

Mapping Possible Meeting-Length for Two Trajectories

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ABSTRACT

Given the pair of objects with trajectory gaps, we aim to find the possible meeting locations where the two objects can meet. The societal applications include improving maritime safety and regulations, and the challenges come from two aspects. If trajectory data are not available around the rendezvous then either linear or shortest-path interpolation may fail to detect the possible meetings of two objects. Furthermore, current methods (e.g., space-time prisms) lack interpretability within the trajectory gaps. In this paper, we proposed a method to map possible meeting-lengths and locations within trajectory gaps in order to more clearly see where and how long two objects with coinciding trajectories gaps meet. Experimental results via visualization based on synthetic trajectory data shows that the proposed approach is able to capture behavior within the trajectory gaps for a variety of different cases.

KEYWORDS: Trajectory Gaps, Space-Time prism, Linear Interpolation, Meeting Lengths

1 Introduction

Given the pair of objects with trajectory gaps, we aim to find the possible meeting locations where the two objects can meet. Figure 3 shows the input as a pair of trajectories with gaps and Figure 4 shows output as the intersection of geo-ellipses with the length of meeting.

For the motivation of our work, global security is a major concern. As we have seen from various articles in the past, events such as illegal fishing or illegal oil transfers in international waters have concerned many countries and international institutions. Hence, we need a way to be able to know where and how long entities or people may meet at a

location. Analyzing possible meeting lengths also have a variety of use cases such as finding potential criminals, working with epidemiologies, and working with astrophysics.

The problem is challenging since the gaps may cause traditional trajectory mining approaches [27] to underperform or fail where they assume the availability and preciseness of trajectories. Such methods do not capture overall object moving capabilities. A second challenge is that it is hard to define trajectory data gaps based on behavioral patterns, and the behavior of the entities is not uniform.

Current literature considers methods which have models that are harder to train due to the lack of trajectory data within trajectory gaps. In addition, current trajectory mining methods assume Linear Interpolation Methods which may miss possible meetings of two objects within trajectory gaps. Finally, other time geography model assume uniformity (i.e., an object is equally likely to present within a deterministic region) within its spatial geo-ellipse estimation. In this work, we proposed a method to map possible meeting-lengths and locations within trajectory gaps for more informative representation of space-time prisms. Such visualization may help domain experts to conclude more informative decisions.

Contributions:

- We defined trajectory gap behavior based on meeting time.
- We calculated the potential meeting times and locations for a pair of trajectories.
- We provided visualizations of experimental results based on synthetic dataset.

2 Related Work

For many years, modeling system dynamics with only partially observable states has been a

significant research subject, and several schools of thought have arisen. Space time prism in time geography is one of them and provides a powerful and reliable tool in the real world applications.

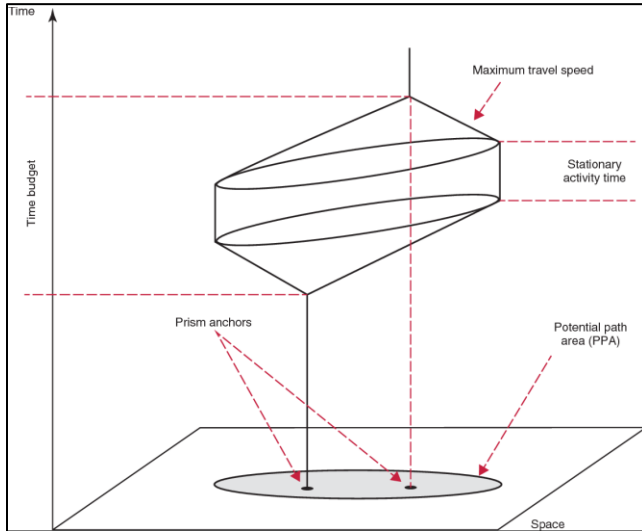


Figure 1. A 3D representation of a Space-Time Prism as depicted by Miller.

According to Miller, Time geography and space-time prism are traced back to the 1970s as a conceptual framework developed by Torsten Hagerstrand. The introduced concept asserts the presence of the uniform local velocity of motion in an anisotropic space simultaneously, ignoring the fact that the transport network limits the movement of individuals and that their speed of locomotion is determined by how the transport networks are connected [1]. The Swedish geographer is credited for nurturing practical ideas on time and space.

He was concerned that the livable world of time geography and regional science had been ignored for a long time despite its essential nature. He was convinced of the power of regional geography in providing a counterbalance in enhancing access to geographic information. It is important to appreciate the fact that this analytic approach has continued to emerge as the response to growing needs for scientific data collection and statistical analysis of the collected data on mobile devices and human activities. Since then, there have been comprehensive data collection measures and the best procedures for analyzing space-

time paths and constrained networks based on velocity fields.

In [1], the authors introduced the space-time prism model into the community of Geographic Information Systems (GIS). It is also important to appreciate that Phaser and Jensen enhanced the spatio-temporal and Moving Object Databases (MOD). This work was an eye-opener in enhancing the applicability of the space-time prism model. On the other hand, Egenhofer and Hornsby continued expounding on the space-time prism concept, coming up with more scientific discoveries that have improved geometric data analysis.

As of 1969, time geography gained a unique competitive advantage at the international community level, especially the Regional Science Association presidential address. Extensive research and development programs have enabled experts to develop other advanced analytical and computational tools for statistical analysis of the prisms of planar space and their accompanied transport networks. Current applications of these innovations include but are not limited to dispatch planning, analysis of risks associated with environmental accidents, understanding disease prevalence patterns, and criminal forensics.

3 Problem Formulation

In this section, we provide some basic concepts and problem definition.

3.1 Basic Concepts

- 1) Time to Site:** Length of time taken for an entity to go from start anchor to meeting location.
- 2) Time from Site:** - Length of time taken for an entity to go from meeting location to end anchor
- 3) Dwelling Time:** Time during the gap that does not consist of the Time to Site and the Time from Site.
- 4) Waiting Time** - Time the first object spends waiting for the second object.

5) Meeting Time: Length of time two entities are able to meet.

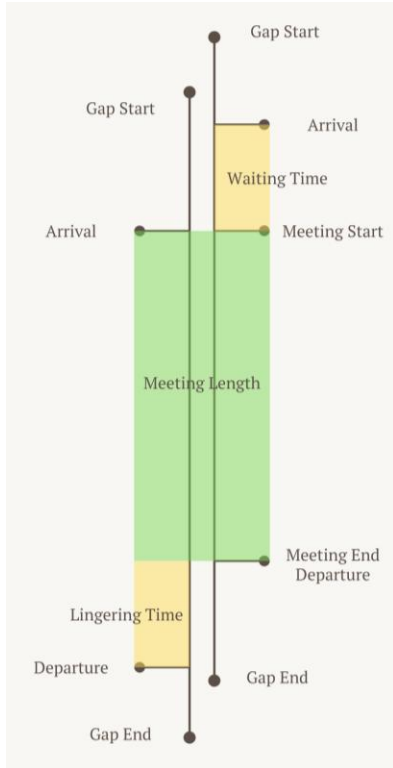


Figure 2. Illustration of basic concepts.

3.2 Problem Definition

Input: A pair of trajectories with gaps

Output: Intersection of geo-ellipses with length of meeting

Objective: Find possible meeting locations and lengths of two objects.

Constraints: Maximum Acceleration not Available.
Behavior within trajectories: Object goes straight to the meeting site.

In order to map out possible meeting-lengths that exist within a trajectory gap, we require some information about the speed of the moving object, the start position, the end position, and the length of the time gap for a trajectory as the input. As the output, we obtain a mapping of the meeting-time lengths in the intersection of the dwelling-time intervals. This output is under the constraint that no acceleration data

is available. The objective is to find the possible meeting locations and lengths of the two objects.

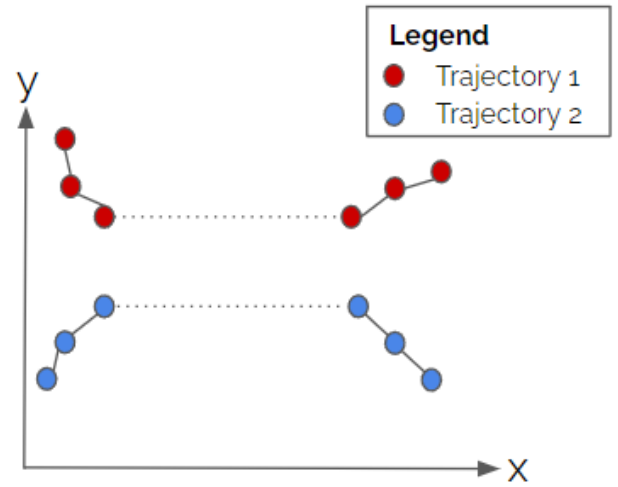


Figure 3. Input: A pair of trajectory gaps.

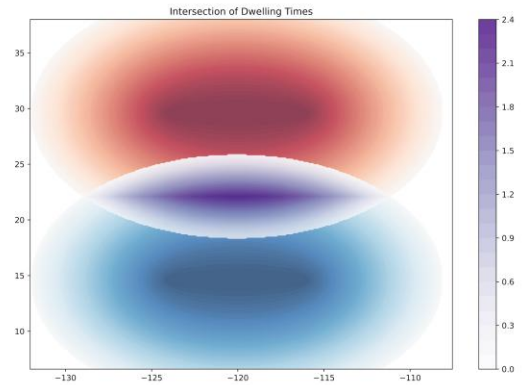


Figure 4. Output: Visualization of the meeting-length, which is calculated by intersecting dwelling-time intervals for both trajectories.

4 Proposed Approach

We designed our method such that a grid is created around the location of interest and calculate the distances from the start. From there, our method follows the steps below in order to obtain our result.

Step 1: Choose the center of a grid to be a potential meeting site.

- $tt_site = \text{hypot}(\text{length1}, \text{length2}) / \text{speed}$

Step 2: Calculate the time to site and time from site for both trajectories.

- $tt_site = \text{hypot}(\text{length1}, \text{length2}) / \text{speed}$
- $tf_site = \text{hypot}(\text{length3}, \text{length4}) / \text{speed}$

Step 3: Calculate the dwelling-time.

- $DT = \text{gap_length} - tt_site - tf_site$

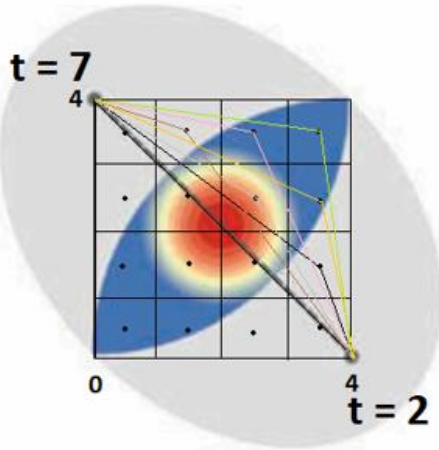
Step 4: Calculate the dwelling-time interval.

- $DS = \text{start} + tt_site$
- $DE = DS + DT$

Step 5: Calculate the meeting-length.

- $MT = \text{INTERSECT}(\text{interval}(DS1, DE1), \text{interval}(DS2, DE2))$

speed = 2



t_diff = 5

Figure 5. Illustration of calculations performed on an example trajectory.

5 Validation

In this section, we validated our proposed approach via visualizing dwelling time on synthetic dataset. The following sections describe synthetic data generation and experimental results.

5.1. Synthetic Data Description

For the cases used in the following section, these parameters are used:

Case 1: Identical Parallel Trajectories

- Same parameters of both Red and Blue:
 - $\text{start_time} = 0$
 - $\text{time_gap} = 8$
 - $\text{speed} = 3$

Case 2: Blue Delayed Parallel Trajectories

- Blue is delayed by 1 unit of time:
 - **Red**
 - $\text{start_time} = 0$
 - **Blue**
 - $\text{start_time} = 1$

Case 3: Blue Delayed and Increase Time Gap

- Reduced potential meeting areas due to parameters
- Multiple parameters are altered:
 - **Red**
 - $\text{start_time} = 0$
 - $\text{time_gap} = 8$
 - $\text{speed} = 3$
 - **Blue**
 - $\text{start_time} = 1$
 - $\text{time_gap} = 10$
 - $\text{speed} = 2$

5.2 Experimental Results

After using our method on the parameters listed above, we obtain Figures 6-8, which all depict the meeting-length as the intersection between dwelling-time intervals.

In Figure 6 we present Case 1, which are parallel trajectory gaps with identical parameters. The trend that we saw from this case was a symmetrical meeting-length, where the darker regions represent a longer possible meeting-length.

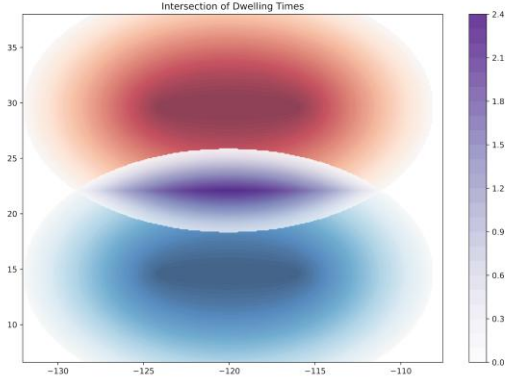


Figure 6. Mapping of meeting-lengths generated by using case 1 parameters.

Figure 7 represents Case 2, which has the blue trajectory gap's start time delay by 1 unit of time. The trend that we saw in this case was that the possible meeting locations were reduced due to the blue trajectory being delayed. Since the blue trajectory was delayed by 1 unit, at the locations that Case 1 could have met that Case 2 can no longer meet at, the red trajectory must head toward its end anchor before the blue trajectory can arrive at that location. As for the meeting-lengths in the intersection, the duration is reduced.

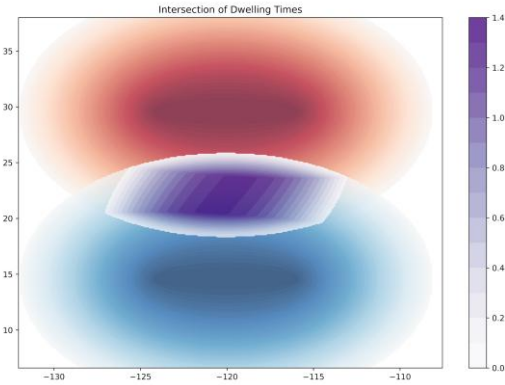


Figure 7. Mapping of meeting-lengths generated by using case 2 parameters.

In Figure 8, the mapping of meeting-lengths is reduced in area and time due to the parameters mentioned in Case 3. Since the speed was decreased, the total possible area covered by the blue trajectory would decrease, but since the time gap is increased, it can still

cover a decent amount of area, but this may not be enough for a trajectory to maintain its intersection. As a result, we see a narrower possible meeting locations as well as a lower possible meeting-length.

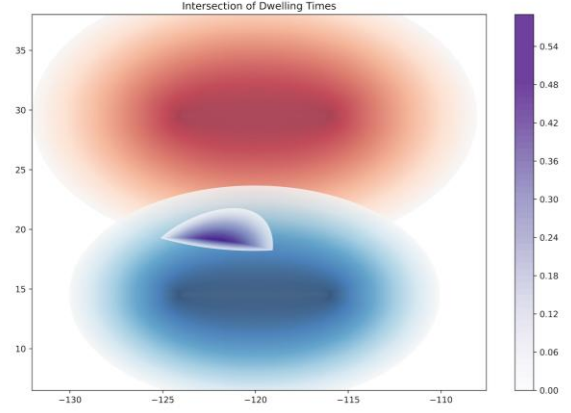


Figure 8. Mapping of meeting-lengths generated by using case 3 parameters.

5 Conclusion and Future Work

In this work, we defined trajectory gap behavior based on meeting time. In addition, we calculated the potential meeting times and locations for the pair of trajectories. Finally, we provided experimental results based on visualizations.

In future work, we are considering more realistic physics based models. We also plan to include acceleration while modeling trajectory gaps. We plan a real-world dataset and analyze other behaviors of entities. Finally, we plan to add probabilistic interpretation in trajectory gaps via Gaussian Processes.

REFERENCES

- [1] Miller, H.J. (2017). Time Geography and space-time-prism. The Ohio State University, USA.
- [2] Archer, E., Park, I. M., Buesing, L., Cunningham, J., & Paninski, L. (2015). Black box variational inference for state-space models. arXiv preprint arXiv:1511.07367.
- [3] Åström, K. J., & Murray, R. M. (2010). Feedback systems. In Feedback Systems. Princeton university press.
- [4] Bayer, J., & Osendorfer, C. (2014). Learning recurrent stochastic networks. arXiv preprint arXiv:1411.7610.
- [5] Billings, S. A. (2013). Nonlinear system identification: CARMAX methods in the time, frequency, and Spatio-temporal domains. John Wiley & Sons.
- [6] Blei, D. M., Kucukelbir, A., & McAuliffe, J. D. (2017). Variational inference: A review for statisticians. Journal of the American Statistical Association, 112(518), 859-877.
- [7] Camacho, E. F., & Alba, C. B. (2013). Model predictive control. Springer Science & business media.
- [8] Chung, J., Kastner, K., Dinh, L., Goel, K., Courville, A. C., & Bengio, Y. (2015). A recurrent latent variable model for sequential data. Advances in neural information processing systems, 28.
- [9] Doerr, A., Daniel, C., Nguyen-Tuong, D., Marco, A., Schaal, S., Marc, T., & Trimpe, S. (2017, October). Optimizing long-term predictions for model-based policy search. In Conference on Robot Learning (pp. 227-238). PMLR.
- [10] Eleftheriadis, S., Nicholson, T., Deisenroth, M., & Hensman, J. (2017). Identification of Gaussian process state-space models. Advances in neural information processing systems, 30.
- [11] Frigola, R., Lindsten, F., Schön, T. B., & Rasmussen, C. E. (2013). Bayesian inference and learning in Gaussian process state-space models with particle MCMC. Advances in neural information processing systems, 26.
- [12] Girard, A., Rasmussen, C. E., Quinonero-Candela, J., Murray-Smith, R., Winther, O., & Larsen, J. (2002). Multiple-step ahead prediction for nonlinear dynamic systems—a gaussian process treatment with uncertainty propagation. Advances in neural information processing systems, 15, 529-536.
- [13] Hensman, J., Fusi, N., & Lawrence, N. D. (2013). Gaussian processes for big data. arXiv preprint arXiv:1309.6835.
- [14] Hochreiter, S., & Schmidhuber, J. (1996). LSTM can solve hard long, time lag problems. Advances in neural information processing systems, 9.
- [15] Kalman, R. E. (1960). A new approach to linear filtering and prediction problems.
- [16] Karl, M., Soelch, M., Bayer, J., & Van der Smagt, P. (2016). Deep Variational Bayes Filters: Unsupervised learning of state-space models from raw data. arXiv preprint arXiv:1605.06432.
- [17] Murray-Smith, R., & Girard, A. (2001, June). Gaussian Process priors with ARMA noise models. In Irish Signals and Systems Conference, Maynooth (Vol. 147, p. 152).
- [18] Nguyen, K., Krumm, J., Shahabi, C. (2021). GAUSSIAN PROCESS FOR TRAJECTORIES
- [19] Wang, J. M., Fleet, D. J., & Hertzmann, A. (2007). Gaussian process dynamical models for human motion. IEEE transactions on pattern analysis and machine intelligence, 30(2), 283-298.
- [20] Mattos, C. L. C., Damianou, A., Barreto, G. A., & Lawrence, N. D. (2016). Latent autoregressive Gaussian processes models for robust system identification. IFAC-PapersOnLine, 49(7), 1121-1126.
- [21] Van Overschee, P., & De Moor, B. (2012). Subspace identification for linear systems: Theory—Implementation—Applications. Springer Science & Business Media.
- [22] Littman, M., & Sutton, R. S. (2001). Predictive representations of state. Advances in neural information processing systems, 14.
- [23] Singh, S. P., Littman, M. L., Jong, N. K., Pardoe, D., & Stone, P. (2003). Learning predictive state representations. In Proceedings of the 20th International Conference on Machine Learning (ICML-03) (pp. 712-719).