Advanced Plasma – Plasma Physics for Fusion : Assignment 1 with solutions

1 Question 1

Consider a current, I, being carried by a copper rod in the z direction. This will produce a magnetic field

$$\underline{B} = \frac{\mu_0 I}{2\pi R} \underline{e}_\Phi$$

in the toroidal direction. This system is shown in figure 1:

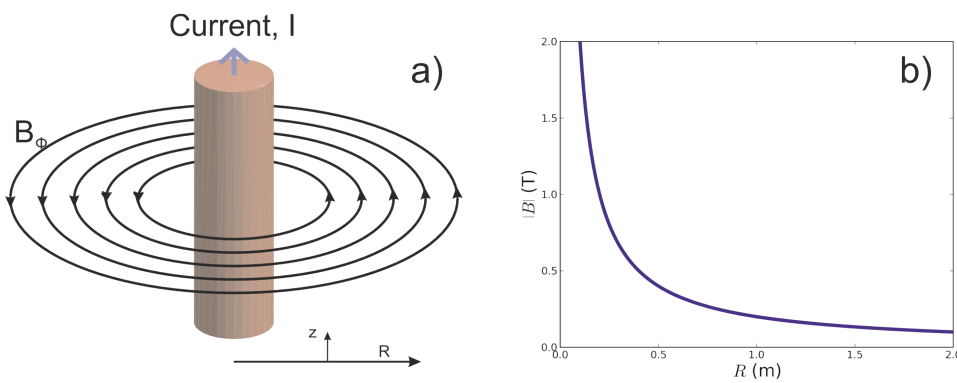

Figure 1: a) A current in the $\underline{e}_z$ direction at $R = 0$ leads to a toroidal field. b) Illustration of the toroidal field strength dependence on R.

a) Suppose 1 MA of current flows through the central rod, find the magnetic field strength at $R = 1$ m and use this to calculate the Larmor radius, r_L, and gyro-frequency, Ω, for an electron with temperature 10 keV and an alpha particle with temperature 3.5 MeV. You may assume $v_\perp = v_{th} = \sqrt{2k_B T_k/m}$ and $m_\alpha = 4m_p$, where m_p is the proton mass.

[2 marks]

b) In this system the ∇-B and curvature drift velocities may by combined and written as:

$$\underline{v}_D = \frac{m}{RZeB}\left(v_\parallel^2 + \frac{v_\perp^2}{2}\right)\underline{e}_z$$

(for the derivation see the solution for PPQ 4 on the VLE). This combined drift, $\underline{v}_D$, is often referred to as simply the magnetic drift.

i) Assuming $v_\parallel^2 + v_\perp^2/2 = v_{th}^2$ calculate this drift velocity for electrons and Deuterium ions ($m_D = 2m_p$) at $R = 1$ m with 10 keV thermal energy.

[1 mark]

ii) In a terrestrial system, such as the one under consideration here, there will be a gravitational force, $m\underline{g}$, acting on the plasma species in the $-\underline{e}_z$ direction. This force will lead to a time independent particle drift. Starting from the equation of motion:

$$m\frac{d\underline{v}}{dt} = -mg\underline{e}_z + Ze\left(\underline{v} \times \underline{B}\right)$$

derive an expression for the perpendicular velocity of the drift due to the gravitational force, $\underline{v}_{g,\perp}$. You may find the following vector identity useful

$$\left(\underline{A} \times \underline{B}\right) \times \underline{C} = \left(\underline{A} \cdot \underline{C}\right)\underline{B} - \left(\underline{B} \cdot \underline{C}\right)\underline{A}$$

[3 marks]

iii) Calculate the magnitude of the gravitational drift velocity for electrons and Deuterium ions on the surface of Earth (i.e. $g = 9.81$ ms^{-2}). Compare these values to the magnetic drift velocity calculated in part i, and briefly discuss the importance of the gravitational drift here.

[2 marks]

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2 Question 2

The system described in question 1 uses a purely toroidal magnetic field. In this question we will consider introducing a poloidal component of the magnetic field, which can be achieved by driving a toroidal current, I_T, at $R = 1$ m. The result of this is that the field lines become helical, orbiting about the location of the current as shown in figure 2.

As particles travel along the now helical field lines they also move in R and Z. To desrcibe this it is convenient for us to introduce the poloidal angle, θ, as shown in figure 2. As a particle moves along the field line it will move in both the toroidal and poloidal angles.

a) Describe why the magnetic drift means a purely toroidal field, such as that discussed in question one, cannot confine a plasma indefinitely.

[2 marks]

b) The magnitude of the toroidal component of the magnetic field, B_T, depends on R and as such the total field magnitude, B, will vary along the field line. Let us assume that this variation is described by $B = B_c\left(1 - a\cos\theta\right)$, where B_c and a are some constants and θ is the poloidal angle. Starting from the conservation of energy and magnetic moment show

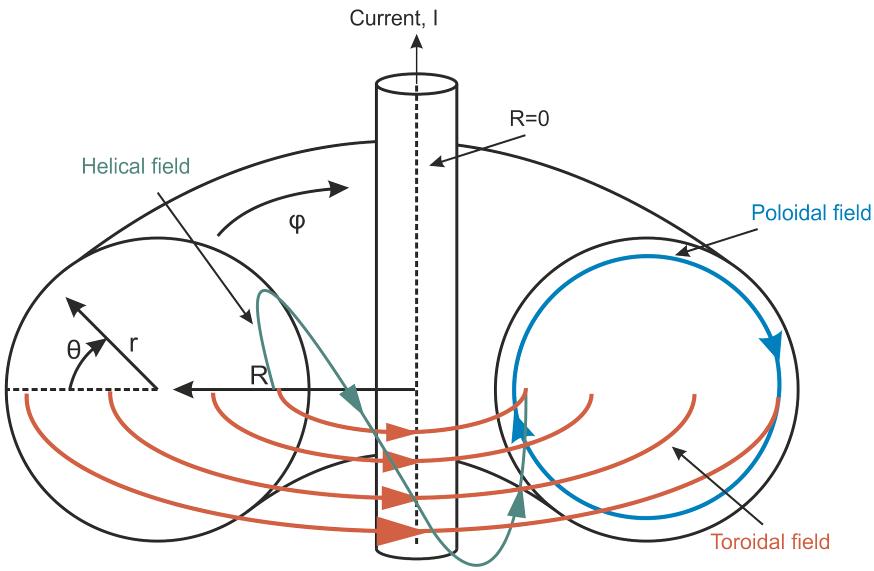

Figure 2: A current in the $-\underline{e}_\phi$ direction at $R = 1$ leads to an additional poloidal field (blue). This adds to the existing toroidal field (red), to give a net helical field (green). It is convenient to define the poloidal angle, θ, as shown in the figure.

that a particle at $\theta = 0$ with pitch angle λ (defined through $\sin\lambda = v_{\perp,0}/v_0 \neq 0$) will be reflected at a poloidal angle, θ, satisfying

$$\cos\theta = \frac{1}{a}\left[1 - \frac{1-a}{\sin^2\lambda}\right]$$

You may neglect the electrostatic potential, ϕ.

[4 marks]

c) A particle is considered trapped if it is reflected at some point along the field line (i.e. somewhere between $\theta = 0$ and $\theta = \pi$). Assuming a Maxwellian velocity distribution, F_M use the result of 2b) with $a \ll 1$ to show that the fraction of particles which are trapped, f_t is given by:

$$f_t = \sqrt{2a}$$

[Hints: You may note that $(1-a)/(1+a) \approx 1 - 2a$ for $a \ll 1$. As a first step try to find an expression for the critical pitch angle (i.e. the pitch angle for which particles are just trapped). The most appropriate co-ordinate system for integrating a Maxwellian is usually the spherical co-ordinates, ϑ, φ and v and you may find it helpful to relate ϑ to the pitch angle, λ.]

[6 marks]
