Proofs About Numbers

Mathematical Writing

This Chapter:

1) Divisibility

2) Rational numbers

3) Modulus

4) More proofs

Definitions

Definition 1:

An integer n is divisible by a nonzero integer k if there is an integer q such that $n = kq$.

Other ways of saying this is,
“k divides n”, “k is a factor of n”,
“n is a multiple of k”.

Definitions

Definition 2:

A real number *r* is rational (aka a fraction) if there exists integers *a* and *b* (b is not 0) with r = a/b.

$$\frac{2}{8}=0.25$$

Definitions

Definition 3:

A real number is *irrational* if it is not rational.

$$\sqrt{(6)} \approx 2.449489743\ldots$$

Definitions

The Division Theorem (page 103)

For all integers *a* and *b* (b > 0), there is an integer *q* (called "the quotient when a is divided by b") and an integer *r* (called "the remainder when a is divided by b") such that:

1. $a = bq + r$
2. $0 \leq r < b$

Definitions

Definition 4:

Modulus (aka “mod”, “%” in some computer languages) is used to describe the remainder when one integer is divided by another.

Thus: $a \bmod b = r$
Means that $\underline{r}$ is the remainder when $\underline{a}$ is divided by $\underline{b}$.

Calculating Modulus

2a. 73 mod 6

12 r1

$6\overline{)73}$

72

1

= 1

2c. -1,234 mod 15

82 r4

$15\overline{)-1234}$

1230

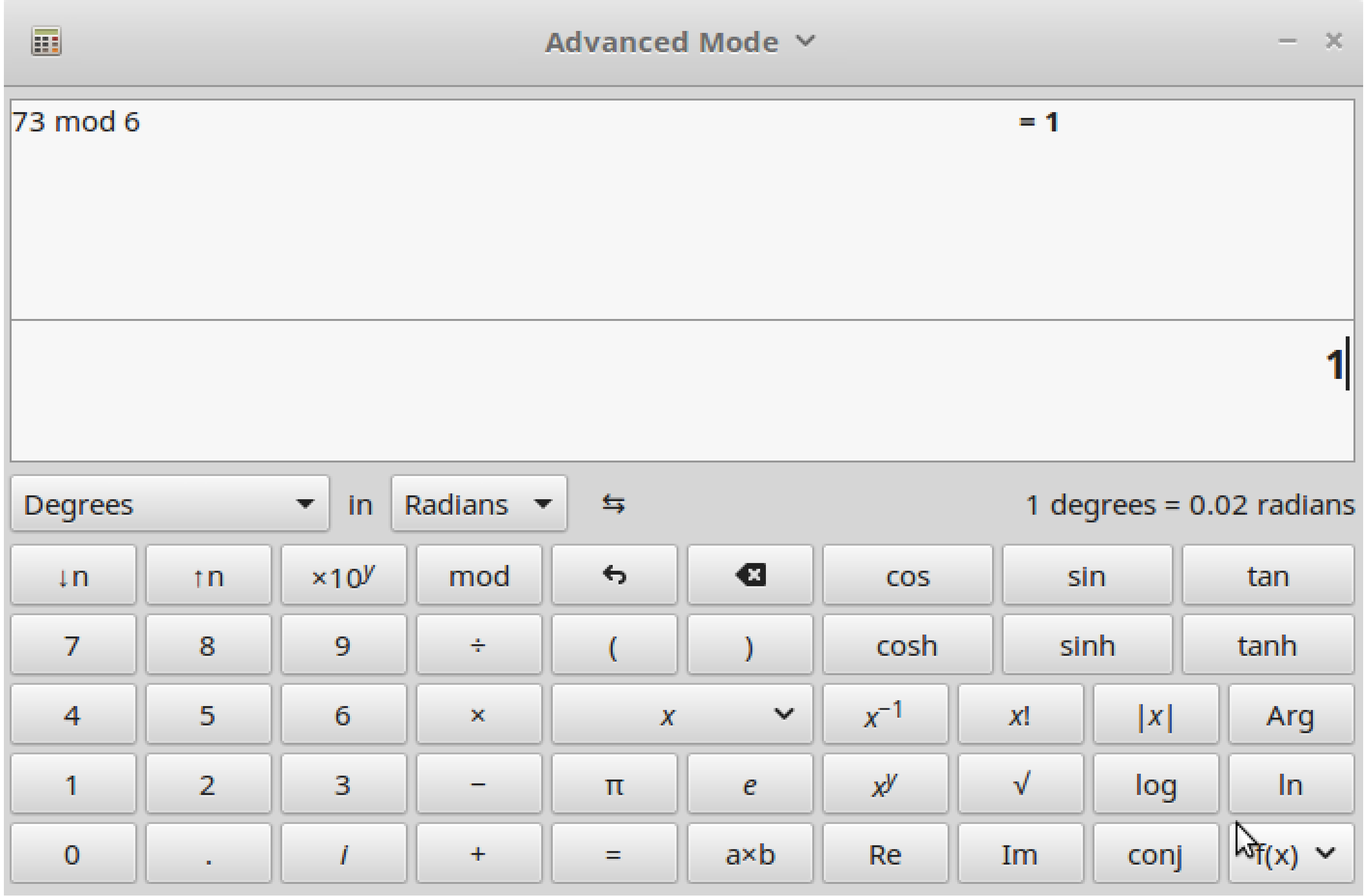
4

= 4

Calculating Modulus

2a. 73 mod 6

Most computer-based calculators also have the mod operator available.



Proofs

<u>7a. Prove that:</u>

"If a divides b and a divides c, then a divides b + c"

We need to express b and c as *divisible by a*, so we will restate them as:

b = Ka , c = La

1. Restate *b+c* with K,L terms: $Ka + La$
2. Factor out the common *a*: $a\left(K+L\right)$
3. Now we can see that
   K+L is still a factor of a.