# Mathematical Writing

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**This Chapter:**

1) Disproving statements by example

2) Writing simple proofs

3) Tracing Proofs

# 1. Disproving Statements

To disprove a statement, we simply need *one* example where the statement's result is false.

Remember that…

$$\neg(p \to q) \equiv p \wedge \neg q$$

For an implication to be **false** (i.e., invalid):

- the hypothesis must be true and
- the conclusion must be false.

# 1. Disproving Statements

Proving a statement requires more than just plugging in numbers to see what works out, because we would have to check *the entire domain* – and usually this is infinite.

But, at least, for disproving a statement, we can plug in values to show an example where the statement is invalid.

# 1. Disproving Statements

Statement:

For every integer , if *n* is odd,
then $n^2+4$ is a prime number.

Disprove:

Plugging in the value *9* results in

$$\begin{aligned} &9^2+4 \\ &=81+4 \\ &=85 \end{aligned}$$

And 85 is divisible by 5 and 17, so the statement is **false.**

# 2. Writing simple proofs

## For these simple proofs, we use the definitions:

<u>Divisible by 4</u>

"An integer *n* is divisible by 4 if it can be written in the form $n = 4M$ for some integer M"

<u>Even</u>

"An integer *n* is even if it can be written in the form $n = 2K$ for some integer K"

<u>Odd</u>

"...an integer *m* is odd if it can be written in the form $m = 2L+1$ for some integer L"

# 2. Writing simple proofs

Using the definitions of even, odd, and divisible by some number, we prove statements by subbing out simple variables like “x” with a statement like “2K+1” for odd, “2L” for even, or “4M” for divisible by 4.

# 2. Writing simple proofs

Example: Show that 10 is even with a proof.

Answer: Show that 10 is even by using
“2L” as the definition of an even number…

$2 \times 5 = 10$

# 2. Writing simple proofs

## Statement:

"The result of summing any odd integer with any even integer is an odd integer."

## Proof:

We can use "2K+1" to represent an odd integer and "2L" to represent an even integer.

The expected output should come out to
2n+1
(where n could be some combination of K and L together.)

# 2. Writing simple proofs

Statement:

"The result of summing any odd integer with any even integer is an odd integer."

Proof:

$$\left(2K+1\right)+\left(2L\right)$$
$$=2K+2L+1$$
$$=2\left(K+L\right)+1$$

The result is 2 x *some integer* + 1,
Which is the definition of an odd number.

# 3. Tracing Proofs

We will also be tracing proofs.

This means taking a proof and plugging in some numbers to “test it out”.

This is similar to how we were trying to disprove statements, but not with the intent to disprove – merely in order to *familiarize* ourselves with the proof.

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# 3. Tracing Proofs

<u>Statement:</u>

For all integers $n > 4$, if $n$ is a perfect square, then $n - 1$ is not a prime number.

<u>Tracing:</u>

Try 1 $n = 9 ; 9 - 1 = 8$ (2 and 4 are factors)

Try 2 $n = 16 ; n - 1 = 15$ (5 and 3 are factors)

Try 3 $n = 144 ; n - 1 = 143$ (11 and 13 are factors)