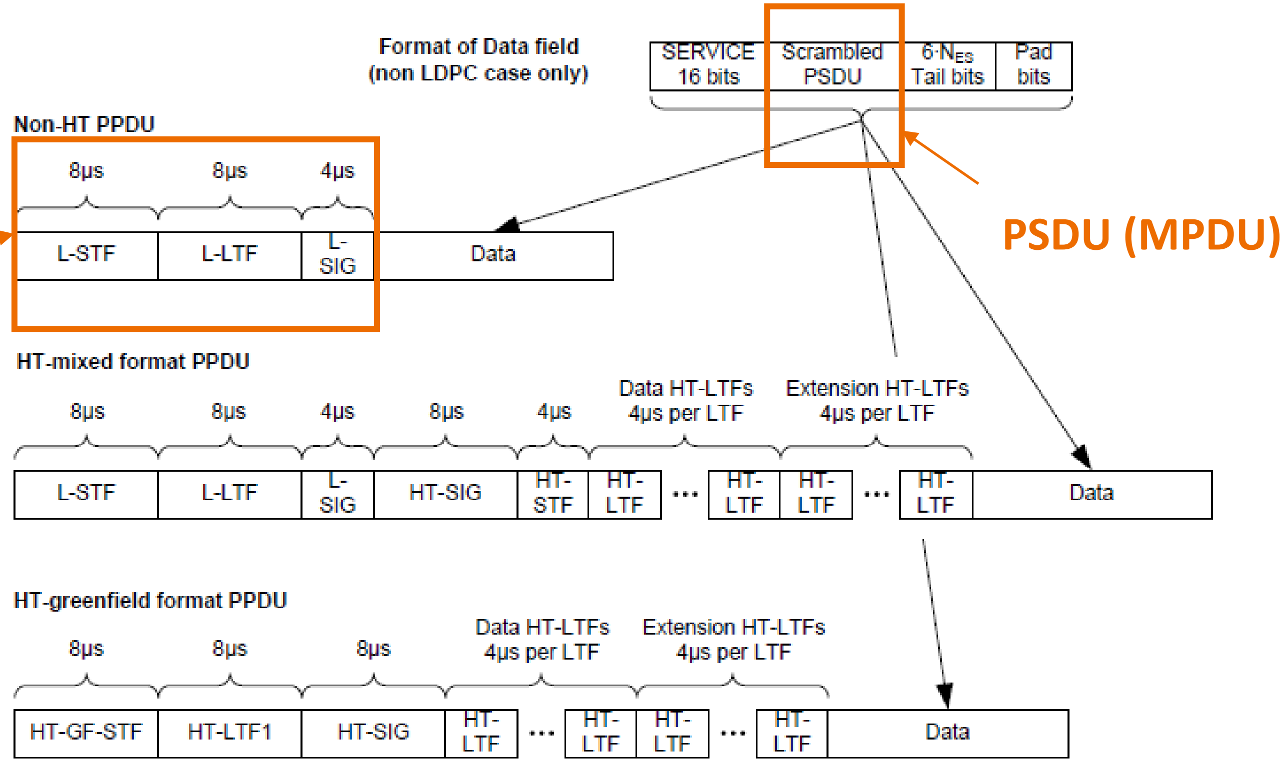
Physical-Layer Synchronization

Lecturer: Dr. Rui Wang

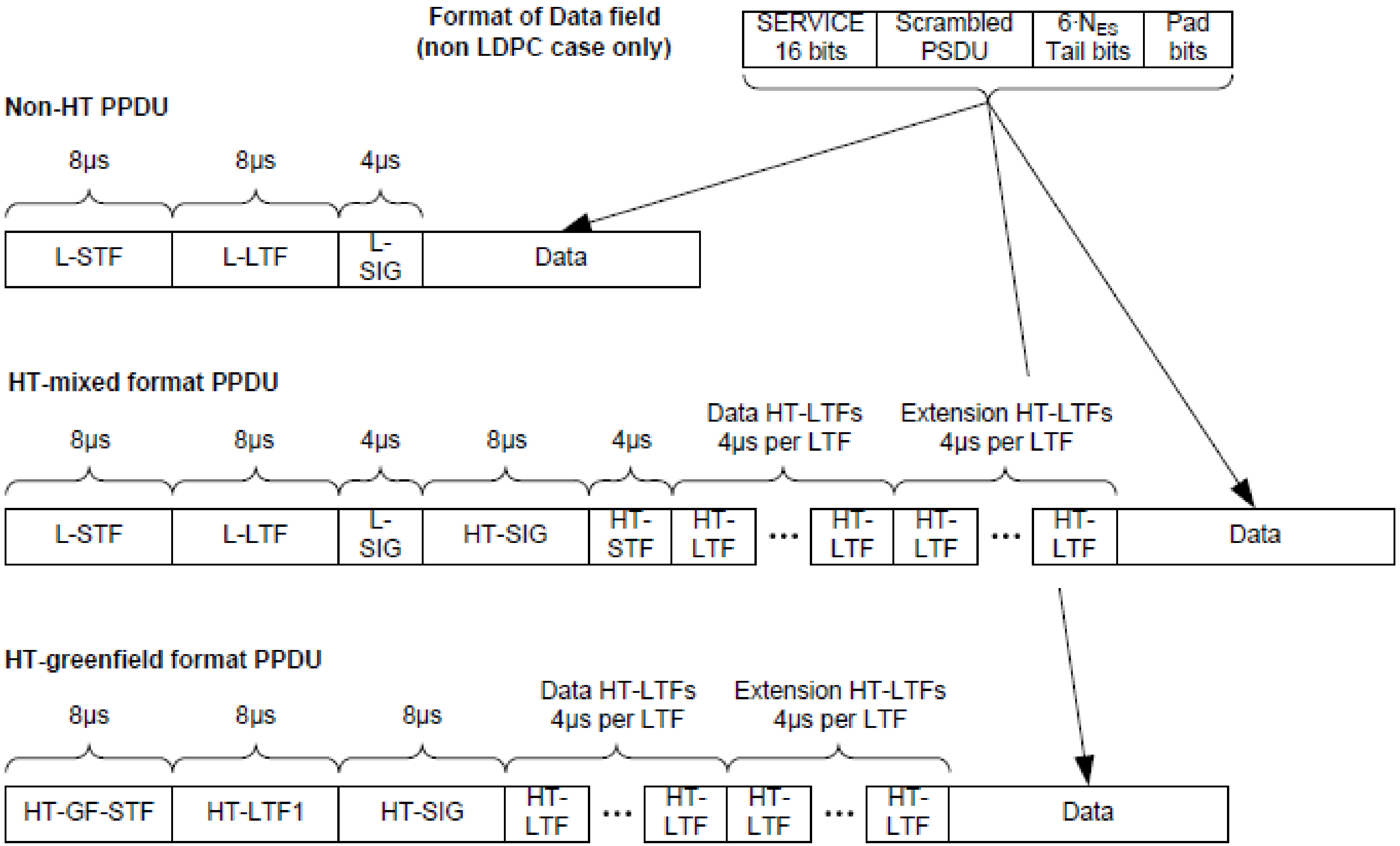
Synchronization and Signaling

- When a WiFi adaptor finds a wireless signal, it should detect
  - What is this? Is this a WiFi signal or from micro oven? --- answered by the header of PPDU
  - Where does it from? What is the source MAC address? --- answered by the header of PSDU
  - What is its destination? Should I further decode this signal? --- answered by the header of PSDU
- **PPDU:** physical protocol data unit
- **PSDU:** physical service data unit
- **MPDU:** MAC protocol data unit

PPDU Header

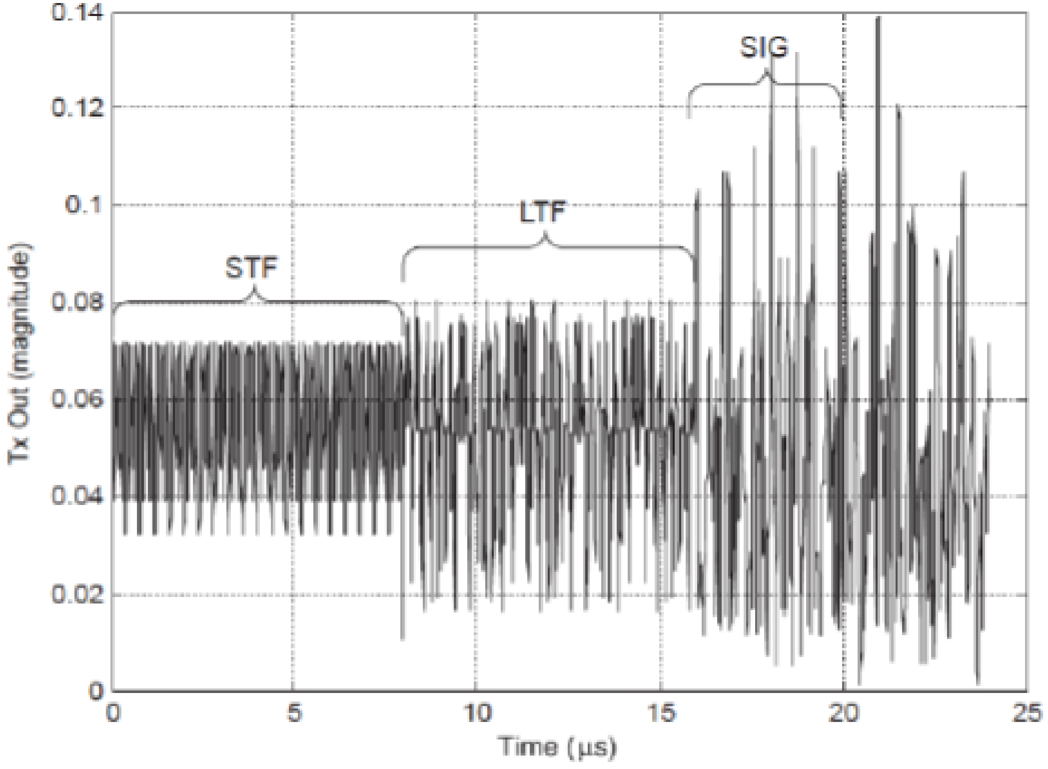


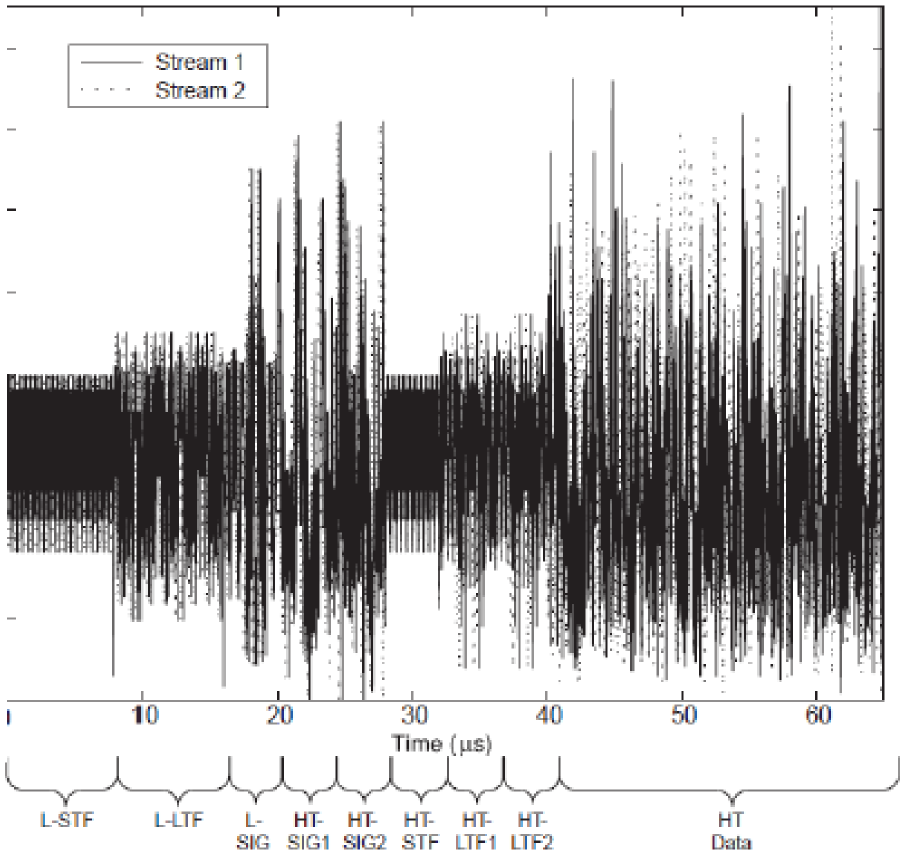
Three PPDU Formats in 11n



- **Non-HT：** either the AP or the STA does not support HT
- **HT-mixed:** AP and STA support HT, however, other STAs may not support HT
- **HT-greenfield:** all the STAs support HT
- L-STF, L-LTF, HT-STF, HT-LTF are known to the Rx
- **L-STF:** mainly for coarse synchronization
- **L-LTF:** mainly for fine synchronization and channel estimation
- **L-SIG, HT-SIG:** basic information about the PPDU

Time Domain Waveform





Function of Short Training Field

- Detection of packet arrival
- Coarse time synchronization
- Coarse frequency offset estimation
- Adaptive gain control (AGC)

Generation of STF - IDFT

$$S_{-26,26} = \sqrt{1/2}\,\{0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 0, 0, 0, 0, -1-j, 0, 0, 0, -1-j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0, 0, 1+j, 0, 0\}$$

$$r_{L-STF}^{(i_{TX})}(t) = \frac{1}{\sqrt{N_{TX} \cdot N_{L-STF}^{Tone}}} w_{T_{L-STF}}(t) \sum_{k=-N_{SR}}^{N_{SR}} \Upsilon_k S_k \exp(j2\pi k \Delta_F (t - T_{CS}^{i_{TX}}))$$

Scaling Factor

“1” in this case

Generation of STF - IDFT

$$r(t) = \sum_{k=-N_{SR}}^{N_{SR}} \Upsilon_k S_k \exp(j2\pi k\Delta_F(t - T_{CS}^{i_{TX}}))$$

Bandwidth of one WiFi channel

Subcarrier Spacing: $\Delta_F = \dfrac{20M}{64}\ Hz$

Number of subcarriers

Generation of STF - IDFT

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$$= \sum_{-N_{SR}}^{N_{SR}} S_k e^{j2\pi k \times \frac{20M}{64} t}$$

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Because of 20MHz bandwidth, the sampling period = 1/20M second

n-th sample of $r(\frac{n}{20M}) = \sum_{-N_{SR}}^{N_{SR}} S_k e^{j2\pi k \times \frac{20M}{64} \times \frac{n}{20M}} = \sum_{-N_{SR}}^{N_{SR}} S_k e^{j\frac{2\pi kn}{64}}$

Generation of STF - IDFT

$$r(t) = \sum_{k=-N_{SR}}^{N_{SR}} \Upsilon_k S_k \exp(j2\pi k \Delta_F (t - T_{CS}^{i_{TX}}))$$

$$= \sum_{-N_{SR}}^{N_{SR}} S_k e^{j2\pi k \times \frac{20M}{64} t}$$

r(t) is the interpolation of 64-IDFT of $S_{-26,26}$

Because of 20MHz bandwidth, the sampling period = 1/20M second

n-th sample of $r(\frac{n}{20M})$ = $\sum_{-N_{SR}}^{N_{SR}} S_k e^{j2\pi k \times \frac{20M}{64} \times \frac{n}{20M}} = \sum_{-N_{SR}}^{N_{SR}} S_k e^{j\frac{2\pi kn}{64}}$

64-IDFT of $S_{-26,26}$

Generation of STF - Repetition

$$r_{L-STF}^{(i_{TX})}(t) = \frac{1}{\sqrt{N_{TX} \cdot N_{L-STF}^{Tone}}} w_{T_{L-STF}}(t) \sum_{k=-N_{SR}}^{N_{SR}} \Upsilon_k S_k \exp(j2\pi k \Delta_F (t - T_{CS}^{i_{TX}}))$$

$$r_{L-STF}^{(i_{TX})}(t)$$

64 samples => Duration = 64 $\times \frac{1}{20M} = 3.2us$

Generation of STF - Repetition

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$r_{L-STF}^{(i_{TX})}(t)$

64 samples => Duration = $64 \times \frac{1}{20M} = 3.2us$

Take the first 16 samples => 0.8 us

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Generation of STF - Repetition

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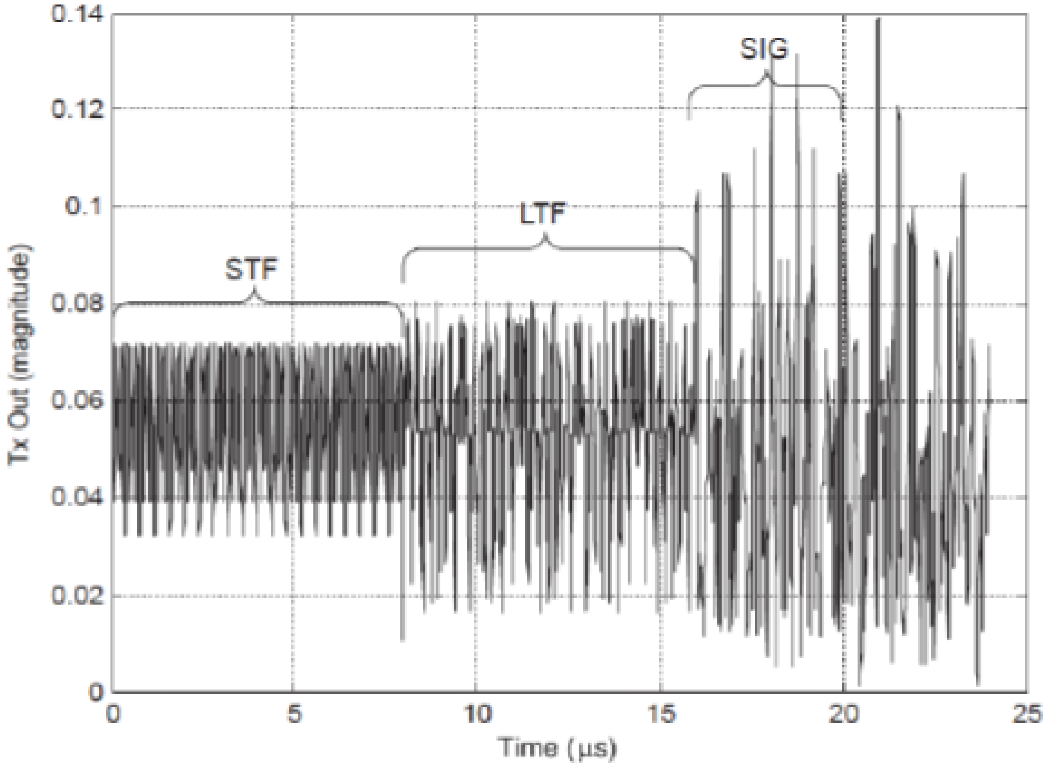
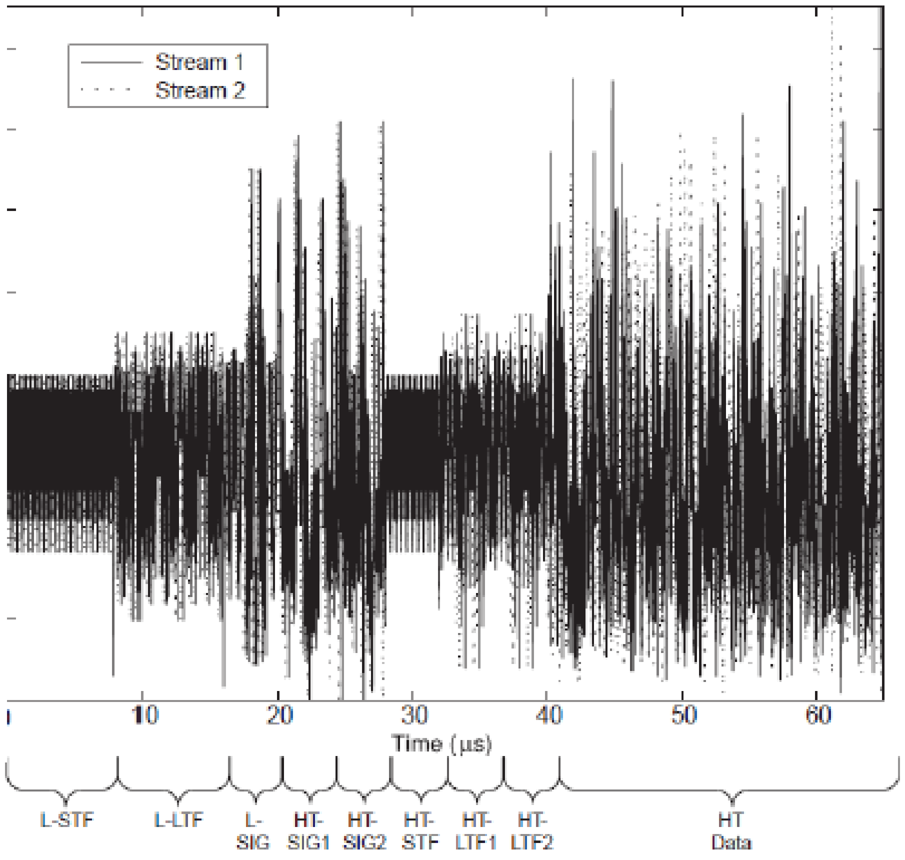
$r_{L-STF}^{(i_{TX})}(t)$	$r_{L-STF}^{(i_{TX})}(t)$	$r_{L-STF}^{(i_{TX})}(t)$	$r_{L-STF}^{(i_{TX})}(t)$	… …	$r_{L-STF}^{(i_{TX})}(t)$

Repeat 10 times => 8 us

Recap: Time Domain Waveform





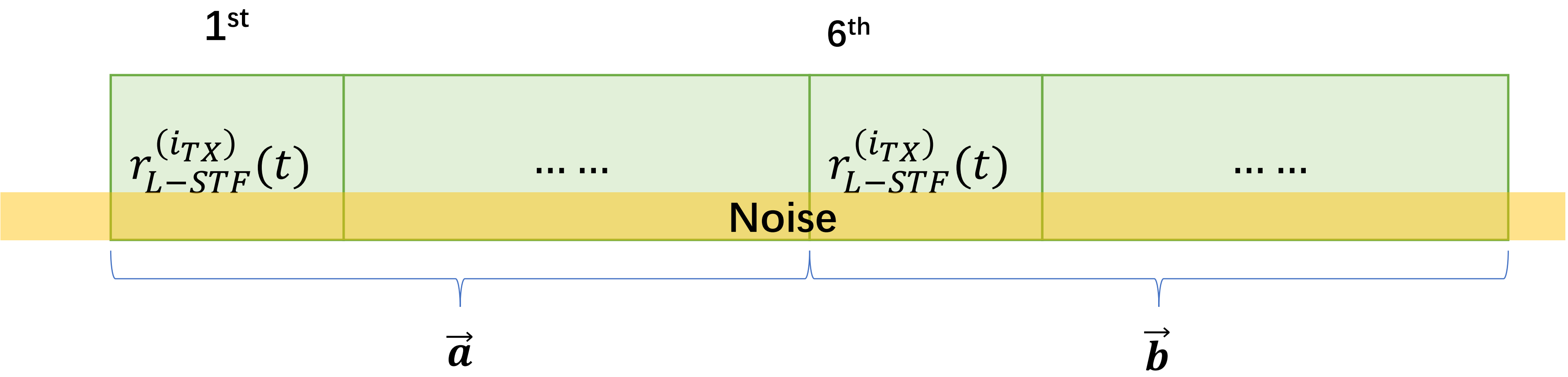
Arrival Detection

- How to find the arrival of one PPDU?
- Threshold based detection
  - Suppose the signal power is p, noise power is σ^2
  - If there is no signal, the mean and variance of noise are 0 and σ^2
  - If there is signal, the mean and variance of received signal are p and σ^2
  - Thus higher SNR $\frac{p}{\sigma^2}$, less detection error
  - We can further improve the detection SNR by auto-correlation of the received signal

Autocorrelation – To Accumulate the Signal Power

- First half of STF: $\vec{\boldsymbol{a}} = [x_1 + n_1, x_2 + n_2, \dots, x_{80} + n_{80}]$
- Second half of STF: $\vec{\boldsymbol{b}} = [x_1 + m_1, x_2 + m_2, \dots, x_{80} + m_{80}]$
- At the receiver, for each sample in $\vec{\boldsymbol{a}}$ or $\vec{\boldsymbol{b}}$, the SNR is $\frac{p}{\sigma^2}$
- The receiver can save $\vec{\boldsymbol{a}}$ and $\vec{\boldsymbol{b}}$, and calculate correlation $\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}^*/\mathbf{80}$

$$\frac{\vec{\boldsymbol{a}}\vec{\boldsymbol{b}}^*}{\mathbf{80}} = \sum_{i=1}^{80} \frac{|x_i|^2}{80} + \sum_{i=1}^{80} \frac{x_i m_i^* + x_i^* n_i}{80} + \sum_{i=1}^{80} \frac{n_i m_i^*}{80}$$



Autocorrelation - Analysis

Suppose the average signal power is p, noise power is σ^2, the average power of auto-correlation is $p^2 + \frac{1}{40}p\sigma^2 + \frac{\sigma^4}{80}$, compared with $\frac{\sigma^4}{80}$ without STF.

Proof:

$$\textit{The first noise}\ \text{term has variance}\ (\textit{average}\ \text{power}):$$

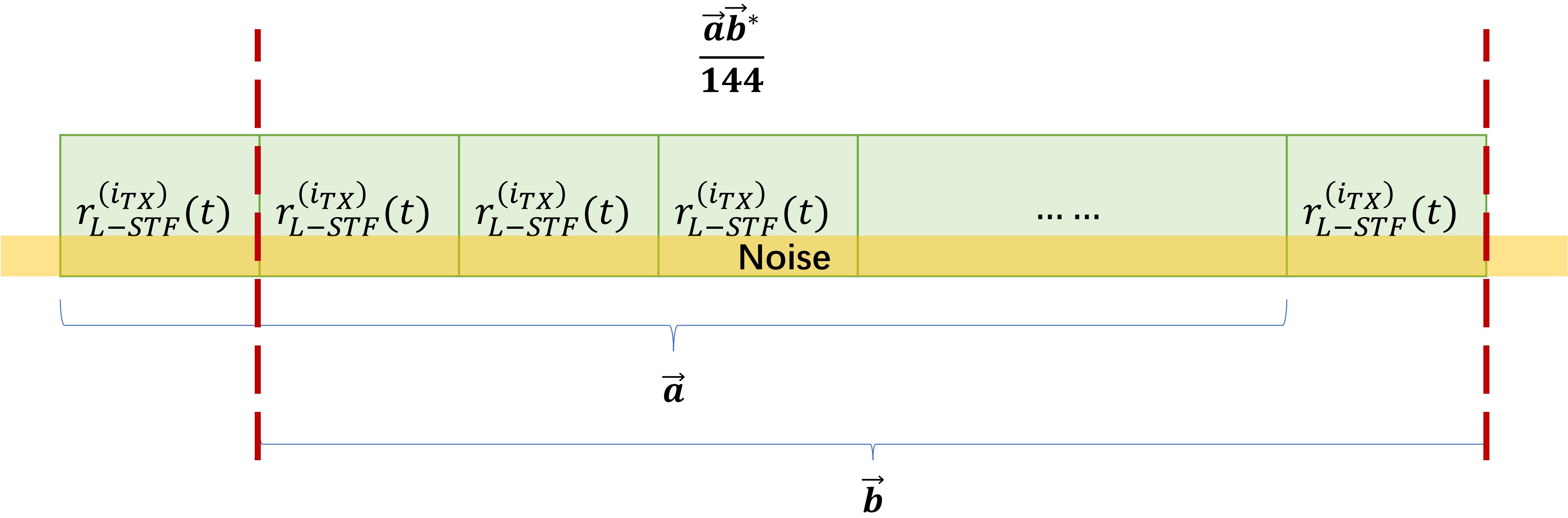
$$var\left(\sum_{i=1}^{80}\frac{x_i m_i^* + x_i^* n_i}{80}\right) = \frac{1}{80^2}\sum_{i=1}^{80}|x_i|^2 var(m_i^*) + |x_i|^2 var(n_i) = \frac{1}{40}p\sigma^2$$

$$\textit{The second noise term has variance (average power)}:$$

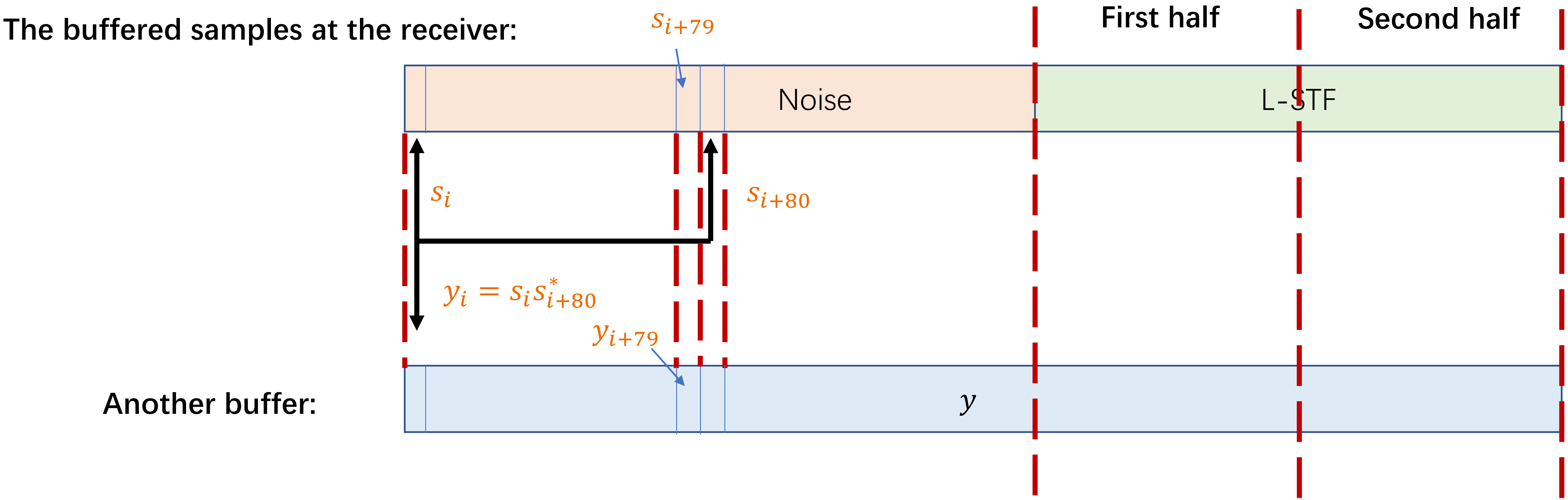
$$var\left(\sum_{i=1}^{80}\frac{n_i m_i^*}{80}\right) = \frac{1}{80^2}\sum_{i=1}^{80} var(n_i m_i^*) = \frac{1}{80} var(n_i m_i^*) = \frac{\sigma^4}{80}$$

Autocorrelation – Can We Do Better

More significant difference between with and without STF can be found if we calculate the auto-correlation as follows



Arrival Detection



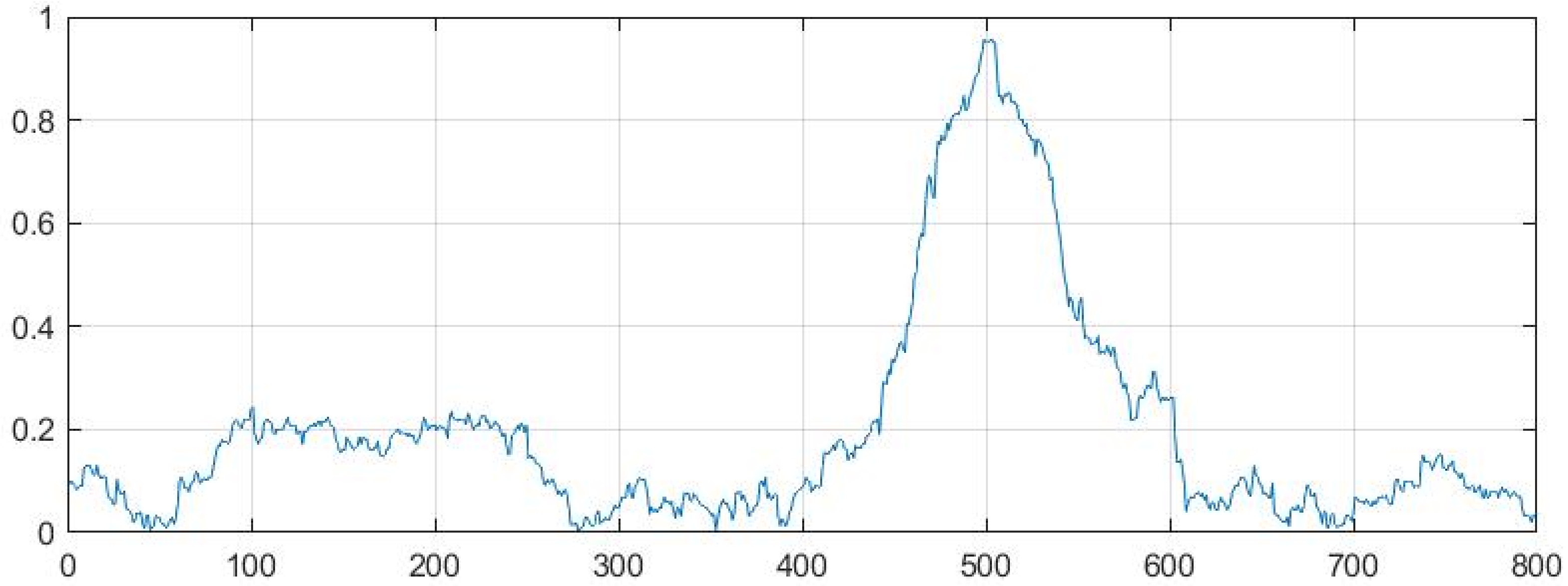
$$\rho_i = \frac{y_i + \cdots + y_{i+79}}{80} = \frac{[s_i \; s_{i+1} \; \dots s_{i+79}][s_{i+80} \; s_{i+81} \; \dots s_{i+159}]^*}{80}$$

$$\rho_{i+1} = \frac{y_{i+1} + \cdots + y_{i+80}}{80} = \frac{[s_{i+1} \; s_{i+2} \; \dots s_{i+80}][s_{i+81} \; s_{i+81} \; \dots s_{i+160}]^*}{80} = \rho_i - \frac{y_i}{80} + \frac{y_{i+80}}{80}$$

…

Simulations

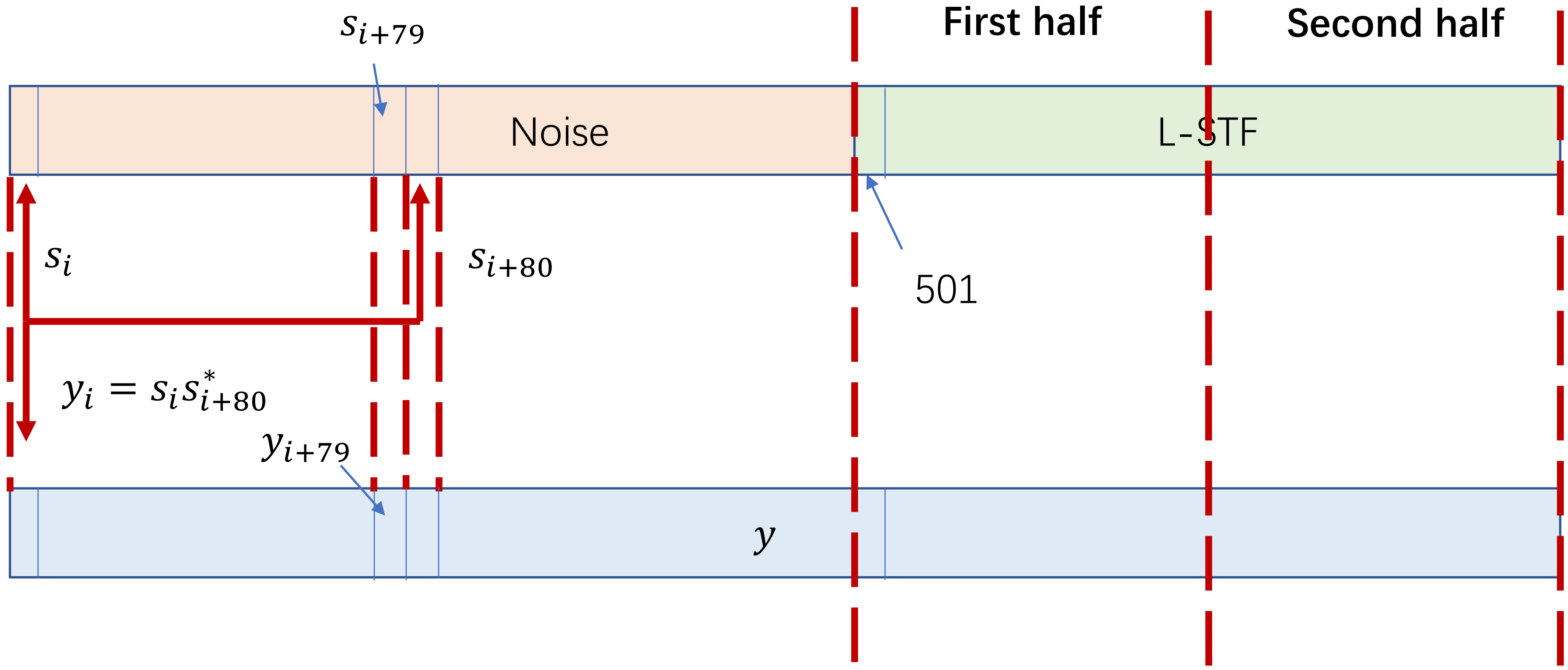
- The STF is added in a noise sequence
- Autocorrelation is used to detection the arrival of STF
- STF is added from the 501's sample
- With 0dB SNR, **we can easily find the arrival of STF**
- However, **the accurate starting point of STF cannot be detected**, as the autocorrelation at [499, 500, 501] are [0.9559, 0.9529, 0.9531]




```
clear;
NoisePower = 1;
cfgHT =
wlanHTConfig('ChannelBandwidth',
'CBW20');
STF = wlanLSTF(cfgHT);
s = sqrt(NoisePower/2) * (randn(1,1000)
+ 1j* randn(1,1000));
s(501:660) = s(501:660) + STF.';
rho = zeros(1, 800);
for i = 1:length(rho)
    rho(i) = s(i:i+79) *
s(i+80:i+159)' / 80;
end
plot(abs(rho));
grid;
```

Discussion – Time Synchronization

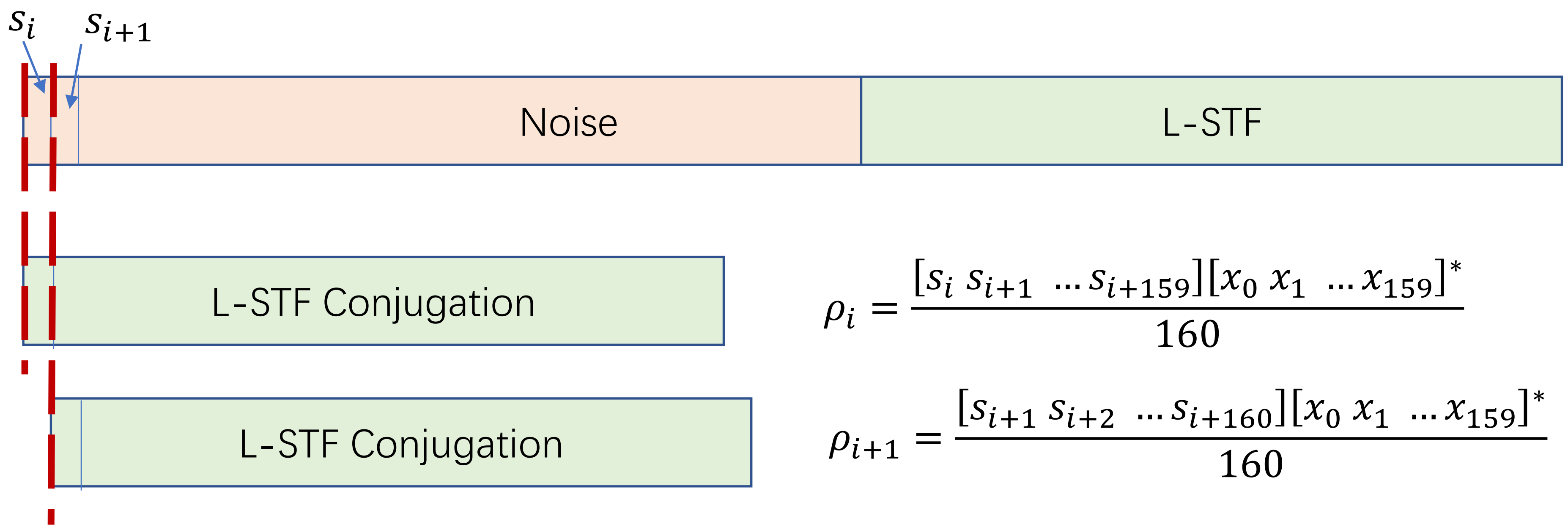
- Is it possible to determine the starting time of one PPDU accurately via auto-correlation?



$$\rho_{502} = \rho_{501} - \frac{y_{501}}{80} + \frac{y_{581}}{80} = \rho_{501} - \frac{s_{501}s_{581}^*}{80} + \frac{s_{581}s_{661}^*}{80}$$

Cross-correlation

- STF signal: $[x_0\ x_1\ \ldots x_{159}]$
- From the i-th received sample, pick up 160 consecutive samples and calculate their correlation with $[x_0\ x_1\ \ldots x_{159}]$. Repeat it for all i.

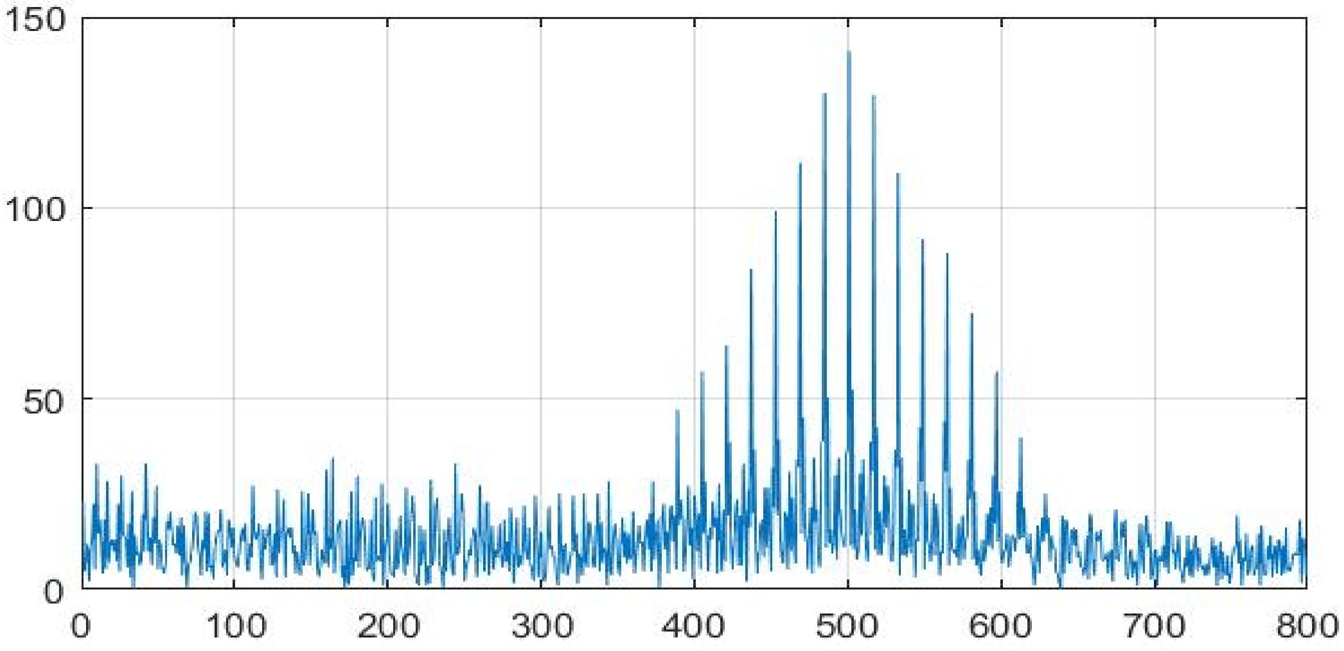


$$\rho_i = \frac{[s_i\ s_{i+1}\ \ldots s_{i+159}][x_0\ x_1\ \ldots x_{159}]^*}{160}$$

$$\rho_{i+1} = \frac{[s_{i+1}\ s_{i+2}\ \ldots s_{i+160}][x_0\ x_1\ \ldots x_{159}]^*}{160}$$

Simulations

- The STF is added in a noise sequence
- Cross-correlation is used to detection the position of STF
- STF is added from the 501's sample
- With 0dB SNR, **we can easily find the sharp highest peak is at position of 501**
- **However, this is still not enough. Low SNR makes the detection of highest peak difficult**

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clear;
NoisePower = 1;
cfgHT = wlanHTConfig('ChannelBandwidth',
'CBW20');
STF = wlanLSTF(cfgHT);
s = sqrt(NoisePower/2) * (randn(1,1000)
+ 1j* randn(1,1000));
s(501:660) = s(501:660) + STF.';
rho = zeros(1, 800);
for i = 1:length(rho)
    rho(i) = s(i:i+159) * conj(STF);
end
plot(abs(rho));
grid;
```

Frequency Offset

- **Frequency offset** refers to the oscillators' frequency difference between the transmitter and receiver.

- Suppose the STF signal is $[x_0\ x_1\ x_2 \ldots x_{159}]$

- Without noise, multipath and attenuation, the received STF signal is still not perfect due to frequency offset. Thus,

$$\left[s_0 = x_0, s_1 = x_1 e^{-j\alpha}, s_2 = x_2 e^{-j2\alpha}, \ldots, s_{159} = x_{159} e^{-j159\alpha}\right]$$

where α is the constant phase rotation due to the frequency offset.

Suppose the frequency offset is $f \Rightarrow \alpha = f \times \frac{1}{20M}$

Frequency Offset Estimation

- In STF, we know $x_i = x_{i+16} = x_{i+32} = \cdots$

- Notice that

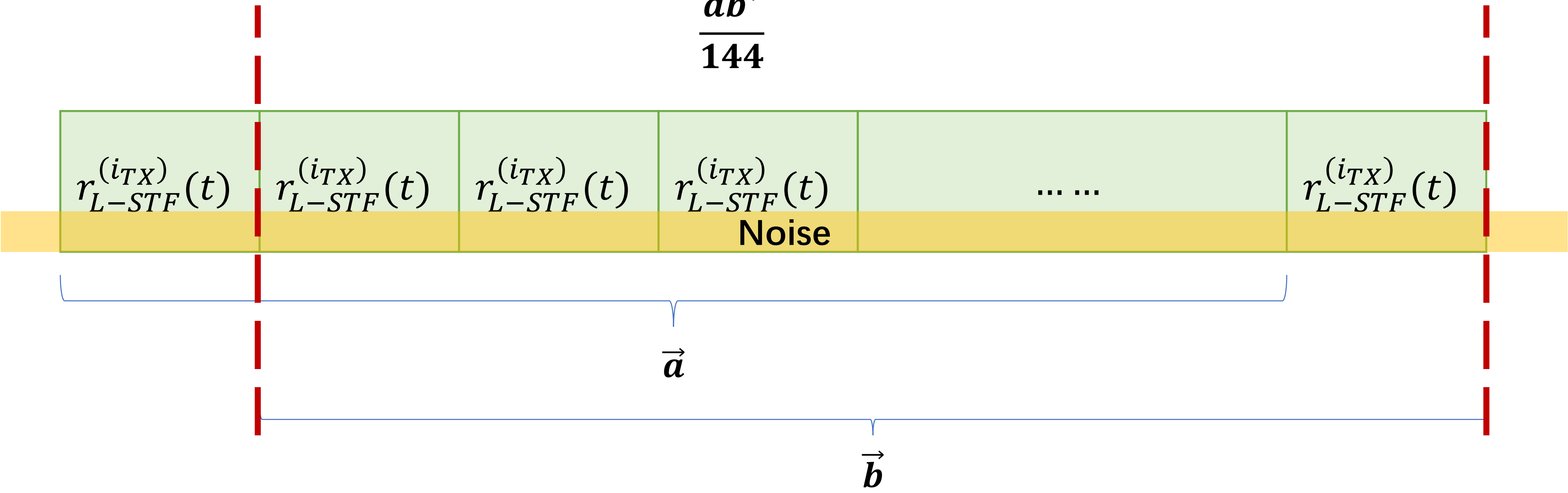
$$\sum_{i=0}^{143} s_i s_{i+16}^* = \sum_{i=0}^{143} s_i \left(s_i^* e^{j16\alpha}\right) = \sum_{i=0}^{143} |s_i|^2 e^{j16\alpha} = e^{j16\alpha} \sum_{i=0}^{143} |s_i|^2$$

We can divide the phase of $\sum_{i=0}^{143} s_i s_{i+16}^*$ by 16 to estimate α

Recap: Autocorrelation – Can We Do Better

More significant difference between with and without STF can be found if we calculate the auto-correlation as follows

$$\frac{\vec{a}\vec{b}^*}{144}$$



Frequency Offset Estimation

$$\sum_{i=0}^{143} s_i s_{i+16}^* = e^{j16\alpha} \sum_{i=0}^{143} |s_i|^2$$

Let $\angle$ denote the phase of a complex number within $[-\pi, \pi)$

$$\angle \sum_{i=0}^{143} s_i s_{i+16}^* = \angle e^{j16\alpha} \sum_{i=0}^{143} |s_i|^2 = 16\alpha + 2k\pi$$

Frequency Offset Estimation

Suppose that $\angle \sum_{i=0}^{143} s_i s_{i+16}^* \in [-\pi, \pi)$

$$16\alpha + 2k\pi = \angle \sum_{i=0}^{143} s_i s_{i+16}^*$$

$$\alpha = \frac{\angle \sum_{i=0}^{143} s_i s_{i+16}^*}{16} - \frac{k\pi}{8}$$

If we can make sure $\alpha \in [-\frac{\pi}{16}, \frac{\pi}{16})$, then $\alpha = \frac{\angle \sum_{i=0}^{143} s_i s_{i+16}^*}{16}$

Reading requirement

IEEE Std 802.11™-2020

- Section 9.1, 9.2

Homework

- Please complete the Assignment 1