Exercise 1.3

Your Name:

¬ = NOT (Negation) Highest precedence

∧ = AND (Conjugation) 2nd highest precedence

∨ = OR (Compound) 3rd highest precedence

Team Name:

Problem 1

Fill in the following truth table:

p	q	¬p ∧ q

Problem 2

Build a truth table for the following statement: p ∧ (q ∨ r)

p	q	r	q ∨ r	p ∧ (q ∨ r)

Problem 3

Translate the following from English into logical terms, using the propositions provided.

Propositions:

- `a = The class has more than 10 students`
- `b = The class has less than 20 students`

1. The class has more than 10 students and less than 20 students	$a \wedge b$
2. The class has does not have more than 10 students	
3. The class has 10 students or less	
4. The class has 20 students or more	

Problem 5

There are two men. One of them is wearing a red shirt, and one is wearing a blue shirt. The two men are named Andrew and Bob, but we do not know which is Andrew and which is Bob.

The guy in the blue shirt says, “I am Andrew.” The guy in the red shirt says, “I am Bob.”

If we know that at least one of them lied, then what color shirt is Andrew wearing?

(Truth-Tellers and Liars. Brilliant.org. Retrieved from https://brilliant.org/wiki/truth-tellers-and-liars/)

Given the propositions:

B	R	L
Blue is telling the truth	Red is telling the truth	At least one is lying

In order to figure out the truth, we need to realize that,
(1) There is exactly one bob, (2) There is exactly one Andrew, and (3) There is at least one lie.
And we have to look for contradictions.

(1) and (2): There can only be one bob if Blue is telling the truth and Red is telling the truth, OR if Blue is lying and Red is lying: $B \wedge R$ OR $\neg B \wedge \neg R$.

Fill out the following truth table to analyze the situation:

Blue is telling the truth B	**Red is telling the truth** R	**At least one lied** L	**1 Bob AND 1 Andrew AND 1 Lie** $[(B \wedge R) \vee (\neg B \wedge \neg R)] \wedge L$

What color shirt is Andrew wearing?

Problem 6

There are 3 boxes, exactly one of which has a car. You can keep the car if you pick the correct box! On each box there is a statement, exactly one of which is true. Which box has the car?

Box A: The car is in this box. Box B: The car is not in this box. Box C: The car is not in box A.

(Truth-Tellers and Liars. Brilliant.org. Retrieved from https://brilliant.org/wiki/truth-tellers-and-liars/)

Think of this problem in terms of the propositions:

1. p: A (car in box A) 2. q: ¬B (car not in box B) 3. r: ¬A (car not in box A)

And since only one note can be true, we will write each scenario in terms of the each note assuming it is true, and the others are false:

4. p ∧ ¬ (q ∨ r)
(p is true,
but q and r are not)

5. q ∧ ¬ (p ∨ r)
(q is true,
but p and r are not)

6. r ∧ ¬ (p ∨ q)
(r is true,
but p and q are not)

Since there is only one car, we will test only these statements:

A: Car is in box A B: Car is in box B C: Car is in box C

Fill out the following truth table to figure out which box the car is in.

A Car in box A	**B** Car in box B	**C** Car in box C	**p** p = A	**q** q = ¬B	**r** r = ¬A	**p ∧ ¬ (q ∨ r)**	**q ∧ ¬ (p ∨ r)**	**r ∧ ¬ (p ∨ q)**
T	F	F						
F	T	F						
F	F	T						

Which box has the car in it?