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ECE4300

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Homework 1

1.3 Describe the steps that transform a program written in a high-level language such as C into a representation that is directly executed by a computer processor.

Answer: There are two kinds of programs/steps that contribute to transforming a program written in a high-level language into a representation that is directly executed by a computer processor. The first being a compiler, which reads the high level codes and translates it to a program in assembly. The second program is an assembler, which takes that assembly program and transforms it into a machine language program, which the computer understands and can execute.

1.4 Assume a color display using 8 bits for each of the primary colors (red, green, blue) per pixel and a frame size of 1280 x 1024.

a. What is the minimum size in bytes of the frame buffer to store a frame?

Answer:

$$1 \text{ byte} = 8 \text{ bits}$$

$$\text{One pixel each for 3 colors: } 3 \times 1 = 3 \text{ bytes}$$

$$\text{Frame Size: } 1280 \times 1024 = 1,310,720 \text{ pixels}$$

$$\text{Minimum Size of the Frame Buffer} = 3 \times 1,310,720 = 3,932,160 \text{ bytes}$$

b. How long would it take, at a minimum, for the frame to be sent over a 100 Mbit/s network?

Answer:

$$1 \text{ Mbit/s} = 10^6 \text{ bit/s}$$

$$100 \text{ Mbit/s} = 100 \times 10^6 \text{ bit/s} = 10^2 \times 10^6 \text{ bit/s} = 10^8 \text{ bit/s}$$

$$\text{Memory Size of the Frame (in bits)} = 3,932,160 \times 8 = 31,457,280 \text{ bits}$$

$$\text{Speed} = \frac{\text{size}}{\text{time}}$$

$$\text{Time} = \frac{\text{size}}{\text{speed}} = \frac{31,457,280 \text{ bits}}{10^8 \text{ bit/s}} \approx 0.315 \text{ seconds}$$

1.5 Consider three different processors P1, P2, and P3 executing the same instruction set. P1 has a 3 GHz clock rate and a CPI of 1.5. P2 has a 2.5 GHz clock rate and a CPI of 1.0. P3 has a 4.0 GHz clock rate and has a CPI of 2.2

a. Which processor has the highest performance expressed in instructions per second?

Answer:

$$\begin{aligned}\text{CPU Time} &= \frac{\text{instructions} \times \text{CPI}}{\text{clock rate}} \\ \text{IPS} &= \frac{\text{instructions}}{\text{CPU time}} = \frac{\text{clock rate}}{\text{CPI}} \\ 1 \text{ GHz} &= 10^9 \text{ Hz} \\ \text{IPS}_1 &= \frac{\text{clock rate}_1}{\text{CPI}_1} = \frac{3 \text{ GHz}}{1.5} = 2 \times 10^9 \\ \text{IPS}_2 &= \frac{\text{clock rate}_2}{\text{CPI}_2} = \frac{2.5 \text{ GHz}}{1} = 2.5 \times 10^9 \\ \text{IPS}_3 &= \frac{\text{clock rate}_3}{\text{CPI}_3} = \frac{4 \text{ GHz}}{2.2} = 1.82 \times 10^9\end{aligned}$$

After calculating the three different processor's instructions per second, it can be found that processor 2 has the highest performance out of the 3.

b. If the processors each execute a program in 10 seconds, find the number of cycles and the number of instructions.

Answer:

$$\begin{aligned}\text{CPU time} &= 10 \text{ seconds} \\ \text{Clock Cycles} &= \text{CPU time} \times \text{Clock Rate} \\ \text{Instructions} &= \text{IPS} \times \text{CPU Time}\end{aligned}$$

Processor 1

$$\begin{aligned}\text{Clock Cycles}_1 &= 10 \times 3 \text{ GHz} = 10 \times 3 \times 10^9 = 3 \times 10^{10} \\ \text{Instructions}_1 &= 2 \times 10^9 \times 10 = 2 \times 10^{10}\end{aligned}$$

Processor 2

$$\begin{aligned}\text{Clock Cycles}_2 &= 10 \times 2.5 \text{ GHz} = 10 \times 2.5 \times 10^9 = 2.5 \times 10^{10} \\ \text{Instructions}_2 &= 2.5 \times 10^9 \times 10 = 2.5 \times 10^{10}\end{aligned}$$

Processor 3

$$\begin{aligned}\text{Clock Cycles}_3 &= 10 \times 4 \text{ GHz} = 10 \times 4 \times 10^9 = 4 \times 10^{10} \\ \text{Instructions}_3 &= 1.82 \times 10^9 \times 10 = 1.82 \times 10^{10}\end{aligned}$$

c. We are trying to reduce the execution time by 30% but this leads to an increase of 20% in the CPI. What clock rate should we have to get this time reduction?

Answer:

$$\text{Execution Time} = \frac{\text{clock cycles}}{\text{clock rate}}$$

$$\text{Clock Cycles} = \text{instructions} \times \text{CPI}$$

$$\text{Execution Time} = \frac{\text{instructions} \times \text{CPI}}{\text{clock rate}}$$

$$\text{Execution Time}_{\text{new}} = 0.7 \text{ Execution Time}_{\text{old}}$$

$$\frac{\text{instructions}_{\text{new}} \times \text{CPI}_{\text{new}}}{\text{clock rate}_{\text{new}}} = 0.7 \times \frac{\text{CPI}_{\text{old}}}{\text{clock rate}_{\text{old}}} \text{ where } \text{CPI}_{\text{new}} = 1.2 \text{ CPI}_{\text{old}}$$

$$\frac{1.2}{\text{clock rate}_{\text{new}}} = \frac{0.7}{\text{clock rate}_{\text{old}}}$$

$$\text{clock rate}_{\text{new}} = \frac{1.2}{0.7} \times \text{clock rate}_{\text{old}} = 1.71 \times \text{clock rate}_{\text{old}}$$

From here, it can be seen that the clock rate must increase by about 71%.

1.6 Consider two different implementations of the same instruction set architecture. The instructions can be divided into four classes according to their CPI (class A, B, C, and D). P1 with a clock rate of 2.5 GHz and CPIs of 1, 2, 3, and 3, and P2 with a clock rate of 3 GHz and CPIs of 2, 2, 2, and 2. Given a program with a dynamic instruction count of 1.0E6 instructions divided into classes as follows: 10% class A, 20% class B, 50% class C, and 20% class D, which implementation is faster?

a. What is the global CPI for each implementation?

Answer:

$$\text{CPU time} = \frac{\text{CPU clock cycle}}{\text{Clock rate}}$$

$$\text{CPU Clock Cycles} = \sum_{i=1}^n (\text{CPI}_i) \times C_i$$

$$\text{Program Executes } 10^6 \times 10\% = 10^5 \text{ instructions of class A}$$

$$\text{Class B: } 10^6 \times 20\% = 2 \times 10^5$$

$$\text{Class C: } 10^6 \times 50\% = 5 \times 10^5$$

$$\text{Class D: } 10^6 \times 20\% = 2 \times 10^5$$

$$CPU \text{ clock cycles}_{p1} = (1 \times 10^5) + (2 \times 2 \times 10^5) + (3 \times 5 \times 10^5) + (3 \times 2 \times 10^5) = 2.6 \times 10^6$$

$$CPU \text{ clock cycles}_{p2} = (2 \times 10^5) + (2 \times 2 \times 10^5) + (2 \times 5 \times 10^5) + (2 \times 2 \times 10^5) = 2 \times 10^6$$

Execution Times

$$CPU \text{ Times}_{p1} = \frac{2.6 \times 10^6}{2.5 \times GHz} = 1.04ms$$

$$CPU \text{ Times}_{p2} = \frac{2 \times 10^6}{3 \times GHz} = 666.67ms$$

Upon doing the calculations, it can be seen that processor P2 is the faster processor.

CPI

$$CPI_{p1} = \frac{CPU \text{ clock cycles}_{p1}}{\text{Number of instructions}} = \frac{2.6 \times 10^6}{10^6} = 2.6$$

$$CPI_{p2} = \frac{CPU \text{ clock cycles}_{p2}}{\text{Number of instructions}} = \frac{2 \times 10^6}{10^6} = 2$$

b. Find the clock cycles required in both cases.

Answer:

Clock Cycles

$$CPU \text{ clock cycles}_{p1} = (1 \times 10^5) + (2 \times 2 \times 10^5) + (3 \times 5 \times 10^5) + (3 \times 2 \times 10^5) = 2.6 \times 10^6$$

$$CPU \text{ clock cycles}_{p2} = (2 \times 10^5) + (2 \times 2 \times 10^5) + (2 \times 5 \times 10^5) + (2 \times 2 \times 10^5) = 2 \times 10^6$$

1.7 Compilers can have a profound impact on the performance of an application. Assume that for a program, compiler A results in a dynamic instruction count of 1.0E9 and has an execution time of 1.1 s, while compiler B results in a dynamic instruction count of 1.2E9 and an execution time of 1.5 s.

a. Find the average CPI for each program given that the processor has a clock cycle time of 1 ns.

Answer:

$$CPU \text{ time} = \text{instructions} \times CPI \times \text{cycle time}$$

$$CPI = \frac{CPU \text{ time}}{\text{instructions} \times \text{cycle time}}$$

$$\text{Cycle time} = 1 \text{ ns}$$

$$CPI_A = \frac{CPU \text{ time}_A}{\text{instructions}_A \times \text{cycle time}} = \frac{1.1 \text{ s}}{10^9 \times 10^{-9} \text{ s}} = 1.1$$

$$CPI_B = \frac{CPU\ time_B}{instructions_B \times cycle\ time} = \frac{1.5\ s}{1.2 \times 10^9 \times 10^{-9}\ s} = 1.25$$

b. Assume the compiled programs run on two different processors. If the execution times on the two processors are the same, how much faster is the clock of the processor running compiler A's code versus the clock of the processor running compiler B's code?

Answer:

$$\text{Execution time} = \text{CPU time} = \frac{\text{instructions} \times CPI}{\text{clock rate}}$$

$$\text{Execution time}_A = \text{Execution time}_B$$

$$\frac{instructions_A \times CPI_A}{\text{clock rate}_A} = \frac{instructions_B \times CPI_B}{\text{clock rate}_B}$$

$$\text{clock rate}_A = \frac{instructions_A \times CPI_A}{instructions_B \times CPI_B} \times \text{clock rate}_B = \frac{10^9 \times 1.1}{1.2 \times 10^9 \times 1.25} \times \text{clock rate}_B = 0.73 \text{ clock rate}_B$$

Therefore, the clock rate for processor A is about 27% slower than processor B.

c. A new compiler is developed that uses only 6.0E8 instructions and has an average CPI of 1.1. What is the speedup of using this new compiler versus using compiler A or B on the original processor?

Answer:

$$CPU\ time_{new} = instructions_{new} \times CPI_{new} \times cycle\ time = 6 \times 10^8 \times 1.1 \times 10^{-9}\ s = 0.66\ s$$

$$\frac{performance_{new}}{performance_A} = \frac{CPU\ time_A}{CPU\ time_{new}} = \frac{1.1\ s}{0.66\ s} = 1.67$$

$$\frac{performance_{new}}{performance_B} = \frac{CPU\ time_B}{CPU\ time_c} = \frac{1.5\ s}{0.66\ s} = 2.27$$

Here we can see that the new compiler will be about 1.67 times faster than compiler A and about 2.27 times faster than compiler B.

1.11 The results of the SPEC CPU2006 bzip2 benchmark running on an AMD Barcelona has an instruction count of 2.389E12, an execution time of 750 s, and a reference time of 9650 s.

1.11.1 Find the CPI if the clock cycle time is 0.333 ns.

Answer:

$$\text{Execution time} = \text{Clock cycles} \times \text{Cycle time}$$

$$750\ s = \text{clock cycles} \times 0.333\ ns = \text{clock cycles} \times 0.333 \times 10^{-9}\ s$$

$$\text{clock cycles} = \frac{750}{0.333 \times 10^{-9}} = 2.25 \times 10^{12}$$

$$\text{clock cycles} = \text{number of instructions} \times \text{CPI}$$

$$\text{CPI} = \frac{\text{clock cycles}}{\text{number of instructions}} = \frac{2.25 \times 10^{12}}{2.389 \times 10^{12}} = 0.94$$

1.11.2 Find the SPECratio.

Answer:

$$\text{SPEC ratio} = \frac{\text{Reference time}}{\text{Execution time}} = \frac{9650}{750} = 12.87$$

1.11.3 Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% without affecting the CPI.

Answer:

$$\text{New Number of Instructions} = 1.1 \times \text{old number} = 2.6279 \times 10^{12}$$

$$\text{clock cycles} = \text{new number of instructions} \times \text{CPI} = 2.6279 \times 10^{12} \times 0.94 = 2.47 \times 10^{12}$$

$$\text{CPU time} = \text{clock cycles} \times \text{cycle time} = 2.47 \times 10^{12} \times 0.33 \times 10^{-9} \text{ s} = 822.51 \text{ s}$$

1.11.4 Find the increase in CPU time if the number of instructions of the benchmark is increased by 10% and the CPI is increased by 5%.

Answer:

$$\text{New CPI} = 1.05 \times 0.94 = 0.987$$

$$\text{clock cycles} = \text{new number of instructions} \times \text{CPI} = 2.6279 \times 10^{12} \times 0.987 = 2.59 \times 10^{12}$$

$$\text{CPU time} = \text{clock cycles} \times \text{cycle time} = 2.59 \times 10^{12} \times 0.333 \times 10^{-9} \text{ s} = 862.47 \text{ s}$$

1.11.5 Find the change in the SPECratio for this change.

Answer:

$$\text{SPECratio} = \frac{\text{Reference time}}{\text{Execution time}} = \frac{9650}{862.47} = 11.19$$

1.11.6 Suppose that we are developing a new version of the AMD Barcelona processor with a 4 GHz clock rate. We have added some additional instructions to the instruction set in such a way that the number of instructions has been reduced by 15%. The execution time is reduced to 700 s and the new SPECratio is 13.7. Find the new CPI.

Answer:

$$\text{cycle time} = \frac{1}{\text{clock rate}} = \frac{1}{4 \text{ GHz}} = 0.25 \times 10^{-9} \text{ s}$$

$$\begin{aligned}
\text{execution time} &= \text{clock cycles} \times \text{cycle time} \\
\text{clock cycles} &= \frac{\text{execution time}}{\text{cycle time}} = \frac{700}{0.25 \times 10^{-9}} = 2.8 \times 10^{12} \\
2.389 \times 10^{12} \times 0.85 &= 2.03 \times 10^{12} \\
\text{clock cycles} &= \text{number of instructions} \times \text{CPI} \\
\text{CPI} &= \frac{\text{clock cycles}}{\text{number of instructions}} = \frac{2.8 \times 10^{12}}{2.03 \times 10^{12}} = 1.38
\end{aligned}$$

1.11.7 This CPI value is larger than obtained in 1.11.1 as the clock rate was increased from 3 GHz to 4 GHz. Determine whether the increase in the CPI is similar to that of the clock rate. If they are dissimilar, why?

Answer:

$$\text{Increased clock rate: } \frac{4}{3} = 1.333$$

$$\text{Increased CPI} = \frac{1.38}{0.94} = 1.468$$

We can see that the CPI has increased by more than 1.333 times given the reduced number of instructions.

1.11.8 By how much has the CPU time been reduced?

Answer:

$$\text{Ratio of new and old CPU time} = \frac{700}{750} = 0.9333$$

From this, it is found that the CPU time has decreased by 6.7%

1.11.9 For a second benchmark, libquantum, assume an execution time of 960 ns, CPI of 1.61, and clock rate of 3 GHz. If the execution time is reduced by an additional 10% without affecting the CPI and with a clock rate of 4 GHz, determine the number of instructions.

Answer:

$$\text{execution time} = \text{clock cycles} \times \text{cycle time} = \frac{\text{clock cycles}}{\text{clock rate}} = \frac{\text{number of instructions} \times \text{CPI}}{\text{clock rate}}$$

$$\text{CPI} = \frac{\text{execution time} \times \text{clock rate}}{\text{number of instructions}}$$

$$\text{number of instructions} = \frac{\text{execution time} \times \text{clock rate}}{\text{CPI}}$$

$$\text{execution time} = 0.9 \times 960 \text{ ns} = 864 \times 10^{-9} \text{ s}$$

$$\text{number of instructions} = \frac{864 \times 10^{-9} \times 4 \times 10^9}{1.61} = 2147$$

1.11.10 Determine the clock rate required to give a further 10% reduction in CPU time while maintaining the number of instructions and with the CPI unchanged.

Answer:

$$CPI = \frac{\text{execution time} \times \text{clock rate}}{\text{number of instructions}}$$
$$\text{clock rate} = \frac{CPI \times \text{number of instructions}}{\text{execution time}}$$

$$\text{New execution time} = 0.9 \times 864 \times 10^{-9} \text{ s} = 777.6 \times 10^{-9} \text{ s}$$

$$\text{New clock rate} = \frac{1.61 \times 2147}{777.6 \times 10^{-9} \text{ s}} = 4.445 \text{ GHz}$$

1.11.11 Determine the clock rate if the CPI is reduced by 15% and the CPU time by 20% while the number of instructions is unchanged.

Answer:

$$CPI = 0.85 \times 1.61 = 1.3685$$

$$\text{Execution time} = 0.8 \times 777.6 \times 10^{-9} \text{ s} = 622.08 \times 10^{-9} \text{ s}$$

$$\text{New clock rate} = \frac{1.3685 \times 2147}{622.08 \times 10^{-9} \text{ s}} = 4.72 \text{ GHz}$$

1.12 Section 1.10 cites as a pitfall the utilization of a subset of the performance equation as a performance metric. To illustrate this, consider the following two processors. P1 has a clock rate of 4 GHz, average CPI of 0.9, and requires the execution of 5.0E9 instructions. P2 has a clock rate of 3 GHz, an average CPI of 0.75, and requires the execution of 1.0E9 instructions.

1.12.1 One usual fallacy is to consider the computer with the largest clock rate as having the largest performance. Check if this is true for P1 and P2.

Answer:

$$CPU \text{ time} = \frac{\text{number of instructions} \times CPI}{\text{clock rate}}$$
$$CPU \text{ time}_{P1} = \frac{5 \times 10^9 \times 0.9}{4 \text{ GHz}} = \frac{4.5 \times 10^9}{4 \times 10^9 \text{ Hz}} = 1.125 \text{ s}$$
$$CPU \text{ time}_{P2} = \frac{1 \times 10^9 \times 0.75}{3 \text{ GHz}} = \frac{0.75 \times 10^9}{3 \times 10^9 \text{ Hz}} = 0.25 \text{ s}$$

From these calculations of the CPU times of both P1 and P2, we can see that P2 is faster than P1.

1.12.2 Another fallacy is to consider that the processor executing the largest number of instructions will need a larger CPU time. Considering that processor P1 is executing a sequence of 1.0E9 instructions and that the CPI of processors P1 and P2 do not change, determine the number of instructions that P2 can execute in the same time that P1 needs to execute 1.0E9 instructions.

Answer:

$$CPU\ time_{P1} = \frac{10^9 \times 0.9}{4\ GHz} = \frac{0.9 \times 10^9}{4 \times 10^9\ Hz} = 0.225\ s$$

$$CPU\ time_{P2} = 0.225\ s$$

$$clock\ rate_{P2} = 3\ GHz$$

$$CPI_{P2} = 0.75$$

$$\frac{\text{number of instructions}_{P2} \times 0.75}{3\ GHz} = 0.225\ s$$

$$\text{number of instructions}_{P2} = \frac{0.225 \times 3 \times 10^9}{3\ GHz} = 9 \times 10^8$$

From the calculations above it can be seen that P1 can process more instructions than P2 given the same time period.

1.12.3 A common fallacy is to use MIPS (millions of instructions per second) to compare the performance of two different processors, and consider that the processor with the largest MIPS has the largest performance. Check if this is true for P1 and P2.

Answer:

$$MIPS = \frac{\text{clock rate}}{CPI \times 10^6}$$

$$MIPS_{P1} = \frac{4\ GHz}{0.9 \times 10^6} = \frac{4 \times 10^9}{0.9 \times 10^6} = 4444$$

$$MIPS_{P2} = \frac{3\ GHz}{0.75 \times 10^6} = \frac{3 \times 10^9}{0.75 \times 10^6} = 4000$$

With these calculations it can be seen that P1 has a bigger MIPS than P2.

1.12.4 Another common performance figure is MFLOPS (millions of floating-point operations per second), defined as MFLOPS = No. FP operations / (execution time \times 1E6) but this figure has the same problems as MIPS. Assume that 40% of the instructions executed on both P1 and P2 are floating-point instructions. Find the MFLOPS figures for the programs.

Answer:

$$MFLOPS = \frac{\text{FLOP instructions}}{\text{time} \times 10^6} = \frac{0.4 \times \text{instructions}}{\text{time} \times 10^6} = 0.4MIPS$$

$$MFLOPS_{P1} = 0.4(4444) = 1777.6$$

$$MFLOPS_{P2} = 0.4(4000) = 1600$$

Like MIPS, P1 has bigger MFLOPS in comparison to P2.

