Applied Data Analysis (CS401)

ada

Lecture 12
Handling networks
2 Dec 2020

EPFL

Robert West

dlab

Announcements

- Project milestone P4 (freestyle analysis + 2nd replication) due in 2.5 weeks (Fri 18 Dec 2020)
  - Video presentation due on Wed 23 Dec 2020
- Friday's lab session:
  - Exercises on handling networks
  - Project office hours
- Looking for MS semester or thesis projects?
  - We have some openings
  - Cave canem: the projects will be intense, so we're very picky in selecting students

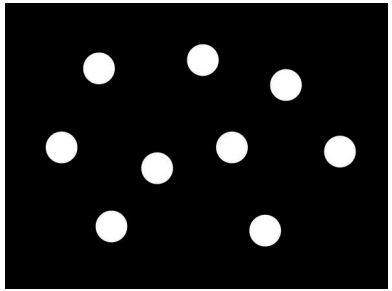
Give us feedback on this lecture here:
https://go.epfl.ch/ada2020-lec12-feedback

- What did you (not) like about this lecture?
- What was (not) well explained?
- On what would you like more (fewer) details?
- …

Beyond flat tables

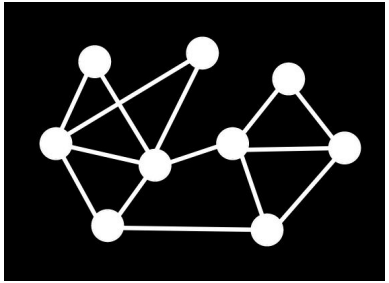
People

id	name	age
1	Bob	36
2	Willy	32
…	…	…



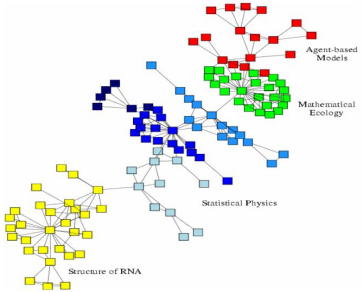
Marriages

husband_id	wife_id
1	34
2	5
2	87
…	…

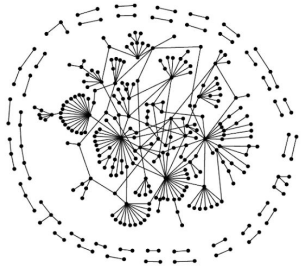


Examples

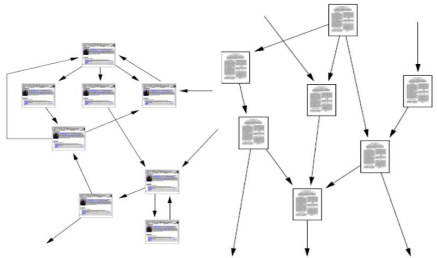
Social networks



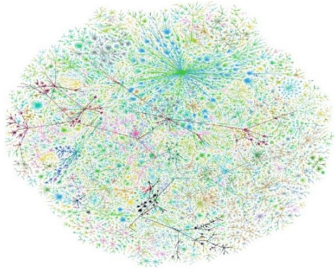
Economic networks



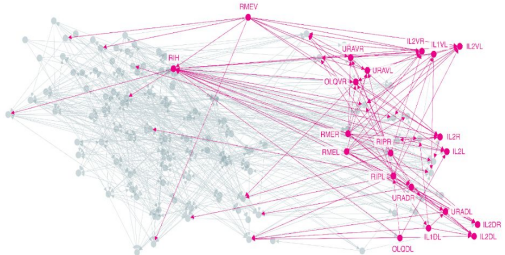
Communication graphs



Information networks:
Web & citations

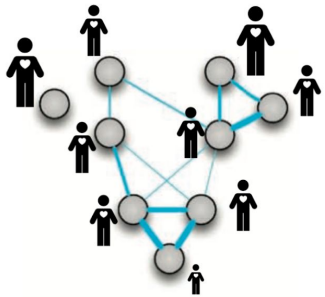


Internet

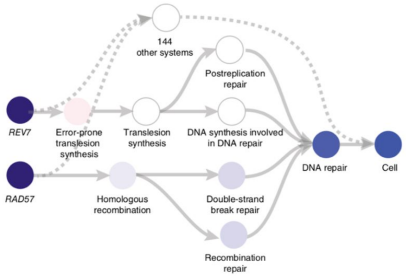


Networks of neurons

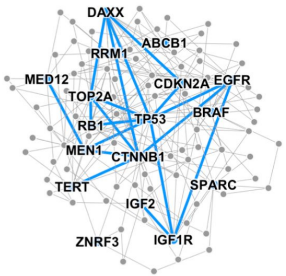
Examples



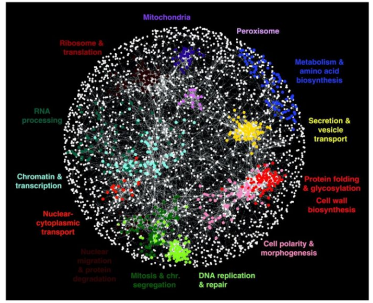
Patient networks



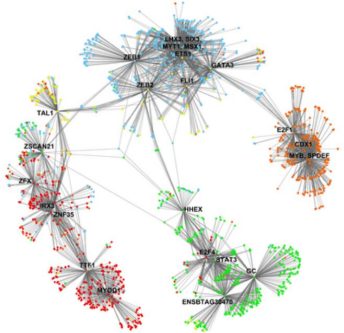
Hierarchies of cell systems



Disease pathways



Genetic interaction networks



Gene co-expression networks

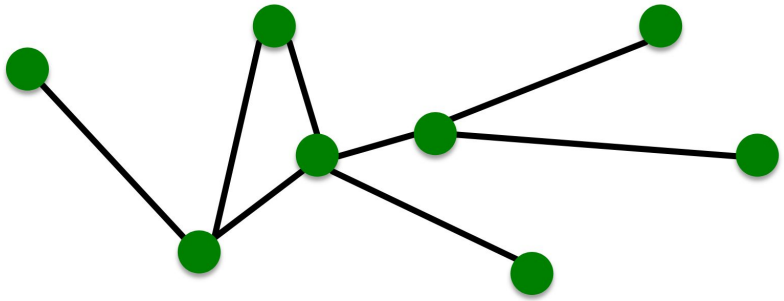
Cell-cell similarity networks

Networks as graphs

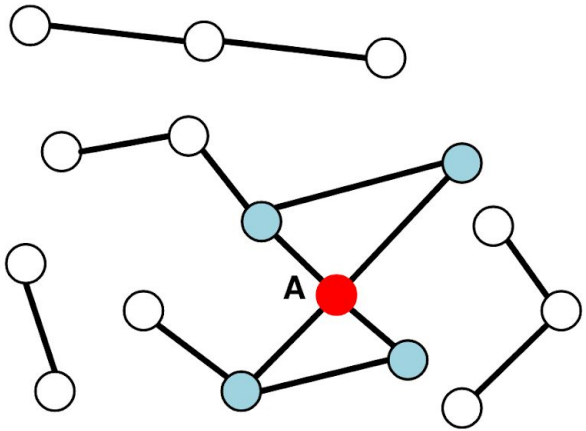
- **Network:** a real-world system of dependent variables, e.g.,
  - WWW is a network of hyperlinked documents
  - Society is a network of individuals linked by family, friendship, professional ties
  - Metabolic network is sum of all chemical reactions in a cell
- **Graph:** mathematical abstraction for describing networks
- In practice, “network” and “graph” are used interchangeably
- You can make a graph out of almost anything (e.g., connect all people whose name starts with the same letter), so must ask: Does this graph correspond to a meaningful network?

Most basic type: undirected graphs

- Entities:
  nodes/vertices V
- Relationships/interactions:
  edges/links E
- Entire system:
  network/graph $G = (V, E)$



Node degree



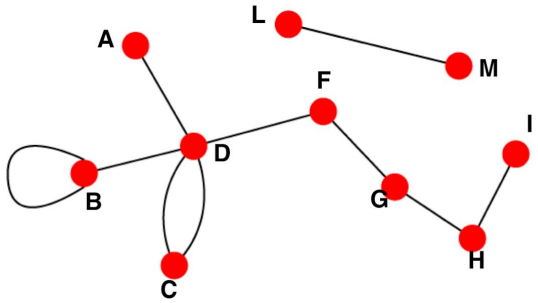
Node degree, k_i: the number of edges adjacent to node $\boldsymbol{i}$

$$k_A = 4$$

Types of graphs: undirected vs. directed

Undirected

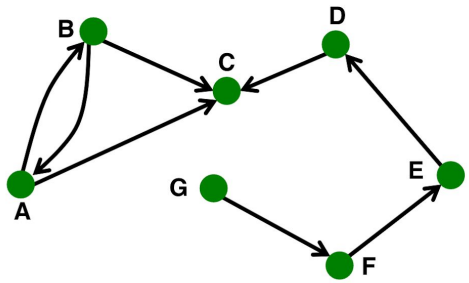
- **Links:** undirected (symmetrical, reciprocal)



- **Examples:**
  - Collaborations
  - Friendship on Facebook

Directed

- **Links:** directed (arcs)

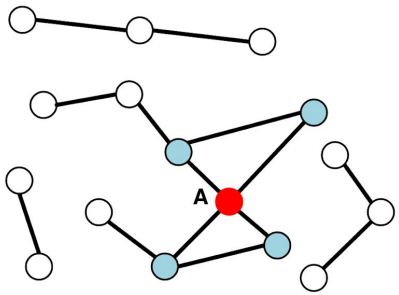


- **Examples:**
  - Phone calls
  - Following on Twitter

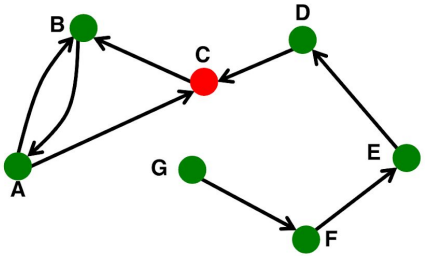
Average node degree

POLLING TIME!

Undirected



Directed



Source: Node with $k^{in} = 0$
Sink: Node with $k^{out} = 0$

Node degree, k_i: the number of edges adjacent to node i

$k_A = 4$

Avg. degree: $\bar{k} = \langle k \rangle = \frac{1}{N}\sum_{i=1}^{N} k_i =$

In directed networks we define an **in-degree** and **out-degree.**

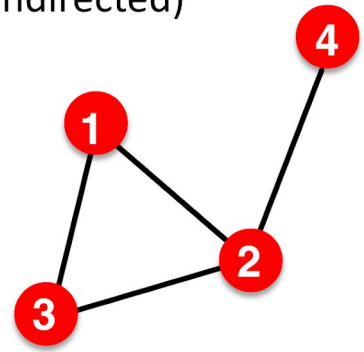
The (total) degree of a node is the sum of in- and out-degrees.

$k_C^{in} = 2 \qquad k_C^{out} = 1 \qquad k_C = 3$

$\bar{k} =$

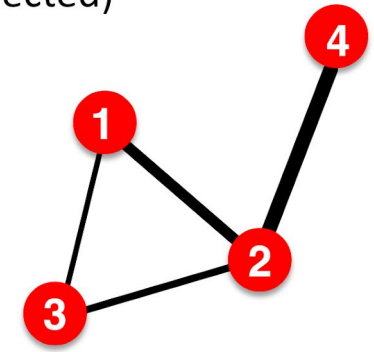
Types of graphs: weighted

- **Unweighted**
  (undirected)



Examples: Friendship, Hyperlink

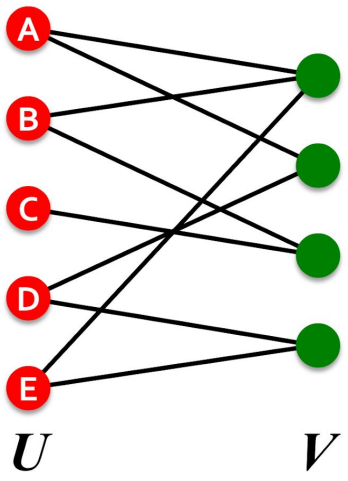
- **Weighted**
  (undirected)



Examples: Collaboration, Internet, Roads

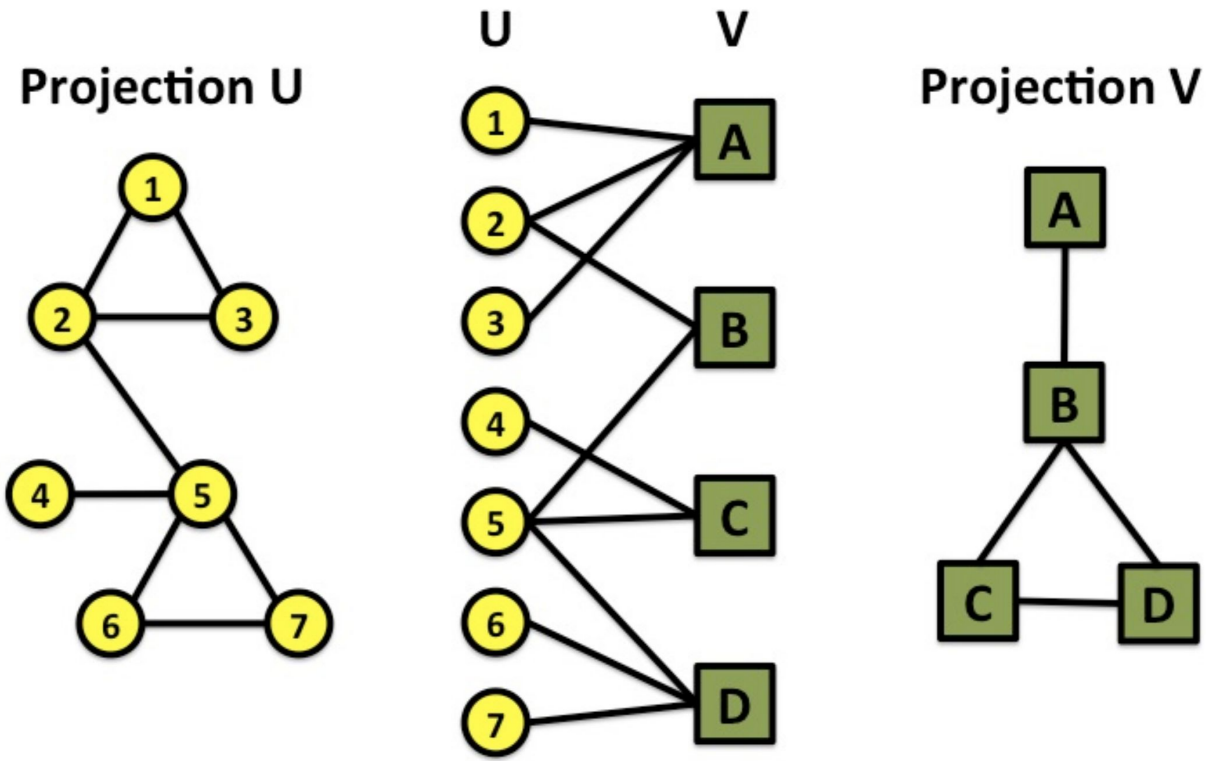
Types of graphs: bipartite

- **Bipartite graph** is a graph whose nodes can be divided into two disjoint sets U and V such that every link connects a node in U to one in V; that is, U and V are **independent sets**

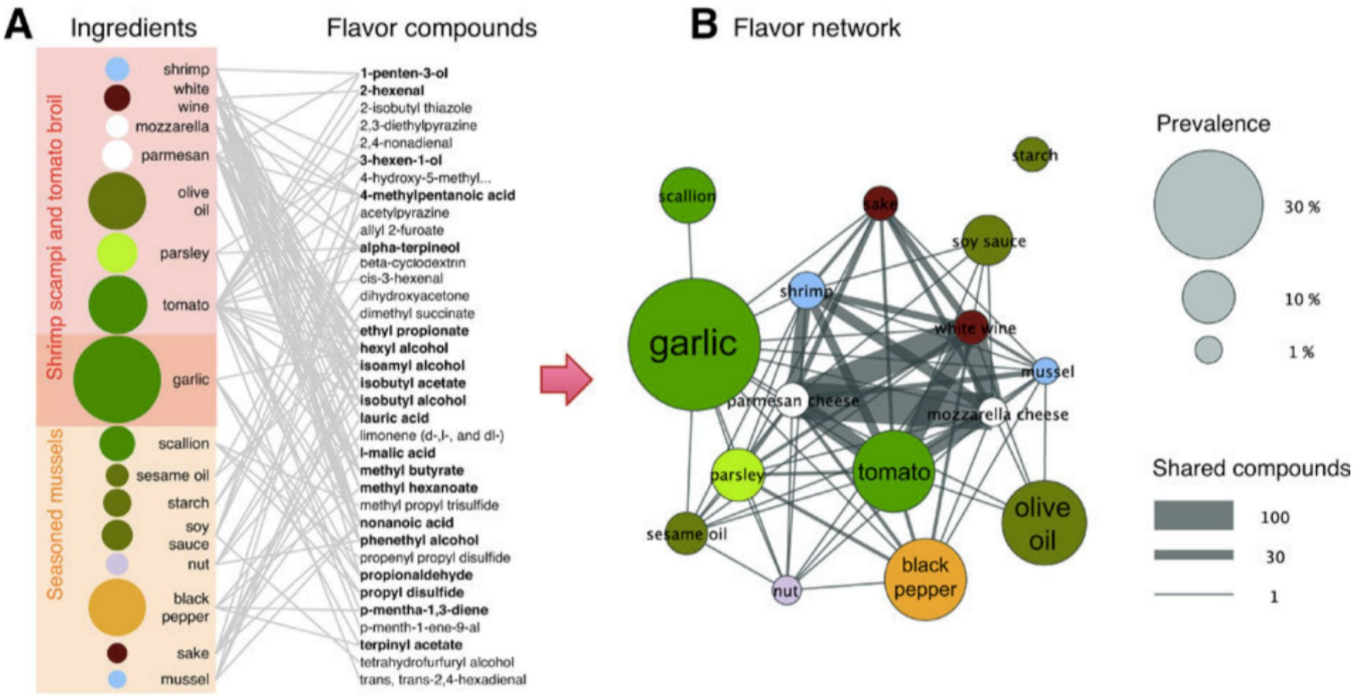


- **Examples:**
  - Authors-to-Papers (they authored)
  - Actors-to-Movies (they appeared in)
  - Users-to-Movies (they rated)
  - Recipes-to-Ingredients (they contain)
- **"Folded" networks:**
  - Author collaboration networks
  - Movie co-rating networks

Projections of bipartite graph



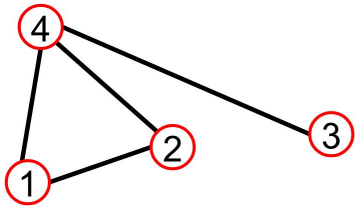
Example: flavor networks

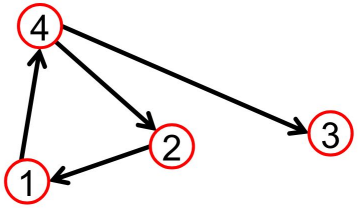


Y.-Y. Ahn, S. E. Ahnert, J. P. Bagrow, A.-L. Barabási
Flavor network and the principles of food pairing , Scientific Reports 196, (2011).

[paper]

Representing graphs on computers: adjacency matrix





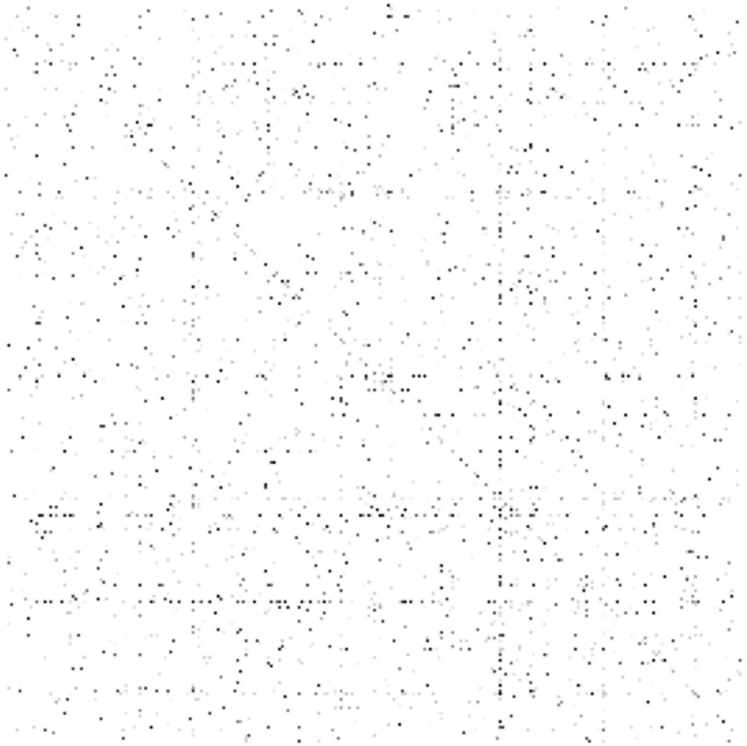
$A_{ij} = 1$ if there is a link from node ***i*** to node ***j***

$A_{ij} = 0$ otherwise

$$A = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \qquad A = \begin{pmatrix} 0 & 0 & 0 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \end{pmatrix}$$

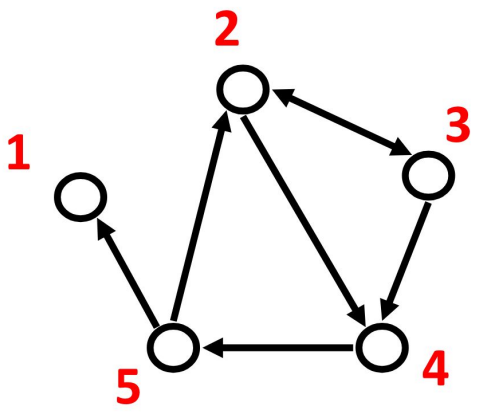
Note that for a directed graph (right) the matrix is not symmetric.

Adjacency matrix usually sparse



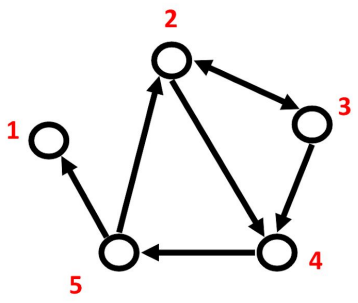
Representing graphs on computers: edge list

- **Represent graph as a set of edges:**
  - (2, 3)
  - (2, 4)
  - (3, 2)
  - (3, 4)
  - (4, 5)
  - (5, 2)
  - (5, 1)



Representing graphs on computers: adjacency list

- **Adjacency list:**
  - Easier to work with if network is
    - Large
    - Sparse
  - Allows us to quickly retrieve all neighbors of a given node
    - 1:
    - 2: 3, 4
    - 3: 2, 4
    - 4: 5
    - 5: 1, 2

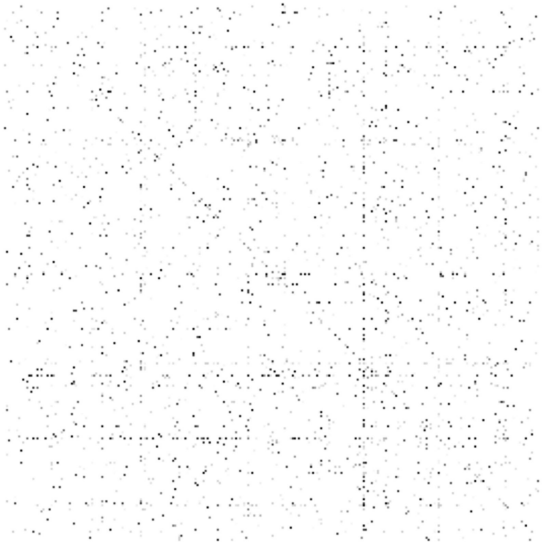


Properties of real-world networks

- Real networks are different from arbitrary graphs
- Real networks tend to share certain properties
- Remarkable, given the diversity of networks
  - Information networks (e.g., Web graph, knowledge graphs)
  - Social networks (e.g., Facebook, sexual networks)
  - Biological networks
  - …

Properties of real-world networks: sparsity

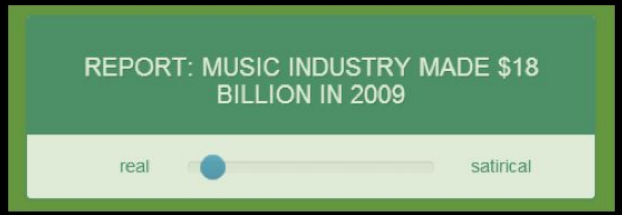
- Every node connected to only small fraction of all other nodes
- i.e., $k_i \ll N$
- Often bounded by a constant
  - e.g., social networks: Dunbar's number (cognitive limit to the number of people with whom one can maintain stable social relationships; allegedly 150)



Commercial break: Help us understand humor better!

source:

source:



http://unfun.me

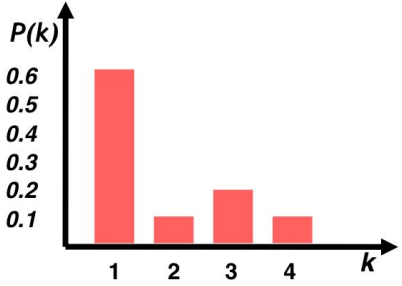
Properties of real-world networks: degree distribution

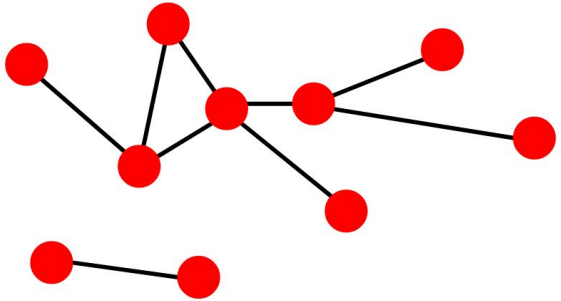
- **Degree distribution $P(k)$:** Probability that a randomly chosen node has degree k

  N_k = # nodes with degree k

- Normalized histogram:

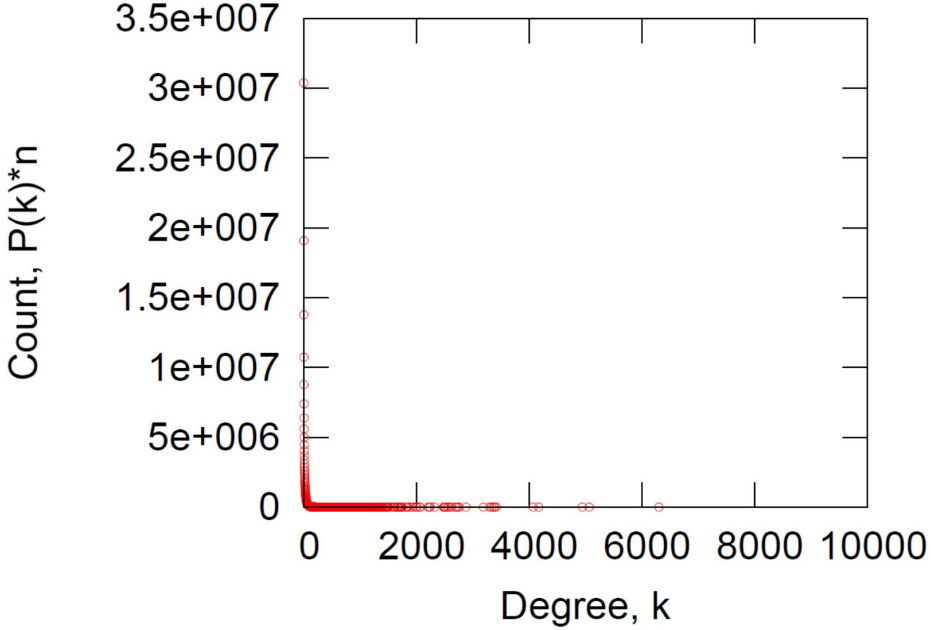
  $P(k) = N_k / N$ ➔ **plot**



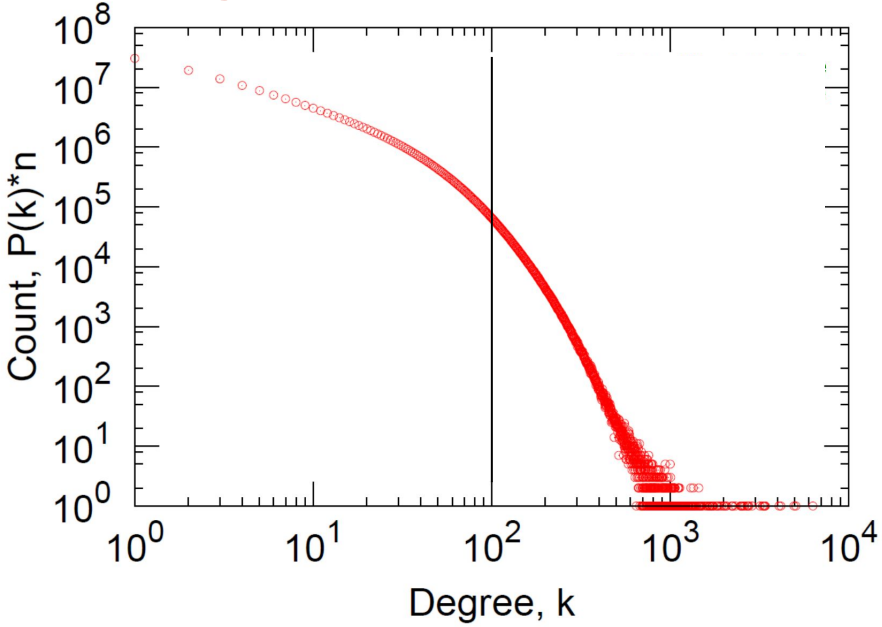


Properties of real-world networks: degree distribution

Linear axes

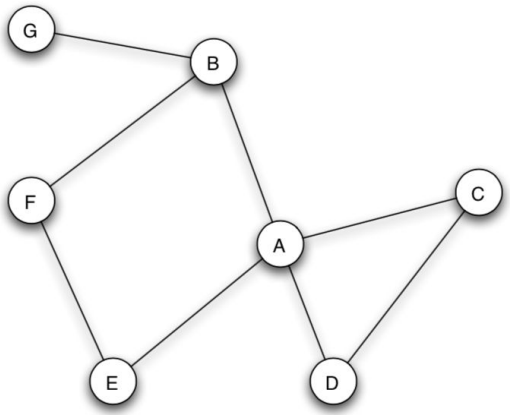


Logarithmic axes



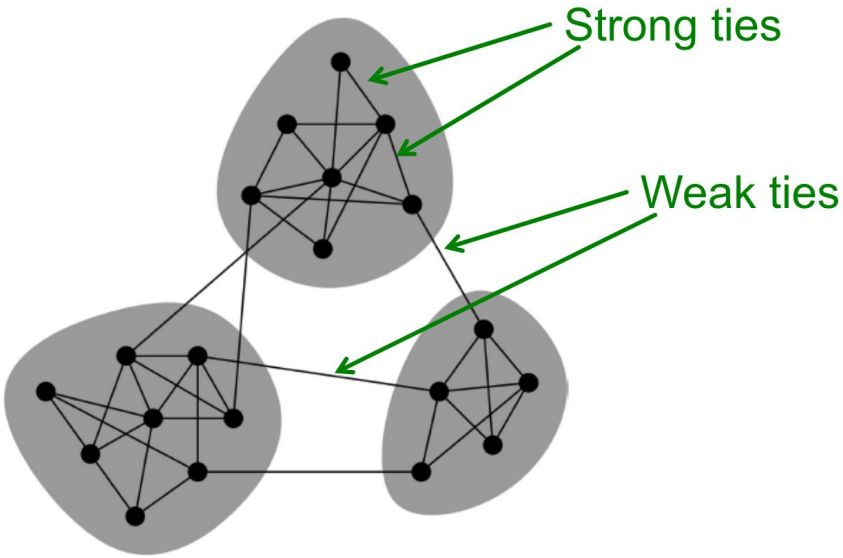
Properties of real-world networks: triadic closure

- "A friend of my friend is my friend"
- Measured via clustering coefficient C_i of node i:
  C_i = (#edges among neighbors of i) / (#potential edges among neighbors of i)
- #potential edges among neighbors of i in undirected graph: $k_i (k_i - 1) / 2$
- $C_A = 1 / (4*3/2) = 1/6$
- In real networks, nodes have large clustering coefficients



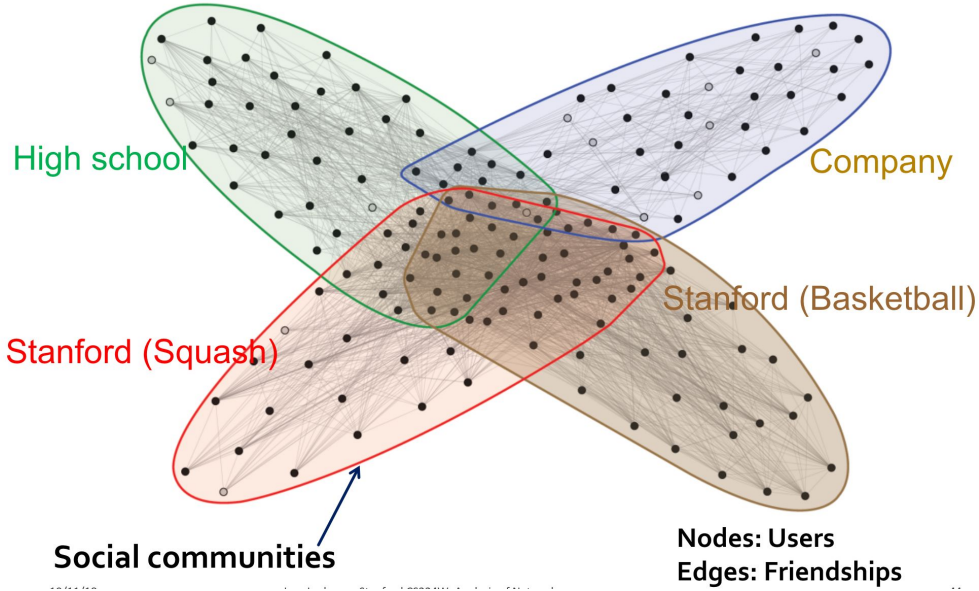
Properties of real-world networks: community structure

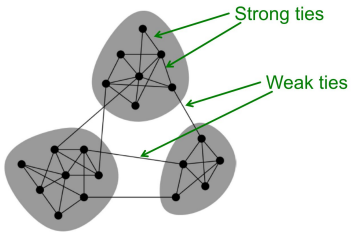
- Triadic closure makes real networks cluster into locally dense "communities"
- Communities connected via "weak ties"
- "Strength of weak ties" (Granovetter)
- "Structural holes"



Properties of real-world networks: community structure

- In real life, communities are often not disjoint, But overlapping:





[George Costanza's ideal world; unfortunately not realistic]

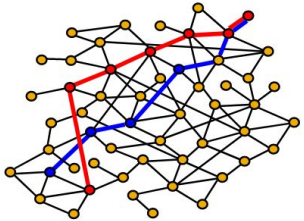
Extra homework: Start watching *Seinfeld,*

so you understand the [“Worlds Collide” theory]

Properties of real-world networks: average shortest-path length

- **What is the typical shortest path length between any two people?**
  - **Experiment on the global friendship network**
    - Can't measure, need to probe explicitly
- **Small-world experiment** [Milgram '67]
  - Picked 300 people in Omaha, Nebraska and Wichita, Kansas
  - Ask them to get a letter to a stock-broker in Boston by passing it through friends
- **How many steps did it take?**

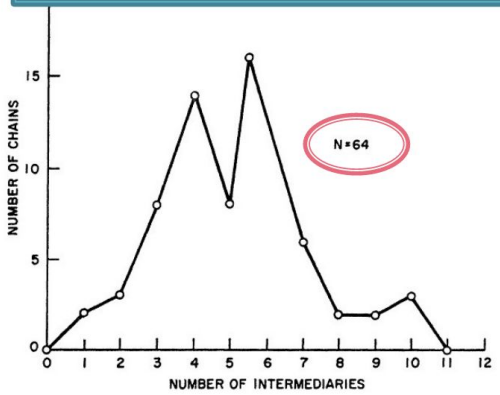
[Movie]



Properties of real-world networks: average shortest-path length

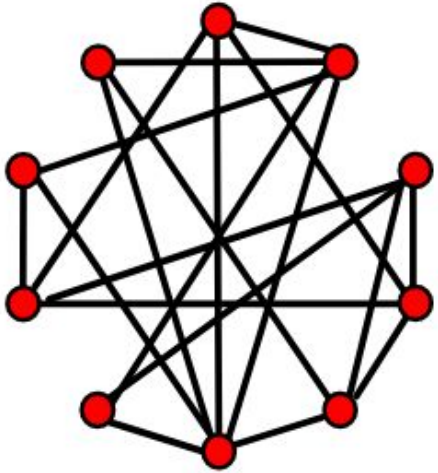
- **64 chains completed:**
  (i.e., 64 letters reached the target)
  - It took 6.2 steps on the average, thus **"6 degrees of separation"**
- **Further observations:**
  - People who owned stock had shorter paths to the stockbroker than random people: 5.4 vs. 6.7
  - People from the Boston area have even closer paths: 4.4

Milgram's small world experiment



Properties of real-world networks: navigability

- For decades, people focused on the fact that short paths exist in social networks
- But this is true even in random graphs (e.g., Erdős-Rényi model)
- The truly amazing fact is not that short paths exist, but that they are discoverable via greedy decentralized routing (as in Milgram's experiment)
- Intrigued? Read Jon Kleinberg: "The Small-World Problem: An Algorithmic Perspective"

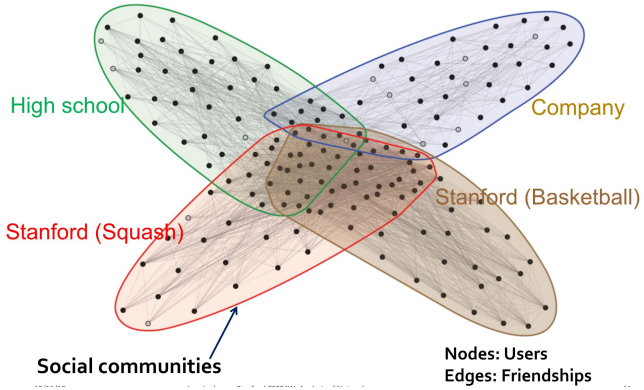


Don’t believe me?

- Play a game and see for yourself how well you can navigate an a-priori unknown network:
  - Wikispeedia.net

Properties of real-world networks: homophily/heterophily

- "Birds of a feather flock together"
- Especially in social networks
- Big confound and cause of debate: Influence vs. homophily
- E.g., obese people's friends are more likely to also be obese
  - Influence: I copy eating behavior of those around me [video]
  - Homophily: people with similar eating behavior prone to become friends



Properties of real-world networks: summary

Real-world networks (across many types)

- are sparsely connected,
- but some nodes are much more connected than most others (i.e., skewed degree distribution);
- form locally dense clusters via triadic closure,
- which leads to community structure;
- have short paths between random node pairs (partly due to “hubs” [skewed degree distribution!]),
- and the short paths are easily discoverable.

How to measure “importance” of a node?

- Formalized via *centrality measures*
- Map each node i to a scalar value $C(i)$ capturing its importance in the overall network

Degree centrality

- Simplest centrality measure
- Many neighbors → important node
  - $C(i)$ = number of neighbors of i
- Very brittle, easy to “rig”
  - E.g., Twitter: scam account, followed by 100,000 other scam accounts

Closeness centrality

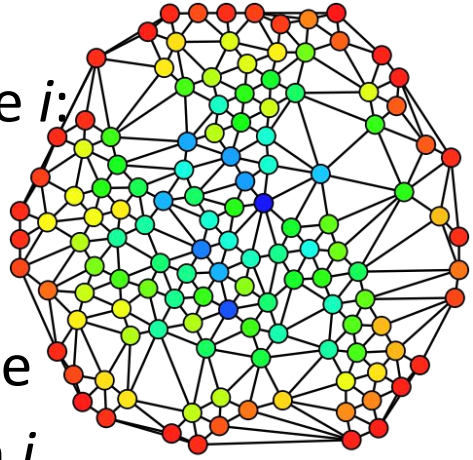
- Farness(x) = average (or cumulative) distance to x from other nodes
- $C(x)$ = 1 / Farness(x)
  - Reciprocal to turn farness into closeness
  - Only defined for connected graphs (otherwise d infinite)
- Under closeness centrality, nodes that are easy to reach from anywhere in the network are considered important
- Variant: harmonic centrality: switch sum and reciprocal
  - $C(x)$ = average reciprocal distance of x to other nodes
  - Defined even for disconnected graphs (define $1/d$ of disconnected nodes as 0)

$$C(x) = \frac{1}{\sum_y d(y,x)}$$

$$C(x) = \sum_{y \neq x} \frac{1}{d(y,x)}$$

Betweenness centrality

- $C(i)$ = fraction of all shortest paths in the network that pass through node i
- Computation of betweenness centrality of node i:
  - For each pair of vertices (s, t), compute all shortest paths between them.
  - For each pair of vertices (s, t), determine the fraction of shortest paths that pass through i.
  - Average this fraction over all pairs of vertices (s, t).
- Expensive to compute



Katz centrality

- Generalization of degree centrality
- Degree centrality counts only number of direct neighbors (i.e., neighbors at distance 1)
- Katz centrality also counts neighbors at distances 2, 3, …
- More precisely, number of paths from other nodes to *i*, giving less weight to larger distances:

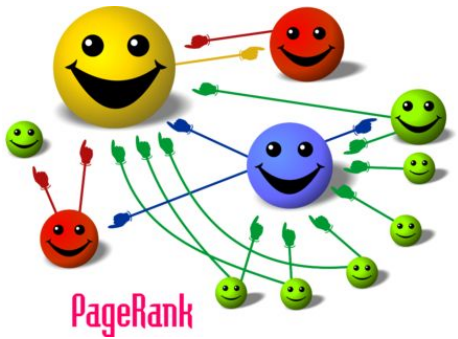
$$C(i) = \sum_{k=1}^{\infty} \sum_{j=1}^{N} \alpha^k (A^k)_{ji}$$

k-th power of adjacency matrix *A* contains number of length-*k* paths for each node pair

- More robust than degree centrality

PageRank centrality

- Recursive definition: my centrality $C(i)$ =: x_i is high if I receive inlinks from many other central nodes:



$$x_i = \sum_j a_{ji} \frac{x_j}{L(j)}$$

$$L(j) = \sum_i a_{ji}$$

a_{ji}: entry (j, i) of adjacency matrix A (1 if j links to i, else 0)
$L(j)$: out-degree of j

- Some extra tweaks to make it work with any graph (e.g., disconnected)

PageRank centrality

$$x_i = \sum_j a_{ji} \frac{x_j}{L(j)}$$

- Matrix notation: $x = M\,x$
  (where M is computed from adjacency matrix A)
- Do you recognize this?
  - x is eigenvector of M with eigenvalue 1 → we're in linear-algebra land (home sweet home)
  - x is the steady state the Markov chain induced by the network: x_i is fraction of time a random walker will spend in node i

PageRank centrality

- The technology that made Google huge
  - “Page” in PageRank for Larry Page
  - MapReduce (next lecture!) was invented to compute PageRank on full Google Web crawl
  - “The $25,000,000,000 eigenvector” [link]
- Bottom line:
  Pay attention in your linear algebra class

Give us feedback on this lecture here:
https://go.epfl.ch/ada2020-lec12-feedback

- What did you (not) like about this lecture?
- What was (not) well explained?
- On what would you like more (fewer) details?
- …

Credits

- Some slides borrowed from CS224W class (Jure Leskovec, Stanford)