

## Discrimination of Measurement Paths for a Three-Qubit System

Sing Poh Toh\*

*Faculty of Engineering, The University of Nottingham Malaysia Campus Jalan Broga,  
43500 Semenyih, Selangor Darul Ehsan, Malaysia*

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A method of discerning measurement paths in quantum mechanics was conceived by Sala Mayato and Muga [Phys. Lett. A 375 (2011) 3167] and applied to a two-qubit system. There are some calculational errors in that paper. We adopt the method to a three-qubit system where we choose two measurement contexts, i.e.,  $\{A, B, C, D\}$  and  $\{A', B', C', D\}$ , that constitute three measurement paths, i.e.,  $D, ABC$  and  $A'B'C'$ , for measuring the quantum observable  $D$  in such a way that  $D = ABC = A'B'C'$ . The observables  $A, B, C, D$  and  $A', B', C', D$  are pairwise commuting, but  $A, B, C$  do not commute with  $A', B', C'$ . The expectation values of the observables for the final states produced after performing the Lüders' measurement allow us to determine the measurement path that has been taken.

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## I. INTRODUCTION

Noncontextuality and locality are two properties that are incompatible with quantum mechanics. Quantum contextuality is a more general concept than quantum nonlocality, in the sense that quantum contextuality reveals itself in a single system and does not depend on the entanglement of the state, but the conflict between measurement due to quantum nonlocality exists in the entangled state. The Bell theorem excludes the local hidden variable theories which are the subset of noncontextual hidden variable theories that are disproved by the Kochen-Specker theorem [1].

Two observables  $A$  and  $B$  are said to be compatible when the outcome of a measurement of  $A$  performed on a system does not depend on the measurement of  $B$ . A maximal set of mutually compatible observables define a context. Although some observables can be associated to two different contexts, observables in two different contexts are not necessarily compatible. A measurement is said to be noncontextual when the outcome of the measurement of an observable is independent of the context, otherwise it is said to be contextual. Noncontextuality is a classical property, but quantum mechanics is contextual. All observables in any noncontextual hidden variable theory have definite values determined by some hidden variables the system carries prior to and independent of observation. The Kochen-Specker theorem states that the predictions of quantum mechanics cannot be reproduced by noncontextual hidden-variable theories that are based on the assumption of

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\*Electronic address: SingPoh.Toh@nottingham.edu.my; singpoh@gmail.com

noncontextuality. The first proof of the Kochen-Specker theorem was given in 1967 [2]. A qutrit is the simplest quantum system in which contextuality occurs [3, 4].

Many experiments have been performed to test quantum contextuality. Bartosik *et al.* used a single neutron system [5] in an experiment to disprove the noncontextuality hidden variable theories. Kirchmair *et al.* observed the quantum contextuality in an experiment using a trapped ion system [6]. Amselem *et al.* carried out a state-independent single photo experiment in 2009 [7]. Contextuality occurs even in non-entangled states, the experiment of Liu *et al.* [8] reveals the quantum contextuality of a product state. In 1999 a discussion was started on the experimental testability of quantum contextuality if the finite precision of measurements is taken into account [9, 10]. One of the fruitful results of this discussion is that inequalities based on noncontextual hidden-variable theories violated by any quantum states have been put forward [11] and used in experimental tests, including the experiment by Moussa *et al.* that shows the violation of the KS inequality with a nuclear magnetic resonance system [12].

Consider two measurement contexts  $\{A, B, C\}$  and  $\{A', B', C\}$ , where  $C$  is a common observable and  $AB = A'B' = C$ . Note that observables from the same context are mutually commuting but  $A, B$  do not commute with  $A', B'$ . The commutation relation allow us to write the above condition as  $BA = B'A' = C$ . The condition of  $AB = A'B' = C$  shows three different ways to measure  $C$ , i.e., measure  $C$  directly, via  $AB$  or  $A'B'$ , that will give the same statistical results for  $C$ . If contextuality is compatible with quantum mechanics and someone performs a measurement of  $C$ , how can we know which way the measurement was taken? Sala Mayato and Muga [13] set forth a method using Lüders' measurement to answer this question for the two-qubit system. A Lüders' measurement produces a state

$$\rho' = \sum_n P_n \rho P_n, \quad (1)$$

where  $P_n$  is the projection operator

$$P_n = \sum_{i=1}^{g_n} |o_n^i\rangle \langle o_n^i|, \quad (2)$$

that represents the observable  $O$  having discrete eigenvalues  $o_1, o_2, o_3, \dots$  with degeneracies  $g_1, g_2, g_3, \dots$ , respectively. When all the degeneracies are 1, i.e., the eigenvalues are not degenerate, (1) is the same as the final state produced by von Neumann's measurement,

$$\rho' = \sum_{n,i} P_n^i \rho P_n^i, \quad (3)$$

where

$$P_n^i = |o_n^i\rangle \langle o_n^i|. \quad (4)$$

Although the basic idea is right, there are some calculational errors in [13], due to the wrong chosen common eigenvectors. It is thus worthy to show in exact detail calculations via an extension of the method to a higher dimensional system. In Sec. II, a long calculation is presented for an eight dimensional three-qubit system. The results obtained have a structure like the case of the two-qubit system. We present our conclusions in Sec. III.

## II. DISCERNING MEASUREMENT PARTS: THREE-QUBIT SYSTEM

In this section, we extend the method of discrimination of measurement paths explicated in [13] to the eight dimensional state space. In a paper published in 1993 [14], Mermin proposed a simple proof of the Kochen-Specker theorem using ten observables of a three-qubit system with dichotomic eigenvalues  $\pm 1$ . Figure 1 is a modified pentagram given by Aravind [15] depicting the arrangement of the ten real matrices representing observables into five sets that lie along the five lines of a pentagram. Each observable belongs to two contexts. Four observables in the same set are mutually commuting. The product of the four observables in each set is  $+1$ , except for the set which lies on the horizontal line, where the product is  $-1$ .

$$\begin{array}{ccccc}
 & & \sigma_x^3 & & \\
 & \sigma_z^1 \sigma_z^2 \sigma_z^3 & \sigma_z^1 \sigma_x^2 \sigma_x^3 & \sigma_x^1 \sigma_z^2 \sigma_z^3 & \sigma_x^1 \sigma_x^2 \sigma_z^3 \\
 & \sigma_z^1 & & \sigma_x^1 & \\
 & & \sigma_z^3 & & \\
 \sigma_x^2 & & & & \sigma_z^2
 \end{array}$$

FIG. 1: Mermin's Pentagram

We choose  $A = \sigma_z^2$ ,  $B = \sigma_z^1$ ,  $C = \sigma_z^1 \sigma_z^2 \sigma_z^3$ ,  $A' = \sigma_x^2$ ,  $B' = \sigma_x^1$ ,  $C' = \sigma_x^1 \sigma_x^2 \sigma_x^3$ , and  $D = \sigma_z^3$ , where, for example,  $\sigma_z^2$  is a shorthand for  $I \otimes \sigma_z \otimes I$  and  $I$  is the  $2 \times 2$  identity matrix. We thus have two measurement contexts  $\{A, B, C, D\}$  and  $\{A', B', C', D\}$ . Note that  $A$ ,  $B$ , and  $C$  do not commute with  $A'$ ,  $B'$ , and  $C'$ . As  $D = ABC = A'B'C'$ , measurement of  $D$  can be done via measuring  $D$  directly,  $ABC$ , or  $A'B'C'$ , that will bring the same statistical result for  $D$ . By constructing a table resembling the two-qubit system (labeled as (7) in [13]) for predetermined values of observables  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $A'$ ,  $B'$  and  $C'$ , it is straightforward to show the impossibility of the determination of the measurement paths, namely whether it is measured via  $D$ ,  $ABC$ , or  $A'B'C'$ . However, the contextuality property of quantum mechanics makes the determination of measurement paths become possible, as will be shown in detail in the following.

As  $A$ ,  $B$ ,  $C$ , and  $D$  are diagonal matrices with non-zero entries  $1$  or  $-1$ , it is obvious that their common eigenvectors are  $|e_i\rangle$  in the  $z$ -basis, i.e.,  $|e_1\rangle = |+++ \rangle, \dots, |e_8\rangle = |--- \rangle$ . Specifically, (5) shows the common eigenvectors and associated eigenvalues  $a$ ,  $b$ ,  $c$ , and  $d$

for  $A$ ,  $B$ ,  $C$ , and  $D$ ,

$$\begin{array}{cccc}
 & a & b & c & d \\
 |e_1\rangle & +1 & +1 & +1 & +1, \\
 |e_2\rangle & +1 & +1 & -1 & -1, \\
 |e_3\rangle & -1 & +1 & -1 & +1, \\
 |e_4\rangle & -1 & +1 & +1 & -1, \\
 |e_5\rangle & +1 & -1 & -1 & +1, \\
 |e_6\rangle & +1 & -1 & +1 & -1, \\
 |e_7\rangle & -1 & -1 & +1 & +1, \\
 |e_8\rangle & -1 & -1 & -1 & -1.
 \end{array} \tag{5}$$

On the other hand, the common eigenvectors of  $A'$ ,  $B'$ ,  $C'$ , and  $D$  expressed in terms of  $|e_i\rangle$  are

$$\begin{aligned}
 |\psi_1\rangle &= \frac{1}{2}[|e_1\rangle + |e_3\rangle + |e_5\rangle + |e_7\rangle], \\
 |\psi_2\rangle &= \frac{1}{2}[|e_2\rangle + |e_4\rangle + |e_6\rangle + |e_8\rangle], \\
 |\psi_3\rangle &= \frac{1}{2}[-|e_2\rangle - |e_4\rangle + |e_6\rangle + |e_8\rangle], \\
 |\psi_4\rangle &= \frac{1}{2}[-|e_1\rangle - |e_3\rangle + |e_5\rangle + |e_7\rangle], \\
 |\psi_5\rangle &= \frac{1}{2}[-|e_2\rangle + |e_4\rangle - |e_6\rangle + |e_8\rangle], \\
 |\psi_6\rangle &= \frac{1}{2}[-|e_1\rangle + |e_3\rangle - |e_5\rangle + |e_7\rangle], \\
 |\psi_7\rangle &= \frac{1}{2}[|e_1\rangle - |e_3\rangle - |e_5\rangle + |e_7\rangle], \\
 |\psi_8\rangle &= \frac{1}{2}[|e_2\rangle - |e_4\rangle - |e_6\rangle + |e_8\rangle],
 \end{aligned} \tag{6}$$

with the associated eigenvalues  $a'$ ,  $b'$ ,  $c'$ , and  $d$  as

$$\begin{array}{cccc}
 & a' & b' & c' & d \\
 |\psi_1\rangle & +1 & +1 & +1 & +1, \\
 |\psi_2\rangle & +1 & +1 & -1 & -1, \\
 |\psi_3\rangle & +1 & -1 & +1 & -1, \\
 |\psi_4\rangle & +1 & -1 & -1 & +1, \\
 |\psi_5\rangle & -1 & +1 & +1 & -1, \\
 |\psi_6\rangle & -1 & +1 & -1 & +1, \\
 |\psi_7\rangle & -1 & -1 & +1 & +1, \\
 |\psi_8\rangle & -1 & -1 & -1 & -1.
 \end{array} \tag{7}$$

Let us consider an initial state of a physical system that is completely specified by

the state vector,

$$|\psi\rangle = \sum_{i=1}^8 \alpha_i |e_i\rangle, \quad (8)$$

where  $\alpha_i$  are complex numbers satisfying  $\sum |\alpha_i|^2 = 1$ , and  $|\alpha_i|^2$  is the probability that a suitable measurement will find the system to be in the states  $|e_i\rangle$ . The density matrix for the system is given as

$$\begin{aligned} \rho &= |\psi\rangle\langle\psi| \\ &= \sum_{i,j=1}^8 r_{ij} |e_i\rangle\langle e_j|. \end{aligned} \quad (9)$$

The relationship between the complex coefficients  $r_{ij}$  and  $\alpha_i$  can be easily obtained as

$$r_{ij} = \begin{cases} |\alpha_{ii}|^2 & \text{for } i = j, \\ \alpha_i \alpha_j^* & \text{for } i < j, \\ (\alpha_i \alpha_j^*)^* & \text{for } i > j. \end{cases} \quad (10)$$

Based on (5) and applying Lüders' measurement (1) on state (9), the final state produced after the direct measurement of  $D$  is

$$\begin{aligned} \rho_D &= (P_{11} + P_{33} + P_{55} + P_{77})\rho(P_{11} + P_{33} + P_{55} + P_{77}) \\ &\quad + (P_{22} + P_{44} + P_{66} + P_{88})\rho(P_{22} + P_{44} + P_{66} + P_{88}) \\ &= \sum_{i=1}^8 r_{ii} P_{ii} + \sum_{i=1}^8 r_{i,j=i+2} P_{ij}, \end{aligned} \quad (11)$$

where

$$P_{ij} = |e_i\rangle\langle e_i|. \quad (12)$$

Carrying out the similar steps for the measurement  $ABC$  yields

$$\rho_{ABC} = \sum_{i=1}^8 |\alpha_{ii}|^2 P_{ii}. \quad (13)$$

The evaluation of the final state  $\rho_{A'B'C'}$  as a result of measurement of  $A'B'C'$  involves a lengthy but straightforward calculation. Applying Lüder's measurement (1) three times

based on (7), the consecutive states produced are

$$\begin{aligned}\rho_{C'} &= (P'_{11} + P'_{33} + P'_{55} + P'_{77})\rho(P'_{11} + P'_{33} + P'_{55} + P'_{77}) \\ &\quad + (P'_{22} + P'_{44} + P'_{66} + P'_{88})\rho(P'_{22} + P'_{44} + P'_{66} + P'_{88}),\end{aligned}\tag{14}$$

$$\begin{aligned}\rho_{B'C'} &= (P'_{11} + P'_{22} + P'_{55} + P'_{66})\rho_{C'}(P'_{11} + P'_{22} + P'_{55} + P'_{66}) \\ &\quad + (P'_{33} + P'_{44} + P'_{77} + P'_{88})\rho_{C'}(P'_{33} + P'_{44} + P'_{77} + P'_{88}),\end{aligned}\tag{15}$$

$$\begin{aligned}\rho_{A'B'C'} &= (P'_{11} + P'_{22} + P'_{33} + P'_{44})\rho_{B'C'}(P'_{11} + P'_{22} + P'_{33} + P'_{44}) \\ &\quad + (P'_{55} + P'_{66} + P'_{77} + P'_{88})\rho_{B'C'}(P'_{55} + P'_{66} + P'_{77} + P'_{88}) \\ &= \sum_{i=j=1}^8 P'_{ij}\rho P'_{ij},\end{aligned}\tag{16}$$

respectively, where

$$P'_{ij} = |\psi_i\rangle\langle\psi_j|,\tag{17}$$

and we have applied

$$P'_{ii}P'_{jj} = \begin{cases} P'_{ii}, & \text{for } i = j, \\ 0, & \text{for } i \neq j. \end{cases}\tag{18}$$

Using (6), the state (16) can be expressed in terms of  $P_{ij}$  to give

$$\begin{aligned}\rho_{A'B'C'} &= k_o(P_{11} + P_{33} + P_{55} + P_{77}) + l_o(P_{13} + P_{31} + P_{57} + P_{75}) \\ &\quad + r_o(P_{15} + P_{51} + P_{37} + P_{73}) + s_o(P_{17} + P_{71} + P_{35} + P_{53}) \\ &\quad + k_e(P_{22} + P_{44} + P_{66} + P_{88}) + l_e(P_{24} + P_{42} + P_{68} + P_{86}) \\ &\quad + r_e(P_{26} + P_{62} + P_{48} + P_{84}) + s_e(P_{28} + P_{82} + P_{46} + P_{64}),\end{aligned}\tag{19}$$

where the subscripts ‘ $o$ ’ and ‘ $e$ ’ stand for odd and even, respectively, that indicate the

entries of the matrix (19) having the following values,

$$\begin{aligned}
k_o &= \frac{1}{4}[r_{11} + r_{33} + r_{55} + r_{77}], \\
l_o &= \frac{1}{2}[\Re(r_{13}) + \Re(r_{57})], \\
r_o &= \frac{1}{2}[\Re(r_{15}) + \Re(r_{37})], \\
s_o &= \frac{1}{2}[\Re(r_{17}) + \Re(r_{35})], \\
k_e &= \frac{1}{4}[r_{22} + r_{44} + r_{66} + r_{88}], \\
l_e &= \frac{1}{2}[\Re(r_{24}) + \Re(r_{68})], \\
r_e &= \frac{1}{2}[\Re(r_{26}) + \Re(r_{48})], \\
s_e &= \frac{1}{2}[\Re(r_{28}) + \Re(r_{46})].
\end{aligned} \tag{20}$$

Note that we have applied (10) and

$$\begin{aligned}
\Re(r_{ij}) &= \frac{1}{2}(r_{ij} + r_{ij}^*), \\
\Im(r_{ij}) &= \frac{1}{2i}(r_{ij} - r_{ij}^*),
\end{aligned} \tag{21}$$

where the asterisk denotes complex conjugation, with  $i, j = 1, 2, \dots, 8$  to obtain (20).

The expectation values of  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $A'$ ,  $B'$ , and  $C'$  for the final states (11), (13), and (19) are

$$\begin{aligned}
\langle A \rangle_D &= \langle A \rangle_{ABC} = (r_{11} + r_{22} + r_{55} + r_{66}) - (r_{33} + r_{44} + r_{77} + r_{88}), \\
\langle A \rangle_{A'B'C'} &= 0,
\end{aligned} \tag{22}$$

$$\begin{aligned}
\langle B \rangle_D &= \langle B \rangle_{ABC} = (r_{11} + r_{22} + r_{33} + r_{44}) - (r_{55} + r_{66} + r_{77} + r_{88}), \\
\langle B \rangle_{A'B'C'} &= 0,
\end{aligned} \tag{23}$$

$$\begin{aligned}
\langle C \rangle_D &= \langle C \rangle_{ABC} = (r_{11} + r_{44} + r_{66} + r_{77}) - (r_{22} + r_{33} + r_{55} + r_{88}), \\
\langle C \rangle_{A'B'C'} &= 0,
\end{aligned} \tag{24}$$

$$\begin{aligned}
\langle A' \rangle_D &= \langle A' \rangle_{A'B'C'} = 2[\Re(r_{13}) + \Re(r_{24}) + \Re(r_{57}) + \Re(r_{68})], \\
\langle A' \rangle_{ABC} &= 0,
\end{aligned} \tag{25}$$

$$\begin{aligned}
\langle B' \rangle_D &= \langle B' \rangle_{A'B'C'} = 2[\Re(r_{15}) + \Re(r_{26}) + \Re(r_{37}) + \Re(r_{48})], \\
\langle B' \rangle_{ABC} &= 0,
\end{aligned} \tag{26}$$

$$\begin{aligned}
\langle C' \rangle_D &= \langle C' \rangle_{A'B'C'} = 2[\Re(r_{17}) + \Re(r_{35}) - \Re(r_{28}) - \Re(r_{46})], \\
\langle C' \rangle_{ABC} &= 0,
\end{aligned} \tag{27}$$

$$\begin{aligned}
\langle D \rangle_D &= \langle D \rangle_{ABC} = \langle D \rangle_{A'B'C'} \\
&= (r_{11} + r_{33} + r_{55} + r_{77}) - (r_{22} + r_{44} + r_{66} + r_{88}).
\end{aligned} \tag{28}$$





$$\rho_{ABC} = \frac{1}{120} \begin{pmatrix} 21 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 17 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 17 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 9 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 17 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 9 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 9 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 21 \end{pmatrix}, \quad (31)$$

$$\rho_{A'B'C'} = \frac{1}{60} \begin{pmatrix} 8 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 7 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 7 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 7 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 7 \end{pmatrix}, \quad (32)$$

respectively. The expectation values of the observables  $A$ ,  $B$ ,  $C$ ,  $D$ ,  $A'$ ,  $B'$ , and  $C'$  associated to the states (30), (31), and (32) are

$$\begin{array}{ccc} & \rho_D & \rho_{ABC} & \rho_{A'B'C'} \\ \langle A \rangle & \frac{1}{15} & \frac{1}{15} & 0, \\ \langle B \rangle & \frac{1}{15} & \frac{1}{15} & 0, \\ \langle C \rangle & -\frac{1}{5} & -\frac{1}{5} & 0, \\ \langle A' \rangle & 0 & 0 & 0, \\ \langle B' \rangle & 0 & 0 & 0, \\ \langle C' \rangle & \frac{2}{15} & 0 & \frac{2}{15}, \\ \langle D \rangle & \frac{1}{15} & \frac{1}{15} & \frac{1}{15}. \end{array} \quad (33)$$

It is obvious from (33) that only the values of  $\{\langle A \rangle, \langle C' \rangle\}$ ,  $\{\langle B \rangle, \langle C' \rangle\}$  and  $\{\langle C \rangle, \langle C' \rangle\}$  can be used to discern measurement paths.

### III. CONCLUSION

As there are calculation errors in [13], it is worth to show in detail the method of discriminating measurement paths proposed in the paper applied to an eight-dimensional state space of a three-qubit system. Two measurement contexts,  $\{A, B, C, D\}$  and  $\{A', B', C', D\}$ , that measure observable  $D$  on a three-qubit system were chosen from Mermin's pentagram to form three measurement paths, i.e.,  $ABC$ ,  $A'B'C'$ , and  $D$ , in such a way that  $ABC = A'B'C' = D$ . The statistical results for measuring  $D$  via the three measurement paths are the same, but as the produced states after the execution of Lüders' measurement are different, the measurement paths can be discerned by checking the expectation values of  $A$ ,  $B$ ,  $C$ ,  $A'$ ,  $B'$ , and  $C'$  for the final states. As expected, the calculation for a three-qubit

system is far more complicated compared to a two-qubit system. Nonetheless, the results for both systems carries the same patterns, and thus the same path-checking argument is applicable to both.

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