Evolution of the Universe with Time Varying Constants

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In this paper, we discuss the evolution of the universe by considering Λ and G as time varying constants with different choices of Λ. We choose the polytropic equation of state as the total energy content and find approximate values of the constant parameters for the cold dark matter and radiation components of the fluid of the universe. We also mention different possibilities for the constant parameters in order to interpret quintessence, vacuum, and phantom dark energy phases of the universe. Also, we find the conditions on the different constant parameters under which the behavior of G and Λ is consistent with the present observations. Finally, we calculate the statefinder parameters in this scenario.

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I. INTRODUCTION

Modern cosmology inherits an interesting discovery of the accelerating expansion of the present universe [1]. Recent observations [2]–[7] suggest that most of the energy is in the form of dark energy (DE) which contains 2/3 of the entire energy content in the universe. The dark energy has a large negative pressure in order to accelerate the expansion of the universe. Despite the wealth of observational schemes, cosmology still bears the unknown identity of DE.

In order to identify this unknown quantity, many approaches and models have been proposed. The equation of state parameter ω describes different dominant components of the universe, e.g., $\omega = \frac{1}{3},\ \omega = 0,\ \omega = -1,\ -1 < \omega < -1/3$, and $\omega < -1$ corresponding to radiation, cold dark matter (CDM), vacuum, quintessence, and phantom, respectively. The polytropic EoS is also used to explain the acceleration of the expanding universe. Also, it is consistent with relativistic models in stellar astrophysics [8]. This model possesses two applications: firstly pressure is dominated by degenerate electrons in white dwarfs or degenerate neutrons in neutron stars; secondly, pressure and density are linked adiabatically in main sequence stars [9]. It has been used for various astrophysical aspects like in the Lane-Emden models [10].

The idea of the variation of fundamental physical constants was first suggested by Dirac [11] through the "Large Number Hypothesis" (LNH). A time dependent G was also

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a natural consequence of this hypothesis. Brans-Dicke theory [12] also originated from the variable-G theories in which the gravitational constant is replaced by a scalar field coupling to gravity through a new parameter. Lau and Prokhovnik [13] reconciled Dirac's LNH with general relativity by introducing a time dependent cosmological term Λ in the field equations. Some people [14] argued that G can be taken as varying, being a function of time or equivalently of the scale factor. Further, a varying G is useful in exploring the dark matter (DM) problem [15], the controversies of the Hubble parameter value [16] and the cosmic coincidence problem [17].

On the other hand, the variation of Λ has a major role in the evolution of the universe. Its cosmic history shows that it decreases with the passage of time [18]. Belinchon [19] used dimensional analysis and showed that Λ and G decrease with time. It was argued that the evolution of variable cosmic constants are dependent on each other [20]. Arbab [21] proved that G increases and Λ decreases with the passage of time. He discussed these variables in a phantom universe (in which DE violates the strong energy conditions). Sharif and Kausar [22] discussed the behavior of G and Λ by using a symmetry approach in different anisotropic models. Sharif and Khanum [23] examined the cosmological parameters by taking variable G and Λ in a Kaluza-Klein cosmology. Recently, Sharif and Jawad [24, 25] explored different cosmological parameters in the presence of a modified holographic DE by taking G as a variable in flat and non-flat Kaluza-Klein universes.

Chakraborty and Debnath [26] investigated the behavior of G, Λ, and statefinder in the scenario of a modified Chaplygin gas by assuming three different forms of Λ. They found that the statefinder parameters describe the universe from the radiation era to the ΛCDM model in the presence of time varying constants. Jamil *et al.* [27] considered Λ in terms of a modified Chaplygin gas and found that Λ and G exhibit decreasing and increasing behavior with time, respectively. It would be interesting to explore the consistency of behavior of time varying constants and statefinder parameters with the present status of the universe.

In this paper, we explore the behavior of time varying constants G and Λ by taking a polytropic EoS in order to describe the total energy density. The scheme of the paper is as follows: Section II provides the basic formalism and discussion of three models for Λ. Section III is devoted to the discussion of the statefinder parameters for the three models of Λ only. The last section summarizes the results.

II. EINSTEIN FIELD EQUATIONS AND COSMOLOGICAL PARAMETERS

The spacetime for a spatially flat, isotropic and homogeneous FRW universe is given by

$$ds^2 = dt^2 - a(t)^2[dr^2 + r^2(d\theta^2 + \sin^2\theta d\phi^2)], \tag{1}$$

where $a(t)$ is a dimensionless scale factor representing the relative expansion of the universe. The corresponding equations of motion are obtained as follows:

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G(t)}{3}\rho_{tot} + \frac{\Lambda(t)}{3}, \tag{2}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G(t)}{3}(\rho_{tot} + 3p_{tot}) + \frac{\Lambda(t)}{3}. \tag{3}$$

Here ρ_{tot} represents the total energy density, which includes the DM and DE densities of the universe while p_{tot} indicates corresponding total pressure. Also, a dot indicates derivative with respect to time and $H \equiv \dot{a}/a$ is the Hubble parameter which estimates the expansion rate of the universe. The corresponding energy conservation yields

$$\dot{\rho}_{tot} + 3H(\rho_{tot} + p_{tot}) = 0. \tag{4}$$

The Bianchi identities along with the above expression lead to the following relation between Λ and G:

$$\rho_{tot}\dot{G} + \frac{\dot{\Lambda}}{8\pi} = 0. \tag{5}$$

We assume a polytropic gas model describing the total energy content whose EoS is given by

$$p_{tot} = K\rho_{tot}^{1+\frac{1}{n}}, \tag{6}$$

where K is a constant and n is any real number termed the polytropic index. Using Eq. (6) in (4), it follows that

$$\rho_{tot} = (Ba^{\frac{3}{n}} - K)^{-n}, \tag{7}$$

where B is an integration constant. For positivity of the total energy density, we choose n to be an even positive number. The corresponding pressure takes the following form

$$p_{tot} = K(Ba^{\frac{3}{n}} - K)^{-n-1}. \tag{8}$$

Inserting the values of ρ_{tot} and p_{tot} in the EoS $p_{tot} = \omega_{tot}\rho_{tot}$, we get the dimensionless EoS parameter w_{tot} as

$$\omega_{tot} = \frac{p_{tot}}{\rho_{tot}} = -1 - \frac{Ba^{\frac{3}{n}}}{K - Ba^{\frac{3}{n}}}. \tag{9}$$

or

$$\omega_{tot} = -1 - \frac{B(1+z)^{-\frac{3}{n}}}{K - B(1+z)^{-\frac{3}{n}}} = \frac{K}{B(1+z)^{\frac{-3}{n}} - K}, \tag{10}$$

where we have used $a = (1+z)^{-1}$. In general, this is an EoS parameter for the total energy density which characterizes different phases of the universe. For $K = 0$, we obtain an era where the universe is filled with CDM.

Blake *et al.* [28] have discussed the cosmic expansion history by combining distant supernovae observations with a geometrical analysis of large-scale galaxy clustering within the WiggleZ DE Survey. They used the Alcock-Paczynski test to measure the distortion of standard spheres. They constituted a robust and non-parametric measurement of the Hubble expansion rate as a function of time under $10-15\%$ precision in four bins within the redshift range $0.1 < z < 0.9$. It was found that their results about the cosmic expansion is consistent with a cosmological-constant DE. Also, they established the result of total DM density, i.e., $\Omega_{m0} = 0.29 \pm 0.03$ at four values of redshift $z = 0.22,\ 0.41,\ 0.6,\ 0.78$, which is consistent with the result of Komatsu *et al.* [29] by using CMB radiation with $H_0 = 70.2 \pm 1.4$. For the radiation phase of the universe, the total matter density Ω_m is defined as

$$\Omega_m = \frac{8\pi G}{3H^2}\rho_m = \frac{8\pi G}{H^2}p_m = \frac{8\pi GK}{H^2}(B(1+z)^{-\frac{3}{n}} - K)^{-n-1}, \tag{11}$$

where ρ_m and p_m are the energy density and pressure for the radiation component of the fluid. Also $w_{tot} = w_m = \frac{p_m}{\rho_m} = \frac{1}{3}$ for this case.

We cannot find the values of the constant parameters $B,\ K,\ n$ from the above expression because G is varying with time. However, we can find the approximated values of these parameters from Eq. (10) (with $w_{tot} = w_m = \frac{1}{3}$) for the radiation phase in view of four redshift values $z = 0.22,\ 0.41,\ 0.6,\ 0.78$ suggested by Blake *et al.* [28]. The values are given in the Table I.

TABLE I: Parameters in the radiation-dominated era.

n	$z = 0.22$	$z = 0.41$	$z = 0.6$	$z = 0.78$
2	$B = 0.65,\ K = 0.12$	$B = 0.8,\ K = 0.12$	$B = 0.97,\ K = 0.12$	$B = 1.13,\ K = 0.12$
2	$B = 0.5,\ K = 0.093$	$B = 0.5,\ K = 0.0.075$	$B = 0.5,\ K = 0.062$	$B = 0.5,\ K = 0.053$
4	$B = 0.56,\ K = 0.12$	$B = 0.62,\ K = 0.12$	$B = 0.685,\ K = 0.12$	$B = 0.739,\ K = 0.12$
4	$B = 0.4,\ K = 0.086$	$B = 0.4,\ K = 0.077$	$B = 0.4,\ K = 0.07$	$B = 0.4,\ K = 0.0.065$

The polytropic EoS is also used for the DE scenario. In this regard, Karami *et al.* [30] have discussed its interacting scenario with CDM. They have found that its effective EoS gives phantom behavior (i.e., $\omega_\Lambda^{\text{eff}} < -1$ and Λ denotes the DE symbol) for $K > Ba^n$ and $n = 2, 3, 4, \ldots$ for positivity of the polytropic DE density. Further, Karami and Ghaffari [31] explored the validity of the generalized second law of thermodynamics in a non-flat FRW universe for an interacting polytropic DE with CDM. The reconstruction of scalar field DE models has also been found with interacting [32] and non-interacting [33] polytropic DE models. It is also found [34] that the polytropic DE model has a type III singularity i.e., as $a \to a_s = \left(\frac{K}{B}\right)^{\frac{n}{3}}$ then $\rho_\Lambda \to \infty$ and $p_\Lambda \to \infty$. Also, Capozziello *et al.* [35] discussed the dynamics of the universe through general DE EoS containing the polytropic part $A\rho_\Lambda^\alpha$ (A and α are constants) in general relativity, scalar-tensor theory, and modified F(R) gravity.

Motivated from the DE interpretation of the polytropic EoS, we can also find the corresponding DE eras of the universe. For example, from Eq. (10), we can achieve the quintessence phase of the universe for $K < 0$ and $B > 0$ with small values of z. For $B = 0$, it gives the vacuum region of the universe. For $K > B$ with $z \to 0$ and $B > 0$, we have a phantom phase of the universe. Recent study about the universe predicts that the universe is expanding with acceleration. In the following, we connect the behavior of the time varying constants G and Λ with the accelerated expansion of the universe. We obtain expressions for G and Λ by taking three choices of $\Lambda(t)$.

The cosmological constant Λ is the simplest candidate of DE but it suffers two major problems like 'fine tuning problem' and 'coincidence problem' [36]. In order to alleviate these problems macroscopically, Chen and Wu [37] proposed the dependence of Λ on the universal scale factor $a(t)$ and its first derivative. Carvalho *et al.* [38] generalized this proposal to the form

$$\Lambda = 3\beta H^2 + 3\alpha a^{-2}, \tag{12}$$

where α and β are dimensionless constants, and discussed different cosmological parameters of the universe. Arbab and Rahman [39] used this form in Ozer-Taha cosmology [40] and explored different phases of the universe. After that, a lot of parametrizations of Λ have been made, including $\Lambda \propto t^{-l},\ a^{-m},\ H^n,\ (\frac{\ddot{a}}{a})^r$ [41], and $\Lambda \propto \rho_{tot}$ [42]. Arbab [43] used three forms of $\Lambda \propto \rho_{tot},\ H^2,\ (\frac{\ddot{a}}{a})$ and investigated different cosmological parameters of the universe. Recently, Uzan [44] has explained various prospects for the existence of time varying constants.

Case I

First we take $\Lambda(t)$ in the form [43]

$$\Lambda = \frac{\beta_1}{\rho_{tot}^\gamma}, \tag{13}$$

where β_1 and γ are constants. It is interesting to mention here that for $\gamma = -1$, it reduces to the form used by Chakraborty and Debnath [26]. Using Eqs. (7) and (13) in (5), the parameters G and Λ take the form

$$G = -\frac{\beta_1\gamma}{8\pi(1+\gamma)}\left(Ba^{\frac{3}{n}} - K\right)^{n(1+\gamma)}, \quad \gamma \neq -1 \tag{14}$$

$$\Lambda = \beta_1\left(Ba^{\frac{3}{n}} - K\right)^{n\gamma}. \tag{15}$$

It is observed that G is positive and exhibits increasing behavior for $-1 < \gamma < 0$. On the other hand, Λ is also positive and shows decreasing behavior for $\gamma < 0$.

Case II

Here we use the ansatz [43]

$$\Lambda = \beta_2 H^2, \tag{16}$$

where β_2 is an arbitrary constant. Making use of Eqs. (16), (2), (7), and (5), we obtain

$$G = c_1 \left(Ba^{\frac{3}{n}} - K\right)^{\frac{n\beta_2}{3}}, \tag{17}$$

$$\Lambda = \frac{8\pi\ c_1\beta_2}{3-\beta_2} \left(Ba^{\frac{3}{n}} - K\right)^{-n(1-\frac{\beta_2}{3})}, \tag{18}$$

where c_1 is an integration constant and Λ is discontinuous at $\beta_2 = 3$. Also, $G > 0$ and increasing (for $\beta_2 > 3$) while $\Lambda > 0$ and decreasing for $\beta_2 < 3$.

Case III

In this case, we assume the cosmological constant as follows [43]:

$$\Lambda = \beta_3 \frac{\ddot{a}}{a}, \tag{19}$$

where β_3 is constant. Substituting Eqs. (19), (3) and (7) in (5), it gives

$$\begin{aligned} G &= c_2 \left(2 - \beta_3 - 3b_1\beta_3 \left(Ba^{\frac{3}{n}} - K\right)^{-1}\right)^{-\frac{n\beta_3}{6-3\beta_3} - \frac{n}{3}(1+\frac{1}{n})} \\ &\times \left(b_2 a^{\frac{3}{n}} - K\right)^{-\frac{n\beta_3}{6-3\beta_3}}, \end{aligned} \tag{20}$$

$$\begin{aligned} \Lambda &= -\frac{4\pi\beta_3 c_2}{3-\beta_3} \left(Ba^{\frac{3}{n}} - K\right)^{-\frac{n\beta_3}{6-3\beta_3}} \left[\left(Ba^{\frac{3}{n}} - K\right)^{-n} + 3b_1 \left(Ba^{\frac{3}{n}} - K\right)^{-n-1}\right] \\ &\times \left(2 - \beta_3 - 3K\beta_3 \left(Ba^{\frac{3}{n}} - K\right)^{-1}\right)^{-\frac{n\beta_3}{6-3\beta_3} - \frac{n}{3}(1+\frac{1}{n})}, \end{aligned} \tag{21}$$

with c_2 being another integration constant and Λ is discontinuous at $\beta_3 = 3$. It is noted that G is positive as n is even and exhibits increasing behavior for $\beta_3 > 2$ (since this value of β_3 gives $\frac{\beta_3}{6-3\beta_3} \gg \frac{1}{3}(1 + \frac{1}{n})$). Further, Λ is positive for $\beta_3 > 3$ and one cannot judge its behavior due to its complicated nature.

III. STATEFINDER PARAMETERS

Here, we discuss the cosmological diagnostic pair (r, s) known as the statefinder parameters [45]. These parameters are dimensionless and generalize the geometrical parameters like Hubble and the deceleration parameter. Due to their dependence on the cosmic

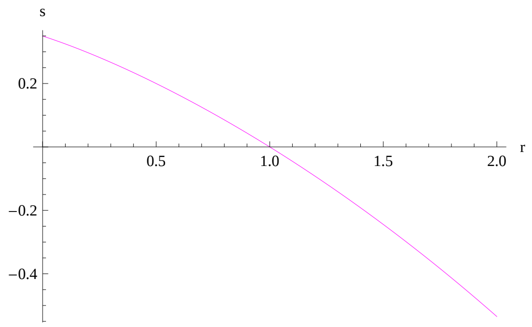

FIG. 1: Plot of s versus r for case I.

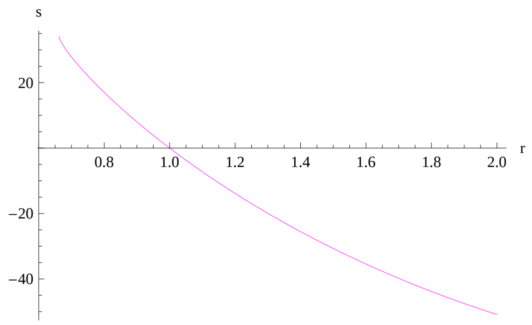

FIG. 2: Plot of s versus r for case II.

scale factor, they have great geometrical significance. The statefinder parameters are defined as

$$r = \frac{\dddot{a}}{aH^3}, \quad s = \frac{r-1}{3(q-1)}. \tag{22}$$

The trajectories in the (r, s) plane define different cosmological models, e.g., $(r, s) = (1, 0)$ corresponds to the ΛCDM model. For case I, these parameters are obtained by using Eqs. (2), (4), (5), (13), and (22) as

$$r = 1 - \frac{9}{2}\gamma(1+\omega_{tot})[K(1+\frac{1}{n})\omega_{tot} - (1+\omega_{tot})(1+\gamma)], \tag{23}$$

$$s = \frac{3\gamma(1+\omega_{tot})[(K(1+\frac{1}{n})\omega_{tot} - (1+\omega_{tot})(1+\omega_{tot})]}{(3+\gamma(1+\omega_{tot})}. \tag{24}$$

For the case II, we obtain

$$r = 1 + \frac{(3-\beta_2)(1+\omega_{tot})[3K(1+\frac{1}{n})\omega_{tot} - \beta_1(1+\omega_{tot})]}{2}, \tag{25}$$

$$s = \frac{(3-\beta_2)(1+\omega_{tot})[3K(1+\frac{1}{n})\omega_{tot} - \beta_1(1+\omega_{tot})]}{3(-3+(3-\beta_2)(1+\omega_{tot}))}. \tag{26}$$

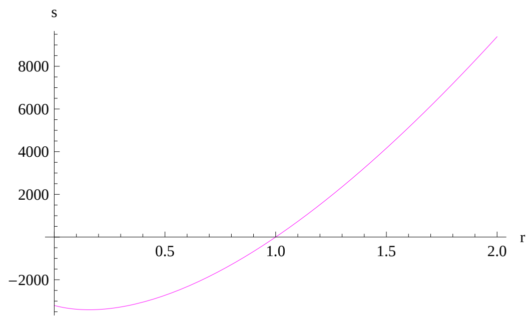

FIG. 3: Plot of s versus r for case III.

Case III leads to the following form:

$$r = 1 + \frac{2(3-\beta_3)(1+\omega_{tot})[6K(1+\frac{1}{n})\omega_{tot} + 1 + \beta_3(1+\omega_{tot})]}{2[3-\beta_3(1+\omega_{tot})]^2}, \tag{27}$$

$$s = \frac{2(3-\beta_3)(1+\omega_{tot})[6K(1+\frac{1}{n})\omega_{tot} + 1 + \beta_3(1+\omega_{tot})]}{3[3-\beta_3(1+\omega_{tot})][\beta_3 + (6+\beta_3)\omega_{tot}]}. \tag{28}$$

Here we can write s in terms of r which can be plotted in the $r-s$ plane with $\gamma = -0.5,\ \beta_2 = 1,\ \beta_3 = 3.5,\ K = 1,\ n = 2$ as shown in Figures (1)–(3). The $r-s$ plane for the first two cases is shown in Figures (1) and (2), indicating that these models represent that the universe started from a matter dominated era to the ΛCDM model. In the third case, s increases with respect to r and represents the universe starting from the ΛCDM limit to the matter dominated era, as shown in Figure (3).

IV. SUMMARY AND DISCUSSION

Recent observations through different aspects indicate that the universe is flat, expanding, and accelerating. There is exotic DE which acts like a driving force for the accelerated expansion of the universe. This provides the behavior of the Newton gravitational constant G and cosmological constant Λ (vacuum DE), whether these still remain constant or not. In a paper [46], it was found that G varies inversely with respect to cosmic time and possible different ranges for G were estimated. On the other hand, Milne [47] showed that G directly varies with cosmic time t. Belinchon [48] obtained results consistent with Milne [47] through dimensional analysis. Recently, some people [21, 26, 27] obtained results in favor of growing G and decreasing Λ with time.

The aim of the present work is also a chain of the above discussion in which we have used a polytropic gas and the results are summarized as follows. The total energy density remains positive for even polytropic index n. We have found different values of the constant parameters for which the EoS parameter lies in the CDM and radiation phases of the universe. We have also found a DE description of the polytropic EoS through its involving parameters. The cosmological parameter G remains positive and shows increasing behavior for $-1 < \gamma < 0$, $\beta_2 > 3$, and $\beta_3 > 2$ in three different cases of Λ, respectively. However, $\Lambda > 0$ and exhibits decreasing behavior for $\gamma < 0$ and $\beta_2 < 3$ in the first and second cases, respectively. In the third case, it is positive for $\beta_3 > 3$. Also, the trajectory of r and s in this scenario of DE passes through the ΛCDM fixed point in all the cases. The first case predicts a transition of the universe from radiation to ΛCDM and then goes towards the phantom DE era. However, in the third case, we attain the ΛCDM limit. We would like to mention here that a polytropic EoS leads to consistent results of G, Λ with [26, 27] and statefinders with [26].

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