

Definitions and Diagrams

4.1

Topics:

- 1) Functions
- 2) Relations
- 3) Arrow diagrams

Function Notation

The notation $f: A \rightarrow B$ is used for a function, simply called f , with a set of inputs A (called the *domain*), and a set B (called the *codomain*) that includes all the *outputs*. The function f associates with each input in A one and only one output in B .

$f: A \rightarrow B$ is read “ f is a function from A to B ”.

Functions

$f : A \rightarrow B$ Is read “ f is a function from A to B .”

The domain is A – the inputs.

The codomain is B – the outputs.

And we add a rule (like $f(x) = x^2$)

Description of a Function

To completely describe a function, we must do four things:

- 1) Give the function a name. f , g , and h are popular names for functions, but it's always okay to be creative and descriptive.
- 2) Describe the domain.
- 3) Describe the codomain.
- 4) Describe the rule.

Arrow Diagrams

Arrow diagrams list all the elements of the domain and codomain, with arrows pointing from each input, going to each output.

Functions

Function Name: f

Domain: $\{1, 2, 3\}$

Codomain: \mathbb{N}

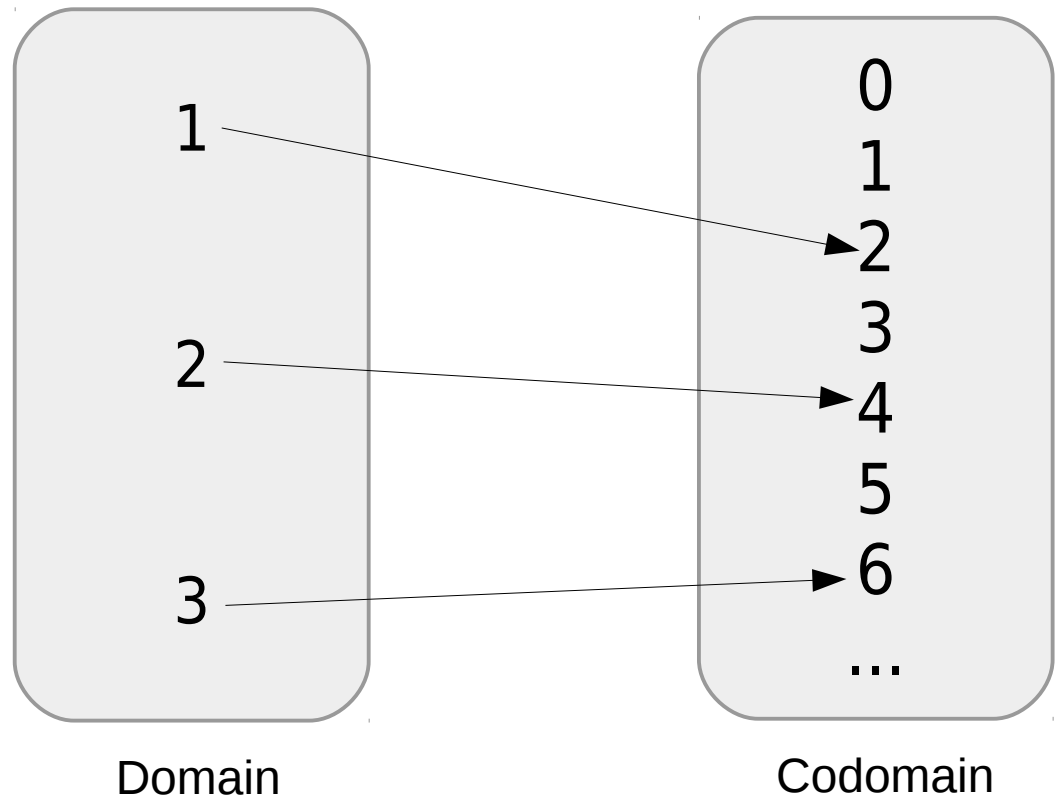
Rule: $2k$

We can diagram each input to an output with an arrow diagram.

$$f(1) = 2$$

$$f(2) = 4$$

$$f(3) = 6$$



Binary Relation

A binary relation R consists of three components:
a domain A , a codomain B ,
and a subset of $A \times B$ called the “rule” for the relation.

Relation

For Relation R:

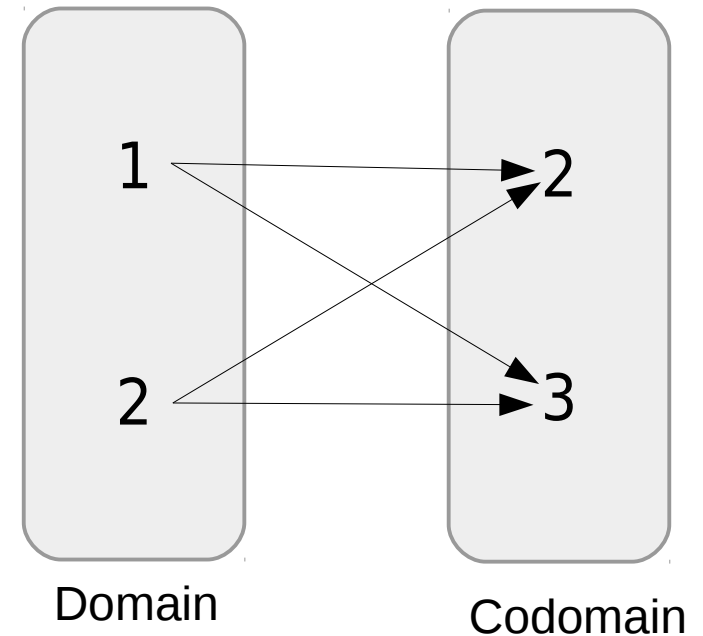
Domain: $A = \{1, 2\}$

Codomain: $B = \{2, 3\}$

Rule:

$L = \{ (1, 2), (1, 3), (2, 2), (2, 3) \}.$

We can diagram each input to an output with an arrow diagram.



A function that is a binary relation

A function F from A to B is a binary relation with the domain A and the codomain B with the property that for every $x \in A$, there is exactly one element $y \in B$ for which

$$(x, y) \in F$$

A function that is a binary relation

Let R be the relation whose domain is $\{a, b, c, d\}$ and whose codomain is \mathbb{N} , and whose rule is given by $\{(a, 1), (b, 3), (a, 5), (c, 3), (d, 7)\}$. Is this relation also a function?

1. Domain: $\{a, b, c, d\}$

Codomain: \mathbb{N}

2. Does each input in the domain have an output in the subdomain? **YES**

3. Does each input in the domain have one and only one output in the codomain? **NO: $(a, 1)$ and $(1, 5)$**

So it is not a function

A function that is a binary relation

Let R be the relation whose domain is $\{a, b, c, d\}$ and whose codomain is \mathbb{N} , and whose rule is given by $\{(a, 1), (b, 3), (a, 5), (c, 3), (d, 7)\}$. Is this relation also a function?

If the rule were $\{(a, 1), (b, 3), (c, 3), (d, 7)\}$ instead, then it would be a function as well as a relation.

Relations

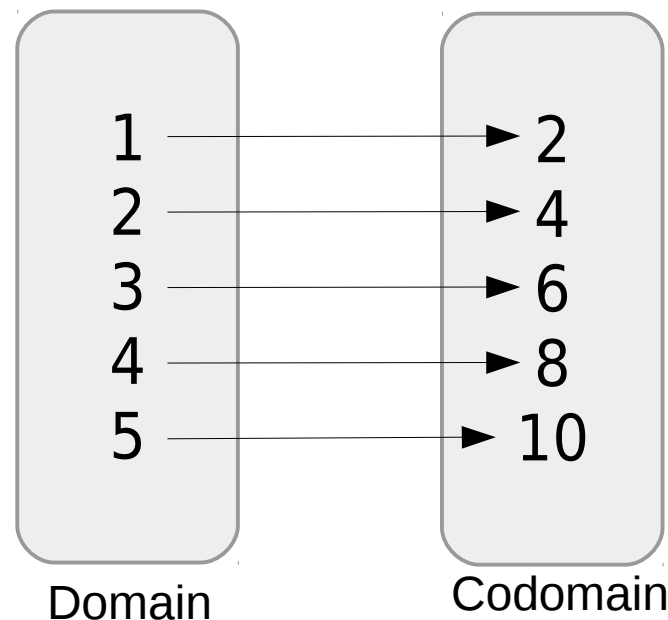
Example: Draw a portion of the arrow diagram showing at least five elements of the domain and codomain:

$$f : \mathbb{N} \rightarrow \mathbb{N} \text{ with } f(n) = 2n$$

Relations

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Relations

Example: Draw the two-set arrow diagram for the relation R where the Domain is the power set $\wp(\{a, b\})$, the Codomain is the set $B = \{0, 1, 2\}$, and the Rule is $(S, n) \in R$ where n is the sum of the elements in S .

Relations

Example: Draw the two-set arrow diagram for the relation R where the Domain is the power set $\wp(\{a, b\})$, the Codomain is the set $B = \{0, 1, 2\}$, and the Rule is $(S, n) \in R$ where n is the sum of the elements in S .

Hint:
Here is the domain

$\{\}$

$\{a\}$

$\{b\}$

$\{a, b\}$

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Hint:
Here is the
codomain

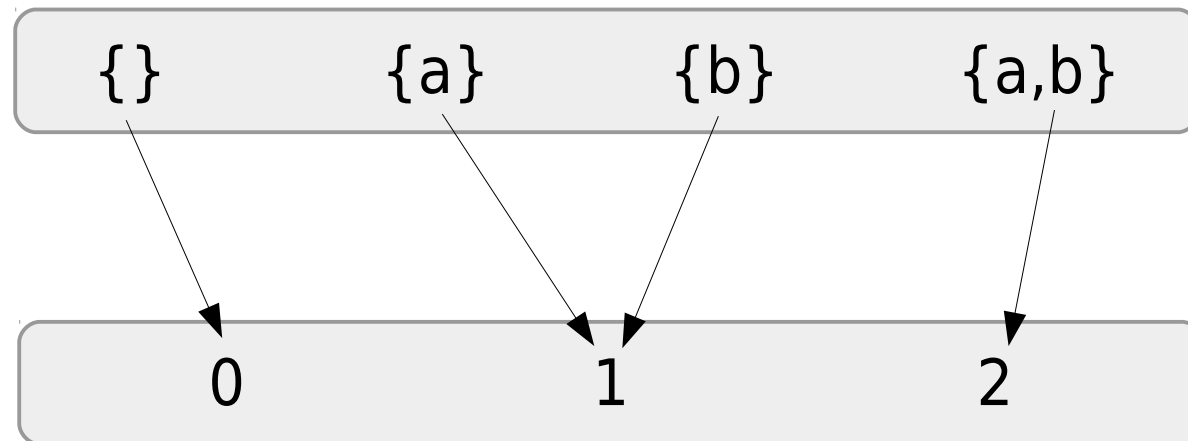
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Relations

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Relations

Example: Is the relation R also a function?

Domain & codomain: $\{1, 2, 3, 4, 5\}$.

Rule: $\{ (1, 5), (2, 3), (3, 3), (4, 2), (5, 1) \}$

Relations

Example: Is the relation R also a function?

Domain & codomain: $\{1, 2, 3, 4, 5\}$.

Rule: $\{ (1, 5), (2, 3), (3, 3), (4, 2), (5, 1) \}$

1. Does every input have an output in the codomain?

YES

2. Does each input in the domain have one and only one output in the codomain?

YES

So it is a function.