

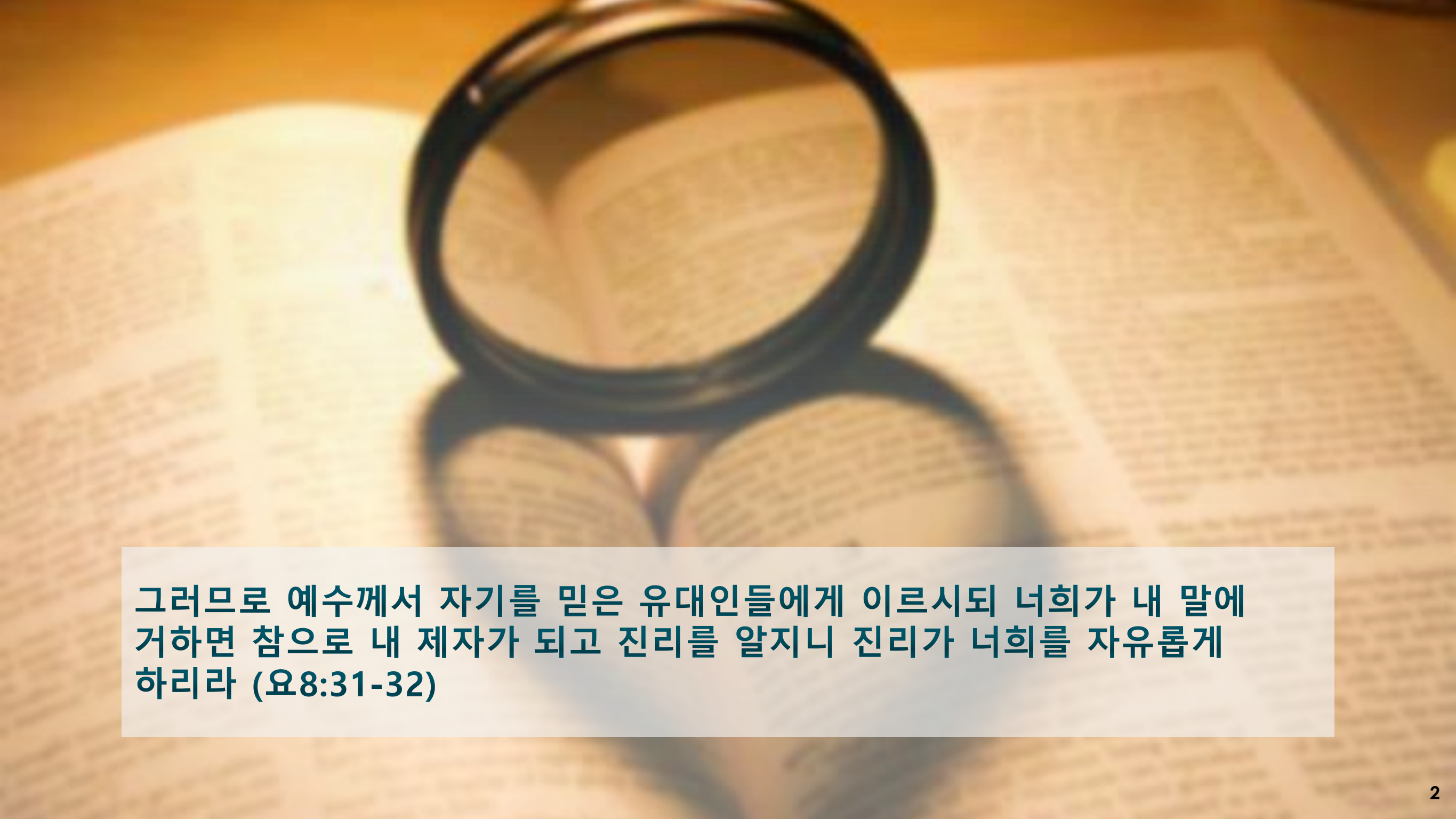
Data Structures

Chapter 3

1. Stack

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - Using Dynamic Array
 - Using Vector
 - Using Template

2. Queue



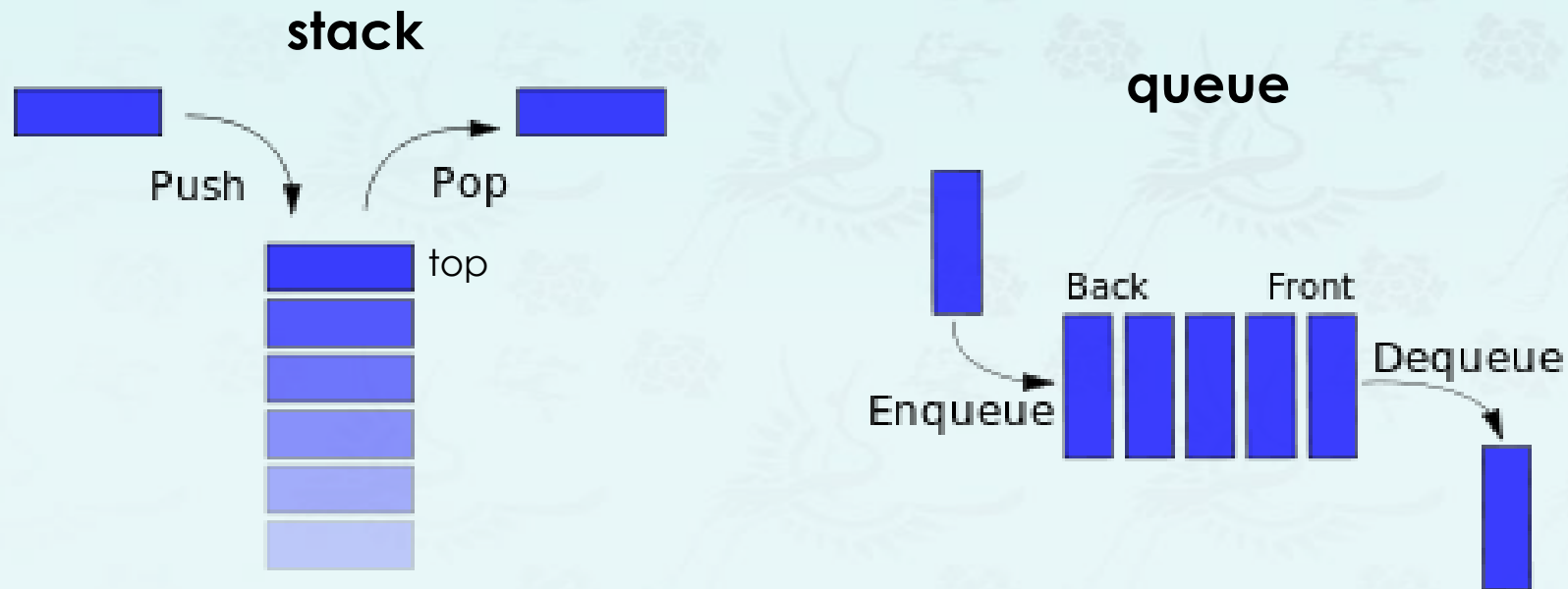
그러므로 예수께서 자기를 믿은 유대인들에게 이르시되 너희가 내 말에
거하면 참으로 내 제자가 되고 진리를 알지니 진리가 너희를 자유롭게
하리라 (요8:31-32)

Stack

- **Stack** is a linear data structure represented by a real physical stack or pile, a structure where insertion and deletion of items takes place at one end called top of the stack.

Stack

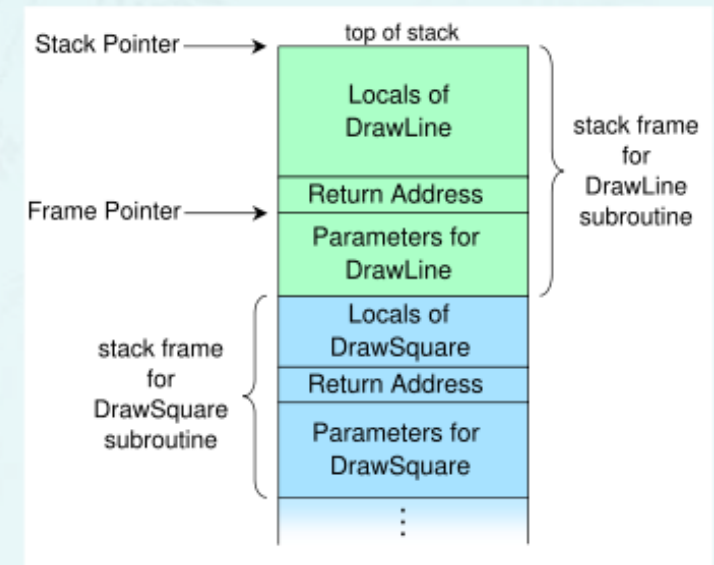
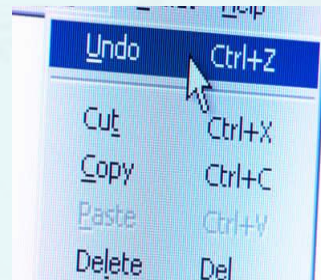
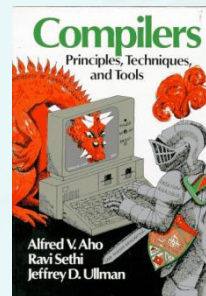
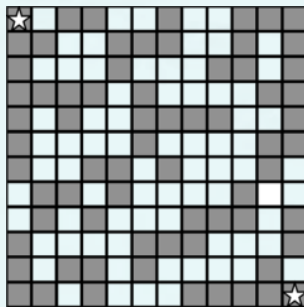
- **Stack** is a linear data structure represented by a real physical stack or pile, a structure where insertion and deletion of items takes place at one end called top of the stack.
- This particular order of the operation is called **LIFO(Last In First Out)**.



FIFO = “First in First out”
Remove the item
least recently added.

Stack Applications

- Parsing in a compiler.
- Undo in a word processor.
- Back button in a Web browser.
- PostScript language for printers.
- Backtracking as in a maze
- Implementing function calls in a compiler.
- ...



Stack – ADT (Abstract Data Type)

ADT Stack is

- **Objects:** a finite ordered list with zero or more elements
- **Operations (or Functions):**

```
Stack newStack(maxStackSize)
```

```
bool empty()
```

```
void push(item)
```

```
void pop()
```

```
int top()
```

```
int size;
```

Stack - Why ADT?

- **Separate interface and implementation.**
 - Ex: stack, queue, bag, priority queue, symbol table, union-find,
- **Benefits.**
 - **Driver (or Client)** can't know details of implementation
 - Client has many implementations from which to choose.
 - Program using operations defined in **interface**.
 - **Interface** is description of data type, basic operations.
 - **Implementation** is actual code implementing operations.
 - **Design**: creates modular, reusable libraries.
 - **Performance**: use optimized implementation where it matters.

Stack: Example in C++

STL	#include <stack>	Stack class in C++ STL
	<code>stack<value_type></code>	<i>creates an empty stack of <value_type></i>
<code>void</code>	<code>push(const value_type& item)</code>	<i>inserts a new item onto stack</i>
<code>void</code>	<code>pop()</code>	<i>removes top item from stack (which is most recently added)</i>
<code>value_type&</code>	<code>top()</code>	<i>returns a reference to the top item</i>
<code>bool</code>	<code>empty()</code>	<i>is the stack empty?</i>
<code>int</code>	<code>size()</code>	<i>returns the number of items in the stack</i>

<https://cplusplus.com/reference/stack/stack/>

Warm-up client: Reverse sequence of strings using stack.

Stack: Driver/Client using stack class in C++ STL

- Read strings from a collection using a range-for loop.
 - If string equals "-", pop string from stack and print.
 - Otherwise, push string onto stack.

```
int main () { // stack initialization using range-based for
    string list[] = {"to", "be", "or", "not", "to", "-", "be",
                    "-", "-", "that", "-", "-", "-", "is"};

    stack<string> s;
    for (auto item : list) {    // to be not that or be
        if (item != "-")
            s.push(item);
        else {
            cout << s.top() << ' ';
            s.pop();
        }
    }
    cout << "\nsize(): " << s.size();
    cout << "\ntop() : " << s.top();
}
```

```
void printStack(stack<string> s) {
    while (!s.empty()) {
        cout << s.top() << ' ';
        s.pop();
    }
    cout << endl; // now, s is empty
}
```

Stack: Implementation

Let's implement our own stack in several different ways.

- Array implementation
 - fixed size array
 - dynamic array
- Vector implementation
- Using Template
 - Array implementation
 - Vector implementation

Stack: Array implementation

Let's implement our own stack in several different ways.

- Array implementation of a stack:
 - Use array `s[]` to store `N` items on stack.
 - **push()**: add new item at `s[N]`.
 - **top()**: return item from `s[N-1]`.
 - **pop()**: remove item from `s[N-1]`, it just decrements `N` by one.

<code>s[]</code>	to	be	or	not	to	be	null	null	null	null
	0	1	2	3	4	5	6	7	8	9

`N` capacity = 10

Defect. Stack overflows when `N` exceeds capacity. *[stay tuned]*

Stack: Things to consider

- Underflow:
 - Throw exception if pop from an empty stack or return null;

```
string top(stack s) {  
    return s->item[s->N - 1];  
}
```

```
string top(stack s) {  
    if (empty())  
        throw std::out_of_range("underflow");  
  
    return s->item[s->N - 1];  
}
```

- Overflow:
 - Use resizing array for array implementation. [stay tuned]
 - Use successive doubling method
- Generic programming using Template in C++
 - It makes the stack data(item) type-independent
 - **template<typename T>**

Stack: version.2 – using a fixed size array

stack2_arr.cpp

```
struct Stack {
    string *item;
    int N;
    int capacity;
};
using stack = Stack *;

stack newStack(int capacity) {
    stack s = new Stack;
    s->item = new string[capacity];
    s->N = 0;
    s->capacity = capacity;
    return s;
}

void free(stack s) {
    delete[] s->item;
    delete s;
}
```

a shortcoming
(stay tuned)

item[N] is next to be filled if any.

```
int size(stack s)    { return s->N; }

bool empty(stack s) { return s->N == 0; }

void pop(stack s)    { s->N--; }

string top(stack s) {
    return s->item[s->N - 1];
}

void push(stack s, string item) {
    s->item[s->N++] = item;
}

void printStack(stack s) {
    while (!empty(s)) {
        cout << top(s) << ' ';
        pop(s);
    }
    cout << endl;    // stack is empty now
}
```

N is not decremented

use N and incremented
N points an empty slot

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - **Using Dynamic Array**
 - Using Vector
 - Using STL Template

14

Data Structures

Chapter 3

1. Stack

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - **Using Dynamic Array**
 - Using Vector
 - Using STL Template

2. Queue

Stack: Using dynamic arrays

- Problem:
 - Requiring client to provide **capacity** (size of stack) is inappropriate.
 - Question: How to grow and shrink array?
- First try.
 - **push()**: increase size of array **s[]** by 1.
 - **pop()**: decrease size of array **s[]** by 1.
- **Too expensive.**
 - Need to copy all items to a new array.
 - Inserting first N items takes time proportional to $1 + 2 + 3 + \dots + N \approx N^2/2$.


infeasible for large N

Challenge: Ensure that array resizing happens infrequently.

Stack: Using dynamic arrays

Q. How to grow and shrink array?

A. If array is full, create a new array of **twice** the size, and copy items.

"successive doubling"
↓

```
stack newStack(int capacity = 1) {  
    stack s = new Stack;  
    s->item = new string[capacity];  
    s->capacity = capacity;  
    s->N = 0;  
    return s;  
}
```

```
void resize(stack s, int new_capacity) {  
    string *copied = new string[new_capacity];  
    for (int i = 0; i < s->N; i++)  
        copied[i] = s->item[i];  
    delete[] s->item;  
    s->item = copied;  
    s->capacity = new_capacity;  
}
```

```
copy(s->item, s->item + s->N, copied);
```

Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(s.length + 10)`?

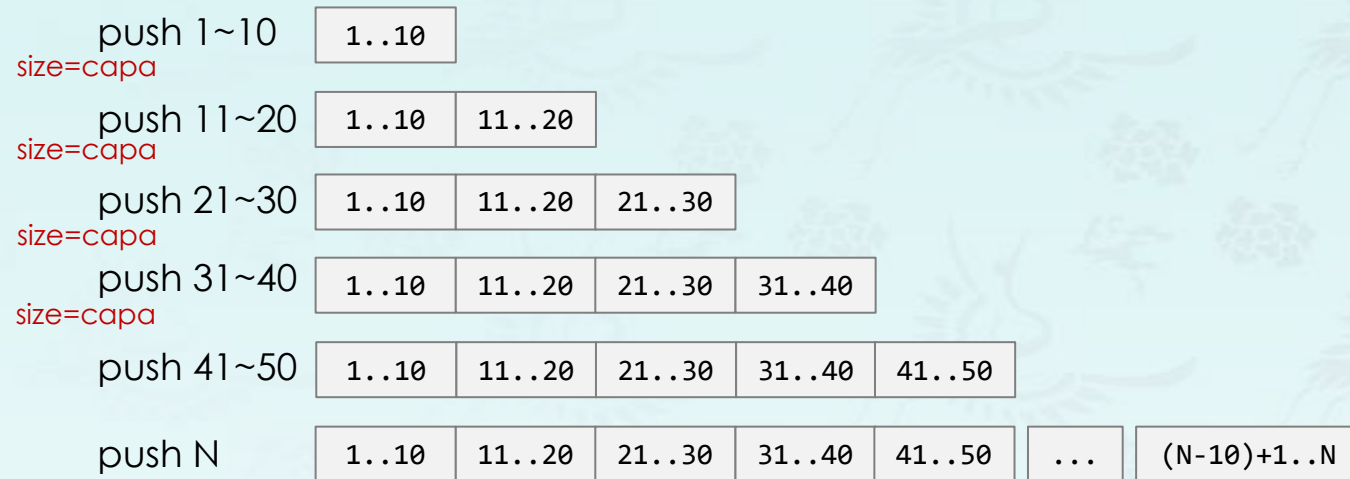
- **A.** $T(N) = 10N + (10 + 20 + 30 + \dots + N)$

↑
1 array access per push

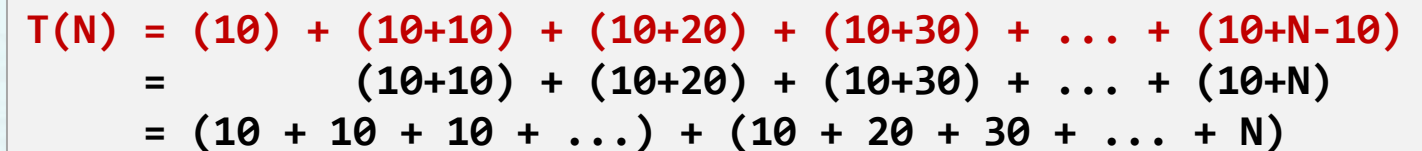
↑
k array accesses when memory is resized by increment of 10
(ignoring cost to create new array)
(assuming `new()` costs copying each item one by one)

Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(s.length + 10)`?
- Let us consider an example of a simple array stack pushes.



- Q. Cost of inserting first N items by `resize(s.length + 10)`?
- Let us consider an example of a simple array stack pushes.



Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(s.length + 10)`?

- **A.** $T(N) = 10N + (10 + 20 + 30 + \dots + N)$

↑
1 array access per push

↑
k array accesses when memory is resized by increment of 10
(ignoring cost to create new array)
(assuming new() costs copying each item one by one)

Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(s.length + 10)`?
- **A.** $T(N) = 10N + (10 + 20 + 30 + \dots + N)$

How many terms? k terms, then $N = 10k$

$$T(N) = 10N + (10 + 20 + 30 + \dots + N)$$

Let $N = 10k$, then it becomes

$$\begin{aligned} T(N) &= 10N + (10 + 20 + 30 + \dots + 10k) \\ &= 10N + 10(1 + 2 + 3 + \dots + k) \end{aligned}$$

$$= 10N + 10 \frac{k(k+1)}{2}$$

$$= 10N + 10 \frac{\frac{N}{10}(\frac{N}{10} + 1)}{2}$$

$$\text{Therefore, } T(N) = 10N + \frac{N}{2} \left(\frac{N}{10} + 1 \right)$$

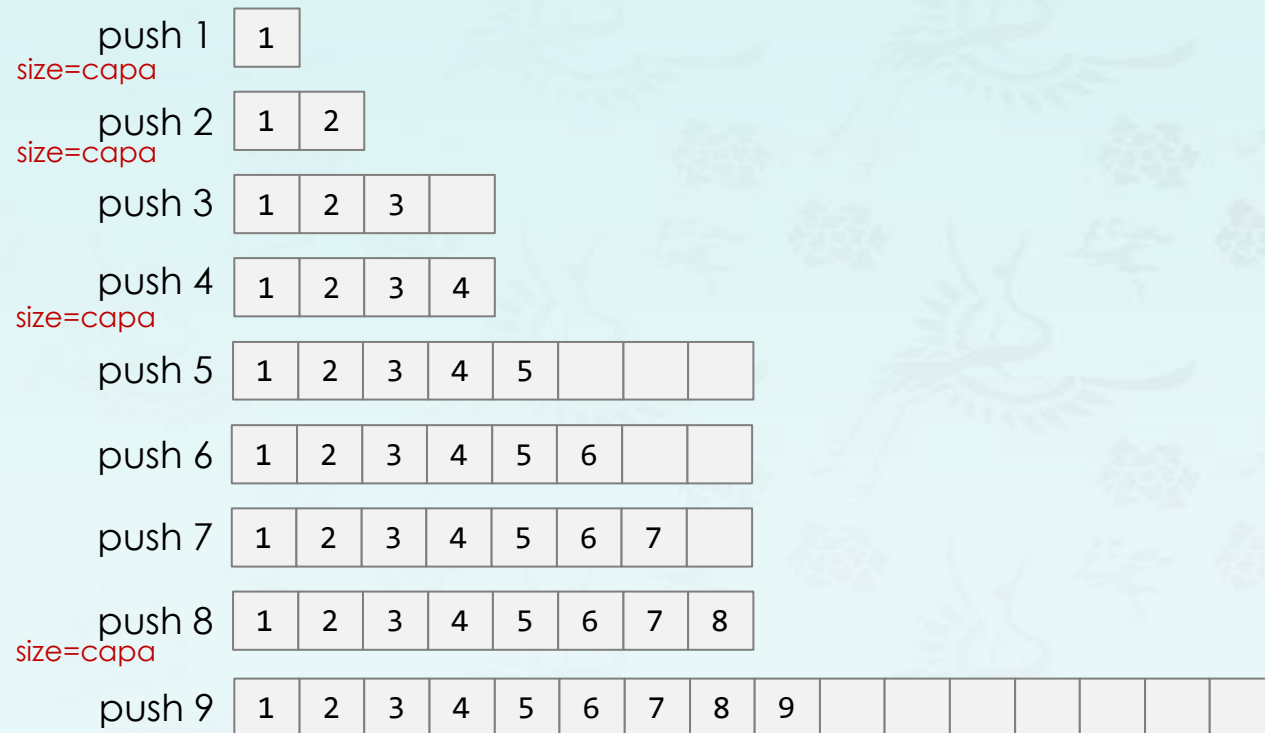
→ The time complexity of the algorithm is $O(n^2)$.

Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(capacity * 2)`?
- **A.** $T(N) = N + (1 + 2 + 4 + 8 + \dots + N)$

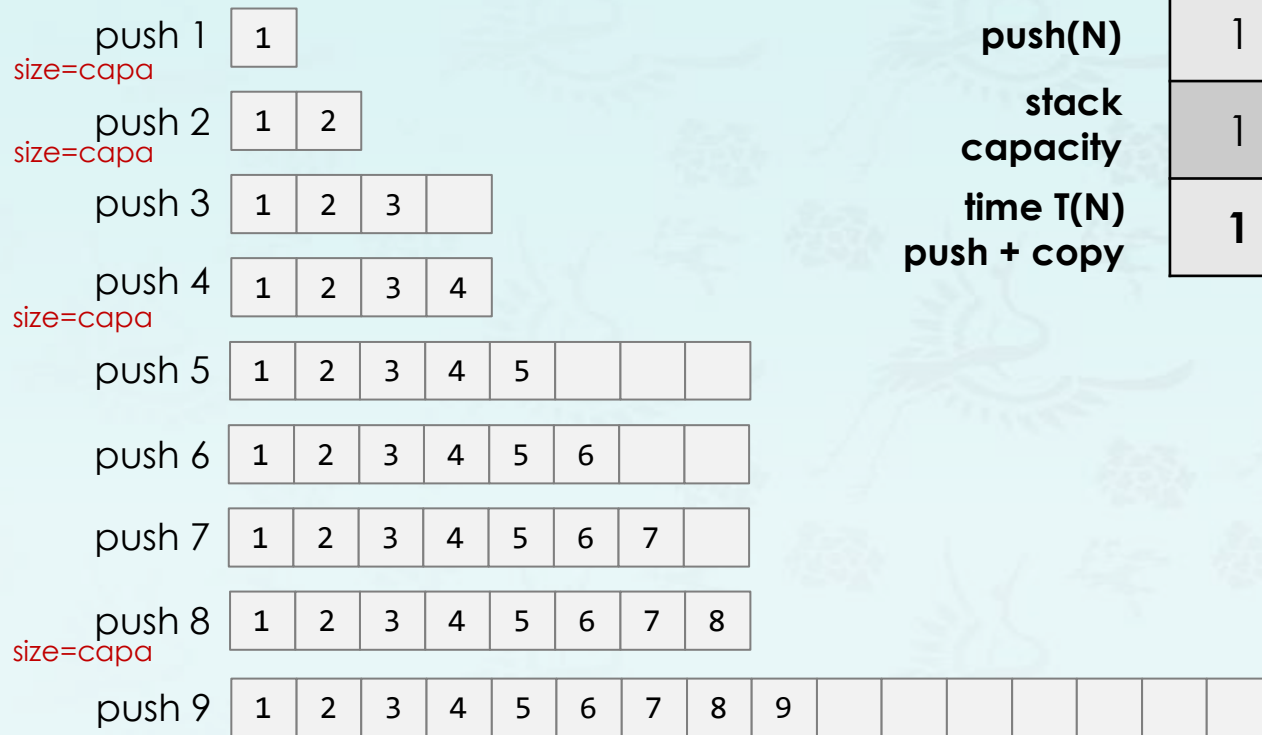
Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(capacity * 2)`?
- Let us consider an example of a simple array stack pushes.



Stack: Using dynamic arrays

- Q. Cost of inserting first N items by `resize(capacity * 2)`?
- Let us consider an example of a simple array stack pushes.



push(N)	1	2	3	4	5	6	7	8	9	10
stack capacity	1	2	4	4	8	8	8	8	16	16
time T(N) push + copy	1	1+1	1+2	1	1+4	1	1	1	1+8	1

$$\begin{aligned}
 & (1+1) + (1+2) + (1+4) + (1+8) + \dots + (1 + 2^k) \\
 &= (1 + 1 + 1 + \dots) + (1 + 2 + 4 + 8 + \dots + 2^k) \\
 &= (k + 1) + (1 + 2 + 4 + 8 + \dots + 2^k)
 \end{aligned}$$

For example, $N = 9$ (or push 9, $k = 3$):

$$\begin{aligned}
 T'(9) &= (1+1) + (1+2) + (1+4) + (1+8) \\
 &= (1+1+1+1) + (1+2+4+8) \\
 &= (3+1) + (1+2+4+8) \\
 &= 19 \quad // \text{resizing steps only}
 \end{aligned}$$

How many 1's in the time for N pushes?

Then $T(N)$ can be expressed

$$\begin{aligned}
 T(N) &= x + (k + 1) + (1 + 2 + 4 + 8 + \dots + 2^k) \\
 &= N + (1 + 2 + 4 + 8 + \dots + 2^k) \\
 &= N + (1 + 2 + 4 + 8 + \dots + N)
 \end{aligned}$$

Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(capacity * 2)`?
- Let us consider an example of a simple array stack pushes.

push(N)	1	2	3	4	5	6	7	8	9	10
stack capacity	1	2	4	4	8	8	8	8	16	16
time T(N) push + copy	1	1+1	1+2	1	1+4	1	1	1	1+8	1

When $(N - 1)$ is not a power of 2,
we can use $k = \text{floor}(\log(N - 1))$

For example:

For $N = 8$, $k = 2$

$$\begin{aligned} T(N) &= N + (1 + 2 + 4) \\ &= 8 + (1 + 2 + 4) \\ &= 15 \end{aligned}$$

For $N = 10$, $k = 3$

$$\begin{aligned} T(N) &= 10 + (1 + 2 + 4 + 8) \\ &= 25 \end{aligned}$$

$$\begin{aligned} &(1+1) + (1+2) + (1+4) + (1+8) + \dots + (1 + 2^k) \\ &= (1 + 1 + 1 + \dots) + (1 + 2 + 4 + 8 + \dots + 2^k) \\ &= (k + 1) + (1 + 2 + 4 + 8 + \dots + 2^k) \end{aligned}$$

For example, $N = 9$ (or push 9, $k = 3$):

$$\begin{aligned} T'(9) &= (1+1) + (1+2) + (1+4) + (1+8) \\ &= (1+1+1+1) + (1+2+4+8) \\ &= (3+1) + (1+2+4+8) \\ &= 19 \quad // \text{resizing steps only} \end{aligned}$$

How many 1's in the time for N pushes?

Then $T(N)$ can be expressed

$$\begin{aligned} T(N) &= (k + 1) + (1 + 2 + 4 + 8 + \dots + 2^k) \\ &= N + (1 + 2 + 4 + 8 + \dots + 2^k) \\ &= N + (1 + 2 + 4 + 8 + \dots + N) \end{aligned}$$

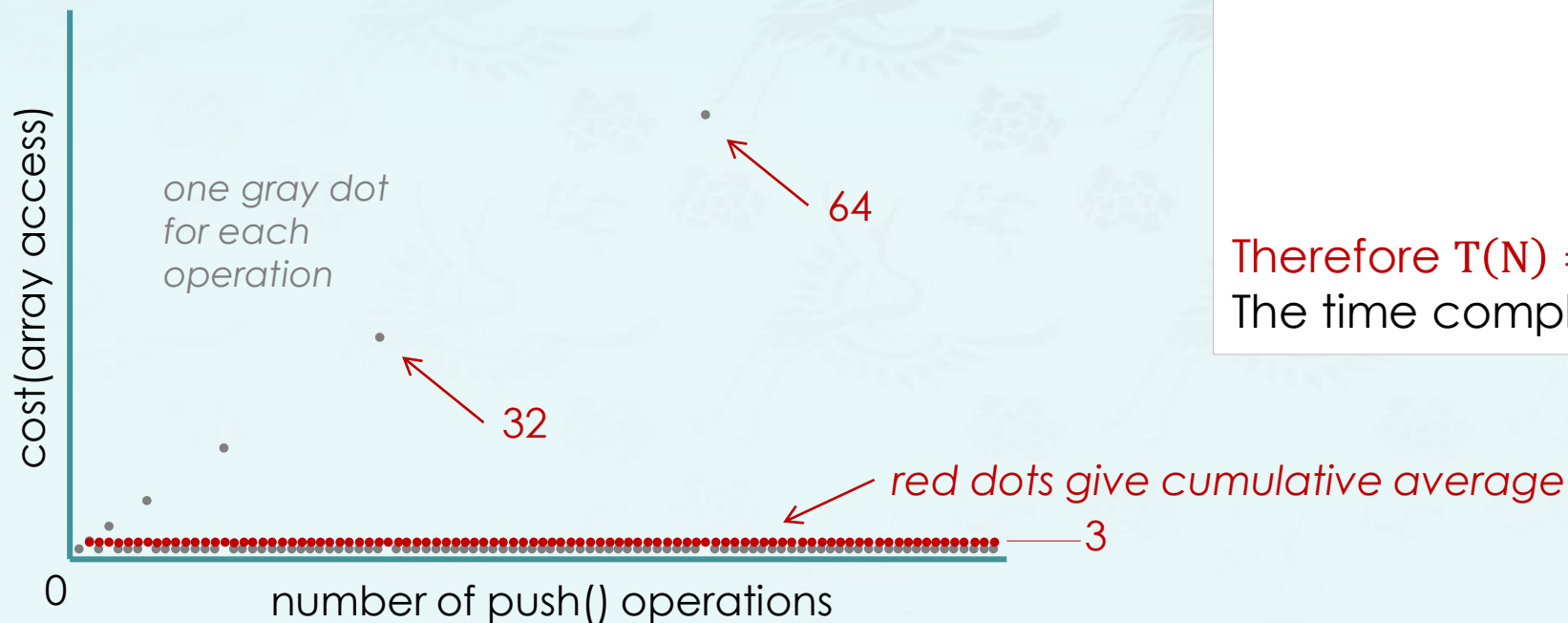
Stack: Using dynamic arrays

- **Q.** Cost of inserting first N items by `resize(capacity * 2)`?
- **A.** $T(N) = N + (1 + 2 + 4 + 8 + \dots + N)$

$$1 + a + a^2 + a^3 + \dots + a^n = \frac{a^{n+1} - 1}{a - 1}$$

$$1 + 2 + 4 + \dots + 2^n = \frac{2^{n+1} - 1}{2 - 1} = 2^{n+1} - 1$$

Therefore $T(N) = N + (1 + 2 + 4 + \dots + N) = ?$
The time complexity of the algorithm is $O(n)$.



Stack: Using dynamic arrays

Q. How to grow and shrink array?

A. If array is full, create a new array of **twice** the size, and copy items.

"successive doubling"
↓

```
stack newStack(int capacity = 1) {
    stack s = new Stack;
    s->item = new string[capacity];
    s->capacity = capacity;
    s->N = 0;
    return s;
}

void resize(stack s, int new_capacity) {
    string *copied = new string[new_capacity];
    for (int i = 0; i < s->N; i++)
        copied[i] = s->item[i];
    delete[] s->item;
    s->item = copied;
    s->capacity = new_capacity;
}
```

```
void push(stack s, string item) {
    // your code here
    s->item[s->N++] = item;
}
```


Stack: Using dynamic arrays

- **Q:** How to shrink array?
- **First try.**
 - **push():** double size of array **s[]** when array is full
 - **pop():** halve size of array **s[]** when array is one-half full.
- **Too expensive in worst case.**
 - Consider push-pop-push-pop- ... sequence when array is full
 - Each operation takes time proportional to N .

N= 5 to be or not to be null null

N= 4 to be or not

N= 5 to be or not to be null null

N= 4 to be or not

Stack: Using dynamic arrays

- **Q:** How to shrink array?
- **Efficient solution**
 - **push():** double size of array **s[]** when array is full
 - **pop():** **halve** size of array **s[]** when array is **one-quarter full**.

```
void pop(stack s) {  
    s->N--;  
    // your code here  
}
```

❖ **Invariant.** Array is between 25% and 100% full.

Stack: Using dynamic arrays

- **Amortized analysis:**

- Average running time per operation over a worst-case sequence of operations.

- **Proposition:**

- Starting from an empty stack, any sequence of N push and pop operations takes time proportional to N .

	best	worst	amortized
construct	$O(1)$	$O(1)$	$O(1)$
push	$O(1)$	$O(n)$	$O(1)$
pop	$O(1)$	$O(n)$	$O(1)$
size	$O(1)$	$O(1)$	$O(1)$

doubling and
halving operations

order of growth of running time
for resizing stack with N items

Stack: Using dynamic arrays

Q. How to grow and shrink array?

A. If array is full, create a new array of **twice** the size, and copy items.

"successive doubling"
↓

```
stack newStack(int capacity = 1) {
    stack s = new Stack;
    s->item = new string[capacity];
    s->capacity = capacity;
    s->N = 0;
    return s;
}

void resize(stack s, int new_capacity) {
    string *copied = new string[new_capacity];
    for (int i = 0; i < s->N; i++)
        copied[i] = s->item[i];
    delete[] s->item;
    s->item = copied;
    s->capacity = new_capacity;
}
```

```
bool empty(stack s) { return s->N == 0; }

string top(stack s) {
    return s->item[s->N - 1];
}

void push(stack s, string item) {
    // your code here
    s->item[s->N++] = item;
}

void pop(stack s) {
    s->N--;
    // your code here
}
```

Data Structures

Chapter 3

1. Stack

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - Using Dynamic Array
 - Using Vector**
 - Using Template**

2. Queue



Data Structures

Chapter 3

1. Stack

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - Using Dynamic Array
 - **Using Vector**
 - **Using Template**

2. Queue

Stack: **Vector** implementation

Let's implement our own stack in several different ways.

- Array implementation
 - fixed size array
 - dynamic array
- **Vector implementation**
 - A good tutorial in Korean at <https://codechacha.com/ko/cpp-stl-vector/>
- Using Template
 - Array implementation
 - Vector implementation

Vector Container

- An alternative to the built-in array.
- A vector is self-grown.
 - No allocation/free is necessary!
- Template implementation advantage!!
- For example:
 - vector<int>** - vector of integers.
 - vector<string>** - vector of strings.
 - vector<int *>** - vector of pointers to integers.
 - vector<Shape>** - vector of Shape objects. **Shape is a user defined class.**

Operations on vector

- `iterator begin();`
- `iterator end();`
- `bool empty();`
- `void push_back(const T& x);`
- `void pop_back();`
- `void back();`
- `void clear();`
- `size_type size();`
- `size_type capacity();`

Vector Container Example

```
#include<iostream>
#include<vector>
using namespace std;
int main(){
    vector<int> v(5);
    for(int i=0; i < v.size(); i++)
        cin >> v[i];

    for(int i=0; i < v.size(); i++)
        cout << v[i] << ' ';
    cout << endl;
}
```

Vector Container Example

```
#include<iostream>
#include<vector>
using namespace std;
int main(){
    vector<int> v(5);
    for(int i=0; i < v.size(); i++)
        cin >> v[i];

    for(int i=0; i < v.size(); i++)
        cout << v[i] << ' ';
    cout << endl;
}
```

```
for(int x: v)
    cout << x << ' ';
cout << endl;

for(auto x: v)
    cout << x << ' ';
cout << endl;

vector<int>::iterator it;
for(it = v.begin(); it!=v.end(); it++)
    cout << *it << ' ';
cout << endl;
```

Operations on vector

- `iterator begin();`
- `iterator end();`
- `bool empty();`
- `void push_back(const T& x);`
- `void pop_back();`
- `const_reference back();`
- `void clear();`
- `size_type size();`
- `size_type capacity();`

```
int main() {  
    int count = 0;  
    vector<int> vec;  
    vec.push_back(1);  
    vec.push_back(2);  
    vec.push_back(3);  
    while (!vec.empty()) {  
        count++;  
        vec.pop_back();  
    }  
    cout << count;  
    return 0;  
}
```


Operations on vector

- `iterator begin();`
- `iterator end();`
- `bool empty();`
- `void push_back(const T& x);`
- `void pop_back();`
- **`const_reference back();`**
- `void clear();`
- **`size_type size();`**
- `size_type capacity();`

```
int main () {  
    vector<int> vec;  
    vec.push_back(10);  
    while (vec.back() != 0) {  
        vec.push_back ( vec.back() - 1 );  
    }  
  
    cout << "vec contains: ";  
    for (unsigned i=0; i<vec.size() ; i++)  
        cout << vec[i] << ' ';  
    cout << endl;  
    return 0;  
}
```

Operations on vector

- `iterator begin();`
- `iterator end();`
- `bool empty();`
- `void push_back(const T& x);`
- `void pop_back();`
- **`const_reference back();`**
- `void clear();`
- **`size_type size();`**
- `size_type capacity();`

```
int main () {  
    vector<int> vec;  
    vec.push_back(10);  
    while (vec.back() != 0) {  
        vec.push_back ( vec.back() - 1 );  
    }  
  
    cout << "vec contains: ";  
    for (unsigned i=0; i<vec.size() ; i++)  
        cout << vec[i] << ' '  
    cout << endl;  
    return 0;  
}
```

vec contains: 10 9 8 7 6 5 4 3 2 1 0

Operations on vector

- `iterator begin();`
- `iterator end();`
- `bool empty();`
- `void push_back(const T& x);`
- `void pop_back();`
- `const_reference back();`
- `void clear();`
- **`size_type size();`**
- **`size_type capacity();`**

```
int main () {  
    vector<int> vec;  
    for (int i=0; i<100; i++) vec.push_back(i);  
    cout << "size: " << vec.size() << endl;  
    cout << "capa: " << vec.capacity() << endl;  
    return 0;  
}
```

```
size: 100  
capa: 128
```

Stack: version.4 – using a vector in C++ STL

stack4_vec.cpp

```
struct Stack {  
    vector<string> item;  
};  
using stack = Stack *;  
  
void free(stack s) {  
    delete s;  
}  
  
int size(stack s) {  
    return s->item.size();  
}  
  
bool empty(stack s) {  
    return s->item.empty();  
}
```

```
void pop(stack s) {  
    // your code here  
}  
string top(stack s) {  
    // your code here  
}  
void push(stack s, string item) {  
    // your code here  
}  
void printStack(stack s) {  
    while (!empty(s)) {  
        cout << top(s) << ' ' ;  
        pop(s);  
    }  
    cout << endl; // stack is empty now  
}
```

Stack: Using template

- A **template** is a mechanism that allows a programmer to use types as parameters for a class or a function. The compiler then generates a specific class or function when we **later** provide specific types as arguments.
- A function/class defined using **template** is called a **generic function/class**. This is one of the key features of C++.
- Use **templates** when we need functions/classes that apply the same algorithm to a several types. So we can use the same function/class regardless of the types of the argument or result.
- The syntax is:
 - `template <class T> function_declaration;`
 - or
 - `template <typename T> function_declaration;`

Stack: Using template

- A **template** is a mechanism that allows a programmer to use types as parameters for a class or a function. The compiler then generates a specific class or function when we **later** provide specific types as arguments.
- A function/class defined using **template** is called a **generic function/class**. This is one of the key features of C++.
- Use **templates** when we need functions/classes that apply the same algorithm to a several types. So we can use the same function/class regardless of the types of the argument or result.
- The syntax is:
 - `template <class T> function_declaration;`
 - or
 - `template <typename T> function_declaration;`

Stack: Using template

Pros and Cons of Templates

■ Pros:

- It provides us **type-safe, efficient** generic containers and generic algorithms
- The main reason for using C++ and templates is the trade-offs in performance and maintainability outweigh the bigger size of the resulting code and longer compile times.
- The drawbacks of not using them are likely to be much greater.

■ Cons:

- Templates can lead to **slower compile-times** and possibly larger executable.
- Compilers often produce incomprehensible poor error diagnostics and **poor error messages**.

Stack: version.4T – using a vector<> in C++ STL

stack4_vecT.cpp

```
struct Stack {  
    vector<string> item;  
};  
  
using stack = Stack *;  
  
void free(stack s) {  
    delete s;  
}  
  
string top(stack s) {  
    return s->item.back();  
}
```

stack4_vec.cpp



```
template<typename T>  
struct Stack {  
    vector<T> item;  
};  
  
template<typename T>  
using stack = Stack<T> *;  
  
template<typename T>  
void free(stack<T> s) {  
    delete s;  
}  
  
template<typename T>  
T top(stack<T> s) {  
    return s->item.back();  
}
```

Compare these two program segments and see how to use **Templates** in C++ for generic programming.

Data Structures

Chapter 3

1. Stack

- Stack Concept
 - STL stack class
- Stack Implementations
 - Using Fixed Array
 - Using Dynamic Array
 - Using Vector
 - Using Template

2. Queue