Section 1: Propositional Logic

Definitions: Propositional Logic

We call a sentence a **proposition** if it is unambiguously true or false. A *propositional variable* is simply a variable name that stands for a proposition. A *formal proposition* will mean a proposition written using propositional logic notation according to the following rules:

1. Any propositional variable alone is a formal proposition.
2. The proposition $\boldsymbol{p \wedge q}$ stands for "Both p and q are true", and we read this as "p ***and*** q".
3. The proposition $\boldsymbol{p \vee q}$ stands for "Either p or q is true", and we read this as "p ***or*** q".
4. The proposition $\boldsymbol{\neg p}$ stands for "It is not the case that p is true", and we read this as "***not*** p". $\neg p$ is known as the **negation** of p.

As an example,

"p" symbolizes the proposition "x > 0", "q" symbolizes the proposition "x < 10".

The result of p can either be **true** or **false**, and the same is true of q. We can also combine p and q into another proposition, such as: "Is p true, or is q true?" or "Is p true, and is q true?".

To make it shorter, we can write these with symbols: "$p \wedge q$", "$p \vee q$".

Exercise 1 **18%**

A proposition does not need to be *true* in order to be a proposition. For the following propositions, mark whether they are true or false.

(a) p: "10 + 20 = 30"

(b) q: "12 + 12 = 20"

(c) r: "x > 10", when x = 0.

(d) s: "x > 10", when x = 15.

(e) t: "x < 5", when x = 5.

For the following combinations of propositions, mark whether the entire statement is true or false.

(f) p: "a equals 1", q: "b equals 2" $p \wedge q$, when a = 1 and b = 2

(g) p: "a equals 1", q: "b equals 2" $p \wedge q$, when a = 1 and b = 3

(h) p: "a equals 1", q: "b equals 2" $p \vee q$, when a = 5 and b = 2

(i) p: "a equals 1", q: "b equals 2" $p \vee q$, when a = 3 and b = 3

Exercise 2 **32%**

Sometimes we have propositions where we don't know the exact answer. When programming, our if statements might be looking at these propositions to figure out whether to branch or not.

Given the following propositions:

p: "The printer is offline", *q*: "The printer is out of paper", *r*: "The document has finished printing"

Write propositions that are described by the following sentences, using $\wedge$, $\vee$, and $\neg$:

(a) The printer is not out of paper.

(b) The printer is online.

(c) The printer is offline and it is out of paper.

(d) The printer is online and it is not out of paper.

(e) Either the printer is offline, or it is out of paper.

(f) The printer is online, but it is out of paper.

(g) The printer is offline or it is out of paper, but not both.

(h) The printer is online and the printer has paper, and the document has not finished printing.

Section 2: Truth Tables

We can build truth tables to show all possible outcomes of a proposition. We list out all possible combinations of the *propositional variables* on the left side, and then build out the result of the full proposition on the right side.

Examples:

$p \wedge q$

p	q		$p \wedge q$
T	T		T
T	F		F
F	T		F
F	F		F

$p \vee q$

p	q		$p \wedge q$
T	T		T
T	F		F
F	T		F
F	F		F

$\neg p$

p		$\neg p$
T		F
F		T

Exercise 3 **12%**

Finish the following truth tables.

(a)

p	q		$\neg q$	$p \wedge \neg q$
T	T		F	
T	F		T	
F	T			
F	F			

(b)

p	q		$\neg p$	$\neg q$	$\neg p \vee \neg q$
T	T				
T	F				
F	T				
F	F				

Exercise 4 **24%**

When building truth tables, there is a specific order you should write out the “T” and “F” values of the propositional variables: Start with “T” first, and as you go, change the *right-most* possible value. Then, move leftward from there. So: “TT”, “TF”, “FT”, “FF”.

Or for three variables: “TTT”, “TTF”, “TFT”, “TFF”, “FTT”, “FTF”, “FFT”, “FFF”.

Build truth tables for the following propositions.

(a) $p \wedge (\neg p \vee q)$

(Note: it might help to add columns for $\neg p$ and $\neg p \vee q$ before doing $p \wedge (\neg p \vee q)$)

(b) $p \wedge (q \vee r)$

Exercise 5 **14%**

Write out a truth table to show that the following two propositions are *logically* equivalent.

(a) $p \wedge (\neg p \vee q)$ with $\neg p \wedge (p \vee q)$

(b) $(\neg q \wedge p) \vee (\neg p \wedge q)$ with $\neg p \vee \neg q$