Evolution of Fairness in Different Networks

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The Ultimatum Game

- The ultimatum game is a mathematical framework that models the conflict of rationality and fairness.
- A player has a certain amount of money. It is this player’s job then to split this amount of money with another player. The player on the receiving end decides whether or not if the offer will be accepted:
    - If the receiver rejects the first player’s offer, according to its *acceptance threshold*, then neither of the players receives anything.
    - If the receiver accepts the first player’s offer then the money will be split between the players as per the first player’s proposal.

Our Objective

- In this presentation we analyse the evolution of fairness throughout different typologies.
- We also explore the idea of applying penalties to low offer values to see how they affect the behavior of each player.
- We chose to work on this topic because we wished to study the behavior of players in a game when presented with either a fair or unfair choice and verify if it replicates what happens in a real world environment.

Our Approach

- We replicate the multi-agent-system model of *The spatial ultimatum game* [1] and *The Ultimatum Game in complex networks* [2].
- We considered 4 settings:
  - The baseline (every player interchanges plays with every other player) [1]
  - A scale-free network (Albert-Barabasi)
  - A random network
  - And a small-world network (Watts-Strogatz)
- Offer and acceptance values are between 0 and 1.
- In order to evaluate the fairness of a model, we consider a fair play to be when a player's offer has a value between 0,4 and 0,5.
- If penalties are being applied, players with offers with values between 0,1 and 0,2 are penalised.

Our Model

- Our model receives as input 7 variables:
  - **N**: Number of players.
  - **m**: Number of initial links of a player.
  - **generations:** Determines how many times the model is executed.
  - **prob**: Probability of creating links between players.
  - **network_type**: Determines which type of network the model will have.
  - **mutation_error**: Decimal value ε (between 0 and 1) that adds more random values to a player's *offer* and *acceptance threshold.*
  - **low_offers_penalty_flag**: Flag that determines if the model will apply penalties (it is either 0 or 1).

Our Model

- The graph has the following features: [2]
  - **Offer_Matrix**: Each cell represents the difference $1-p$ of a player's offer.
  - **Reward_Matrix**: Each cell represents the Reward value received by a proposal of a player of value p.
  - **Payoff_Matrix**: Each cell is the sum of the respective offers and reward values and represents the overall payoff of a player.
- Each player has the following features:
  - **p_Value**: Value of a player's offer p.
  - **q_Value**: Player's acceptance value q.
  - **Play_Flag**: Indicates whether a player is eligible to be proposed an offer.

Our Model

- Natural Selection and Random Strategies:
  - At the end-step of each generation every player updates its p and q strategy according to the payoff proportion of the fittest players of the previous generation.
  - This update will also take into account a given mutation error ε in order to add more random strategies
- Low Offer Penalties:
  - Our model has the possibility of applying penalties to players that propose low offer values.
  - Penalties consists of marking the player as not being suitable to perform a proposal with.
  - Once a player is marked as not fit to play with, other players will refuse to play with this marked player and no proposal will be made.

Our Implementation

- At the beginning of every simulation all players have random offer and acceptance values. [1]
- In each generation, each player will play, as both the proposer and the receiver, either:
  - With every other player (in the baseline model).
  - With all its neighbours (in a network environment).
- Every time an offer is accepted: [2]
  - Offer_Matrix[proposer][receiver] = 1 − p.
  - Reward_Matrix[receiver][proposer] = p.
- After all players have exchanged with each other:
  - The payoff of each player is calculated: obtained by adding the Offer Matrix with the Reward Matrix [2].
- At the end of each generation the strategies (the p and q values of each player) will be propagated through the next generation proportionally to their payoff levels.
  - The update of these strategies will also take into account the mutation error that will choose the p and q values randomly within the given error interval centered around the previous generation's p and q values. [1]
- At the end of the simulation,, the results will be printed:
  - The average p and q value of all players for each generation.
  - The logarithmic timestamp of each generation.
  - The percentage of the number of fair plays executed of each generation

Results

- The following table shows the average offer and acceptance values for different mutation errors in the baseline model.
  - Population of N = 100 players with 10^5 generations.
  - These values were obtained only considering values after the point of convergence of value $\log_{10}(t) = 2$ of the noisy convergence scale.
  - These values replicate the values of the *Table 1* of the work done in *The spatial ultimatum game.* [1]

ε	p	q
0.001	0.0638	0.0424
0.01	0.1164	0.0529
0.1	0.3245	0.1376
0.2	0.3384	0.1269

Results: Simulation Parameters

- All the following simulations were executed with
  - 10^4 generations, due to time constraints
  - 0.1 mutation error ε
  - Population of N = 100 players.
  - The time convergence scale is logarithmic ($\log_{10}(t)$ where t is the timestamp of each generation)
- The fairness percentage is calculated for each generation.

Results: No-Penalty Simulations

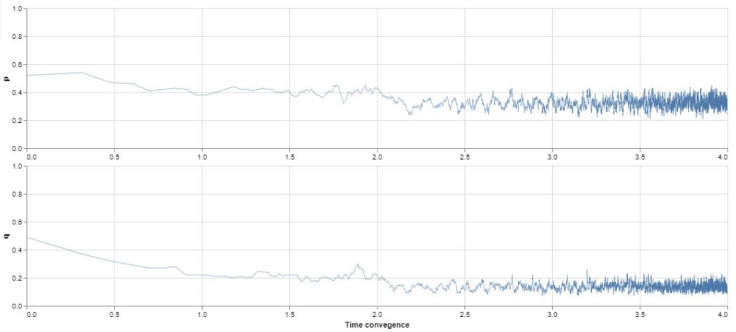

Figure 1. Evolution of p and q values in the Baseline model. Every player plays with every other player as both a proposer and a receiver. These values replicate the values of the *Figure 1* of the work done in *The spatial ultimatum game.* [1]

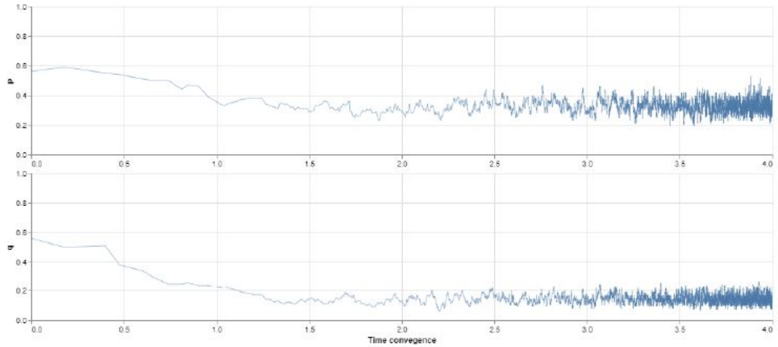

Figure 2. Evolution of p and q values in a Albert-Barabasi network for $m = 5$. Every player plays with its neighbors as both a proposer and a receiver.

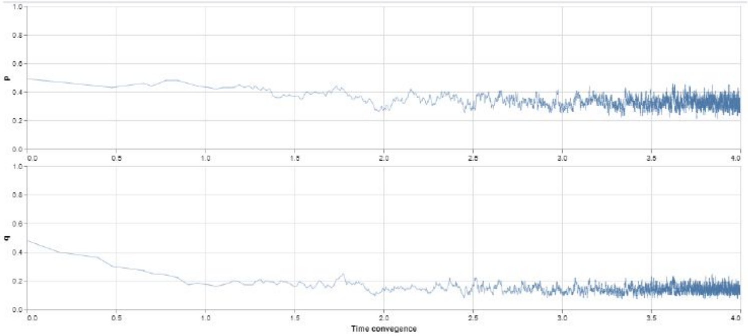

Figure 3. Evolution of p and q values in a Random network for $prob = 0.1$. Once again every player plays with its neighbors as both a proposer and a receiver.

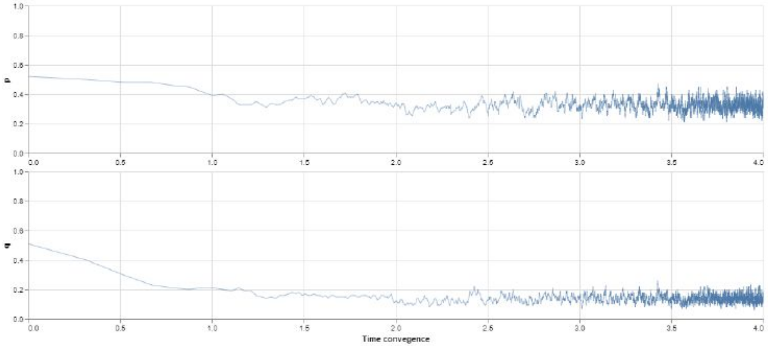

Figure 4. Evolution of p and q values in a Watts-Strogatz network with $m = 5$ and $prob = 0.5$. Once again every player plays with its neighbors as both a proposer and a receiver.

Results: No-Penalty Simulations

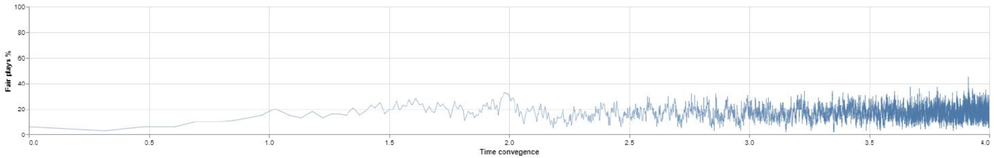

Figure 5. Evolution of the fairness percentage values in in the Baseline model.

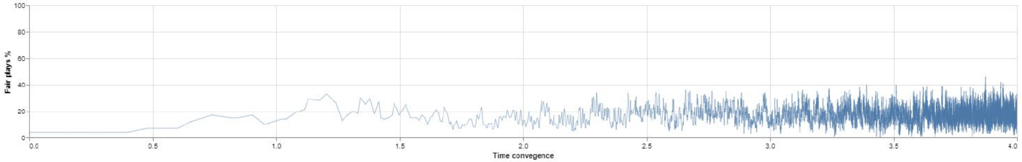

Figure 6. Evolution of the fairness percentage values in a Albert-Barabasi network for $m = 5$.

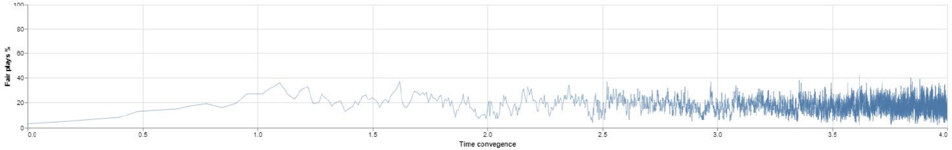

Figure 7. Evolution of the fairness percentage values in a Random network for $prob = 0.1$.

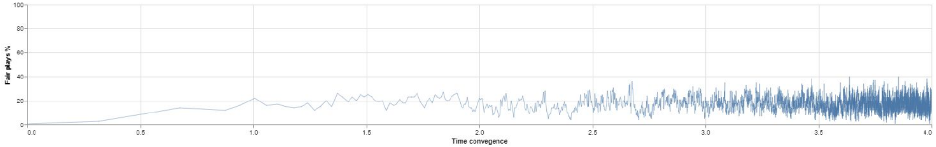

Figure 8. Evolution of fairness percentage values in a Watts-Strogatz network for $m = 5$ $prob = 0.5$.

Results: No-Penalty Simulations

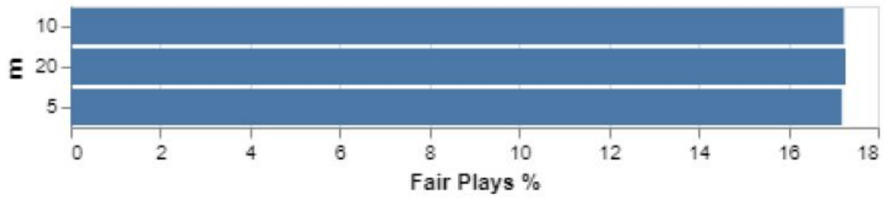

Figure 9. Evolution of the fairness percentage values in a Albert-Barabasi network for different values of *m*.

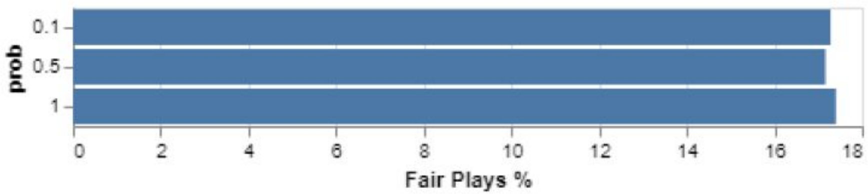

Figure 10. Evolution of the fairness percentage values in a Random network for different values of *prob*.

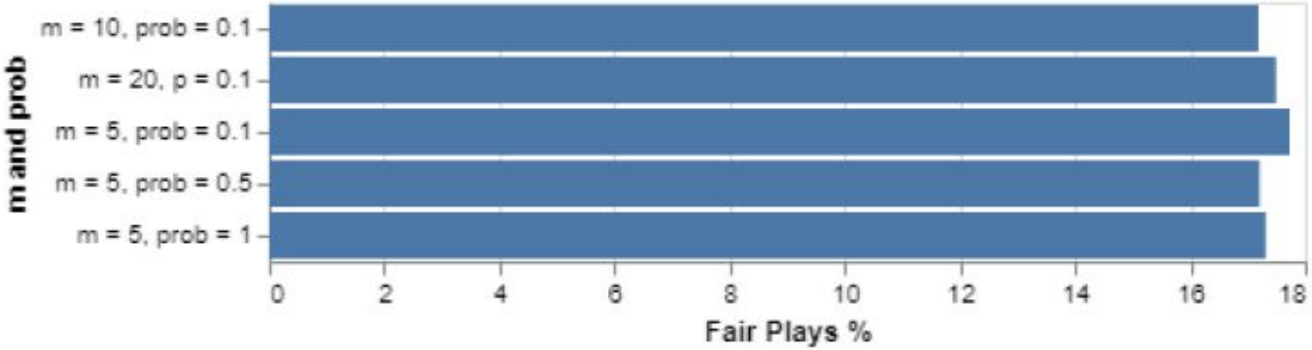

Figure 11. Evolution of the fairness percentage values in a Watts-Strogatz network for different values of *m* and *prob*.

Results: Penalty Simulations

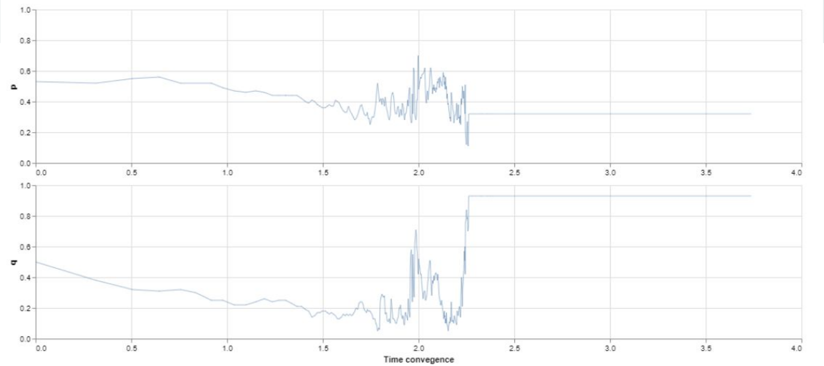

Figure 12. Evolution of fairness percentage values in the Baseline model while applying penalties to low offer values ($lowOffersPenaltyFlag = 1$).

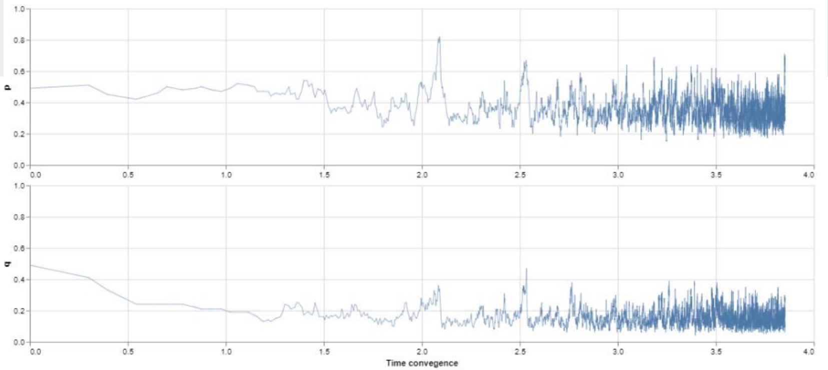

Figure 13. Evolution of p and q values in a Albert-Barabasi for $m = 5$ while applying penalties to low offer values ($lowOffersPenaltyFlag = 1$).

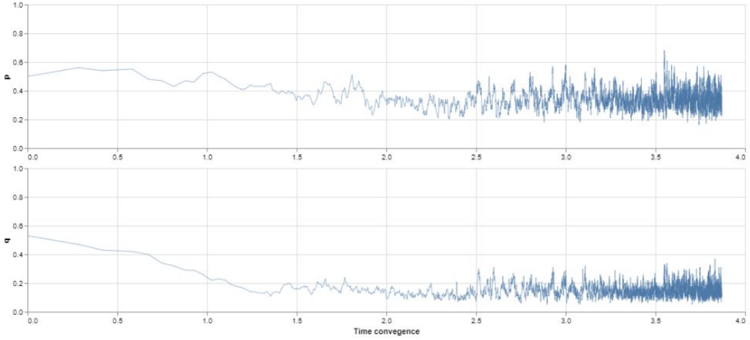

Figure 14. Evolution of p and q values in a Random network for $prob = 0.1$ while applying penalties to low offer values ($lowOffersPenaltyFlag = 1$).

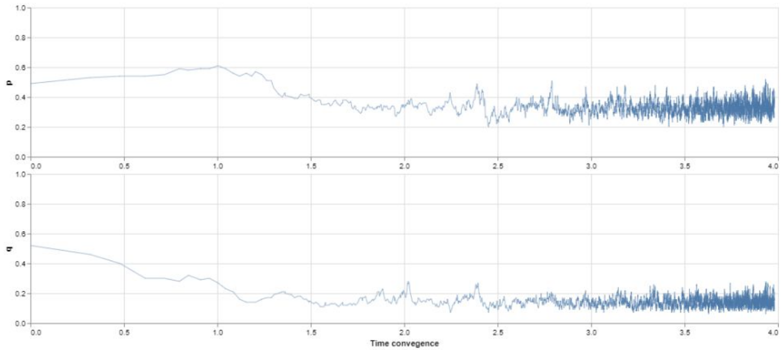

Figure 15. Evolution of p and q values in a Watts-Strogatz network for $m = 5$ and $prob = 0.5$ while applying penalties to low offer values ($lowOffersPenaltyFlag = 1$).

Results: Penalty Simulations

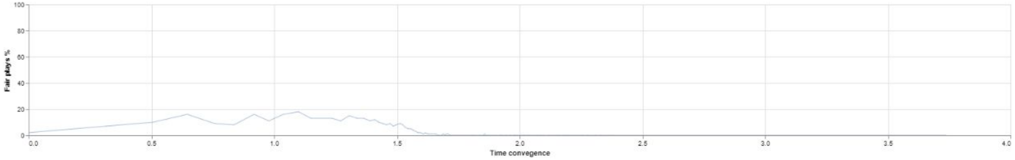

Figure 16. Evolution of fairness percentage values in in the Baseline model while applying penalties ($lowOffersPenaltyFlag = 1$).

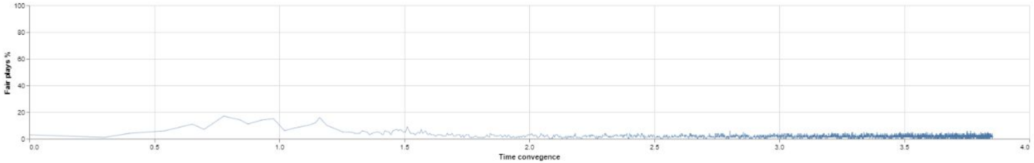

Figure 17. Evolution of fairness percentage values in a Albert-Barabasi for $m = 5$ while applying penalties ($lowOffersPenaltyFlag = 1$).

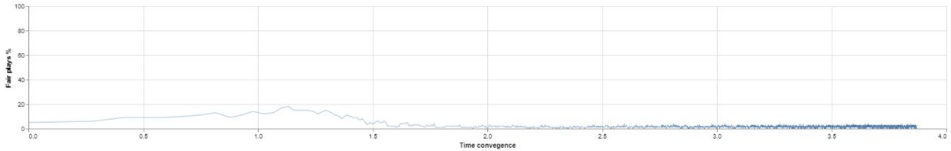

Figure 18. Evolution of fairness percentage values in a Random network for $prob = 0.1$ while applying penalties ($lowOffersPenaltyFlag = 1$).

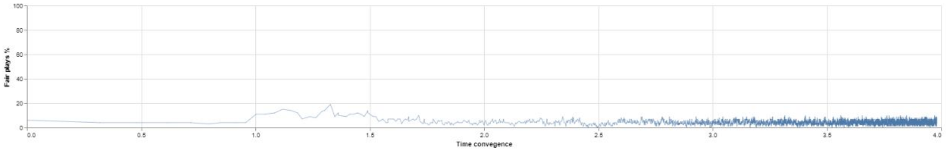

Figure 19. Evolution of fairness percentage values in a Watts-Strogatz network for $m = 5$ and $prob = 0.5$ ($lowOffersPenaltyFlag = 1$).

Conclusion

- In short, we conclude that:
  - In no-penalty simulations:
    - All of the environments showed a tendency towards the same type of behavior when penalties were not being applied.
    - The players began to gravitate towards more fair and empathetic solutions until they reached their convergence point.
  - In penalty simulations:
    - Players had to deal with being penalized for making a low offer.
    - Players started having more rational behaviors and decreased the general level of fairness of the model.
  - The evolution of fairness in this system does not depend on how many players play with each other but rather on how the player's strategies are updated.
- In future work we would:
  - Try to alter the way the model updates the strategies of each player
  - Explore if and how strategy updates depending on neighbours instead of all players of the population would affect the model.
  - Execute the system with more generations in order to get more accurate results.

Thank you!

Any questions?

References:

[1] K. Page, M. Nowak, and K. Sigmund, "The spatial ultimatum game," *Proceedings. Biological sciences / The Royal Society*, vol. 267, pp. 2177–82, 12 2000.

[2] R. Sinatra, J. Iranzo, J. Gómez-Gardeñes, L. Floría, V. Latora, and Y. Moreno, "The ultimatum game in complex networks," *Journal of Statistical Mechanics: Theory and Experiment*, vol. 2009, p. P09012, 09 2009.