Advanced Artificial Intelligence

Lab 05

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The adjacency representation

- The adjacency representation is designed to facilitate the manipulation of edges.
- The crossover operators based on this representation generate offspring that inherit most of their edges from the parent chromosomes.

The adjacency representation

The adjacency representation can be described as follows:

- city j occupies position i in the chromosome if there is an edge from city i to city j in the tour.
- An example
  - the chromosome 38526417 encodes the tour 13564287.
  - City 3 occupies position 1 in the chromosome because edge (1, 3) is in the tour.
  - Similarly, city 8 occupies position 2 because edge (2, 8) is in the tour, etc.

The adjacency representation

- Alternate edges crossover *(Grefenstette et al. [1])*
  - A starting edge (i,j) is selected at random in one parent.
  - Then, the tour is extended by selecting the edge (j, k) in the other parent.
  - The tour is progressively extended in this way by alternatively selecting edges from the two parents.
  - When an edge introduces a cycle, the new edge is selected at random (and is not inherited from the parents).

The adjacency representation

- Alternate edges crossover *(Grefenstette et al. [1])*
  - In the following figure, an offspring is generated from two parent chromosomes that encode the tours 13564287 and 14236578
  - Here, edge (1, 4) is first selected in parent 2, and city 4 in position I of parent 2 is copied at the same position in the offspring.
  - Then, the edges (4, 2) in parent 1, (2, 3) in parent 2, (3, 5) in parent 1 and (5, 7) in parent 2 are selected and inserted in the offspring.

parent 1	:	**3**	**8**	**5**	**2**	**6**	**4**	**1**	**7**
parent 2	:	4	3	6	2	7	5	8	1
offspring	:	4	3	**5**	**2**	7	*8*	*6*	1

The adjacency representation

- Alternate edges crossover *(Grefenstette et al. [1])*
  - Then, edge (7, 1) is selected in parent 1. However, this edge introduces a cycle, and a new edge leading to a city not yet visited is selected at random. Assume that (7, 6) is chosen.
  - Then, edge (6, 5) is selected in parent 2, but it also introduces a cycle. At this point, (6, 8) is the only selection that does not introduce a cycle.
  - Finally, the tour is completed with edge (8, 1).
  - The final offspring encodes the tour 14235768, and all edges in the offspring are inherited from the parents, apart from the edges (7, 6) and (6, 8).

parent 1	:	**3**	**8**	**5**	**2**	**6**	**4**	**1**	**7**
parent 2	:	4	3	6	2	7	5	8	1
offspring	:	4	3	**5**	**2**	7	*8*	*6*	1

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - The alternate edge operator introduces many random edges in the offspring, particularly the last edges, when the choices for extending the tour are limited.
  - Since the offspring must inherit as many edges as possible from the parents, the introduction of random edges should be minimized.
  - The edge recombination operator reduces the myopic behavior of the alternate edge approach with a special data structure called the "edge map".

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - The edge map maintains the list of edges that are incident to each city in the parent tours and that lead to cities not yet included in the offspring.
  - These edges are still available for extending the tour and are said to be active.
  - The strategy is to extend the tour by selecting the edge that leads to the city with the minimum number of active edges.
  - In the case of equality between two or more cities, one of these cities is selected at random.
  - The approach is less likely to get trapped in a "dead end", namely, a city with no remaining active edges (thus requiring the selection of a random edge).

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - For the tours 13564287 and 14236578 (path representation), the initial edge map is shown in the following figure.

city 1 has edges to	:	3 4 7 8
city 2 has edges to	:	3 4 8
city 3 has edges to	:	1 2 5 6
city 4 has edges to	:	1 2 6
city 5 has edges to	:	3 6 7
city 6 has edges to	:	3 4 5
city 7 has edges to	:	1 5 8
city 8 has edges to	:	1 2 7

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - The operation of the edge recombination crossover operator will now be illustrated on this initial edge map.

city 1 has edges to : 3 4 7 8
city 2 has edges to : 3 4 8
city 3 has edges to : 1 2 5 6
city 4 has edges to : 1 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 1 5 8
city 8 has edges to : 1 2 7

(a) City 1 is selected
city 2 has edges to : 3 4 8
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5 8
city 8 has edges to : 2 7

(b) City 8 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5

(c) City 7 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6
city 6 has edges to : 3 4 5

(d) City 5 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 6
city 4 has edges to : 2 6
city 6 has edges to : 3 4

(e) City 6 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2
city 4 has edges to : 2

(f) City 4 is selected
city 2 has edges to : 3
city 3 has edges to : 2 6

(g) City 2 is selected
city 3 has edges to :

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - Assume that city 1 is selected as the starting city.
  - Accordingly, all edges incident to city 1 must first be deleted from the initial edge map.
  - From city 1, we can go to cities 3, 4, 7 or 8.
  - City 3 has three active edges, while cities 4, 7 and 8 have two active edges.
  - Hence, a random choice is made between cities 4, 7 and 8.

city 1 has edges to	: 3 4 7 8
city 2 has edges to	: 3 4 8
city 3 has edges to	: 1 2 5 6
city 4 has edges to	: 1 2 6
city 5 has edges to	: 3 6 7
city 6 has edges to	: 3 4 5
city 7 has edges to	: 1 5 8
city 8 has edges to	: 1 2 7

City 1 is selected

(a)

city 2 has edges to	: 3 4 8
city 3 has edges to	: 2 5 6
city 4 has edges to	: 2 6
city 5 has edges to	: 3 6 7
city 6 has edges to	: 3 4 5
city 7 has edges to	: 5 8
city 8 has edges to	: 2 7

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - Assume that city 8 is selected.
  - From 8, we can go to cities 2 and 7.

City 1 is selected

(a)
city 2 has edges to : 3 4 8
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5 8
city 8 has edges to : 2 7

City 8 is selected

(b)
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - As indicated in edge map (b), city 2 has two active edges and city 7 only one, so city 7 is selected.
  - From city 7, there is no choice but to go to city 5.

City 8 is selected

(b)	city 2 has edges to :	3 4
	city 3 has edges to :	2 5 6
	city 4 has edges to :	2 6
	city 5 has edges to :	3 6 7
	city 6 has edges to :	3 4 5
	city 7 has edges to :	5

→

City 7 is selected

(c)	city 2 has edges to :	3 4
	city 3 has edges to :	2 5 6
	city 4 has edges to :	2 6
	city 5 has edges to :	3 6
	city 6 has edges to :	3 4 5

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - City 5 is selected.
  - From this point, edge map (d) offers a choice between cities 3 and 6 with two active edges.

City 7 is selected

(c)
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6
city 6 has edges to : 3 4 5

City 5 is selected

(d)
city 2 has edges to : 3 4
city 3 has edges to : 2 6
city 4 has edges to : 2 6
city 6 has edges to : 3 4

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - Assume that city 6 is randomly selected.
  - From city 6, we can go to cities 3 and 4, and edge map (e) indicates that both cities have one active edge.

City 5 is selected

(d)
city 2 has edges to : 3 4
city 3 has edges to : 2 6
city 4 has edges to : 2 6
city 6 has edges to : 3 4

City 6 is selected

(e)
city 2 has edges to : 3 4
city 3 has edges to : 2
city 4 has edges to : 2

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - assume that city 4 is randomly selected.
  - from city 4 we can only go to city 2.

City 6 is selected

(e) city 2 has edges to : 3 4
city 3 has edges to : 2
city 4 has edges to : 2

City 4 is selected

(f) city 2 has edges to : 3
city 3 has edges to : 2 6

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - City 2 is selected.
  - From city 2 we must go to city 3.
  - The final tour is 18756423 and all edges are inherited from both parents.

(f) City 4 is selected

city 2 has edges to : <u>3</u>

city 3 has edges to : 2 6

→

(g) City 2 is selected

city <u>3</u> has edges to :

The adjacency representation

- Edge recombination crossover (ER) *(Whitley et al. [2])*
  - To this end, the edge map will be updated after each city selection. In these edge maps, cities of particular interest are underlined.

city 1 has edges to : 3 4 7 8
city 2 has edges to : 3 4 8
city 3 has edges to : 1 2 5 6
city 4 has edges to : 1 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 1 5 8
city 8 has edges to : 1 2 7

(a) City 1 is selected
city 2 has edges to : 3 4 8
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5 8
city 8 has edges to : 2 7

(b) City 8 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6 7
city 6 has edges to : 3 4 5
city 7 has edges to : 5

(c) City 7 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 5 6
city 4 has edges to : 2 6
city 5 has edges to : 3 6
city 6 has edges to : 3 4 5

(d) City 5 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2 6
city 4 has edges to : 2 6
city 6 has edges to : 3 4

(e) City 6 is selected
city 2 has edges to : 3 4
city 3 has edges to : 2
city 4 has edges to : 2

(f) City 4 is selected
city 2 has edges to : 3
city 3 has edges to : 2 6

(g) City 2 is selected
city 3 has edges to :

The adjacency representation

- Heuristic crossover (HX) *(Grefenstette et al. [1], Grefenstette [3])*
  - The previous crossover operators did not exploit the distances between the cities (i.e., the length of the edges).
  - It is a characteristic of the genetic approach to avoid any heuristic information about a specific application domain, apart from the overall evaluation or fitness of each chromosome.
  - This characteristic explains the robustness of the genetic search and its wide applicability.

The adjacency representation

- Heuristic crossover (HX) *(Grefenstette et al. [1], Grefenstette [3])*
  - Step 1. Choose a random starting city from one of the two parents.
  - Step 2. Compare the edges leaving the current city in both parents and select the shorter edge.
  - Step 3. If the shorter parental edge introduces a cycle in the partial tour, then extend the tour with a random edge that does not introduce a cycle.
  - Step 4. Repeat steps 2 and 3 until all cities are included in the tour.

Mutation operators

- Mutation operators for the TSP aim at randomly generating new permutations of the cities.
- Permutation operators for the TSP often greatly modifies the original tour.
    - Swap
    - Local hill-climbing
    - Scramble

Mutation operators

- Swap
  - Two cities are randomly selected and swapped (i.e., their positions are exchanged).
- Local hill-climbing
  - A local edge exchange heuristic is applied to the tour (e.g., 2-opt).
- Scramble
  - Two cut points are selected at random on the chromosome, and the cities within the two cut points are randomly permuted.

Exercise

For the tours 13564287 and 14236578 (path representation), the initial edge map is shown in the following figure. Here, we consider the Edge recombination crossover (ER) in The adjacency representation.

- Exercise 1:

  What is the final tour if city 2 is selected as the starting city? Write down the derivation process and justify your conclusion.

- Exercise 2:

  What is the final tour if city 3 is selected as the starting city? Write down the derivation process and justify your conclusion.

city 1 has edges to	:	3 4 7 8
city 2 has edges to	:	3 4 8
city 3 has edges to	:	1 2 5 6
city 4 has edges to	:	1 2 6
city 5 has edges to	:	3 6 7
city 6 has edges to	:	3 4 5
city 7 has edges to	:	1 5 8
city 8 has edges to	:	1 2 7

References

- [1] J. Grefenstette, R. Gopal, B.J. Rosmaita and D.V. Gucht, Genetic algorithms for the traveling salesman problem, in: Proc. 1st. Int. Conf. on Genetic Algorithms (ICGA '85), Carnegie-Mellon University, Pittsburgh, PA (1985) pp. 160-168.
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- [3] J. Grefenstette, Incorporating problem specific knowledge into genetic algorithms, in: Genetic Algorithms and Simulated Annealing, ed. L. Davis (Morgan Kaufmann, 1987) pp. 42-60.