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Unsteady Viscous Fluid Flow with Porous Medium in the Presence of Radiation and Chemical Reaction

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In this paper we consider the problem of unsteady, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of a porous medium with radiation and chemical reactions. Using perturbation techniques the expressions for the velocity distribution, temperature distribution, concentration distribution, shear stress, Nusselt number, and Sherwood number are obtained. The velocity increases with an increase in K, G_r, and G_m.

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I. NOMENCLATURE

T and C - dimensional temperature and concentration
u and v - velocity in the direction of x and y
x, y - Cartesian coordinates
g - acceleration due to gravity
ρ - density of the fluid
σ - electrical conductivity
β and β' - coefficients of volume expansion due to temperature and concentration
C_p - specific heat at constant pressure
k - thermal conductivity
U - mean velocity
t - time
B_0 - magnetic induction
G_r - Grashof number
G_m - modified Grashof number
K - porosity parameter
P_r - Prandtl number
S_c - Schmidt number
N_R - radiation parameter
ε - small reference parameter $\lll 1$
A - suction parameter
n - a constant exponential index

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D - molar diffusivity
w - conditions at the wall
∞ - denotes free stream conditions, primes denote dimensional quantities

II. INTRODUCTION

A porous medium (or a porous material) is a material containing pores (voids). The porous media is formed by a solid, which contains many holes, with the constraint that the size of holes is relatively small as compared to the size of the solid. Fluid flow through porous media has gained considerable attention in recent years due to its relevance in a wide range of applications such as biological, electronics cooling, thermal insulation engineering, water movement in geothermal reservoirs, heat pipes, underground spreading of chemical waste, nuclear waste repository, geothermal engineering, grain storage, and enhanced recovery of petroleum reservoirs. Radiative heat transfer and multiphase transport processes in porous media, both with and without phase change, have gained extensive attention in recent years. This is due to the wide range of applicability of these research areas in contemporary technology. These applications include, but are not restricted to, areas such as geothermal engineering, building thermal insulation, chemical catalytic reactors, packed cryogenic microsphere insulation, petroleum reservoirs, direct contact heat exchangers, coal combusters, nuclear waste repositories, and heat pipe technology.

Chauhan and Kumar [1] discussed the effects of slip conditions on forced convection and entropy generation in a circular channel occupied by a highly porous medium: the Darcy extended Brinkman-Forchheimer model. Prakash and Ogulu [2] investigated unsteady two dimensional flow of a radiating and chemically reacting MHD fluid with time-dependent suction. Adrian [3] studied the influence of a magnetic field on heat and mass transfer by natural convection from vertical surfaces in porous media considering the Soret and Dufour effects. Singh and Gupta [4] considered the MHD free convective flow of a viscous fluid through a porous medium bounded by an oscillating porous plate in the slip flow regime with mass transfer. Bhargava *et al.* [5] investigated finite element solutions for non-Newtonian pulsatile flow in a non-Darcian porous medium conduit. Cortell [6] studied flow and heat transfer of a fluid through a porous medium over a stretching surface with internal heat generation/absorption and suction/blowing. Krishnambal and Anuradha [7] discussed the effect of radiation on the flow of a viscoelastic fluid and heat transfer in a porous medium over a stretching sheet. Jen and Yan [8] considered developing fluid flow and heat transfer in a channel partially filled with a porous medium. Tong and Subramaniam [9] considered natural convection in rectangular enclosures partially filled with a porous medium. Sharma *et al.* [10] investigated using a numerical study of a two dimensional MHD forward staganation-point flow in the presence of the Hall current. Chang and Chang [11] considered mixed convection in a vertical parallel plate channel partially filled with a porous media of high permeability. Kim [12] studied unsteady convection flow of micro polar fluids past a vertical porous plate embedded in a porous medium.

In the present study we investigated the influence of a porous medium on a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of radiation and chemical reactions.

III. PROBLEM FORMULATION

We consider the problem of unsteady, laminar, boundary layer flow of a viscous, incompressible, electrically conducting fluid along a semi-infinite vertical plate in the presence of a porous medium with radiation and chemical reactions. A time-dependent suction is assumed and the governing equations are as follows:

$$\frac{\partial v'}{\partial y'} = 0, \tag{1}$$

$$\frac{\partial u'}{\partial t'} + v'\frac{\partial u'}{\partial y'} = \nu\frac{\partial^2 u'}{\partial y'^2} + g\beta\left(T - T_\infty\right) + g\beta'\left(C - C_\infty\right) - \frac{\nu}{K'}u', \tag{2}$$

$$\left(\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'}\right) = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} + \frac{1}{\rho C_p}\frac{\partial q_r}{\partial y'}, \tag{3}$$

$$\frac{\partial C}{\partial t'} + v'\frac{\partial C}{\partial y'} = D\frac{\partial^2 C}{\partial y'^2} - k_r'\left(C - C_\infty\right). \tag{4}$$

By using the Rosseland approximation for the radiation, we take

$$q_r = -\frac{4\sigma^*}{3k^*}\frac{\partial T^4}{\partial y'}. \tag{5}$$

T^4 can be expanded in a taylor series about the free stream temperature T_∞, so that after neglecting higher order terms we have

$$T^4 \cong 4T_\infty^3 T - 3T_\infty^4. \tag{6}$$

The energy equation after the substitution of Equations (5) and (6) can now be written as

$$\left(\frac{\partial T}{\partial t'} + v'\frac{\partial T}{\partial y'}\right) = \frac{k}{\rho C_p}\frac{\partial^2 T}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\rho C_p k^*}\frac{\partial^2 T}{\partial y'^2}. \tag{7}$$

From Equation (1) we can see that the suction is a function of time only, so we assume it in the form

$$v' = -V_0\left(1 + \varepsilon A e^{n't'}\right) \tag{8}$$

where $\varepsilon A \ll 1$, the minus sign indicates that the suction is towards the plane. It is now convenient to introduce the following dimensionless parameters:

$$
\begin{aligned}
&u=\frac{u'}{U_0}, \quad y=\frac{U_0}{\nu}y', \quad t=\frac{U_0^2}{\nu}t', \quad n=\frac{\nu n'}{U_0^2}, \quad k_r=\frac{k_r'\nu}{U_0^2}, \quad P_r=\frac{\rho\nu C_p}{k},\\
&S_c=\frac{\nu}{D}, \quad \theta=\frac{T-T_\infty}{T_w-T_\infty}, \quad \phi=\frac{C-C_\infty}{C_w-C_\infty},\\
&G_r=\frac{\nu g\beta\left(T_w-T_\infty\right)}{U_0^3}, \quad G_m=\frac{\nu g\beta'\left(C_w-C_\infty\right)}{U_0^3}, \quad N_R=\frac{16\rho' T_\infty^3}{3k'k}, \quad K=\frac{K'U_0^2}{\nu^2}.
\end{aligned}
$$

On substituting these into Equations (2), (4), and (7), our governing equations become

$$
\frac{\partial u}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial u}{\partial y}=\frac{\partial^2 u}{\partial y^2}+G_r\theta+G_r\phi-\frac{1}{K}u, \tag{9}
$$

$$
\frac{\partial \theta}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial \theta}{\partial y}=\left(\frac{1+N_R}{P_r}\right)\frac{\partial^2 \theta}{\partial y^2}, \tag{10}
$$

$$
\frac{\partial \phi}{\partial t}-\left(1+\varepsilon A e^{nt}\right)\frac{\partial \phi}{\partial y}=\frac{1}{S_c}\frac{\partial^2 \theta}{\partial y^2}-k_r\phi, \tag{11}
$$

with the boundary conditions

$$
\left.
\begin{array}{llll}
u=1, & \theta=1+\varepsilon e^{nt}, & \phi=1+\varepsilon e^{nt} & \text{at y}=0,\\
u\to U_0, & \theta\to 0, & \phi\to 0 & \text{as y}\to\infty.
\end{array}
\right] \tag{12}
$$

IV. ANALYSIS

Since ε is small, so let us assume u, θ, ϕ as

$$
\left.
\begin{array}{l}
u(y,t)=u_0(y)+\varepsilon e^{nt}u_1(y)+0(\varepsilon^2)\\
\theta(y,t)=\theta_0(y)+\varepsilon e^{nt}\theta_1(y)+0(\varepsilon^2)\\
\phi(y,t)=\phi_0(y)+\varepsilon e^{nt}\phi_1(y)+0(\varepsilon^2)
\end{array}
\right]. \tag{13}
$$

We now substitute Eq. (13) into Eqs. (10)–(12), and, equating the non-harmonic terms and harmonic terms neglecting higher order terms in ε, obtain

$$
u_0''+u_0'-\frac{1}{K}u_0=-G_r\theta_0-G_m\phi_0, \tag{14}
$$

$$
u_1''+u_1'-\left(\frac{1}{K}+n\right)u_1=-Au_0'-G_r\theta_1-G_m\phi_1, \tag{15}
$$

$$\theta_0'' + h\theta_0' = 0, \tag{16}$$

$$\theta_1'' + h\theta_1' - nh\theta_1 = -Ah\theta_0', \tag{17}$$

$$\phi_0'' + S_c\phi_0' - k_r S_c\phi_0 = 0, \tag{18}$$

$$\phi_1'' + S_c\phi_1' - (k_r + n)\, S_c\phi_1 = -AS_c\phi_0', \tag{19}$$

where $h = \frac{P_r}{1+N_R}$ and primes denote differentiation with respect to y. Now the boundary conditions are

$$\left.\begin{array}{llllll} u_0 = 1, & u_1 = 0, & \theta_0 = 1 = \theta_1, & & \phi_0 = 1 = \phi_1 & \text{at y} = 0 \\ u_0 \to 0, & u_1 \to 0, & \theta_0 \to 0, & \theta_1 \to 0, & \phi_0 \to 0, \quad \phi_1 \to 0 & \text{as y} \to \infty \end{array}\right] . \tag{20}$$

Integrating (15) to (20), subject to the conditions in Equations (21), we have

$$u_0 = C_3 e^{-m_1 y} - Z_1 e^{-hy} - Z_2 e^{-m_4 y}, \tag{21}$$

$$u_1 = C_4 e^{-m_2 y} + Z_3 e^{-m_1 y} - Z_4 e^{-hy} - Z_5 e^{-m_4 y} - Z_6 e^{-m_3 y} - Z_7 e^{-m_5 y}, \tag{22}$$

$$\theta_0 = e^{-hy}, \tag{23}$$

$$\theta_1 = C_2 e^{-m_3 y} - \frac{Ahe^{-hy}}{n}, \tag{24}$$

$$\phi_0 = e^{-m_4 y}, \tag{25}$$

$$\phi_1 = C_1 e^{-m_5 y} + \frac{AS_c m_4 e^{-m_4 y}}{m_4^2 + S_c m_4 - (k_r + n)\, S_c}, \tag{26}$$

where

$$m_1 = \frac{1 + \sqrt{1 + 4\frac{1}{K}}}{2}, \quad m_2 = \frac{1 + \sqrt{1 + 4\left(\frac{1}{K} + n\right)}}{2}, \quad m_3 = \frac{h + \sqrt{h^2 + 4nh}}{2},$$

$$m_4 = \frac{S_c + \sqrt{S_c^2 + 4S_c k_r}}{2}, \quad m_5 = \frac{S_c + \sqrt{S_c^2 + 4S_c\,(k_r + n)}}{2},$$

$$C_1 = 1 - \frac{AS_c m_4}{m_4^2 + S_c m_4 - (k_r + n)S_c}, \quad C_2 = 1 - \frac{Ahe^{-hy}}{n},$$

$$Z_1 = \frac{G_r}{h^2 - h - \frac{1}{K}}, \quad Z_2 = \frac{G_m}{m_4^2 - m_4 - \frac{1}{K}}, \quad C_3 = 1 + Z_1 + Z_2, \quad Z_3 = \frac{Am_1 C_1}{m_1^2 - m_1 - \left(\frac{1}{K} + n\right)},$$

$$Z_4 = \frac{\left(AhZ_1 + \frac{Ah}{n}\right)}{h^2 - h - \left(\frac{1}{K} + n\right)}, \; Z_5 = \left[\left(Am_4 Z_2 + \frac{AS_c m_4}{m_4^2 - m_4 - (k_r + n)\, S_c}\right)\frac{1}{m_4^2 - m_4 - \left(\frac{1}{K} + n\right)}\right],$$

$$Z_6 = \frac{C_3 m_3}{m_3^2 - m_3 - \left(\frac{1}{K} + n\right)}, \quad Z_7 = \frac{G_m C_1}{m_5^2 - m_5 - \left(\frac{1}{K} + n\right)}, \quad C_4 = -Z_3 + Z_4 + Z_5 + Z_6 + Z_7.$$

Such that the velocity, temperature, and concentration distributions can be expressed as

$$
\begin{aligned}
u(y,t) \;=\; & C_3 e^{-m_1 y} - Z_1 e^{-hy} - Z_2 e^{-m_4 y} + \varepsilon e^{nt}\Big\{ C_4 e^{-m_2 y} + Z_3 e^{-m_1 y} \\
& - Z_4 e^{-hy} - Z_5 e^{-m_4 y} - Z_6 e^{-m_3 y} - Z_7 e^{-m_5 y} \Big\},
\end{aligned}
\tag{27}
$$

$$
\theta(y,t) = e^{-hy} + \varepsilon e^{nt}\left\{ C_2 e^{-m_3 y} - \frac{Ahe^{-hy}}{n} \right\},
\tag{28}
$$

$$
\phi(y,t) = e^{-m_4 y} + \varepsilon e^{nt}\left\{ C_1 e^{-m_5 y} + \frac{AS_c m_4 e^{-m_4 y}}{m_4^2 + S_c m_4 - (k_r + n)\, S_c} \right\}.
\tag{29}
$$

From the technological point of view the most important parameters of problems of this nature are the shear stress, Nusselt number, and Sherwood number, which can be defined respectively in non-dimensional form as:

$$
\tau = \left.\frac{\partial u}{\partial y}\right|_{y=0} = -m_1 C_3 + hZ_1 + m_4 Z_2 + \varepsilon e^{nt}\left\{ -m_2 C_4 - m_1 Z_3 + hZ_4 + m_4 Z_5 + m_3 Z_6 + m_5 Z_7 \right\},
\tag{30}
$$

$$
\frac{N_u}{Re_x} = \left.\frac{\partial \theta}{\partial y}\right|_{y=0} = -h + \varepsilon e^{nt}\left\{ -m_3 C_2 + \frac{Ah^2}{n} \right\},
\tag{31}
$$

$$
\frac{Sh}{Re_x} = \left.\frac{\partial \phi}{\partial y}\right|_{y=0} = -m_4 - \varepsilon e^{nt}\left\{ m_5 C_1 + \frac{AS_c m_4^2}{m_4^2 + S_c m_4 - (k_r + n)\, S_c} \right\}.
\tag{32}
$$

V. RESULT AND DISCUSSION

The velocity distribution u is plotted in Fig. 1 having Graph-1 to 4 at $n = 0.1$, $t = 0.1$, $P_r = 0.71$, $N_R = 0.5$, $S_c = 0.2$, $k_r = 0.5$, $\varepsilon = 0.001$, and $A = 0.3$, with different values of K, G_r, and G^m.

It is observed from Figure 1 that all the velocity graphs are increasing sharply up to $y = 1.2$, after that the velocity in each graph begins to decrease and tends to zero with an increase in y. It is also observed from Figure 1 that the velocity increases with the increase in K, G_r, and G_m.

Now if we consider the significance of the porous media then one can easily observe that when $1/K \downarrow$, then $v \uparrow$. i.e., as the porosity factor decreases ($\downarrow$), the velocity factor increases ($\uparrow$) [13].

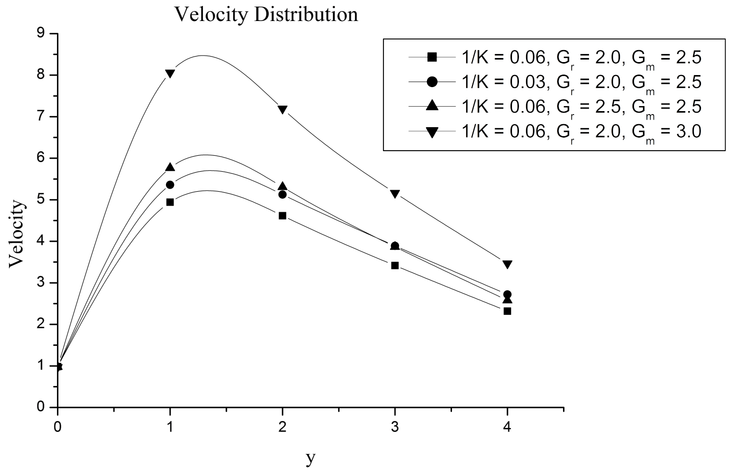

FIG. 1

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