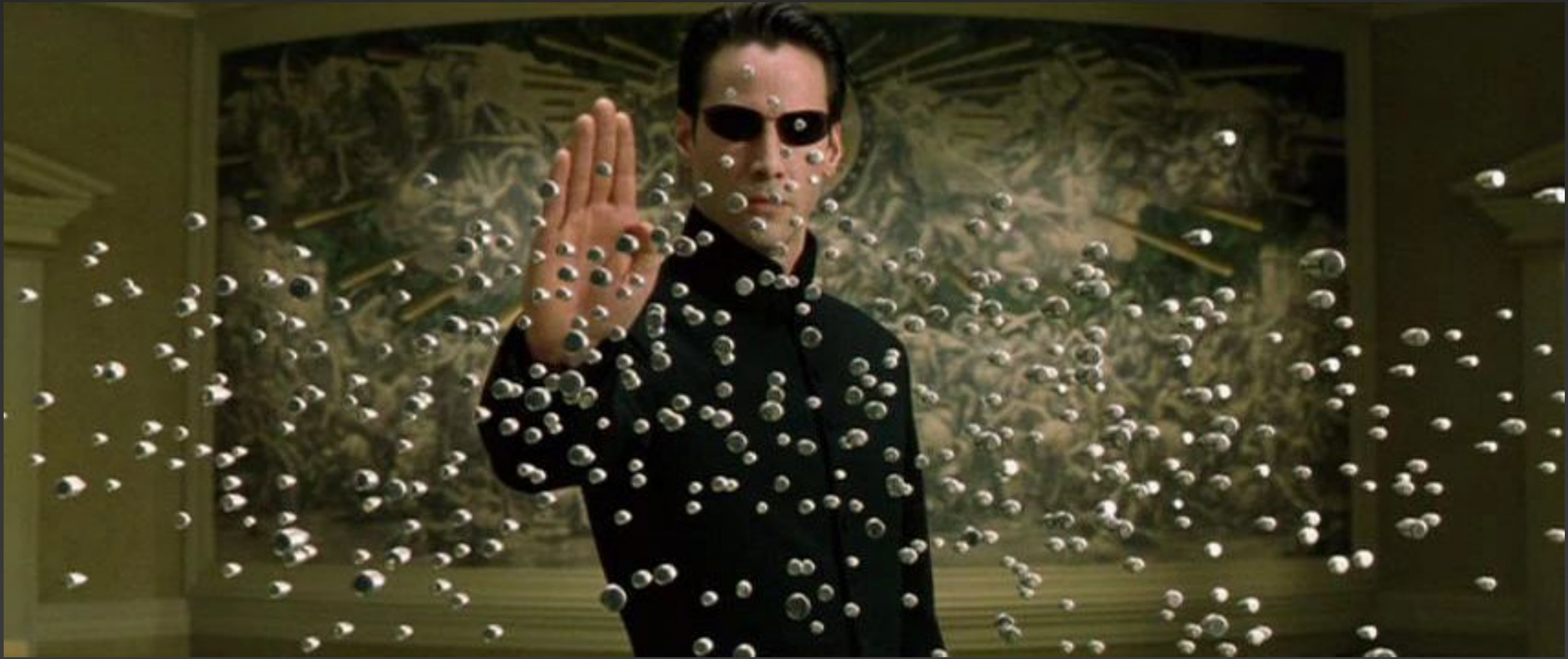
Pattern Recognition

CS129.18 - Bautista
Section B



Intro to Linear Algebra

CS129.18 - Bautista
Section B

Linear Algebra is a **continuous form of mathematics** and is applied throughout science and engineering because it allows you to **model natural phenomena** and to compute them efficiently.

In Linear Algebra, data is represented by linear equations, which are presented in the form of **matrices and vectors**.

34

Scalar

Row Vector

[34, 5, -4]

$$\begin{bmatrix} \mathbf{34} \\ \mathbf{5} \\ \mathbf{-4} \end{bmatrix}$$

Column Vector

Vector

Row(s) x Column(s)

$$\begin{bmatrix} 34, & 5, & -4, & 2 \\ 20, & 2, & 0, & 1 \end{bmatrix}$$

Matrix

Matrices are denoted by **CAPITAL** letters

Vectors are denoted by **lowercase** letters

Vector

Magnitude + Direction

v is a 3-dimensional vector

$$v = [\, 2, 5, -4 \,]$$

$v_0 = 2$

$v_1 = 5$

$v_2 = -4$

A Vector is an ordered array of numbers and can be in a row or a column.

$$\text{Matrix } A := [\, a_{ij} \,]_{mn}$$

A_{ij} = ith row, jth column

m rows, n columns

$m = 2,\ n = 4$

Row(s) x Column(s)

$$M = \begin{bmatrix} 34, & 5, & -4, & 2 \\ 20, & 2, & 0, & 1 \end{bmatrix}$$

$M_{0,0} = 34$

$M_{1,2} = 0$

$M_{0,1} = 5$

A Matrix is an ordered 2D array of numbers and it has two indices. The first one points to the row and the second one to the column.

Uppercase variables map to **matrices**
Lowercase variables map to **vectors**

Matrix

2D ordered array of numbers with rows and columns

$$c = \mathbf{2}$$

$$v = [\ 3, 5, -4\ ]$$

$$c * v = [\ (2 \times 3), (2 \times 5), (2 \times -4)\ ]$$

$$c * v = [\ (c \times v_1), (c \times v_2), (c \times v_3) \ldots c \times v_n\ ]$$

A Matrix is an ordered 2D array of numbers and it has two indices. The first one points to the row and the second one to the column.

Matrix Operations

Scalar x Vector Math

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix} + \begin{bmatrix} e & f \\ g & h \end{bmatrix} = \begin{bmatrix} a+e & b+f \\ c+g & d+h \end{bmatrix}$$

Cannot add unequal dimension matrices

Matrix Operations

Matrix + Matrix Math

$$C \times \begin{bmatrix} a & b \\ e & d \end{bmatrix} = \begin{bmatrix} a*c & b*c \\ e*c & d*c \end{bmatrix}$$

Same process with vector * scalar multiplication

Matrix Operations

Scalar x Matrix Math

$$\begin{bmatrix} a \\ b \end{bmatrix} \times \begin{bmatrix} e & f \\ g & h \\ i & j \end{bmatrix} = \begin{bmatrix} a*e + b*f \\ a*g + b*h \\ a*i + b*j \end{bmatrix}$$

Multiplying a Matrix by a Vector can be thought of as multiplying each row of the Matrix by the column of the Vector. The output will be a Vector that has the same number of rows as the Matrix.

Matrix Operations

Vector x Matrix Math

x = n x 1 matrix

A = m x n matrix

$$A * x = y$$

y = m-dimensional vector

Multiplying a Matrix by a Vector can be thought of as multiplying each row of the Matrix by the column of the Vector. The output will be a Vector that has the same number of rows as the Matrix.

Matrix Operations

Vector x Matrix Math

B = i x j matrix

A = m x n matrix

A * B = AB

AB = m x j dimensional matrix

Note that you can only multiply Matrices together **if the number of the first Matrix’s columns matches the number of the second Matrix’s rows.** The result will be a Matrix with the same number of rows as the first Matrix and the same number of columns as the second Matrix.

$$\begin{bmatrix} a & b & c \\ d & e & f \end{bmatrix} \times \begin{bmatrix} z & k \\ g & h \\ i & j \end{bmatrix} = \begin{bmatrix} a*z + b*g + c*i, & a*k + b*h + c*j \\ d*z + e*g + f*i, & d*k + e*h + f*j \end{bmatrix}$$

3x2 matrix　　2x3 matrix　　2x2 matrix

Note that you can only multiply Matrices together **if the number of the first Matrix's columns matches the number of the second Matrix's rows.** The result will be a Matrix with the same number of rows as the first Matrix and the same number of columns as the second Matrix.

Matrix Operations

Matrix x Matrix Math

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}$$

Multiplying two Matrices together isn’t that hard either if you know how to multiply a Matrix by a Vector.

Matrix Operations

Matrix x Matrix Math

$$\begin{bmatrix} 1 & 3 & 2 \\ 4 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 3 \\ 0 & 1 \\ 5 & 2 \end{bmatrix} = \begin{bmatrix} 11 & 10 \\ 9 & 14 \end{bmatrix}$$

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Multiplying two Matrices together isn't that hard either if you know how to multiply a Matrix by a Vector.

Matrix Operations

Matrix x Matrix Math

$3 \times 7 = 7 \times 3$ $A \times B \mathrel{!=} B \times A$

Scalar Multiplication is commutative but Matrix Multiplication is not.

Matrix Properties

Things to note

$$5(3 \times 7) = (5 \times 3)7 \qquad A(B \times C) = (A \times B)C$$

Scalar and Matrix Multiplication are both associative.

Matrix Properties

Things to note

$$5(3 + 7) = (5*3 + 5*7)$$

$$C(A + B) = (A*C + B*C)$$

Scalar and Matrix Multiplication are also both distributive.

Matrix Properties

Things to note

$$\begin{bmatrix} 1 & 20 & 2 \\ 10 & 1 & 5 \\ 0 & 42 & 10 \end{bmatrix}$$

$$\begin{bmatrix} 0.2 & 0.4 & 0.1 & 0.9 \\ 0.1 & 1.0 & 0.8 & 0.01 \\ 0.2 & 0.2 & 0.4 & 0.7 \\ 0\ .0 & 0.1 & 0.01 & 1.0 \end{bmatrix}$$

A square matrix is where the number of columns and rows are the same, and is defined by the ff:

$$\text{Square matrix} = A_{mxm}$$

Square Matrix

Things to note

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

You can spot an Identity Matrix by the fact that it has ones along its diagonals and that every other value is zero. It is also a “squared matrix,” meaning that its number of rows matches its number of columns.

Identity Matrix

Things to note

Thank You