The Composition Operation

4.2

Topics:

1) The compositions of functions

2) Arrow diagrams of compositions

Compositions

If $f : A \rightarrow B$ **and** $g : B \rightarrow C$ **, then we can build a new function called** $(g \circ f)(x)$ **that has domain A and codomain C, and that follows the rule . We call , read "g of f", the composition of g with f.**

Compositions

Say that $f(x)=2x$ **and** $g(x)=x+3$**,**
then

f of g

$(f \circ g)(x)$

$= f(g(x))$

$= f(x+3)$

$= 2(x+3)$

$(f \circ g)(x) = 2x+6$

g of f

$(g \circ f)(x)$

$= g(f(x))$

$= g(2x)$

$(g \circ f)(x) = 2x+3$

Compositions

Say that $f(x)=2x$ **and** $(f \circ g)(x)=2x+6$, and we need to find g(x) from this information...

1) Create an alias for g(x): a = g(x).

2) $f(a)=2a$ **, and** $2a=2x+6$

3) Solve for a $a=x+3$

4) So, $g(x)=x+3$

Compositions

Say that $g(x)=x+3$ **and** $(f \circ g)(x)=2x+6$, **and we need to find f(x) from this information…**

1) Create an alias for g(x): a = g(x), or $a=x+3$

2) Solve for x: $x=a-3$
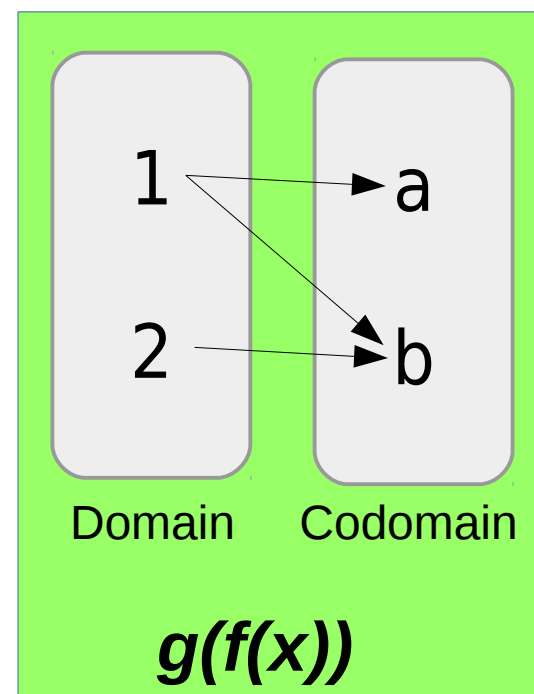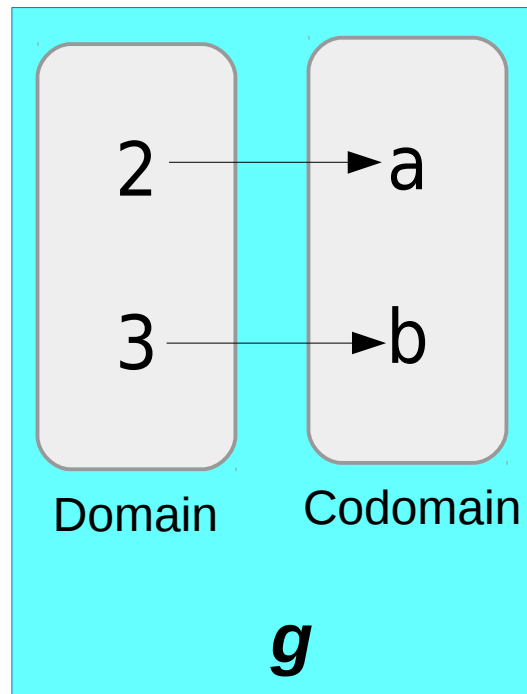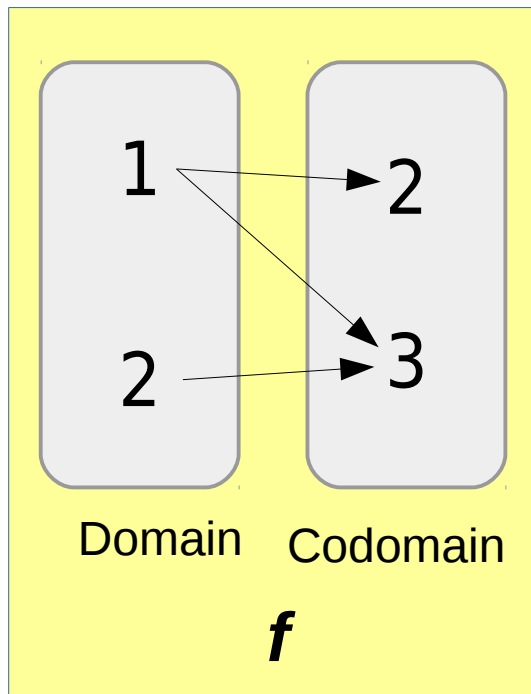
3) For f(g(x)), write out f(a) then sub in x = a – 3.
$$f(g(x))=2x+6=2(a-3)+6$$

4) Simplify $2a-6+6=2a$ **, so we found that**
$$f(x)=2x$$

Arrow diagrams

We can also draw diagrams for these compositions...

$$g(f(x)) = \left(g \circ f \right)\left(x \right)$$

Practice

Example: Given $f(x)=x+1$ $g(x)=2x+3$

Find $(f \circ g)(x)$

Practice

Example: Given $f(x) = x + 1$ $g(x) = 2x + 3$

Find $(f \circ g)(x)$

$$f(g(x)) = f(2x + 3)$$
$$= (2x + 3) + 1$$
$$= 2x + 4$$

Practice

Example: Given $f(x) = x + 4$ $(f \circ g)(x) = 4x + 4$

Find $g(x)$

Practice

Example: Given $f(x) = x + 4$ $\quad (f \circ g)(x) = 4x + 4$

Find $g(x)$

$$a = g(x)$$
$$f(g(x)) = f(a)$$
$$a + 4 = 4x + 4$$
$$a = 4x$$
$$\boxed{g(x) = 4x}$$

Practice

Example: Given $g(x) = x + 2$ $(f \circ g)(x) = 3x + 7$

Find $f(x)$

Practice

Example: Given $g(x)=x+2$ $(f \circ g)(x)=3x+7$

Find $f(x)$

$a=g(x)$
$a=x+2$
$x=a-2$

$f(g(x))=f(a)=3x+7$
$\qquad =3(a-2)+7$
$\qquad =3a-6+7$
$\qquad =3a+1$
$f(a)=3a+1$
$\boxed{f(x)=3x+1}$