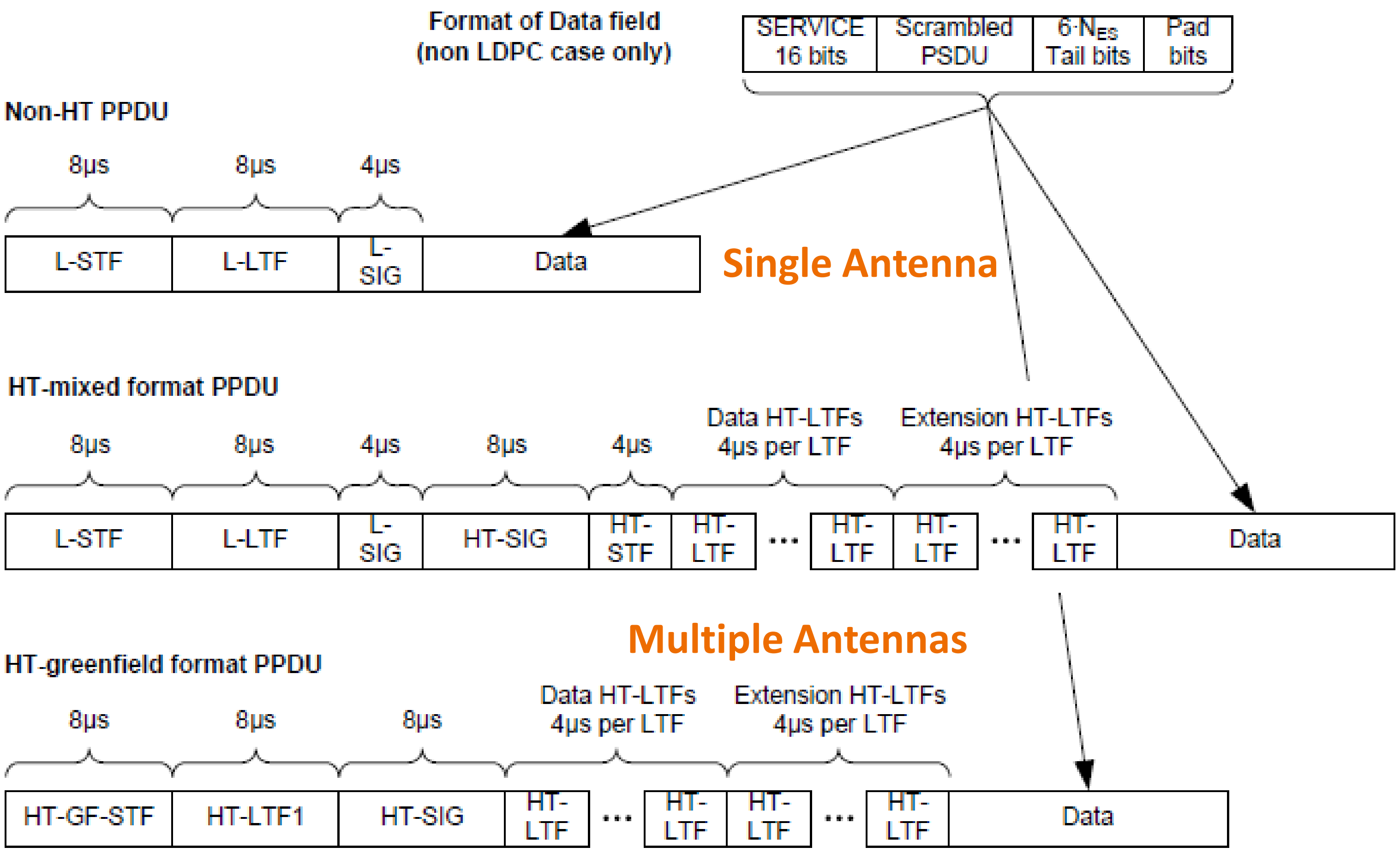
MIMO Transmitter in IEEE802.11

Lecturer: Dr. Rui Wang

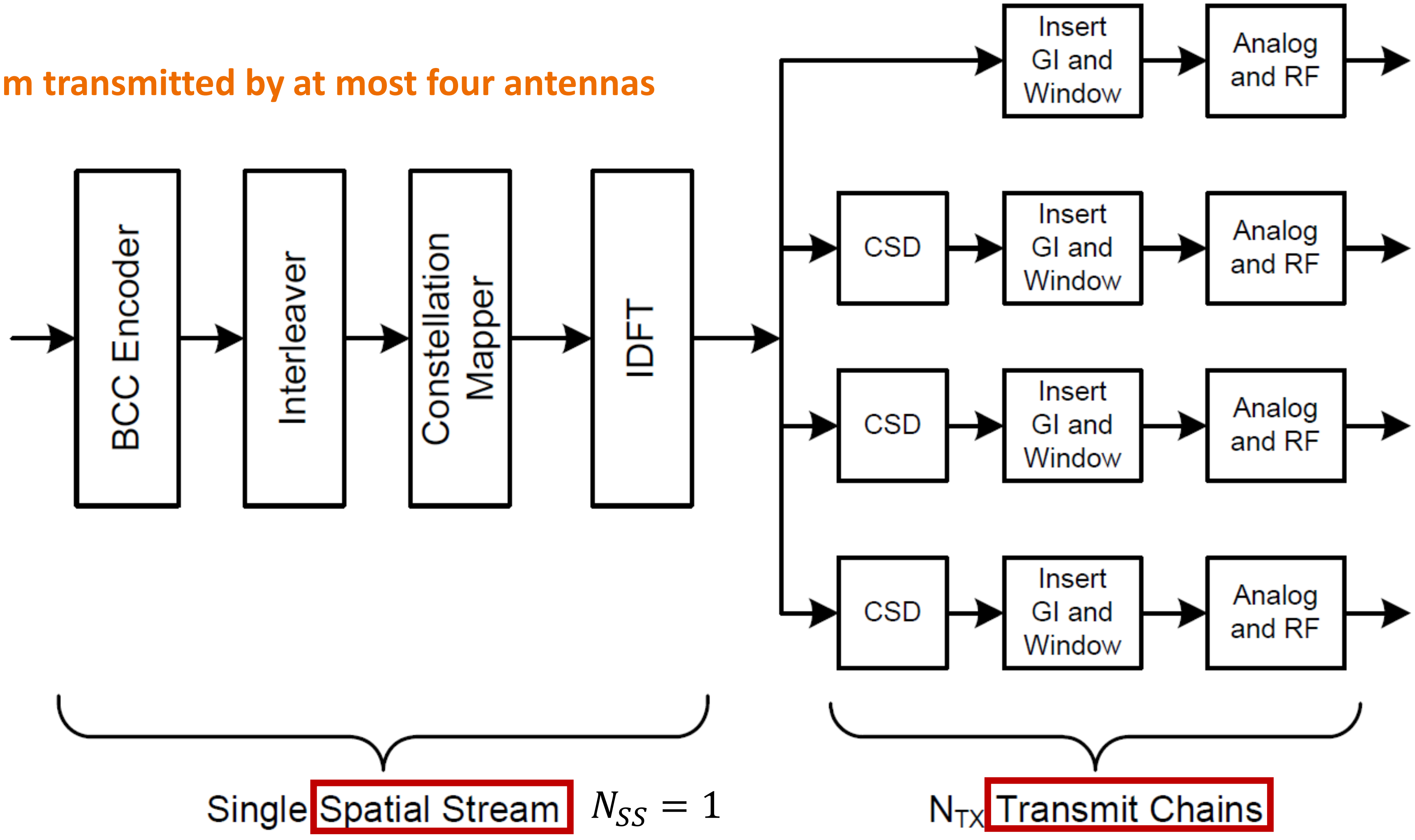
Recap: Three PPDU Formats



- Non-HT： either the AP or the STA does not support HT
- HT-mixed: AP and STA support HT, however, other STAs may not support HT
- HT-greenfield: all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- L-SIG, HT-SIG: basic information about the PPDU

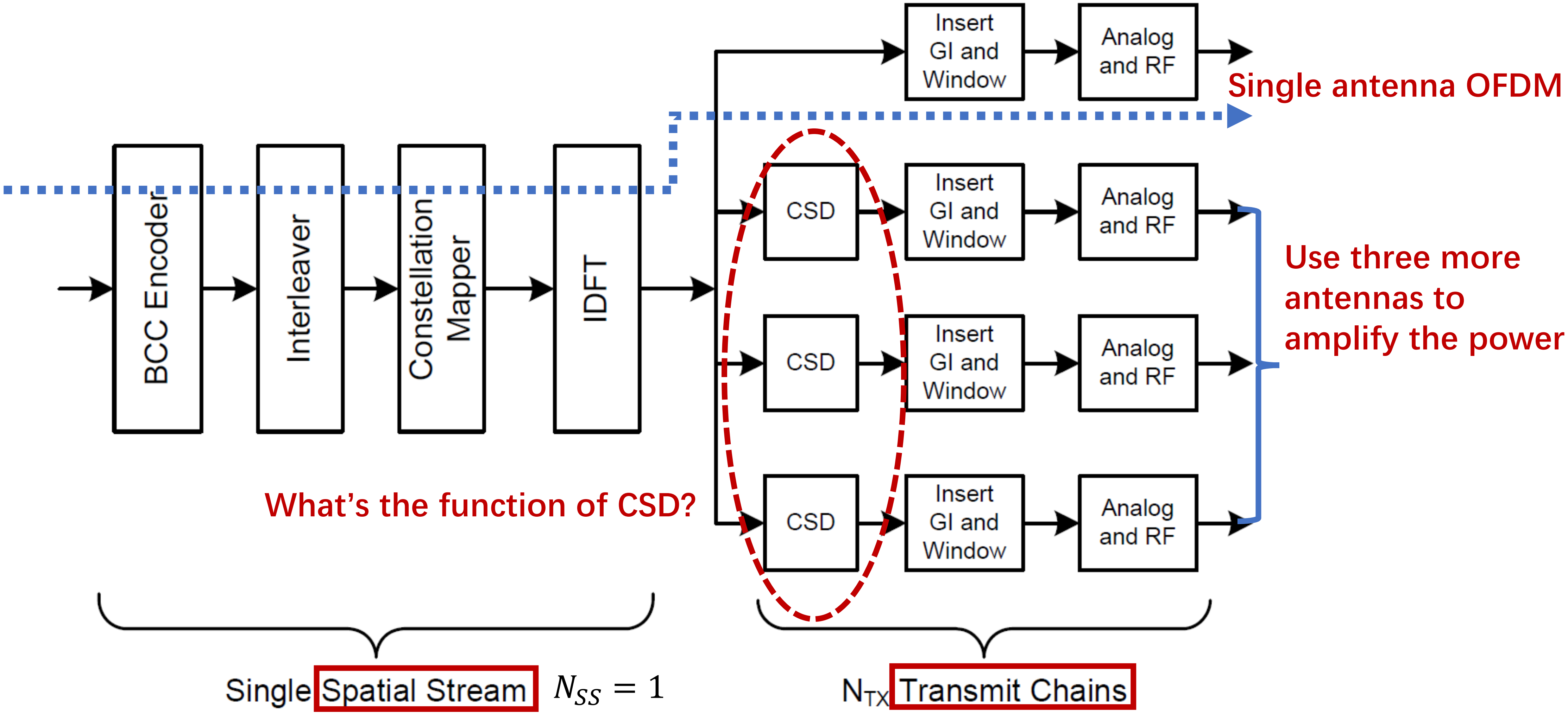
Single Spatial Stream (19.3.3)

One data stream transmitted by at most four antennas



"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

Single Spatial Stream (19.3.3)



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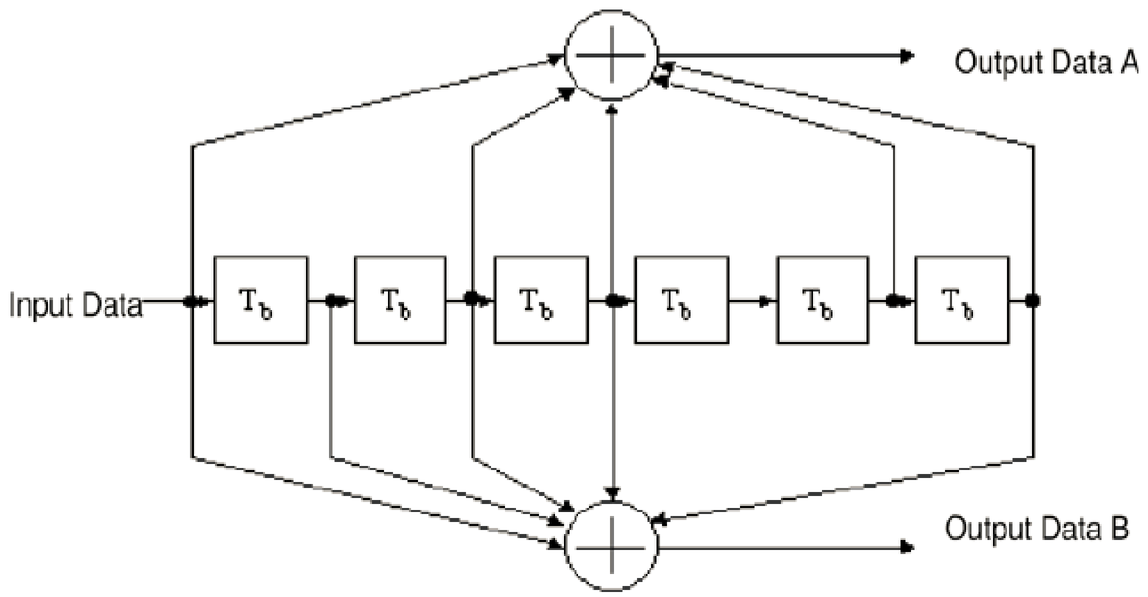
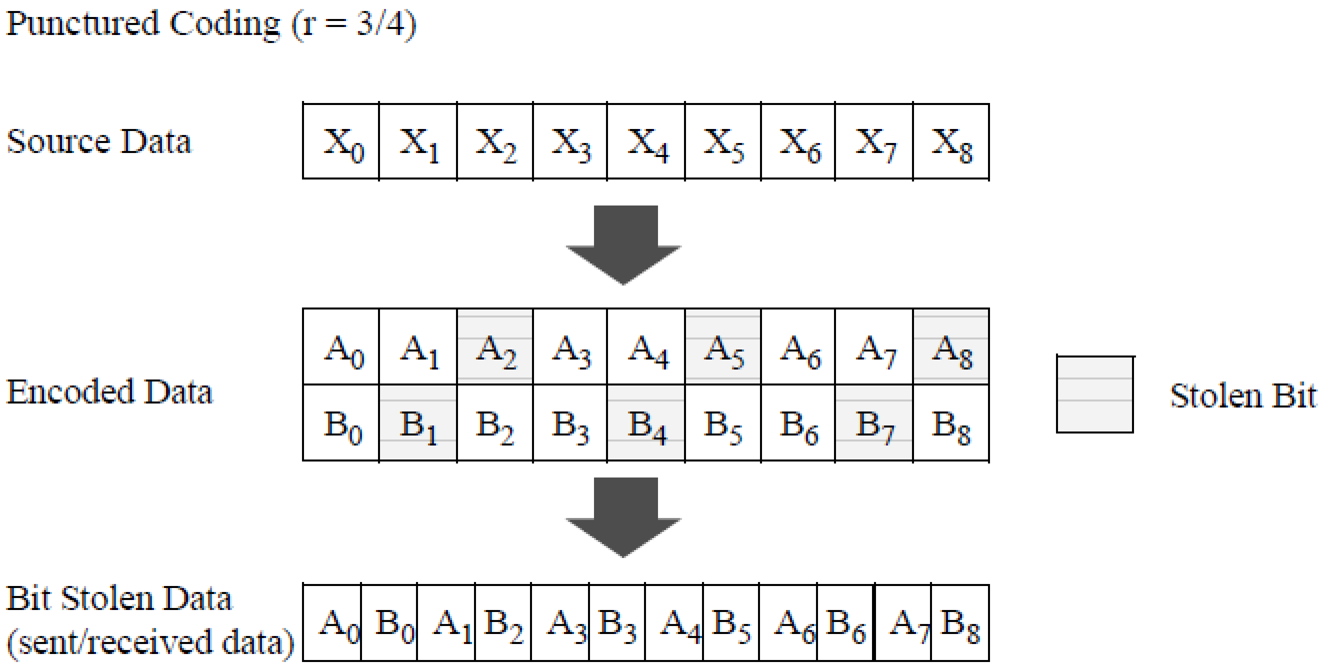
Channel Coding

- **Binary convolutional code (BCC) – 17.3.5.6**





- **Data interleaving – 17.3.5.7**

Interleaves the bits of each spatial stream (changes order of bits) to prevent long sequences of adjacent noisy bits.

Modulation and Coding Scheme (MCS, 19.5)

Table 19-27—MCS parameters for mandatory 20 MHz, N_{SS} = 1, N_{ES} = 1

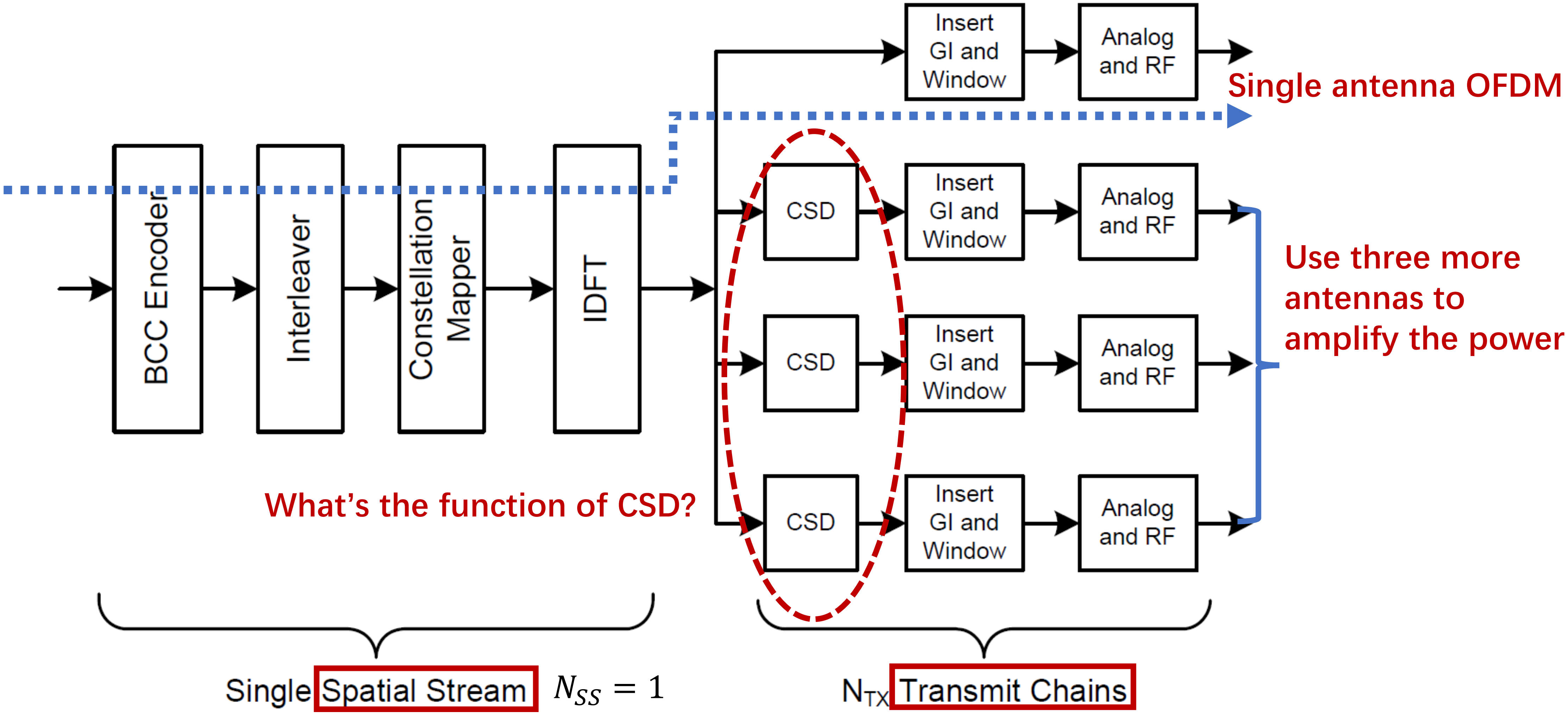
MCS Index	Modulation	R	$N_{BPSCS}(i_{SS})$	N_{SD}	N_{SP}	N_{CBPS}	N_{DBPS}	Data rate (Mb/s)	
								800 ns GI	400 ns GI (see NOTE)
0	BPSK	1/2	1	52	4	52	26	6.5	7.2
1	QPSK	1/2	2	52	4	104	52	13.0	14.4
2	QPSK	3/4	2	52	4	104	78	19.5	21.7
3	16-QAM	1/2	4	52	4	208	104	26.0	28.9
4	16-QAM	3/4	4	52	4	208	156	39.0	43.3
5	64-QAM	2/3	6	52	4	312	208	52.0	57.8
6	64-QAM	3/4	6	52	4	312	234	58.5	65.0
7	64-QAM	5/6	6	52	4	312	260	65.0	72.2

NOTE—Support of 400 ns GI is optional on transmit and receive.

N_{BPSCS}: Number of coded bits per subcarrier
N_{SD}: Number of data subcarriers
N_{SP}: Number of pilot subcarriers

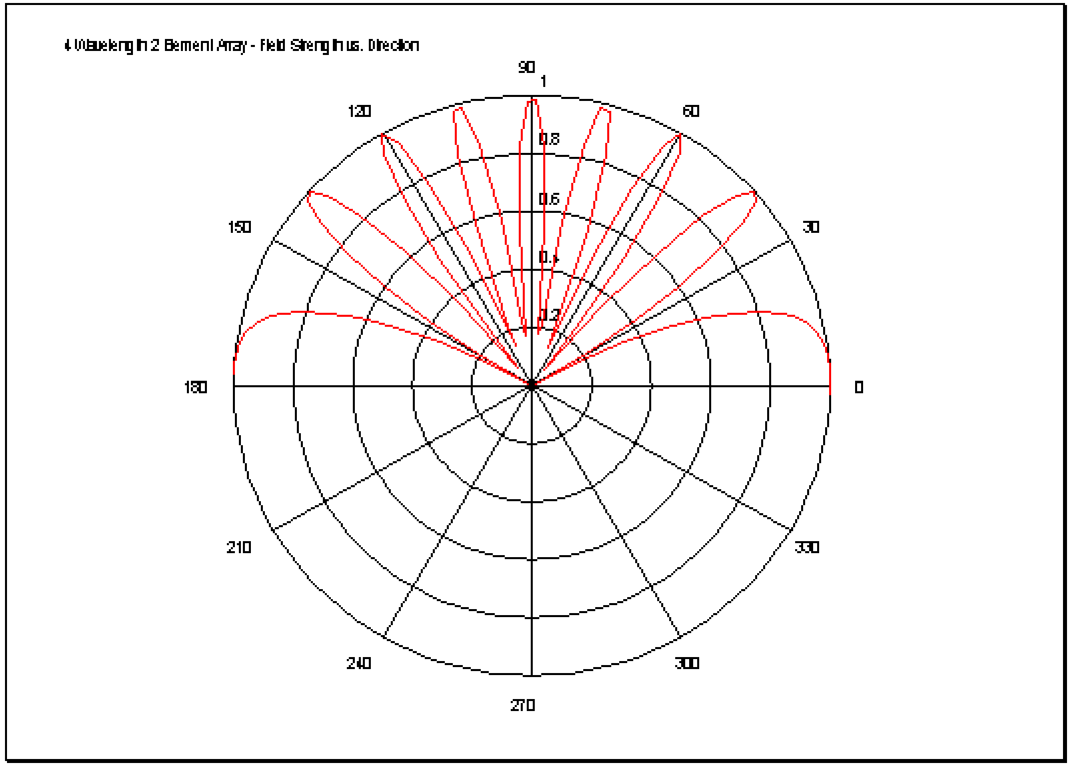
N_{CBPS}: Number of coded bits per symbol
N_{DBPS}: Number of data bits per symbol

Recap: Single Spatial Stream (19.3.3)



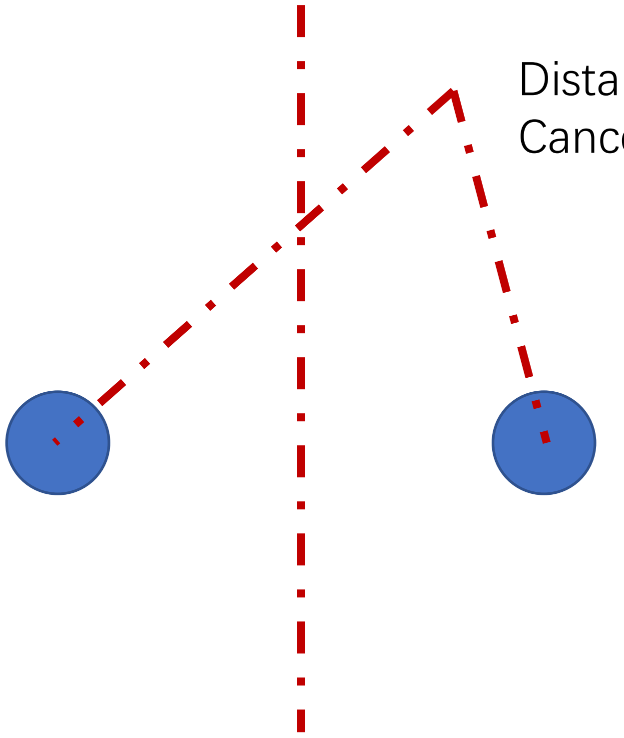
"... generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

Array Pattern

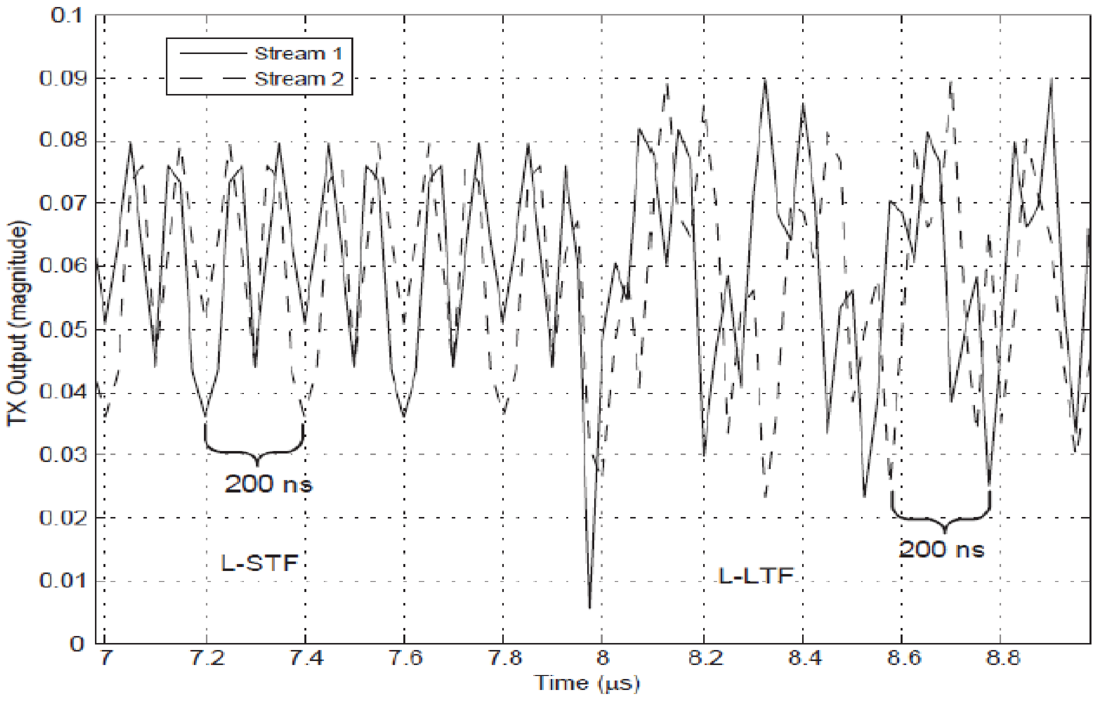


- Why antenna specific delay (cyclic shift)?
- If two antenna ports transmit the same signal at the same time, there might be effect of beamforming
- Not desired at the phase of packet synchronization
- Cyclic shift is applied to avoid the beamforming effect

Array Pattern



Cyclic Shift (19.3.9.3.2)



"Cyclic shifts are used to prevent **unintentional beamforming** when the same signal or scalar multiples of one signal are transmitted through different spatial streams or transmit chains."

Table 19-9—Cyclic shift for non-HT portion of packet

$T_{CS}^{i_{TX}}$ values for non-HT portion of packet				
Number of transmit chains	Cyclic shift for transmit chain 1 (ns)	Cyclic shift for transmit chain 2 (ns)	Cyclic shift for transmit chain 3 (ns)	Cyclic shift for transmit chain 4 (ns)
1	0	—	—	—
2	0	–200	—	—
3	0	–100	–200	—
4	0	–50	–100	–150

50ns is the duration of one sample

Let s(t) be the signal without cyclic shift, TCS is the cyclic shift value, then the cyclic shift is as follows.

$$s_{CS}(t;T_{CS})\big|_{T_{CS}<0} = \begin{cases} s(t-T_{CS}) & 0 \le t < T + T_{CS} \\ s(t-T_{CS}-T) & T+T_{CS} \le t \le T \end{cases}$$

Cyclic Shift is different from the Cyclic Prefix of OFDM, which is referred to Guard Interval (GI) in standard

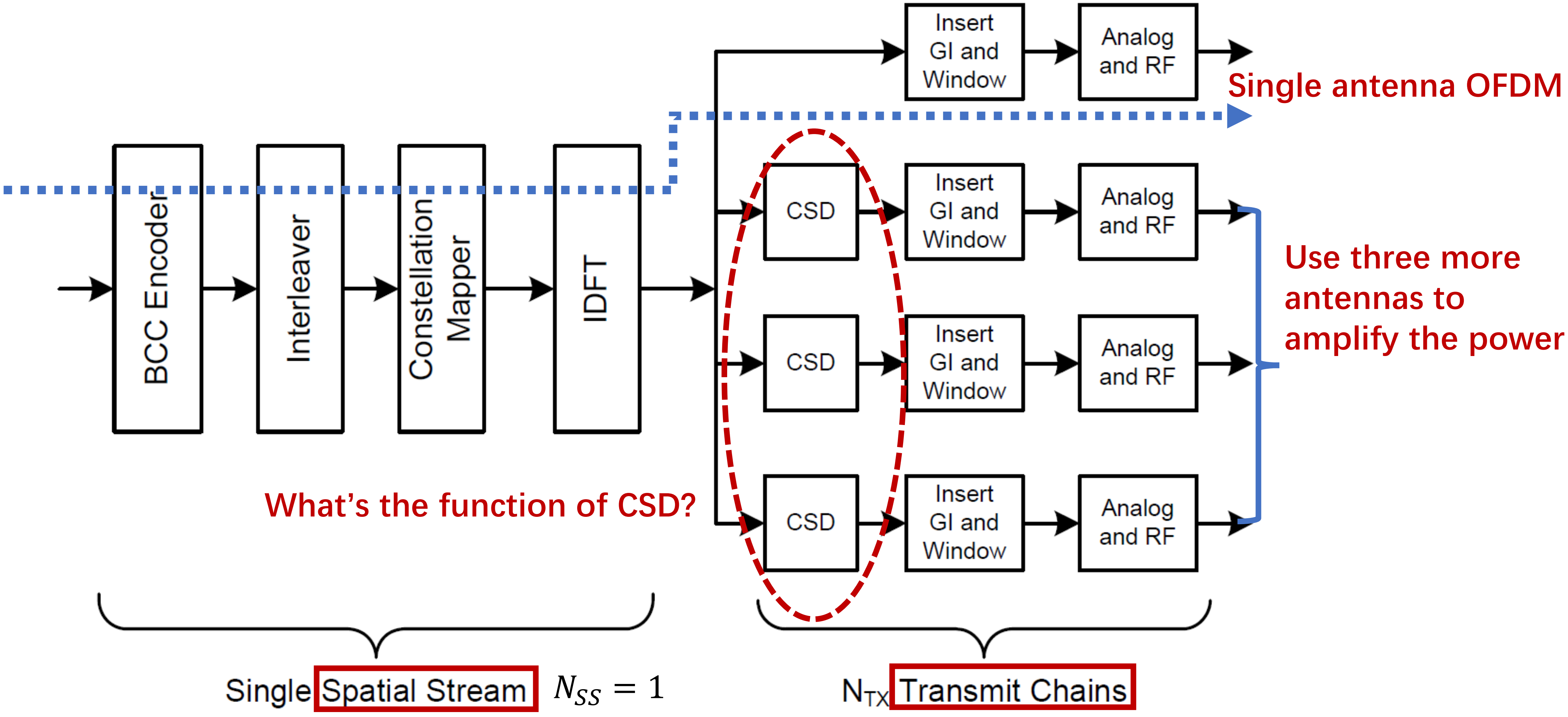
Cyclic Shift - Example

- Suppose non-HT mode, two antennas transmit the same signal, CSD happens after IFFT
- Suppose cyclic shift is 200ns, which include 4 samples

Tx Antenna 1: S1, S2, S3, S4, S5, ⋯, S64

Tx Antenna 2: S61, S62, S63, S64, S1, ⋯, S60

Recap: Single Spatial Stream (19.3.3)



"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

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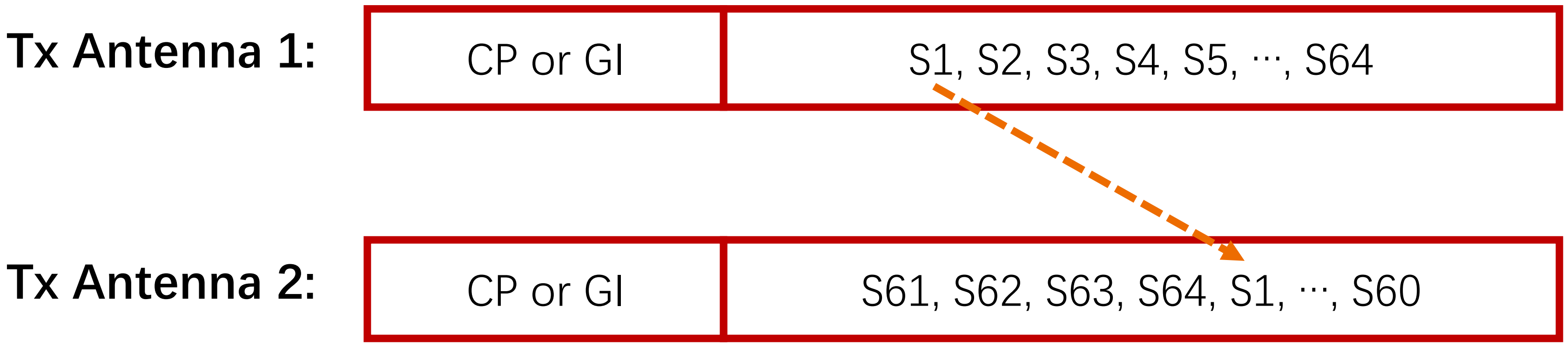

Analysis of Cyclic Shift

CP or GI	S1, S2, S3, S4, S5, ⋯, S64

Tx Antenna 1

Impulse response h_1 => after DFT $\{H_1^1, H_1^2, \dots, H_1^{64}\}$

Rx Antenna 1

Tx Antenna 2

Impulse response h_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

CP or GI	S61, S62, S63, S64, S1, ⋯, S60

Analysis of Cyclic Shift

$$\begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_{64} \end{bmatrix} \text{IDFT} \Rightarrow \begin{bmatrix} S_1 \\ S_2 \\ \dots \\ S_{64} \end{bmatrix}$$

Add CP => Convolution with I_1 => Remove CP => DFT =>

$$\begin{bmatrix} H_1^1 D_1 \\ H_1^2 D_2 \\ \dots \\ H_1^{64} D_{64} \end{bmatrix}$$

Tx Antenna 1

Impulse response I_1 => after DFT $\{H_1^1, H_1^2, \dots, H_1^{64}\}$

Rx Antenna 1

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Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

$$[?]\ \text{IDFT} \Rightarrow \begin{bmatrix} S_{61} \\ S_{62} \\ \dots \\ S_{60} \end{bmatrix}$$

Analysis of Cyclic Shift

Tx Antenna 1

$$D_k = \sum_{n=1}^{64} S_n e^{-\frac{2\pi j}{64}kn} = S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}$$

Tx Antenna 2

$$S_{61} e^{-\frac{2\pi j}{64}k} + S_{62} e^{-\frac{2\pi j}{64}2k} + \cdots + S_1 e^{-\frac{2\pi j}{64}5k} + S_2 e^{-\frac{2\pi j}{64}6k} + \cdots + S_{60}\, e^{-\frac{2\pi j}{64}64k}$$

$$= \left[S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}\right] e^{-\frac{2\pi j}{64}4k}$$

$$= D_k e^{-\frac{2\pi j}{64}4k}$$

Analysis of Cyclic Shift

$$\begin{bmatrix} D_1 \\ D_2 \\ \cdots \\ D_{64} \end{bmatrix} \text{IDFT} \Rightarrow \begin{bmatrix} S_1 \\ S_2 \\ \cdots \\ S_{64} \end{bmatrix} \quad \text{Add CP} \Rightarrow \text{Convolution with } I_1 \Rightarrow \text{Remove CP} \Rightarrow \text{DFT} \Rightarrow \begin{bmatrix} H_1^1 D_1 \\ H_1^2 D_2 \\ \cdots \\ H_1^{64} D_{64} \end{bmatrix}$$

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Rx Antenna 1

Tx Antenna 2

Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

$$\begin{bmatrix} D_1 e^{-\frac{2\pi j}{64}4} \\ D_2 e^{-\frac{2\pi j}{64}4\times 2} \\ \cdots \\ D_{64} e^{-\frac{2\pi j}{64}4\times 64} \end{bmatrix} \text{IDFT} \Rightarrow \begin{bmatrix} S_{61} \\ S_{62} \\ \cdots \\ S_{60} \end{bmatrix} \quad \text{Add CP} \Rightarrow \text{Convolution with } I_1 \Rightarrow \text{Remove CP} \Rightarrow \text{DFT} \Rightarrow \begin{bmatrix} H_2^1 e^{-\frac{2\pi j}{64}4} D_1 \\ H_2^2 e^{-\frac{2\pi j}{64}4\times 2} D_2 \\ \cdots \\ H_2^{64} e^{-\frac{2\pi j}{64}4\times 64} D_{64} \end{bmatrix}$$

Analysis of Cyclic Shift

$$\begin{bmatrix} (H_1^1+H_2^1 e^{-\frac{2\pi j}{64}4})D_1 \\ (H_1^2+H_2^2 e^{-\frac{2\pi j}{64}4\times 2})D_2 \\ \dots \\ (H_1^{64}+H_2^{64} e^{-\frac{2\pi j}{64}4\times 64})D_{64} \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_{64} \end{bmatrix} \text{IDFT} => \begin{bmatrix} S_1 \\ S_2 \\ \dots \\ S_{64} \end{bmatrix}$$

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Rx Antenna 1

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Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

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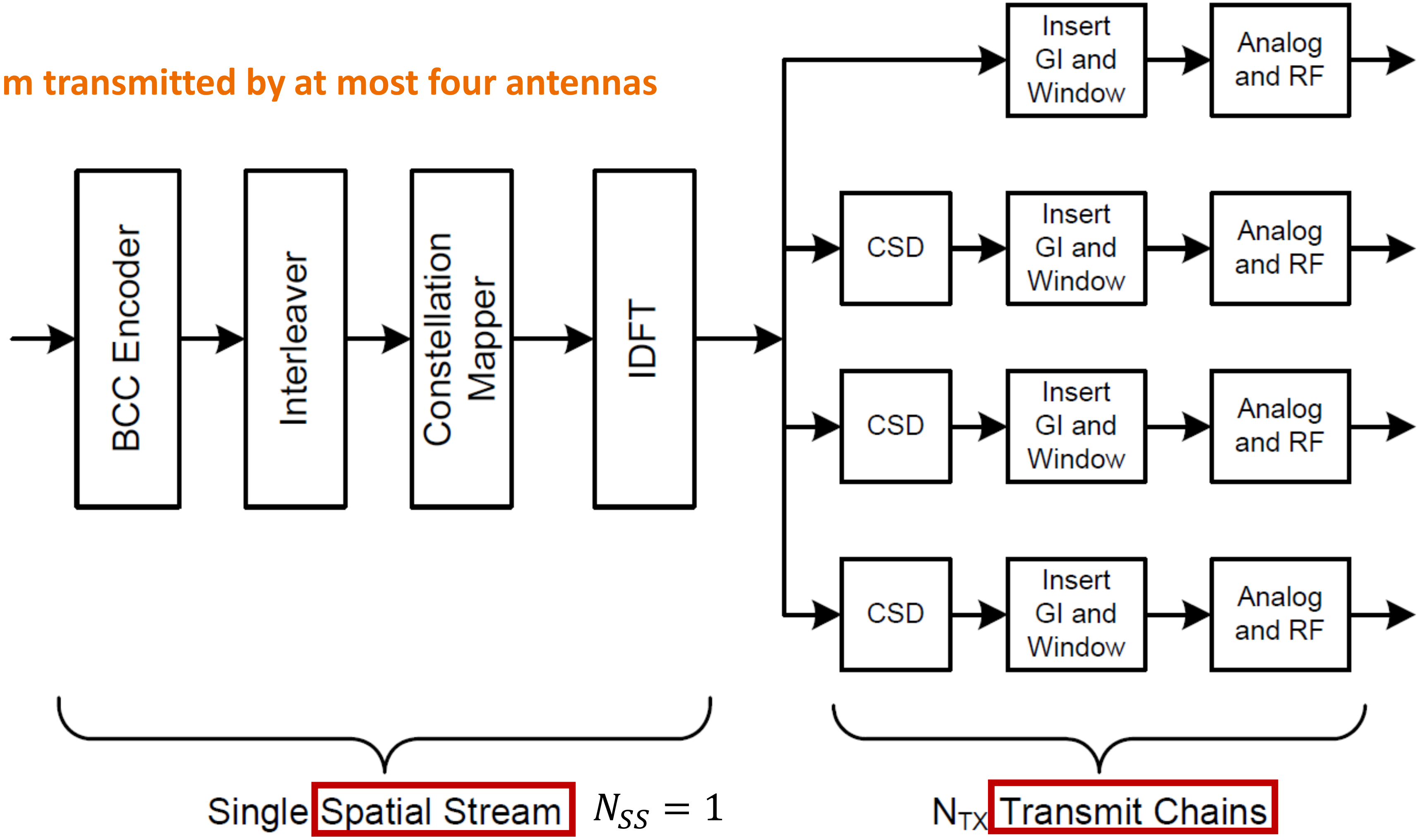

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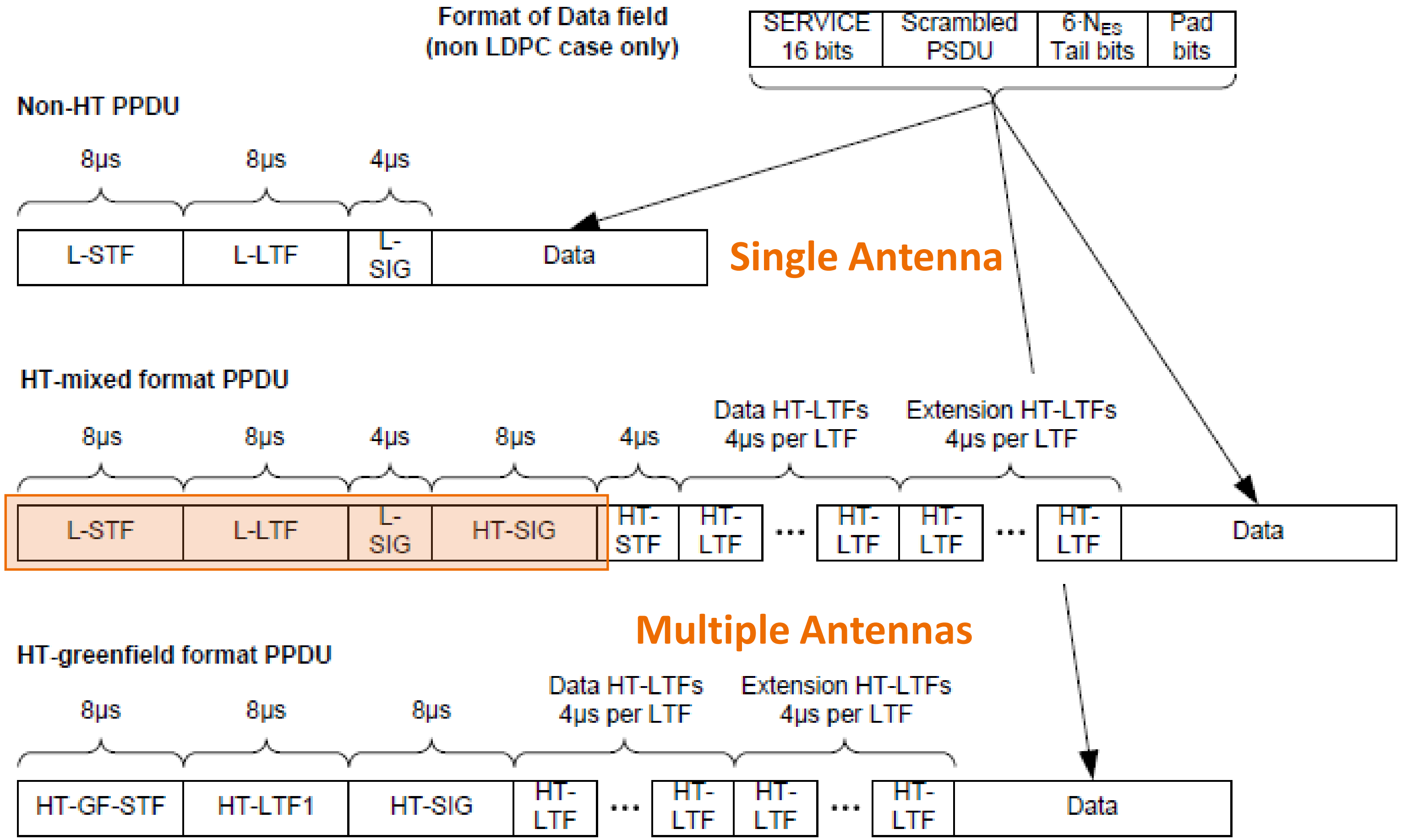

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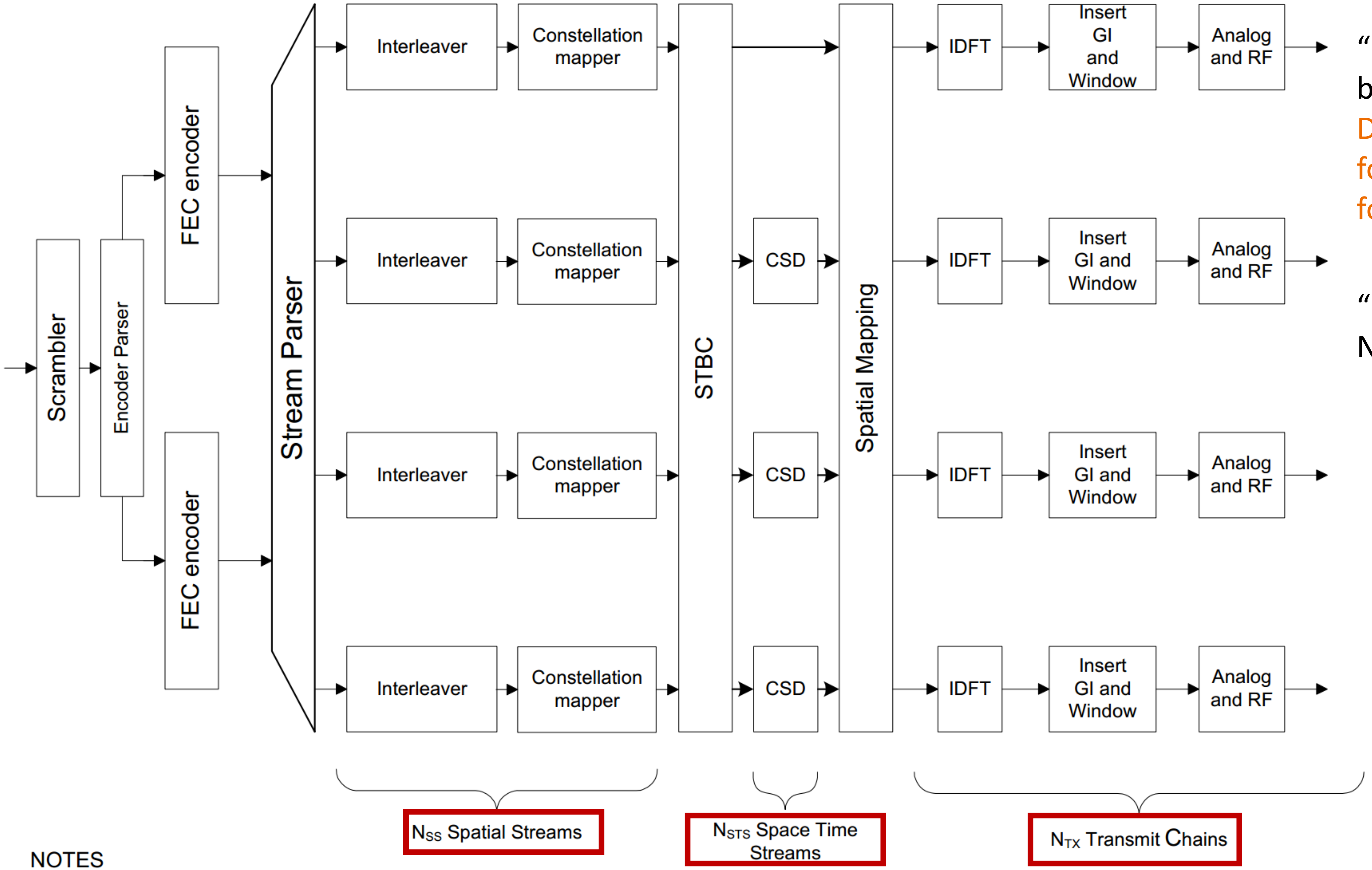
- L-SIG, HT-SIG: basic information about the PPDU

Multiple Spatial Stream (19.3.3)

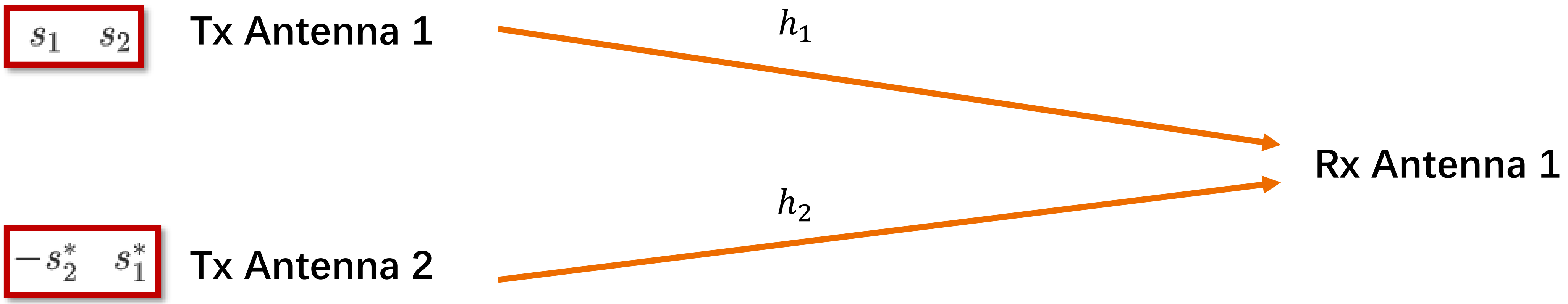


"…shows the transmitter blocks used to generate the Data field of the HT-mixed format and HT-greenfield format PPDUs."

" STBC is used only when $N_{STS}>N_{SS}$ "

NOTES

STBC Example: Alamouti Code



Diversity is increased to 2, less packet outage probability

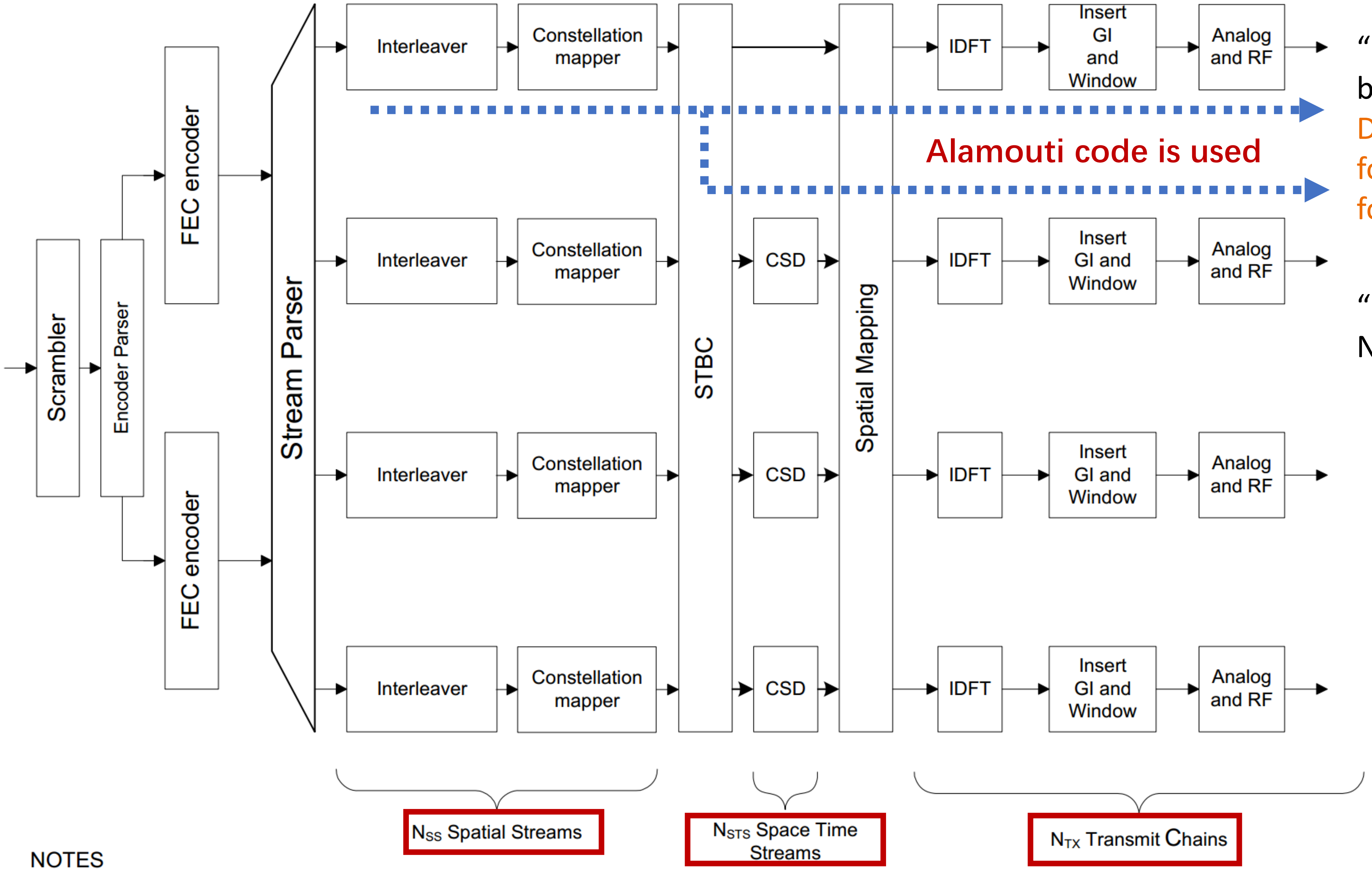
Require CSIR only, CSIT is not necessary

$$[y_1\ \ y_2] = [h_1\ h_2]\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} + [w_1\ w_2]$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & -h_{12} \\ h_{12}^* & h_{11}^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix}$$

$$\begin{cases} \hat{s}_1 = \frac{1}{|h_1|^2+|h_2|^2}(h_1^* y_1 + h_2 y_2^*) \\ \hat{s}_2 = \frac{1}{|h_1|^2+|h_2|^2}(-h_2 y_1^* + h_1^* y_2) \end{cases}$$

Multiple Spatial Stream (19.3.3)



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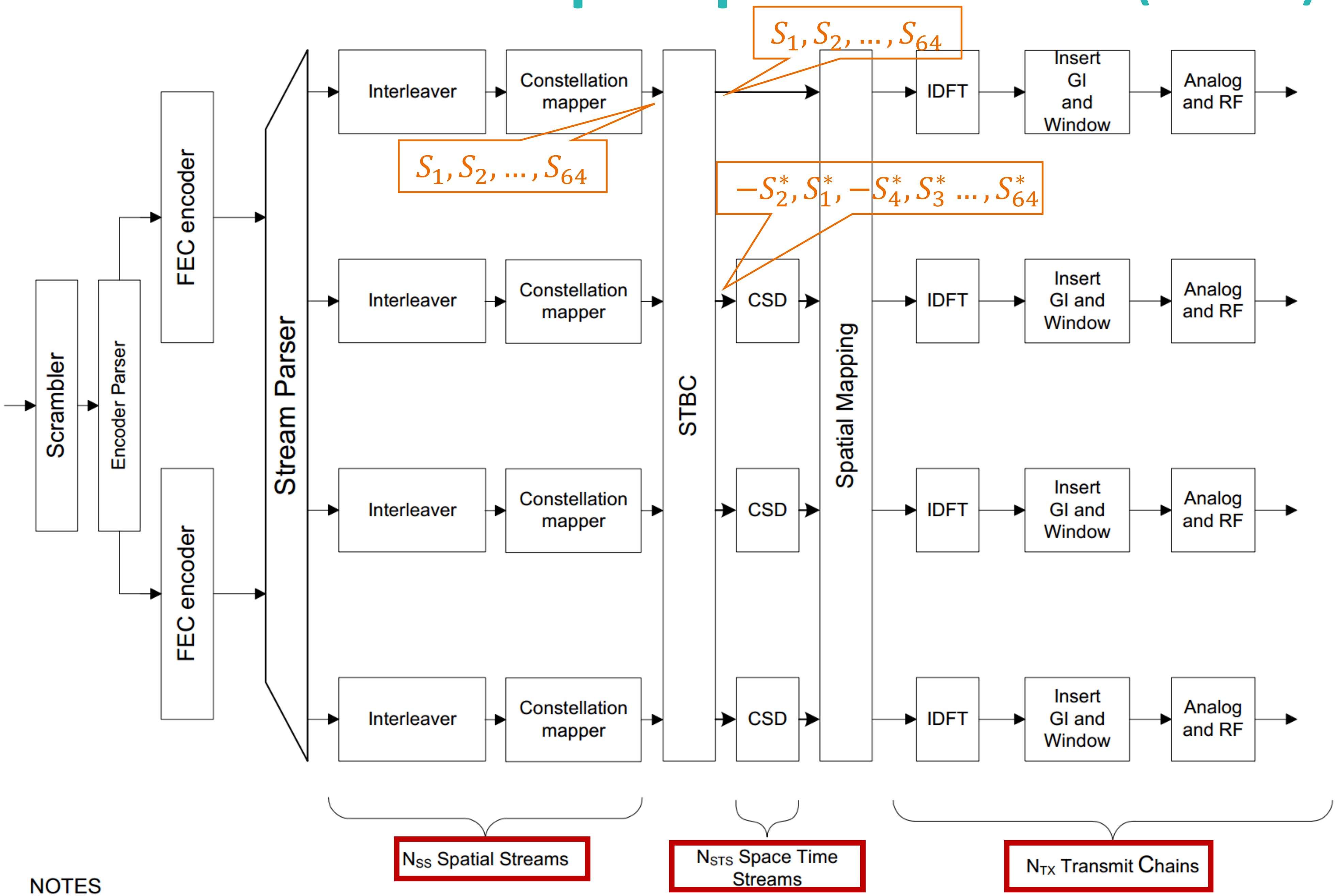

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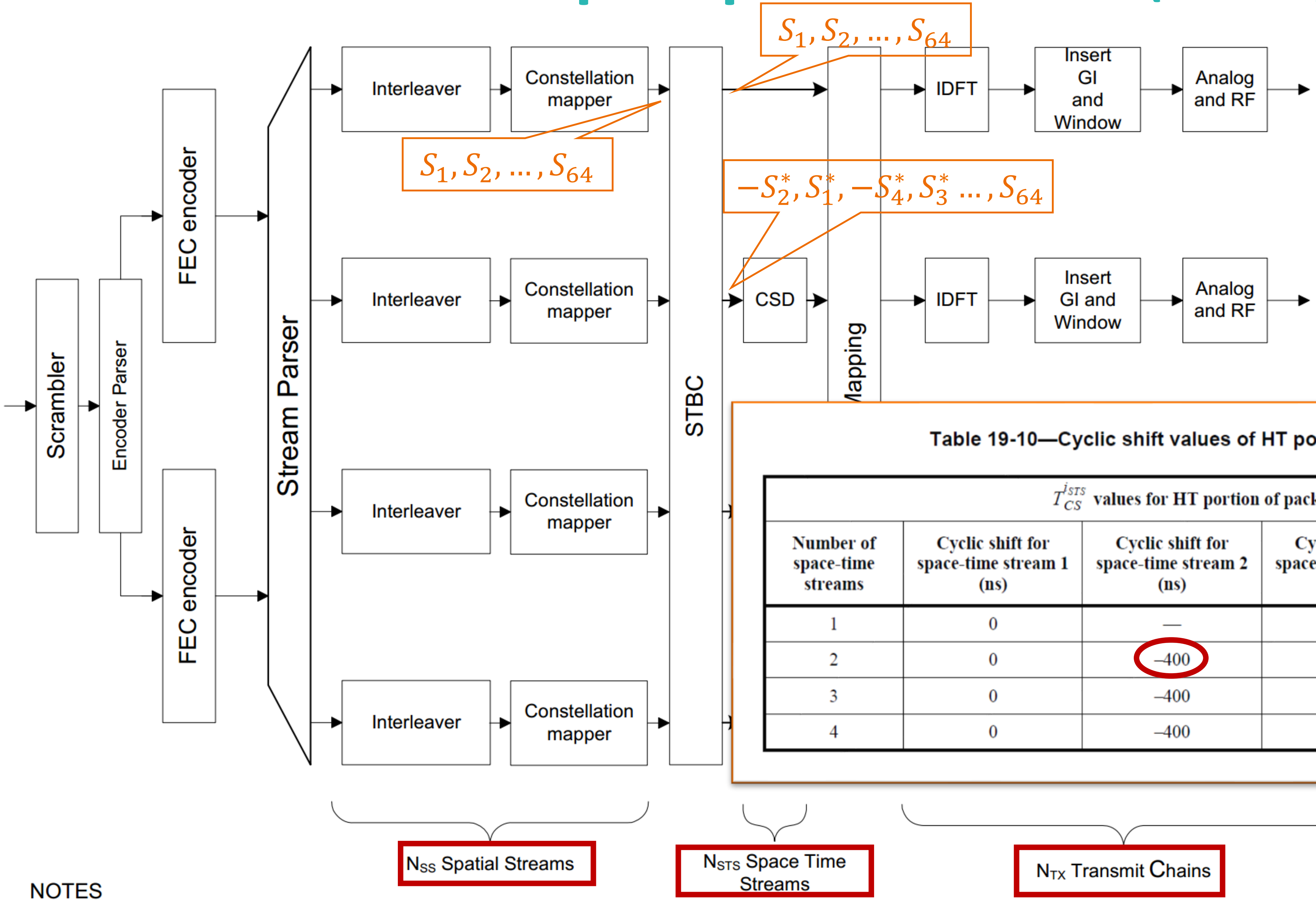
Multiple Spatial Stream (19.3.3)



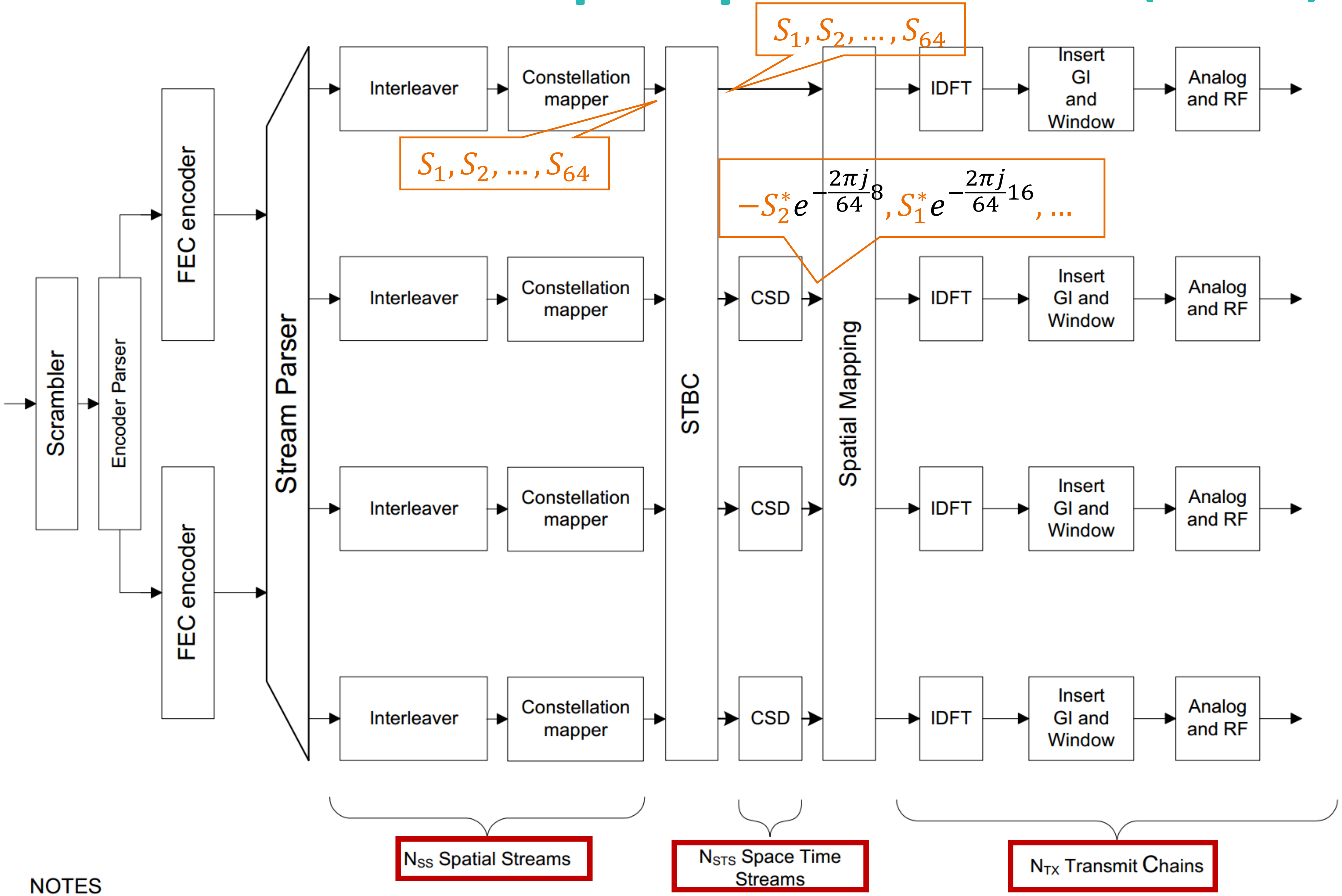
400ns = 8 samples

Table 19-10—Cyclic shift values of HT portion of packet

$T_{CS}^{i_{STS}}$ values for HT portion of packet				
Number of space-time streams	Cyclic shift for space-time stream 1 (ns)	Cyclic shift for space-time stream 2 (ns)	Cyclic shift for space-time stream 3 (ns)	Cyclic shift for space-time stream 4 (ns)
1	0	—	—	—
2	0	–400	—	—
3	0	–400	–200	—
4	0	–400	–200	–600

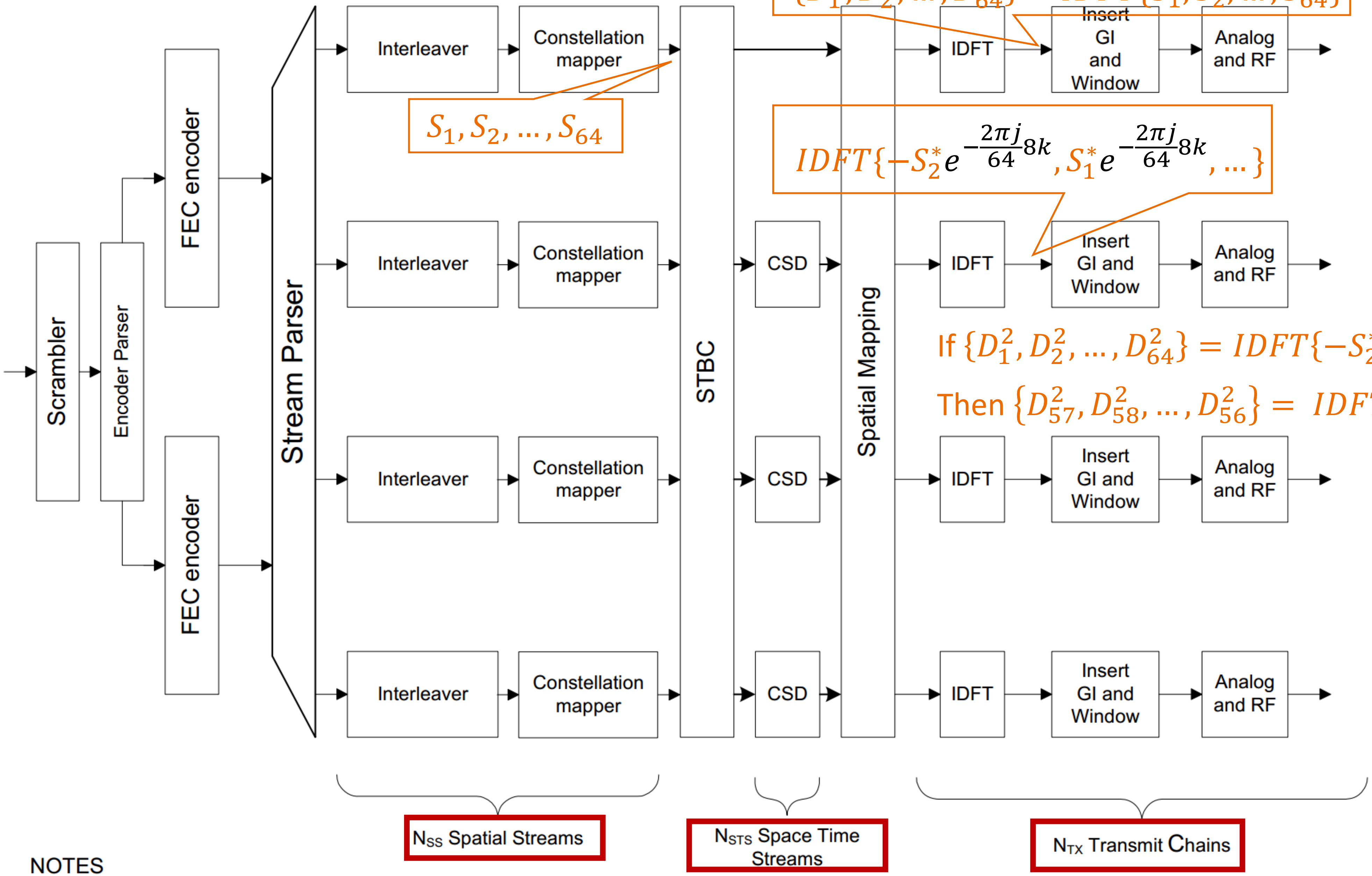
NOTES

Multiple Spatial Stream (19.3.3)



NOTES

Multiple Spatial Stream (19.3.3)



$\{D_1^1, D_2^1, \dots, D_{64}^1\} = IDFT\{S_1, S_2, \dots, S_{64}\}$

$S_1, S_2, \dots, S_{64}$

$IDFT\{-S_2^* e^{-\frac{2\pi j}{64}8k}, S_1^* e^{-\frac{2\pi j}{64}8k}, \dots\}$

If $\{D_1^2, D_2^2, \dots, D_{64}^2\} = IDFT\{-S_2^*, S_1^*, \dots\}$

Then $\{D_{57}^2, D_{58}^2, \dots, D_{56}^2\} = IDFT\{-S_2^* e^{-\frac{2\pi j}{64}8}, S_1^* e^{-\frac{2\pi j}{64}16}, \dots\}$

NOTES

Space-Time Block Code (19.3.11.9.2)

- Support 4 space-time block codes

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
2	0–7	1	1	1	$d_{k,1,2m}$ S_1	$d_{k,1,2m+1}$ S_2
				2	$-d^*_{k,1,2m+1}$ $-S_2^*$	$d^*_{k,1,2m}$ S_1^*
3	8–15, 33–38	2	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
4	8–15	2	2	1	$d_{k,1,2m}$ S_1	$d_{k,1,2m+1}$ S_2
				2	$-d^*_{k,1,2m+1}$ $-S_2^*$	$d^*_{k,1,2m}$ S_1^*
				3	$d_{k,2,2m}$ S_3	$d_{k,2,2m+1}$ S_4
				4	$-d^*_{k,2,2m+1}$ $-S_4^*$	$d^*_{k,2,2m}$ S_3^*

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
4	16–23, 39, 41, 43, 46, 48, 50	3	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
				4	$d_{k,3,2m}$	$d_{k,3,2m+1}$

NOTE—the '*' operator represents the complex conjugate.

Example: 4×2 STBC

$$\begin{pmatrix} y_1 & y_3 \\ y_2 & y_4 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{pmatrix} \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \\ s_3 & s_4 \\ -s_4^* & s_3^* \end{pmatrix} + \begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} h_{11} & -h_{12} & h_{13} & -h_{14} \\ h_{21} & -h_{22} & h_{23} & -h_{24} \\ h_{12}^* & h_{11}^* & h_{14}^* & h_{13}^* \\ h_{22}^* & h_{21}^* & h_{24}^* & h_{23}^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2^* \\ s_3 \\ s_4^* \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3^* \\ w_4^* \end{pmatrix}$$

At least **2 Rx antennas** are required

MIMO Detector

- Given $Y = HX + Z$ and (Y,H), how to estimate vector X?

- **Approach 1: Zero-forcing (ZF)**
  - $\hat{X} = H^{-1}Y = X + H^{-1}Z$

- **Approach 2: Minimum mean square error (MMSE)**
  - $\hat{X} = QY = QHX + QZ \implies Error: \Delta = \hat{X} - X = (QH - I)X + QZ$
  - $E\,[\Delta\Delta^H] = E[(QH - I)XX^H(QH - I)^H + QZZ^HQ^H]$
  - $E\,[\Delta\Delta^H] = \sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H$
  - Thus, we should $\min_Q trace[\sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H]$

- **Approach 3: Maximum likelihood (ML)**
  - The choices of X is finite
  - For each possible X, calculate vector distance $|Y - HX|$
  - Choose the X, which minimize the distance
  - Thus, $\hat{X} = \min_X |Y - HX|$

Reading & Assignment (3.16)

Reading

- IEEE Std 802.11™-2020: Section 19.3.9 – 19.3.11
- Reference Paper

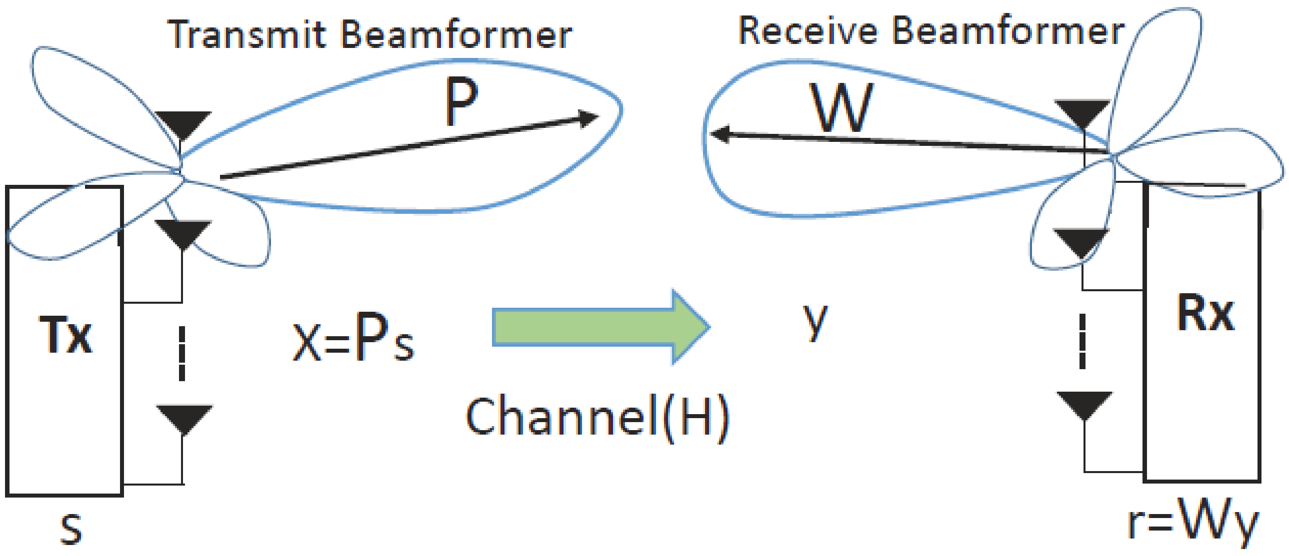
Assignment 3

Difference of MIMO Modes

- STBC requires CSIR only
- It can transmit single or multiple spatial streams
- Multiple streams with equal powers
- Single receiver

- MIMO beamforming requires both CSIR and CSIT
- It can transmit single or multiple spatial streams
- Multiple streams with power adaptation
- Support multiple receivers

MIMO Beamforming



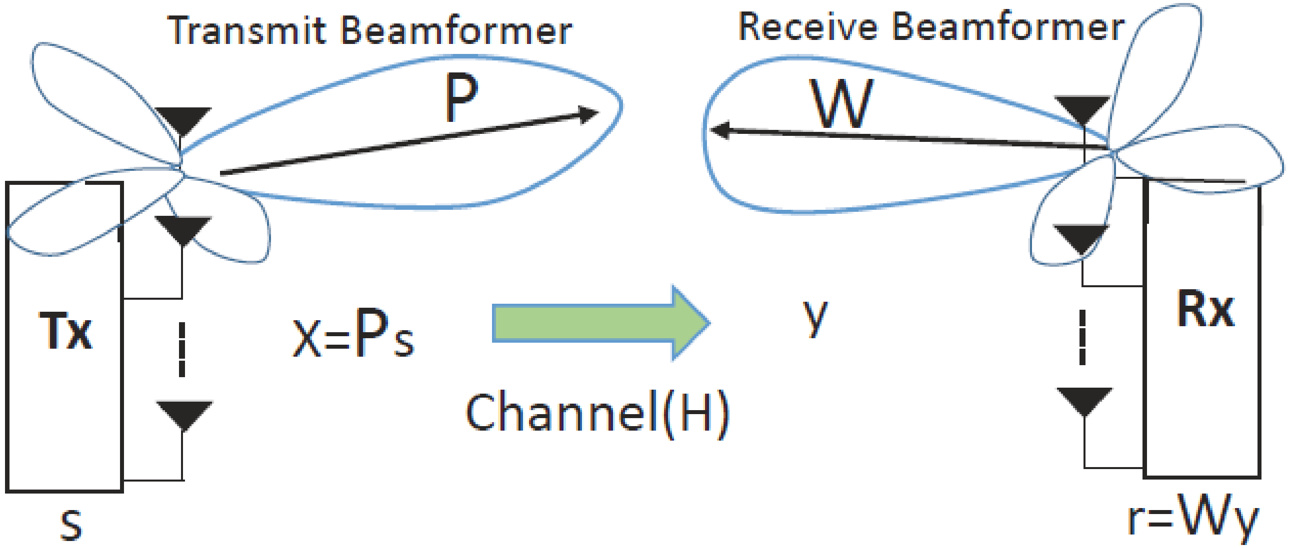
- The general model for a MIMO wireless communication is given as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}; \quad \mathbf{H} \in \mathbb{C}^{N_R \times N_T} \tag{1}$$

$$= \mathbf{HPs} + \mathbf{n} \tag{2}$$

- $\mathbf{x} = \mathbf{Ps}$ is the precoded and $\mathbf{r} = \mathbf{Wy}$ is the filtered output

MIMO Beamforming



$$\mathbf{r} = \mathbf{WHPs} + \mathbf{Wn}$$

A lot of choices of (W,P)
Typical one is SVD

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- $\mathbf{x} = \mathbf{Ps}$ is the precoded and $\mathbf{r} = \mathbf{Wy}$ is the filtered output

SU-MIMO: SVD Decomposition

- Consider singular value decomposition(**SVD**) of $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}\mathbf{\Sigma}\mathbf{V}^H \tag{3}$$

- where $\mathbf{\Sigma}$ is a diagonal matrix with entries σ_i (singular values)
- Choosing $\mathbf{P} = \mathbf{V}$ and $\mathbf{W} = \mathbf{U}^H$ in eq. (2) gives

$$\mathbf{r} = \mathbf{\Sigma}\,\mathbf{s} + \mathbf{U}^H\mathbf{n} \tag{4}$$

$$= \mathbf{\Sigma}\,\mathbf{s} + \tilde{\mathbf{n}} \tag{5}$$

- Then, one data stream per singular value can be transmitted as

$$\mathbf{r}_i = \sigma_i \mathbf{s}_i + \tilde{\mathbf{n}}_i;\;\; 1 \leq i \leq \min\{N_R, N_T\} \tag{6}$$

E. Telatar, "Capacity of multiantenna Gaussian channels, European Transactions on Telecommunications", 1999

SU-MIMO: Power Adaptation

Summation of Channel Capacity

SNR per Spatial Stream

$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

$$subject\ to\ \sum_i P_i \leq P$$

Total Transmission Power Constraint

SU-MIMO: Power Adaptation

$$E\,|\tilde{n}_i|^2 = E\,u_i^H n n^H u_i = \sigma_z^2\,u_i^H I u_i = \sigma_z^2$$

Summation of Channel Capacity

SNR per Spatial Stream

$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

$$subject\ to \sum_i P_i \le P$$

Total Transmission Power Constraint

SU-MIMO: Power Adaptation

$$E\,|\tilde{n}_i|^2$$
$$= E\, u_i^H n n^H u_i$$
$$= \sigma_z^2\, u_i^H I u_i$$
$$= \sigma_z^2$$

Summation of Channel Capacity

SNR per Spatial Stream

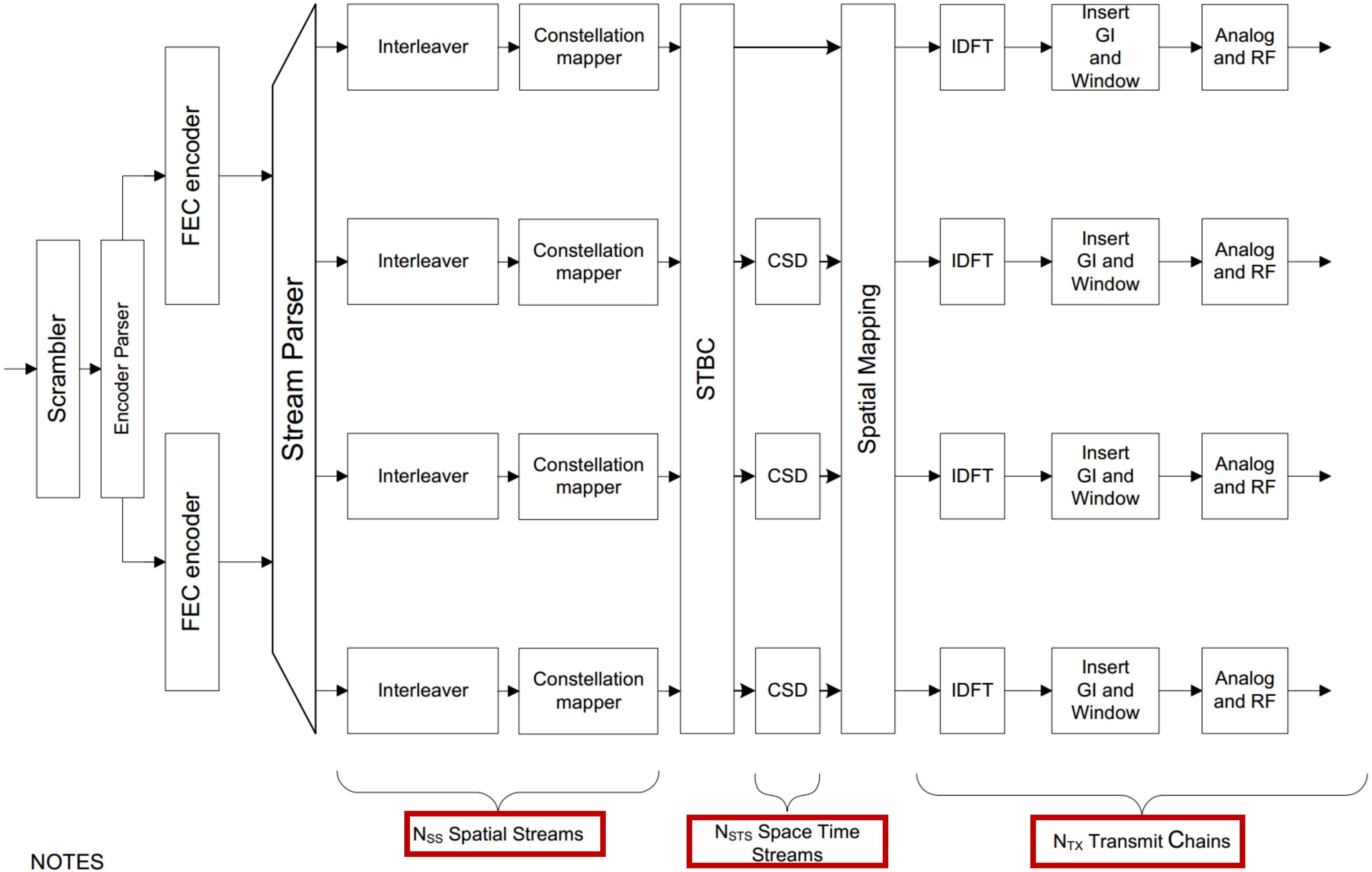
$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

$$subject\ to \sum_i P_i \le P$$

Total Transmission Power Constraint

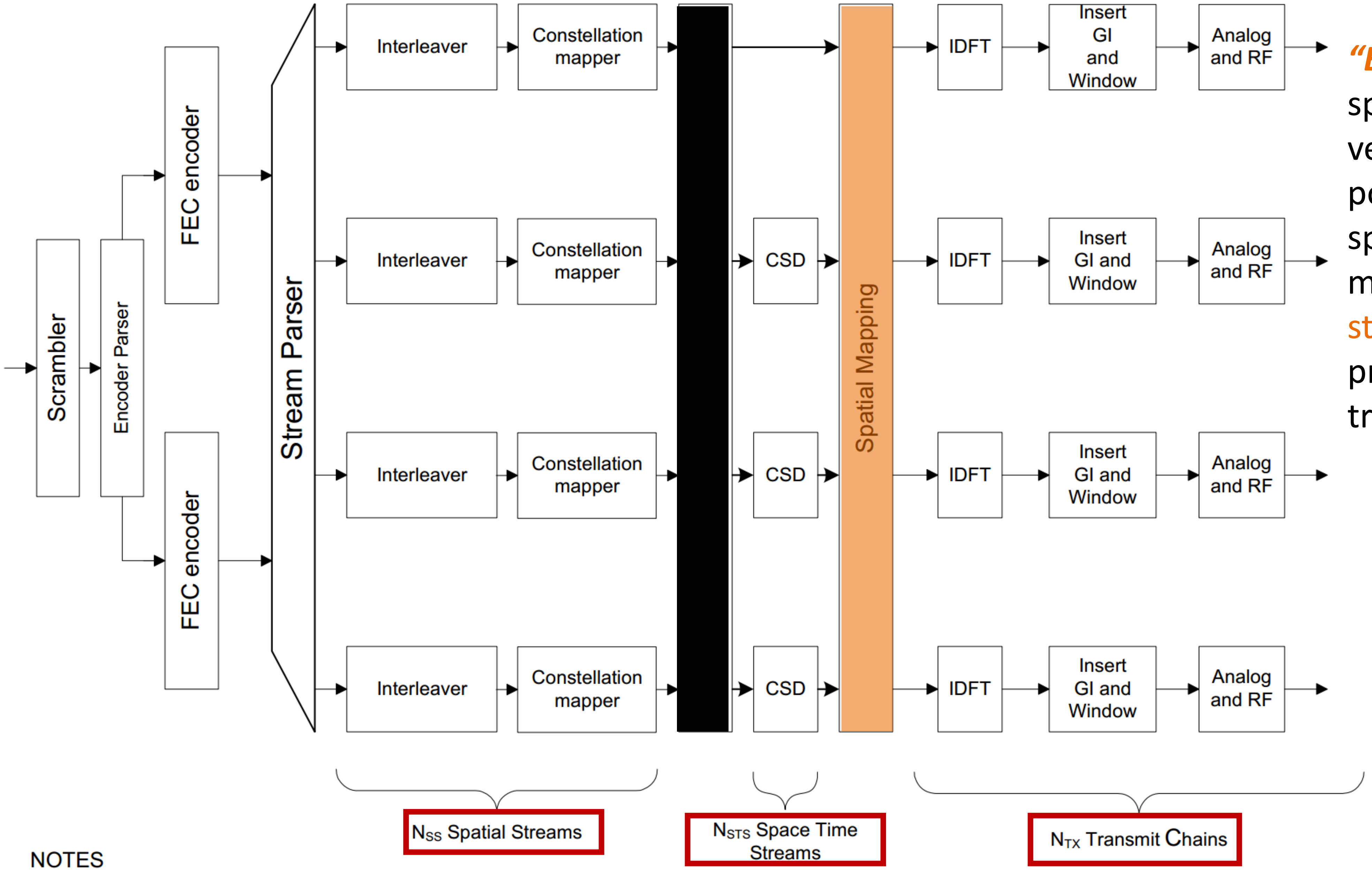
With water-filling algorithm, the total throughput can be maximized
STBC cannot do this

Recap: Multiple Spatial Stream (19.3.3)



NOTES

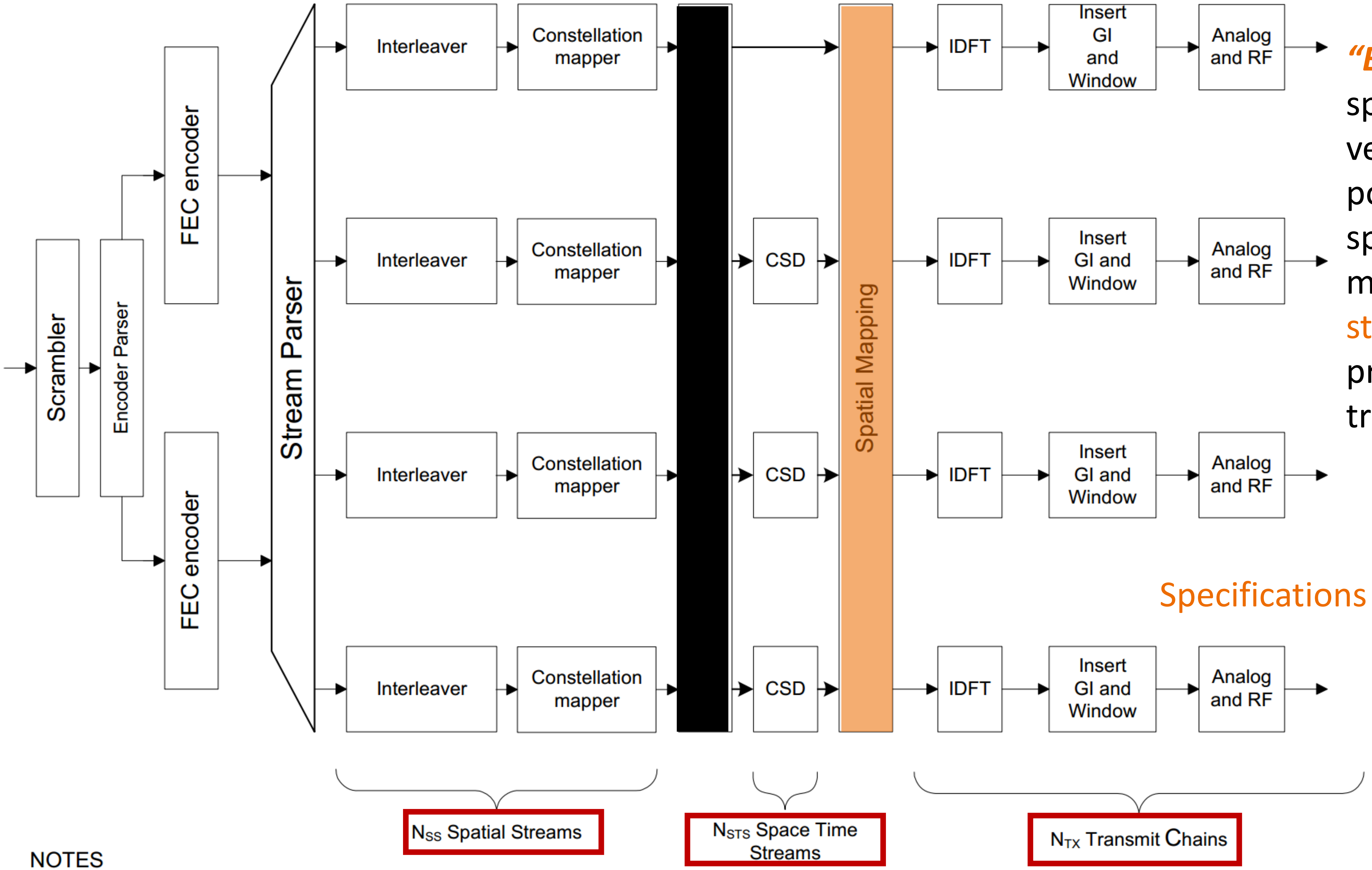
Recap: Multiple Spatial Stream (19.3.3)



"***Beamforming***: Similar to spatial expansion, each vector of constellation points from all of the space-time streams is multiplied by a matrix of steering vectors to produce the input to the transmit chains."

NOTES

Recap: Multiple Spatial Stream (19.3.3)



"Beamforming: Similar to spatial expansion, each vector of constellation points from all of the space-time streams is multiplied by a matrix of steering vectors to produce the input to the transmit chains."

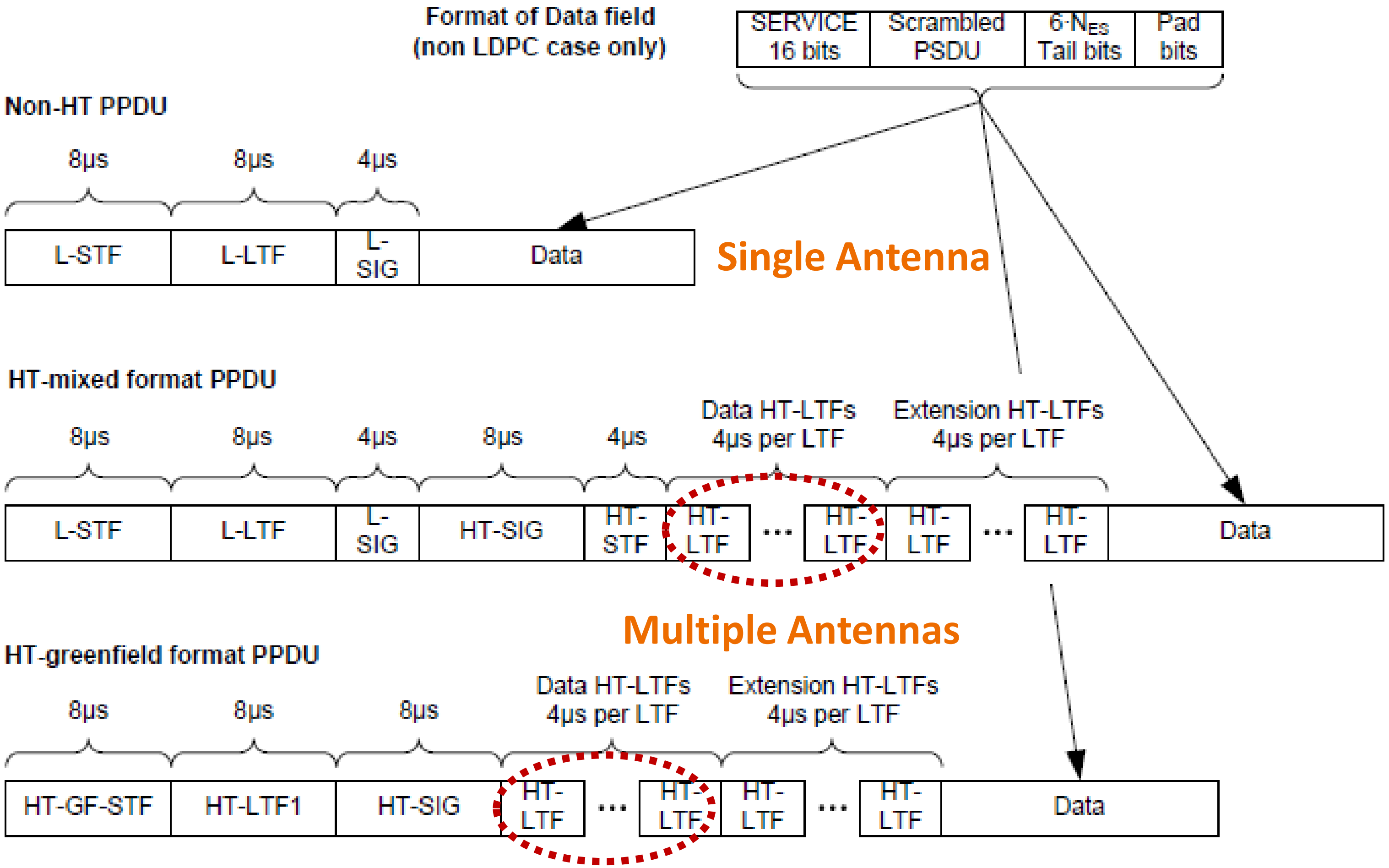
Specifications do not specify receiver!

NOTES

Remaining Questions

- How to estimate channel at the receiver?
- How to estimate channel at the transmitter?

Recap: Three PPDU Formats



- Non-HT： either the AP or the STA does not support HT
- HT-mixed: AP and STA support HT, however, other STAs may not support HT
- HT-greenfield: all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- L-SIG, HT-SIG: basic information about the PPDU

HT-LTF(19.3.9.4.6)

- "The HT-LTF provides a means for the receiver to estimate the MIMO channel between **the set of QAM mapper outputs** (or, if STBC is applied, the STBC encoder outputs) and **the receive chains**."
- "The HT-LTF portion has one or two parts. The first part consists of one, two, or four HT-LTFs that are **necessary for demodulation of the HT-Data portion** of the PPDU. These HT-LTFs are referred to as ***HT-DLTFs***."
- "The optional second part consists of zero, one, two, or four HT-LTFs that may be used to **sound extra spatial dimensions of the MIMO channel** that are not utilized by the HT-Data portion of the PPDU. These HT-LTFs are referred to as ***HT-ELTFs***."

$$N_{HT\text{-}LTF} = N_{HT\text{-}DLTF} + N_{HT\text{-}ELTF}$$

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \quad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain:

$$r_{HT-LTF}^{n, i_{TX}}(t) = \frac{1}{\sqrt{N_{STS} \cdot N_{HT-LTF}^{Tone}}} w_{T_{HT-LTFs}}(t) \quad (19\text{-}25)$$

$$\cdot \sum_{k=-N_{SR}}^{N_{SR}} \sum_{i_{STS}=1}^{N_{STS}} [Q_k]_{i_{TX}, i_{STS}} [P_{HT\text{-}LTF}]_{i_{STS}, n} \Upsilon_k HT\text{-}LTF_k \exp(j2\pi k \Delta_F (t - T_{GI} - T_{CS}^{i_{STS}}))$$

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \quad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain:

$$r_{HT-LTF}^{n, i_{TX}}(t) = \frac{1}{\sqrt{N_{STS} \cdot N_{HT-LTF}^{Tone}}} w_{T_{HT-LTFs}}(t) \quad (19\text{-}25)$$

$$\cdot \sum_{k=-N_{SR}}^{N_{SR}} \boxed{\sum_{i_{STS}=1}^{N_{STS}} [Q_k]_{i_{TX}, i_{STS}} [P_{HT\text{-}LTF}]_{i_{STS}, n}} \Upsilon_k HT\text{-}LTF_k \exp(j2\pi k \Delta_F (t - T_{GI} - T_{CS}^{i_{STS}}))$$

$\boldsymbol{Q_k}$ is the precoding matrix

$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \quad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain and k-th subcarrier:

$$[Q_k \, P_{HT-LTF}\,]_{i_{Tx},n} \, HT-LTF_k$$

Scalar

Generation of HT-LTF

$$\begin{aligned} HT\text{-}LTF_{-28,28} = \{&1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ &1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \end{aligned} \tag{19-23}$$

n-th HT-LTF on all the transmit chains and k-th subcarrier:

$$[Q_k \, P_{HT-LTF}]_n \, HT-LTF_k$$ **Column Vector**

Generation of HT-LTF

$$\begin{aligned} HT\text{-}LTF_{-28,28} = \{&1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ &1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \end{aligned} \quad (19\text{-}23)$$

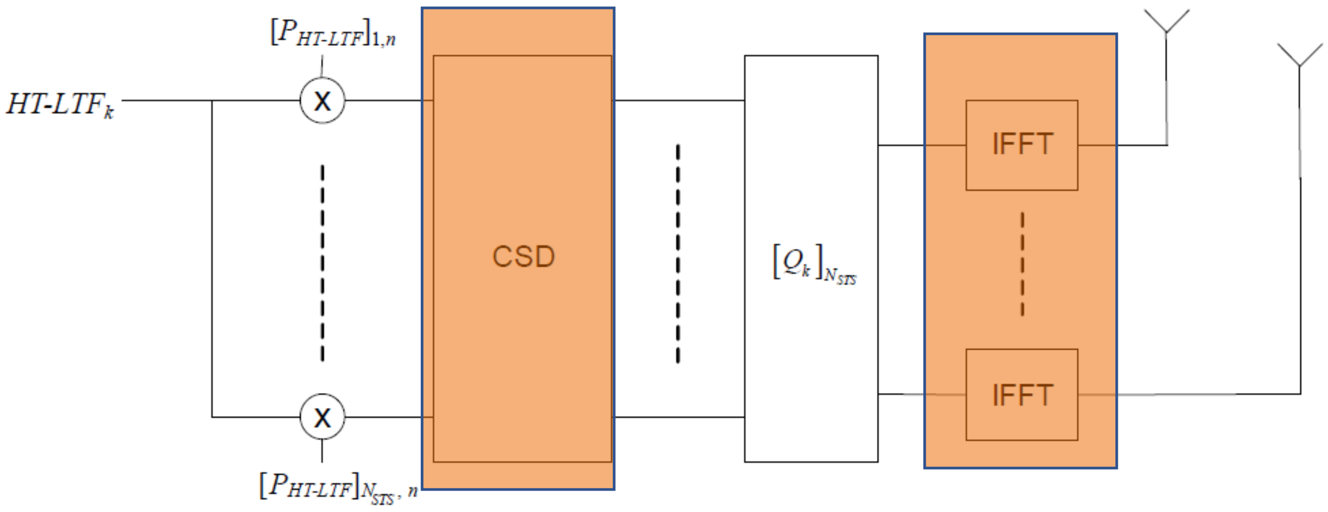
All 4 HT-LTFs on all the transmit chains and k-th subcarrier:

$$Q_k \, P_{HT-LTF} \, HT - LTF_k$$

Matrix

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:

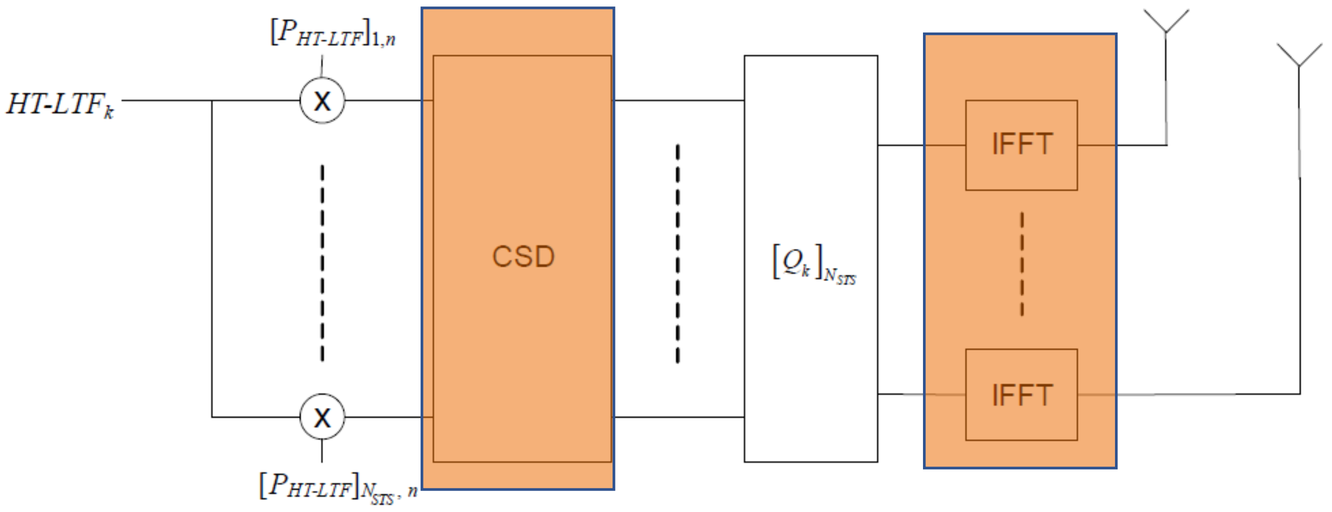


$$[\boldsymbol{Y}_{t_1}^{(k)}, \boldsymbol{Y}_{t_2}^{(k)}, \cdots \boldsymbol{Y}_{N_{LTF}}^{(k)}] =$$

$$\underbrace{\begin{bmatrix} \tilde{h}_{11}^{(k)} & \tilde{h}_{12}^{(k)} & \cdots & \tilde{h}_{1N_{SS}}^{(k)} \\ \tilde{h}_{21}^{(k)} & \tilde{h}_{22}^{(k)} & & \tilde{h}_{2N_{SS}}^{(k)} \\ & \vdots & \ddots & \vdots \\ \tilde{h}_{N_{RX}1}^{(k)} & \tilde{h}_{N_{RX}2}^{(k)} & \cdots & \tilde{h}_{N_{RX}N_{SS}}^{(k)} \end{bmatrix}}_{\boldsymbol{H_k Q_k}} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + [\boldsymbol{Z}_{t_1}^{(k)}, \boldsymbol{Z}_{t_2}^{(k)}, \cdots \boldsymbol{Z}_{N_{LTF}}^{(k)}].$$

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:



$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

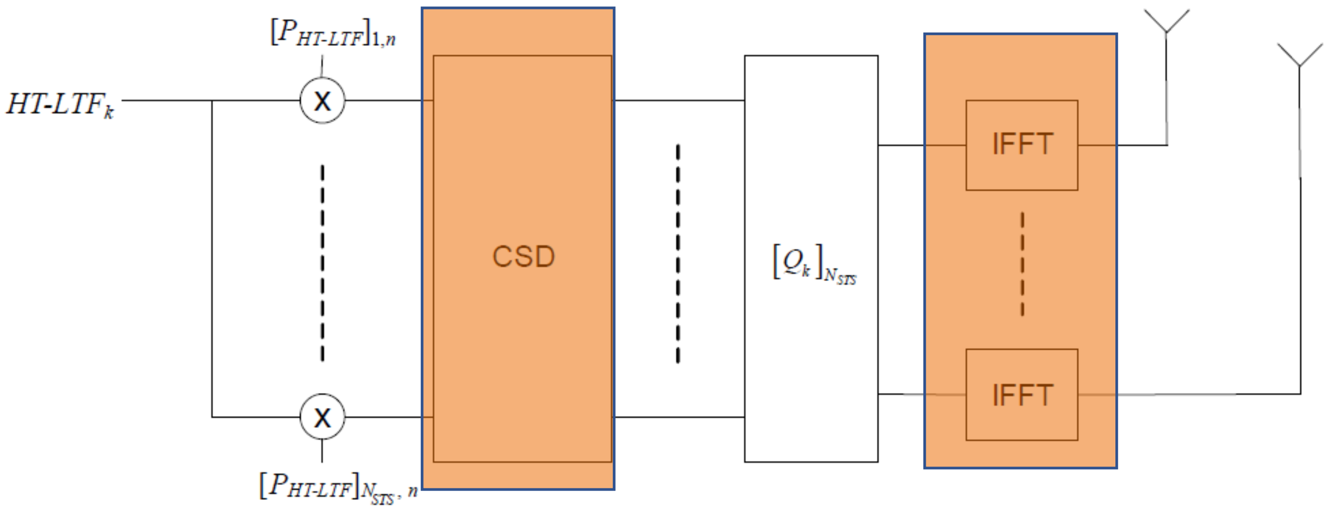
$$\boldsymbol{P}_{HTLTF}\boldsymbol{P}'_{HTLTF} = 4\boldsymbol{I} \Rightarrow \boldsymbol{P}_{HTLFT}^{-1} = \frac{1}{4}\boldsymbol{P}'_{HTLTF}$$

$$\left[\boldsymbol{Y}_{t_1}^{(k)}, \boldsymbol{Y}_{t_2}^{(k)}, \cdots \boldsymbol{Y}_{N_{LTF}}^{(k)}\right] =$$

$$\underbrace{\begin{bmatrix} \tilde{h}_{11}^{(k)} & \tilde{h}_{12}^{(k)} & \cdots & \tilde{h}_{1N_{SS}}^{(k)} \\ \tilde{h}_{21}^{(k)} & \tilde{h}_{22}^{(k)} & & \tilde{h}_{2N_{SS}}^{(k)} \\ & \vdots & \ddots & \vdots \\ \tilde{h}_{N_{RX}1}^{(k)} & \tilde{h}_{N_{RX}2}^{(k)} & \cdots & \tilde{h}_{N_{RX}N_{SS}}^{(k)} \end{bmatrix}}_{\boldsymbol{H}_k\boldsymbol{Q}_k} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + \left[\boldsymbol{Z}_{t_1}^{(k)}, \boldsymbol{Z}_{t_2}^{(k)}, \cdots \boldsymbol{Z}_{N_{LTF}}^{(k)}\right].$$

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:



$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

$$\boldsymbol{P}_{HTLTF}\boldsymbol{P}'_{HTLTF} = 4\boldsymbol{I} \Rightarrow \boldsymbol{P}^{-1}_{HTLFT} = \frac{1}{4}\boldsymbol{P}'_{HTLTF}$$

$$\left[\boldsymbol{Y}^{(k)}_{t_1}, \boldsymbol{Y}^{(k)}_{t_2}, \cdots \boldsymbol{Y}^{(k)}_{N_{LTF}}\right] = \underbrace{\begin{bmatrix} \tilde{h}^{(k)}_{11} & \tilde{h}^{(k)}_{12} & \cdots & \tilde{h}^{(k)}_{1N_{SS}} \\ \tilde{h}^{(k)}_{21} & \tilde{h}^{(k)}_{22} & & \tilde{h}^{(k)}_{2N_{SS}} \\ & \vdots & \ddots & \vdots \\ \tilde{h}^{(k)}_{N_{RX}1} & \tilde{h}^{(k)}_{N_{RX}2} & \cdots & \tilde{h}^{(k)}_{N_{RX}N_{SS}} \end{bmatrix}}_{\boldsymbol{H}_k\boldsymbol{Q}_k} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + \left[\boldsymbol{Z}^{(k)}_{t_1}, \boldsymbol{Z}^{(k)}_{t_2}, \cdots \boldsymbol{Z}^{(k)}_{N_{LTF}}\right].$$

- IEEE 802.11 does not define Q_k
- Transmitter determines it locally, but the receiver may not know

Channel Estimation

- $Y = HX + Z$, estimate H with the knowledge of X and Y

- **Approach 1: Zero-forcing**
  - $\hat{H} = YX^{-1} = H + ZX^{-1}$

- **Approach 2: MMSE**
  - $\hat{H} = YP = HXP + ZP \Longrightarrow \quad Error: \Delta = \hat{H} - H = H(XP - I) + ZP$
  - $E\,[\Delta^H \Delta] = E[(XP - I)^H H^H H(XP - I) + P^H Z^H ZP]$
  - $E\,[\Delta^H \Delta] = N\sigma^2 (XP - I)^H (XP - I) + N\sigma_z^2 P^H P$
  - Thus, we should $\min_{P} trace[\, N\sigma^2 (XP - I)^H (XP - I) + N\sigma_z^2 P^H P]$

$$\min_P trace[\, N\sigma^2 (XP - I)^H (XP - I) + N\sigma_z^2 P^H P]$$
$$= \min_P trace[\, P^H (N\sigma^2 X^H X + N\sigma_z^2 I)P - N\sigma^2 P^H X^H - N\sigma^2 XP + N\sigma^2 I]$$

$\mathbf{A}$ is not a function of $\mathbf{X}$	[7] $\frac{\partial \operatorname{tr}(\mathbf{AX})}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{XA})}{\partial \mathbf{X}} =$	$\mathbf{A}$	$\mathbf{A}^\top$
$\mathbf{A}$ is not a function of $\mathbf{X}$	[5] $\frac{\partial \operatorname{tr}(\mathbf{AX}^\top)}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{X}^\top\mathbf{A})}{\partial \mathbf{X}} =$	$\mathbf{A}^\top$	$\mathbf{A}$
$\mathbf{A}$ is not a function of $\mathbf{X}$	[5] $\frac{\partial \operatorname{tr}(\mathbf{X}^\top\mathbf{AX})}{\partial \mathbf{X}} =$	$\mathbf{X}^\top(\mathbf{A}+\mathbf{A}^\top)$	$(\mathbf{A}+\mathbf{A}^\top)\mathbf{X}$
$\mathbf{A}$ is not a function of $\mathbf{X}$	[5] $\frac{\partial \operatorname{tr}(\mathbf{X}^{-1}\mathbf{A})}{\partial \mathbf{X}} =$	$-\mathbf{X}^{-1}\mathbf{A}\mathbf{X}^{-1}$	$-(\mathbf{X}^{-1})^\top\mathbf{A}^\top(\mathbf{X}^{-1})^\top$
$\mathbf{A}$, $\mathbf{B}$ are not functions of $\mathbf{X}$	$\frac{\partial \operatorname{tr}(\mathbf{AXB})}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{BAX})}{\partial \mathbf{X}} =$	$\mathbf{BA}$	$\mathbf{A}^\top\mathbf{B}^\top$
$\mathbf{A}$, $\mathbf{B}$, $\mathbf{C}$ are not functions of $\mathbf{X}$	$\frac{\partial \operatorname{tr}(\mathbf{AXBX}^\top\mathbf{C})}{\partial \mathbf{X}} =$	$\mathbf{BX}^\top\mathbf{CA}+\mathbf{B}^\top\mathbf{X}^\top\mathbf{A}^\top\mathbf{C}^\top$	$\mathbf{A}^\top\mathbf{C}^\top\mathbf{XB}^\top+\mathbf{CAXB}$

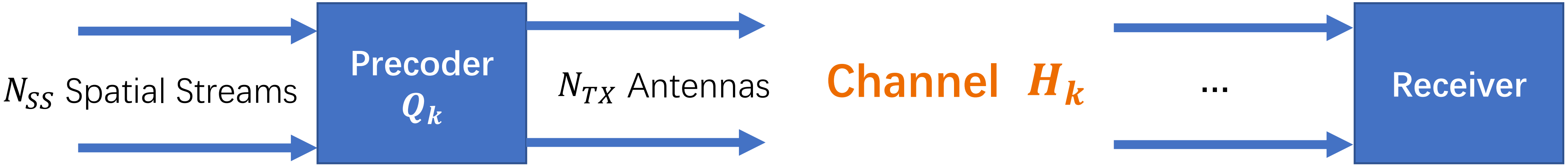
Applying derivative w.r.t. P, we can obtain the optimal P for channel estimation

Can we use the ML approach to estimate the channel?

D. P. Palomar and Y. Jiang, “MIMO Transceiver Design via Majorization Theory”, 2006

Explicit feedback beamforming (19.3.12.3.1)

- "In explicit beamforming, in order for STA A to transmit a beamformed packet to STA B, STA B measures the channel matrices and sends STA A either **the effective channel** or the **beamforming feedback matrix**"

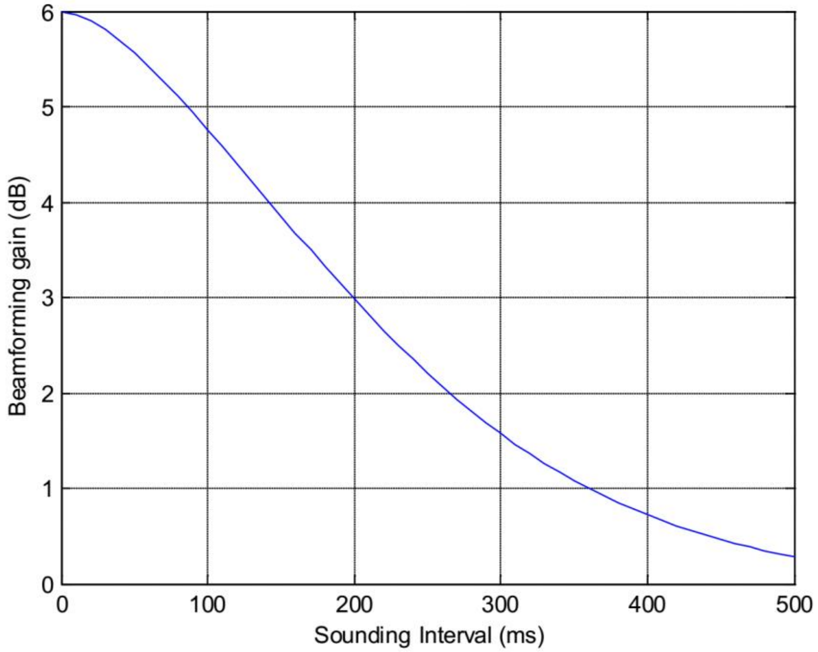


- Receiver can estimate $H_{eff,k} = H_k Q_k$ and
  - Feedback $H_{eff,k}$ directly to the transmitter, the transmitter calculates the new precoder Q_k
  - Or feedback an update matrix V_k, the transmitter use $Q_k V_k$ as the new precoder matrix

Encoding of Feedback Matrix（19.3.12.3.3）

- The channel feedback consists of the following three parts:
  - Real part of each element in the channel matrix
  - Imaginary part of each element in the channel matrix
  - A common scaling factor
- Beamforming feedback matrix: noncompressed and compressed

Implementation – Sounding Frequency

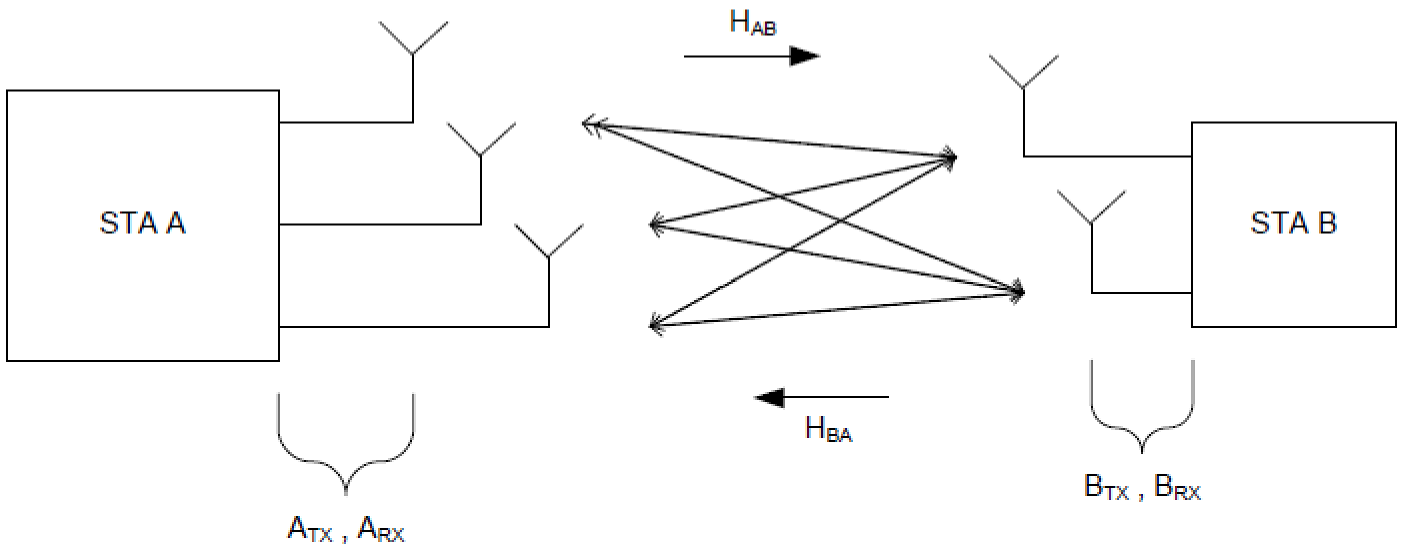


- Feedback can incur significant overhead

- Tradeoff between sounding frequency and beamforming gain
  - Indoor channel environment is benign

Implicit Feedback Beamforming (19.3.12.2)

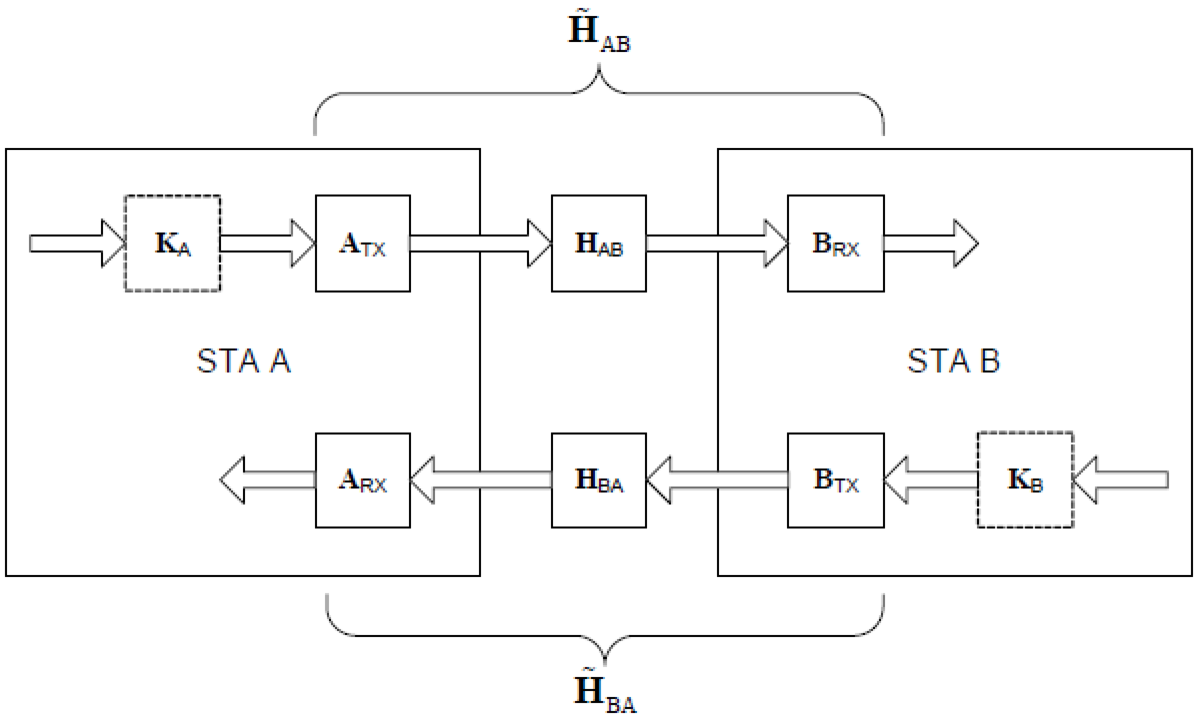
- Transmitter relies on reciprocity in the time division duplex channel to estimate the channel
- The forward and reverse channels are reciprocal, **the channel from STA A to STA B** in subcarrier k is the **matrix transpose** of **the channel from STA B to STA A** in subcarrier k to within a complex scaling factor



$$H_{AB,\,k} = \rho\,[H_{BA,\,k}]^T .$$

"The amplitude and phase responses of the transmit and receive chains can be expressed as **diagonal matrices with complex valued diagonal entries**"

Calibration



Although the wireless channel is reciprocal, the baseband-to-baseband channel is not.

$$\left.\begin{aligned}\tilde{H}_{AB,k} &= B_{RX,k} H_{AB,k} A_{TX,k} \\ \tilde{H}_{BA,k} &= A_{RX,k} H_{BA,k} B_{TX,k}\end{aligned}\right\} \text{Not reciprocal}$$

Solution: choose

$$K_{A,k} = \alpha_{A,k} [A_{TX,k}]^{-1} A_{RX,k}$$

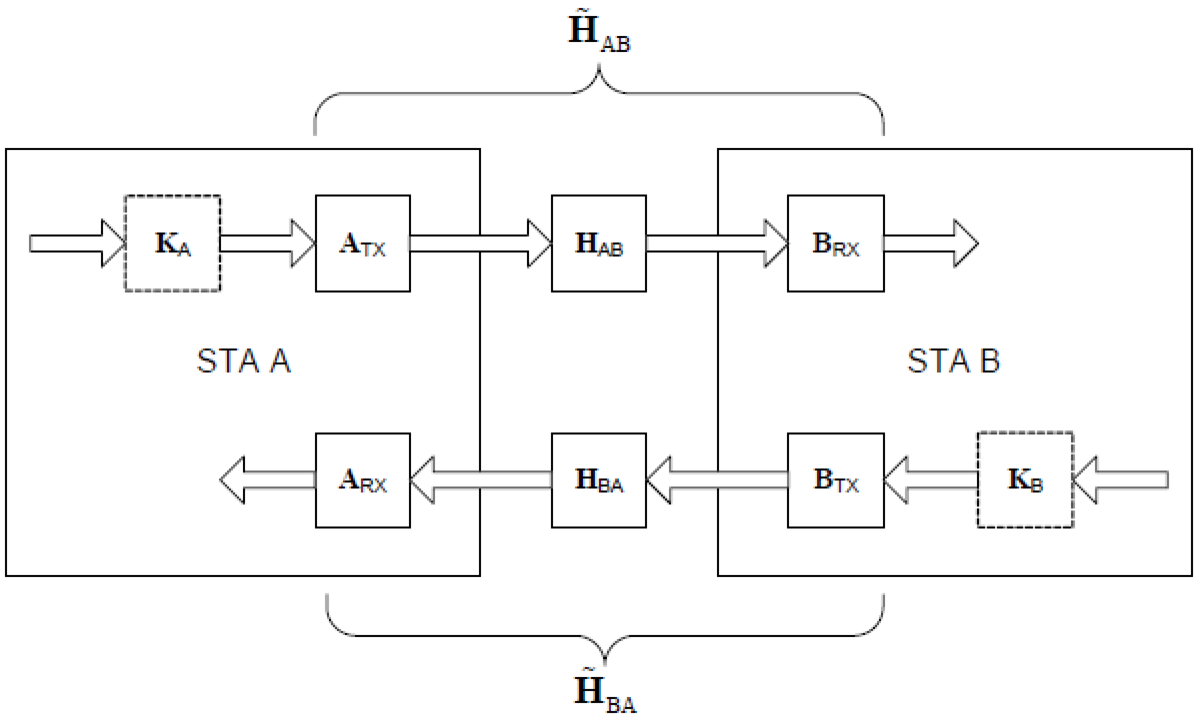
and

$$K_{B,k} = \alpha_{B,k} [B_{TX,k}]^{-1} B_{RX,k}$$

Such that

$$\tilde{H}_{AB,k} K_{A,k} = \rho [\tilde{H}_{BA,k} K_{B,k}]^{T}$$

Calibration



Although the wireless channel is reciprocal, the baseband-to-baseband channel is not.

Change frequently

$$\tilde{H}_{AB,k} = B_{RX,k} \boxed{H_{AB,k}} A_{TX,k}$$

$$\tilde{H}_{BA,k} = A_{RX,k} \boxed{H_{BA,k}} B_{TX,k}$$

Not reciprocal

Solution: choose

$$K_{A,k} = \alpha_{A,k}[A_{TX,k}]^{-1}A_{RX,k}$$

and

Change slowly

$$K_{B,k} = \alpha_{B,k}[B_{TX,k}]^{-1}B_{RX,k}$$

Such that

$$\tilde{H}_{AB,k}K_{A,k} = \rho[\tilde{H}_{BA,k}K_{B,k}]^{T}$$

How to calibrate

- Calibration procedure:
  - STA A sends STA B a sounding PPDU, the reception of which allows STA B to estimate the channel matrices $\widetilde{H}_{AB,k}$.
  - STA B sends STA A a sounding PPDU, the reception of which allows STA A to estimate the channel matrices $\widetilde{H}_{BA,k}$.
  - STA B sends the quantized estimation of $\widetilde{H}_{AB,k}$ to STA A.
  - STA A uses its local estimation of $\widetilde{H}_{BA,k}$ and the quantized estimation of $\widetilde{H}_{AB,k}$ received from STA B to compute the correction matrices

Homework (3.23)

Reading

IEEE Std 802.11™-2020 : Section 19.3.9, 19.3.12

Paper 1 – Chapter 3

Paper 2 *

Assignment 4