The Effect of Radiation on the Jeans Instability of a Viscous Quantum Plasma with Finite Electrical Resistivity

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(Received October 4, 2012; Revised May 6, 2013)

The effect of the radiative heat-loss function on the Jeans instability of a quantum plasma, in the presence of finite electrical resistivity and viscosity, is investigated. A general dispersion relation is derived from the linearized perturbation equations using normal mode analysis and discussed for the longitudinal and transverse direction of wave propagation. On the basis of the Routh-Hurwitz criterion, the dynamic stability of the system is discussed. The condition for the Jeans instability of a quantum plasma is discussed in the different cases of our interest. It is found that, owing to the inclusion of a radiative heat-loss function and thermal conductivity, the Jeans criterion of gravitational instability changes into a radiative instability criteria. For the transverse mode, in the case of a resistive medium the expression of the Jeans instability is independent of the magnetic field, while for a perfectly conducting medium the magnetic field gives a stabilizing effect. From the curves it is apparent that the viscosity, temperature dependent heat-loss function, and magnetic field stabilize the system, while the electrical resistivity has a destabilizing effect.

I. INTRODUCTION

It is an established fact that the problem of self-gravitational instability is a broad area of research in plasma physics, astrophysics, and in many other crucial phenomena of the interstellar medium (ISM). It plays an important role in star formation in magnetic dusty clouds due to the gravitation collapsing process. Jeans [1] has discussed the condition under which a fluid becomes gravitationally unstable under the action of self gravitation. Chandrashekhar [2] has presented a comprehensive survey on the gravitational instability problem under various assumptions. In many investigations [3–6], the gravitational instability of a homogeneous plasma, considering the effect of various parameters, was studied.

The above mentioned studies are based on the problem of the self-gravitational instability of a homogeneous plasma; none of the authors have incorporated the radiative effect in their studies. The radiative effect plays an important role in astrophysics and plasma physics. Several authors investigated the phenomena of radiative instability arising due to the heat-loss mechanism in plasma. Field [7] has investigated the normal modes of perturbation in a homogeneous gas under the local thermal balance, and established the

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criteria for thermal instability which is widely used in theoretical astrophysics. In the last decade, Hennebelle and Perault [8] have investigated a dynamical condensation process in a magnetized and thermally conducting medium, and stressed its importance in the thermal condensation modes with magnetic fields. Inutsuka *et al.* [9] have investigated the propagation of shock waves into a warm neutral medium taking into account radiative terms. Menou *et al.* [10] have shown the importance of the radiative effect in the sun's upper radiative zone. Stiele *et al.* [11] have discussed clump formation due to thermal instability in a weakly ionized plasma. Fukue and Kamaya [12] have explored the problem of thermal instability considering the effects of ion-neutral friction, the radiative cooling function, and the magnetic field.

The problem of the self-gravitational instability with radiative effect was first investigated by Aggrawal and Talwar [13], and they found a criterion of radiative instability. Again this problem was considered by Bora and Talwar [14] taking the effect of the Hall current and electron inertia for a non-rotating plasma. In the past few years there was a lot of work carried out on this problem: Prajapati *et al.* [15] have discussed the radiative instability problem for a self-gravitating rotating Hall plasma considering the effect of electron inertia, while Kaothekar and Chhajlani [16] have corrected the radiative instability criterion with finite Larmor radius (FLR) corrections for a non-rotating viscous medium. The effects of the radiative heat-loss function and electron inertia on the self gravitational instability of a partially ionized plasma have been investigated by Dangarh *et al.* [17]. Thus from the above mentioned studies we find that the problem of the thermal instability of the self-gravitating medium is an important phenomena for understanding the gravitational collapse of the protostar.

It is an established fact that the viscous force provides the damping effect on the growth rate of the system in the ISM Structure. The kinematic viscosity of the fluid is important in many astrophysical objects, namely the dwarf star, nebulae, and H_{II} regions. Also the effect of finite electrical resistivity has been considered in the discussion of the instability problem of the ISM due to its importance for understanding the magnetic reconnection processes and plasma confinement problem. In this direction, the effects of electrical resistivity and viscosity in the calculation of the time evolution of an incompressible magnetic turbulence in the ISM have been studied by Cho *et al.* [18]. Recently, Kaothekar and Chhajlani [19] discussed the self-gravitational instability of a plasma in connection with the FLR correction, a radiative heat-loss function, thermal conductivity, and finite electrical resistivity for viscous and non-viscous systems.

It is a well known fact that the study of quantum plasma has a wide range of astrophysical and laboratory applications, such as low temperature plasma, white dwarf stars, planetary nebulae, supernovas, and micro and nano-electronic devices. In this direction, Haas [20] predicted a modified quantum magnetohydrodynamic (QMHD) model. Using this QMHD model, the Jeans instability of a magnetized quantum plasma including electron spin effects has been investigated by Lundin *et al.* [21]; they have found that intrinsic magnetization of the plasma enhances the Jeans instability and affects the structure of the instability spectrum. Also Ren *et al.* [22] used the QMHD model to investigate the problem of the Jeans instability of a quantum magneto plasma considering finite electrical resistiv-

ity. Masood *et al.* [23] investigated the self-gravitational instability of a multi-component quantum plasma using the Bohm potential and statistical terms for the electrons and ions. The Jeans instability in a homogeneous cold quantum dusty plasma in the presence of a magnetic field and quantum correction was examined by Salimullah *et al.* [24]. Recently, Prajapati and Chhajlani [25] found the quantum corrected criterion for the self-gravitational instability of a Hall plasma.

In the light of all the above studies, it is of interest to study the Jeans instability with a radiative heat-loss function in quantum magneto plasma. We have investigated the simultaneous effect of a radiative heat-loss function, finite electrical resistivity, thermal conductivity, viscosity and quantum correction on the Jeans instability of an infinite homogeneous gaseous magnetized plasma.

II. LINEARIZED PERTURBATION EQUATIONS

Consider an infinite homogeneous, viscous, radiative and thermally conducting, self-gravitating quantum plasma including electrical resistivity. It is assumed that the above medium is permeated with a uniform magnetic field $\mathbf{B}(0, 0, B)$, in the z-direction. In the present study we have introduced the quantum effect through the Bohm potential term in the momentum transfer equation. The magnetic field is taken here classically, and it is supposed that it is unaffected by quantum magnetization. We consider the following set of linearized perturbation equations.

The momentum transfer equation:

$$\rho \frac{\partial \delta \mathbf{v}}{\partial t} = -\nabla \delta p + \rho \nabla \delta \phi + \rho \nu \left(\nabla^2 \delta \mathbf{v} \right) + \frac{1}{4\pi} \left(\nabla \times \delta \mathbf{B} \right) \times \mathbf{B} + \frac{\hbar^2}{4 m_e m_i} \nabla \left(\nabla^2 \delta \rho \right). \quad (1)$$

The equation of continuity:

$$\frac{\partial \delta \rho}{\partial t} = -\rho \nabla \cdot \delta \mathbf{v}. \quad (2)$$

Poisson's equation for a self-gravitational potential:

$$\nabla^2 \delta \phi = -4\pi G \delta \rho. \quad (3)$$

The induction equation for the magnetic field with finite electrical resistivity:

$$\frac{\partial \delta \mathbf{B}}{\partial t} = \nabla \times (\mathbf{v} \times \mathbf{B}) + \eta \nabla^2 \delta \mathbf{B}. \quad (4)$$

$$\nabla \cdot \delta \mathbf{B} = 0. \quad (5)$$

The heat equation for a perfect gas including the radiative effect and thermal conduction:

$$\frac{1}{(\gamma - 1)} \frac{\partial}{\partial t} \delta \rho - \frac{\gamma}{(\gamma - 1)} \frac{p}{\rho} \frac{\partial}{\partial t} \delta \rho + \rho \left[\left(\frac{\partial \mathcal{L}}{\partial \rho} \right)_T \delta \rho + \left(\frac{\partial \mathcal{L}}{\partial T} \right)_\rho \delta T \right] - \lambda \nabla^2 \delta T = 0. \quad (6)$$

The gas equation:

$$\frac{\delta p}{p} = \left(\frac{\delta T}{T} + \frac{\delta \rho}{\rho} \right). \tag{7}$$

where $\mathbf{v}(v_x, v_y, v_z)$, p, γ, ϕ, G, λ, ρ, T, η, ν, and $\hbar$ denote the gas velocity, gas pressure, adiabatic index, gravitational potential, gravitational constant, coefficient of thermal conductivity, density of gas, temperature, finite electrical resistivity, kinematic viscosity, and plank constant divided by 2π, respectively. The perturbations in the fluid pressure, fluid density, magnetic field, gravitational potential, and temperature are denoted by the symbols δp, $\delta \rho$, $\delta \mathbf{B}(\delta B_x, \delta B_y, \delta B_z)$, $\delta \phi$, and δT, respectively. The parameter $\mathcal{L}(\rho, T)$ in Equation (6) is the heat-loss function per gram of the material per second exclusive of thermal conduction, and in general it is a function of the local density and temperature of the gas.

III. DISPERSION RELATION

Let us assume that all the perturbed quantities vary as

$$\exp \left\{ i \left(k_x x + k_z z + \sigma t \right) \right\}.$$

where the k's are the wave numbers relating $k_x^2 + k_z^2 = k^2$, and σ is the growth rate of the perturbations. Combining Equation (6) and (7), we obtain the expression for δp as

$$\delta p \left\{ \omega + P_1 \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{P_2}{p} \right) \right\} = \delta \rho \left\{ P_1 \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{P_2}{\rho} \right) + \omega c^2 \right\}. \tag{8}$$

Using Equations (2)–(8) in (1), we obtain the following algebraic equations for the components:

$$\left[R_1 + \frac{k^2 V^2}{d} \right] v_x + \frac{i k_x}{k^2} \left(\Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) s = 0. \tag{9}$$

$$\left[R_1 + \frac{k_z^2 V^2}{d} \right] v_y = 0. \tag{10}$$

$$R_1 v_z + \frac{i k_z}{k^2} \left(\Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) s = 0. \tag{11}$$

The divergence of (1) with the aid of (2)–(8) gives

$$\left(\frac{i k_x V^2 k^2}{d} \right) v_x - \left(\omega R_1 + \Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) s = 0, \tag{12}$$

where $V = B/\sqrt{4\pi\rho}$ is the Alfvén velocity, $c = (\gamma p/\rho)^{1/2}$ and $c' = (p/\rho)^{1/2}$ are the adiabatic and isothermal velocities of sound, respectively. $\mathcal{L}_{\rho,T}$ are the partial derivatives of the density dependent $(\partial\mathcal{L}/\partial\rho)_T$ and temperature dependent $(\partial\mathcal{L}/\partial T)_\rho$ heat-loss function, respectively. Also we have assumed the following substitutions:

$$\omega = i\sigma, \; P_1 = (\gamma - 1), \; P_2 = \lambda k^2 T, \; \alpha = P_1\left(\mathcal{L}_T T - \mathcal{L}_\rho\rho + \frac{P_2}{\rho}\right), \; \beta = P_1\left(\frac{\mathcal{L}_T T\rho}{p} + \frac{P_2}{p}\right),$$

$$M_1 = \left[R_1 + \frac{k^2 V^2}{d}\right], \; M_2 = \left[R_1 + \frac{k_z^2 V^2}{d}\right], \; M_3 = \left(\omega R_1 + \Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i}\right), \; d = (\omega + \Omega_m),$$

$$R_1 = (\omega + \Omega_v), \; \Omega_I^2 = \left(k^2\alpha - 4\pi G\rho\beta\right), \; \Omega_j^2 = \left(k^2 c^2 - 4\pi G\rho\right), \; \Omega_T^2 = \left(\frac{\omega\Omega_j^2 + \Omega_I^2}{\omega + \beta}\right).$$

Now we can write Equation (9)–(12) in a matrix form, to obtain the dispersion relation, as

$$[X][Y] = 0. \tag{13}$$

where $[X]$ is a fourth order matrix and $[Y]$ is the single column matrix of elements $[v_x, v_y, v_z, s]$. The vanishing of $[X]$ gives the following equation:

$$-R_1 M_1 M_2 M_3 + \left[\frac{k_x^2}{k^2}\left(\Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i}\right)\left\{\frac{k^2 V^2 R_1 M_2}{d}\right\}\right] = 0. \tag{14}$$

Equation (14) is the general dispersion relation of the system under consideration, which shows the effect of finite electrical resistivity, thermal conductivity, radiative heat-loss function, and viscosity on the magnetized quantum plasma. The dispersion relations obtained by Ren *et al.* [22] and Lundin *et al.* [21] may be derived from Equation (14) as its special cases.

Equation (14) may be discussed separately for the longitudinal propagation $(k_z = k)$ and transverse propagation $(k_x = k)$ modes of propagations. Further, in these modes, the effect of various parameters will be exhibited distinctly.

IV. DISCUSSION

IV-1. Longitudinal propagation
For longitudinal propagation we have $k_x = 0$ and $k_z = k$; on substituting this in Equation (14), we get the following dispersion relation:

$$R_1 M_3 M_1^2 = 0. \tag{15}$$

This reduced dispersion relation (15) for longitudinal propagation has three independent factors discussed separately.

The first factor gives

$$\left(\omega + \nu k^2\right) = 0. \tag{16}$$

This equation (16) is similar to Kaothekar and Chhajlani [16] representing a stable damped mode modified due to the presence of viscosity of the quantum fluid plasma. From Equation (16) it is clear that the viscosity has a stabilizing effect on the system.

The second factor may be represented as follows:

$$\omega^2 + \omega \left(\eta k^2 + \nu k^2\right) + V^2 k^2 + \eta k^2 \nu k^2 = 0. \tag{17}$$

The dispersion relation (17) shows the influences of viscosity, finite electrical resistivity, and magnetic field on the plasma medium, but does not show the influence of thermal conductivity, the radiative heat-loss function, self-gravitation, and quantum correction. This dispersion relation represents a stable non-gravitating Alfvén mode modified by finite electrical resistivity and viscosity. The dynamic stability of the above system may be discussed by applying the Routh-Hurwitz criterion. According to which, it is a necessary condition that all the coefficients of the polynomial equation must be positive. As for the sufficient condition: all the principal diagonal minors of the Hurwitz matrix must be positive for the system to be stable. We calculate the minors and get

$$\Delta_1 = \left[\eta k^2 + \nu k^2\right] > 0 \quad \text{as} \ \ \gamma > 1,$$

$$\Delta_2 = \left[\nu k^2 \eta k^2 + V^2 k^2\right] \Delta_1 > 0,$$

$$\Delta_3 = 0.$$

Since all the diagonal minors (Δ) of Hurwitz matrices are positive for the above system, hence it is stable in nature.

The third factor of the dispersion relation (15) gives

$$\omega^3 + \omega^2 \left(\nu k^2 + \beta\right) + \omega \left(\nu k^2 \beta + \Omega_j^2 + \frac{\hbar^2 k^4}{4 m_e m_i}\right) + \Omega_I^2 + \beta \left(\frac{\hbar^2 k^4}{4 m_e m_i}\right) = 0. \tag{18}$$

This dispersion relation represents a modified gravitating condensation mode due to the presence of viscosity and the quantum effect. This dispersion relation is similar to that of Ren *et al.* [22] when the effects of viscosity and radiative heat loss function have been neglected. In the absence of radiative effects this dispersion relation is reduced to that of Prajapati and Chhajlani [25] excluding the Hall current and permeability. As we know, the necessary condition for instability can be seen only from the constant term of the dispersion relation. Thus when the constant term is less than zero, one of the roots must be real, and hence it shows an unstable mode, which gives the condition of instability as

$$(\gamma - 1) \left\{ \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho}\right) + \frac{\hbar^2 k^2}{4 m_e m_i} \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p}\right) \right\} < \frac{4\pi G \rho}{k^2}. \tag{19}$$

Equation (19) represents a corrected condition of radiative instability due to the quantum effect. Here we find that electrical resistivity, a magnetic field, and the viscosity parameter do not affect the condition for radiative instability of the quantum plasma in the longitudinal direction of propagation. Now, in the absence of the quantum effect, the dispersion relation (18) reduces to

$$\omega^3 + \omega^2 \left(\nu k^2 + \beta\right) + \omega \left(\nu k^2 \beta + \Omega_j^2\right) + \Omega_I^2 = 0. \tag{20}$$

The condition of instability, for the dispersion relation (20), is given by

$$\left(k^2 \alpha - 4\pi G \rho \beta\right) < 0. \tag{21}$$

It is evident from the condition of instability (21) that the Jeans criterion of instability is modified due to the inclusion of thermal conductivity and the radiative heat-loss function. This condition is similar to that of Dangarh et $al.$ [17] for a radiating partially ionized plasma.

For a thermally non-conducting, non-radiating classical plasma medium, we have $\alpha = \beta = Q = 0$, then the dispersion relation (18) reduces to

$$\omega^2 + \Omega_j^2 = 0. \tag{22}$$

It is clear from Equation (22) that when $\Omega_j{}^2 < 0$ the product of the roots of Equation (22) must, therefore, be negative. This implies that at least one root of ω is positive. Hence, the system is unstable. Thus for Equation (22) the condition of instability is

$$\left(c^2 k^2 - 4\pi G \rho\right) < 0. \tag{23}$$

or

$$k < k_j = \left(\frac{4\pi G \rho}{c^2}\right)^{1/2}, \tag{24}$$

where k_j is the Jeans wave number. The above criterion of instability is identical to that of Chandrasekhar [2]. Thus, from condition (21) and (23), it is clear that, due to the presence of the radiative heat-loss function and thermal conductivity, the fundamental Jeans criterion of gravitational instability changes into a radiative instability criteria.

The dispersion relation (18) for a non-radiating and thermally non-conducting quantum plasma may be written in the form

$$\omega^2 + \omega \nu k^2 + \left(\Omega_j^2 + \frac{\hbar^2 k^4}{4 m_e m_i}\right) = 0. \tag{25}$$

For the constant term of Equation (25), the condition of the Jeans instability can be easily obtained, which is given by

$$\frac{4\pi G \rho}{k^2} > c^2 + \frac{\hbar^2 k^4}{4 m_e m_i}. \tag{26}$$

This is a condition of instability similar to that obtained by Ren *et al.* [22]. On comparing Equation (19) and (26), we notice that the quantum corrected Jeans criterion is modified into a quantum corrected radiative instability criterion, due to the inclusion of thermal conductivity and the radiative heat-loss function. The system represented by Equation (26) will be unstable if the above condition holds.

IV-2. Transverse propagation

For transverse propagation we have $k_z = 0$ and $k_x = k$. On substituting it in Equation (14), we get the following dispersion relation:

$$R_1^3 \left\{ \omega R_1 + \left(\Omega_T^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) + \frac{\omega k^2 V^2}{d} \right\} = 0. \tag{27}$$

The dispersion relation (27) shows two independents factor; the first factor gives

$$\omega + \nu k^2 = 0. \tag{28}$$

Equation (28) is similar to Equation (16) of longitudinal propagation. Thus we conclude that the damping effect of viscosity is independent of the direction of propagation.

The second factor of the dispersion relation (27) gives

$$\left\{ \omega^4 + \omega^3 \left(\beta + \nu k^2 + \eta k^2 \right) + \omega^2 \left(\eta k^2 \beta + \beta \nu k^2 + \Omega_j^2 + V^2 k^2 + \eta k^2 \nu k^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) \right.$$
$$\left. + \omega \left(\Omega_I^2 + \eta k^2 \Omega_j^2 + \beta \frac{\hbar^2 k^4}{4 m_e m_i} + \eta k^2 \nu k^2 \beta + V^2 k^2 \beta + \eta k^2 \frac{\hbar^2 k^4}{4 m_e m_i} \right) + \left(\Omega_I^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \beta \right) \eta k^2 \right\}$$
$$= 0. \tag{29}$$

This dispersion relation shows the combined influence of the radiative heat-loss function, thermal conductivity, viscosity, finite electrical resistivity, and quantum correction on the Jeans instability of the magnetized plasma. The condition of instability is given as

$$(\gamma - 1) \left\{ \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho} \right) + \frac{\hbar^2 k^2}{4 m_e m_i} \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right) \right\} < \frac{4 \pi G \rho}{k^2}. \tag{30}$$

The above condition of instability is the same as the condition of instability given in Equation (19) for the longitudinal direction of propagation. It means that the condition of radiative instability for a quantum plasma with finite electrical resistivity is the same in both the longitudinal and transverse direction of propagation. If we ignore the effect of resistivity, taking $\eta = 0$, i.e., for a perfectly conducting medium, we get

$$\left\{ \omega^3 + \omega^2 \left(\beta + \nu k^2 \right) + \omega \left(\beta \nu k^2 + \Omega_j^2 + V^2 k^2 + \frac{\hbar^2 k^4}{4 m_e m_i} \right) + \Omega_I^2 + \beta \left(\frac{\hbar^2 k^4}{4 m_e m_i} + V^2 k^2 \right) \right\} = 0. \tag{31}$$

From the constant term of Equation (31), we get the criterion of radiative instability for the quantum plasma in the transverse direction of propagation as

$$(\gamma - 1)\left\{ \left(\mathcal{L}_T T - \mathcal{L}_\rho \rho + \frac{\lambda k^2 T}{\rho} \right) + \left(\frac{\hbar^2 k^2}{4 m_e m_i} + V^2 \right) \left(\frac{\mathcal{L}_T T \rho}{p} + \frac{\lambda k^2 T}{p} \right) \right\} < \frac{4\pi G \rho}{k^2}. \quad (32)$$

This is a modified condition of radiative instability for a quantum plasma in the transverse direction of propagation. This modification in the radiative instability criterion is due to the Alfvén velocity, which represents the effect of the magnetic field. On comparing Equation (30) and (32), we notice that the effect of the magnetic field appears in the condition of radiative instability when the system is infinitely conducting, in other words we can say that finite resistivity takes out the Alfvén velocity term from the condition of instability.

In order to analyze the influence of various physical parameters on the growth rate of the unstable mode, we have performed numerical $\overline{\omega}$ calculations of the dispersion relation (29) to locate the roots of the non-dimensional growth rate against the non-dimensional wave number $\overline{k}$ for different values of the parameters. For the numerical calculations we have taken the value of γ as $5/3$. Dispersion relation (29) may be written in non-dimensional form as follows:

$$\left\{ \overline{\omega}^4 + \overline{\omega}^3 \left(\overline{\beta} + \overline{\nu}\overline{k}^2 + \overline{\eta}\overline{k}^2 \right) + \overline{\omega}^2 \left(\overline{\eta}\overline{k}^2 \overline{\beta} + \overline{\beta}\overline{\nu}\overline{k}^2 + \left(\overline{k}^2 - 1 \right) + \overline{V}^2 \overline{k}^2 + \overline{\eta}\overline{k}^2 \overline{\nu}\overline{k}^2 + \overline{Q}\overline{k}^2 \right) \right.$$

$$+ \overline{\omega} \left\{ \left(\overline{k}^2 \overline{\alpha} - \overline{\beta} \right) + \overline{\eta}\overline{k}^2 \left(\overline{k}^2 - 1 \right) + \overline{Q}\overline{k}^2 \overline{\beta} + \overline{\eta}\overline{k}^2 \overline{\nu}\overline{k}^2 \overline{\beta} + \overline{V}^2 \overline{k}^2 \overline{\beta} + \overline{\eta}\overline{k}^2 \overline{Q}\overline{k}^2 \right\}$$

$$\left. + \left(\overline{k}^2 \overline{\alpha} - \overline{\beta} \right) \overline{\eta}\overline{k}^2 + \overline{\beta}\overline{\eta}\overline{k}^2 \overline{Q}\overline{k}^2 \right\} = 0, \quad (33)$$

where $\overline{\omega} = \frac{\omega}{\sqrt{4\pi G\rho}}$, $\overline{\nu} = \frac{\nu\sqrt{4\pi G\rho}}{c^2}$, $\overline{\mathcal{L}}_\rho = \frac{(\gamma-1)\rho\mathcal{L}_\rho}{c^2\sqrt{4\pi G\rho}}$, $\overline{\mathcal{L}}_T = \frac{(\gamma-1)\rho T\mathcal{L}_T}{p\sqrt{4\pi G\rho}}$, $\overline{\alpha} = \frac{1}{\gamma}\left(\overline{\mathcal{L}}_T + \overline{\lambda}\overline{k}^2 \right) - \overline{\mathcal{L}}_\rho$, $\overline{\beta} = \overline{\mathcal{L}}_T + \overline{\lambda}\overline{k}^2$, $\overline{k} = \frac{kc}{\sqrt{4\pi G\rho}}$, $\overline{\lambda} = \frac{(\gamma-1)T\lambda\sqrt{4\pi G\rho}}{pc^2}$, $\overline{Q} = \frac{\hbar^2 k_j^2}{4m_e m_i}$, $\overline{\eta} = \frac{\eta\sqrt{4\pi G\rho}}{c^2}$, $\overline{V} = \frac{V\sqrt{4\pi G\rho}}{c}$. The variation of the growth rate $\overline{\omega}$ with wave number $\overline{k}$ are shown in Figures 1–4.

From Fig. 1 we find that the viscosity shows a negative growth rate with increasing wave number, and this negative growth rate is increased with an increasing value of viscosity. Hence, we can conclude that the viscosity parameter has a stabilizing effect on the system.

We notice that Figure 2 shows similar curves as in Figure 1, i.e., the increasing value of the temperature dependent radiative heat-loss function decreases the value of the growth rate. Hence, we conclude that the temperature dependent radiative heat-loss function has a stabilizing effect on the growth rate of instability of the system, similar to viscosity.

In Figure 3 the growth rate of the system is increased with increasing values of electrical resistivity. This means that for the higher values of electrical resistivity, the system tends to get an instability. In other words, an infinitely conducting system will be more stable than a finitely conducting medium.

In Figure 4, line (1) shows a non-magnetized medium while the lines (2, 3, and 4) are showing an increasing magnetization of thhe medium. So, we notice that the growth rate of the instability for a non-magnetized medium (line 1) is higher in comparison with that of
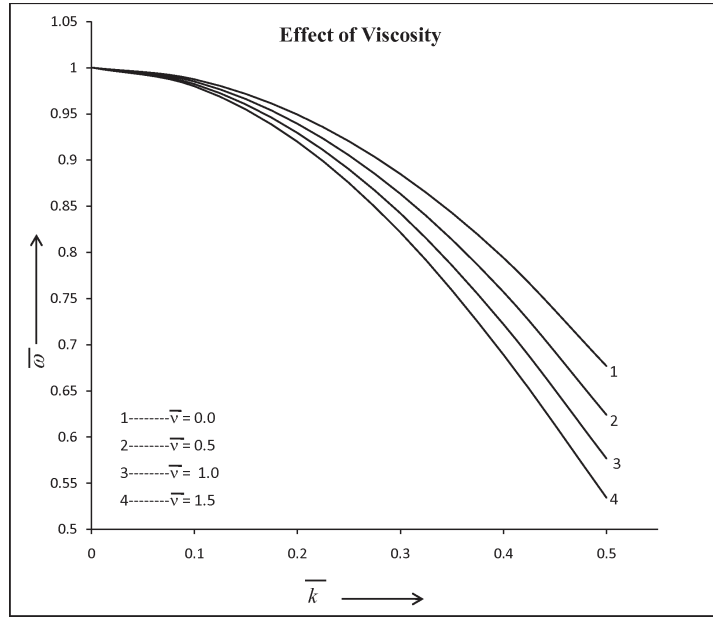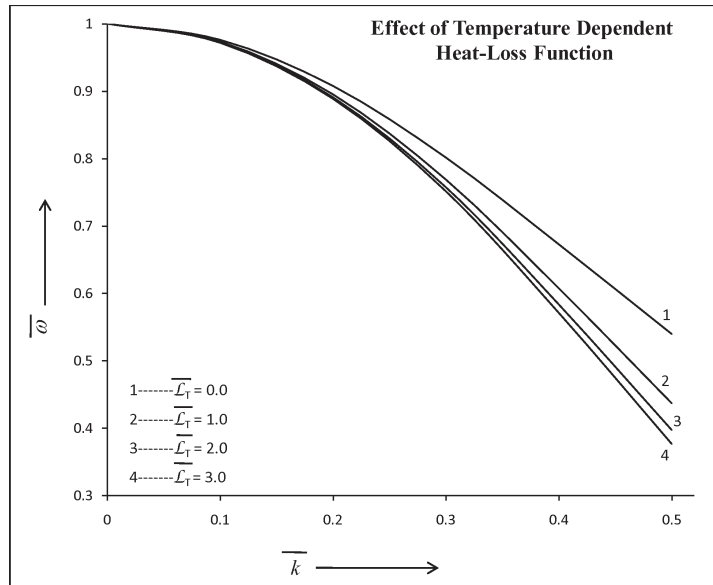
FIG. 1: The non-dimensional growth rate $\overline{\omega}$ is plotted against the non-dimensional wave number $\overline{k}$ with different values of viscosity $\overline{\nu} = 0.0, 0.5, 0.1, 1.5$; the values of the other parameters are fixed: $\overline{\mathcal{L}_\rho} = \overline{\mathcal{L}_T} = 0.5$ and $\overline{\lambda} = \overline{Q} = \overline{V} = \overline{\eta} = 1.0$.
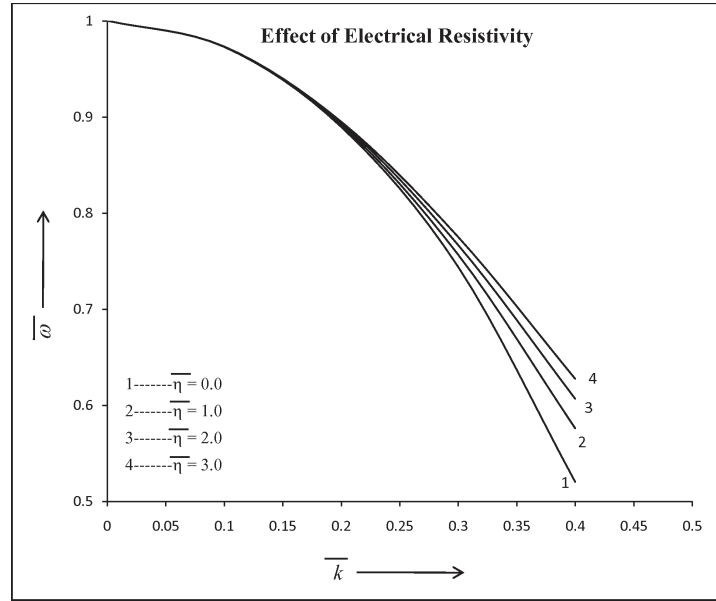

FIG. 2: The non-dimensional growth rate $\overline{\omega}$ is plotted against the non-dimensional wave number $\overline{k}$ with different values of the temperature dependent radiative heat-loss function $\overline{\mathcal{L}_T} = 0.0, 1.0, 2.0, 3.0$; the values of the other parameters are fixed: $\overline{\mathcal{L}_\rho} = \overline{\lambda} = \overline{Q} = \overline{\nu} = \overline{\eta} = \overline{v} = 1.5$.

FIG. 3: The non-dimensional growth rate $\overline{\omega}$ is plotted against the non-dimensional wave number $\overline{k}$ with different values of the finite electrical resistivity $\overline{\eta} = 0.0$, 1.0, 2.0, 3.0; the values of the other parameters are fixed: $\overline{\mathcal{L}_\rho} = \overline{\mathcal{L}_T} = \overline{\nu} = \overline{\lambda} = \overline{Q} = \overline{v} = 1.5$.
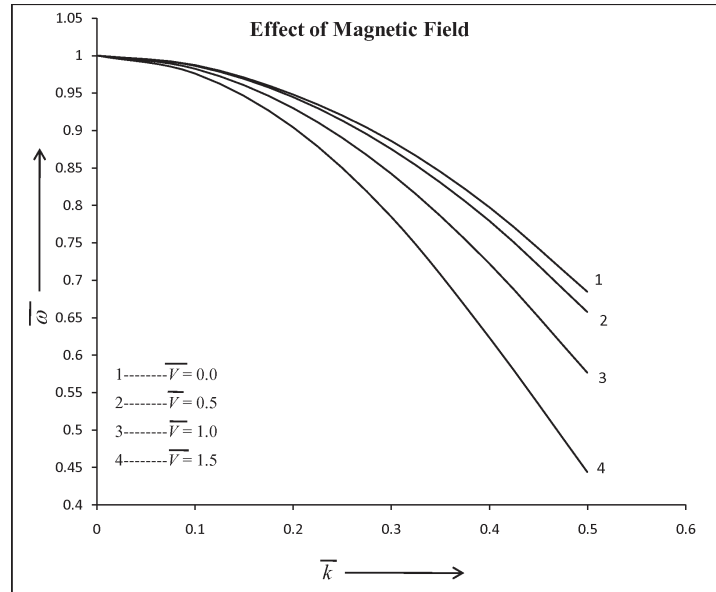

FIG. 4: The non-dimensional growth rate $\overline{\omega}$ is plotted against the non-dimensional wave number $\overline{k}$ with different values of the magnetic field $\overline{v} = 0.0$, 0.5, 1.0, 1.5; the values of other parameters are fixed $\overline{\mathcal{L}_\rho} = \overline{\mathcal{L}_T} = 0.5$, and $\overline{\lambda} = \overline{Q} = \overline{\nu} = \overline{\eta} = 1.0$.

a magnetized medium (line 2, 3, 4). It is also noted that the growth rate is decreased with increasing magnetization of the medium. Hence, we conclude that the increasing magnetic field tends to stabilize the system.

V. CONCLUSION

In the present study, we have investigated the problem of the Jeans instability of a viscous quantum plasma, considering the effect of finite electrical resistivity, thermal conductivity, and the radiative heat-loss function. The general dispersion relation is obtained, which is modified due to the presence of these parameters. This dispersion relation is reduced for longitudinal and transverse modes of propagation. It is found that viscosity has a damping effect and stabilizes the system in both the longitudinal and transverse modes of propagation.

In the case of longitudinal wave propagation, it is found that the Alfvén mode is modified due to electrical resistivity and viscosity. The condition of the Jeans instability is modified due to thermal conductivity, the radiative heat-loss function, and the quantum correction.

In the transverse mode of propagation, the radiative instability of a self-gravitating quantum plasma is affected by the magnetic field only when the medium is perfectly conducting. For a resistive medium the condition of instability remains unaffected by a magnetic field.

From the curves, it is observed that viscosity, the magnetic field, and the temperature dependent heat-loss function have a stabilizing effect, while finite electrical resistivity shows a destabilizing effect on the growth rate.

References

[1] J. H. Jeans, *Astronomy and Cosmogony* (Cambridge university press, Cambridge, UK, 1929).
[2] S. Chandrashekhar, *Hydrodynamics and Hydromagnetic Stability* (Clarendon Press, Oxford, 1961).
[3] R. K. Chhajlani, and R. K. Snaghavi, Astrophy. Space Sci. **142**, 341 (1985).
[4] W. D. Langer, Astrophys. J. **225**, 95 (1978). doi: 10.1086/156471
[5] A. A. Mamun, Physics of Plasma **5**, 3542 (1998). doi: 10.1063/1.873071
[6] M. KaKati, and K. S. Goswami, Pramana **54**, 771 (2000). doi: 10.1007/s12043-000-0122-1
[7] G. B. Field, Astrophys. J. **142**, 531 (1965). doi: 10.1086/148317
[8] P. Hennebelle, and M. Perrault, Astron. Astrophys. **359**, 1124 (2000).
[9] S. Inutsuka, H. Koyama, and T. Inoue, AIP conf. Proc. **784**, 318 (2005).
[10] K. Menou, A. S. Balbus, and C. H. Spruit, Astrophys. J. **607**, 564 (2004). doi: 10.1086/383463
[11] H. Stiele, H. Lesch, F. and Heitsch, Mon. Not. Roy. Astron. Soc. **372**, 862 (2006). doi: 10.1111/j.1365-2966.2006.10909.x
[12] T. Fukue, H. and Kamaya, Astrophys. J. **669**, 363 (2007). doi: 10.1086/521268
[13] M. Aggarwal and S. P. Talwar, Mon. Not. Roy. Astro. Soc. **146**, 235 (1969).
[14] M. P. Bora and S. P. Talwar, Phys. Fluids **B5(3)**, 950 (1993).

[15] R. P. Prajapati, R. K. Pensia, S. Kaothekar, and R. K. Chhajlani, Astrophys. Space Sci. **327**, 139 (2010). doi: 10.1007/s10509-010-0273-6

[16] S. Kaothekar and R. K. Chhajlani, J. Phys.: Conf. Ser. **365,** 012053 (2012). doi: 10.1088/1742-6596/365/1/012053

[17] B. K. Dangarh, R. K. Pensia, V. Shrivastava, and V. Prajapat, Adv. Studies Theor. Phys. **5(16)**, 755 (2011).

[18] J. Cho, A. Lazarian, and E. T. Vishniac, Astrophysical J. **595**(2), 812 (2003).

[19] S. Kaothekar, and R. K. Chhajlani, ISRN Astronomy and Astrophysics, Vol. **2012**, Article ID 420938, 14 pages. doi: 10.5402/2012/420938

[20] F. Haas, Phys. Plasmas **12**, 062117 (2005). doi: 10.1063/1.1939947

[21] J. Lundin, M. Marklund, and G. Brodin, Phys. Plasma **15**, 072116 (2008). doi: 10.1063/1.2956641

[22] H. Ren, Z. Wu, J. Cao, and P. K. Chu, Phys. Plasmas **16**, 072101 (2009). doi: 10.1063/1.3168612

[23] W. Masood, M. Sallimullah, and H. A. Shah, Phys. Lett. A **372**, 6757 (2008). doi: 10.1016/j.physleta.2008.09.034

[24] M. Sallimullah, M. Jamil, H. A. Shah, and G. Murtaza, Phys. Plasmas **16,** 014502 (2009). doi: 10.1063/1.3070664

[25] R. P. Prajapati and R. K. Chhajlani, Phys. Scr. **82,** 055003 (2010). doi: 10.1088/0031-8949/82/05/055003