The Composition Operation

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4.2

Topics:

1) The compositions of functions

2) Arrow diagrams of compositions

Compositions

If $f: A \to B$ **and** $g: B \to C$ **, then we can build a new function called** $(g \circ f)(x)$ **that has domain A and codomain C, and that follows the rule . We call , read "g of f", the composition of g with f.**

Compositions

Say that $f(x)=2x$ **and** $g(x)=x+3$ **, then**

f of g

$$(f\circ g)(x)$$
$$=f(g(x))$$
$$=f(x+3)$$
$$=2(x+3)$$
$$(f\circ g)(x)=2x+6$$

g of f

$$(g\circ f)(x)$$
$$=g(f(x))$$
$$=g(2x)$$
$$(g\circ f)(x)=2x+3$$

Compositions

Say that $f(x)=2x$ **and** $(f\circ g)(x)=2x+6$ **, and we need to find g(x) from this information...**

1) Create an alias for g(x): a = g(x).

2) $f(a)=2a$ **, and** $2a=2x+6$

3) Solve for a $a=x+3$

4) So, $g(x)=x+3$

Compositions

Say that $g(x)=x+3$ **and** $(f\circ g)(x)=2x+6$ **, and we need to find f(x) from this information...**

1) Create an alias for g(x): a = g(x), or $a=x+3$

2) Solve for x: $x=a-3$

3) For f(g(x)), write out f(a) then sub in x = a – 3.

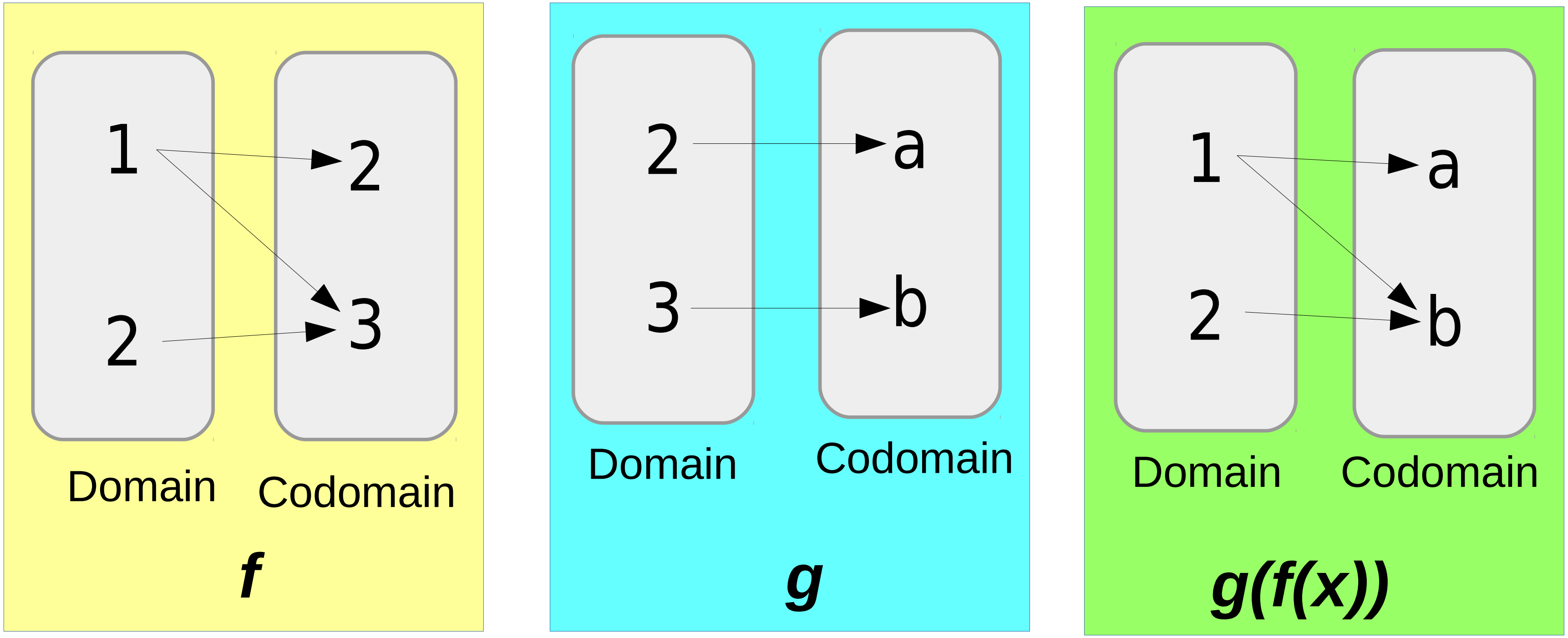
$$f(g(x))=2x+6=2(a-3)+6$$

4) Simplify $2a-6+6=2a$ **, so we found that**

$$f(x)=2x$$

Arrow diagrams

We can also draw diagrams for these compositions...



$$g(f(x)) = \left(g \circ f\right)\left(x\right)$$

Practice

<u>Example:</u> Given $f(x) = x + 1 \quad g(x) = 2x + 3$

Find $(f \circ g)(x)$

Practice

<u>Example:</u> Given $f(x) = x + 1 \quad g(x) = 2x + 3$

Find $(f \circ g)(x)$

$$\begin{aligned} f(g(x)) &= f(2x+3) \\ &= (2x+3)+1 \\ &= 2x+4 \end{aligned}$$

Practice

<u>Example:</u> Given $f(x) = x + 4$ $(f \circ g)(x) = 4x + 4$

Find $g(x)$

Practice

<u>Example:</u> Given $f(x)=x+4$ $\quad (f\circ g)(x)=4x+4$

Find $g(x)$

$$a=g(x)$$

$$f(g(x))=f(a)$$

$$a+4=4x+4$$

$$a=4x$$

$$\boxed{g(x)=4x}$$

Practice

<u>Example:</u> Given $g(x)=x+2$ $(f\circ g)(x)=3x+7$

Find $f(x)$

$$a=g(x)$$
$$a=x+2$$
$$x=a-2$$

$$f(g(x))=f(a)=3x+7$$
$$=3(a-2)+7$$
$$=3a-6+7$$
$$=3a+1$$
$$f(a)=3a+1$$
$$\boxed{f(x)=3x+1}$$