Exact Space-Time Gauge Symmetry of Gravity, Its Couplings and Approximate Internal Symmetries in a Total-Unified Model

Jong-Ping Hsu*

*Department of Physics, University of Massachusetts Dartmouth,
North Dartmouth, MA 02747-2300, USA*
(Received May 28, 2013)

The gravitational field is the manifestation of space-time translational (T_4) gauge symmetry, which enables the gravitational interaction to be unified with the strong and the electroweak interactions. Such a total-unified model is based on a generalized Yang-Mills framework in flat space-time. Following the idea of Glashow-Salam-Ward-Weinberg, we gauge the groups $T_4 \times (SU_3)_{color} \times SU_2 \times U_1 \times U_{1b}$ on equal-footing, so that we have the total-unified gauge covariant derivative $\delta_\mu = \partial_\mu - ig\phi_\mu^\nu p_\nu + ig_s G_\mu^a(\lambda^a/2) + if W_\mu^k t^k + if'U_\mu t_o + ig_b B_\mu$. The generators of the external T_4 group have the representation $p_\mu = i\partial_\mu$, which differs from the other generators of all the other internal groups, which have constant matrix representations. Consequently, the total-unified model leads to the following new results: (a) All internal $(SU_3)_{color}$, SU_2, U_1, and baryonic U_{1b} gauge symmetries have extremely small violations due to the gravitational interaction. (b) The T_4 gauge symmetry remains exact and dictates the universal coupling of gravitons. (c) Such a gravitational violation of internal gauge symmetries leads to modified eikonal and Hamilton-Jacobi type equations, which are obtained in the geometric-optics limit and involve effective Riemann metric tensors. (d) The rules for Feynman diagrams involving new couplings of the photon-graviton, gluon-graviton, and quark-graviton are obtained.

I. INTRODUCTION

In previous papers, we have discussed the electroweak and strong sectors of a unified model involving Yang-Mills gravity with the space-time translational gauge group T_4 based on flat space-time [1, 2]. We showed that an experimentally consistent model, which unifies Yang-Mills gravity and other forces in nature, can be formulated on the basis of a gauge invariant action under local space-time translations. For the motion of classical objects, eikonal (or Hamilton-Jacobi type) equations are derived from T_4 gauge invariant wave equations of bosons and fermions in the geometric-optics limit, and they contain 'effective Riemannian metric tensors.' The emergence of 'effective curved space-time with Riemannian metric tensors' in (and only in) the geometric-optics limit of wave equations is essential for Yang-Mills gravity to be formulated in flat space-time and to be consistent with experiments. This approach brings gravity back to the arena of gauge field theory and

---

*Electronic address: jhsu@umassd.edu

quantum mechanics in flat space-time [3, 4].

In this paper, we step back and look at the big picture of all possible interaction forces in nature and explore the most general implications of a total-unified model based on a generalized Yang-Mills framework, which can accommodate both internal and external gauge groups in flat space-time. We know that the electromagnetic and weak interactions have been unified based on a gauge invariant action with the internal gauge groups $SU_2 \times U_1$ [5]. And the strong interaction can also be described a gauge field theory, i.e., by quantum chromodynamics with internal color SU_3 groups. Apart from these well-known forces, there may be other forces of gauge fields related to, say, accelerated cosmic expansion [6, 7] and quark confinement [8]. For example, the cosmic repulsive force due to a gauge field works if and only if there is not much anti-matter in the observable universe. Moreover, such a force should be much weaker than gravity within the solar system and our galaxy, but its repulsive force between two galaxies should become stronger than the gravitational force. We have discussed a modified Lee-Yang U_{1b} baryonic gauge field, which satisfies a gauge invariant fourth-order differential equation and has a linear static potential [8]. Such a modified Lee-Yang model predicts a constant acceleration for the expansion of the universe, provided the observable universe is dominated by baryon matter. Thus, it appears to suggest that all known interactions can be understood on the basis of gauge symmetries and, hence, can be unified, similar to the unification of the electromagnetic and weak interactions.

At the first glance of the total-unified model by gauging the groups, $T_4 \times (SU_3)_{color} \times SU_2 \ldots$, one may get the impression that, in the total covariant derivative, various gauge fields with their coupling constants and group generators are simply juxtaposed, without any true integration. However, such a total-unified model without any true integration is probably all what one can do in the physical world as we know it. The reasons are as follows: (i) Mathematically, it appears to be very unlikely that one can further unify these external and internal gauge symmetries in one single Lie group. (ii) The usual asymptotic freedom and running coupling constants in non-Abelian gauge theories are no longer true in the total-unified model, which is not renormalizable by power counting because the gravitational interaction has the dimensional coupling constant g. The asymptotic freedom of non-Abelian gauge theories holds only when one ignores gravity. But no physical particle in nature can escape from the gravitational interaction. Therefore, it is very unlikely that all the different interactions can be unified at extremely high energies with one single coupling constant. Nevertheless, the total-unified model involving Yang-Mills gravity [9, 10] is interesting, because it leads to a new understanding of physics that gauge symmetries dictate all particle interactions [11] and can make new predictions to be tested experimentally.

II. TOTAL UNIFICATION OF ALL INTERACTIONS AND GAUGE CURVATURES

Based on previous discussions of Yang-Mills gravity [9], a total-unified group G_{tu} in flat space-time, consistent with experimentally established conservation laws, appears to

be [1, 2]

$$G_{tu} = T_4 \times [SU_3]_{color} \times \{SU_2 \times U_1\} \times U_{1b}, \tag{1}$$

where, if one wishes, the leptonic conservation law can also be included in (1) with an additional leptonic U_1 group. The mathematical structure of the leptonic gauge field is identical to that of the baryonic gauge field, so we shall not consider it here. In the total-unified model, the total gauge covariant derivative δ_μ in general frames (inertial and non-inertial) of reference [4, 9, 12] (with the Poincaré metric tensor $P_{\mu\nu}$ and the associated covariant derivative D_μ) is assumed to take the form (c=$\hbar$=1)

$$\delta_\mu = D_\mu + g\phi_\mu^\nu D_\nu + ig_s G_\mu^a \frac{\lambda^a}{2} + if W_\mu^m t^m + if' U_\mu t_o + \frac{ig_b}{3} B_\mu, \tag{2}$$

where t^m, $m = 1, 2, 3$, are SU_2 generators in general and t_o (weak hypercharge) is the $U(1)$ generator [5]. In the limit of zero acceleration, all non-inertial frames reduce to inertial frames [4], and the T_4 generators have the familiar representation $p_\mu = i\partial_\mu$. The T_4 generator differs from the generators of all other internal gauge groups, which have constant matrix representations.

There are five independent gauge coupling constants: g_s, g_b, f, and f' are dimensionless, and g has the dimension of length because the T_4 generator $p_\mu = i\partial/\partial x^\mu$ has the dimension of (1/length) in natural units. A basic difference between gravity and, say, electrodynamics is as follows: The electron and the positron have the same attractive gravitational force characterized by one coupling constant g. However, they have different electromagnetic forces (i.e., attractive and repulsive), which are characterized by two dimensionless coupling constants $\pm e$. This difference stems from the absence of an 'i' in the T_4 term involving $-ig\phi_\mu^\nu p_\nu = g\phi_\mu^\nu \partial_\nu$ in (2). This property also indicates that gravity described by $\phi_{\mu\nu}$ is the manifestation of the translational gauge symmetry in flat space-time.

The gauge curvature associated with each individual group can be obtained by calculating the commutator of the covariant derivative defined in (2). The commutator of the total-unified gauge covariant derivative δ_μ defined in (2) is given by

$$[\delta_\mu, \delta_\nu] = C_{\mu\nu\sigma} D^\sigma + ig_s \mathbf{G}_{\mu\nu}^a \frac{\lambda^a}{2} + if W_{\mu\nu}^k t^k + if' U_{\mu\nu} t_o + i\frac{g_b}{3} B_{\mu\nu}, \tag{3}$$

where the T_4 gauge curvature $C_{\mu\nu\sigma}$ is given by

$$C_{\mu\nu\alpha} = J_{\mu\sigma} D^\sigma J_{\nu\alpha} - J_{\nu\sigma} D^\sigma J_{\mu\alpha} = \Delta_\mu J_{\nu\alpha} - \Delta_\nu J_{\mu\alpha} = -C_{\nu\mu\alpha}, \tag{4}$$

$$\Delta_\mu = J_{\mu\nu} D^\nu, \quad J_{\mu\nu} = P_{\mu\nu} + g\phi_{\mu\nu}, \quad \phi_{\mu\nu} = \phi_{\nu\mu} = P_{\nu\lambda}\phi_\mu^\lambda,$$

where Δ_μ is the T_4 gauge covariant derivative in arbitrary coordinates with metric tensors $P_{\mu\nu}$ within flat space-time. The modified color SU_3 curvature $\mathbf{G}_{\mu\nu}^a$ is given by

$$\mathbf{G}_{\mu\nu}^a = J_\mu^\sigma D_\sigma G_\nu^a - J_\nu^\sigma D_\sigma G_\mu^a - g_s f^{abc} G_\mu^b G_\nu^c, \tag{5}$$

where $a, b, c = 1, 2, ...8$, and the structure constants f_{abc} are completely anti-symmetric [5]. The modified $SU(2) \times U(1)$ gauge curvatures $W_{\mu\nu i}$ and $U_{\mu\nu}$, in the presence of the gravitational gauge potential, are given by

$$W_{\mu\nu}^i = J_\mu^\sigma D_\sigma W_\nu^i - J_\nu^\sigma D_\sigma W_\mu^i - f\epsilon^{ijk} W_\mu^j W_\nu^k, \tag{6}$$

$$U_{\mu\nu} = J_\mu^\sigma D_\sigma U_\nu - J_\nu^\sigma D_\sigma U_\mu, \quad J_\mu^\sigma = \delta_\mu^\sigma + g\phi_\mu^\sigma, \tag{7}$$

where $i, j, k = 1, 2, 3$ for the SU(2) group. The modified U_{1b} gauge curvature $B^{\mu\nu}$ associated with the baryon charge is found to be

$$B_{\mu\nu} = J_\mu^\sigma D_\sigma B_\nu - J_\nu^\sigma D_\sigma B_\mu, \tag{8}$$

in the presence of gravity.

Let us consider the physical implications of the total-unified model due to the presence of gravity. For simplicity, we use inertial frames with the metric tensor $P_{\mu\nu} = \eta_{\mu\nu}$ and $D_\mu = \partial_\mu$. We first note that all these modified gauge curvatures (5)–(8) reduced to the usual gauge curvatures in quantum chromodynamics, electroweak theory, and U_{1b} gauge theory in the limit

$$g \to 0, \text{ or } J_\mu^\sigma \to \delta_\mu^\sigma. \tag{9}$$

In particular, since the SU_2 and U_1 gauge transformations are the same as those in the Weinberg-Salam theory [5], the local gauge transformations for $W_\mu^i(x)$ and $W_{\mu\nu}^i(x)$ are given by

$$W_\mu^{'i}(x) = W_\mu^i(x) + \frac{1}{f}\partial_\mu\omega^i(x) + C^{ijk}\,\omega^j(x)W_\mu^k(x), \tag{10}$$

$$W_{\mu\nu}^{'i}(x) = W_{\mu\nu}^i(x) + C^{ijk}\,\omega^j(x)W_{\mu\nu}^k(x), \text{ for } g = 0, \tag{11}$$

where

$$W_{\mu\nu}^i = \partial_\mu W_\nu^i - \partial_\nu W_\mu^i - f\epsilon^{ijk} W_\mu^j W_\nu^k \tag{12}$$

for SU_2 gauge curvature [5].

III. VERY SMALL VIOLATIONS OF U_1, SU_2, AND SU_3 GAUGE SYMMETRIES DUE TO GRAVITY

First, we note that, under the U_{1b} gauge transformation, we have

$$B_\mu'(x) = B_\mu(x) + \partial_\mu\Lambda(x),$$

$$B'_{\mu\nu}(x) = B_{\mu\nu}(x), \quad \partial_\lambda B'_{\mu\nu}(x) = \partial_\lambda B_{\mu\nu}(x), \text{ for } g = 0. \tag{13}$$

Thus, in the absence of gravity, i.e., $g = 0$, both $B_{\mu\nu}(x)$ and $\partial_\lambda B_{\mu\nu}(x)$ are gauge invariant. Both the quadratic forms $L_1 = -(1/4)B_{\mu\nu}(x)B^{\mu\nu}(x)$ and $L_2 = (-L_s^2/4)\partial_\lambda B_{\mu\nu}(x)\partial^\lambda B^{\mu\nu}(x)$ can be used as the invariant Lagrangian of the baryonic gauge field. The Lagrangian L_1 was originally discussed by Lee and Yang to understand conservation of the baryon number [6]. Such a baryonic field is identical to the electromagnetic field, except that the original Lee-Yang force for baryons is much weaker than the gravitational force, so that it has no other observable effects in physics. We shall assume the modified gauge invariant Lagrangian L_2 for the baryonic gauge field, because it leads to a linear static potential for the baryonic gauge field, and it can have cosmological implications for the accelerated expansion of the universe involving baryon matter.

However, in the presence of gravity, the gauge curvature $B_{\mu\nu}$ is modified to have the form (8) when $g \neq 0$. Under the U_{1b} gauge transformation, instead of (13), we obtain

$$B'_{\mu\nu} = B_{\mu\nu} + J_\mu^\sigma \partial_\sigma \partial_\nu \Lambda(x) - J_\nu^\sigma \partial_\sigma \partial_\mu \Lambda(x) \neq B_{\mu\nu}. \tag{14}$$

Thus, the modified gauge curvature $B_{\mu\nu}$ in (8) has a very small violation of U_{1b} gauge invariance due to gravity. Similar non-gauge invariance happens to the SU_2 and SU_3 gauge curvature. As a result, there are very small violations of the internal gauge symmetries in all the gauge curvatures (5)–(8) due to gravity.

Such an extremely small violation of internal gauge symmetries due to gravity is intimately related to the property that all gauge fields and generators associated with the groups T_4, SU_3, SU_2, and U_1 are on equal footing in the total gauge covariant derivative δ_μ in (2), just like that in the $SU_2 \times U_1$ electroweak theory. This 'equal footing' of all gauge fields in (2) leads to the simplest coupling between gauge fields, and should be considered as a basic postulate of total-unification of external and internal gauge symmetries in the model.

The gauge transformations of the internal groups SU_2 and U_1, etc. are well-known [5]. Some of them are discussed in Equations (10)–(13). However, the T_4 gauge transformations are more involved because the (infinitesimal) local and arbitrary space-time translations,

$$x'^\mu = x^\mu + \Lambda^\mu(x), \tag{15}$$

are, simultaneously, also the general coordinate transformations with arbitrary infinitesimal function $\Lambda^\mu(x)$. Thus, the T_4 gauge transformations for, say, vector and tensor fields are formally the same as the Lie variation of tensors. For the rest of this section, we shall consider arbitrary coordinates with the Poincare metric tensors. Under the infinitesimal T_4 gauge transformations in flat space-time [9], we have

$$S^\$(x) = S(x) - \Lambda^\sigma(x)\partial_\sigma S(x),$$

$$\Phi_\mu^\$(x) = \Phi_\mu(x) - \Lambda^\sigma(x)\partial_\sigma \Phi_\mu(x) - \Phi_\sigma(x)\partial_\mu \Lambda^\sigma(x), \tag{16}$$

$$\phi^{\$}_{\mu\nu}(x) = \phi_{\mu\nu}(x) - \Lambda^{\sigma}(x)\partial_{\sigma}\phi_{\mu\nu}(x) - \phi_{\mu\sigma}(x)\partial_{\nu}\Lambda^{\sigma}(x) - \phi_{\sigma\nu}(x)\partial_{\mu}\Lambda^{\sigma}(x),$$

and so on, where $\Lambda_{\mu}(x)$ denotes infinitesimal and arbitrary vector functions. Furthermore, we assume that the tensor gauge field $\phi^{\nu}_{\mu}(x)$ is an electrically neutral field and an iso-scalar, so that the T_4 gauge potential ϕ^{ν}_{μ} does not transform under SU_3, SU_2, and U_1 gauge transformations.

To see the exact invariance of the total-unified action functional $S_{tu} = \int L_{tu}\sqrt{-P}d^4x$, where $P = detP_{\mu\nu}$, in a general reference frame (inertial and non-inertial) with arbitrary coordinates associated with Poincaré metric tensor $P_{\mu\nu}$ [9, 12], we first note that the total-unified Lagrangian L_{tu} is a scalar function by construction, with the gauge curvatures given by Equations (4)–(8) in arbitrary coordinates. Thus it transforms the same as a scalar $S(x)$ in (16). One can show that $\sqrt{-P}$ transforms as follows:

$$\sqrt{-P^{\$}} = [(1 - \Lambda^{\sigma}\partial_{\sigma})\sqrt{-P}](1 - \partial_{\alpha}\Lambda^{\alpha}), \tag{17}$$

where we have used the last relation in (16) [1]. We have

$$S^{\$}_{tu} = S_{tu} - \int [\partial_{\lambda}(\Lambda^{\lambda}L_{tu}\sqrt{-P})d^4x = S_{tu}, \tag{18}$$

where the divergent term inside the four-dimensional volume integral does not contribute, because it can be transformed into the integral of a vector over a hypersurface on the boundaries of the volume of integral where fields and their variation vanish. Therefore, the total-unified action S_{tu} is exactly invariant under the infinitesimal T_4 gauge transformations (16). In contrast, all other internal gauge symmetries have a very small non-invariance in their gauge curvatures due to the presence of gravity in the total-unified model, as we have discussed previously.

IV. FEYNMAN-DYSON RULES IN THE STRONG SECTOR (CHROMO-GRVITY)

The rules for Feynman diagrams without involving gravitons in the usual electroweak theory and quantum chromodynamics (QCD) are well-known [5]. The rules in pure Yang-Mills gravity were obtained and discussed [13].

In the following discussions, we shall consider the total-unified model in inertial frames for simplicity. Within the generalized Yang-Mills framework in inertial frames with $P_{\mu\nu} = \eta_{\mu\nu}$, all covariant derivatives D_{μ} in (2)–(8) are replaced by ∂_{μ}. As usual, the total-unified Lagrangian is assumed to be quadratic in gauge curvatures,

$$L_{tu} = L_{\phi} + L_G - \frac{1}{4}(W^{\mu\nu k}W^k_{\mu\nu} + U^{\mu\nu}U_{\mu\nu})$$

$$-\frac{1}{4}L_s^2\partial_{\lambda}B_{\mu\nu}\partial^{\lambda}B^{\mu\nu} + L_{matter} + L_H, \tag{19}$$

$$L_\phi = \frac{1}{4g^2} \left(C_{\mu\nu\alpha} C^{\mu\nu\alpha} - 2C_{\mu\alpha}{}^\alpha C^{\mu\beta}{}_\beta \right), \tag{20}$$

$$L_G = -\frac{1}{4} \mathbf{G}^{a\mu\nu} \mathbf{G}^a_{\mu\nu}, \tag{21}$$

where the Lagrangian L_H in (18) involves Higgs scalar doublets and their couplings. The Lagrangian L_{matter} will be discussed later in Section VI. It contains spinor quarks with three colors and six flavors and their interactions with gauge fields [1, 2, 5].

We now use the total-unified Lagrangians (19) to write down the Feynman-Dyson rules for the interactions of gravitons, gluons, and quarks. Although these are conventionally referred to as the Feynman rules [14], we choose to refer to them as the Feynman-Dyson rules in this total unified model, because the intuitive Feynman rules for calculating the invariant amplitude (a Lorentz scalar) are inadequate for higher-orders. It was Dyson's rigorous derivation and elaboration of the rules that allowed their application to complicated and higher-order interactions in quantum Yang-Mills gravity and in Kinoshita's higher-order corrections to the magnetic moment of the electron in quantum electrodynamics [15, 16].

Let us consider the general propagator for the graviton corresponding to the general gauge-fixing terms:

$$L_{\xi\zeta} = \frac{\xi}{2g^2} \left[\partial^\mu J_{\mu\alpha} - \frac{\zeta}{2} \partial_\alpha J^\lambda_\lambda \right] \left[\partial^\nu J_{\nu\beta} - \frac{\zeta}{2} \partial_\beta J^\lambda_\lambda \right] \eta^{\alpha\beta}. \tag{22}$$

From (19) and (22) with $\zeta = 1$ and $\xi = 2$, we obtain the following simple propagator for the graviton $G_{\alpha\beta\rho\sigma}$:

$$G_{\alpha\beta\rho\sigma} = \frac{-i}{2k^2} \left[(\eta_{\alpha\beta}\eta_{\rho\sigma} - \eta_{\rho\alpha}\eta_{\sigma\beta} - \eta_{\rho\beta}\eta_{\sigma\alpha}) \right], \tag{23}$$

where the usual prescription of $i\epsilon$ for the Feynman propagator (23) is understood. In this paper, it should always be understood as being done, even when the $i\epsilon$ is not explicitly included in the propagators. The propagator (23) has been discussed by DeWitt, and the gauge condition specified by (22) with $\zeta = 1$ and $\xi = 2$ may be called the DeWitt gauge [17]. The result (23) is consistent with that obtained in previous works by Leibbrandt, Capper, and others in general relativity [18–20]. As an aside, the overall factor of (1/2) in the graviton propagators (23) is necessary for Yang-Mills gravity to be consistent with its T_4 gauge identity—a generalized Ward-Takahasi identity for the Abelian group T_4 with ghosts, and to be consistent with the Slavnov-Taylor identity [21].

We now consider the Feynman-Dyson rules for graviton interactions corresponding to iL_G in (21). The gluon and graviton momenta in the Feynman-Dyson rules are in-coming to the vertices. Suppose the 3-vertex for gluon-gluon-graviton is denoted by $(G^{\mu a}(p)G^{\nu b}(q)\phi^{\alpha\beta}(k))$. The gluon-gluon-graviton 3-vertex is

$$ig\delta^{ab} \left(p^\alpha q^\beta \eta^{\mu\nu} + p^\beta q^\alpha \eta^{\mu\nu} - p^\nu q^\beta \eta^{\mu\alpha} - p^\beta q^\mu \eta^{\alpha\nu} \right)_{(\alpha\beta)}. \tag{24}$$

We use the symbol $(\alpha\beta)$ to denote that a symmetrization is to be performed on the index pair $(\alpha\beta)$.

The 4-vertex for gluon-graviton coupling, $(G^{\mu a}(p)G^{\nu b}(q)G^{\sigma c}(k)\phi^{\alpha\beta})$, is given by

$$igg_s f^{abc}\left(p^\beta\eta^{\mu\sigma}\eta^{\nu\alpha} - p^\beta\eta^{\alpha\sigma}\eta^{\mu\nu} - q^\beta\eta^{\mu\alpha}\eta^{\nu\sigma} + q^\beta\eta^{\alpha\sigma}\eta^{\mu\nu} + k^\beta\eta^{\mu\alpha}\eta^{\nu\sigma} - k^\beta\eta^{\mu\sigma}\eta^{\alpha\nu}\right)_{(\alpha\beta)}, \tag{25}$$

where f^{abc} is the completely anti-symmetric structure constant of the color SU_3 group. There is another 4-vertex for 2-gluon and 2-graviton coupling, denoted by $(G^{\mu a}(p)G^{\nu b}(q)\phi^{\alpha\beta}\phi^{\gamma\delta})$, which is given by

$$ig^2\delta^{ab}\left(p^\beta q^\delta\eta^{\mu\nu}\eta^{\alpha\gamma} + p^\delta q^\beta\eta^{\mu\nu}\eta^{\alpha\gamma} - p^\beta q^\delta\eta^{\mu\gamma}\eta^{\nu\alpha} - p^\delta q^\beta\eta^{\alpha\mu}\eta^{\nu\gamma}\right)_{(\alpha\beta)(\gamma\delta)}, \tag{26}$$

where both index pairs $(\alpha\beta)$ and $(\gamma\delta)$ are to be symmetrized.

V. NEW PHOTON-GRAVITON COUPLINGS

In this section, let us consider the Feynman-Dyson rules for new graviton-photon interactions in the QED sector implied by the matter Lagrangian L_{matter} in (15). The Lagrangian L_γ with new graviton-photon coupling can be derived from the gauge boson terms involving $W^{\mu\nu a}$ and $U^{\mu\nu}$ in (19) and is given by [1, 5]

$$L_\gamma = \frac{-1}{4}F_{\mu\nu}F^{\mu\nu}, \tag{27}$$

$$F_{\mu\nu} = J_{\mu\sigma}\partial^\sigma A_\nu - J_{\nu\sigma}\partial^\sigma A_\mu, \quad J_{\mu\sigma} = \eta_{\mu\sigma} + g\phi_{\mu\sigma},$$

where A_μ is the electromagnetic potential and $F_{\mu\nu}$ is the modified electromagnetic field strength in the presence of gravity. The 3-vertex for the photon-graviton coupling is denoted by $(A^\mu(p)A^\nu(q)\phi^{\alpha\beta}(k))$ and is given by

$$ig\left(p^\alpha q^\beta\eta^{\mu\nu} + p^\beta q^\alpha\eta^{\mu\nu} - p^\nu q^\beta\eta^{\mu\alpha} - p^\beta q^\mu\eta^{\alpha\nu}\right)_{(\alpha\beta)}. \tag{28}$$

We use the symbol $(\alpha\beta)$ to denote that a symmetrization is to be performed on the index pair $(\alpha\beta)$. All photon and graviton momenta are incoming to the vertex.

The 4-vertex for the photon-graviton coupling, $(A^\mu(p)A^\nu(q)A^\sigma(k)\phi^{\alpha\beta})$, is given by

$$ig^2\left(p^\beta q^\delta\eta^{\mu\nu}\eta^{\alpha\gamma} + p^\delta q^\beta\eta^{\mu\nu}\eta^{\alpha\gamma} - p^\beta q^\delta\eta^{\mu\gamma}\eta\nu\alpha + p^\delta q^\beta\eta^{\alpha\mu}\eta^{\nu\gamma}\right)_{(\alpha\beta)(\gamma\delta)}, \tag{29}$$

where both index pairs $(\alpha\beta)$ and $(\gamma\delta)$ are to be symmetrized.

VI. THE MATTER LAGRANGIAN FOR QUARKS, LEPTONS WITH GRAVITONS

Let us now consider the interactions of gravitons with quarks and leptons. As usual, we denote left-handed and right-handed fermions (weak interaction or gauge eigenstates) as follows:

$$L : \begin{pmatrix} \nu_e \\ e \end{pmatrix}_L \quad \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L \quad \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \cdots,$$

$$\overline{R} : \overline{e_R} \;\; \overline{\mu_R} \;\; \overline{\tau_R} \cdots,$$

$$Q_L : \begin{pmatrix} u \\ d \end{pmatrix}_L \quad \begin{pmatrix} c \\ s \end{pmatrix}_L \quad \begin{pmatrix} t \\ b \end{pmatrix}_L \cdots,$$

$$\overline{Q_R} : \overline{u_R} \;\; \overline{c_R} \;\; \overline{t_R} \cdots,$$

$$\overline{Q'_R} : \overline{d_R} \;\; \overline{s_R} \;\; \overline{b_R} \cdots .$$

By definition, the SU_3 generator $g_s \lambda_3 / 2$ has eigenvalues: $+g_s/2$ for the red quark doublet and the green antiquark singlets; $-g_s/2$ for the green quark doublet and the red antiquark singlets; and zero for all other left-handed fermions [22, 23].

In this total-unified model in inertial frames, the action S_{tot} is assumed to be

$$S_{tu} = \int d^4x \; L_{tu}, \tag{30}$$

where L_{tu} is given by (19), in which the matter Lagrangian L_{matter} involving quarks and leptons is given by

$$L_{matter} = \overline{L} \; i\gamma^\mu \delta_\mu L + \overline{R} \; i\gamma^\mu \delta_\mu R + \overline{Q_L} \; i\gamma^\mu \delta_\mu Q_L + \overline{Q_R} \; i\gamma^\mu \delta_\mu Q_R + \overline{Q'_R} \; i\gamma^\mu \delta_\mu Q'_R, \tag{31}$$

where we have suppressed the family index. Let us concentrate on the coupling of gravitons and fermions in the total-unified model. The total gauge covariant derivatives δ_μ applied to fermion fields give the results,

$$(\delta_\mu Q_L)_{fm} = \left[\delta_{fh}\delta_{mj}(\partial_\mu + g\phi_\mu^\nu \partial_\nu + if'U_\mu t^o + ig_b B_\mu) + \delta_{fh}(ifW_\mu^k t^k)_{mj} + (ig_s G_\mu^a \frac{\lambda^a}{2})_{fh}\delta_{mj} \right] (Q_L)_{hj}, \tag{32}$$

$$(\delta_\mu Q_R)_f = \left[\delta_{fh}(\partial_\mu + g\phi_\mu^\nu \partial_\nu + if'U_\mu t^o + ig_b B_\mu) + (ig_s G_\mu^a \frac{\lambda^a}{2})_{fh} \right] (Q_R)_h, \tag{33}$$

$$(\delta_\mu Q'_R)_f = \left[\delta_{fh}(\partial_\mu + g\phi_\mu^\nu \partial_\nu + if'U_\mu t^o + ig_b B_\mu) + (ig_s G_\mu^a \frac{\lambda^a}{2})_{fh} \right] (Q'_R)_h, \tag{34}$$

$$(\delta_\mu L)_m = \left[\delta_{mj}(\partial_\mu + g\phi_\mu^\nu \partial_\nu + if'U_\mu t^o + ig_b B_\mu) + (ifW_\mu^k t^k)_{mj} \right] (L)_j, \tag{35}$$

$$\delta_\mu R = \left(\partial_\mu + g\phi_\mu^\nu \partial_\nu + if'U_\mu t^o + ig_b B_\mu \right) R, \tag{36}$$

where the eight $[SU_3]_{color}$ generators are represented by $\lambda^a/2$, a=1,2,...8, (i.e., the Gell-Mann matrices) with the normalization $Tr(\lambda^a \lambda^b) = 2\delta^{ab}$, t^k are the SU_2 generators in general (while their fundamental representation is denoted by $\tau^k/2$), and t^o is the hypercharge generator of the U_1 group [5, 23].

In the absence of gravity and baryon field (i.e., $g = g_b = 0$), the matter Lagrangian (31) reduces to the corresponding results in the Weinberg-Salam theory and quantum chromodynamics [24, 25]. In general, it is lengthy to write down the results in terms of physical leptons and quarks. Let us consider the Feynman-Dyson rules for the couplings of graviton-leptons and graviton-quarks in the matter Lagrangian (31). For example, the Feynman-Dyson rules for the quark-graviton 3-vertex $(\bar{q}(k)q(p)\phi_{\mu\nu})$, is given by

$$\frac{i}{2}g\gamma_\mu(p_\nu + k_\nu)_{(\mu\nu)}, \tag{37}$$

where $q(p)$ may be considered as an annihilation operator of a quark with momentum p_ν, and $\bar{q}(k)$ may be a creation operator of a quark with the momentum k_ν.

The electron-graviton 3-vertex with $(\bar{e}(q)e(p)\phi_{\mu\nu}(k))$ has the same form as (37),

$$\frac{i}{2}g\gamma_\mu(p_\nu + q_\nu)_{(\mu\nu)}, \tag{38}$$

where the electron in the Lagrangian has been symmetrized [13]. Note that $e(p)$ may be considered as an annihilation operator of an electron with momentum p_ν, and $\bar{e}(q)$ is considered as a creation operator of an electron with the momentum q_ν.

We may remark that the total Lagrangian (19) is gauge invariant under the $SU_3 \times SU_2 \times U_1 \times U_{b1}$ only in the limit $g \to 0$, i.e., the absence of gravitational interaction. In general, the total action (30) is exactly gauge invariant under the local space-time translational gauge group. All internal gauge symmetries such as SU_3, SU_2, and U_1 are only approximate in the presence of the gravitational interaction. The physical effects of these violations are in general too small to be detected in laboratory, unless a macroscopic object such as the sun or a galaxy is involved, as we shall see in the next sections.

VII. EXPERIMENTAL TESTS OF LEE-YANG FORCE FOR ACCELERATED EXPANSION OF THE UNIVERSE

An interesting question is: Can the accelerated expansion of the universe be understood within the total-unified model? This great enigma challenges the power of the Yang-Mills idea of gauge symmetry.

A linear potential and its effect on the accelerated expansion of the universe have been discussed [7]. Here we summarize the result in the context of the total-unified model. First, we observe that the result (13) implies that the gauge invariance of $\partial_\lambda B_{\mu\nu}$ enables us to construct an alternative gauge invariant Lagrangian L_{AE} for involving up and down quarks and the baryonic gauge field B_μ:

$$L_{AE} = -\frac{L_s^2}{4}\partial_\lambda B_{\mu\nu}\partial^\lambda B^{\mu\nu} + L_{ud}, \tag{39}$$

$$L_{ud} = i\overline{u}_a\gamma_\mu\left(\partial^\mu - \frac{ig_b}{3}B_\mu\right)u_a - m_u\overline{u}_a u_a + (u \to d),$$

where the color index a is summed from 1 to 3.

For accelerated expansion, let us ignore the gravitational fields for simplicity and concentrate on the baryonic gauge fields and the associated Lee-Yang force. The new gauge-invariant field equation derived from (39) is a fourth-order partial differential equation, $\partial^2\partial_\nu B^{\nu\mu} - g_b' J_q^\mu = 0$, where $g_b' = g_b/(3L_s^2)$, and the source of the gauge field is $J_q^\mu = \overline{u}_n\gamma^\mu u_n + \overline{d}_n\gamma^\mu d_n$. For the static case, the potential field B_0 satisfies the fourth order differential equation

$$\nabla^2\nabla^2 B^0 = -\frac{g_b}{3L_s^2}J^0 \equiv \rho_B. \tag{40}$$

Suppose we impose a 'Coulomb-like gauge' $\partial_k B^k = 0$, the static exterior potential satisfies the equation $\nabla^2\nabla^2 B^0 = 0$. The solution for such a potential is not unique, one has $B^o = A'/r$ or $B'r$ or $C'r^2$. It turns out that only the solution $B'r$ can be understood as due to the exchange of virtual quanta described by the fourth order field equation. The reason is as follows.

From the viewpoint of quantum field theory, the fermion (baryon) source ρ_B in (40) is represented by the usual delta-function, because these fermions satisfy the Dirac equations. Based on this property, the solution for the potential B_0 should be proportional to r rather than r^2 or $1/r$. This result can be seen explicitly by substituting $\rho_B(\mathbf{r}) = g_b'\delta^3(\mathbf{r})$ in (40), where $g' = g_b/(3L_s^2)$ for quarks (and $g' = g_b/(L_s^2)$ for protons and neutrons). We obtain a linear potential in the static limit,

$$B^0(\mathbf{r}) = g_b'\int_{-\infty}^{\infty}\frac{1}{(\mathbf{k}^2)^2}e^{i\mathbf{k}\cdot\mathbf{r}}d^3k = -\frac{g_b'}{8\pi}|\mathbf{r}|, \tag{41}$$

which can be understood as a generalized function [26]. This linear potential (41) could produce a constant repulsive Lee-Yang force between baryonic galaxies.

The equation of motion of a freely moving test particle in the Newtonian limit is changed to the following form [7]:

$$\frac{d^2\mathbf{r}}{dt^2} = \mathbf{g} + \mathbf{g}_{LY}, \tag{42}$$

where **g** is the gravitational acceleration produced by the distribution of ordinary matter [27], while the constant acceleration $\mathbf{g}_{LY}$ is due to the modified Lee-Yang baryon gauge field [7]. This constant repulsive force between baryon matter in the universe is the characteristic feature of the modified cosmic Lee-Yang force. If one uses Einstein's field equation with a cosmological constant one will have the result that the acceleration of expansion depends linearly on the distance between two galaxies [7, 27]. Thus, a crucial question is whether the acceleration in the expansion of the universe is a constant or dependent on distance. Since we have accelerated Wu transformations of space-time, which gives new Doppler shift for radiation source with constant acceleration [12], it is hoped that experimental test of (42) can be carried out in the near future.

Because of the extreme weakness of the coupling strength of the baryonic gauge field B_μ, its existence cannot be detected in the solar system and in our galaxy, it may be fitting to call it a "dark field." However, the sources of such a dark field are the ubiquitous baryonic matter rather than some mysterious 'dark energy' in the universe. This dark field model suggests that the accelerated cosmic expansion could be understood in terms of a modified Lee-Yang force produced by a baryonic gauge field. Thus, the Lagrangian (39) for the "dark field" B_μ has a field-theoretic interpretation and can be included in the total unified model.

VIII. EXPERIMENTS FOR GEOMETRIC-OPTICS LIMITS OF WAVE EQUATIONS FOR QUARKS AND GAUGE BOSONS

Next, let us consider the relation between the classical equation of motion and the massive fermion wave equation. The fermion wave equation can be derived from the Lagrangian (19) in inertial frames [9],

$$i\gamma_\mu J^{\mu\nu}\partial_\nu\psi - m\psi + \frac{i}{2}\gamma_\mu[\partial_\nu J^{\mu\nu}]\psi = 0, \tag{43}$$

where the fermion fields have been symmetrized. Using the expression for the field $\psi = \psi_o exp(iS)$ [28], we can derive the equation for the motion of a classical object in the presence of the gravitational tensor field $\phi^{\mu\nu}$ in flat space-time,

$$G^{\mu\nu}(\partial_\mu S)(\partial_\nu S) - m^2 = 0, \quad G^{\mu\nu} = P_{\alpha\beta}J^{\alpha\mu}J^{\beta\nu}. \tag{44}$$

It is formally the same as the corresponding Hamilton-Jacobi equation in general relativity [29]. Within the framework of Yang-Mills gravity in flat space-time, we shall call (44) the Einstein-Grossmann equation of motion for classical objects [30]. It appears that the classical equation of motion must have this particular form, (44), in order to describe accurately the motion of big objects in the presence of gravity, no matter whether the theory is based on curved or flat space-time.

In Yang-Mills gravity, the fundamental equation of geometrical optics can be derived. After some calculations, we find a new eikonal equation, which involves an effective metric tensor $G_L^{\mu\nu}$ for a light ray,

$$G_L^{\mu\nu}\partial_\mu\Psi\partial_\nu\Psi = 0, \tag{45}$$

$$G_L^{\mu\nu} = G^{\mu\nu} - \frac{g}{4}\phi_\lambda^\mu J^{\lambda\nu} = \eta_{\alpha\beta}(\eta^{\mu\alpha} + g\phi^{\mu\alpha})\left(\eta^{\nu\beta} + \frac{3g}{4}\phi^{\nu\beta}\right), \tag{46}$$

in the geometric-optics (or classical) limit.

Equations (44) for fermions and (45) for gauge bosons are not exactly the same, due to their different gauge invariant interactions and different properties of fields.

If the smaller term $-(g/4)\phi_\lambda^\mu J^{\lambda\nu}$ in the effective metric tensors $G_L{}^{\mu\nu}$ for the eikonal equations (46) is neglected, then both fermions and bosons have the same effective Riemannian metric tensors in the geometric-optics limit. In view of the good agreement between the effective metric tensor $G^{\mu\nu}$ in (44) and the perihelion shift [9], the small additional term in $G_L{}^{\mu\nu}$ given by (46) suggests a possible experimental test of Yang-Mills gravity.

Let us estimate the contribution of the smaller term $(g/4)\phi_\lambda^\mu J^{\lambda\nu}$ in (46) to the bending of a light ray by the sun. In the presence of gravity, the electromagnetic wave equation with the usual gauge condition $\partial_\mu A^\mu = 0$, leads to the eikonal equation for a light ray (45) in the short-wavelength (or geometric-optics) limit. Following the usual procedure, Yang-Mills gravity predicts a smaller angle for the bending of a light ray,

$$\Delta\phi_{light} = \frac{7Gm}{2R_o} \approx 1.53'', \tag{47}$$

where R_o is the distance from sun's center [4, 31]. The result (47) is smaller than the usual value $1.75''$ in Einstein gravity by about 12%. So far, the experimental accuracy for the bending of a light ray (with optical frequencies) by the sun is no better than 10%–20% [32].

If the experimental accuracy can be further improved to a few percent, the difference between Yang-Mills gravity and Einstein gravity can be tested. The significance of this experiment cannot be over emphasized, because it will indicate whether the Yang-Mills gravity with space-time translational gauge group dictates the gravitational interaction.

We may remark that previous radar echo experiments are not suitable to test Yang-Mills gravity with the new eikonal equation (45). The eikonal equation has the simple effective metric tensor $G_L^{\mu\nu}$ if and only if one takes the geometric-optics (or high-frequency) limit. The radar echo experiments used the frequencies 7840 MHz and 2300 MHz [33], which are roughly 10^9 (1/sec) and are too small in comparison with the frequencies of visible light ray $\approx 10^{14}$ (1/sec.). The eikonal equation (45) is expected to be valid for high-frequency waves such as visible light rays.

IX. DISCUSSIONS AND REMARKS

In connection with linear potentials, we note that quark confinement could also be understood in terms of a linear potential with attractive force at low energies [8]. For a linear potential to produce only attractive force between quarks and quarks, quarks and anti-quarks, and anti-quarks and anti-quarks, the fields must be a tensor of the second rank or a scalar field. But a scalar field cannot be a gauge field. This property suggests that the relevant gauge symmetry is likely to involve both internal and external gauge

symmetries. [34] It would be very interesting if the potentials at the smallest distance scale inside hadrons between confined quarks and at the large cosmic scale between galaxies with accelerated expansion are both due to linear potentials associated with the fourth-order differential field equations in the low energy approximation [7, 8]. If this turns out to be the case, then 'particle cosmology' would be very interesting in the physics of the future.

We may remark that an important difference between Yang-Mills gravity and Einstein gravity is that the simple 'effective Riemann metric tensor' $G_L^{\mu\nu}$ given by (45) is only for a high frequency light ray, while the inherent space-time metric tensor in Einstein gravity can be applied to an electromagnetic wave with any frequency. Thus, if the radar wave in the echo experiment can be replaced by electromagnetic waves with higher frequencies, say, laser light, then it could also test Yang-Mills gravity versus Einstein gravity.

The total-unified model is not renormalizable by naive power counting. It is not clear whether the T_4 gauge symmetry could change the situation at higher orders. The usual argument based on the running coupling constant may not be air-tight, because the gravitational coupling has not been included. Thus, the usual properties and conclusions in gauge field theories cannot be taken for granted in the total-unified model. Furthermore, the most complicated couplings of the graviton with itself and other particles are the 4-vertex, as shown in (25), (26), (29) and Ref. [13], in the total-unified model. In contrast, in the (Feynman) quantization of Einstein gravity, the graviton can have an N-vertex of self-coupling, where N is an arbitrarily large number.

In the total-unified model, Yang-Mills gravity reveals the flat space-time translational origin of the gravitational field $\phi_{\mu\nu}$ and leads to an *effective curved space-time* for the motion of classical objects. It is a logically and experimentally consistent gauge theory, which represents an alternative to Einstein gravity. Furthermore, Yang-Mills gravity with flat space-time translational gauge symmetry paves the way to unify all interactions based on a generalized Yang-Mills framework. In this sense, the total-unified model is a satisfactory framework that could accommodate all fundamental laws of physics in a unified manner and could be consistent with the accelerated expansion of the universe. These properties could be considered as an advantage of Yang-Mills idea of gauge symmetry, and, furthermore, the total-unified model supports the idea that gauge symmetries dictate interactions, as advocated by Utiyama, Yang, and others [11].

References

[1] J. P. Hsu, Mod. Phys. Lett. A **26**, 1707 (2011), (arXiv:1106.2121).
doi: 10.1142/S021773231103619X

[2] J. P. Hsu, Chinese Phys. C **36**(5), 403 (2012), (arXiv:1108.2423).

[3] J. P. Hsu, Eur. Phys. J. Plus **126**, 24 (2011), (arXiv:1102.2253). doi: 10.1140/epjp/i2011-11024-x

[4] J. P. Hsu and L. Hsu, *Space-Time Symmetry and Quantum Yang-Mills Gravity* (World Scientific, 2013), pp. 23–27, p. 157, part II.

[5] K. Huang, *Quarks, Leptons and Gauge Field* (World Scientific, 1982), pp. 103-121 and pp. 241-242.

[6] T. D. Lee and C. N. Yang, *in 100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang-Mills*, eds. J. P. Hsu and D. Fine, (World Scientific, 2005), p. 155.

[7] J. P. Hsu, Mod. Phys. Lett. A **20**, 2855 (2005). It is assumed that the observable portion of the universe is dominated by baryonic matter, which is consistent with observations.

[8] The quark confinement force should be attractive for all quarks and anti-quarks, and such a force may be related to a linear potential. For a discussion, see J. P. Hsu, Nuovo Cimento B **88**, 140 (1985). doi: 10.1007/BF02728896

[9] J. P. Hsu, Int. J. Mod. Phys. A **21**, 5119 (2006), (arXiv:1102.2010).
doi: 10.1142/S0217751X06034082

[10] J. P. Hsu, Int. J. Mod. Phys. A **24**, 5217 (2009), (arXiv:1005.3250).
doi: 10.1142/S0217751X0904765X

[11] This interesting and, perhaps, profound idea was discussed and stressed by Utiyama, Yang, and others. See *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang-Mills*, eds. J. P. Hsu and D. Fine, (World Scientific, 2005), p. 157 and 527.

[12] L. Hsu and J. P. Hsu, Nuovo Cimento B **112**, 1147 (1997), appendix.

[13] J. P. Hsu, Euro. J. Phys. Plus **126(3)**, 1 (2011), arXiv:1102.2253 [gr-qc].

[14] See, for example, R. P. Feynman, *Theory of Fundamental Processes* (W. A. Benjamin, 1962) p. 168.

[15] F. J. Dyson, in *Lorentz and Poincaré Invariance*, eds. J. P. Hsu and Y. Z. Zhang (World Scientific, 2001) p. 316.

[16] T. Kinoshita, talk at Nishina Hall, RIKEN (2010). (google search: Kinoshita, Magnetic moment.)

[17] B. S. DeWitt, in *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang-Mills*, eds. J. P. Hsu and D. Fine, (World Scientific, 2005) p. 309.

[18] D. M. Capper and M. Ramon Medrano, Phys. Rev. **D 9**, 1641 (1974). doi: 10.1103/PhysRevD.9.1641

[19] G. Leibbrandt, Rev. Mod. Phys. **47**, 849 (1975). doi: 10.1103/RevModPhys.47.849

[20] D. M. Capper and M. A. Namazie, Nucl. Phys. **B 142**, 535 (1978).
doi: 10.1016/0550-3213(78)90229-8

[21] J. P. Hsu, Euro. J. Phys. Plus **127(3)** 1 (2012), arXiv:1203.4546 [hep-th].

[22] S. Weinberg, *The Quantum Theory of Fields* (Cambridge Univ. Press, 1996), Vol. II, pp. 305–318, pp. 328–329.

[23] L. Lusanna and P. Valtancoli, Int. J. Mod. Phys. **A13**, 4605 (1998), arXiv:hep-th/9707072v1.
doi: 10.1142/S0217751X98002213

[24] R. E. Marshak, *Conceptual Foundations of Modern Particle Physics* (World Scientific, 1993) Ch. 2.

[25] T.-Y. Wu and W.-Y. Pauchy Hwang, *Relativistic Quantum Mechanics and Quantum Fields* (World Scientific, 1991), Chpts. 12, 13, and 14.

[26] I. M. Gel'fand and G. E. Shilov, *Generalized Functions, Vol. 1* (Academic Press, New York, 1964), p. 363.

[27] P. J. E. Peebles and B. Ratra, Rev. Mod. Phys. **75**, 559 (2003) and references therein. doi: 10.1103/RevModPhys.75.559

[28] L. Landau and E. Lifshitz, *The Classical Theory of Fields*, Trans. by M. Hamermesh, (Addison-Wesley, Cambridge, Mass. 1951), pp. 314–316.

[29] J. P. Hsu, Phys. Rev. Lett. **42**, 934 (1979). doi: 10.1103/PhysRevLett.42.934

[30] Marcel Grossmann was associated with Einstein in elucidating the mathematical foundation of general relativity.

[31] J. P. Hsu, Euro. Phys. J. Plus **128**, 31 (2013), arXiv:1302.6956 [gr-qc]. doi: 10.1140/epjp/i2013-

13031-3

[32] W. T. Ni, in *100 Years of Gravity and Accelerated Frames, The Deepest Insights of Einstein and Yang-Mills*, eds. J. P. Hsu and D. Fine, (World Scientific, 2005) p. 481.

[33] S. Weinberg, *Gravitation and Cosmology* (John Wiley and Sons, 1972), pp. 201–207.

[34] It appears that gauge fields associated with only internal gauge symmetry will lead to both attractive and repulsive forces, just like the electromagnetic force.