More Operations on Sets

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This Chapter:

1) Identifying and specifying partitions of a set.
2) Finding the Cartesian Product of two sets.
3) Finding the Power Set of a set

Cartesian Product

Given two sets A and B, the Cartesian Product of A and B ($A \times B$, or "A cross B") is defined as

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

So the result of A cross B is a set of coordinate pairs, which are combinations of the elements from A and B.

Additionally, $A \times A$ is written as A^2

Cartesian Product

A = { 1, 2 }   and   B = { 3 }

The result of $A \times B$ is
{ (1, 3), (2, 3) }

And the result of $B \times A$ is
{ (3, 1), (3, 2) }

So note that
$A \times B \neq B \times A$

Cartesian Product

It might be helpful to first write out all the coordinate pairs in a table before writing it out as a set…

Cartesian Product

A = { 2, 4, 6, 8 }   B = { 1, 2, 3, 4, 5 }

List all elements of $A \times B$

Cartesian Product

A = { 2, 4, 6, 8 }   B = { 1, 2, 3, 4, 5 }

List all elements of $A \times B$

B → A ↓	***1***	***2***	***3***	***4***	***5***
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)
8	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)

Remember that A's elements make up the x coordinate and B's elements make up the y coordinate!

Power Set

In mathematics, the power set (or powerset) of any set S is the set of all subsets of S, including the empty set and S itself.

https://en.wikipedia.org/wiki/Power_set

If S is the set {x, y, z}, then the subsets are:
{ } (empty set $\emptyset$), {x}, {y}, {z},
{x, y}, {x, z}, {y, z}, {x, y, z}

and hence the power set of
S is { $\emptyset$, {x}, {y}, {z}, {x, y}, {x, z}, {y, z}, {x, y, z}}

Power Set

1) What is the Power Set of { 1, 2, 3 }?

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{ Ø , {1}, {2}, {3}, {1, 2}, {1, 3}, {2, 3}, {1, 2, 3} }

Power Set

Given the sets A = {1, 2} and B = { 3 },

1) What is the Power Set $\wp(A \times B)$ **?**

Power Set

Given the sets A = {1, 2} and B = { 3 },

1) What is the Power Set $\wp(A \times B)$ **?**

A x B = { (1, 3), (2, 3) }

Power set of A x B is { ∅ , { (1, 3) }, { (2, 3) }, { (1, 3), (2, 3) } }

It might help to substitute each ordered pair with another variable, like j = (1, 3) and k = (2, 3). This way, you can see that the set has two elements, and the Power Set will have some combination of these.

Power Set

Given the sets A = {1, 2} and B = { 3 },

2) What is the result of $\wp(A) \times \wp(B)$ **?**

Power Set

Given the sets A = {1, 2} and B = { 3 },

2) What is the result of $\wp(A)\times\wp(B)$ **?**

The Power Set of A is { Ø , {1}, {2}, {1, 2} }.
The Power Set of B is { Ø , {3} }

The cross of these two would be pairs of all the elements…:

{ (∅, ∅), (∅, {3}), ({1}, ∅), ({1}, {3}), ({2}, ∅), ({2}, {3}), ({1, 2}, ∅), ({1, 2}, {3}) }

Again, it might be helpful to come up with substitutions if you're finding it hard to keep all the subsets straight.

Partitions

For each set A, a *partition* of A is a set of subsets of A, such that…:

1. For all i, $s_i \neq \emptyset$
(Each part is non-empty)

2. For all i and j, if $S_i \neq S_j$, then $S_i \cap S_j = \emptyset$
(Different parts have nothing in common)

3. $S_1 \cup S_2 \cup S_3 \cup \ldots = A$
(Every element in A is in some part)

Partitions

Given some set A = { 1, 2, 3, 4 },
There are many valid partitions, so long as each element shows up in the partition set.

{ {1}, {2}, {3, 4} }

{ {1, 2}, {3, 4} }

{ {1, 4}, {2, 3} }

{ {1, 2, 3}, {4} },

{ {1, 2, 3, 4 } }

Power Set

For the set { 1, 2, 3, 4, 5, 6 }, find a partition that meets the criteria:

a) Every part has the same size

b) No two parts have the same size

Power Set

For the set { 1, 2, 3, 4, 5, 6 }, find a partition that meets the criteria:

a) Every part has the same size
One answer: { {1, 3}, {2, 4}, {5, 6} }

b) No two parts have the same size
One answer: { {2}, {3, 6}, {4, 1, 5} }

Look at the in-class exercises
and the homework assignments
for practice with each of these!