University of Missouri
Department of Statistics

instats

AI and extremes

Likun Zhang[1] Jordan Richards[2] Reetam Majumder[3]

[1]Department of Statistics, University of Missouri

[2]School of Mathematics, University of Edinburgh

[3]Department of Mathematical Sciences, University of Arkansas

Crash course on extreme-value theory

Sample averages

3 GHCN stations, 1949∼2017, **DJF mean precipitation**

Block maxima

3 GHCN stations, 1949∼2017, **DJF maximum precipitation**

Block maxima

3 GHCN stations, 1949∼2017, **DJF maximum precipitation**

Classic extreme value theory

CLT is for averages, and EVT is for tails

Suppose $Y_1, Y_2, \ldots$ are iid with distribution function F, there exists sequences of $a_n > 0$ and $b_n \in \mathbb{R}$ such that as $n \to \infty$,

$$\max_{i=1,\cdots,n} \frac{Y_i - b_n}{a_n} \xrightarrow{d} M,$$

where M subject to a generalized extreme value(GEV) distribution.

A parametric family w.r.t $\boldsymbol{\theta} = (\sigma, \mu, \xi)$.

Peaks over threshold (POT)

3 GHCN stations, 1949∼2017, **DJF maximum precipitation**

Classic extreme value theory

Peaks over threshold (POT)

Pickands–Balkema–de Haan

Let $Y_1, Y_2, \ldots$ be iid with distribution function F. For a high threshold u_n and normalizing constants $a_n > 0$,

$$\lim_{n\to\infty} \Pr\left(\frac{Y-u_n}{a_n} \le x \mid Y > u_n\right) = 1 - \left(1+\xi x\right)^{-1/\xi}, \quad x \ge 0,$$

where the limiting distribution is called Generalized Pareto distribution (**GPD**).

A parametric family w.r.t $\boldsymbol{\theta} = (\mu, \sigma, \xi)$.

Block maxima

Single-station extreme value analysis

$$\Rightarrow \quad (\sigma_1, \mu_1, \xi_1) = (21.02, 48.18, \mathbf{0.10})$$

$$\Rightarrow \quad (\sigma_2, \mu_2, \xi_2) = (14.64, 23.31, \mathbf{0.19})$$

$$\Rightarrow \quad (\sigma_3, \mu_3, \xi_3) = (15.39, 30.69, \mathbf{0.21})$$

Block maxima

Single-station extreme value analysis

Risser et al. (2019). *A probabilistic gridded product for daily precipitation extremes over the United States*. Climate Dynamics, 53(5), pp.2517.

The climatological coefficients at s_i:

$$(\mu_0(s_i) + \mu_1(s_i)t, \sigma(s_i), \xi(s_i)), \qquad (1)$$

where $i = 1, \ldots, 5202$, t=1949, $\ldots$, 2017.

Block maxima

Single-station extreme value analysis?

Single-station analyses

- have lots of noise in trend and shape parameters;
- make detection of significant trend meaningless;
- fail to account for dependence of **individual storms**;
- cannot estimate **areal exceedance** probabilities.

To model dependence and borrow strength from space, Cooley et al. (2007) modeled

$$\theta(s) = X(s)\beta + u(s), \quad \theta \in \{\mu_0, \mu_1, \log\sigma, \xi\},$$

where $u(s)$, $s \in \mathcal{S}$ is a Gaussian process with a nonstationary covariance function.

Spatial dependence

Dependence in the bulk and tail

DJF maximum precipitation from 1950 to 2017.

$36km$ apart

$851km$ apart

Spatial/Multivariate dependence

Gaussian process

Definition

A **Gaussian Process (GP)** is a collection of random variables, any finite subset of which has a joint Gaussian distribution.

$$Y(\mathbf{s}) \sim \mathcal{GP}\big(m(\mathbf{s}),\, k(\mathbf{s}, \mathbf{s}')\big),\ \mathbf{s} \in \mathcal{D},$$

where

$$m(\mathbf{s}) = \mathbb{E}[Y(\mathbf{s})] \quad \text{(mean function)},$$
$$k(\mathbf{s}, \mathbf{s}_1') = \mathrm{Cov}[Y(\mathbf{s}), Y(\mathbf{s}')] \quad \text{(covariance/kernel)}.$$

Spatial/Multivariate dependence

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$$m(\mathbf{s}) = \mathbb{E}[Y(\mathbf{s})] \quad \text{(mean function)},$$
$$k(\mathbf{s}, \mathbf{s}_1') = \mathrm{Cov}[Y(\mathbf{s}), Y(\mathbf{s}')] \quad \text{(covariance/kernel)}.$$

- No matter how close two stations are,

$$\chi_{12}(u) = P[F_1(Y(\mathbf{s}_1)) > u \mid F_2(Y(\mathbf{s}_2)) > u] \to 0,\ u \to 1,$$

that is, the probability of co-occurrence of very extreme events is always close to 0.

Spatial dependence

Dependence in the tail

Definition

Consider $\{Y(\boldsymbol{s}) : \boldsymbol{s} \in \mathcal{D}\}$ with continuous sample paths. The limiting process from the renormalized pointwise maxima of its independence copies is a max-stable process:

$$\left\{ \max_{i=1,\cdots,n} \frac{Y_i(\boldsymbol{s}) - b_n(\boldsymbol{s})}{a_n(\boldsymbol{s})} \right\}_{\boldsymbol{s}\in\mathcal{D}} \to \{M(\boldsymbol{s})\}_{\boldsymbol{s}\in\mathcal{D}}, \ n \to \infty.$$

By de Haan (1984), any unit-Fréchet max-stable process admits

$$M(\mathbf{s}) \ = \ \max_{i\geq 1} P_i \, W_i(\mathbf{s}),$$

where

- $\{P_i\}$ are points of a Poisson process on $(0, \infty)$ with intensity $p^{-2}dp$,
- $\{W_i(\cdot)\}$ are iid nonnegative functions with $\mathbb{E}[W(\mathbf{s})] = 1$.

This yields *exponent measure* for any finite set $\{\mathbf{s}_1, \ldots, \mathbf{s}_k\}$:

$$\Pr\big\{M(\mathbf{s}_j) \leq z_j, \ j = 1, \ldots, k\big\} = \exp\big\{-V(z_1, \ldots, z_k)\big\}.$$

Max-stable process

Challenges

Derived from the joint CDF

$$\Pr\{M(\mathbf{s}_j) \leq z_j,\ j = 1, \ldots, k\} = \exp\{-V(z_1, \ldots, z_k)\},$$

the (full) likelihood function for independent replicates simply has too many terms for likelihood inference in practice.

- **Pairwise (composite) likelihood** (Huser and Davison, 2013): maximize $\sum_{i<j} \log f_{ij}\big(M(\mathbf{s}_i), M(\mathbf{s}_j); \theta\big)$.
- More natural pairwise extremal covariance function? **Tail pairwise dependence function** (TPDF; Cooley and Thibaud, 2019; Wixson and Cooley, 2023)
- The dependence structure of max-stable processes is very rigid:

More flexible spatial extremes models

There has been **tremendous** progress in the spatial extremes literature:

	Block Maxima	Peaks-over-threshold
Asymptotic models	Max-stable processes e.g., Padoan et al., 2010 Davison et al., 2012, 2019	Generalized Pareto processes e.g., Ferreira and de Haan, 2014; Thibaud and Opitz, 2015 ↓ risk functional r r-Pareto processes e.g., Dombry and Ribatet, 2015; de Fondeville and Davison, 2018
Sub-asymptotic models	Max-infinitely divisible (max-id) processes e.g., Reich and Shaby, 2012 Padoan, 2013 Huser et al., 2021 Bopp et al., 2021	Inverted max-stable processes e.g., Wadsworth & Tawn, 2012 Scale- or location-mixture models e.g., Opitz, 2016 Krupskii et al., 2018 Huser & Wadsworth, 2019 Zhang et al., 2022 (MVM) Single-station conditioning models e.g., Wadsworth & Tawn, 2019 Richards et al., 2022 Simpson et al., 2023

Hands-on Keras tutorial

Outline

- We follow Richards, J. and Huser, R. (2025). Extreme Quantile Regression with Deep Learning. Chapter 21 of Handbook (left).
- Deep extreme quantile regression models built and trained in the R interface to Keras v2, using TensorFlow.
- Vignette at `https://github.com/Jbrich95/WACE_shortcourse`. Handbook specific code at `https://github.com/Jbrich95/deepExtQuantReg_code`.

Regression: General Setting

Given:

- Response variable Y (often in $\mathbb{R}$, but could also be discrete classes);
- Covariates $X \in \mathbb{R}^q$.

We are interested in learning some **estimand** that describes the conditional distribution of $Y \mid \mathbf{X} = \mathbf{x}$.

Typical estimands of interest are:

- Class probabilities: $\Pr\{Y = k | \mathbf{X} = \mathbf{x}\}$ for $k \in \mathbb{N}$;
- Expectation: $\mathbb{E}[Y | \mathbf{X} = \mathbf{x}]$;
- Quantiles: $Q_x(\tau) = \inf\{y : F_{Y|\mathbf{X}}(y|x) \geq \tau\}$ for $\tau \in (0, 1)$;
- "Parameters" of $F_{Y|\mathbf{X}}$. Can include parameters in the standard finite-dimensional sense, but may also be interested in semi-/non-parametric model for $F_{Y|\mathbf{X}}$.

Regression: How do we estimate these parameters?

Set up an estimable model $\mathbf{m} : \mathbb{R}^q \mapsto \boldsymbol{\Theta}$:

- Let $\boldsymbol{\theta} \in \boldsymbol{\Theta}$ contain your estimands of interest.
- Setup a function $\mathbf{m}$ that maps your covariates $\mathbf{x}$ to $\boldsymbol{\theta}$, i.e., $\boldsymbol{\theta}(\mathbf{x}) := \mathbf{m}(\mathbf{x})$.
- Get yourself some data $\{(y_i, \mathbf{x}_i)\}_{i=1}^n$.

Estimate $\mathbf{m}$ via minimisation of some loss or cost function:

- Class probabilities: binary cross-entropy;
- Expectation: MSE $n^{-1} \sum_{i=1}^n (y_i - \theta(\mathbf{x}_i))^2$
- Quantiles: Pinball loss $n^{-1} \sum_{i=1}^n (y_i - \theta(\mathbf{x}_i))(\tau - \mathbb{1}\{y_i < \theta(\mathbf{x}_i)\})$;
- $F_{Y|\mathbf{X}}$. Associated negative log-likelihood.

For example, an estimate for the conditional expectation function, say $\widehat{\theta(\mathbf{x})}$, is the minimiser

$$\widehat{\theta(\mathbf{x})} \in \arg\min_{\mathbf{m}\in\mathcal{M}} \frac{1}{n}\sum_{i=1}^{n}(y_i - \mathbf{m}(\mathbf{x}_i))^2,$$

where $\mathcal{M}$ is some appropriate space of estimable functions.

But what should $\mathcal{M}$ look like?

Machine learning

- The **fundamental** difference between statistics and AI/ML/DL is the complexity of $\mathbf{m}(\cdot)$.
- Traditional statistics want $\mathbf{m}(\cdot)$ simple, e.g., linear, parametric, splines, so that estimates are interpretable.
- ML/DL wants flexibility in $\mathbf{m}(\cdot)$.
- Other key ideas - optimisation, regularisation, testing - tend to be the same.

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Deep learning

The **deep** in Deep learning comes from the depth of **m**:

- We represent **m** as a neural network. This is just a composition of **layers** of simple, differentiable operations.
- There are lots of different choices of operations, which give rise to different types of neural network. Today, we focus on the standard feed-forward multi-layered perceptron - baby's first neural network.

Created with DeepAI text2img https://deepai.org/machine-learning-model/text2img

Deep learning

Multi-layer Perceptron (MLP)

The neural network $\mathbf{m}_\psi$ is constructed as a composition of $L \in \mathbb{N}$ *hidden layers*, $\mathbf{m}^{(l)}$ for $l = 1, \ldots, L$, and an *output/final* layer, $\mathbf{m}^{(L+1)}$, such that $\mathbf{m}_\psi(\cdot) := \mathbf{m}^{(L+1)} \circ \cdots \circ \mathbf{m}^{(1)}(\cdot)$.

- ψ contains all the estimable parameters (weights and biases);
- the neural network has the hierarchical structure

$$
\begin{aligned}
\mathbf{x}^{(1)} &:= \mathbf{m}^{(1)}(\mathbf{x}) = \mathbf{a}^{(1)}\left(\mathbf{W}^{(1)}\mathbf{x} + \mathbf{b}^{(1)}\right), \\
\mathbf{x}^{(2)} &:= \mathbf{m}^{(2)}(\mathbf{x}^{(1)}) = \mathbf{a}^{(2)}\left(\mathbf{W}^{(2)}\mathbf{x}^{(1)} + \mathbf{b}^{(2)}\right), \\
&\vdots \\
\mathbf{x}^{(L)} &:= \mathbf{m}^{(L)}(\mathbf{x}^{(L-1)}) = \mathbf{a}^{(L)}\left(\mathbf{W}^{(L)}\mathbf{x}^{(L-1)} + \mathbf{b}^{(L)}\right), \\
\theta(\mathbf{x}) &:= \mathbf{m}^{(L+1)}(\mathbf{x}^{(L)}) = \mathbf{a}^{(L+1)}\left(\mathbf{W}^{(L+1)}\mathbf{x}^{(L)} + \mathbf{b}^{(L+1)}\right).
\end{aligned}
$$

Activation functions

Activation functions provide flexibility by making the layers non-linear.

Popular choices includes:

- ReLU: $\mathbf{a}(\mathbf{x})_j = \max(x_j, 0)$
- sigmoid: $\mathbf{a}(\mathbf{x})_j = \exp(x_j)/(1 + \exp(x_j))$
- tanh
- eLU and leaky ReLU
- softmax: $\mathbf{a}(\mathbf{x}) = (\frac{\exp(x_1)}{\sum_i \exp(x_i)}, \frac{\exp(x_2)}{\sum_i \exp(x_i)}, \dots)$

Training models

Models are trained by solving the empirical risk minimisation problem

$$\widehat{\boldsymbol{\psi}} \in \arg\min_{\boldsymbol{\psi}} \frac{1}{n} \sum_{i=1}^{n} \ell\{y_i; \mathbf{m}_{\boldsymbol{\psi}}, (\mathbf{x}_i)\},$$

where ℓ is our loss function.

In **statistics**, this would be achieved using gradient descent, via the update

$$\boldsymbol{\psi} \leftarrow \boldsymbol{\psi} - \frac{\lambda}{n} \sum_{i=1}^{n} \boldsymbol{\nabla}_{\boldsymbol{\psi}} \ell\{y_i, \mathbf{m}_{\boldsymbol{\psi}}(\mathbf{x}_i)\}, \tag{2}$$

where $\lambda > 0$ is a tunable learning rate and $\boldsymbol{\nabla}_{\boldsymbol{\psi}}$ denotes the differential operator with respect to entries in $\boldsymbol{\psi}$.

Training deep models

In **deep learning**, where n is typically large, a more economical approach is to update over individual observations (SGD), using the rule

$$\boldsymbol{\psi} \leftarrow \boldsymbol{\psi} - \lambda \boldsymbol{\nabla}_{\boldsymbol{\psi}} \ell\{y_i, \mathbf{g}_{\boldsymbol{\psi}}(\mathbf{x}_i)\}, \qquad i = 1, \ldots, n. \tag{3}$$

Neural networks are usually trained using an algorithm that falls in-between gradient descent and true SGD, which is called *mini-batch SGD*. In this case, all samples are split into n_b mini-batches of equal size, and n_b updates are performed for every **epoch** of the training scheme.

Usually, $\boldsymbol{\nabla}_{\boldsymbol{\psi}}$ is tractable and fast-to-compute via **backpropagation**. This is because the neural network model is a composition of simple differentiable operations.

Regularisation: Training/validation/testing

Due to their large number of parameters, neural networks are prone to overfitting. It's necessary to use **validation/testing** to assess overfitting.

- Subset the data into training, validation, and testing.
- Each set has a different purpose:
  - The model is **trained** on the training data. This is used to update the weights and biases.
  - We **validate** the model on the validation data. Typically used for model selection.
  - To get a truly unbiased evaluation of the model fit and to compare amongst different models, we **test** the model on test data.

0.95
0.90
0.85
0.80
loss
0
200
400
600
800
1000
epoch
data
training
validation

Regularisation/numerics

Other forms of regularisation to consider:

- Weight penalties (think LASSO or ridge)
- Random weight initialization
- Dropout
- Sharing weights*
- Early-stopping* and checkpoints*

Some numerical aspects:

- Pre-training*
- Standardising input data*
- Parameter dependent support
- Optimizing architecture/training scheme (please don't ask about this)

Deep GPD regression

We now use these ideas to perform deep GPD regression. We perform a **two-stage** estimation procedure:

- Use the tilted loss to estimate $u(\mathbf{x})$ as the τ-quantile of $Y \mid \mathbf{x} = \mathbf{x}$ (for τ close to one).
- Model conditional excesses $(Y - u(\mathbf{x})) \mid (\mathbf{X} = \mathbf{x})$ as $\mathrm{GPD}(\sigma_u(\mathbf{x}), \xi(\mathbf{x}))$.

The latter stage follows via maximum likelihood and we consider three different models for $(\sigma_u(\mathbf{x}), \xi(\mathbf{x}))$.

Semi-Parametric Quantile Regression (SPQR)

Estimating entire densities

- Our goal almost always is to make probability statements based on a fitted model
- However, sometimes we *don't* want to assume a parametric form of the density
- In that case, we need:
  - A non/semi-parametric way to represent the density
  - A loss function which evaluates the (usual negative log-likelihood) loss between the response y_i and the estimates $\pi(x_i)$
- SPQR does this using an $M-$spline representation of continuous density functions

SPQR

SPQR represents a continuous distribution $Y|X$ as:

$$f(y|x) = \sum_{k=1}^{K} \pi_k(x) M_k(y)$$

- $\{\pi_k\}_{k=1}^{K}$ is a vector of probabilities output from a NN
- The loss function is $-\log f(y|x)$
- $M-$splines are rescaled $B-$splines
- $I-$splines are just integrated $M-$splines such that:

$$F(y|x) = \sum_{k=1}^{K} \pi_k(x) I_k(y)$$

- Representation is scale-invariant

The good, the bad, and the next.

While the demo uses `keras`, there *is* a package based on `torch` at `https://github.com/stevengxu/SPQR` (Xu et al., 2022).

Good.

- Supports multi-modal input and various architectures
- Quantile function can be estimated using interpolation
- Can easily make probability statements

Bad.

- $M-$splines are bounded - extrapolation outside the observed range of y not possible

Next up.

- SPQRx (Majumder and Richards, 2025)

Further topics

Further topics

Machine learning for extreme quantile/extremal regression:

- NN-based methods:
  - GEV: (Cannon, 2010; Vasiliades et al., 2015; Bennett et al., 2015; Richards and Huser, 2024)
  - GPD: (Carreau and Bengio, 2007; Carreau et al., 2009; Carreau and Vrac, 2011; Richards et al., 2023; Allouche et al., 2024; Dahal et al., 2024; Pasche and Engelke, 2024; Majumder and Richards, 2025; Wilson et al., 2022)
  - Other: (Richards and Huser, 2022; Cisneros et al., 2024)
- Classical ML:
  - Random forests: (Taillardat et al., 2019; Gnecco et al., 2024)
  - Regression trees: (Farkas et al., 2021, 2024)
  - Boosting: (Velthoen et al., 2023; Koh, 2023; Koh et al., 2025)

Further topics

Multivariate and spatial models:

- Geometric methods: (Murphy-Barltrop et al., 2024; Mackay et al., 2025; De Monte et al., 2025)
- d-max-decreasing NNs: (Hasan et al., 2022)
- Deep compositional spatial models: (Shao et al., 2025)
- Generative methods:
  - Excess-GAN :(Allouche et al., 2025)
  - Ex-GAN: (Bhatia et al., 2021)
  - EV-GAN: (Allouche et al., 2022)
  - GPDflow: (Hu and Castro-Camilo, 2025)
  - EVT-GAN: (Boulaguiem et al., 2022)
  - HT-GAN: (Girard et al., 2024)
  - GP-GAN (Li et al., 2024)
  - Angular simulation: (Wessel et al., 2025)
  - WA-GAN: (Lhaut et al., 2025)
  - VAEs: (Lafon et al., 2023; Zhang et al., 2023)

Further topics

Statistical (unsupervised) learning:

- Clustering: (Janßen and Wan, 2020; Bernard et al., 2013; Saunders et al., 2021; Bador et al., 2015; Rohrbeck and Tawn, 2021; Vignotto et al., 2021; Boulin et al., 2023)
- Dimension reduction: (Chautru, 2015; Drees and Sabourin, 2021; Cooley and Thibaud, 2019)
- Anomaly detection: (Clifton et al., 2014; Rudd et al., 2017; Vignotto and Engelke, 2020)
- Review by Clémençon and Sabourin (2025)

Further topics

Fast inference:

- Neural estimation:
  - Spatio-temporal: (Lenzi et al., 2023; Sainsbury-Dale et al., 2024a,b; Richards et al., 2024; Sainsbury-Dale et al., 2025; Dell'Oro and Gaetan, 2024; Rai et al., 2025; Hector and Lenzi, 2024)
  - Multivariate: (André et al., 2025; Hua, 2025)
  - Univariate: (Rai et al., 2024; Richards et al., 2025)
- Intractable likelihood approximation: (Walchessen et al., 2024; Majumder and Reich, 2023; Majumder et al., 2024)
- Variational inference: (Maceda et al., 2024)
- Dependence classification: (Ahmed et al., 2022; Wixson and Cooley, 2024)

Further topics

Other things to look out for:

- KANs: (de Carvalho et al., 2025)
- Conformal prediction: (Pasche et al., 2025)

Special issue

Home > Collection

Bridging Heavy Tails and Artificial Intelligence

Participating journal: Extremes

Closed for submissions

The increasing frequency and severity of extreme events—such as catastrophic flooding, record-breaking temperatures, and unprecedented heatwaves—has highlighted the urgent need for innovative approaches in risk assessment and modeling. Modern advancements in data-collection techniques have provided increasingly large and complex datasets, which can only be processed used fast and scalable algorithms and computational software. This special issue aims to bridge the gap between Artificial Intelligence (AI) and Extreme Value Theory (EVT) to harness the strengths of both fields and address the growing challenges posed by these extreme events.

Participating journal

Journal

Extremes

Extremes is a dedicated platform for the publication of original research in statistical extreme value theory and its applications across various fields.

Publishing model	Hybrid
Journal Impact Factor	1.1 (2023)
Downloads	58k (2024)
Submission to first decision (median)	7 days

https://link.springer.com/collections/haghbdfdhb

Handbook chapters

- Ch11 - Principal Component Analysis for Multivariate Extremes. Cooley, Sabourin, and Wixson.
- Ch12 - Clustering Methods for Multivariate Extremes. Wan and Janßen.
- Ch20 - On the Simulation of Extreme Events with Neural Networks. Allouche, Girard, and Gobet.
- Ch21 - Extreme Quantile Regression with Deep Learning. Richards and Huser.
- Ch26 - Statistics of Extremes for Wildfires. Koh.

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