Predicates

Definitions

Predicate

In mathematical logic, a **predicate** is commonly understood to be a Boolean-valued function $P: X \to$ {true, false}, called the predicate on X.

Informally, a **predicate** is a statement that may be true or false depending on the values of its variables. [1] It can be thought of as an operator or function that returns a value that is either true or false.[2] For example, predicates are sometimes used to indicate set membership: when talking about sets, it is sometimes inconvenient or impossible to describe a set by listing all of its elements. Thus, a predicate *P(x)* will be true or false, depending on whether x belongs to a set.

For instance, $\{x \mid x$ is a positive integer less than $4\}$ is the set $\{1,2,3\}$.

If t is an element of the set $\{x \mid P(x)\}$, then the statement $P(t)$ is *true*.

Here, $P(x)$ is referred to as the *predicate*, and x the *subject* of the *proposition*. Sometimes, $P(x)$ is also called a propositional function, as each choice of x produces a proposition.

From https://en.wikipedia.org/wiki/Predicate_(mathematical_logic)

Predicates can be **always true** for all possible inputs, **sometimes true** given certain inputs, or **never true** for all possible inputs.

For these problems, a **domain** must also be specified.

Domain

In mathematics, and more specifically in naive set theory, the **domain of definition** (or simply the **domain**) of a function is the set of "input" or argument values for which the function is defined. That is, the function provides an "output" or value for each member of the domain.

From https://en.wikipedia.org/wiki/Domain_of_a_function

Existential quantification ∃, "There Exists"

In predicate logic, an **existential quantification** is a type of quantifier, a logical constant which is interpreted as "there exists", "there is at least one", or "for some". Some sources use the term **existentialization** to refer to existential quantification.[1]

From https://en.wikipedia.org/wiki/Existential_quantification

Universal quantification ∀, "For All"

In predicate logic, a **universal quantification** is a type of quantifier, a logical constant which is interpreted as "given any" or "for all". It expresses that a propositional function can be satisfied by every member of a domain of discourse. In other words, it is the predication of a property or relation to every member of the domain. It asserts that a predicate within the scope of a universal quantifier is true of every value of a predicate variable.

From https://en.wikipedia.org/wiki/Universal_quantification

The symbols $\forall$ and $\exists$ are **quantifiers**.

Examples

For the domain D = { 1, 2, 3, 4, 5 }, which inputs make the following predicates true? After analyzing these, does is the predicate **true for all** $d \in D$ **, or false for all** $d \in D$ **,** or **does there exist some *(at least one)*** $d \in D$ **that makes the predicate true?**

a. $P(x)$ is $d > 0$ True for all… $\forall d \in D, P(x)$	b. $Q(x)$ is $d < 2$ True for only Q(1), $\exists d \in D, Q(x)$
c. $R(x)$ is d is even True for R(2) and R(4), $\exists d \in D, R(x)$	d. $S(x)$ is d is negative Not true for any, $\neg(\exists d \in D, S(x))$ $= \forall d \in D, d$ is positive

Negations

Propositions

- $\neg(p \vee q) \equiv \neg p \wedge \neg q$
- $\neg(p \wedge q) \equiv \neg p \vee \neg q$

Predicates

- $\neg(\forall x \in D, P(x)) \equiv \exists x \in D, \neg P(x)$
- $\neg(\exists x \in D, Q(x)) \equiv \forall x \in D, \neg Q(x)$

Examples

For the domain D = { 1, 2, 4, 8, 16 }…

e. *P(n)* is the predicate, “n > 8”.

What is the negation? $\neg(n > 8) \equiv n \leq 8$

This negation is true for which elements of the domain? 1, 2, 4, 8

f. *Q(n)* is the predicate, “n is even”.

What is the negation? “n is odd”

This negation is true for which elements of the domain? None