Stochastic Resonance for a Cancer Growth System Subjected to Correlated Multiplicative and Additive Noises

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In the present paper, the phenomena of stochastic resonance (SR) for a stochastic cancer development system that is driven by correlated multiplicative and additive noises is investigated. By using the fast descent method and the two-state theory, the expression of the signal-to-noise ratio (SNR) is obtained. Numerical results show that conventional SR occurs in the cancer growth model under different values of the system parameters. If the correlation strength between the two noises is positive or negative, the effects of the addictive noise intensity, the multiplicative noise intensity, and the correlated noise strength on the SNR are different.

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I. INTRODUCTION

In recent years, much attention has been directed towards nonlinear physics and its application to uncover biological complexities. Studies have confirmed the important role of noise in nonlinear stochastic systems [1]. In particular, many studies have focused on the growth law of tumor cells via dynamics, especially stochastic dynamics [2–8]. Many noise-induced effects, such as noise-induced transitions [2, 3], stochastic resonance [4–6], noise-enhanced stability, and resonant activation [7–9], have been observed and studied in tumor cell growth systems. These studies further deepen the understanding of the growth law of the tumor cells. Cancer research is now an interdisciplinary field, and so far the investigation of the growth law of tumor cells is still a challenging subject.

Since the stochastic resonance (SR) phenomenon was first found in the study of the periodic changes of the ancient climate [10, 11], it has been extensively investigated both theoretically and experimentally in many scientific fields including biophysics due to its potential applications [1]. For example, the SR phenomenon has been investigated in bistable systems [12–14], bias linear systems [15, 16], single mode laser systems [17–20], biological systems [4–6, 21], and so on. SR is a typical example for the constructive role of noise. Adiabatic elimination theory [22], linear response theory [23, 24], and perturbation theory [25] were employed to characterize the SR phenomenon. The typical signature of SR

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is the existence of a maximum of the output signal and of the signal-to-noise ratio (SNR) as a function of noise intensity. The residence-time distribution offers another possibility to characterize SR [26–28]. Collins *et al.* introduced a multiplicative periodic signal into a symmetric bistable potential with the presence of additive noise, and found a periodic SR [29, 30]. Recently, Nicolis and Nicolis studied the SR in the presence of slowly varying control parameters and a multiplicative periodic signal in a bistable system [31]. Their results may give rise to a new method for the control of the transition rates [32]. Chapeau *et al.* gave the SR phenomenon for an isolated dynamical saturating system for processing a noisy sinusoidal noise [39]. Duan *et al.* investigated stochastic resonance in a parallel array of nonlinear dynamical elements, considering the effects of Fisher information as a metric of locally optimal processing on weak-periodic-signal stochastic resonance [40–42]. Ma *et al.* [43] studied the weak period stochastic resonance in a parallel array of static nonlinearities. Yao *et al.* [44] investigated the SR phenomena in a monostable system driven by a periodic rectangular signal and uncorrelated noises. Guo *et al.* [45] discussed the effect of entropic stochastic resonance in a confined structure driven by a square-wave signal.

In this paper, based on the stochastic cancer development model, the stochastic resonance phenomenon with a consideration of additive noise, multiplicative noise, and correlation noise is put forward. To describe the effect of the periodic treatment on the tumor cell growth, a periodic signal is introduced into the system. The effects of the noises and noise correlation on SR induced by the additive signal are studied in terms of the SNR technique. The paper aims to contribute to a better understanding of the behavior of the tumor cells under the influences of additive and multiplicative noises, correlation noise, and a periodic signal. In the next section, the approximate analytical expression of the SNR of the system is derived, and the effects of the correlated noises on the SNR of the stochastic system are analyzed. A brief conclusion and some discussion are given in the final section.

II. MODEL

We apply the predator-prey model [36, 38] to depict the cancer cells population growth in the face of cytotoxic cells. The population dynamics can be represented as follows: To begin with, the cytotoxic cells bind to the tumor cells at a rate proportional to the kinetic constant k_1; in the next place, the cancer cells which have been bound are killed and the complex dissociates at a rate proportional to k_2. The process can be described schematically:

$$X + Y \xrightarrow{k_1} Z \xrightarrow{k_2} Y + P. \tag{1}$$

Here X denotes the population of tumor cells. Similarly, Y, Z, and P stand for active cytotoxic cells, bound cells, and dead tumor cells, respectively. In a given (small) volume element, there is an upper limit M to the number of cells which may be present, taking into account that each cell has a typical diameter equal to a. From now on, we will use normalized cellular densities: ($x = X/N$ instead of X, etc.). Following the original presentation [38], we assume that: (1) cancer cells undergo replication at a rate proportional to the time constant λ; (2) as a result of cellular replication in a limited volume, a diffusive propagation of cancer cells is possible, with transport coefficient λa^2; (3) the local cytotoxic cell population remains constant, i.e., $Y + Z = E$ (E is the constant).

In the limit when the effector cells diffuse much faster than the cancer cells propagate by cellular replication and in which the dead cells are rapidly eliminated, the spatial distribution of Y and Z cells equilibrates rapidly with respect to the local density of living tumor cells, and the above scheme of kinetics can be recast in the form of the scalar problem [37, 38]:

$$\frac{dx(t)}{dt} = (1-\theta x)x - \beta\frac{x}{1+x}, \tag{2}$$

where

$$\theta = \frac{k_2}{k_1}, \quad \beta = \frac{k_1 E}{\lambda}.$$

In our paper, we will consider the spatially homogeneous form of Eq. (2).

Under the Michaelis-Menten theory, considering all kinds of disturbances from the outside environment, we can give the stochastic cancer growth model as follows:

$$\frac{dx(t)}{dt} = (1-\theta x)x - [\beta + \xi(t)]\frac{x}{1+x} + \eta(t) + A\cos(\omega t), \tag{3}$$

where x represents the number of tumor cells, θ and β are the influence coefficients which control the varying number of tumor cells. $\theta > 0$, $\beta > 0$. In fact, the tumor cells are always influenced by some external factors, such as temperature, drugs, radiotherapy, and so on. Taking the stochastic properties of these factors into account, physically we consider it reasonable to introduce a multiplicative noise $\xi(t)$ and an additive noise $\eta(t)$ into the tumor cell model to describe the effects of these external factors on the cancer development. The fluctuation of some external factors generates a multiplicative noise $\xi(t)$. Some factors such as drugs, radiotherapy, and emigration or immigration of tumors may change the number of tumor cells in a local area and give rise to an additive noise $\eta(t)$. $\xi(t)$ and $\eta(t)$ are both Gaussian white noise and are characterized by their mean and variance, i.e.,

$$\begin{aligned} &\langle \xi(t) \rangle = \langle \eta(t) \rangle = 0, \quad \big\langle \xi(t)\xi(t') \big\rangle = 2Q\delta(t-t'), \\ &\big\langle \eta(t)\eta(t') \big\rangle = 2M\delta(t-t'), \quad \big\langle \xi(t)\eta(t') \big\rangle = \big\langle \eta(t)\xi(t') \big\rangle = 2\lambda\sqrt{QM}\delta(t-t'), \end{aligned} \tag{4}$$

where both Q and M are the intensities of the multiplicative noise and the additive noise, respectively, λ denotes the intensity of the correlated noise. In the case of $-1 < \lambda < 0$, the correlation strength between the two noises is negative; in the case of $0 < \lambda < 1$, the correlation strength between the two noises is positive. $A\cos(\omega t)$ is a weak low-frequency signal, i.e., $A \ll 1$ and $\omega \ll 1$. A is the amplitude of the input periodic signal and ω is its frequency. Eq. (2) can be considered as describing an overdamped motion of the state variable moving in a quasi-'free energy potential':

$$V(x) = \frac{\theta}{3}x^3 - \frac{1}{2}x^2 + \beta x - \beta\ln(1+x). \tag{5}$$

In the absence of external periodic force and noise terms, the fixed points of Eq. (2) strongly depend on θ and β: (1) For $\theta > 1$ and $0 < \beta < 1$, a stable point $x_- = 0$ and an unstable point $x_u = \frac{1-\theta+\sqrt{(1+\theta)^2-4\beta\theta}}{2\theta}$; (2) For $0 < \theta < 1$ and $0 < \beta < (1+\theta)^2/4\theta$, two stable points $x_- = 0$, $x_+ = \frac{1-\theta+\sqrt{(1+\theta)^2-4\beta\theta}}{2\theta}$ and an unstable point $x_u = \frac{1-\theta-\sqrt{(1+\theta)^2-4\beta\theta}}{2\theta}$; (3) For $0 < \theta < 1$ and $\beta = (1+\theta)^2/4\theta$, a stable point $x_- = 0$ and an unstable point $x_u = (1-\theta)/2\theta$. In the following section, we will consider only the parameters in the region of $0 < \theta < 1$ and $0 < \beta < (1+\theta)^2/4\theta$, in which case the model possesses two stable states.

III. OUTPUT SNR

By applying Novikov's theorem [33] and Fox's approach [34], the approximate Fokker-Planck equation (FPE) corresponding to Eq. (3) can be written as

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}\mu(x)P(x,t) + \frac{\partial^2}{\partial x^2}\sigma^2(x)P(x,t). \tag{6}$$

The drift coefficient $\mu(x)$ and the diffusion coefficient $\sigma^2(x)$ are

$$\mu(x) = f(x) + Qg(x)g'(x) + \lambda\sqrt{QM}g'(x) + A\cos(\omega t), \tag{7}$$

$$\sigma^2(x) = Qg^2(x) + 2\lambda\sqrt{QM}g(x) + M, \tag{8}$$

where

$$f(x) = (1-\theta x)x - \beta\frac{x}{1+x}, \quad g(x) = -\frac{x}{1+x}. \tag{9}$$

The steady-state probability distribution function corresponding to the Fokker-Planck equation (6) can be written as [40, 41, 43, 44]

$$P_{st}(x,t) = \frac{N}{\sigma(x)}\exp\left[\int_x \frac{\mu(x')}{\sigma^2(x')}dx'\right] = \frac{N}{\sigma(x)}\exp\left[U(x)\right], \tag{10}$$

where N is the normalization constant ensuring $\int_0^\infty P_{st}(x,t)dx = 1$, and $U(x)$ is the modified generalized potential function, which is described as

$$\begin{aligned} U(x) \;=\; & \frac{A_1x^2}{2(Q-2\lambda\sqrt{QM}+M)^2} - \frac{\theta x^3}{3(Q-2\lambda\sqrt{QM}+M)} \\ & + \frac{A_2x}{(Q-2\lambda\sqrt{QM}+M)^3} - \frac{A_3\ln(1+x)}{(Q-2\lambda\sqrt{QM}+M)^3} \\ & + \frac{A_4\ln(Qx^2-2\lambda\sqrt{QM}x-2\lambda\sqrt{QM}x^2+M+2Mx+Mx^2)}{2(Q-2\lambda\sqrt{QM}+M)^4} \\ & + \frac{\arctan\left(\frac{(Q-2\lambda\sqrt{QM}+M)x+M-\lambda\sqrt{QM}}{\sqrt{(\lambda^2-1)QM}}\right)}{\sqrt{(\lambda^2-1)QM}} \\ & \times\left(\frac{A_5}{(Q-2\lambda\sqrt{QM}+M)^3} + \frac{A_6}{(Q-2\lambda\sqrt{QM}+M)^4}\right), \end{aligned} \tag{11}$$

where

$$A_1 = M + Q - 2\lambda\sqrt{QM} - 2\theta Q + x^2\theta\lambda\sqrt{QM},$$

$$\begin{aligned} A_2 \;=\; & 2Q^2 + 2MQ + A\cos(\omega t)Q^2 + A\cos(\omega t)M^2 - \beta Q^2 - \beta M^2 - \theta Q^2 \\ & +3M\theta Q - 2\beta MQ - 6Q\lambda\sqrt{QM} - 2\lambda\sqrt{QM}M + 2A\cos(\omega t)QM \\ & +4A\cos(\omega t)\lambda^2 MQ - 4\beta\lambda^2 MQ + 4\lambda^2 MQ - 4A\cos(\omega t)\lambda Q\sqrt{QM} \\ & -4A\cos(\omega t)\lambda M\sqrt{QM} + 4\beta Q\lambda\sqrt{QM} + 4\beta M\lambda\sqrt{QM} - 2M\theta\lambda\sqrt{QM}, \end{aligned}$$

$$A_3 = M^3 + 8\lambda^3(MQ)^{3/2} + 3QM^2 + 3Q^2M,$$

$$\begin{aligned} A_4 \;=\; & M^4 + Q^4 + Q^3 - 3QM^2 - 2Q^2M - Q^3\beta + 4Q^3M + 4QM^3 + 6Q^2M^2 \\ & +2Q^3A\cos(\omega t) + \beta M^3 + 2M^2\theta\lambda\sqrt{QM} + 2M^2\lambda\sqrt{QM} - 4\lambda\beta\sqrt{QM}M^2 \\ & +8\lambda Q\sqrt{QM}M - 2\lambda Q^2\sqrt{QM} - 4\lambda^2M^2Q - 8\lambda^3M^3\sqrt{QM} \\ & +8\lambda^3MA\cos(\omega t)(QM)^{3/2} - 8\lambda\sqrt{QM}M^3 + 16\lambda^3M(QM)^{3/2} \\ & -4Q^2\lambda^2\beta M - 24Q^2\lambda\sqrt{QM}M - 24Q\lambda\sqrt{QM}M^2 - 10Q^2A\cos(\omega t)\lambda\sqrt{QM} \\ & +8QA\cos(\omega t)\lambda^2M^2 + 4Q\lambda^2\beta M^2 - 2Q^2\theta\lambda\sqrt{QM} + 16Q^2A\cos(\omega t)\lambda^2M \\ & +4Q^2\lambda\beta\sqrt{QM} - 48\lambda^3M^2Q\sqrt{QM} - 40Q^2\lambda^3\sqrt{QM}M + Q\beta M^2 - 4QM^2\theta \\ & +2QA\cos(\omega t)M^2 + 4Q^2A\cos(\omega t)M - 8\lambda Q^3\sqrt{QM} + 8\lambda^3Q(QM)^{3/2} \\ & +4Q^2M\theta + 16Q^2\lambda^4M^2 + 24Q^3\lambda^2M + 24Q\lambda^2M^3 + 48\lambda^2M^2Q^2 \\ & +\frac{8}{Q}\lambda^3(QM)^{3/2}M^2 - Q^2M\beta - 12QA\cos(\omega t)\lambda\sqrt{QM}M \\ & -16QA\cos(\omega t)\lambda^3\sqrt{QM}M - 2A\cos(\omega t)\lambda\sqrt{QM}M^2 - 8A\cos(\omega t)\lambda^3\sqrt{QM}M^2 \\ & +8A\cos(\omega t)\lambda^3(QM)^{3/2}, \end{aligned}$$

$$\begin{aligned} A_5 \;=\; & 4\lambda^2QM^2 - M^4 - Q^2M\theta + 8Q\lambda\sqrt{QM}MA\cos(\omega t) + 6\lambda Q\sqrt{QM}M \\ & +Q^3A\cos(\omega t) + 2Q^2M + 2QM^2 - 3Q^2M^2 - 3QM^3 - Q^3M - 8\lambda^3(QM)^{3/2}M^2 \\ & +8QA\cos(\omega t)\lambda\sqrt{QM}M + 2A\cos(\omega t)\lambda\sqrt{QM}M^2 + 4\beta\lambda\sqrt{QM}M^2 \\ & -2M^2\theta\lambda\sqrt{QM} + 8A\cos(\omega t)\lambda^3(QM)^{3/2} + 15\lambda Q\sqrt{QM}M^2 + 9\lambda Q^2\sqrt{QM}M \\ & +16\lambda^3M^2\sqrt{QM}Q + 7\lambda M^3\sqrt{QM} - 6Q\lambda\sqrt{QM}M + 8A\cos(\omega t)\lambda^3\sqrt{QM}M^2 \\ & +8\lambda^3\sqrt{QM}M^3 - 2\lambda\sqrt{QM}M^2 - 4Q\beta\lambda^2M^2 - 8QA\cos(\omega t)\lambda^2M^2 \\ & +6Q^2A\cos(\omega t)\lambda\sqrt{QM} - 12Q^2A\cos(\omega t)\lambda^2M - 2Q^2A\cos(\omega t)M - Q^2\beta M \\ & -2Q\beta M^2 - Q^2\theta M - \beta M^3 - 6Q^3\lambda^2M - 24Q^2\lambda^2M^2 + Q^3\lambda\sqrt{QM} - 18Q\lambda^2M^3, \end{aligned}$$

$$
\begin{aligned}
A_6 \; = \; & M^5 + 16\lambda^3(MQ)^{3/2}M^2 + 2Q^3\lambda^2 M - 4Q\beta\lambda^3 M^2\sqrt{QM} + 48Q^2\lambda^4 M^3 \\
& +2\lambda\sqrt{QM}M^3 - Q^3 M\beta + 2\beta M^4 - 2Q^2\lambda^2 + 2Q\beta M^3 - 2Q^2\beta M^2 - 4Q^2\beta\lambda^2 M^2 \\
& +4Q^2\beta\lambda^3 M\sqrt{QM} + 8M^2 A\cos(\omega t)\lambda^3(QM)^{3/2} - 14QA\cos(\omega t)\lambda M^2\sqrt{QM} \\
& -24Q^3\lambda^3 M\sqrt{QM} + 8Q\beta\lambda^2 M^3 - 4QM^3\theta + 2QA\cos(\omega t)M^3 + 4Q^2 A\cos(\omega t)M^2 \\
& +4Q^2M^2\theta + 72Q^2\lambda^2 M^3 + 32\lambda^2 QM^4 - Q^4\lambda\sqrt{QM} + 8\lambda^3(QM)^{3/2}M^3 \\
& -Q^3\lambda\sqrt{QM} + 11Q\lambda\sqrt{QM}M^2 + 6Q^2M^3 + 4QM^4 - 2Q^2M^2 - 3QM^3 + Q^3M \\
& +4Q\lambda^3M^2\sqrt{QM} + 2\lambda M^3\sqrt{QM} - 2Q^3 A\cos(\omega t)\lambda\sqrt{QM} + Q^3\beta\lambda\sqrt{QM} \\
& -16Q^3\lambda^5M^2\sqrt{QM} - 9\lambda\sqrt{QM}M^4 - 72\lambda^3 QM^3\sqrt{QM} - 88\lambda^3Q^2M^2\sqrt{QM} \\
& -30\lambda Q^2M^2\sqrt{QM} - 28\lambda QM^3\sqrt{QM} - 2A\cos(\omega t)\lambda\sqrt{QM}M^3 + 2\lambda\theta M^3\sqrt{QM} \\
& +10QA\cos(\omega t)\lambda^2M^3 - 14\lambda Q^2 A\cos(\omega t)M\sqrt{QM} + 28Q^2 A\cos(\omega t)\lambda^2 M^2 \\
& +8Q\lambda^3(QM)^{3/2}M - 12\lambda Q^3 M\sqrt{QM} - 6Q^2\theta\lambda M\sqrt{QM} + 4QM^2\theta\lambda\sqrt{QM} \\
& -6Q\lambda^2M^3 + 48Q^2\lambda^4M^3 + 2Q^3 A\cos(\omega t)M + 32Q^3\lambda^4M^2 + 8Q^4\lambda^2M + 2Q^3\lambda^2\theta M \\
& +2Q^3\lambda^2 M + 8Q^2\lambda^4 A\cos(\omega t)M^2 - 2\lambda^2 Q\theta M^3 - 4Q^3\beta\lambda^2 M + 10Q^3 A\cos(\omega t)\lambda^2 M).
\end{aligned}
\tag{12}
$$

Since the frequency ω is very small, there is enough time for the system to reach a local equilibrium during the period of $1/\omega$. On the other hand, assuming that the amplitude of the input period signal is small enough ($A \ll 1$), it cannot make the particle transit from one well to another. Using the definition of the mean first-passage time and steepest descent method, one can obtain the expression of the transition rates $W_\pm$ in terms of $x_\pm$:

$$W_+ = \frac{\sqrt{|V''(x_+)V''(x_u)|}}{2\pi}\exp[U(x_u) - U(x_+)], \tag{13}$$

$$W_- = \frac{\sqrt{|V''(x_-)V''(x_u)|}}{2\pi}\exp[U(x_u) - U(x_-)], \tag{14}$$

in which $V(x)$ and $U(x)$ have been defined by Eqs. (5) and (11), respectively.

In the general asymmetric nonlinear dynamical system, the SR phenomenon has been found, and the related theory has been developed. Here, we only simply give this method for calculating the signal-to-noise ratio (SNR).

For the general asymmetric case we define the SNR as the ratio of the strength of the output signal and the broadband noise output evaluated at the signal frequency. Finally, the expression of SNR is given by the two-state approach [35]:

$$SNR = \frac{A^2\pi(\mu_1\alpha_2 + \mu_2\alpha_1)^2}{4\mu_1\mu_2(\mu_1 + \mu_2)}, \tag{15}$$

where

$$
\begin{aligned}
& \mu_1 = W_-|_{A\cos(\omega t)=0}\,, \quad \mu_2 = W_+|_{A\cos(\omega t)=0}\,, \\
& \alpha_1 = \left.\frac{dW_-}{dA\cos(\omega t)}\right|_{A\cos(\omega t)=0}, \quad \alpha_2 = \left.\frac{dW_+}{dA\cos(\omega t)}\right|_{A\cos(\omega t)=0}.
\end{aligned}
\tag{16}
$$

IV. DISCUSSION

In this section, by virtue of Eq. (15), the effects of the external force and the noises as well as the system parameters upon the SNR are discussed according to the cross-correlation noise intensity $\lambda > 0$ and $\lambda < 0$.

Fig. 1 is a plot of the SNR as a function of the multiplicative noise intensity Q for different values of M. No matter whether the correlative noise intensity λ is positive or negative, the SNR is always a non-monotonic function on the multiplicative noise intensity Q. As seen from Fig. 3, the SNR decreases monotonically as the very small additive noise intensity M increases. From Fig. 1 (a) and (b), we can see that the peaks of the SNR both become lower quickly with an increase of small value of M, but the position of the peak of the SNR on the multiplicative noise intensity Q stays invariant.

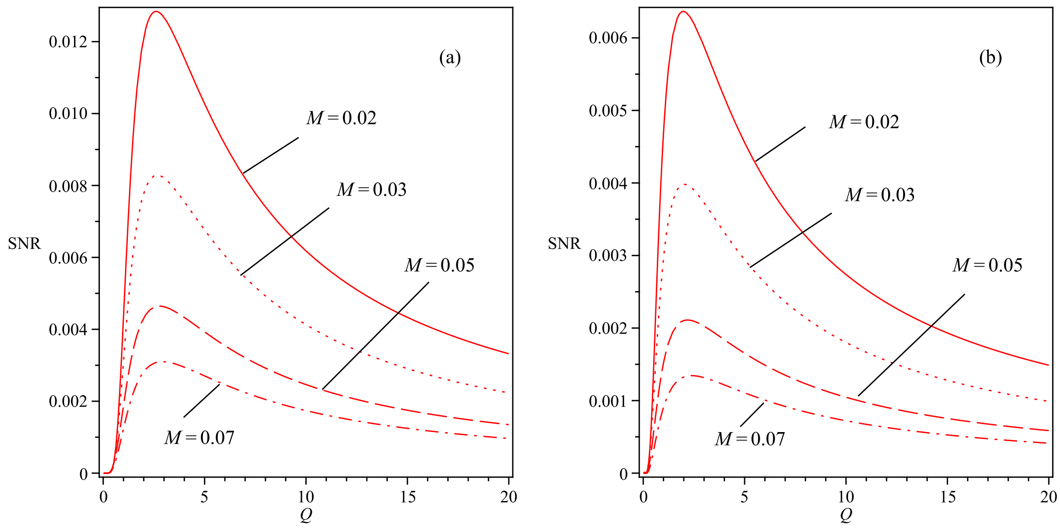

FIG. 1: The SNR versus multiplicative noise intensity Q for different values of M, and the other parameters take $\theta = 0.1$, $\beta = 2.6$, $A = 0.01$, $\omega = 0.01$, and (a) $\lambda = 0.3$, (b) $\lambda = -0.3$.

Fig. 2 reflects the effect of the multiplicative noise intensity Q on the SNR with different values of M, when the additive noise intensity M takes larger values. Comparing Fig. 2 (a) with Fig. 2 (b), we can easily see that their behaviors are similar. With an increase of Q, there firstly appears a suppression phenomenon in the SNR plot. Afterwards, the SNR begins to rise and arrives at the summit finally with Q increasing. The resonance peak of the SNR descends step by step as the additive noise intensity M increases, and the SR phenomenon converges to disappearance. Combining Fig. 1 and Fig. 2, we can find that the additive noise intensity M always plays a role in restraining the stochastic resonance phenomenon.

Fig. 3 illustrates the effect of the additive noise intensity M on the SNR with different values of Q. We can be aware that Fig. 3 (a) is similar to Fig. 3 (b). We can discover that there exists a single peak of the SNR in the plot. The peak of the SNR in Fig. 3 (a) descends

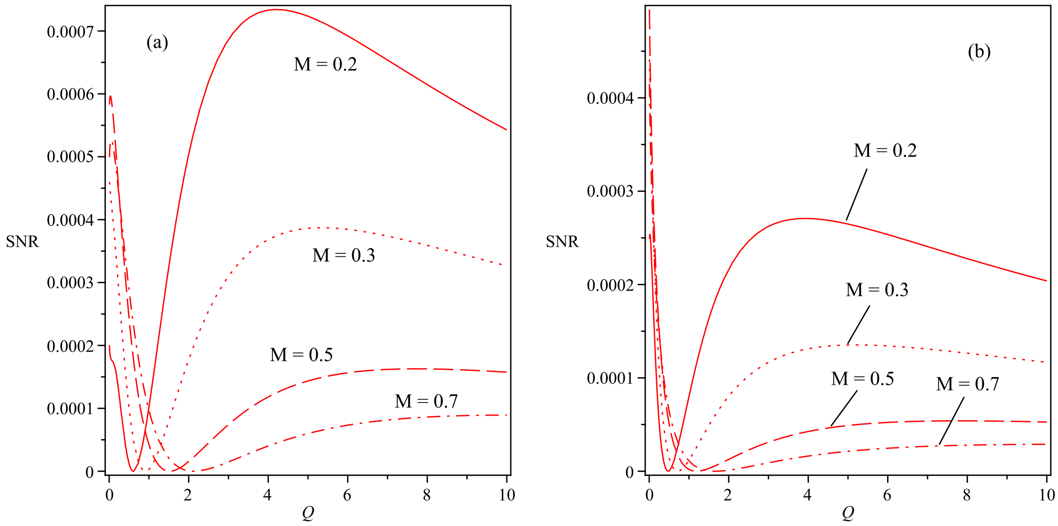

FIG. 2: The SNR versus multiplicative noise intensity Q for different values of M, and the other parameters take $\theta = 0.1$, $\beta = 2.6$, $A = 0.01$, $\omega = 0.01$, and (a) $\lambda = 0.3$, (b) $\lambda = -0.3$.

gradually, but the resonance peak of the SNR in Fig. 3 (b) drops quickly with an increase of the very small value of Q. This shows that a small multiplicative noise intensity Q can restrain the phenomenon of the stochastic resonance.

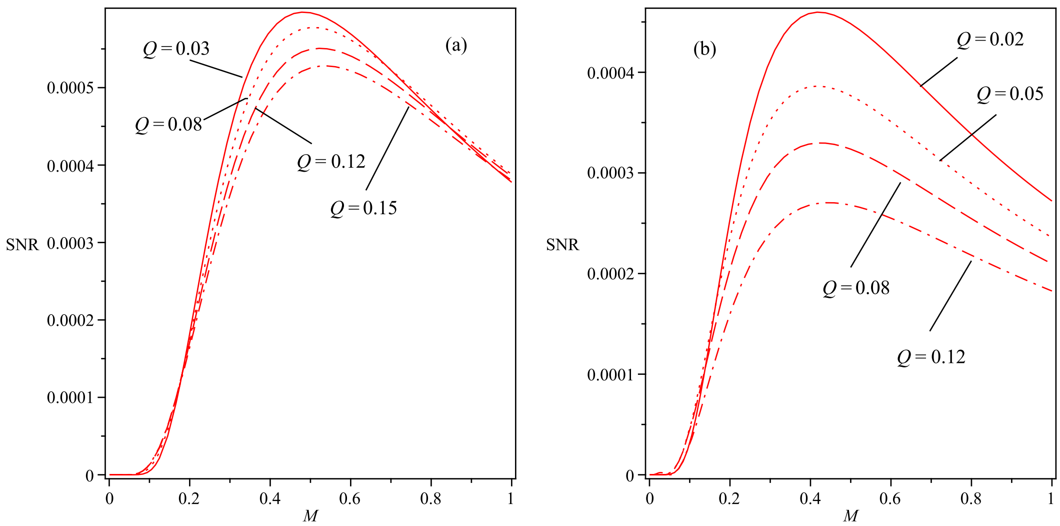

FIG. 3: The SNR versus additive noise intensity M for different values of Q, and the other parameters take $\theta = 0.1$, $\beta = 2.6$, $A = 0.01$, $\omega = 0.01$, and (a) $\lambda = 0.3$, (b) $\lambda = -0.3$.

Fig. 4 shows that the curves for the SNR versus the additive noise intensity M with different large values of Q. As seen in Fig. 4 (a), at the beginning there is only a single peak of the SNR in the SNR-M plot, but as the multiplicative noise intensity Q increases, two resonance peaks appear in the plot. During the process of Q increasing, the first resonance peak rises quickly, but the second resonance peak falls sharply, and its position moves to the right gradually. When the correlation noise intensity λ is negative, from Fig. 4(b), we can find that there exists a gradual peak of the SNR. With an increase of the multiplicative noise intensity Q, the peak rises slowly to the left. Finally, the SNR degenerates into a monotonic decreasing function on the additive noise intensity M. A phase transformation appears in the stochastic system.

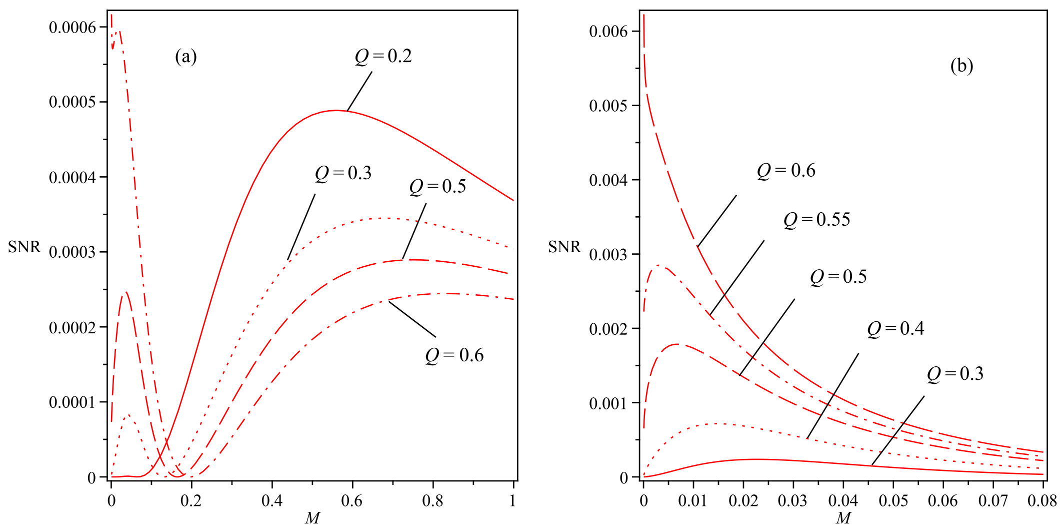

FIG. 4: The SNR versus additive noise intensity M for different values of Q, and the other parameters take $\theta = 0.1$, $\beta = 2.6$, $A = 0.01$, $\omega = 0.01$, and (a) $\lambda = 0.3$, (b) $\lambda = -0.3$.

In Fig. 5, we analyze the influence of the correlation noise intensity λ on the SNR when the SNR is shown to be a function of the additive noise intensity M and the multiplicative noise intensity Q. As we observe from Fig. 5 (a), the SNR is a non-monotonic function on the additive noise intensity M. There appears a single peak in the SNR-M plot. We can also see that the peaks of the SNR are enhanced rapidly, and its position shifts to the right slowly with an increase of λ. The behavior shown in Fig. 5 (b) is almost in agreement with that in Fig. 5 (a). This shows that correlation noise intensity λ can enhance the SR phenomenon greatly.

Fig. 6 is a plot of the SNR as a function of the correlation noise intensity λ for different values of Q. From Fig. 6 (b), we can see that in the case of $M = 0.3$, the SNR is a non-monotonic function of λ. With an increase of small values of Q, the peak of the SNR descends slowly. Meanwhile, the SNR is also a monotonically decreasing function on small Q values. In contrast, Fig. 6 (a) shows that for the case of $M = 0.01$, the SNR is a monotonic increasing function on large Q values. At the beginning, the SNR is a monotonic increasing function on λ. As Q increases, the resonance peak grows up and rises quickly.

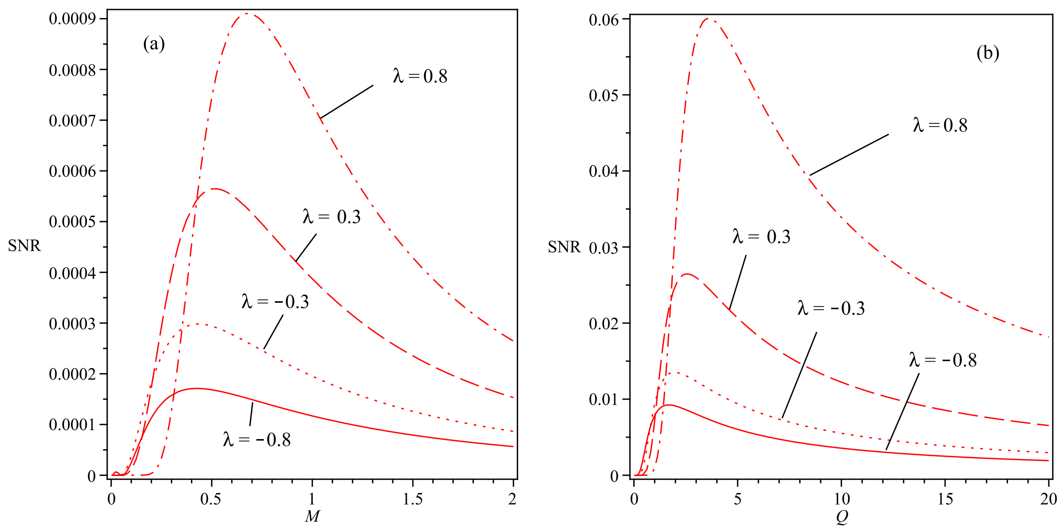

FIG. 5: The SNR versus additive noise intensity M and multiplicative noise intensity Q for different values of λ, and the other parameters take (a) $Q = 0.1$, (b) $M = 0.01$ and $A = 0.01$, $\omega = 0.01$, $e = 0.2$, $c = 0.8$.

Its position moves to the right gradually. The results show that small Q values can restrain the SR phenomenon, while large Q values can stimulate the SR phenomenon.

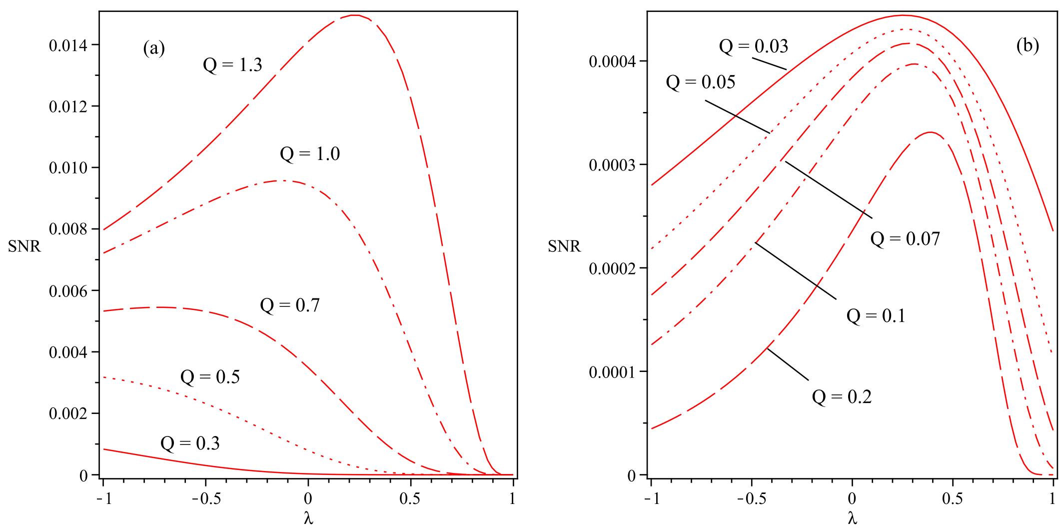

FIG. 6: The SNR versus correlation noise intensity λ for different values of Q, and the other parameters take $e = 0.2$, $c = 0.8$, $A = 0.01$, $\omega = 0.01$, and (a) $M = 0.01$, (b) $M = 0.3$.

From Fig. 7 (b), we can find that when Q takes a small value, 0.05, the SNR decreases monotonically with an increase of λ at the beginning. As the small additive noise M

increases, a single peak of the SNR arises and shoots up. Fig. 7 (a) exhibits that if Q takes a large value, 0.2, with an increase of the large additive noise M, a single peak appears and rises quickly, its position shifts to the right step by step. In the end, the curve of SNR degenerates into a monotonically increasing function on λ. This shows that M plays an important role in exciting the SR phenomenon in the SNR-λ plot.

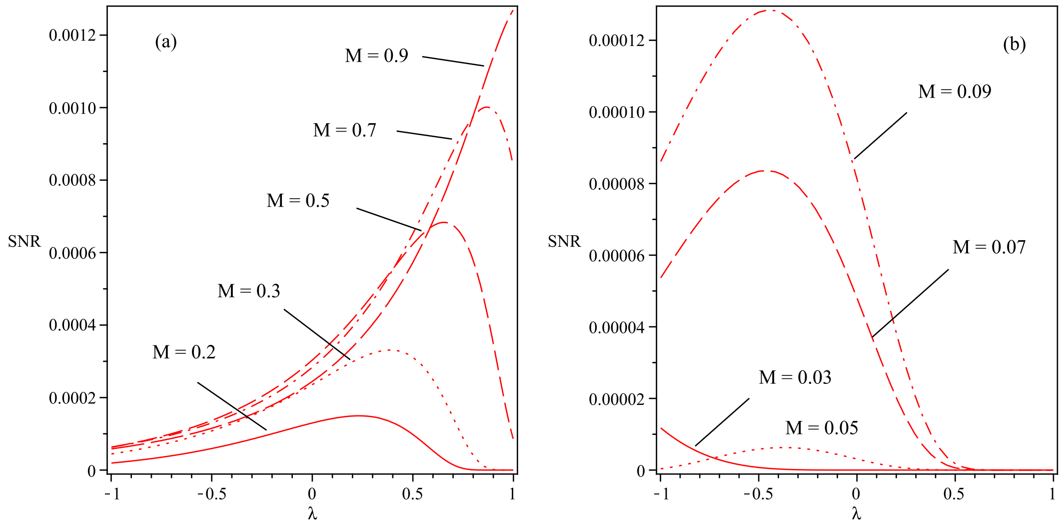

FIG. 7: The SNR versus correlation noise intensity λ for different values of M, and the other parameters take $e = 0.2$, $c = 0.8$, $A = 0.01$, $\omega = 0.01$, and (a) $Q = 0.2$, (b) $Q = 0.05$.

V. CONCLUSION

In summary, by means of applying the SR theory proposed by McNamara and Wiesenfeld [22], the SR phenomenon in a cancer growth system subjected to both correlated multiplicative and additive noises and a periodic signal is investigated. For this system, the SNR is found to be dependent on the additive noise intensity M, the multiplicative noise intensity Q, and the correlated noise intensity λ. By virtue of the expression of the SNR and through numerical computation, the following conclusions can be drawn from the present study.

The effects of the multiplicative noise intensity, the additive noise intensity, and the correlated noise intensity on SNR are different:

(1) Fig. 1 and Fig. 2 show that no matter whether the correlation noise intensity λ is positive or negative, the small additive noise intensity M always plays the same role in restraining the SR phenomenon. Fig. 3 exhibits that small values of Q can always suppress the SR phenomenon. Fig. 4 exhibits that in the case of $\lambda > 0$, large values of Q can excite the phenomenon of double resonance peaks; in the case of $\lambda < 0$, large values of Q can enhance the SR phenomenon at first, but finally take away the SR phenomenon.

(2) It is illustrated that the correlated noise intensity λ always plays a key role in influencing the SR phenomenon in Fig. 5. According to Fig. 6, we can discover that for the case of small M, big values of Q can stimulate the stochastic resonance; for the case of large M, small values of Q can restrain the stochastic resonance. Fig. 7 displays that if Q takes the appropriate values, M always plays a key role in exciting the SR phenomenon.

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