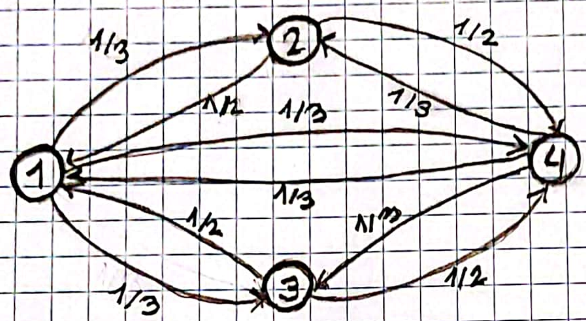
Homework 1

Group 20: João Antunes - nº 87668, Maria Inês Morais - nº 83609

Exercise 1:

(a) State space: $\mathcal{X} = \{1, 2, 3, 4\}$, where 1 = cell 1
2 = cell 2
3 = cell 3
4 = cell 4



Transition diagram

Transition probabilities

	1	2	3	4
1	0	1/3	1/3	1/3
2	1/2	0	0	1/2
3	1/2	0	0	1/2
4	1/3	1/3	1/3	0

(b) We can compute the transition probability matrix to calculate the transition probability:

$$[P]^3 = P(X_3 = y \mid X_0 = 1)$$

$$P = \begin{bmatrix} 0 & 1/3 & 1/3 & 1/3 \\ 1/2 & 0 & 0 & 1/2 \\ 1/2 & 0 & 0 & 1/2 \\ 1/3 & 1/3 & 1/3 & 0 \end{bmatrix}$$

transition probability matrix

$$P^3 = \begin{bmatrix} \boxed{2/9 \quad 7/27 \quad 7/27 \quad 7/27} \\ 7/18 \quad 1/9 \quad 1/9 \quad 7/18 \\ 7/18 \quad 1/9 \quad 1/9 \quad 7/18 \\ 7/27 \quad 7/27 \quad 7/27 \quad 2/9 \end{bmatrix}$$

At time step $t=3$ the probabilities of the ghost being in each cell are:

cell 1 : $22,\bar{2}\%$ cell 2 = $25,93\%$

cell 3 : $25,93\%$ cell 4 = $25,93\%$

(c) For this exercise we consired the transations between cells to be the duration of 1 timestep

Variable G_{ij} = how many time steps it takes to go from cell i to cell j

Variable R_{ij} = how many time steps it takes to return from cell i to cell j

$$E(1\to1) = \tfrac{1}{3}(\overset{=1}{G_{14}} + R_{41}) + \tfrac{1}{3}(\overset{=1}{G_{12}} + R_{21}) + \tfrac{1}{3}(\overset{=1}{G_{13}} + R_{31})$$

$$= \frac{1}{3}(3 + R_{21} + R_{31} + R_{41}) = \frac{3}{3} + \frac{R_{21}+R_{31}+R_{41}}{3}$$

$$= 1 + \frac{R_{21}+R_{31}+R_{41}}{3}$$

$$R_{21} = \tfrac{1}{2}(\overset{=1}{G_{21}} + \overset{=0}{R_{11}}) + \tfrac{1}{2}(\overset{=1}{G_{24}} + R_{41}) = \tfrac{1}{2} + \tfrac{1}{2} + \tfrac{R_{41}}{2}$$

$$= 1 + \tfrac{R_{41}}{2}$$

$$R_{31} = \tfrac{1}{2}(\overset{=1}{G_{31}} + \overset{=0}{R_{11}}) + \tfrac{1}{2}(\overset{=1}{G_{34}} + R_{41}) = \tfrac{1}{2} + \tfrac{1}{2} + \frac{R_{41}}{2}$$

$$= 1 + \tfrac{R_{41}}{2}$$

$$R_{41} = \tfrac{1}{3}(\overset{=1}{G_{41}} + \overset{=0}{R_{11}}) + \tfrac{1}{3}(\overset{=1}{G_{42}} + R_{21}) + \tfrac{1}{3}(\overset{=1}{G_{43}} + R_{31})$$

$$= \tfrac{1}{3} + \tfrac{1}{3} + \tfrac{R_{21}}{3} + \tfrac{1}{3} + \tfrac{R_{31}}{3} = 1 + \tfrac{R_{21}}{3} + \tfrac{R_{31}}{3}$$

$$\begin{cases} R_{21} = 1 + R_{41}/2 \\ R_{31} = 1 + R_{41}/2 \\ R_{41} = 1 + R_{21}/3 + R_{31}/3 \end{cases} \Leftrightarrow \cancel{\begin{cases} R_{31} = R_{21} \\ R_{41} = 1 + R_{21}/3 + R_{31}/3 \end{cases}}$$

$$\Leftrightarrow \cancel{\begin{cases} R_{31} = 1 + R_{41}/2 \end{cases}} \Leftrightarrow \begin{cases} R_{41} = 1 + \dfrac{\left(1 + \frac{R_{41}}{2}\right)}{3} + \dfrac{\left(1 + \frac{R_{41}}{2}\right)}{3} \end{cases}$$

$$\Leftrightarrow \begin{cases} R_{41} = 1 + 1/3 + \dfrac{R_{41}}{6} + 1/3 + R_{41}/6 \end{cases} \Leftrightarrow \begin{cases} R_{41} = 5/3 + R_{41}/3 \end{cases}$$

$$\Leftrightarrow \begin{cases} R_{41} - R_{41}/3 = 5/3 \end{cases} \Leftrightarrow \begin{cases} \frac{2}{3} R_{41} = 5/3 \end{cases} \Leftrightarrow \begin{cases} R_{41} = 5/2 \end{cases}$$

$$\Leftrightarrow \begin{cases} R_{21} = 1 + \frac{5/2}{2} \\ R_{31} = 1 + \frac{5/2}{2} \\ \end{cases} \Leftrightarrow \begin{cases} R_{21} = 9/4 \\ R_{31} = 9/4 \\ R_{41} = 1 + \frac{9/4}{3} + \frac{9/4}{3} \end{cases} \Leftrightarrow \begin{cases} R_{21} = 9/4 \\ R_{31} = 9/4 \\ R_{41} = 5/2 \end{cases}$$

$$E(1 \to 1) = 1 + \frac{9/4 + 9/4 + 5/2}{3} = 10/3 = 3{,}333$$

In expectation, it takes around 3 steps to return to cell 1 again