THE HONG KONG POLYTECHNIC UNIVERSITY
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AAE6102 – Satellite Communication and Navigation

PPP

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RTK

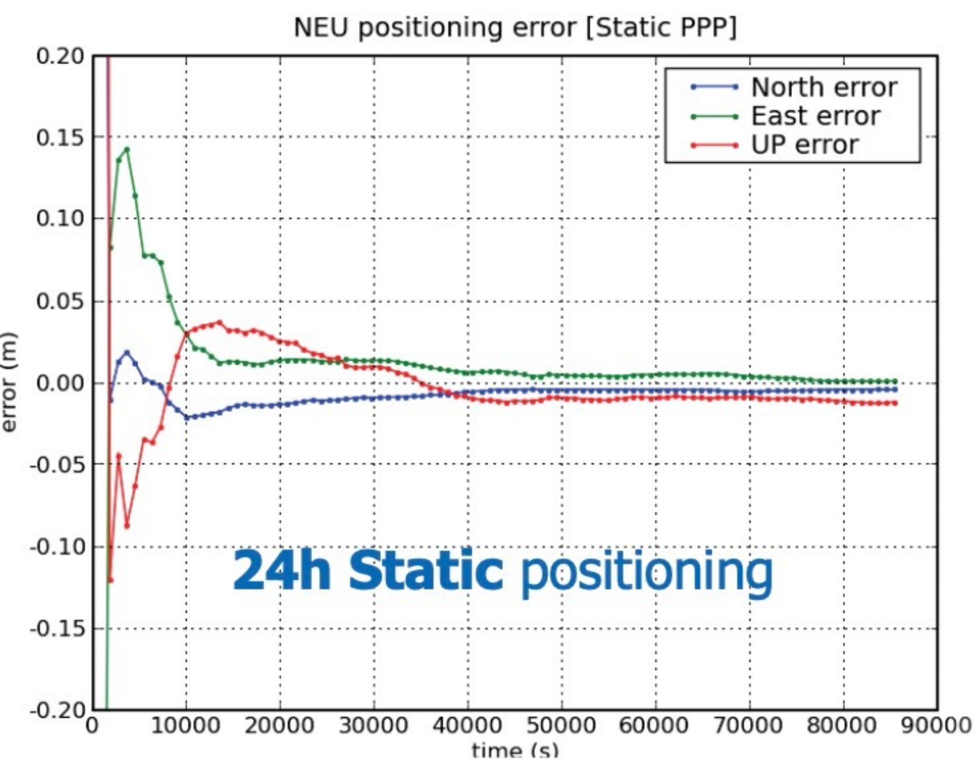
- Real-Time Kinematic (RTK) positioning is positioning that is based on at least two GPS receivers—a base receiver and one or more rover receivers. The base receiver takes measurements from satellites in view and then broadcasts them, together with its location, to the rover receiver(s). The rover receiver also collects measurements to the satellites in view and processes them with the base station data. The rover then estimates its location relative to the base.
- The key to achieving centimeter-level positioning accuracy with RTK is the use of the GPS carrier phase signals. Although carrier phase measurements are highly precise, they contain an unknown bias, termed the *integer cycle ambiguity*, or *carrier phase ambiguity*. The rover has to resolve, or initialize, the carrier phase ambiguities at power-up and every time that the satellite signals are interrupted.

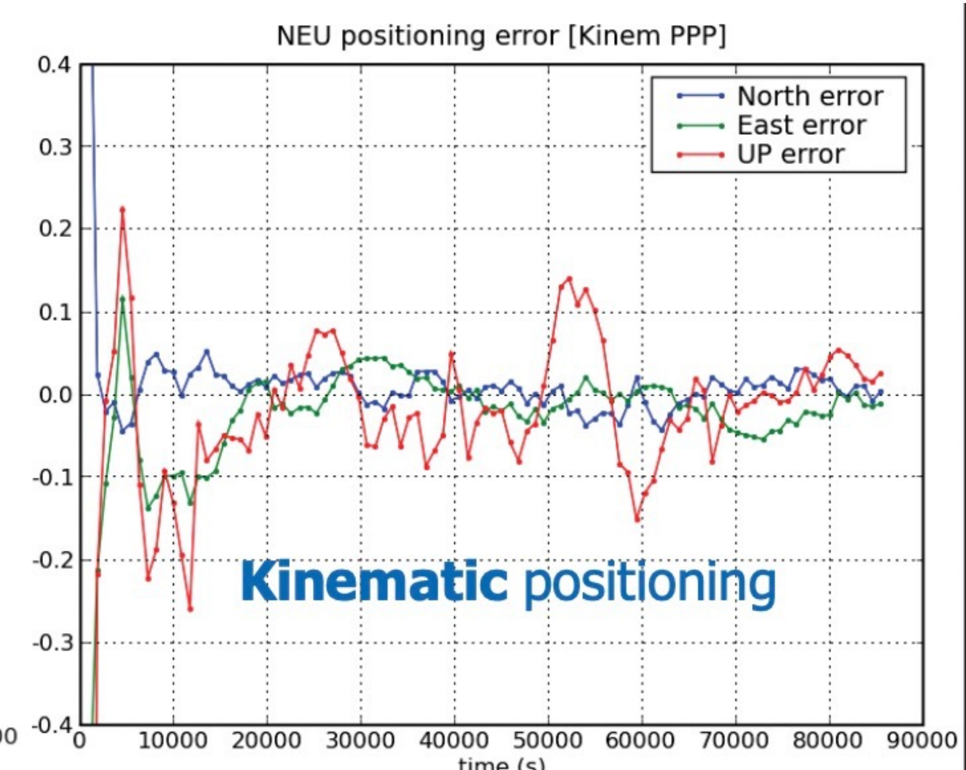
PPP

- Precise Point Positioning (PPP) is a positioning technique that removes or models GNSS system errors to provide a high level of position accuracy from a single receiver. A PPP solution depends on GNSS satellite clock and orbit corrections, generated from a network of global reference stations. These corrections are used by the receiver, resulting in decimetre-level or better positioning with no base station required.
- Compare RTK and PPP?

Precise Point Positioning

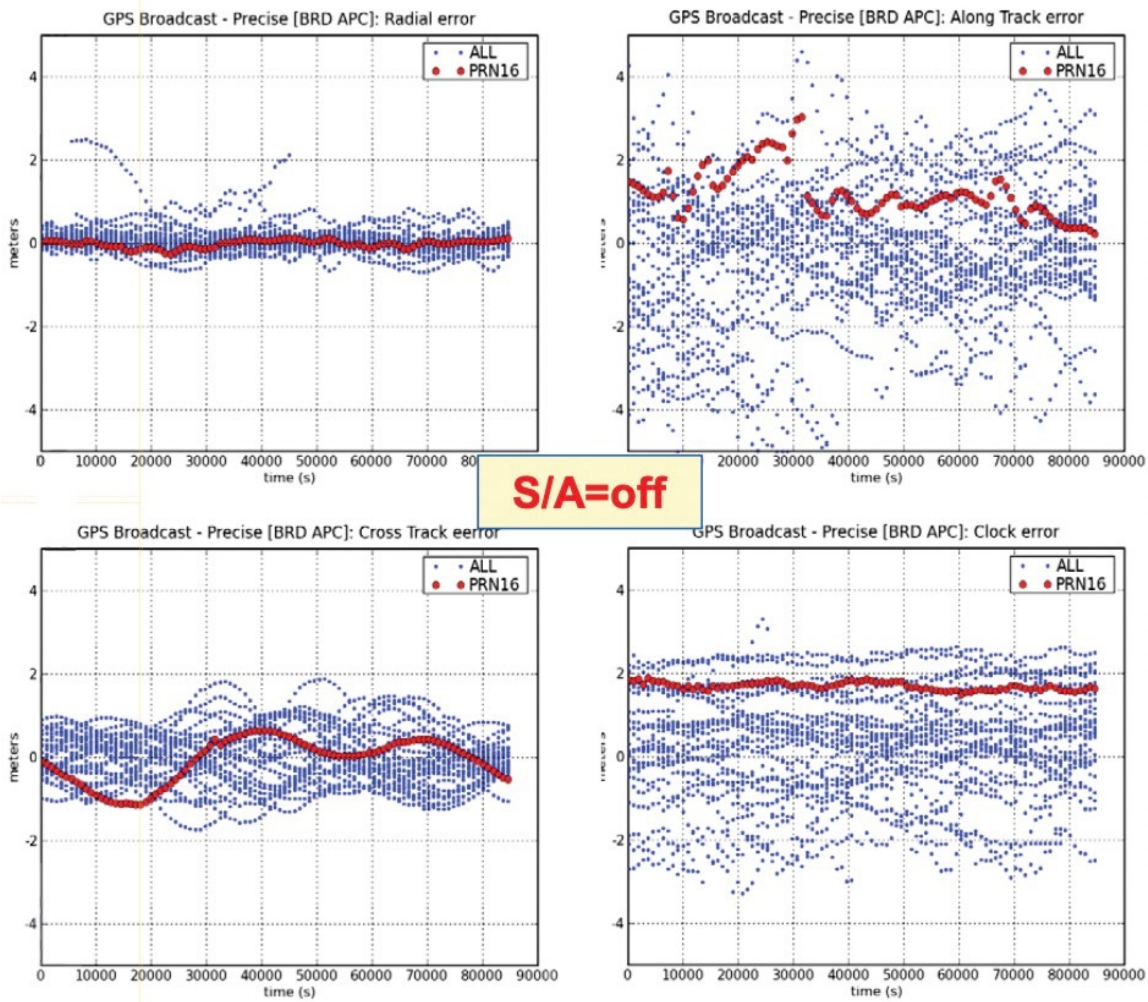
- PPP technique allows centimeter level accuracy to be achieved for static positioning and decimeter level for kinematic positioning



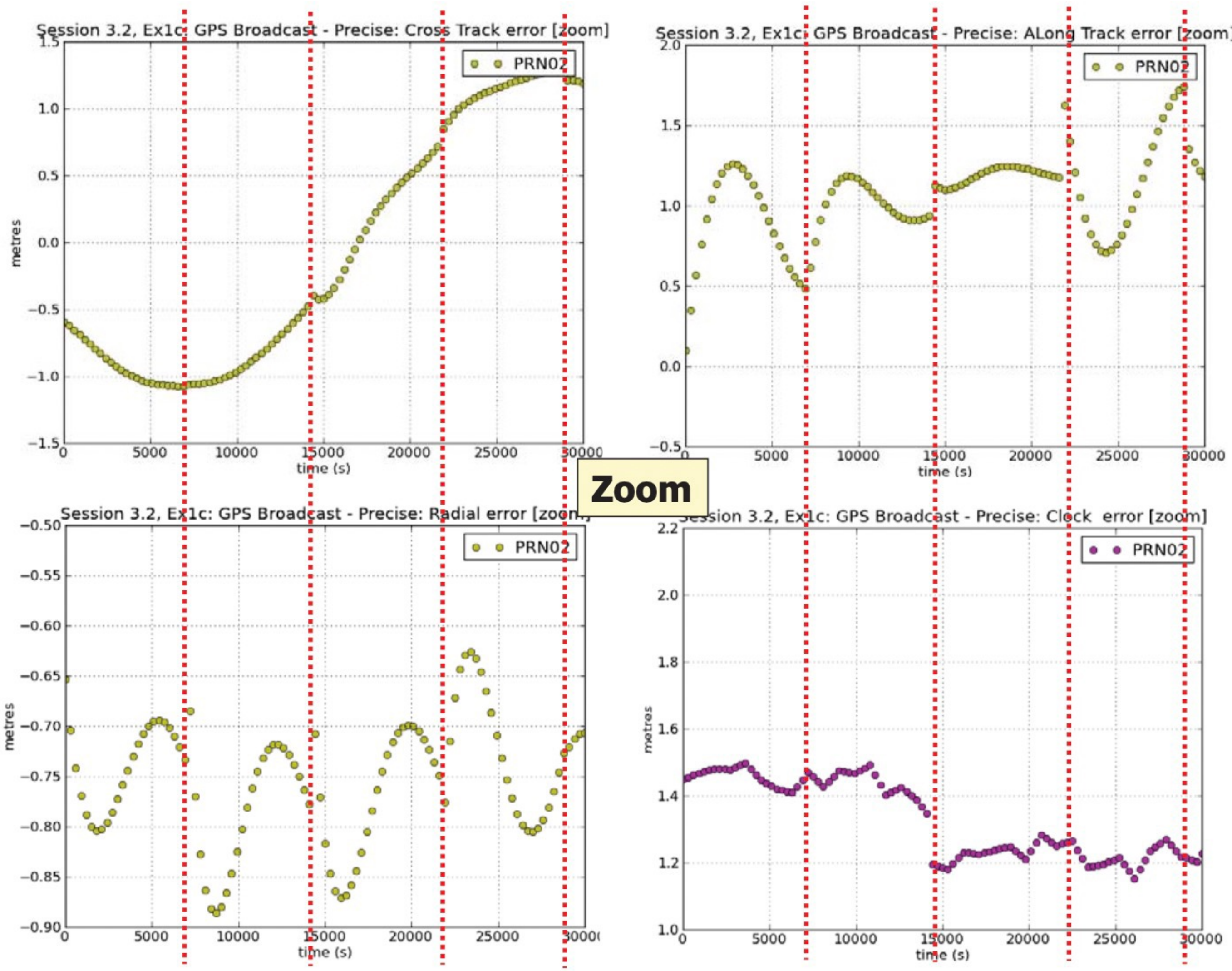


Precise Orbit and Clocks

- Broadcast versus precise
- Broadcast orbits and clocks are accurate at few meters level
- IGS precise orbit/clocks are as accurate as a few cm level



Broadcast Ephemeris Updates



IGS Precise Orbit and Clock Products

Products (delay)	Broadcast (real time)	Ultra-rapid Predicted (real time)	Ultra-rapid Observed (3–9 h)	Rapid (17–41 h)	Final (12–18 d)
Orbit GPS (sampling)	~100 cm (~2 h)	~5 cm (15 min)	~3 cm (15 min)	~2.5cm (15 min)	~ 2.5 cm (15 min)
Glonass (sampling)					~5 cm (15 min)
Clock GPS (sampling)	~5 ns (daily)	~3 ns (15 min)	~150 ps (15 min)	~75 ps (5 min)	~75 ps (30 s)
Glonass (sampling)					~ TBD (15 min)

http://igscb.jpl.nasa.gov/components/prods.html

Computation of satellite coordinates from precise products

- Precise orbits for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**
- Orbits are given by *(x,y,z)* coordinates with a sampling rate of 15 minutes. The satellite coordinates between epochs can be computed by polynomial interpolation. A 10th-order polynomial is enough for a centimetre level of accuracy with 15 min data.

$$
\begin{aligned}
P_n(x) &= \sum_{i=1}^{n} y_i \frac{\prod_{j\neq i}(x-x_j)}{\prod_{j\neq i}(x_i-x_j)} \\
&= y_1 \frac{x-x_2}{x_1-x_2}\cdots\frac{x-x_n}{x_1-x_n}+\cdots \\
&\quad + y_i \frac{x-x_1}{x_i-x_1}\cdots\frac{x-x_{i-1}}{x_i-x_{i-1}}\frac{x-x_{i+1}}{x_i-x_{i+1}}\cdots\frac{x-x_n}{x_i-x_n}+\cdots \\
&\quad + y_n \frac{x-x_1}{x_n-x_1}\cdots\frac{x-x_{n-1}}{x_n-x_{n-1}}
\end{aligned}
$$

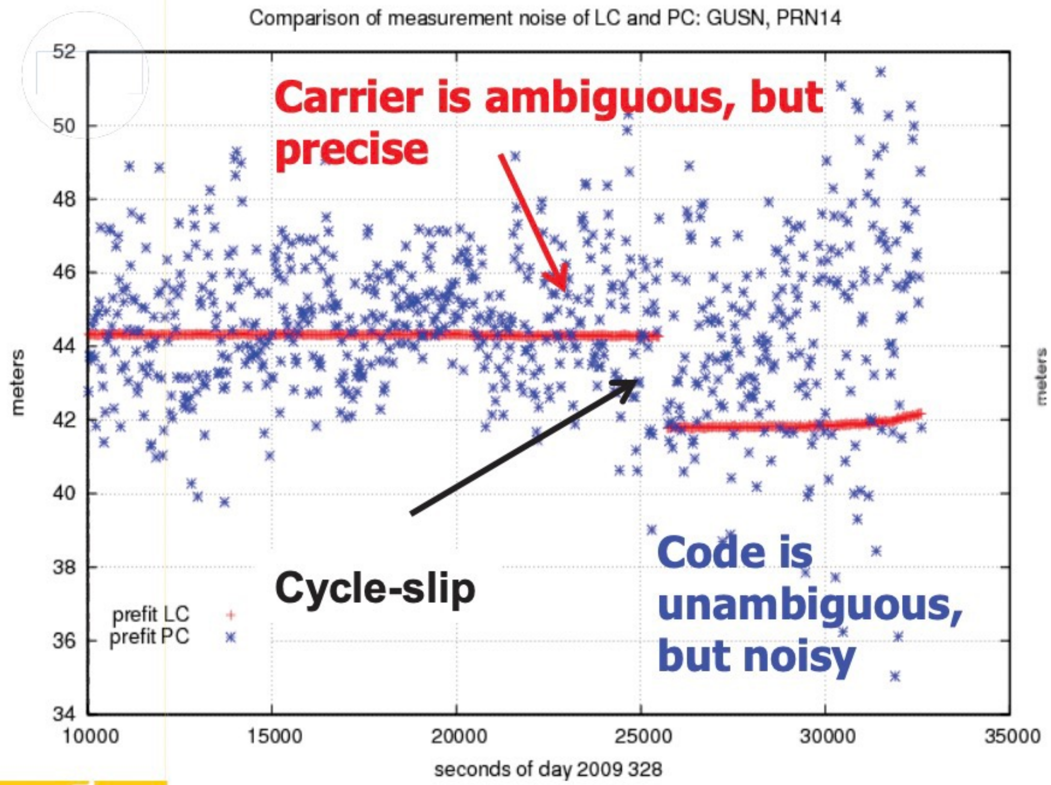
Computation of satellite clocks from precise products

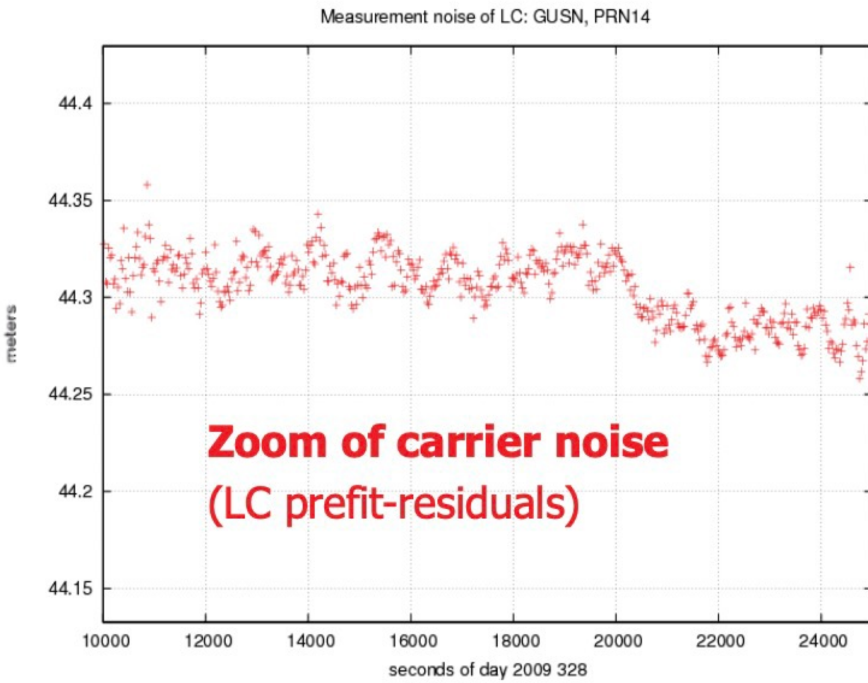
- Precise clocks for GPS satellites can be found on the International GNSS Service (IGS) server **http://igscb.jpl.nasa.gov**
- They are providing precise orbits and clock files with a sampling rate of 15 min, as well as precise clock files with a sample rate of 5 min and 30 s in SP3 format.
- Some centres also provide GPS satellite clocks with a 5 s sampling rate, like the les obtained from the Crustal Dynamics Data Information System (CDDIS) site.
- Stable clocks with a sampling rate of 30 s or higher can be interpolated with a first-order polynomial to a few centimetres of accuracy. Clocks with a lower sampling rate should not be interpolated, because clocks evolve as random walk processes.

Measurements: Code and carrier

- For high-accuracy positioning, the carrier phase must be used, besides the code pseudorange.
- As commented before, the carrier measurements are very precise, typically at the level of a few millimetres, but contain unknown ambiguities which change every time the receiver locks the signal after a cycle slip.
- Nevertheless, such ambiguities can be estimated in the navigation solution, together with the coordinates and other parameters.

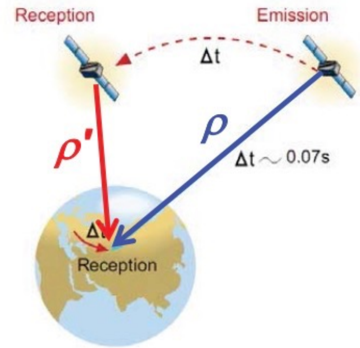
A cycle slip causes a jump in carrier-phase measurements when the receiver phase tracking loops experience a temporary loss of lock due to signal blockage or some other disturbing factor. On the other hand, pseudoranges remain unaffected.

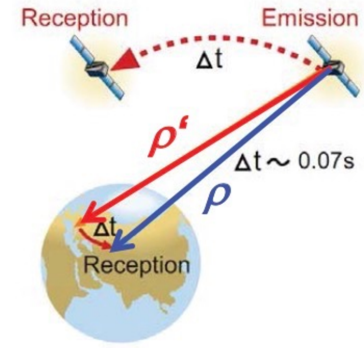


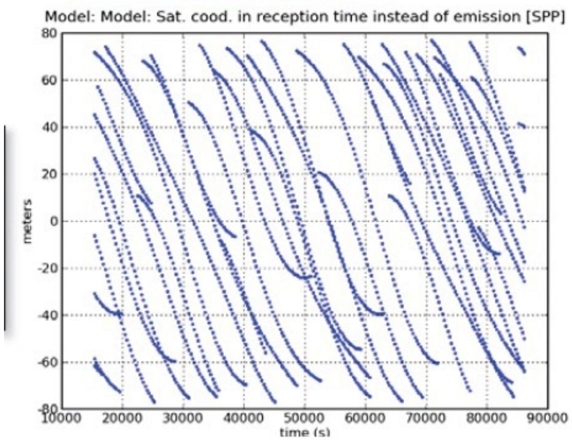


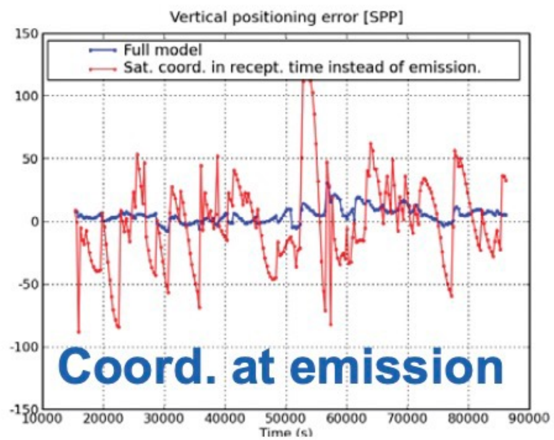
- **Carrier phase biases are estimated in the navigation filter** along with the other parameters (coordinates, clock offsets, etc.). If these biases were fixed, measurements accurate to the level of few millimetres would be available for positioning.

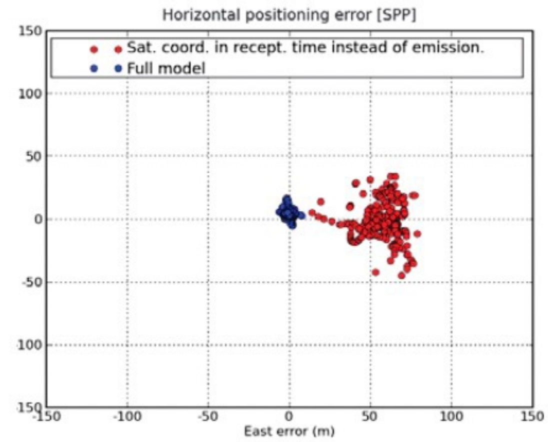
Satellite coordinates computation at signal emission time

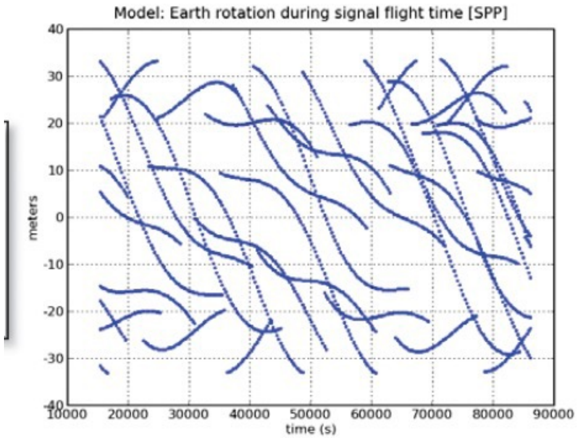


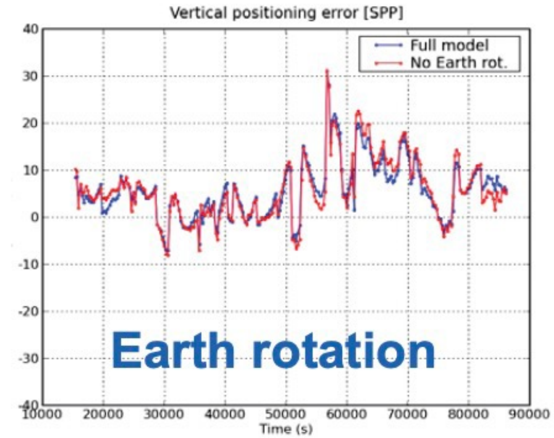


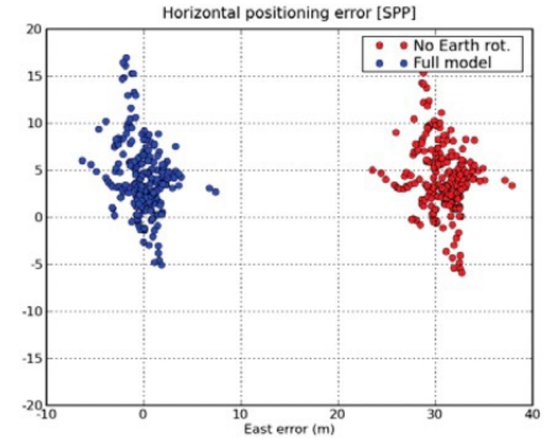




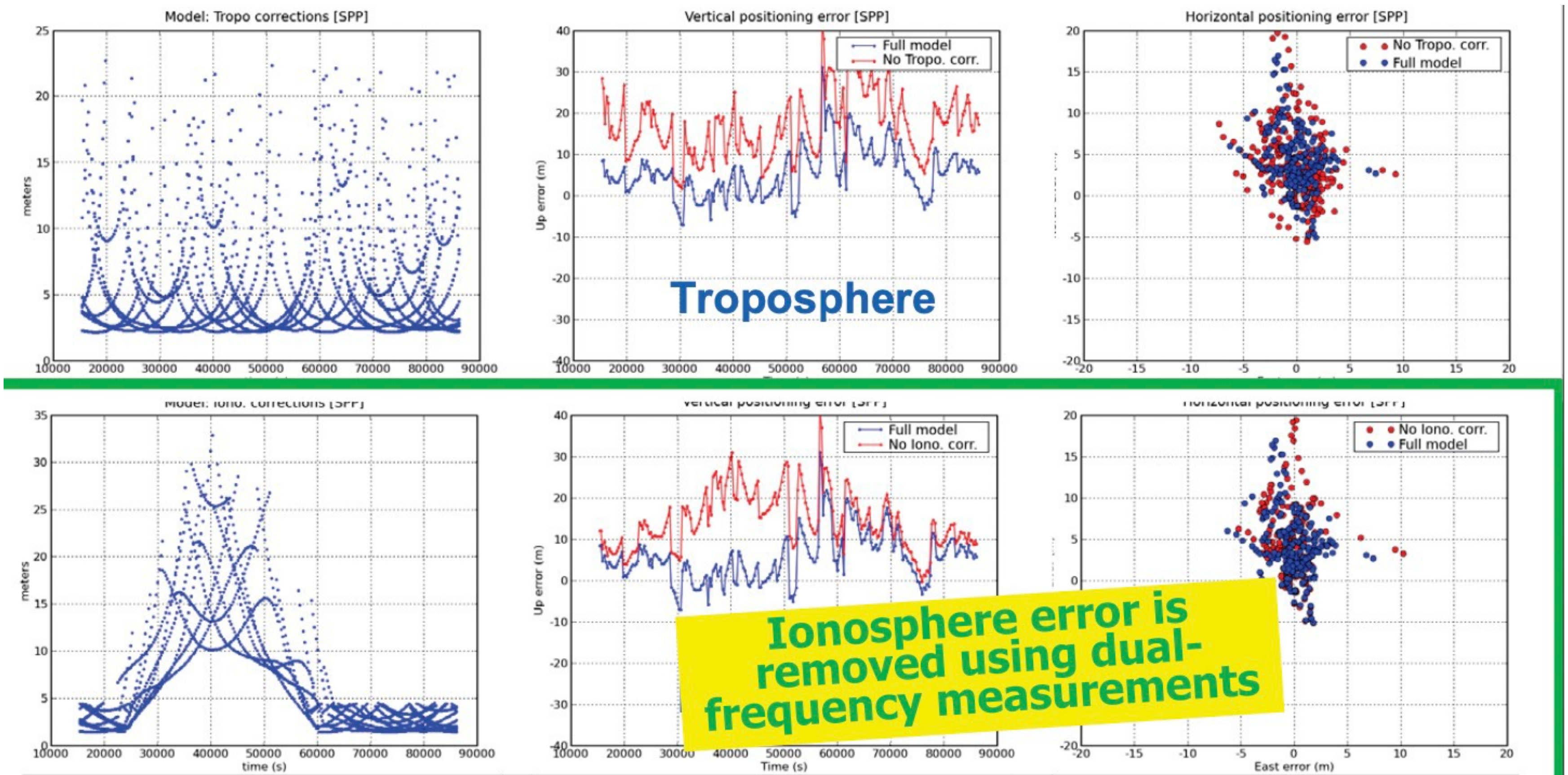








Signal propagation errors on the Atmosphere



Ionosphere Free Combinations

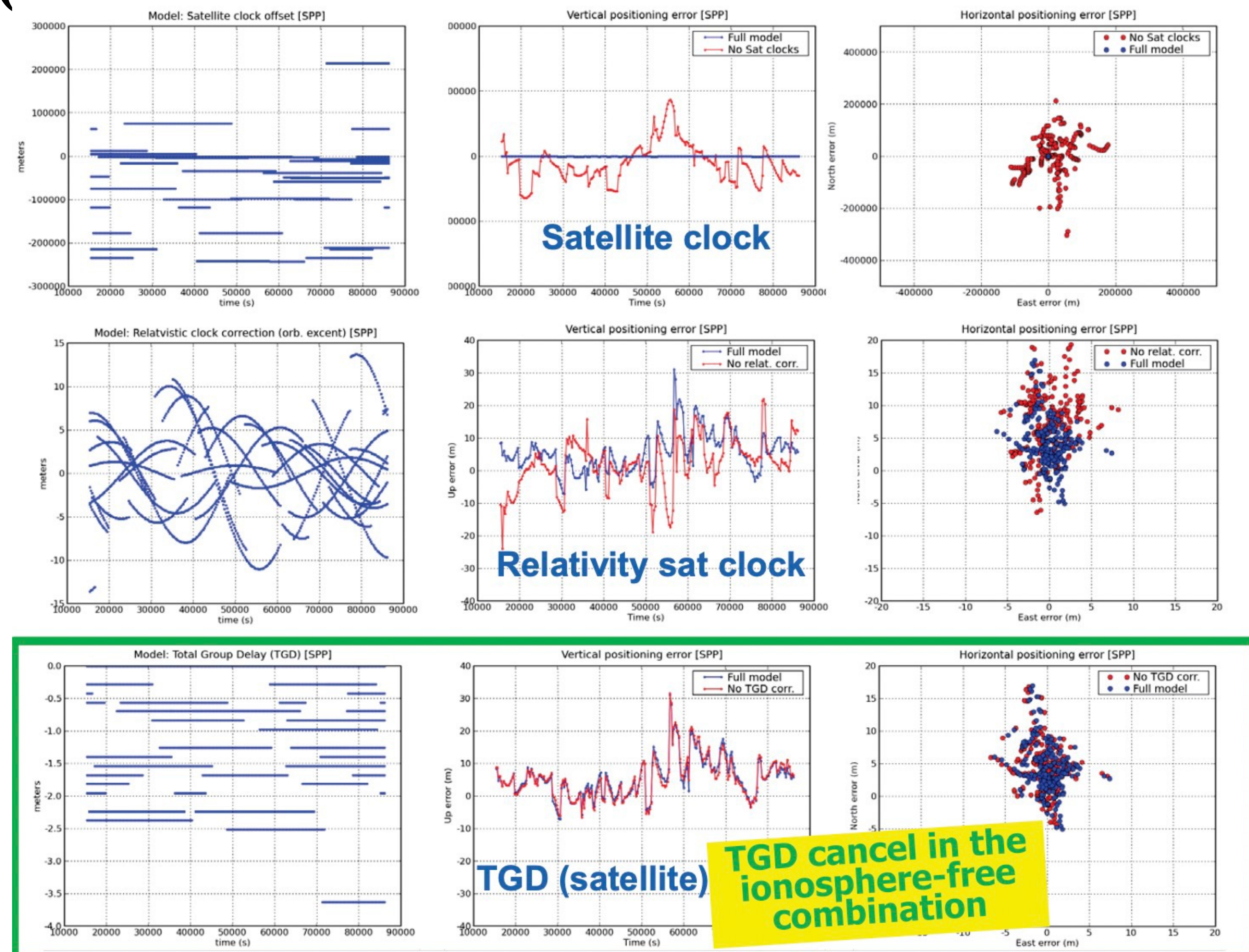
$$n_{ph} = 1 - \frac{40.3}{f^2} N_e \qquad n_{gr} = 1 + \frac{40.3}{f^2} N_e$$

- It removes the first order (up to 99.9%) ionospheric effect, which depends on the inverse square of the frequency

$$\Phi_{LC} = \frac{f_1^2 \ \Phi_{L1} - f_2^2 \ \Phi_{L2}}{f_1^2 - f_2^2} \quad ; \quad R_{PC} = \frac{f_1^2 \ R_{P1} - f_2^2 \ R_{P2}}{f_1^2 - f_2^2}$$

- Satellite clocks are defined relative to R_{PC} combination

Satellite clocks and Total Group Delay (TGD)



Instrumental Delay

- Possible sources of instrumental delays are antennas, cables, as well as different filters used in receivers and satellites. These instrumental delays affect both, code and carrier measurements.
- The receiver instrumental delay is assimilated in the receiver clock. That is, being common for all satellites, it is assumed as zero and is included in the receiver clock estimate.
- Satellite clocks (broadcast or precise) are referred to the ionosphere-free combination of codes (R_{PC}) and, thence, the instrumental delays cancel in such combination of two frequency signals

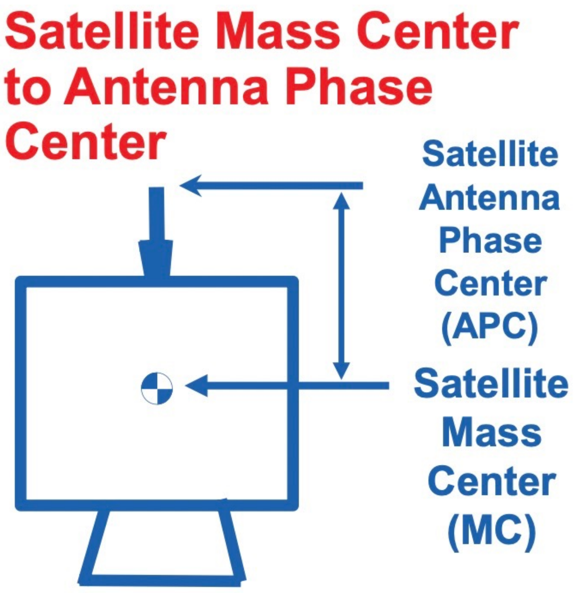
TGD and DCB

- For single frequency users, the satellites broadcast in their navigation messages the Timing Group Delay or Total Group Delay (TGD)
- The interfrequency bias between the code instrumental delays at the frequencies f1 and f2, is called Differential Code Biases (DCB)
- The DCBs of the satellites are broadcast in the GNSS navigation messages as the TGD. TGD is proportional to the Differential Code Bias (DCB).

$$TGD_{P1} = \frac{-1}{\gamma_{12}-1}(K_{P2}^{sat} - K_{P1}^{sat}) = -\hat{\alpha}_1 K_{P21}^{sat} \qquad TGD_{P2} = \gamma_{12}\, TGD_{P1}$$

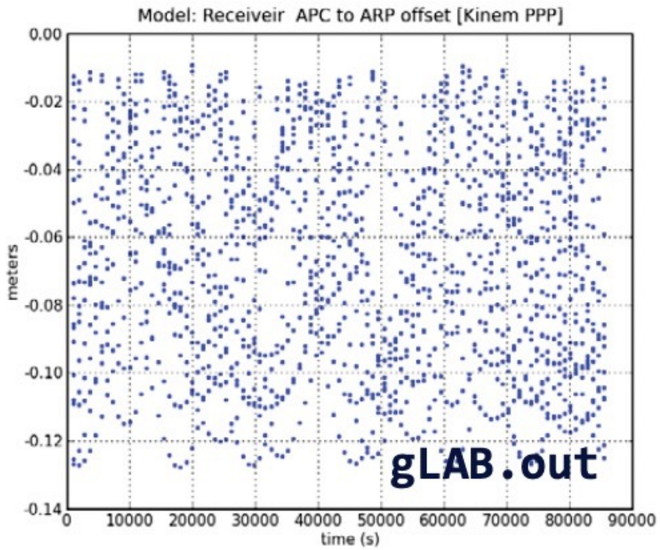
Additional Modelling for PPP

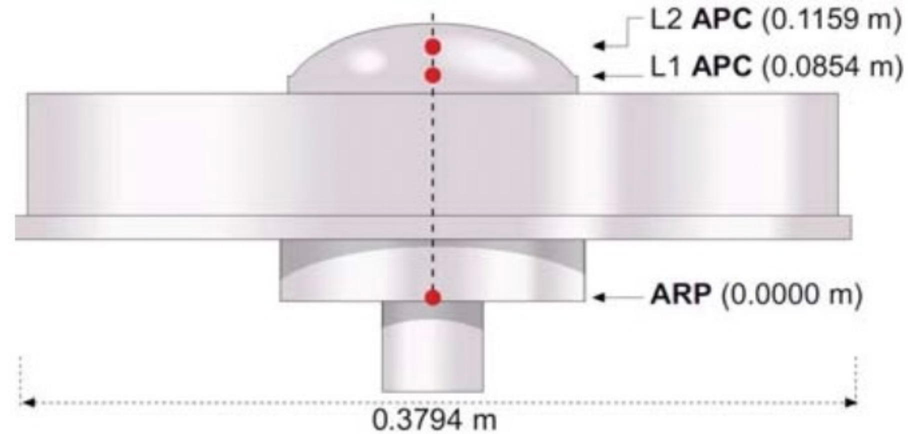
- GPS Broadcast orbits are referred to the antenna phase center, but IGS precise orbits are referred to the satellite mass center.
- The satellite MC to APC eccentricity vector depends on the satellite. The APC values used in the IGS orbits and clocks products are referred to the iono-free combination. They are given in the IGS ANTEX files. (e.g., igs05.atx).



Additional Modelling for PPP

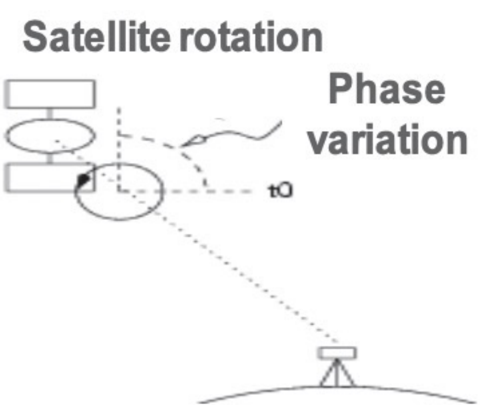
- GNSS measurements are referred to the **Receiver Antenna Phase center (APC)**. This is not necessarily the geometric center of the antenna, and it depends on the signal frequency and the incoming radio signal direction. For geodetic positioning a reference tied to the antenna reference point (ARP) is used

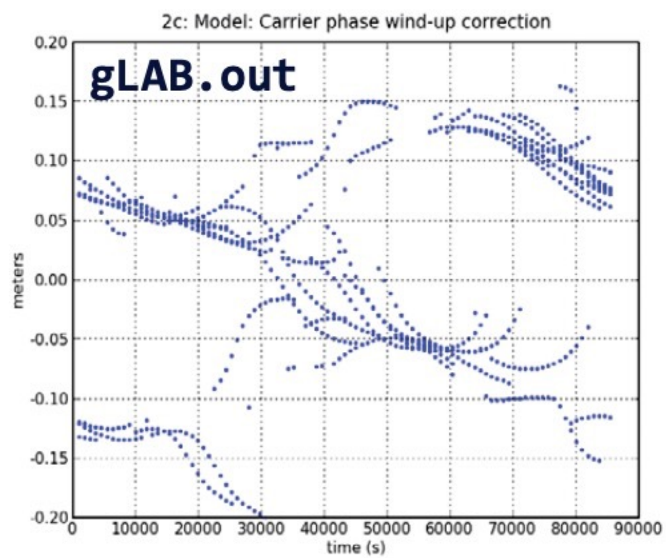




Additional Modelling for PPP

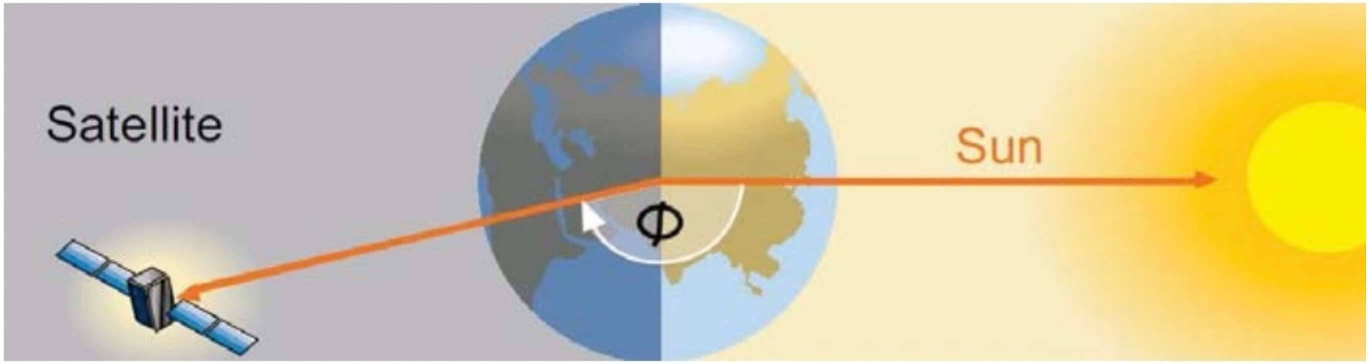
- **Wind-up** affects only carrier phase. It is due to the electromagnetic nature of circularly polarized waves of GNSS signals.
- As the satellite moves along its orbital path, it performs a rotation to keep its solar panels pointing to the Sun direction. This rotation causes a carrier variation, and thence, a range measurement variation.





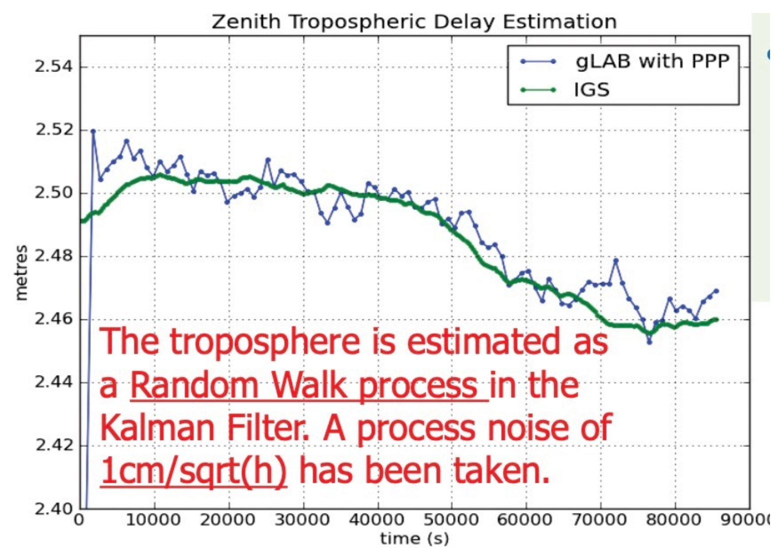
Additional Modelling for PPP

- High-accuracy GNSS positioning degrades during the GNSS satellites eclipse seasons.
  - Once the satellite goes into shadow, the radiation pressure vanishes. This effect introduces errors in the satellite dynamics due to the difficulty of properly modelling the solar radiation pressure.
  - On the other hand, the satellite's solar sensors lose sight of the Sun and the attitude control (trying to keep the panels facing the Sun).
- As a consequence, the orbit during shadow and eclipse periods may be considerably degraded and the removal of satellites under such conditions can improve the high-precision positioning results.



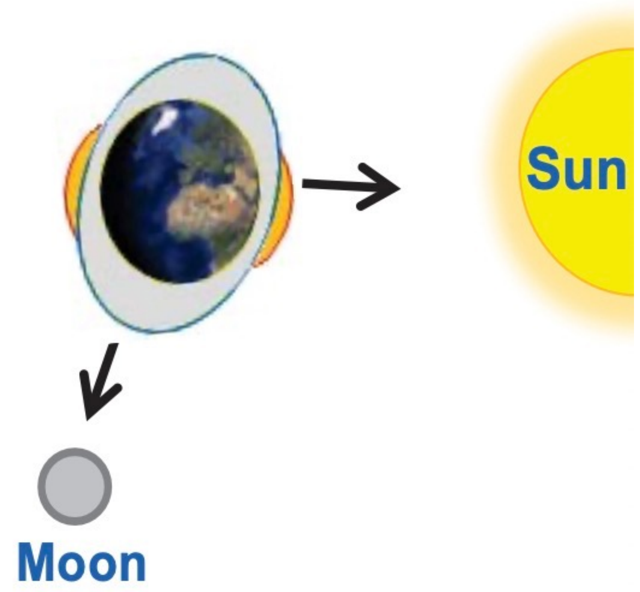
Additional Modelling for PPP

- The ionospheric refraction and TGDs are removed using the ionosphere-free combination of code and carrier measurements
- The tropospheric refraction can be modelled by Dry and Wet components
- A residual tropospheric delay is estimated (as wet ZTD delay) in the Kalman filter, together with the coordinates, clock and carrier phase biases.



Additional Modelling for PPP

- **Earth Deformation**
- **Solid Tides** concern the movement of Earth's crust (and thus the variation in the receiver's location coordinates) due to gravitational attractive forces produced by external bodies, mainly the Sun and Moon.
- Solid tides produce vertical and horizontal displacements



Linear observation model for PPP

- It is based code and carrier measurements in the ionosphere-free combination (Pc, Lc), which are modelled as follows:

$$P_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + \mathcal{M}_{Pc} + \varepsilon_{P_C}$$

$$L_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c\cdot(dt_{rec} - dt^{sat}) + Trop_{rec}^{sat} + \lambda_N \omega_{rec}^{sat} + B_{C\,rec}^{\;sat} + m_{L_C} + \varepsilon_{L_C}$$

where

$$P_C = \frac{f_1^2 P_1 - f_2^2 P_2}{f_1^2 - f_2^2}; \quad L_C = \frac{f_1^2 L_1 - f_2^2 L_2}{f_1^2 - f_2^2}$$

Ionosphere is removed

$$P_{C\,rec}^{\;sat} = \rho_{rec}^{sat} + c \cdot (dt_{rec} - dt^{sat}) + Trop + \varepsilon$$

Linearising ρ around an 'a priori' receiver position $(x_{rec,0}, y_{rec,0}, z_{rec,0})$

$$= \rho_{rec,o}^{sat} + \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c\left(dt_{rec} - dt^{sat}\right) + Trop$$

where:

$$\Delta x_{rec} = x_{rec} - x_{rec,o} \quad ; \quad \Delta y_{rec} = y_{rec} - y_{rec,o} \quad ; \quad \Delta z_{rec} = z_{rec} - z_{rec,o}$$

and taking: $Trop_{rec}^{sat} = Trop_{0\,rec}^{\;sat} + M_{wet,rec}^{\;sat}\, \Delta Tr_{Z,wet}$

Prefit-residuals (Prefit)

$$\boxed{P_{C\,rec}^{\;sat} - \rho_{rec,o}^{sat} + c\,dt^{sat} - Trop_{0rec}^{\;sat}} = \frac{x_{rec,o} - x^{sat}}{\rho_{rec,o}^{sat}} \Delta x_{rec} + \frac{y_{rec,o} - y^{sat}}{\rho_{rec,o}^{sat}} \Delta y_{rec} + \frac{z_{rec,o} - z^{sat}}{\rho_{rec,o}^{sat}} \Delta z_{rec} + c\,dt_{rec} + M_{wet}^{\;sat} \Delta Tr_{Z,wet}$$

measurement

computed

unknown

The same for carrier, but adding the ambiguity as an unknown

$$\mathbf{y} = \mathbf{G}\,\mathbf{x}$$

$$\begin{bmatrix} Prefit(Pc)^1 \\ Prefit(Lc)^1 \\ \cdots\cdots \\ Prefit(Pc)^n \\ Prefit(Lc)^n \end{bmatrix} = \begin{bmatrix} \frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 0 & \cdots & \cdots & 0 \\ \frac{x_{o,rec}-x^1}{\rho^1_{0,rec}} & \frac{y_{o,rec}-y^1}{\rho^1_{0,rec}} & \frac{z_{o,rec}-z^1}{\rho^1_{0,rec}} & 1 & M^1_{wet} & 1 & \cdots & \cdots & 0 \\ \vdots & \vdots & \vdots & \vdots & \cdots & \vdots & \cdots & \vdots \cdots & \vdots \\ \frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \cdots & \cdots & 0 \\ \frac{x_{o,rec}-x^n}{\rho^n_{0,rec}} & \frac{y_{o,rec}-y^n}{\rho^n_{0,rec}} & \frac{z_{o,rec}-z^n}{\rho^n_{0,rec}} & 1 & M^n_{wet} & 0 & \cdots & \cdots & 1 \end{bmatrix} \begin{bmatrix} \Delta x_{rec} \\ \Delta y_{rec} \\ \Delta z_{rec} \\ cdt_{rec} \\ \Delta Tr_{Z,wet} \\ B_C{}^1 \\ \vdots \\ B_C{}^n \end{bmatrix}$$

Carrier ambiguities

$$Prefit(P_C)^k = P_C{}^k - \rho_0^k + cdt^k - Trop_0^k$$

$$Prefit(L_C)^k = L_C{}^k - \rho_0^k + cdt^k - Trop_0^k - \lambda_N \omega^k$$

Floating Ambiguities

- The linear observation model **y = Gx** can be solved using the **Kalman filter**. The stochastic model are defined as:
- **Carrier phase biases** are taken as `constant´ along continuous phase arcs, and as `white noise´ when a cycle slip happens
- **Wet tropospheric delay** is taken as a random walk process
- **Receiver clock** is taken as a white-noise process
- **Receiver coordinates**
  - For **static positioning** the coordinates are taken as constants.
  - For **kinematic positioning** the coordinates are taken as white noise or a random walk process.

Floating Ambiguities

- This solution procedure is called **floating ambiguities**. `Floating´ in the sense that the ambiguities are estimated by the filter `as real numbers´.
- The bias estimations will converge to a solution after a transition time that depends on the observation geometry, model quality and data noise.
- In general, one must expect errors at the decimetre level, or better, in pure kinematic positioning (after the best part of one hour) and at the centimetre level in static PPP over 24h data.