A Rumor Spreading Model with Control Mechanism on Social Networks

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In this paper, considering a control mechanism that uses an anti-rumor spread to combat a rumor spread, a novel susceptible-infected (SI) model is developed. By means of mean-field theory, the early stage propagation dynamics of the rumor and the anti-rumor on homogeneous networks and inhomogeneous networks are investigated, respectively. Theoretical analysis and simulation results show that, in both types of networks, as the immunity of the anti-rumor or the curing rate of infected nodes increase, the velocity of rumor spreading obviously decreases, which significantly reduces the probabilities of rumor outbreaks. Also, we find that shortening the time delay between the rumor starting and the anti-rumor starting can improve the control effect of rumor transmission. Our results may shed new light on the research of rumor control strategies.

I. INTRODUCTION

Rumors generally bring to people's daily lives anxiety, fear, and panic [1, 2], which may lead to economic loss and even violence, such as the iodized salt shortage panic induced by fear of radiation in the Japanese earthquake in 2011 and the recent doomsday panic all over the world. Initially, rumors spread on social networks through relationships between different individuals. With the emergence of social web and other media, rumors can propagate at a faster velocity and generate greater influence on people's lives [3]. Therefore understanding the transmission mechanisms of rumors and then devising effective measures to suppress their spreading are of considerable importance [4, 5].

In fact, the investigation of the propagation mechanisms of rumors has had a long history; researchers coming from many fields have paid much attention to it [6]. Up to now, several rumor propagation models on social networks have been well studied. The most popular transmission model of rumors, conceptually similar to the susceptible-infected-removed (SIR) epidemic spreading model, was presented by Daley and Kendall [7]. Zanette [8] investigated the transmission dynamics of a rumor on a small-world network [9]. Moreno *et al.* [10] discussed in detail the process of rumor spreading on random scale-free networks [11]. Zhang *et al.* [12] established a model to describe the interplay between rumor propagation and emergency development. For a possible application in disaster management, Rabajante *et al.* [13] introduced a conceptual mathematical model for studying the

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spread of rumors. Xing *et al.* [14] discussed rumor spreading dynamics on a connecting multi-small-world network. Trpevski *et al.* [15] investigated the spreading dynamics of two different types of rumors on complex social networks. Zhao *et al.* [16] studied the impact of new media on the propagation characteristics of rumors.

The above investigations mainly focused on the propagation dynamics of rumors. However, few works have investigated the control mechanisms that can effectively suppress rumor transmission on social networks. Obviously, the control mechanisms are quite necessary and practically meaningful for reducing the negative influences on the society [17]. Recently, by introducing epidemic-like immunization strategies, Huang *et al.* [18] presented a modified SIR model in order to investigate the control of rumor spreading on a small-world network. However, the situations of combating the rumor transmission and epidemic transmission are evidently different [19, 20]. Specifically, we can immunize a certain proportion of nodes in the networks to suppress epidemic spreading; these immune nodes cannot participate in the epidemic spreading process. But people who have accepted the true messages can also spread them to other people, which is unlike vaccines for epidemics that can only be implemented to individuals.

In this paper, we propose a novel epidemic-like susceptible-infected (SI) model with control mechanism in order to further investigate the dynamics of rumor spreading; this is because the SI model can be used to expediently describe the dynamics in the very early stage of rumor outbreaks, which is a critical period for taking measures to control the rumor spreading [21]. In our SI model, when a rumor begins to spread on the social network, the strategy of combating the rumor is broadcasting a true message (called the anti-rumor) by the authorities to people to debunk the rumor. As many people may perceive that the authorities are vested interests in revealing the rumor, in fact, initially only few people in the social network can accept the anti-rumor. Therefore, although the source of the anti-rumor is objectively more trusted than that of the rumor, the whole transmission process of the anti-rumor depends primarily on the contacts (physical contact or via the Internet) among different individuals. In the process of the rumor spread and the anti-rumor spread, since the source of the anti-rumor is more authoritative, the anti-rumor spread has a dominant position [15]. We will discuss two cases of homogeneous networks and heterogeneous networks to address the influence of the anti-rumor spreading on the rumor spreading [22]. As will be shown below, the trust degree of the authorities plays an important role in the control of rumor spreading. The smaller the time delay between the rumor starting and the anti-rumor starting, the less the influence that the rumor can exert.

The rest of this paper is organized as follows. Considering the control medium, a novel SI rumor transmission model is presented in Section II. In Section III, by using the mean-field theory, we investigate in detail the transmission characteristics of our SI rumor model on homogeneous networks and heterogeneous networks, respectively. Then in Section IV, the predictions obtained in Section III are confirmed by sufficient numerical simulations. Finally, in Section V, we draw some conclusions from the present studies.

II. NOVEL SI RUMOR MODEL

Assume that the social network consisting of N individuals is represented by an undirected graph $G = (V, E)$ with node set V and edge set E. In our SI model, the total nodes (the network size) N is considered as constant, each node is in one of the three states corresponding to ignorant (similar to susceptible of the SI epidemic model), infected, and vaccinated, respectively. Ignorant means that the nodes have not heard the rumor or the anti-rumor but can be infected or vaccinated. Infected denotes that the nodes carry the rumor and are willing to pass the rumor on to ignorant nodes. Vaccinated means that the nodes have been vaccinated by the anti-rumor, and can transmit the anti-rumor to ignorant nodes or infected nodes. Since the source of the anti-rumor is usually more credible or more reputable than that of the rumor, ignorant nodes always consider the anti-rumor first, i.e., the anti-rumor has been prebuilt as a preference in the present SI model. Specifically, when an ignorant node contacts an infected node and a vaccinated node simultaneously, the ignorant node first accepts the anti-rumor, i.e., the ignorant is vaccinated, with a certain probability. If the ignorant node has not been vaccinated successfully, then it will be infected by the rumor at a certain rate. In addition, when contacting a vaccinated node, an infected node can abandon the rumor with a certain probability, and convert from the infected state to the vaccinated state, i.e., the infected node is cured. In this way, both ignorant nodes and infected nodes can accept the anti-rumor and become vaccinated nodes. The rumor spreading process of the novel model is shown in Figure 1.
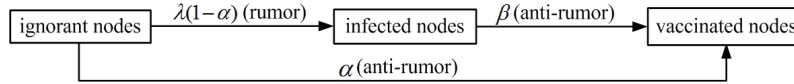

FIG. 1: Spreading process of the novel SI rumor model.

As shown in Figure 1, we can generalize the transmission rules of our SI rumor model as follows.

(1) When an ignorant node contacts a vaccinated node at time t, the ignorant node will be vaccinated at time $t + 1$ with a rate of α, namely the immunity of the anti-rumor, which is directly related to the trust degree of the authorities.

(2) When an ignorant node contacts an infected node with probability $1 - \alpha$ at time t, the ignorant node will be infected at time $t + 1$ with a rate of λ, namely the rumor spreading rate.

(3) When an infected node contacts a vaccinated node at time t, the infected node will become a vaccinated node at time $t + 1$ with probability β, namely the curing rate of infected nodes, which also depends on the trust degree of the authorities.

III. ANALYSIS OF THE NOVEL SI MODEL

In this part, in order to evaluate the efficacy of our SI rumor model, using the mean-field theory [23] we study the dynamical behavior of the model on homogeneous networks and heterogeneous networks, respectively.

III-1. HOMOGENEOUS NETWORKS

We first analyze the spreading characteristics of the SI model on homogeneous networks, in which all nodes are deemed to be equivalent and the degree correlations are not to be considered. Thus the degree of each node can be assumed to roughly equal to the network average degree, i.e., $k \simeq \langle k \rangle$.

In the following let $I(t)$, $S_1(t)$, and $S_2(t)$ denote the densities of ignorant, infected, and vaccinated nodes in the social network at time t, respectively, then

$$I(t) + S_1(t) + S_2(t) = 1. \tag{1}$$

Considering the spreading mechanisms of the rumor and the anti-rumor in our SI model, and by applying the mean-field theory, the evolution equations for these variables $I(t)$, $S_1(t)$, and $S_2(t)$ are established as follows:

$$\frac{dI(t)}{dt} = -\lambda(1 - \alpha) \langle k \rangle S_1(t)I(t) - \alpha \langle k \rangle S_2(t)I(t), \tag{2}$$

$$\frac{dS_1(t)}{dt} = \lambda(1 - \alpha) \langle k \rangle S_1(t)I(t) - \beta \langle k \rangle S_1(t)S_2(t), \tag{3}$$

$$\frac{dS_2(t)}{dt} = \alpha \langle k \rangle S_2(t)I(t) + \beta \langle k \rangle S_1(t)S_2(t). \tag{4}$$

Combining with Eq. (1) and expanding the rhs of Eq. (3) yields

$$\frac{dS_1(t)}{dt} = \lambda(1 - \alpha) \langle k \rangle S_1(t) - \lambda(1 - \alpha) \langle k \rangle S_1^2(t) - (\lambda(1 - \alpha) + \beta) \langle k \rangle S_1(t)S_2(t). \tag{5}$$

When the spreading time of the rumor and the anti-rumor is small, the densities of both infected nodes and vaccinated nodes are very small. Hence, we can neglect the terms of order $O\left(S_1^2\right)$ and $O\left(S_1 S_2\right)$ in Eq. (5) and have

$$\frac{dS_1(t)}{dt} \simeq \lambda(1 - \alpha) \langle k \rangle S_1(t). \tag{6}$$

Let $S_1(0)$ denote the initial density of infected nodes. Eq. (6) can be exactly integrated for $S_1(t)$ to yield

$$S_1(t) \simeq S_1(0)e^{t/\tau_1}, \tag{7}$$

where

$$\tau_1 = \frac{1}{\lambda(1-\alpha)\langle k \rangle} \tag{8}$$

is the time scale governing the spreading velocity of the rumor on homogeneous networks. Obviously, τ_1 is not related to the curing rate of infected nodes β. When we do not consider the control mechanism, i.e., $\alpha = 0$, Eq. (8) can be rewritten as

$$\tau_1 = \frac{1}{\lambda \langle k \rangle}, \tag{9}$$

which is just the same as for the standard SI epidemic model [20]. Comparing Eqs. (8) with (9), it can be found that in the present SI model, the introduction of the control mechanism remarkably alleviates the rumor spreading on homogeneous networks, and thus reduces the density of infected nodes.

Similar to Eq. (3), Eq. (4) can be expressed as

$$\frac{dS_2(t)}{dt} \simeq \alpha \langle k \rangle S_2(t). \tag{10}$$

Denote by $S_2(0)$ the initial density of vaccinated nodes, and neglect the terms of order $O(S_1 S_2)$ and $O(S_2^2)$. We can solve Eq. (10) explicitly to derive

$$S_2(t) \simeq S_2(0)e^{t/\tau_2}, \tag{11}$$

where

$$\tau_2 = \frac{1}{\alpha \langle k \rangle} \tag{12}$$

is the growth time scale of the anti-rumor on homogeneous networks. Eq. (12) shows that the time scale τ_2 is inversely proportional to the product of the network average degree and the immunity of the anti-rumor. Comparing Eqs. (8) with (12), we find that the spread of the anti-rumor has a great influence on the rumor spreading, which is mainly because the source of the anti-rumor has a dominant position on the homogeneous networks.

III-2. HETEROGENEOUS NETWORKS

We next discuss the propagation features of the SI rumor model on heterogeneous networks. In heterogeneous networks, the connectivity distributions are obviously fluctuating, thus we cannot use the network average degree $\langle k \rangle$ to characterize the network property. Here, we denote by $I_k(t)$, $S_{1,k}(t)$, and $S_{2,k}(t)$ the relative densities of ignorant, infected, and vaccinated k-degree nodes at time t, respectively. By using the mean-field theory, we can obtain the following differential equations for the SI model:

$$\frac{dI_k(t)}{dt} = -\lambda(1-\alpha)kI_k(t)\sum_l S_{1,l}(t)P(l/k) - \alpha k I_k(t)\sum_l S_{2,l}(t)P(l/k), \tag{13}$$

$$\frac{dS_{1,k}(t)}{dt} = \lambda(1-\alpha)kI_k(t)\sum_l S_{1,l}(t)P(l/k) - \beta k S_{1,k}(t)\sum_l S_{2,l}(t)P(l/k), \tag{14}$$

$$\frac{dS_{2,k}(t)}{dt} = -\alpha k I_k(t)\sum_l S_{2,l}(t)P(l/k) + \beta k S_{1,k}(t)\sum_l S_{2,l}(t)P(l/k). \tag{15}$$

In the initial rumor and anti-rumor stages, the terms of order $O\left(S_1^2\right)$ and $O\left(S_1 S_2\right)$ can be neglected, and Eq. (14) simplifies to

$$\frac{dS_{1,k}(t)}{dt} \simeq \lambda(1-\alpha)k\Theta(t), \tag{16}$$

where

$$\Theta(t) = \sum_l S_{1,l}(t)P(l/k) \tag{17}$$

is the probability that a randomly chosen edge of an ignorant node points to an infected node with degree l, $P(l/k)$ denotes the probability that a randomly selected edge emanating from a k-degree node is pointing to a l-degree node. For simplicity, here only the uncorrelated heterogeneous networks are considered; we can now obtain

$$P(l/k) = \frac{lP(l)}{\langle k \rangle}. \tag{18}$$

Substituting Eq. (18) into Eq. (17), one can derive

$$\Theta(t) = \frac{\sum_k S_{1,k}(t)kP(k)}{\langle k \rangle}. \tag{19}$$

Combining Eq. (16) and Eq. (19), one can calculate the time derivative of $\Theta(t)$ to obtain that

$$\frac{d\Theta(t)}{dt} = \frac{\lambda(1-\alpha)\left\langle k^2 \right\rangle}{\langle k \rangle}\Theta(t). \tag{20}$$

When the rumor begins to spread on heterogeneous networks, we assume that the infected nodes obey the uniform distribution [24], thus we have

$$S_{1,k}(t=0) = S_1(0). \tag{21}$$

Solving Eq. (20) under the uniform initial condition Eq. (21), we obtain

$$\Theta(t) = S_1(0)\exp\left(\frac{\lambda(1-\alpha)\left\langle k^2 \right\rangle}{\langle k \rangle}t\right). \tag{22}$$

Let τ_1' denote the time scale of rumor spreading on heterogeneous networks, that is

$$\tau_1' = \frac{\langle k \rangle}{\lambda(1-\alpha)\langle k^2 \rangle}. \tag{23}$$

Hence

$$\Theta(t) = S_1(0)e^{t/\tau_1'}. \tag{24}$$

By averaging over the various connectivity classes of infected nodes, we have

$$\begin{aligned}
S_1(t) &= \sum_k P(k)S_{1,k}(t) = \lambda(1-\alpha)\langle k \rangle \int_0^t \Theta(u)\mathrm{d}u + S_1(0) \\
&= S_1(0)\left(\frac{\langle k \rangle^2}{\langle k^2 \rangle}\left(e^{t/\tau_1'} - 1\right) + 1 \right).
\end{aligned} \tag{25}$$

Eqs. (23) and (25) show that the existence of a control mechanism can apparently slow down the spreading velocity of the rumor on heterogeneous networks, and thus reduce the number of infected nodes. From Eq. (25) we also find that the spreading behavior of the rumor is not determined by the curing rate of infected nodes β.

Similar to Eq. (14), neglecting the terms of order $O\left(S_1 S_2\right)$ and $O\left(S_2^2\right)$, we can obtain from Eq. (15) that

$$S_2(t) = S_2(0)\left(\frac{\langle k \rangle^2}{\langle k^2 \rangle}\left(e^{t/\tau_2'} - 1\right) + 1 \right), \tag{26}$$

where τ_2' is time scale that has the form

$$\tau_2' = \frac{\langle k \rangle}{\alpha \langle k^2 \rangle}. \tag{27}$$

When $\alpha = \lambda$, the time scale τ_2' has exactly the same form as that of the standard epidemic SI model [20]. However, Eqs. (23) and (27) show that the anti-rumor propagation can effectively control the rumor spreading.

IV. NUMERICAL SIMULATIONS

In order to validate the above analytical results, in this section, we perform numerical simulations of the present SI rumor model on the WS network and the BA network, separately. Through the course of the simulations, the WS network and BA network are of size $N = 2000$ with average degree $\langle k \rangle = 6$, the rewiring probability of the WS network is given as $p = 0.1$. Generally speaking, the rumor appears earlier than the anti-rumor, i.e., between the rumor starting and the anti-rumor starting, there will always exist a time

delay represented by T. The spreading process starts with randomly selecting a certain number of infected nodes and vaccinated nodes. All the simulations are performed for at least 20 different initial configurations of our SI rumor mode.

In Figures 2(a) and 2(b), we report the densities of infected nodes and vaccinated nodes for different values of α on the WS network and the BA network. In both types of networks, as the immunity of the anti-rumor α increases, the density of infected nodes remarkably decreases, but the density of vaccinated nodes clearly increases. Thus, the anti-rumor propagation can greatly reduce the spreading velocity of the rumor. From this, we know that in order to enhance the rumor control effect, the authorities should improve the immunity of the anti-rumor, i.e., improve their trust degree. When the value of α is small, $\alpha = 0.007$, in the early stage, the spreading velocity of the rumor is larger than that of the anti-rumor; however, as time goes on, the latter exceeds the former, and finally the density of infected nodes can be brought down to zero. This is mainly because that, with the spreading of the rumor and the anti-rumor, the probability that vaccinated nodes contact infected nodes increases and more infected nodes can be cured. The inserts in both plots show that the densities of infected nodes and vaccinated nodes grow in an exponential form in the early stage, which strongly supports the analytical results obtained in the previous section.

In Figures 3(a) and 3(b), we display the dependence of the densities of infected nodes and vaccinated nodes on time for different values of β on the WS network and the BA network. It can be seen that in both types of networks, increasing β can slow down the transmission velocity of the rumor, but speed up the anti-rumor spreading, for which the reason is that after being cured many nodes transform from the infected state to the vaccinated state. Therefore, we can effectively inhibit the rumor spreading through improving the ability of the anti-rumor to cure the rumor. Comparing Figs. 3(a) with 3(b), one can see that both the rumor and the anti-rumor spread faster on the BA network than on the WS network. This is because the hubs of the BA network can accelerate the rumor spreading and the anti-rumor spreading.

Figures 4(a) and 4(b) show the densities of infected nodes and vaccinated nodes as a function of time for different initial densities of vaccinated nodes on the WS network and the BA network. As can be seen in Figs. 4(a) and 4(b), the more the number of initial anti-rumor spreaders, the more slowly the rumor transmits. From this, it follows that when there is a rumor spread on both types of networks, increasing the number of initial anti-rumor spreaders can obviously reduce the propagation velocity of the rumor. Thus, when the rumor begins to spread on a social network, in order to combat it, the authorities should do their best to make more people know the truth. Figs. 4(a) and 4(b) also show that the influence of the initial density of vaccinated nodes on the rumor spread on the WS network is smaller than that on the BA network.

Figures 5(a) and 5(b) illustrate the time evolution of the densities of infected nodes and vaccinated nodes for different time delay T on the WS network and the BA network. As shown in Figs. 5(a) and 5(b), in both types of networks, the larger the value of T, the larger the number of infected nodes and the longer the rumor spreads, which is in good agreement with actual rumor propagation situations. Thereby, the effect of the anti-rumor combating
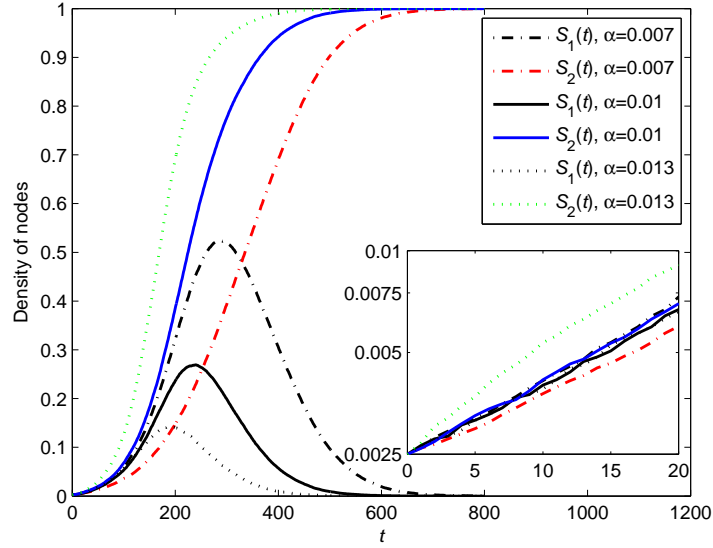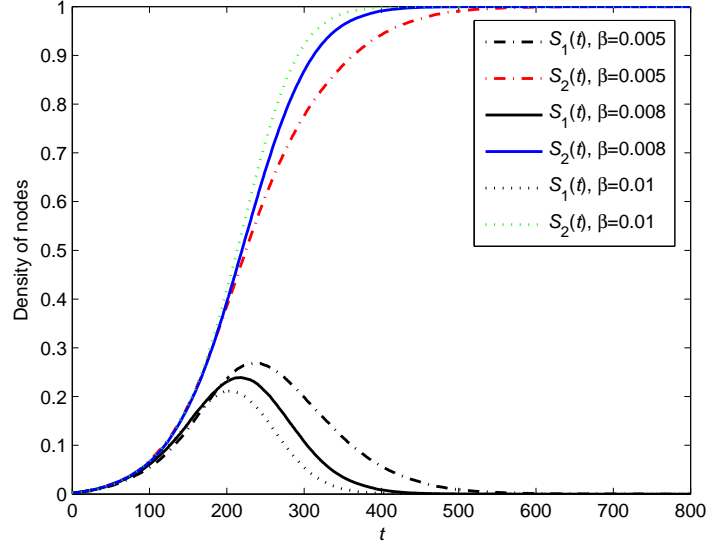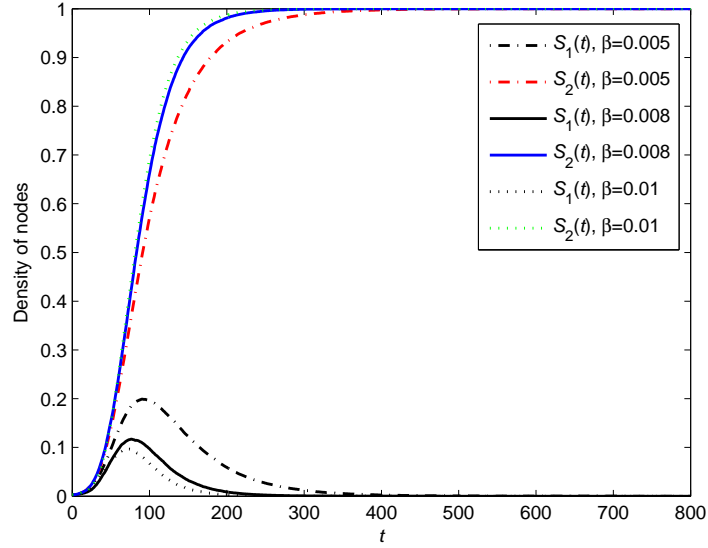
(a) WS network.
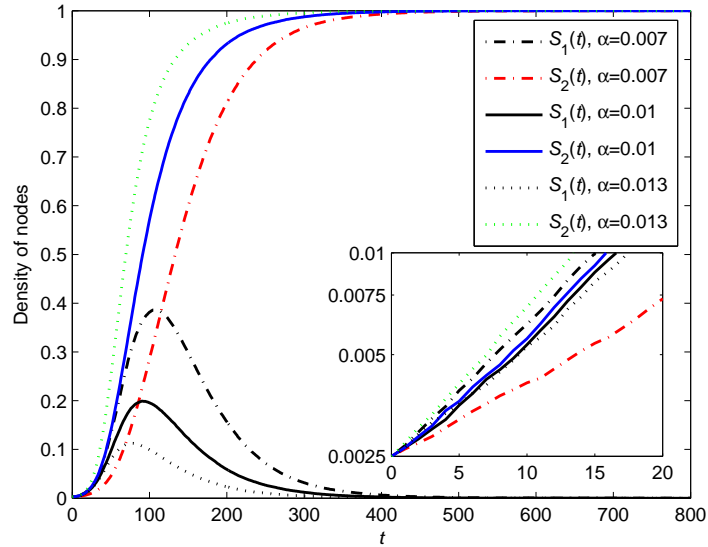
(b) BA network.

FIG. 2: The densities of infected nodes $S_1(t)$ and vaccinated nodes $S_2(t)$ versus time: (a) on the WS network, (b) on the BA network. The insert on the semi-logarithmic scale in each plot shows details in the small time part. The green dot, blue solid, and red dash-dot curves, from top to bottom, represent the cases of $\alpha = 0.013$, 0.01, and 0.007, respectively. The black dash-dot, black solid, and black dot curves, from top to bottom, correspond to $\alpha = 0.007$, 0.01, and 0.013, respectively. The rumor spreading rate $\lambda = 0.01$, the curing rate of infected nodes $\beta = 0.005$, the initial densities of infected nodes and vaccinated nodes are $S_1(0) = S_2(0) = 5/N$, and the time delay $T = 0$.
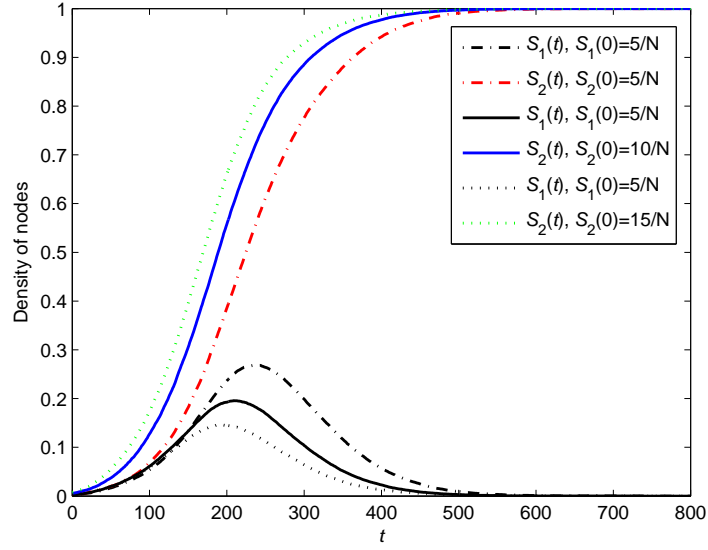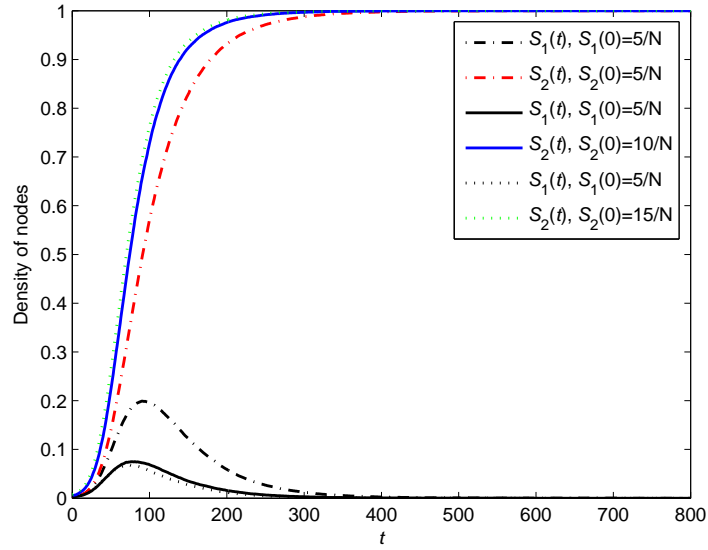
(a) WS network.

(b) BA network.

FIG. 3: The dependence of the densities of infected nodes $S_1(t)$ and vaccinated nodes $S_2(t)$ on time: (a) on the WS network, (b) on the BA network. The green dot, blue solid, and red dash-dot curves, from top to bottom, correspond to $\beta = 0.01$, 0.008, and 0.005, respectively. The black dash-dot, black solid, and black dot curves, from top to bottom, correspond to $\beta = 0.005$, 0.008, and 0.01, respectively. The rumor spreading rate $\lambda = 0.01$, the immunity of the anti-rumor $\alpha = 0.01$, the initial densities of infected nodes and vaccinated nodes are $S_1(0) = S_2(0) = 5/N$, and the time delay $T = 0$.
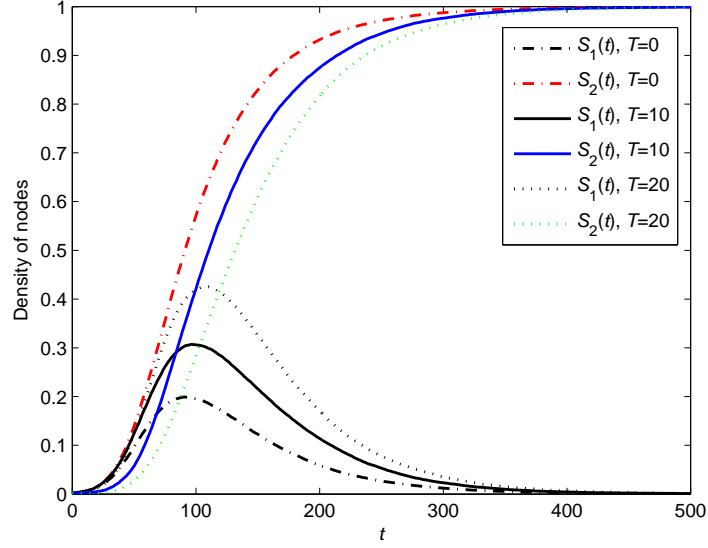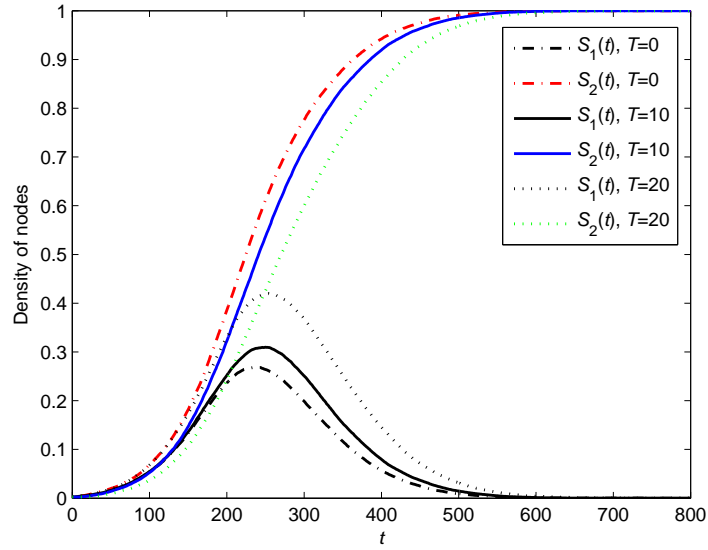
(a) WS network.

(b) BA network.

FIG. 4: The densities of infected nodes $S_1(t)$ and vaccinated nodes $S_2(t)$ as a function of time: (a) on the WS network, (b) on the BA network. The green dot, blue solid, and red dash-dot curves, from top to bottom, correspond to $S_2(0) = 15/N$, $10/N$, and $5/N$, respectively. The black dash-dot, black solid, and black dot curves, from top to bottom, all correspond to $S_1(0) = 5/N$. The rumor spreading rate $\lambda = 0.01$, the immunity of the anti-rumor $\alpha = 0.01$, the curing rate of infected nodes $\beta = 0.005$, and the time delay $T = 0$.

(a) WS network.

(b) BA network.

FIG. 5: Time evolution of the densities of infected nodes $S_1(t)$ and vaccinated nodes $S_2(t)$: (a) on the WS network, (b) on the BA network. The red dash-dot, blue solid, and green dot curves, from top to bottom, correspond to $T = 0$, 10, and 20, respectively. The black dot, black solid, and black dash-dot curves, from top to bottom, correspond to $T = 20$, 10, and 0, respectively. The rumor spreading rate $\lambda = 0.01$, the immunity of the anti-rumor $\alpha = 0.01$, the curing rate of infected nodes $\beta = 0.005$, and the initial densities of infected nodes and vaccinated nodes are $S_1(0) = S_2(0) = 5/N$.

the rumor is significantly weakened. It can be concluded that in order to effectively prohibit rumors spreading on social networks, the authorities should detect the rumors as soon as possible and take effective measures to deal with their spread.

V. DISCUSSION AND SUMMARY

Rumor propagation can cause serious consequences; thus the study on how to take effective measures to control its spreading is of great practical significance. In this paper, we use anti-rumor transmission to suppress the rumor transmission, and propose a novel SI model to discuss the dynamics of rumor spreading and anti-rumor spreading on homogeneous networks and heterogeneous networks, separately. Some novel results about rumor spreading control are obtained. It is found that the spreading velocity of the rumor apparently decreases with an increase of the immunity of the anti-rumor or the curing rate of infected nodes, but the velocity of the anti-rumor spreading significantly increases. We also show the control effect of rumor propagation can be obviously improved by shortening the time delay between the start of the rumor and the start of the anti-rumor. Our results could shed some new light on the control of rumor spreading. From such understanding, we know that all levels of authorities can effectively control rumor spreading by improving their degree of trust. The results obtained in this paper only considered constant immunity of the anti-rumor, rumor spreading rate, and curing rate of infected nodes; in fact, these parameters may vary from person to person, which is another problem worthy of future study.

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