Propositional Logic

Definitions

Propositions

A proposition in logical is a statement that is either true or false, and it cannot be both.

Propositional Variables

In mathematical logic, a propositional variable (also called a sentential variable or sentential letter) is a variable which can either be true or false. Propositional variables are the basic building-blocks of propositional formulas, used in propositional logic and higher logics.

Logical Connectives

We use symbols called **logical connectives** to build a proposition...:

- ¬ = NOT (Negation)
  - Highest precedence
- ∧ = AND (Conjugation)
  - Second highest precedence
- ∨ = OR (Compound)
  - Third highest precedence

Example, having some propositional variable represent a true/false (or yes/no) question, similar to a boolean variable in programming…

- p: Is x > 10?
- q: Is x < 100?
- r: Is password more than 6 characters?

Given a boolean, each of these have only two states: True or False. But, if we combine them with ANDs and ORs, the amount of resulting states from the combination is 2^n, where n is the amount of propositional variables in the statement.

If (x > 10 && x < 100)   = $\left(p \wedge q \right)$

Examples:

    a. Create a variable for "class is full", create a variable for "prerequisites are met", write out different combinations and what they translate to, using AND and OR and NOT.

Diagramming it out with a truth table…

Examples:

    b. $\left(p \wedge \neg q\right)$
    c. $p \vee \neg q$
    d. $\left(p \wedge q\right) \vee \left(p \wedge \neg q\right)$

Otherwise, maybe diagram "p or q, but not both".

Multiple variables… when is the result true? False?

    e. $p \wedge q \wedge r$
    f. $p \vee q \vee r$

Negation

-    $\neg(p \wedge q) \equiv \neg p \vee \neg q$
-    $\neg(p \vee q) \equiv \neg p \wedge \neg q$

Prove via truth table, showing logical equivalence.