## Section 1: Predicates

A statement of the form "if $p$ is true, then $q$ is true" is called an **implication**.

We write an implication as $p \rightarrow q$, which is read "$p$ **implies** $q$".

In the statement "if $p$, then $q$" ( $p \rightarrow q$ ), we call $p$ the **hypothesis** and $q$ the **conclusion**.

**Example:**
$p$ represents "$x$'s age is less than 13", $q$ represents "ticket price is discounted"

$p \rightarrow q$ means "if $x$'s age is less than 13, then they get a discounted ticket price."

**Exercise 1** **15%**

Write the following as a statement of formal propositional logic. Assign **variable names** to the simple phrases, and write the statements with the logical connectives $\neg$, $\wedge$, $\vee$, and $\rightarrow$ .

(a) If you don't attend the concert, you will get an F for the course.

(b) If age is greater than or equal to 18, and is a veteran, then ticket price is discounted.

(c) If age is greater than or equal to 18, and is not a veteran, then ticket price is not discounted.

**Truth for implications**

For a statement of the form "if **hypothesis**, then **conclusion**" to be FALSE, it must be the case that the **hypothesis** is true while the **conclusion** is false. Otherwise, the statement is TRUE.

The truth table is as follows:

| $p$ | $q$ | $p \rightarrow q$ |
|---|---|---|
| T | T | T |
| T | F | F |
| F | T | T |
| F | F | T |

Does this seem weird? The only *false* result is if the **hypothesis is true** and the **conclusion is false**. Think of this as, if the hypothesis is false, then our question is pointless anyway – it doesn't affect the conclusion at all.

**Exercise 2** **30%**

Complete the truth tables for the given compound expressions.

(a) $(p \wedge q) \rightarrow q$ (b) $(p \vee q) \rightarrow q$ (c) $p \wedge (q \rightarrow r)$

**Exercise 3** **24%**

Write each of the following predicate using the simple predicates x > 0 and y > 0 along with the propositional connectives $\neg$ , $\wedge$ , $\vee$ , and $\rightarrow$ .

(a) If x is positive, then y is positive.

(b) If x is positive, then y is not positive.

(c) If x is not positive, then y is positive.

(d) If x is not positive, then y is not positive.

## Section 2: Negations of Implications

***Proposition 1***
The negation of the implication $p \rightarrow q$ is the statement $p \wedge (\neg q)$ .

We can see this through a truth table:

| $p$ | $q$ | | $p \rightarrow q$ | $\neg(p \rightarrow q)$ | | $\neg q$ | $p \wedge \neg q$ |
|---|---|---|---|---|---|---|---|
| T | T | | T | **F** | | F | **F** |
| T | F | | F | **T** | | T | **T** |
| F | T | | T | **F** | | F | **F** |
| F | F | | T | **F** | | T | **F** |

Note that the negation of an implication is **not an implication!**

**Example**
If Bob has an 8:00 class today, then it is a Tuesday.
We can think of *p* as “Bob has an 8:00 class today”, and *q* as “it is a Tuesday”.

$$\neg(p \rightarrow q) \equiv p \wedge \neg q$$

So the negation would be: “Bob has an 8:00 class today and it is not Tuesday.”

**Exercise 4** **15%**

Write the negation of each of the following statements:

(a) If Jessica gets chocolate, then she has a happy birthday.

(b) For all real numbers *x*, if *x* > *2*, then $x^2 > 4$

(c) For all real numbers x > 0, if $x^2 = 1$ , then $x^3 = 1$

## Section 3: Implications and Quantifiers

Consider the implication $\forall x \in D, P(x) \rightarrow Q(x)$ .

1. The **converse** of the implication is $\forall x \in D, Q(x) \rightarrow P(x)$
2. The **inverse** of the implication is $\forall x \in D, \neg P(x) \rightarrow \neg Q(x)$
3. The **contrapositive** of the implication is $\forall x \in D, \neg Q(x) \rightarrow \neg P(x)$

**Example:**
*P(n)* stands for "n ends in a digit 2", *Q(n)* stands for "n is divisible by 2".

- $P(n) \rightarrow Q(n)$ means "If *n* ends in a digit 2, then *n* is divisible by 2."
- $Q(n) \rightarrow P(n)$ means "If *n* is divisible by 2, then *n* ends in a digit 2."
- $\neg P(n) \rightarrow \neg Q(n)$ means "If *n* does not end in a digit 2, then *n* is not divisible by 2."
- $\neg Q(n) \rightarrow \neg P(n)$ means "If *n* is not divisible by 2, then *n* does not end in a digit 2."

**Exercise 5** **12%**

Form the contrapositive of the following:

(a) If you don't attend the concert, you will get an F for the course

(b) If you don't eat your breakfast, you will be hungry.