

More Operations on Sets

More Operations on Sets

This Chapter:

- 1) Identifying and specifying partitions of a set.
- 2) Finding the Cartesian Product of two sets.
- 3) Finding the Power Set of a set

Cartesian Product

Given two sets A and B, the Cartesian Product of A and B ($A \times B$, or “A cross B”) is defined as

$$A \times B = \{(a, b) : a \in A, b \in B\}$$

So the result of A cross B is a set of coordinate pairs, which are combinations of the elements from A and B.

Additionally, $A \times A$ is written as A^2

Cartesian Product

$$A = \{1, 2\} \quad \text{and} \quad B = \{3\}$$

The result of $A \times B$ is
 $\{(1, 3), (2, 3)\}$

And the result of $B \times A$ is
 $\{(3, 1), (3, 2)\}$

So note that
 $A \times B \neq B \times A$

Cartesian Product

It might be helpful to first write out all the coordinate pairs in a table before writing it out as a set...

Cartesian Product

$$A = \{ 2, 4, 6, 8 \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

List all elements of $A \times B$

Cartesian Product

$$A = \{ 2, 4, 6, 8 \}$$

$$B = \{ 1, 2, 3, 4, 5 \}$$

List all elements of $A \times B$

$B \rightarrow$ $A \downarrow$	1	2	3	4	5
2	(2,1)	(2,2)	(2,3)	(2,4)	(2,5)
4	(4,1)	(4,2)	(4,3)	(4,4)	(4,5)
6	(6,1)	(6,2)	(6,3)	(6,4)	(6,5)
8	(8,1)	(8,2)	(8,3)	(8,4)	(8,5)

*Remember that A's elements make up the x coordinate
and B's elements make up the y coordinate!*

Power Set

In mathematics, the power set (or powerset) of any set S is the set of all subsets of S , including the empty set and S itself.

https://en.wikipedia.org/wiki/Power_set

If S is the set $\{x, y, z\}$, then the subsets are:

$\{\}$ (empty set \emptyset), $\{x\}$, $\{y\}$, $\{z\}$,
 $\{x, y\}$, $\{x, z\}$, $\{y, z\}$, $\{x, y, z\}$

and hence the power set of
 S is $\{\emptyset, \{x\}, \{y\}, \{z\}, \{x, y\}, \{x, z\}, \{y, z\}, \{x, y, z\}\}$

Power Set

1) What is the Power Set of $\{ 1, 2, 3 \}$?

Power Set

1) What is the Power Set of $\{ 1, 2, 3 \}$?

$\{ \emptyset , \{1\}, \{2\}, \{3\}, \{1, 2\}, \{1, 3\}, \{2, 3\}, \{1, 2, 3\} \}$

Power Set

Given the sets $A = \{1, 2\}$ and $B = \{3\}$,

1) What is the Power Set $\wp(A \times B)$?

Power Set

Given the sets $A = \{1, 2\}$ and $B = \{3\}$,

1) What is the Power Set $\mathcal{P}(A \times B)$?

$$A \times B = \{(1, 3), (2, 3)\}$$

Power set of $A \times B$ is $\{\emptyset, \{(1, 3)\}, \{(2, 3)\}, \{(1, 3), (2, 3)\}\}$

It might help to substitute each ordered pair with another variable, like $j = (1, 3)$ and $k = (2, 3)$. This way, you can see that the set has two elements, and the Power Set will have some combination of these.

Power Set

Given the sets $A = \{1, 2\}$ and $B = \{3\}$,

2) What is the Power Set $\wp(A) \times \wp(B)$?

Power Set

Given the sets $A = \{1, 2\}$ and $B = \{3\}$,

2) What is the Power Set $\wp(A) \times \wp(B)$?

The Power Set of A is $\{\emptyset, \{1\}, \{2\}, \{1, 2\}\}$.

The Power Set of B is $\{\emptyset, \{3\}\}$

The cross of these two would be:

$\{\emptyset, (\{1\}, \{3\}), (\{2\}, \{3\}), (\{1, 2\}, \{3\})\}$

Again, it might be helpful to come up with substitutions if you're finding it hard to keep all the subsets straight.

Partitions

For each set A , a *partition* of A is a set of subsets of A , such that...:

1. For all i , $s_i \neq \emptyset$
(Each part is non-empty)
2. For all i and j , if $s_i \neq s_j$, then $s_i \cap s_j = \emptyset$
(Different parts have nothing in common)
3. $s_1 \cup s_2 \cup s_3 \cup \dots = A$
(Every element in A is in some part)

Partitions

Given some set $A = \{ 1, 2, 3, 4 \}$,
There are many valid partitions, so long as
each element shows up in the partition set.

$\{ \{1\}, \{2\}, \{3, 4\} \}$

$\{ \{1, 2\}, \{3, 4\} \}$

$\{ \{1, 4\}, \{2, 3\} \}$

$\{ \{1, 2, 3\}, \{4\} \},$

$\{ \{1, 2, 3, 4\} \}$

Power Set

For the set $\{ 1, 2, 3, 4, 5, 6 \}$, find a partition that meets the criteria:

- a) Every part has the same size**
- b) No two parts have the same size**

Power Set

For the set $\{ 1, 2, 3, 4, 5, 6 \}$, find a partition that meets the criteria:

a) Every part has the same size

One answer: $\{ \{1, 3\}, \{2, 4\}, \{5, 6\} \}$

b) No two parts have the same size

One answer: $\{ \{2\}, \{3, 6\}, \{4, 1, 5\} \}$



Let's do some more examples...