Mathematical Writing

Written by Rachel J. Morris, last updated 2016-09-17

Mathematical Writing

This Chapter:

1) Disproving statements by example

2) Writing simple proofs

3) Tracing Proofs

1. Disproving Statements

To disprove a statement, we simply need *one* example where the statement's result is false.

Remember that…

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

For an implication to be **false** (i.e., invalid):

- the hypothesis must be true and
- the conclusion must be false.

1. Disproving Statements

Proving a statement requires more than just plugging in numbers to see what works out, because we would have to check *the entire domain* – and usually this is infinite.

But, at least, for disproving a statement, we can plug in values to show an example where the statement is invalid.

1. Disproving Statements

Statement:

For every integer , if *n* is odd, then n^2+4 is a prime number.

Disprove:

Plugging in the value *9* results in

$$\begin{aligned} & 9^2+4 \\ = & 81+4 \\ = & 85 \end{aligned}$$

And 85 is divisible by 5 and 17, so the statement is **false.**

2. Writing simple proofs

For these simple proofs, we use the definitions:

Divisible by 4

"An integer n is divisible by 4 if it can be written in the form $n = 4M$ for some integer M"

Even

"An integer n is even if it can be written in the form $n = 2K$ for some integer K"

Odd

"...an integer m is odd if it can be written in the form $m = 2L+1$ for some integer L"

2. Writing simple proofs

Using the definitions of even, odd, and divisible by some number, we prove statements by subbing out simple variables like “x” with a statement like “2K+1” for odd, “2L” for even, or “4M” for divisible by 4.

2. Writing simple proofs

Example: Show that 10 is even with a proof.

Answer: Show that 10 is even by using "2L" as the definition of an even number…

$$2 \times 5 = 10$$

2. Writing simple proofs

Statement:

“The result of summing any odd integer with any even integer is an odd integer.”

Proof:

We can use “2K+1” to represent an odd integer and “2L” to represent an even integer.

The expected output should come out to
2n+1
(where n could be some combination of K and L together.)

2. Writing simple proofs

Statement:

"The result of summing any odd integer with any even integer is an odd integer."

Proof:

$$(2K+1)+(2L)$$
$$=2K+2L+1$$
$$=2(K+L)+1$$

The result is 2 x *some integer* + 1,
Which is the definition of an odd number.

3. Tracing Proofs

We will also be tracing proofs.

This means taking a proof and plugging in some numbers to “test it out”.

This is similar to how we were trying to disprove statements, but not with the intent to disprove – merely in order to *familiarize* ourselves with the proof.

3. Tracing Proofs

We will also be tracing proofs.

This means taking a proof and plugging in some numbers to “test it out”.

This is similar to how we were trying to disprove statements, but not with the intent to disprove – merely in order to *familiarize* ourselves with the proof.

3. Tracing Proofs

Statement:

For all integers *n > 4*, if *n* is a perfect square, then *n – 1* is not a prime number.

Tracing:

Try 1 $n=9 ; 9-1=8$ (2 and 4 are factors)

Try 2 $n=16 ; n-1=15$ (5 and 3 are factors)

Try 3 $n=144 ; n-1=143$ (11 and 13 are factors)