CMPT 440 – Spring 2019: Quantum Finite Automata

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Introduction and Background

A Quantum Finite Automaton(QFA) is similar to a Deterministic Finite Automaton(DFA) in a few ways and understanding DFAs is crucial to understanding QFAs. There are however a multitude of properties which make QFAs especially interesting and potentially extremely useful. A 1-way QFA is definied by Ambainis and Freivalds (1998) with the six-tuple: $M = (Q, \Sigma, \delta, q_0, Q_{acc}, Q_{rej})$ where

- Q = Finite set of states in M,
- $\Sigma = Thealphabetofsymbolsaccepted$,
- $\delta = Thetransitionfunction$,
- $q_0 = TheinitialstateofM$,
- $Q_{acc} = TheacceptingstatesofM$,
- $Q_{rej} = TherejectingstatesofM$

Constructing a QFA to accept the same language as a DFA can yield a QFA with exponentially less states. Ambainis and Freivalds (1998) first showed that a language $L_n = a^i | iisdivisiblebyn$ can be recognized with O(log n) states. Specifically, QFAs can accept a periodic language L_n over a one-letter alphabet with a period n in O($\sqrt{n}$)$.So,$a$^i \in L$ if and only if $a^{(}i + n) \in L$. Ambainis (2011).

The transition function for a QFA on a symbol in alphabet M maps $Q \times \Gamma \times Q$ to C. Where Γ is the working alphabet of M such that $\Gamma = \Sigma \cup \{c, \$\}$ where c and $ represent the left and right bounds of the input string. Ambainis and Freivalds (1998)

References

A. Ambainis. Quantum finite automata. pages 9–13, 01 2011.

A. Ambainis and R. Freivalds. 1-way quantum finite automata: strengths, weaknesses and generalizations. In *Proceedings 39th Annual Symposium on Foundations of Computer Science (Cat. No. 98CB36280)*, pages 332–341. IEEE, 1998.