Mathematical Induction

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Mathematical Induction

This Chapter:

1) Showing that a recursive formula and a closed formula are equivalent via a proof

Don’t worry; we’re not generating formulas from sequences of numbers.

Previously...

In Chapter 2.1 and 2.2, the idea was to prove that a statement was true or false by turning the variables of the statement

("n is even", "m is odd")

into more mathematical terms that we can work with.

("represent n as 2K,
represent m as 2L+1")

This time...

For the proofs in Chapter 2.3, we are essentially doing the same thing, except that we do not care about *even*, *odd*, and *divisible by…*, but instead we will prove that two formulas are equivalent.

This time…

This mostly means that we're going to:

(1)
Swap out the original variable
with a different equation we can work with

(2)
After substituting the variable out,
simplifying the math until we get the
result, which ought to prove the statement.

Which is exactly what we were doing last time.

Let’s review a simple proof

It might be handy to try to make a “to-do” list of the steps we take in order to prove something. Let’s go back to a 2.1 proof:

Statement:

“The result of summing any odd integer with any even integer is an odd integer.”

Let's review a simple proof

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First Step:

Can we write this as a simple math equation?

Let's review a simple proof

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"The result of summing any odd integer
with any even integer is an odd integer."

First Step:

Can we write this as a simple math equation?

$$x+y=z$$

Where x is odd, y is even, and z is odd.

Let’s review a simple proof

Statement:

“The result of summing any odd integer with any even integer is an odd integer.”

$$x+y=z$$

Where x is odd, y is even, and z is odd.

Second Step:

How can we explicitly state that x is odd, y is even, and z is odd, in math terms?

Let's review a simple proof

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"The result of summing any odd integer with any even integer is an odd integer."

$$x + y = z$$

Where x is odd, y is even, and z is odd.

Second Step:

How can we explicitly state that x is odd, y is even, and z is odd, in math terms?

$$x = 2K + 1 \qquad y = 2L \qquad z = 2M + 1$$

Each variable needs to have its own "alternative" variable; don't keep reusing "K"!!

Let’s review a simple proof

Statement:

“The result of summing any odd integer with any even integer is an odd integer.”

$$x + y = z$$

Where x is odd, y is even, and z is odd.

$x = 2K + 1$ $y = 2L$ $z = 2M + 1$

Third Step:

Now that we have “aliases” for x, y, and z, replace them in the original equation.

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Where x is odd, y is even, and z is odd.

$$x=2K+1 \qquad y=2L \qquad z=2M+1$$

Third Step:

Now that we have “aliases” for x, y, and z, replace them in the original equation.

$$\underbrace{(2K+1)}_{x}+\underbrace{(2L)}_{y}=\underbrace{(2M+1)}_{z}$$

Let’s review a simple proof

Statement:

“The result of summing any odd integer with any even integer is an odd integer.”

$$(2K+1)+(2L)=(2M+1)$$

Fourth Step:

Use our rules of algebra to tweak the left side in order to match the same form as the right side.

Note: The left side will not result in an “M” variable. However, if we get K and L together, we can represent these two together as ”M”.

Let's review a simple proof

Statement:

"The result of summing any odd integer with any even integer is an odd integer."

$$(2K+1)+(2L)=(2M+1)$$

$$2K+1+2L=2M+1$$ Simplify

$$2K+2L+1=2M+1$$ Rearrange

$$2(K+L)+1=2M+1$$ Factor out common terms

Note that

2(K+L)+1

Means "2 times some integer plus 1", which is our definition of an *odd number*. We can treat "(K+L)" as being "Just some integer" because any two integers added together results in another integer.

Let's review a simple proof

<u>Statement:</u>

"The result of summing any odd integer with any even integer is an odd integer."

$$(2K+1)+(2L)=(2M+1)$$

$$2(K+L)+1=2M+1$$

2 (Some Integer) + 1 = 2 (Some Integer) + 1

Another way we can think about it, is to say that K+L = M, which is just an arbitrary alias.

The result of summing is any **odd integer.**
2(K+L)+1 is some odd integer,
resulting from the addition of an odd and an even.

Let’s review a simple proof

These proofs are all about writing our proposition as a mathematical formula that we can do operations on.

Just take it a step at a time – *don’t skip steps* – and they won’t give you too much trouble.

The only “guesswork” involved is just figuring out how to rearrange numbers and variables around.

Mathematical Induction

OK, back to chapter 2.3

Mathematical Induction

The Principle of Mathematical Induction

Let P(n) be a statement about the positive integers. If one can prove that…

1) P(1) is true, and that
2) for every integer $m \geq 2$, whenever P(1), P(2), …, P(m-1) have all been checked to be true, it follows that P(m) is true.

Then we can conclude that P(n) is true for every positive integer n.

Mathematical Induction

Definition

A *statement about the positive integers* is a predicate P(n) with the set of positive integers as its domain.

That is, when any positive integer is substituted for *n* in statement P(n), the result is a proposition that is unambiguously either true or false.

Mathematical Induction

Let’s work some problems