THE HONG KONG POLYTECHNIC UNIVERSITY
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AAE6102 – Satellite Communication and Navigation

Error Terms and DGPS

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Measurement Errors

- The accuracy of the position/time solution determined by GPS is ultimately expressed as the product of a geometry factor and a pseudorange error factor.

$$\hat{\mathbf{x}} = \left(\mathbf{H}^T\mathbf{H}\right)^{-1}\mathbf{H}^T\mathbf{y}$$

(error in GPS solution) = (geometry factor) × (pseudorange error factor)

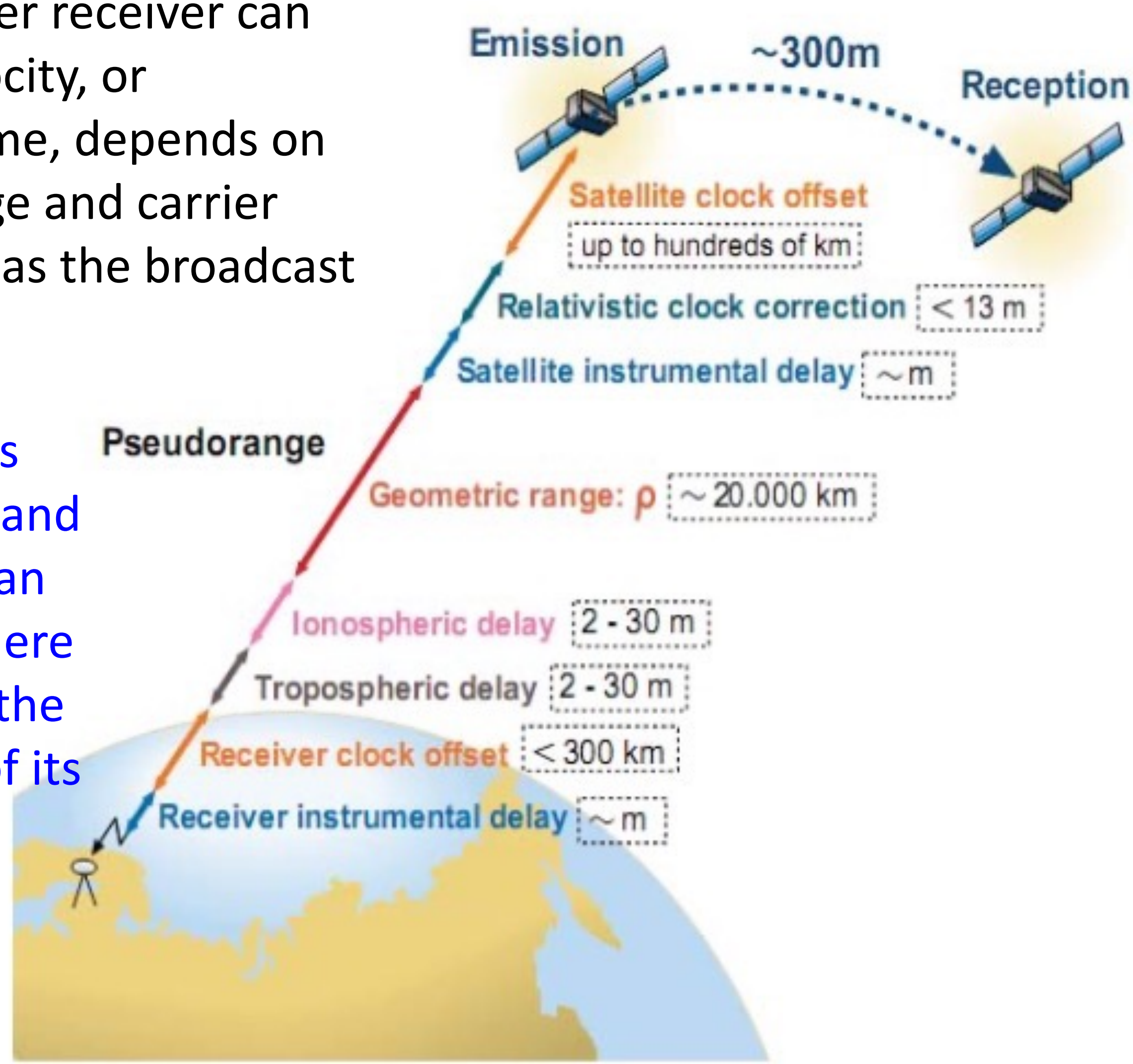
- here are a number of sources of error that corrupt the measurements including atmosphere, receiver noise and interference, multipath and receiver hardware offsets

$$\delta t_D = \delta t_{atm} + \delta t_{noise\&int} + \delta t_{mp} + \delta t_{hw}$$

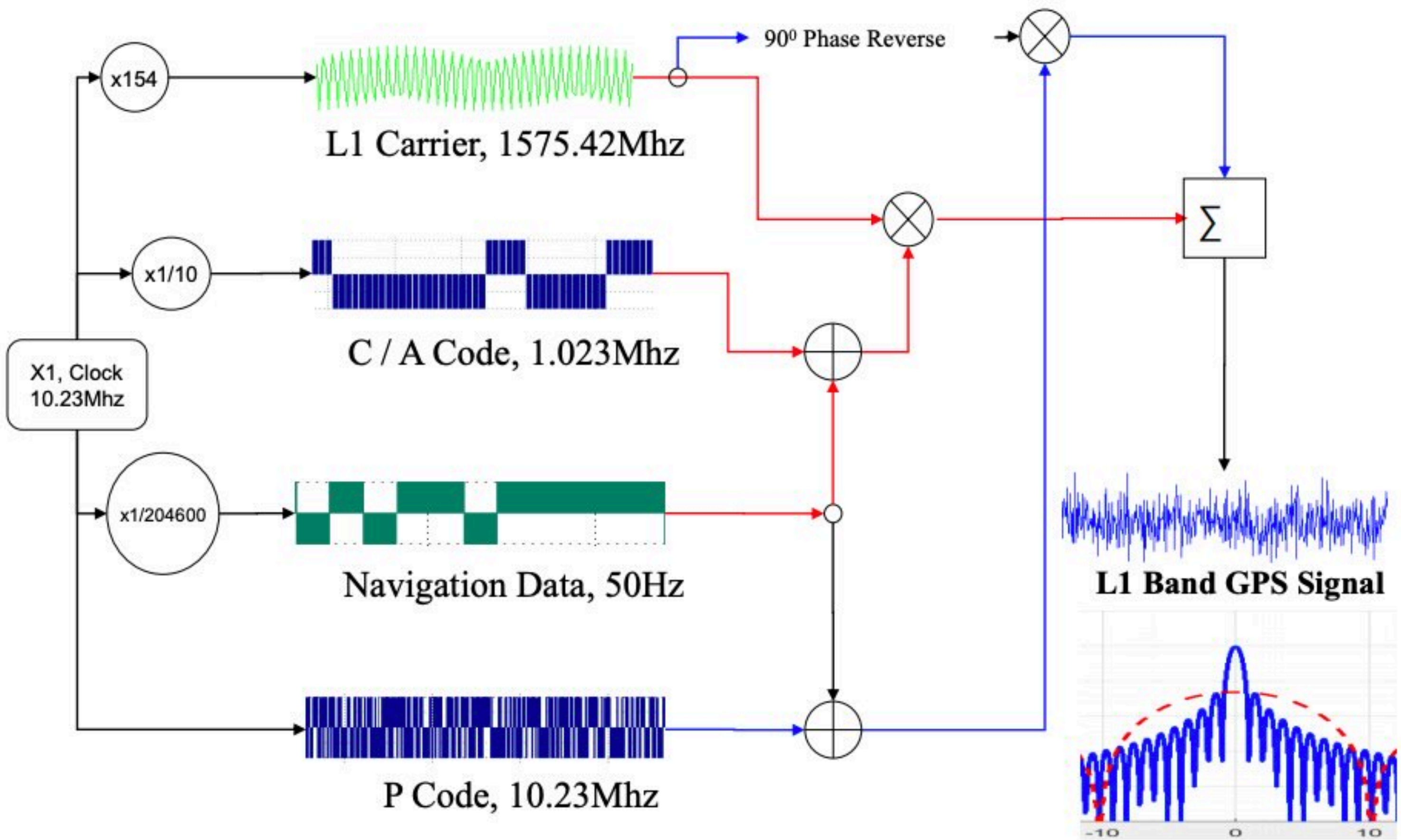
Performance of Stand-Alone Positioning

The accuracy with which a user receiver can determine its position or velocity, or synchronize to GPS system time, depends on the quality of the pseudorange and carrier phase measurements as well as the broadcast navigation data.

Usually, the error components are considered independent, and is approximated as a zero mean Gaussian random variable where its variance is determined as the sum of the variance of each of its components.



Satellite Clock in signal generation



Satellite Clock Error

- The satellites contain atomic clocks that control all onboard timing operations, including broadcast signal generation. Although these clocks are highly stable, the clock correction fields in the navigation data message are sized such that the deviation between SV time and GPS time may be as large as 1 ms, which translates to a 300 km pseudorange error.

$$\delta t_{clk} = a_{f0} + a_{f1}(t - t_{oc}) + a_{f2}(t - t_{oc})^2 + \Delta t_r$$

The control segment determines and transmits clock correction parameters to the satellites for rebroadcast in the navigation message

where:

a_{f0} = clock bias (s)

a_{f1} = clock drift (s/s)

a_{f2} = frequency drift (i.e., aging) (s/s^2)

t_{oc} = clock data reference time (s)

t = current time epoch (s)

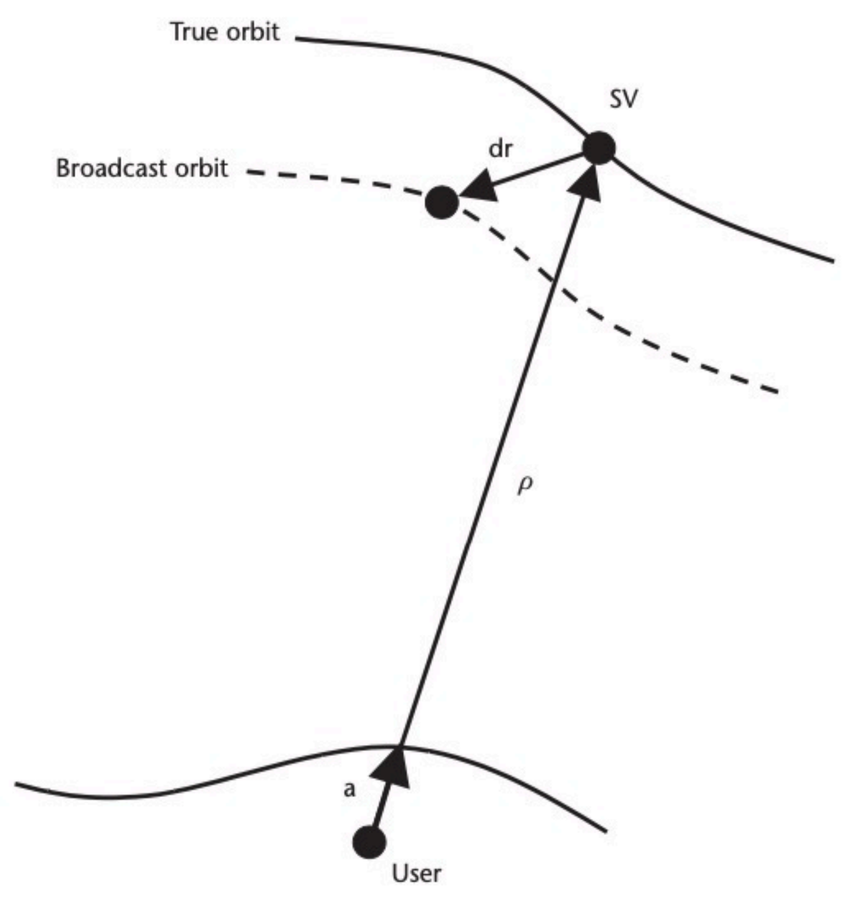
Δt_r = correction due to relativistic effects (s)

Satellite Clock Error

- Since these parameters are computed using a **curve-fit** to predicted estimates of the actual satellite clock errors, some residual error remains. This residual clock error, δt, results in ranging errors that typically vary from 0.3–4m, depending on the type of clock (Rubidium, Hydrogen) and age of the broadcast data.
- The stability of the rubidium clock is so good that it would lose only three seconds in one million years, while the passive hydrogen maser is even more stable, and it would lose only one second in three million years.
- At zero age of data (ZAOD), clock errors for a typical satellite are on the order of 0.8m. Errors 24 hours after an upload are generally within the range of 1–4m.

Ephemeris Error

- Estimates of ephemerides for all satellites are computed and uplinked to the satellites with other navigation data message parameters for rebroadcast to the user. Similar as the satellite clock corrections, these corrections are generated using a **curve fit** of the control segment's best prediction of each satellite's position at the time of upload.
- The effective pseudorange and carrier-phase errors due to ephemeris prediction errors can be computed by projecting the satellite position error vector onto the satellite-to-user Line Of Sight (LOS) vector.

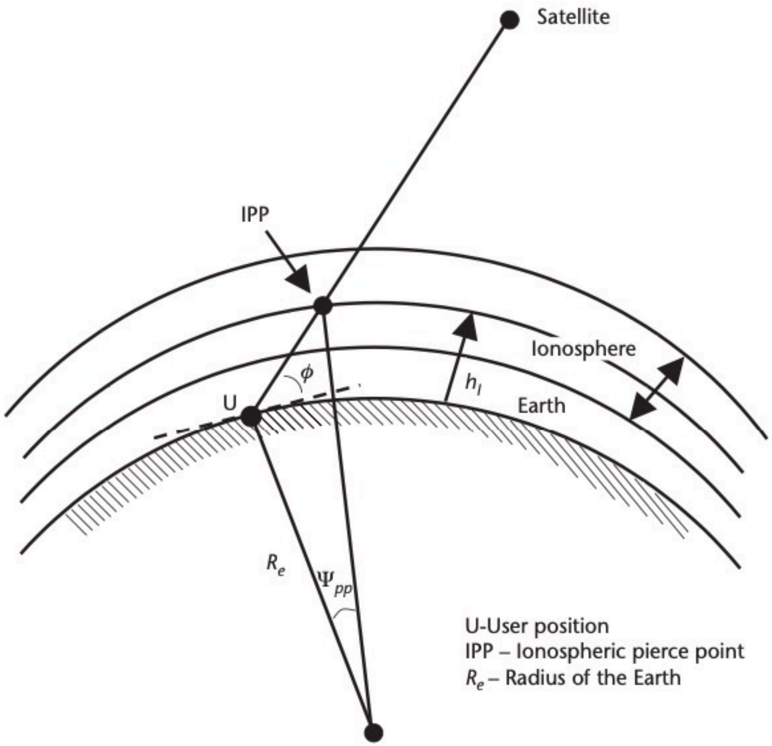


Ionosphere Delay

The propagation speed of a wave in a medium v can be expressed in terms of the index of refraction for the medium n.

$$n = \frac{c}{v}$$

where *c* is the speed of light equal to 299,792,458 m/s as defined within the WGS-84 system.



- The ionosphere is a dispersive medium located primarily in the region of the atmosphere between about 70 km and 1,000 km above the Earth’s surface. Within this region, ultraviolet rays from the sun ionize a portion of gas molecules and release free electrons. These free electrons influence electromagnetic wave propagation, including the GPS satellite signal broadcasts.
- The medium is **dispersive** if the propagation speed (or, equivalently, the index of refraction) is a function of the wave’s frequency.

Phase and Group Velocity

- In a dispersive medium, the propagation velocity v_p of the signal's carrier phase (Phase Velocity) differs from the Group Velocity v_g associated with the waves carrying the signal information. The information-carrying aspect can be thought of as *a group of waves* traveling at slightly different frequencies.

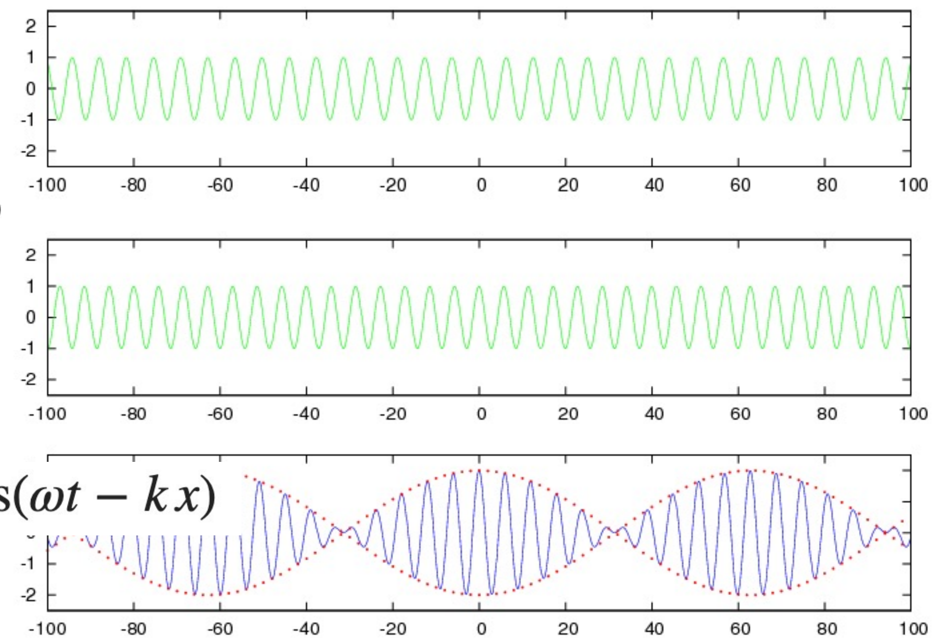
$$s(x,t) = s_0 \cos(\omega t - kx + \phi_0)$$

$$\omega = 2\pi/T,\ k = 2\pi/\lambda \text{ and } T = 1/f = 2\pi/\omega$$

$$s_1(x,t) = \cos((\omega + \Delta\omega)t - (k + \Delta k)x),$$

$$s_2(x,t) = \cos((\omega - \Delta\omega)t - (k - \Delta k)x)$$

$$s_1(x,t) + s_2(x,t) = 2\cos(\Delta\omega t - \Delta k x)\cos(\omega t - kx)$$



$$v_{gr} = \frac{d\omega}{dk} \qquad v_{ph} = \frac{\omega}{k}$$

Ionosphere Delay

- A medium where the angular frequency ω and the wave number k are not proportional, is a dispersive media (i.e., the wave propagation speed and thence, the refractive index depends on the frequency). This is the case with the ionosphere where ω and k are related, in a first approximation, by *Relation of Dispersion* of the ionosphere $$\omega^2 = c^2 k^2 + \omega_p^2$$

- ω_p is called the critical frequency of the ionospheric plasma, in the sense that signals with $\omega<\omega_p$ will be reflected and signals with $\omega>\omega_p$ will cross through the plasma

$$\omega_p = 2\pi f_p \qquad \text{with} \qquad f_p = 8.98\sqrt{N_e} \qquad \text{in Hz}$$

Ionosphere Delay

- The electron density in the ionosphere changes with the height having maximum of $Ne \simeq 10^{11}-10^{12} e^{-}/m^{3}$ around 300−500km.
- Electromagnetic signals with $f > f_p \simeq 10^6$Hz will be able to cross the ionosphere. This is the case of GNSS signals which frequencies are at the order of $1\ GHz = 10^9\ Hz$. Radio signals with frequencies under f_p will be reflected in the ionosphere.
- The index of refraction is

$$n_{ph} = \frac{c}{v_{ph}} \qquad , \qquad n_{gr} = \frac{c}{v_{gr}}$$

$$n_{ph} = \sqrt{1 - \left(\frac{f_p}{f}\right)^2} \simeq 1 - \frac{1}{2}\left(\frac{f_p}{f}\right)^2 = 1 - \frac{40.3}{f^2} N_e$$

$$n_{gr} = 1 + \frac{40.3}{f^2} N_e$$

Ionosphere Delay

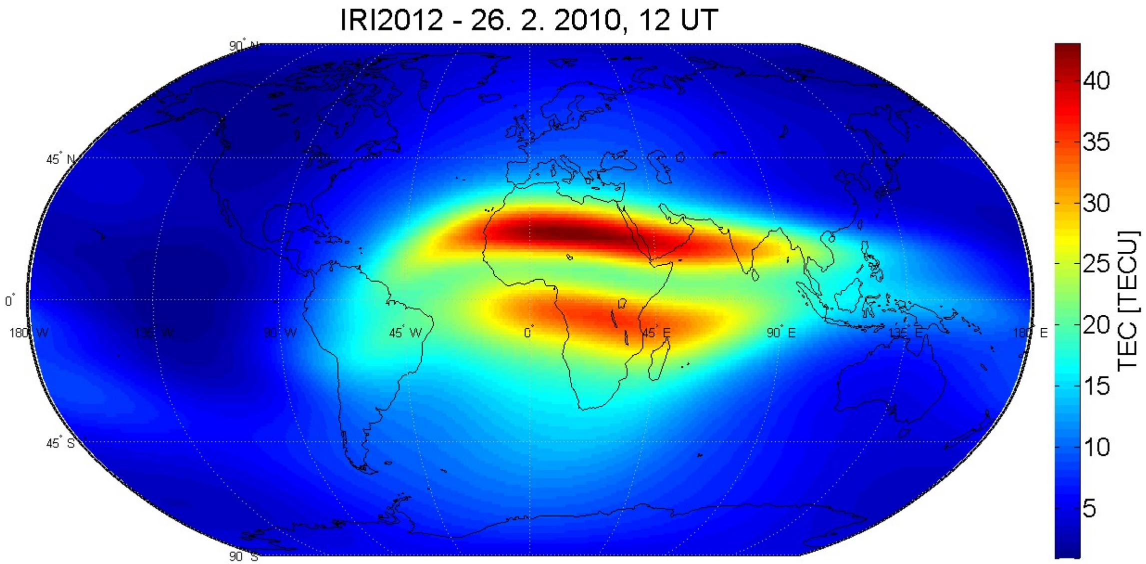
- It can be observed that the phase velocity will exceed that of the group velocity. The amount of retardation of the group velocity is equal to the advance of the carrier phase with respect to free-space propagation. In the case of GPS, this translates to the signal information (e.g., PRN code and navigation data) being delayed and the carrier phase experiencing an advance, a phenomenon referred to as ***ionospheric divergence***.
- The path length difference due to ionospheric refraction is the phase and code ionospheric refraction

$$\Delta_{ph,f}^{iono} = -\frac{40.3}{f^2}\int N_e\, dl \qquad \Delta_{gr,f}^{iono} = +\frac{40.3}{f^2}\int N_e\, dl$$

- the integral is defined as the Slant TEC (STEC). *The total electron count* (TEC) is expressed as electrons/m^2. The TEC is a function of time of day, user location, satellite elevation angle, season, ionizing flux, magnetic activity, sun-spot cycle, and scintillation.

$$STEC = \int N_e\, dl$$

Ionosphere Delay



- Since the ionospheric delay is frequency dependent, it can be virtually eliminated by making ranging measurements with a dual-frequency receiver. An *ionospheric-free* pseudorange may be formed as:

$$\rho_{ionospheric-free} = \frac{\rho_{L2} - \gamma\rho_{L1}}{1-\gamma} \qquad \text{where } \gamma = (f_{L1}/f_{L2})^2.$$

Troposphere Delay

- The troposphere is the lower part of the atmosphere that is **nondispersive** for frequencies up to 15 GHz
- Within this medium, the phase and group velocities associated with the GPS carrier and signal information (PRN code and navigation data) on both L1 and L2 are equally delayed with respect to free-space propagation. The path length difference can also be expressed in terms of **refractivity index N**,

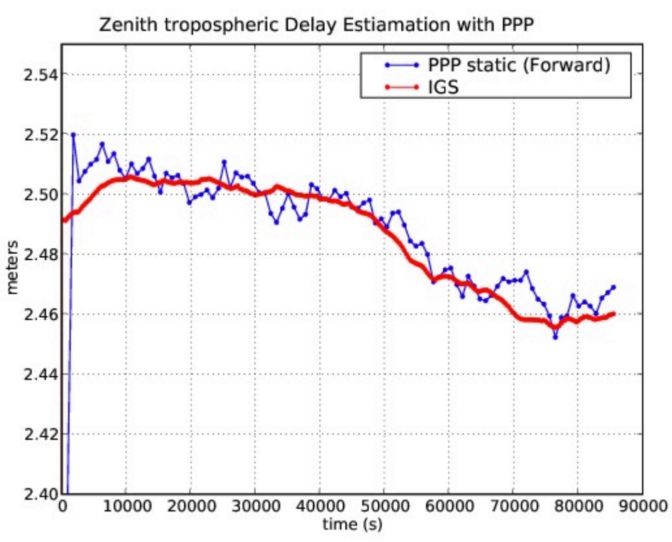
$$\Delta S_{tropo} = 10^{-6} \int_{SV}^{User} N ds$$

- The refractivity is often modeled as including both a dry (hydrostatic) and wet (nonhydrostatic) component, which is dependent on the local temperature, pressure, and relative humidity.

Troposphere Delay

- The dry component, which arises from the dry air, gives rise to about 90% of the tropospheric delay and can be predicted very accurately. The wet component, which arises from the water vapor, is more difficult to predict due to uncertainties in the atmospheric distribution.
- Both components extend to different heights in the troposphere; the dry layer extends to a height of about 40 km, while the wet component extends to a height of about 10 km.
- Troposphere models are developed to

compensate for this effect.

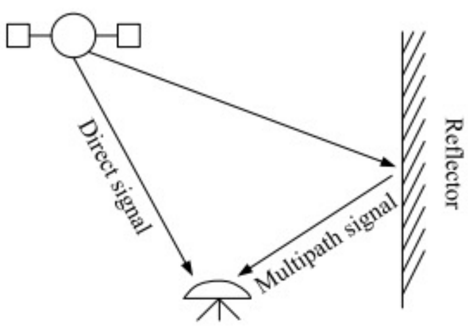


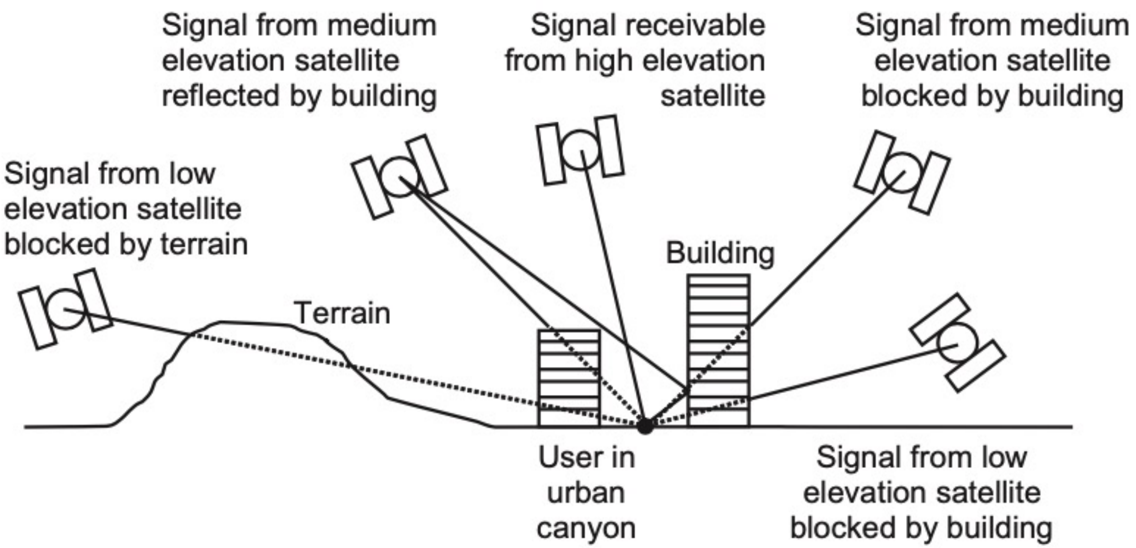
Receiver Noise

- Measurement errors are also induced by the receiver tracking loops, where the dominant sources of pseudorange measurement error are thermal **noise jitter**, the effects of **interference/jamming** and **multipath**.
- Receiver noise and resolution errors affect carrier phase measurements, which are on the order of 1.2 mm (1σ) when tracking the C/A code and 1.6 mm (1σ) when tracking the P(Y) code in nominal conditions.
- Intentional Jamming: A transmitter, tuned to the same frequency as the opponents' receiving equipment and with the same type of modulation, can, with enough power, override any signal at the receiver.

Multipath

- One of the most significant errors incurred in the receiver measurement process is multipath.
- Multipath errors vary significantly in magnitude depending on the environment within which the receiver is located, satellite elevation angle, receiver signal processing, antenna gain pattern, and signal characteristics.





Choke ring antenna

Multipath

How to improve?

- Signal to noise density Ratio C/N0: C/N_0 values follow a nominal curve with respect to the satellite elevation that is mainly determined by the antenna gain pattern. Thus, deviations from this pattern can be easily identified and attributed to reflected and/or diffracted signals.
- Signal Processing: reduce the spacing of early and late correlator
- Antenna: increases the directivity of the antenna for the upper hemisphere and reduces reflections from below the antenna horizon.

Chock ring antenna

Hardware Bias

- 1. **Satellite Bias**: Upon signal transmission, the GPS signals on each carrier frequency and among frequencies are imperfectly synchronized due to the different digital and analog signal paths corresponding to each signal.
- 2. **User Equipment Bias**: GPS signals are delayed as they travel through the antenna, analog hardware (e.g., RF and IF filters, low-noise amplifiers, and mixers) and digital processing until the point where pseudorange and carrier-phase measurements are physically made within the digital receiver channels.

Although the absolute delay values for propagation from the antenna phase center until the digital channels may be quite large, for similar signals on the same carrier frequency the delays experienced for the set of visible signals are nearly exactly equal.

The absolute delay is important for timing applications and must be calibrated out. For many applications, however, the common delay does not affect performance, since it does not influence positioning accuracy, but rather directly appears only in the least-squares estimate of receiver clock bias.

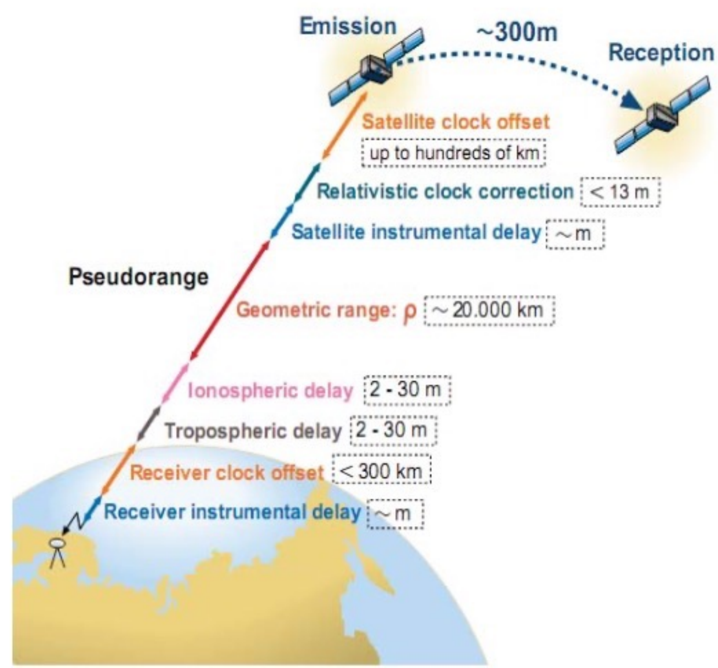
Pseudorange Error Budgets

- For a single-frequency user, the dominant pseudorange error source is the residual ionospheric delay after applying the broadcast ionospheric delay corrections. Dual frequency users can remove the error due to ionospheric delays.

Segment Source	*Error Source*	*1σ Error (m)*
Space/control	Broadcast clock	1.1
	L1 P(Y)-L1 C/A group delay	0.3
	Broadcast ephemeris	0.8
User	Ionospheric delay	7.0*
	Tropospheric delay	0.2
	Receiver noise and resolution	0.1
	Multipath	0.2
System UERE	Total (RSS)	7.1*

Spatially Correlated Errors

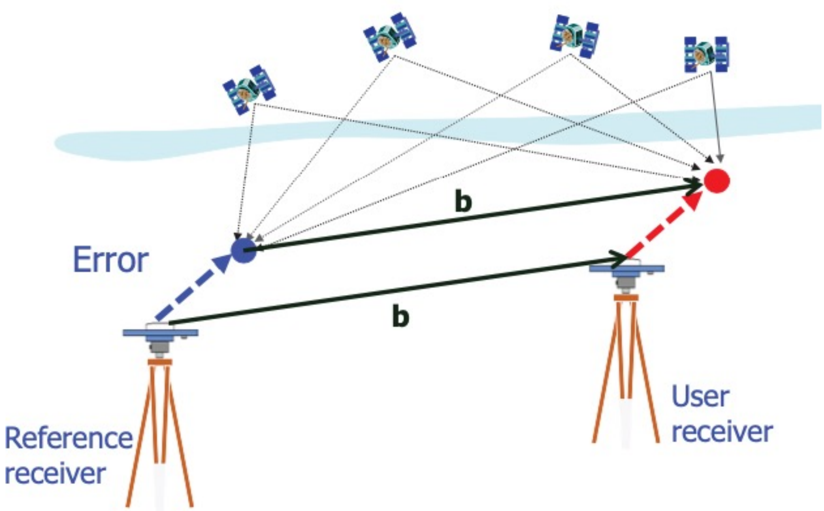
- Many of the GPS error sources discussed before are highly correlated over space, with two receivers
- Based on what you learned from standalone positioning, can you identify which errors are spatially correlated? e.g. satellite clock and ephemeris error, ionosphere and troposphere, multipath and noise.



Positioning with multiple receivers

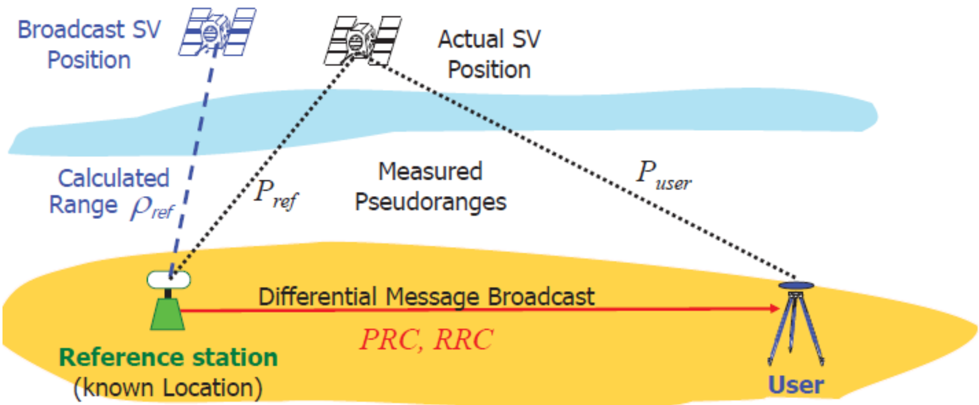
- The determination of the vector between the receivers (i.e. the baseline “b”) is more accurate than the single receiver solution, because common errors cancel out.
- The reference receiver (not necessarily at rest) broadcast its time-tagged measurements to the user. The user applies these measurements to compute its “relative position” to the reference station. With the reference station coordinates as known, the user can estimate its absolute position.

DGPS is a method to improve the positioning or timing performance of GPS using one or more reference stations at known locations, each equipped with at least one GPS receiver.



Differential GPS (DGPS)

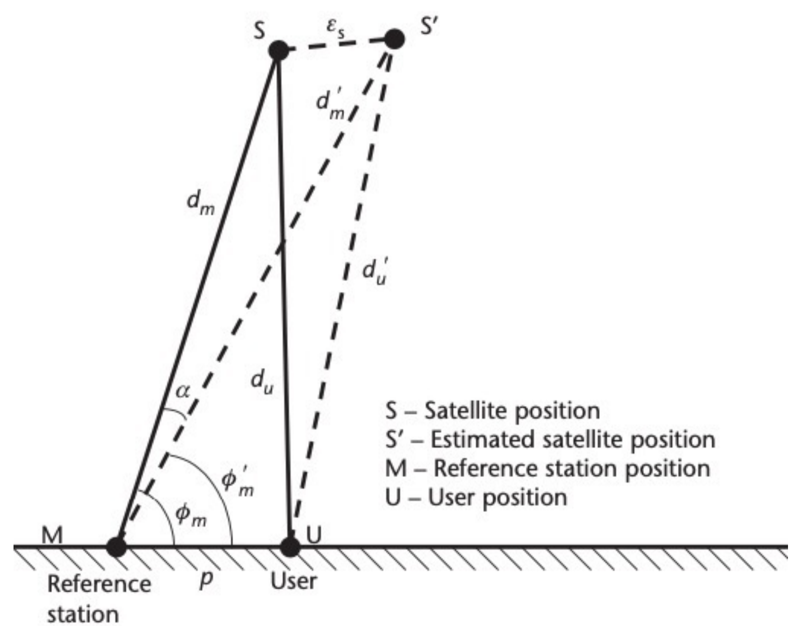
- The reference station(s) provides information to the end user via a data link that may include:
  - Corrections to the raw end user's pseudorange measurements, corrections to GPS satellite-provided clock and ephemeris data, or data to replace the broadcast clock and ephemeris information;
  - Raw reference station measurements (e.g., pseudorange and carrier phase);
  - Integrity data (e.g., "use" or "don't use" indications for each visible satellite, or statistical indicators of the accuracy of provided corrections);
  - Auxiliary data including the location, health, and meteorological data of the reference station(s).



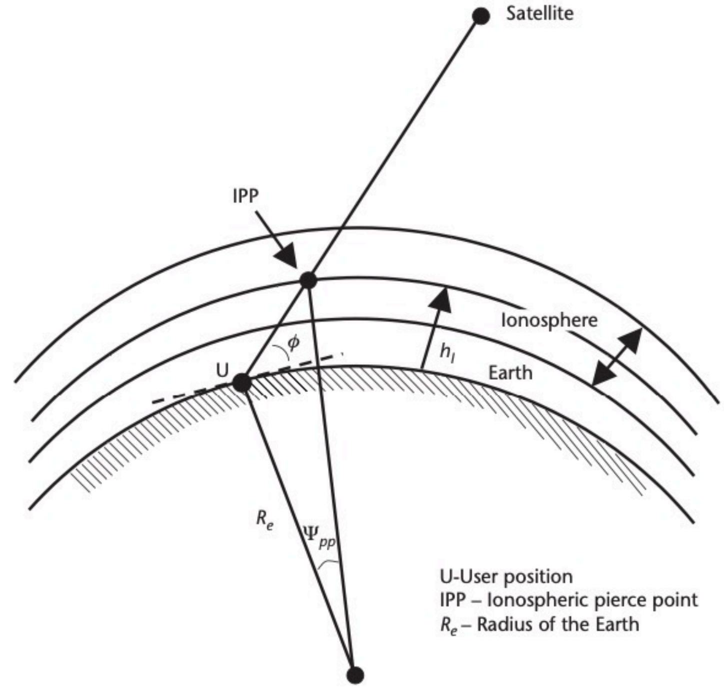
Satellite Clock and Ephemeris Errors

- Satellite clock error causes the same effect on pseudorange and carrier-phase measurements, regardless of the location of the user.
- The magnitude of ephemeris-induced pseudorange or carrier-phase errors are dependent on the LOS between the user and the satellite, these errors change with user location.

The ephemeris error increases directly with the separation between the reference station and the user receiver.

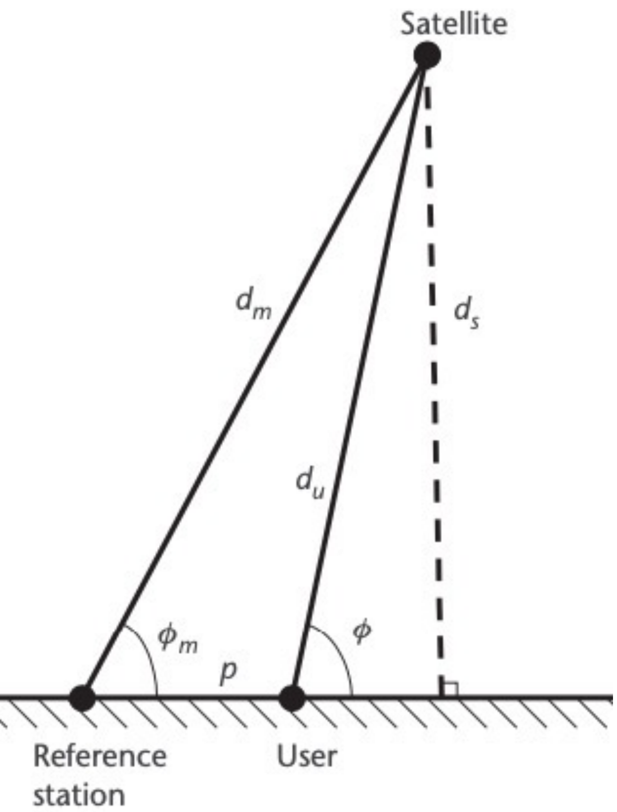


Ionospheric Error



- The ionosphere delay can be expressed by the frequency, *f*, of the signal, the elevation angle, φ′, at the ionospheric pierce point, and the total electron content, *TEC*, along the path of the signal.
- Spatial variations in TEC within the ionosphere typically lead to much greater differences in ionospheric delay than those attributable to elevation angle.
- Therefore, the residual ionosphere error after differential corrections from reference stations depends on the different locations of the user and reference antennas

Troposphere



- The troposphere delay or the speed of electromagnetic radiation varies, depending on temperature, pressure, and relative humidity, as it passes through the troposphere.
- The residual troposphere error after differential correction by the reference stations is therefore determined by the different weather parameters at two locations
- The dry component in troposphere is more easily to be modeled than the ionosphere, and the wet component also varies and not easy to be predicted.

Receiver Noise and Multipath

- Unlike the other error sources considered thus far, receiver noise and multipath result in pseudorange and carrier-phase errors that are uncorrelated between receivers separated by even very short baselines.
- Multipath often dominates error budgets for short-baseline code- and carrier-based DGPS systems. It is generally larger than those caused by receiver noise. The magnitude of multipath errors varies significantly depending on the type of receiver and environment.