Section 1: Predicates

Definition: Predicate

A **predicate** *P(x)* is a statement that incorporates a variable *x*, such that whenever *x* is replaced by a value, the resulting proposition is unambiguously true or false.

Example:

p is the proposition, "x > 10". It can either be true or false, but we cannot tell without additional information, so we build truth tables to show the possible results.

P(x) is the predicate, "x > 10". Now, we can tell whether it is true or false based on what is plugged into P. P(2) is false, but P(12) is true.

Exercise 1 **15%**

For each of the predicates, replace *x* with the values 2, 23, -5, and 15. Specify whether the result is true or false.

(a) *P(x)* is " $x>15$ " (b) *Q(x)* is " $x\leq 15$ " (c) *R(x)* is " $(x>5)\wedge(x<20)$ "

When we're working with predicates, we will also define the domain.

The **domain** is a set of numbers that are possible inputs for our predicate, *P(x)*. In other words, when we plug a value into *x*, it will be a number from the domain.

Exercise 2 **20%**

For the following predicates and domain sets given, specify whether the predicate is true for **all members of the domain**, **some members of the domain**, or **no members of the domain**.

(a) *P(x)* is " $x>15$ "
Domain is { 10, 12, 14, 16, 18 }

(b) *Q(x)* is " $x\leq 15$ "
Domain is { 0, 1, 2, 3 }

(c) *R(x)* is " $(x>5)\wedge(x<20)$ "
Domain is { 0, 1, 2 }

(c) *R(x)* is " $(x>1)\wedge(x<5)$ "
Domain is { 2, 3, 4 }

Section 2: Quantifiers

When we're working with predicates and domains, we can specify that the variable *x* is some number in the domain, symbolically, this way: $x \in D$. This is read as, "x exists in domain D".

We can further symbolize questions like in Exercise 2, with symbols $\forall$ and $\exists$.

Definitions

1. The symbol **∈** indicates membership in a set. For example, "*k* ∈ *D*" means that *k* is a member of the set *D*.
2. The symbol **∀** means "for all" or "for every".
3. The symbol **∃** means "there is (at least one)" or "there exists (at least one)".
4. The symbols **∀** and **∃** are called **quantifiers**. When we use quantifiers with a predicate, we refer to the resulting statement as a **quantified predicate.**

Example

For the following predicate, rewrite the sentence symbolically:

P(x) is " $x > 15$ ", Domain D is { 16, 17, 18 }, and as we can see, for every value of *x*, the predicate is true.

$$\forall x \in D, P(x) \quad \text{(“For all x in D, x is greater than 15.”)}$$

Exercise 3 **12%**

For the following predicates, rewrite the sentence symbolically. D = { 3, 4, 5, 10, 20, 25 }. After writing, determine whether the statement is **true** or **false**. You can write the predicate as-is; it does not need to be specified as *P(s)*.

(a) For every *n* that is a member of domain *D*, $n < 20$.

(b) For all *n* in the set *D*, $n < 5$ or *n* is a multiple of 5.

Exercise 4 **14%**

For the following predicates, rewrite the sentence symbolically. D = { 3, 4, 5, 10, 20, 25 }. After writing, determine whether the statement is **true** or **false**. You can write the predicate as-is; it does not need to be specified as *P(s)*.

(a) There is (at least one) *k* in the set *D* with the property that k^2 is also in the set *D*.

(How can you specify the predicate " k^2 is also in the set *D*" symbolically?)

(b) There exists *m*, a member of *D*, such that $m \geq 3$.

Section 3: Negating Quantifiers

Proposition 1

For any predicates *P* and *Q* over a domain *D*,

- The negation of **∀x∈D, P(x)** is **∃x∈D, ¬P(x)**
- The negation of **∃x∈D, Q(x)** is **∀x∈D, ¬Q(x)**

When negating a predicate that uses an equal sign, the negation would be "not equals":

$$\neg(a=b)\equiv a\neq b$$

Exercise 5 **15%**

Write the negation of each of these statements, simplified so as not to require the $\neg$ symbol to the left of any quantifier.

(a) $\forall x\in\mathbb{Z}, \exists y\in\mathbb{Z}, x+2y=3$

(b) $\exists x>0, \forall y>0, x\cdot y<x$

(c) $\exists x\in\mathbb{Z}, \exists y\in\mathbb{Z}, x+y=13, \text{ and } x\cdot y=36$

Exercise 6 **24%**

Which elements of the set D = { 2, 4, 6, 8, 10, 12 } make the **negation** of each of these predicates true?

(a) *Q(n)* is the predicate, “n > 10”.

(b) *R(n)* is the predicate, “n is even”.

(c) *S(k)* is the predicate, “ $k^2 < 1$ ”

(d) *T(m)* is the predicate, “m – 2 is an element of D”.