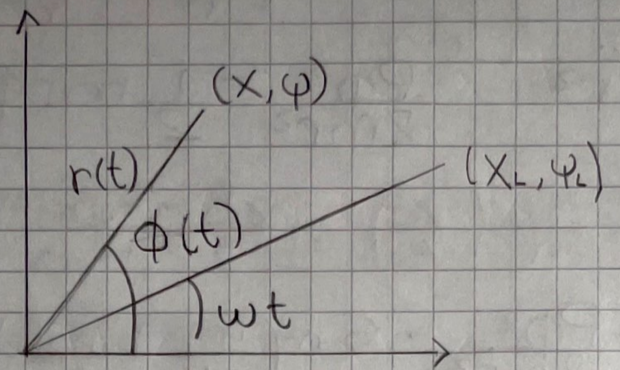
LUNAR ROCKET

c)



C. Polares

$x = r\cos(\phi)$

$\psi = r\,\mathrm{sen}(\phi)$

$x_L = d\cos(\omega t)$

$\psi_L = d\,\mathrm{sen}(\omega t)$

→ Me piden demostrar $r_L(r, \phi, t)$

$$r_L^2 = [r\cos(\phi) - d\cos(\omega t)]^2 + [r\,\mathrm{sen}(\phi) - d\,\mathrm{sen}(\omega t)]^2$$

$$= r^2[\cos^2(\phi) + \mathrm{sen}^2(\phi)] - 2rd[\cos(\phi)\cos(\omega t) + \mathrm{sen}(\phi)\,\mathrm{sen}(\omega t)] + d^2[\cos^2(\omega t) + \mathrm{sen}^2(\omega t)]$$

$$r_L^2 = r^2 + d^2 - 2rd\cos(\phi - \omega t)$$

$$r_L(r, \phi, t) = \sqrt{r^2 + d^2 - 2rd\cos(\phi - \omega t)}$$

d) Hamiltoniano

$\mathcal{L} = T - V$

$\vec{v} = \dot{r}^2 + r\dot{\phi}^2$

$T = \frac{1}{2} m v^2$

$T = \frac{1}{2} m(\dot{r}^2 + r\dot{\phi}^2)$

$V = -\frac{G m_T m}{\sqrt{x_R^2 + \psi_R^2}} - \frac{G m_L m}{r_L}$

$V = -Gm\left(\frac{m_T}{r} + \frac{m_L}{r_L}\right.$

$$\mathcal{L} = \frac{1}{2} m(\dot{r}^2 + r\dot{\phi}^2) + Gm\left(\frac{m_T}{r} + \frac{m_L}{r}\right)$$

$$\frac{P_r^2}{2m} = \frac{1}{2} m\dot{r}^2 \qquad \frac{P_\phi^2}{2mr^2} = \frac{1}{2} mr\dot{\phi}^2$$

$$H = T + V$$

$$H = \frac{P_r^2}{2m} + \frac{P_\phi^2}{2mr^2} - Gm\left(\frac{m_T}{r} + \frac{m_L}{r_L}\right)$$

e) $$\frac{\partial}{\partial r}(r_L^{-1}) = -r_L^{-2}\frac{\partial r_L}{\partial r}$$

$$= \frac{-1}{r_L^2} \frac{2r - 2d\cos(\phi - \omega t)}{2r_L}$$

$$\frac{\partial}{\partial \phi}(r_L^{-1}) = -\frac{1}{r_L^2} \frac{[2rd\,\mathrm{sen}(\phi - \omega t)]}{2r_L}$$

$$\dot{P}_r = \frac{P_\phi^2}{mr^3} - \frac{Gmm_T}{r^2} - \frac{Gmm_L}{r_L^3}[r - d\cos(\phi - \omega t)]$$

$$\dot{P}_\phi = -\frac{Gmm_L}{r_L^3}[rd\,\mathrm{sen}(\phi - \omega t)]$$

f) $$\bar{r} = \frac{r}{d} \to \dot{r} = d\dot{\bar{r}}$$

$$\bar{P_r} = \frac{P_r}{md} \to P_r = md\dot{\bar{r}}$$

$$md\bar{P_r} = md\dot{\bar{r}}$$

$$\bar{P_\phi} = \frac{P_\phi}{md^2}, \quad P_\phi = mr^2\dot{\phi} \qquad \boxed{\frac{1}{\bar{r}^2} = \frac{d^2}{r^2}}$$

$$\bar{P_\phi} = \frac{r^2\dot{\phi}}{d^2}$$

$$\dot{\phi} = \frac{\bar{P_\phi}}{\bar{r}^2}$$

$$p_\phi = md^2 \bar{p}_\phi$$

$$r = \bar{r} d$$

$$\dot{p}_r = md \dot{\bar{p}}_r$$

$$\cancel{md}\,\dot{\bar{p}}_r = \frac{\cancel{m^2 d^2}\,\bar{p}_\phi^2}{\cancel{m}\,\bar{r}^3 d^3} - \frac{\Delta \cancel{m}}{\bar{r}^2 d^2} - \frac{6 m_L \cancel{m}\,[\bar{r}\cancel{d} - \cancel{d}\cos(\phi-\omega t)]}{\sqrt{(d^2+\bar{r} d^2 - 2\bar{r} d^2 \cos(\phi-\omega t))^3}}$$

$$\bar{r}'^2 = 1+\bar{r}^2 - 2\bar{r}\cos(\phi-\omega t) \qquad \Delta N = \frac{6 m_L}{d^3}$$

$$\dot{\bar{p}}_r = \frac{\bar{p}_\phi^2}{\bar{r}^3} - \Delta\left\{\frac{1}{\bar{r}^2} + \frac{N}{\bar{r}'^3}[\bar{r} - \cos(\phi-\omega t)]\right\}$$

$$\cancel{m}\cancel{d^2}\,\dot{\bar{p}}_\phi = -\frac{6 \cancel{m} m_L\, \bar{r}\cancel{d^2}\,\mathrm{sen}(\phi-\omega t)}{\sqrt{(d^2+\bar{r}^2 d^2 - 2\bar{r} d^2\cos(\phi-\omega t))^3}}$$

$$\dot{\bar{p}}_\phi = -\frac{6 m_L}{d^3}\frac{\bar{r}\,\mathrm{sen}(\phi-\omega t)}{\bar{r}'^3}$$

$$\dot{\bar{p}}_\phi = \frac{\Delta N \bar{r}}{\bar{r}'^3}\,\mathrm{sen}(\phi-\omega t)$$

g) $\bar{p}_{r0} = \frac{\dot{r}}{d} \to \frac{1}{d}\frac{dr}{dt}$

$$\bar{p}_{r0} = \frac{1}{d}\frac{d\sqrt{x^2+\varphi^2}}{dt}$$

$$\bar{p}_{r0} = \frac{x\dot{x}+\varphi\dot{\varphi}}{rd}$$

$$\bar{r}_{r0} = \frac{\dot{x}}{d}\cos(\phi) + \frac{\dot{\varphi}}{d}\,\mathrm{sen}(\phi)$$

$$\bar{p}_{r0} = \bar{V}_0\cos(\theta)\cos(\phi) + \bar{V}_0\,\mathrm{sen}(\theta)\,\mathrm{sen}(\phi)$$

$$\bar{p}_{r0} = \bar{V}_0\cos(\theta-\phi)$$

$$\bar{p}_{\phi 0} = \frac{mr^2\dot{\phi}}{md^2}$$

$$\phi = \tan^{-1}\left(\frac{\varphi}{x}\right)$$

$$\dot{\phi} = \frac{d}{dt}\left[\tan^{-1}\left(\frac{\varphi}{x}\right)\right]$$

$$\bar{r}^2\dot{\phi} = \frac{1}{1+\left(\frac{\varphi}{x}\right)^2}\frac{d}{dt}\left(\frac{\varphi}{x}\right) = \left(\frac{\dot{\varphi}x - \dot{x}\varphi}{x^2+\varphi^2}\right)\bar{r}^2$$

$$r^2 = x^2 + \varphi^2$$

$$\left(\frac{\bar{r}}{r}\right)^2(\dot{\varphi}x - \dot{x}\varphi) = \bar{r}^2\dot{\phi}$$

$$\frac{\bar{r}^2}{r}\left(V_0 \operatorname{sen}(\theta)\cos(\phi) - V_0\cos(\theta)\operatorname{sen}(\phi)\right)$$

$$= \frac{\bar{r}^2}{r} V_0 \operatorname{sen}(\theta - \phi)$$

$$\frac{\bar{r}^2}{r} = \frac{d}{dr} = \bar{r}$$

$$P_{\phi^\circ} = \bar{r}_0 V_0 \operatorname{sen}(\theta - \phi)$$