CLASS NOTES FOR ABE 498

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BY

JUSTIN WASSERMAN

CLASS NOTES

Urbana, Illinois

TABLE OF CONTENTS

LIST OF TABLES

LIST OF FIGURES

LIST OF ABBREVIATIONS

GPS	Global Positioning System
$\mathrm{c}(\theta)$	$= \cos(\theta)$
$\mathrm{s}(\theta)$	$= \sin(\theta)$

CHAPTER 1

REVIEW

1.1 Control Systems

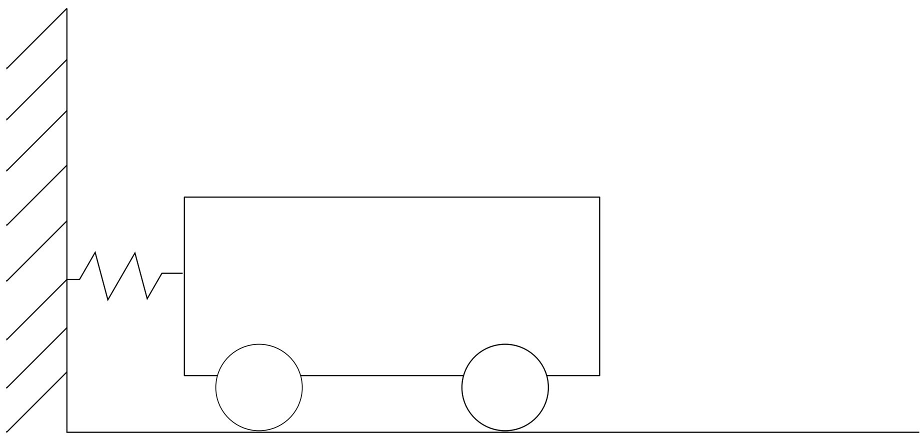

Figure 1.1: Example Car-Spring System

For the purposes of this class a control system is a system with dynamics and an input that the user can select. In the example given in Figure 1.1, the input to the cart is done by pulling on it away from the wall. The general formula for the state space model is given by $\dot{X}_1(t) = f_1(x_1, x_2, \ldots, x_n, u_1, u_2, \ldots, , u_m)$

In the example given in Figure 1.1, x_1 is the displacement of the cart relative to its starting position, and x_2 is the velocity of the cart. This gives the kinematics relationship $\dot{x_1} = x_2$. The dynamic relationship of the cart can be observed by observing Figure 1.2.

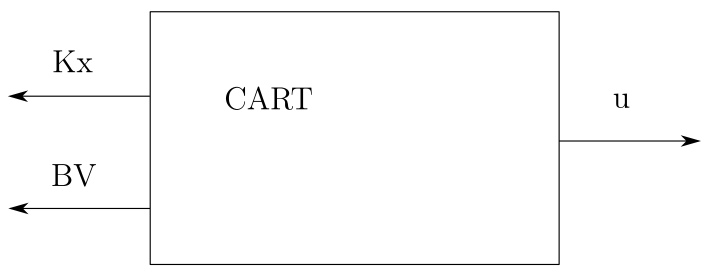

Figure 1.2: forces

CHAPTER 2

INTRODUCTION TO ROTATION

2.1 Introduction

Rotation is important for understanding the kinematics of a robot and to understand what sensor date may look like with different frames of references. A frame of reference may be an arbitrary point on the robot that makes the data more useful. This section will first introduce 2D rotation and then discuss 3D rotation.

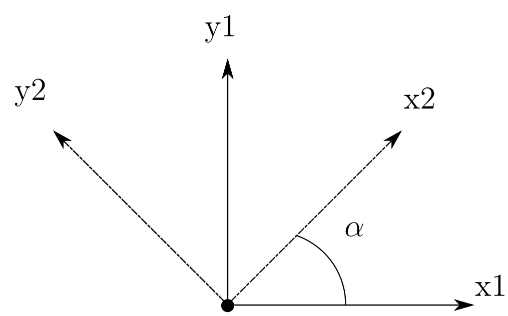

Figure 2.1: Two Reference Frames (x_1, y_1) and (x_2, y_2)

A 2D frame can be offset from another frame in terms of x position, y position, and rotation. For a 2D system the positional offset of a frame can be described by (x,y) (as can been seen in Figure 2.2). Figure 2.1 shows reference frame 2 being offset from reference frame 1 by a rotation α.

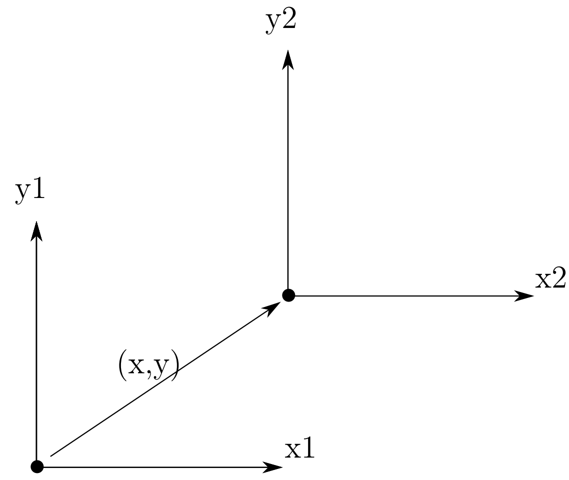

Figure 2.2: Two Frames Offset by (x,y)

2.2 SO(2) Matrix

$$\begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix} \tag{2.1}$$

The SO(2) matrix (which is short for Special Orthogonal Matrix of order 2), is a matrix that can describe rotation in a 2D space, it can be seen in Equation 2.1. The SO(2) matrix has a few important properties (R is the SO(2) matrix).

1. det(R) = 1
2. $RR^T = R^T R = I$

Property 1 of the SO(2) matrix implies that when a vector does not scale or become skewed when a dot product is performed with it and a SO(2) matrix. Property 2 of the SO(2) matrix implies that $R^{-1} = R^T$ and that the matrix is orthogonal. Talk about what an orthogonal matrix is and why its important

A vector x can be rotated by an angle α by creating a SO(2) matrix with α and then performing a dot product with x.

$$x_2 = R_\alpha x \tag{2.2}$$

Another rotation can be performed.

$$x_3 = R_\beta x_2 \tag{2.3}$$

However, this is equivalent to

$$x_3 = R_\beta R_\alpha x \tag{2.4}$$

Further simplification can be performed for SO(2) matrices

$$R_\beta R_\alpha = \begin{bmatrix} c\beta & s\beta \\ -s\beta & c\beta \end{bmatrix} \begin{bmatrix} c\alpha & s\alpha \\ -s\alpha & c\alpha \end{bmatrix} = \begin{bmatrix} c(\alpha+\beta) & s(\alpha+\beta) \\ -s(\alpha+\beta) & c(\alpha+\beta) \end{bmatrix} = R_{\alpha+\beta} \tag{2.5}$$

So, if 2 or more rotations were to be performed on a vector, it could be done in a single dot product instead of many.

2.3 3D rotation

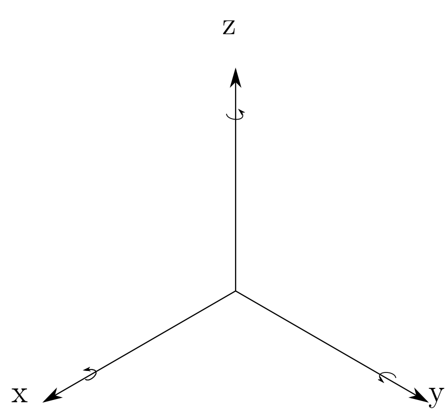

Figure 2.3: Rotation in 3 Dimensions

Rotation can also be given in 3D, but to do so relies on introduces the SO(3) matrix.

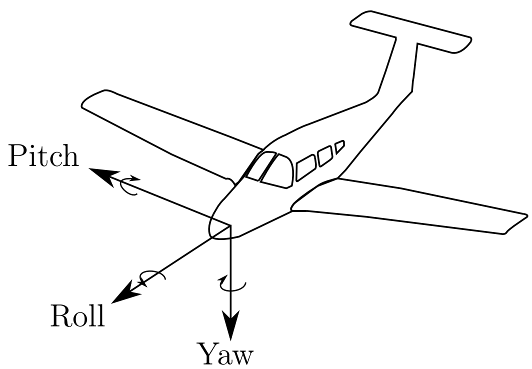

Figure 2.4: planeDOF

REFERENCES