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Short communication

Uncertainty analysis of positional deviations of CNC machine tools

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Abstract

The assessment of the measurement uncertainty is an indispensable task in all calibration procedures. By international accord, the evaluation is to be done in accordance with the ISO Guide to the Expression of Uncertainty in Measurement (GUM). To calibrate the positional deviations of computer numerically controlled (CNC) machine tools, calibration laboratories will usually follow the guidelines in ISO 230-2 International Standard. However, that standard does not address uncertainty. In this paper, we present an uncertainty evaluation scheme that is firmly grounded in the GUM, and can therefore be of use as a guide to develop appropriate uncertainty calculations in this and similar types of calibrations.

Keywords: Uncertainty analysis; Machine tools; Calibration; Dimensional metrology

1. Introduction

The ISO 230-2 International Standard [1] is accepted and used worldwide to determine the accuracy and repeatability of positioning of computer numerically controlled (CNC) machine tool axes; it may be used for type testing, acceptance tests, comparison testing, periodic verification and machine compensation.

The test method standardized in ISO 230-2 involves repeated measurements at predefined positions of the axis under test. The results are used to determine various parameters intended to quantify the performance of the machine. Some of these parameters depend only on the values measured at a given position; other parameters characterize the global behavior of the axis. The local parameters are the mean positional deviations, the reversal values and the repeatabilities. The global parameters are the reversal value, the mean reversal value, the repeatability, the systematic positional deviation, the mean positional deviation and the accuracy.

The repeatability and accuracy depend upon a further local parameter that ISO 230-2 defines as the standard uncertainty of positioning, s. For example, the unidirectional repeatability at position i is defined as $4s_i$. The intention is clearly to provide a range derived from an expanded uncertainty using a coverage factor of 2. The latter terminology is taken from the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [2], which ISO 230-2 declares to follow. However, the GUM defines the parameter s as the "sample standard deviation"; under no circumstances it should be taken as an estimator of standard uncertainty. It follows that there is a conflict in terminology between these two documents.

Moreover, the International Vocabulary of Basic and General Terms in Metrology (VIM) [3] defines calibration as "the set of operations that establish the relationship between values of quantities indicated by a measuring system and the corresponding values realized by measurement standards." Since the test code specified by ISO 230-2 clearly complies with this (abridged) definition, it follows that the test report can be interpreted as a calibration certificate of the machine. In that case, however, the uncertainty of measurement should be reported together with the measurement results [4]. ISO 230-2 does not address this crucial point.

The purpose of this paper is, therefore, to propose a procedure by which to evaluate the uncertainty associated with the measured positional deviations of machine tools. The procedure follows strictly the recommendations of the GUM and uses the measurement results obtained with the ISO 230-2 test method. We propose that future revisions of the standard should include this procedure. In this way, the testing laboratory would be able to issue a calibration certificate instead of just a test report. Future revisions of ISO 230-2 should also alter slightly its terminology with regard to the parameter s, so as to comply with the GUM.

The outline of the paper is as follows. In Section 2 we summarize the ISO 230-2 test method. In Section 3 we present

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the measurement model on which we base the uncertainty evaluation. From this model all relevant input quantities are identified. The standard uncertainties of these quantities are derived following the recommendations of the GUM. An example is presented in Section 4 and conclusions are given in Section 5. Appendix A treats the case of nonlinear measurement models.

2. The ISO 230-2 test method

In the ISO 230-2 test method the machine is programmed to locate its moving part at a series of predefined target positions along the axis under test. Fig. 1 shows the standard test cycle. It consists of n unidirectional approaches in both the positive (increasing) and negative (decreasing) directions to m target positions. The symbols $\uparrow$ and $\downarrow$ are used to represent each of these directions of movement.

The measured quantities are the positional deviations. For point i in unidirectional approach j, these are defined as the differences between the actual positions p_{ij} and the target positions p_i:

$$x_{ij}| = p_{ij}| - p_i, \qquad (1)$$

where the symbol | stands for either $\uparrow$ or $\downarrow$. In this expression the index j runs from 1 to n and the index i runs from 1 to m. In the standard test cycle, $n = 5$. The value of m depends on the length of the axis; ISO 230-2 recommends selecting a minimum of five unevenly spaced target positions per meter of measurement travel.

Local and global parameters are derived from the positional deviations; they are given in Tables 1 and 2, respectively. It is recommended that these parameters be shown in graphical form. Fig. 2 shows a typical plot; it is taken from the example provided in ISO 230-2.

The test report should also provide information regarding the test equipment, the test conditions, the built-in compensation routines used (if any), the feed rate, the dwell time at the target positions, the warm-up operations that precede the test, etc. In particular, the test report should detail the location of the temperature sensors, the indicated temperatures and the thermal expansion coefficient used to calculate the nominal differential expansion (NDE) correction for measurements performed at temperatures other than 20 °C. Although the standard provides a typical value of uncertainty equal to $\pm 2\ \mu$m/(m °C) resulting from temperature departures from 20 °C, this is left for the user of the test report to assess.

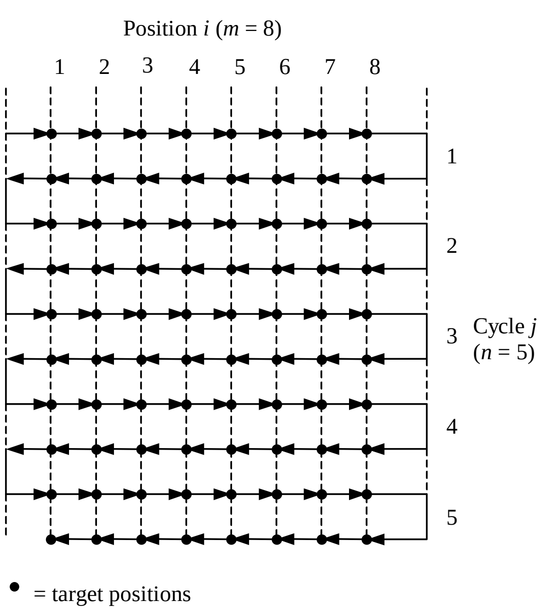

Fig. 1. Standard test cycle for $n = 5$ bidirectional approaches to $m = 8$ calibrated points.

Table 1
Local parameters defined in ISO 230-2

Parameter	Symbol	Definition
Mean positional deviations	$\bar{x}_i \uparrow$	$(1/n)\sum_j x_{ij}\uparrow$
	$\bar{x}_i \downarrow$	$(1/n)\sum_j x_{ij}\downarrow$
	$\bar{x}_i$	$(\bar{x}_i\uparrow + \bar{x}_i\downarrow)/2$
Reversal value	B_i	$\bar{x}_i\uparrow - \bar{x}_i\downarrow$
Experimental variances[a]	$s_i^2\uparrow$	$\sum_j (x_{ij}\uparrow - \bar{x}_i\uparrow)^2/(n-1)$
	$s_i^2\downarrow$	$\sum_j (x_{ij}\downarrow - \bar{x}_i\downarrow)^2/(n-1)$
Repeatabilities	$R_i\uparrow$	$4s_i\uparrow$
	$R_i\downarrow$	$4s_i\downarrow$
	R_i	$\mathrm{Max}[2s_i\uparrow + 2s_i\downarrow + \lvert B_i\rvert;\ R_i\uparrow;\ R_i\downarrow]$

[a] ISO 230-2 does not actually define the experimental variances. It defines their square roots as "estimators of the unidirectional standard uncertainty of positioning."

Table 2
Global parameters defined in ISO 230-2

Parameter	Symbol	Definition
Reversal value	B	$\mathrm{Max}[\lvert B_i\rvert]$
Mean reversal value	$\bar{B}$	$(1/m)\sum_i B_i$
Repeatabilities	$R\uparrow$	$\mathrm{Max}[R_i\uparrow]$
	$R\downarrow$	$\mathrm{Max}[R_i\downarrow]$
	R	$\mathrm{Max}[R_i]$
Systematic positional deviations	$E\uparrow$	$\mathrm{Max}[\bar{x}_i\uparrow] - \min[\bar{x}_i\downarrow]$
	$E\downarrow$	$\mathrm{Max}[\bar{x}_i\downarrow] - \min[\bar{x}_i\downarrow]$
	E	$\mathrm{Max}\,[\bar{x}_i\uparrow;\ \bar{x}_i\downarrow] - \min\,[\bar{x}_i\uparrow;\ \bar{x}_i\downarrow]$
Mean positional deviation	M	$\mathrm{Max}[\bar{x}_i] - \min[\bar{x}_i]$
Accuracies	$A\uparrow$[a]	$\mathrm{Max}[\bar{x}_i\uparrow + 2s_i\uparrow] - \min[\bar{x}_i\uparrow - 2s_i\uparrow]$
	$A\downarrow$	$\mathrm{Max}[\bar{x}_i\downarrow + 2s_i\downarrow] - \min[\bar{x}_i\downarrow - 2s_i\downarrow]$
	A	$\mathrm{Max}[\bar{x}_i\uparrow + 2s_i\uparrow;\ \bar{x}_i\downarrow + 2s_i\downarrow] - \min[\bar{x}_i\uparrow - 2s_i\uparrow;\ \bar{x}_i\downarrow - 2s_i\downarrow]$

[a] In ISO 230-2 the parameter $A\uparrow$ is given as $\max[\bar{x}_i\uparrow + 2s_i\uparrow] - \min[\bar{x}_i\uparrow + 2s_i\uparrow]$.

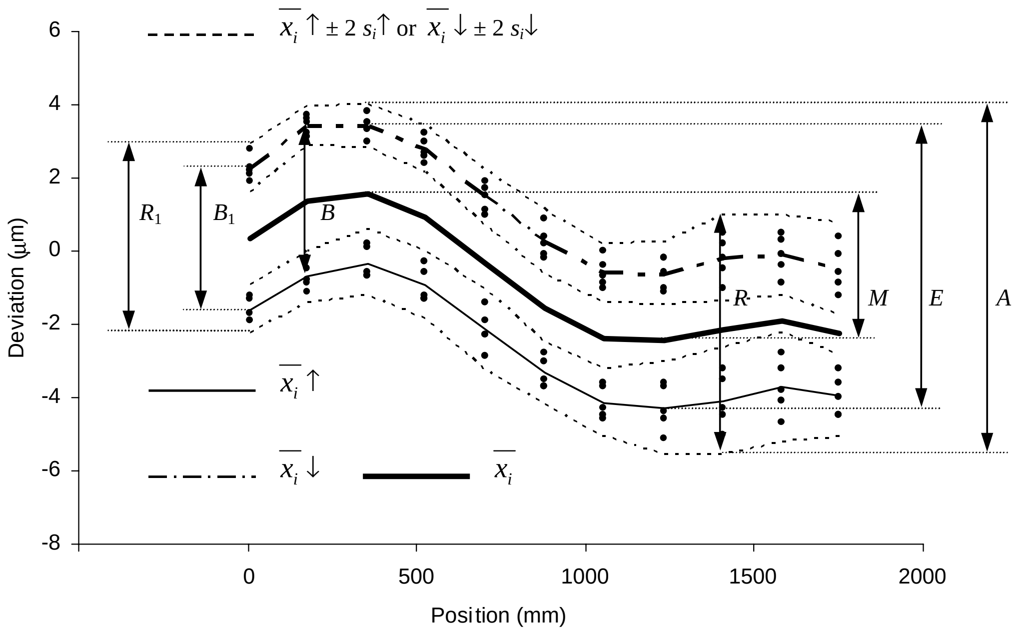

Fig. 2. Example of results obtained with the test code in ISO 230-2.

It is then clear that the main contribution of ISO 230-2 is the standardization of the various parameters that characterize the behavior of machine tools, such as repeatability and accuracy. However, it does not address the calculation of the measurement uncertainty. It does not consider resolution, misalignment, and calibration of the measuring equipment. In addition, as mentioned above, its terminology does not comply with that in the GUM.

3. A proposal to evaluate the uncertainty in the test of CNC machines

Reference [4] indicates that "the statement of the result of a measurement is complete only if it contains both the value attributed to the measurand and the uncertainty of measurement associated with that value." In order to evaluate measurement uncertainties, the measurands must be defined through appropriate measurement models. When the models are linear or weakly nonlinear, they allow to express the standard uncertainties of the output quantities in terms of the standard uncertainties of the input quantities by using the so-called law of propagation of uncertainties, LPU [2,5]. This law says that for a quantity modeled as

$$z = f(x, y), \tag{2}$$

the standard uncertainty is obtained as the square root of

$$u^2[z] = c_x^2 u^2[x] + c_y^2 u^2[y] + 2 c_x c_y r_{xy} u[x] u[y], \tag{3}$$

where $c_x = \partial f/\partial x$, $c_y = \partial f/\partial y$ and r_{xy} is the correlation coefficient between the input quantities x and y. If these are independent, the correlation coefficient is zero. The generalization of this law to more than two input quantities is straightforward.

3.1. Basic model

In the calibration of machine tools, the measurands are the positional deviations. These are modeled as

$$x_i| = p_i| - p_i, \tag{4}$$

where $p_i|$ is the corrected indication of the measurement system at position i and p_i is the corrected target position. As in Section 2, the symbol | stands for either ↑ or ↓.

The models for the quantities $p_i|$ and p_i should in turn include all conceivable random and systematic effects that influence them. While this depends on the particulars of the measurement system, it is reasonable to assume that measurements in testing machine tools are in general affected or limited by the characteristics of the test equipment and also by the conditions at which the test is carried out. The latter include temperature, alignment and resolution. Therefore, we write

$$p_i| = f_\mathrm{t} f_\mathrm{a} p_{\mathrm{m}i}| + c_\mathrm{s} \tag{5}$$

and

$$p_i = p_{\mathrm{n}i} + c_\mathrm{r}, \tag{6}$$

where

$$p_{\mathrm{m}i}| = \frac{\sum_j p_{ij}|}{n} \tag{7}$$

is the arithmetic mean of the actual positions $p_{ij}|$ in the n unidirectional approaches to the nominal (programmed) target position $p_{\mathrm{n}i}$, f_t is a correction factor to compensate for thermal expansion of the positioning scale, f_a is a correction factor to compensate for possible misalignment of the measurement system, c_s is a correction associated with the measurement system, and c_r is a correction associated with the resolution of the axis scale.

(Note the difference between our definition of $x_i|$ and the parameter $\bar{x}_i|$ in ISO 230-2 standard. In terms of our notation, the latter is $\bar{x}_i| = p_{mi}| - p_{ni}$.)

Replacing (5) and (6) in (4), using the LPU, assuming that all input quantities are independent, and recognizing that no uncertainty should be attached to the given nominal positions, we obtain

$$u^2[x_i|] = (f_t f_a u[p_{mi}|])^2 + (f_a p_{mi}|u[f_t])^2 + (f_t p_{mi}|u[f_a])^2 + u^2[c_r] + u^2[c_s]. \quad (8)$$

Finally, the model for the mean bidirectional positional deviation at position i is

$$\bar{x}_i = \tfrac{1}{2}(x_i \uparrow + x_i \downarrow). \quad (9)$$

The uncertainty associated with this quantity is then

$$u^2[\bar{x}_i] = \tfrac{1}{4}(u^2[x_i \uparrow] + u^2[x_i \downarrow] + 2u[x_i \uparrow]u[x_i \downarrow]). \quad (10)$$

This last expression assumes a correlation coefficient between $x_i\uparrow$ and $x_i\downarrow$ equal to one. This is reasonable, since all influence quantities that are likely to affect $x_i\uparrow$, such as barometric pressure, ambient temperature and humidity, are likely to affect $x_i\downarrow$ in the same sense.

3.2. *Type A uncertainties*

According to the GUM, the squares of the standard uncertainties associated with the unidirectional position means $p_{mi}|$ should be evaluated as the sample variances divided by the number n of unidirectional measurements. Thus, we set

$$u^2[p_{mi}|] = \frac{S_i|^2}{n}, \quad (11)$$

where

$$S_i|^2 = \frac{\sum_j (p_{ij}| - p_{mi}|)^2}{(n-1)}. \quad (12)$$

Note the difference between the sample variances $S_i|^2$ and $s_i|^2$ (see the ISO 230-2 definition in Table 1). The former uses the actual positions $p_{ij}|$ while the latter uses the deviations $x_{ij}|$. If the measuring instrument indicates directly the deviations, the actual positions must be obtained as $p_{ij}| = x_{ij}| + p_{ni}$.

Eq. (11) is the well-known type A evaluation of standard uncertainty [2]. In this method of evaluation, the sample standard deviation S_i has to be divided by the square root of the number of measurements n to obtain the standard uncertainty $u[p_{mi}|]$. This is due to the fact that $u[p_{mi}|]$ is associated with the *average* of the measurements, not to any one single result $p_{ij}|$. For this reason we believe that the quantities $s_i|$ in Table 1 should *not* be called "estimators of standard uncertainties." For further discussion on this point, see Section 3.8 of Reference [5].

3.3. *Type B uncertainties*

The type A evaluation of standard uncertainty applies only to quantities that are measured directly several times under repeatability conditions. The uncertainty of input quantities that are measured only once, that are evaluated from models that involve further quantities, or that are imported from other sources, should be evaluated by other means. The GUM refers to these "other means" as the type B method of evaluation. In many cases this involves obtaining an uncertainty as the standard deviation of the probability density function (pdf) that is assumed to apply to the quantity involved.

Consider first the NDE correction. The model for this correction is derived from the equation that defines the linear coefficient of thermal expansion α. This equation is

$$L - L_o = L_o\alpha(T - T_o), \quad (13)$$

where L_o is the length of an object at a reference temperature T_o and L is the length of the same object at the actual temperature T.

In our case, L_o is the position p_{oi} at temperature $T_o = 20\,^\circ$C and L is the measured position $p_{mi}|$ at the temperature of the test. Therefore, the thermal correction factor is

$$f_t = (1 + \alpha\Delta T)^{-1}, \quad (14)$$

where $\Delta T = T - 20\,^\circ$C. Application of the LPU then gives

$$u^2[f_t] = (\Delta T^2 u^2[\alpha] + \alpha^2 u^2[\Delta T])f_t^4. \quad (15)$$

The temperature T should be measured at at least two places along the scale and at various times, for example, at the beginning and at the end of the cycle. Doing this allows to establish a range of values for ΔT. This range should be increased to both sides by including the expanded uncertainty of the temperature measurement device. In this way one obtains a range w_T over which a uniform pdf is a reasonable assumption. Thus, from Equation 3.19 in Reference [5] we set

$$u^2[\Delta T] = \frac{w_T^2}{12}. \quad (16)$$

The coefficient α is normally not measured; its value is imported from the manufacturer of the machine or from tabulated values in handbooks for the material one believes the scale to be made off. The standard uncertainty $u[\alpha]$ should then be evaluated from a uniform pdf whose width w_α has to be assigned depending on how well one knows the material and on how well that material's coefficient is known. In other words, w_α corresponds to twice the maximum possible error in the assignment of the value of α. With this assumption we get

$$u^2[\alpha] = \frac{w_\alpha^2}{12}. \quad (17)$$

It should be noted, however, that in the vicinity of $\Delta T = 0$ Eq. (15) yields a value for $u[f_t]$ that is independent of the uncertainty in knowing α. This is obviously an unreasonable result; it is due to the nonlinearity of the model for the thermal expansion correction factor in the vicinity of vanishing values of the input quantities. In the Appendix A we address this problem.

(Note that by writing Eq. (13) in the form $L - L_o = L\alpha\Delta T$, one gets $f_t = 1 - \alpha\Delta T$. An almost equal numerical value for the correction factor is obtained, but the expression for the standard uncertainty becomes $u^2[f_t] = \Delta T^2 u^2[\alpha] + \alpha^2 u^2[\Delta T]$. We feel that Eq. (15) is to be preferred. In any case, however, if $\Delta T = 0$ the uncertainty of f_t is independent of the uncertainty of α.)

Consider next the alignment correction factor f_a. This is modeled as

$$f_a = \cos\theta, \tag{18}$$

where θ is the angle between the actual measurement direction and the axis direction. Because of the form of (18) this correction factor is usually referred-to as the cosine error. Application of the LPU gives

$$u[f_a] = \sin\theta. \tag{19}$$

However, normally the best estimate for the misalignment angle will be $\theta = 0$, yielding $f_a = 1$ and $u[f_a] = 0$ irrespective of the uncertainty associated with θ. This is again an unreasonable result, and is due to the strong nonlinearity of $\cos\theta$ in the vicinity of $\theta = 0$. Therefore, we recommend to estimate the uncertainty of f_a from

$$u^2[f_a] = \frac{1}{2}(f_a \cos\theta_{max} + 1) - f_a^2, \tag{20}$$

where θ_{max} is an upper bound for the maximum deviation angle, and where we must use

$$f_a = \frac{\sin\theta_{max}}{\theta_{max}}. \tag{21}$$

The angle θ_{max} has to be established based on the particulars of the measurement system. Since normally this angle should be small, $\cos\theta_{max} \approx 1$ and $u[f_a] \approx 0$. Eqs. (20) and (21) are derived in the Appendix A. In the Appendix A we also discuss why a uniform pdf is not appropriate to represent the cosine error.

The evaluation of the resolution correction is straightforward. Its value is equal to zero and its standard uncertainty is obtained from a uniform pdf of width equal to the resolution r of the axis scale. Thus,

$$u^2[c_r] = \frac{r^2}{12}. \tag{22}$$

The measurement system correction c_s depends on the instrument that is used to perform the measurements. An appropriate model for this correction is

$$c_s = c_{sr} + c_c + c_e,$$

where c_{sr} is a resolution correction, c_c is a calibration correction and c_e is a correction arising from use of the instrument at non-standard environmental conditions. From this model we get

$$u^2[c_s] = u^2[c_{sr}] + u^2[c_c] + u^2[c_e].$$

The value of c_{sr} is zero, and its uncertainty is obtained as $u^2[c_{sr}] = r_s^2/12$, where r_s is the resolution of the measurement system. Since this resolution should normally be much smaller than r, the uncertainty $u[c_{sr}]$ can be neglected.

The value and uncertainty of c_c should be obtained from the calibration certificate of the instrument, thus providing the traceability for the calibration of the machine.

Finally, if the instrument performs environmental corrections automatically, the value of the correction c_e may be taken as zero. However, the uncertainty $u[c_e]$ is normally not given in the calibration certificate of the measuring system. A reasonable assumption is to take this uncertainty as being proportional to the nominal position p_{ni} and to the temperature difference $\Delta T_e = T_e - T_o$, where T_e is the ambient temperature. In other words, we may write $u[c_{ei}] = Kp_{ni}\Delta T_e$, where the value for the proportionality constant K should be establish based on the characteristics of the measurement system.

Although this procedure to estimate $u[c_e]$ may be questionable, it is the most one can do if there is no information about the uncertainty of the environmental compensation. We must remember that uncertainties are parameters that are not measured, they only express the "doubt" that we have on the results of measurements [2]. Therefore, subjective criteria for establishing standard uncertainties do not contradict the GUM.

3.4. Expanded uncertainties

To obtain the expanded uncertainties $U[\bar{x}_i]$, a coverage factor $k = 2$ is appropriate to define intervals estimated to have a level of confidence of about 0.95 for the true value of the mean bidirectional deviations $\bar{x}_i$. This is because the standard uncertainties $u[\bar{x}_i]$ are obtained through models that involve several input quantities whose standard uncertainties are derived mostly from type B analysis. Therefore, the number of degrees of freedom should be rather large (see Chapter 4 in Reference [5]).

4. Example

In this section, we calculate the uncertainty associated with the mean bidirectional deviations reported in the example given in ISO 230-2. The example refers to the test of a linear axis at $m = 11$ positions. Tables 3 and 4 show the unidirectional deviations $x_{ij}\uparrow$ and $x_{ij}\downarrow$, respectively. Table 5 shows the numerical values of each uncertainty term in the right-hand side of (8) for the mean unidirectional positional deviations $x_i\uparrow$. The corresponding values for the decreasing deviations $x_i\downarrow$ are very similar. The dominant contribution arises from the NDE correction. Table 6 shows the mean bidirectional positional deviations $\bar{x}_i$ and their expanded uncertainties $U[\bar{x}_i]$. These values are plotted in Fig. 3, and were obtained as follows.

The test report given in ISO 230-2 mentions that NDE correction was performed, but it does not state the temperature

Table 3
Deviations (in micrometers) for a series of 5 unidirectional approaches to 11 nominal positions in the increasing direction

Position i	$x_{i1}\uparrow$	$x_{i2}\uparrow$	$x_{i3}\uparrow$	$x_{i4}\uparrow$	$x_{i5}\uparrow$
1	−1.2	−1.7	−1.9	−1.3	−1.9
2	−0.5	−0.9	−1.1	−0.2	−0.8
3	0.2	−0.6	−0.7	0.1	−0.7
4	−0.6	−1.2	−1.3	−0.3	−1.3
5	−1.9	−2.3	−2.9	−1.4	−2.3
6	−3.0	−3.5	−3.7	−2.8	−3.7
7	−3.7	−4.3	−4.6	−3.6	−4.5
8	−3.7	−4.4	−5.1	−3.6	−4.6
9	−3.5	−4.3	−5.0	−3.2	−4.5
10	−3.2	−3.8	−4.7	−2.8	−4.1
11	−3.6	−4.0	−4.5	−3.2	−4.5

The nominal positions are given in Table 6.

Table 4
Deviations (in μm) for a series of 5 unidirectional approaches to 11 nominal positions in the decreasing direction

Position i	$x_{i1}\downarrow$	$x_{i2}\downarrow$	$x_{i3}\downarrow$	$x_{i4}\downarrow$	$x_{i5}\downarrow$
1	2.3	2.1	1.9	2.8	2.2
2	3.6	3.5	3.1	3.7	3.2
3	3.5	3.3	3.0	3.8	3.5
4	3.0	2.7	2.4	3.2	2.6
5	1.7	1.5	1.0	1.9	1.1
6	0.4	0.2	−0.2	0.9	−0.1
7	−0.4	−0.7	−1.0	0.0	−0.9
8	−0.2	−0.6	−1.0	−0.2	−1.1
9	0.2	−0.2	−1.0	0.5	−0.5
10	0.3	−0.1	−0.9	0.5	−0.4
11	−0.1	−0.6	−1.2	0.4	−0.9

The nominal positions are given in Table 6.

Table 5
Components of the standard uncertainty (in μm) associated with the positional deviations in the increasing direction

Position i	$f_t f_a u[p_{mi}\uparrow]$	$f_a p_{mi}\uparrow u[f_t]$	$f_t p_{mi}\uparrow u[f_a]$	$u[c_r]$	$u[c_s]$
1	0.148	0.030	0.003	0.289	0.003
2	0.158	0.772	0.079	0.289	0.019
3	0.201	1.561	0.161	0.289	0.038
4	0.206	2.319	0.239	0.289	0.056
5	0.248	3.106	0.320	0.289	0.075
6	0.186	3.890	0.400	0.289	0.094
7	0.206	4.658	0.479	0.289	0.113
8	0.282	5.445	0.560	0.289	0.132
9	0.330	6.213	0.640	0.289	0.150
10	0.334	6.971	0.718	0.289	0.169
11	0.254	7.724	0.795	0.289	0.187

Table 6
Mean bidirectional positional deviation and expanded uncertainties ($k = 2$)

p_{ni} (mm)	$\bar{x}_i$ (μm)	$U[\bar{x}_i]$ (μm)
6.711	0.3	0.7
175.077	1.4	1.7
353.834	1.5	3.2
525.668	0.9	4.7
704.175	−0.4	6.3
881.868	−1.6	7.9
1055.890	−2.4	9.4
1234.304	−2.4	11.0
1408.462	−2.1	12.5
1580.269	−1.9	14.0
1750.920	−2.2	15.6

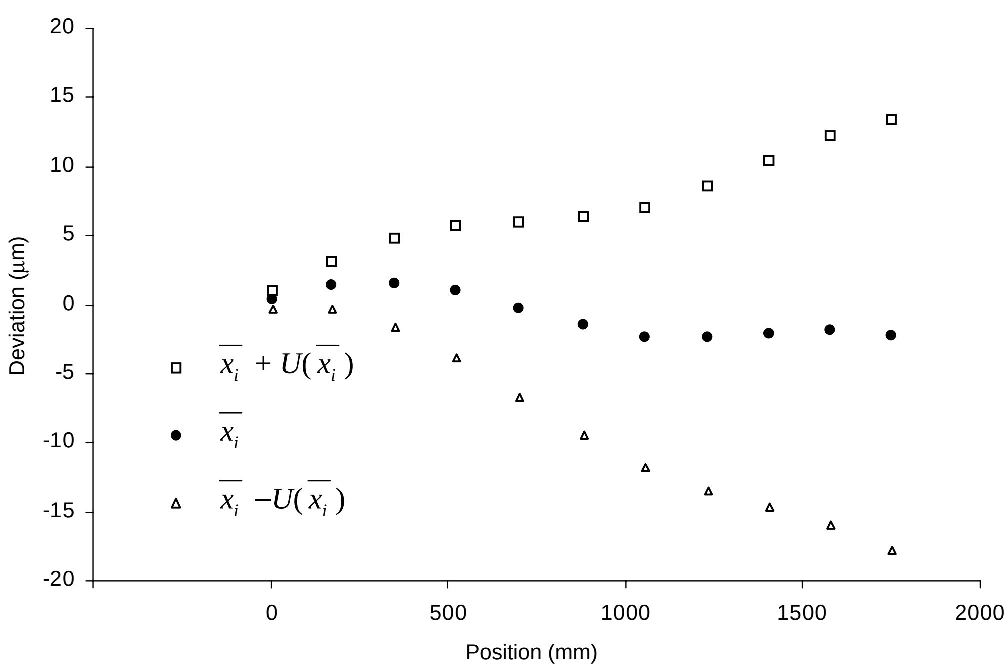

Fig. 3. Mean bidirectional positional deviations and expanded uncertainties for the example of ISO 230-2.

difference used. It indicates a minimum temperature of 21.8 °C at the start of the test at position 50 mm, and a maximum temperature of 23.1 °C at the end of the test at position 1700 mm. Therefore, we assume an average temperature $T = 22.45\,°\mathrm{C}$ and a temperature difference $\Delta T = 2.45\,°\mathrm{C}$. To simulate the actual (uncorrected) positions, we take $p_{ij}| = (x_{ij}| + p_{ni})/f_t$ with $f_t = 0.99997$. For the uncertainty of ΔT we take $w_T = (23.1 - 21.8)\,°\mathrm{C} = 1.3\,°\mathrm{C}$. Since this temperature spread is likely to be much greater than the uncertainty of the thermometer, we ignore the latter. This gives $u[\Delta T] = 0.38\,°\mathrm{C}$.

The thermal expansion coefficient is reported as being $\alpha = 11 \times 10^{-6}\,°\mathrm{C}^{-1}$. For the associated uncertainty we use $w_\alpha = 0.2\alpha$, that is, we assume a maximum error of 10% of the nominal value to each side. This gives $u[\alpha] = 0.635 \times 10^{-6}\,°\mathrm{C}^{-1}$.

For the maximum misalignment angle, we use $\theta_{\max} = 0.1°$, giving $u[f_a] = 0.45 \times 10^{-6}$. For consistency, in (8) we use the value $f_a = 0.99999949$ obtained from (21). However, to evaluate the deviations $x_i\!\uparrow$ and $x_i\!\downarrow$ we take $f_a = 1$ in order not to modify the measured values given in the example of ISO 230-2.

From the nominal positions in the example we assume that the scale resolution is $r_s = 1\,\mu\mathrm{m}$. This gives $u[c_r] = 0.289\,\mu\mathrm{m}$.

The example does not give details on the measurement system used to obtain the data in Tables 3 and 4. We assume that a laser interferometer was used with $r_s = 0.01\,\mu\mathrm{m}$ resolution, giving $u[c_{sr}] = 0.0029\,\mu\mathrm{m}$. We assume also that the calibration certificate of the interferometer states a negligible correction c_c at standard conditions with a typical relative expanded uncertainty $U[c_c] = 2 \times 10^{-7}$ and a coverage factor $k = 2$. We then take $u[c_{ci}] = p_{ni} \times 10^{-7}$, giving $u[c_c] = 0.175\,\mu\mathrm{m}$ at nominal position 1750.92 mm.

Finally, the ambient temperature is reported as being 20.6 °C at the start of the test and 20.9 °C at the end. These temperatures provide only an indication of the environmental conditions during the test, but are not used for NDE correction. In our model we use $c_e = 0$, $T_e = 20.75\,°\mathrm{C}$, $\Delta T_e = 0.75\,°\mathrm{C}$ and $K = 0.05\,°\mathrm{C}^{-1}$. These values give a maximum uncertainty $u[c_e] = 0.066\,\mu\mathrm{m}$ at nominal position 1750.92 mm.

5. Conclusions

ISO 230-2 standardizes the various parameters that characterize the behavior of machine tools, such as repeatability and accuracy. However, it does not address the calculation of uncertainty, which limits its use as reference for issuing calibration certificates based on the test method. In this paper we have presented a detailed procedure to evaluate the uncertainty in the calibration of the positional deviations of linear axes of numerically controlled machine tools. The method complies strictly with the GUM's recommendations: it is derived from a model for the measurand, whose uncertainty is obtained through application of the law of propagation of uncertainties together with types A and B evaluation methods. We propose to include this uncertainty evaluation procedure in a future revision of the standard. The revision should also alter slightly the terminology, so as to comply with the GUM.

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Appendix

Both the thermal expansion and alignment correction factors arise frequently in dimensional measurements. Since the models for these quantities are nonlinear, one may encounter problems when applying the LPU to derive the corresponding uncertainties. Possible solutions are developed in this appendix.

Consider first the thermal expansion correction factor. For generality we write x instead of ΔT, y instead of α and z instead of f_t, giving

$$z = \frac{1}{(1 + xy)}. \tag{A.1}$$

Assume that x and y are independent and that the state of knowledge about these quantities is represented by pdfs $f_x(x)$ and $f_y(y)$, respectively. Use of the tools in Section 6.9 of Reference [5] gives the following pdf for z

$$f_z(z) = z^{-2} \int f_x(x) f_y(y) |y^{-1}|\, \mathrm{d}y, \tag{A.2}$$

where

$$x = y^{-1}(z^{-1} - 1). \tag{A.3}$$

Assume now that x and y are distributed uniformly over the intervals (x_m, x_M) and (y_m, y_M), respectively. The integrand in (A.2) is then equal to the constant $(w_x w_y)^{-1}$, where $w_x = (x_M - x_m)$ and $w_y = (y_M - y_m)$, when the following two inequalities are satisfied simultaneously

$$x_m \le x \le x_M, \tag{A.4}$$

$$y_m \le y \le y_M. \tag{A.5}$$

Careful consideration of (A.4) and (A.5) yields the pdf $f_z(z)$. It is of the form

$$f_z(z) = (w_x w_y)^{-1} z^{-2} \ln(R), \tag{A.6}$$

where R depends on z and on whether x_m and x_M are positive or negative (y is always positive). The function R is given in Table A.1.

Unfortunately from the pdf $f_z(z)$ we cannot obtain an analytical expression for the standard uncertainty $u[z]$; it has to be obtained numerically. Let us assume that the calibration temperature is $(18 \pm 0.5)\,°\mathrm{C}$, such that $\Delta T = x = -2\,°\mathrm{C}$, $x_m =$

Table A.1
Function R to be inserted in Eq. (A.6)

R	Range for z	Applies if
$zx_M y_M/(1-z)$	(z_{MM}, z_{Mm})	$x_m y_M < x_M y_m$ and $0 < x_m$
y_M/y_m	(z_{Mm}, z_{mM})	
$(1-z)/(zx_m y_m)$	(z_{mM}, z_{mm})	
$zx_M y_M/(1-z)$	(z_{MM}, z_{mM})	$x_M y_m < x_m y_M$ and $0 < x_m$
x_M/x_m	(z_{mM}, z_{Mm})	
$(1-z)/(zx_m y_m)$	(z_{Mm}, z_{mm})	
$zx_M y_M/(1-z)$	(z_{MM}, z_{Mm})	$x_m < 0 < x_M$
y_M/y_m	(z_{Mm}, z_{mm})	
$zx_m y_M/(1-z)$	(z_{mm}, z_{mM})	
$(1-z)/(zx_M y_m)$	(z_{Mm}, z_{MM})	$x_M y_M < x_m y_m$ and $x_M < 0$
y_M/y_m	(z_{MM}, z_{mm})	
$zx_m y_M/(1-z)$	(z_{mm}, z_{mM})	
$(1-z)/(zx_M y_m)$	(z_{Mm}, z_{mm})	$x_m y_m < x_M y_M$ and $x_M < 0$
x_m/x_M	(z_{mm}, z_{MM})	
$zx_m y_M/(1-z)$	(z_{MM}, z_{mM})	

In this table, z_{ab} is defined as $1/(1+x_a y_b)$, where subscripts a and b are both either m or M.

$-2.5\,^\circ$C and $x_M = -1.5\,^\circ$C. Assume also that $\alpha = y = 11.7\times10^{-6}\,^\circ\mathrm{C}^{-1}$ with $w_\alpha = 2.34\times10^{-6}\,^\circ\mathrm{C}^{-1}$, such that $y_m = 10.53\times10^{-6}\,^\circ\mathrm{C}^{-1}$ and $y_M = 12.87\times10^{-6}\,^\circ\mathrm{C}^{-1}$. The standard deviation of $f_z(z)$ turns out to be $u[f_t] = 3.64\times10^{-6}$, the same result one gets from (15). If instead the temperature difference ΔT is estimated to be zero with $x_m = -1\,^\circ$C, $x_M = 1\,^\circ$C and the same values for y_m and y_M, the standard deviation of $f_z(z)$ is $u[f_t] = 6.77\times10^{-6}$, whereas from (15) one gets $u[f_t] = 6.75\times10^{-6}$. Finally, if again $x_m = -1\,^\circ$C and $x_M = 1\,^\circ$C but the spread of α is increased to 50% from its nominal value, $u[f_t]$ increases to 7.03×10^{-6} while the value one obtains from (15) remains equal to 6.75×10^{-6}. Therefore, we conclude that in most cases the use of the LPU is appropriate for the NDE correction, unless one assigns a very large uncertainty to the coefficient α.

Consider now the cosine error. This is treated in paragraph F.2.4.4 of the GUM, which analysis is rather complicated. A simpler procedure, taken from Section 3.5 of Reference [5], consists of obtaining the uncertainty $u[f_a]$ as

$$u^2[f_a] = \mathrm{V}(f_a) = \mathrm{E}(\cos^2\theta) - [\mathrm{E}(\cos\theta)]^2, \tag{A.7}$$

where V and E represent the variance and expectation operators, respectively. Now, one can assume that the pdf associated with θ is uniform, centered at $\theta = 0$ and of width $2\theta_{max}$. This gives

$$\begin{aligned}\mathrm{E}(\cos^2\theta) &= \frac{1}{(2\theta_{max})}\int \cos^2\theta\,\mathrm{d}\theta \\ &= \frac{(\sin\theta_{max}\cos\theta_{max})}{2\theta_{max}} + \frac{1}{2}\end{aligned} \tag{A.8}$$

and

$$\mathrm{E}(\cos\theta) = \frac{1}{(2\theta_{max})}\int \cos\theta\,\mathrm{d}\theta = \frac{\sin\theta_{max}}{\theta_{max}}. \tag{A.9}$$

To be consistent, the value of f_a should then be taken not as 1, but as the expectation of $\cos\theta$. This gives Eq. (21). Substitution of (A.8) and (A.9) in (A.7) gives Eq. (20). Evidently, other formulas would apply if another pdf is used for the misalignment angle.

We note finally that one might think of treating the cosine error with a uniform pdf. Indeed, it is possible to model the measurand as $L_o = L - c_a$, where L is the measured value and $c_a = L(1-\cos\theta)$ is an additive alignment correction. One then gets $u^2[L_o] = u^2[L] + u^2[c_a]$. A uniform pdf of width w_a for c_a gives $u^2[c_a] = w_a^2/12$. This would be incorrect, however, because it would mean centering the pdf around the value zero for a quantity that is always positive.

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