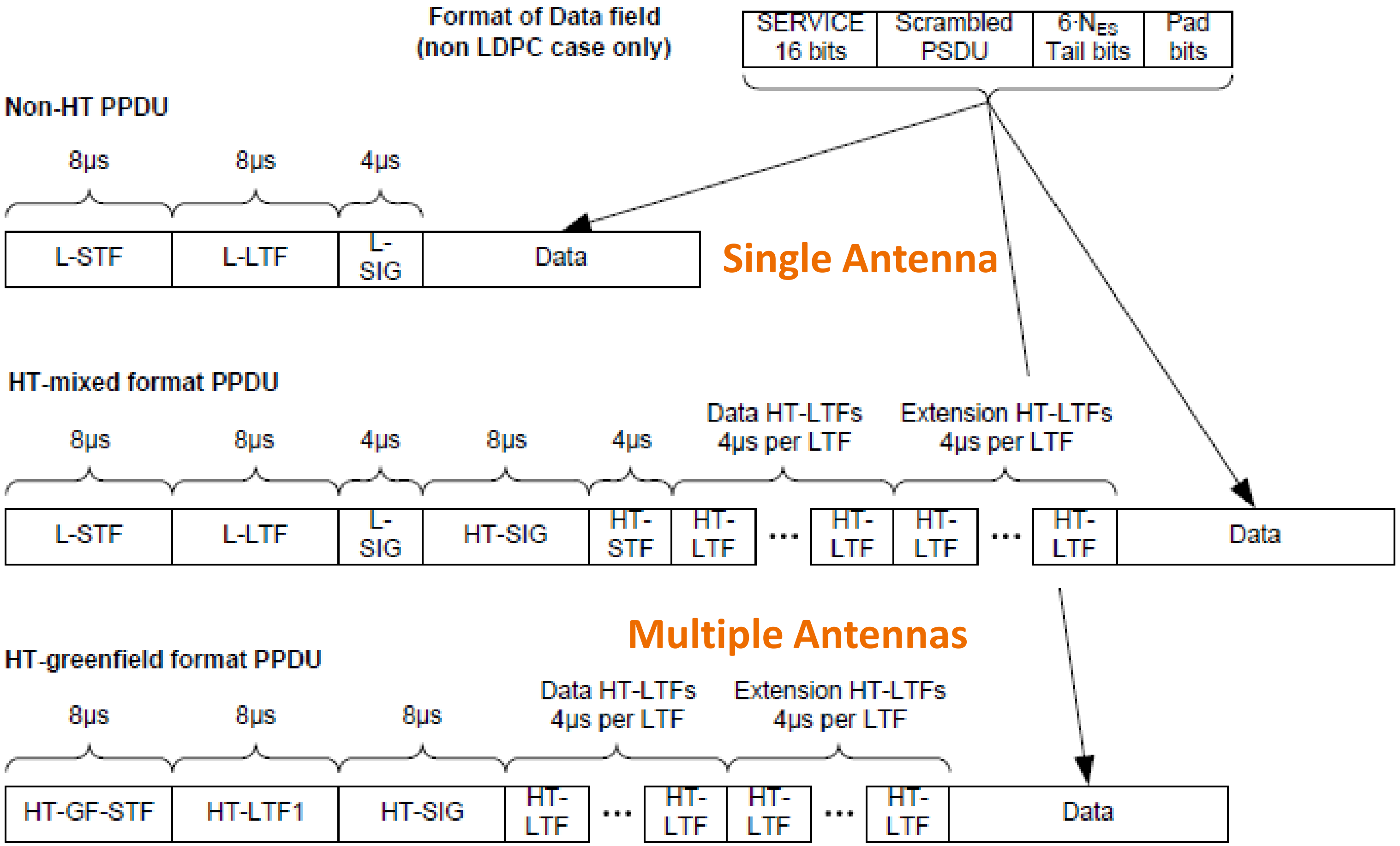
MIMO Transmitter in IEEE802.11

Lecturer: Dr. Rui Wang

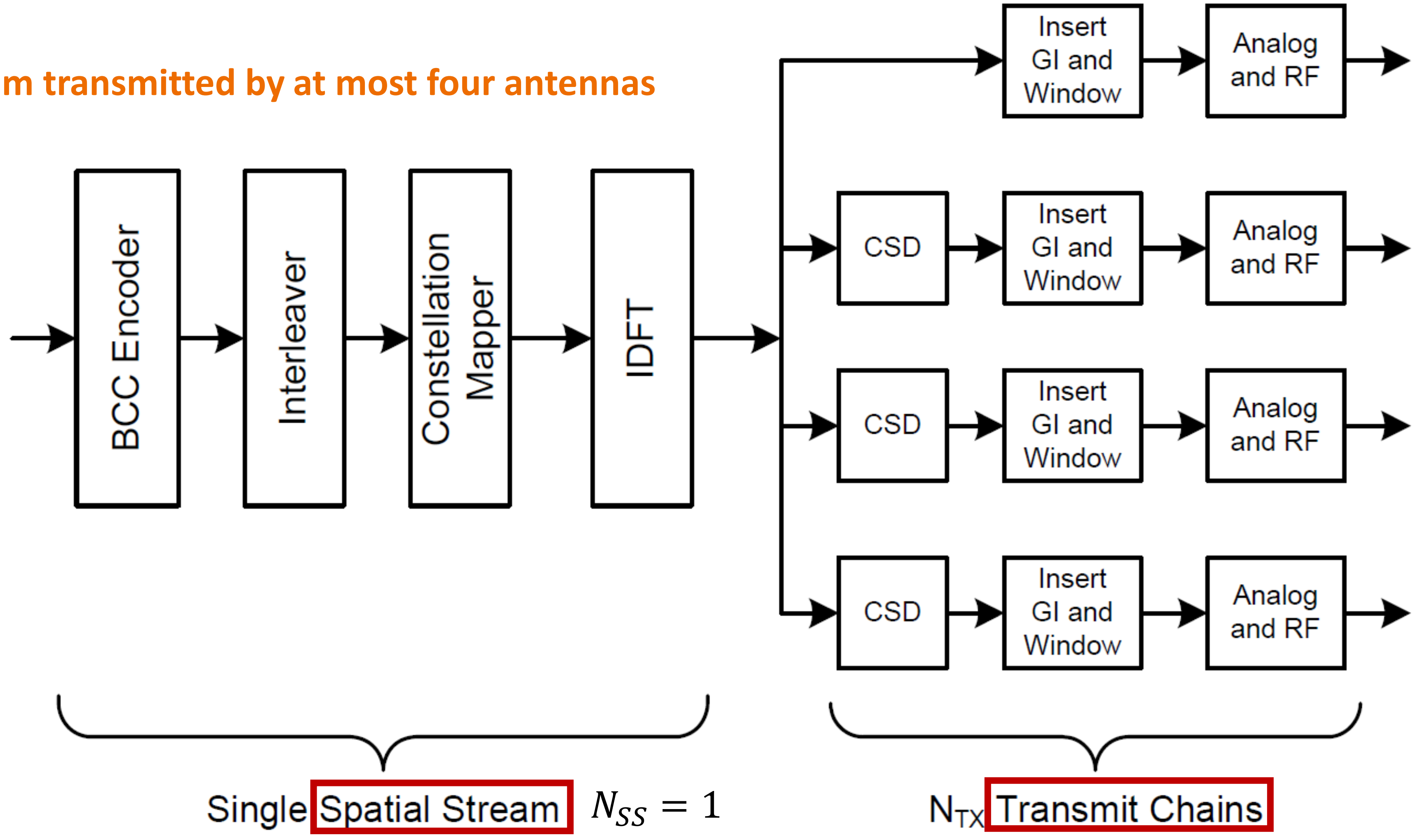
Recap: Three PPDU Formats



- Non-HT： either the AP or the STA does not support HT
- HT-mixed: AP and STA support HT, however, other STAs may not support HT
- HT-greenfield: all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- L-SIG, HT-SIG: basic information about the PPDU

Single Spatial Stream (19.3.3)

One data stream transmitted by at most four antennas



"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

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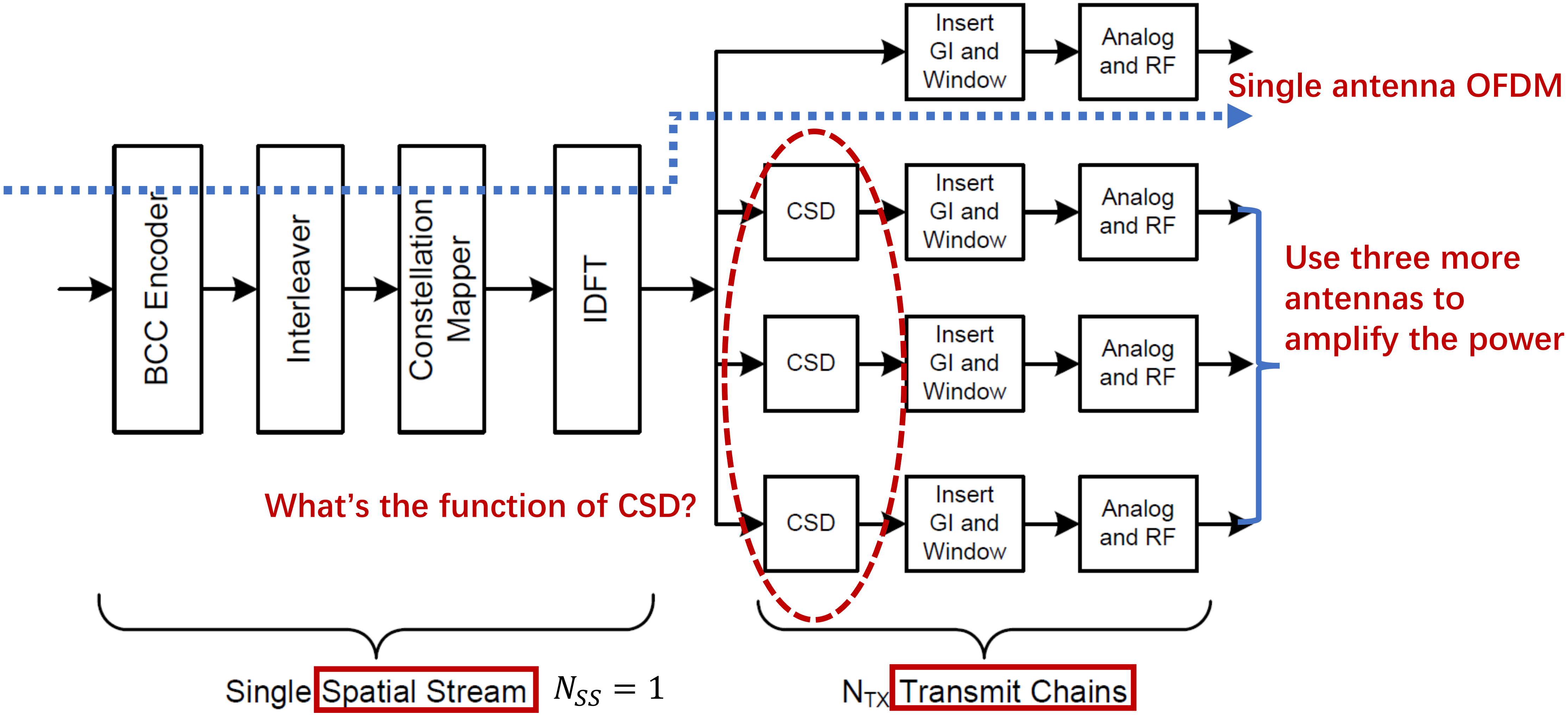

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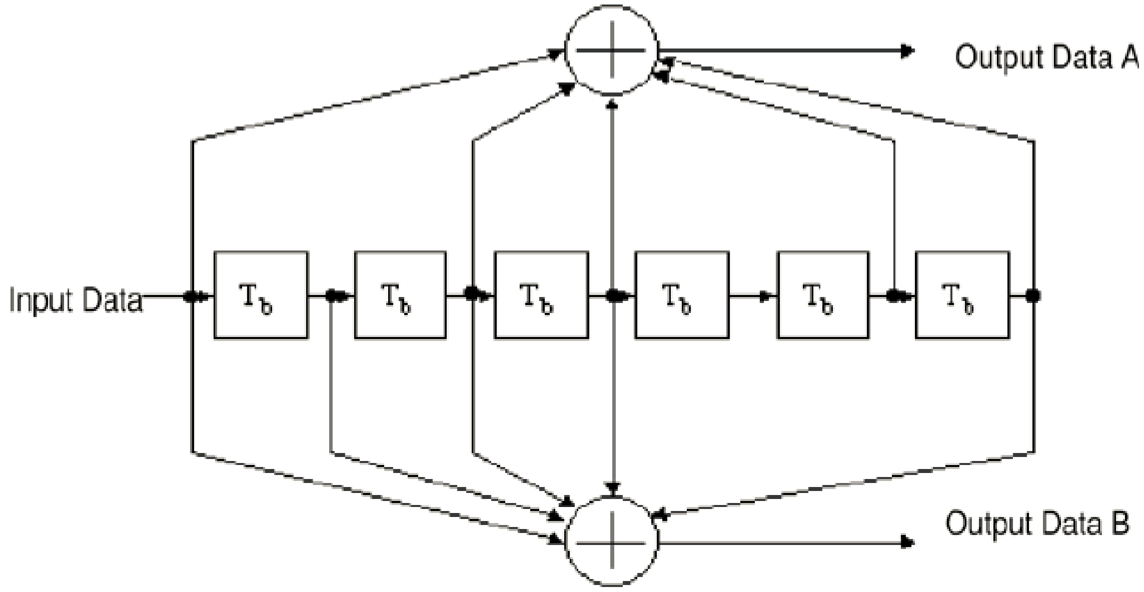
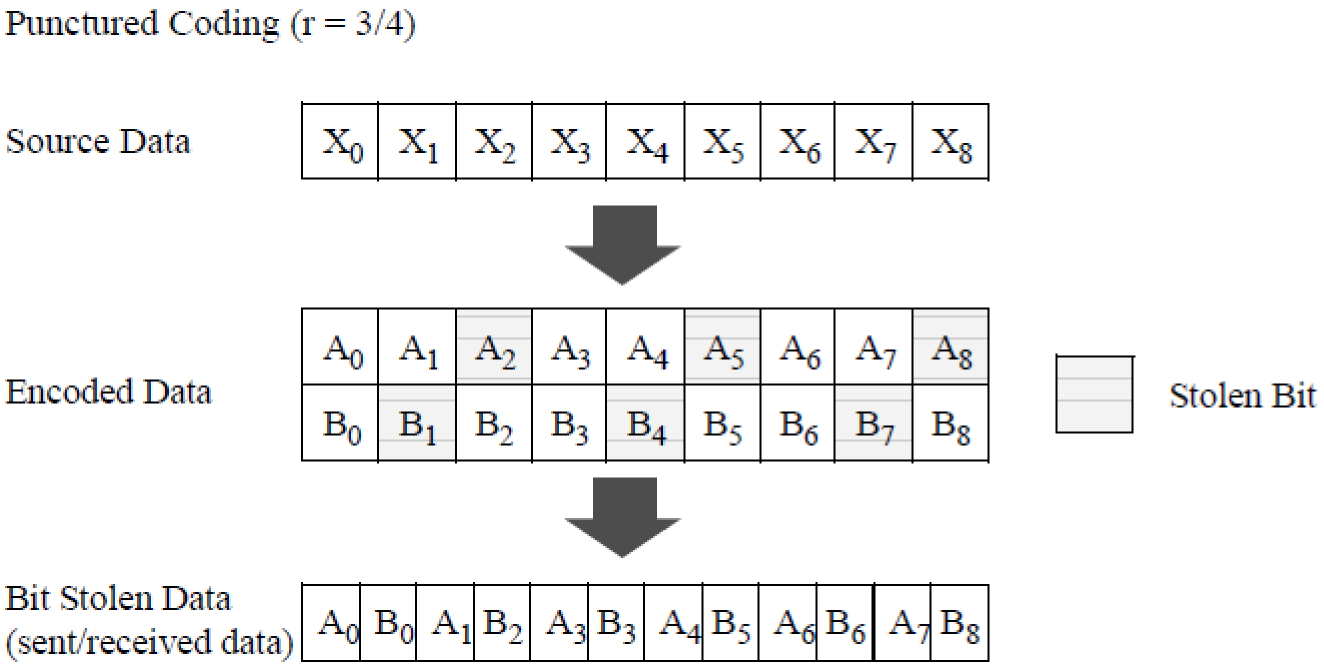
Channel Coding

- **Binary convolutional code (BCC) – 17.3.5.6**





- **Data interleaving – 17.3.5.7**

Interleaves the bits of each spatial stream (changes order of bits) to prevent long sequences of adjacent noisy bits.

Modulation and Coding Scheme (MCS, 19.5)

Table 19-27—MCS parameters for mandatory 20 MHz, N_{SS} = 1, N_{ES} = 1

MCS Index	Modulation	R	$N_{BPSCS}(i_{SS})$	N_{SD}	N_{SP}	N_{CBPS}	N_{DBPS}	Data rate (Mb/s)	
								800 ns GI	400 ns GI (see NOTE)
0	BPSK	1/2	1	52	4	52	26	6.5	7.2
1	QPSK	1/2	2	52	4	104	52	13.0	14.4
2	QPSK	3/4	2	52	4	104	78	19.5	21.7
3	16-QAM	1/2	4	52	4	208	104	26.0	28.9
4	16-QAM	3/4	4	52	4	208	156	39.0	43.3
5	64-QAM	2/3	6	52	4	312	208	52.0	57.8
6	64-QAM	3/4	6	52	4	312	234	58.5	65.0
7	64-QAM	5/6	6	52	4	312	260	65.0	72.2
NOTE—Support of 400 ns GI is optional on transmit and receive.									

N_{BPSCS}: Number of coded bits per subcarrier

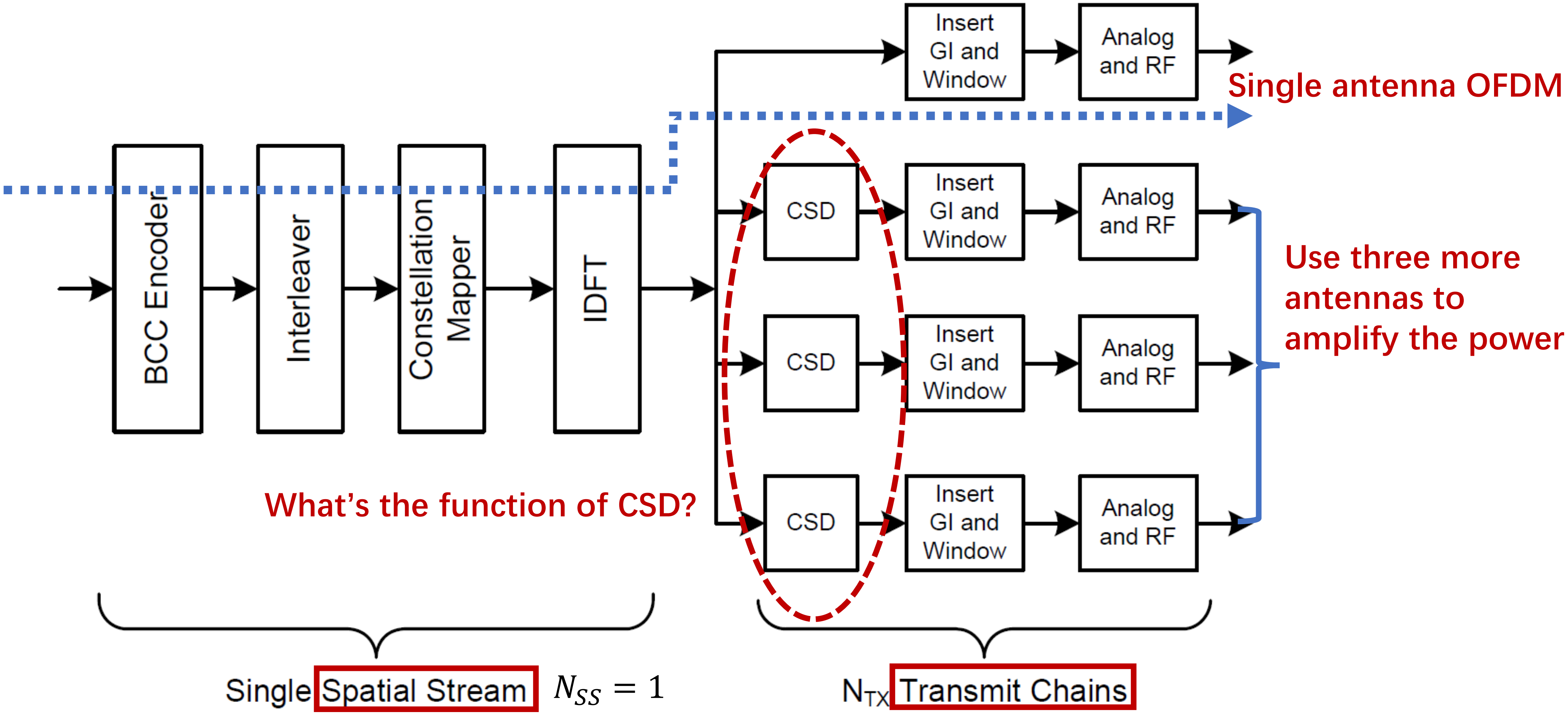
N_{SD}: Number of data subcarriers

N_{SP}: Number of pilot subcarriers

N_{CBPS}: Number of coded bits per symbol

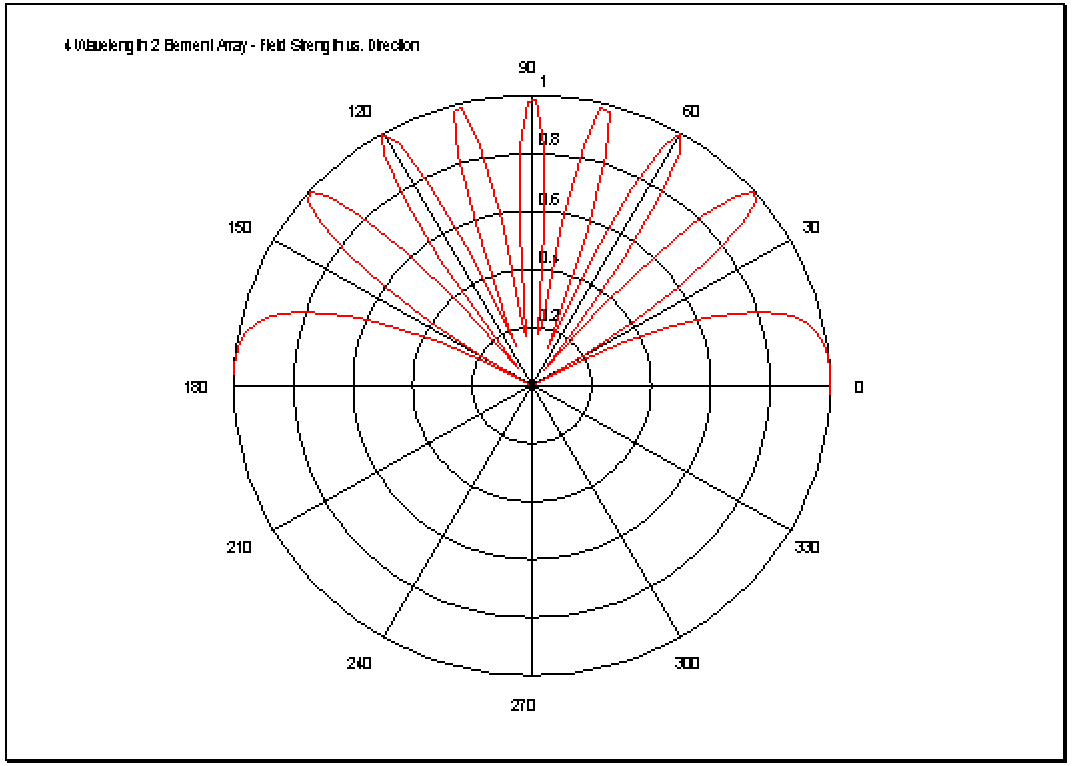
N_{DBPS}: Number of data bits per symbol

Recap: Single Spatial Stream (19.3.3)



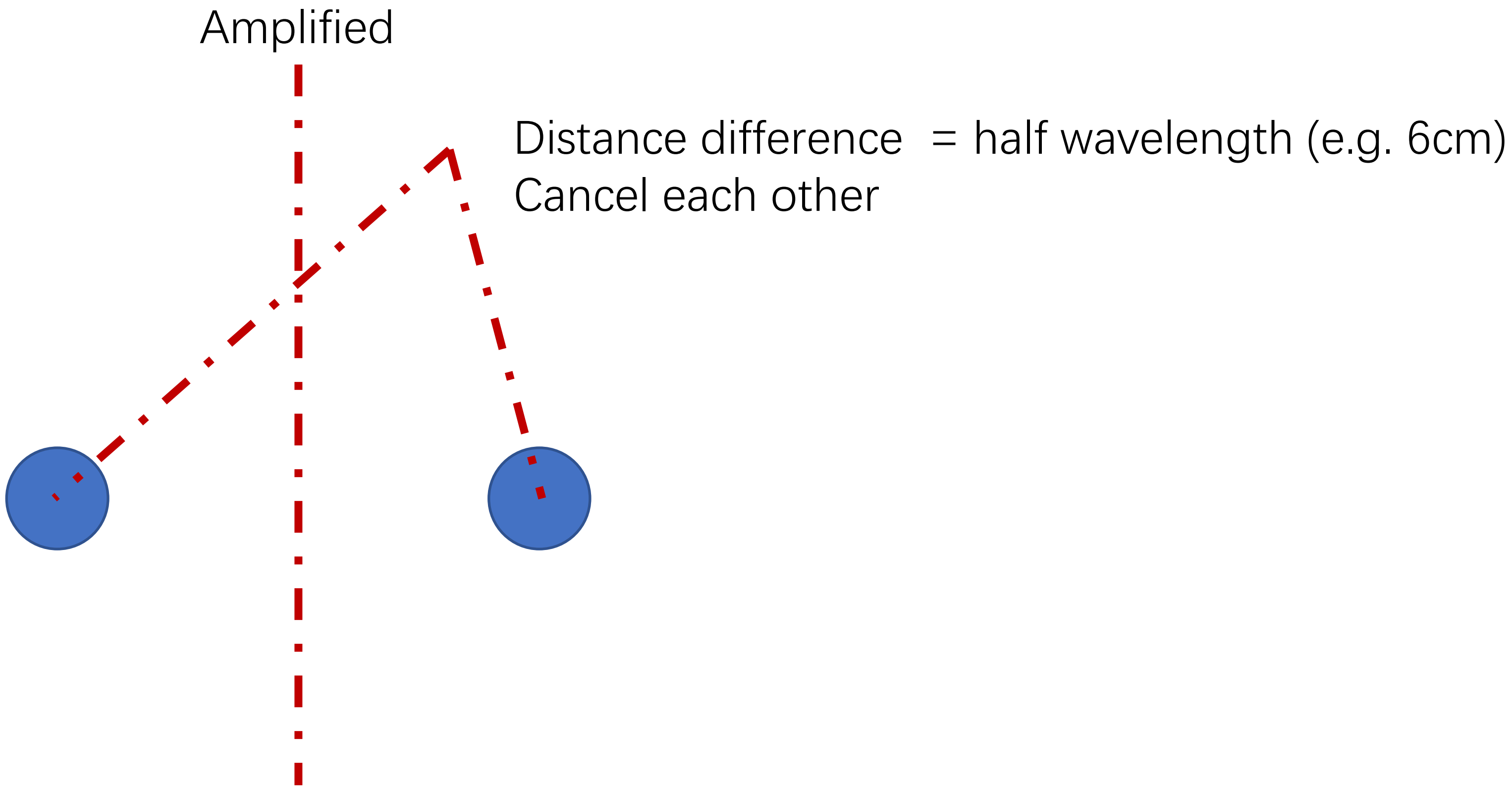
"… generate the HT-SIG of the HT-mixed format PPDU. These transmitter blocks are also used to generate the non-HT portion of the HT-mixed format PPDU, except that the BCC encoder and interleaver are not used when generating the L-STF and L-LTFs."

Array Pattern



- Why antenna specific delay (cyclic shift)?
- If two antenna ports transmit the same signal at the same time, there might be effect of beamforming
- Not desired at the phase of packet synchronization
- Cyclic shift is applied to avoid the beamforming effect

Array Pattern



Cyclic Shift (19.3.9.3.2)

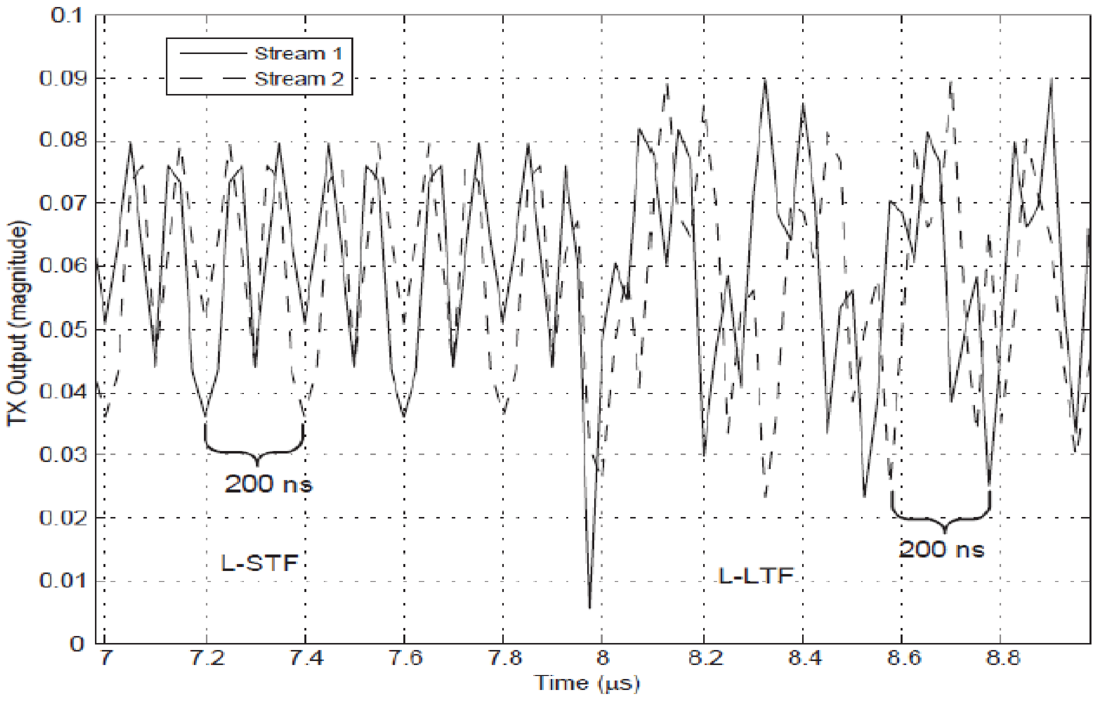


Table 19-9—Cyclic shift for non-HT portion of packet

$T_{CS}^{i_{TX}}$ values for non-HT portion of packet				
Number of transmit chains	Cyclic shift for transmit chain 1 (ns)	Cyclic shift for transmit chain 2 (ns)	Cyclic shift for transmit chain 3 (ns)	Cyclic shift for transmit chain 4 (ns)
1	0	—	—	—
2	0	–200	—	—
3	0	–100	–200	—
4	0	–50	–100	–150

50ns is the duration of one sample

Let s(t) be the signal without cyclic shift, TCS is the cyclic shift value, then the cyclic shift is as follows.

$$s_{CS}(t;T_{CS})\Big|_{T_{CS}<0} = \begin{cases} s(t-T_{CS}) & 0 \le t < T+T_{CS} \\ s(t-T_{CS}-T) & T+T_{CS} \le t \le T \end{cases}$$

"Cyclic shifts are used to prevent **unintentional beamforming** when the same signal or scalar multiples of one signal are transmitted through different spatial streams or transmit chains."

Cyclic Shift is different from the Cyclic Prefix of OFDM, which is referred to Guard Interval (GI) in standard

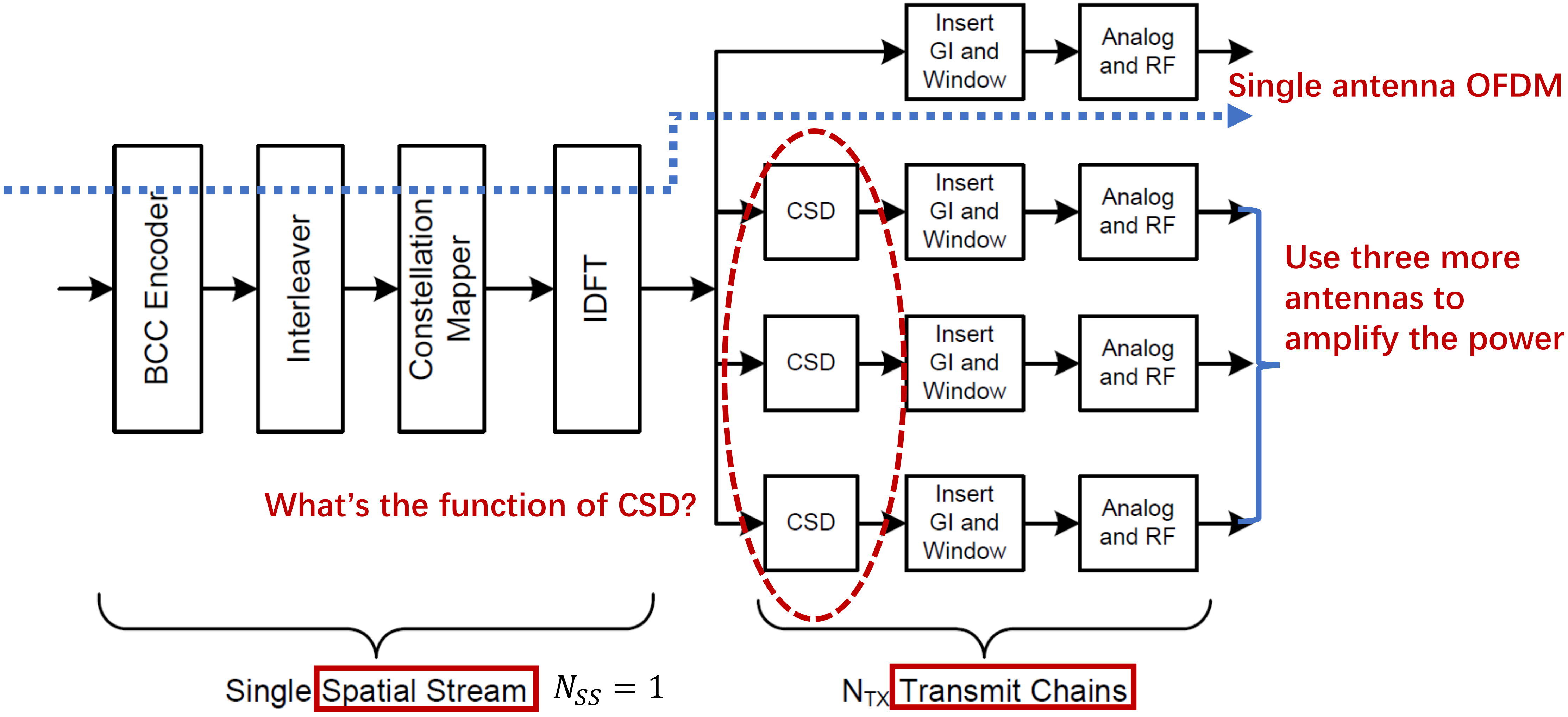
Cyclic Shift - Example

- Suppose non-HT mode, two antennas transmit the same signal, CSD happens after IFFT
- Suppose cyclic shift is 200ns, which include 4 samples

Tx Antenna 1: S1, S2, S3, S4, S5, ⋯, S64

Tx Antenna 2: S61, S62, S63, S64, S1, ⋯, S60

Recap: Single Spatial Stream (19.3.3)



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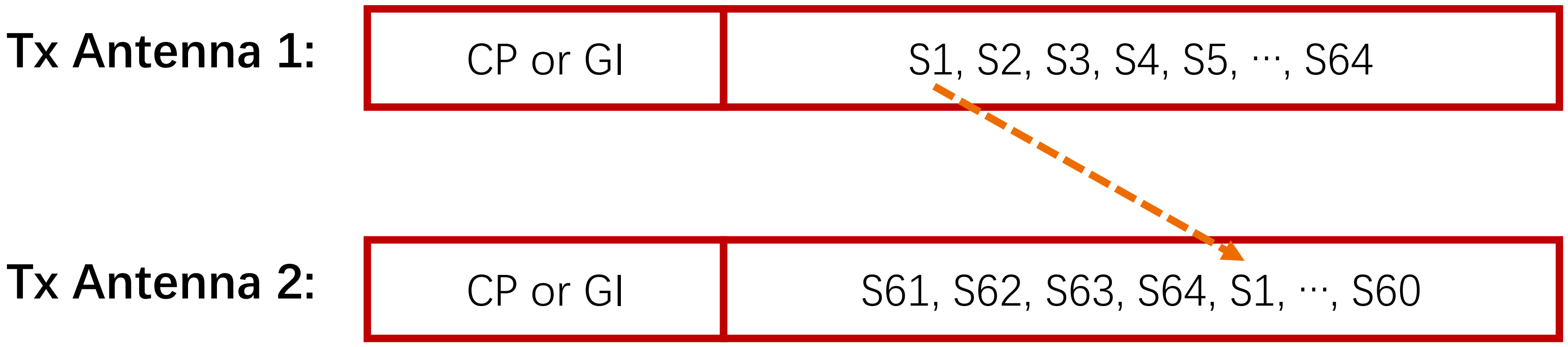
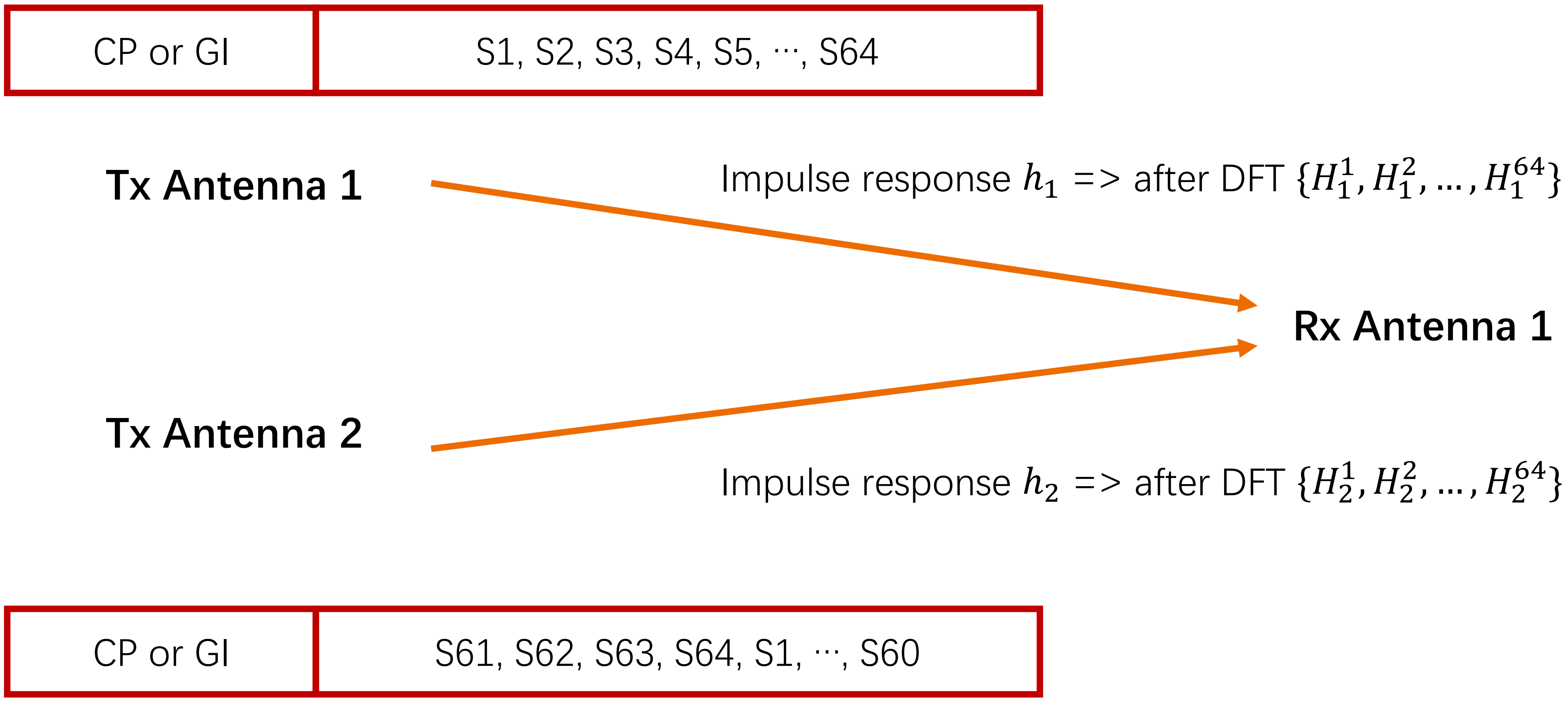


Analysis of Cyclic Shift



Analysis of Cyclic Shift

$$\begin{bmatrix} D_1 \\ D_2 \\ \ldots \\ D_{64} \end{bmatrix} \text{IDFT} \Rightarrow \begin{bmatrix} S_1 \\ S_2 \\ \ldots \\ S_{64} \end{bmatrix}$$

Add CP => Convolution with I_1 => Remove CP => DFT =>

$$\begin{bmatrix} H_1^1 D_1 \\ H_1^2 D_2 \\ \ldots \\ H_1^{64} D_{64} \end{bmatrix}$$

Tx Antenna 1

Impulse response I_1 => after DFT $\{H_1^1, H_1^2, \ldots, H_1^{64}\}$

Rx Antenna 1

Tx Antenna 2

Impulse response I_2 => after DFT $\{H_2^1, H_2^2, \ldots, H_2^{64}\}$

$$[?] \text{ IDFT} \Rightarrow \begin{bmatrix} S_{61} \\ S_{62} \\ \ldots \\ S_{60} \end{bmatrix}$$

Analysis of Cyclic Shift

Tx Antenna 1

$$D_k = \sum_{n=1}^{64} S_n e^{-\frac{2\pi j}{64}kn} = S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}$$

Tx Antenna 2 $S_{61} e^{-\frac{2\pi j}{64}k} + S_{62} e^{-\frac{2\pi j}{64}2k} + \cdots + S_1 e^{-\frac{2\pi j}{64}5k} + S_2 e^{-\frac{2\pi j}{64}6k} + \cdots + S_{60}\, e^{-\frac{2\pi j}{64}64k}$

$$= \left[S_1 e^{-\frac{2\pi j}{64}k} + S_2 e^{-\frac{2\pi j}{64}2k} + \cdots + S_{64}\, e^{-\frac{2\pi j}{64}64k}\right] e^{-\frac{2\pi j}{64}4k}$$

$$= D_k e^{-\frac{2\pi j}{64}4k}$$

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$$\begin{bmatrix} D_1 e^{-\frac{2\pi j}{64}4} \\ D_2 e^{-\frac{2\pi j}{64}4\times 2} \\ \cdots \\ D_{64} e^{-\frac{2\pi j}{64}4\times 64} \end{bmatrix} \text{IDFT} \Rightarrow \begin{bmatrix} S_{61} \\ S_{62} \\ \cdots \\ S_{60} \end{bmatrix}$$

Add CP => Convolution with I_1 => Remove CP => DFT =>

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Analysis of Cyclic Shift

$$\begin{bmatrix} (H_1^1 + H_2^1 e^{-\frac{2\pi j}{64}4}) D_1 \\ (H_1^2 + H_2^2 e^{-\frac{2\pi j}{64}4\times 2}) D_2 \\ \dots \\ (H_1^{64} + H_2^{64} e^{-\frac{2\pi j}{64}4\times 64}) D_{64} \end{bmatrix}$$

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Rx Antenna 1

Tx Antenna 2

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$$\begin{bmatrix} D_1 e^{-\frac{2\pi j}{64}4} \\ D_2 e^{-\frac{2\pi j}{64}4\times 2} \\ \dots \\ D_{64} e^{-\frac{2\pi j}{64}4\times 64} \end{bmatrix} \text{IDFT} => \begin{bmatrix} S_{61} \\ S_{62} \\ \dots \\ S_{60} \end{bmatrix}$$

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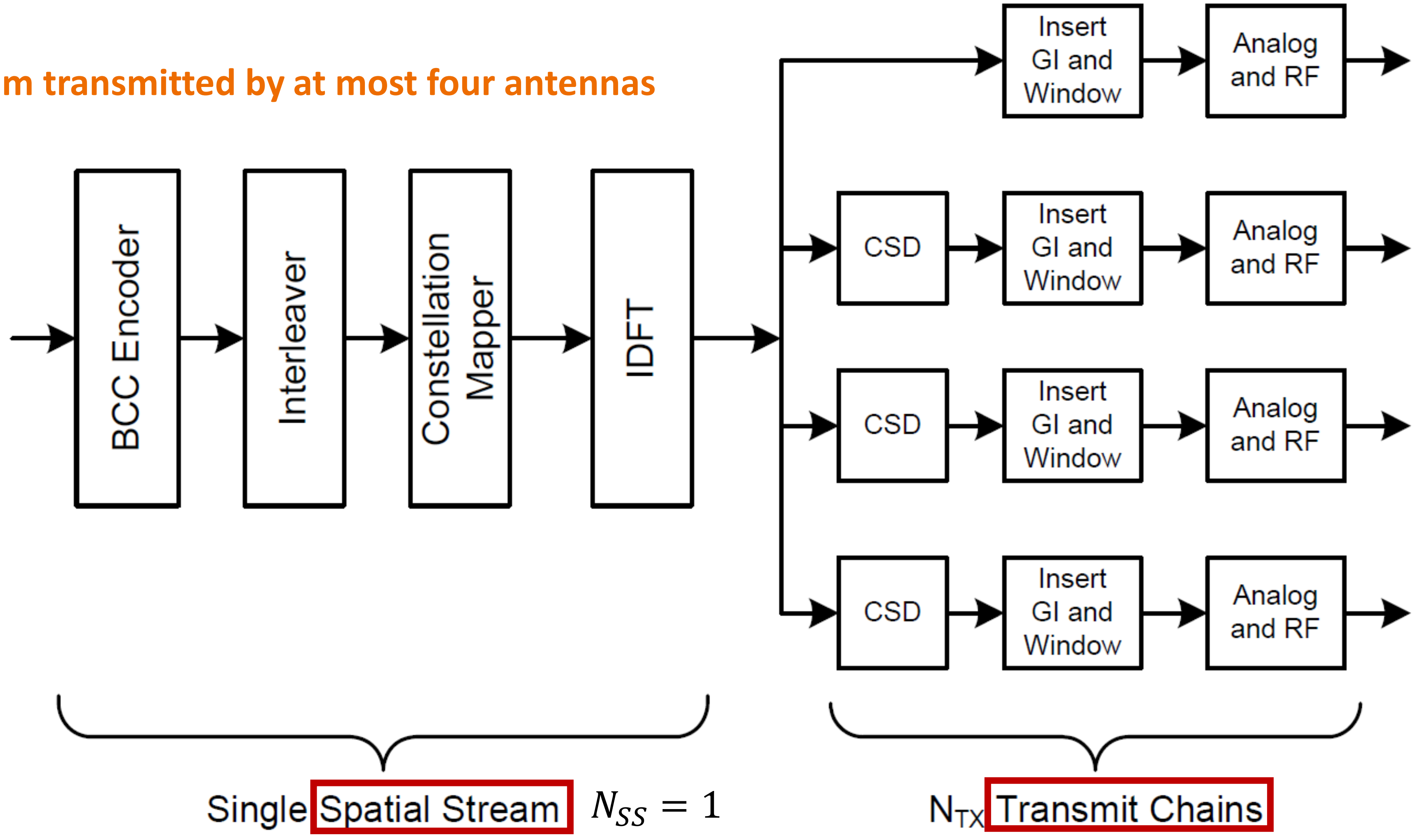

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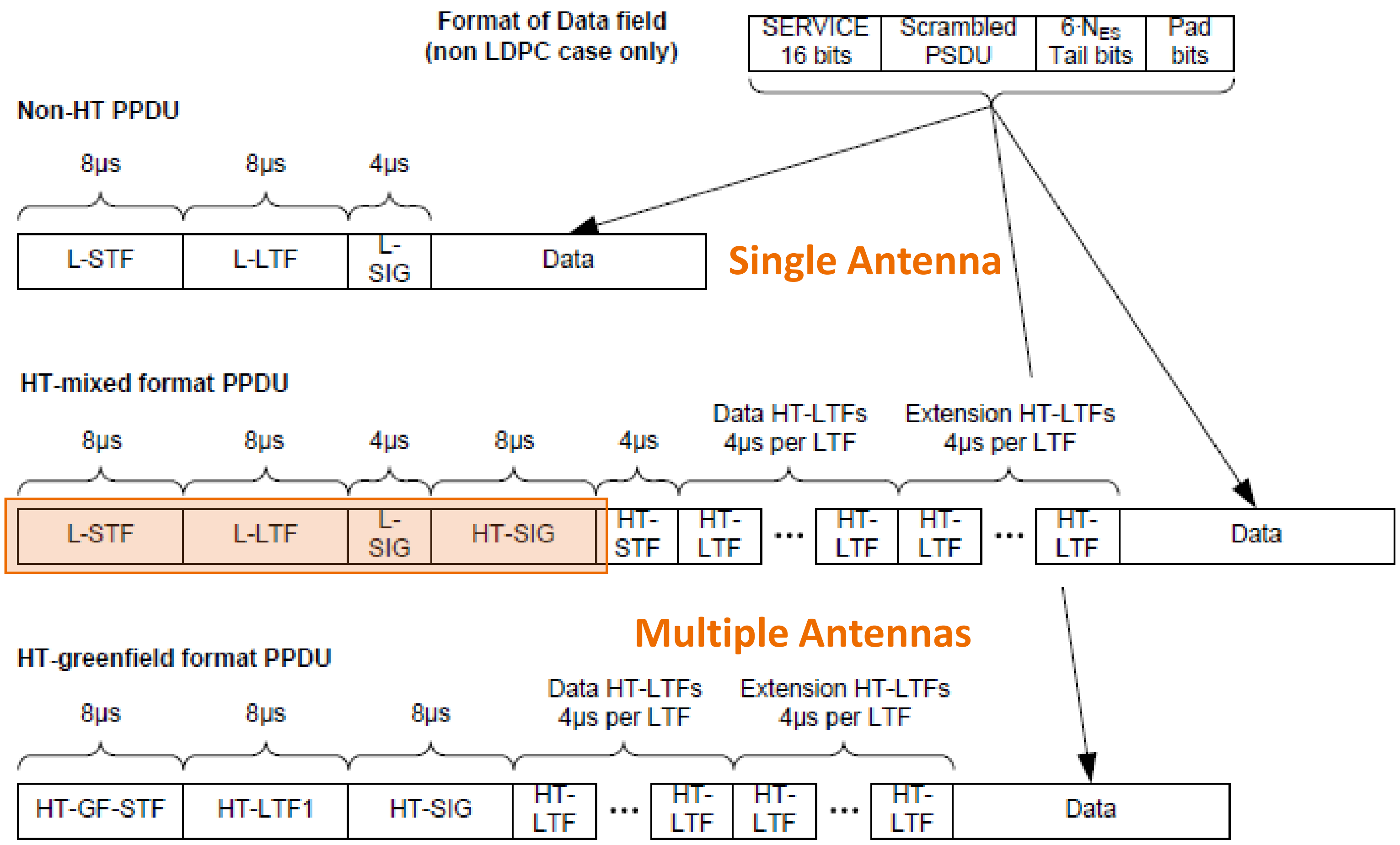

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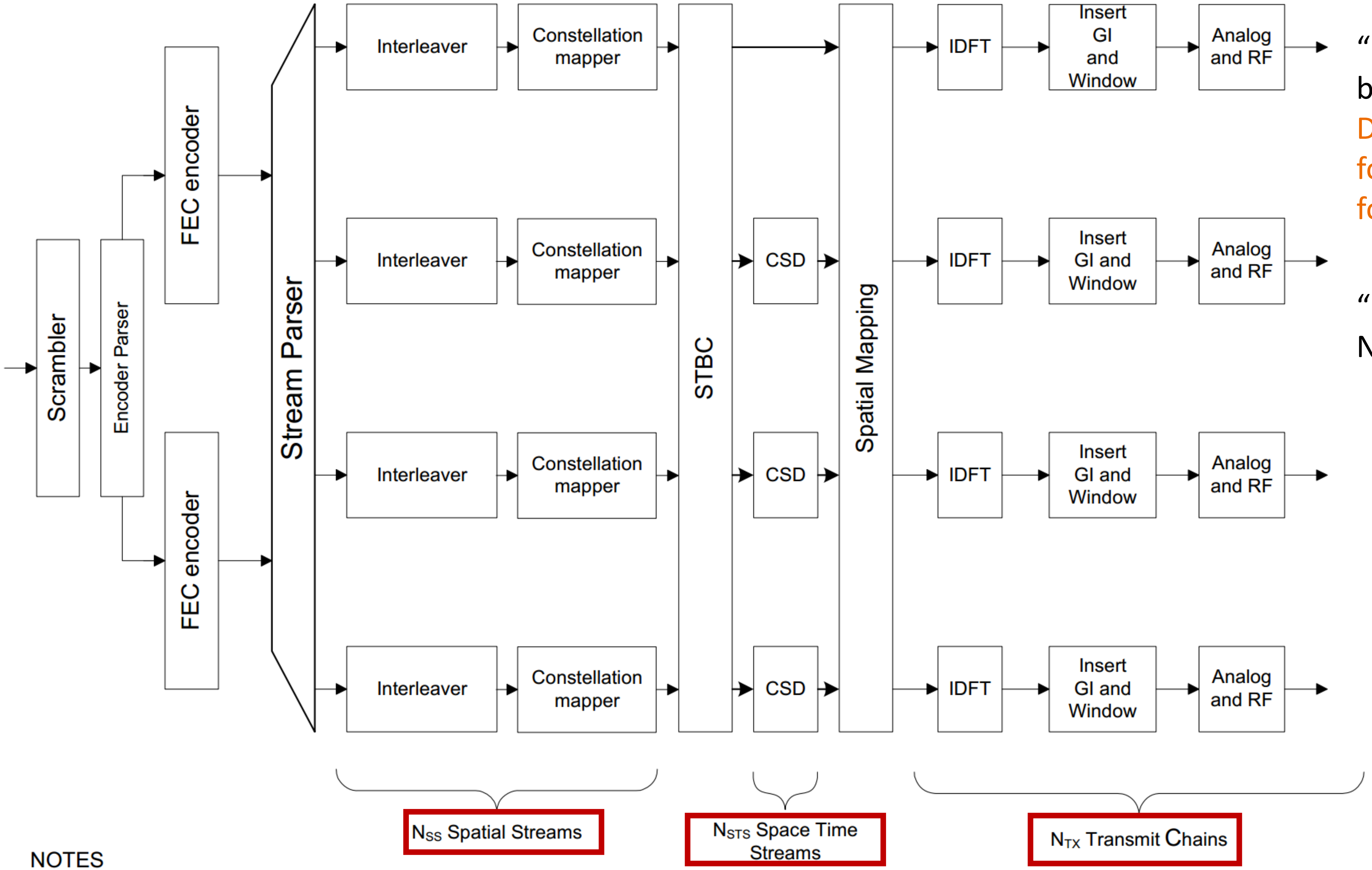
Multiple Spatial Stream (19.3.3)

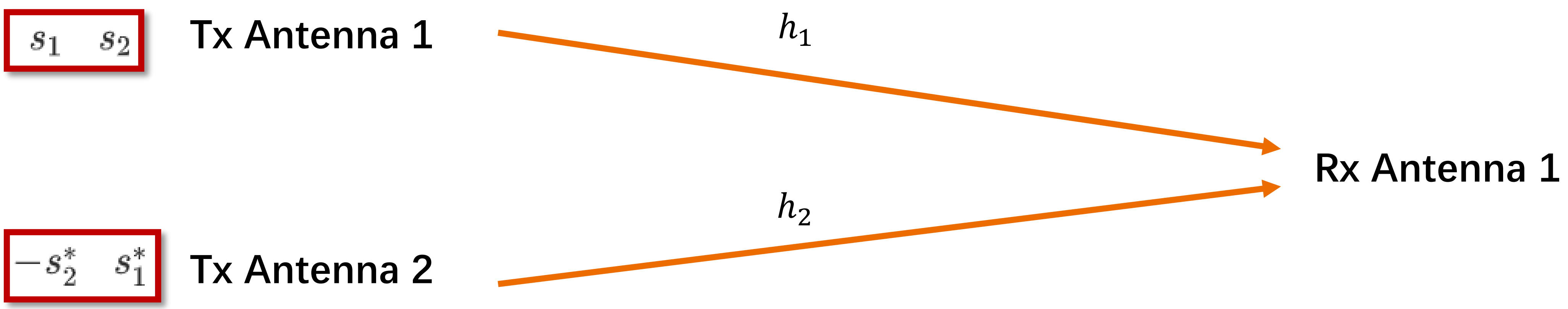


"…shows the transmitter blocks used to generate the Data field of the HT-mixed format and HT-greenfield format PPDUs."

" STBC is used only when N_{STS}>N_{SS} "

NOTES

STBC Example: Alamouti Code



Diversity is increased to 2, less packet outage probability

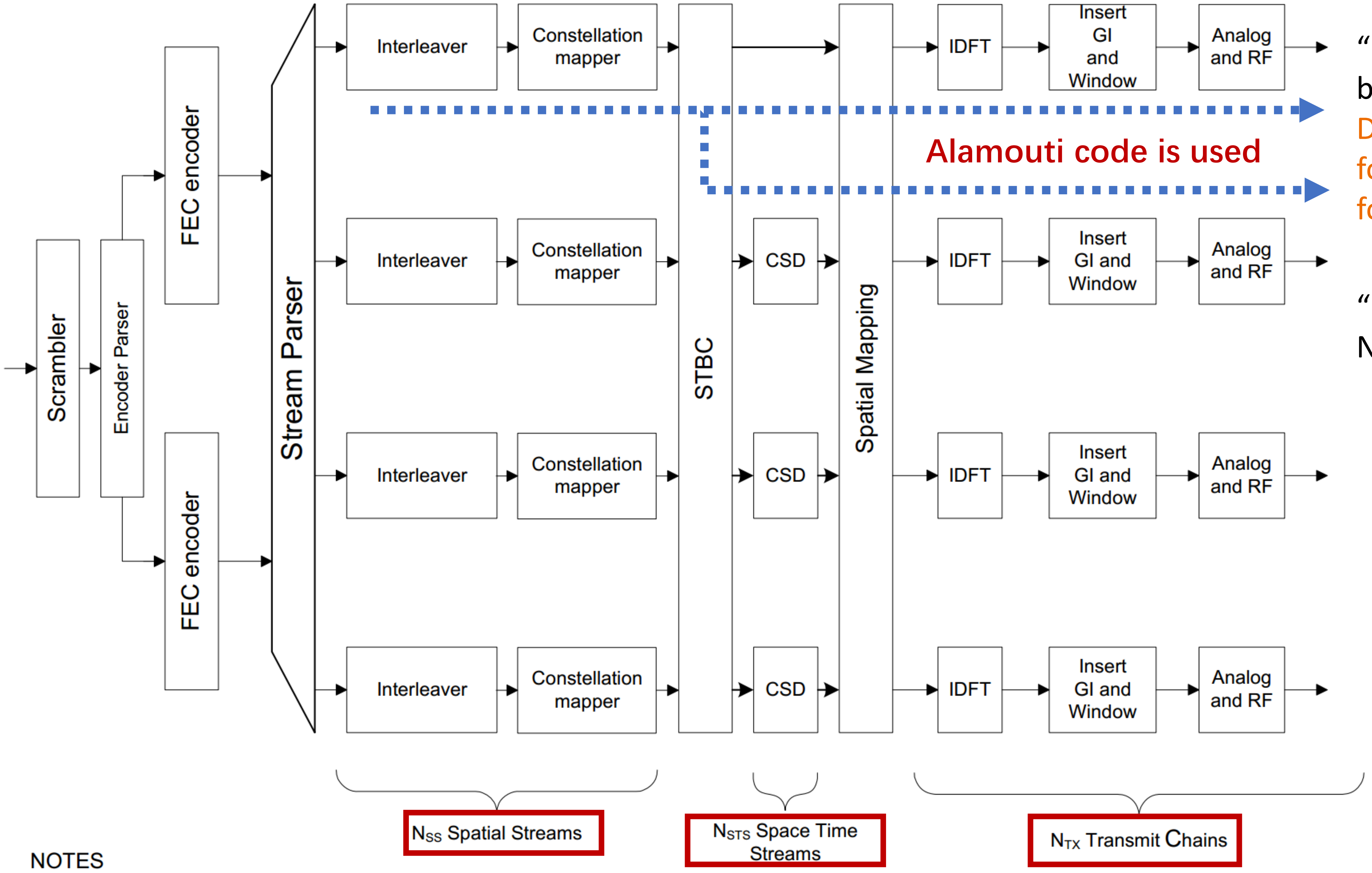
Require CSIR only, CSIT is not necessary

$$[y_1 \;\; y_2] = [h_1 \; h_2]\begin{bmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \end{bmatrix} + [w_1 \; w_2]$$

$$\Downarrow$$

$$\begin{bmatrix} y_1 \\ y_2^* \end{bmatrix} = \begin{bmatrix} h_{11} & -h_{12} \\ h_{12}^* & h_{11}^* \end{bmatrix} \begin{bmatrix} s_1 \\ s_2^* \end{bmatrix} + \begin{bmatrix} w_1 \\ w_2^* \end{bmatrix} \Rightarrow \begin{cases} \hat{s}_1 = \dfrac{1}{|h_1|^2 + |h_2|^2}(h_1^* y_1 + h_2 y_2^*) \\ \hat{s}_2 = \dfrac{1}{|h_1|^2 + |h_2|^2}(-h_2 y_1^* + h_1^* y_2) \end{cases}$$

Multiple Spatial Stream (19.3.3)



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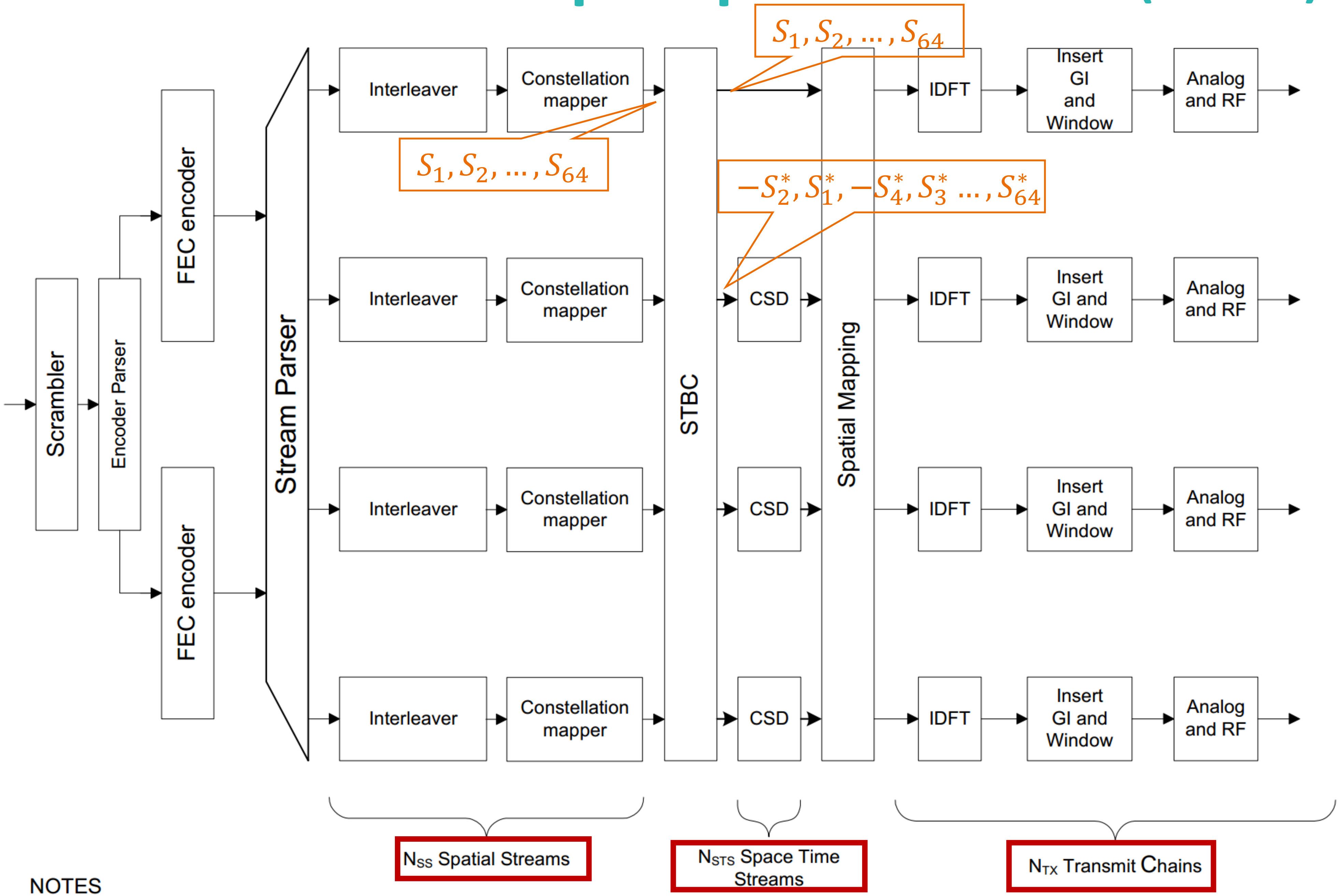
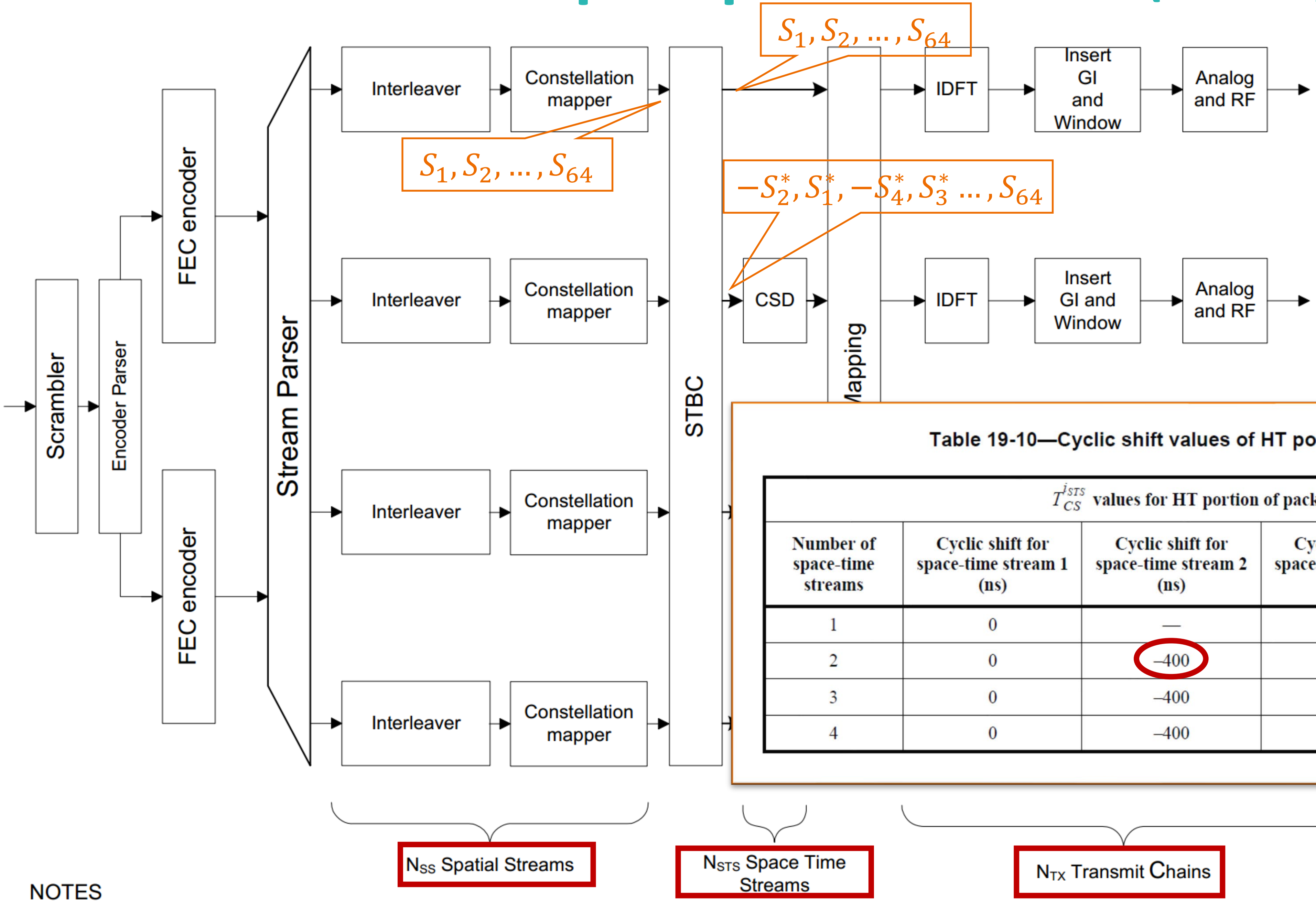


NOTES

Multiple Spatial Stream (19.3.3)



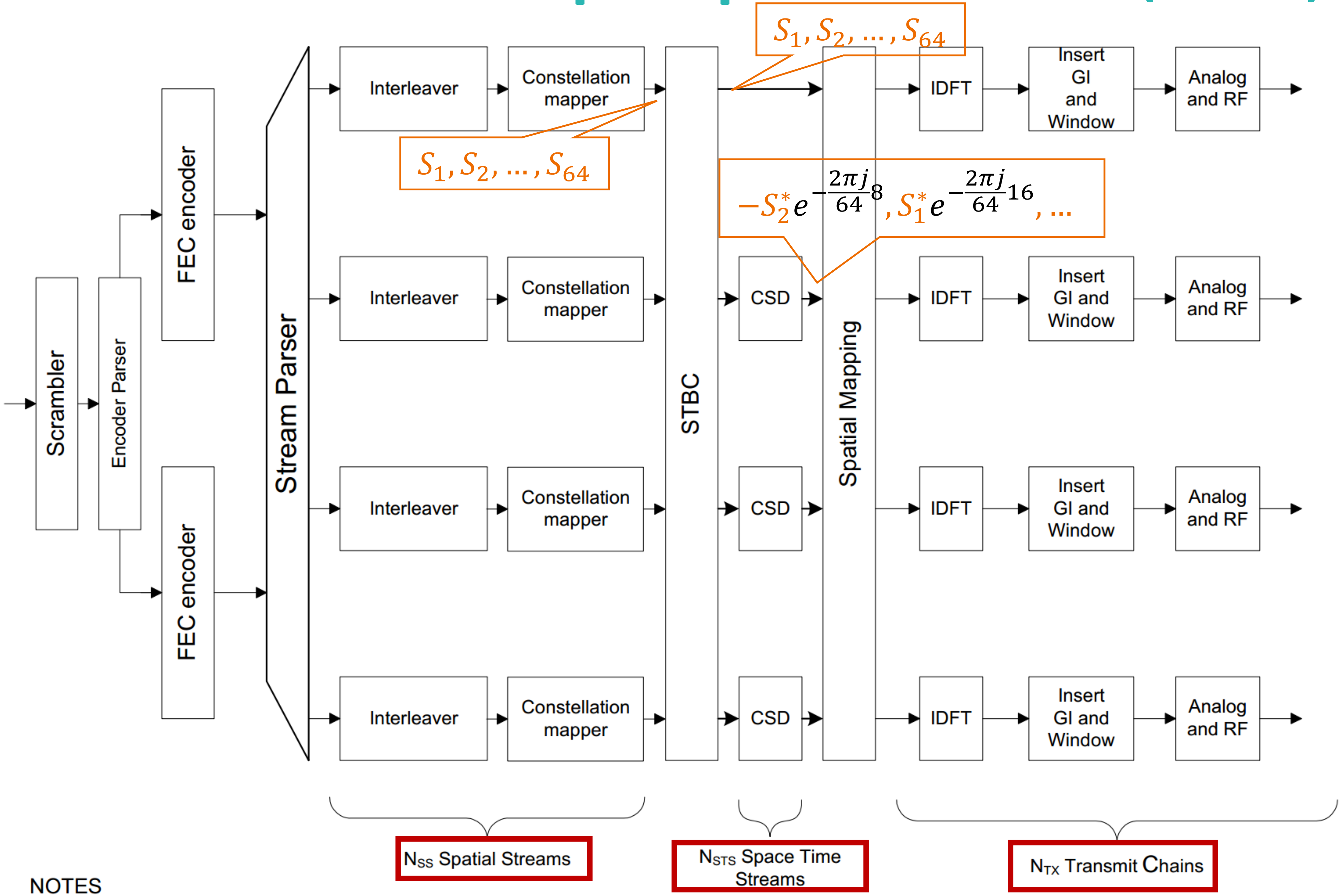
400ns = 8 samples

Table 19-10—Cyclic shift values of HT portion of packet

$T_{CS}^{i_{STS}}$ values for HT portion of packet				
Number of space-time streams	Cyclic shift for space-time stream 1 (ns)	Cyclic shift for space-time stream 2 (ns)	Cyclic shift for space-time stream 3 (ns)	Cyclic shift for space-time stream 4 (ns)
1	0	—	—	—
2	0	–400	—	—
3	0	–400	–200	—
4	0	–400	–200	–600

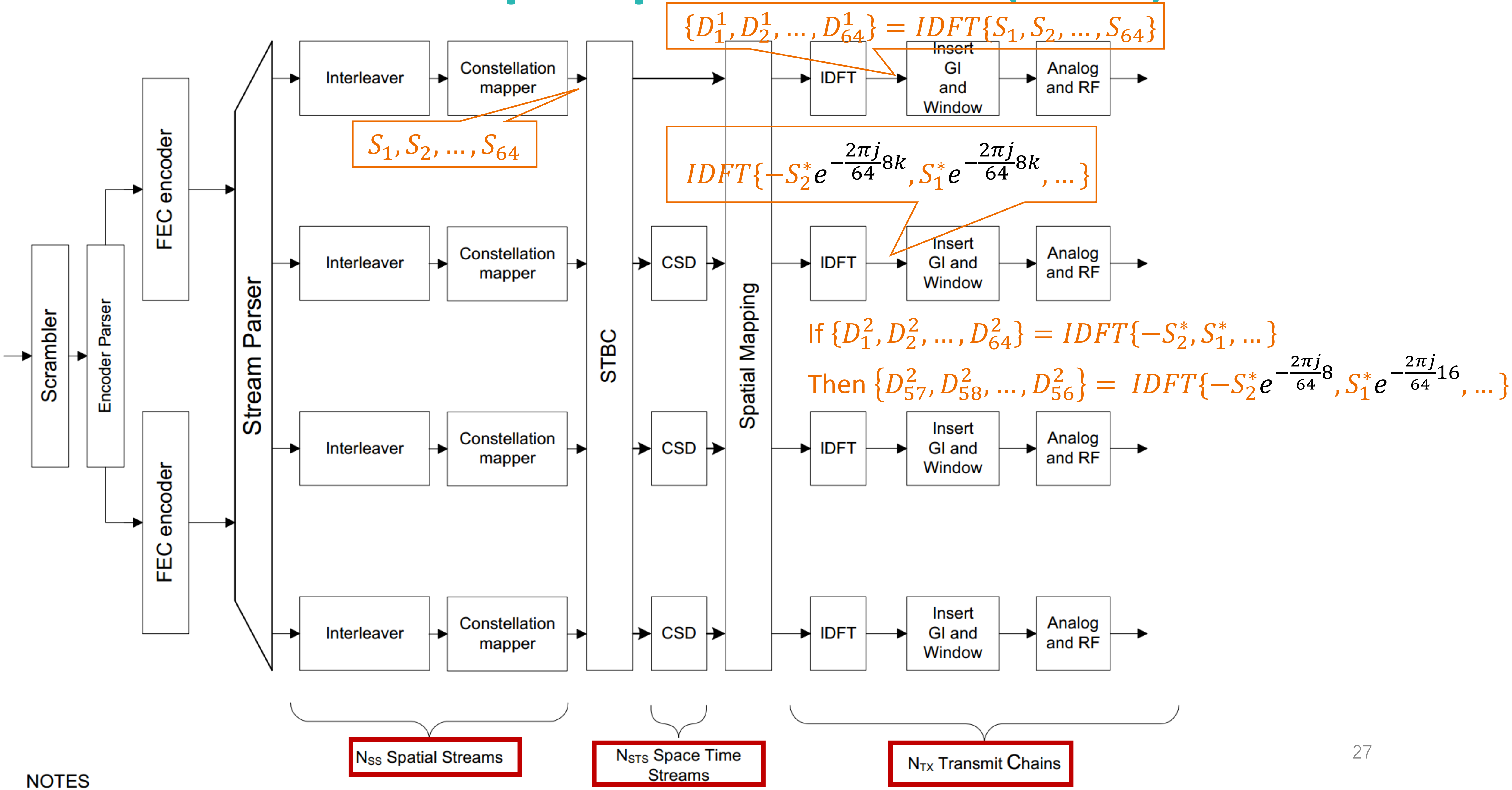
NOTES

Multiple Spatial Stream (19.3.3)



NOTES

Multiple Spatial Stream (19.3.3)



NOTES

Space-Time Block Code (19.3.11.9.2)

- Support 4 space-time block codes

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
2	0–7	1	1	1	$d_{k,1,2m}$ s_1	$d_{k,1,2m+1}$ s_2
				2	$-d^*_{k,1,2m+1}$ $-s_2^*$	$d^*_{k,1,2m}$ s_1^*
3	8–15, 33–38	2	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
4	8–15	2	2	1	$d_{k,1,2m}$ s_1	$d_{k,1,2m+1}$ s_2
				2	$-d^*_{k,1,2m+1}$ $-s_2^*$	$d^*_{k,1,2m}$ s_1^*
				3	$d_{k,2,2m}$ s_3	$d_{k,2,2m+1}$ s_4
				4	$-d^*_{k,2,2m+1}$ $-s_4^*$	$d^*_{k,2,2m}$ s_3^*

N_{STS}	HT-SIG MCS field (bits 0–6 in HT-SIG$_1$)	N_{SS}	HT-SIG STBC field (bits 4–5 in HT-SIG$_2$)	i_{STS}	$\tilde{d}_{k,i,2m}$	$\tilde{d}_{k,i,2m+1}$
4	16–23, 39, 41, 43, 46, 48, 50	3	1	1	$d_{k,1,2m}$	$d_{k,1,2m+1}$
				2	$-d^*_{k,1,2m+1}$	$d^*_{k,1,2m}$
				3	$d_{k,2,2m}$	$d_{k,2,2m+1}$
				4	$d_{k,3,2m}$	$d_{k,3,2m+1}$

NOTE—the '*' operator represents the complex conjugate.

Example: 4 × 2 STBC

$$\begin{pmatrix} y_1 & y_3 \\ y_2 & y_4 \end{pmatrix} = \begin{pmatrix} h_{11} & h_{12} & h_{13} & h_{14} \\ h_{21} & h_{22} & h_{23} & h_{24} \end{pmatrix} \begin{pmatrix} s_1 & s_2 \\ -s_2^* & s_1^* \\ s_3 & s_4 \\ -s_4^* & s_3^* \end{pmatrix} + \begin{pmatrix} w_1 & w_3 \\ w_2 & w_4 \end{pmatrix}$$

$$\begin{pmatrix} y_1 \\ y_2 \\ y_3^* \\ y_4^* \end{pmatrix} = \begin{pmatrix} h_{11} & -h_{12} & h_{13} & -h_{14} \\ h_{21} & -h_{22} & h_{23} & -h_{24} \\ h_{12}^* & h_{11}^* & h_{14}^* & h_{13}^* \\ h_{22}^* & h_{21}^* & h_{24}^* & h_{23}^* \end{pmatrix} \begin{pmatrix} s_1 \\ s_2^* \\ s_3 \\ s_4^* \end{pmatrix} + \begin{pmatrix} w_1 \\ w_2 \\ w_3^* \\ w_4^* \end{pmatrix}$$

At least **2 Rx antennas** are required

MIMO Detector

- Given $Y = HX + Z$ and (Y,H), how to estimate vector X?

- **Approach 1: Zero-forcing (ZF)**
  - $\hat{X} = H^{-1}Y = X + H^{-1}Z$

- **Approach 2: Minimum mean square error (MMSE)**
  - $\hat{X} = QY = QHX + QZ \implies Error\!: \Delta = \hat{X} - X = (QH - I)X + QZ$
  - $E\,[\Delta\Delta^H] = E[(QH - I)XX^H(QH - I)^H + QZZ^HQ^H]$
  - $E\,[\Delta\Delta^H] = \sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H$
  - Thus, we should $\min\limits_{Q} trace[\sigma^2(QH - I)(QH - I)^H + \sigma_Z^2 QQ^H]$

- **Approach 3: Maximum likelihood (ML)**
  - The choices of X is finite
  - For each possible X, calculate vector distance $|Y - HX|$
  - Choose the X, which minimize the distance
  - Thus, $\hat{X} = \min\limits_{X} |Y - HX|$

Reading & Assignment (3.16)

Reading

- IEEE Std 802.11™-2020: Section 19.3.9 – 19.3.11
- Reference Paper

Assignment 3