file access model not applicable
  - relationship between allocation and query processing
- Cost of integrity enforcement should be considered
- Cost of concurrency control should be considered

Allocation – Information Requirements

- Database Information
  - selectivity of fragments
  - size of a fragment
- Application Information
  - number of read accesses of a query to a fragment
  - number of update accesses of query to a fragment
  - A matrix indicating which queries updates which fragments
  - A similar matrix for retrievals
  - originating site of each query
- Site Information
  - unit cost of storing data at a site
  - unit cost of processing at a site
- Network Information
  - communication cost/frame between two sites
  - frame size

Allocation Model

General Form

min(Total Cost)

subject to

response time constraint

storage constraint

processing constraint

Decision Variable

$$x_{ij} = \begin{cases} 1 \text{ if fragment } F_i \text{ is stored at site } S_j \\ 0 \text{ otherwise} \end{cases}$$

Allocation Model

- Total Cost

$$\sum_{\text{all queries}} \text{query processing cost} + \sum_{\text{all sites}} \sum_{\text{all fragments}} \text{cost of storing a fragment at a site}$$

- Storage Cost (of fragment F_j at S_k)

$$(\text{unit storage cost at } S_k) * (\text{size of } F_j) * x_{jk}$$

- Query Processing Cost (for one query)

processing component + transmission component

Allocation Model

- Query Processing Cost

  Processing component

  access cost + integrity enforcement cost + concurrency control cost

  ➞ Access cost

$$\sum_{\text{all sites}} \sum_{\text{all fragments}} (\text{no. of update accesses} + \text{no. of read accesses}) * x_{ij} * \text{local processing cost at a site}$$

  ➞ Integrity enforcement and concurrency control costs

  - ✦ Can be similarly calculated

Allocation Model

- Query Processing Cost

  Transmission component

  cost of processing updates + cost of processing retrievals

  → Cost of updates

$$\sum_{\text{all sites}} \sum_{\text{all fragments}} \text{update message cost} + \sum_{\text{all sites}} \sum_{\text{all fragments}} \text{acknowledgment cost}$$

  → Retrieval Cost

$$\sum_{\text{all fragments}} \min_{\text{all sites}} (\text{cost of retrieval command} + \text{cost of sending back the result})$$

Allocation Model

- Constraints
  - Response Time

    execution time of query $\leq$ max. allowable response time for that query
  - Storage Constraint (for a site)

$$\sum_{\text{all fragments}} \text{storage requirement of a fragment at that site} \leq \text{storage capacity at that site}$$

  - Processing constraint (for a site)

$$\sum_{\text{all queries}} \text{processing load of a query at that site} \leq \text{processing capacity of that site}$$

Allocation Model

- Solution Methods
  - FAP is NP-complete
  - DAP also NP-complete
- Heuristics based on
  - single commodity warehouse location (for FAP)
  - knapsack problem
  - branch and bound techniques
  - network flow

Allocation Model

- Attempts to reduce the solution space
  - ➡ assume all candidate partitionings known; select the “best” partitioning
  - ➡ ignore replication at first
  - ➡ sliding window on fragments