Section 1: Introduction

For this section, we are analyzing sequences of numbers in order to build *closed* and/or *recursive* formulas to describe them.

It can be a bit challenging at first to figure out the equation based on a list of numbers, so make sure to take note of some techniques for analyzing these sequences!

Let's start off simply…

Exercise 1 **4%**

For the given sequence of numbers: 2, 4, 6, 8, 10

(a) What is the next number in the sequence?
If you can tell just by looking at it, *how* can you tell?

(b) If we assign numbers to each of these, for example:

Item 1	Item 2	Item 3	Item 4	Item 5
2	4	6	8	10

Come up with a way to tell what the value is based on the item #. Item number *n* is:

(c) If we're describing Item 2 in terms of Item 1 as a math equation, we can say that…
Item 2 = Item 1 + ?

(d) If we're describing Item 3 in terms of Item 2 as a math equation, we can say that…
Item 3 = ?

(e) If we want to generalize this and describe any item *n* in terms of the previous item, *n-1*, we can say that… Item *n* = ?

Section 2: Sequences

Definition: Recursive Formula

A **recursive formula** for a sequence is a formula where each term is described in relation to a previous term (or terms) of the sequence. This type of description must include enough information on how the list begins for the recursive relationship to determine every subsequent term in the list. This is sometimes called a **recurrence relation**.

Definition: Closed Formula

A **closed formula** for a sequence is a formula where each term is described only in relation to its position in the list.

Definition: Sequence Notation

We usually use lowercase letters (a, b, etc.) to name sequences, and we use subscripting to indicate position in a sequence. The notation a_n indicates the n-th term of the sequence we are writing as *a*. We read a_n as "a subscript n", or "a *sub* n". (Note, with computers it might be written a[n].)

Exercise 2 **12%**

Write out the first 5 *elements* of the following equations:

(a) The closed formula $a_n = n + 1$

(b) The closed formula $a_n = 2n + 1$

(c) The recursive formula $a_1 = 1, a_n = a_{n-1} + 2$

(d) The recursive formula $a_1 = 2, a_n = 2 \cdot a_{n-1} + 1$

Tips for finding equations
If it isn't immediately obvious what a sequence's function is, here are a few tips:

1. Write out each element, like: $a_1 = 2$, $a_2 = 5$, $a_3 = 10$, $a_4 = 17$, etc. Try to see the relationship between the **index** and the **element**.
2. Compare the difference between each element. (5 – 2 = **3**, 10 – 5 = **5**, 17 – 10 = **7**). Is the difference the same?
3. Compare the difference between the *differences*. Above, each iteration shows that the difference **increases** by 2 each time.

Exercise 3 **12%**

Figure out **closed formulas** for the following sequences. For these sequences, *n* will not be multiplied by anything, but will have something added to it.

(a) 3, 4, 5, 6, 7

(b) 6, 7, 8, 9, 10

Exercise 4 **12%**

Figure out **closed formulas** for the following sequences. For these sequences, *n* will have something multiplied to it.

(a) 2, 4, 6, 8, 10

(b) 3, 6, 9, 12, 15

(c) 5, 10, 15, 20, 25

(d) 1, 4, 9, 16, 25

Exercise 5 **12%**

Figure out **closed formulas** for the following sequences. For these sequences, *n* will have something multiplied to it and added to the product.

(a) 1, 3, 5, 7, 9

(b) 4, 7, 10, 13, 16

(c) 7, 12, 17, 22, 27

(d) 2, 5, 10, 17, 26

Exercise 6 **12%**

Figure out **recursive formulas** for the following sequences. For these sequences, a_{n-1} will not be multiplied by anything, but will have something added to it. Be sure to specify a_1 first.

(a) 1, 3, 5, 7, 9

(b) 1, 5, 9, 13, 17

(c) 2, 4, 6, 8, 10

(d) 2, 6, 10, 14, 18

Exercise 7 **12%**

Figure out **recursive formulas** for the following sequences. For these sequences, a_{n-1} will have something multiplied to it, but nothing added to it.

(a) 2, 4, 8, 16, 32

(b) 1, 3, 9, 27, 81

(c) 3, 6, 12, 24, 48

(d) 2, 4, 16, 256, 65536

Exercise 8 **12%**

Figure out **recursive formulas** for the following sequences. For these sequences, a_{n-1} will have something multiplied to it and added to it.

(a) 1, 3, 7, 15, 31

(b) 2, 5, 11, 23, 47

(c) 1, 5, 17, 53, 161

(d) 1, 4, 10, 22, 46

Section 3: Summations

For a sequence of numbers (denoted a_k) with $k \geq 1$, we use the notation

$$\sum_{k=1}^{n} a_k$$

to denote the sum of the first *n* terms of the sequence. This is called *sigma notation* for the sum.

Example:

Evaluate the sum $\sum_{k=1}^{3}(2k-1)$. First, we need to find elements 1, 2, and 3.

$$a_1=(2\cdot 1-1)=1 \quad , \qquad a_2=(2\cdot 2-1)=3 \quad , \qquad a_3=(2\cdot 3-1)=5$$

Then we add the values:

$$\sum_{k=1}^{3}(2k-1)=a_1+a_2+a_3=1+3+5=9$$

Exercise 9 **12 %**

Evaluate the following summations.

(a) $\sum_{k=1}^{7} 3k$

(b) $\sum_{k=1}^{9} 4$