

Laboratorio de Programación

2016 -2

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Noviembre de 2016

Outline

1 Graphs

- Basics
- Terminology

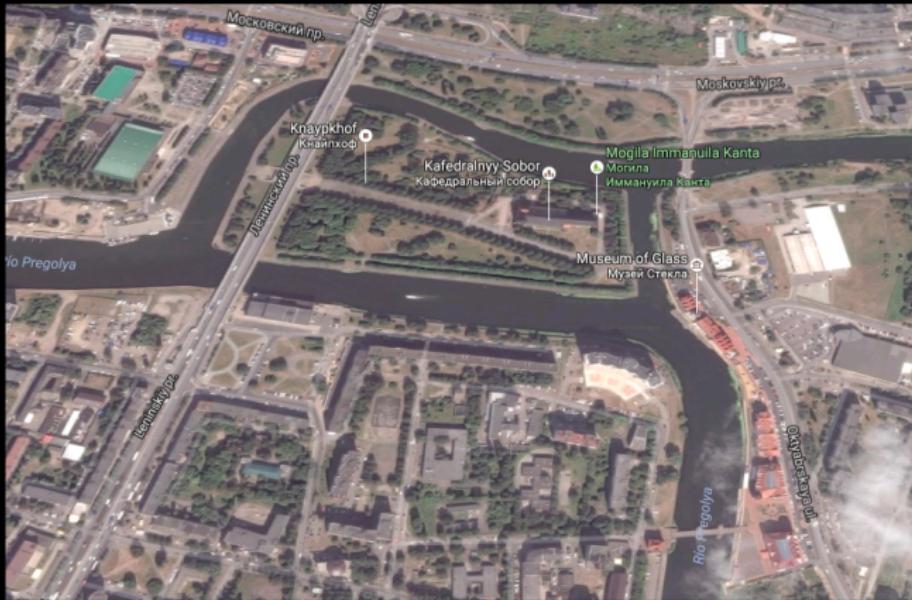
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- Terminology

The Königsberg bridges problem

Königsberg (Kaliningrad - Калининград)



The Königsberg bridges problem

The problem (Euler 1736)

- Königsberg was the capital of the old East Prussia, now part of the Russian Federation and renamed as Kaliningrad

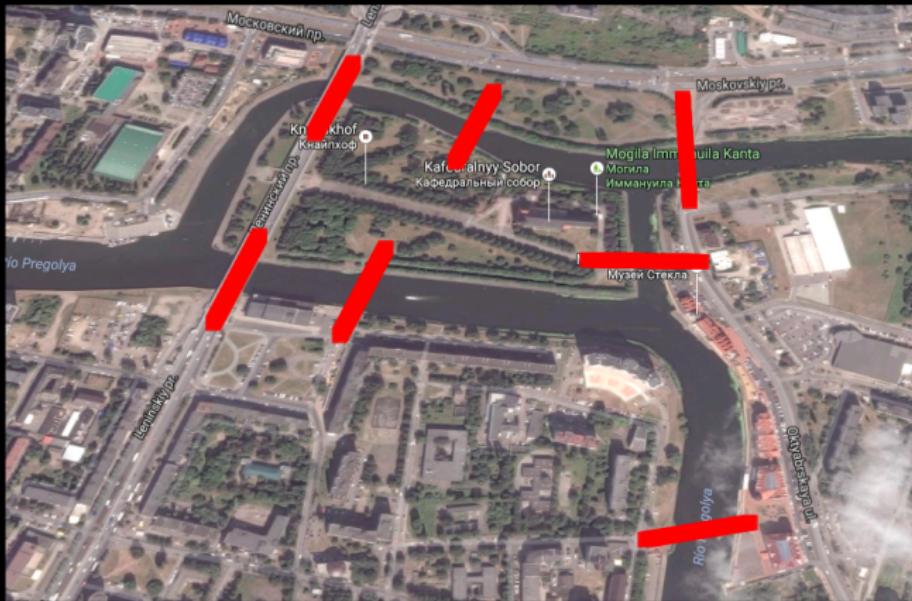
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- Königsberg was the capital of the old East Prussia, now part of the Russian Federation and renamed as Kaliningrad
- Königsberg had 7 bridges over the Pregel River

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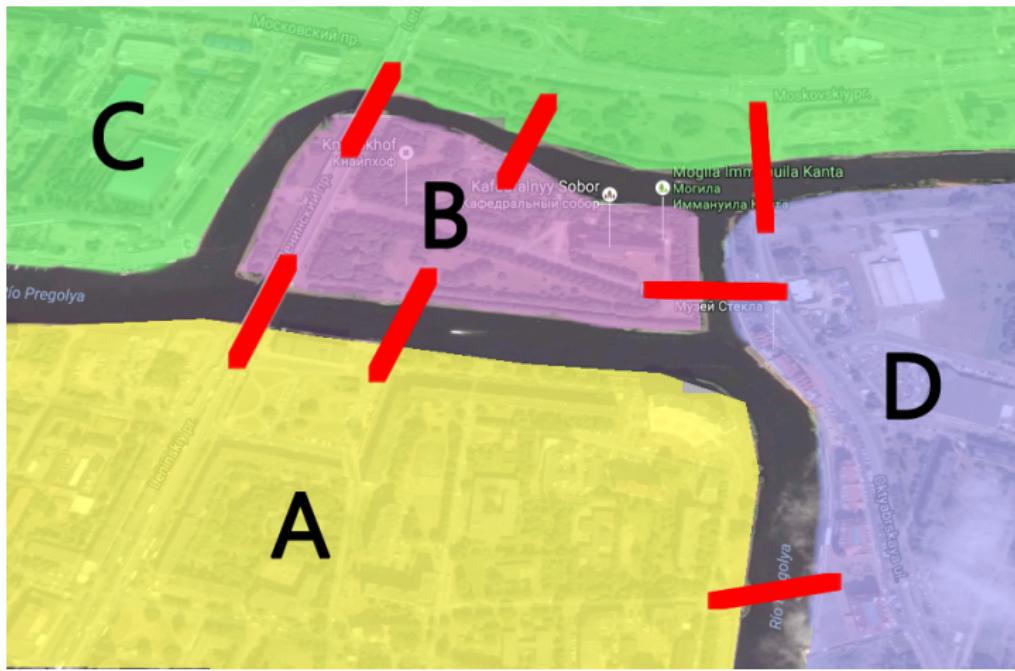
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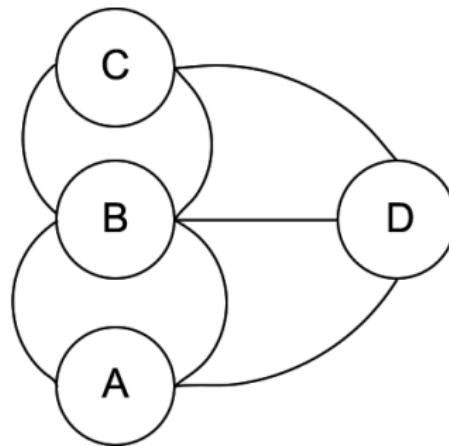
The problem (Euler 1736)

- Königsberg was the capital of the old East Prussia, now part of the Russian Federation and renamed as Kaliningrad
- Königsberg had 7 bridges over the Pregel River
- How can we walk through the city, crossing each bridge once and only once?

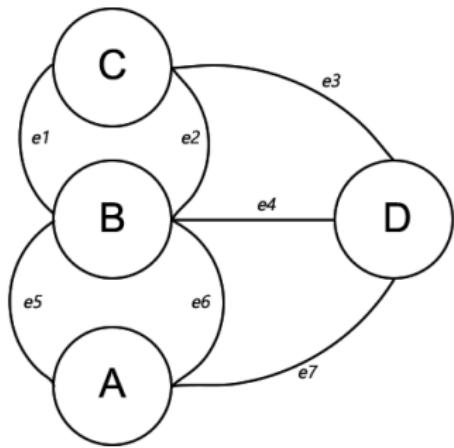
The Königsberg bridges problem



The Königsberg bridges problem



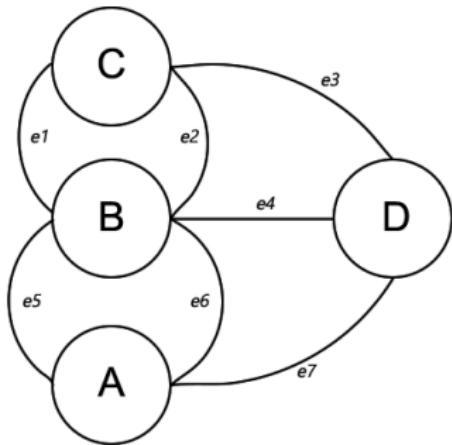
Graphs



Graphs

Definition

A graph $G = \{V, E\}$ is a collection of Vertexes (V), and Edges (E) which connects the vertexes.



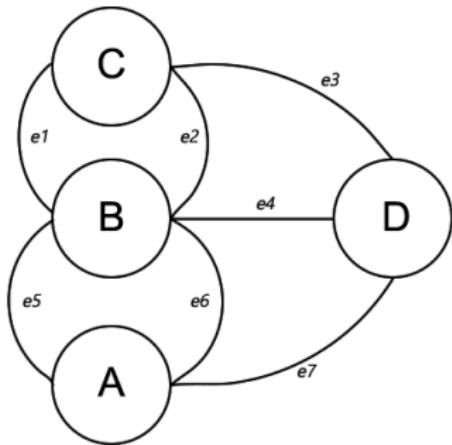
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Vertexes

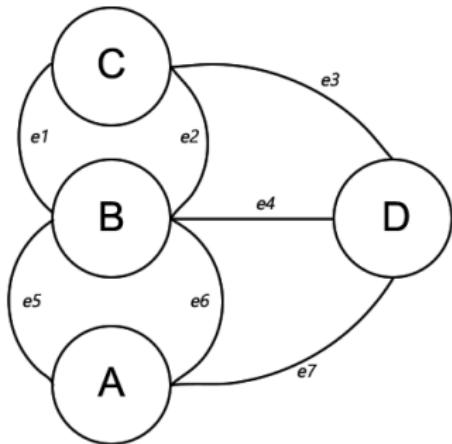
$$V = \{A, B, C, D\}$$



Graphs

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Vertexes

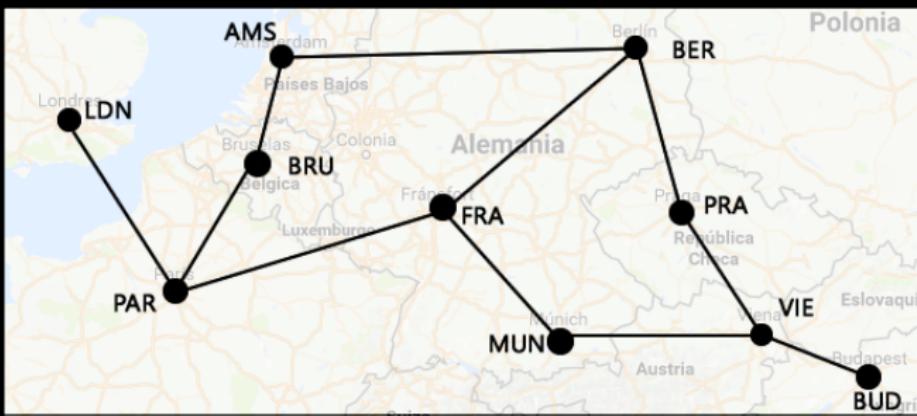
$$V = \{A, B, C, D\}$$

Edges

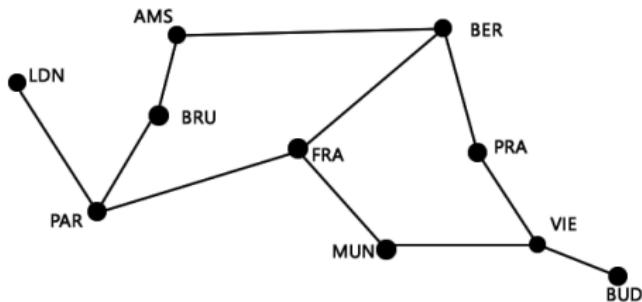
$$\begin{aligned}e1 &= \{C, B\} & e2 &= \{C, D\} & e3 &= \{C, D\} \\e4 &= \{D, B\} & e5 &= \{B, A\} & e6 &= \{B, A\} \\e7 &= \{A, D\}\end{aligned}$$

Graphs

High-Speed Railway Network



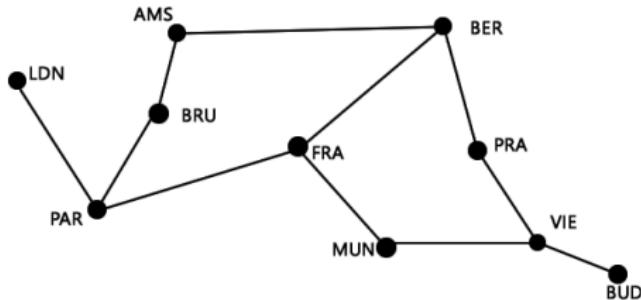
Graphs



Simple graph

A *Simple Graph* $G = (V, E)$ contains:

Graphs

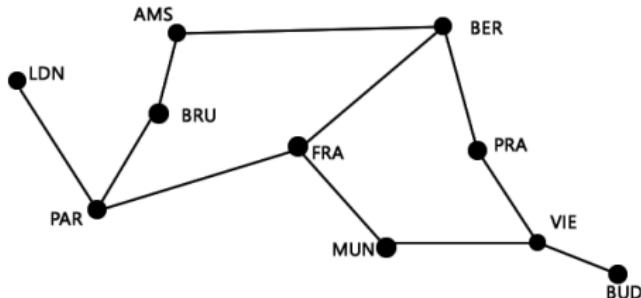


Simple graph

A *Simple Graph* $G = (V, E)$ contains:

- V : A non-empty set of vertexes

Graphs

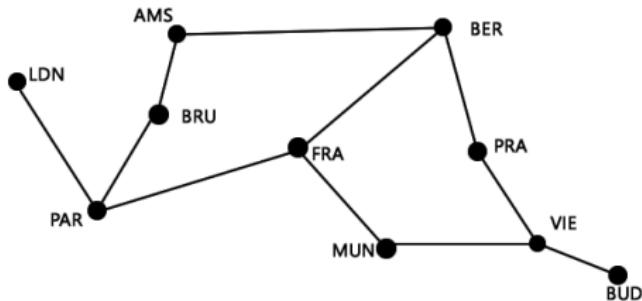


Simple graph

A *Simple Graph* $G = (V, E)$ contains:

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- E : Set of unsorted pairs of different vertexes

Graphs



Simple graph

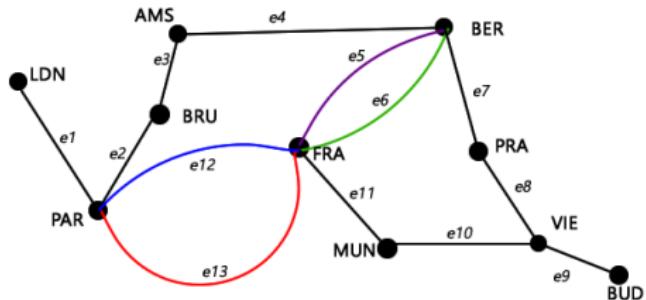
A *Simple Graph* $G = (V, E)$ contains:

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E.g:

$$E = \{\{LDN, PAR\}, \{FRA, BER\}, \dots\}$$

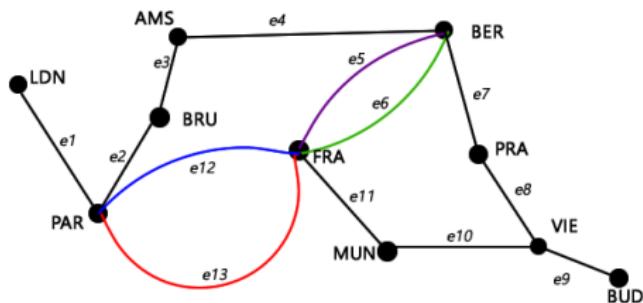
Graphs



Multigraph

A *Multigraph* $G = (V, E)$ contains:

Graphs

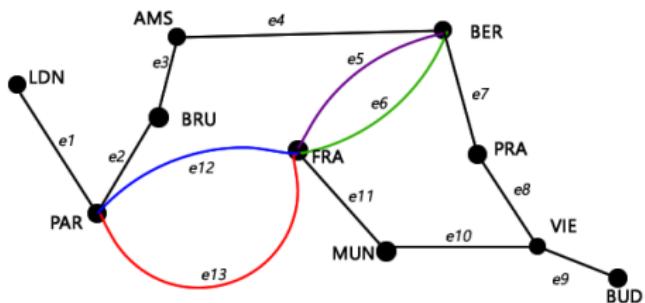


Multigraph

A *Multigraph* $G = (V, E)$ contains:

- V : A set of vertexes and E : A set of edges

Graphs



Multigraph

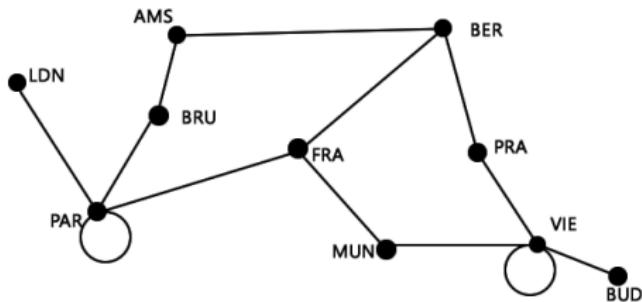
A *Multigraph* $G = (V, E)$ contains:

- V : A set of vertexes and E : A set of edges
- **Multiple edges** where $f(e_1) = f(e_2)$ and $e_1 \neq e_2$

E.g:

$$\begin{aligned}f(e_{12}) &= \{FRA, PAR\} \\f(e_{13}) &= \{FRA, PAR\}\end{aligned}$$

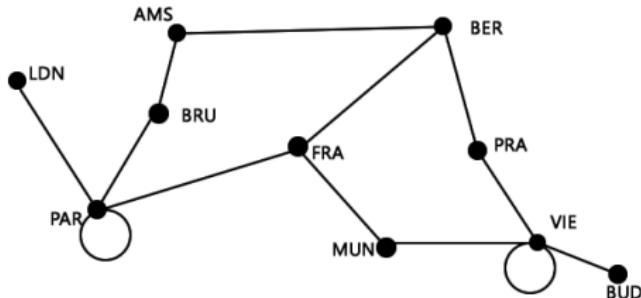
Graphs



Pseudograph

A *Pseudograph* $G = (V, E)$ contains:

Graphs

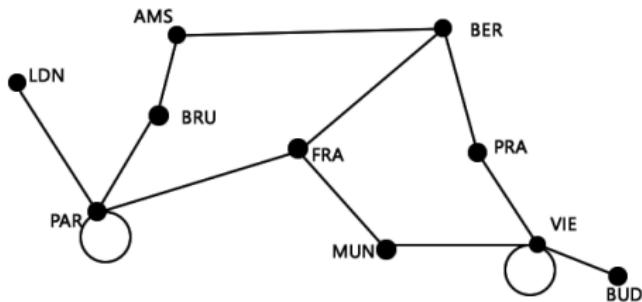


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Pseudograph

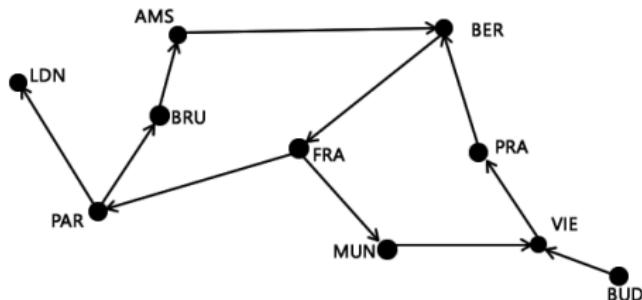
A *Pseudograph* $G = (V, E)$ contains:

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- E : A set of edges
- **Loops** where $f(e) = \{u, u\}$

E.g:

$\{PAR, PAR\} \{VIE, VIE\}$

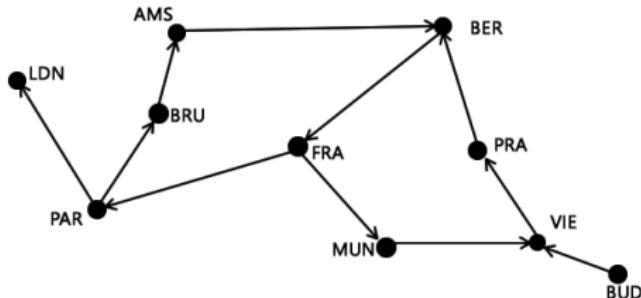
Graphs



Directed graph

A *Pseudograph* $G = (V, E)$ contains:

Graphs

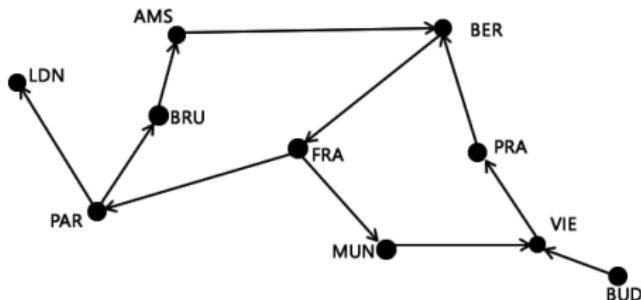


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Graphs

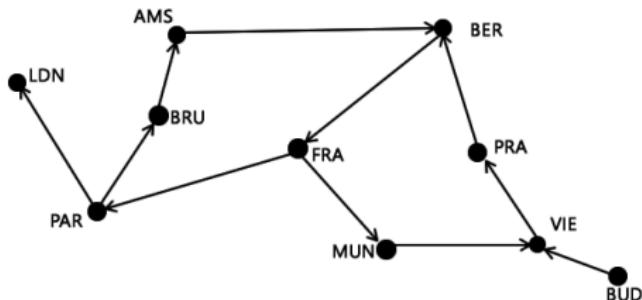


Directed graph

A *Pseudograph* $G = (V, E)$ contains:

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- E : Set of **sorted** pairs of vertexes

Graphs



Directed graph

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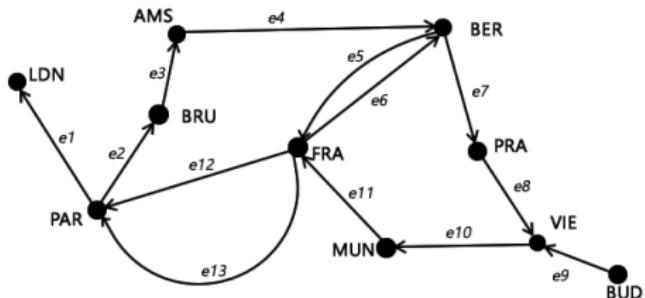
E.g:

$$E = \{(PAR, LDN), (VIE, PRA)\dots\}$$

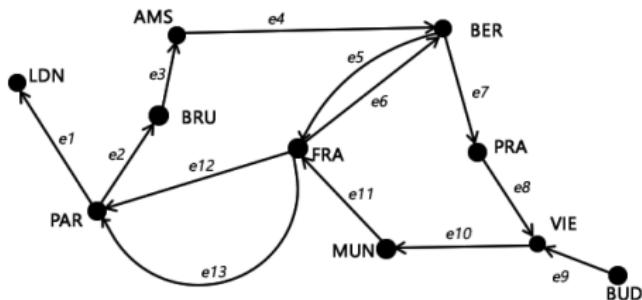
Graphs

Directed Multigraph

A *Directed Multigraph* is a directed graph $G = (V, E)$ which contains:



Graphs

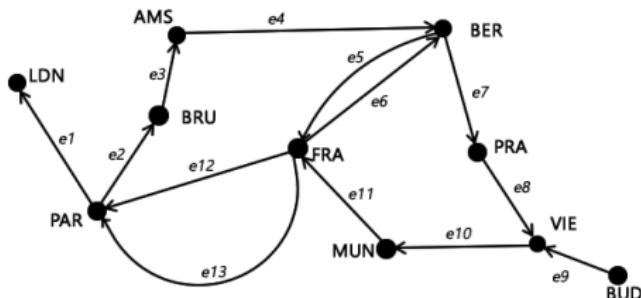


Directed Multigraph

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Graphs



Directed Multigraph

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E.g:

$$f(e_{12}) = (FRA, PAR)$$

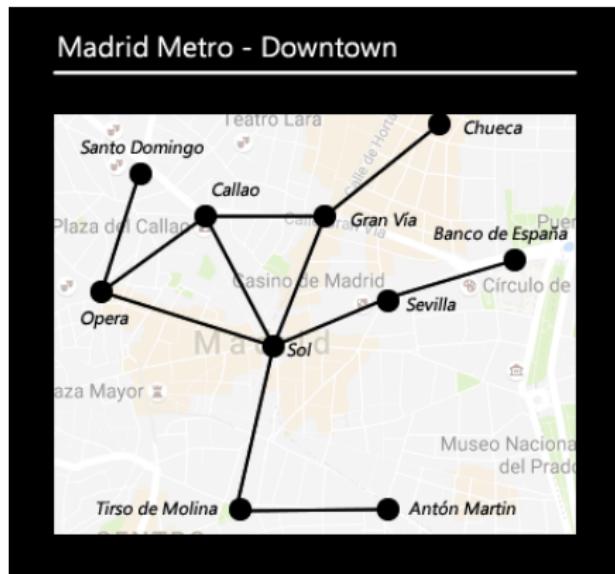
$$f(e_{13}) = (FRA, PAR)$$

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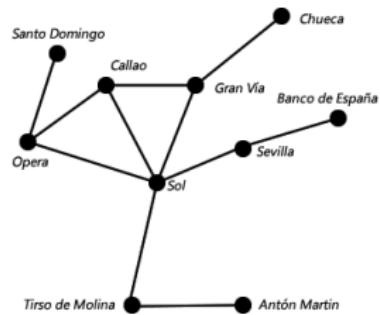
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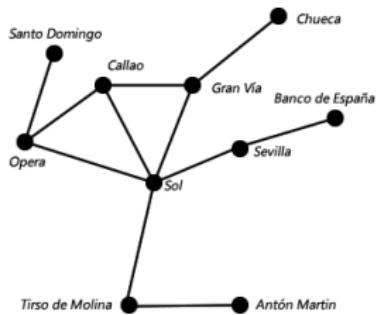
Graphs - Terminology



Graphs - Terminology

Adjacency

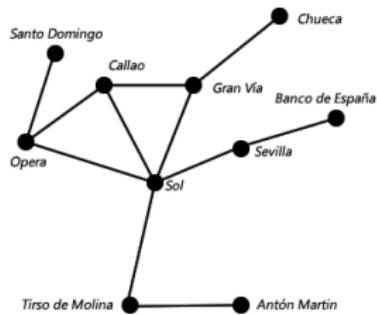
Vertexes u, v are **adjacent**
Opera and *Sol* are adjacent



Graphs - Terminology

Adjacency

Vertexes u, v are **adjacent**
Opera and *Sol* are adjacent



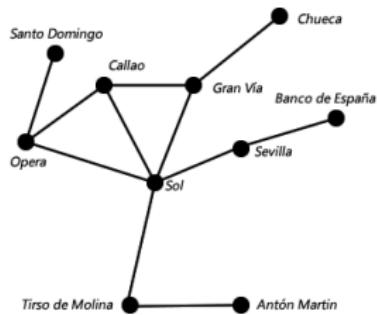
Incidence

Let's say $e = \{Sol, Sevilla\}$. The edge e is **incidence** with *Sol* and *Sevilla* so, e connects *Sol* and *Sevilla*

Graphs - Terminology

Adjacency

Vertexes u, v are **adjacent**
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Degree

On a non-directed graph is the number of incidence Edges related to a vertex.
 $\delta(Sol) = 5$, $\delta(Opera) = 3$

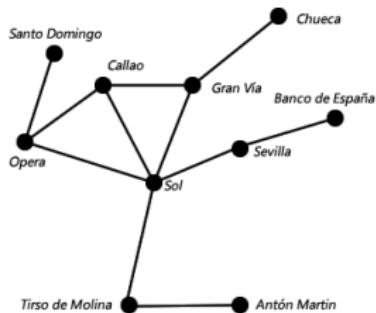
A **loop** has double **incidence**!

Graphs - Terminology

Path

Are the edges we need to traverse to go from a vertex (u) to another one (v).

Problem: Go from Callao to Sevilla
Paths:



- {Callao, Sol} {Sol, Sevilla}
- {Callao, Gran Vía} - {Gran Vía, Sol} - {Sol, Sevilla}
- {Callao, Opera} - {Opera, Sol} - {Sol, Sevilla}
- {Callao, Opera} - {Opera, Sol} - {Sol, Gran via} - {Gran via, Chueca} - {Chueca, Gran via} - {Gran via - Sol} - {Sol - Sevilla}