Set Definitions and Operations

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Set Definitions and Operations

This Chapter:

1) Common sets of numbers,
2) Specifying elements of a set
3) Specifying subsets of a set
4) Difference, Intersection, & Union of sets
5) Venn diagrams
6) Set-builder notation

Set Definitions and Operations

There are a lot of things to cover for this first chapter! Let's take it a step at a time...

Common Sets

$\mathbb{N}$ The set of natural numbers

These are numbers that can answer counting problems.

$\mathbb{Q}$ The set of rational numbers

These are characterized as ratios of integers such as 1/2, -17/4, or 3/1

$\mathbb{R}$ The set of real numbers

These can be thought of as decimal numbers with possibly unending strings of digits after the decimal point.

$\mathbb{Z}$ The set of integers

Numbers (positive, negative, and zero) without a fractional/decimal portion.

Common Sets

We might also specify these sets with additional attributes:

$\mathbb{R}^{+}$ **The set of positive real numbers.**

$\mathbb{R}^{\geq 0}$ **The set of nonnegative real numbers.**

$\mathbb{Q}^{+}$ **The set of positive rationals.**

$\mathbb{Q}^{\geq 0}$ **The set of nonnegative rationals.**

$\mathbb{Z}^{+}$ **The set of positive integers.**

$\mathbb{Z}^{\geq 0}$ **The set of nonnegative integers, same as** $\mathbb{N}$

Our Sets

When working through problems, often we will explicitly define the sets that we are working with.

These generally are represented with a capital letter.

$A = \{ 1, 2, 3 \}$ $B = \{ 2, 4, 6, 8 \}$, ETC.

Universal & Empty Sets

We will also encounter the **universal set** and the **empty set** when working with sets.

When defining a problem and specifying sets, we must also include the universal set U, which the other sets are contained within.

If we look at some combinations of sets together, it is possible that the result will be an *empty set*, which is denoted with $\emptyset$

Example...

We have two sets, A = { 1, 2, 3 }, B = { 2, 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) Which elements do A and B have in common?

Example...

We have two sets, A = { 1, 2, 3 }, B = { 2, 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) Which elements do A and B have in common?

The set of elements that A and B have in common is { 2, 3 }

This is known as the <u>intersection</u> of A and B, denoted as:

$$A \cap B = \{2, 3\}$$

Example...

We have two sets, $A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 4 \}$

Our universal set is $U = \{ 1, 2, 3, 4, 5 \}$

1) Which elements does A have, that B doesn't have?

2) Which elements does B have, that A doesn't have?

Example...

We have two sets, A = { 1, 2, 3 }, B = { 2, 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) Which elements does A have, that B doesn't have?

The difference between A and B is that A has { 1 }

This is known as the <u>difference</u> of A and B, denoted as:

$$A - B = \{1\}$$

2) Which elements does B have, that A doesn't have?

The difference between B and A is that B has { 4 }

This is known as the <u>difference</u> of B and A, denoted as:

$$B - A = \{4\}$$

Example...

We have two sets, $A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 4 \}$

Our universal set is $U = \{ 1, 2, 3, 4, 5 \}$

1) What is the set if we combine all elements of A and B?

Example...

We have two sets, A = { 1, 2, 3 }, B = { 2, 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) What is the set if we combine all elements of A and B?

Combining A and B gives us { 1, 2, 3, 4 }.

This is known as the <u>union</u> of A and B, denoted as:

$$A \cup B = \{1, 2, 3, 4\}$$

Example…

We have two sets, A = { 1, 2, 3 }, B = { 2, 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) What is the complement of A?

The complement of A would be everything in the Universe that is not in A: { 4, 5 }

This is known as <u>A's complement</u>, denoted as A-prime:

$$A' = \{4, 5\}$$

Note that A' is equivalent to:

$$A' = U - A$$

Summary

$A \cap B$ **The intersection of A and B**

Elements common to both A and B

$A \cup B$ **The union of A and B**

Elements that are either in set A or in set B

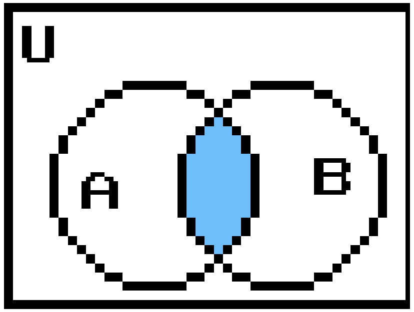
$A - B$ **The difference of A and B**

Elements in A which are not in B

A' **The complement of A**

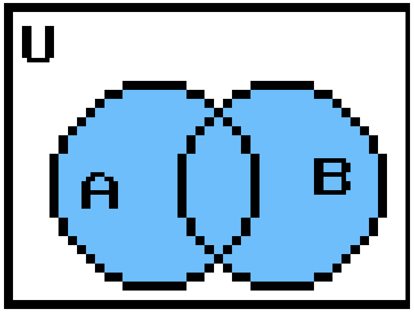
Elements of the universal set U which are not in A

As Venn Diagrams…



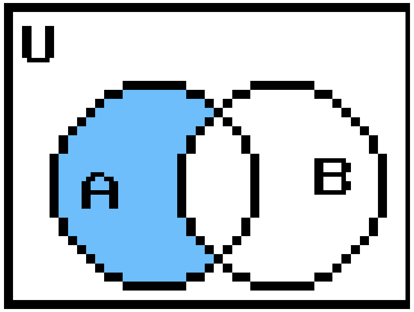
$A \cap B$

Intersection



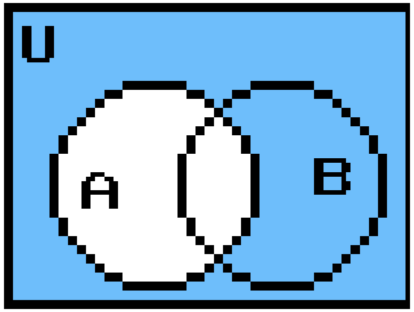
$A \cup B$

Union



$A - B$

Difference



A'

Complement

Subsets of a set

$A \subseteq B$ **A is a subset of B**

A is a subset of B if every element in A is also an element in B.

Formally, this means that for every *x*,
If $x \in A$ then $x \in B$

$A = B$ **A is equal to B**

A is equal to B if they have exactly the same members.

Formally, this means $A \subseteq B$ and $B \subseteq A$

Example...

We have the sets,
A = { 1, 2, 3 }, B = { 2, 3, 4 }, C = { 1, 2 }, D = { 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) **Is C a subset of A?**

2) **Is C a subset of B?**

3) **Is D a subset of A?**

4) **Is D a subset of B?**

Example...

We have the sets,
A = { 1, 2, 3 },   B = { 2, 3, 4 },   C = { 1, 2 }, D = { 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) **Is C a subset of A?** All elements of C are also in A, so $C \subseteq A$

2) **Is C a subset of B?** C contains 1, which is not in B, so $C \not\subseteq B$

3) **Is D a subset of A?** D contains 4, which is not in A, so $D \not\subseteq A$

4) **Is D a subset of B?** All elements of D are also in B, so $D \subseteq B$

Example...

We have the sets,
$A = \{ 1, 2, 3 \}$, $B = \{ 2, 3, 4 \}$, $C = \{ 1, 2 \}$, $D = \{ 3, 4 \}$

Our universal set is $U = \{ 1, 2, 3, 4, 5 \}$

1) If C is a subset of A, is A also a subset of C?

Example...

We have the sets,
A = { 1, 2, 3 }, B = { 2, 3, 4 }, C = { 1, 2 }, D = { 3, 4 }

Our universal set is U = { 1, 2, 3, 4, 5 }

1) If C is a subset of A, is A also a subset of C?

No, A has 3 in its set, and C does not.

They are only subsets of each other if they are EQUAL.

Set-builder Notation

With sets, often we cannot write out the entire set if the set is something like, “all even numbers”.

Therefore, we use **set-builder notation**
To define our sets.

There is the **property description set-builder notation**
And the **form description set-builder notation.**

Set-builder Notation

Property Description

The set of all even integers:

$$\{\ x \in \mathbb{Z}\ :\ x = 2y \text{ for some } y \in \mathbb{Z}\ \}$$

We define the set that *x* belongs to.

We define x as some even number

We define the set that y is in as well.
(y must be an integer for the definition of even integer)

Set-builder Notation

Form Description

The set of all even integers:

$$\{\ 2y\ :\ y \in \mathbb{Z}\ \}$$

We define an even number. [x =] 2y

We define the set that *y* belongs to.

Note that we don’t even mention *x* in this form.

Let’s work through some examples