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Dynamics of the Quantum Correlation of Qubits Dissipating into a Common Environment

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In this paper, we have investigated the dynamics of the quantum correlation between two qubits of a system composed of an arbitrary number of qubits dissipating into a common environment. Our results imply that, when the initial state of the system contains one excitation, the quantum correlation of the quantum discord and the entanglement of formation can be generated between two qubits which have no interaction with each other but are interacting with a common environment. From our analysis, we find that the dynamics of the quantum discord and the entanglement of formation depends not only on the scaled time but also on the size of the system. The larger the size of the system is, the less the quantum discord and the entanglement of formation is generated. By using a comparison between the quantum discord and the entanglement of formation, we find that the quantum discord is more robust to the size of the system than the entanglement of formation is, in the sense that the former decreases more slowly with an increase of the size of the system than the latter does. In addition, the case of the initial state with two excitations is discussed.

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Entanglement plays an important role in quantum information processing [1], because of its unique property in the sense that it has no classical counterpart, and it has been applied to quantum teleportation [2] and quantum communication [3]. A lot of work concerning entanglement has been done [4–10]. The authors in Ref. [4] studied the entanglement dynamics in a double Jaynes-Cummings model, and the entanglement generation in multi-atom systems was investigated in Ref. [5]. The entanglement of a three-particle system and the entanglement of a high-dimension bipartite system was studied in Refs. [6] and [7], respectively. The authors in Ref. [8] made an analysis of entanglement measure with matrix representations, and the problem of quantum correlation as witness of quantum phase transition is studied in Refs. [9] and [10]. Even though the entanglement has been studied and applied to quantum information processing extensively, yet there are some exceptions. Quantum computation based on one pure qubit state does not need entanglement but it employs other quantum correlation resources. For instance, quantum discord (QD) is one kind of such quantum correlation [11–17]. Quantum discord was proposed by Zurek first [11] and has gained much attention from physicists recently [12–18], because some quantum information processing can be performed without entanglement. As an extension of the difference of two definitions of the mutual information from classical

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information theory to a quantum version, quantum discord was demonstrated as one basic quantum correlation measure in the quantum information theory. The basic difference of the quantum discord from entanglement is evident from that fact that the quantum discord is non-zero in certain separable states which are unentangled by definition.

In order to study the quantum discord of quantum states, we first give the definition of quantum discord. The quantum version of the mutual information for the correlation between the two parties of the density matrix ρ_{XY} is given by the following expressions, as noted in Ref. [11]:

$$I(\rho_X : \rho_Y) = S(\rho_X) + S(\rho_Y) - S(\rho_{XY}), \tag{1}$$
$$J(\rho_{XY}) = S(\rho_X) - S(\rho_X|\rho_Y), \tag{2}$$

where $S(\rho) = -\mathrm{Tr}(\rho \log \rho)$ is the von Neumann entropy of ρ and $\rho_X(\rho_Y)$ is the reduced density matrix of ρ_{XY} obtained by tracing out Y (X), and $S(\rho_X|\rho_Y)$ is the quantum conditional entropy. The classical counterparts of these two expressions of mutual information are equivalent to using Bayes's rule. It should be noted that the conditional entropy of $S(\rho_X|\rho_Y)$ depends on the choice of measurement from a set of projectors $\hat{\Pi}_i$. Performing the measurement locally on the party Y, we obtain the state of the system $\rho_i = \hat{\Pi}_i^Y i \rho_{XY} \hat{\Pi}_i^Y / p_i$ and $p_i = \mathrm{tr}(\hat{\Pi}_i^Y i \rho_{XY} \hat{\Pi}_i^Y)$. With the definition of the quantum conditional entropy, we obtain the quantum mutual information:

$$J(\rho_{XY} : \{\hat{\Pi}_i\}) = S(\rho_X) - S(\rho_{XY}\{\hat{\Pi}_i\}). \tag{3}$$

Henderson and Vedral [12] have shown that the maximum of $J(\rho_{XY} : \{\hat{\Pi}_i\})$ is a measure of the classical correlations $C(\rho_{XY})$, that is $C(\rho_{XY}) = \max_{\{\hat{\Pi}_i\}} J(\rho_{XY} : \{\hat{\Pi}_i\})$. The quantum discord is defined by the difference between the total correlations $I(\rho_{XY})$ and the classical correlations $C(\rho_{XY})$ in the following expression:

$$Q(\rho_{XY}) = I(\rho_{XY}) - C(\rho_{XY}). \tag{4}$$

Usually, due to decoherence, entanglement and quantum discord are considered fragile under the interaction between the environment and system. However, in recent years, the authors in [19] have shown that the entanglement can persist for a long time up to stationary conditions, and the long-living entanglement experimental realization was made by the authors in [20]. Specially, the authors in [21, 22] have proved that the environment can play a constructive role in establishing quantum entanglement. More recently, the authors in [23] considered the entanglement dynamics for qubits dissipating into a common environment, and found that the entanglement can be established between two qubits which have no interaction with each other but are interacting with a common environment. Beyond the entanglement, what will happen to the quantum discord when the systems are dissipating into a common environment? In this paper, we will examine the dynamics of the quantum discord for qubits dissipating into a common environment. The content is arranged as follows. In Sec. I, we introduce the model and derive the time evolution of the quantum states. In Sec. II, the main results are presented and analyzed in detail. Finally, we conclude our results in Sec. III.

I. DYNAMICS OF QUANTUM STATES

In order to study the dynamics of the quantum correlation measured by the quantum discord and entanglement, we should know the time evolution of the quantum states. Consider a system of n qubits dissipating into a common environment at zero temperature, we get the differential equation as the Lindblad master equation [24]:

$$\dot{\rho}(t) = \gamma\left(2\sigma\rho(t)\sigma^\dagger - \sigma^\dagger\sigma\rho(t) - \rho(t)\sigma^\dagger\sigma\right) \equiv \hat{D}\rho(t), \tag{5}$$

where $\sigma \equiv \sum_{i=1}^{n} \sigma_i$, $\sigma_i = |0\rangle\langle 1|$ for the ith qubit, $|0\rangle, |1\rangle$ are the ground and excited states for a single qubit, and γ is the dissipation rate. This master equation can be solved analytically with an appropriate choice of the initial state, as given in [23]. Our consideration is focused on the case that the initial state of the system is $|1_k\rangle$, where $|1_k\rangle$ denotes that the kth qubit is in the excited state while all the other qubits are in the ground state. Then $\rho(0) = |1_k\rangle\langle 1_k|$. Applying $\hat{D}$ to the initial state $\rho(0)$, we get the following closed relation in the following expressions with the relations for the ith qubit: $\sigma_i|0\rangle_i = 0, \sigma_i|1\rangle_i = |0\rangle_i, \sigma^\dagger|1\rangle_i = 0$.

$$\begin{aligned}
&\hat{D}|G\rangle\langle G| = 0, \quad \hat{D}|1_k\rangle\langle 1_k| = \gamma\left[2|G\rangle\langle G| - 2|1_k\rangle\langle 1_k| - (|A_k\rangle\langle 1_k| + |1_k\rangle\langle A_k|)\right],\\
&\hat{D}|A_k\rangle\langle A_k\rangle = \gamma\left[2(n-1)^2|G\rangle\langle G| - 2(n-1)|A_k\rangle\langle A_k| - (n-1)(|A_k\rangle\langle 1_k| + |1_k\rangle\langle A_k|)\right],\\
&\hat{D}(|A_k\rangle\langle 1_k| + |1_k\rangle\langle A_k|) = \gamma\left[4(n-1)|G\rangle\langle G| - 2|A_k\rangle\langle A_K| - 2(n-1)|1_k\rangle\langle 1_k|\right.\\
&\qquad\qquad\qquad\qquad\qquad\left. -n(|A_k\rangle\langle 1_k| + |1_k\rangle\langle A_k|)\right],
\end{aligned} \tag{6}$$

where the state of $|G\rangle$ is defined as that where all of the qubits are in the ground state and the state of $|A_k\rangle$ is defined by $|A_k\rangle = \sum_{i\neq k}^{n} |1_i\rangle$ with $|1_i\rangle$ as the state in which the ith qubit is in the excited state and all the others are in the ground state. With the above expressions, we can obtain the time evolution of the n-qubit system in the following equations.

$$\rho(t) = c_1(t)|G\rangle\langle G| + c_2(t)|1_k\rangle\langle 1_k| + c_3(t)|A_k\rangle\langle A_k| + c_4(t)(|A_k\rangle\langle 1_k| + |1_k\rangle\langle A_k|). \tag{7}$$

With Equations (5-7), we obtain the following differential equations:

$$\begin{aligned}
\dot{c}_1(t) &= \gamma[2c_2(t) + 2(n-1)^2c_3(t) + 4(n-1)c_4(t)],\\
\dot{c}_2(t) &= \gamma[-2c_2(t) - 2(n-1)c_4(t)], \quad \dot{c}_3(t) = \gamma[-2(n-1)c_3(t) - 2c_4(t)],\\
\dot{c}_4(t) &= \gamma[-c_2(t) - (n-1)c_3(t) - nc_4(t)].
\end{aligned} \tag{8}$$

As we have set that the initial conditions have $c_2(0) = 1$ and the other coefficients of $c_1(0), c_3(0), c_4(0)$ being 0, we get the time evolution of the coefficients in the following expressions as analyzed in [23].

$$\begin{aligned}
c_1(t) &= \lambda(t)[2 - n\lambda(t)], \quad c_2(t) = [1-\lambda(t)]^2,\\
c_3(t) &= \lambda(t)^2, \quad c_4(t) = \lambda(t)^2 - \lambda(t),
\end{aligned} \tag{9}$$

where the parameter of $\lambda(t)$ is the time evolution parameter characterizing the time evolution of the quantum correlation measured by the quantum discord and quantum entanglement; it takes the expression $\lambda(t) = \frac{1}{n}\left(1 - \mathrm{e}^{-n\gamma t}\right)$. In order to understand the dependence of the parameter of $\lambda(t)$ on the parameters $n, \gamma t$ we plot Fig. 1. From the Fig.1, $\lambda(t)$ is zero

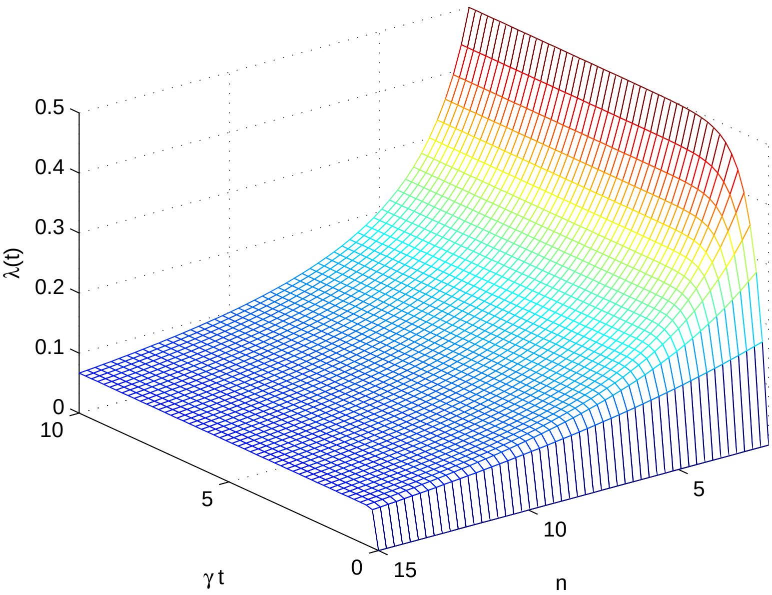

FIG. 1: $\lambda(t)$ versus the number of the system n and the time γt that is plotted.

initially and increases to a stable value with increasing time γt for a certain number of the system n. The larger the number of the system n is, the smaller the parameter of $\lambda(t)$ is.

II. MAIN RESULTS

In order to study the dynamics of the quantum discord, we should first calculate the quantum discord. For a given state with the density matrix in the following expression,

$$\rho(t) = \begin{pmatrix} \rho_{11} & 0 & 0 & 0 \\ 0 & \rho_{22} & \rho_{23} & 0 \\ 0 & \rho_{32} & \rho_{33} & 0 \\ 0 & 0 & 0 & \rho_{44} \end{pmatrix} \tag{10}$$

the quantum discord of the above density matrix can be obtained with the analytical results in [25, 26].

$$Q = \min\{Q_1, Q_2\}, \tag{11}$$

where Q_1, Q_2 take the following expressions:

$$\begin{aligned} Q_1 &= h[\rho_{11}+\rho_{33}] + \sum_{k=1}^{4} \delta_k \log_2 \delta_k + h[\tau], \\ Q_2 &= \sum_{1}^{k=4} \delta_k \log_2 \delta_k - \sum_{k=1}^{4} \rho_{kk} \log_2 \rho_{kk}, \end{aligned} \tag{12}$$

with δ_k being the eigenvalues of $\rho(t)$, $h[\tau] \equiv -\tau \log_2 \tau - (1-\tau)\log_2(1-\tau)$, and $\tau = [1+\sqrt{[1-2(\rho_{33}+\rho_{44})]^2 + 4|\rho_{23}|^2}]/2$. Beyond the quantum discord, the entanglement of formation should be introduced here. For the given state in Eq. (10), the entanglement of formation (EoF) as a function of concurrence can be calculated as [27].

$$E(\rho) = -\Theta \log_2 \Theta - (1-\Theta)\log_2(1-\Theta) \equiv h[\Theta], \tag{13}$$

where $\Theta = (1+\sqrt{1-C^2})/2$, with the concurrence C taking the expression $C = 2\max\{0, |\rho_{23}| - \sqrt{\rho_{11}\rho_{44}}\}$. In order to understand the relation between the quantum discord and the entanglement of formation, we employ an example to illustrate their behavior on a state which is defined as $\rho = (1-\alpha)|00\rangle\langle 00| + \alpha|\psi\rangle\langle\psi|$ with α being the parameter to characterize the noise and $|\psi\rangle$ reading $|\psi\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$. After a careful calculation, we obtain the quantum discord and the entanglement of formation of the state in the following expression with the above definition of the quantum correlation measures.

$$Q_w = \min\{Q_1, Q_2\}, \tag{14}$$

$$Q_1 = h\left[1-\frac{\alpha}{2}\right] - h[\alpha] + h[\tau_0], \qquad Q_2 = \alpha, \tag{15}$$

$$E_w = h\left[\frac{1+\sqrt{1-\alpha^2}}{2}\right], \tag{16}$$

where α ranges from 0 to 1 and $\tau_0 = \left[1+\sqrt{2\alpha^2-2\alpha+1}\right]/2$. To compare the behaviors of the quantum discord and the entanglement of formation, we plot Fig. 2. The results presented in Fig. 2 can shed some light on the relation between the quantum discord and the entanglement of formation in an explicit way by using an explicit example. From Fig. 2, we can find that when the parameter of α ranges from 0 to a value of about 0.6664, the quantum discord is larger than the entanglement of formation. While when the parameter of α ranges from 0.6664 to 1, the entanglement of formation is larger than the quantum discord. When the parameter of $\alpha = 0.6664$, both the quantum discord and the entanglement of formation take the same value, about 0.5497. Specially, when α takes a value 0 or 1, the quantum discord and the entanglement of formation coincide with each other.

With the analytical expressions of the quantum correlation measured by the quantum discord and the entanglement of formation, we can analyze the dynamics of the quantum

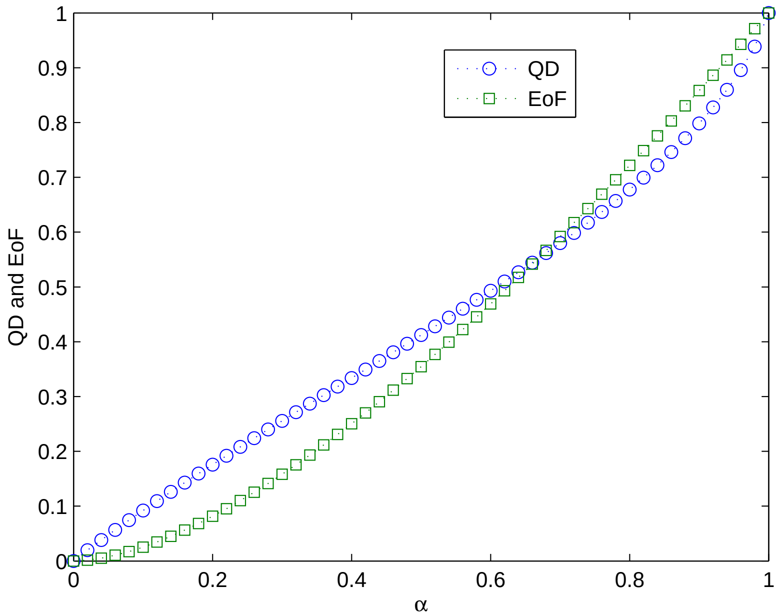

FIG. 2: Quantum correlations (QD, EoF) versus the parameter α is plotted.

correlation of the states dissipating into a common environment. Here, we firstly consider the case of the quantum correlation between the kth qubit in the excited state initially and the jth qubit in the ground state initially. With time evolution of the density matrix of the n-qubit system, the reduced density matrix of two qubits (k, j) can be obtained with Eqs. (7–9).

$$\begin{aligned}\rho_{k,j}(t) &= 2\lambda(t)(1-\lambda(t))|00\rangle\langle 00| + \lambda(t)^2|01\rangle\langle 01| \\ &+ (1-\lambda(t))^2|10\rangle\langle 10| - \lambda(t)(1-\lambda(t))(|01\rangle\langle 10| + |10\rangle\langle 01|). \end{aligned} \tag{17}$$

Through calculation, the quantum discord can be obtained as

$$Q_a(t) = \min\{Q_1(t), Q_2(t)\}, \tag{18}$$

where $Q_1(t), Q_2(t)$ read

$$\begin{aligned}Q_1(t) &= h[1-\lambda(t)^2] - h[2\lambda(t) - 2\lambda(t)^2] + h[\tau_1], \\ Q_2(t) &= (1-2\lambda(t)+2\lambda(t)^2)\log_2(1-2\lambda(t)+2\lambda(t)^2) \\ &\quad -\lambda(t)^2\log_2\lambda(t)^2 - (1-\lambda(t))^2\log_2(1-\lambda(t))^2, \end{aligned} \tag{19}$$

with $\tau_1 = \left[1+\sqrt{[1-2(1-\lambda(t))^2]^2 + 4\lambda(t)^2(1-\lambda(t))^2}\right]/2$. The entanglement of formation takes the following expression

$$E_a(\rho_{k,j}(t)) = h\left[\frac{1+\sqrt{1-4\lambda(t)^2(1-\lambda(t))^2}}{2}\right]. \tag{20}$$

It should be noted that the index of a denotes the first case considered by us. To under-

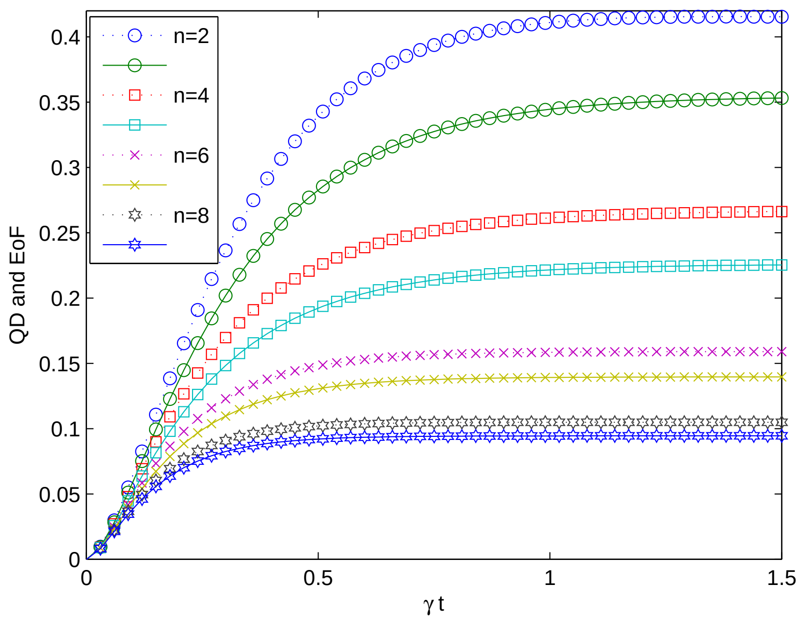

FIG. 3: Quantum correlations (QD, EoF) versus the time γt for different sizes of the system is plotted with dashed lines for the QD and solid lines for the EoF, respectively.

stand the behaviors of the quantum correlation measured by the quantum discord and the entanglement of formation, we plot Fig. 3. From Fig. 3, we can find that the quantum correlation increases with an increase of the scaled time γt to a stable value for a certain number of the size of the system. The quantum discord increases faster and reaches a larger stable value than the entanglement of formation does with an increase of time. For the effect of the size of the system on the quantum correlation, the larger the size of the system is, the smaller is the stable value which the quantum correlation can reach. By a comparison between the quantum discord and the entanglement of formation, the stable values of the quantum discord and the entanglement of formation are different for a given size of the system. The value of the quantum discord is larger than that of the entanglement of formation. This point is consistent with the analysis after Fig. 2, as the reduced density matrix takes a similar form to the example in Fig. 2 and the largest stable value is less than 0.5. The difference between the stable values of the quantum discord and the entanglement of formation become small with an increase of the size of the system. When the size of the system takes a value large enough, the quantum discord and the entanglement of formation will coincide with each other. This point can also be found from the Fig. 3, in which we consider the case where the time takes an infinite value and the function of $\lambda(t)$ takes the expression that $\lambda(\infty) = 1/n$. This corresponds to the case that the parameter of α in Fig. 2 takes a value small enough. The effect of size on the quantum correlation between the kth qubit and jth qubit is illustrated in Fig. 4. From Fig. 4, we can get more

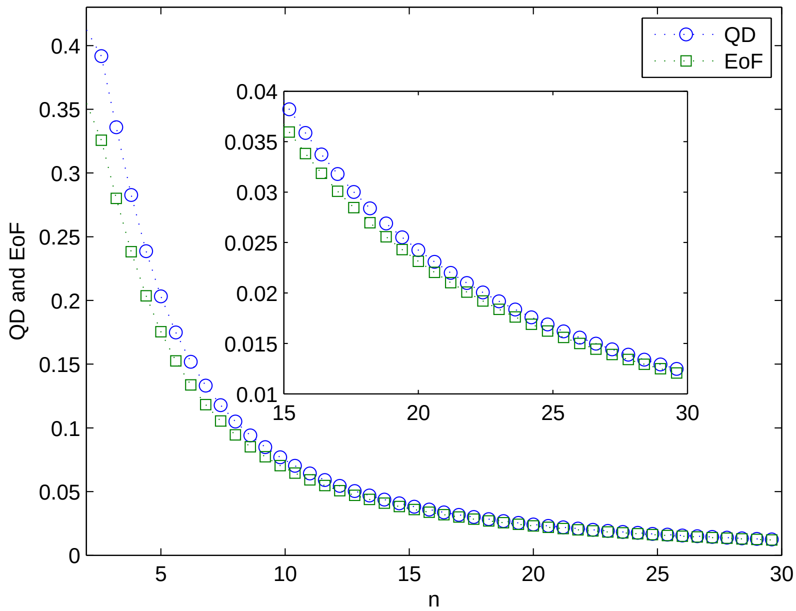

FIG. 4: Quantum correlations (QD, EoF) versus the size of n is plotted when the time takes an infinite value.

information on the effect of size on the quantum correlations of the quantum discord and the entanglement of formation. The value of the quantum discord is larger than that of the entanglement of formation for a given size of the system. Even though the quantum discord and the entanglement of formation will coincide with each other when the size of the system is large enough. The quantum discord and the entanglement of formation become close with the size becoming large. This is consistent with the analysis in Fig. 2, that when the parameter α takes a value small enough, the quantum discord and the entanglement of formation become close. In fact, when the size of the system is larger than 30, the values of the quantum discord and the entanglement of formation can be negligible as they are too small. In this sense, we can say that the quantum discord is more robust against the size of the system than the entanglement of formation.

Secondly, we consider the quantum correlations between the jth qubit and the mth qubit ($j \neq k, m \neq k$) when the initial state is $|1_k\rangle$, which is the state that the kth qubit is in the excited state and all the others qubits are in the ground state. After tracing the $n-2$ qubits including the kth qubit off the density matrix $\rho(t)$, we obtain the reduced density matrix $\rho_{j,m}(t)$ in the following expression with Eq. (7–9):

$$\rho_{j,m}(t) = [1 - 2\lambda(t)^2]|00\rangle\langle 00| + \lambda(t)^2(|01\rangle + |10\rangle)(\langle 01| + \langle 10|). \tag{21}$$

The time evolution of the quantum correlation measured by the quantum discord and the entanglement of formation can be calculated, and the expressions of quantum correlation read as follows:

$$\begin{aligned} Q_b &= \min\left\{2\lambda(t)^2, h[\lambda(t)^2] - h[2\lambda(t)^2] + h[\tau_2]\right\}, \\ E_b(\rho_{j,m}(t)) &= h\left[\frac{1+\sqrt{1-4\lambda(t)^4}}{2}\right], \end{aligned} \tag{22}$$

where the parameter of τ_2 reads $\tau_2 = \left[1 + \sqrt{8\lambda(t)^4 - 4\lambda(t)^2 + 1}\right]/2$. To understand the

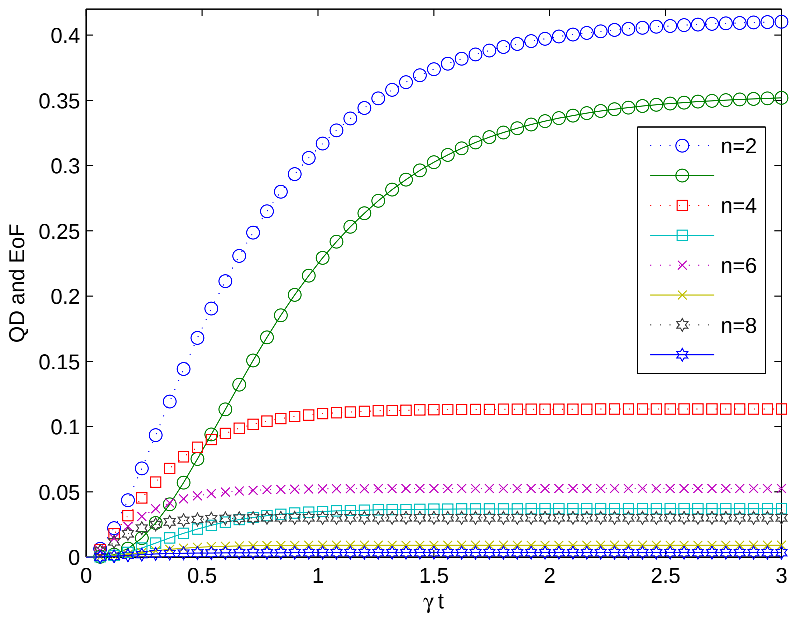

FIG. 5: Quantum correlations (QD, EoF) versus the time γt for different sizes of the system is plotted with dashed lines for the QD and solid lines for the EoF, respectively.

effect of time and the size of the system on the quantum correlation between the (j, m) qubits, we plot Fig. 5. From Fig. 5, we can find that the quantum correlation of the quantum discord and the entanglement of formation between the (j, m) qubits can be established by the common environment. The behavior of the quantum correlation between the (j, m) qubits is similar to that of the quantum correlation between the (k, j) qubits to a large extent. However, there are some differences. The first point is the stable values which the quantum correlation can reach are different for a given size n of the system when the size of n is larger than 2. The value of the quantum correlation between the (k, j) qubits is larger than that between the (j, m) qubits for a given size larger than 2. The second point is that the time needed for the quantum correlation reaching the stable values are different. The time of the quantum correlation between the (j, m) qubits reaching the stable values is longer than that of the quantum correlation between the (k, j) qubits for a given size of the system. Combining these two points, we can conclude that the quantum correlation between the (k, j) qubits is more robust against the size of the system than that between the (j, m) qubits.

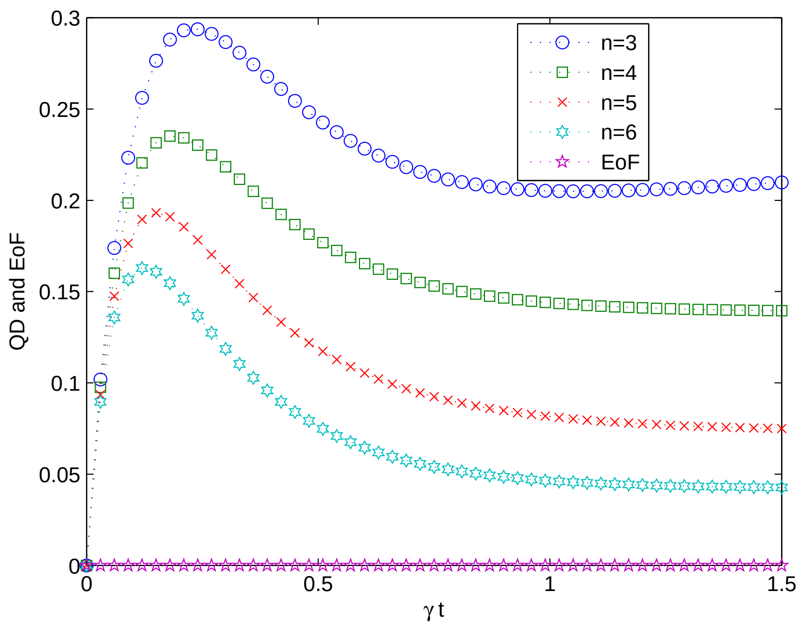

FIG. 6: Quantum correlations (QD, EoF) versus the time γt for different sizes of the system are plotted.

Thirdly, we consider the case that the initial state consists of two excitations. That is, the kth qubit and the lth qubit are in the excited state, while all the other qubits are in the ground state. The initial state is denoted by $\rho(0) = |1_k, 1_l\rangle\langle 1_k, 1_l|$. Under this condition, three circumstances including the quantum correlation between (k, l) qubits, the (k, j) qubits, and the (j, m) qubits should be considered here. Due to the troublesome algebra and complicated expressions involved in the calculation here the expressions of the time evolution of the quantum correlation are omitted for convenience, and we will employed the results obtained in [23] to examine the quantum correlation measured by the quantum discord and the entanglement of formation between any two-qubit subsystem of the system. Now, we consider the quantum correlation between the (k, l) qubits when the initial state of the system is $\rho(0) = |1_k, 1_l\rangle$ and the results are presented in Fig. 6. From Fig. 6, we can find that there is no entanglement of formation between the kth qubit and the lth qubit at any time. This can also be said that the kth qubit is separable with the lth qubit. However, the quantum discord between the kth qubit and the lth qubit can be generated due to the common environment. This further confirms the fact that some untangled states contain quantum discord [11]. The quantum discord increases from zero initially to a maximum and then decreases to a stable value for a given size of the system. The larger the size of the system is, the smaller the maximum is, and the smaller the stable value is. With regard to the quantum correlation measured by the quantum discord and the entanglement of formation of the (k, j) qubits and the (j, m) qubits when the initial state consists of two excitations, our simulation results imply that the behavior of the quantum correlation of the qubits (k, j) and the (j, m) qubits are similar to the case of the initial state with only one qubit in the excited state as analyzed before to a large extent. The difference lies in the stable values of quantum correlation. The stable values of quantum correlation for the case of the initial state with only one qubit in the excited state is larger than that of the case of the initial state with two qubits in the excited state. Therefore, we will omit the discussion here.

III. DISCUSSION AND CONCLUSIONS

To conclude, we have investigated the dynamics of the quantum correlation measured by the quantum discord and the entanglement of formation for the qubits dissipating into a common environment. Our results imply that both the quantum discord and the entanglement of formation can be established between the qubits which have no interaction with each other but interact with the environment independently when the initial state is appropriately chosen. For the quantum correlation of the two qubits which are initially in the excited state, we find that there is no entanglement at any time, but there is quantum discord which can be generated during the evolution. Our analysis could be researched experimentally in the context of trapped ions [28] and can also be relevant for the driving cavity quantum electric dynamics (QED) experiments [29]. In a word, our study can contribute to some understanding of the dynamics of the quantum correlation of qubits dissipating into a common environment.

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