

- (2) For two-side diffraction induced by a small-size object, the signal amplitude during the process of crossing the FFZ is non-monotonous and varies in the shape of “W”. The local peak at the bottom appears when the gravity center of the object is on the LoS.

### 3.2 Doppler-Velocity Model

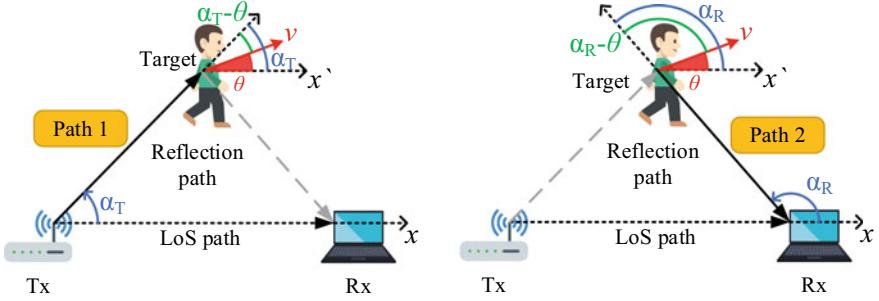
Velocity is one of the most important parameters in wireless sensing to support applications such as trajectory tracking, activity recognition, and fall detection. The principle of velocity estimation using radio frequency signals is that there is a relationship between the velocity of a moving target and the induced Doppler frequency shift (DFS). In this subsection, we introduce the Doppler-velocity model to quantify the relationship between the target’s velocity and DFS. We summarize the essential properties of Doppler-velocity model when it is applied for velocity estimation in real-world applications.

#### 3.2.1 Doppler-Velocity Model

*Doppler effect* is defined as the frequency change of a signal caused by the motion of a signal transmitter, receiver, or even a reflector. In WiFi/4G/5G system, signals arrive at a receiver through multiple paths. If a signal gets reflected from a target and the target moves, the movement changes the frequency of the signal due to Doppler effect. The frequency difference between the transmitted signal and received signal is called Doppler frequency shift (DFS). Fundamentally, the frequency shift is caused by signal path length change. If the signal path length increases, Doppler frequency shift is negative and when the path length decreases, Doppler frequency shift is positive. To establish the relationship between Doppler frequency shift and the reflection path length change, we divide the target reflection path into two parts, i.e., (1) *Path 1: from transmitter (Tx) to target* and (2) *Path 2: from target to receiver (Rx)*, as shown in Fig. 7. The total Doppler frequency shift can be obtained by adding the Doppler frequency shifts induced by Path 1 and Path 2.

- (1) *Signal path length change from Tx to target.* If the target moves in the direction of  $\theta$  and  $\alpha_T$  is signal direction, the angle between Path 1 and the target moving direction is  $\alpha_T - \theta$  as shown in Fig. 7. Then the target’s velocity component in the direction of Path 1 is  $v\cos(\alpha_T - \theta)$ . The relative speed of signal Path 1 viewed at the target can thus be represented as  $c - v\cos(\alpha_T - \theta)$ . When the target’s speed is much less than the speed of light (i.e.,  $v \ll c$ ), we can obtain the following relationship [29] between the signal frequency at the transmitter ( $f_{tx}$ ) and at the target ( $f_{tar}$ )

$$\frac{c}{f_{tx}} = \frac{c - v\cos(\theta - \alpha_T)}{f_{tar}}, \quad (19)$$



**Fig. 7** Doppler-velocity model in WiFi/4G/5G based wireless sensing system

where  $c$  is the speed of light. Therefore, we can derive the following formula

$$\frac{c}{f_{tx}} = \frac{c - v\cos(\theta - \alpha_T)}{f_{tar}} = \frac{c - v\cos(\theta - \alpha_T) - c}{f_{tar} - f_x} = \frac{-v\cos(\theta - \alpha_T)}{\Delta f_{p1}}, \quad (20)$$

where  $\Delta f_{p1}$  is the Doppler frequency shift ( $f_{tar} - f_{tx}$ ) for Path 1 which can be represented as

$$\Delta f_{p1} = f_{tx} \frac{-v\cos(\theta - \alpha_T)}{c}. \quad (21)$$

- (2) *Signal path length change from target to Rx:* The signal reflected by the moving target continues to propagate to Rx. The moving target can be viewed as the new signal source and the signal frequency is  $f_{tar}$ . The angle between target moving direction  $\theta$  and the signal propagation direction  $\alpha_R$  is now  $\alpha_R - \theta$ . The target's velocity component in the direction of Path 2 is  $v\cos(\alpha_R - \theta)$ . The relative speed of signal Path 2 viewed at the receiver can be represented as  $c - v\cos(\alpha_R - \theta)$ . Similarly, we can obtain the Doppler frequency shift of Path 2 as

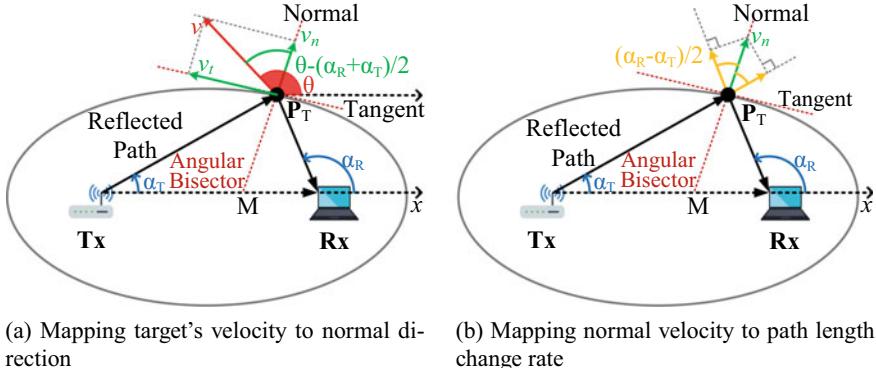
$$\Delta f_{p2} = f_{tx} \frac{-v\cos(\theta - \alpha_R)}{c}. \quad (22)$$

The total Doppler frequency shift  $f_D$  of the reflection path can be calculated by adding the two parts (i.e.,  $\Delta f_{p1}$  and  $\Delta f_{p2}$ )

$$f_D = \Delta f_{p1} + \Delta f_{p2} = f_{tx} \frac{-v\cos(\theta - \alpha_T) - v\cos(\theta - \alpha_R)}{c}. \quad (23)$$

As the target's speed  $v$  is much smaller than the speed of light  $c$ , Eq. (23) can be simplified as

$$f_D = -\frac{f_{tx}}{c} \cdot \underbrace{v\cos(\theta - \frac{\alpha_R + \alpha_T}{2})}_{\textcircled{1}} \cdot \underbrace{2\cos(\frac{\alpha_R - \alpha_T}{2})}_{\textcircled{2}}. \quad (24)$$



**Fig. 8** The meanings of Doppler-velocity model

The sign of  $f_D$  is related to the movement direction of the target. To fully understand Eq. (24), let us consider an ellipse with the two foci at Tx and Rx as shown in Fig. 8. Now, let  $M$  be a point on the LoS path such that the line  $P_T M$  is the angular bisector of  $\angle T x P_T R x$ . Based on the property of ellipse [30],  $P_T M$  is a normal line of the ellipse. Now we depict the two angles in Eq. (24), i.e.,  $\theta - \frac{\alpha_R + \alpha_T}{2}$  and  $\frac{\alpha_R - \alpha_T}{2}$  in Fig. 8, and present two important observations as below:

- *Term ① of Eq. (24) projects the target's velocity to the normal line of the ellipse as  $v_n = v \cos(\theta - \frac{\alpha_R + \alpha_T}{2})$ .* The projected speed  $v_n$  depends on the target's movement direction  $\theta$  and location. For a same target speed  $v$ , if the target moves along the normal line, the Doppler shift is maximum. When the target moves along the tangent line, the Doppler shift is 0.
- *Term ② of Eq. (24) further projects the normal velocity component at the two signal path directions, i.e.,  $T x P_T$  and  $P_T R x$ .* The rate of path length change is the sum of change rate at  $T x P_T$  and  $P_T R x$ . Since the normal line is also the angular bisector of  $\angle T x P_T R x$ , the rates of path length change along lines  $T x P_T$  and  $P_T R x$  are both  $v_n \cos(\frac{\alpha_R - \alpha_T}{2})$ . Therefore, the sum rate is  $2v_n \cos(\frac{\alpha_R - \alpha_T}{2})$ .

### 3.2.2 Velocity Estimation Sensitivity Analysis

**Sensitivity Analysis of Speed Estimation.** Based on Eq. (24), the first-order derivative of the moving speed with respect to Doppler frequency shift is given by

$$\frac{dv}{df_D} = -\frac{c}{f_{tx} \cos(\theta - \frac{\alpha_T + \alpha_R}{2}) \cdot 2 \cos \frac{\alpha_T - \alpha_R}{2}}. \quad (25)$$

The larger the gradient  $\frac{dv}{df_D}$ , the more sensitive the speed estimate is related to Doppler shift. In other words, a small change of Doppler shift can lead to a large change of  $v$ .