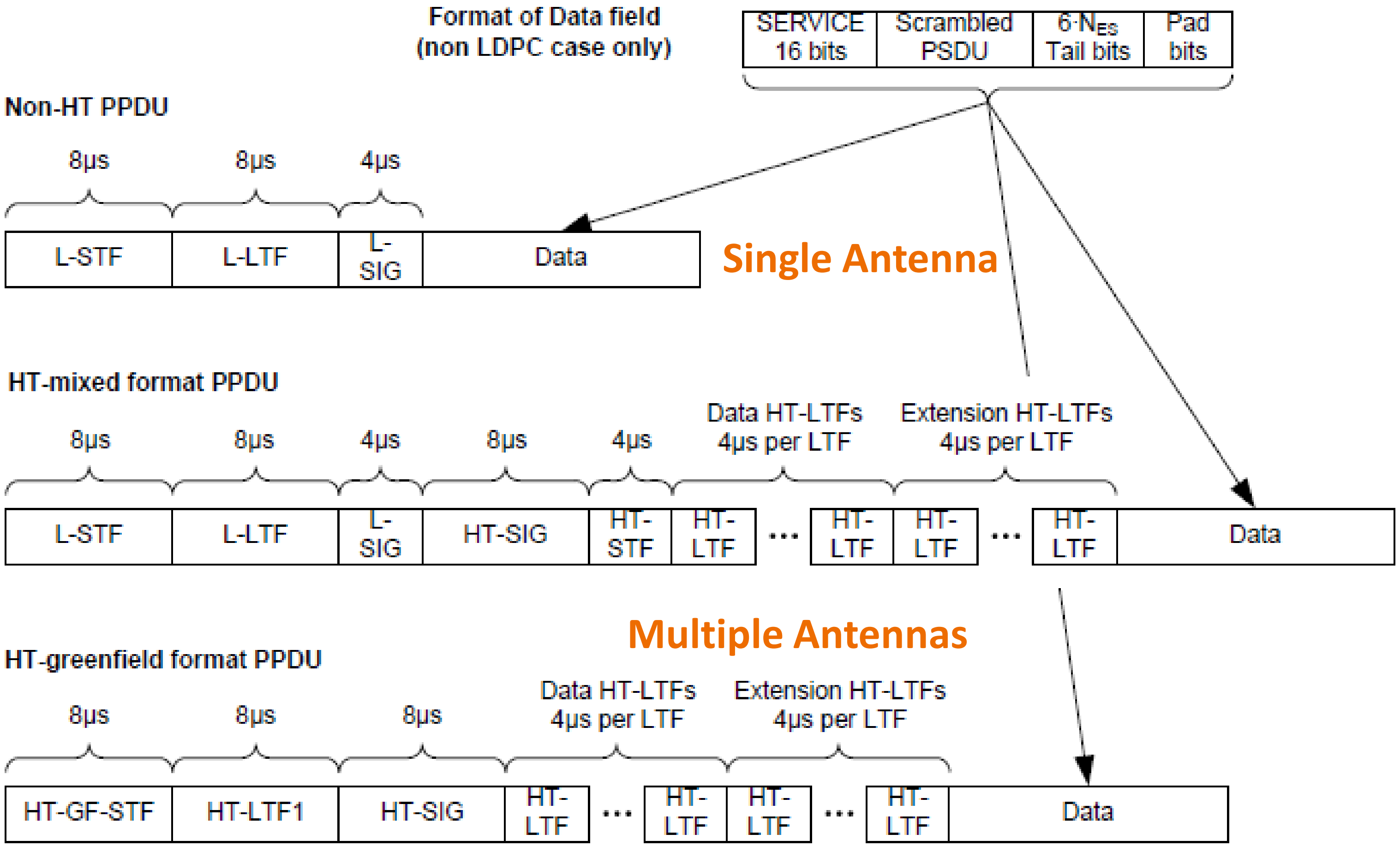
MIMO Transmitter in IEEE802.11

Lecturer: Dr. Rui Wang

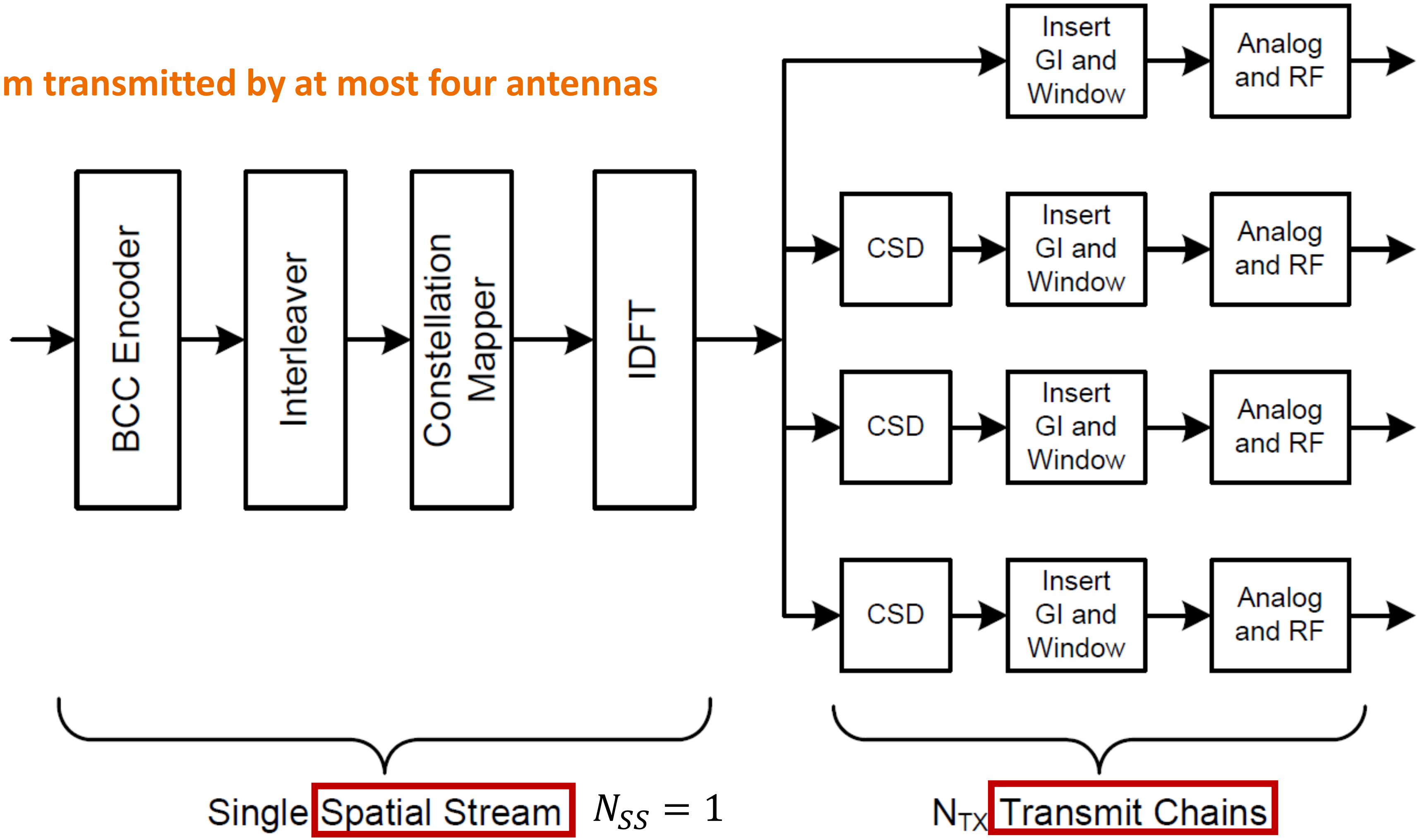
Recap: Three PPDU Formats



- Non-HT： either the AP or the STA does not support HT
- HT-mixed: AP and STA support HT, however, other STAs may not support HT
- HT-greenfield: all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- L-SIG, HT-SIG: basic information about the PPDU

Single Spatial Stream (19.3.3)

One data stream transmitted by at most four antennas



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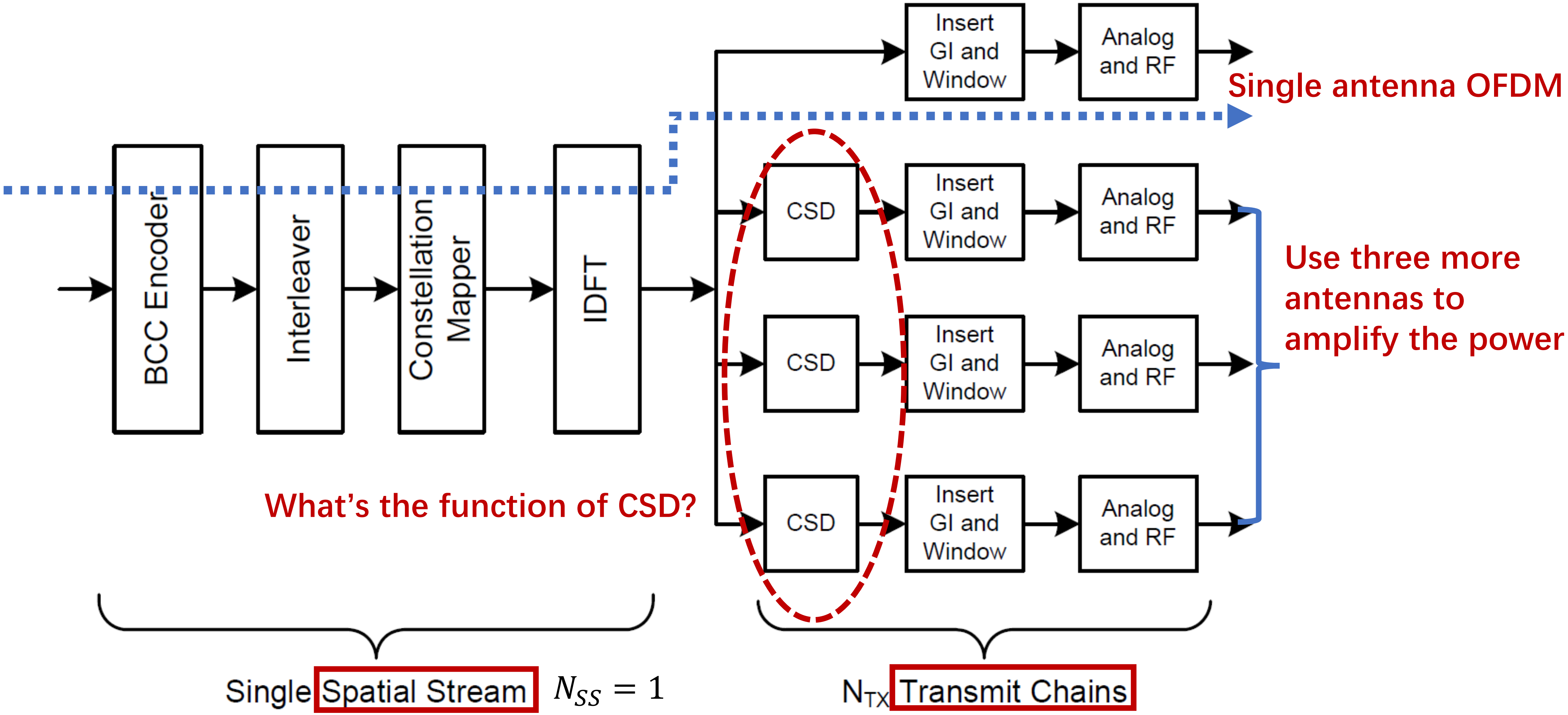

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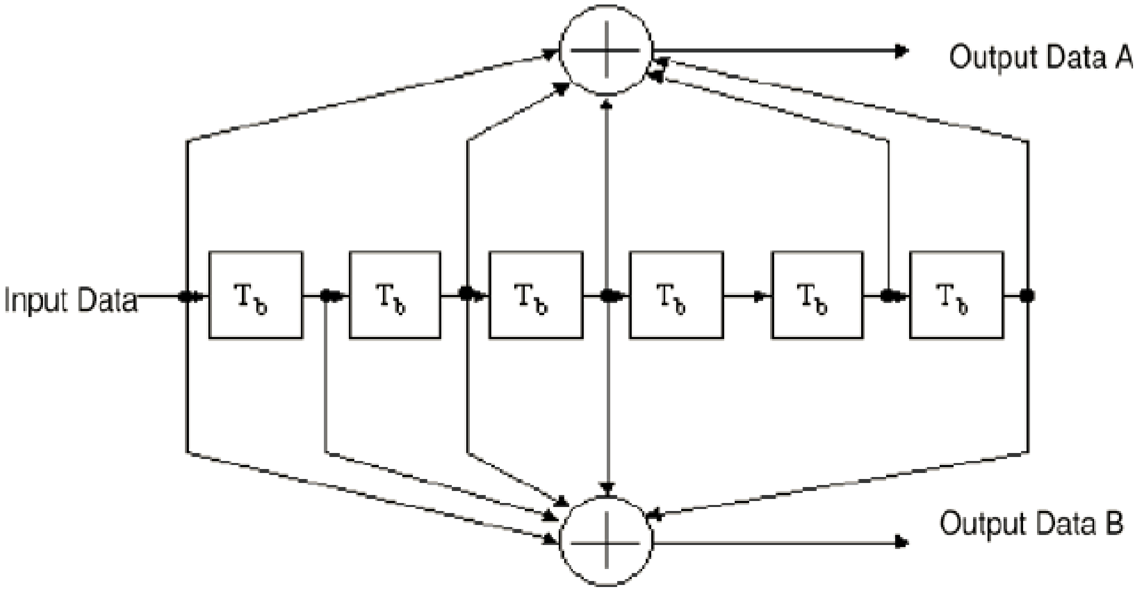
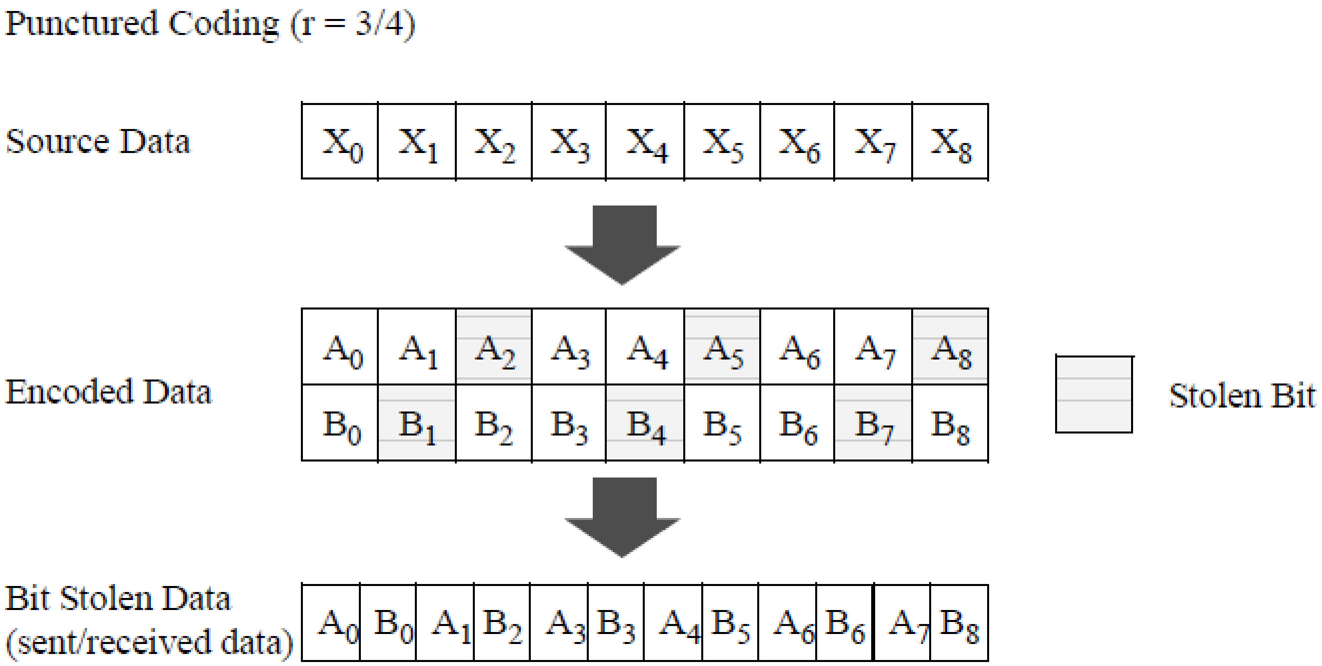
Channel Coding

- **Binary convolutional code (BCC) – 17.3.5.6**





- **Data interleaving – 17.3.5.7**

Interleaves the bits of each spatial stream (changes order of bits) to prevent long sequences of adjacent noisy bits.

Modulation and Coding Scheme (MCS, 19.5)

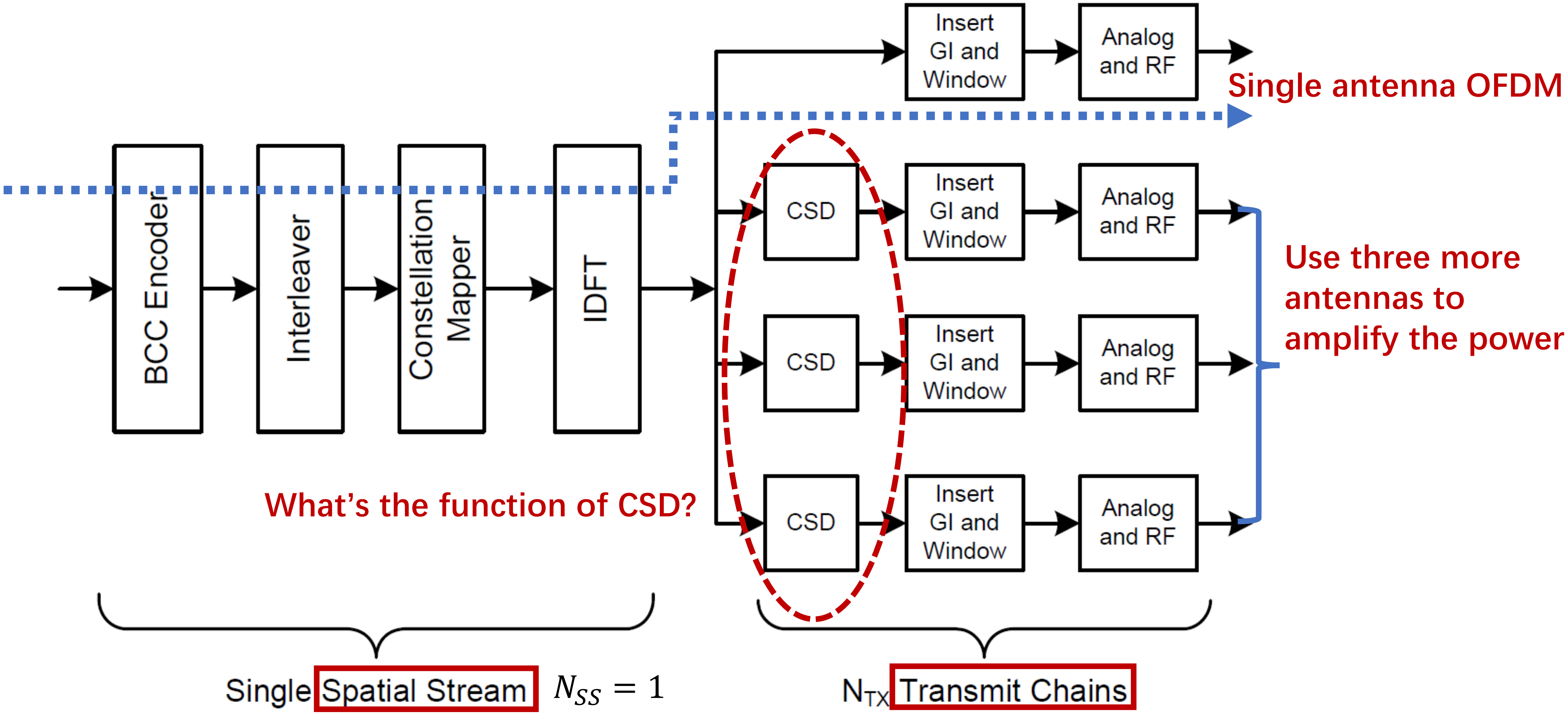
Table 19-27—MCS parameters for mandatory 20 MHz, N_{SS} = 1, N_{ES} = 1

MCS Index	Modulation	R	$N_{BPSCS}(i_{SS})$	N_{SD}	N_{SP}	N_{CBPS}	N_{DBPS}	Data rate (Mb/s)	
								800 ns GI	400 ns GI (see NOTE)
0	BPSK	1/2	1	52	4	52	26	6.5	7.2
1	QPSK	1/2	2	52	4	104	52	13.0	14.4
2	QPSK	3/4	2	52	4	104	78	19.5	21.7
3	16-QAM	1/2	4	52	4	208	104	26.0	28.9
4	16-QAM	3/4	4	52	4	208	156	39.0	43.3
5	64-QAM	2/3	6	52	4	312	208	52.0	57.8
6	64-QAM	3/4	6	52	4	312	234	58.5	65.0
7	64-QAM	5/6	6	52	4	312	260	65.0	72.2
NOTE—Support of 400 ns GI is optional on transmit and receive.									

NBPSCS: Number of coded bits per subcarrier
NSD: Number of data subcarriers
NSP: Number of pilot subcarriers

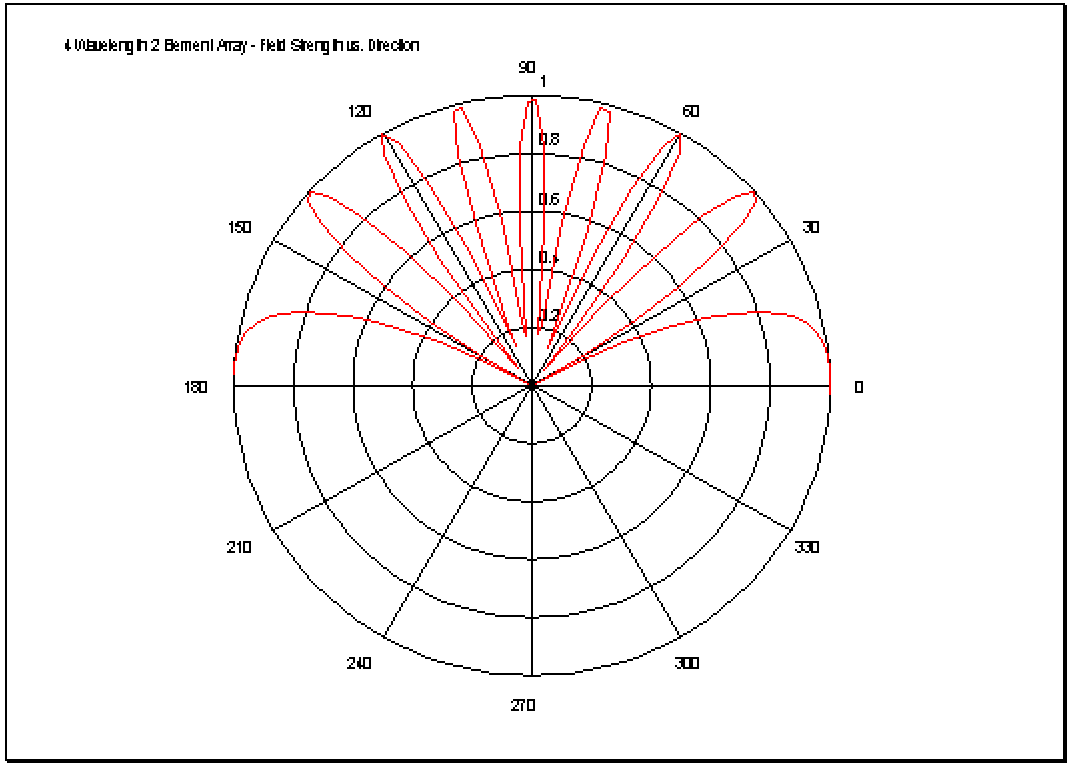
NCBPS: Number of coded bits per symbol
NDBPS: Number of data bits per symbol

Recap: Single Spatial Stream (19.3.3)



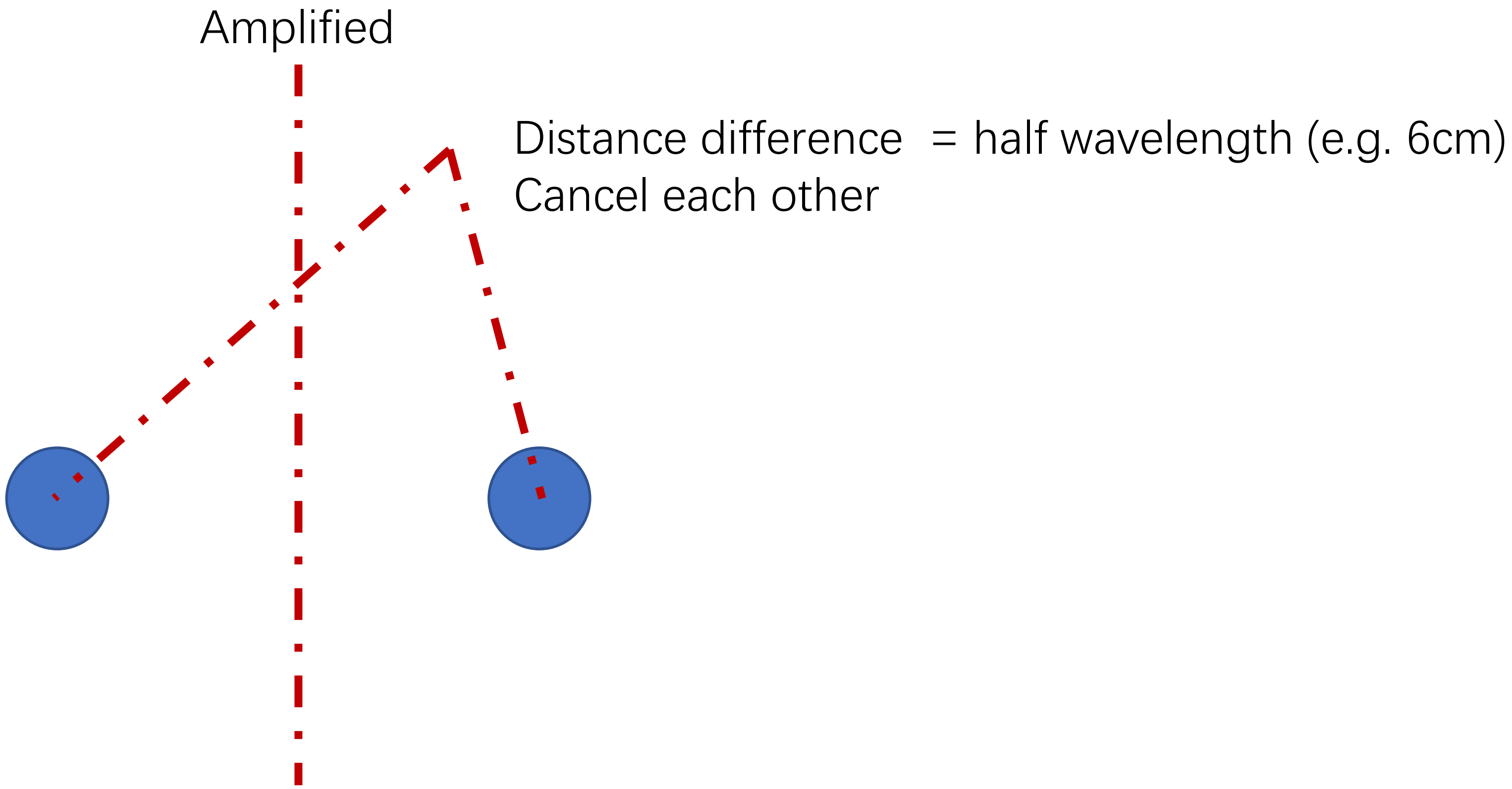
"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

Array Pattern



- Why antenna specific delay (cyclic shift)?
- If two antenna ports transmit the same signal at the same time, there might be effect of beamforming
- Not desired at the phase of packet synchronization
- Cyclic shift is applied to avoid the beamforming effect

Array Pattern



Cyclic Shift (19.3.9.3.2)

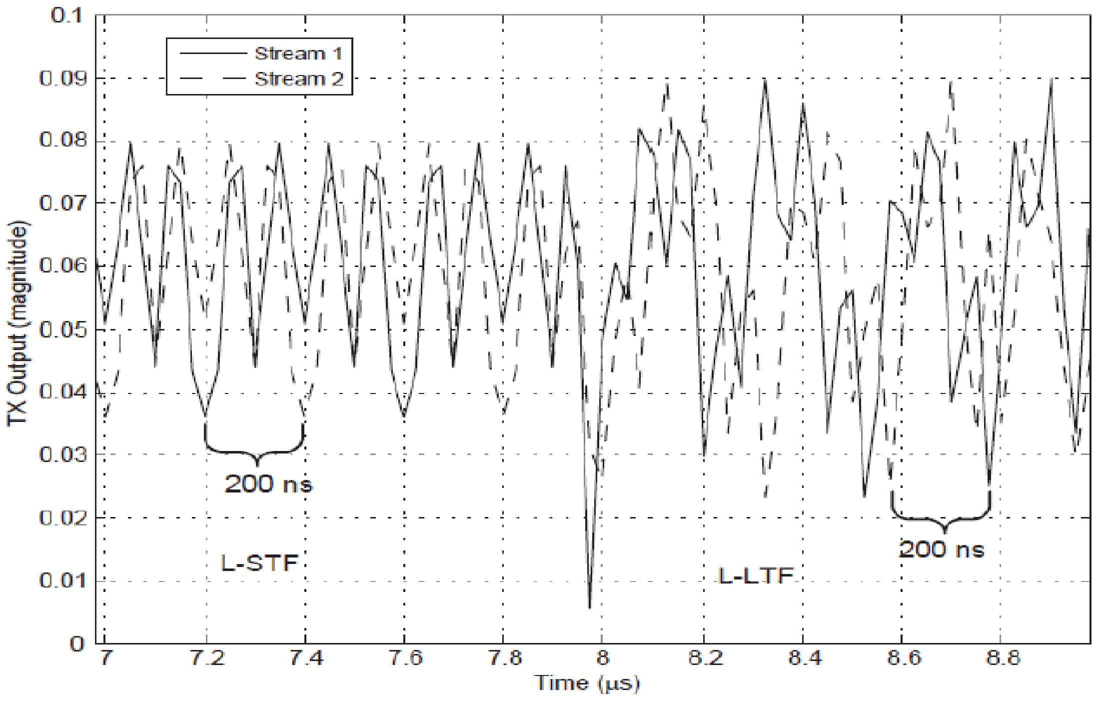


Table 19-9—Cyclic shift for non-HT portion of packet

$T_{CS}^{i_{TX}}$ values for non-HT portion of packet				
Number of transmit chains	Cyclic shift for transmit chain 1 (ns)	Cyclic shift for transmit chain 2 (ns)	Cyclic shift for transmit chain 3 (ns)	Cyclic shift for transmit chain 4 (ns)
1	0	—	—	—
2	0	–200	—	—
3	0	–100	–200	—
4	0	–50	–100	–150

50ns is the duration of one sample

Let s(t) be the signal without cyclic shift, TCS is the cyclic shift value, then the cyclic shift is as follows.

$$s_{CS}(t;T_{CS})\big|_{T_{CS}<0} = \begin{cases} s(t-T_{CS}) & 0 \le t < T+T_{CS} \\ s(t-T_{CS}-T) & T+T_{CS} \le t \le T \end{cases}$$

"Cyclic shifts are used to prevent **unintentional beamforming** when the same signal or scalar multiples of one signal are transmitted through different spatial streams or transmit chains."

Cyclic Shift is different from the Cyclic Prefix of OFDM, which is referred to Guard Interval (GI) in standard

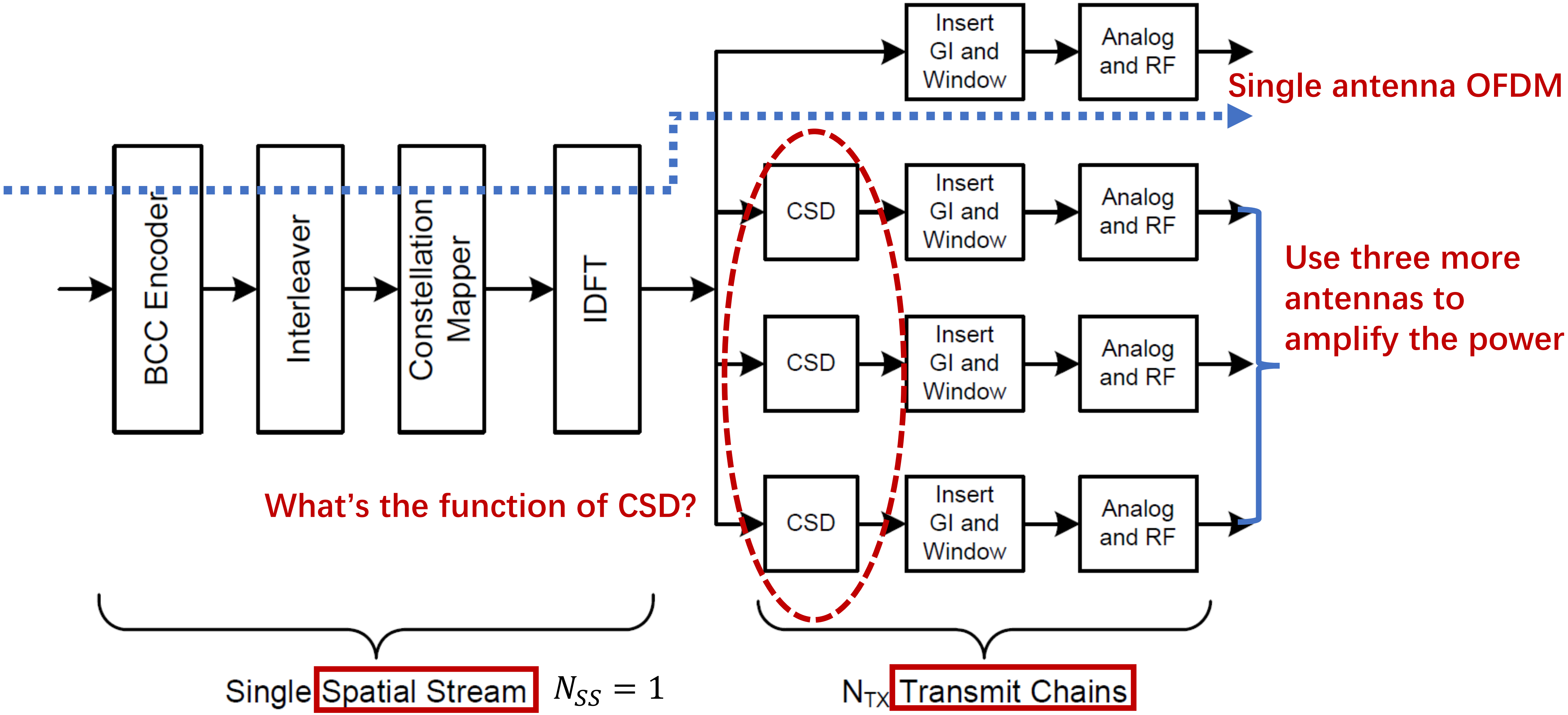
Cyclic Shift - Example

- Suppose non-HT mode, two antennas transmit the same signal, CSD happens after IFFT
- Suppose cyclic shift is 200ns, which include 4 samples

Tx Antenna 1: S1, S2, S3, S4, S5, ···, S64

Tx Antenna 2: S61, S62, S63, S64, S1, ···, S60

Recap: Single Spatial Stream (19.3.3)



"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

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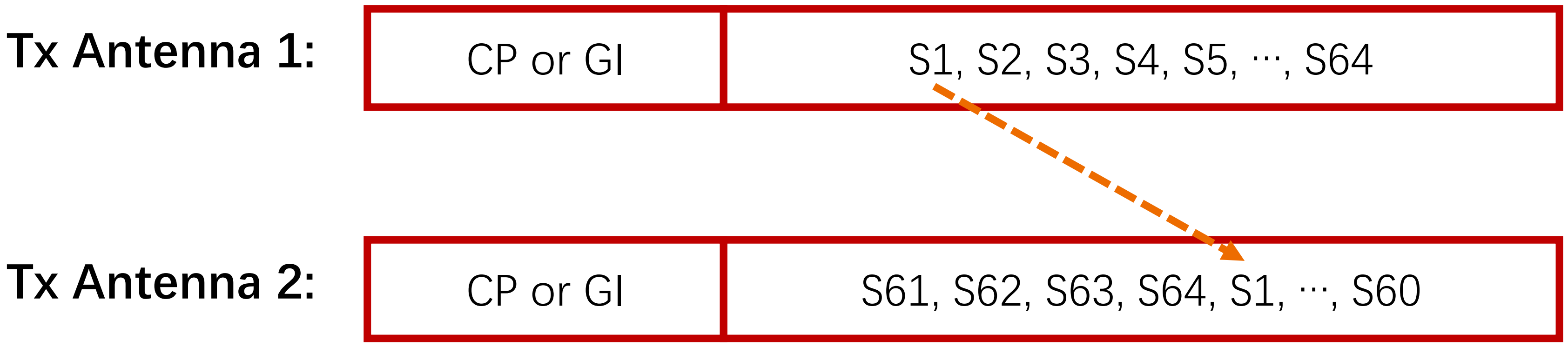
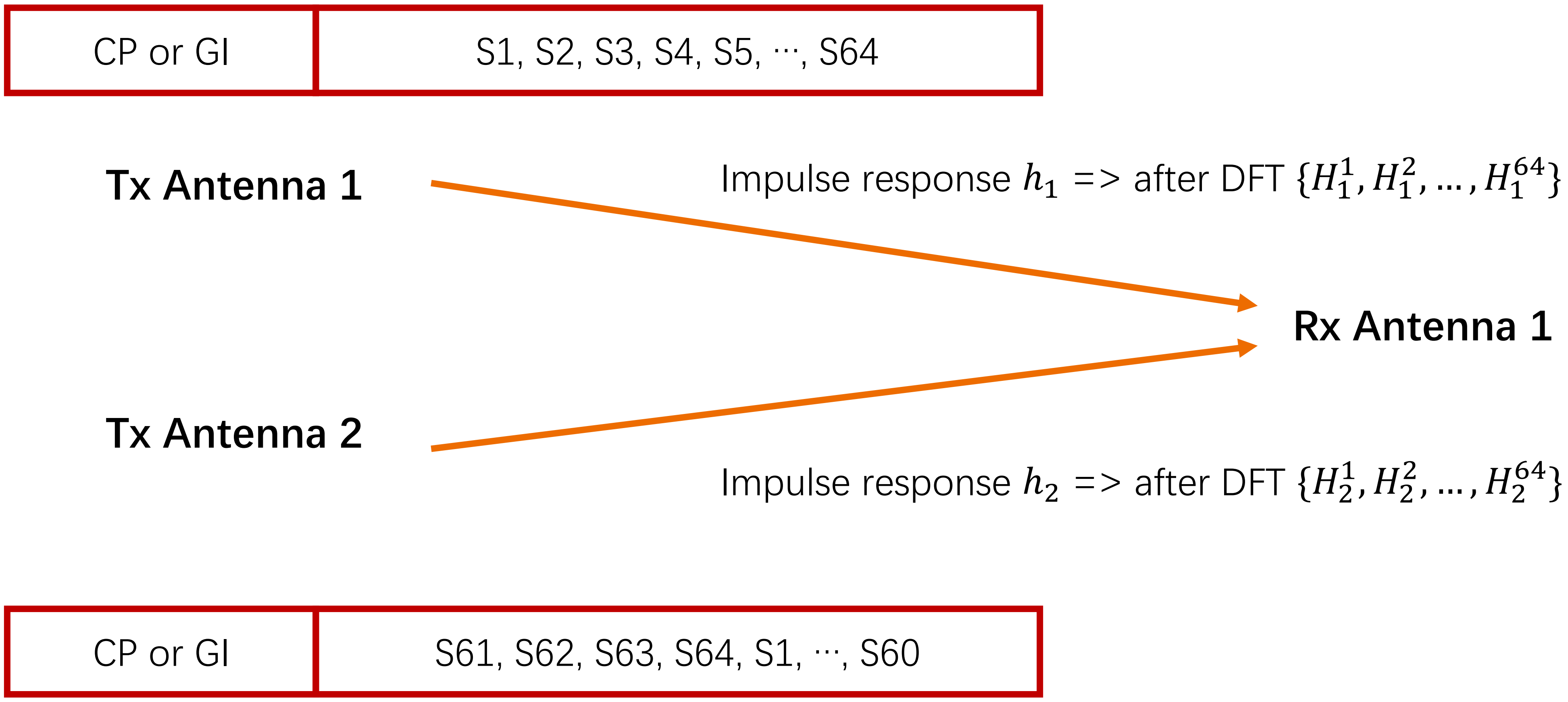


Analysis of Cyclic Shift



Analysis of Cyclic Shift

$\begin{bmatrix} D_1 \\ D_2 \\ \dots \\ D_{64} \end{bmatrix}$ IDFT => $\begin{bmatrix} S_1 \\ S_2 \\ \dots \\ S_{64} \end{bmatrix}$ Add CP => Convolution with I_1 => Remove CP => DFT => $\begin{bmatrix} H_1^1 D_1 \\ H_1^2 D_2 \\ \dots \\ H_1^{64} D_{64} \end{bmatrix}$

Tx Antenna 1

Impulse response I_1 => after DFT $\{H_1^1, H_1^2, \dots, H_1^{64}\}$

Rx Antenna 1

Tx Antenna 2

Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

[?] IDFT => $\begin{bmatrix} S_{61} \\ S_{62} \\ \dots \\ S_{60} \end{bmatrix}$

Analysis of Cyclic Shift

Tx Antenna 1

$$D_k = \sum_{n=1}^{64} S_n e^{-\frac{2\pi j}{64}kn} = S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}$$

Tx Antenna 2 $S_{61} e^{-\frac{2\pi j}{64}k} + S_{62} e^{-\frac{2\pi j}{64}2k} + \cdots + S_1 e^{-\frac{2\pi j}{64}5k} + S_2 e^{-\frac{2\pi j}{64}6k} + \cdots + S_{60}\, e^{-\frac{2\pi j}{64}64k}$

$$= \left[S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}\right] e^{-\frac{2\pi j}{64}4k}$$

$$= D_k e^{-\frac{2\pi j}{64}4k}$$

Analysis of Cyclic Shift

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$\begin{bmatrix} D_1 e^{-\frac{2\pi j}{64}4} \\ D_2 e^{-\frac{2\pi j}{64}4\times 2} \\ \dots \\ D_{64} e^{-\frac{2\pi j}{64}4\times 64} \end{bmatrix}$ IDFT => $\begin{bmatrix} S_{61} \\ S_{62} \\ \dots \\ S_{60} \end{bmatrix}$ Add CP => Convolution with I_1 => Remove CP => DFT => $\begin{bmatrix} H_2^1 e^{-\frac{2\pi j}{64}4} D_1 \\ H_2^2 e^{-\frac{2\pi j}{64}4\times 2} D_2 \\ \dots \\ H_2^{64} e^{-\frac{2\pi j}{64}4\times 64} D_{64} \end{bmatrix}$

Analysis of Cyclic Shift

$$\begin{bmatrix} (H_1^1+H_2^1 e^{-\frac{2\pi j}{64}4})D_1 \\ (H_1^2 + H_2^2 e^{-\frac{2\pi j}{64}4\times 2})D_2 \\ \cdots \\ (H_1^{64} + H_2^{64} e^{-\frac{2\pi j}{64}4\times 64})D_{64} \end{bmatrix}$$

$$\begin{bmatrix} D_1 \\ D_2 \\ \cdots \\ D_{64} \end{bmatrix} \text{IDFT} => \begin{bmatrix} S_1 \\ S_2 \\ \cdots \\ S_{64} \end{bmatrix}$$

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Rx Antenna 1

Tx Antenna 2

Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \dots, H_2^{64}\}$

$$\begin{bmatrix} D_1 e^{-\frac{2\pi j}{64}4} \\ D_2 e^{-\frac{2\pi j}{64}4\times 2} \\ \cdots \\ D_{64} e^{-\frac{2\pi j}{64}4\times 64} \end{bmatrix} \text{IDFT} => \begin{bmatrix} S_{61} \\ S_{62} \\ \cdots \\ S_{60} \end{bmatrix}$$

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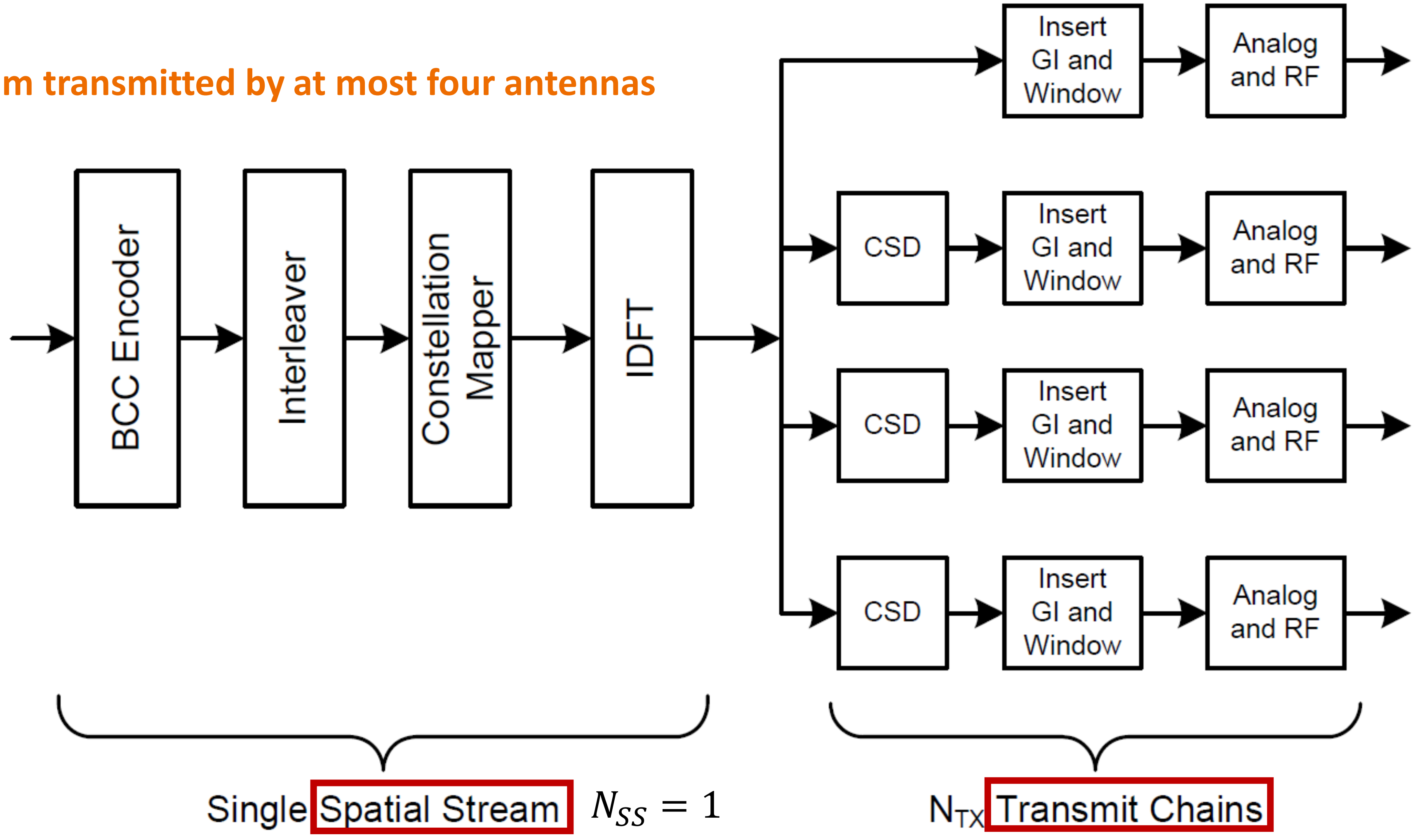
One data stream transmitted by at most four antennas



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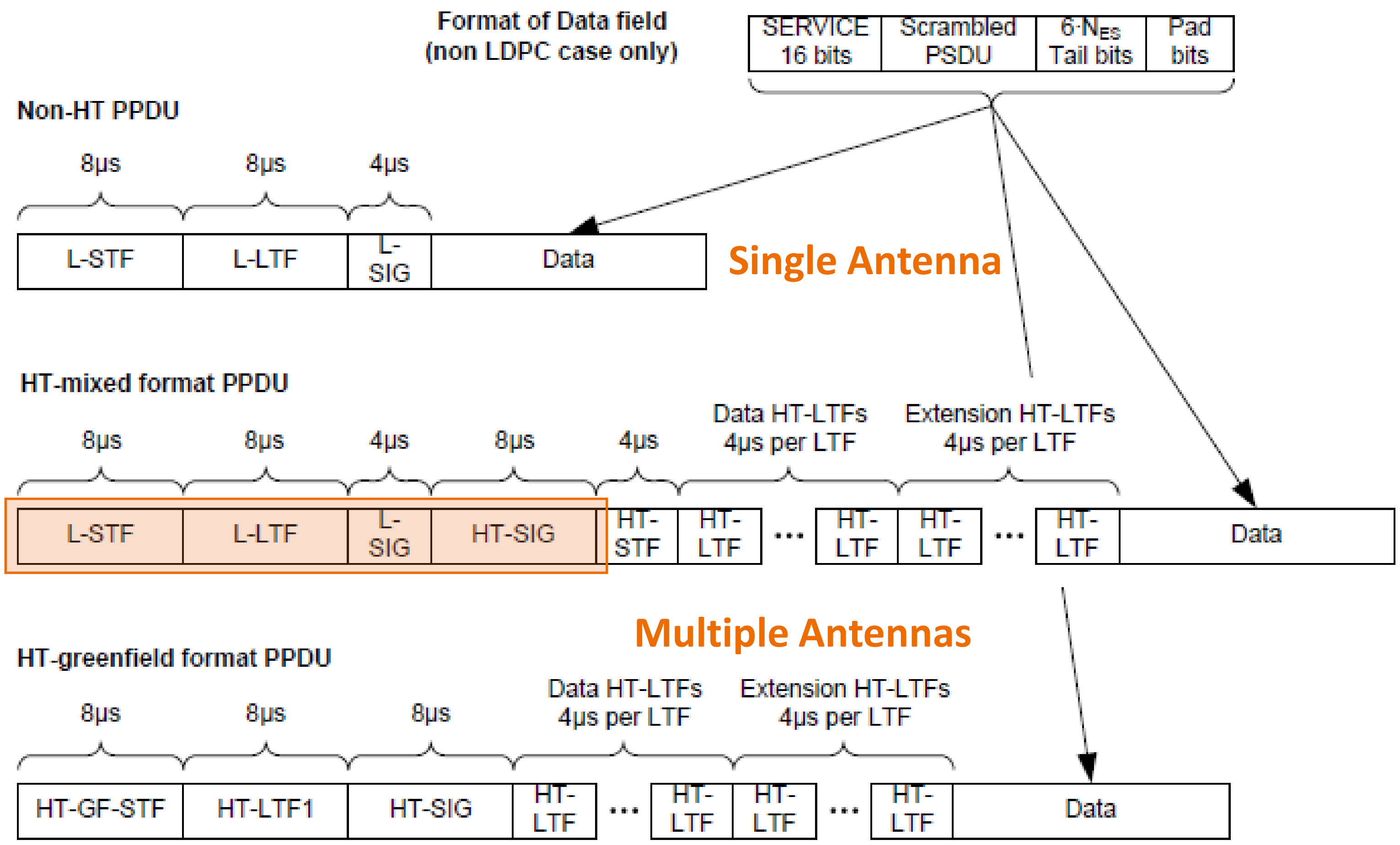

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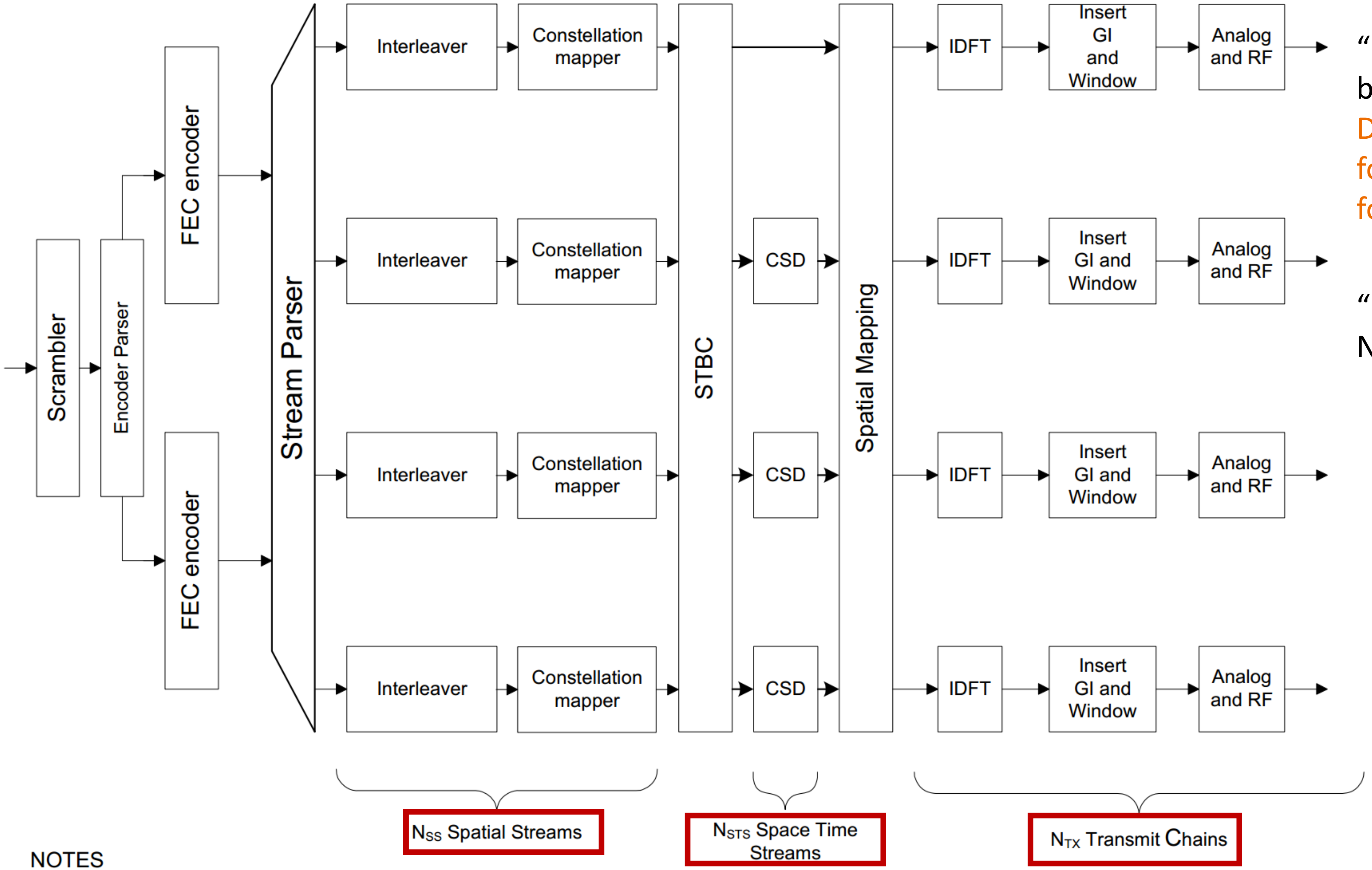
Multiple Spatial Stream (19.3.3)

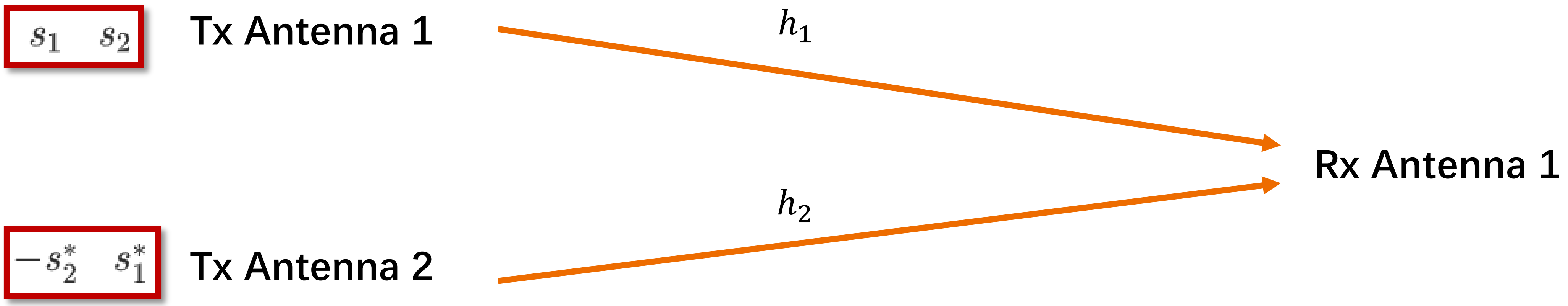


"…shows the transmitter blocks used to generate the Data field of the HT-mixed format and HT-greenfield format PPDUs."

" STBC is used only when N_{STS}>N_{SS} "

NOTES

STBC Example: Alamouti Code



Diversity is increased to 2, less packet outage probability

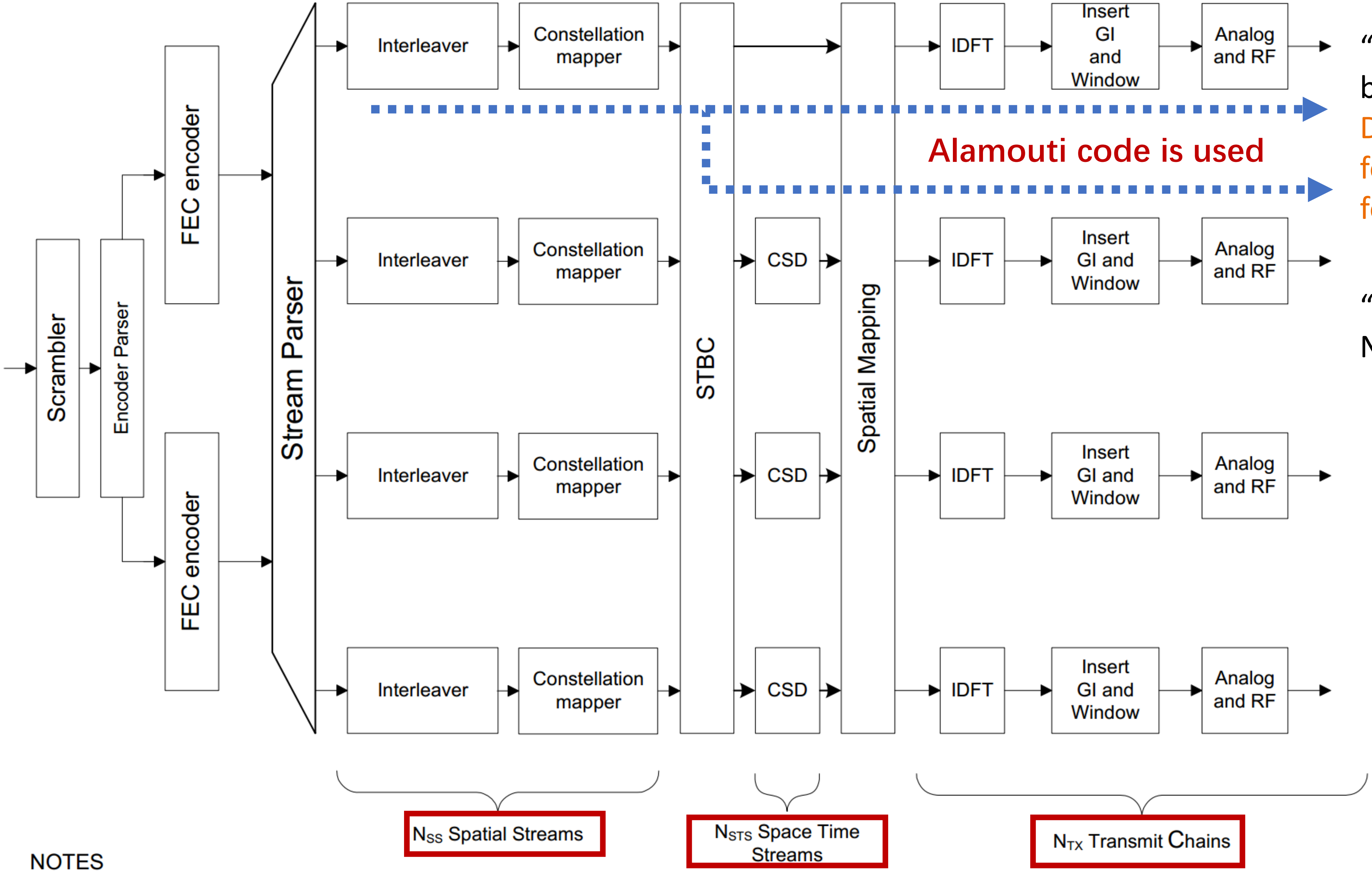
Require CSIR only, CSIT is not necessary

$$[y_1 \;\; y_2] = [h_1 \; h_2]\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} + [w_1 \; w_2]$$

$$\Downarrow$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & -h_{12} \\ h_{12}^* & h_{11}^* \end{bmatrix}\begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \quad \Rightarrow \quad \begin{cases} \hat{s}_1 = \frac{1}{|h_1|^2+|h_2|^2}(h_1^* y_1 + h_2 y_2^*) \\ \hat{s}_2 = \frac{1}{|h_1|^2+|h_2|^2}(-h_2 y_1^* + h_1^* y_2) \end{cases}$$

Multiple Spatial Stream (19.3.3)



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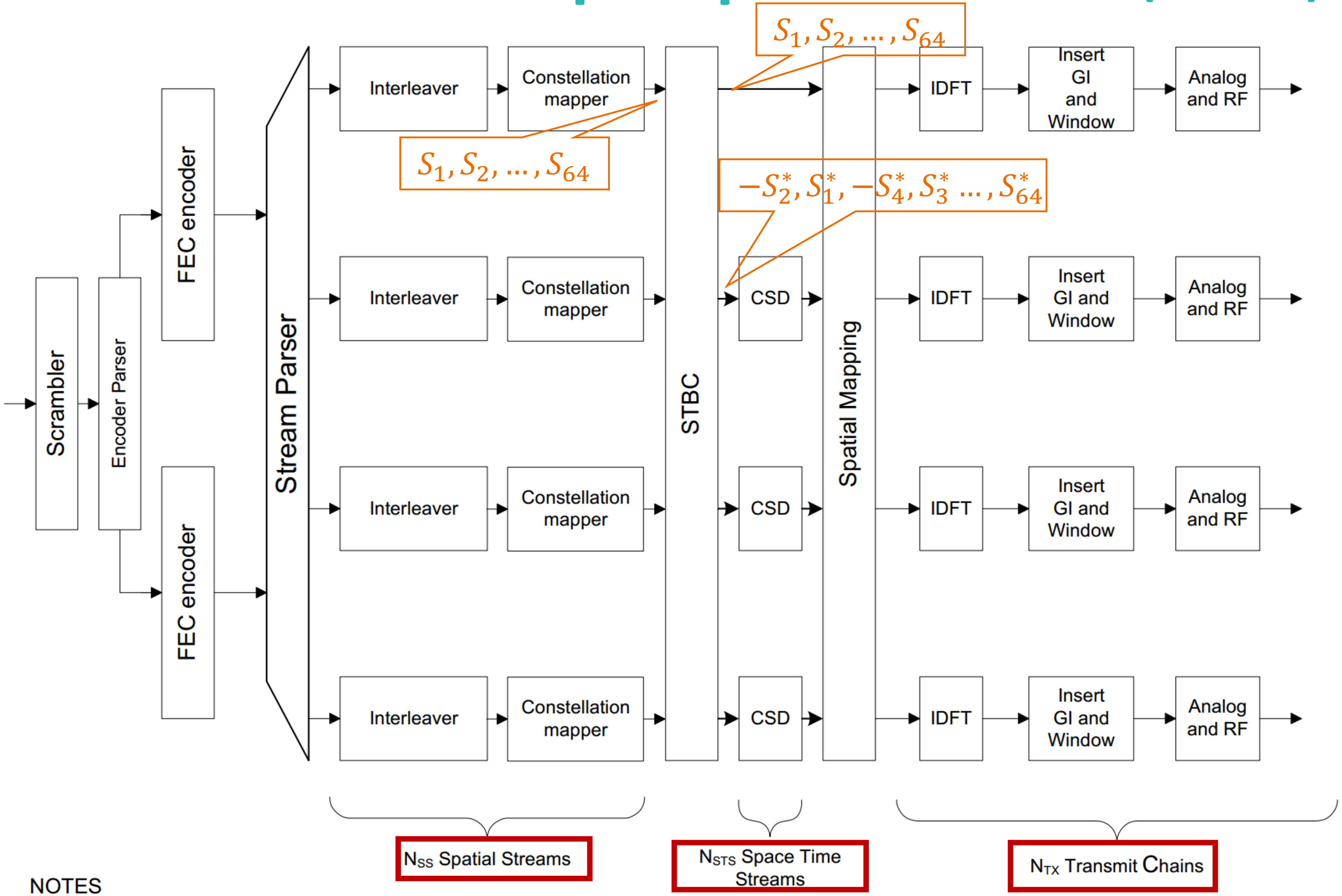

NOTES

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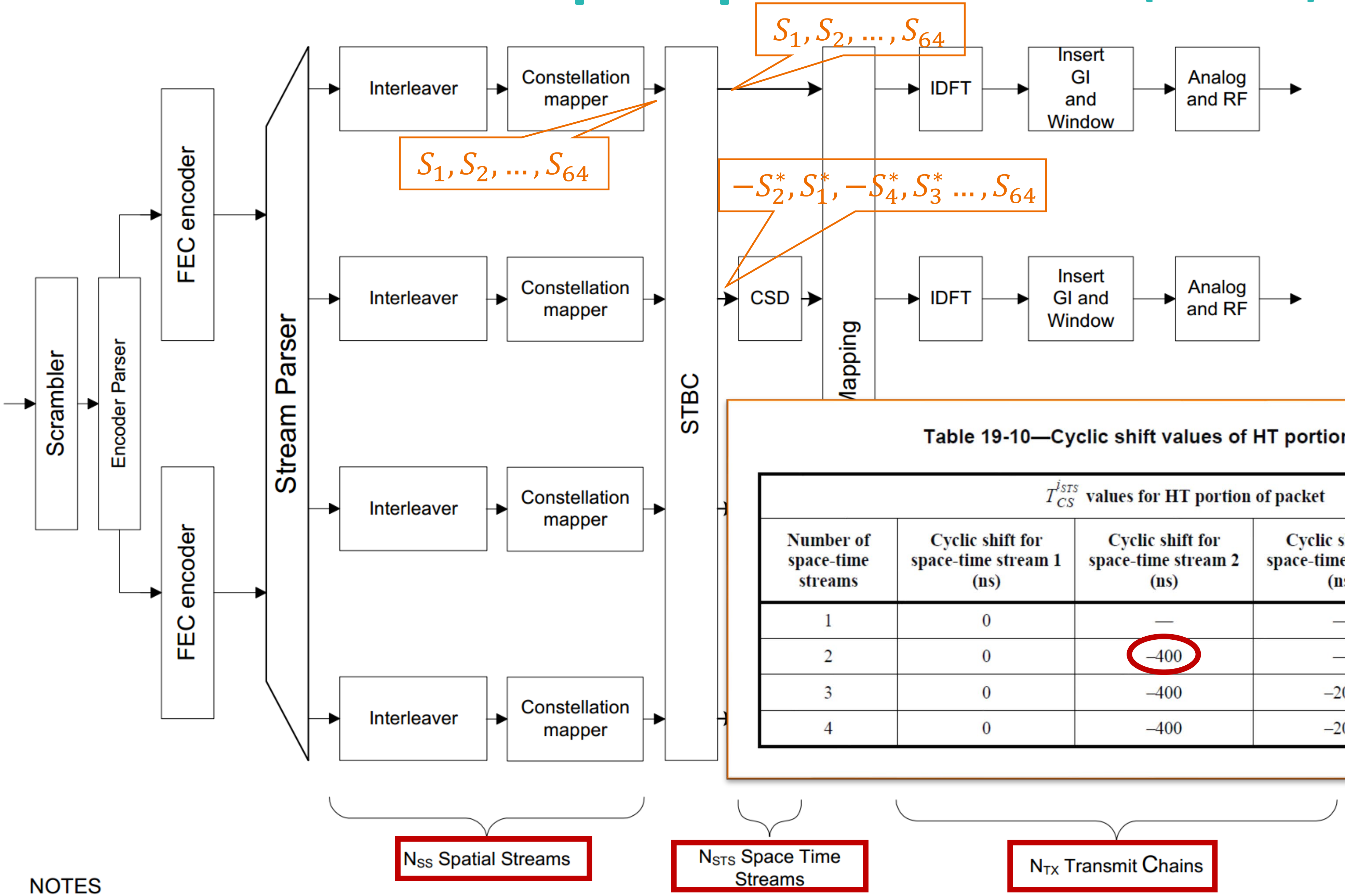


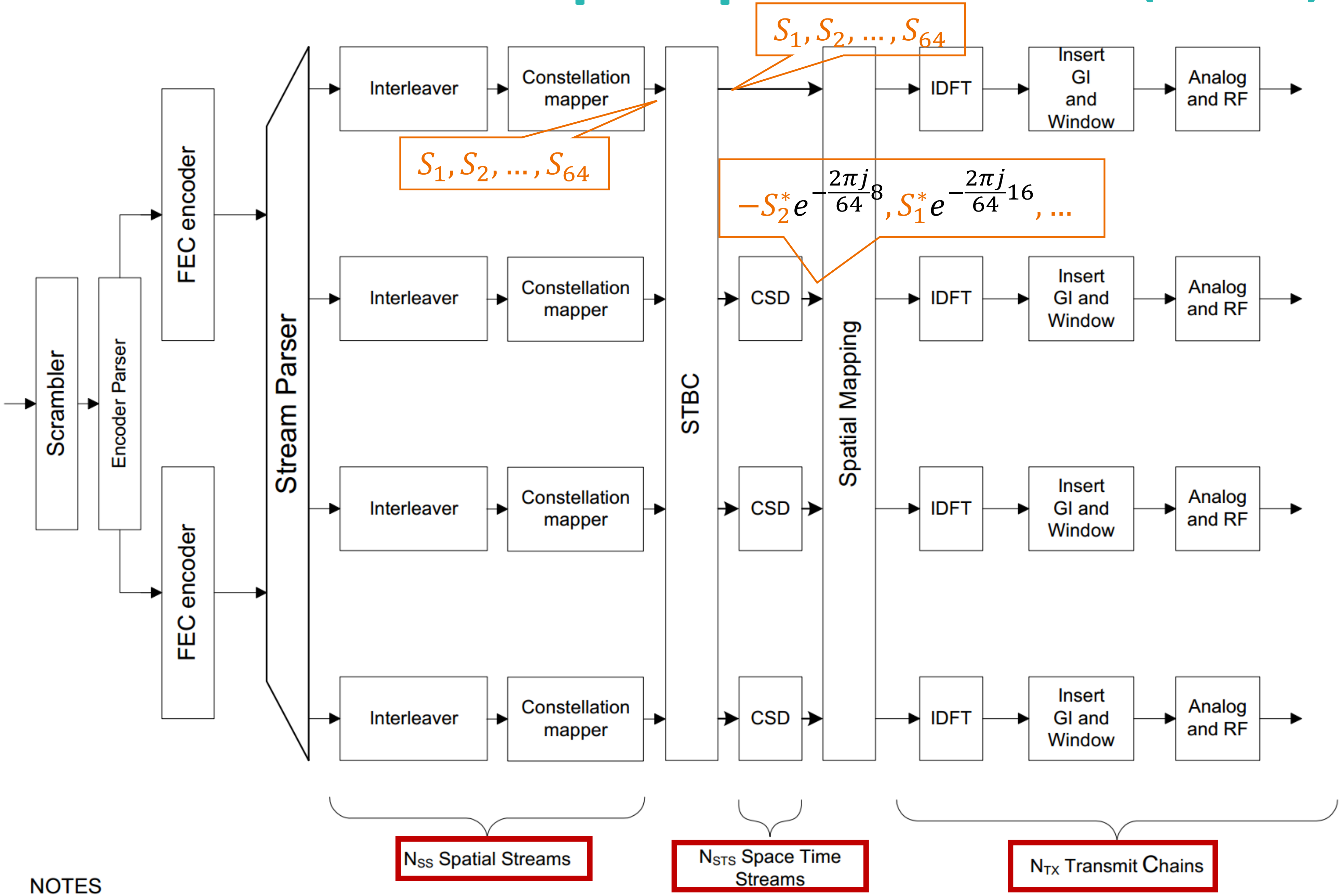
400ns = 8 samples

Table 19-10—Cyclic shift values of HT portion of packet

$T_{CS}^{i_{STS}}$ values for HT portion of packet				
Number of space-time streams	Cyclic shift for space-time stream 1 (ns)	Cyclic shift for space-time stream 2 (ns)	Cyclic shift for space-time stream 3 (ns)	Cyclic shift for space-time stream 4 (ns)
1	0	—	—	—
2	0	–400	—	—
3	0	–400	–200	—
4	0	–400	–200	–600

NOTES

Multiple Spatial Stream (19.3.3)



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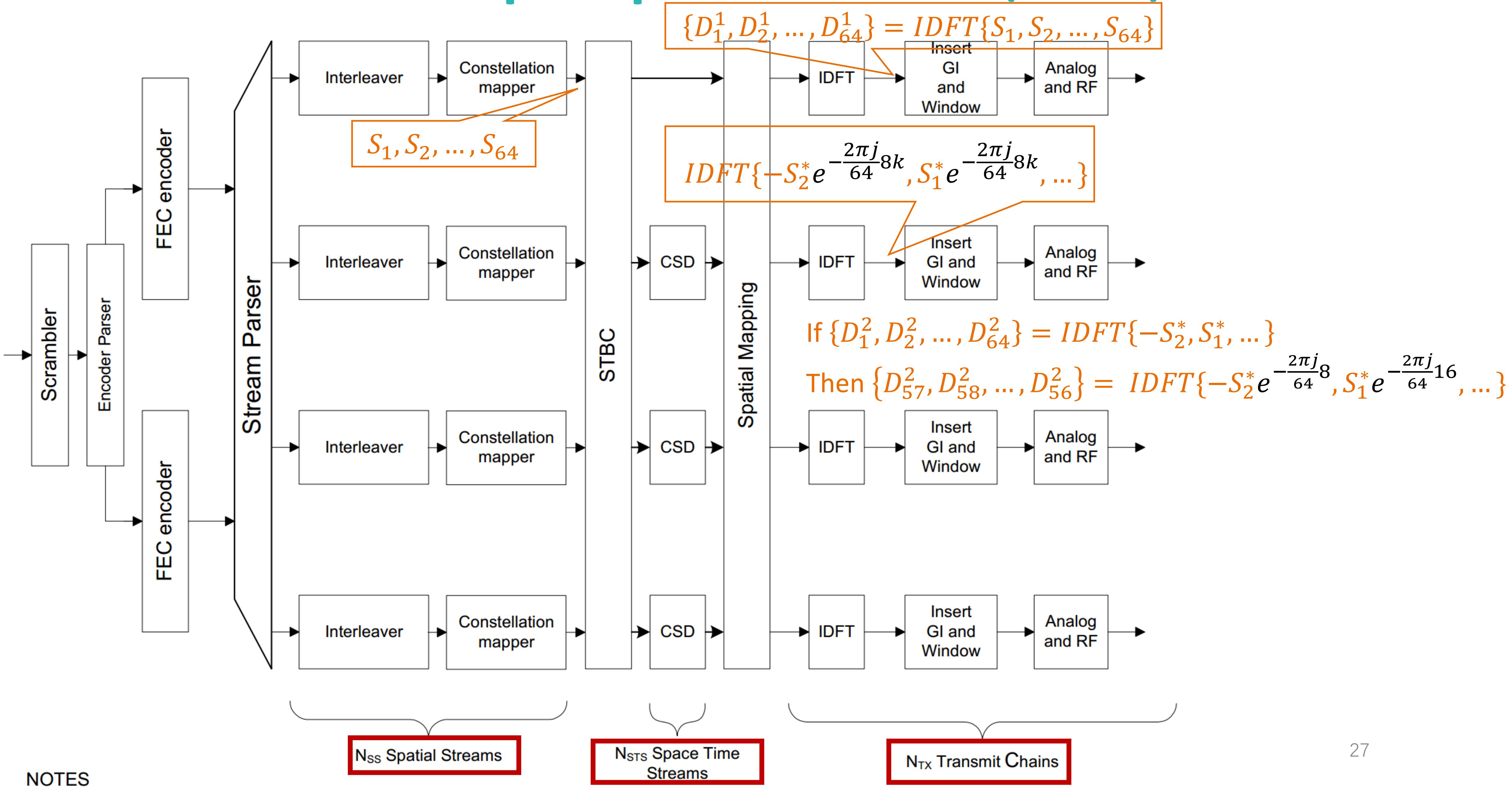
Multiple Spatial Stream (19.3.3)



NOTES

Space-Time Block Code (19.3.11.9.2)

- Support 4 space-time block codes

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
2	0–7	1	1	1	$d_{k,1,2m}$ S_1	$d_{k,1,2m+1}$ S_2
				2	$-d^*_{k,1,2m+1}$ $-S_2^*$	$d^*_{k,1,2m}$ S_1^*
3	8–15, 33–38	2	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
4	8–15	2	2	1	$d_{k,1,2m}$ S_1	$d_{k,1,2m+1}$ S_2
				2	$-d^*_{k,1,2m+1}$ $-S_2^*$	$d^*_{k,1,2m}$ S_1^*
				3	$d_{k,2,2m}$ S_3	$d_{k,2,2m+1}$ S_4
				4	$-d^*_{k,2,2m+1}$ $-S_4^*$	$d^*_{k,2,2m}$ S_3^*

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
4	16–23, 39, 41, 43, 46, 48, 50	3	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
				4	$d_{k,3,2m}$	$d_{k,3,2m+1}$

NOTE—the '*' operator represents the complex conjugate.

Example: 4×2 STBC

$$\begin{pmatrix} y_1 & y_3 \\ y_2 & y_4 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{pmatrix} \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \\ s_3 & s_4 \\ -s_4^* & s_3^* \end{pmatrix} + \begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} h_{11} & -h_{12} & h_{13} & -h_{14} \\ h_{21} & -h_{22} & h_{23} & -h_{24} \\ h_{12}^* & h_{11}^* & h_{14}^* & h_{13}^* \\ h_{22}^* & h_{21}^* & h_{24}^* & h_{23}^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2^* \\ s_3 \\ s_4^* \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3^* \\ w_4^* \end{pmatrix}$$

At least **2 Rx antennas** are required

MIMO Detector

- Given $Y = HX + Z$ and (Y,H), how to estimate vector X?

- **Approach 1: Zero-forcing (ZF)**
  - $\hat{X} = H^{-1}Y = X + H^{-1}Z$

- **Approach 2: Minimum mean square error (MMSE)**
  - $\hat{X} = QY = QHX + QZ \implies Error\colon \Delta = \hat{X} - X = (QH - I)X + QZ$
  - $E\,[\Delta\Delta^H] = E[(QH - I)XX^H(QH - I)^H + QZZ^HQ^H]$
  - $E\,[\Delta\Delta^H] = \sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H$
  - Thus, we should $\min\limits_{Q} trace[\sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H]$

- **Approach 3: Maximum likelihood (ML)**
  - The choices of X is finite
  - For each possible X, calculate vector distance $|Y - HX|$
  - Choose the X, which minimize the distance
  - Thus, $\hat{X} = \min\limits_{X} |Y - HX|$

Reading & Assignment (3.16)

Reading

- IEEE Std 802.11™-2020: Section 19.3.9 – 19.3.11
- Reference Paper

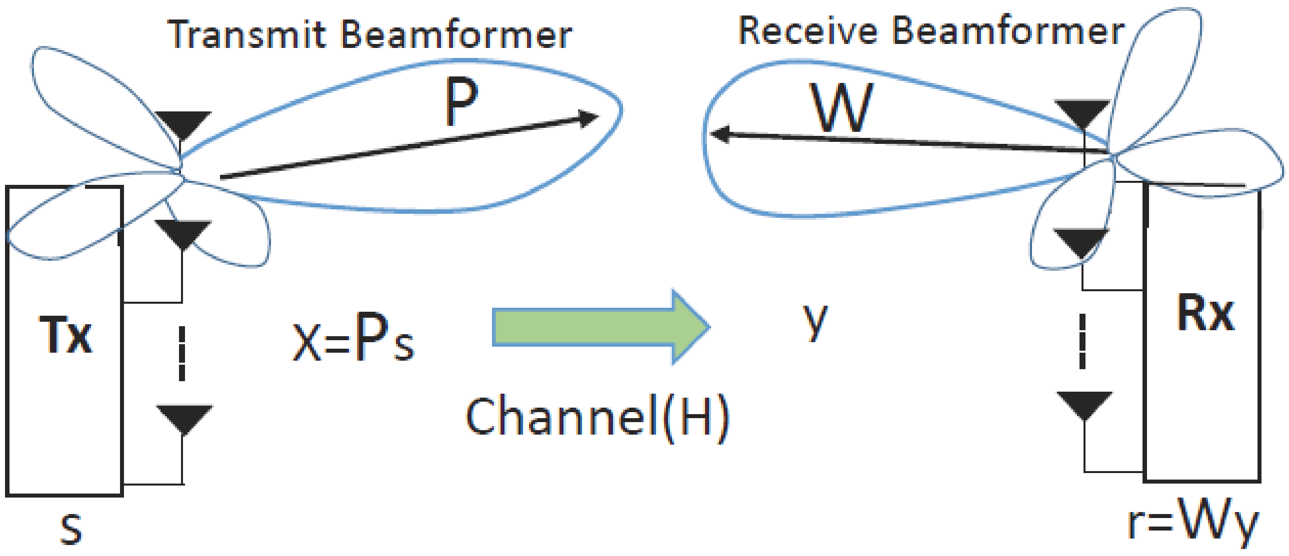
Assignment 3

Difference of MIMO Modes

- STBC requires CSIR only
- It can transmit single or multiple spatial streams
- Multiple streams with equal powers
- Single receiver

- MIMO beamforming requires both CSIR and CSIT
- It can transmit single or multiple spatial streams
- Multiple streams with power adaptation
- Support multiple receivers

MIMO Beamforming

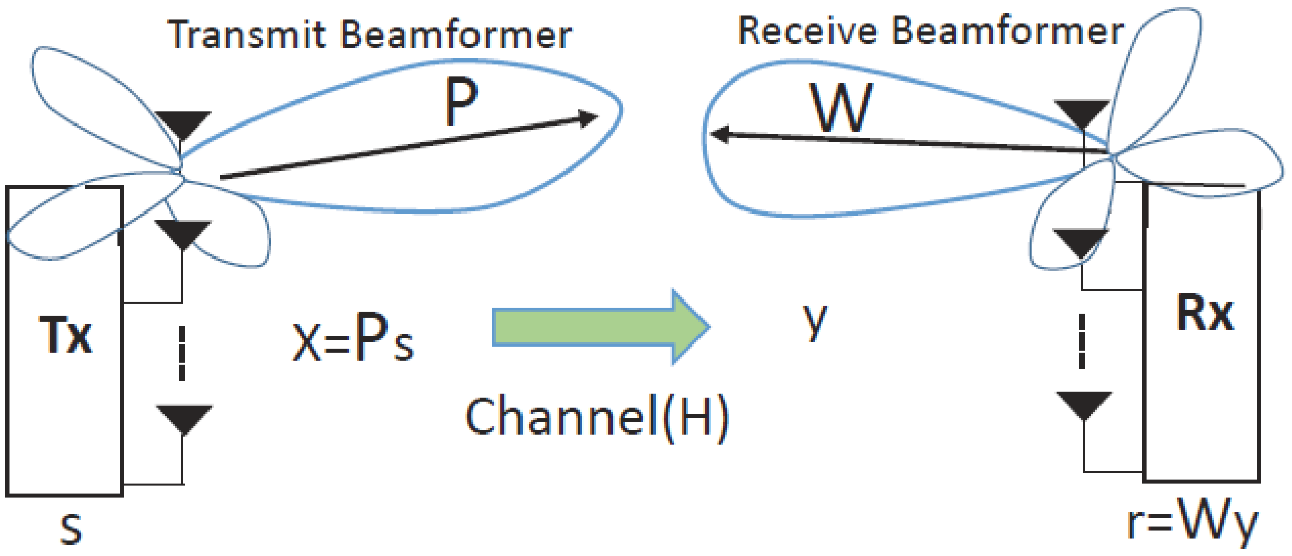


- The general model for a MIMO wireless communication is given as

$$\begin{aligned} \mathbf{y} &= \mathbf{H}\mathbf{x} + \mathbf{n}; \quad \mathbf{H} \in \mathbb{C}^{N_R \times N_T} & (1) \\ &= \mathbf{H}\mathbf{P}\mathbf{s} + \mathbf{n} & (2) \end{aligned}$$

- $\mathbf{x} = \mathbf{P}\mathbf{s}$ is the precoded and $\mathbf{r} = \mathbf{W}\mathbf{y}$ is the filtered output

MIMO Beamforming



$$\mathbf{r} = \mathbf{WHPs} + \mathbf{Wn}$$

A lot of choices of (W,P)
Typical one is SVD

- The general model for a MIMO wireless communication is given as

$$\mathbf{y} = \mathbf{Hx} + \mathbf{n}; \quad \mathbf{H} \in \mathbb{C}^{N_R \times N_T} \tag{1}$$
$$= \mathbf{HPs} + \mathbf{n} \tag{2}$$

- $\mathbf{x} = \mathbf{Ps}$ is the precoded and $\mathbf{r} = \mathbf{Wy}$ is the filtered output

SU-MIMO: SVD Decomposition

- Consider singular value decomposition(**SVD**) of $\mathbf{H}$

$$\mathbf{H} = \mathbf{U}\boldsymbol{\Sigma}\mathbf{V}^H \tag{3}$$

- where $\boldsymbol{\Sigma}$ is a diagonal matrix with entries σ_i (singular values)
- Choosing $\mathbf{P} = \mathbf{V}$ and $\mathbf{W} = \mathbf{U}^H$ in eq. (2) gives

$$\begin{aligned} \mathbf{r} &= \boldsymbol{\Sigma}\,\mathbf{s} + \mathbf{U}^H\mathbf{n} \qquad (4) \\ &= \boldsymbol{\Sigma}\,\mathbf{s} + \tilde{\mathbf{n}} \qquad (5) \end{aligned}$$

- Then, one data stream per singular value can be transmitted as

$$\mathbf{r}_i = \sigma_i \mathbf{s}_i + \tilde{\mathbf{n}}_i; \quad 1 \leq i \leq \min\{N_R, N_T\} \tag{6}$$

E. Telatar, "Capacity of multiantenna Gaussian channels, European Transactions on Telecommunications", 1999

SU-MIMO: Power Adaptation

Summation of Channel Capacity

SNR per Spatial Stream

$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

$$subject\ to \sum_i P_i \le P$$

Total Transmission Power Constraint

SU-MIMO: Power Adaptation

Summation of Channel Capacity

SNR per Spatial Stream

$$E\,|\tilde{n}_i|^2 = E\, u_i^H n n^H u_i = \sigma_z^2\, u_i^H I u_i = \sigma_z^2$$

$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

$$subject\ to \sum_i P_i \leq P$$

Total Transmission Power Constraint

SU-MIMO: Power Adaptation

Summation of Channel Capacity

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$$E\,|\tilde{n}_i|^2 = E\, u_i^H n n^H u_i = \sigma_z^2\, u_i^H I u_i = \sigma_z^2$$

$$\max \sum_i \log_2\left(1 + \boxed{\frac{|\sigma_i|^2 P_i}{E\,|\tilde{n}_i|^2}}\right)$$

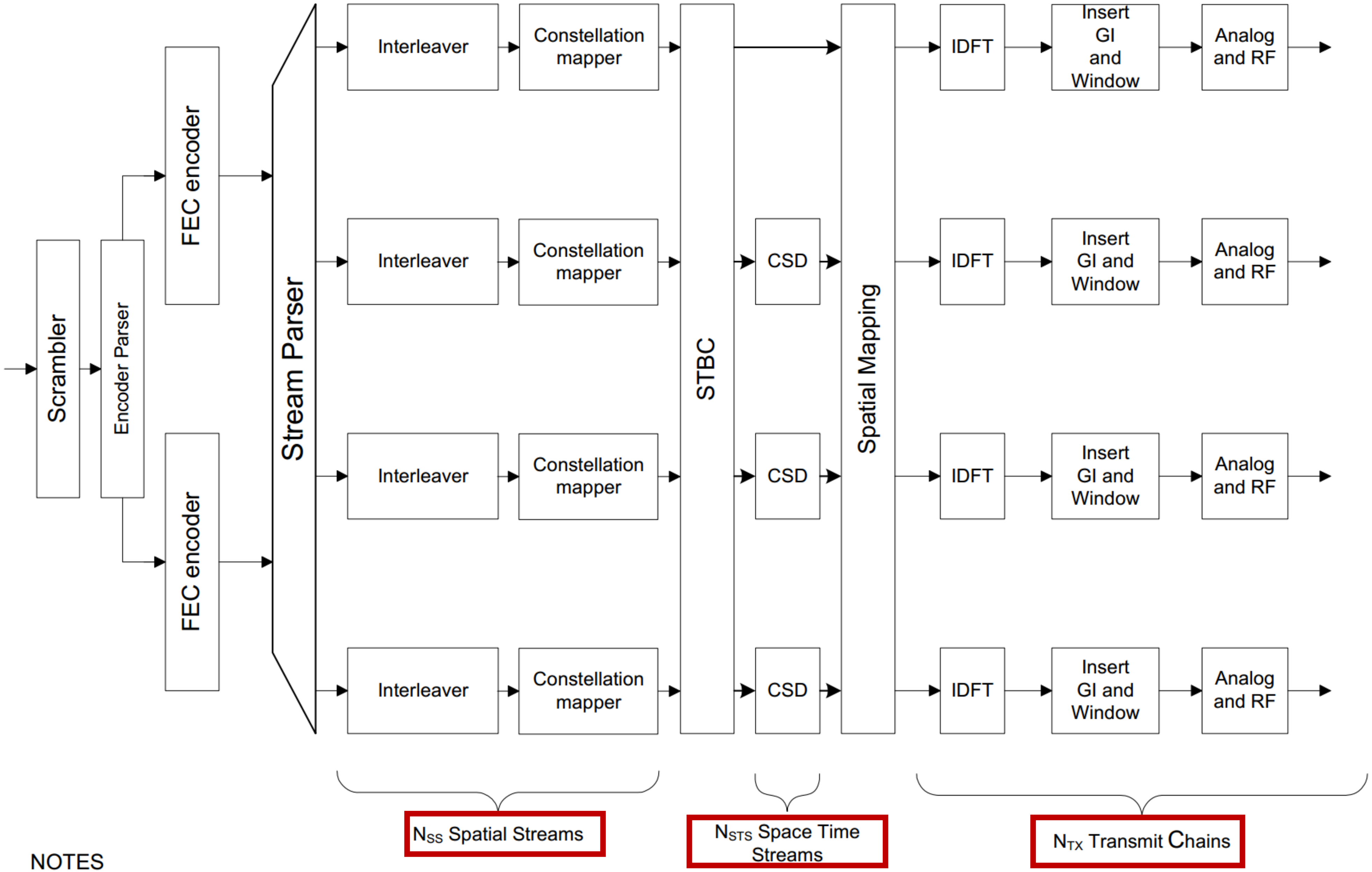
$$subject\ to \sum_i P_i \leq P$$

Total Transmission Power Constraint

With water-filling algorithm, the total throughput can be maximized

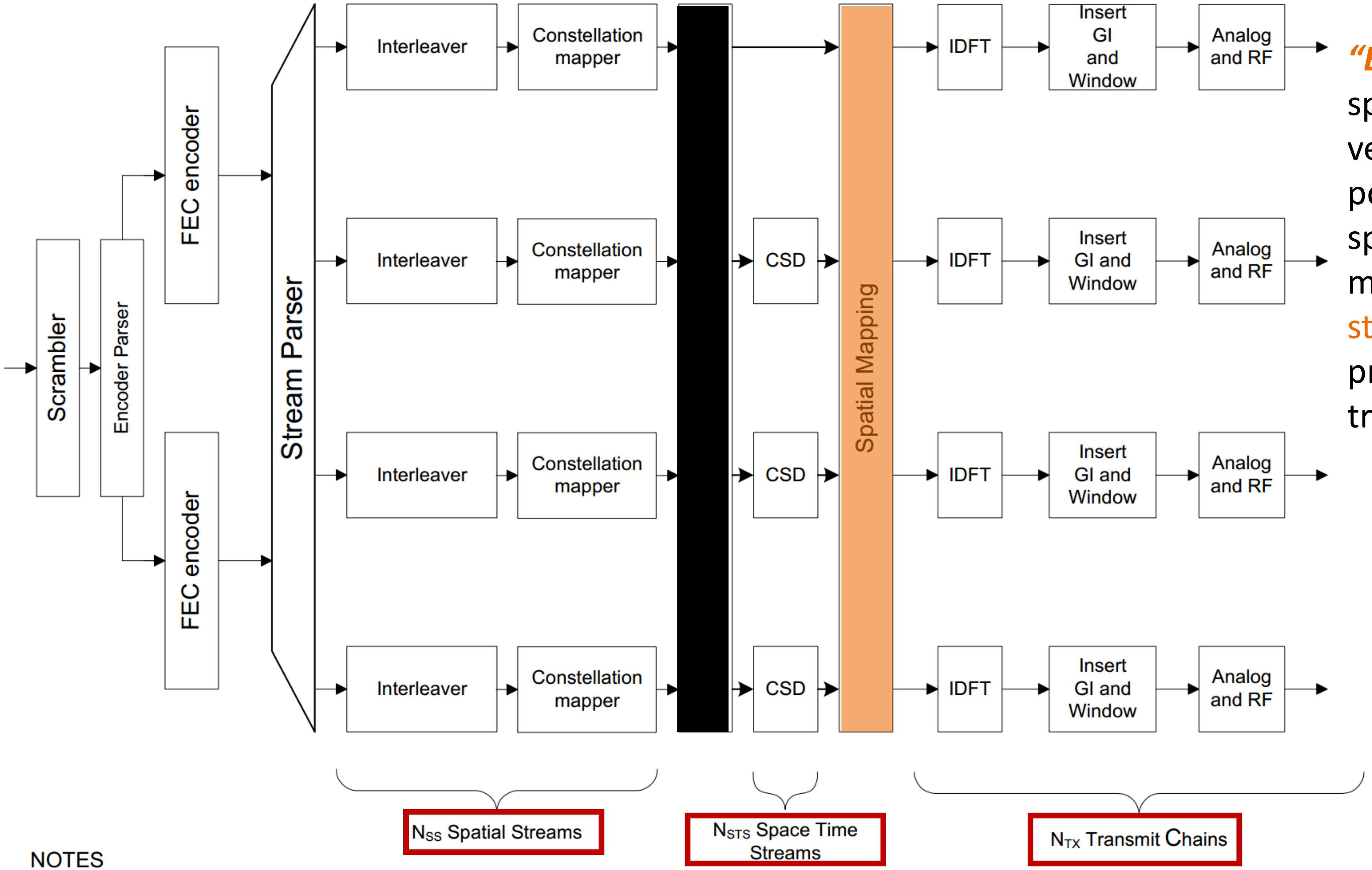
STBC cannot do this

Recap: Multiple Spatial Stream (19.3.3)



NOTES

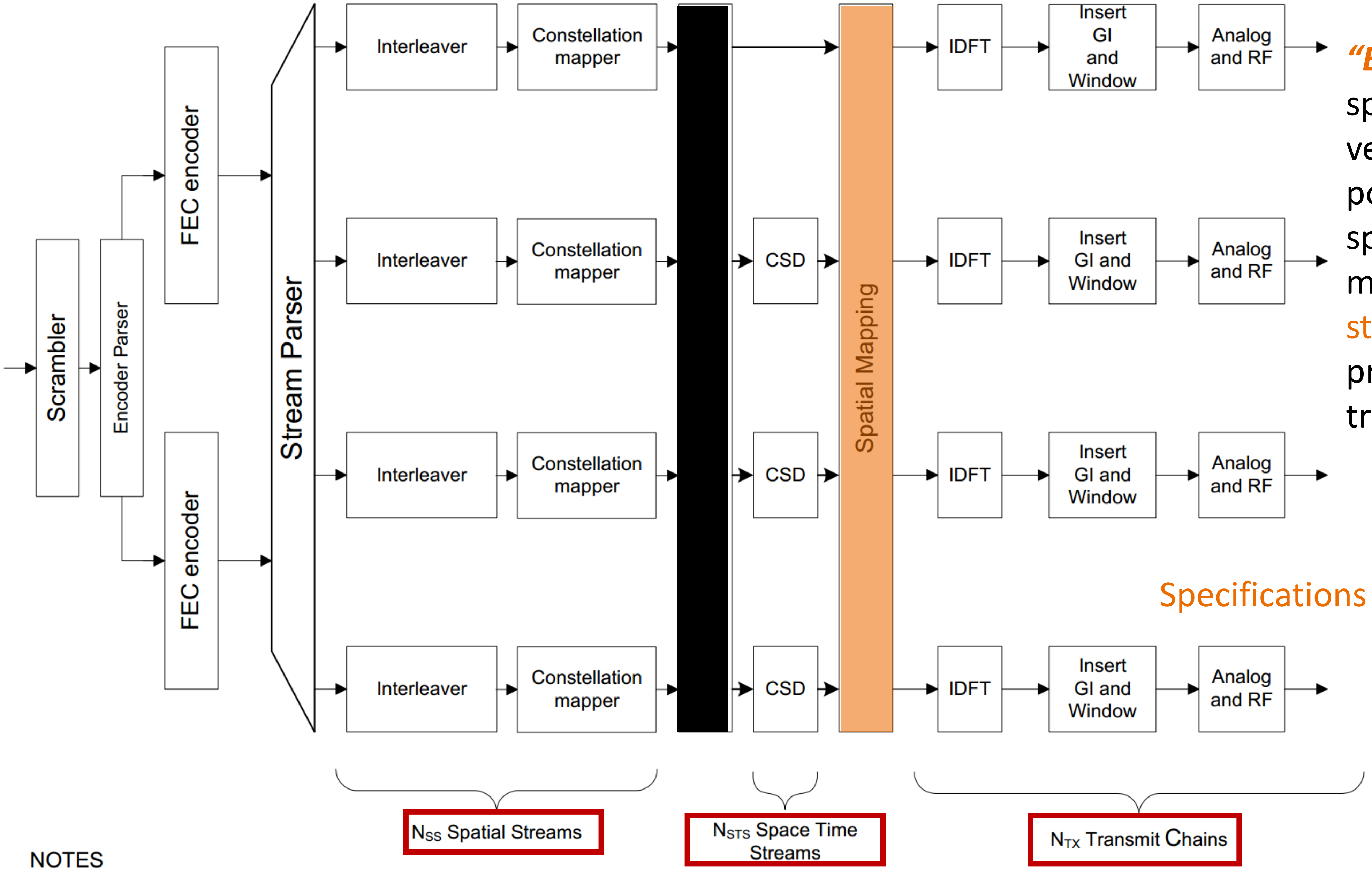
Recap: Multiple Spatial Stream (19.3.3)



*"**Beamforming**"*: Similar to spatial expansion, each vector of constellation points from all of the space-time streams is multiplied by a matrix of steering vectors to produce the input to the transmit chains."

NOTES

Recap: Multiple Spatial Stream (19.3.3)



"Beamforming: Similar to spatial expansion, each vector of constellation points from all of the space-time streams is multiplied by a matrix of steering vectors to produce the input to the transmit chains."

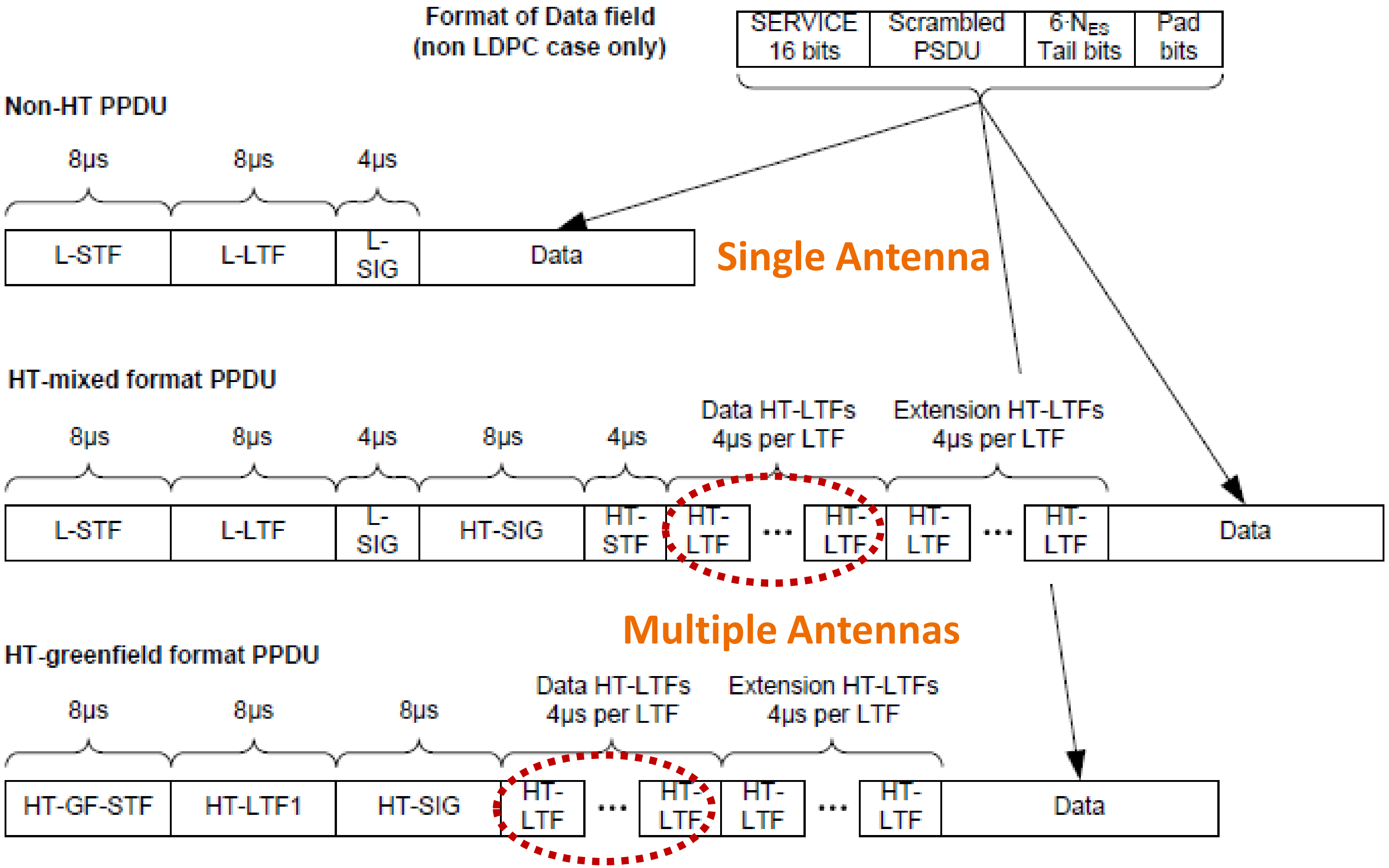
Specifications do not specify receiver!

NOTES

Remaining Questions

- How to estimate channel at the receiver?
- How to estimate channel at the transmitter?

Recap: Three PPDU Formats



- Non-HT： either the AP or the STA does not support HT
- HT-mixed: AP and STA support HT, however, other STAs may not support HT
- HT-greenfield: all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- L-SIG, HT-SIG: basic information about the PPDU

HT-LTF(19.3.9.4.6)

- "The HT-LTF provides a means for the receiver to estimate the MIMO channel between **the set of QAM mapper outputs** (or, if STBC is applied, the STBC encoder outputs) and **the receive chains**."
- "The HT-LTF portion has one or two parts. The first part consists of one, two, or four HT-LTFs that are **necessary for demodulation of the HT-Data portion** of the PPDU. These HT-LTFs are referred to as ***HT-DLTFs***."
- "The optional second part consists of zero, one, two, or four HT-LTFs that may be used to **sound extra spatial dimensions of the MIMO channel** that are not utilized by the HT-Data portion of the PPDU. These HT-LTFs are referred to as ***HT-ELTFs***."

$$N_{HT\text{-}LTF} = N_{HT\text{-}DLTF} + N_{HT\text{-}ELTF}$$

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0,\\ 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \qquad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain:

$$r_{HT-LTF}^{n, i_{TX}}(t) = \frac{1}{\sqrt{N_{STS} \cdot N_{HT-LTF}^{Tone}}} w_{T_{HT-LTFs}}(t) \qquad (19\text{-}25)$$

$$\cdot \sum_{k=-N_{SR}}^{N_{SR}} \sum_{i_{STS}=1}^{N_{STS}} [Q_k]_{i_{TX}, i_{STS}} [P_{HT\text{-}LTF}]_{i_{STS}, n} \Upsilon_k HT\text{-}LTF_k \exp(j 2\pi k \Delta_F (t - T_{GI} - T_{CS}^{i_{STS}}))$$

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \quad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain:

$$r_{HT-LTF}^{n,i_{TX}}(t) = \frac{1}{\sqrt{N_{STS} \cdot N_{HT-LTF}^{Tone}}} w_{T_{HT-LTFs}}(t) \quad (19\text{-}25)$$

$$\cdot \sum_{k=-N_{SR}}^{N_{SR}} \boxed{\sum_{i_{STS}=1}^{N_{STS}} [Q_k]_{i_{TX}, i_{STS}} [P_{HT\text{-}LTF}]_{i_{STS}, n}} \Upsilon_k HT\text{-}LTF_k \exp(j2\pi k \Delta_F(t - T_{GI} - T_{CS}^{i_{STS}}))$$

$\boldsymbol{Q_k}$ is the precoding matrix

$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

Generation of HT-LTF

$$\begin{aligned} HT\text{-}LTF_{-28,28} = \{&1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ &1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \end{aligned} \quad (19\text{-}23)$$

n-th HT-LTF on i_{Tx}-th transmit chain and k-th subcarrier:

$$[Q_k \, P_{HT-LTF}\,]_{i_{Tx},n} \, HT - LTF_k$$

Scalar

Generation of HT-LTF

$$HT\text{-}LTF_{-28,28} = \{1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ 1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \tag{19-23}$$

n-th HT-LTF on all the transmit chains and k-th subcarrier:

$$[Q_k\, P_{HT-LTF}\,]_n\, HT - LTF_k$$ **Column Vector**

Generation of HT-LTF

$$\begin{aligned} HT\text{-}LTF_{-28,28} = \{&1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 1, 1, -1, -1, 1, 1, -1, 1, -1, 1, 1, 1, 1, 0, \\ &1, -1, -1, 1, 1, -1, 1, -1, 1, -1, -1, -1, -1, -1, 1, 1, -1, -1, 1, -1, 1, -1, 1, 1, 1, 1, -1, -1\} \end{aligned} \tag{19-23}$$

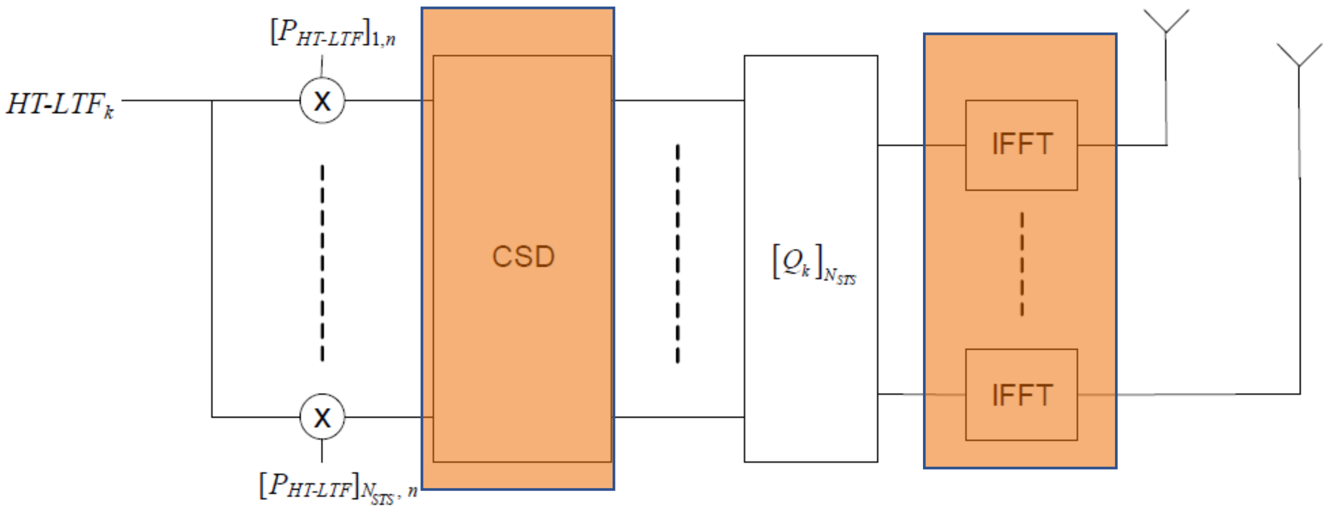
All 4 HT-LTFs on all the transmit chains and k-th subcarrier:

$$Q_k \, P_{HT-LTF} \, HT-LTF_k$$

Matrix

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:

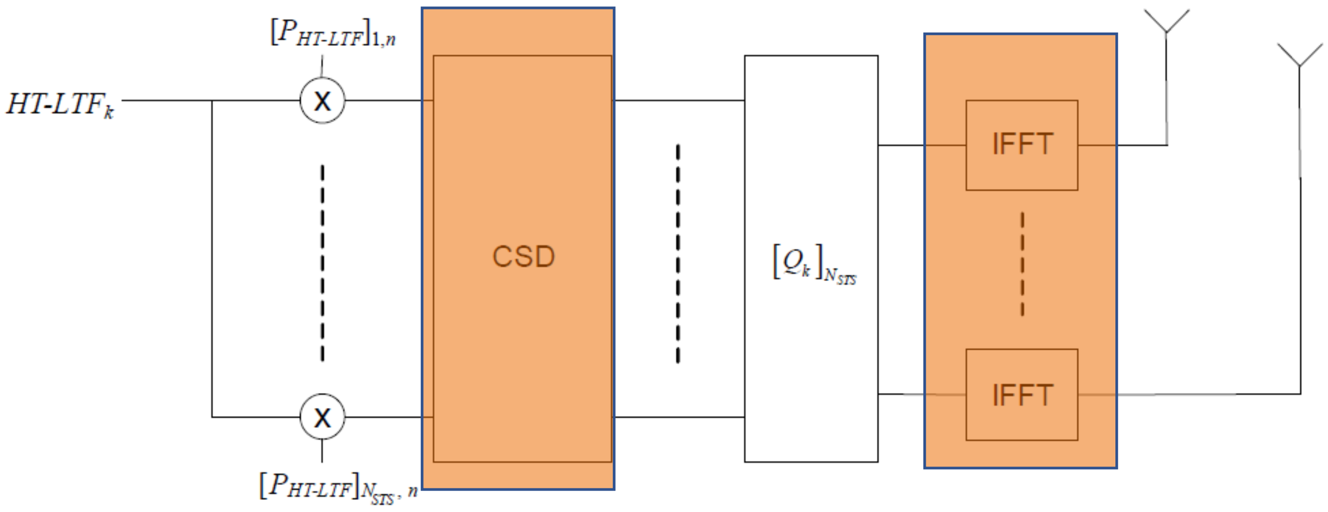


$$
\left[\boldsymbol{Y}_{t_1}^{(k)}, \boldsymbol{Y}_{t_2}^{(k)}, \cdots \boldsymbol{Y}_{N_{LTF}}^{(k)}\right] =
$$

$$
\underbrace{\begin{bmatrix} \tilde{h}_{11}^{(k)} & \tilde{h}_{12}^{(k)} & \cdots & \tilde{h}_{1N_{SS}}^{(k)} \\ \tilde{h}_{21}^{(k)} & \tilde{h}_{22}^{(k)} & & \tilde{h}_{2N_{SS}}^{(k)} \\ & \vdots & \ddots & \vdots \\ \tilde{h}_{N_{RX}1}^{(k)} & \tilde{h}_{N_{RX}2}^{(k)} & \cdots & \tilde{h}_{N_{RX}N_{SS}}^{(k)} \end{bmatrix}}_{\boldsymbol{H_k Q_k}} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + \left[\boldsymbol{Z}_{t_1}^{(k)}, \boldsymbol{Z}_{t_2}^{(k)}, \cdots \boldsymbol{Z}_{N_{LTF}}^{(k)}\right].
$$

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:



$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

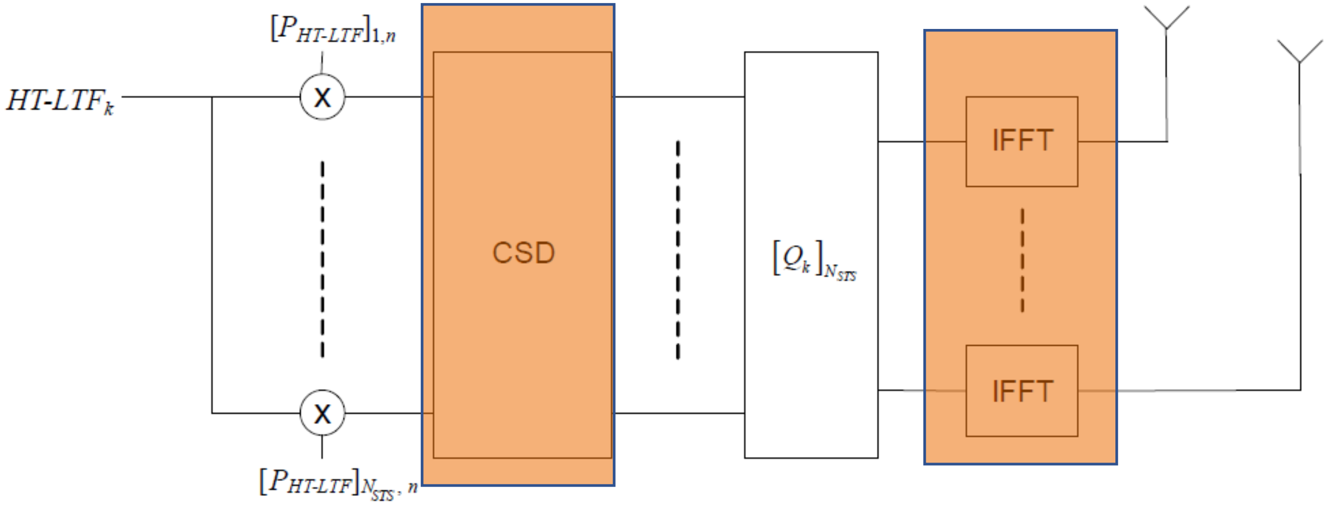
$$\boldsymbol{P}_{HTLTF}\boldsymbol{P}'_{HTLTF} = 4\boldsymbol{I} \Rightarrow \boldsymbol{P}_{HTLFT}^{-1} = \frac{1}{4}\boldsymbol{P}'_{HTLTF}$$

$$\left[\boldsymbol{Y}_{t_1}^{(k)}, \boldsymbol{Y}_{t_2}^{(k)}, \cdots \boldsymbol{Y}_{N_{LTF}}^{(k)}\right] =$$

$$\underbrace{\begin{bmatrix} \tilde{h}_{11}^{(k)} & \tilde{h}_{12}^{(k)} & \cdots & \tilde{h}_{1N_{SS}}^{(k)} \\ \tilde{h}_{21}^{(k)} & \tilde{h}_{22}^{(k)} & & \tilde{h}_{2N_{SS}}^{(k)} \\ \vdots & & \ddots & \vdots \\ \tilde{h}_{N_{RX}1}^{(k)} & \tilde{h}_{N_{RX}2}^{(k)} & \cdots & \tilde{h}_{N_{RX}N_{SS}}^{(k)} \end{bmatrix}}_{\boldsymbol{H}_k\boldsymbol{Q}_k} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + \left[\boldsymbol{Z}_{t_1}^{(k)}, \boldsymbol{Z}_{t_2}^{(k)}, \cdots \boldsymbol{Z}_{N_{LTF}}^{(k)}\right].$$

Generation of HT-LTF

All 4 HT-LTFs on all the transmit chains and k-th subcarrier:



$$P_{HTLTF} = \begin{bmatrix} +1 & -1 & +1 & +1 \\ +1 & +1 & -1 & +1 \\ +1 & +1 & +1 & -1 \\ -1 & +1 & +1 & +1 \end{bmatrix}$$

$$\boxed{\boldsymbol{P}_{HTLTF}\boldsymbol{P}'_{HTLTF} = 4\boldsymbol{I} \Rightarrow \boldsymbol{P}^{-1}_{HTLFT} = \frac{1}{4}\boldsymbol{P}'_{HTLTF}}$$

- IEEE 802.11 does not define Q_k
- Transmitter determines it locally, but the receiver may not know

$$\left[\boldsymbol{Y}^{(k)}_{t_1}, \boldsymbol{Y}^{(k)}_{t_2}, \cdots \boldsymbol{Y}^{(k)}_{N_{LTF}}\right] =$$

$$\underbrace{\begin{bmatrix} \tilde{h}^{(k)}_{11} & \tilde{h}^{(k)}_{12} & \cdots & \tilde{h}^{(k)}_{1N_{SS}} \\ \tilde{h}^{(k)}_{21} & \tilde{h}^{(k)}_{22} & & \tilde{h}^{(k)}_{2N_{SS}} \\ & \vdots & \ddots & \vdots \\ \tilde{h}^{(k)}_{N_{RX}1} & \tilde{h}^{(k)}_{N_{RX}2} & \cdots & \tilde{h}^{(k)}_{N_{RX}N_{SS}} \end{bmatrix}}_{\boldsymbol{H}_k\boldsymbol{Q}_k} \cdot \mathbf{P_{HTLF}} \cdot \mathrm{HTLF_k} + \left[\boldsymbol{Z}^{(k)}_{t_1}, \boldsymbol{Z}^{(k)}_{t_2}, \cdots \boldsymbol{Z}^{(k)}_{N_{LTF}}\right].$$

Channel Estimation

- $Y = HX + Z$, estimate H with the knowledge of X and Y

- **Approach 1: Zero-forcing**
  - $\widehat{H} = YX^{-1} = H + ZX^{-1}$

- **Approach 2: MMSE**
  - $\widehat{H} = YP = HXP + ZP \Longrightarrow \ Error\colon \Delta = \widehat{H} - H = H(XP - I) + ZP$
  - $E\ [\Delta^H \Delta] = E[(XP - I)^H H^H H(XP - I) + P^H Z^H ZP]$
  - $E\ [\Delta^H \Delta] = N\sigma^2 (XP - I)^H (XP - I) + N\sigma_z^2 P^H P$
  - Thus, we should $\min_{P} trace[\, N\sigma^2 (XP - I)^H (XP - I) + N\sigma_z^2 P^H P]$

$$\min_P trace[\, N\sigma^2 (XP-I)^H (XP-I) + N\sigma_z^2 P^H P]$$
$$= \min_P trace[\, P^H (N\sigma^2 X^H X + N\sigma_z^2 I) P - N\sigma^2 P^H X^H - N\sigma^2 XP + N\sigma^2 I]$$

A is not a function of **X**	[7] $\frac{\partial \operatorname{tr}(\mathbf{AX})}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{XA})}{\partial \mathbf{X}} =$	$\mathbf{A}$	$\mathbf{A}^\top$
A is not a function of **X**	[5] $\frac{\partial \operatorname{tr}(\mathbf{AX}^\top)}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{X}^\top\mathbf{A})}{\partial \mathbf{X}} =$	$\mathbf{A}^\top$	$\mathbf{A}$
A is not a function of **X**	[5] $\frac{\partial \operatorname{tr}(\mathbf{X}^\top\mathbf{AX})}{\partial \mathbf{X}} =$	$\mathbf{X}^\top(\mathbf{A}+\mathbf{A}^\top)$	$(\mathbf{A}+\mathbf{A}^\top)\mathbf{X}$
A is not a function of **X**	[5] $\frac{\partial \operatorname{tr}(\mathbf{X}^{-1}\mathbf{A})}{\partial \mathbf{X}} =$	$-\mathbf{X}^{-1}\mathbf{AX}^{-1}$	$-(\mathbf{X}^{-1})^\top\mathbf{A}^\top(\mathbf{X}^{-1})^\top$
A, **B** are not functions of **X**	$\frac{\partial \operatorname{tr}(\mathbf{AXB})}{\partial \mathbf{X}} = \frac{\partial \operatorname{tr}(\mathbf{BAX})}{\partial \mathbf{X}} =$	$\mathbf{BA}$	$\mathbf{A}^\top\mathbf{B}^\top$
A, **B**, **C** are not functions of **X**	$\frac{\partial \operatorname{tr}(\mathbf{AXBX}^\top\mathbf{C})}{\partial \mathbf{X}} =$	$\mathbf{BX}^\top\mathbf{CA}+\mathbf{B}^\top\mathbf{X}^\top\mathbf{A}^\top\mathbf{C}^\top$	$\mathbf{A}^\top\mathbf{C}^\top\mathbf{XB}^\top+\mathbf{CAXB}$

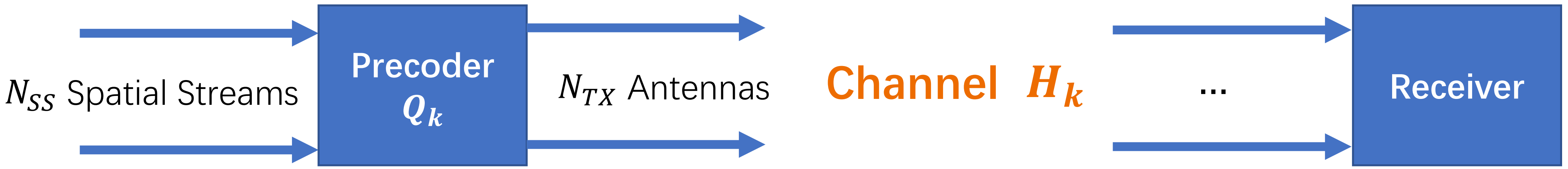
Applying derivative w.r.t. P, we can obtain the optimal P for channel estimation

Can we use the ML approach to estimate the channel?

D. P. Palomar and Y. Jiang, “MIMO Transceiver Design via Majorization Theory”, 2006

Explicit feedback beamforming (19.3.12.3.1)

- "In explicit beamforming, in order for STA A to transmit a beamformed packet to STA B, STA B measures the channel matrices and sends STA A either **the effective channel** or the **beamforming feedback matrix**"

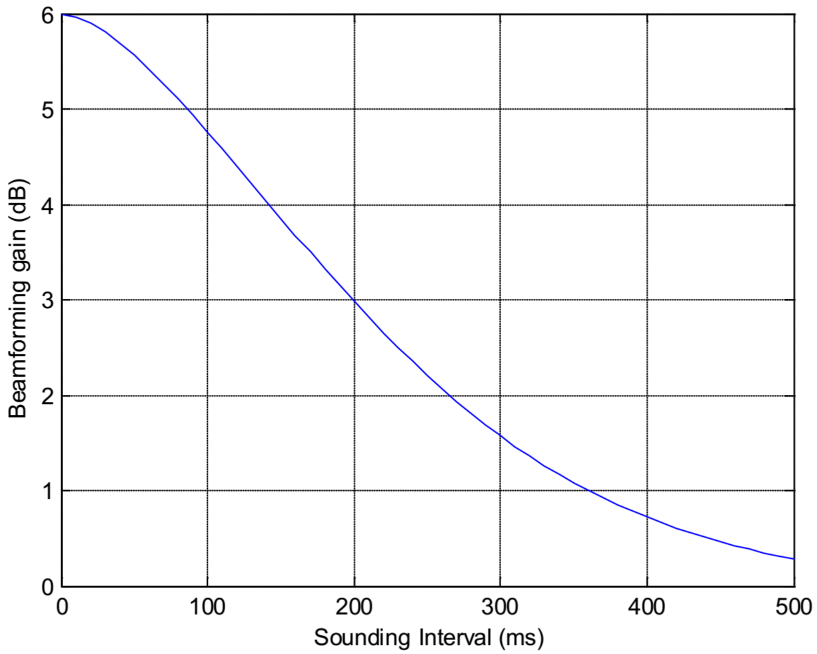


- Receiver can estimate $H_{eff,k} = H_k Q_k$ and
  - Feedback $H_{eff,k}$ directly to the transmitter, the transmitter calculates the new precoder Q_k
  - Or feedback an update matrix V_k, the transmitter use $Q_k V_k$ as the new precoder matrix

Encoding of Feedback Matrix （19.3.12.3.3）

- The channel feedback consists of the following three parts:
  - Real part of each element in the channel matrix
  - Imaginary part of each element in the channel matrix
  - A common scaling factor
- Beamforming feedback matrix: noncompressed and compressed

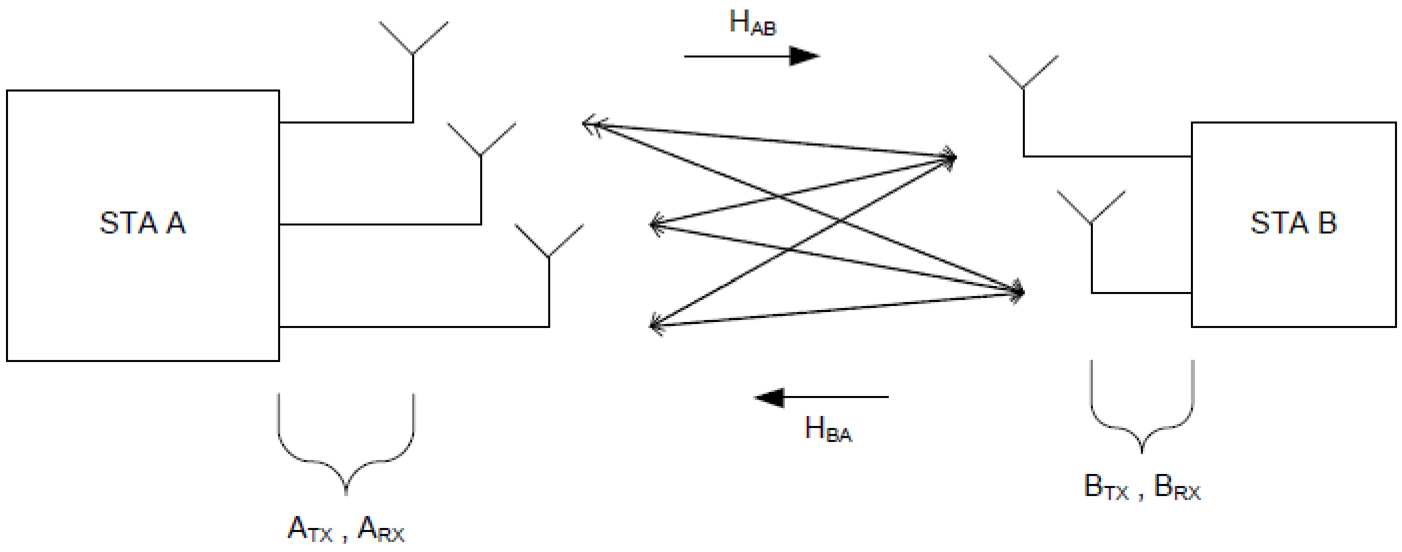
Implementation – Sounding Frequency



- Feedback can incur significant overhead
- Tradeoff between sounding frequency and beamforming gain
  - Indoor channel environment is benign

Implicit Feedback Beamforming (19.3.12.2)

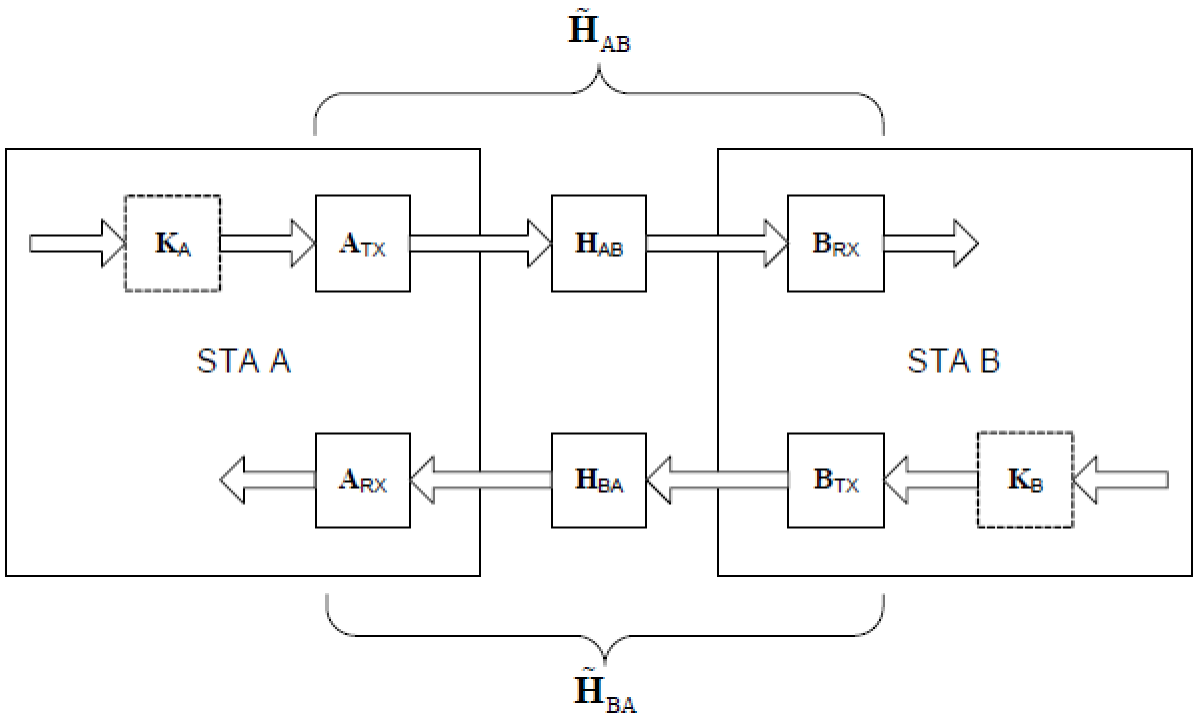
- Transmitter relies on reciprocity in the time division duplex channel to estimate the channel
- The forward and reverse channels are reciprocal, **the channel from STA A to STA B** in subcarrier k is the **matrix transpose** of **the channel from STA B to STA A** in subcarrier k to within a complex scaling factor



$$H_{\mathrm{AB},k} = \rho[H_{\mathrm{BA},k}]^T .$$

"The amplitude and phase responses of the transmit and receive chains can be expressed as **diagonal matrices with complex valued diagonal entries**"

Calibration



Although the wireless channel is reciprocal, the baseband-to-baseband channel is not.

$$\tilde{H}_{AB,k} = B_{RX,k} H_{AB,k} A_{TX,k}$$

$$\tilde{H}_{BA,k} = A_{RX,k} H_{BA,k} B_{TX,k}$$

Not reciprocal

Solution: choose

$$K_{A,k} = \alpha_{A,k}[A_{TX,k}]^{-1} A_{RX,k}$$

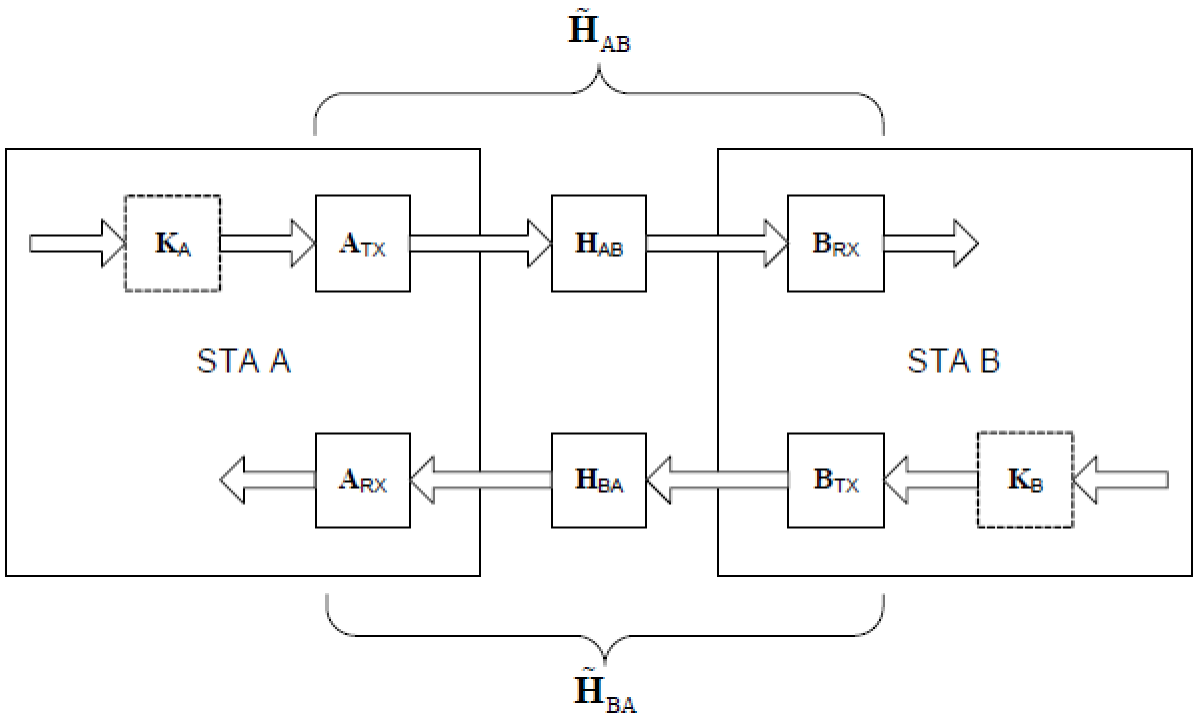
and

$$K_{B,k} = \alpha_{B,k}[B_{TX,k}]^{-1} B_{RX,k}$$

Such that

$$\tilde{H}_{AB,k} K_{A,k} = \rho[\tilde{H}_{BA,k} K_{B,k}]^{T}$$

Calibration



Although the wireless channel is reciprocal, the baseband-to-baseband channel is not.

Change frequently

$$\tilde{H}_{AB,k} = B_{RX,k} H_{AB,k} A_{TX,k}$$

$$\tilde{H}_{BA,k} = A_{RX,k} H_{BA,k} B_{TX,k}$$

Not reciprocal

Solution: choose

$$K_{A,k} = \alpha_{A,k}[A_{TX,k}]^{-1} A_{RX,k}$$

and

Change slowly

$$K_{B,k} = \alpha_{B,k}[B_{TX,k}]^{-1} B_{RX,k}$$

Such that

$$\tilde{H}_{AB,k} K_{A,k} = \rho[\tilde{H}_{BA,k} K_{B,k}]^{T}$$

How to calibrate

- Calibration procedure:
  - STA A sends STA B a sounding PPDU, the reception of which allows STA B to estimate the channel matrices $\widetilde{H}_{AB,k}$.
  - STA B sends STA A a sounding PPDU, the reception of which allows STA A to estimate the channel matrices $\widetilde{H}_{BA,k}$.
  - STA B sends the quantized estimation of $\widetilde{H}_{AB,k}$ to STA A.
  - STA A uses its local estimation of $\widetilde{H}_{BA,k}$ and the quantized estimation of $\widetilde{H}_{AB,k}$ received from STA B to compute the correction matrices

Homework (3.23)

Reading

IEEE Std 802.11™-2020 : Section 19.3.9, 19.3.12

Paper 1 – Chapter 3

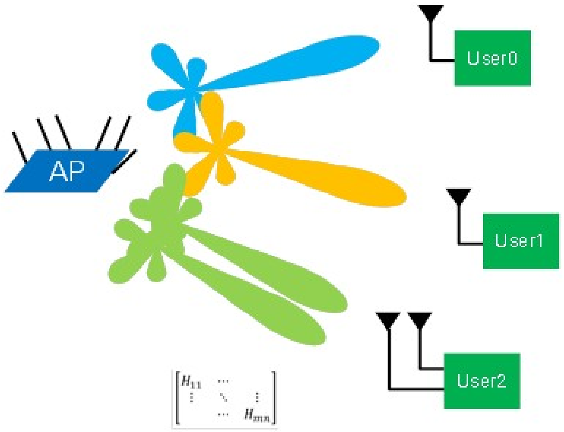
Paper 2 *

Assignment 4

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \cdots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \cdots \\ H_k \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \cdots \\ x_m \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ \cdots \\ z_k \end{pmatrix}$$



m antennas at the AP, single antenna at each STA, k STAs

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{pmatrix} = (P_1\ P_2\ \dots P_k) \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_k \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ \dots \\ x_m \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

m antennas at the AP, single antenna at each STA, k STAs

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} (P_1 \; P_2 \; \dots P_k) \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_k \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

m antennas at the AP, single antenna at each STA, k STAs

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \sum_{i=1}^{k} P_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

m antennas at the AP, single antenna at each STA, k STAs

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \sum_{i=1}^{k} P_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

$$y_j = H_j \sum_{i=1}^{k} P_i s_i + z_j = H_j P_j s_j + z_j + H_j \sum_{i \neq j, i=1}^{k} P_i s_i$$

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \sum_{i=1}^{k} P_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

$$y_j = H_j \sum_{i=1}^{k} P_i s_i + z_j = H_j P_j s_j + z_j + H_j \sum_{i \neq j, i=1}^{k} P_i s_i$$

We must guarantee $H_j P_i = 0, when\ i \neq j$

MU-MIMO

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \sum_{i=1}^{k} P_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

$$y_j = H_j \sum_{i=1}^{k} P_i s_i + z_j = H_j P_j s_j + z_j + H_j \sum_{i \neq j, i=1}^{k} P_i s_i$$

We must guarantee $H_j P_i = 0, when\ i \neq j$

To find P_1:

1. Find the null space bases of $\begin{pmatrix} H_2 \\ H_3 \\ \dots \\ H_k \end{pmatrix}$
2. n=m-k+1. Let $E_1, E_2, \dots, E_n$ be the bases
3. $P_1 = (H_1 E_1)\, E_1 + \ \dots + (H_1 E_n)\, E_n$

MU-MIMO

To find P_1:

1. Find the null space bases of $\begin{pmatrix} H_2 \\ H_3 \\ \dots \\ H_k \end{pmatrix}$
2. n=m-k-1. Let $E_1, E_2, \dots, E_n$ be the bases
3. $P_1 = (H_1 E_1)\, E_1 + \dots + (H_n E_n)\, E_n$

$m \geq k$

- The AP should isolate the inter-cell interference

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_k \end{pmatrix} = \begin{pmatrix} H_1 \\ H_2 \\ \dots \\ H_k \end{pmatrix} \sum_{i=1}^{k} P_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_k \end{pmatrix}$$

$$y_j = H_j \sum_{i=1}^{k} P_i s_i + z_j = H_j P_j s_j + z_j + H_j \sum_{i \neq j, i=1}^{k} P_i s_i$$

We must guarantee $H_j P_i = 0, when\ i \neq j$

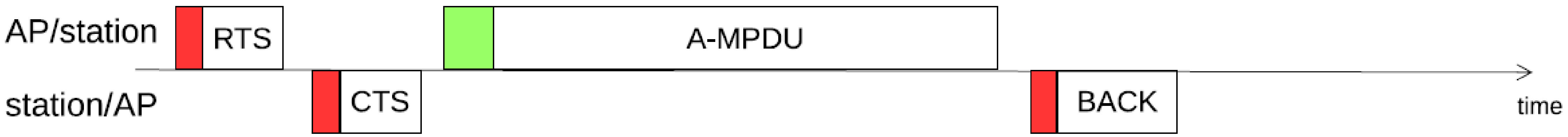
UL MU-MIMO

- The AP can do joint detection --- like SU-MIMO

$$\begin{pmatrix} y_1 \\ y_2 \\ \dots \\ y_m \end{pmatrix} = \sum_{i=1}^{k} H_i s_i + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_m \end{pmatrix} = (H_1\ H_2\ \dots H_k) \begin{pmatrix} s_1 \\ s_2 \\ \dots \\ s_k \end{pmatrix} + \begin{pmatrix} z_1 \\ z_2 \\ \dots \\ z_m \end{pmatrix}$$

MU-MIMO in IEEE802.11

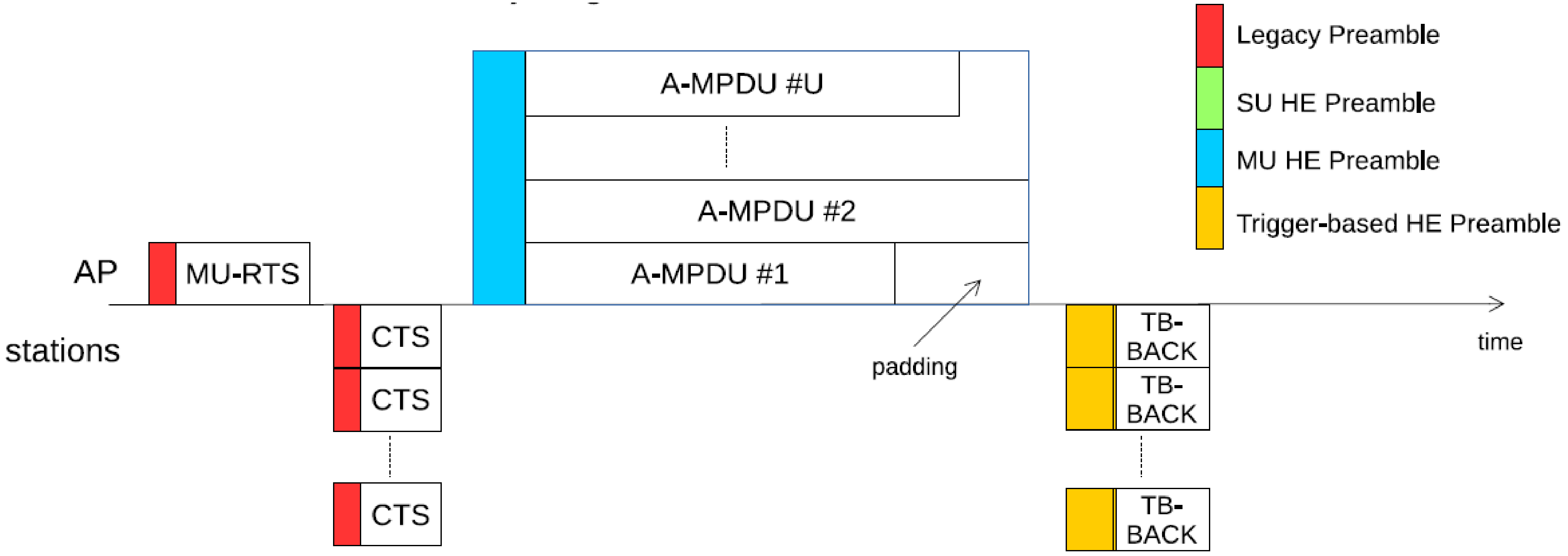
- 802.11ac (Very High Throughput, VHT) supports downlink MU-MIMO
- 802.11ax (High Efficiency, HE) supports both downlink and uplink MU-MIMO



a) Single-user transmission

MU-MIMO in IEEE802.11

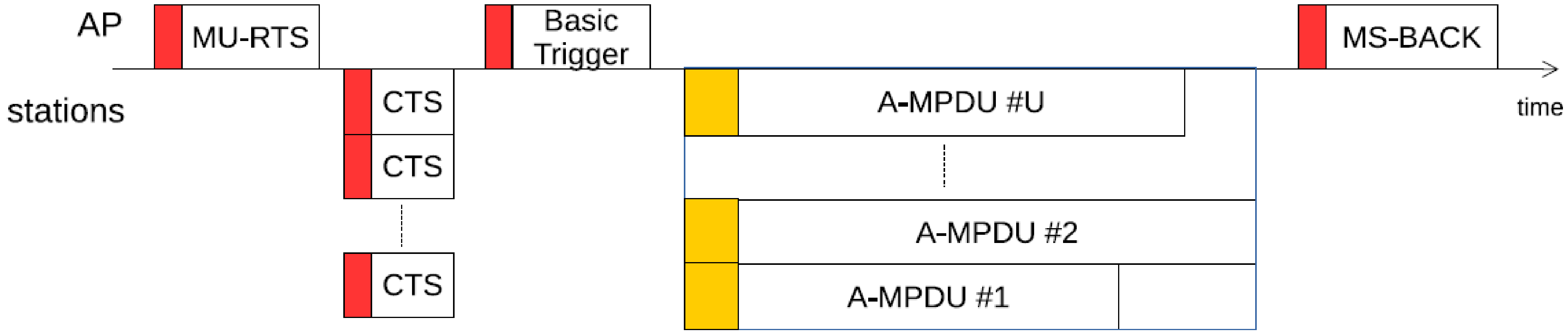
- 802.11ac supports downlink MU-MIMO
- 802.11ax supports both downlink and uplink MU-MIMO



b) Downlink multiuser transmission

MU-MIMO in IEEE802.11

- 802.11ac supports downlink MU-MIMO
- 802.11ax supports both downlink and uplink MU-MIMO



c) Uplink multiuser transmission

Homework (3.30)

Assignment 4