Homework 1. Markov chains

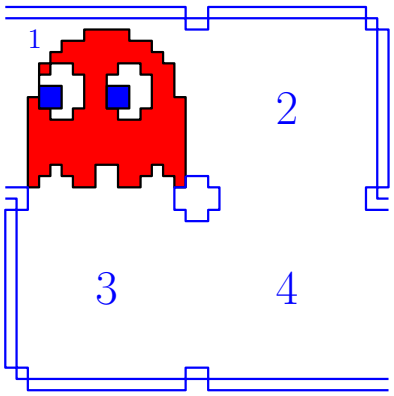

Figure 1: Pacman ghost moving in a 2×2 grid.

Consider the snapshot depicted in Fig. 1, representing a ghost from the Pacman game moving in a 2×2 grid. In this homework, you will describe the motion of the ghost using a Markov chain. To that purpose, consider the following information:

- The cells are numbered from 1 to 4, as indicated by the blue numbers;
- At each time step, the ghost is in one of the four cells; in the next time step, it will move to one of the adjacent cells with equal probability;
- The cell in the top left corner (cell 1) is adjacent, to the left, to the cell in the lower right corner (cell 4). In other words, if the ghost "moves left" at cell 1, it will end up in cell 4, and vice-versa.

Exercise 1.

(a) Write down the Markov chain model (state space and transition probabilities) describing the motion of the ghost.

(b) Suppose that the ghost is in cell 1 at time step $t = 0$. Compute the probability of the ghost being in each cell at time step $t = 3$.

(c) Suppose that the ghost is in cell 1 at time step $t = 0$. *In expectation*, how many time steps will it take to return to cell 1 again?