1. Resolver analíticamente la ecuación diferencial no lineal:

$$\frac{du}{dt} = u^q, \ t \in [0, 10] \tag{1}$$

La solución exacta es: $u(t) = e^t$ para $q = 1$ y $u(t) = (t(1-q)+1)^{\frac{1}{1-q}}$ para $q < 1$ y $t(1-q)+1 > 0$.

→ $q = 1$

$$\frac{du}{dt} = v'$$

$$\int \frac{dv}{v} = \int dt$$

$$\ln|v| = t + c$$

$$v = e^{t+c} \qquad c = 1$$

$$v = e^t$$

→ $q < 1$

$$\frac{du}{dt} = v^q \qquad v = \varphi^{\frac{1}{1-q}} \qquad \varphi = v^{1-q}$$

$$v' = \frac{\varphi^{\frac{q}{1-q}}}{1-q}\varphi'$$

$$v' = v^q$$

$$\frac{\varphi^{\frac{q}{1-q}}}{1-q}\varphi' = \left(\varphi^{\frac{1}{1-q}}\right)^q$$

$$\varphi' = (1-q)\,dt$$

$$\varphi = (1-q)t + C$$

$$v^{1-q} = (1-q)t + C$$

$$v = ((1-q)t + C)^{\frac{1}{1-q}} \qquad C = 1$$

$$v = ((1-q)t + 1)^{\frac{1}{1-q}}$$

$$(1-q)t + 1 > 0$$