Definitions and Diagrams

4.1

Topics:

1) Functions

2) Relations

3) Arrow diagrams

Function Notation

The notation $f : A \rightarrow B$ is used for a function, simply called *f*, with a set of inputs *A* (called the *domain*), and a set B (called the *codomain*) that includes all the *outputs*. The function *f* associates with each input in *A* one and only one output in *B*.

$f : A \rightarrow B$ is read "*f* is a function from A to B".

Functions

$f : A \rightarrow B$ **Is read "*f* is a function from A to B."**

The __domain__ is A – the inputs.
The __codomain__ is B – the outputs.
And we add a __rule__ (like $f(x) = x^2$ **)**

Description of a Function

To completely describe a function, we must do four things:

1) Give the function a name. *f*, *g*, and *h* are popular names for functions, but it's always okay to be creative and descriptive.
2) Describe the domain.
3) Describe the codomain.
4) Describe the rule.

Arrow Diagrams

Arrow diagrams list all the elements of the domain and codomain, with arrows pointing from each input, going to each output.

Functions
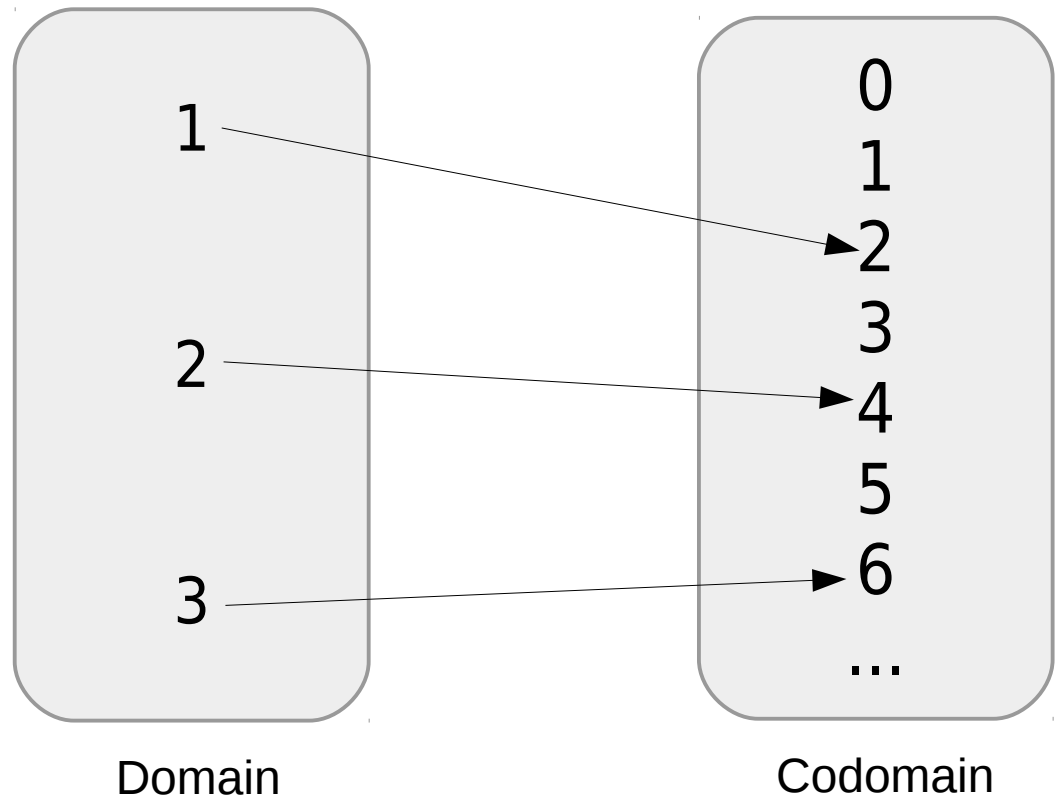
Function Name: *f*
Domain: {1, 2, 3}
Codomain: $\mathbb{N}$
Rule: *2k*

We can diagram each input to an output with an arrow diagram.

f(1) = 2
f(2) = 4
f(3) = 6

Domain

Codomain

Binary Relation

A *binary relation R* consists of three components:
a domain A, a codomain B,
and a subset of $A \times B$ called the "rule" for the relation.
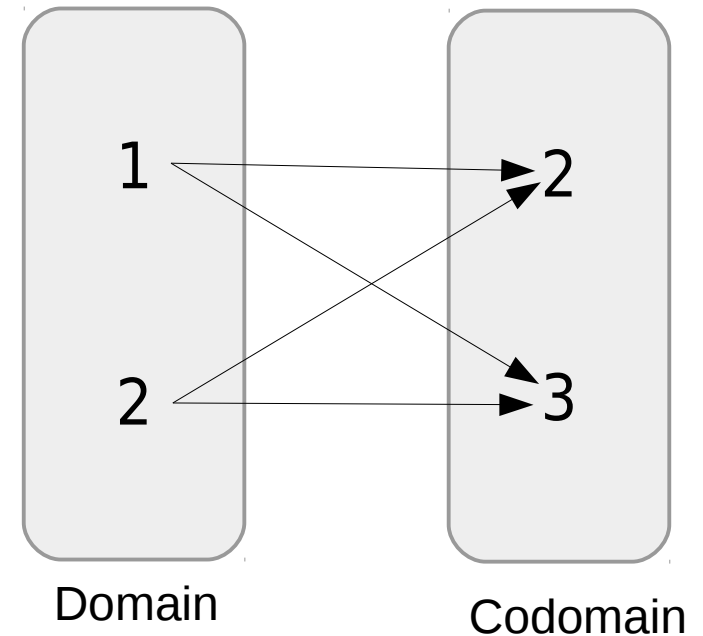
Relation

For Relation R:

Domain: A = {1,2}

Codomain: B = {2,3}

Rule:
L = { (1, 2), (1, 3), (2, 2), (2, 3) }.

We can diagram each input to an output with an arrow diagram.

Domain   Codomain

A function that is a binary relation

A *function F* from *A to B* is a binary relation with the domain *A* and the codomain *B* with the property that for every $x \in A$, there is exactly one element $y \in B$ for which

$$(x, y) \in F$$

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A function that is a binary relation

Let R be the relation whose domain is {a, b, c, d} and whose codomain is $\mathbb{N}$, and whose rule is given by { (a, 1), (b, 3), (a, 5), (c, 3), (d, 7) }. Is this relation also a function?

1. Domain: {a, b, c, d}   Codomain: $\mathbb{N}$

2. Does each input in the domain have an output in the subdomain?   **YES**

3. Does each input in the domain have <u>one and only one</u> output in the codomain?   **NO: (a,1) and (1,5)**

So it is <u>not</u> a function

A function that is a binary relation

Let R be the relation whose domain is {a, b, c, d} and whose codomain is $\mathbb{N}$, and whose rule is given by { (a, 1), (b, 3), (a, 5), (c, 3), (d, 7) }. Is this relation also a function?

If the rule were { (a, 1), (b, 3), (c, 3), (d, 7) } instead, then it <u>would</u> be a function as well as a relation.
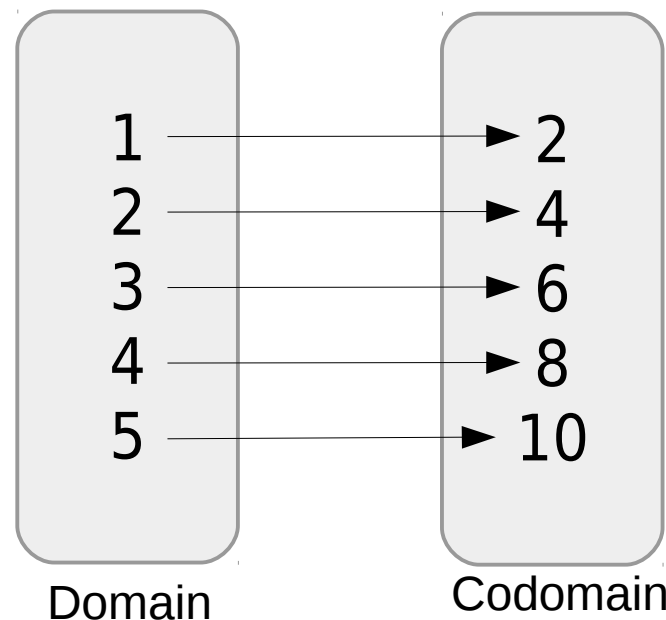
Relations

Example: Draw a portion of the arrow diagram showing at least five elements of the domain and codomain:

$$f : \mathbb{N} \rightarrow \mathbb{N} \text{ with } f(n) = 2n$$

Relations

Example: Draw a portion of the arrow diagram showing at least five elements of the domain and codomain:

$$f : \mathbb{N} \rightarrow \mathbb{N} \text{ with } f(n) = 2n$$

Domain   Codomain

Relations

Example: Draw the two-set arrow diagram for the relation R where the Domain is the power set $\wp(\{a,b\})$, the Codomain is the set B = { 0, 1, 2 }, and the Rule is $(S, n) \in R$ where *n* is the sum of the elements in S.

Relations

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Hint:
Here is the domain

{ }   {a}   {b}   {a,b}

Relations

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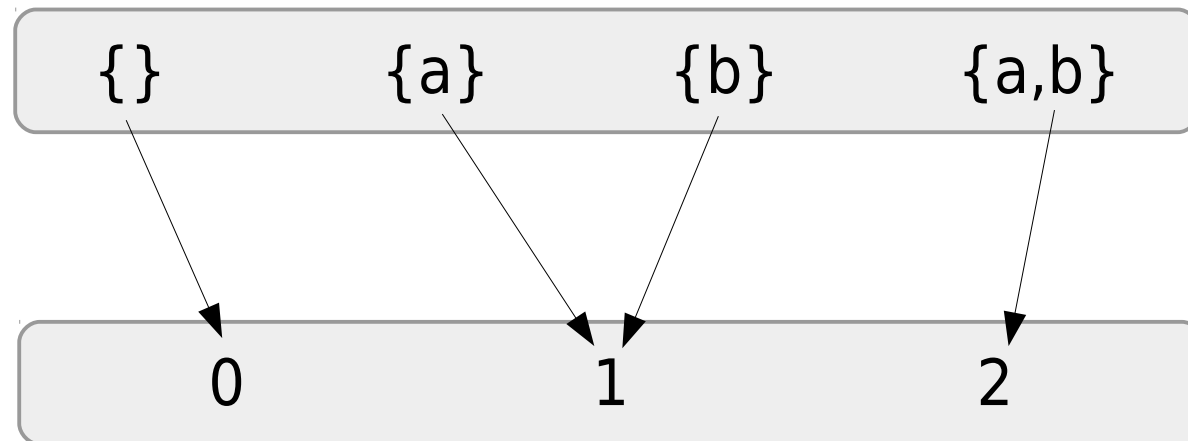| {} | {a} | {b} | {a,b} |

Hint: Here is the codomain

| 0 | 1 | 2 |

Relations

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Relations

Example: **Is the relation R also a function?**
Domain & codomain: {1, 2, 3, 4, 5}.
Rule: { (1, 5), (2, 3), (3, 3), (4, 2), (5, 1) }

Relations

Example: Is the relation R also a function?
Domain & codomain: {1, 2, 3, 4, 5}.
Rule: { (1, 5), (2, 3), (3, 3), (4, 2), (5, 1) }

1. Does every input have an output in the codomain?
   YES

2. Does each input in the domain have one and only one output in the codomain?
   YES

       So it is a function.