

Wheat Yield exercise

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Dataset

- ▶ Exercise developed by Chris Wikle and Dan Pagendam (2019).
- ▶ Dataset created by Dan Pagendam and Josh Bowden at CSIRO.
- ▶ The data consists of simulated wheat yields from a farm in Dalby, Queensland using the model APSIMX.
- ▶ The dataset was created from 10,000 simulations of wheat growth under randomly generated meteorological conditions and management (planting and fertiliser application).

Predictors

- ▶ The predictors are:
 - ▶ summary statistics for the amount of rainfall in each of 52 weeks in the year.
 - ▶ degree days in each of the 52 weeks in the year.
 - ▶ cumulative evaporation in each of the 52 weeks of the year.
 - ▶ the “thermal time” of the wheat at each of the 52 weeks in the year.
 - ▶ the amount of Nitrogen fertiliser applied at planting.
 - ▶ the amount of Nitrogen fertiliser applied as top-up.
 - ▶ the day of year that the crop was planted.
 - ▶ the planting density of seed.

Response

- ▶ There are four response variables:
 - ▶ wheat grain yield.
 - ▶ grain size.
 - ▶ grain protein content.
 - ▶ wheat total weight.

Load dataset

```
#For those using Colab run: install.packages("keras")  
load("../Data/Wheat.Rdata")
```

- ▶ The dataset contains both training and validation data, with response denoted by Y and predictors denoted by \mathbf{X}
- ▶ The training and validation sets were determined by randomly allocating approximately 10% of the simulations to the validation set and the remaining 90% to the training set.

```
dim(trainData_X)
```

```
## [1] 8993 212
```

```
dim(validationData_X)
```

```
## [1] 1007 212
```

Data normalisation

- ▶ We first scale the input data to improve the numerical stability of training

```
rescaleCols <- function(rowX, colMins, colMaxs)
{
  r <- (rowX - colMins)/(colMaxs - colMins)
  r[is.nan(r)] <- 0
  return(r)
}
```

```
colMinsX <- apply(trainData_X, 2, min)
colMaxsX <- apply(trainData_X, 2, max)
```

```
trainData_X_scaled <- t(apply(trainData_X, 1,  
    rescaleCols, colMinsX, colMaxsX))  
validationData_X_scaled <- t(apply(validationData_X, 1,  
    rescaleCols, colMinsX, colMaxsX))
```

Response

- ▶ We focus on modelling a single output response Y : Wheat total weight

```
trainData_Y = matrix(trainData_Y[, "wheatTotalWeight"],  
                      ncol = 1)  
validationData_Y = matrix(  
  validationData_Y[, "wheatTotalWeight"],  
                    ncol = 1)
```

- ▶ We will begin by 'predicting' Y given \mathbf{X} , i.e., we estimate the expectation $\mathbb{E}[Y|\mathbf{X}]$

Building a Keras prediction model

- ▶ We will build a feed-forward neural network with densely-connected layers and ReLU activations
- ▶ Begin by defining the input layer:

```
library(keras)
input.lay <- layer_input(shape = ncol(trainData_X_scaled),
                          name = 'input_layer')
```


Hidden layers

- ▶ The model will have three hidden layers, each with 64 nodes

```
hidden.lay<- input.lay %>%  
  layer_dense(units = 64, activation = "relu") %>%  
  layer_dense(units = 64, activation = "relu") %>%  
  layer_dense(units = 64, activation = "relu")
```

- ▶ The output layer has a single unit; we estimate one value, i.e., the expectation, for each input vector

```
output.lay <- hidden.lay %>% layer_dense(units = 1)
```

Define a Keras model

► Model definition and summary

```
model <- keras_model(  
  inputs = c(input.lay),  
  outputs = c(output.lay)  
)
```

```
summary(model)
```

Model compilation

- ▶ We now compile the model with a loss function and an optimiser
- ▶ Here we use the MSE loss, which targets the mean of the response Y . If we wanted to target the median, we could use MAE instead
- ▶ We will use the RMSProp optimisation algorithm

```
model %>% compile(  
  loss = "mse",  
  optimizer = optimizer_rmsprop(learning_rate = 0.0001)  
)
```

Fitting the model

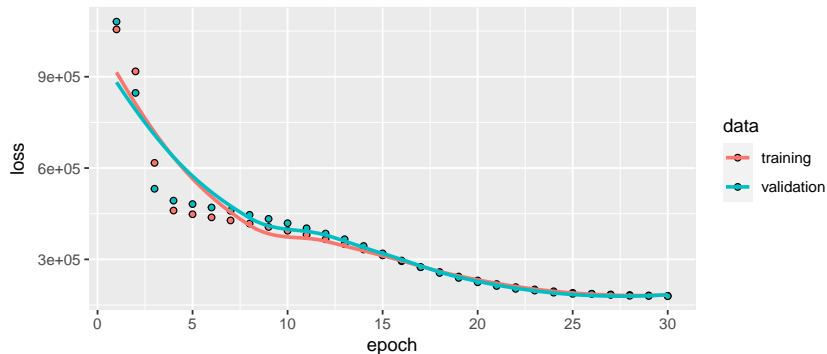
- ▶ We are now ready to fit the model.
- ▶ We define the training data, but also out-of-sample validation data that we can use to check how well the model generalises.
- ▶ When training the neural network, we use batches of data (32 samples here) to estimate the gradient of the loss function w.r.t the parameters.
- ▶ We also need to specify the number of epochs.
- ▶ Because we chose a small learning rate, we'll probably need to use more epochs to find the optimal fit.

```
history <- model %>% fit(  
  x = trainData_X_scaled, y = trainData_Y,  
  epochs = 30, batch_size = 32,  
  validation_data = list(  
    validationData_X_scaled, validationData_Y),  
  verbose=0  
)
```

Fitting the model

- Check for overfitting

```
plot(history)
```



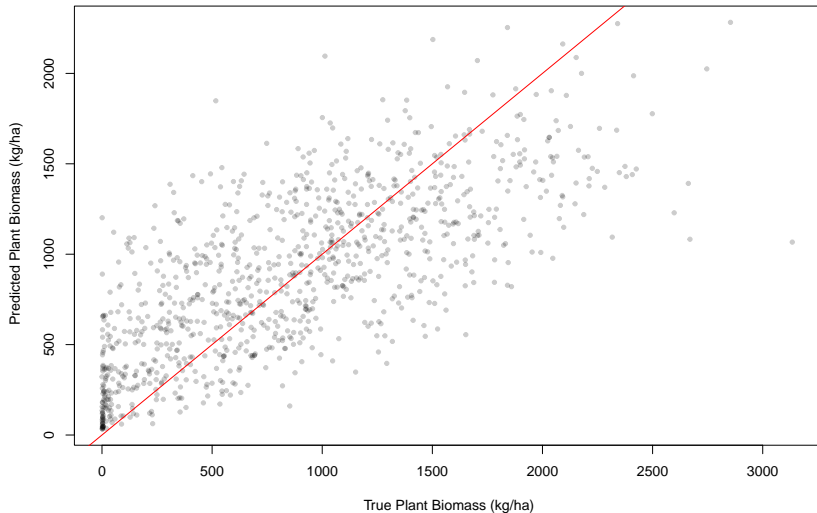
Predictions

► Check predictions

```
pred_validation <- predict(model, validationData_X_scaled)
plot(validationData_Y, pred_validation,
      xlab = " True Plant Biomass (kg/ha)",
      ylab = " Predicted Plant Biomass (kg/ha)",
      pch = 20,
      col = adjustcolor("black", 0.2))
abline(0, 1, col = "red")
```

Predictions

32/32 - 0s - 166ms/epoch - 5ms/step



Improvements

- ▶ We just trained a model using the built in “mse” loss function. This minimises mean squared error and so is analogous to linear regression where we assumed the error to be homoscedastic.
- ▶ Instead, let's fit a log-normal regression model with varying scale parameter, i.e., a heteroscedastic model
- ▶ Do so by minimising the negative log-likelihood, i.e., MLE
- ▶ Not implemented as standard in Keras → We're going to create a custom loss function that is equal to the negative of the log-likelihood under the assumption that our predicted wheat yields can be modelled as having a log-normal distribution.

Custom loss functions

- ▶ See “keras_build.rmd” for help writing custom loss functions
- ▶ All custom loss functions have the same broad structure

```
custom_loss=function(y_true,y_pred){  
  K <- backend()  
  #Code here  
}
```

Custom loss functions

Important notes:

- ▶ when writing your code, you need to remember that `y_true` and `y_pred` are tensors.
- ▶ `y_true` and `y_pred` don't necessarily have to have the same dimension.
- ▶ the first dimension of both the input tensors is equal to the batch size.
- ▶ there is very little documentation on how to write custom loss functions.
- ▶ try to use the Keras backend functions as much as possible and avoid using R functions (they may not work).
- ▶ if a Keras function has the "axis" argument, it is asking you which dimension you want to "apply" the function to.

Custom loss functions

```
negLL_logNormal <- function(y_true, y_pred)
{
  K <- backend()
  # Extract the first and second columns of predictions
  mu <- (y_pred[,1])
  sigma <- K$exp(y_pred[,2])

  # Extract first column of y_true to ensure same dimensions
  y <- y_true[,1]

  # Use mu and sigma as parameters
  # describing log-normal distributions
  logLike <- -1*(K$log(y) + K$log(sigma)) -
    0.5*K$log(2*pi) - (K$log(y) - mu)^2/(2*(sigma)^2)
  return( -(K$sum(logLike)))
}
```

Building a custom model

- ▶ Based on the custom loss function we just created, our model requires two outputs:
 - ▶ one for μ (location parameter of the log-normal).
 - ▶ one for σ (scale parameter of the log-normal).
- ▶ Let's use 3 hidden layers, with 64 nodes per layer and stick with the rectified linear unit (ReLU) activation function.
- ▶ Notice, that we now have two nodes / units in the output layer of the network.

```
input.lay <- layer_input(shape = ncol(trainData_X_scaled),  
                        name = 'input_layer')  
hidden.lay2<- input.lay %>%  
  layer_dense(units = 64, activation = "relu") %>%  
  layer_dense(units = 64, activation = "relu") %>%  
  layer_dense(units = 64, activation = "relu")  
output.lay2 <- hidden.lay2 %>% layer_dense(units = 2)
```

Compiling

- ▶ We will use the same optimiser as previously
- ▶ ... but must specify the custom loss function

```
model2 <- keras_model(  
  inputs = c(input.lay),  
  outputs = c(output.lay2)  
)  
model2 %>% compile(  
  loss = negLL_logNormal,  
  optimizer = optimizer_rmsprop(learning_rate = 0.0001)  
)
```

Fitting the model

- ▶ We're also going to introduce a callback into our training procedure.
- ▶ This will reduce the learning rate when we stop seeing a reduction in the validation loss.
- ▶ A smaller learning rate means smaller changes to the parameters, so we can think of this as fine-tuning our parameters with more and more epochs.
- ▶ Other callbacks include checkpoints and early-stopping

```
callback <- list(  
  callback_reduce_lr_on_plateau(  
    monitor = "val_loss",  
    factor = 0.25, patience = 5  
  )  
)
```

Fitting the model

```
history <- model2 %>% fit(  
  x = trainData_X_scaled, y = trainData_Y,  
  epochs = 50, batch_size = 32,  
  validation_data = list(  
    validationData_X_scaled, validationData_Y),  
  callbacks = callback,  
  verbose=0  
)
```

Predictions

- ▶ The model outputs two quantities for each input vector
- ▶ These are the μ and $\log(\sigma)$ parameters for a log-normal predictive density.
- ▶ We can compare the mean of the log-normal to the true yield.

```
yhat <- predict(model2, validationData_X_scaled)
```

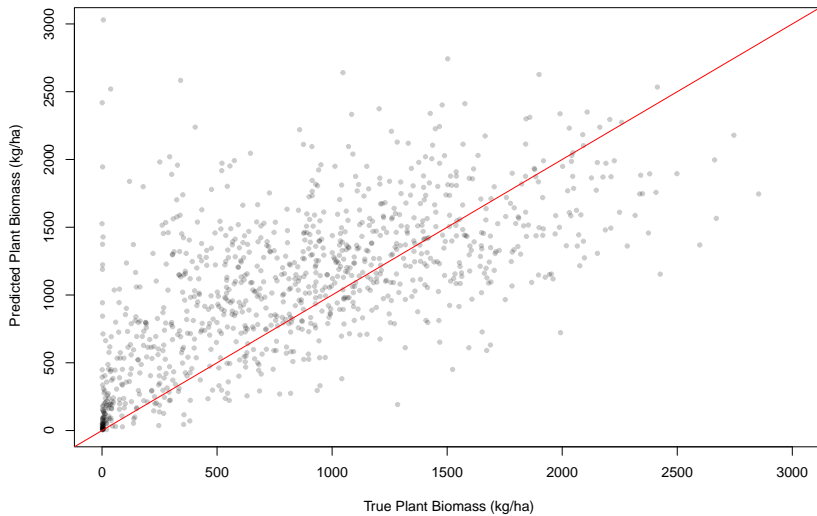
```
## 32/32 - 0s - 117ms/epoch - 4ms/step
```

```
mu <- yhat[, 1]
```

```
sigma <- exp(yhat[, 2])
```

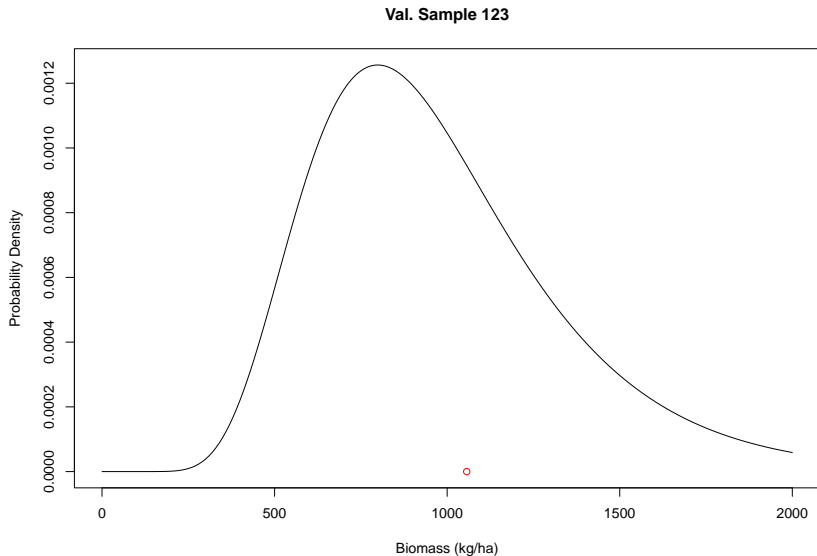
```
pred_mean <- exp(mu + 0.5*sigma^2)
```


Predictions



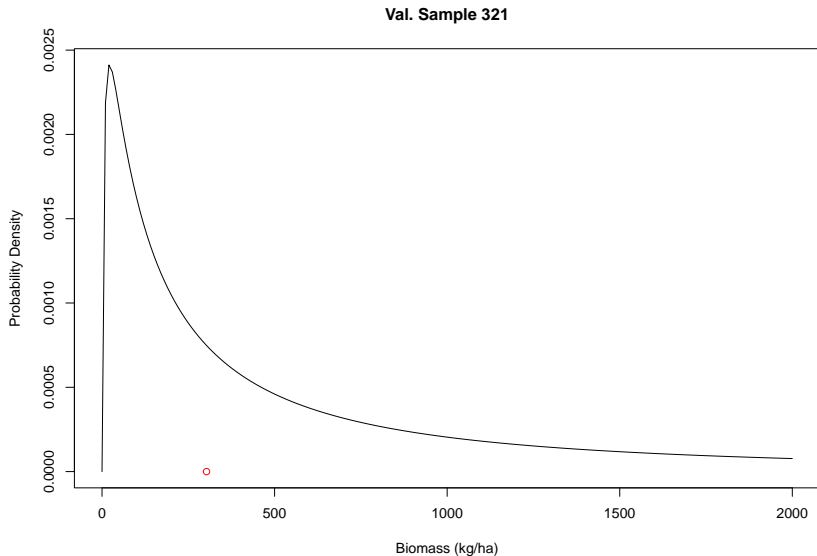
Predicted densities

- We can also look at the predicted density for specific samples



Predicted densities

- We can also look at the predicted density for specific samples



Extensions

- ▶ How do your results change if you include more nodes in the hidden layers?
- ▶ How do your results change if you make the network deeper?
- ▶ Try including some form of regularisation (e.g. Dropout or L1 regularisation) in the hidden layers to help prevent overfitting.
- ▶ There's some extra details in `keras_build.html`, including how to save/load models (can be difficult with custom loss functions), multi input/output models, homogeneous model parameters