Sometimes, it can be easy to tell what number comes next in a sequence of numbers:

a. 1, 3, 5, 7, ___
b. 10, 20, 30, 40, ___
c. 5, 10, 15, 20, ___

However, sometimes it may not be so obvious, so we need to analyze the sequence in order to figure out the relationship…

d. 1, 2, 6, 24, 120, 720, ___
e. 1, 9, 17, 25, 33, 41, ___
f. 1, 4, 9, 16, 25, 36, ___

There are two ways we can write out a formula to represent a sequence of numbers:

- Recursive formula:
  - A recursive formula for a sequence is a formula where each term is described in relation to a previous term (or terms) of the sequence.
- Closed formula:
  - A closed formula for a sequence is a formula where each term is described only in relation to its position in the list.

Example, writing out the terms:

g. $a_1 = 2;\quad a_n = a_{n-1} + 2$, write out the first 4 terms. (2, 4, 6, 8)

h. $a_n = 2n$, write out the first 4 terms. (2, 4, 6, 8)

With a recursive formula, each term is based on the previous, so the **very first item** must be given a value as part of the formula's definition.

With the closed formula, the value of the element is based on the position *n*, such as *n=1*, *n=2,* etc.

Analyzing the values by brute-force…

- Is addition involved?
- Is subtraction involved?
- Is multiplication involved?
- Are exponent involved?

Example:

i. Find the closed formula and the recursive formula for the sequence: 1, 3, 5, 7, 9, …

Recursive: What is the difference between each number? (2)

Closed: What sequence is this similar to? (2, 4, 6, 8, 10), what is it offset by? (-1).

Another way to solve… $a_n = m \cdot n + b$

- Write out the formula for two elements:
  - $a_1 = m \cdot 1 + b = 1$ $a_2 = m \cdot 2 + b = 3$
  - Rewrite so that we can solve for a variable by substitution, or elimination (old algebra topic!):
  - $m + b = 1$ $2m + b = 3$
  - Solve for b, or solve for m, then solve for the other.
  - $m + b = 1 \rightarrow b = 1 - m$
  - Plug in to the other formula:
  - $2m + b = 3 \rightarrow 2m + (1 - m) = 3$
  - Solve for *m*:
  - $2m + (1 - m) = 3 \rightarrow 2m + 1 - m = 3 \rightarrow m + 1 = 3 \rightarrow m = 3 - 1 \rightarrow m = 2$
  - Solve for *b*:
  - $2m + b = 3 \rightarrow 2 \cdot 2 + b = 3 \rightarrow b = 3 - 4 \rightarrow b = -1$
  - Write out the resulting formula:
  - $a_n = m \cdot n + b \rightarrow a_n = 2 \cdot n - 1$

Check your work by plugging in values of *n* into $a_n = 2 \cdot n - 1$ to see if it matches the sequence.

Example:

j. Find the recursive and closed formulas for the sequence: 1, 9, 17, 25, 33, 41, ___

Recursive Formula:

- First value is 1.
- Difference between each is 8.
- $a_1 = 1$, $a_n = a_{n-1} + 8$

Closed Formula:

- The difference between each number is 8. $a_n = m \cdot n + b$, $m = 8$.
- $a_1 = 8 \cdot 1 + b = 1$ → $b = 1 - 8$ → $b = -7$
- $a_n = 8n - 7$

Examples:

k. 1, 4, 9, 16, 25, 36	
l. 2, 4, 8, 16, 32, 64	
m. 1, 2, 6, 24, 120, 720	

Summations, definition:

For a sequence of numbers a_k with $k \geq 1$, we use the notation $\sum_{k=1}^{n} a_k$ to denote the sum of the first *n* terms of the sequence. This is called *sigma notation* for the sum.

Examples, evaluate the following:

o. $\sum_{k=1}^{6} (2k-1)$

p. $\sum_{k=0}^{4} (3^k)$

q. $\sum_{k=3}^{3} (k^2)$

r. $\sum_{k=1}^{5} \frac{1}{k(k+1)}$