Data Structures
Chapter 5
Tree

1. **introduction**
2. **Binary tree**
3. Binary search tree
4. Tree balancing

Prof. Youngsup Kim, idebtor@gmail.com, CSEE Dept., Grace School Rm204, Handong Global University

다윗은 당시에 하나님의 뜻을 따라 섬기다가 잠들어 그 조상들과 함께 묻혀 썩음을 당하였으되. 행13:36

For when David had served God's purpose in his own generation, he fell asleep; he was buried with his fathers and his body decayed. Acts 13:36

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하나님이 우리를 구원하사 거룩하신 소명으로 부르심은 우리의 행위대로 하심이 아니요 오직 자기의 뜻과 영원 전부터 그리스도 예수 안에서 우리에게 주신 은혜대로 하심이라 (딤후1:9)

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   - **Definition and Terminology**
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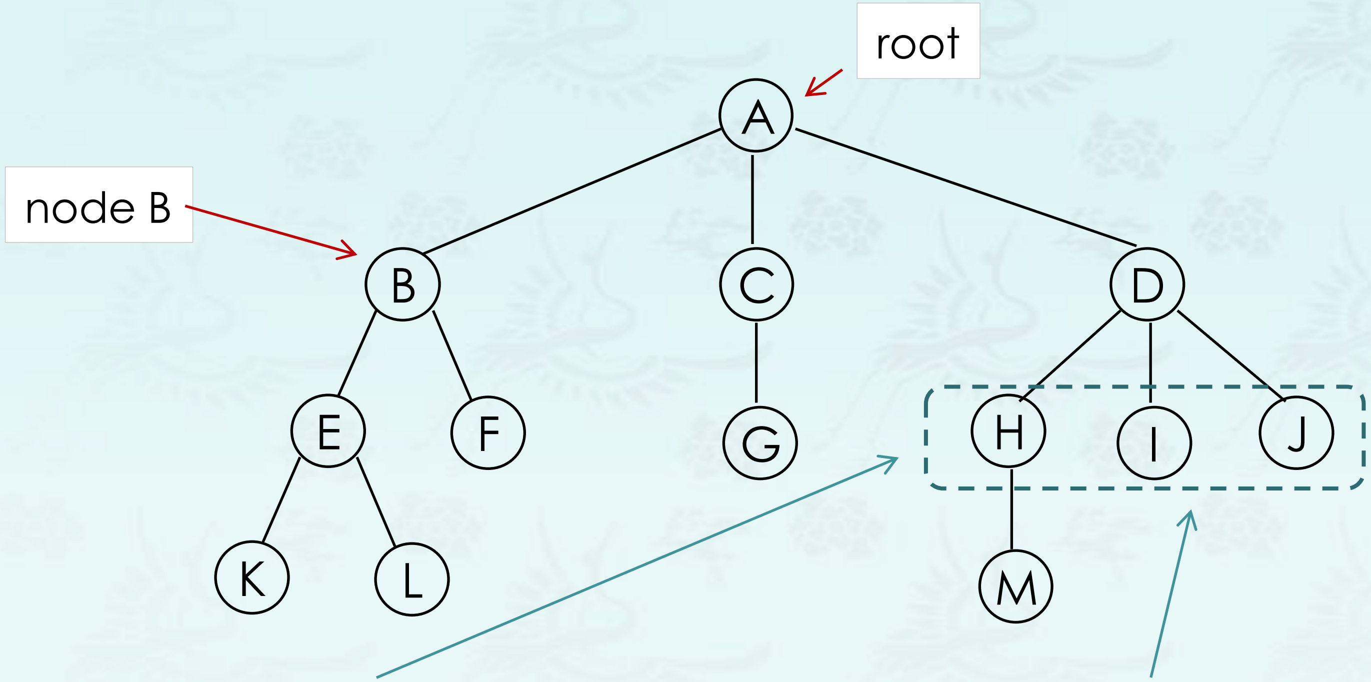
Introduction - Terminology

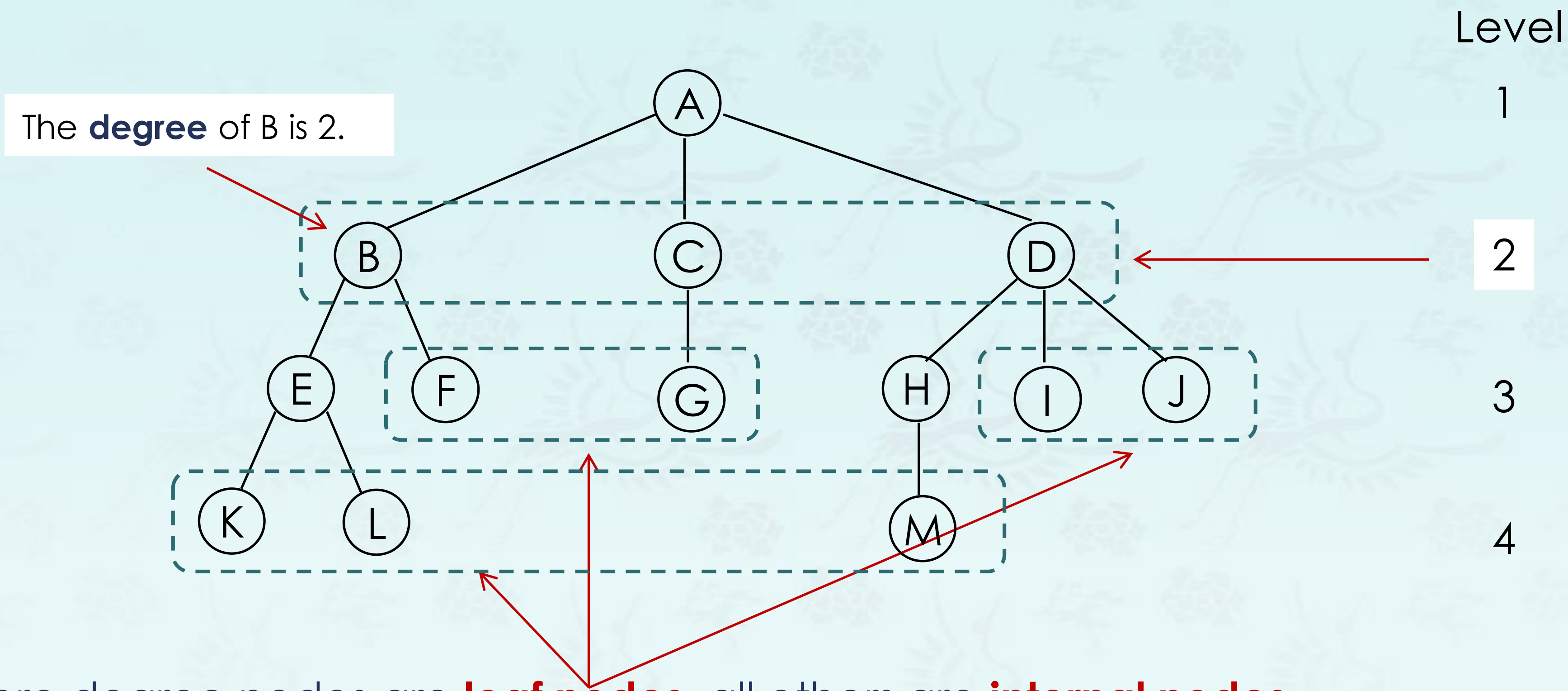
- **A tree data structure:** it is like a linked list that has a **first** node, this node is called as the **root** of the tree.
- **Example.** A **tree** with a root storing the value 'A'



- The **children** of **D** are **H, I, and J**; **H, I, and J** are **siblings**.
- The **parent** of **D** is **A**.

Introduction - Terminology

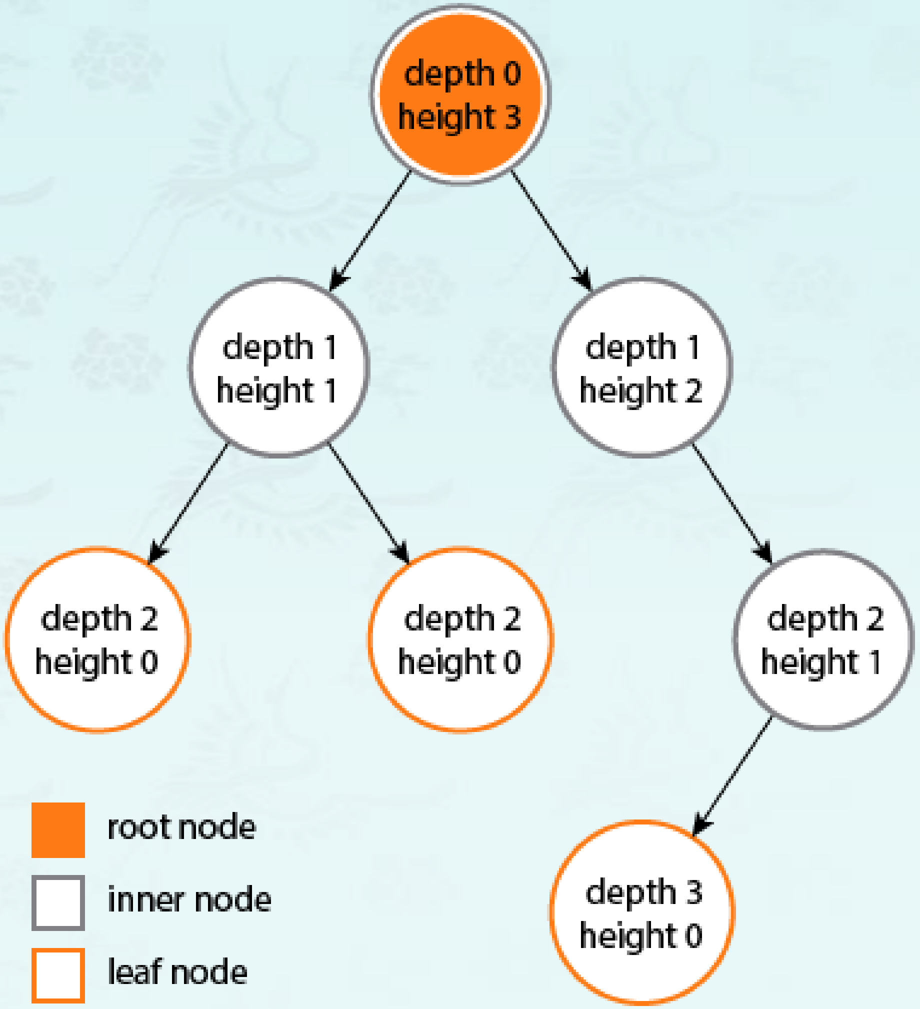
- **Definition.** child, parent, sibling, degree, leaf nodes, level, and internal node



- Zero degree nodes are **leaf nodes**, all others are **internal nodes.**
  - An ***internal node*** is any node that has at least one non-empty child.
- The **degree** of a node is the number of children.
- The **degree of a tree** is the **maximum of the degree of the nodes** in the tree.

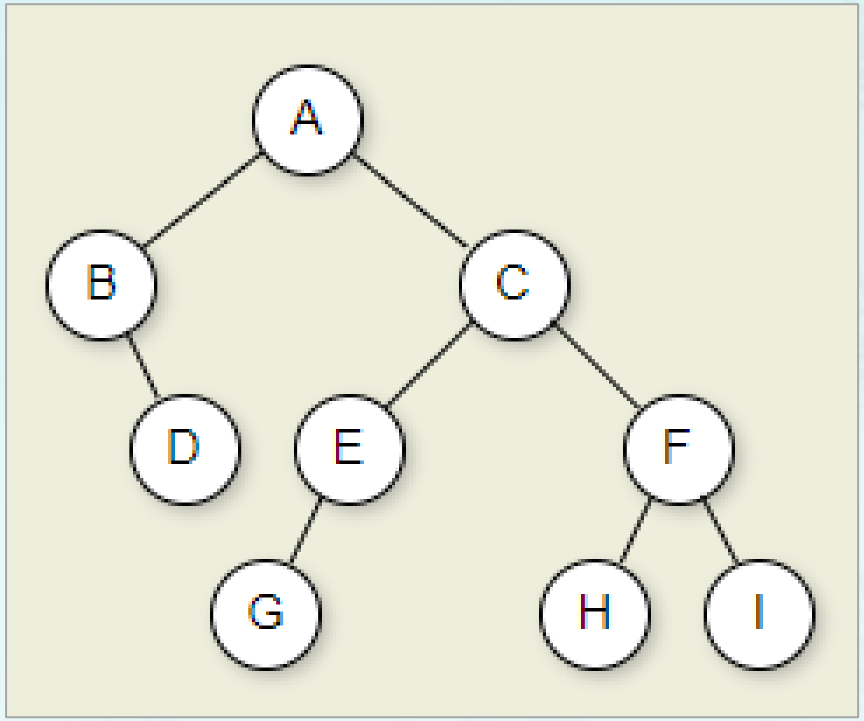
Introduction - Terminology

- **Definition.** height, depth, and level
- The **height** of a node is **the number of edges** on the *longest path* from the node to a leaf.
  - A leaf node will have a height of 0.
  - The height of a tree is the height of root.
  - The height of a tree with 1 node is 0.
  - The height of a tree is the maximum depth.
  - **The height of a tree is the depth of the deepest node in the tree.**
- The **depth** of a node is the number of edges from the node to the tree's root node.
  - A root node will have a depth of 0.
- The **level** of a node is defined by 1 + the number of connections between the node and the root.



Introduction - Terminology

- **Review**



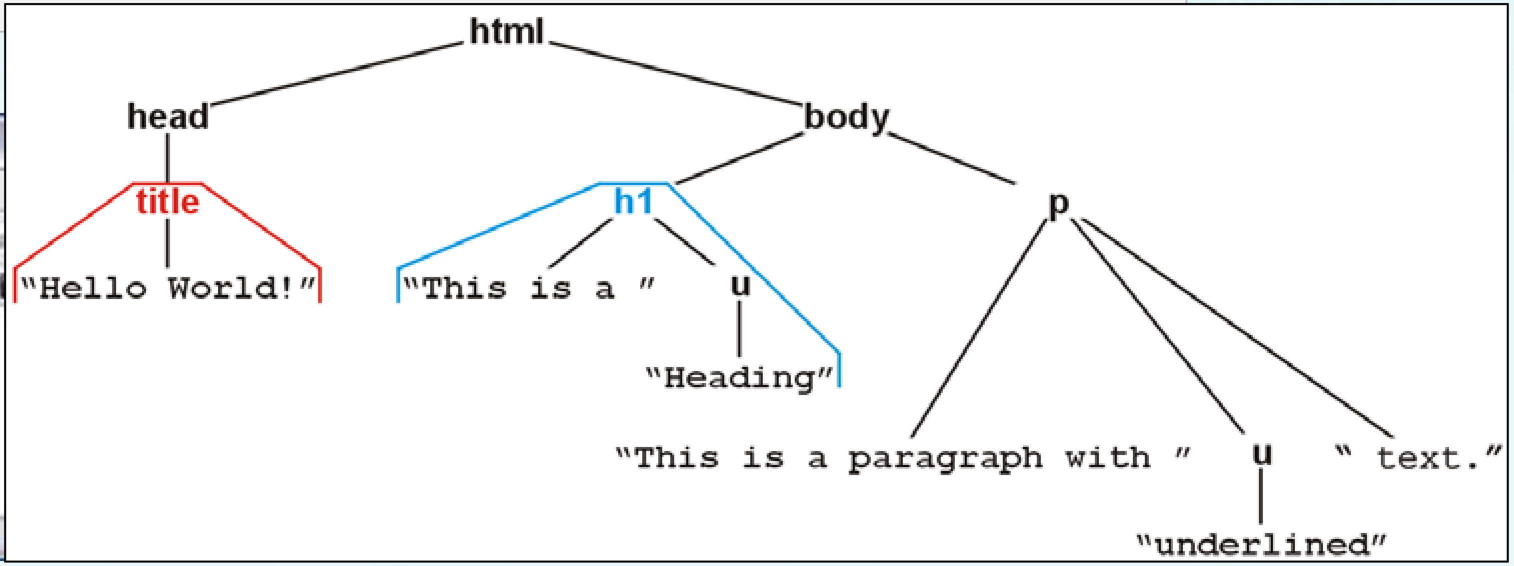
- A binary tree.
- Node A is the root.
- Nodes B and C are A's children.
- Nodes B and D together form a subtree.
- Node B has two children: Its left child is the empty tree and its right child is D.
- Nodes A, C, and E are ancestors of G.
- Nodes D, E, and F make up **level** 2 + 1 of the tree; node A is at **level** 0 + 1.
- The edges from A to C to E to G form a path of length 3.
- Nodes D, G, H, and I are leaves.
- Nodes A, B, C, E, and F are internal nodes.
- The depth of I is 3.
- **The height of this tree is 3.**

https://opendsa-server.cs.vt.edu/ODSA/Books/CS3/html/BinaryTree.html#definitions-and-properties

Introduction – Representation of trees

- **Exercise.** The tree representing the HTML document below:

```
<html>
    <head>
        <title>Hello World!</title>
    </head>
    <body>
        <h1>This is a <u>Heading</u></h1>
        <p>This is a paragraph with some <u>underlined</u> text.</p>
    </body>
</html>
```



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2. **Binary tree**
   - **Definition and Properties**
   - **Traversal**
   - Coding
3. Binary search tree
4. Tree balancing

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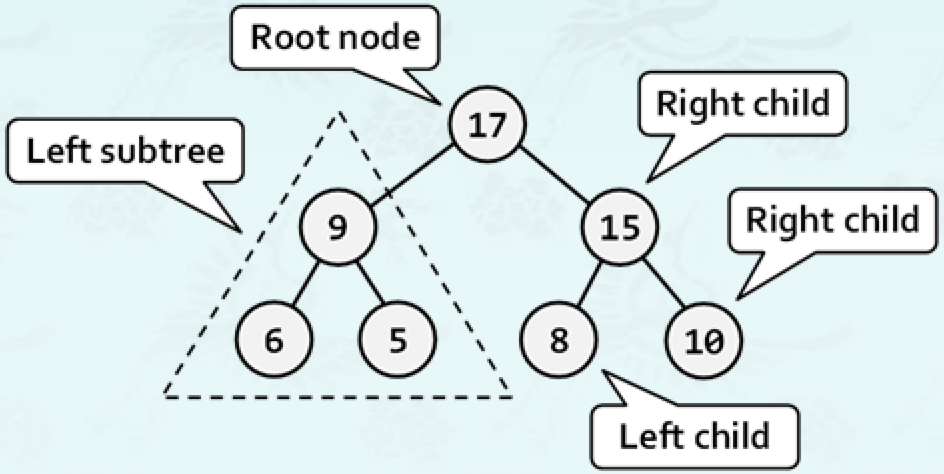
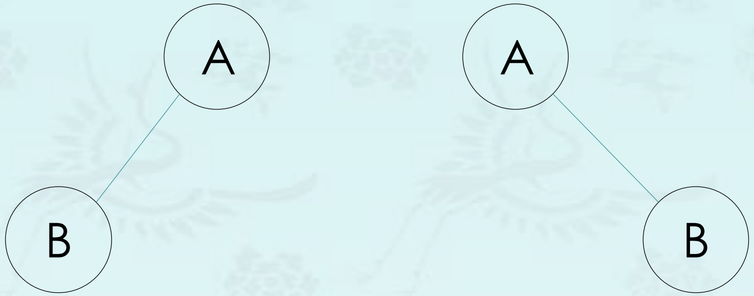
Binary trees

- **Definition:** A tree such that each node has *exactly* two children.
  - Notice, exactly two children - not up to two children! Because *exactly* two children means a left child **and/or** right child, no middle child.
  - Each child is either empty or another binary tree.
  - Given this constraint, we can label the two children as left and right nodes or subtrees.



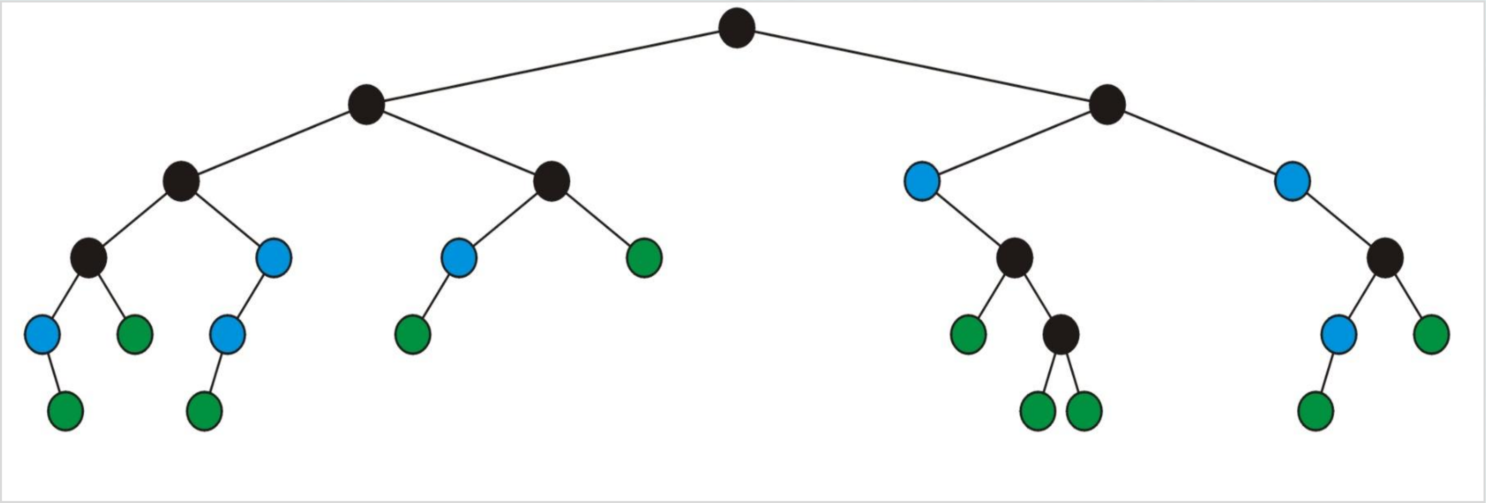
Binary trees

- **Example:** two binary trees with two nodes
  - Q: Are they two different **binary** trees?
  - A: Yes!



Binary trees

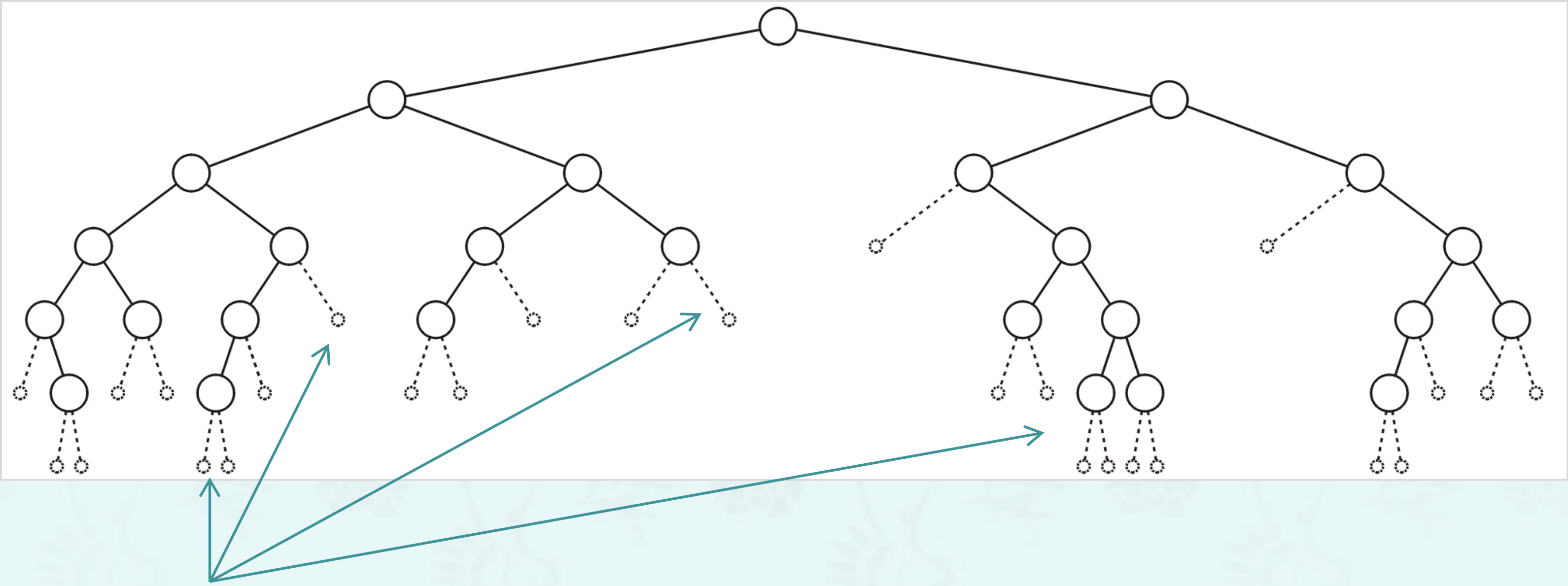
- **Definition:** A **full node** is a node where both left **and** right sub-trees are non-empty trees:
  - Q: How many full nodes are there?
  - Q: How many leaf nodes are there?
  - Q: What is the height of the tree?
  - Q: What is the degree of the tree?



full nodes leaf nodes neither

Binary trees

- **Definition:** An ***empty node*** or ***null sub-tree*** is a location where a new leaf node (or a sub-tree) could be inserted.



null link, graphically, the missing branches

Binary trees - ADT

- Objects: a finite set of nodes either empty or consisting of a root node, leftBinaryTree, and rightBinaryTree.
- Functions:

  boolean empty(bt)

  binaryTree new Node{key, left, right}

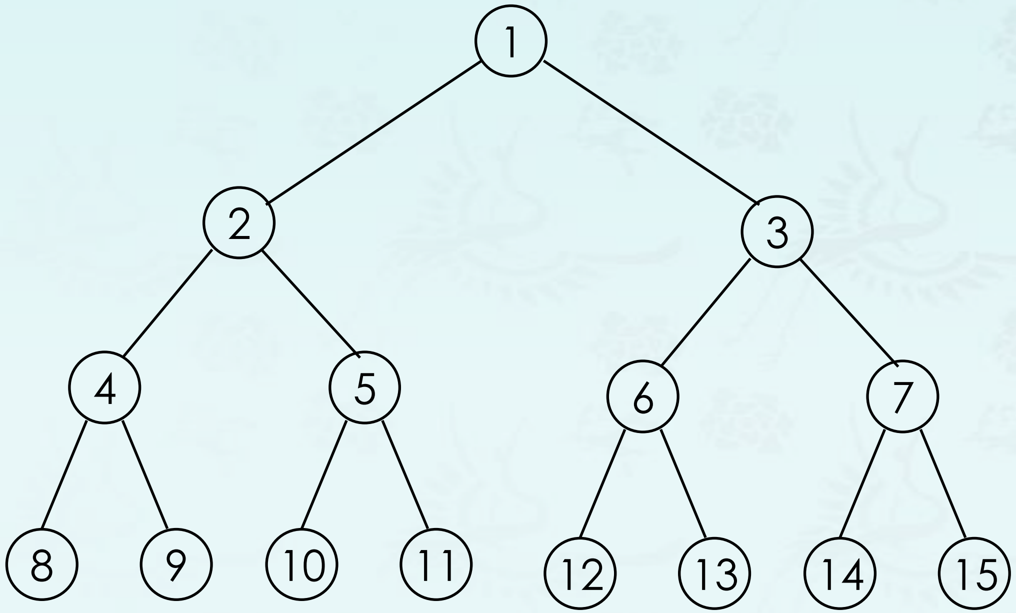
  binaryTree left(bt)

  element getKey(bt)

  binaryTree right(bt)

Binary trees - Properties

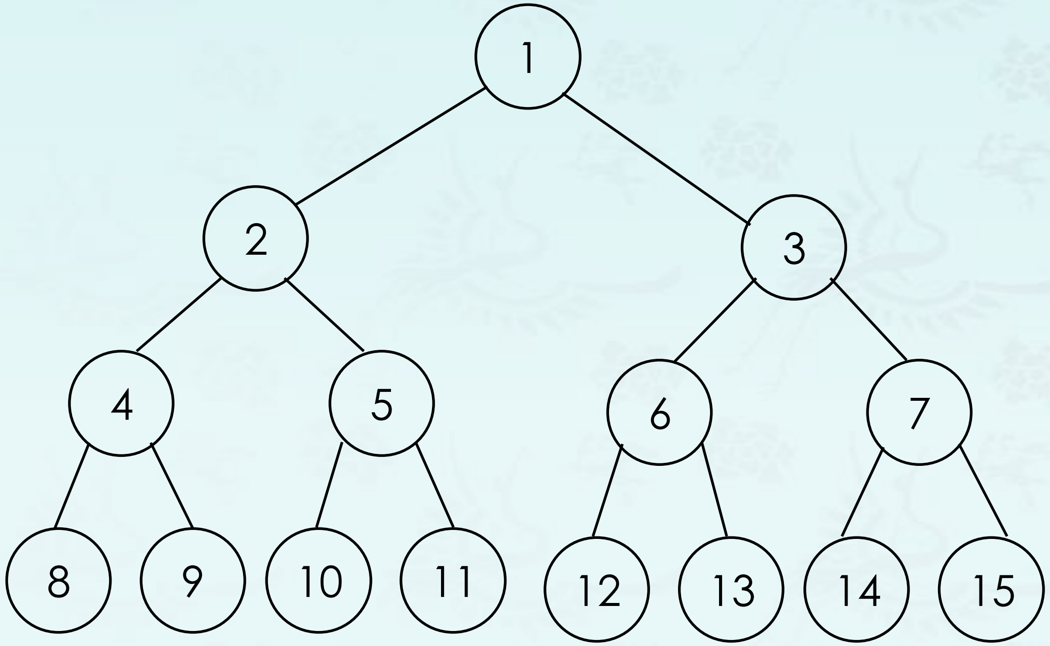
- **Observation:**
  - **Q:** Maximum number of nodes in binary trees in each level and all levels?
  - **Q:** What is the max level k if there are n nodes? k(n) = ?



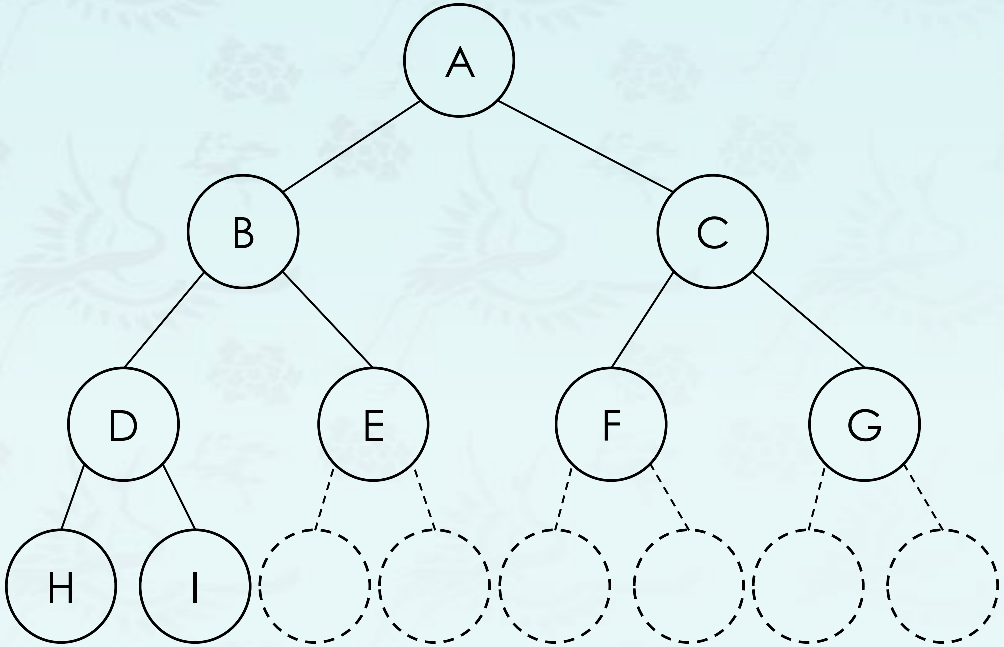
Level	Node Numbers at Each Level	Total Numbers of Nodes
1	$1 = 2^0$	$1 = 2^1 - 1$
2	$2 = 2^1$	$3 = 2^2 - 1$
3	$4 = 2^2$	$7 = 2^3 - 1$
4	$8 = 2^3$	$15 = 2^4 - 1$
.	.	.
11	$1024 = 2^{10}$	$2047 = 2^{11} - 1$
.	.	
k	$\mathbf{2^{k-1}}$	$\mathbf{2^k - 1}$
h	$\mathbf{2^h}$	$\mathbf{2^{h+1} - 1}$

Binary trees - Properties

- **Definition:** *A **full** binary tree* of level k is a binary tree having **2^k – 1 nodes**, $k \geq 0$.
- **Definition**: A binary tree with n nodes and level k is **complete** iff its nodes correspond to the nodes numbered from 1 to n in the full binary tree of level k.



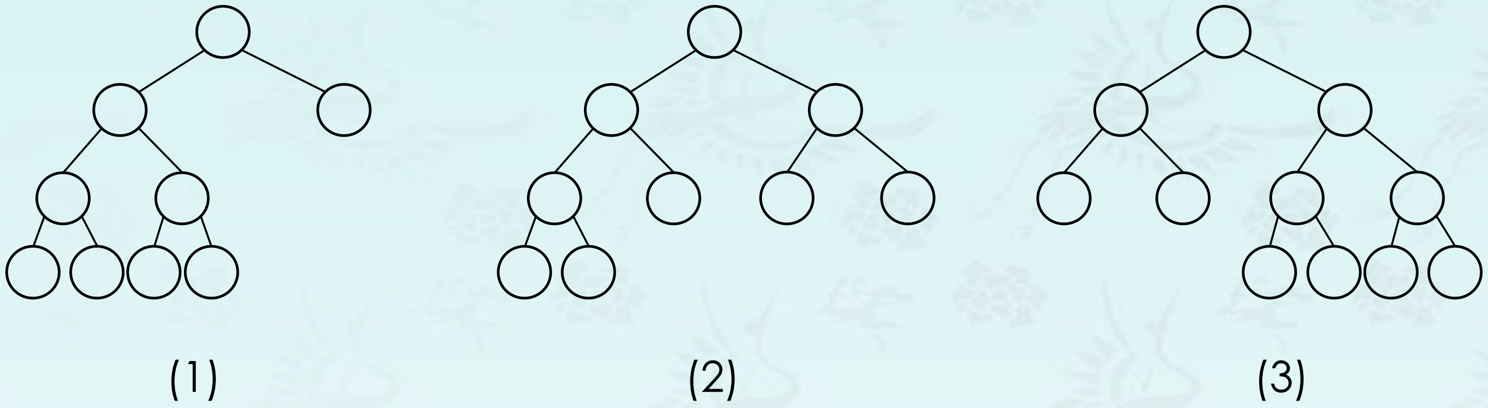
*A **full** binary tree*



*A **complete** binary tree*

Binary trees - Properties

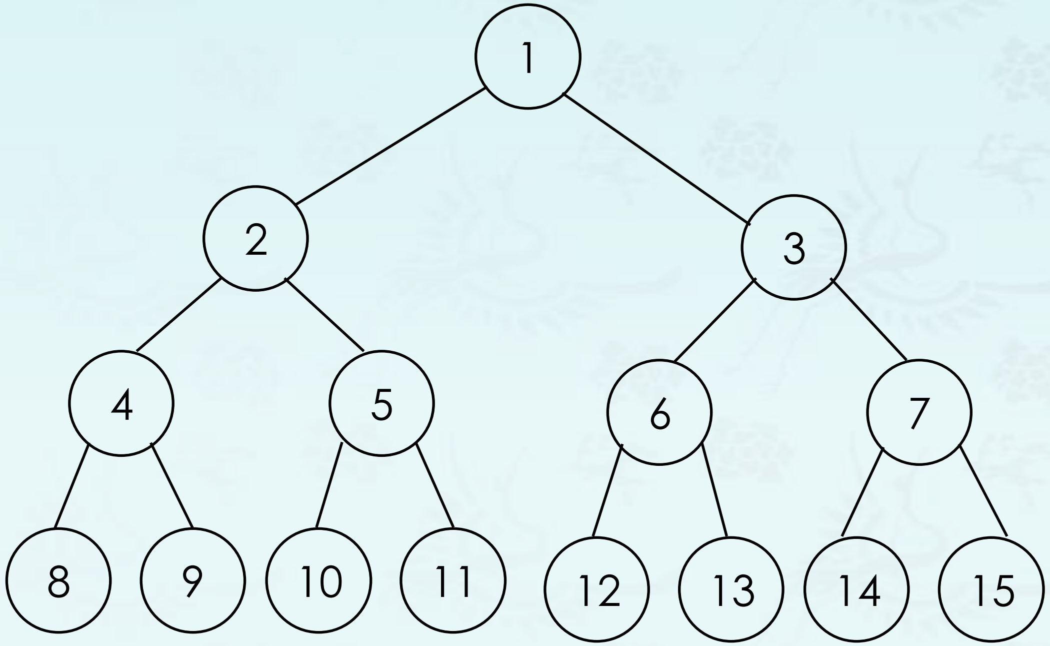
- **Q:** Identify a **complete** binary tree.



- **Q: Meanings of a complete tree in terms of ADT?**
  A: Removals of a node are only allowed from the "last" position.
  There is one position available to insert a node every time!

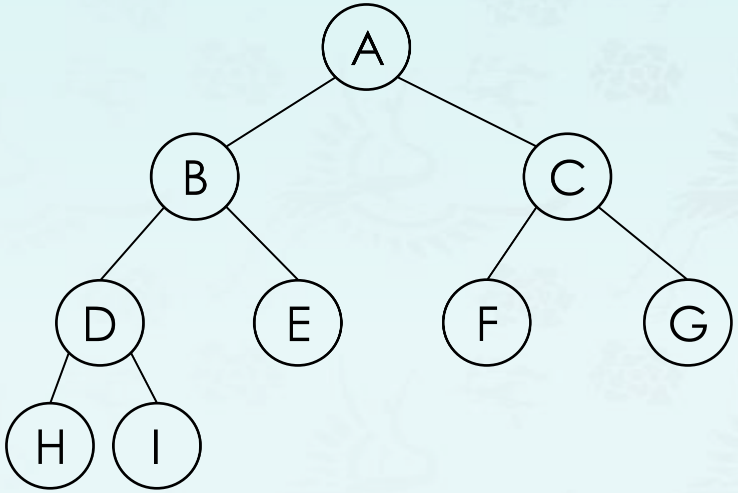
Binary trees - Array representation

- **Q:** What is the potential problem when you use one-dimensional array to represent a binary tree in memory?
- A: **It is** good for a full binary tree, but not good memory usage for a skewed or complete binary tree.



Binary trees - Array representation

- **Q:** Let's suppose that you have a **complete binary tree** in an array. Find its parent, left child and right child at node D.



[0]	-
[1]	A
[2]	B
[3]	C
[4]	D
[5]	E
[6]	F
[7]	G
[8]	H
[9]	I

Solution:

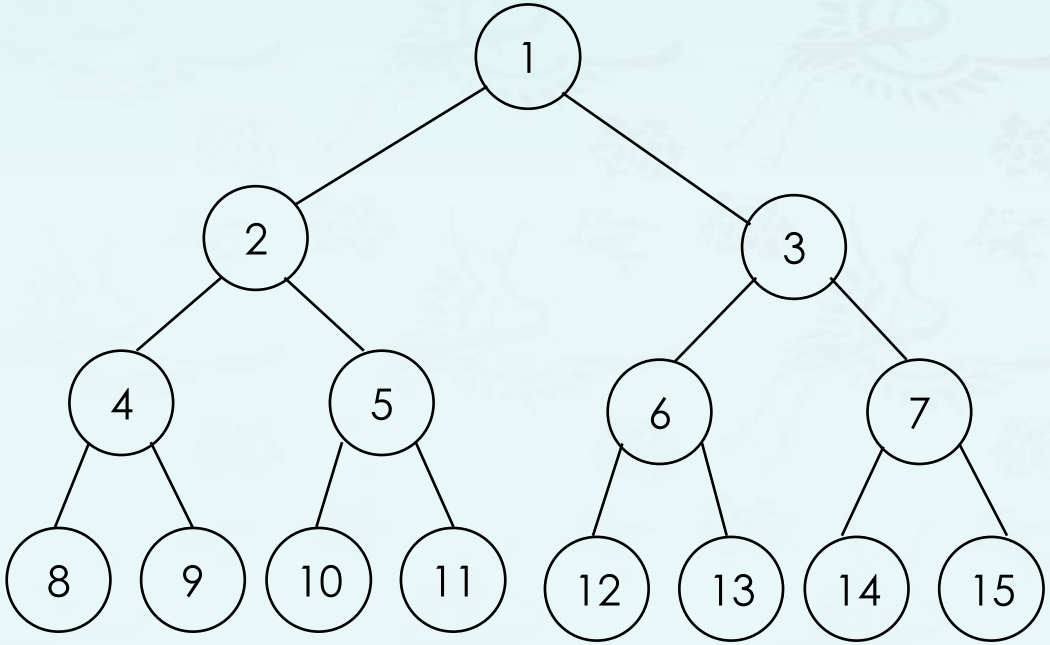
$parent(x = 4)$ is at 4/2 = 2

$leftChild(4)$ is at 2x4 = 8

$rightChild(4)$ is at 2x4 + 1 = 9

Binary trees - Array representation

- **Q:** Let's suppose that you have a **complete binary tree** in an array, how can we locate node x's parent or child?
- A **complete** binary tree with n nodes, any node index i, $1 \leq i \leq n$, we have
  - `parent(i) is at` $\lfloor i/2 \rfloor$ `If i = 1, i is at the root and has no parent`
  - `leftChild(i) is at 2i  if 2i <= n. If 2i > n, then i has no left child.`
  - `rightChild(i) is at 2i+1 if 2i+1 <= n. If 2i+1 > n, then i has no right child.`

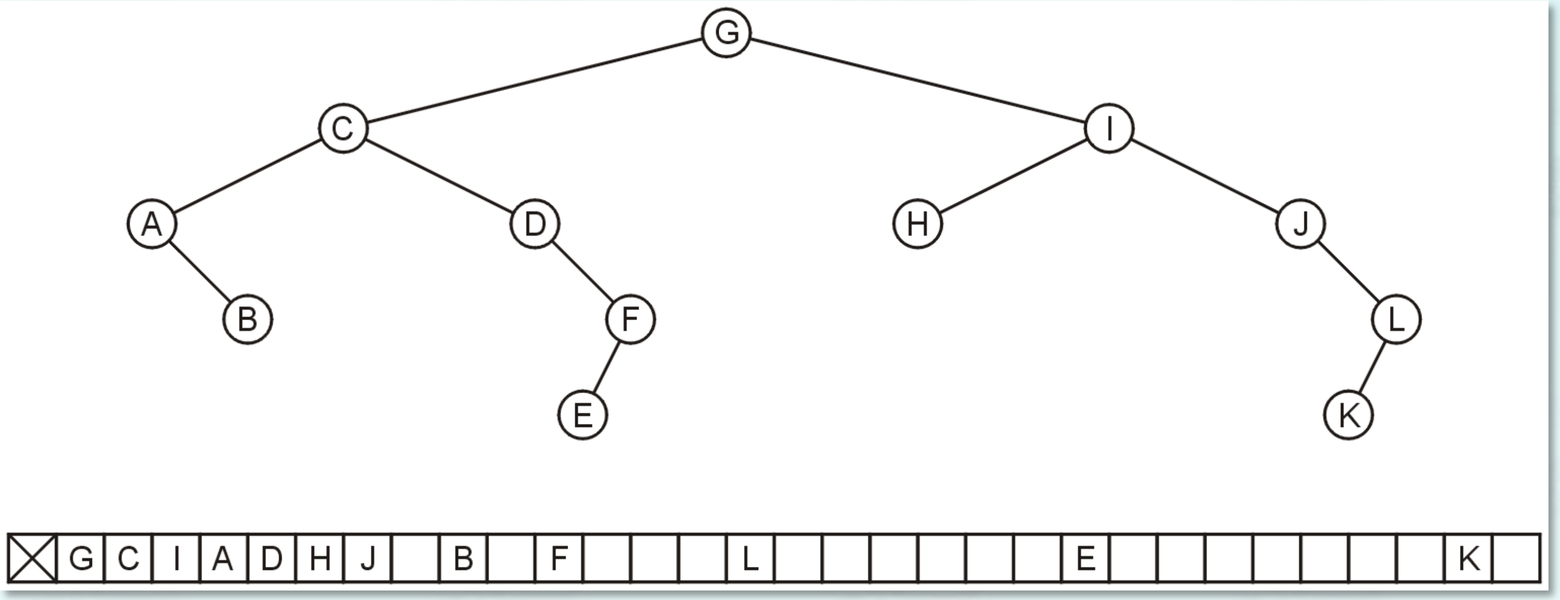


Wow! Can we use this to all binary trees?
Why not?

Problem remains:
The problem with storing an arbitrary binary tree using an array is the inefficiency *in memory usage*.

Binary trees - Array representation

- **Q:** Can we use this array rep. to store all binary trees? Why not?
- **A: For example,** the tree has 12 nodes, and requires an array of 32 elements. Adding one extra node, as a child of node K or E **doubles** the required memory for the array!



A. In the worst case a skewed tree of level k will require **$2^k - 1$** space which is O(**2^k**).
Of these, **only k** will be used.

Q. What happens when k = n? (Is there such a tree?)

Binary trees - Properties

(1) The maximum number of **nodes on level k** of a binary tree is

$$2^{k-1}, \qquad k \geq 1$$

(2) The maximum number of **nodes in a binary tree of level k** is

$$2^k - 1, \quad k \geq 1$$

(3) The maximum level of a **complete binary tree** with **n** nodes is

$k(n) = \lceil log_2 (n+1) \rceil$, $\lceil x \rceil$ is the smallest integer $\geq x$.

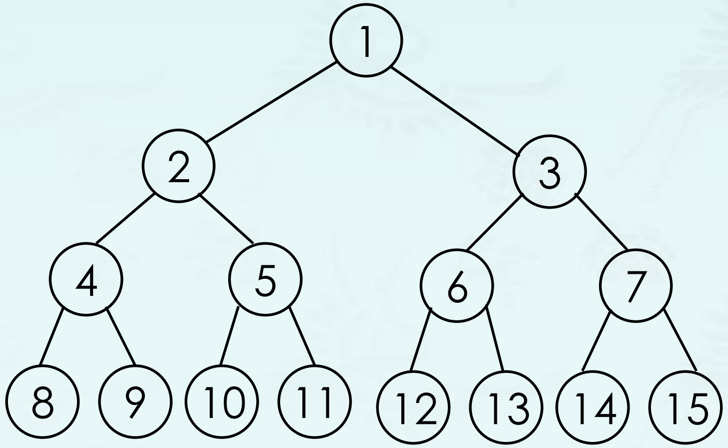
$$\begin{aligned} n &= 2^k - 1 \\ n+1 &= 2^k \\ \log(n+1) &= log2^k \\ \log(n+1) &= k \\ k(n) &= \lceil \log(n+1) \rceil \\ k(n) &= \lfloor \log(n) \rfloor + 1 \end{aligned}$$

n is the maximum number of nodes at level k

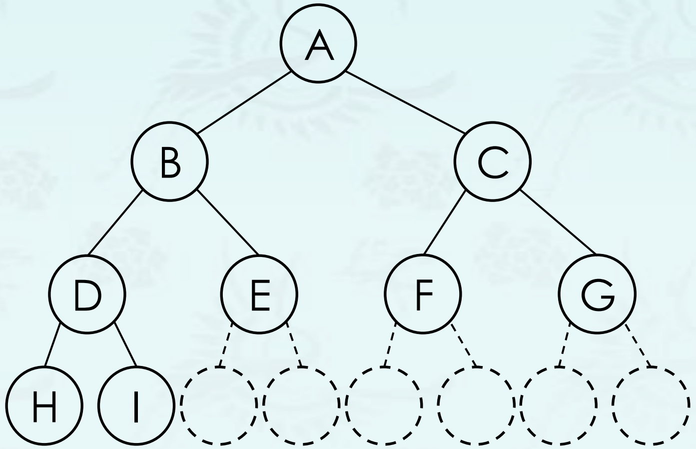
since k is an integer, and includes the max level of complete binary tree.

Binary trees - Properties

- **Observation:** The max level of a full binary tree of n nodes is k = $\lfloor \log(n) \rfloor$+1 :
  - Many operations with trees have a run time that goes with the max level of some path within the tree;
  - If we have a full binary tree (or something *close* to it), we know that those operations run in $\boldsymbol{O}(\log \boldsymbol{n})$.



*A **full** binary tree*



*A **complete** binary tree*

Binary trees – Linked representation

- **Node representations:**

```
struct TreeNode{
    int       key;
    TreeNode* left;
    TreeNode* right;
};
using tree = TreeNode*;
```

```
TreeNode* t = new TreeNode(9);
tree t = new TreeNode(9);
```

Recursion & Tree Structure

```
struct TreeNode{
    int       key;
    TreeNode* left;
    TreeNode* right;
};
using tree = TreeNode*;
```

```
struct TreeNode{
  int       key;
  TreeNode* left;
  TreeNode* right;

  TreeNode(int k, TreeNode* l, TreeNode* r) {
    key = k; left = l; right = r;
  }
  TreeNode(int k) : key(k), left(nullptr), right(nullptr) {}

  ~TreeNode(){}
};
using tree = TreeNode*;
```

Binary trees – Linked representation

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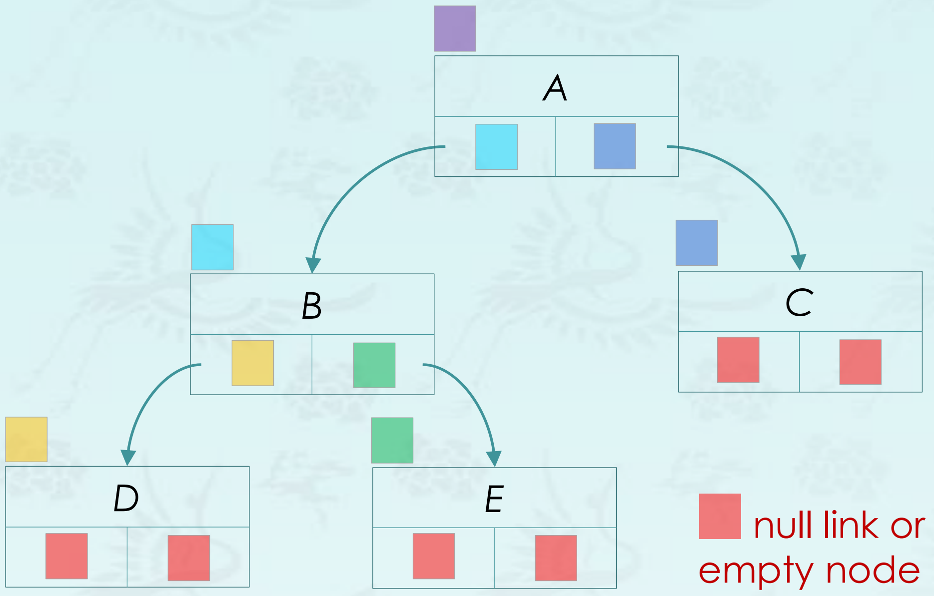

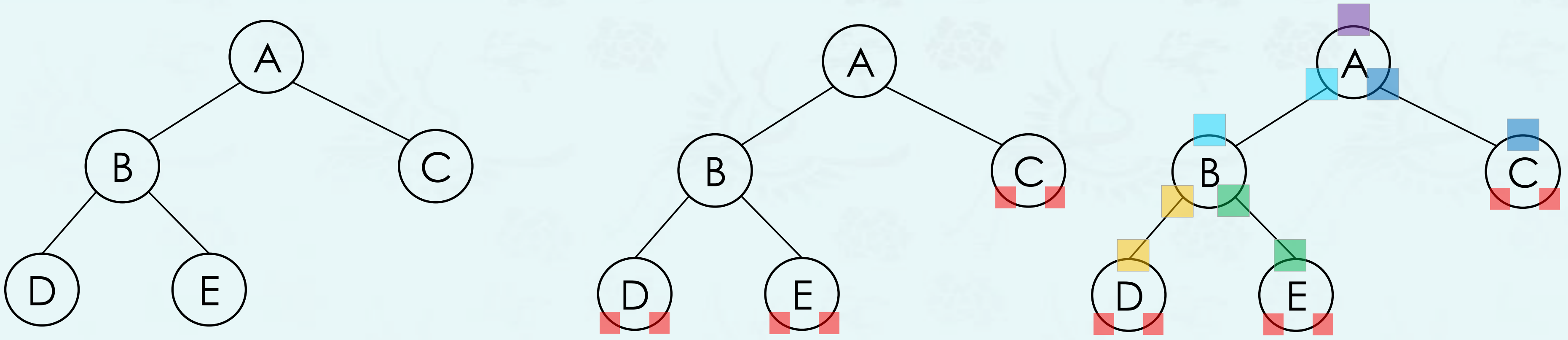

Binary trees – Linked representation

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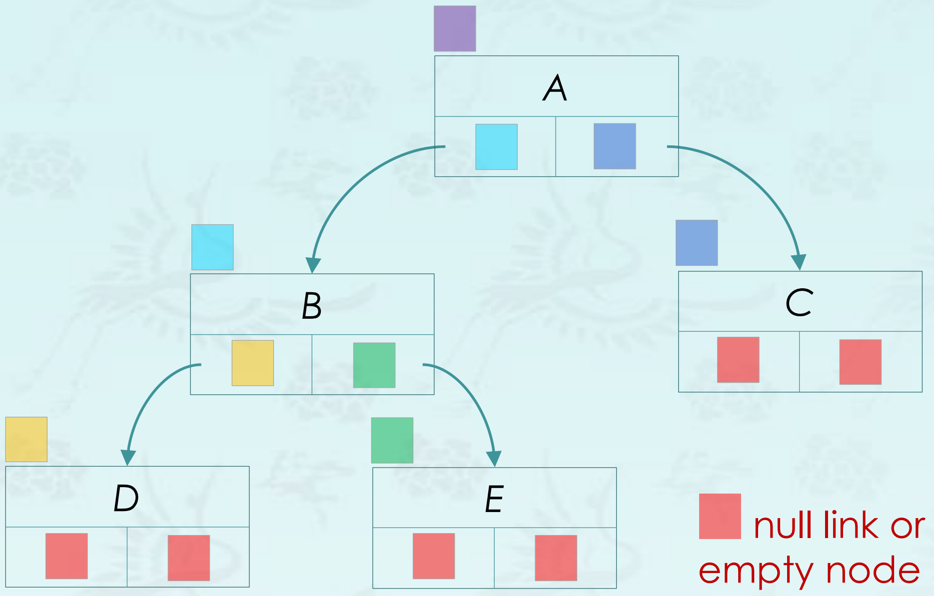

- **Q.** Is this node structure good enough?
  - Not easy to find its parent node. Parent field could be added if necessary.
- **Q.** It is similar to a doubly-linked list(DLL). What is different?
  - One head, but many tails. Null points empty node conceptually.

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