**DOA(Direction Of Arrival) Introduction**

Suppose the system have M antennas, N signal packages, K targets.

Begin with time difference, if the signal arrives at ULA with angle , from the figure we can notice that there are different s, which causes phase difference , where c is the propagation speed of light, m is the number of arrays.

图表, 雷达图

描述已自动生成

We can induce the formula of arrived signal , assume there is only one signal package

We can simplify

When it comes to N packages, they come from N different directions:

**Traditional: MVDR(Minimum Variance Distortionless Response) Method etc**

First introducing weight vector , this vector helps us coordinate a specific direction to receive signals. It also make a great contribution in constraining the variance.

图示

描述已自动生成

The beam formed signal can be written as:

From the formula above we can calculate the beam formed signal power

If we take out the original signal :

Obviously, we want to minimize the noise and makes the signal go through the gateway completely, so we have our mathematic expression:

**Conventional Subspace-Based: MUSIC, ESPRIT**

**MUSIC(Multiple Signal Classification)**

The MUSIC algorithm is also called the decomposition subspace algorithm. The MUSIC algorithm has good angle measurement performance when performing DOA estimation on non-coherent signal sources. Since the MUSIC algorithm breaks through the performance bottleneck of the linear prediction algorithm, it can distinguish multiple target signal sources existing in a beam.

The mathematical model of the target signal source is:

Assuming that the noise is spatially ideal white noise and the noise power is , the received data covariance matrix of the antenna array can be obtained from above:

Eigenvalue decomposition of **:**

Where is a subspace formed by eigenvector corresponding to large eigenvalues, which also becomes a signal subspace, and is a subspace formed by eigenvector corresponding to small eigenvalues, and also becomes a noise subspace. Under ideal conditions, the steering vector in the signal subspace is orthogonal to the noise subspace:

Considering that the actual received data matrix is limited, the maximum likelihood estimate of the covariance matrix is:

The MUSIC algorithm is implemented with minimum optimized search:

The spatial spectral of MUSIC algorithm is:

**ESPRIT(Estimating Signal Parameters Via Rotational Invariance Techniques)**

Step 1

Calculate autocorrelation , apply eigenvalues decomposition to obtain eigenvectors

***[V,D] = eig(A) produces a diagonal matrix D of eigenvalues and a full matrix V whose columns are the corresponding eigenvectors so that A\*V = V\*D.***

Step 2

Construct matrix and , they are the first M-1 columns and last M-1 columns of respectively.

Step 3

Calculate the eigenvalues of

Step 4

Calculate the

***angle(H) returns the phase angles, in radians, of a matrix with complex elements.***

**Decorrelation: Forward Smooth, Backward Smooth, FB Smooth**