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## Purpose

## Model the distribution of observables $\mathbf{x}$ in terms of

- Physical parameters of interest p
- Other parameters **q** to describe detector effects (resolution, efficiency,...)



# Probability density function F(x;p,q)

 normalized over allowed range of the observables x w.r.t the parameters p and q

## **Implementation**

- Add-on package to ROOT
  - ROOT is an object-oriented analysis environment
  - C++ command line interface & macros
  - Graphics interface
  - I/O support ('persisted objects')

- RooFit is collection of classes that augment the ROOT environment
  - Object-oriented data modeling
  - Integration in existing analysis environment
    - · Interfaces with existing data formats
    - No need to learn new language



#### RooFit @ BaBar

- Successor of RooFitTools
  - RooFitTools no longer maintained
  - RooFit is a nearly complete rewrite (~95%) of RooFitTools
    - Class structure redesigned from scratch, having learned from RooFitTools evolution
    - Key class names and functionality identical to enhance macro portability
- Code split in two SRT packages
  - RooFitCore
    - Core code, base classes, interface to MINUIT, plotting logic, integrators, PDF operator classes, ...
    - Everything except the PDFs
    - · Maintained exclusively by Wouter & David for code stability and design overview
  - RooFitModels
    - PDF implementations (Gauss, Argus etc)
    - · Contributed by BaBar users
- No code dependence on other BaBar software
  - Uses SoftRelTools for BaBar builds, but standalone Makefile provided
    - Some work still in progress...
  - Compiles clean & tested on Linux, Solaris, OSF
  - You can run it on your laptop, at home,...

# The basics



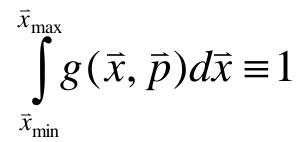
Probability density functions & likelihoods

The basics of OO data modeling

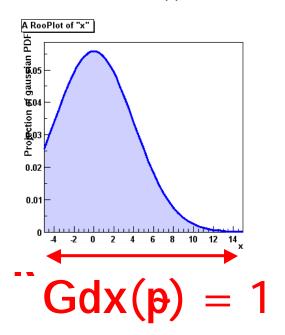
The essential ingredients: PDFS, datasets, functions

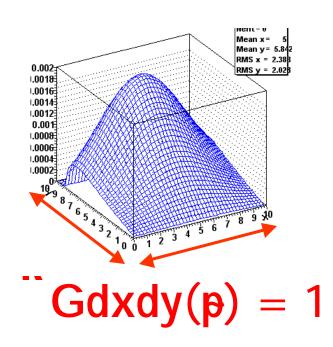
## Probability density functions

- Fundamental property of any probability density function g(x,p):
  - Easy to construct for 1-dim. PDF much more effort for >1 dim.



- RooFit automatically takes care of this
  - User supplied function need not be normalized





## Likelihood fits & ToyMC generation

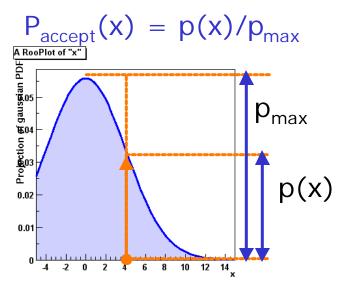
- Likelihood fit
  - Likelihood is product of probabilities given by g(x) for all data points in a given dataset D[x]

$$L(\vec{p}) = \prod_{D} g(\vec{x}_i, \vec{p})$$

- Fit find  $\vec{p}$  for which  $-\log(L(\vec{p}))$  is smallest

$$-\log(L(\vec{p})) = -\sum_{D}\log(g(\vec{x}_i, \vec{p}))$$

- ToyMC generation
  - Accept/reject method ------
  - 'Direct' method (e.g. gauss)



## Object-oriented data modeling

- In RooFit every variable, data point, function, PDF represented in a C++ object
  - Objects classified by data/function type they represent, not by their role in a particular setup
  - All objects are self documenting
- Objects
  representing a 'real' value.

  RooRealVar mb0("mb0","B0 mass",5.2794,"GeV");

  RooRealVar mb0("mb0","B0 mass",5.2794,"GeV");

  RooRealVar mb0("mb0","B0 mass",5.2794,"GeV");

  RooRealVar mb0("mb0","B0 mass",5.2794,"GeV");

References to variables

## Object-oriented data modeling

Elementary operations on value holder objects

```
mass.Print()
     Print value and attributes
                               RooRealVar::mass: 5.2500 L(5.2 - 5.3)
                               mass = 5.27;
                                                             Error: new value
                               mass.setVal(5.27);
            Assign new value
                                                             out of allowed range
                               mass = 9.0
                               RooAbsRealLValue::inFitRange(mass):
                                    value 9 rounded down to max limit 5.3
            Retrieve contents
                               Double_t massVal = mass.getVal();
                               b0sig.Print()
Print works for all RooFit objects
                               RooGaussian::b0sig(mass,mb0,width) = 0
getVal() works for all real-valued
                               Double_t val = b0sig.getVal()
objects (variables and functions)
```

## Elementary operations with a PDF

Setup gaussian PDF and plot

```
// Build Gaussian PDF
RooRealVar x("x","x",-10,10);
RooRealVar mean("mean", "mean of gaussian", 0, -10, 10);
RooRealVar sigma("sigma", "width of gaussian", 3);
RooGaussian gauss("gauss", "gaussian PDF", x, mean, sigma);
// Plot PDF
                                     A RooPlot of "x"
                                    gaussian PDF
0.00
0.00
0.00
RooPlot* xframe = x.frame()
gauss.plotOn(xframe);
xframe->Draw();
                                    Jo uoi 6.015
      Axis label from gauss title-
                                     0.01
                                                           Unit
A RooPlot is an empty frame
                                                       normalization
                                     0.005
capable of holding anything
plotted versus it variable
                                                                            8
                                       -10
                                                                2
                                                                        6
                        Plot range taken from limits of x
                                                                Wouter Verkerke, UCSB
```

Correct axis label for data

Events / ( 0.2 )

200

A RooPlot of "x"

- 1) Generate 10K events from PDF
- 2) Fit PDF to event sample
- 3) Plot PDF on data

```
150
                                                      PDF
// Generate a toy MC set
                                      100
                                                   automatical
RooDataSet* data =
                                                    normalized
         gauss.generate(x,10000)
                                                    to dataset
// Fit pdf to toy
gauss.fitTo(*data);
// Plot PDF and toy data overlaid Once the model is built,
                                     Generating ToyMC, fitting, plotting
RooPlot* xframe2 = x.frame();
data->plotOn(xframe2) ;
                                     are mostly one-line operations!
gauss.plotOn(xframe2,"L");
xframe2->Draw();
                                                         Wouter Verkerke, UCSB
```

 PDF objects have no intrinsic notion of a variable begin a parameter or observable

```
RooGaussian b0sig("b0sig","B0 sig PDF", mass, mb0, width);
```

But, PDF normalization depends on parameter/observable interpretation of variables

$$\int_{x_{\min}}^{x_{\max}} g(x, p) dx \equiv 1 \qquad \begin{array}{l} x = \text{observable} \\ p = \text{parameter} \end{array}$$

- Parameter/observable interpretation is automatic and implicit when a PDF is used together with a dataset
  - All PDF variables that are member of the dataset are observables
  - All other PDF variables are parameters
  - Limits are normalization range if variable is observable Limits are MINUIT bounds if variable is parameter

#### Variables → Parameter or observable?

- Example of dynamic variable interpretation
  - BMixingPDF(dt,mixState,...) + data(dt)
    - mixState is parameter.
    - Data is fitted with pure mixed or unmixed PDF depending on value of mixState
  - BMixingPDF(dt,mixState,...) + data(dt,mixState)
    - mixState is observable.
    - PDF is normalized explicitly over the 2 states of mixState and behaves like a 2-dimensional PDF
- Determining the parameters/observables of a given PDF

```
getDependents:
Make list of common variables
between data and gauss
```

getParameters:

Make list of variables of gauss
that do not occur in data

```
RooArgSet* paramSet = gauss.getDependents(data);
paramSet.Print("v");
RooArgSet::dependents:
    1) RooRealVar::x : 0 L(-10 - 10)

RooArgSet* paramSet = gauss.getParameters(data);
paramSet.Print("v");
RooArgSet::parameters:
    1) RooRealVar::mean : -0.940910 +/- 0.0304
    2) RooRealVar::sigma : 3.0158 +/- 0.0222
```

#### Lists and sets

 RooFit has two collection classes that are frequently passed as arguments or returned as argument

- **RooArgSet** Set semantics
  - Each element may appear only once
  - No ordering of elements
- RooArgList List semantics
  - Elements may be inserted multiple times
  - Insertion order is preserved

```
RooArgSet s1(x,y,z);
RooArgSet s2(x,x,y); //ERROR!
```

```
RooArgList l1(z,y,x);
RooArgList l2(x,x,y);
l2.Print();
RooArgList:::
   1) RooRealVar::x: "x"
   2) RooRealVar::x: "x"
   3) RooRealVar::y: "y"
```



Basic PDFs

Combining building blocks via addition, multiplication

Generic real-valued functions

Plug-and-play parameters

## The building blocks

RooFitModels provides a collection of 'building block' PDFs

- Argus background shape RooArgusBG **ROOBCPEffDecay** - BO decay with CP violation RoobmixDecay - B0 decay with mixing RooBifurGauss - Bifurcated Gaussian RooBreitWigner - Breit-Wigner shape - Crystal Ball function RooCBShape - Chebychev polynomial RooChebychev - Simple decay function RooDecay RooDircPdf - DIRC resolution description - D\* background description RooDstD0BG RooExponential - Exponential function - Gaussian function RooGaussian

RooKeysPdfNon-parametric data descriptionNon-parametric data description

RooPolynomial - Generic polynomial PDFRooVoigtian - Breit-Wigner (X) Gaussian

- More will PDFs will follow
  - Easy to for users to write/contribute new PDFs

## Generic expression-based PDFs

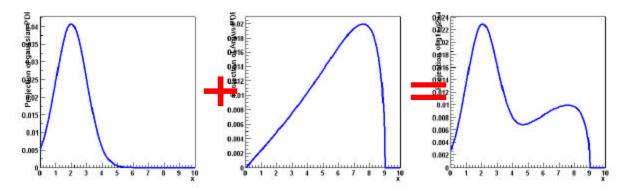
If your favorite PDF isn't there
 and you don't want to code a PDF class right away
 → USE RooGenericPdf

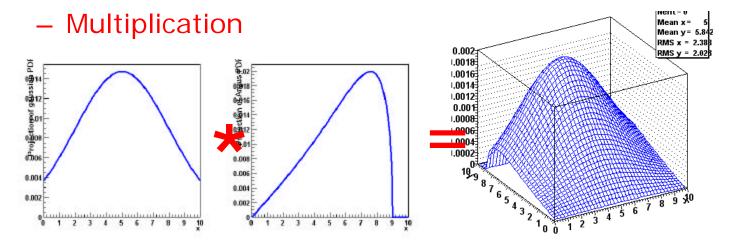
Just write down the PDFs expression as a C++ formula

- Automatic normalization
  - Expression divided by numerical integral of expression

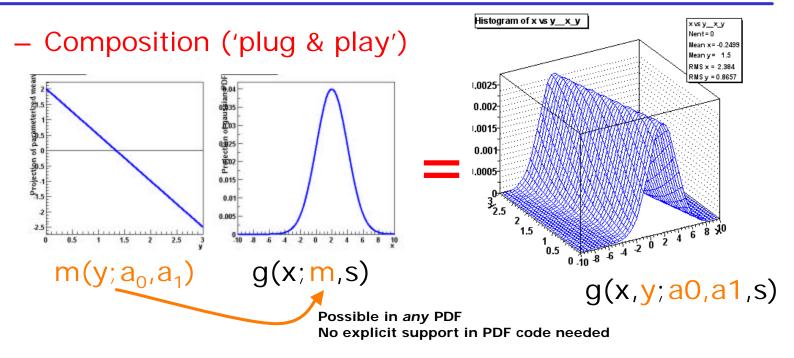
## Building realistic models

- Complex PDFs be can be trivially composed using operator classes
  - Addition

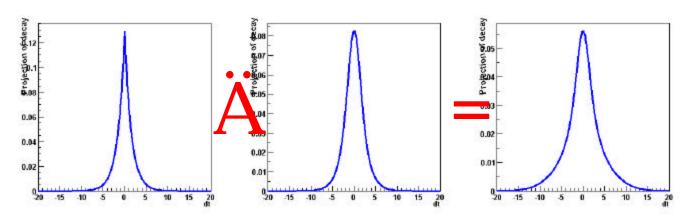




## Building realistic models



#### Convolution

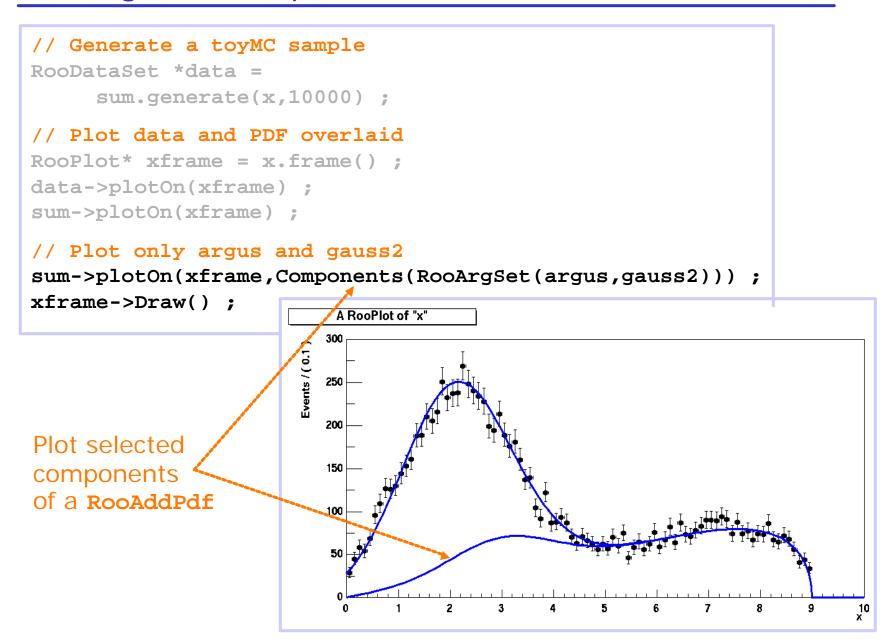


#### **RooAddPdf** constructs the sum of N PDFs with N-1 coefficients:

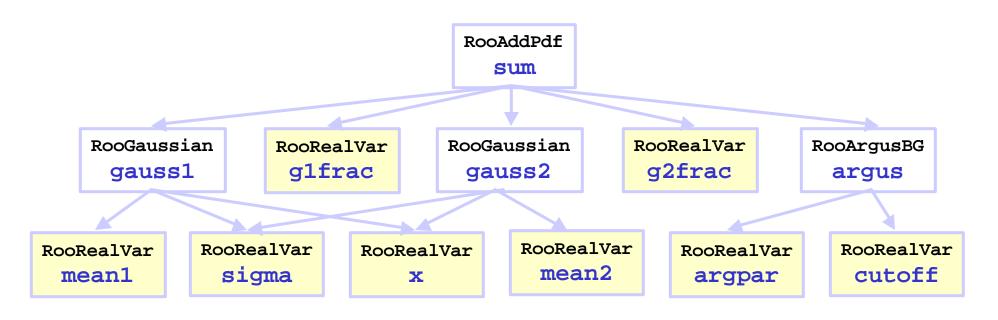
$$S = c_0 P_0 + c_1 P_1 + c_2 P_2 + \dots + c_{n-1} P_{n-1} + \left(1 - \sum_{i=0, n-1} c_i\right) P_n$$

```
// Build two Gaussian PDFs
             RooRealVar x("x","x",0,10);
             RooRealVar mean1("mean1", "mean of gaussian 1", 2);
Build 2
             RooRealVar mean2("mean2", "mean of gaussian 2", 3);
Gaussian
             RooRealVar sigma("sigma", "width of gaussians", 1);
 PDFs
             RooGaussian gauss1("gauss1", "gaussian PDF", x, mean1, sigma);
             RooGaussian gauss2("gauss2", "gaussian PDF", x, mean2, sigma);
             // Build Argus background PDF
  Build
             RooRealVar argpar("argpar", "argus shape parameter", -1.0);
ArgusBG
             RooRealVar cutoff("cutoff", "argus cutoff", 9.0);
  PDF
             RooArgusBG argus("argus","Argus PDF",x,cutoff,argpar);
             // Add the components
             RooRealVar glfrac("glfrac", "fraction of gauss1", 0.5); List of PDFs
             RooRealVar g2frac("g2frac", "fraction of gauss2", 0.1);
             RooAddPdf sum("sum", "g1+g2+a", RooArgList(gauss1, gauss2, argus),
                                             RooArgList(glfrac,g2frac));
```

## Adding PDF components



## Parameters of composite PDF objects



```
RooArgSet *paramList = sum.getParameters(data) ;
paramList->Print("v") ;
RooArgSet::parameters:
  1) RooRealVar::argpar : -1.00000 C
  2) RooRealVar::cutoff: 9.0000 C
                                        The parameters of sum
  3) RooRealVar::glfrac: 0.50000 C
                                        are the combined
  4) RooRealVar::g2frac : 0.10000 C
  5) RooRealVar::mean1 : 2.0000 C
                                        parameters
  6) RooRealVar::mean2 : 3.0000 C
                                        of its components
  7) RooRealVar::sigma
                        : 1.0000 C
                                                     Wouter Verkerke, UCSB
```

#### **RooProdPdf** constructs the product of N PDFs:

$$P = P_0(x_1, x_2) \cdot P_1(y_1, y_2,...) \cdot P_2(z_1, z_2,...) \cdot ... P_n(w_1, w_2,...)$$

Build 2 Gaussian PDFs

# Component PDFs may not share dependents e.g. $pdf_1(\mathbf{x}, \mathbf{y}) * pdf_2(\mathbf{x}, \mathbf{z})$ not allowed

Such forms are not very common, but can be performed with RooGenericPdf Shared parameters no problem

Normalization more complicated

## Plotting multi-dimensional PDFs

```
RooPlot* xframe = x.frame();

data->plotOn(xframe);

prod->plotOn(xframe);

xframe->Draw();

f(x) = \int pdf(x,y)dy
c->cd(2);

RooPlot* yframe = y.frame();

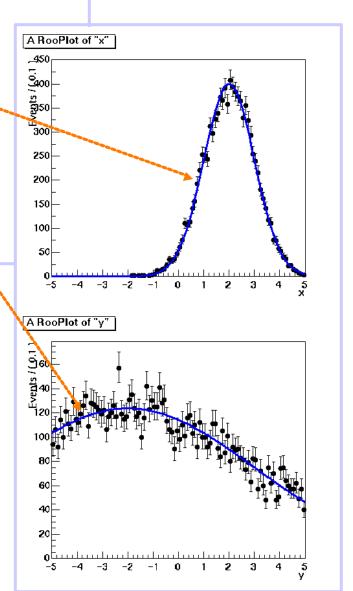
data->plotOn(yframe);

prod->plotOn(yframe);

yframe->Draw();

f(y) = \int pdf(x,y)dx
```

- -Plotting a dataset D(x,y) versus x represents a *projection over y*
- -To overlay PDF(x,y), you must plot Int(dy)PDF(x,y)
- -RooFit automatically takes care of this!
  - •RooPlot remembers dimensions of plotted datasets



Suppose you want to build a PDF like this

```
PDF(x,y) = gauss(x,m(y),s)

m(y) = m_0 + m_1 \cdot sqrt(y)
```

How do you do that? Just like that:

```
RooRealVar x("x","x",-10,10);
RooRealVar y("y","y",0,3);

**Muild a parameterized mean variable for gauss**
RooRealVar mean0("mean0", "mean offset",0.5);
RooRealVar mean1("mean1", "mean slope",3.0);
RooFormulaVar mean("mean", "mean0+mean1*y",

**RooArgList(mean0,mean1,y));
RooRealVar sigma("sigma", "width of gaussian",3);
RooGaussian gauss("gauss", "gaussian", x, mean, sigma);
```

Plug-and-play parameters!

PDF expects a real-valued object as input, not necessarily a variable

#### Generic real-valued functions

- RooFormulaVar makes use of the ROOT TFormula technology to build interpreted functions
  - Understands generic C++ expressions, operators etc
  - Two ways to reference RooFit objects
     By name:

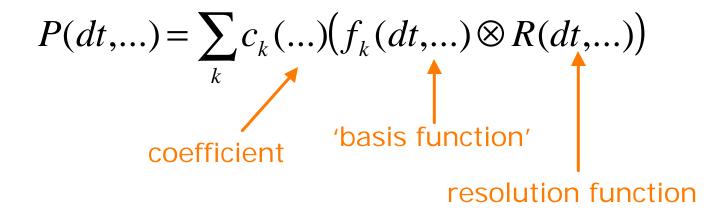
```
RooFormulaVar f("f","exp(foo)*sqrt(bar)", RooArgList(foo,bar));

By position:
RooFormulaVar f("f","exp(@0)*sqrt(@1)",RooArgList(foo,bar));
```

- You can use RooFormulaVar where ever a 'real' variable is requested
- RooPolyVar is a compiled polynomial function

```
RooRealVar x("x","x",0.,1.);
RooRealVar p0("p0","p0",5.0);
RooRealVar p1("p1","p1",-2.0);
RooRealVar p2("p2","p2",3.0);
RooFormulaVar f("f","polynomial",x,RooArgList(p0,p1,p2));
```

 Convoluted PDFs that can be written if the following form can be used in a very modular way in RooFit



Example: Bo decay with mixing

$$c_0 = 1 \pm \Delta w,$$
  $f_0 = e^{-|t|/t}$   $c_1 = \pm (1 - 2w),$   $f_1 = e^{-|t|/t} \cos(\Delta m \cdot t)$ 

#### Convoluted PDFs

Physics model and resolution model are implemented separately in RooFit

Implements  $f_i(dt,...) \otimes R(dt,...)$ Also a PDF by itself

RooResolutionModel

$$P(dt,...) = \sum_{k} c_{k}(...) (f_{k}(dt,...) \otimes R(dt,...))$$

RooConvolutedPdf (physics model)

Implements  $c_k$ Declares list of  $f_k$  needed



User can choose combination of physics model and resolution model at run time

(Provided resolution model implements all f<sub>k</sub> declared by physics model)

#### Convoluted PDFs

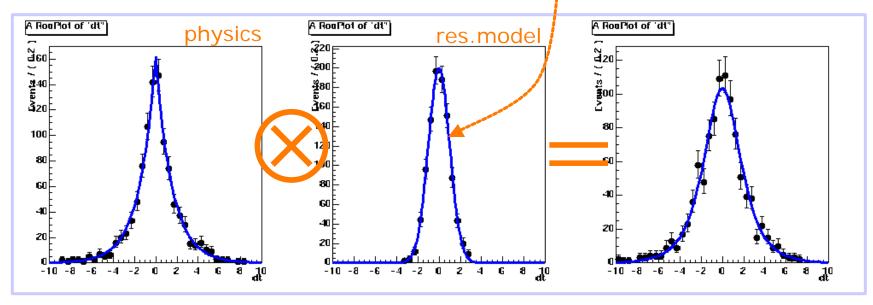
```
RooRealVar dt("dt", "dt", -10,10);
RooRealVar tau("tau","tau",1.548);
                                               A RouPlot of 'dt"
                                                                    decay
                                               30.6
// Truth resolution model
RooTruthModel tm("tm","truth model 1,dt)
                                               .05
8.05
                                               0.04
// Unsmeared decay PDF
                                                0.03
RooDecay decay tm("decay tm", "decay",
   dt,tau,tm,RooDecay::DoubleSided);
                                                0.02
                                                0.01
// Gaussian resolution model
                                                     -6 -4 -2 0 2 4 G 8 10
RooRealVar bias1("bias1","bias1",0);
RooRealVar sigma1("sigma1","sigma1",1);
                                               A RoaPlot of "dt"
                                                              decay ⊗ gm1
RooGaussModel gm1("gm1", "gauss model",
                                               9.04
                        dt,bias1,sigma1)/;
                                               02035
                                               9.03
// Construct a decay (x) gauss PDF
                                               0-025
RooDecay decay_gm1("decay_gm1", "decay",
                                               0.02
    dt,tau,gm1,RooDecay::DoubleSided);
                                               0.015
                                                0.01
                                               0.005
```

### Composite Resolution Models: RooAddModel

```
//... (continued from last page)
// Wide gaussian resolution model
                                              A RouPlot of 'dt"
                                                            decay ⊗ gm1
                                              20.04
RooRealVar bias2("bias2","bias2",0);
RooRealVar sigma2("sigma2","sigma2",5);
                                              RooGaussModel gm2("gm2", "gauss model 2"
                                              9.03
                                              0.025
                    ,dt,bias2,sigma2);
                                              0.02
                                              0.015
// Build a composite resolution model
                                               0.01
RooRealVar f("f", "fraction of gm1", 0.5)
                                              0.005
RooAddModel gmsum("gmsum", "gm1+gm2",
                                                RooArgList(gm1,gm2),f);
                                                 decay \otimes (f \cdot gm1 + (1-f) \cdot gm2)
// decay (x) (gm1 + gm2)
                                              29.03 F
RooDecay decay_gmsum("decay_gmsum",
                                              07025
                "decay", dt, tau, gmsum,
                                              ≨.02
                RooDecay::DoubleSided) ;
                                              0.015
                                               0.01
                                              0.005
 →RooAddModel works like RooAddPdf
```

#### Resolution models

- Currently available resolution models
  - RooGaussModel Gaussian with bias and sigma
  - RooGExpModel Gaussian (X) Exp with sigma and lifetime
  - RooTruthModel Delta function
- A RooResolutionModel is also a PDF
  - You can use the same resolution model you use to convolve your physics PDFs to ft to MC residuals



Extended PDFs add extra term to global likelihood

$$-\log(L(\vec{p})) = -\sum_{D}\log(g(\vec{x}_i, \vec{p})) + N_{\exp} - N_{obs}\log(N_{\exp})$$

- Building extended PDFs
  - Any PDF can be turned into an extended PDF by wrapping it in a RooExtendPdf object

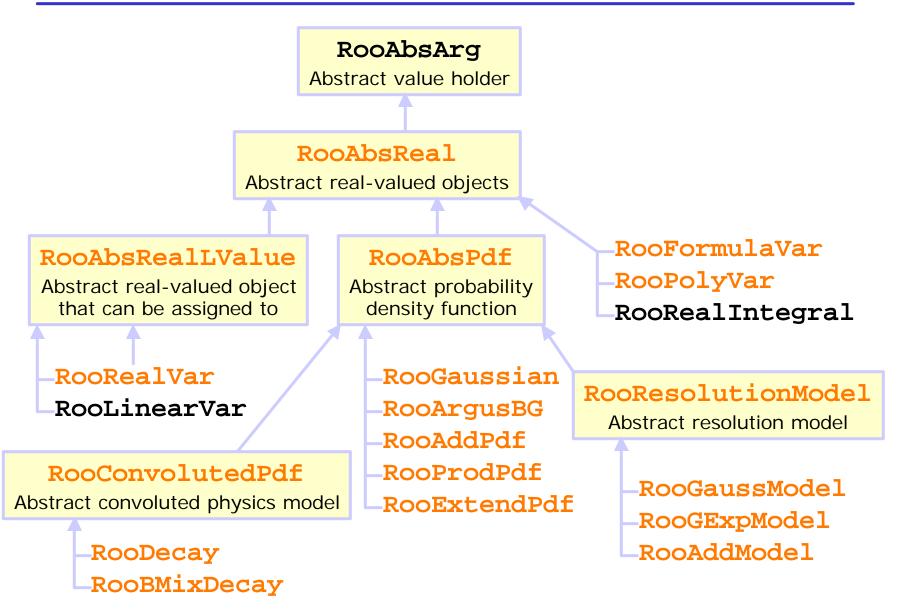
```
RooGaussian gauss("gauss","Gaussian",x,mean,sigma);
RooRealVar nsig("nsig","number of signal events",5,0,100);
RooExtendPdf gausse("gausse","Extended Gauss",gauss,nsig);
```

nsig is now a parameter of gausse -----and represents the number of expected events

#### Extended likelihood PDFs

- Composition rules for extended PDFs
  - A ROOAddPdf of all extendable PDFs is extendable
    - No coefficients needed (fractions calculated from components Nexpected)
  - A ROOProdPdf with a single extendable component is extendable
  - A RooSimultaneous with any extendable component is extendable
    - Can do mixed extended/regular MLL fits in various data subsets
- RooAddPdf short-hand form for branching fraction fits
  - If RooAddPdf is given N coefficients instead of N-1 fractions
    - → RooAddPdf is automatically extended
    - → coefficients represent the expected #events for each PDF comp.

## Class tree for real-valued objects



# Discrete variables



Organizing and classifying your data with discrete functions

Discrete-valued functions

Tabulating discrete data

- So far we have expressed all models purely in terms of real-valued variables
  - RooFit also has extensive support for discrete variables
  - Discrete variables are called categories
- Properties of RooFit categories
  - Finite set of named states → self documenting
  - Optional integer code associated with each state

```
At creation.
                      // Define a cat. with explicitly numbered states
        a category
                      RooCategory b0flav("b0flav", "B0 flavour");
      has no states
                      b0flav.defineType("B0",-1);
                      b0flav.defineType("B0bar",1);
        Add states
with a label and index
                      // Define a category with labels only
                      RooCategory tagCat("tagCat", "Tagging category") ;
       Add states
                      tagCat.defineType("Lepton");
   with a label only.
                      tagCat.defineType("Kaon");
     Indices will be
                      tagCat.defineType("NetTagger-1");
      automatically
                      tagCat.defineType("NetTagger-2");
          assigned
```

# When to use discrete variables

- Discrete valued observables
  - B0 flavour
  - Rec/tag mixing state
- Event classification
  - tagging category
  - run block
  - B0 reconstruction mode
- Cuts
  - Mass window / sideband window
- In general, anything that you would use integer codes for in FORTRAN
  - RooFit makes your life easier:
     all states are labeled by name → no codes to memorize
  - Optional integer code associated with category states allows to import existing integer encoded data
    - Self-documenting: category state definitions provide single and easily understandable integer—state name conversion point

# Managing data subsets / RooSimultaneous

- Simultaneous fit to multiple data samples
  - E.g. to fit  $PDF_A$  to dataset  $D_A$  and  $PDF_B$  to dataset  $D_B$  simultaneously, the NLL is

$$NLL = \sum_{i=1,n} -\log(PDF_{A}(D_{A}^{i})) + \sum_{i=1,m} -\log(PDF_{B}(D_{B}^{i}))$$

 Use categories to split a master dataset D into subsets D<sub>A</sub>, D<sub>B</sub> etc

Dataset A	4				
5.0			Dataset A+B		
3.7			5.0	Α	
1.2			3.7	Α	
4.3			1.2	А	
			4.3	А	
Dataset F	3		5.0	В	
5.0			3.7	В	
3.7			1.2	В	
1.2					

# Using categories: RooSimultaneous

RooSimultaneous implements 'switch' PDF:

```
case (indexCat) {
   A: return pdfA;
   B: return pdfB;
}
Effectively fitting
pdfA to dataA
pdfB to dataB
```

```
Create dataset indexing category  

Create dataset indexing category  

Associate created  
   PDFs with appropriate index  
   category state  

// Define a category with labels only  
RooCategory tagCat("tagCat", "Tagging category");  
tagCat.defineType("Lepton");  

// Build PDFs for Lepton and Kaon data subsets

// Construct simultaneous PDF for lep and kao  
RooSimultaneous simPdf("simPdf", "simPdf", tagCat);  
simPdf.addPdf(pdfLep, "Lepton");  
simPdf.addPdf(pdfKao, "Kaon");
```

Sig Sideband

You can use discrete variables to describe cuts, e.g.

<del>~</del>800 ⊏

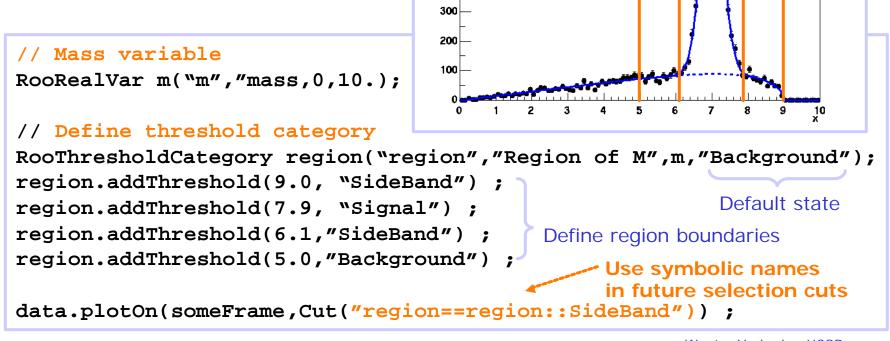
Events / (0

500

400

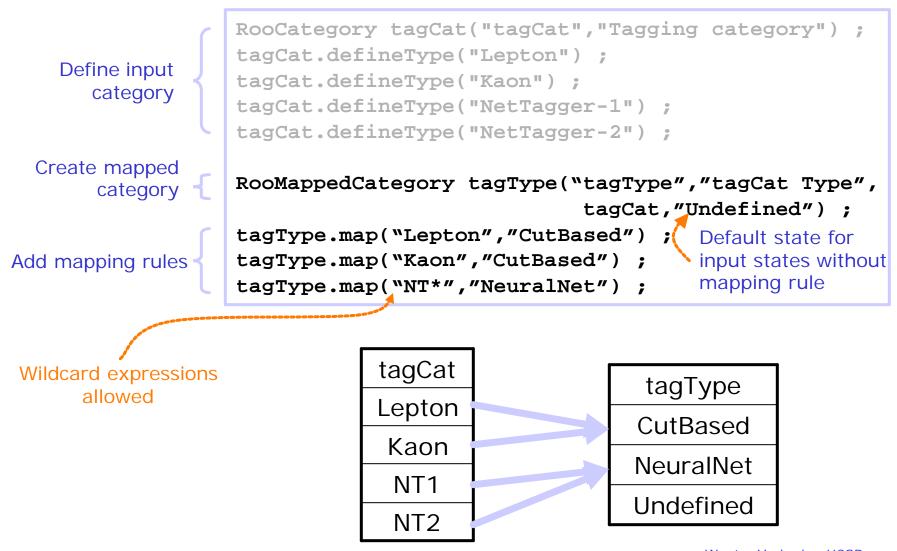
background

- Signal, sideband mass windows
- RooThresholdCategory
  - Defines regions of a real variable



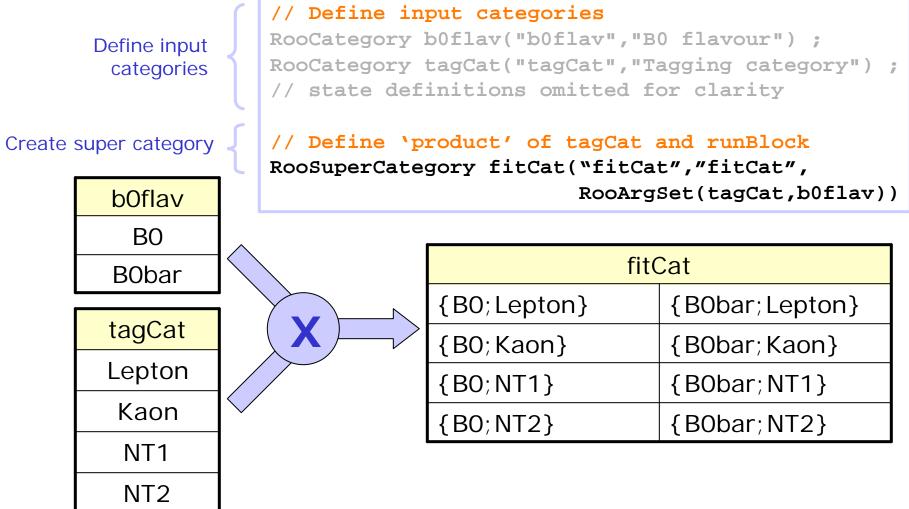
# Discrete functions

RooMappedCategory provides cat → cat mapping



# Discrete functions

 RooSuperCategory/RooMultiCategory provides category multiplication



# Exploring discrete data

 Like real variables of a dataset can be plotted, discrete variables can be tabulated

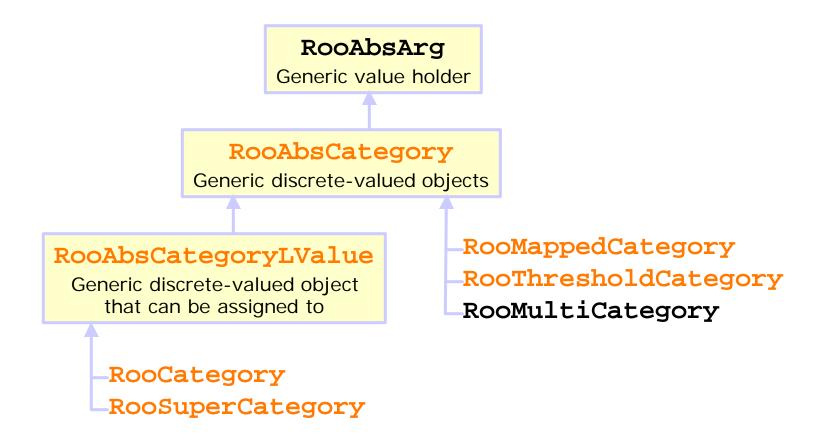
```
RooTable* table=data->table(b0flav) ;
                               table->Print();
  Tabulate contents of dataset
                               Table b0flav : aData
           by category state
                                    B0bar
      Extract contents by label
                               Double_t nB0 = table->get("B0");
                               Double_t b0Frac = table->getFrac("B0");
Extract contents fraction by label
                               data->table(tagCat, "x>8.23")->Print();
                                  Table tagCat : aData(x>8.23)
         Tabulate contents of
                                          Lepton
                                                    668
       selected part of dataset
                                            Kaon
                                                    717
                                    NetTagger-1
                                                    632
                                    NetTagger-2
```

# Exploring discrete data

 Discrete functions, built from categories in a dataset can be tabulated likewise

data->table(b0Xtcat)->Print(); Table b0Xtcat : aData {B0;Lepton} {B0bar;Lepton} Tabulate RooSuperCategory states {B0;Kaon} {B0bar; Kaon} {B0; NetTagger-1} {B0bar; NetTagger-1} {B0; NetTagger-2 1223 {B0bar; NetTagger-2 data->table(tcatType)->Print(); Table tcatType : aData Tabulate **RooMappedCategory** states Unknown Cut based Neural Network

# Class tree for discrete-valued objects



# **Datasets**

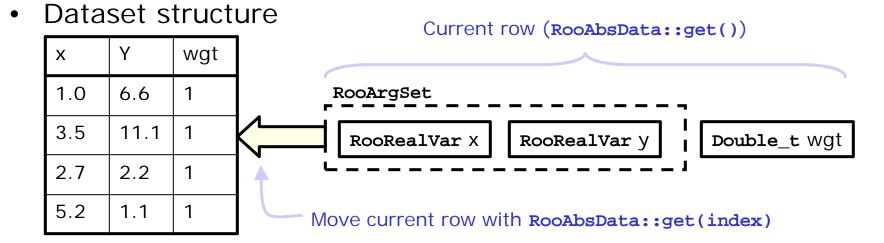


Binned vs unbinned datasets

Importing data from outside sources

Operations on datasets

- A dataset is a collection of points in N-dimensional space
  - Dimensions can be either real or discrete
  - Two dataset implementations:
    - RooDataSet unbinned (weighted & unweighted)
    - RooDataHist binned
  - Common abstract base class RooAbsData
  - Nearly all RooFit classes/functions (including fitting)
     take RooAbsData objects
    - Operations universally supported for binned and unbinned data



# Unbinned dataset basics

Create empty dataset with fields x,y,c.
Dataset row representation will be a **clone** of (x,y,c).
Original (x,y,c) will no longer be referenced after ctor.

To add a datapoint value holders x,y,c must be passed

```
// Create dataset variables
RooRealVar x("x","x",-10,10);
RooRealVar y("y","y", 0, 40);
RooCategory c("c", "c");
c.defineType("Plus",+1);
c.defineType("Minus",-1);
RooDataSet
data("data","data",RooArgSet(x,y,c));
// Fill d with dummy values
Int t i;
for (i=0; i<1000; i++) {
 x = i/50 - 10;
 y = sqrt(1.0*i);
  c = (i\%2)?"Plus":"Minus";
 d.add(RooArgSet(x,y,c));
d.Print("v") ;
RooDataSet::d: "d"
  Contains 1000 entries
  Defines RooArgSet::Dataset Variables:
    1) RooRealVar::x:
    2) RooRealVar::y:
    3) RooCategory::c: "c"
  Caches RooArgSet::Cached Variables:
```

# Unbinned dataset basics

Access the pointer to the RooArgSet holding the current row

Load row #900 in the RooArgSet holding the current row

Find value holder for x in the current row

```
// Retrieve the 'current' row
RooArgSet* row = data.get() ;
row->Print("v");
RooArgSet::Dataset Variables: (Owning contents)
  1) RooRealVar::x : 9.0000 L(-10 - 10)
  2) RooRealVar::y: 31.607 L(0 - 40)
  3) RooCategory::c : Plus
// Retrieve a specific row
row = data.get(900);
row->Print("v");
RooArgSet::Dataset Variables: (Owning contents)
  1) RooRealVar::x: 8.0000 L(-10 - 10)
  2) RooRealVar::y: 30.000 L(0 - 40)
  3) RooCategory::c : Minus
// Retrieve a specific field of the row
RooRealVar* xrow = (RooRealVar*) row->find("x") ;
cout << xrow->getVal() << endl ;</pre>
8.0000
```

# Weighting unbinned datasets

```
// Print current row and weight of dataset
                     row->Print("v");
                     RooArgSet::Dataset Variables: (Owning contents)
                        1) RooRealVar::x : 8.0000 L(-10 - 10)
                        2) RooRealVar::y: 30.000 L(0 - 40)
                        3) RooCategory::c : Minus
                     cout << data.weight() << endl ;</pre>
                      1.0000
  Instruct dataset
                      // Designate variable y as the event weight
  to interpret y as
                     data.setWeightVar(y)
  the event weight
                      // Retrieve same row again
                     row = data.get(900);
                     row->Print("v");
                     RooArgSet::Dataset Variables: (Owning contents)
   Variable y is
                        1) RooRealVar::x: 8.0000 L(-10 - 10)
   no longer in the
                        2) RooCategory::c : Minus
   current row
    Current value
                     cout << data.weight() << endl ;</pre>
   of y is returned
                      30.0000
as the event weight
```

# Importing data

- Unbinned datasets (RooDataSet) can be constructed from
  - ROOT TTree objects
    - RooRealVar dataset rows are taken /D /F /I tree branches with equal names
    - RooCategory dataset rows are taken from /I /b tree branches with equal names

```
Ttree* tree = <someTFile>.Get("<someTTree>");
RooDataSet data("data","data",tree,RooArgSet(x,c));
```

- ASCII data data files
  - ASCII file fields are interpreted in order of supplied RooArgList

```
RooDataSet* data =
   RooDataSet::read("ascii.file",RooArgList(x,c));
```

# Implicit selection: External data may contain entries that exceed limits set on RooFit value holder objects

- If a loaded value of a **RooRealVar** exceeds the RRVs limits, the entire tree row is not loaded
- If a loaded index of a RooCategory is not defined,
   the entire tree row is not loaded

# Importing data

- Binned dataset (RooDataHist) can be constructed from
  - ROOT TH1/2/3 objects
    - TH dimensions are matched in order to supplied list of RooFit value holders

```
TH2* histo = <yourTHistogram> ;
RooDataHist bdata("bdata","bdata",RooArgList(x,y),histo);
```

- RooDataSet unbinned datasets
  - Binning for each dimension is specified by setFitRange(lo,hi),setFitBins(nbins)
  - The unbinned dataset may have more dimensions than the binned dataset.

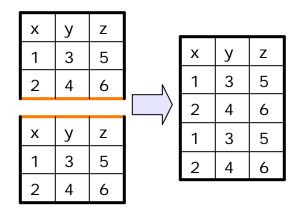
Dimensions not specified are automatically projected

```
RooDataSet* data = <yourUnbinnedData> ;
RooDataHist bdata("bdata","bdata",RooArgList(x,y),data) ;
```

# Extending and reducing unbinned datasets

# Appending

```
RooDataSet d1("d1","d1",RooArgSet(x,y,z));
RooDataSet d2("d2","d2",RooArgSet(x,y,z));
d1.append(d2);
```



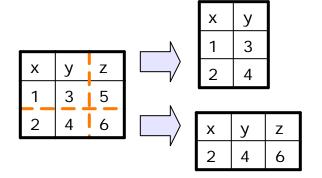
# Merging

```
RooDataSet d1("d1","d1",RooArgSet(x,y);
RooDataSet d2("d2","d2",RooArgSet(z));
d1.merge(d2);
```

х	у	Z	,	х	У	Z
1	3	5		1	3	5
2	4	6	V	2	4	6

# Reducing

```
RooDataSet d1("d1","d1",RooArgSet(x,y,z);
RooDataSet* d2 = d1.reduce(RooArgSet(x,y));
RooDataSet* d3 = d1.reduce("x>1");
```

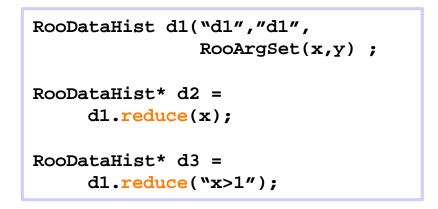


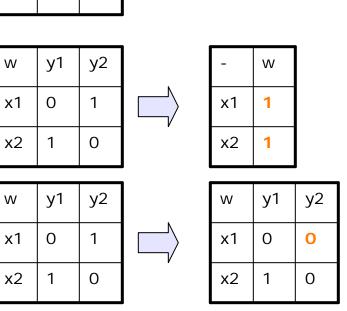
# Adding and reducing binned datasets

# Adding

# w y1 y2 x1 0 1 x2 1 0 w y1 y2 x1 1 1 x2 1 0

# Reducing





# Fitting & Generating

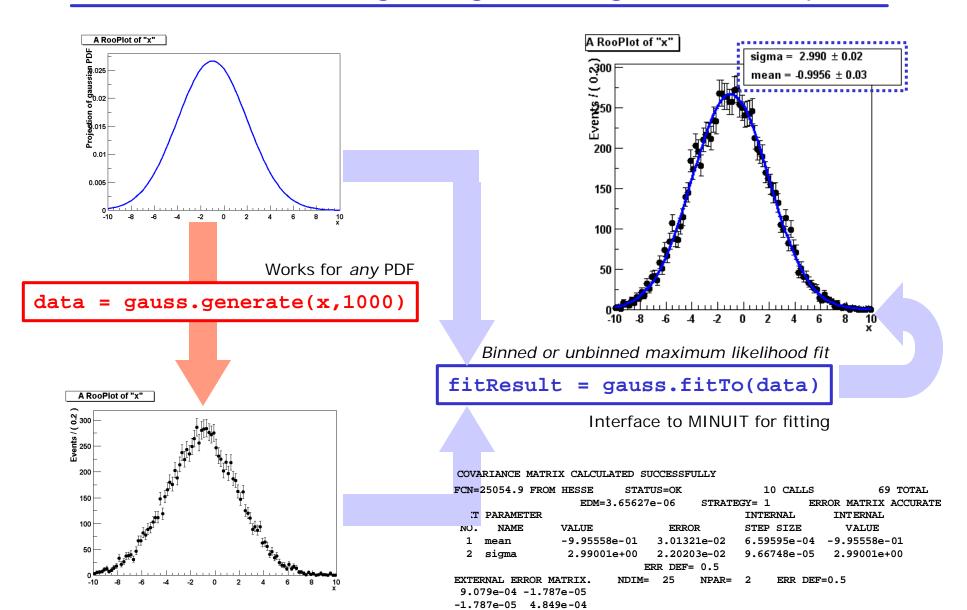


Fitting

Browsing your fit results
Generating toy MC

Putting it all together

# Given a model, fitting and generating are 1-line operations



Wouter Verkerke, UCSB

# Fitting

```
RooAbsData* data ;
RooAbsPdf* pdf ;
RooFitResult* fitres = pdf->fitTo(*data,"<options>") ;
```

- Binned/unbinned fit performed depending on type of dataset (RooDataHist/RooDataSet)
- Fitting options:

```
"m" = MIGRAD only, i.e. no MINOS
MINUIT
           "s" = estimate step size with HESSE before starting MIGRAD
control •
           "h" = run HESSE after MIGRAD
           "e" = Perform extended MLL fit
options
           "0" = Run MIGRAD with strategy MINUIT 0 (faster, but no corr. matrix at end)
                 Does not apply to HESSE or MINOS, if run afterwards.
            "q" = Switch off verbose mode
           "1" = Save log file with parameter values at each MINUIT step
output
           "v" = Show changed parameters at each MINUIT step
options
           "t" = Time fit
           "r" = Save fit output in RooFitResult object (return value is object RFR pointer)
```

# Automatic fit optimization

- RooFit analyzes PDF objects prior to fit and applies several optimizations
  - Actual fit performed on copy of PDF and dataset
    - · Allows case-specific non-reversible optimizations
  - Components that have all constant parameters are pre-calculated
  - Dataset variables not used by the PDF are dropped
  - Simultaneous fits: When a parameters changes only parts of the total likelihood that depend on that parameter are recalculated
  - PDF normalization integrals are only recalculated when the ranges of their observables or the value of their parameters are changed
    - · Lazy evaluation: calculation only done when intergral value is requested
- Little or no need for 'hand-tuning' of user PDF code
  - Easier to code and code is more readable
- 'Typical' large-scale fits see significant speed increase
  - Factor of 3x 10x not uncommon.

# Browsing fit results with RooFitResult

- As fits grow in complexity (e.g. 45 floating parameters), number of output variables increases
  - Need better way to navigate output that MINUIT screen dump
- RooFitResult holds complete snapshot of fit results
  - Constant parameters
  - Initial and final values of floating parameters
  - Global correlations & full correlation matrix
  - Returned from RooAbsPdf::fitTo() when "r" option is supplied
- Compact & verbose printing mode

**Compact Mode** 

```
Constant
                fitres->Print();
 parameters
                  RooFitResult: min. NLL value: 1.6e+04, est. distance to min: 1.2e-05
 omitted in
compact mode
                                       FinalValue +/- Error
                    Floating Parameter
                               argpar -4.6855e-01 +/- 7.11e-02
 Alphabetical
                               g2frac 3.0652e-01 +/- 5.10e-03
  parameter
                                       7.0022e+00 +/- 7.11e-03
                                mean1
    listing
                                mean2
                                        1.9971e+00 +/- 6.27e-03
                                        2.9803e-01 +/- 4.00e-03
                                sigma
```

# Browsing fit results with RooFitResult

### Verbose printing mode

```
fitres->Print("v");
 RooFitResult: min. NLL value: 1.6e+04, est. distance to min: 1.2e-05
 Constant Parameter
                      Value
                                     Constant parameters
              cutoff 9.0000e+00
                                     listed separately
              glfrac 3.0000e-01
                                     FinalValue +/- Error
   Floating Parameter
                      InitialValue
                                                             GblCorr.
              argpar -5.0000e-01 -4.6855e-01 +/- 7.11e-02 0.191895
              g2frac 3.0000e-01 3.0652e-01 +/- 5.10e-03 0.293455
               mean1 7.0000e+00 7.0022e+00 +/- 7.11e-03 0.113253
               mean2 2.0000e+00
                                     1.9971e+00 +/- 6.27e-03 0.100026
               sigma 3.0000e-01
                                     2.9803e-01 +/- 4.00e-03 0.276640
```

Initial, final value and global corr. listed side-by-side

Correlation matrix accessed separately

# Browsing fit results with RooFitResult

- Easy navigation of correlation matrix
  - Select single element or complete row by parameter name

```
r->correlation("argpar","sigma")
(const Double_t)(-9.25606412005910845e-02)

r->correlation("mean1")->Print("v")
RooArgList::C[mean1,*]: (Owning contents)
   1) RooRealVar::C[mean1,argpar]: 0.11064 C
   2) RooRealVar::C[mean1,g2frac]: -0.0262487 C
   3) RooRealVar::C[mean1,mean1]: 1.0000 C
   4) RooRealVar::C[mean1,mean2]: -0.00632847 C
   5) RooRealVar::C[mean1,sigma]: -0.0339814 C
```

- RooFitResult persistable with ROOT I/O
  - Save your batch fit results in a ROOT file and navigate your results just as easy afterwards

# Visualize errors and correlation matrix elements

```
RooFitResult* r = pdf->fitTo(data,"mhvr") ;
     RooPlot* f = new RooPlot(tau, sigma1, 1.35, 1.6, 0.85, 1.20);
      r->plotOn(f,tau,sigma1,"ME12VHB");
      f->Draw();
                               1.2
E 1.2
1.15
Works on any RooFitResult,
                                 1.1
Also after persistence
                                1.05
                                0.95
                                 0.9
 MINUIT contour scan
                                0.85
1.35
 is also possible with
                                          1.4
                                                  1.45
                                                          1.5
                                                                  1.55
                                                                           1.6
                                                                         tau
 a separate interface
```

# **Generating ToyMC**

- Normal generator run
  - Just specify set of observables to generate and #events

```
RooAbsPdf* pdf ;
RooDataSet* toyMCdata = pdf->generate(RooArgSet(dt,mixState),10000);

#events
Observables to generate
```

- Generator run with prototype data
  - Specify set of observables to generate and a prototype dataset

```
RooDataSet* protoData
RooAbsPdf* pdf ;
RooDataSet* toyMCdata = pdf->generate(RooArgSet(dt,mixState),*protoData);
```

Observables to generate

Prototype dataset

- Generated dataset will replicate exactly the prototype dataset except for observables generated by the PDF
- Ideal for per-event errors, tagging breakdown, ...

# Automatic generator optimizations

- Most efficient generator technique automatically selected
  - PDF components can advertise a smarter generation technique (direct generation, e.g. gauss) which is used when appropriate
  - RooProdPdf delegates generation of observables to component PDFs (1 x N-dim generation → N x 1-dim generation)
  - RooAddPdf components generated separately
     Accept/reject method very inefficient when broad and narrow distributions are summed
  - RooConvolutedPdf generates physicsPDF and smearing model separately if both support 'direct' generation (convolution integrals not evaluated during generation)

# Putting it all together: generating and fitting a decay PDF

```
// Build a simple decay PDF
RooRealVar dt("dt", "dt", -20,20);
RooRealVar tau("tau","tau",1.548,-10,10);
// Build a gaussian resolution model
RooRealVar bias("bias","bias",0,-5,5);
RooRealVar sigma("sigma", "sigma", 1, 0.1, 2.0);
RooGaussModel gm("gm", "gauss model", dt, bias, sigma) ;
// Construct a decay (x) gm
RooDecay decay("decay", "decay", dt, tau, gm, RooDecay::DoubleSided);
// Generate BMixing data with above set of event errors
RooDataSet *data = decay.generate(dt,2000) ;
// Fit the generated data to the model
                                                A RooPlot of "dt"
                                                                        tau = 1.561 ± 0.06
RooFitResult* r = decay.fitTo(*data,"mhr")
                                                                        sigma1 = 0.98 ± 0.1
r->correlation(sigma,tau);
                                                                        bias1 = 0.007 \pm 0.05
                                               ≃ 250
-0.818443
// Make a plot of the data and PDF
                                                 150
RooPlot* dtframe = dt.frame(-10,10,30);
data->plotOn(dtframe) ;
                                                 100
pdf.plotOn(dtframe) ;
                                                 50
pdf.paramOn(dtframe) ;
dtframe->Draw();
```

# Plotting & Saving



Adding statistics, parameter boxes

Changing colors and styles

Plotting in 2 and 3 dimensions

Persisting plots & fit results

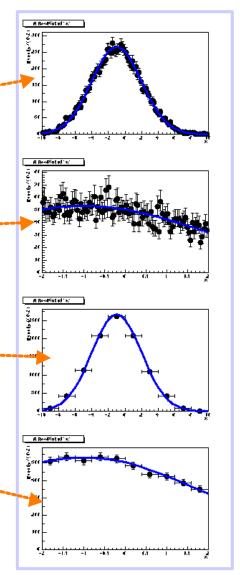
# Changing the plot range / histogram binning

By default a Rooplot frame takes the limits and the

number of bins from its plot variable

Can be overridden by frame() arguments

```
data->plotOn(frame1);
pdf->plotOn(frame1);
frame1->Draw();
RooPlot* frame2 = x.frame(-2,2);
data->plotOn(frame1) ;
pdf->plotOn(frame1);
frame2->Draw();
RooPlot* frame3 = x.frame(10) ---
data->plotOn(frame1) ;
pdf->plotOn(frame1);
frame3->Draw();
RooPlot* frame3 = x.frame(-2,2,10)
data->plotOn(frame1) ;
pdf->plotOn(frame1);
frame3->Draw() :
```



# Decoration

- A RooPlot is an empty frame that can contain
  - RooDataSet projections
  - PDF and generic real-valued function projections
  - Any ROOT drawable object (arrows, text boxes etc)
- Adding a dataset statistics box / PDF parameter box

```
RooPlot* frame = x.frame() ;
data.plotOn(xframe) ;
                                             A RooPlot of "x"
pdf.plotOn(xframe) ;
                                                                               sigma = 3.043 \pm 0.02
                                            300
pdf.paramOn(xframe,data)
                                                                               mean = -1.0112 \pm 0.03
                                            Events / (
data.statOn(xframe)
                                                                               N = 10000
                                                                               \langle x \rangle = -0.9975 \pm 0.03
xframe->Draw();
                                                                               x_{RMS} = 3.017 \pm 0.02
                                              150
                                              100
```

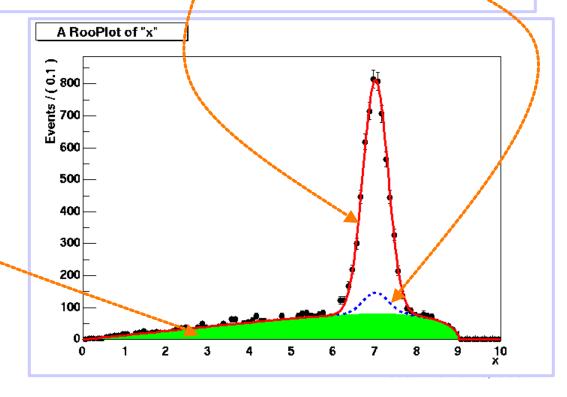
# **Decoration**

Adding generic ROOT text boxes, arrows etc.

```
TPaveText* tbox = new TPaveText(0.3,0.1,0.6,0.2,"BRNDC");
tbox->AddText("This is a generic text box") ;
TArrow* arr = new TArrow(0,40,3,100);
xframe2->addObject(arr)
xframe2->addObject(tbox);
                               A RooPlot of "x"
                                                                      sigma = 3.043 \pm 0.02
                              Events / ( 0.2 )
                                                                     mean = -1.0112 \pm 0.03
                                                                      N = 10000
                                                                      \langle x \rangle = -0.9975 \pm 0.03
                                                                     x_{RMS} = 3.017 \pm 0.02
                                200
                                150
                                100
                                 50
                                               This is a generic text box
                                                      -2
```

# Customization

Changing colors and styles of histograms and curves



# Plotting in more than 2,3 dimensions

- No equivalent of RooPlot for >1 dimensions
  - Usually >1D plots are not overlaid anyway
  - Methods provided to produce 2/3D ROOT histograms from datasets and PDFs/functions

```
TH2* ph2 = x.createHistogram("x vs y pdf",y,0,0,0,bins);
prod.fillHistogram(ph2,RooArgList(x,y));
ph2->Draw("SURF") ;
TH2* dh2 = x.createHistogram("x vs y data",y,0,0,0,bins);
data->fillHistogram(dh2,RooArgList(x,y));
dh2->Draw("LEGO") ;
                 Histogram of x vs y data | x y
                                                          Histogram of x vs y pdf x y
                                                x va y dete x y
                                                                                        x va y pdf x
                                                RMS y - 2.632
                                                                                        RMS y - 2.664
                                                          0.008
                                                          0.007
                   70
                                                          0.006
                                                          0.005
                                                          0.004
                                                          0.003
                   30
                                                          0.002
                                                          0.001
```

# Persisting and reviving Rooplots

Persisting ROOT Object Browser File View Options <u>H</u>elp RooFitResult\* r : demo2.root RooPlot\* xframe ; All Folders Contents of "/ROOT Files/demo2.root" Iroot Tfile f("demo2.root" /afs/slac.stanford.edu/u/ec/verk "RECREATE" ROOT Files fit(sum,sumData);1 frame(089da940);1 r->Write(); all demo2.root xframe->Write(); f.Close(); 2 Objects. Reviving A Roo lot of "x" 2 800 Throwswer the 설 700 600 E RooFitResult\* r = f.Get("fit(data,sum)"); 500 r->Print("v"); 300 RooFitResult: min. NLL value: 1.6e+04, ... 200 -Floating Parameter FinalValue +/- Error argpar -4.6855e-01 +/- 7.11e-0Wouter Verkerke, UCSB

## Storing configuration data in ASCII files

- RooArgLists can be written to and read from ASCII file
  - Convenient to load initial values of fit parameters

```
set.Print("v");
RooArgSet::parameters:
    1) RooRealVar::argpar : -0.468507 +/- 0.0711 (-0.0713, 0.0710) L(-2 - 0)
    2) RooRealVar::cutoff : 9.0000 C
    3) RooRealVar::g1frac : 0.30000 C
    4) RooRealVar::g2frac : 0.30652 +/- 0.00510 (-0.00509, 0.00511)
    5) RooRealVar::mean1 : 7.0022 +/- 0.00711 (-0.00712, 0.00710) L(0 - 10)
    6) RooRealVar::mean2 : 1.9971 +/- 0.00627 (-0.00628, 0.00626) L(0 - 10)
    7) RooRealVar::sigma : 0.29803 +/- 0.00400 (-0.00396, 0.00403)
set.writeToFile("config.txt");
```

#### config.txt

```
argpar = -0.468507 +/- 0.0711 (-0.0713, 0.0710) L(-2 - 0)
cutoff = 9.0000 C
g1frac = 0.30000 C
g2frac = 0.30652 +/- 0.00510 (-0.00509, 0.00511)
mean1 = 7.0022 +/- 0.00711 (-0.00712, 0.00710) L(0 - 10)
mean2 = 1.9971 +/- 0.00627 (-0.00628, 0.00626) L(0 - 10)
sigma = 0.29803 +/- 0.00400 (-0.00396, 0.00403)
```

```
set.readFromFile("config.txt");
```

## Documentation



RooFit home page
Tutorial macros
Inline code documentation

## How to get started / documentation

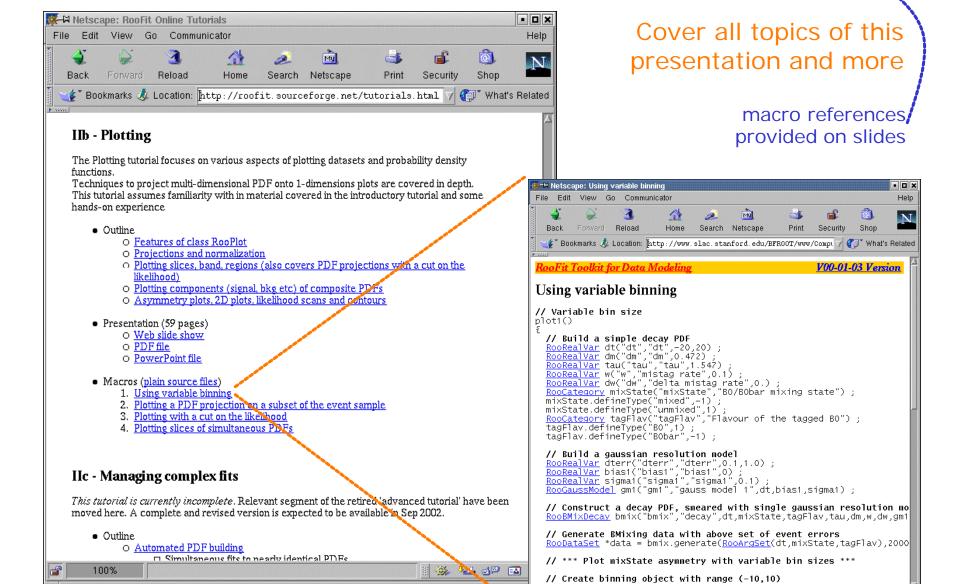
Starting point for all documentation is the RooFit homepage

http://roofit.sourceforge.net

#### Online tutorials

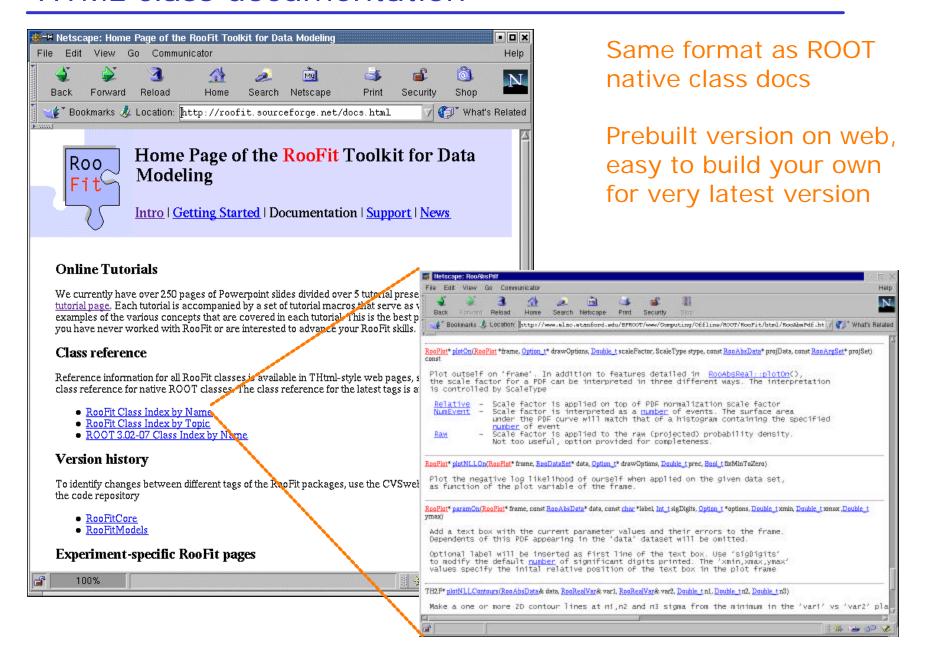


🔆 🛂 🔞 🖼 🤣



Rinning abins(-10.10)

#### HTML class documentation





## **RooFit Tutorial – Fitting and Generating**

Wouter Verkerke (UC Santa Barbara) David Kirkby (UC Irvine)

#### Overview

- WARNING This tutorial is incomplete
  - Relevant parts of the retired 'advanced' tutorial have been moved here.

## Generating Toy MC events



Prototyping your ToyMC off real data
Using RooMCStudy to efficiently generate (and fit) many samples

#### PDF generator interface has two invocation methods

#### Regular

All observables generated by the PDF

```
genData = pdf.generate(<observables>,numEvent) ;
```

#### With prototype data

Some observables modeled by a prototype dataset, other generated by PDF

```
RooDataSet protoData("D","D",cprotoObservables>");
// fill protoDataSet;
genData = pdf.generate(<genObservables>,protoData);
```

- Generator cycles through dataset, loading proto-observables in PDF.
- PDF generates other observables → all correlations preserved
- By default #events is also taken from proto dataset.
   If overruled, generated dataset may not exactly reproduce proto data, unless #events = n \* #proto-events

## Generating ToyMC for sin2beta fit

- CP/mixing PDF doesn't model per-event errors & tagCats
  - Must use prototype data when generating
- Where to get the prototype data for per-event errors etc...?
  - Pre-generate proto data with separate PDF

```
// Generate tagging categories
              RooDataSet proto("pD","pD",tagCat) ;
Add desired
              tagCat = "Lep"; for (i=0;i<400;i++) proto.add(tagCat);</pre>
#entries for
each tagCat
              tagCat = "Kao"; for (i=0;i<1600;i++) proto.add(tagCat);
  by hand
              tagCat = "NT1"; for (i=0;i<250;i++) proto.add(tagCat);
              tagCat = "NT2"; for (i=0;i<1500;i++) proto.add(tagCat);
              // Generate a set of event errors
 Generate
              RooBifurGauss gerr("gerr", "error distribution", dterr,
 per-event
                                  RooRealConstant::value(0.1),
errors from
                                  RooRealConstant::value(0.3),
separate PDF
                                  RooRealConstant::value(0.8));
(same #evts)
              RooDataSet *errdata = gerr.generate(dterr,proto.numEntries());
Add per-event
              // Add per-event error column to protoData
errors column
              proto.merge(errdata) ;
to proto data
```

## Generating ToyMC for sin2beta fit

- Where to get the prototype data for per-event errors etc...?
  - Use actual distribution from data

```
// Select per-event errors and tagCat from actual data
RooDataSet* protoData = cpData->reduce(RooArgSet(dterr,tagCat));
```

- Prototype data can be provided for <u>any</u> observable, <u>overriding the PDF</u>s intrinsic distribution
  - Example: force B<sup>0</sup>/B<sup>0</sup>-bar distribution of actual data

- Generate ToyMC dataset that mimic all properties of data
  - Use prototype data for everything except dt and mB
  - Tagging breakdown, #B0/B0bar, per-event errors, run numbers all taken from data

#### How to *efficiently* generate multiple sets of ToyMC?

- Use ROOMCStudy class to manage generation and fitting
- Generating features
  - Generator overhead only incurred once
     ® Efficient for large number of small samples
  - Optional Poisson distribution for #events of generated experiments
  - Optional automatic creation of ASCII data files

#### Fitting

- Fit with generator PDF or different PDF
- Fit results (floating parameters & NLL)
   automatically collected in summary dataset

#### Plotting

 Automated plotting for distribution of parameters, parameter errors, pulls and NLL

Generating and fitting a simple PDF

```
// Setup PDF
RooRealVar x("x","x",-5,15);
RooRealVar mean("mean", "mean of gaussian", -1);
RooRealVar sigma("sigma", "width of gaussian", 4);
RooGaussian gauss("gauss", "gaussian PDF", x, mean, sigma);
                   Generator PDF
                                     Generator Options
// Create manager
RooMCStudy mgr(gauss,gauss,x,"","mhv");
       Fitting PDF
                                      Fitting Options
                  Observables
// Generate and fit 1000 experiments of 100 events each
mgr.generateAndFit(1000,100);
RooMCStudy::run: Generating and fitting sample 999
RooMCStudy::run: Generating and fitting sample 998
RooMCStudy::run: Generating and fitting sample 997
```

Plot the distribution of the value, error and pull of mean

```
// Plot the distrution of the value
RooPlot* mframe = mean.frame(-2,0);
mgr.plotParamOn(mframe);
mframe->Draw();
// Plot the distrution of the error
RooPlot* meframe = mgr.plotError(mean, 0., 0.1);
meframe->Draw()-----
                                                                              Add Gaussian fit
// Plot the distrution of the pull
RooPlot* mpframe = mgr.plotPull(mean/-3,3,40,kTRUE) ;
mpframe->Draw();
                                                                       A RooPlot of "mean of gaussian Pull"
         A RooPlot of "mean of gaussian"
                                        A RooPlot of "mean of gaussian Error
                                                                                       pullSigma = 0.969 ± 0.02
         Events / (0.5)
                                                                                       pullMean = 0.051 \pm 0.03
                                         40
                                                                        20
          20
                                         20
                                         0.1 0.12 0.14 0.16 0.18 0.2 0.22 0.24 0.26 0.28 0.3 mean of gaussian Error
           1 -0.8 -0.6 -0.4 -0.2 -0 -0.2 -0.4 -0.6 -0.8 -1
                                                                                     mean of gaussian Pull
```

Plot the distribution of –log(L)

```
// Plot the distribution of the NLL

mgr.plotNLL(mframe);

mframe->Draw();

20
40
40
40
20
2450
2550
2550
2600
1og(Likelihood)**
```

- All plots are regular RooPlots and can be further decorated
  - E.g add arrow indicating value of fit to actual data

 For other uses, use summarized fit results in RooDataSet form

Pulls and errors have separate entries for easy access and plotting

#### A Roomcstudy example

 If the "r" fit option is supplied the RooFitResult output of each fit is be saved

```
mgr.fitResult(10)->Print("v");
  RooFitResult: minimized NLL value: 2585.13, estimated distance to minimum: 3.18389e-06
   Floating Parameter InitialValue FinalValue +/- Error
                                                             GblCorr.
                        0.0000e+00 1.4814e-01 +/- 1.91e-01 0.597596
                                                                      <none>
                sigma 4.0000e+00 4.0619e+00 +/- 1.43e-01 0.597596 <none>
mgr.fitResult(10)->correlation("sigma")->Print("v") ;
RooArgList::C[sigma,*]: (Owning contents)
                                                  A RooPlot
  1) RooRealVar::C[sigma, mean] : -0.597596 C
                                                  width of gaussian x
  2) RooRealVar::C[sigma, sigma]: 1.0000 C
RooPlot* frame = new RooPlot(...)
mgr.fitResult(10)->plotOn(frame,meanx,
                         sigmax,"ME12VHB");
                                                  0.95
                                                                     2.1 2.15 2
mean of gaussian x
```

## Using RooMCStudy for sin2b

- sin2b fit is relatively expensive (~30 minutes)
  - Don't want do 1000 consecutive fits on 1 processor...
  - Use only generator part
- Example: generating 1000 toys for goodness-of-fit

```
// Create manager
RooMCStudy mgr(cpmixPdf,cpmixPdf,RooArgSet(dt,mB),
                 "","" ,protoData) ;
                                              Observables to
                                             generate with PDF
                Prototype data
            (with all other observables
// Generate 1000 ToyMC sets in memory
mgr.generate(1000,0,kTRUE);
                                                 Write out each dataset
                                                immediately to ASCII file.
         Keep generated datasets in memory
                                                Do not retain in memory
                Warning: may be large!
// Generate 1000 ToyMC sets in memory
mgr.generate(1000,0,kFALSE,"output/cpfit %04d.dat");
```

## General accept/reject generating caveats

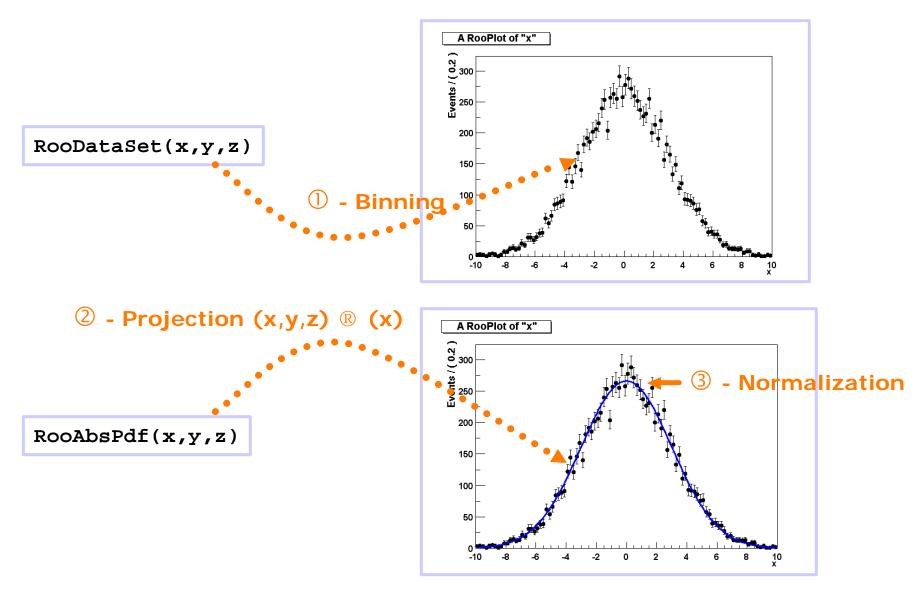
- Monolithic multi-dimensional PDFs have high startup overhead
  - Need to find maximum function value empirically
  - Large number of samples needed
    - 1D 1000, 2-D 100,000, 3-D 10,000,000
    - Initial samples will be reused for generation, but usually requests need < 10M samples</li>
  - Potential solution:
     If more efficient generation technique exists,
     implement that method in PDF and advertise it
     (see 'writing PDF yourself' section')
- Pathological distributions may be sampled incorrectly
  - Narrow peak may be missed by initial sampling (<1% of phase-space)</li>
  - Otherwise generation is very inefficient in nearly all of phase space
  - Solution: generate your dataset in parts
    - Generate peak and non-peak areas separately, append afterwards



#### **RooFit Data Visualization Tutorial**

Wouter Verkerke (UC Santa Barbara) David Kirkby (UC Irvine)

#### Data Visualization in RooFit - Overview



# 1-Dimensional plots



The basics

### 1-Dimensional plots – class Rooplot

1-Dimensional plots are most frequently used and have special support in RooFit via the RooPlot class:

- Derives from TH1 for implementation of graphics, axes etc...
  - Container class for plotable objects: doesn't contain any data itself,
     TH1 member functions operating on data are non-functional
  - Persistable with ROOT I/O (including contents)
- Hold a list of objects to be plotted
  - Datasets (represented as histograms)
  - PDF projections (represented as curves)
  - Any other TObject that can be drawn (e.g. TArrow, TPaveText)
- Takes care of normalization PDF projection curves
  - Unit-normalized curve is automatically multiplied by number of events of last plotted dataset
- Facilitates automatic projection of PDFs onto plotted observable
  - RooPlot knows plotted observable and all observables of last plotted dataset.
  - PDF are automatically
    - Normalized over all known observables
    - Projected over all known observables except the plotted observables

#### Using **RooPlot** – the basics

A RooPlot class is easiest created from a RooRealVar

```
RooRealVar x("x","magic x",-10,10);
       RooPlot* xframe = x.frame() ;
        xframe->Draw();
                                    A RooPlot of "magic x"
 // To change title
 xframe->SetName("blah");
                                     -0.5
                                                       Title of RooRealVar
// Alternate frame() methods
                                       Default plot range = limits of x
// change default range, binning
RooPlot* xframe = x.frame(-5,5);
RooPlot* xframe = x.frame(40);
                                                           Wouter Verkerke, UCSB
```

## Using **RooPlot** – Adding datasets

```
// d contains x,y,z
                                    A RooPlot of "x"
                                                              Poisson
RooDataSet *d;
                                   300
2
300
                                                               errors
d->plotOn(frame);
frame->Draw();
                                    200
                                    150
                                    100
                                    50
// list frame contents
frame->Print("v");
RooPlot::frame(088aa410): "A RooPlot of "magic x""
 Plotting RooRealVar::x: "magic x"
 Plot contains 1 object(s)
   (Options="P") RooHist::gData plot x: "Histogram of gData plot x"
     // Adding a dataset also updates
     // the set of normalization observables
     frame->getNormVars()->Print("1");
     (x,y,z)
```

PDFs added after this dataset that depend on y,z will be normalized & projected over y,z

### Using **RooPlot** – Datasets and binning

#### Default binning

```
RooDataSet *d ;
d->plotOn(frame) ;
frame->Draw() ;
```

#### Custom (non-uniform) binning

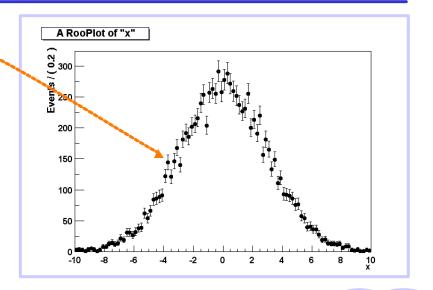
```
// Create binning object
RooBinning b(-10,10);

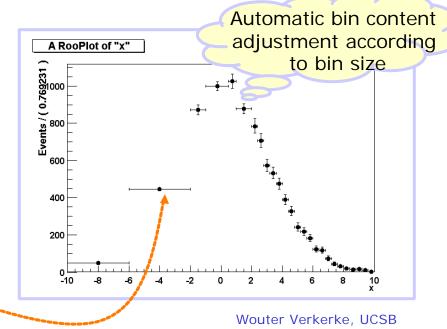
// Add single boundary
b.addBoundary(0.5);

// Add (x,-x) pairs of boundaries
b.addBoundaryPair(1);
b.addBoundaryPair(2);

// Add uniform patterns
b.addUniform(2,-10,-2);
b.addUniform(20,2,10);

RooDataSet *d;
d->plotOn(frame),Binning(b));
frame->Draw();
```





## Using **RooPlot** – Adding PDF projections

```
// pdf depends on x,y,z
RooAbsPdf* p
p->plotOn(frame);---
RooAbsReal::plotOn(f) plot on x integrates over variables (y,z)
frame->Draw();
                                                            Adaptive spacing
                                                             of curve points
                                                             to achieve 1%
                                       A RooPlot of "magic x"
     Automatic because RooPlot
                                                                precision
                                      Events / (0.2)
    remembers dimensions of
     last plotted dataset (x,y,z)
                                       200
                                       150
                                       100
// list frame contents
frame->Print("v");
RooPlot::frame(088aa410): "A RooPlot of "magic x""
 Plotting RooRealVar::x: "magic x"
 Plot contains 2 object(s)
    (Options="P") RooHist::gData_plot__x: "Histogram of gData_plot__x"
    (Options="L") RooCurve::curve gProjected: "Projection of g"
```

### Using **RooPlot** – Adding PDF projections

Change p->plotOn(xframe, DrawOption("F")) draw option (e.g. 'Fill') // Correction w.r.t default normalization p->plotOn(xframe, Norm(0.7)); // Override number of events for PDF normalization p->plotOn(xframe, Norm(RooAbsReal::NumEvent, 10000)); Modify the default // Use expected number of events of extended PDF normalization p->plotOn(xframe, Norm(RooAbsReal::RelativeExpected, 1.0)) in various ways // Raw scale factor (no bin width correction is applied) p->plotOn(xframe, Norm(RooAbsReal::Raw, 5.27)); // No variables are projected by default when PDF (Re)define // is plotted on an empty frame manually which of the // Enter custom definition of observables PDF variables xframe->updateNormVars(RooArgSet(x,y,z)); are observables p->plotOn(xframe);

#### Other **RooPlot** features

Change attributes of last added plot elements

```
frame->getAttLine()->setLineColor(kRed)
frame->getAttMarker()->setMarkerType(22)
```

Change the plotting order of contained objects

```
frame->drawAfter("objectName1","objectName2") ;
```

Add non-RooFit objects

```
TArrow *a = new TArrow(0,0,5,7);
frame->addObject(a);
```

Use
frame->Print("v")
to see list of object
names

Merge contents from another RooPlot

```
frame->merge(frame2) ;
```

Curve/histogram χ<sup>2</sup> calculation

```
frame->chiSquare();
frame->chiSquare("curveName","histName");
```

## Projecting out dimensions



Projection via Integration

Projecting discrete vs real observables

Projection via data averaging

Mixing projection methods

### Projecting out hidden dimensions - Integration

- PDF is always normalized over all observables
  - Normalization set n = variables PDF and dataset have in common
- PDF is projected over all unplotted observables
  - The plot variable set  $\mathbf{x}$  = the plotted dimensions of the PDF (for a 1-D RooPlot this is always 1 variable)
  - The projection set *p* is *n x*
  - The projected PDF function is

$$P_{f}(\vec{x}) = \frac{\int f(\vec{x}, \vec{p}) d\vec{p}}{\int f(\vec{x}, \vec{p}) d\vec{x} d\vec{p}}^{Projected}$$
Plotted observables

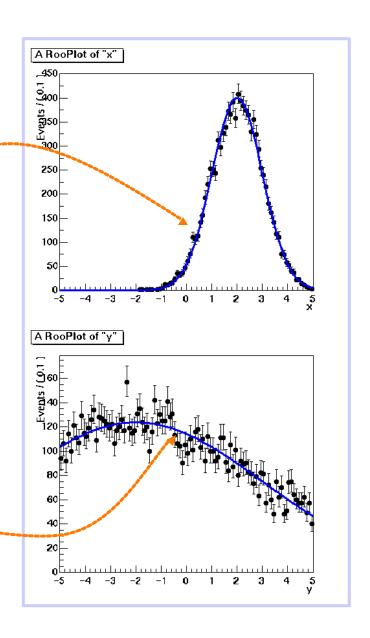
### Projecting out hidden dimensions

- Example in 2 dimensions
  - 2-dim dataset D(x,y)
  - 2-dim PDF P(x,y)=gauss(x)\*gauss(y)
- 1-dim plot versus x

$$P_p(x) = \frac{\int p(x, y)dy}{\int p(x, y)dxdy}$$

1-dim plot versus y

$$P_p(y) = \frac{\int p(x, y) dx}{\int p(x, y) dx dy}$$



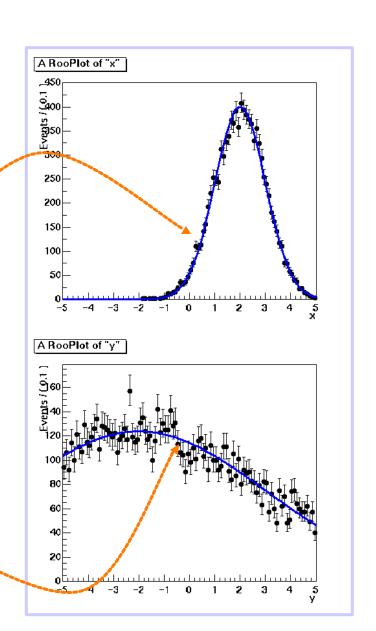
## RooProdPdf automatic optimization

- Example in 2 dimensions
  - 2-dim dataset D(x,y)
  - 2-dim PDF P(x,y) = gaus(x)\*gauss(y)
- 1-dim plot versus x

$$P_{p}(x) = \frac{\int g(x)g(y)dy}{\int g(x)g(y)dxdy} = \frac{g(x)\int g(y)dy}{\int g(x)dx\int g(y)dy} = \frac{g(x)}{\int g(x)dx}$$

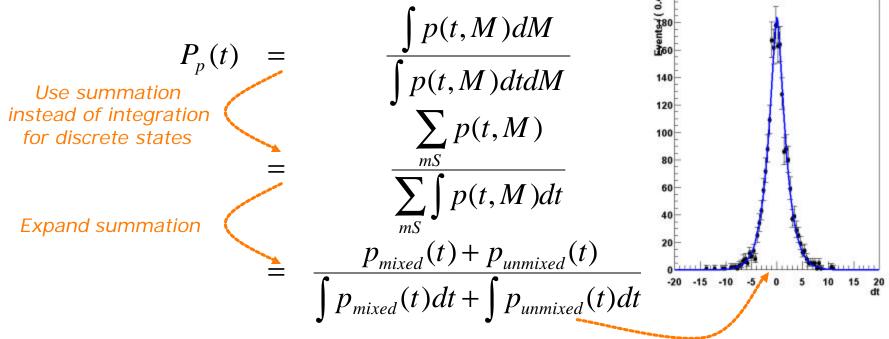
1-dim plot versus y

$$P_{p}(y) = \frac{\int g(x)g(y)dx}{\int g(x)g(y)dxdy} = \frac{\int g(x)dx \cdot g(y)}{\int g(x)dx \int g(y)dy} = \frac{g(y)}{\int g(y)dy}$$



### Projecting out discrete observables

- Works the same way as for real observables
  - Projected discrete dimension is summed over all its states
- Example: B-Decay with mixing
  - dataset(dt,mixState) & PDF(dt,mixState)
  - 1-dim plot versus dt:





Projection works universally for real and discrete observables

## Projecting out observables - Data averaging

 An alternative method to project out observables is to construct a data weighted average function:

Integrate over 
$$y$$

$$P_{p}(x) = \frac{\int p(x, y) dy}{\int p(x, y) dx dy}$$
Sum over all  $y_{i}$  in dataset  $D$ 

$$P_{p}(x) = \frac{1}{N} \sum_{D}^{i=1,N} \frac{p(x, y_{i})}{\int p(x, y_{i}) dx}$$

- The summed variable (y) is treated as a parameter
  - PDF is not normalized over y in above example
- Can be used to cancel the effect of a disagreement between data and PDF in a projected observable
  - Example: per-event errors:
     PDF is usually flat in dtErr, distribution in data is usually peaked.

## Selecting data averaging as the projection method

```
The ProjWData() modifier
// PDF and data defined elsewhere,
                                                    overrides
// observables:dt,dtErr,mixState
                                              the projection method of
RooAbsData* data ;
                                                selected variables:
RooAbsPdf* bmixPdf:
                                              Observable dterr will be
                                              averaged over the values
// Create frame and plot data as usual
                                                in dataset projData
RooPlot* dtframe = dt.frame() :
data->plotOn(dtframe) ;
// Plot bmixPdf, projecting dterr with data
bmixPdf->plotOn(dtframe, ProjWData(dterr, projData));
RooAbsReal::plotOn(bmixPdf) plot on dt integrates
over variables (mixState)
RooAbsReal::plotOn(bmixPdf) plot on dt averages
using data variables (dtErr)
```

**ProjWData** only controls **how** observables are projected. It does **not** override **which** observables are projected

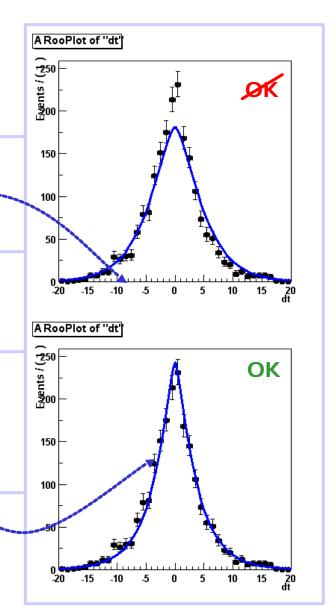
#### Example: integration vs. data averaging on per-event errors

Special property of per-event errors: Distribution in data and PDF do not agree

#### Integrating out per-event errors

```
RooPlot* dtFrame = dt.frame();
data->plotOn(dtFrame);
bmixPdf.plotOn(dtFrame)-;
dtFrame->Draw();
```

#### Projecting per-event errors with data



## Selecting data averaging as the projection method

- Projection via data averaging may be applied to any observable
  - Also discrete valued observables
- Choosing data averaging instead of integration changes the meaning of the projected function
  - The theoretical model / experimental data distinction is blurred:
     the plotted curve takes part of its behavior from the dataset
  - Often applied to non-physics observables (e.g. per-event errors)
    - Shape of per-event error distribution irrelevant to physics and may be hard to model correctly in a PDF
  - Can also to applied to well-modeled physics observable:

Example: plot  $\delta t$  distribution of B-mixing PDF while

- projecting the mix state via integration –
   True model/experimental data comparison
- projecting the mix state with data averaging Compare only dt shape aspect of model with data

Any effects that purely arise from PDF/data discrepancy in B<sup>0</sup>/B<sup>0</sup> counter are taken out Wouter Verkerke, UCSB

#### Data average projection - Performance

- Data-averaged projections can be computationally expensive
  - Effectively the sum of N curves is plotted (N = #evts in projection dataset)
- Projections with large datasets can be accelerated enormously by using binned projection data sets
  - Works the same way, just provide a binned dataset

```
RooPlot* dtFrame = dt.frame();
data->plotOn(dtFrame);
dterr.setFitBins(50);
RooDataHist projData("projData","projData",dtErr,data);
bmixPdf.plotOn(dtFrame,ProjWData(projData));
```

- Minor loss of precision may occur, but with sufficient data and a prudent binning net loss may be less than plotting precision
- Example: unbinned projection with 20K events: 51.2 sec
   binned projection with 100 bins: 0.2 sec
- Also possible when projecting >1 dimensions, and/or discrete dimensions
  - Simply create a multi-dimensional binned dataset

## Integration vs. data averaging - Summary

- Default projection method for all observables is integration
- To override integration method with data averaging method, provide a projection dataset with observables to be averaged
  - Projection dataset only controls method of projection, not which variables are projection
  - Projection dataset may contain both discrete and real observables
  - Projection dataset may be binned (speed vs accuracy tradeoff)
- Any projected PDF observable may be averaged with data instead of integrated
- Final projection may be combination of data-averaging & integration

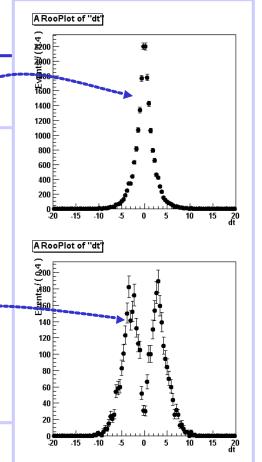
# Slicing & Cutting



Plotting a slice in real & discrete dimensions Understanding normalization in slicing

## Plotting a slice of a dataset

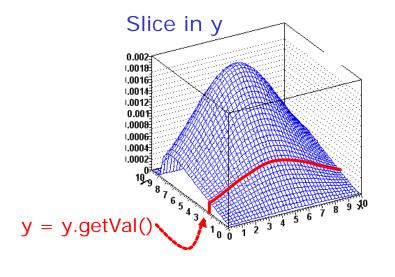
Use the optional cut string expression

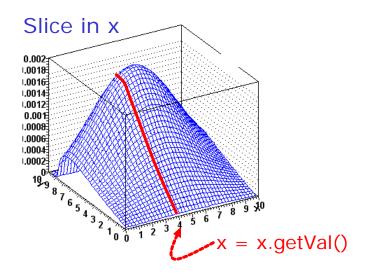


- Works the same for binned data sets
- The target RooPlot will retain the total number of events for future PDF normalizations (not the number of events in the slice)
  - More about this later

## Plotting a slice of a PDF - plotSliceOn()

- To plot a (projection of a) slice of a PDF use slice()
  - RooAbsReal::plotOn(frame,Slice(sliceSet),...)
     overrides default set of observables to project out
  - Argument sliceSet specifies the set of observables that should not be projected out
  - Position of slice is determined by the current value of slice observable





- Slicing can be done in real and discrete dimensions
- Slice set can have an any number of dimensions

## Example: plotting mixed-only slice of data and PDF

```
RooPlot* dtframe = dt.frame();
data->plotOn(dtframe,Cut("mixState==mixState::mixed"));
mixState = "mixed" ;
bmix.plotOn(dtframe,Slice(mixState));
dtframe->Draw();
                          Events / ( 0.4 )
                            120
                            100
                            80
                            60
                            40
                            20
                             -20
                                       -10
                                             -5
                                                                15
                                  -15
```

#### Understanding the normalization for PDF/data slices

f<sub>mixed</sub>

A PDF plotted with plotSliceOn() is normalized to *all* observables, *including the sliced observables*, therefore

$$\int P_{f}(t,M)dt \neq 1 = \int dt \left(\frac{p(t,M)}{\sum_{M} \int p(t,M)dt}\right)$$
The integral of a PDF slice projection is not 1!
$$= \frac{\int p(t,M)dt}{\sum_{M} \int p(t,M)dt} = f_{M}$$

Integral of PDF projection = Fraction of mixed events predicted by PDF

N<sub>total</sub>

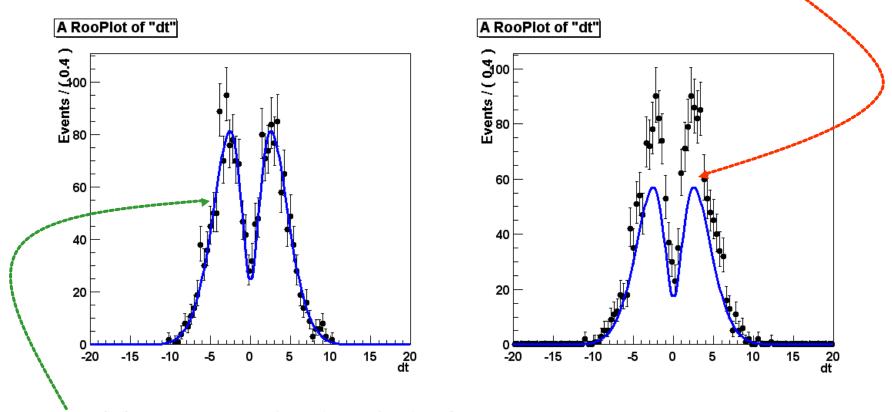
The RooAbsData::plotOn() function with cut gives the *full* (uncut) number of events to the RooPlot so that the final normalization comes out as

$$\begin{split} C_f(\vec{x}) &= P_f(\vec{x}) \cdot N_{\text{data}}^{\text{total}} \cdot f_{mixed}^{PDF} \cdot V_{bin} \\ &= P_f(\vec{x}) \cdot N_{\text{PDF}}^{\text{slice}} \cdot V_{bin} \\ &\neq N_{\text{data}}^{\text{slice}} \end{split}$$

The normalization of the PDF slice curve reflects the PDFs prediction of the slice fraction

#### Understanding the normalization for PDF/data slices

Data has large fraction of mixed events than PDF predicts



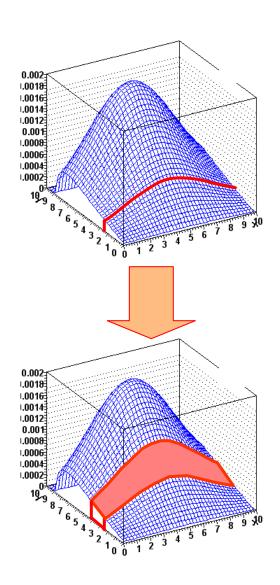
#### Slices in a real-valued observable

- Real-valued slices have no width
  - Usually not that useful (equivalent slices in data are usually empty)
  - Finite width slices can be made with different technique (see later)
- Example plot: effect of per-event error

```
RooPlot* dtframe = dt.frame();
data->plotOn(dtframe); // not a slice
dtErr=0.1; bmix.plotOn(dtframe,Slice(dtErr));
dtErr=0.5; bmix.plotOn(dtframe,Slice(dtErr));
dtErr=1.0; bmix.plotOn(dtframe,Slice(dtErr));

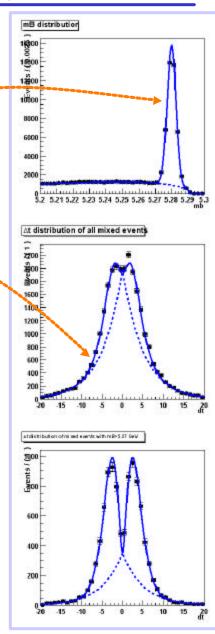
dtframe->Draw();
```

#### Plotting slices with finite width - Introduction



- Problem: analytic calculation of the projection of a 'band' of a PDF often very hard or impossible
- Solution: Numeric solution via ToyMC approach
  - Construct finite width slice as weighted average of no-width slices:
- Generate a sufficiently large ToyMC sample to be plotted
- 2) Reduce the ToyMC data to the band to be plotted
- 3) Plot the PDF the usual way, projecting out all unplotted observables via data averaging. Use the reduce ToyMC set as weighting dataset

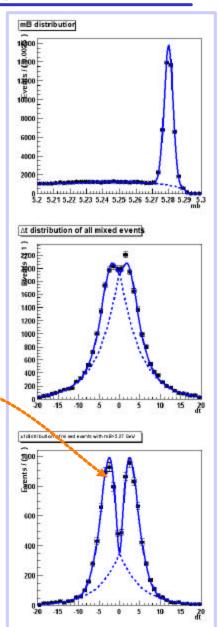
#### Plotting slices with finite width - Example



## Plotting slices with finite width - Example

```
Example setup:
Argus(mB)*Decay(dt) + (background)
Gauss(mB)*BMixDecay(dt) (signal)
```

- Reduce dataset before plotting
- Generate a sufficiently large ToyMC sample to be plotted
- Reduce the toyMC data to the band to be plotted
- Plot the PDF the usual way, projecting out all unplotted observables via data averaging.



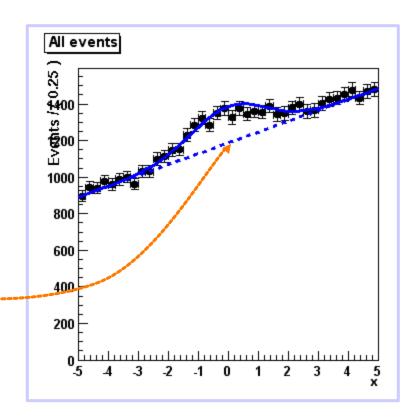
## Plotting non-rectangular PDF regions

- The ToyMC projection technique makes no assumptions on the shape of the selected region
  - Regions of arbitrary size, shape and dimension can be selected
- Example: Likelihood projection plot
  - Common technique in rare decay analyses
  - PDF typically consist of N-dimensional event selection PDF, where N is large (e.g. 6.)
  - Projection of data & PDF in any of the N dimensions doesn't show a significant excess of signal events
  - To demonstrate purity of selected signal, plot data distribution (with overlaid PDF) in one dimension, while selecting events with a cut on the likelihood in the remaining N-1 dimensions

- Simple example
  - 3 observables (x,y,z)
  - Signal shape: gauss(x)·gauss(y)·gauss(z)
  - Background shape:  $(1+a\cdot x)(1+b\cdot y)(1+c\cdot z)$
  - Plot distribution in x with cut on likelihood in (y,z)

```
// Plot x distribution of all events
RooPlot* xframe1 = x.frame(40);
data->plotOn(xframe1);
sum.plotOn(xframe1);
```

Integrated projection of data/PDF on X doesn't reflect signal/background discrimination power of PDF in y,z



Automatic optimization: If f factorizes as g(x)\*h(y,z):

$$P_{f}(y,z,\vec{p}) = \frac{\int g(x,p_{g})h(y,z,\vec{p}_{h})dx}{\int g(x,\vec{p}_{g})h(y,z,\vec{p}_{h})dxdydz}$$
$$= \frac{h(y,z,\vec{p}_{h})}{\int h(y,z,\vec{p}_{h})dydz}$$

Construct per-event likelihood and add as pre-calculated column to the dataset

```
let(x,y,z),50000);
                                         cion(RooArgSet(y,z),x);
RooFormulaVar nllFunc("nll","-log(likelihood)","-log(@0)",*pdfProj);
RooRealVar* nll = data->addColumn(nllFunc) ;
RooPlot* pframe = nll-\frac{4.5}{7.5},100);
data->plotOn(pframe) ;
                                     B00
RooDataSet* sliceData = data->r
                                     Edents) ((
RooPlot* xframe2 = x.frame(40)
sliceData->plotOn(xframe2) ;
sum.plotOn(xframe2,"L",1.0, Roo
                                     1200
                                     1000
                                     800 F
                                     600
  Plot per-event likelihood
                                     400 ⊢
  distribution to tune cut
                                     200
                                                              .5 7 7.
-log(likelihood)
                                                  5.5
```

```
RooDataSet* data = sum.generate(RooArgSet(x,y,z),50000) ;
 RooAbsReal* pdfProj = sum.createProjection(RooArgSet(y,z),x) ;
 RooFormulaVar nllFunc("nll","-log(likelihood)","-log(@0)",*pdfProj);
 RooRealVar* nll = data->addColumn(
                                         Reduce ToyMC projection dataset
                                          with cut on per-event likelihood
 RooPlot* pframe = nll->frame(4.5,7
 data->plotOn(pframe) ;
 RooDataSet* sliceData = data->reduce(RooArgSet(x,y,z),"nll<5.2") ;</pre>
 RooPlot* xframe2 = x.frame(40) ;
                                              S 20
 sliceData->plotOn(xframe2) ;
 sum.plotOn(xframe2,ProjWData(sliceData))
                                              Events / (
Plot PDF with selected ToyMC events
```

#### Summary of slice plotting

To project category slices (or no-width real slices) use

```
RooAbsData::plotOn(frame,Cut("slice_cut_expr"),...)
RooAbsPdf::plotOn(frame,Slice(sliceSet),...);
```

- Normalization of PDF slice projection will reflect the PDF information on  $f_{slice}$ , not the  $f_{slice}$  of the data
- To plot bands, likelihood slices or arbitrarily shaped regions
  - Use ToyMC projection technique
  - If the number of projected observables is low (<=2) binning the ToyMC projection dataset can speed up the plotting process.
  - Can be used in combination with slice()
     to slice in observables no participating in the region cut

# Component plotting



Selecting components to be plotted Slices vs components

## Component plotting - Introduction

 A PDF that is explicitly constructed as a sum of components via RooAddPdf can plot its components separately

Use Method Components()

Example: Argus + Gaussian PDF

```
A RooPlot of "x"
Events / ( 0.15)
  200
  150
  100
```

#### Component plotting – Selecting components

There are various ways to select single or multiple components to plot

```
// Single component selection
pdf->plotOn(frame,Components(argus));
pdf->plotOn(frame,Components("gauss"));

// Multiple component selection
pdf->plotOn(frame,Components(RooArgSet(pdfA,pdfB)));
pdf->plotOn(frame,Components("pdfA,pdfB"));

// Wild card expression allowed
pdf->plotOn(frame,Components("bkgA*,bkgB*"));
```

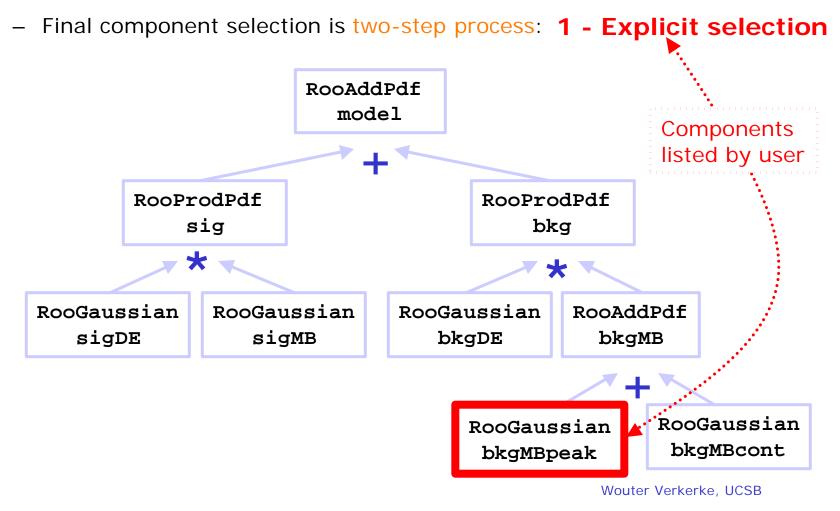
Wildcard option particularly useful for simultaneous PDFs built by RooSimPdfBuilder.

Example: simultaneous Gauss+Argus fit over 4 tagging categories

```
// plot data and full PDF
data->plotOn(frame);
pdf->plotOn(frame, Components("Argus_*"));
Plots sum of all background PDFs
Syntax independent of number and
names of index category states
```

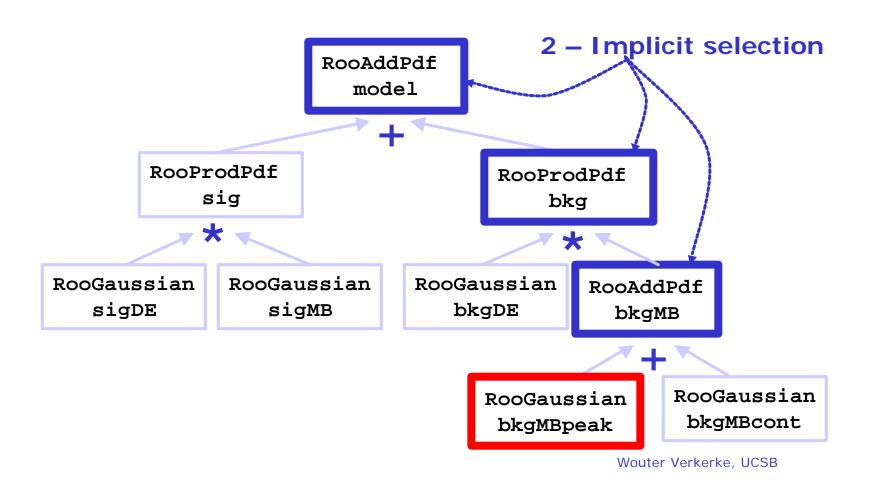
#### Component plotting – Multi layer selection

- Method plotCompOn() can be called on any PDF, and also works for nested RooAddPdf structures
  - Selection mechanism works recursively



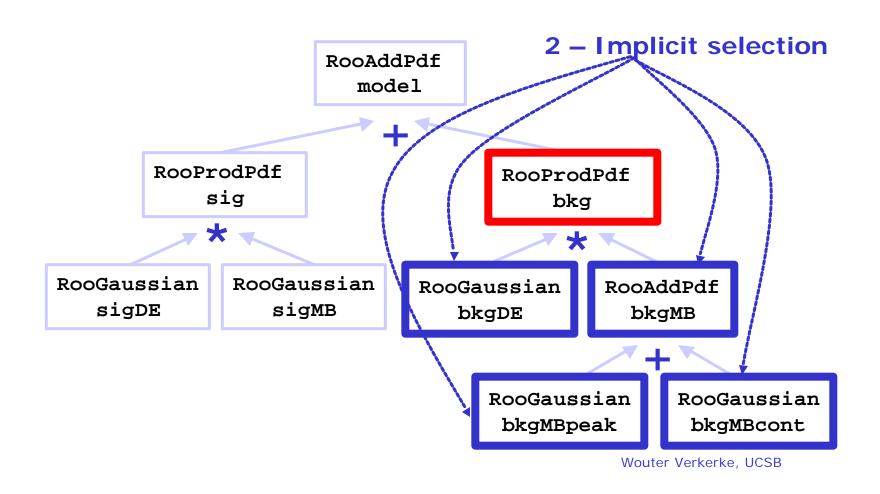
#### Component plotting – Implicit selection

 All nodes in the path between each selected node and the top-level node is implicitly selected



## Component plotting – Implicit selection

All nodes below each selected node is implicitly selected



#### Component plotting – Code example

Component selection gives feedback on explicit/implicit selection

```
RooAddPdf
                                                 model
                                                   +
                                   RooProdPdf
                                                           RooProdPdf
                                       siq
                                                              bka
                             RooGaussian
                                         RooGaussian
                                                      RooGaussian
                                                                  RooAddPdf
                               siqDE
                                           siqMB
                                                        bkqDE
                                                                   bkaMB
                                                           RooGaussian
                                                                       RooGaussian
pdf->plotOn(frame, Components("bkg"));
                                                           bkgMBpeak
                                                                        bkgMBcont
RooAbsPdf::plotCompOn(model)
   directly selected PDF components: (bkg)
RooAbsPdf::plotCompOn(model) indirectly selected
   PDF components: (bkgMBPeak,bkgMBCont,bkg,model)
```

- Component selection in a PDF slice projection
  - Use plotOn(frame, Components("compList"),Slice(sliceSet),...)
  - No special issues, just combine features of Slice() and Components()

## RooSimultaneous



Projecting and slicing RooSimultaneous PDFs

#### Plotting RooSimultaneous PDFs

 Plotting of RooSimultaneous PDFs is not different from any other PDF



- Everything works the same as for regular PDFs, except that the index category cannot be projected out via integration
- Always provide a projection dataset for the index category (or its components if the index category is composite)
- Otherwise,
   treat the RooSimultaneous index category as a regular observable

#### Simultaneous PDF for (A,B) – plot sum of A,B

Needed to project out cat

## Plotting Roosimultaneous PDFs

Projection (=summation) View of RooSimultaneous in 2D over index category Roosim index category Component PDF observable(s)

#### Plotting a component PDF of a RooSimultaneous

- A component PDF of a RooSimultaneous
  is a slice of the RooSimultaneous in the index category.
  - Use slice() not Components()!

Simultaneous PDF for (A,B) – plot A only

```
// Plot data/PDF for A only
RooPlot *frame = x.frame();
data->plotOn(frame,Cut("cat==cat::A"));
cat="A";
sim->plotOn(frame,Slice(cat),ProjWData(cat,data));
```

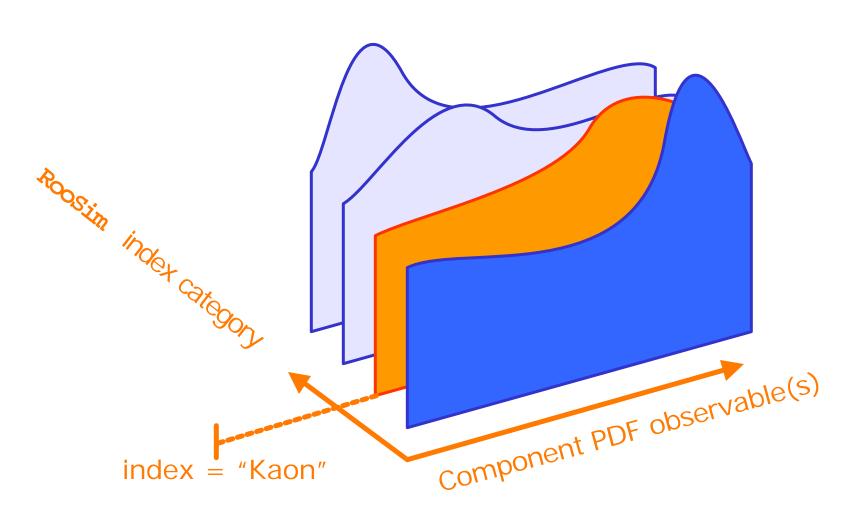
– Why does plotsliceOn() need data?

Normalization works like in regular plotSliceOn()

- RooAbsData::plotOn(frame,Cut("cutExpr"))
   stores total number of events without cut
- RooAbsPdf::plotOn(frame,Slice()) normalizes projection to 1  $^{*}$  f<sub>slice</sub>
- RooSimultaneous needs projection dataset to calculate f<sub>slice</sub>

#### Plotting Roosimultaneous PDFs

A slice in the RooSimultaneous index category selects a component PDF



#### RooSimultaneous - Projection a slice with data averaging

- RooSimultaneous, Slice() and component data averaging
  - RooSimultaneous needs projection dataset for entire dataset
  - Component PDF needs projection dataset for events in slice only

A	0.12	
A	0.23	
A	0.17	
A	0.43	
В	0.34	
В	0.07	
В	0.19	
В	0.13	
В	0.22	
В	1.05	,

- Apparent problem: need 2 projection dataset with different sizes
- Solution: RooSimultaneous::plotOn automatically trims the dataset when passing it on to the components plotOn()

#### RooSimultaneous - Projection a slice with data averaging

RooSimultaneous index projection uses entire dataset

Component dterr projection uses subset of dataset with cat==A ◆

## Miscellaneous



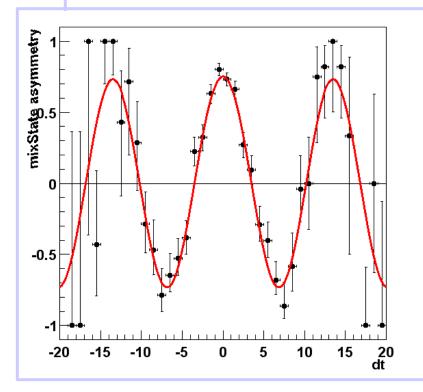
Asymmetry plots

Likelihood plots

Plots in more 1 dimension

#### Asymmetry plots

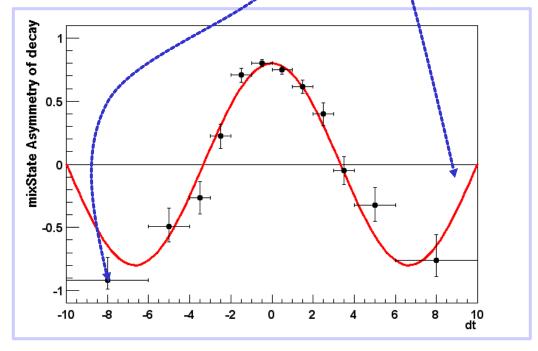
- RooFit supports generic asymmetry plotting
  in any RooCategory with (+1,-1) or (+1,0,-1) states
  - Example: mixState asymmetry of BMixing PDF & data



Can be combined with other plot arguments

# Asymmetry plots - Features

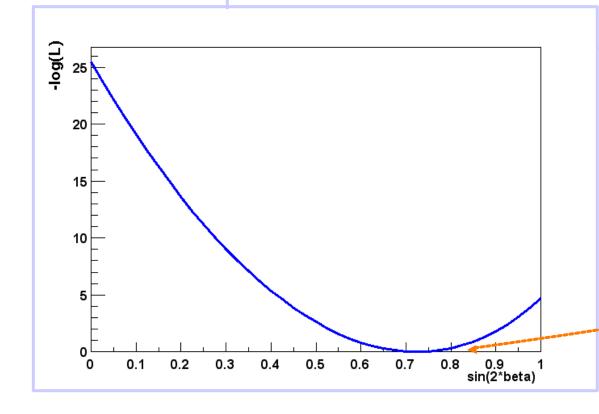
- RooAbsData::plotOn(Asymmetry())
  - Non-uniform bin sizes OK
  - Points have binomial errors instead of Poisson errors
- RooAbsReal::plotOn(Asymmetry())
  - All regular PDF projection techniques work:
    - Projection via integration
    - · Projection with data averaging
    - Slice plotting
    - ToyMC region plotting
    - ...



### Likelihood scans in 1 dimension

Plot — log(L) for a PDF/dataset on a frame

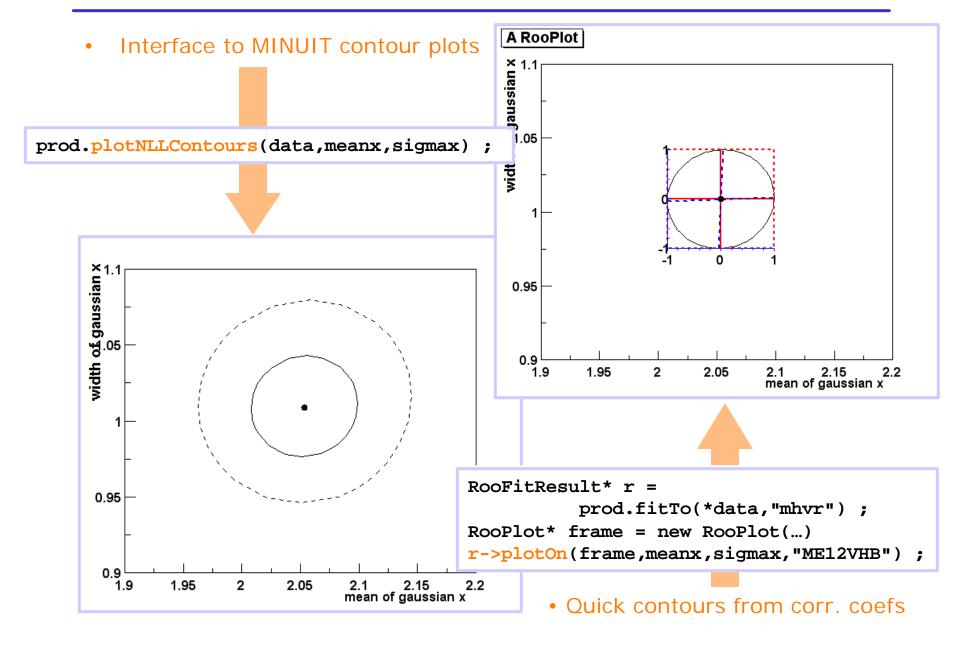
```
// cpmixPdf and cpmixData previously defined
RooPlot* frame = sin2b.frame(0,1,20);
cpmixPdf->plotNLLOn(frame,cpmixData,1.0,kTRUE);
```



Adaptive NLL sampling used (standard for all RooPlot curves) Explicit control over resolution tunes CPU/precision tradeoff

Optional automatic baseline shift to zero

### Likelihood contours in 2 dimensions



### Plotting in more than 2,3 dimensions

- No equivalent of RooPlot for >1 dimensions
  - Usually >1D plots are not overlaid anyway
  - Methods provided to produce 2/3D ROOT histograms from datasets and PDFs/functions

```
TH2* ph2 = x.createHistogram("x vs y pdf",y,0,0,0,bins);
prod.fillHistogram(ph2,RooArgList(x,y));
ph2->Draw("SURF") ;
TH2* dh2 = x.createHistogram("x vs y data",y,0,0,0,bins);
data->fillHistogram(dh2,RooArgList(x,y));
dh2->Draw("LEGO") ;
                 Histogram of x vs y data | x y
                                                          Histogram of x vs y pdf x y
                                                x va y dete x y
                                                                                        x va y pdf x
                                                RMS y - 2.632
                                                                                        RMS y - 2.664
                                                          0.008
                                                          0.007
                   70
                                                          0.006
                                                          0.005
                                                          0.004
                                                          0.003
                   30
                                                          0.002
                                                          0.001
```



# **RooFit Tutorial – Managing complex fits**

Wouter Verkerke (UC Santa Barbara) David Kirkby (UC Irvine)

### Overview

- WARNING This tutorial is incomplete
  - Relevant parts of the retired 'advanced' tutorial have been moved here.
- Scope of this tutorial
  - Issues arising in building and managing a large scale fit project in RooFit
  - Use  $sin2\beta$  fit frame work as illustration: Focus on topics such as
    - Blinding
    - Managing and building a large number of similar PDFs
    - How to deal with per-event errors
    - Non-trivial plots (e.g. projecting out per-event errors)
    - Setting up ToyMC studies for e.g. goodness-of-fit determination
    - · Writing new PDF classes

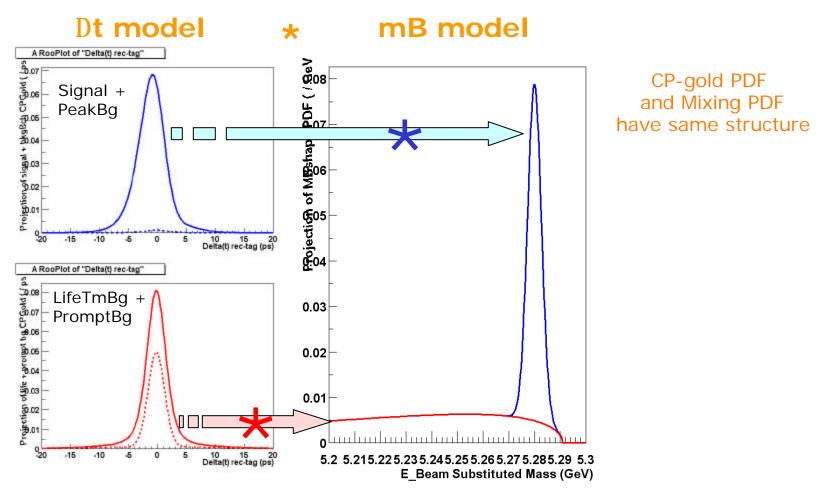
# Managing many PDFs



The sin2β fit as example
Automating PDF replication
RooSimPdfBuilder

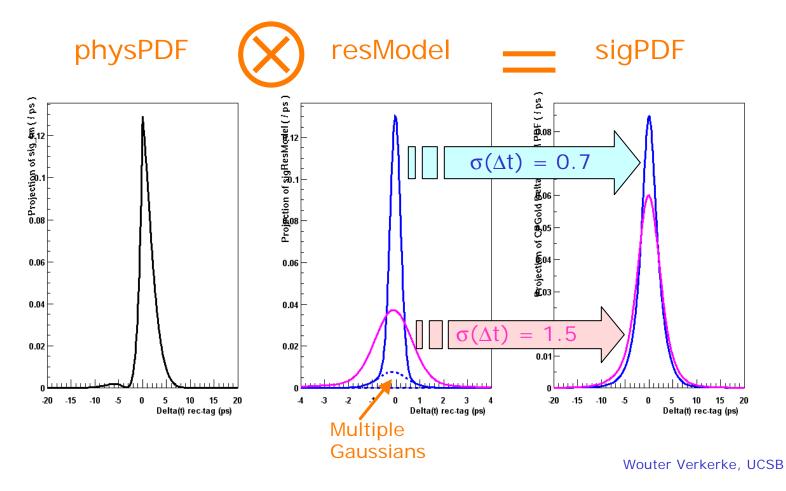
### The sin2beta fit model in two slides

- Simultaneous fit of
  - CP data to CP model and BReco data to mixing model
  - Split data by tagging category and CP event type (Gold, Klong[IFR/EMC][ee/μμ])



### The sin2beta fit model in two slides

- Each Δt PDF is a convolution of a physics PDF with a 2 or 3 gauss resolution model.
  - The bias and width of the resolution model are scaled with the per-event error



### Building complex simultaneous fits

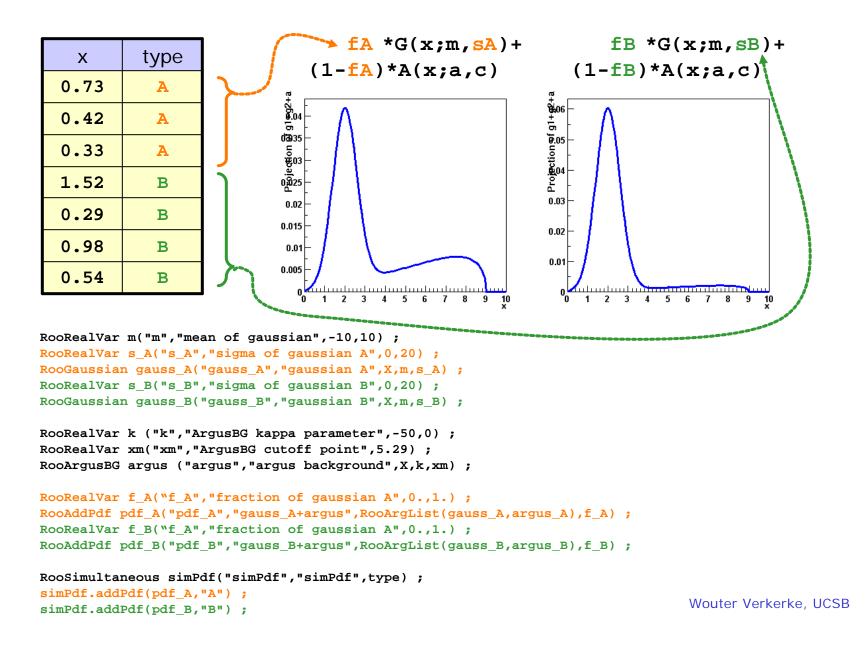
- The essence of the sin2β fit is simple
  - For each physics event type (CP-gold/CP-klong/Breco-mix) there is one 'prototype' PDF
  - Simultaneous fit would look like this

```
// Build Simultaneous CP/Mixing PDF
RooSimultaneous simPdf("simPdf","CP/mixing PDF",physCat);
simPdf.addPdf(cpgoldPDF,"CP-gold");
simPdf.addPdf(klongPDF,"CP-klong");
RooCategory in dataset
simPdf.addPdf(mixingPDF,"BR-mix");

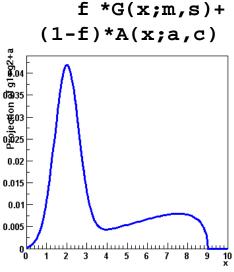
// Perform the fit
simPDF.fitTo(data,"fit options")
```

- Complications arise from further subdivision of data
  - Data is subdivided by tagCat, KL-reco type etc
  - Need to create many PDFs that trivially differ from each other:
    - CP-goldPdf\_Kao, CP-goldPdf\_Lep, CP-goldPdf\_NT1, CP-goldPdf\_NT2 CP-klongPdf\_Kao\_EMC, CP-klongPdf\_Kao\_IFR, CP-klongPdf\_Lep\_EMC,...
  - Lots of bookkeeping involved

### PDF replication: manual approach



## PDF replication: automated approach



```
RooSimPdfBuilder builder(pdf);
RooArgSet* config = builder.createProtoBuildConfig();
  (*config)["physModels"] = "pdf";
  (*config)["splitCats"] = "type";

  (*config)["pdf"] = "type : f,s";
  RooSimultaneous* simPdf = builder.buildPdf(*config,&D);
```

#### **Customization prescription**

```
f ® f[type]
s ® s[type]
```

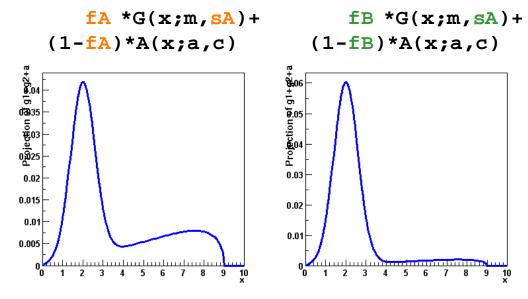


### Prototype PDF

```
RooRealVar m("m","mean",-10,10);
RooRealVar s("s","sigma",0,20);
RooGaussian gauss("gauss","g",X,m,s);

RooRealVar k("k","kappa",-50,0);
RooRealVar xm("xm","ebeam",5.29);
RooArgusBG argus("argus","a",X,k,xm);

RooRealVar f("f","f(gauss)",0.,1.);
RooAddPdf pdf("pdf","gauss+argus",
RooArgList(gauss,argus),gfrac);
```



Customized PDF copies Wouter Verkerke, UCSB

- RooSimPdfBuilder is a factory class
  - takes a collection of prototype PDFs
  - builds a Roosimultaneous given a set of parameter splitting prescriptions
  - Configuration supplied by a RooArgSet of RooStringVars
    - Can be filled in a macro, or read from a file
- Simple build: One prototype PDF

```
Specifies which prototype
will be used in this build
                                      prototype is gauss+argus of previous example
                                  physModel = pdf
                                   splitCat
                                               = type
                                  pdf
                                               = type : f,s
Specifies which categories
of the dataset will be used
to define data subsets
                                      Specifies how prototype 'pdf' should
                                      be tailored for each data subset:
                                      Parameters f and s should individual
                                      per state of type
                                                                Wouter Verkerke, UCSB
```

```
// Build the simultanous PDF
RooSimultaneous* simPdf = builder.buildPdf(config,data) ;
RooSimPdfBuilder::buildPdf: list of physics models (pdf)
RooSimPdfBuilder::buildPdf: list of splitting categories (type)
RooSimPdfBuilder::buildPdf: processing physics model pdf
RooSimPdfBuilder::buildPdf: configured customizers for all physics models
RooCustomizer for sum
  Splitting rules:
  f is split by type
   s is split by type
RooSimPdfBuilder::buildPdf: Customizing physics model sum for mode {A}
RooSimPdfBuilder::buildPdf: Customizing physics model sum for mode {B}
// Print the parameters of the built PDF
simPdf->getParameters(data)->Print("v");
RooArgSet::parameters:
  1) RooRealVar::k : -1.00000 C
  2) RooRealVar::xm : 9.0000 C
                                     Individual by type, as specified
  3) RooRealVar::f_A : 0.50000 C
 4) RooRealVar::f_B : 0.50000 C
  5) RooRealVar::m : 2.0000 C
                                      Individual by type, as specified
 6) RooRealVar::s A : 0.60000 C
  7) RooRealVar::s B : 0.60000 C
```

- Build with >1 prototype
  - To be used when prototype itself RooSimultaneous (e.g.  $sin2\beta$ )

Split data in subsets of tagCat runBlock (physCat)

Proto-Roosimultaneous prescription:

- Index category is physCat
- Associate physCat state CPGold with PDF GoldPDF
- Associate physCat state BMix with PDF BRMixPDF

Specifies how prototype 'CPGoldPdf' should be tailored:

- Parameters x,y,z should individual per state of tagCat
- Parameter foo should be individual per state of runBlock
- Parameter zaza should be individual per state of tagCat and runBlock

Specifies how prototype 'BRMixPDF' should be tailored

- Using the same prototype for multiple physCat states
  - Split parameters by **physCat** to distinguish PDFs

The same prototype is assigned to 2 physCat states

The parameter **foo** will be individual for **physCat=KlEmc** and **physCat=KlIfr**, distinguishing the PDF for the two **physCat** states

```
foo_KlEmc
foo_KlIfr
bar_{KlEmc,Kao}
bar_{KlIFR,Kao}
```

•••

- Non-trivial parameters splits
  - Sofar only 'simple' splits were used (grid structure)

tagCat

Lep

Kao

NT1

NT2

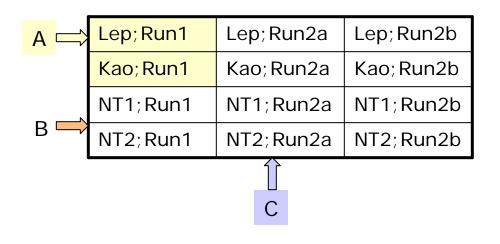
runBlock

	Run1	Run2a	Run2b
•			

tagCat ´ runBlock

Lep;Run1	Lep;Run2a	Lep;Run2b
Kao; Run1	Kao; Run2a	Kao; Run2b
NT1;Run1	NT1;Run2a	NT1;Run2b
NT2;Run1	NT2;Run2a	NT2;Run2b

Q: How do split a parameter in regions {A,B,C}?



A: Define a category function that implements the transformation (tagCat,runBlock) → {A,B,C}

#### Options:

- RooGenericCategory
- RooMappedCategory

- Given a category function superCat as function of tagCat, runBlock
- If splitting only in superCat and not in tagCat or runBlock
  - Add superCat as precalculated column to the dataset

```
data->addColumn(superCat) ;
```

Proceed as usual, specifying superCat as splitting category

```
physModel = pdf
splitCat = superCat
CPGoldPdf = superCat : zaza
```

- If splitting in both superCat and it's ingredients (tagCat etc)
  - Must take correlations into account, specify superCat as auxiliary splitting category

```
RooSimultaneous* simPdf =
   builder.buildPdf(config,data,superCat);
```

### PDF replication: putting it together for sin2β

- Using RoosimPdfBuilder the implementation of the sin2β fit is easy.
  - Given the prototype PDFs (CPGold, CPKlong, Bmixing), the following builder configuration will build the full PDF!

```
physModels
             = physCat
                                  : Gold=CPGold BMix=BMixing
                                   KlEmcE=KLong KlEmcM=KLong KlIfrE=KLong KlIfrM=KLong
splitCats
             = tagCat
BMixing
                                  : sig eta, sig deta, bgC eta, bgL eta,
             = tagCat
                                   bgP eta, sigC bias, outl bias
               physCat, tagCat
                                 : bgP f,mbMean,mbWidth,argPar,sigfrac
               physCat
                                 : bgC f
CPGold
             = tagCat
                                 : sig eta, sig deta, sigC bias, outl bias
                                 : mbMean, mbWidth, argPar, bgP f, bgC f
               physCat
                                 : sigfrac
               physCat, tagCat
             = tagCat
                                 : sig eta, sig deta, sigC bias, outl bias
KLong
               physCat,tagCatL : klSig_frac,klBgCcK_frac,klKstBg_frac,klKshBg_frac,
                                   klNPLBg frac,klNPPBg frac,klPsxBg frac
               physCat
                                  : deGMeanSig, deGWidthSig, deACutoffSig, deAKappaSig,
                                   deGFracSig, dePol1IPbg, dePol2IPbg, dePol3IPbg, dePol4IPbg,
                                   deACutoffSBbg, deAKappaSBbg, deGMeanKSBg, deGWidthKSBg,
                                   deACutoffKSBg, deAKappaKSBg, deGFracKSBg, deG2MeanSig,
                                   deG2WidthSig,deG2FracSig,psix cpev
```

# Configuring your fit



How to manage your configuration data

- Complex fits such as sin2β need lots of configuration data
  - Collection of input datasets
  - Structural definition of PDF
  - Blinding string / configuration
  - Initial parameters
- Use standard RooFit classes to store your configuration data
  - Each configuration item can be represented by a
    - RooRealVar Fit parameters etc
    - RooCategory Switches, options etc
    - RooStringVar File names, blinding strings etc
  - A set of configuration values is represented by a RooArgSet
  - RooArgSet provides methods to write its contents to a humanreadable ASCII file and to read it back

```
// Read input data section
                                                       Read file one section at a time
RooStringVar charmDir("charmDir","charmDir","");
RooStringVar charmFiles("charmFiles","charmFiles","");
RooArgSet(charmDir,charmFile).readFromFile("config.txt",0,"Input Files");
                                                                            config.txt
                       [Input Data]
All configuration
                                    = /nfs/farm/babar/AWG/sin2b/data run2/
                         charmDir
  data labeled
                         charmFiles = dt_JpsiKs_run1.fit,dt_JpsiKs_run2.fit
    by name
                       [Blinding]
                         blindingStatus = Blind
                         blindingString = No kstar in this fit
                         blindingSigma = 0.1
                         blindingMPoint = 0.6
                       [CP/mixing-fit structure]
                         physModels
                                      = physCat
                                                        : Gold=CPGold BMix=BMixing
                         splitCats
                                      = tagCat
                                                        : sig eta, sig deta, sigC bias
                         CPGold
                                      = tagCat
                                                        : mbMean, mbWidth, argPar, bgP f
                                        physCat
                                        physCat, tagCat
                                                        : sigfrac
                       [Initial parameter values]
                                            = -24.42 L(-100 - 0)
                         argPar Gold
                         argPar {BMix;Kao}
                                            = -27.88 L(-100 - 0)
                         argPar \{BMix; Lep\} = -41.53 L(-100 - 0)
                         argPar {BMix;NT1}
                                            = -33.86 L(-100 - 0)
                         argPar_{BMix;NT2}
                                            = -36.64 L(-100 - 0)
                                            = 5.291 C
                         mbMax
```

config.txt

RoosimPdfBuilder configuration is already a RooArgSet Of RoostringVars

```
[Input Data]
  charmDir
             = /nfs/farm/babar/AWG/sin2b/data run2/
  charmFiles = dt JpsiKs run1.fit,dt JpsiKs run2.fit
[Blinding]
 blindingStatus = Blind
 blindingString = No kstar in this fit
 blindingSigma = 0.1
 blindingMPoint = 0.6
[CP/mixing-fit structure]
                                 : Gold=CPGold BMix=BMixing
  physModels
              = physCat
  splitCats = tagCat
                                 : sig eta, sig deta, sigC bias
  CPGold
              = tagCat
                                 : mbMean,mbWidth,argPar,bgP f
                physCat
                                 : sigfrac
                physCat, tagCat
[Initial parameter values]
 argPar Gold
                    = -24.42 L(-100 - 0)
 argPar {BMix;Kao}
                    = -27.88 L(-100 - 0)
 argPar {BMix;Lep}
                    = -41.53 L(-100 - 0)
 argPar {BMix;NT1}
                    = -33.86 L(-100 - 0)
 argPar_{BMix;NT2}
                    = -36.64 L(-100 - 0)
 mbMax
                     = 5.291 C
```

A RooArgSet with all the parameters of a PDF can trivially be obtained

```
RooArgSet* allParams = pdf->getParameters(data) ;
      allParams->readFromFile("config.txt","READ","Initial parameter values");
                                                                                    config.txt
                             [Input Data]
            Flag all
                               charmDir
                                          = /nfs/farm/babar/AWG/sin2b/data run2/
            initialized
                               charmFiles = dt JpsiKs run1.fit,dt JpsiKs run2.fit
            parameters
                             [Blinding]
                               blindingStatus = Blind
                               blindingString = No kstar in this fit
                               blindingSigma = 0.1
        Automatically
                               blindingMPoint = 0.6
        list parameters not
        initialized from file
                             [CP/mixing-fit structure]
                                                                 Gold=CPGold BMix=BMixing
         allParams->selectByAttrib("READ", kFALSE)->Print("v");
                                                                  sig eta, sig deta, sigC bias
                                                                : mbMean,mbWidth,argPar,bgP f
                                              physCat
                                                                : sigfrac
                                              physCat, tagCat
                             [Initial parameter values]
                                                  = -24.42 L(-100 - 0)
                               argPar Gold
Contents of list
                               argPar \{BMix; Kao\} = -27.88 L(-100 - 0)
automatically includes
                               argPar \{BMix; Lep\} = -41.53 L(-100 - 0)
all split parameters
                               argPar \{BMix;NT1\} = -33.86 L(-100 - 0)
introduced by
                               argPar_{BMix;NT2} = -36.64 L(-100 - 0)
RooSimPdfBuilder
                               mbMax
                                                  = 5.291 C
```

## Configuration data, advanced options

Limited <u>pre-processing</u> available

```
# coments BaBar style
            BaBar-style and C++ style
                                         // comments C++ style
            inline comments allowed
                                         file = foo.txt // comments C++ style
                                         [Initial parameter values]
  Echo command prints user messsages
                                           echo Now processing parameter section
        while reading the configuration
                                           argPar Gold
                                                                = -24.42 L(-100 - 0)
                                           argPar_{BMix;Kao} = -27.88 L(-100 - 0)
                                           argPar_{BMix;Lep}
                                                                = -41.53 L(-100 - 0)
                                           argPar {BMix;NT1}
                                                                = -33.86 L(-100 - 0)
                                           argPar {BMix;NT2}
                                                                = -36.64 L(-100 - 0)
       Include other configuration files
                                           include common parameters.txt
                    Recursion allowed
                                           if (runMode==runMode::ExtraFancy)
     Conditionals with full C++ syntax.
                                              fancyPar = 5.0 + / - 0.3 L(0-10)
    All variables in the read RooArgSet
                                           else if (runMode==runMode::Normal)
   can be referenced in the expression.
                                              normalPar = 17.0 L(10-20)
               Nested conditionals OK
                                           else
                                              echo You can do this
Abort statement forces RooArgSet::read
                                              abort
                to fail with error states
                                           endif
                                                                   Wouler Verkerke, UCSB
```

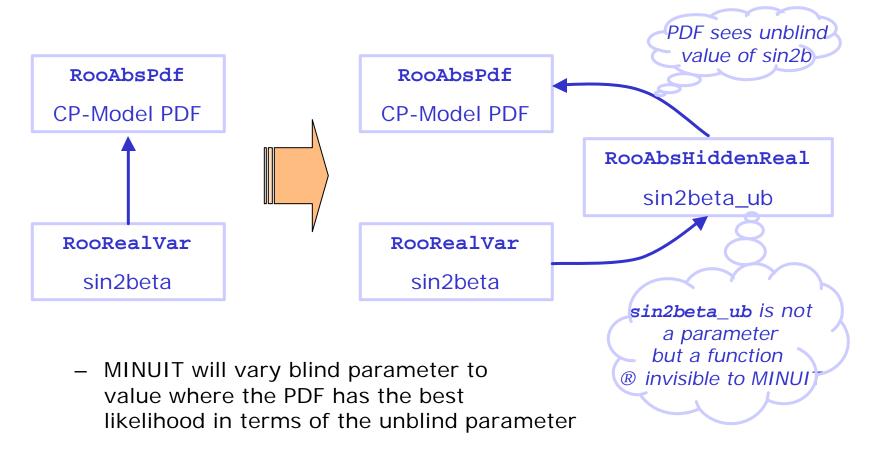
### RooFit users tutorial



Blinding fit parameters

### Parameter blinding

- What does blinding a parameter involve?
  - Insert unblinding transformation between parameter value holder and PDF object using that parameter value



## Parameter blinding

- Blinding-related classes in RooFitModels
  - RooBlindTools
    - replica of BaBar BlindTools class
  - RooUnblindPrecision,
     RooUnblindOffset
     RooUnblindUniform
     RooUnblindCPDeltaT
     RooUnblindCPAsym

Each method can take an optional RooCategory parameter that serves as 'blinding switch' → Disable blinding without structural change

Implementations of RooAbsHiddenReal wrapping 5 different blinding methods from RooBlindTools

- How to keep secrets
  - The purpose of blinding is to avoid accidental exposure of the blinded parameter
    - RooFit is a toolkit, not a monolithic black box
  - Classes derived from RooAbsHiddenReal will never show their value in Print() and other display routines
  - Unblinder functions never show up in any parameter list
  - The RooAbsHiddenReal::getVal() method is protect-ed.
     Cannot be called from the ROOT command line without explicit cast
  - To intentionally unblind call RooAbsHiddenReal::getHiddenVal()

### Parameter blinding

Example implementation

```
// Sin2beta variable
sin2b = new RooRealVar("sin2b", "sin(2*beta)", -1.0, 4.0);
// Blinding switch [ category parameter ]
s2b bs = new RooCategory("s2b bs", "sin2beta blinding state");
s2b bs->defineType("Unblind",0);
s2b bs->defineType("Blind",1);
// Unblinding transformation
sin2b ub = new RooUnblindPrecision("sin2b ub", "Unblinded Sin2beta",
                                   blindString,_cval,_sigma,
                                   *sin2b,*s2b bs,kTRUE);
// Use sin2b ub in fit where ever sin2beta input is needed
// Activate/Deactivate blinding at any moment
s2b bs->setLabel("Blind"/"Unblind") ;
```



# **RooFit Programmers Tutorial**

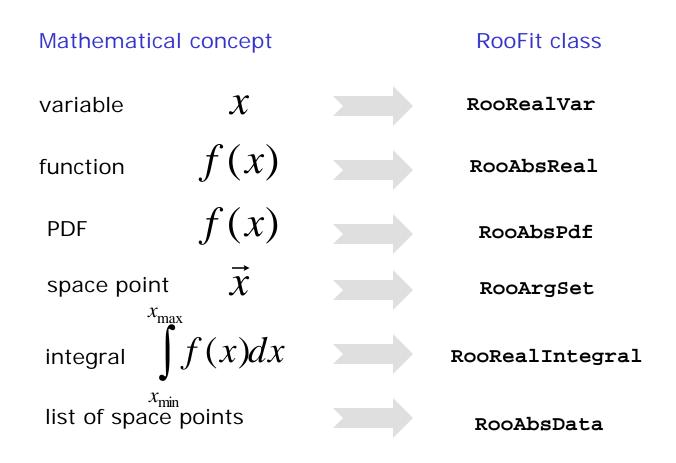
Wouter Verkerke (UC Santa Barbara) David Kirkby (UC Irvine)



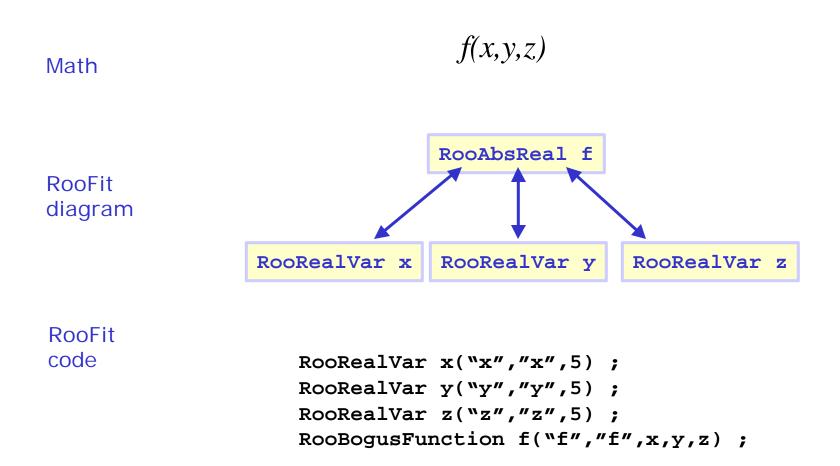
Mathematical concepts as C++ objects

General rules for RooFit classes

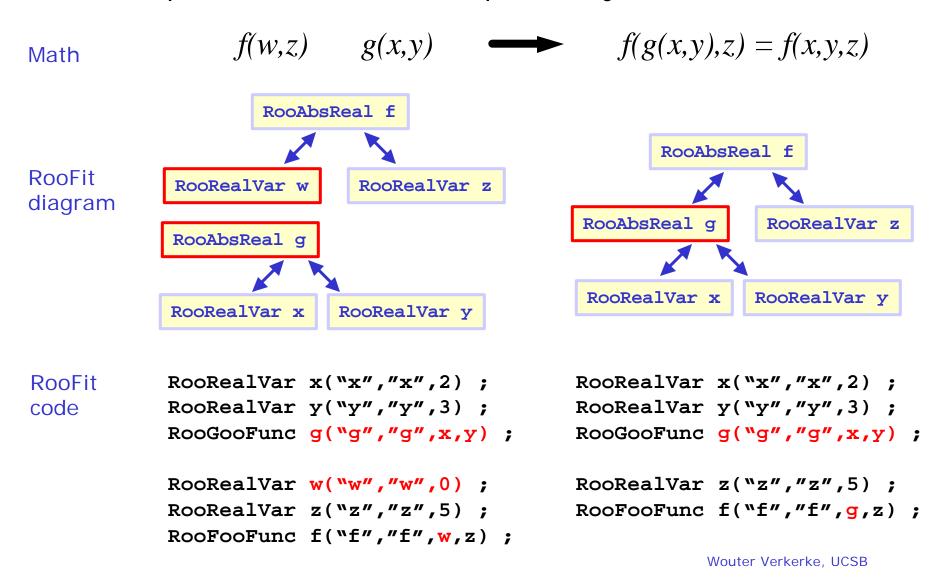
Mathematical objects are represented as C++ objects



 Represent relations between variables and functions as client/server links between objects

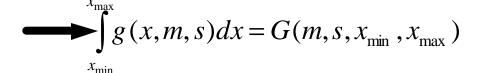


Composite functions → Composite objects

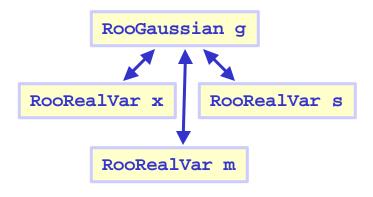


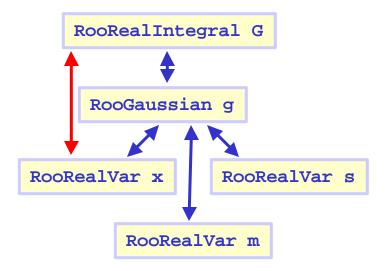
 Represent integral as an object, instead of representing integration as an action

Math



#### RooFit diagram





# RooFit code

```
RooRealVar x("x","x",2,-10,10)
RooRealVar s("s","s",3);
RooRealVar m("m","m",0);
RooGaussian g("g","g",x,m,s)
```

### RooFit designed goals for easy-of-use in macros

- Mathematical concepts mimicked as much as possible in class design
  - Intuitive to use
- Every object that can be constructed through composition should be fully functional
  - No implementation level restrictions
  - No zombie objects

Only current exception: convolutions

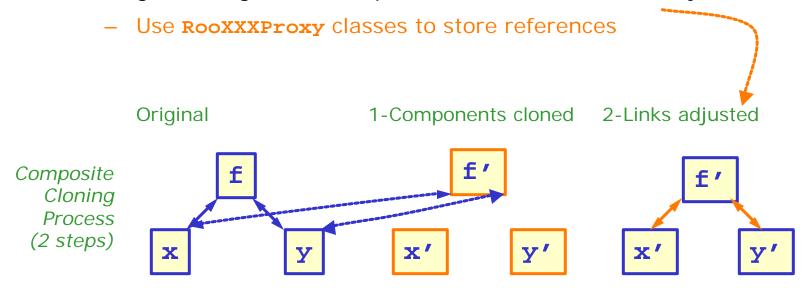
- All methods must work on all objects
  - Integration, toyMC generation, etc
  - No half-working classes

# RooFit designed for easy-of-use in macros

- At the same time, RooFit class structure designed to facilitate lightweight implementation-level classes
  - All value representing classes
     inherit from a common base class: RooAbsArg
- RooAbsArg and other intermediate abstract base classes handle bulk of the logistics
  - In most cases only one method is required: evaluate()
  - Implementation of common techniques such as integral calculation or ToyMC generator not mandatory
  - Base classes provide default numerical/generic methods
- RooAbsArg implementation must follow a minimal set of coding rules

# Coding rules for RooAbsArg derived classes

- Write well-behaved classes.
  - RooAbsArg objects classes are not glorified structs, well-defined copy semantics are essential: write a functional copy constructor
- 2. Every concrete class must have a clone() method
- 3. Do not store pointers to other RooAbsArg objects
  - Many high-level RooFit operations, such as plotting, fitting and generating, clone composite PDFs and need to readjust links



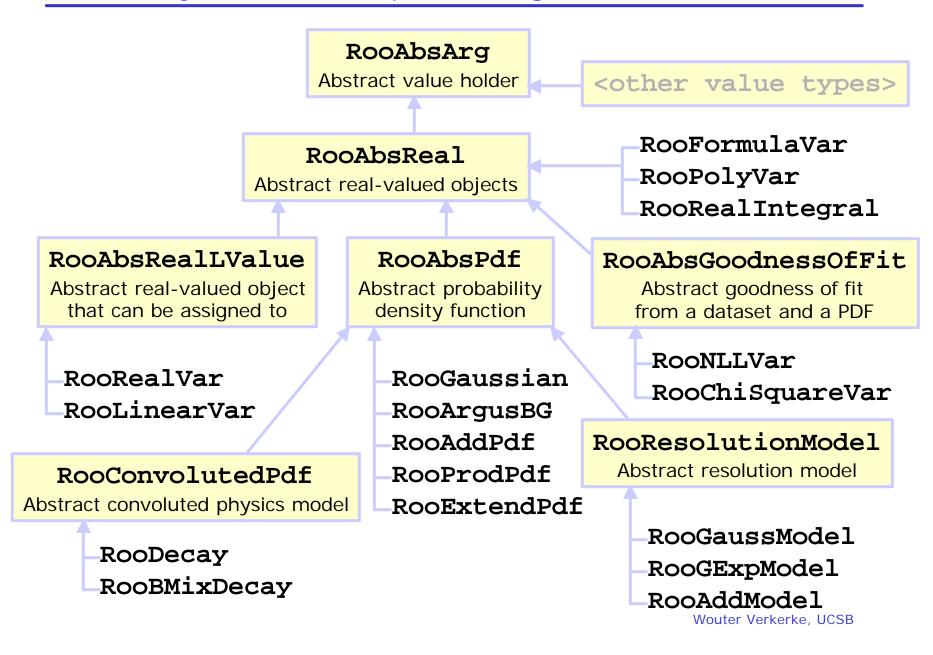
# Class hierarchy



Introduction of various abstract base classes

Coding examples

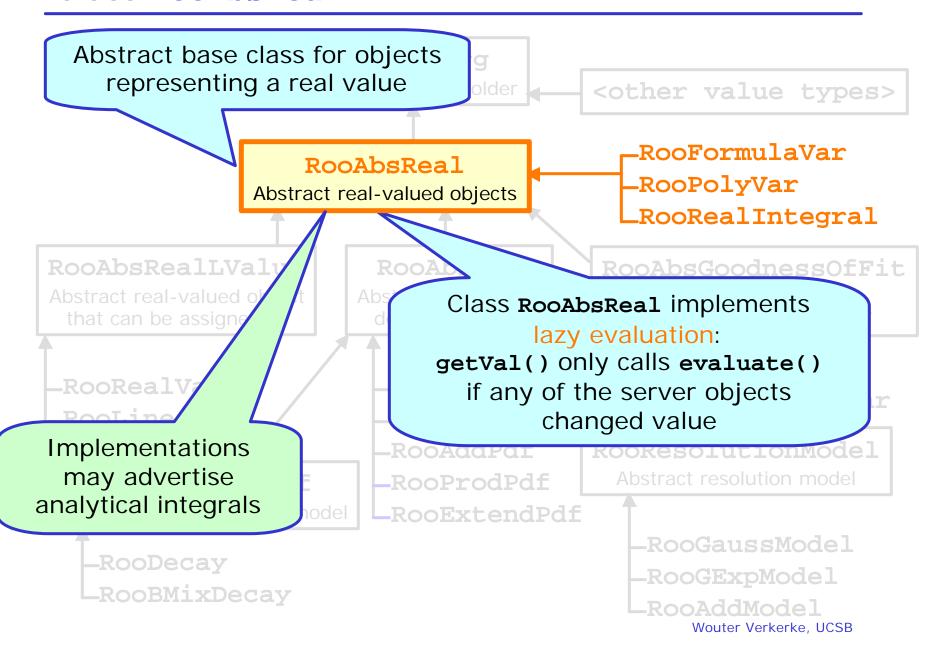
#### Hierarchy of classes representing a value or function



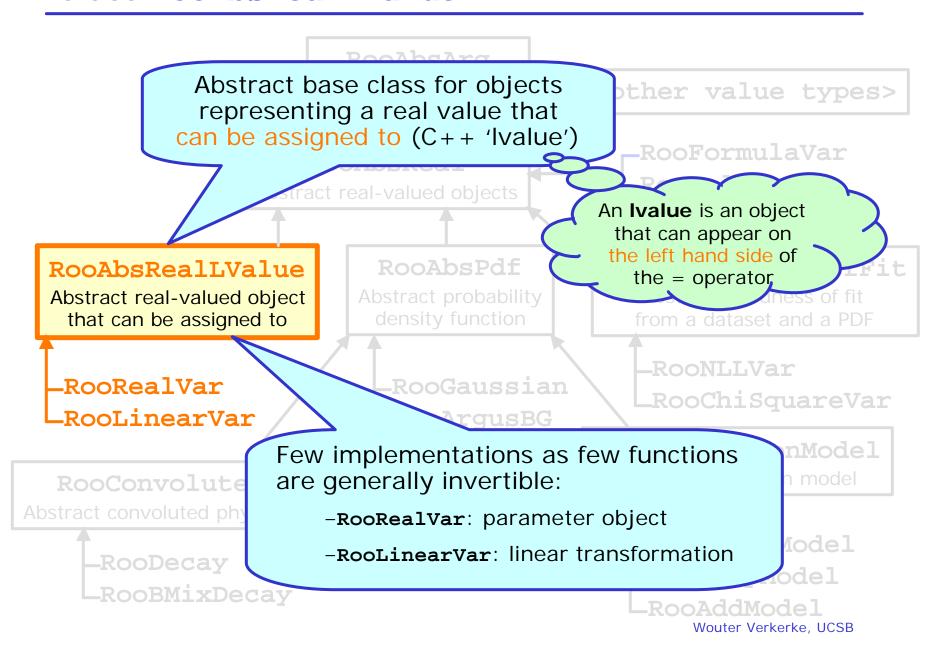
#### Class RooAbsArg

**Implementations** RooAbsArg can represent Abstract value holder any type of data. -RooFormulaVar Top-level class for objects sReal -RooPolyVar representing a value RooRealIntegral RooAbsRealLValue The main role of RooAbsArg is Abstract real-valued object to manage client-server links between RooAbsArg instances that are functionally related to -RooRealVar Roc each other -RooLinearVar -RooArgusbG -RooAddPdf RooResolutionModel Abstract resolution model -RooProdPdf RooConvolutedPdf Abstract convoluted physics model -RooExtendPdf -RooGaussModel -RooDecay -RooGExpModel -RooBMixDecay -RooAddModel Wouter Verkerke, UCSB

#### Class RooAbsReal



#### Class RooAbsRealLValue



#### Class RooAbsPdf

#### RooAbsArg

Abstract value holder

Abstract base class for probability density functions

#### RooAbsRealLValue

Abstract real-valued object that can be assigned to

-RooRealVar -RooLinearVar

#### RooConvolutedPdf

Abstract convoluted physics model

-RooDecay -RooBMixDecay

#### RooAbsPdf

Abstract probability density function

-RooGaussian -RooArgusBG -RooAddPdf -RooProdPdf

objects

-RooExtendPdf

Defining property

$$\int f(\vec{x}, \vec{p}) d\vec{x} \equiv 1$$

Where **x** are the observables and **p** are the parameters

from a dataset and a PDF

-RooNLLVar -RooChiSquareVar

RooResolutionModel

Abstract resolution model

-RooGaussModel -RooGExpModel -RooAddModel

Wouter Verkerke, UCSB

#### Class RooConvolutedPdf

Implements  $f_i(dt,...) \otimes R(dt,...)$ 

RooResolutionModel

$$P(dt,...) = \sum_{k} c_{k}(...) (f_{k}(dt,...) \otimes R(dt,...))$$

RooConvolutedPdf (physics model)

Implements  $c_k$ , declares list of  $f_k$  needed **No convolutions calculated in this class!** 

ue types>

ulaVar

Var

Integral

dnessOfFit

odness of fit

RooConvolutedPdf

Abstract convoluted physics model

\_RooDecay \_RooBMixDecay -RooArgusBG -RooAddPdf -RooProdPdf

RooExten

\_RooChiSquareVar

War

RooResolutionModel

Abstract resolution model

Abstract base class for PDFs that can be convoluted with a resolution model

#### Class RooResolutionModel

RooAbsArg Implementations of RooResolutionModel are RooAbsi regular PDFs with the Abstract real-val added capability to calculate their function convolved with a RooAbsRealLValue Ro series of 'basis' functions Resolution model advertises which basis functions it can RooNLLVar handle Gaussian RooChiSquareVar ArgusBG To be used with a given AddPdf RooResolutionModel RooConvolutedPdf Abstract resolution model ProdPdf implementation, a resolution model must -RooGaussModel support all basis functions used -RooGExpModel by the RooConvolutedPdf RooAddModel

Wouter Verkerke, UCSB

#### Class RooAbsGoodnessOfFit

Provides the framework for efficient calculation of goodness-of-fit quantities.

A goodness-of-fit quantity is a function that is calculated from

A dataset

the PDF value for each point in that dataset

-RooRealVar

-RooLinearVar

from a dataset and

RooRea

-RooNLLVar -RooChiSquareVar

#### Built-in support for

-Automatic constant-term optimization activated when used by RooMinimizer(MINUIT)

RooGaus

RooAbsReal

- -Parallel execution on multi-CPU hosts
- -Efficient calculation of RooSimultaneous PDFs

RooAbsGoodnessOfFit

Abstract goodness of fit from a dataset and a PDF

plutionModel

mulaVar

:ypes>

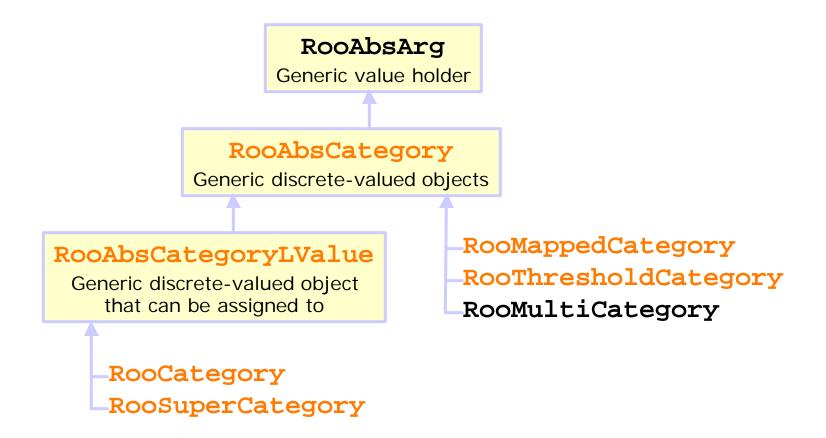
esolution model

aussModel

ExpModel AddModel

Wouter Verkerke, UCSB

# Class tree for discrete-valued objects



# Code examples



Implementing a RooAbsReal
Providing analytical integrals
Implementing a RooAbsPdf
Providing an internal generator
Implementing a RooConvolutedPdf/RooResolutionModel
Implementing a RooAbsGoodnessOfFit

### Writing a real-valued function – class RooAbsReal

Class declaration

```
class RooUserFunc : public RooAbsReal {
                   public:
                     RooUserFunc(const char
                                                e, const char *title,
                                                k, RooAbsReal& mean,
                                                sigma);
                                                        er,
Real-valued functions inherit from RooAbsReal
                     virtual TObject* clone(const char* newname) const {
                             return new RooUserFunc(*this, newname);
                     inline virtual ~RooUserFunc() { }
                   protected:
                     RooRealProxy x ;
                     RooRealProxy mean ;
                     RooRealProxy sigma ;
                     Double t evaluate() const;
                   private:
                     ClassDef(RooUserFunc,0) // Gaussian PDF
```

Mandatory methods

```
class RooUserFunc : public RooAbsPdf {
                      public:
RooAbsReal& x, RooAbsReal& mean,
                                  RooAbsReal& sigma);
• Copy constructor RooUserFunc(const RooUserFunc& other,
                                 const char* name=0);
                       virtual TObject* clone(const char* newname) const {
Clone
                              return new RooUserFunc(*this, newname);

    Destructor

                      inline virtual ~RooUserFunc() { }
                                                     Use copy ctor
                      protected:
                                                      in clone()
                       RooRealProxy x ;
                       RooRealProxy mean ;
                       RooRealProxy sigma ;
evaluate
                     Double_t evaluate() const ;
 Calculates your
                      private:
 PDF return value
                       ClassDef(RooUserFunc,0) // Gaussian PDF
```

Constructor arguments

Try to be as generic as possible, i.e.

Use RooAbsReal& to receive real-valued arguments
Use RooAbsCategory& to receive discrete-valued
arguments

Allows user to plug in either a variable (RooRealVar) or a function (RooAbsReal)

```
private:
   ClassDef(RooUserFunc,0) // Gaussian PDF
};
```

Storing RooAbsArg references

for cloning of

composite objects

```
Always use proxies to store RooAbsArg references:
                                                               r *title,
         RooRealProxy
                            for RooAbsReal
                                                               % mean,
         RooCategoryProxy for RooAbsCategory
                           for a set of RooAbsArgs
         RooSetProxy
                           for a list of RooAbsArgs
         RooListProxy
                                                              name) const {
                                 return new RooUserFunc(*this, newname);
                             .ine virtual ~RooUserFunc() { }
                        pro cted:
                          RooRealProxy x ;
                          RooRealProxy mean ;
                          RooRealProxy sigma ;
Storing references
in proxies allows RooFit
                          Double t evaluate() const ;
to adjust pointers
                        private:
                          ClassDef(RooUserFunc,0) // Gaussian PDF
This is essential
```

ROOT-CINT dictionary methods

```
class RooUserFunc : public RooAbsPdf {
                public:
                  RooUserFunc(const char *name, const char *title,
                              RooAbsReal& x, RooAbsReal& mean,
                              RooAbsReal& _sigma);
                  RooUserFunc(const RooUserFunc& other,
                             const char* name=0);
                  virtual TObject* clone(const char* newname) const {
                         return new RooUserFunc(*this newmame).
                                                  Description here
                  inline virtual ~RooUserFunc(
                                                   will be used in
                                                  auto-generated
                protected:
                  RooRealProxy x ;
                                                       THtml
                                                   documentation
Don't forget ROOT ClassDef macro
   No semi-colon at end of line!
                 civate:
                  ClassDef(RooUserFunc,1) // Gaussian PDF
```

Constructor implementation

```
RooUserFunc::RooUserFunc(const char *name, const char *title,
                                            RooAbsReal& x, RooAbsReal& mean,
                                            RooAbsReal& sigma):
                   RooAbsPdf(name,title),
                   x("x", "Dependent", this, x),
                   mean("mean","Mean",this, mean),
                    sigma("sigma","Width",this, sigma)
                                                      Pointer to
Initialize the proxies
                       rFunc::Roo
                                    rFunc(const
                                                   owning object
from the RooAbsArg
                        osPdf (othe:
                                                    is needed to
                                      ne),
method arguments
                         this other
                                                   register proxy
                   mean("mean",thi
                                        er.mean
                    sigma("sigma",t
                                         her.sigma
                    Name and title are for
                        description only
                                                () const
                   Double_t arg= x - mean;
                   return exp(-0.5*arg*arg/(sigma*sigma));
                                                             Mouter Verkerke IICCR
```

Implement a copy constructor!

```
RooUserFunc::RooUserFunc(const char *name, const char *title,
                              RooAbsReal& x, RooAbsReal& mean,
                              RooAbsReal& sigma):
     RooAbsPdf(name,title),
In the class copy constructor,
call all proxy copy constructors
                          Ac(const RooUserFunc& other,
   RooUserFunc::RooU
                                      const char* name) :
     RooAbsPdf(other name),
     x(this,other.x),
     mean(this,other.mean),
     sigma(this,other.sigma)
                       Pointer to
   Double t RooU
                     owning object
                       is (again)
     Double t arg
                       needed to
     return exp(
                                       gma));
                     register proxy
                                               Moutar Varbarba IICCR
```

Write evaluate function

In evaluate(), calculate and return the function value

```
Double_t RooUserFunc::evaluate() const
{
    Double_t arg= x - mean;
    return exp(-0.5*arg*arg/(sigma*sigma));
}
```

# Working with proxies

- RooRealProxy/RooCategoryProxy objects automatically cast to the value type they represent
  - Use as if they were fundamental data types

```
RooRealProxy x ;
                                RooCategoryProxy c ;
Double t func = x*x;
                                if (c=="bogus") {...}
Use as Double t
                                Use as const char
```

 To access the proxied RooAbsReal/RooAbsCategory Object use the arg() method

```
RooRealProxy x ;
RooCategoryProxy c ;
RooAbsReal& xarg = x.arg()
RooAbsCategory& carg = c.arg() ;
```

*NB*: the value or **arg()** may change during the lifetime of the object (e.g. if a composite cloning operation was performed)

- Set and list proxy operation completely transparent
  - Use as if they were RooArgSet/RooArgList Objects
     Wouter Verkerke, UCSB

# Lazy function evaluation & caching

- Method getVal() does not always call evaluate()
  - Each RooAbsReal object caches its last calculated function value
  - If none of the dependent values changed, no need to recalculate
- Proxies are used to track changes in objects
  - Whenever a RooAbsArg changes value,
     it notifies all its client objects that recalculation is needed
  - Messages passed via client/server links that are installed by proxies
  - Only if recalculate flag is set getVal() will call evaluate()
- Redundant calculations are automatically avoided
  - Efficient optimization technique for expensive objects like integrals
  - No need to hand-code similar optimization in function code: evaluate() is only called when necessary

# Writing a function – analytical integrals

- Analytical integrals are optional!
- Implementation of analytical integrals is separated in two steps
  - Advertisement of available (partial) integrals:
  - Implementation of partial integrals
- Advertising integrals: getAnalyticalIntegral()

Integration is requested over all variables in set allvars

#### Task of getAnalyticalIntegral():

- 1) find the *largest subset* that function can integrate analytically
- 2) Copy largest subset into analVars
- 3) Return unique identification code for this integral

# Writing a function – advertising integrals

#### Task of getAnalyticalIntegral():

- 1) find the *largest subset* that function can integrate analytically
- 2) Copy largest subset into analVars
- 3) Return unique identification code for this integral

Utility method matchArgs() does all the work for you:

If allvars contains the variable held in proxy x variable is copied to analvars and matchArgs() returns kTRUE

If not, it returns kFALSE

# Writing a function – advertising multiple integrals

If multiple integrals are advertised, test for the largest one first

You may advertise analytical integrals for both *real-valued* and *discrete-valued* integrands

# Writing a function – implementing integrals

- Implementing integrals: analyticalIntegral()
  - One entry point for all advertised integrals

Integral identification code
assigned by getAnalyticalIntegral()

Discrete-valued integrands are always summed over *all* states

## Calculating integrals – behind the scenes

- Integrals are calculated by RooRealIntegral
  - To create an RooRealIntegral for a RooAbsReal

```
RooAbsReal* f; // f(x)
RooAbsReal* int_f = f.createIntegral(x);

RooAbsReal* g ; // g(x,y)
RooAbsReal* inty_g = g.createIntegral(y);
RooAbsReal* intxy_g = g.createIntegral(RooArgSet(x,y));
```

- Tasks of RooRealIntegral
  - Structural analysis of composite
  - Negotiate analytical integration with components PDF
  - Provide numerical integration where needed
- RooRealIntegral works universally on simple and composite objects

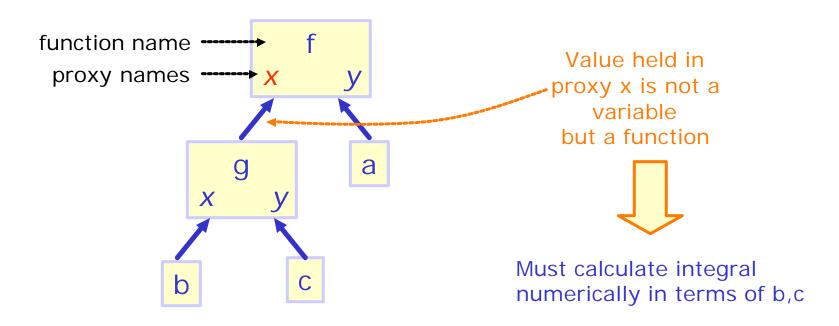
A RooRealIntegral is also a RooAbsReal

is RooFits most complex class!

#### Why advertised analytical integrals are sometimes not used

- Integration variable is not a fundamental
  - Suppose f(x,y) advertises analytical integration over x

$$f(\mathbf{x}, a), g(b, c) \rightarrow f(g(b, c), a)$$

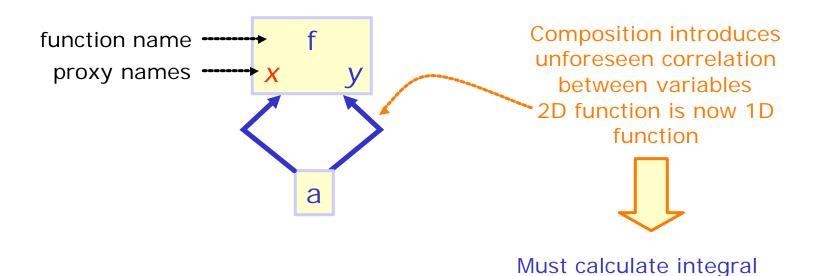


(Exception: g(x,y) is an invertable function (RooAbsRealLValue) with a constant Jacobian term)

#### Why advertised analytical integrals are sometimes not used

- Function depends more than once on integration variable
  - Suppose f(x,y) advertises analytical integration over x

$$f(\mathbf{x}, \mathbf{y}) \to f(a, a)$$

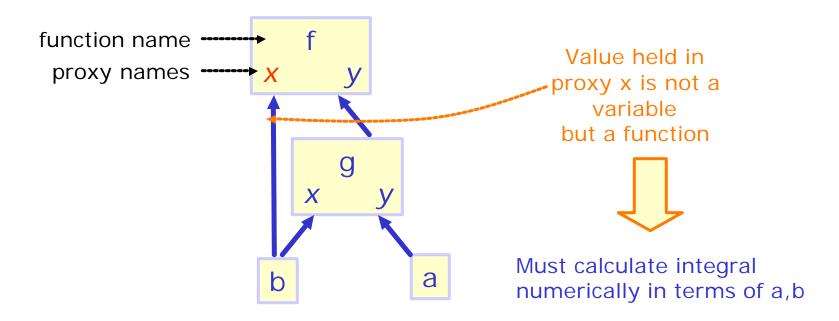


numerically in terms of a

#### Why advertised analytical integrals are sometimes not used

- Function depends more on integration variable via more than one route
  - Suppose f(x,y) advertises analytical integration over x

$$f(\mathbf{x}, y), g(a, x) \rightarrow f(x, g(a, x))$$



#### Class documentation

 General description of the class functionality should be provided at the beginning of your .cc file

```
// -- CLASS DESCRIPTION [PDF] --
// Your description goes here PDF Keyword causes class to be classified as PDF class
```

- First comment block in each function will be picked up by THtml as the description of that member function
  - Put some general, sensible description here

## Writing a PDF – class RooAbsPdf

Class declaration

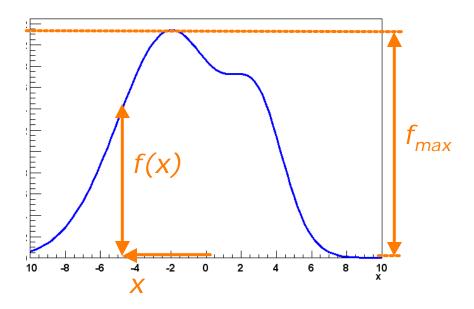
```
class RooUserPdf : public RooAbsPdf {
          public:
            RooUserFunc(const char
                                                 har *title,
PDFs inherit from RooAbsPdf
This is the only difference with a RooAbsReal
RooAbsPdf::getVal()will automatically normalize
your return value by dividing it by the integral of the
PDF. No further action is needed!
            RooRealProxy mean
            RooReal
                    RooRealIntegral used for integral calculation
            Do
                    RooAbsPdf owns RRI configured for last
                    normalization configuration. If normalization set
          priva
                    Changes, new RRI as created on the fly...
```

## Writing a PDF – Normalization

- Do not under any circumstances attempt to normalize your PDF in evaluate() via explicit or implicit integration
- You do not know over what variables to normalize
  - In RooFit, parameter/observable distinction is dynamic,
     a PDF does not have a unique normalization/return value
- You don't even now know how to integrate yourself!
  - Your PDF may be part of a larger composite structure.
     Variables may be functions, your internal representation may have a difference number of dimensions that the actual composite object.
  - RooRealIntegral takes proper care of all this
- But you can help!
  - Advertise all partial integrals that you can calculate
  - They will be used in the normalization when appropriate
    - · Function calling overhead is minimal

# PDF Event generation – Accept/reject method

- By default, toy MC generation from a PDF is performed with accept/reject sampling
  - Determine maximum PDF value by repeated random sample
  - Throw a uniform random value (x) for the observable to be generated
  - Throw another uniform random number between 0 and fmax
     If ran\*f<sub>max</sub> < f(x) accept x as generated event</li>

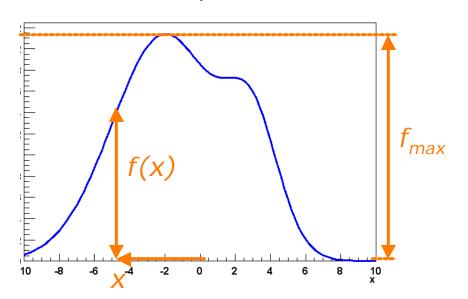


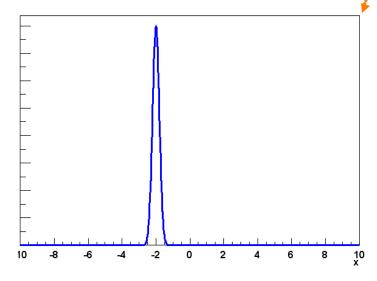
# PDF Event generation – Accept/reject method

- Accept/reject method can be very inefficient
  - Generating efficiency is

$$\frac{\int_{x_{\text{max}}}^{x_{\text{min}}} f(x) dx}{(x_{\text{max}} - x_{\text{min}}) \cdot f_{\text{max}}}$$

- Efficiency is very low for narrowly peaked functions
- Initial sampling for fmax requires very large trials sets in multiple dimension (~10000000 in 3D)





# PDF Event generation – Optimizations

- RooFit 'operator' PDFs provide various optimizations
- RooProdPdf Components PDFs generated separately
  - Breaks down N dimensional problem to n m-dimensional problems
  - Large initial f<sub>max</sub> sampling penalty not incurred
- RooAddPdf Only one component generated at a time
  - RooAddPdf randomly picks a component PDF to generate for each event. Component probabilities weighted according to fractions
  - Helps to avoid accept/reject sampling on narrowly peaked distributions, if narrow and wide component are separately generated
- RooSimultaneous Only one component generated at a time
  - Technique similar to RooAddPdf

# PDF Event generation – Internal generators

- For certain PDFs alternate event generation techniques exist that are more efficient that accept/reject sampling
  - Example: Gaussian, exponential,...
- If your PDF has such a technique, you can advertise it
  - Interface similar to analytical integral methods

```
RooAbsPdf::getGenerator()
RooAbsPdf::initGenerator()
RooAbsPdf::generateEvent()
```

- You don't have to be able to generate all observables
  - Generator context can combine accept/reject and internal methods within a single PDF
- This is an optional optimization
  - PDF can always generate events via accept/reject method

# Writing a PDF – advertising an internal generator

#### Task of getGenerator():

- 1) find the *largest subset* of observables PDF can generate internally
- 2) Copy largest subset into dirVars
- 3) Return unique identification code for this integral

```
Int_t RooUserFunc::getGenerator(
   RooArgSet& allVars, RooArgSet& dirVars, Bool_t staticOK) const
{
   if (matchArgs(allVars,dirVars,x)) return 1 ;
   return 0 ;
}
```

Utility method matchArgs() does all the work for you:

If allvars contains the variable held in proxy x variable is copied to dirvars and matchArgs() returns kTRUE

If not, it returns kFALSE

# Writing a PDF – advertising an internal generator

- For certain internal generator implementations it can be efficient to do a one-time initialization for each set of generated events
  - Example: precalculate fractions for discrete variables
- Caveat: one-time initialization only safe if no observables are generated from a prototype dataset
  - Only advertise such techniques if staticOK flag is true

configurations, try the most extensive one first

# Writing a PDF - implementing an internal generator

- Implementing a generator: generateEvent()
  - One entry point for all advertised event generators

Generator identification code assigned by getGenerator()

```
void RooGaussian::generateEvent(Int_t code)
{
   Double_t xgen ;
   while(1) {
      xgen = RooRandom::randomGenerator()->Gaus(mean,sigma);
      if (xgen<x.max() && xgen>x.min()) {
            x = xgen ;
            break;
      }
            Return generated value
      return;      by assigning it to the proxy
}
```

# Writing a PDF - implementing an internal generator

- Static generator initialization: initGenerator()
  - This function is guaranteed to be call once before each series of generateEvent() calls with the same configuration

Generator identification code assigned by getGenerator()

### Writing a convoluted PDF – physics/resolution factorization

- Physics model and resolution model are implemented separately in RooFit
  - Factorization achieved via a common set 'basis functions' f<sub>k</sub>

Implements 
$$f_i(dt,...) \otimes R(dt,...)$$
 Also a PDF by itself

RooResolutionModel

$$P(dt,...) = \sum_{k} c_{k}(...) (f_{k}(dt,...) \otimes R(dt,...))$$

RooConvolutedPdf (physics model)

- Implements  $c_k$
- Declares list of  $f_k$  needed



**No magic**: You must still calculate the convolution integral yourself, but factorization enhances modularity & flexibility for end user

### Writing a convoluted PDF - class RooConvolutedPdf

Class declaration

```
class RooBMixDecay : public RooConvolutedPdf {
     public:
Convolutable PDF classes inherit from
RooConvolutedPdf instead of RooAbsPdf
       RooBMixDecay(const RooBMixDecay& other, const char* name=0);
       virtual TObject* clone(const char* newname) const ;
       virtual ~RooBMixDecay();
       virtual Double_t coefficient(Int_t basisIndex) const ;
     protected:
         Implement coefficient() instead of evaluate()
```

### Class **RooConvolutedPdf** – Constructor implementation

Constructor must declare all basis functions used

```
RooBMixDecay::RooBMixDecay(const char *name, const char *title,...):
  RooConvolutedPdf(name, title, model, t), ...
                                                           Supply basis
                                                             function
  // Constructor
                                                            parameters
  basisExp = declareBasis("exp(-abs(@0)/@1)",
                                                               here
                             RooArgList(tau)) ;
  basisCos = declareBasis("exp(-abs(@0)/@1)*cos(@0*@2)",
                             RooArgList(tau,dm));
                  Call declareBasis() for
                           each
                      basis functions
                                                Name of basis function is
                     used in this PDF
                                               RooFormulaVar expression
                                                @0 is convolution variable
                                                @1..@n are basis function
    Return code assign
                                                      parameters
  unique integer code to
   each declared basis
```

Wouter Verkerke, UCSB

#### Class **RooConvolutedPdf** – Coefficient implementation

• Method coefficient() implements all coefficient values

```
Requested index is one of the basis function codes returned by declareBasis()
```

```
Double_t RooBMixDecay::coefficient(Int_t basisIndex) const
{
   if (basisIndex==_basisExp) {
     return (1 - _tagFlav*_delMistag);
   }
   if (basisIndex==_basisCos) {
     return _mixState*(1-2*_mistag);
   }
   return 0;
}
```

At this point class is complete and functional

### Class **RooConvolutedPdf** – Analytical integrals

- You can optionally advertise and implement analytical integrals for your coefficient functions
  - Interface similar to analytical integrals in RooAbsReal
- Advertising coefficient integrals
  - Method identical to RooAbsReal::getAnalyticalIntegral(), just the name is different

- Implementing coefficient integrals
  - Method similar to RooAbsReal::analyticalIntegral()
  - One extra argument to identify the coefficient in question

```
Double_t coefAnalyticalIntegral(Int_t coef, Int_t code) const;
```

# Class **RooConvolutedPdf** – Internal generator implementation

- You can optionally advertise and implement an internal generator for the unconvoluted PDF function
  - Methods identical to regular PDF generator implementation
- An internal generator will greatly accelarate toyMC generation from a convoluted PDF
  - If both physics PDF and resolution model provide internal generators, then events can be generated as

$$x_{P\otimes R} = x_P + x_R$$

i.e. no convolutions integrals need to be evaluated

- Only works with internal generator implementations because both  $x_P$  and  $x_R$  must be generated on an unbound domain for this technique to work
  - Accept reject sample doesn't work on unbound domains

### Writing a resolution model – physics/resolution factorization

- Physics model and resolution model are implemented separately in RooFit
  - Factorization achieved via a common set 'basis functions' f<sub>k</sub>

Implements  $f_i(dt,...) \otimes R(dt,...)$ Also a PDF by itself

RooResolutionModel

$$P(dt,...) = \sum_{k} c_{k}(...) (f_{k}(dt,...) \otimes R(dt,...))$$

RooConvolutedPdf (physics model)

- Implements  $c_k$
- Declares list of  $f_k$  needed

# Writing a resolution model PDF - class RooResolutionModel

Class declaration

```
class RooGaussModel : public RooResolutionModel {
     public:
Resolution model classes inherit from
RooResolutionModel instead of RooAbsPdf
       kooGaussmodel(const koobmixDecay& other, const char* name=0);
      virtual TObject* clone(const char* newname) const ;
      virtual ~RooGaussModel();
Method basisCode() advertises supported basis functions
      virtual Int_t basisCode(const char* name) const = 0 ;
      virtual Double_t evaluate() const ;
     protected:
                      evaluate() returns regular or convoluted PDF
       ClassDef(Roo
                      depending on internal state
     };
```

### Class **RooResolutionModel** – Advertising basis functions

 Function basisCode() assigns unique integer code to each supported basis function

```
Int_t RooGaussModel::basisCode(const char* name) const
{
   if (!TString("exp(-@0/@1)").CompareTo(name)) return 1;
   if (!TString("exp(@0/@1)").CompareTo(name)) return 2;
   if (!TString("exp(-abs(@0)/@1)").CompareTo(name)) return 3;
   if (!TString("exp(-@0/@1)*sin(@0*@2)").CompareTo(name)) return 4;
   if (!TString("exp(@0/@1)*sin(@0*@2)").CompareTo(name)) return 5;
   if (!TString("exp(-abs(@0)/@1)*sin(@0*@2)").CompareTo(name)) return 6;
   if (!TString("exp(-@0/@1)*cos(@0*@2)").CompareTo(name)) return 7;
   if (!TString("exp(@0/@1)*cos(@0*@2)").CompareTo(name)) return 8;
   if (!TString("exp(-abs(@0)/@1)*cos(@0*@2)").CompareTo(name)) return 9;
   return 0;
}
```

Return 0 if basis function is not supported

### Class RooResolutionModel - Implementing evaluate()

 evaluate() returns both convoluted and unconvoluted PDF value

the ID of the basis function we're convoluted with.

If zero, not convoluted is requested

```
Double_t RooGaussModel::evaluate() c
{
   Int_t code = currentBasisCode();

   if (code==0) {
        // return unconvoluted PDF value;
   }

   if (code==1) {
        // Return PDF convoluted with basis function #1

        // Retrieve basis function paramater value
        Double_t tau = basis().getParameter(1))->getVal();
   }
}
```

### Class RooResolutionModel - Implementing evaluate()

 evaluate() returns both convoluted and unconvoluted PDF value

```
Double t RooGaussModel::evaluate() const
                    ptBasisCode();
basis() returns a reference
   to the RooFormulaVar
representing the current basis
                            uted PDF value
         function
                                            getParameter(n) returns a
                                           RooAbsReal reference to the
                                               nth parameter of the
       if (code==1)
                                                 RooFormulaVar
                          nvoluted with
         // Return PD.
         // Retrieve basis function param _er value
         Double_t tau = basis().getParameter(1))->getVal();
```

### Class **RooResolutionModel** – Analytical integrals

 Advertising and implementing analytical integrals works the same way as in RooAbsPdf

Advertisement and implementation should reflect the 'current' convolution indicated by currentBasisCode()

```
Int t RooGaussModel::
      getAnalyticalIntegral(Ref
                                   c& allVars,
                                rgSet& analVars) const
  switch(currentBasisCode()) {
  // Analytical integration capability of raw PDF
  case 0:
    if (matchArgs(allVars,analVars,convVar())) return 1;
   break :
  // Analytical integration capability of convoluted PDF
  case 1:
  if (matchArgs(allVars,analVars,convVar())) return 1;
    break ;
                                               Wouter Verkerke, UCSB
```

# Class **RooResolutionModel** – Internal generator implementation

- You can optionally advertise and implement an internal generator for the unconvoluted resolution model
  - Methods identical to regular PDF generator implementation

# Class RooAbsGoodnessOfFit - Goodness of fit

• No time left to write this section... (sorry!)

# Debugging



ROOT and gdb/dbx
Finding memory leaks
Tracing function evaluation
Checking integrals & generators
Profiling

# Using the system debugger

- Compiled applications linked with RooFit
  - Just use 'gdb/dbx <executable>'
- Interactive ROOT
  - You can use gdb/dbx to debug your compiled RooFit class
  - Trick: attach debugger to already running ROOT process
    - 1. Start interactive ROOT the usual way
    - 2. In a separate shell on the same host attach gdb/dbx to the running ROOT session
    - 3. Resume running of ROOT gdb> continue
    - 4. Execute the code you want to test

```
#!/bin/sh
line=`ps -wwfu $USER | grep root.exe | grep -v grep | tail -1`
if [ "$line" = "" ] ; then
  echo "No ROOT session running"
  exit 1
fi
set $line
exec gdb $8 $2
Wouter Verkerke, UCSB
```

# Finding memory leaks

RooTrace utility keeps track of RooFit object allocation

```
RooTrace::active(kTRUE)

RooRealVar x("x","x",-10,10);
RooGaussian g("g","g",x,RooConst(0),RooConst(1));

RooTrace::dump(cout);
List of RooFit objects allocated while trace active:
00086b7118:
RooRealVar - x
00086aa178:
RooArgList - RooRealVar Constants Database
00086b7658:
RooConstVar - 0.000000
00086b7b08:
RooConstVar - 1.000000
00086bc3e8:
RooGaussian - g
```

# Finding memory leaks

You can do incremental leak searches.

```
RooTrace::active(kTRUE)

RooRealVar x("x","x",-10,10);
RooGaussian g("g","g",x,RooConst(0),RooConst(1));

RooTrace::mark(); // mark all objects created sofar

RooGaussian g2("g2","g2",x,RooConst(2),RooConst(1));

// Dump only objects created since last mark
RooTrace::dump(cout,kTRUE);
List of RooFit objects allocated while trace active:
00086c8f50: RooConstVar - 2.000000
00086c9400: RooGaussian - g2
5 marked objects suppressed
```

# Tracing function evaluation

- When you have many instances of a single class it can be more useful to trace function evaluation with printed messages than via debugger
  - Debugger breakpoint will stop in every instance of your class even if you only want to examine a single instance
- RooFit provides system-wide tracing techniques
  - RooAbsArg::setVerboseDirty(kTRUE)
    - Track lazy evaluation logic of RooAbsArg classes
    - May help to understand why your evaluate() doesn't get called
  - RooAbsArg::setVerboseEval(Int\_t level)
    - Level 0 No messages
    - Level 1 Print one-line message each time a normalization integral is recalculated
    - Level 2 Print one-line message each time a PDF is recalculated
    - Level 3 Provide details of convolution integral recalculations

# Tracing function evaluation

- And object-specific tracing techniques
  - pdf->setTraceCounter(Int\_t n, Bool\_t recursive)
  - Prints one-lines messages for the next n times pdf is evaluated
  - If recursive option is set, trace counter is also set for all component PDFs of pdf
  - Useful in fitting/likelihood calculations where is single likelihood evaluation can trigger thousands of PDF evaluations

### Checking analytical integrals and internal generators

- Function integrals and PDF event generators both have a numerical backup solution
  - You can use those as a cross check to validate your function/PDF-specific implementation
- Integrals
  - RooAbsReal::forceNumInt(kTRUE) will disable the use of any advertised analytical integrals
- Generators
  - RooAbsPdf::forceNumGen(kTRUE) will disable the use of any advertised internal PDF generator methods

# **Profiling**

- To run the profiler you must build your test application as a standalone executable
  - compile & link with -pg flag

```
#include "TROOT.h"
#include "TApplication.h"

// Instantiate ROOT system
TROOT root("root", "root");
int main(int argc, char **argv)
{
    // Instantiate graphics event handler
    TApplication app("TAppTest",&argc,argv);

    // User code goes here
}
```

- You cannot have any RooFit classes as global variables
  - Prior instantiation of TROOT needed, but cannot be guaranteed
- Place your driver executable in the RooFitModels directory and list it as a binary target in the GNUMakefile
   Wouter Verkerke, UCSB

### Outlook

- New goodness-of-fit calculation classes will be introduced soon (~1 week)
  - Likelihood and ChiSquare as examples.
  - Complete function optimization support for likelihood fitting now generically available for all goodness of fits
  - Built-in support for handling RooSimultaneous PDFs
  - Support for parallel execution on multi-CPU hosts
    - No support from user code needed except prescription to merge partial results (Default implementation adds partial results)

#### THE ROOFIT TOOLKIT FOR DATA MODELING

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RooFit is a library of C++ classes that facilitate data modeling in the ROOT environment. Mathematical concepts such as variables, (probability density) functions and integrals are represented as C++ objects. The package provides a flexible framework for building complex fit models through classes that mimic math operators, and is straightforward to extend. For all constructed models RooFit provides a concise yet powerful interface for fitting (binned and unbinned likelihood,  $\chi^2$ ), plotting and toy Monte Carlo generation as well as sophisticated tools to manage large scale projects. RooFit has matured into an industrial strength tool capable of running the BABAR experiment's most complicated fits and is now available to all users on SourceForge<sup>1</sup>.

#### 1. Introduction

One of the central challenges in performing a physics analysis is to accurately model the distributions of observable quantities  $\vec{x}$  in terms of the physical parameters of interest  $\vec{p}$  as well as other parameters  $\vec{q}$  needed to describe detector effects such as resolution and efficiency. The resulting model consists of a "probability density function" (PDF)  $F(\vec{x}; \vec{p}, \vec{q})$  that is normalized over the allowed range of the observables  $\vec{x}$  with respect to the parameters  $\vec{p}$  and  $\vec{q}$ .

Experience in the BaBar experiment has demonstrated that the development of a suitable model, together with the tools needed to exploit it, is a frequent bottleneck of a physics analysis. For example, some analyses initially used binned fits to small samples to avoid the cost of developing an unbinned fit from scratch. To address this problem, a general-purpose toolkit for physics analysis modeling was started in 1999. This project fills a gap in the particle physicists' tool kit that had not previously been addressed.

A common observation is that once physicists are freed from the constraints of developing their model from scratch, they often use many observables simultaneously and introduce large numbers of parameters in order to optimally use the available data and control samples.

#### 2. Overview

The final stages of most particle physics analysis are performed in an interactive data analysis framework such as  $PAW^2$  or  $ROOT^3$ . These applica-

tions provide an interactive environment that is programmable via interpreted macros and have access to a graphical toolkit designed for visualization of particle physics data. The RooFit toolkit extends the ROOT analysis environment by providing, in addition to basics visualization and data processing tools, a language to describe data models. The core features of RooFit are:

- A natural and self-documenting vocabulary to build a model in terms of its building blocks (e.g., exponential decay, Argus function, Gaussian resolution) and how they are assembled (e.g., addition, composition, convolution). A template is provided for users to add new PDFs specific to their problem domain.
- A data description language to specify the observable quantities being modeled using descriptive titles, units, and any cut ranges. Various data types are supported including real valued and discrete valued (e.g. decay mode). Data can be read from ASCII files or ROOT ntuples.
- Generic support for fitting any model to a dataset using a (weighted) unbinned or binned maximum likelihood, or  $\chi^2$  approach
- Tools for plotting data with correctly calculated errors, Poisson or binomial, and superimposing correctly normalized projections of a multidimensional model, or its components.
- Tools for creating a event samples from any model with Monte Carlo techniques, with some variables possibly taken from a prototype dataset, e.g. to more accurately model the statistical fluctuations in a particular sample.

- Computational efficiency. Models coded in RooFit should be as fast or faster than hand coded models. An array of automated optimization techniques is applied to any model without explicit need for user support.
- Bookkeeping tools for configuration management, automated PDF creation and automation of routine tasks such as goodness-of-fit studies.

#### 3. Object-Oriented Mathematics

To keep the distance between a physicists' mathematical description of a data model and its implementation as small as possible, the Roofit interface is styled after the language of mathematics. The object-oriented ROOT environment is ideally suited for this approach: each mathematical object is represented by a C++ software object. Table 1 illustrates the correspondence between some basic mathematical concepts and Roofit classes.

$\operatorname{Concept}$	Math Symbol	RooFit class name
Variable	x, p	RooRealVar
Function	$f(ec{x})$	${\tt RooAbsReal}^*$
$\operatorname{PDF}$	$F(ec{x};ec{p},ec{q})$	RooAbsPdf*
Space point	$ec{x}$	RooArgSet
${\rm Integral}$	$\int_{\vec{x}_{min}}^{\vec{x}_{max}} f(\vec{x}) d\vec{x}$	RooRealIntegal
List of points	$ec{x}_k$	RooAbsData*

<sup>\*</sup> Abstract base classes

Composite objects are built by creating all their components first. For example, a Gaussian probability density function with its variables is created as follows:

```
RooRealVar x("x","x",-10,10) ;
RooRealVar m("m","mean",0) ;
RooRealVar s("s","sigma",3) ;
RooGaussian g("g","gauss(x,m,s)",x,m,s) ;
```

Each object has a name, the first argument, and a title, the second argument. The name serves as unique identifier of each object, the title can hold a more elaborate description of each object and only serves documentation purposes.

Function objects are linked to their ingredients: the function object g always reflects the values of its input variables x,m, and s. The absence of any explicit invocation of calculation methods allows for

true symbolic manipulation in mathematical style.

Roofit implements its data models in terms of probability density functions. The normalization of probability density functions, traditionally one of the most difficult aspects to implement, is handled internally by Roofit: all PDF objects are automatically normalized to unity. If a specific PDF class doesn't provide its normalization internally, a variety of numerical techniques are used to calculate the normalization.

Composition of complex models from elementary PDFs is straightforward: a sum of two PDFs is a PDF, the product of two PDFs is a PDF. The Roofit toolkit provides as set of 'operator' PDF classes that represent the sum of any number of PDFs, the product of any number of PDFs and the convolution of two PDFs.

Existing PDF building blocks can be tailored using standard mathematical techniques by substituting a variable with a formula expression. Free-form interpreted C++ function and PDF objects are available to glue together larger building blocks. The universally applicable composition operators and free-style interpreted functions make it possible to write probability density functions of arbitrary complexity in a straightforward mathematical form.

#### 4. Composing and Using Data Models

We illustrate the process of building a model and its various uses with a simple one-dimensional yield fit example.

The RooFit models library provides more than 20 basic probability density functions that are commonly used in high energy physics applications, including basics PDFs such Gaussian, exponential and polynomial shapes, physics inspired PDFs, e.g. decay functions, Breit-Wigner, Voigtian, Argus shape, Crystal Ball shape, and non-parametric PDFs (histogram and KEYS<sup>4</sup>).

In the example below we use two such PDFs: a Gaussian and an ARGUS background function:

```
// Observable
RooRealVar mes("mes","mass_ES",-10,10) ;

// Signal model and parameters
RooRealVar mB("mB","m(B0)",0) ;
RooRealVar w("w","Width of m(B0)",3) ;
RooGaussian G("G","G(meas,mB,width)",mes,mB,w) ;
```

```
// Background model and parameters
RooRealVar m0("m0","Beam energy / 2",-10,10) ;
RooRealVar k("k","ARGUS slope parameter",3) ;
RooArgusBG A("A","A(mes,m0,k)",mes,m0,k) ;

// Composite model and parameter
RooRealVar f("f","signal fraction",0,1) ;
RooAddPdf M("M","G+A",RooArgList(G,A),f) ;
```

The RooAddPdf operator class M combines the signal and background component PDFs with two parameters each into a composite PDF with five parameters:

```
M(m_{ES}; m_B, w, m_0, k, f) = f \cdot G(m_{ES}; w, g) + (1 - f) \cdot A(m_{ES}; m_0, k).
```

Once the model M is constructed, a maximum likelihood fit can be performed with a single function call:

```
M.fitTo(*data) ;
```

Fits performed this way can be unbinned, binned and/or weighted, depending on the type of dataset provided. The result of the fit, the new parameter values and their errors, are immediately reflected in the RooRealVar objects that represent the parameters of the PDF, mB,w,m0,k and f. Parameters can be fixed in a fit or bounded by modifying attributes of the parameter objects prior to the fit:

```
m0.setConstant(kTRUE) ;
f.setRange(0.5,0.9) ;
```

Visualization of the fit result is equally straightforward:

A RooPlot object represents a one-dimensional view of a given observable. Attributes of the RooRealVar object mes provide default values for the properties of this view (range, binning, axis labels). Figure 1 shows the result of the frame->Draw() operation in the above code fragment.

The default error bars drawn for a dataset are asymmetric and correspond to a Poisson confidence interval equivalent to  $1\sigma$  for each bin content. The curve of the PDF is automatically normalized to the number of events of the dataset last plotted in the same frame. The points of the curve are chosen by an

adaptive resolution-based technique: the deviation between the function value and the curve will not exceed a given tolerance regardless of the binning of the plotted dataset.

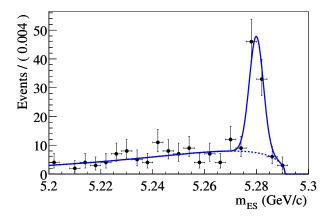


Fig. 1. One dimensional plot with histogram of a dataset, overlaid by a projection of the PDF M. The histogram error are asymmetric, reflecting the Poisson confidence interval corresponding to a  $1\sigma$  deviation. The PDF projection curve is automatically scaled to the size of the plotted dataset.

The plotOn() methods of datasets and functions accept optional arguments that modify the style and contents of what is drawn. The second M.plotOn() call in the preceding example illustrates some of the possibilities for functions: only the A component of the composite model M is drawn and the line style is changed to a dashed style. Similarly, the presentation of datasets can be changed, for example a sum-of-weights error  $(\sqrt{\Sigma_i w_i^2})$  can optionally be selected for use with weighted datasets.

# 5. Efficiency and Optimal Function Calculation

As the complexity of fits increases, efficient use of computing resources becomes increasingly important. To speed up the evaluation of probability density functions, optimization techniques such as value caching and factorized calculations can be used.

Traditionally such optimizations require a substantial programming effort due to the large amount of bookkeeping involved, and often result in incomplete use of available optimization techniques due to lack of time or expertise. Ultimately such optimizations represent a compromise between development cost, speed and flexibility.

Roofit radically changes this equation as the object-oriented structure of its PDFs allows centrally provided algorithms to analyze any PDFs structure and to apply generic optimization techniques to it. Examples of the various optimization techniques are:

- Precalculation of constant terms. In a fit, parts of a PDF may depend exclusively on constant parameters. These components can be precalculated once and used throughout the fit session.
- Caching and lazy evaluation. Functions are only recalculated if any of their input has changed. The actual calculation is deferred to the moment that the function value is requested.
- Factorization. Objects representing a sum, product or convolution of other PDFs, can often be factorized from a single N-dimensional problem to a product of N easier-to-solve 1-dimensional problems.
- Parallelization. Calculation of likelihoods and other goodness-of-fit quantities can, due to their repetitive nature, easily be partitioned in to set of partial results that can be combined a posteriori. Roofit automates this process and can calculate partial results in separate processes, exploiting all available CPU power on multi-CPU hosts.

Optimizations are performed automatically and tailored to each potentially CPU intensive operation. This realizes the maximum available optimization potential for every operation at no cost for the user.

#### 6. Data and Project Management Tools

As analysis projects grow in complexity, users are often confronted with an increasing number of logistical issues and bookkeeping tasks that may ultimately limit the complexity of their analysis. RooFit provides a variety of tools to ease the creation and management of large numbers of datasets and probability density functions such as:

- Discrete variables. A discrete variable in Roofit is a variable with a finite set of named states. The naming of states, instead of enumerating them, facilitates symbolic notation and manipulation.
- Automated PDF building. A common analysis technique is to classify the events of a dataset D into subsets  $D_i$ , and simultaneously fit a set of PDFs  $P_i(\vec{x}, \vec{p}_i)$  to these subsets  $D_i$ . In cases where individually adjusted PDFs  $P_i(\vec{x}, \vec{p}_i)$  can describe the data better than a single global PDF  $P(\vec{x}, \vec{p})$ , a better statistical sensitivity can be obtained in the fit. Often, such

PDFs do not differ in structure, just in the value of their parameters. RooFit offers a utility class to automate the creation the the PDFs  $P_i(\vec{x}, \vec{p_i})$ : given a prototype PDF  $P(\vec{x}, \vec{p})$  and a set of rules that explain how the prototype should be altered for use in each subset (e.g. "Each subset should have its own copy of parameter foo") this utility builds entire set of PDFs  $P_i(\vec{x}, \vec{p_i})$ .

• Project configuration management. Advanced data analysis projects often need to store and retrieve the projection configuration, such as initial parameters values, names of input files and other parameter that control the flow of execution. RooFit provides tools to store such information in a standardized way in easy-to-read ASCII files. The use of standardized project management tools promotes structural similarity between analyses and increases a users' ability to understand other RooFit projects and to exchange ideas and code.

#### 7. Development Status

Roofit was initially released as RoofitTools in 1999 in the BaBar collaboration and has over the years been adopted by virtually all BaBar physics analyses. Analysis topics include searches for rare B decays, measurements of B branching fractions and CP-violating rate asymmetries, time-dependent analyses of B and D decays to measure lifetime, mixing, and symmetry properties, and Dalitz analyses of B decays to determine form factors. Since October 2002 Roofit is available to the entire HEP community: the code and documentation repository has been moved from BaBar to SourceForge, an OpenSource development platform, which provides easy and equal access to all HEP users. (http://roofit.sourceforge.net). Since July 2005 RooFit is also bundled with ROOT releases, starting with ROOT version 5.02-00.

#### References

- 1. http://roofit.sourceforge.net
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- K. Cranmer, Kernel Estimation in High-Energy Physics, Comp. Phys. Comm 136, 198-207 (2001).

# **RooFit Users Manual v2.07**

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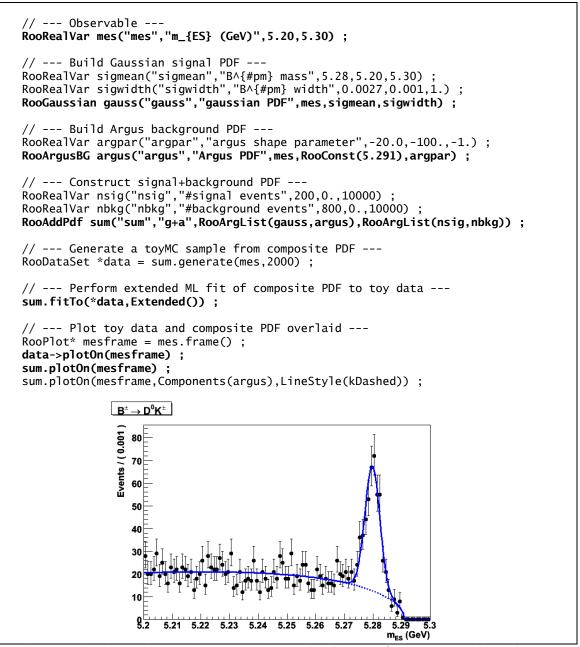
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#### What is RooFit?

The RooFit library provides a toolkit for modeling the expected distribution of events in a physics analysis. Models can be used to perform likelihood fits, produce plots, and generate "toy Monte Carlo" samples for various studies. The RooFit tools are integrated with the object-oriented and interactive ROOT graphical environment.

RooFit was originally developed for the BaBar collaboration, a particle physics experiment at the Stanford Linear Accelerator Center. This software is primarily designed as a particle physics data analysis tool, but its general nature and open architecture make it useful for other types of data analysis also.



Example 1 – A non-trivial code example: Using RooFit to perform an extended unbinned maximum likelihood fit of a Gaussian signal on top of a Argus shaped background to toy Monte Carlo data sampled from the same model.

# 1. Installing RooFit

### ROOT5

RooFit is distributed as external package with ROOT and is integrated into its make system from ROOT version 5.02 onward. Binary distributions of ROOT5 are shipped with pre-compiled RooFIt libraries and for those distributions you do not need to do anything special to obtain roofit. For source distributions of ROOT5, obtained either as source tarball, or from CVS you need to indicate in the makefile configuration that you wish to compile RooFit as well. To enable building of the RooFit library in ROOT5 add the <code>-enable-roofit</code> option to the configure command when you install ROOT.

```
unix> ./configure <platform_id> --enable-roofit
unix> make
```

If you have a full-source installation of ROOT you can also at any moment upgrade RooFit to the latest version in your existing installation of ROOT5 by downloading a new source tarball from http://roofit.sourceforge.net/summary.php. Go to 'File Releases' and down the latest 'RooFit (ROOT integrated)' tarball file. Untar the tarball to your ROOT installation directory and remake ROOT.

```
unix> cp roofit_vXYZ_root5kit.tar $ROOTSYS
unix> cd $ROOTSYS
unix> rm -rf roofit/
unix> tar -xvf roofit_vXYZ_root5kit.tar
unix> make
```

### ROOT3 and ROOT4

RooFit also works with ROOT versions 3 and 4, but you need a full-source version of ROOT, either from a tarball or CVS and a source tarball of RooFit that you can obtain from the RooFit home page, as explained above. Download the roofit\_vXYZ\_root4kit.tar file and untar it in your ROOTSYS directory and issue a make command to rebuild ROOT. The root4kit tarball contains some extra makefile fragments that configure ROOT3/4 to recognize RooFit as a module.

# Loading RooFit in ROOT

Once the RooFit library is available in \$ROOTSYS/1ib, you can use it in ROOT by executing the following commands

```
root> gSystem->Load("libRooFit") ;
root> using namespace RooFit ;
```

Be sure not to forget the second line, otherwise you will not see some of the helper functions that RooFit defines in the global namespace.

# 2. Getting started

In this section we will guide you through a simple exercise of building a model and fitting it to data. The aim is to familiarize you with several basic concepts and get you to a point where you can do something useful yourself quickly. In subsequent sections we will explore several aspects of RooFit in more detail

# **Building a model**

A key concept in RooFit is that models (i.e. functions) are built in a truly object-oriented fashion. Each RooFit class has a one-to-one correspondences to a mathematical object: there is a class to express a variable, RooRealVar, a base class to express a function, RooAbsReal, a base class to express a probability density function, RooAbsPdf, to name a few. As even the simplest mathematical functions consists of multiple objects – i.e. the function itself and its variables – all RooFit models also consist of multiple objects. The following example illustrates this

```
RooRealVar x("x","x",-10,10) ;
RooRealVar mean("mean","Mean of Gaussian",0,-10,10) ;
RooRealVar sigma("sigma","Width of Gaussian",3,-10,10) ;
RooGaussian gauss("gauss","gauss(x,mean,sigma)",x,mean,sigma) ;
```

Example 2 - Construct a Gaussian probability density function

Each variable used in gauss is initialized with several properties: a name, a title, a range and optionally an initial value. Variables described by RooRealVar have more properties that are not visible in this example, for example an (a)symmetric error associated with the variable and a flag that specifies if the variable is constant or floating in a fit. In essence class RooRealVar collects all properties that are usually associated with a variable

The last line of code creates a Gaussian probability density function (PDF), as implemented in RooGaussian. Class RooGaussian is an implementation of the abstract base class RooAbsPdf, which described the common properties of all probability density functions. The PDF gauss has a name and a title, just like the variable objects, and is linked to the variables x, mean and sigma through the references passed in the constructor.

# Visualizing a model

The first thing we usually want to do with a model is to see it. RooFit takes slightly more formal approach to visualization than plain ROOT. First you have to define a 'view', essentially an empty plot frame with one of the RooReaTVar variables along the x-axis. Then, in OO style, you ask your model plot itself on the frame. Finally you draw the view on a ROOT TCanvas:

```
RooPlot* xframe = x.frame() ;
gauss.plotOn(frame) ;
frame->Draw()
```

The result of this example is shown in Figure 1. Note that in the creation of the view we do not have to specify a range, it is automatically taken from the range associated with the RooRealVar. It is of course possible to override this, we'll return to this later. Note also that when gauss draws itself on the frame we don't have to say that we want to plot gauss as function of x, this information is retrieved from the frame.

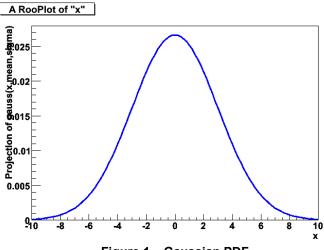


Figure 1 - Gaussian PDF

A frame can contain multiple objects (curves, histograms) to visualize. We can for example draw gauss twice with a different value of parameter sigma.

```
RooPlot* xframe = x.frame() ;
gauss.plotOn(frame) ;
sigma = 2 ;
gauss.plotOn(frame,LineColor(kRed)) ;
frame->Draw()
```

In this example we change the value of RooRealVar sigma after the first plot0n() command using the assignment operator. The color of the second curve is made red through additional LineColor(kRed) argument passed to plot0n()¹. LineColor is an example of a 'named argument'. Named arguments are used throughout RooFit and provide a convenient and readable way to modify the default behavior of methods.Named arguments are covered in more detail in later sections. The output of the second code fragment in shown in Figure 2.

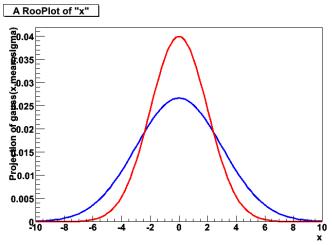


Figure 2 - Gaussian PDF with different widths

<sup>&</sup>lt;sup>1</sup> If you get a ROOT error message at this point because LineColor is not defined, you have forgotten to include 'using namespace RooFit' in your ROOT setup as was explained in Section 1.

The example also demonstrates that method plotOn() make a 'frozen' snapshot of the PDF: if the PDF changes shape after it has been drawn, as happens in the last code fragment, the already drawn curve will not change. Figure 2 also demonstrates that RooGaussian is always normalized to unity, regardless of the parameter values.

### Importing data

Data analysis revolves around, well... data, so the next step is to import some data. Data in general comes in two flavors: unbinned data, represented in ROOT by class TTree and binned data, represented in ROOT by classes TH1, TH2 and TH3. RooFit can work with both.

### Binned data (histograms)

In RooFit, binned data is represented by the RooDataHist class. You can import the contents of any ROOT histogram into a RooDataHist object

```
TH1* hh = (TH1*) gDirectory->Get("ahisto") ;
RooRealVar x("x","x",-10,10) ;
RooDataHist data("data","dataset with x",x,hh) ;
```

Example 3 - Importing data from a TTree and drawing it on a TCanvas

When you import a ROOT histogram the binning of the original histogram is imported as well. A RooDataHist always associates the histogram with a RooFit variable object of type RooRealVar. In this way it always known what kind of data is stored in the histogram.

A RooDataHist can be visualized in the same way as a function can be visualized:

```
RooPlot* xframe = x.frame() ;
data.plotOn(frame) ;
frame->Draw()
```

The result is shown in Figure 3.

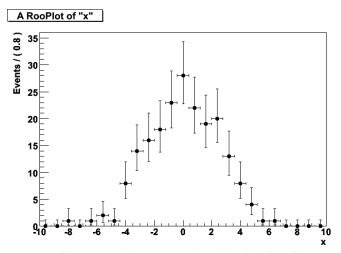


Figure 3 - Histogram visualized in RooFit

If you look closely at Figure 3 you will see that the error bars for entries at low statistics are not symmetric. This is not a mistake but a feature: at low statistics symmetric Gaussian errors of magnitude  $\sqrt{N}$  are only an approximation of the actual statistical uncertainty on a bin with N entries.

RooFit by default shows the 68% confidence interval for Poisson statistics<sup>2</sup>, which is more difficult to calculate but also more accurate. Appendix C includes some basic statistics reading material that covers this and other issues.

### Unbinned data (trees)

Unbinned data can be imported in RooFit much along the same lines and is store in class RooDataSet

```
TTree* tree = (TTree*) gDirectory->Get("atree") ;
RooRealVar x("x","x",-10,10) ;
RooDataSet data("data","dataset with x",x,tree) ;
```

In this example tree is assumed to have a branch named "x" as the RooDataSet constructor will import data from the tree branch that has the same name as the RooRealVar that is passed as argument.

Plotting unbinned data is similar to plotting binned data with the exception that you can now show it in any binning you like.

```
RooPlot* xframe = x.frame() ;
data.plotOn(frame,Binning(25)) ;
frame->Draw()
```

In this example we have overridden the default setting of 100 bins using the Binning() named argument.

### Working with data

In general working with binned and unbinned data is very similar in RooFit as both class RooDataSet (for unbinned data) and class RooDataHist (for binned data) inherit from a common base class, RooAbsData, which defines the interface for a generic abstract data sample. With few exceptions, all RooFit methods take abstract datasets as input arguments, making it easy to use binned and unbinned data interchangeably.

The examples in this section have always dealt with one-dimensional datasets. Both RooDataSet and RooDataHist can however handle data with an arbitrary number of dimensions. In the next sections we will revisit datasets and explain how to work with multi-dimensional data.

# Fitting a model to data

Fitting a model to data can be done in many ways. The most common methods are the  $\chi^2$  fit and the – log(L) fit. The default fitting method in ROOT is the  $\chi^2$  method, the default method in RooFit is the – log(L) method. We prefer the –log(L) method because it is more robust for low statistics fits and because it can also be performed on unbinned data. If you are unfamiliar with the basics of likelihood

<sup>&</sup>lt;sup>2</sup> To be more precise the intervals shown are 'classic central' intervals as described in Table I of Cousins, Am. J. Phys. 63, 398 (1995)

fitting we suggest you read through appendix C, which contains an easy introduction to the statistical theory behind  $\chi^2$  and  $-\log(L)$  fitting and compares their relative advantages and disadvantages.

In practice both fitting techniques work very similar: first you construct the estimator quantity – either  $\chi^2$  or –log(L) – and then you perform the fit by finding the minimum value of the estimator with respect to all the parameters of the model. The errors on the fitted parameters are defined by the variation of the parameters that leads to a unit/half-unit increase of the  $\chi^2$ /-log(L) respectively.

The standard tool in High Energy Physics to perform the minimization and error analysis since decades is MINUIT, and also RooFit delegates the minimization task to the ROOT implementation MINUIT in class TMinuit. RooFit is therefore more of a data modeling package rather than a fitting package.

The high-level interface to model fitting in RooFit packages all of the above in a very easy-to-use interface:

```
gauss.fitTo(data) ;
```

This command builds a  $-\log(L)$  function from the gauss function and the given dataset, passes it to MINUIT, which minimizes it and estimate the errors on the parameters of gauss. The output of the fitTo() method produces the familiar MINUIT output on the screen:

```
** 13 **MIGRAD
                                1000
                                                    1
FIRST CALL TO USER FUNCTION AT NEW START POINT, WITH IFLAG=4.
START MIGRAD MINIMIZATION. STRATEGY 1. CONVERGENCE WHEN EDM .LT. 1.00e-03
FCN=25139.4 FROM MIGRAD STATUS=INITIATE 10 CALLS 11 TOTAL
                                                                 1 NO ERROR MATRIX
STEP
                                      nown STRATEGY= 1
CURRENT GUESS ST
ERROR ST
                         EDM= unknown
 EXT PARAMETER
        NAME
                      VALUE
                                                                               DERIVATIVE
                                                                 SIZE
 NO.
  1 mean
2 sigma
                      -1.00000e+00
3.00000e+00
                                          1.00000e+00
1.00000e+00
                                                             1.00000e+00
1.00000e+00
                                                                              -6.53357e+01
-3.60009e+01
                                      ERR DEF= 0.5
MIGRAD MINIMIZATION HAS CONVERGED.
MIGRAD WILL VERIFY CONVERGENCE AND ERROR MATRIX.
COVARIANCE MATRIX CALCULATED SUCCESSFULLY
FCN=25137.2 FROM MIGRAD STATUS=CONVERGED
EDM=8.3048e-07 STRATI
                                                                 33 CALLS
                                                                 34 TOTAL
L ERROR MATRIX ACCURATE
STEP
                                               STRATEGY= 1
 FXT PARAMETER
                                                                              FIRST
DERIVATIVE
                                           ERROR
                      VALUE
 NO. NAME
                                                                 SIZE
  1 mean
2 sigma
                       -9.40910e-01 3.03997e-02
3.01575e+00 2.22446e-02
                                          3.03997e-02 3.32893e-03 -2.95416e-02
2.22446e-02 2.43807e-03 5.98751e-03
                                       ERR DEF= 0.5
EXTERNAL ERROR MATRIX.
                                  NDTM= 25
                                                                   ERR DEF=0.5
                                                   NPAR= 2
9.241e-04 -1.762e-05
-1.762e-05 4.948e-04
PARAMETER CORRELATION COEFFICIENTS
       NO. GLOBAL
         1 0.02606
                          1.000 -0.026
2 0.02606
                        -0.026 1.000
     18 **HFSSF
                              1000
COVARIANCE MATRIX CALCULATED SUCCESSFULLY
                                                             GY= 1 ERROR MATRIX ACCURATE
INTERNAL INTERNAL
FCN=25137.2 FROM HESSE
                          EDM=8.30707e-07
                                                   STRATEGY= 1
 EXT PARAMETER
                      VALUE
                                             FRROR
                                                             STEP STZE
        NAME
                                                                                   VAI IIF
      mean
                      -9.40910e-01 3.04002e-02
3.01575e+00 2.22449e-02
ERR DEF= 0.5
                                                            6.65786e-04 -9.40910e-01
9.75228e-05 3.01575e+00
      sigma
EXTERNAL ERROR MATRIX.
                                  NDIM= 25
                                                   NPAR= 2
                                                                    ERR DEF=0.5
9.242e-04 -1.807e-05
-1.807e-05 4.948e-04
PARAMETER CORRELATION COEFFICIENTS
       NO. GLOBAL
                          1.000 -0.027
             0.02672
         1
            0.02672
                         -0.027
```

The result of the fit – the new parameter values and their errors – is propagated back to the RooRealVar objects that represent the parameters of gauss, as is demonstrated in the code fragment below:

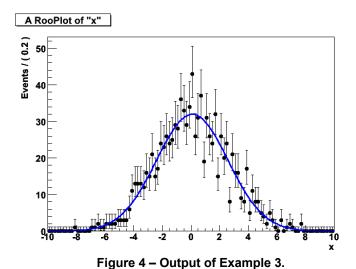
```
mean.Print();
RooRealVar::mean: -0.940910 +/- 0.030400

sigma.Print();
RooRealVar::sigma: 3.0158 +/- 0.022245
```

A subsequent drawing of gauss will therefore reflect the new shape of the function after the fit. We now draw both the data and the fitted function on a frame.

```
RooPlot* xframe = x.frame() ;
data.plotOn(xframe) ;
model.plotOn(xframe) ;
xframe->Draw()
```

The result of this code fragment is shown in Figure 4.



Note that the normalization of the PDF, which has an intrinsic normalization to unity by definition, is automatically adjusted to the number of events in the plot.

A powerful feature of RooFit and one of the main reasons for its inception is that the fit invocation of Example 3 works for both binned *and unbinned* data. In the latter case an unbinned maximum likelihood fit is performed. Unbinned -log(L) fits are statistically more powerful than binned fits (i.e. you will get smaller errors on averages) and avoid any arbitrariness that is introduced by a choice of bin width. These advantages are most visible when fitting small datasets and fitting multidimensional datasets.

The fitting interface to RooFit is highly customizable and easily customizable. For example, if you want fix a parameter in the fit, you just specify that as a property of the RooRealVar parameter object so that this

```
mean.setConstant(kTRUE) ;
gauss.fitTo(data) ;
```

Repeats the fit with parameter mean fixed to its present value. Similarly, you can choose to bound a floating parameter to range of allowed values:

```
sigma.setRange(0.1,3) ;
gauss.fitTo(data) ;
```

All such fit configuration information is automatically passed to MINUIT. Higher level aspects of MINUIT can be controlled through optional named arguments passed to the fitTo() command. This example enables the MINOS method to calculate asymmetric errors and changes the MINUIT verbosity level

```
gauss.fitTo(data, Minos(kTRUE), PrintLevel(-1));
```

The way the likelihood function is constructed can be influenced the same way. To restrict the likelihood (and thus the fit) to a smaller range of x values do

```
gauss.fitTo(data, Range(-5,5)) ;
```

A subsequent plot of this fit will then only show a curve in the fitted range (Figure 5).

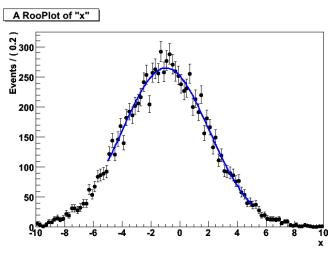


Figure 5 - Fit to a subset of the data

RooFit also supports extended maximum likelihood fits as well as  $\chi^2$  fits. These will be covered in the next sections. The complete range of fitting options as well as ways to do interactive fitting are documented in Appendix A.

# Generating data from a model

RooFit is not just a fitting tool, it is a full-fledged data modeling tool. This means that you can do more with your models that just fit them to data. An important piece of other functionality is the ability to generate 'toy' Monte Carlo data from your model. Generically this is done through sampling your PDF, but smarter techniques may be used behind the scenes for certain shapes, such as a Gaussian. The most efficient technique is automatically selected for you so you don't have worry about this. In it simplest form you can generate a RooDataSet from a pdf as follows:

```
RooDataSet* data = gauss.generate(x,10000) ;
```

This example create a RooDataSet with 10000 events with observable x sampled from pdf gauss.

Sampling datasets from your PDF is often a useful technique to study the stability of your fit, which may become a concern if your are aiming to fit a small number of events or a large sample with a small number of signal events. Sampling allows you to quickly generate similar but statistically independent datasets on which you can exercise your fit. Section 11 has more details on fit stability studies and techniques to automate them in RooFit.

### Putting it all together

At this point we have guided you through various basic procedures in RooFit: defining a model and its variables, importing data, fitting the model to data and generating data from sampling the model. The following macro puts all the knowledge together into a concise exercise that demonstrates all of these abilities and can serve as starting point for your first fitting exercise in RooFit.

```
// Elementary operations on a gaussian PDF
void example(const TH1* histo=0)
  // Build Gaussian PDF
  RooRealVar x("x","x",-10,10);
  RooRealVar mean("mean", "mean of gaussian", 0, -100, 100);
  RooRealVar sigma("sigma", "width of gaussian", 3,0.,10.); RooGaussian gauss("gauss", "gaussian PDF", x, mean, sigma);
  RooAbsData* data = 0 :
  if (histo) {
    // If a histogram is given import it into a RooDataHist - Binned data
    data = new RooDataHist("data", "data", x, histo);
    // If no histogram is given, generate some toy data - Unbinned data
    data = gauss.generate(x,10000);
  }
  // Fit the model to the data
  // Note here that fitTo accepts both binned and unbinned data
  gauss.fitTo(*data);
  // Plot PDF and toy data overlaid
  RooPlot* xframe = x.frame() ;
  data->plotOn(xframe) ;
  gauss.plotOn(xframe) :
  xframe->Draw();
  // Print final value of parameters
  mean.Print();
  sigma.Print();
  // Delete the data
  delete data;
```

In the next section we'll work towards more realistic models: we will explore composite models – multiple PDFs added together – for example a PDF representing your signal and a PDF representing your background.

# 3. Signal and Background - Composite models

### Introduction

One of the most common data analysis scenarios is that you want to determine the amount of signal and background in a given data sample through a fit. The most straightforward approach to such an analysis is to define a composite p.d.f. M(x) as follows

$$M(x) = f \cdot S(x) + (1-f) \cdot B(x)$$

In this formula M(x) is your fit model, S(x) is your signal model, B(x) is your background model and f is the fraction of your event that are part of the signal. RooFit provide a special 'addition operator' p.d.f. in class RooAddPdf to simplify building and using such composite p.d.f.s. A elegant property of adding p.d.f.s in this way is that M(x) does not need to be explicitly normalized to one: if both S(x) and B(x) are normalized to one then M(x) is – by construction – also normalized.

#### The extended likelihood formalism

Often one is not interested in the fraction of signal events in your sample but in the *number* of signal events in your sample.

$$M_E(x) = N_S \cdot S(x) + N_B \cdot B(x)$$

In this formula  $M_E(x)$  is not normalized to 1 but to  $N_S+N_B=N$ , the number of events in the data sample. A model of this type can be fit with equal ease, but an extra piece of information has to be added to the fit (more specifically: to the likelihood function): the number of events in the data sample. With that extra piece of information the fit can relate the number of events expected by the model  $(N_{exp}=N_S+N_B)$ , to the actual number of events in the data  $(N_{obs})$ . The technique that takes care of this extra constraint is called the *extended* maximum likelihood formalism and is described in more detail in Appendix C. All you need to know for now is that RooFit supports both forms of adding p.d.f.s.

### **Building composite models**

Here is a simple example of a composite PDF constructed with RooAddPdf using fractional coefficients.

```
RooRealVar x("x","x",-10,10) ;
RooRealVar mean("mean","mean",0,-10,10) ;
RooRealVar sigma("sigma,"sigma",2,0.,10.) ;
RooGaussian sig("sig","signal p.d.f.",x,mean,sigma) ;

RooRealVar c0("c0","coefficient #0", 1.0,-1.,1.) ;
RooRealVar c1("c1","coefficient #1", 0.1,-1.,1.) ;
RooRealVar c2("c2","coefficient #2",-0.1,-1.,1.) ;
RooChebychev bkg("bkg","background p.d.f.",x,RooArgList(c0,c1,c2)) ;

RooRealVar fsig("fsig","signal fraction",0.5,0.,1.) ;

// model(x) = fsig*sig(x) + (1-fsig)*bkg(x)
RooAddPdf model("model","model",RooArgList(sig,bkg),fsig) ;
```

Example 4 - Adding two pdfs using a fraction coefficient

In this example we first construct a Gaussian p.d.f sig and flat background p.d.f bkg and then add them together with a signal fraction fsig in model.

Note the use the container class RooArgList to pass a list of objects as a single argument in a function. RooFit has two container classes: RooArgList and RooArgSet. Each can contain any number RooFit value objects, i.e. any object that derives from RooAbsArg such a RooRealVar, RooAbsPdf etc. The distinction is that a *list* is ordered, you can access the elements through a positional reference (2<sup>nd</sup>, 3<sup>rd</sup>,...), and can may contain multiple objects with the same name, while a set has no order but requires instead each member to have a unique name. You can read more about the properties of RooArgSet and RooArgList in Section 11.

The number of components a RooAddPdf can sum together is not restricted to 2, you can add any arbitrary number of components. Here is an example that adds three p.d.f.s with two coefficients:

When you build a 'regular' p.d.f, i.e. when you fit for fractions rather than numbers of events, the number of coefficients should always be one less than the number of p.d.f.s.

### The extended likelihood formalism

Here is a revision of the first example that uses the extended likelihood formalism, i.e it implements formula (2) rather than formula (1):

```
RooRealVar x("x","x",-10,10) ;
RooRealVar mean("mean","mean",0,-10,10) ;
RooRealVar sigma("sigma","sigma",2,0.,10.) ;
RooGaussian sig("sig","signal p.d.f.",x,mean,sigma) ;

RooRealVar c0("c0","coefficient #0", 1.0,-1.,1.) ;
RooRealVar c1("c1","coefficient #1", 0.1,-1.,1.) ;
RooRealVar c2("c2","coefficient #2",-0.1,-1.,1.) ;
RooChebychev bkg("bkg","background p.d.f.",x,RooArgList(c0,c1,c2)) ;

RooRealVar nsig("nsig","signal fraction",500,0.,10000.) ;
RooRealVar nbkg("nbkg","background fraction",500,0.,10000.) ;
RooAddPdf model("model","model",RooArgList(sig,bkg),RooArgList(nsig,nbkg)) ;

// shape: model(x) = nsig/(nsig+nbkg)*sig(x) + nbkg/(nsig+nbkg)*bkg(x)
// norm: Nexpect = nsig + nbkg

// Combined: Nexpect*model(x) = nsig*sig(*x) + nbkg*bkg(x)
```

Example 5 - Adding two pdfs using two event count coefficients

The only difference between Example 4 and Example 5 is that you supply RooAddPdf with an equal number of models and coefficients.

In practical terms the difference between the first and the second example is that in the second form the RooAbsPdf object model is capable of predicting the *expected* number of data events (i.e. nsig+nbkg) through its member function expectedEvents(), while model in the first form cannot. This extra functionality provides the information necessary to construct the extended likelihood.

NB: When you fit extended likelihood models such as Example 5 you should explicitly invoke the construction of extended likelihood term in the fitTo() operation using the Extended() option as will explained shortly in the fitting section

### Compose recursively

Note that the input p.d.f.s of RooAddPdf do not need to be basic p.d.f.s, they can be composite p.d.f.s themselves. Take a look at this example that uses sig and bkg from Example 5 as input:

```
RooRealVar mean_bkg("mean_bkg","mean",0,-10,10);
RooRealVar sigma_bkg("sigma_bkg,"sigma",2,0.,10.);
RooGaussian bkg_peak("bkg_peak","peaking bkg p.d.f.",x,mean_bkg,sigma_bkg);

RooRealVar fpeak("fpeak","peaking background fraction",0.1,0.,1.);
RooRealVar fbkg("fbkg","background fraction",0.5,0.,1.);

RooAddPdf sigpeak("sigpeak","sig + peak",RooArgList(bkg_peak,sig),fpeak);
RooAddPdf model("model","bkg + sigpeak",RooArgList(bkg,sigpeak),fbkg);
```

Example 6 - Adding three p.d.f.s through recursive addition of two terms

The code in this example corresponds to the following formula

```
M(x) = [(1-f_1)S(x) + f_1B(x)](1-f_2) + f_2B_2(x)
= (1-f_1)(1-f_2)S(x) + f_1(1-f_2)B_1(x) + f_2B_2(x)
```

schnical Note

Why would you do something like this? First, it might be more intuitive to interpret the fraction parameters this way, but that depends on your specific problem. Second because your fit is more stable this way: If you add two components with a single fraction you can bound the fit to stay in the well defined region by bounding the fraction parameter between 0 and 1. If you have three components and two fraction parameters, the 'well defined' region is the region where the *sum* of the fraction parameters is less than one. If you only require each fraction to be inside the range (0,1) you can still end up with a solution where sum is greater than one and consequently the coefficient of the 3<sup>rd</sup> component is negative. By defining the fractions recursively, as done above, all solutions are well defined as long as each fraction is inside the range (0,1). Another solution is to use the extended likelihood formalism, which sidesteps this problem all together. Also note that if you allow any of the fractions to be exactly zero, the likelihood becomes insensitive to the parameters of the model that is multiplied by this zero fractio. If any of these parameters is floating in the fit, the fit will not converge.

# Plotting composite models

The modular structure of a composite p.d.f. allows you to address the individual components. One can for example plot the individual components of a composite model on top of that model to visualize its structure.

```
RooPlot* frame = x.frame() ;
model.plotOn(frame) ;
model.plotOn(frame, Components(bkg),LineStyle(kDashed)) ;
frame->Draw() ;
```

The output of this code fragment is show in Figure 6. You can reference the components by object reference, as is done above, or by name:

```
model.plotOn(frame, Components("bkg"),LineStyle(kDashed));
```

The latter is convenient when your plotting code has no access to the component objects, for example if your model is built in a separate function that only returns the top-level RooAddPdf object.

If you want to draw the sum of multiple components you can do that in two ways as well:

```
model.plotOn(frame, Components(RooArgSet(bkg1,bkg2)),LineStyle(kDashed)) ;
model.plotOn(frame, Components("bkg1,bkg2"),LineStyle(kDashed)) ;
```

Note that in the latter form wildcards are allowed so that a well chosen component naming scheme allows you for example to do this:

```
model.plotOn(frame, Components("bkg*"),LineStyle(kDashed)) ;
```

If required multiple wildcard expressions can be specified in a comma separated list.

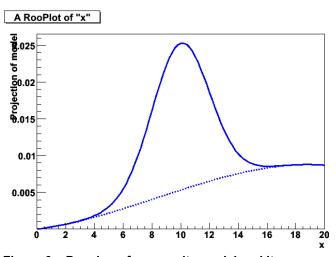


Figure 6 - Drawing of composite model and its components

# Fitting composite models

Fitting composite models with fractional coefficients is no different from fitting any other model:

```
model.fitTo(data) ;
```

But fitting models with event count coefficients *is* essentially different: the 'extended likelihood term', the extra piece of the likelihood that constrains the number of events predicted by the model to be equal to the number of observed events in data must be added to the regular likelihood function for the fit to succeed. You do this with the Extended() named argument in fitTo():

```
model.fitTo(data,Extended(kTRUE)) ;
```

If you forget to do this, no specific warning message will be issued, but the fit will not converge because there is one degree of freedom that cannot be constrained. This is one of the most common mistakes made in extended likelihood fitting.

# Generating data with composite models

Just like you generate toy Monte Carlo data from a simple model you can generate toy data from a composite model:

```
// Generate 10000 events
RooDataSet* x = model.generate(x,10000) ;
```

Sampling data from a composite p.d.f. is often more efficient than sampling data from a monolithic p.d.f. with the same shape as RooFit makes effective use of the component structure of a composite p.d.f.

#### The extended likelihood formalism

Some extra features apply to composite models built for the extended likelihood formalism. Since these model predict a number events one can omit the requested number of events to be generated

```
RooDataSet* x = model.generate(x) ;
```

In this case the number of events predicted by the p.d.f. is generated. You can optionally request to introduce a Poisson fluctuation in the number of generated events trough the Extended() argument:

```
RooDataSet* x = model.generate(x, Extended(kTRUE)) ;
```

This is useful if you generate many samples as part of a study where you look at pull distributions. For pull distributions of event count parameters to be correct, a Poisson fluctuation on the total number of events generated should be present. Fit studies and pull distributions are covered in more detail in section 10.

# General tools for dealing with composite objects

The logistics of creating all your p.d.f. components – and keeping tracking of them – become increasing difficult as your p.d.f. grows in complexity. This section describes some of the tools at your disposal to keep this task as easy possible.

An important feature in the design of RooFit is that all important operations – fitting, generating and plotting – can be performed through the top level p.d.f. object. This means that you can delegate the building of a complex p.d.f. to a designated function that just returns a pointer to the top level p.d.f. component, as is illustrated below

```
{
   RooRealVar x("x","x",-10,10) ;
   RooAbsPdf* model = buildPdf(x) ;

   RooDataSet* data = model->generate(x,1000) ;
   model->fitTo(*data) ;

   RooPlot* frame = x.frame() ;
   data->plotOn(frame) ;
   model->plotOn(frame) ;
   model->plotOn(frame,Components("bkg")) ;
   frame->Draw() ;
}
```

# Example 7 – Building your model in a separate function. (Memory management issues ignored for the moment for clarity)

While all the big operations clearly work fine this way, it is not obvious how one would for example adjust a parameter value in Example 7 in doTheFit(), or print out its value after the fit, so we need some extra tools.

### What are the variables of my model?

Given any composite RooFit value object, the getVariables() method returns you a RooArgSet with all parameters of your model:

```
RooArgSet* params = model->getVariables() ;
params->Print("v") ;
```

This code fragment will output

```
RooArgSet::parameters:
1) RooRealVar::c0: "coefficient #0"
2) RooRealVar::c1: "coefficient #1"
3) RooRealVar::c2: "coefficient #2"
4) RooRealVar::mean: "mean"
5) RooRealVar::nbkg: "background fraction"
6) RooRealVar::nsig: "signal fraction"
7) RooRealVar::sigma: "sigma"
8) RooRealVar::x: "x"
```

If you know the name of a variable, you can retrieve a pointer to the object through the find() method of RooArgSet:

```
RooRealVar* c0 = (RooRealVar*) params->find("c0") ;
c0->setVal(5.3) ;
```

If no object is found in the set with the given name, find() returns a null pointer.

Although sets can contain any RooFit value type (i.e. any class derived from RooAbsArg) one deals in practice usually with sets of all RooRealVars. Therefore class RooArgSet is equipped with some special member functions to simplify operations on such sets. The above example can be shortened to

```
params->setRealValue("c0",5.3) ;
```

Similarly, there also exists a member function getRealValue().

### What is the structure of my composite model?

In addition to manipulation of the parameters one may also wonder what the structure of a given model is. For an easy visual inspection of the tree structure of a composite object use the method printCompactTree():

```
model.printCompactTree() ;
```

The output will look like this:

For each lists object you will see the pointer to the object, following by the class name and object name and finally the object title in parentheses.

A composite object tree is traversed top-down using a depth-first algorithm. With each node traversal the indentation of the printout is increased. This traversal method implies that the same object may appear more than once in this printout if it is referenced in more than one place. See e.g. the multiple reference of observable x in the example above.

Finally we mention the method getComponents(), which returns all the 'branch' nodes of a composite objects and is complementary to getVariables(), which returns the 'leaf' nodes. The example below illustrates the use of getComponents() to only print out the variables of model component "sig":

```
RooArgSet* comps = model.getComponents() ;
RooAbsArg* sig = comps->find("sig") ;
RooArgSet* sigVars = sig->getVariables() ;
sigVars->Print() ;
```

Note that the output of most operations is of type RooAbsArg, the abstract value type in RooFit. Since the tree structure inspection functions are not specific to real-valued positive-definite probability density functions, we can perform all operations with these RooAbsArg\* abstract value type pointers. The output of this example is

```
RooArgSet::parameters:
1) RooRealVar::mean: "mean"
2) RooRealVar::sigma: "sigma"
3) RooRealVar::x: "x"
```

In section 10 will we go into more detail on this subject.

# Putting it all together

In this section you have learned how to add basic p.d.f.s together into a composite p.d.f.s. Adding p.d.f.s can be done in one of two ways: you can add N p.d.f.s with N-1 fractions, or your can N p.d.f.s together with N event counts. The latter form involves the extended likelihood formalism and implies that you fit for the number of events in data as well as the shape of the data. Generating, fitting and plotting composite p.d.f.s is identical to generating, fitting and plotting basic p.d.f.s., except for occasional extra functionality, such as the ability to plot components of a composite p.d.f.

# 4. Choosing & adjusting standard p.d.f. components

We will now have a closer look at what p.d.f.s are provided with RooFit, how you can tailor them to your specific problem and how you can write a new p.d.f.s in case none of the stock p.d.f.s. have the shape you need.

### What p.d.f.s are provided?

RooFit provides a library of about 20 probability density functions that can be used as building block for your model. These functions include basic functions, non-parametric functions, physics-inspired functions and specialized decay functions for B physics.

#### **Basic functions**

The most frequently used basic shapes, the Gaussian, exponential and polynomial functions are all implemented in RooFit. Their shapes are illustrated in Figure 7

Name	Functional form	Class name
Gaussian	$\exp\left(-0.5\left(\frac{x-m}{s}\right)^2\right)$	RooGaussian(name,title,x,m,s)
Exponential	$\exp(a \cdot x)$	<pre>RooExponential(name,title,x,a)</pre>
Polynomial	$1 + \sum_{i=1,n} a_i x^i$	RooPolynomial(name,title,x,alist)
Chebychev polynomial	$1 + \sum_{i=1,n} a_i T_i(x)$	RooChebychev(name,title,x,alist)

Table 1 - Basic functions implemented in RooFit

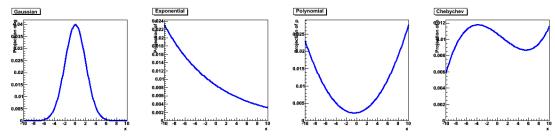


Figure 7 - Basic p.d.f shapes: Gaussian, Exponential, Polynomial and Chebychev polynomial

Note that each functional form in Table 1 has one parameter less than usual form because the degree of freedom that controls the vertical scale is eliminated by the constraint that the integral of the p.d.f. is exactly 1. The formula listed in the table are not normalized to unity for presentation clarity, but each RooAbsPdf-based p.d.f. is internally multiplied by the (analytical) integral of the listed expression to achieve unit normalization.

Practical Tip

We recommend the use of Chebychev polynomials over regular polynomials because of their superior stability in fits. Chebychev polynomials and regular polynomials can describe the same shapes, but a clever reorganization of power terms in Chebychev polynomials results in much lower correlations between the coefficients  $a_i$  in a fit, and thus to a more stable fit behavior. For a definition of the functions  $T_i$  and some background reading, look e.g. at http://mathworld.wolfram.com/ChebyshevPolynomialoftheFirstKind.html

### **Physics inspired functions**

In addition to the basic shapes RooFit also implements a series of shapes that are commonly used to model physical 'signal' distributions.

The Landau function parameterizes energy loss in material and has no analytical form. RooFit uses the parameterized implementation in TMath::Landau.

The Argus function is an empirical formula to model the phase space of multi-body decays near threshold and is frequently used in B physics.

The non-relativistic Breit-Wigner shape models resonance shapes and its cousin the Voigtian – a Breit-Wigner convolved with a Gaussian --- are commonly used to describe the shape of a resonance in the present of finite detector resolution.

The Crystal ball function is a Gaussian with a tail on the low side that is traditionally used to describe the effect of radiative energy loss in an invariant mass.

The decay function differs from the exponential p.d.f in that it can also chosen to be symmetric around 0 and can be convolved analytically with a selection of resolution models.

Their shapes are illustrated in Figure 8.

Name	Functional form	Class name
Landau	TMath::Landau(x,mean,sigma)	RooLandau(name,title,x,mean,sigma)
Argus	$x\left(1-\left(\frac{x}{m}\right)^2\right)^p\cdot\exp\left(c\left(1-\left(\frac{x}{m}\right)^2\right)\right)$	RooArgusBG(name,title,x,m,c,p)
Breit- Wigner	$\frac{1}{\left(x-m\right)^2+\frac{1}{4}g^2}$	RooBreigWigner(name,title,x,m,g)
Voigtian	$\frac{1}{(x-m)^2 + \frac{1}{4}g^2} \otimes \exp\left(-\frac{1}{2}\left(\frac{x}{s}\right)^2\right)$	RooVoigtian(name,title,x,m,g,s)
Crystal Ball	$\frac{\left(\frac{n}{ a }\right)^{n} e^{-\frac{1}{2}a^{2}}}{\left(\frac{n}{ a } -  a  - x\right)^{n}}\bigg _{x < - a },  \exp\left(-\frac{1}{2}\left(\frac{x - m}{s}\right)^{2}\right)\bigg _{x > - a }$	RooCBShape(name,title,x,m,s,a,n)
Decay	$\exp(- x /\tau) \otimes R(x)$	<pre>RooDecay(name,title,x,tau,R)</pre>

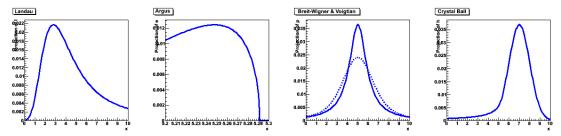


Figure 8 - Physics inspired p.d.f.s: Landau, Argus, Breit-Wigner (Voigtian) and Crystal Ball.

### **Non-parametric functions**

RooFit offers two solutions for modeling distributions that cannot easily be parametrized.

Class RooHistPdf takes an input histogram in the form of a RooDataHist and represents its shape as a probability density. The histogram contents is explicitly scaled to obtain proper normalization. Optionally, the histogram is interpolated (up to 9<sup>th</sup> order).

Class RooKeysPdf is a more elaborate approach to obtain best possible continuous probability density function that aims to describe the parent distribution of an unbinned RooDataSet. The idea behind the KEYS algorithm, documented here<sup>3</sup>, is that the p.d.f. is constructed as a superposition of Gaussians. Each of the events in the input data becomes a Gaussian contribution to the p.d.f. with total weight of 1/N centered at the x value of the data point. The width of the Gaussian is adjusted the local density of events: regions with a low local density of events have a Gaussian with a large width to ensure a smooth function in sparsely populated regions. Areas with a high density of events have Gaussians with a narrow width to preserve details in the structure. Read the original article for further information.

Both classes are shown in Figure 9.

Name	Functional form	Class name
Histogram	Histogram, with optional interpolation	RooHistPdf(name,title,x,hist,intorder)
Empirical density estimate	Superposition of many Gaussians	<pre>RooKeysPdf(name,title,x,data)</pre>

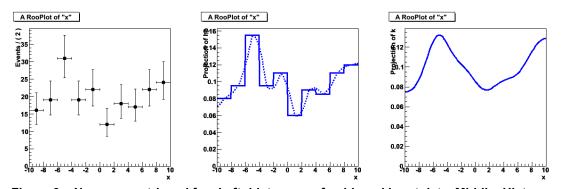


Figure 9 – Non-parametric p.d.f.s: Left: histogram of unbinned input data, Middle: Histogram-based p.d.f (2<sup>nd</sup> order interpolation), Right: KEYS p.d.f from original unbinned input data.

### Specialized functions for B physics

RooFit was originally development for BaBar, the B-factory experiment at SLAC, therefore it also provides a series of specialized B physics p.d.f.s. We will list them here for completeness. A complete description is beyond the scope of this document.

Class Name	Description	
RooBMixDecay	B decay with mixing	
RooBCPEffDecay	B decay with CP violation parameterized as sin(2b) and  I	
RooBCPGenDecay	B decay with CP violation parameterized S and C	
RooNonCPEigenDecay	ecay B decay to non-CP eigenstates with CP violation	
RooBDecay	Generic B decay with mixing, CP violation, CPT violation	

<sup>&</sup>lt;sup>3</sup> 'Kernel Estimation in High-Energy Physics', *K. Cranmer,* Comput.Phys.Commun. 136 (2001) 198-207, hep-ex/0011057

### Plug and play with parameters

You are not stuck with the parameterization of the stock p.d.f.s. that we have chosen. A key feature of the design of RooFit functions and p.d.f.s. is that there is no hard-wired assumption that the parameters of a function are *variables* (i.e. a RooRealVar), so you can modify the parameterization of *any* existing p.d.f. by substituting a *function* for a parameter. The following example illustrates this:

```
RooRealVar x("x","x",-10,10);

RooRealVar mean("mean","mean",0,-10,10);
RooRealVar sigma("sigma_core","sigma (core)",1,0.,10.);
RooGaussian sig_left("sig_left","signal p.d.f.",x,mean,sigma);

RooRealVar shift("shift","shift",1.0);
RooFormulaVar mean_shifted("mean_shifte","mean+shift",RooArgSet(mean,shift));
RooGaussian sig_right("sig_right","signal p.d.f.",x,mean_shifted,sigma);

RooRealVar frac_left("frac_left","fraction (left)",0.7,0.,1.);
RooAddPdf sig("sig","signal",RooArgList(sig_left,sig_right),frac_left);
```

The p.d.f. sig is a sum of two Gaussians in which the position of one Gaussian is shifted by shift with respect to the other one. The mean of the second Gaussian is not specified through a RooRealVar parameter however, but through a RooFormulaVar function objects, which relates the position of the second Gaussian to that of the first Gaussian.

The function that calculates the position of the rightmost Gaussian is an object of type RooFormulaVar, which is a real-valued function that evaluates itself by interpreting the formula expression mean+shift using ROOTs TFormula engine.

While the functional form of the two-Gaussian p.d.f. sig is no different from one constructed of two ordinary Gaussian, each with their own mean, the ability to reparametrize the model like this is that one can now for example fit with a floating mean while keeping the distance between the Gaussians fixed. Figure 10 shows the sig p.d.f. of the above example for mean=-3, mean=3 and shift=3,shift=6 in red and blue respectively.

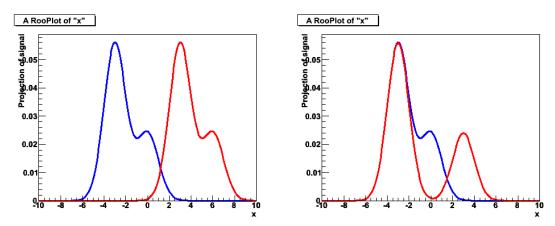


Figure 10 - left: variation of mean variable, right: variation of shift variable

Class RooFormulaVar can handle any C++ expression that ROOT class TFormula can. This includes most math operators (+,-,/,\*,...), nested parentheses and some basic math and trigonometry functions like sin, cos, log, abs etc...Consult the ROOT TFormula documentation for a complete overview of the functionality. The names of the variables in the formula expression are those of the variables

given in the RooArgSet as 3<sup>rd</sup> parameter in the constructor. Alternatively, you can reference the variable through positional index if you pass the variables in a RooArgList:

```
RooFormulaVar mean_shifted("mean_shifte","@0+@1",RooArgList(mean,shift));
```

This form is usually easier if you follow a 'factory-style' approach in your own code where you don't know (or don't care to know) the names of the variables you intend to add in code that declares the RooFormulaVar.

Class RooFormulaVar is explicitly intended for trivial transformations like the one shown above. If you need a more complex transformation you should write a compiled class. The last paragraph of this section shows how you can easily write compilable classes.

### Revisiting the addition of three p.d.f.s.

In the previous section we rewrote the addition of three p.d.f.s. with two fraction coefficients as a recursive addition of two terms to be able to define two fraction parameters that each have a valid range between 0 and 1. The example below accomplishes the same functional form using a single RooAddPdf and a customized coefficient implemented with a RooFormulaVar:

Which form is better – recursive RooAddPdf or using a RooFormulaVar – depends on your specific application. The point here is to demonstrate that you can achieve flexibility in your p.d.f. in more than one way.

# Writing a new p.d.f. class

If none of the existing p.d.f. classes suit your needs, and no one can be customized through use of RooFormulaVar, you can write your own RooFit p.d.f. class. If the formula expression of your model is relatively simple, and performance is not critical you, can use RooGenericPdf which interprets your C++ expression, just like RooFormulaVar:

```
RooRealVar x("x","x",-10,10) ;
RooRealVar alpha("alpha","alpha",1.0,0.,10.) ;
RooGenericPdf g("g","sqrt(abs(alpha*x))+0.1",RooArgSet(x,alpha)) ;

RooPlot* frame = x.frame() ;
g.plotOn(frame) ;
alpha=1e-4 ;
g.plotOn(frame,LineColor(kRed)) ;
frame->Draw() ;
```

The formula expression entered into g is explicitly normalized through numeric integration before it is returned as the value of p.d.f g, so you never have to worry about normalization yourself. The automatic normalization is nicely demonstrated in Figure 11, which shows p.d.f. g for two values of parameter alpha. If your formula expression becomes more complicated than the example shown above, you should write a compiled class that implements your function.

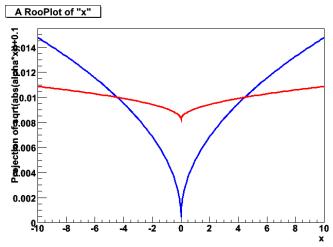


Figure 11 – Generic p.d.f g("sqrt(abs(x\*alpha))+0.1") drawn for alpha=1 (blue) and alpha=0.0001 (red)

### Writing a new p.d.f class using RooClassFactory

A special utility class RooClassFactory greatly simplifies the task of writing a RooFit p.d.f class that is an implementation of RooAbsPdf. The class factory writes a complete working skeleton class for you with the name you specify and with the variable names you specify. Here is an example:

```
RooClassFactory::makePdf("RooMyPdf","x,alpha") ;
```

This example invocation of makePdf creates two files: RooMyPdf.cxx and RooMyPdf.h. The only piece that is missing is actual function expression in terms of the variables you defined. To do so edit the file RooMyPdf.cxx and insert the function expression as return value of the evaluate() method of your class.

```
Double_t RooMyPdf::evaluate() const
{
    // ENTER EXPRESSION IN TERMS OF VARIABLE ARGUMENTS HERE
    return sqrt(abs(alpha*x))+1;
}
```

You are now ready to use your new class: compile the class using ROOTs ACLiC facility

```
root>.L RooMyPdf.cxx++
```

Here is the original example rewritten in terms of your new compiled class RooMyPdf:

```
RooRealVar x("x","x",-10,10);
RooRealVar alpha("alpha","alpha",1.0,0.,10.);
RooMyPdf g("g","compiled class g",x,alpha);
```

Classes that are created through RooClassFactory have an explicit (numeric) normalization step built in, i.e. the return value of evaluate() does not have to be a properly normalized expression. This is done for your convenience, but carries a (small) performance penalty.

If you know how to normalize your expression analytically, you can indicate that in your RooClassFactory invocation and a slightly different skeleton class is built for you that allows to implement the analytical normalization as well. You can find more details in the RooClassFactory HTML class documentation.

### Writing a new function class using RooClassFactory

The code factory class RooClassFactory cannot only write skeleton p.d.f.s, but also skeletons for generic real-valued functions. Generic real-valued function are all classes in RooFit that inherit from RooAbsReal. Class RooFormulaVar is a good example of a generic real-valued function. Unlike p.d.f.s, RooAbsReal are not normalized to unity and can also take negative values.

Compilable custom real-valued functions are a good replacement for RooFormulaVar in cases where the formula expression is less than trivial, or in cases where performance is critical.

Creating a skeleton for a generic function object is done with the makeFunction() method of RooClassFactory

RooClassFactory::makeFunction("RooMyFunction","x,b") ;

# 5. Convolving a p.d.f. or function with another p.d.f.

### Introduction

If you are modeling distribution of an experimental observable you are sometimes faced with a situation where you should explicitly take into account the deformation of the expected signal distributed due to the finite detector resolution. This issue becomes particularly important when the detector resolution is comparable to the structure (width) of your expected signal. The technical aspects combining the effects of detector resolution and your physics model tend to be complicated, which is why we have a separate chapter on this subject.

In general, the observed distribution is described by the convolution of your physics model T(x,a) and your detector response function R(x,b)

$$M(x,a,b) = T(x,a) \otimes R(x,b) = \int_{-\infty}^{+\infty} T(x,a)R(x-x',b)dx'$$

In practice the detector response function R is often a Gaussian, or a superposition of Gaussians. Figure 12 Illustrates the effect of a Gaussian resolution model R with three different widths on a Breit-Wigner function.

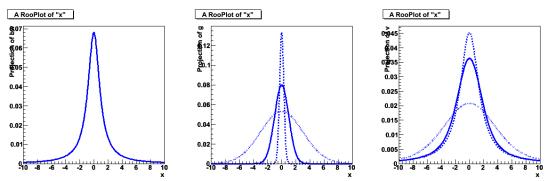


Figure 12 – left: Breit-Wigner, middle Gaussian (σ=0.3, 1, 3) right: Breit-Wigner convolved with Gaussian

You can see from Figure 12 that if R is narrow with respect to T (dotted line), the convolution  $T \otimes R$  is well approximated by T. If R is wide with respect to T (dashed line), the convolution  $T \otimes R$  is well approximated by R, therefore modeling your signal p.d.f. explicitly as  $T \otimes R$  is usually only important if both are comparable width. This is a good thing, since calculation of integral that represents  $T \otimes R$  is generically quite difficult. The normalization condition for p.d.f.s. adds one further difficulty as the final quantity acquires a double integral in the denominator.

$$M(x,a,b) = \frac{\int\limits_{-\infty}^{\infty} T(x,a)R(x-x',b)dx'}{\int\limits_{x_{\min}}^{\infty} \int\limits_{-\infty}^{\infty} T(x,a)R(x-x',b)dx'dx}$$

You are best off if you don't need to perform this calculation, but sometimes you just have to. In the remainder of this section we'll explain how you can deal with convolved p.d.f.s in RooFit.

### **Analytical versus numeric convolution**

A precise and fast calculation of the convolution integral is essential as p.d.f.s are evaluated a large number of times in the course of a fit. Because of that an analytical expression for the convolution integral is therefore strongly preferred. Unfortunately this is not always possible, and a numeric calculation of the integral must sometimes be used as fallback solution.

### **Analytical convolution**

RooFit does not find analytical expressions for convolution integrals for you, but helps you to implement them in a generic and reusable way. It does this by defining two specialize sub-classes of p.d.f.s.: convolvable p.d.f.s, which implement T(x,a) and resolution models, which implement R(x,b). You can combine any R and T at *runtime* into a  $M(x,a,b)=T(x,a)\otimes R(x,b)$  so you are quite flexible in your choice of convolutions when you build your model. RooFit provides the following convolvable p.d.f.s out of the box:

Class Name	Description
RooDecay	Decay function: $\exp(- t /\tau)$ , $\exp(-t/\tau)$ or $\exp(t/\tau)$
RooBMixDecay	B decay with mixing
RooBCPEffDecay	B decay with CP violation parameterized as sin(2b) and  I
RooBCPGenDecay	B decay with CP violation parameterized S and C
RooNonCPEigenDecay	B decay to non-CP eigenstates with CP violation
RooBDecay	Generic B decay with possible mixing, CP violation, CPT violation

And it provides the following resolution models.

Name	Functional form	Class name
Gauss	$\exp\left(-0.5\left(\frac{x-m}{s}\right)^2\right)$	RooGaussModel(name,title,x,m,s)
Gauss⊗Exp	$\exp\left(-0.5\left(\frac{x-m}{s}\right)^2\right) \otimes \exp(-x/\tau)$	RooGExpModel(name,title,x,m,s,tau)
Truth	$\delta(x)$	<pre>RooTruthModel(name,title,x)</pre>
Composite	$\sum_{i=1,n-1} f_i R_i(x,\alpha) + \left(1 - \sum_{i=1,n-1} f_i\right) R_n(x,\alpha)$	<pre>RooAddModel(name,title,Rlist,flist)</pre>

To construct an analytically convolved p.d.f. pass one of the RooResolutionModel implementations to the construct of a convolvable p.d.f. In the example below we construct a decay function convolved with a Gaussian resolution model:

```
RooRealVar x("x","x",-10,10) ;

RooRealVar mean("mean","mean",0) ;
RooRealVar sigma("sigma","sigma",1) ;
RooGaussModel gaussm("gaussm",x,mean,sigma) ;

RooRealVar tau("tau","lifetime",1.54) ;
RooDecay model("model","decay (x) gauss",x,tau,gaussm) ;
```

```
// --- Plot decay (x) gauss ---
RooPlot* frame = x.frame();
model.plotOn(frame);

// --- Overlay with decay (x) truth ---
RooTruthModel truthm("truthm","truth model",x);
RooDecay modelt("modelt","decay (x) delta",x,tau,truthm);
modelt.plotOn(frame,LineStyle(kDashed));

frame->Draw();
```

Figure 133 shows the output of this example.

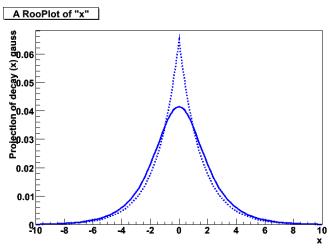


Figure 13 – Decay p.d.f convolved with Gaussian and delta function (dashed)

A realistic detector resolution is often more complex that a simple Gaussian. Class RooAddModel allows you to add multiple resolution models into a single composite resolution model that can be passed to any convolvable p.d.f. Here is an example using RooAddModel to construct a decay function convolved with a double Gaussian resolution.

Class RooAddMode1 works very similar to class RooAddPdf with the restriction that you can only specify fraction coefficients and not event yield coefficients as the extended likelihood formalism doesn't apply to resolution models.

How do classes like RooDecay and RooGaussMode1 divide the work when it comes to performing the analytical integration? First thing to know is that RooAbsAnaConvPdf, the abstract base class for analytically convolvable p.d.f.s decomposes the p.d.f. as follows

 $M(x,a) = \sum_{i} c_i b_i(x)$ 

In this formula  $b_i(x)$  are so-called 'basis functions' and are the common language between a convolvable p.d.f and a resolution model. A resolution model like class RooGaussModel inherits from class RooResolutionModel and implements member functions that advertise if the resolution model can convolve itself with a given b(x). If the resolution model can convolve itself with all of the basis functions  $b_i(x)$  of the p.d.f, the return value of the convolved p.d.f. can the calculated as:

$$M(x,a) = \sum_{i} c_{i} [b_{i}(x) \otimes R(x)]$$

The calculation of the part in the square bracket is delegated to the resolution model object. One of the advantages of this decomposition approach is speed: if parameter model M changes that only affects coefficients  $c_i$ , the convolution integral does not need to be recalculated. Appendix C has additional technical details on the structure and inheritance of classes RooAbsAnaConvPdf and RooResolutionModel and their interaction.

#### **Numeric convolution**

**Technical Note** 

If the convolution of your choice is not available in analytical form, we suggest you first try to calculate it yourself. If an analytical solution exists for your convolution there is a good chance that Wolframs Mathematica can calculate it for you. Try the free web interface on http://integrals.wolfram.com

Numeric integration is computationally intensive as a precision of  $O(10^{-6})$  needs to be reached for the numeric noise not disturb MINUIT in its likelihood minimum finding. In practice this means O(100) evaluations of R and T to calculate M for each data point. Numeric convolution is implemented in class RooNumConvPdf. This class follows the 'operator' formalism: you specify two input p.d.f.s in its construction and its own value is the convolution of the two. Here is an example on how to use RooNumConvPdf.

```
RooRealVar x("x","x",-10,10) ;

RooRealVar meanl("meanl","mean of Landau",2) ;
RooRealVar sigmal("sigmal","sigma of Landau",1) ;
RooLandau landau("landau","landau",x,meanl,sigmal) ;

RooRealVar meang("meang","mean of Gaussian",0) ;
RooRealVar sigmag("sigmag","sigma of Gaussian",2) ;
RooGaussian gauss("gauss","gauss",x,meang,sigmag) ;

RooNumConvPdf model("model","model",x,landau,gauss) ;

RooPlot* frame = x.frame() ;
model.plotOn(frame) ;
landau.plotOn(frame,LineStyle(kDashed)) ;
frame->Draw() ;
```

Example 8 – Numeric convolution of a Landau with a Gaussian

Figure 14 show the result of Example 8.

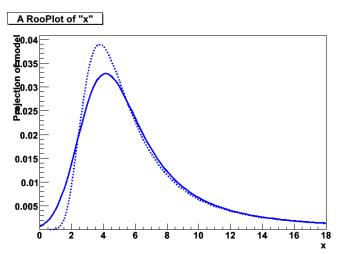


Figure 14 – Output of Example 8 – Numeric convolution of a Landau with a Gaussian, Landau convolved with a Gaussian and the original Landau (dashed line)

### Configuring the numeric convolution integration

By default RooNumConvPdf performs the numeric convolution integral on the full domain of the convolution variable (i.e. from  $-\infty$  to  $+\infty$ ) using a x  $\rightarrow$  1/x transformation to calculate the integrals of the tails extending to infinity. This calculation is difficult, can suffer from stability problems and may be avoided for certain choices of resolution models. For certain resolution models, e.g. a Gaussian, you know a priori that the integrand of the convolution integral is effectively zero when you are far from the core of the resolution model. For such cases one can restrict the domain of the convolution integral to e.g.  $[-5\sigma+\mu,+5\sigma+\mu]$ , where  $\mu$  and  $\sigma$  are the mean and width of the Gaussian resolution model respectively. RooNumConvPdf offers you the option restrict the convolution domain along these lines:

landau.setConvolutionWindow(meang,sigmag,5)

The optional 3<sup>rd</sup> parameter of setConvolutionWindow serves as a multiplier of the width parameter and exists for solely convenience as it saves you a RooFormulaVar: the above example restricts the integration domain to [5\*sigmag+meang,-5\*sigmag+meang].

#### Adjusting numeric integration precision and technique.

If you are going to fit models based on numeric convolutions it is almost inevitable that you will need to fine tune the numeric integration parameters to obtain the right balance between speed and precision. You can access the numeric integration configuration object that is used for the convolution integral from member function convIntConfig(). You can read more about numeric integration configuration in section 11.

*Numeric convolution is an intrinsically difficult problem.* You should expect to spend some time tuning the integration configuration before you obtain a workable configuration (if it is at all possible).

# 6. Using many observables – Multidimensional models

Many data analysis problems deal with more than one observable. Devising a strategy on how to deal with all this information is a central aspect of such analysis. A common strategy is to make a preselection of your data sample using all but one of your observables. Such a preselection can anything ranging from a cut on each of the individual variables to a neural network consolidating the information of many variables into a single observable followed by a cut on that observable. A fit to the distribution of the remaining observable will then determine the number of signal and background events in your sample after preselection and determine the signal properties.

Another – more ambitious – strategy is to use many (or all) of the observables directly in a fit. This has the advantage that the information that is contained in each observable and in the correlation between the observables are optimally exploited and are exploited in a sensible and understood way. This extra power comes at the cost of some additional complications you have to deal with and come in, roughly speaking, two categories: fundamental modeling issues and practicalities. Practical issues usually revolve around your ability to manage the increased complexity of the model and how to visualize multi-dimensional models in an intuitive way. Fundamental issues include your finite ability to truly correctly understand and describe a multidimensional signal and background distribution including all possible correlations between variables.

RooFit has been designed to make working with multi-dimensional models as easy as working with one-dimensional models. Multi-dimensional models have a lot of extra functionality, but working with them is not more cumbersome than working with one-dimensional models. This enables you to design a more ambitious data analysis where you can get the most out of your data. Multidimensional modeling is not an all-or-nothing issue: you can for example combine the information of several variables into a neural network and feed the output of the network along with the remaining observables into a multi-dimensional fit. In this example approach you also let the events that are classified as less-probable by your neural net participate in your analysis and squeeze out some extra statistical power.

This section focuses on the practical aspects of building and using multi-dimensional models at any point in your analysis.

# Building and using multi-dimensional models

A multi-dimensional p.d.f. is a model with more than one observable, but is in all other respects identical to the one-dimensional modes we have covered so far. The normalization condition for multi-dimensional p.d.f.s is identical to that of one-dimensional p.d.f. except that the normalization integral is now performed over all observables:

$$\int F(\vec{x}; \vec{p}) d\vec{x} = 1$$

Here is a very simple example of a two-dimensional p.d.f. constructed with RooFit using RooGenericPdf:

```
RooRealVar x("x","x",-10,10) ;
RooRealVar y("y","y",-10,10) ;

RooRealVar a("a","a",5) ;
RooRealVar b("b","b",2) ;

RooGenericPdf f("f","a*x*x+b*y*y-0.3*y*y*y",RooArgSet(x,y,a,b)) ;
```

Example 9 – A simple two-dimensional p.d.f.

The model f of this example is a 'monolithic' two-dimensional p.d.f, it does cannot be factorized as product of two or more simpler p.d.f.s.

```
// Generate a 2-dimensional dataset data(x,y) from gaussxy
RooDataSet* data = f.generate(RooArgSet(x,y),10000);
// Fit the 2-dimensional model f(x,y) to data(x,y)
f.fitTo(*data) ;
// Plot the x distribution of data(x,y) and f(x,y)
RooPlot* framex = x.frame() ;
data->plotOn(framex) ;
f.plotOn(framex);
// Plot the y distribution of data(x,y) and f(x,y)
RooPlot* framey = y.frame() ;
data->plotOn(framey) ;
f.plotOn(framey) ;
// Draw the x and y frames on a canvas
TCanvas *c = new TCanvas("c","c",800,400);
c->Divide(2);
c->cd(1) ; framex->Draw() ;
c->cd(2) ; framey->Draw() ;
```

Example 10 - A two-dimensional p.d.f. constructed with RooProdPdf

There are several points worth noting about this example. First, generating data works exactly the same for multidimensional p.d.f.s as for one-dimensional datasets, simply supply a RooArgSet of observables instead of single observable as the first argument of generate. Second, fitting also works exactly the same. Lastly, plotting is essentially the same, but have more options now. In the example we created two one-dimensional views: a view in x (framex) and a view in y (framey). Once each view is defined all goes automatic: the two-dimensional dataset data(x,y) plots the appropriate observable in each frame and the two-dimensional p.d.f. fxy(x,y) plots the appropriate projection on the frame. The output of Example 10 is shown in Figure 16.

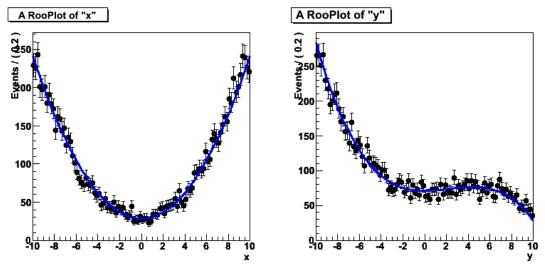


Figure 15 - The x and y projection of p.d.f. f from Example 9.

### A bit more on plotting multi-dimensional p.d.f.s

The fact that the two plots of Example 10 come out as expected is not entirely trivial and reflect some bookkeeping that RooFit does for you in the background. Plotting the data is easy: to obtain the x distribution of data(x,y) you should ignore the y values and fill a histogram with the x values. Plotting a p.d.f. involves a bit more thinking: the x distribution of gaussxy(x,y) is different for each value of y, so we cannot simply plot gauss(x,y) as function of x for a given value of y: we should plot something that matches the x distribution of the data for the given distribution of y values that are in the data. To obtain that shape you need to *integrate* the p.d.f. over y:

$$F_x(x; \vec{p}) = \int F(x, \vec{y}; \vec{p}) d\vec{y}$$

A nice feature of RooFit is that you almost never need to worry about performing such integrals as RooFit keeps track of all the 'projected' observables in any plot that you make. For example, when we plotted data on xframe ('data->plotOn(framex)') not only a histogram representing the distribution in x of data was added to xframe, but also a list of all observables that were stored in data, in this case (x,y). The subsequent call f.plotOn(framex) retrieves this complete list of data observables and compares it to the list of model observables and concludes that both fxy and data have a common observable -y in addition to the plotted x observable. Therefore the RooAbsPdf::plotOn() call automatically integrates fxy over y before adding it to framex to ensure that both data and model represent the same 'view'. Any such transformation in plotting is always announced:

The integrals involved in the creation of p.d.f. projections can be quite cumbersome, e.g. for Example 9 they are

$$f_x(x,a,b) = \frac{\int f(x,y,a,b)dy}{\int \int f(x,y,a,b)dxdy} \quad ; \quad f_y(y,a,b) = \frac{\int f(x,y,a,b)dx}{\int \int f(x,y,a,b)dxdy}$$

where f(x,y,a,b) is the unnormalized expression that was entered in the RooGenericPdf constructor.

# Constructing multi-dimensional p.d.f.s through multiplication

Although it is straightforward to define monolithic multi-dimensional p.d.f.s such as f, they are actually not very common in practice. In many real-life situations you deal with observables that are (nearly) uncorrelated and you construct a multi-dimensional model for such cases by simply multiplying a number of one-dimensional p.d.f.s:

$$F(x, y; p, q) = f(x; p) \cdot g(y; q)$$

The tradeoff between a monolithic p.d.f. and a factorizing product p.d.f is a classic tradeoff between performance and simplicity one side and maximum flexibility and accuracy on the other side. Product p.d.f.s are very elegant in use: if the input p.d.f.s f(x,p) and g(y,q) are both properly normalized than F(x,y;q,p) is automatically normalized too. The interpretation is also straightforward: f(x,p) defines the distribution of the model in x and f(y,q) defines the distribution of the model in y.

The biggest drawback of the product construction is that you cannot introduce correlations between the observables because the product terms are by construction uncorrelated. They can however be introduced in an elegant way through a variant of the product construction: the conditional product. We will come back to this in the next section.

#### Class RooProdPdf

In RooFit the construction of any kind of product p.d.f. is done through class RooProdPdf. Here is a simple example:

```
RooRealVar x("x","x",-10,10) ;
RooRealVar meanx("meanx","meanx",0,-10,10) ;
RooRealVar sigmax("sigmax","sigmax",3,0.,10.) ;
RooGaussian gaussx("gaussx","gaussx",x,meanx,sigmax) ;

RooRealVar y("y","y",-10,10) ;
RooRealVar meany("meany","meany",0,-10,10) ;
RooRealVar sigmay("sigmay","sigmay",2,0.,10.) ;
RooGaussian gaussy("gaussy","gaussy",y,meany,sigmay) ;

RooProdPdf gaussxy("gaussxy","gaussxy",RooArgSet(gaussx,gaussy)) ;

RooDataSet* data = gaussxy.generate(RooArgSet(x,y),10000) ;
gaussxy.fitTo(*data) ;

RooPlot* framex = x.frame() ;
data->plotOn(framex) ;
gaussxy.plotOn(framey) ;
gaussxy.plotOn(framey) ;
gaussxy.plotOn(framey) ;
gaussxy.plotOn(framey) ;
gaussxy.plotOn(framey) ;
gaussxy.plotOn(framey) ;
```

Example 11 - A 2-dimensional p.d.f. constructed as the product of two one-dimensional p.d.f.s

The product p.d.f. gaussxy can be used for fitting and generating in exactly the same way as the monolithic p.d.f. f of Example 9. Note that RooProdPdf can multiply *any* number of components, in this example we multiply two one-dimensional p.d.f.s, but you can equally well multiply e.g. 7 one-dimensional p.d.f.s or 2 five-dimensional p.d.f.s

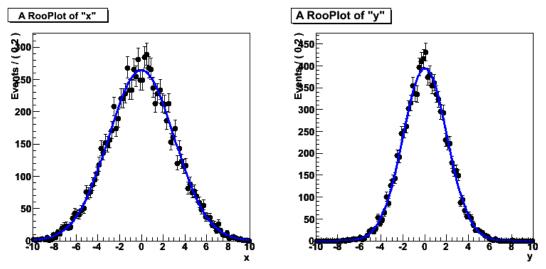


Figure 16 - Output from Example 11

Projection integrals over generic multi-dimensional p.d.f.s such as f are by default created through the createIntegral() method of that p.d.f. and are calculated analytically or numerically depending on the availability of analytical integrals, as advertised by the p.d.f. For multi-dimensional p.d.f.s that are defined as a product of factorizing terms, i.e. RooProdPdf objects, the integral calculation is

automatically factorized as well and often results in a significant simplification of the calculation. For example the integration of gaussxy over y is trivial:

$$F_{x}(x; \vec{p}) = \int f(x; \vec{p})g(\vec{y}; \vec{p})d\vec{y} = f(x; \vec{p}) \int g(\vec{y}; \vec{p})d\vec{y} = f(x; \vec{p}) \cdot 1$$

and comes out to the intuitively expected answer: f(x,p).

### Two-dimensional views

You can also make two-dimensional plots of multi-dimensional p.d.f.s. but he interface to do this is more rudimentary as two-dimensional views lends themselves less to manipulation and layering. It is difficult for example to overlay a two-dimensional view of data and a model and judge by eye if they agree.

In RooFit you can create 2 or 3 dimensional view of datasets and model represented as ROOT TH2 or TH3 objects. The code below creates a two-dimensional histogram of the data and p.d.f. of Example 10 and shows them side by side:

```
TH2* hd = data->createHistogram("hd",x,Binning(20),YVar(y,Binning(20)));
TH2* hf = gaussxy.createHistogram("hf",x,Binning(40),YVar(y,Binning(40)));
TCanvas *c = new TCanvas("c", "c", 800, 400);
c->Divide(2);
c->cd(1) ; hdata->Draw("lego") ;
c->cd(2) ; hpdf->Draw("surf") ;
```

Example 12 - Generating two-dimensional plots of data and p.d.f.

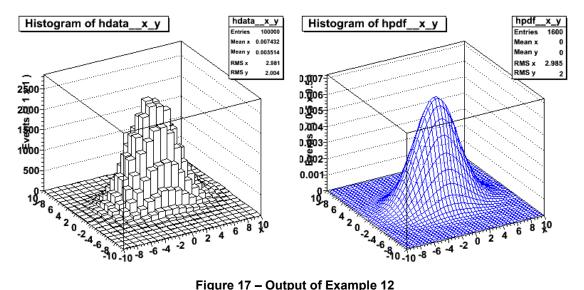


Figure 17 - Output of Example 12

The createHistogram() method of both RooAbsReal and RooAbsData can generate 1,2 and 3 dimensional ROOT histograms depending on the arguments. The option v and z variable are specified through the YVar() and ZVar() named arguments. The binning in each variables can be specified through the Binning() named argument. You can also restrict the range to be histogrammed in each dimension through a Range() named argument. Appendix A documents all options of the createHistogram() method.

### Showing your multi-dimensional signal – slices and projections

Most of the new topics you encounter when going from one-dimensional to multi-dimensional models are in the area of visualization. Instead of a single 'view' of a model, you have multiple views: one for each observable. Alternatively you can make 2- or 3-dimensional views of models, but as mentioned before they are less easy to interpret. In this section we explore other ways to look at you N-dimensional model through 'slice' views: e.g. show the distribution of x in the 'signal region' of y. We will illustrate the concept using this simple two-dimensional model:

```
//--- Observables ---
RooRealVar x("x","x",-10,10);
RooRealVar y("y","y",-10,10);

//--- Signal p.d.f. ---
RooRealVar sigmax("sigmax","sigmax",3,0.,10.);
RooRealVar sigmax("sigmax","sigmax",3,0.,10.);
RooGaussian gaussx("gaussx","gaussx",x,meanx,sigmax);

RooRealVar meany("meany","meany",0,-10,10);
RooRealVar sigmay("sigmay","sigmay",2,0.,10.);
RooGaussian gaussy("gaussy","gaussy",y,meany,sigmay);

RooProdPdf sig("sig","gaussx*gaussy",RooArgSet(gaussx,gaussy));

//--- Background p.d.f. ---
RooPolynomial flatx("flatx","flatx",x);
RooPolynomial flaty("flaty","flaty",y);

RooProdPdf bkg("bkg","flatx*flaty",RooArgSet(flatx,flaty));

//--- Composite model ---
RooRealVar nsig("nsig","nsig",1000,0,10000);
RooRealVar nbkg("nbkg","nbkg",10000,0,100000);
RooAddPdf model("model","sig+bkg",RooArgList(sig,bkg),RooArgList(nsig,nbkg));
```

Example 13 - A composite two-dimensional p.d.f.

We have a 2-dimensional signal and background, the signal is Gaussian in both observables, the background is flat in both observables. If we look at the two-dimensional distribution the signal is nicely visible, but as you can see in Figure 18 the one-dimensional projection and x and y do not do justice to the signal:

Example 14 - Plotting a 2-dimensional composite model

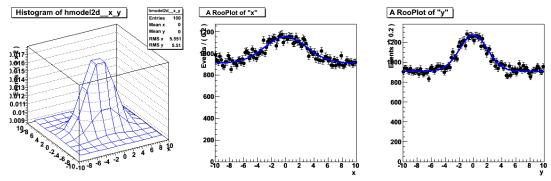


Figure 18 - Output of Example 14

The reason is of course that when you make a projection in either x or y of model you indiscriminately include the regions with a lot of signal – around (0,0) – as well as regions where there is only background. The result is that the nice peak in the 2-dimensional plot is watered down a lot.

You could choose to show the 2-D plot, but that has several disadvantages: it is difficult to overlay data and model for example. Another approach is to show only a *slice* of the data, i.e. when you show the projection in x, you for example only include data in the range -3<y<3.

Show the data with this requirement on y is conceptually straightforward: you only include data points in the histogram that meet your selection criteria in y. The projection of the model with the same requirement is less trivial, but still conceptually easy: the integration limits of the projection integral over y show now be adjusted from the full range to the range that we have selected, i.e.

$$F_x^{a < y < b}(x) = \int_a^b F(x, y) dy$$

Here is how you do it:

Example 15 - Projecting a slice of a two-dimensional p.d.f.

Result: a much more pronounced signal is visible in the slice projection Figure 19 (right) than in the ordinary 'full' projection Figure 19 (left). Lets now look a bit better at the code fragment that produced this plot. The use of ranges in plotting is always a two-step process: first you define a range with a name in one (or more) variables and then you use that range by referencing it by its name. Though it requires you to write one extra line of code this approach has two advantages: 1) you define the requirement in a single location, eliminating the possibility of multiple inconsistent copies and 2) it allows you to refer to ranges in more than one variable through a single name. Also note that we have combined multiples options – to modify the projection range, the line style and the component selection – into a single call.

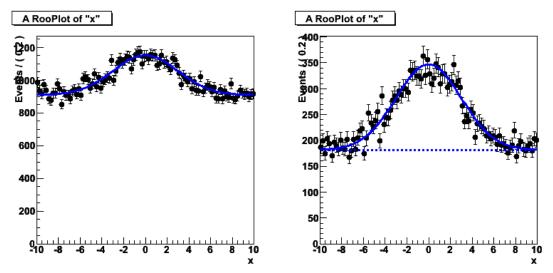


Figure 19 – Regular projection of p.d.f. from Example 13 (left) versus slice projection of same p.d.f (right, |y|<2)

A convenient feature of CutRange() and ProjectionRange() is that they automatically ignore any requirements imposed on the plot variable itself, which allow you to effectively use the 'named range 'concepts ability to refer to multiple range by the same name. This is illustrated in the following example:

```
// Define a range in y named "selection"
x.setRange("selection",-3,3);
y.setRange("selection",-2,2);

// Make plot of data and model in x with "selection" cut on y
RooPlot* xframe2 = x.frame();
data->plotOn(xframe2,CutRange("selection"));
model.plotOn(xframe2,ProjectionRange("selection"));

// Make plot of data and model in y with "selection" cut on x
RooPlot* yframe2 = y.frame();
data->plotOn(yframe2,CutRange("selection"));
model.plotOn(yframe2,ProjectionRange("selection"));
```

#### Dealing with more than 2 dimensions

The technique of projecting slices is easily generalized to p.d.f.s with more than 2 dimensions. Take as an example a three dimensional p.d.f. of the same form as the p.d.f. defined in Example 13, just add a gaussz and a flatz. A projection plot in any dimension now integrates out 2 dimensions and a slice projection plot can, at your choice, make a slice cut in either or both observables that are projected out. You can choose the range that you wish to select in each observable by calling setRange() for each observable:

```
// --- Construct three-dimensional p.d.f. ---
RooRealVar z("z","z",-10,10);
RooRealVar meany("meanz","meanz",0,-10,10);
RooRealVar sigmay("sigmaz","sigmaz",1,0.,10.);
RooGaussian gaussz("gaussz","gaussz",z,meanz,sigmaz);
RooPolynomial flatz("flatz","flatz",z);
```

```
RooProdPdf sig3("sig3","gx*gy*gz",RooArgSet(gaussx,gaussy,gaussz));
RooProdPdf bkg3("bkg3","fx*fy*fz",RooArgSet(flatx,flaty,flatz));
RooAddPdf model3("m3","s3+b3",RooArgList(sig3,bkg3),RooArgList(nsig,nbkg));
RooDataSet* data3 = model3.generate(RooArgSet(x,y,z),100000);

// --- Make straight and slice projection on z ---
x.setRange("selection",-3,3);
y.setRange("selection",-2,2);

RooPlot* zframe = z.frame();
data3->plotOn(zframe);
model3.plotOn(zframe,Components("bkg3"),LineStyle(kDashed));

RooPlot* zframe2 = z.frame();
data3->plotOn(zframe2);
model3.plotOn(zframe2,ProjectionRange("selection"));
model3.plotOn(zframe2,ProjectionRange("selection"));
model3.plotOn(zframe2,ProjectionRange("selection"));
Components("bkg3"),LineStyle(kDashed));
```

Example 16 - Projecting a 2-dimensional slice of a 3-dimensional model

The output is shown in Figure 20.

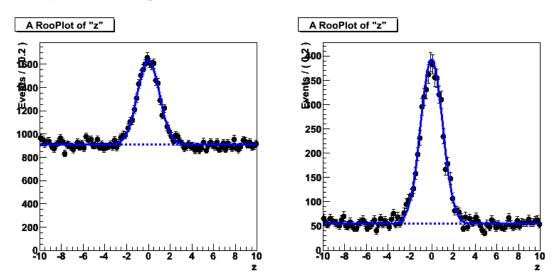


Figure 20 – Projection of 3-dimensional model of Example 16 on z (left) and projection of 2-dimensional slice "selection" in (x,y) on z axis.

#### Plotting multiple or discontinuous ranges

RooFit ranges are 'simple' ranges: each range is defined two parameters: the lower bound and the higher bound. Sometimes though there is good use for discontinuous ranges, for example a lower and higher sideband around a signal region. You effectively construct such discontinues ranges by simultaneously specifying multiple ranges in any command that accepts ranges e.g.

```
x.setRange("sb_lo",-6,-3);
x.setRange("sb_hi", 3, 6);

RooPlot* xframe2 = x.frame();
data->plotOn(xframe2,CutRange("sb_lo,sb_hi"));
model.plotOn(xframe2,ProjectionRange("sb_lo,sb_hi"));
```

Note that when the ranges are active in more than one observable, 'multiplication' amongst observables takes precedence over 'addition' of ranges, i.e. given a range A and a range B defined in both x and y, the definition of (A,B) is  $(A(x) \& A(y)) \mid (B(x) \& B(y))$ .

#### Selecting arbitrarily shaped regions for plotting

The 'range' techniques projects a slice, box or (hyper-)cube shaped region on a one-dimensional plot, but you can also project differently shaped regions. Such projection integrals can in general not be calculated analytically and have to be performed with Monte Carlo-style methods. Before we get into the technical details on how to do that, we will first look a bit better in why you may want to that.

Generally speaking the goal of a 'range' plot is usually to visualize your capability to separate signal from background by showing the data distribution in one dimension but exploiting the discriminating information from all dimensions. In the standard range plot this is accomplished by cutting around the signal region in all dimensions that are projected out. When you try to generalize this concept two questions arise:

- how to you find the cuts that leads to the 'best' plot, and
- what is the optimal shape of the region to select (e.g. a (hyper)ellipsoid is probably better than a (hyper)cube).

The question of what is best somewhat subjective as it revolves around the issue of how to present your data, but generally involves finding a way that fairly represents the signal/background separation that you achieve in a multi-dimensional model in a one-dimensional plot. One of the standard approaches that address both of these issues is the so-called 'likelihood-ratio plot'.

#### The likelihood ratio plot

The idea behind this method is that likelihood of the signal and background component of your model – or rather ratio of these likelihoods – contains all the information you have on signal/background discrimination. A 'constant-likelihood surface' is the optimal cut shape under the assumption that your model is correct. A nice feature of this method is that it reduces the definition of your multi-dimensional signal region to a definition with a single parameter: the likelihood ratio value on the cut surface. This parameter controls the purity versus efficiency balance in your plot. The likelihood ratio plot is not as easily produced as a hyper-cube slice plot because it involves numeric methods.

In practice, a likelihood ratio plot is constructed as follows: for a composite model M(x,y,z) = fS(x,y,z) + (1-f)B(x,y,z) and a dataset D(x,y,z) you

- Formulate a likelihood ratio R(x,y,z) = S(x,y,z)/B(x,y,z)
- Plot the subset of the data D(x,y,z) for which R(x,y,z,) > R<sub>cut</sub>
- Project model M using a Monte Carlo technique for the region of phase space in which it predicts R(x,y,z)>R<sub>cut</sub>

Most of the practical difficulties arise the calculation of the latter, as the over the region defined by  $R(x,y,z)>R_{cut}$  can in all but exceptional cases *not* be performed analytically. In the remainder of this section we will explain how to perform these steps in RooFit.

#### Formulating the likelihood

We illustrate the formulation of the likelihood ratio in RooFit using the model of Example 16 as a starting point:

In this example we do not use the straight signal and background likelihoods as input for the ratio, but rather their integrals over z:

$$S(y,z) = \int S(x,y,z)dx$$
;  $B(y,z) = \int B(x,y,z)dx$ 

You do this because you do *not* want use the information in the z observable in the likelihood ratio if you are plotting the distribution of z: that would be equivalent to showing a distribution on z after a cut on z. The creation of the projection integrals that take out the information in z is facilitated by the createProjection() method of RooAbsReal. Its arguments are the observables over which the resulting integrals should be normalized, (x,y) in the above example, and the observables which should be integrated out, z in the above example. The actual ratio is calculated by a RooFormulaVar, which in the example above, also takes the 10log of the ratio as this compresses long tails in the distribution, and this thus easier to work with in practice.

#### Plotting the data with a likelihood ratio cut

Now we are ready to visualize the data. First we calculate the value of R for each data point and plots its distribution.

```
// --- Calculate llRatio for each point in data3 and add as column to data3 -
RooRealVar* llRatio = data3->addColumn(llRatio_func);

// --- Plot distribution of llRatio values in data3 ---
RooPlot* lframe = llRatio->frame(Range(-10,2));
data3->plotOn(lframe);
```

The first step is accomplished by the addColumn() function of RooAbsData. This function takes one or more RooAbsReal function objects, evaluates their outcome for each data point and adds a column to the dataset with the name of the function object that stores the calculated value of that function. Now the values of llRatio can be simply treated as an observable and we can plot its distribution the usual way with one caveat: we have never associated a default range with llRatio, so give one explicitly here using the Range() argument. Here we have chosen a range bracketing 0 for llRatio as the interesting region of signal-to-background ratios typically hovers around unity. You can also let RooFit do range determination for you using the AutoRange() or AutoSymRange() options:

```
RooPlot* frame = llRatio->frame(AutoRange(*data3)) ;
RooPlot* frame = llRatio->frame(AutoSymRange(*data3)) ;
```

The AutoRange() option chooses a range for you that fits all data in a given dataset with some extra margin of each side. The AutoSymRange() option does the same, but it additionally adjusts the margins such that the mean value of the distribution is by construction in the center of the range.

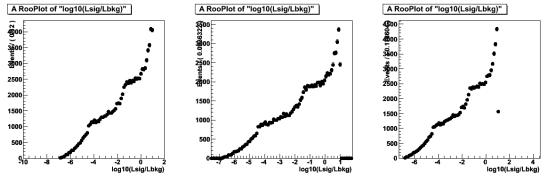


Figure 21 – Distribution of S(x,y)/B(x,y) likelihood ratio, plotted in frames created with Range(-2,10), AutoRange() and AutoSymRange() respectively. (Data is identical all plots)

Figure 21 shows the distribution of 11Ratio as generated by the example code. Based on the distribution of 11Ratio choose a cut 0. Note that this cut value is arbitrary to a certain extent and you should experiment here to achieve the result you like the best. Now we select the subset of events in data3, for which 11Ratio is greater than zero and plot its distribution in z

```
// --- Plot distribution of z values after cut S(x,y)/B(x,y) ratio ---
RooDataSet* selData3 = data3->reduce("llRatio>0") ;
RooPlot* frame = z.frame() ;
selData3->plotOn(frame) ;
```

For the selection step we use the reduce() method of RooAbsData, which takes any Boolean formula expression in terms of the dataset observables. Figure 22 shows the output of this code, as well as the output of a variation where we have required a cut value of -1 and +1 instead.

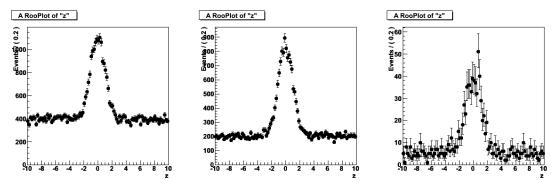


Figure 22 - Distribution of events in z after 11Ratio cut at -1, 0 and +1 respectively

#### Plotting the p.d.f. projection with a likelihood ratio cut

To complete the plots of Figure 22 we should add a projection of the model with an identical requirement, which revolves around calculating the integral

$$\int_{R(x,y)>R_{cut}} M(x,y,z) dxdy$$

As this integral can not be calculated analytically in all but exceptional cases we follow a Monte Carlo inspired numerical approach. The Monte Carlo theorem says that you can approximate any integral by a sum of values randomly sample from the distribution you are trying to integrate, i.e.

$$\lim_{n\to\infty} \left[ \frac{1}{n} \sum_{i=1,n} M(\vec{x}_i) \right]_{x_i \in R} = \int_R M(\vec{x}) d\vec{x}$$

with the values  $x_i$  randomly sample from M, such as is done for example by M.generate(). This theorem is valid for *any* region R to be integrated and provides an easy way to approximate the integral that you need to calculate for the model projection with a likelihood ratio cut. Here is the complete code for the projection of model3 with a likelihood ratio cut:

This code example recycles the addColumn() and reduce() techniques that we used earlier to project the data, but differs in the final plotting call. The ProjWData() argument to RooAbsPdf:plotOn() instructs RooFit to perform the projection integral explicitly through a Monte Carlo approximation using the given dataset, rather than using the default numerical or analytical integration techniques. By giving a dataset with a preselection on llRatio we effectively perform the integral over the region defined by llRatio>0 and thus construct a p.d.f. projection that is consistent with the data. The final result is shown in Figure 23 with a likelihood ratio cut at -1, 0 and +1 respectively.

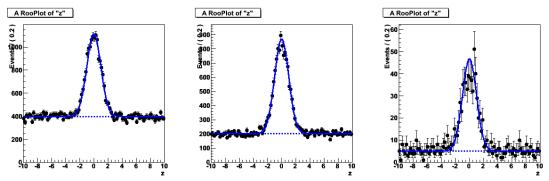


Figure 23 – Distribution of events in z after 11Ratio cut of -1, 0 and +1 respectively overlaid with p.d.f. projection with identical requirement.

#### Likelihood ratio plot - Putting it all together

Here is the complete code to make a likelihood ratio plot starting from the model defined in Example 16.

```
// --- Calculate llRatio for each point in data3 and add as column to data3 -
RooRealVar* 11Ratio = data3->addColumn(11Ratio_func) ;
// --- Plot distribution of llRatio values in data3 ---
RooPlot* Iframe = llRatio->frame(Range(-10,2)) ;
data3->plotOn(lframe) ;
// --- Plot distribution of z values after cut S(x,y)/B(x,y) ratio ---
RooPlot* frame = z.frame() ;
RooDataSet* selData3 = data3->reduce("llRatio>0") ;
selData3->plotOn(frame) ;
// --- Generate toy MC events from model3, calculate 11Ratio for each
                             toy event and add it as column to toyData ---
RooDataSet* toyData = model3.generate(RooArgSet(x,y,z),10000) ;
toyData->addColumn(llRatio_func);
// --- Select subset of toy events that S/B ratio cut ---
RooDataSet* projData = toyData->reduce("11Ratio>0") ;
// --- Draw model projection over z data in frame ---
model3.plotOn(frame,ProjWData(*projData)) ;
model3.plotOn(frame,ProjWData(*projData),Components("bkg3"),
              LineStyle(kDashed));
TCanvas* c = new TCanvas("c", "c", 800, 400) ;
c->Divide(2);
c->cd(1) ; lframe->Draw() ;
c->cd(2) ; frame->Draw() ;
```

Example 17 - Complete code to construct a likelihood ratio plot

# 7. Correlations in multi-dimensional models

We now take the building of multi-dimensional models one step further by introducing explicit correlations between observables. A multi-dimensional model includes correlations if it can not be factorized into a product of one-dimensional p.d.f.s. Dealing with correlations is a central issue in many multivariate analyses. Many standard techniques, for example a neural network, find and exploit correlations between observables automatically for you, but sometimes you may want to explicitly deal with correlations yourself: if you are a priori aware of these correlations and know how to describe them.

In the introduction of the preceding section we looked at a 2-dimensional model with correlations through the construction of a 2-dimensional RooGenericPdf based on a non-factorizable formula involving observables x and y:

```
RooGenericPdf f("f","a*x*x+b*y*y-0.3*y*y*y",RooArgSet(x,y,a,b)) ;
```

While this approach explicitly describes the distribution in x, y and the correlation between x and y, it is not so easy to disentangle the three. If you want to modify the functional form such that the distribution in y changes, but the distribution in x and the correlation between x and y are preserved, it is not obvious how you should do that. The concept of a *conditional* probability density function makes it easier to achieve such a conceptual separation. Before we get into the details of conditional p.d.f.s, we introduce a realistic example of an analysis with known correlations that will serve as illustration and make clear why you want to be able to achieve this type of factorization in your model description.

## Example analysis with a known correlation between observables

Suppose we want to measure the lifetime of the decay of a particle in a generic particle physics experiment. The first step in this measurement is to collect a data sample with observed decays. Each decay is described by a decay time, which is derived from a flight length measurement between the production vertex of the particle and the decay vertex of the particle. For an ideal detector the distribution of observed decay times is an exponential distribution with an exponent that is the inverse of the lifetime  $\tau$  of the particle:



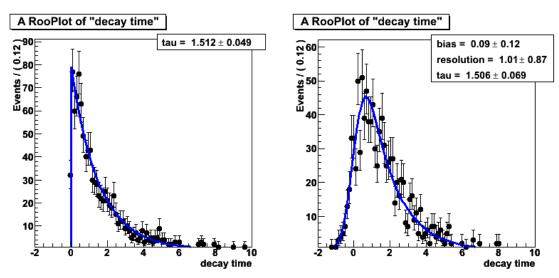


Figure 24 – Distribution of decay times measure with ideal detector (left) and realistic detector(right)

A real-life detector has a finite experimental resolution on each measurement t of the decay time. We adjust our model to incorporate a Gaussian measurement uncertainty on each t by convolving  $F_t$  with a Gaussian:

$$F_R(t) = \exp(-t/\tau) \otimes G(t,\mu,\sigma) \equiv \int dt' \exp(-t'/\tau) G(t-t',\mu,\sigma)$$

In this expression G denotes a Gaussian with mean  $\mu$  and width  $\sigma$ . The width  $\sigma$  expresses the experimental resolution on each measurement of t and the mean  $\mu$  parameterizes the average bias in that measurement. We assume the latter to be zero for the sake of this examples simplicity. Figure 24 shows the ideal and realistic model  $F_l$  and  $F_R$  fit to a sample of toy Monte Carlo events. You can see from the magnitude of error on the fitted value of  $\tau$  that the finite t resolution of the realistic model reduces the precision of the measurement of  $\tau$ .

#### Introducing a second correlated observable

Each measurement of a decay time t in our example is the result of a measurement of the distance between two decay vertices that are each calculated from the intersection of a number of charged particle trajectories. These vertex positions have uncertainties associated to them that are derived from the uncertainties on the participating charged particle trajectories and can be used to assigned an experimental error dt to each measurement t. This means that the detector resolution on t is not really a fixed value, but rather varies from event to event.

Our example of a decay time measurement has not been randomly chosen: it represents a large class of measurements where an observable x is accompanied by an error estimate *dx* that can be treated as a second correlated observable in the model that describes the experimental results.

We modify the model such that each event is characterized by a pair of values (t,dt) rather than a single number t and thereby we acknowledge that certain events – those with small dt – carry more information then others, and use this information to achieve a better measurement of  $\tau$  with the same data. Here is the enhanced p.d.f:

$$F_F(t,dt) = \exp(-t/\tau) \otimes G(t,\mu,dt)$$

It is easy to see that this small modification – replacing the resolution estimate  $\sigma$  by the per-event error dt – accomplishes what you want. Imagine to events A and B with identical observed decay times  $t_A$ = $t_B$ =t and uncertainties that differ by a factor of two dt<sub>A</sub>= $t_B$ /2, the contribution of event A to the total likelihood will differ from the contribution of event B because exponential shape of the model for event A is convolved with a Gaussian that is twice as small as that for event B. A refit of the data sample of Figure 24 to this enhanced model reflects the enhanced statistical power of this model, by reducing the measurement error of  $\tau$  from 0.067 to 0.060, a 10% improvement of the measurement performed on the same data that is equivalent to having 20% more data available<sup>4</sup>.

There is one major caveat in the enhance model  $F_E$ : it assumes that the error estimates dt are correct. If these estimates are too small on average in reality, the error on the physics parameter  $\tau$  will be too small as well. As this is highly undesirable, you should verify the correctness of the errors dt by looking at pull distributions, i.e. comparing the spread of the measured values (the external error) to the distribution of the given errors (the internal error). Fortunately this check an be trivially incorporated in the model  $F_E$  through the following modification:

$$F_E(t,dt) = \exp(-t/\tau) \otimes G(t,\mu,\mathbf{s}\cdot\mathbf{dt})$$

Now the model doesn't make any *absolute* interpretation of the errors dt, it just assumes that the true uncertainty of each t measurement scales linearly with the provided error. The parameter s serves as a global scale factor applied to the per-event errors dt. If you fit this model to the data and the uncertainty estimates dt turn out the be correct on average you will find that  $\sigma$ =1. If the error estimates

<sup>&</sup>lt;sup>4</sup> The actual gain depends on the spread of the per-event errors. The chosen example is typical for BaBar experimental data.

are too high or too low on average, this is apparent from a mismatch in the distribution of values and errors in the data and the fit will steer  $\sigma$  to a value smaller or greater than 1. Effectively one could interpret G as a fit to the pull distribution associated with the vertexing procedure. Thanks to this built-in correction of the per-event errors the improved model  $F_E$  has gained an important quality: the error on the physics parameter  $\tau$  is to first order *independent* of the correctness of the error estimates dt. (A second order dependency comes in when the pull distribution of the dt errors cannot be accurately described by a Gaussian. Also this can be mitigated, for example by replacing G by a sum of two or more Gaussians of different width and mean).

In summary, incorporating the errors dt on each decay t in your model in the form  $F_E$  gives you enhanced statistical sensitivity to  $\tau$ , it gives you an estimate of the correctness of the provided errors, and dt cancels to first order the effects that arise from dt estimates that are too small or too large on average.

#### Some practical caveats

The enhanced decay time model is a great example of a p.d.f with an explicit correlation between observables, but it has some practical caveats that have not been discussed yet and that should be addressed before one can use it in practice: If you were to fit  $F_E$  as written above to data you would effective use it as a two-dimensional model predicting the distribution of t, the correlation between t and dt, and the distribution of dt, as there is nothing in the description of  $F_E$  that warrants a special treatment of dt. In the description of the example we've conveniently left in the middle what the distribution of dt is, but once you start actually using your model with data this becomes acutely relevant as your model must be able to describe the data's dt distribution.

Lets examine our example a bit further: the prediction of  $F_E$  for the distribution of dt is obtained by integrating  $F_E(t,dt)$  over t. You cannot do this analytically, but RooFit can do it numerically, and the result is a more or less flat distribution in dt as shown in Figure 25. (NB: The slight drop-off in Figure 25 towards high values of dt is caused by the finite range of t in the definition of  $F_E$ )

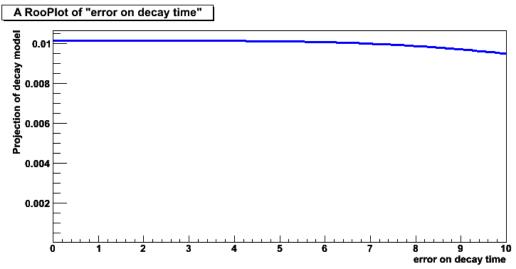


Figure 25 - Prediction of F<sub>E</sub> for the distribution of the per-event error dt

Your actual distribution of dt in data is likely to be very different, so fitting  $F_E$  to data would result in a bad fit. Even worse, you don't have any knobs to turn to modify the shape of the dt distribution predicted by  $F_E$  without altering its behavior in t and in the correlation between t and dt. So here were are back at the opening question of the chapter 'If you want to modify the functional form such that the distribution in y changes, but the distribution in x and the correlation between x and y are preserved, it is not obvious how you should do that.' We have now seen why you want that, next we will talk about how you do it. The key to this lies in the concept of conditional p.d.f.s

## Conditional probability density functions

The premise behind a *conditional* probability density function F(x|y) is that it describes the distribution of a set of observables x *given* the values of a set of other observable y. In effect F(x|y) describes the distribution of x, the correlation between x and y, but not the distribution of y. Conditional p.d.f.s differs in only one respect from ordinary p.d.f.s: the normalization condition. Whereas a regular two-dimensional p.d.f F(x,y) meets the normalization condition

$$\int F(x,y)dxdy \equiv 1$$

A conditional p.d.f. F(x|y) meets the normalization condition

$$\int F(x,y)dx \equiv 1 \text{ for all values of } y$$
 (1)

A conditional p.d.f F(x|y) has no predictive power in y, it just takes the distribution of y as a given and predicts the distribution of x for that value of y. This is precisely the way we want to use our enhanced life time model  $F_E$ : rather than fitting  $F_E(t,dt)$  to data we want to fit  $F_E(t|dt)$  to data. How can we do this in RooFit? In RooFit you can perform both fits, regular and conditional, from the *same* RooAbsPdf object: each RooFit p.d.f is always constructed from a function expression that is explicitly normalized by dividing that expression by its integral

$$F(x,y) = \frac{f(x,y)}{\int f(x,y)dxdy}, \quad F(x \mid y) = \frac{f(x,y)}{\int f(x,y)dx}$$

The only issue is that you have to indicate in the use context that you want a RooAbsPdf to represent the conditional form that than the regular form. The next section will explain how to do that.

#### Using conditional p.d.f.s for fitting, plotting and generating

We first code the enhanced life time p.d.f. of the opening section to be able to concretely illustrate the various uses of conditional p.d.f.s.:

```
// Observables
RooRealVar t("t","decay time",0,20) ;
RooRealVar dt("dt","error on decay time",0,1) ;

// Gaussian resolution model Gauss(t,0,s*dt) ;
RooRealVar s("s","resolution",3,0,20) ;
RooGaussModel res("res","det. resol.",t,RooConst(0),s,dt) ;

// NB: Convenient special ctor of RooGaussModel with 4 arguments
// defines gaussian width as product of 3rd and 4th argument and saves you
// the effort of a separate RooFormulaVar object

// Decay (x) res model
RooRealVar tau("tau","lifetime",1.5,0,20) ;
RooDecay decay("decay","decay model",t,tau,res,RooDecay::SingleSided) ;
```

As was just explained, the definition of decay – like any other p.d.f – has not, and needs not, to have any clauses relating to possible use as a conditional p.d.f

Given a dataset data with observables t and dt we now explicitly fit decay as conditional p.d.f using the ConditionalObservables() directive in the fitTo() command:

```
// Fit decay(t|dt) as conditional p.d.f. to data(t,dt)
decay.fitTo(data,ConditionalObservables(dt));
```

The effect of the ConditionalObservables() argument is that the likelihood function that it is constructed using decay in its conditional form:

$$\begin{split} \textit{NLL}_{\textit{reg}}(\tau, \sigma) &= - \sum_{\textit{D}} \log \left( \textit{decay}(t_i, \textit{dt}_i; \tau, \sigma) \right) \\ \textit{NLL}_{\textit{cond}}(\tau, \sigma) &= - \sum_{\textit{D}} \log \left( \textit{decay}(t_i; \tau, \sigma \mid \textit{dt}_i) \right) \end{split}$$

It is an instructive exercise to compare the output of a regular fitTo() operation with the Verbose() argument with that of a fitTo() operation with both the Verbose() and the Conditional-Observables() arguments: you will see that the absolute value of the likelihood printed by the Verbose() option is very different. This is a direct consequence of the different normalization conditions illustrated in Eq. 1. You will also notice that the performance differs: in the fit to the conditional form of decay the normalization integral needs to be evaluated for every event, as it has a different value for each event. The normalization term of decay when used as a regular fit on the other hand depends only on parameters, and is thus only evaluated when MINUIT changes those parameter values, a much less frequent occurrence.

The visualization of conditional models is conceptually more complicated as the shape of a conditional p.d.f. is partly dictated by the data it is aiming to describe. Lets look concretely at our example analysis: there are two data plots you can make: the distribution of t and the distribution of dt. Since a conditional p.d.f. decay(t|dt) has by construction no predictive power in dt we cannot plot it as function of dt. That leaves us with a plot of the distribution of t that can be overlaid with a matching projection of decay over observable dt. As integration makes no sense here, again because decay(t|dt) has no predictive power in dt, we project out dt by summing the distribution of decay(t|dt) over each dt value in the data, i.e.

$$\frac{1}{n} \sum_{i=1,n} \frac{d(t \mid dt_i)}{\int d(t \mid dt_i) dt}$$

This technique is identical to the Monte Carlo integration technique described in the preceding chapter, it is only applied with different data: for Monte Carlo integration we summed over the values of a simulated dataset sampled from the p.d.f. itself, whereas here we sum over the values of the experimental data:

```
RooPlot* frame = t.frame() ;
data.plotOn(frame) ;
decay.plotOn(frame, ProjWData(dt, data)) ;
```

In the ProjWData() we specify the dataset with dt values over which should be summed – here we use the actual data – and we also specify that we only wish to override the projection method for the observable dt. The latter is not strictly necessary in this example as dt is the only observable to be projected in data, but it is good practice to do spell that out explicitly. In cases where data contains additional observables that you still want to be projected out through integration, this specification is essential. Figure 26 shows the t and dt distributions of our example data and the properly projected t distribution of the conditional p.d.f. is always a 'hybrid' object: it is not a pure model prediction, but a conditional prediction tailored to the data it is being compared to.

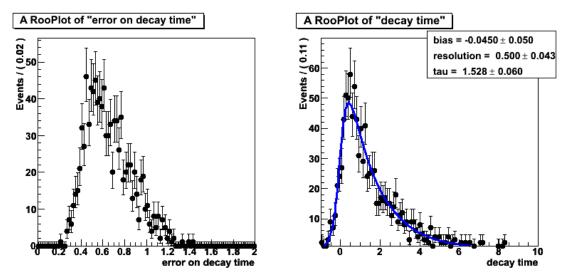


Figure 26 – Distribution of decay time errors (left) and distribution of decay times, overlaid with conditional p.d.f. decay(t|dt) projected with dt values of the data (right).

Generating events using p.d.f. in conditional form requires external input on the conditional variables, just like in plotting conditional p.d.f.s. The simplest way to do this is to pass the generator an existing set of *dt* values and ask it to generate the corresponding t values according to the model

```
RooDataSet* toyData = decay.generate(t,ProtoData(data)) ;
```

The result of this operation is a two-dimensional dataset with values of t and dt. The dt values are identical to those of the input dataset passed through the ProtoData argument, the t values and their correlation with dt are generated from the decay p.d.f. You do not need to specify a number of events to be generated when you use ProtoData(), the number of events in the prototype dataset is the implicit default, but you still can change this through an explicit NumEvents() argument. Beware aware though that when you require more events to be generated then are available in data, certain data entries will be used more than once.

If you wish to describe the distribution in *dt* with a p.d.f. rather than with a collection of values, the event generation becomes a two-step process: First you sample a *dt* distribution from a regular p.d.f, then you sample the t distribution from a conditional p.d.f.

```
RooAbsPdf* dtModel ;
RooDataSet* dtData = dtModel->generate(dt,1000) ;
RooDataSet* allData = decay.generate(t,ProtoData(*dtData)) ;
```

# Multiplying conditional p.d.f.s with regular p.d.f.s.

The ability to use any p.d.f. in conditional form in RooFit unlocks essential new ways to use a p.d.f in describing certain classes of problems, but their direct use is less elegant because information on the distribution of the conditional observables needs to be externally supplied in many operations. We will now look at another way to use conditional p.d.f.s that mitigates these practical problems: conditional products. The essence of the idea is that the final 'high level' p.d.f. is a p.d.f that can be used in regular mode even though it internally contains a conditional p.d.f. We can achieve this idea through a simple multiplication step: we multiply a conditional p.d.f.s with a supplemental p.d.f.s that describes the conditional observables to form a full p.d.f. For the initial example of this chapter this amounts to defining a new p.d.f as follows

```
F(t,dt) = decay(t|dt) \cdot vtx(dt)
```

It easy to convince yourself that F(t,dt) is regular p.d.f by explicitly proving that F is normalized over both t and dt, i.e. Int F(t,dt) dt ddt = 1

```
\iint decay(t \mid dt)vtx(dt)dtddt = \int \left(\int decay(t \mid dt)dt\right)vtx(dt)ddt = \int 1vtx(dt)ddt = 1
```

While F(t,dt) is now a regular p.d.f in all respects we have retained the advantage of the conditional p.d.f: we have separated the description of the 'physics' part of F, the decay model and its correlation with the vertex error from the description of the 'empirical' part of F, the description of the distribution of vertex errors. This leaves us with the task of describing the distribution of dt one way or another, but this could – worst case – be done with a non-parametric p.d.f such as a RooHistPdf or a RooKeysPdf. Here is the reworked example of the introduction that use an – arbitrarily chosen – bifurcated Gaussian as a toy model for the distribution of vertex errors

```
// Bifurcated Gaussian p.d.g. as model for per-event vertex errors
RooRealVar m("m","mean of dt",0.5,0,1);
RooRealVar sl("sl","low-side sigma of dt gauss",0.1,0.,1.);
RooRealVar sr("sr","higth-side sigma of dt gauss",1,0.,10.);
RooBifurGauss vtx("vtx","vtx error dist",dt,m,sl,sr);

// Full model: product of conditional decay model with vtx toy model
RooProdPdf F("F","decay(t|dt)*vtx(dt)",Conditional(decay,t),vtx);
```

The Conditional() modifier in the constructor instruct RooProdPdf to interpret decay as a conditional p.d.f that only describes the observable t. Any other observable referenced in decay – in this case dt – is treated as a conditional observable. The RooProdPdf constructor is in fact the *only* place in which we will make this declaration of conditional use.

The net result is a p.d.f. *F* that has all the necessary information to describe the distribution of *t* and *dt* and we therefore can proceed as usual and work with *F* to generate events, fit it to data and plot it:

```
// Generate events from F(t,dt)
RooDataSet* data = F.generate(RooArgSet(t,dt),10000) ;

// Fit F to data
F.fitTo(*data) ;

// Plot t and dt distributions of data with F overlaid
RooPlot* tframe = t.frame() ;
data->plotOn(tframe) ;
F.plotOn(tframe) ;

RooPlot* dtframe = dt.frame() ;
data->plotOn(dtframe) ;
F.plotOn(dtframe) ;
```

The data samples shown in Figure 26 were in fact produced with this conditional product p.d.f. F.

It is instructive to understand what happens behind scenes in *F* when you plot, fit or generate conditional product p.d.f.s.

- Fitting is straightforward as the probability for each event is simply defined by F(d,dt) = decay(t|dt)vtx(dt), which we already proved is properly normalized.
- Generating events from F is again a two-step process, as was the case for standalone use of conditional p.d.f.s, except that F now has all the information it needs to complete both

generation step in a single command: first a value of dt is generated from vtx, then a value of t is generated by decay given the value of dt.

• Finally, for plotting the projection of F over dt is calculated as a standard numeric integral rather than as an implicit Monte Carlo approximation

$$F_t(t) = \int decay(t \mid dt)vtx(dt)ddt$$

Even though above projection integral can only be evaluated numerically, its evaluation is faster than a Monte Carlo approximation as specialized one-dimensional integration techniques such as the adaptive Gauss-Kronrod rule converge to a controllable precision in O(30) function evaluations.

## 8. Discrete variables

This section is scheduled for the next version of the manual

Representing discrete information – class RooCategory

Representing selection criteria – real-to-discrete functions

**Tabulating discrete information** 

# 9. Multiple datasets and simultaneous fitting

This section is scheduled for the next version of the manual

The problem

The solution

Automating the solution

# Organizational tools – Setting up a complex analysis

This section is scheduled for the next version of the manual

Using sets and list to manage user configuration

Automated function building and customization

# 11. Common issues, pitfalls and their solutions

This section is scheduled for the next version of the manual

# **Integrating PDFs**

How to use createIntegral()

# Tuning numeric integration parameters and methods

How to use defaultIntegratorConfig() etc...

# Using weighted data

Describe ways to use weighted data and problems that may arise in likelihood fits

# Adding penalty terms to a likelihood or $\chi 2$

How to use RooFormulaVar to add penalty terms to a likelihood or  $\chi 2$ 

### Interactive fitting

How to use class RooMinuit to do an interactive fit session

## Blinding parameters

How to blind parameters in your fit

## Rearranging the contents of a RooPlot, adding arrows, boxes etc...

How to rearrange the contents of a RooPlot and beautify it

## Merging and concatenating, reducing data

How to merge and concatenate datasets

## Fit instability due to strongly correlated parameters

### Effects of bounding parameters

#### Observables and parameters

#### Distinguishing parameters from observables

There are tools to distinguish the 'parameter' variables of a p.d.f. from the 'observable' variables of a p.d.f. Lets first start with the definition of these terms:

- An observable is a variable of a p.d.f. that also occurs in the data. A p.d.f. is also normalized
  to unity with respect to all its observables.
- A parameter is any remaining variable of a p.d.f.s.

RooFit p.d.f.s have no intrinsic or static designation which variable is a observable and which variable is a parameter. This designation always arises dynamically in the context of data and is a fundamental design consideration. You can read more about this in appendix C. A consequence is that any routine that identifies a variable as either parameter or observable needs to be passed a dataset to be able to make the distinction:

```
RooAbsData* data ; // A sample dataset containing 'x'
RooArgSet* params = model.getParameters(data) ;
RooArgSet* observables = model.getObservables(data) ;
params->Print("1") ;
observables->Print("1") ;

RooArgSet::parameters:
    1) RooRealVar::c0: "coefficient #0"
    2) RooRealVar::c1: "coefficient #1"
    3) RooRealVar::c2: "coefficient #2"
    4) RooRealVar::mean: "mean"
    5) RooRealVar::nbkg: "background fraction"
    6) RooRealVar::nsig: "signal fraction"
    7) RooRealVar::sigma: "sigma"

RooArgSet::dependents:
    1) RooRealVar::x: "x"
```

# Appendix A – Quick reference guide

This appendix summarizes the most core *named argument* methods of RooFit for plotting, fitting and data manipulation. The named argument formalism chief advantage is that it is a flexible and self-documenting way to call methods that have a highly variable functionality. Here is the list of methods that is documented in this section

Action	Method	Page#
Make a plot frame	RooAbsRealLValue::frame()	57
Draw a PDF on a frame	RooAbsPdf::plotOn()	58
Draw the parameters of a PDF on a frame	RooAbsPdf::paramOn()	60
Draw data on a frame	RooAbsData::plotOn()	60
Draw data statistics on a frame	RooAbsData::statOn()	61
Fill a 2D or 3D root histogram from a dataset	RooAbsData::createHistogram()	62
Fill a 2D or 3D root histogram from a pdf	RooAbsReal::createHistogram()	63
Fit a PDF to data	RooAbsPdf::fitTo()	64
Print fit results as a LaTeX table	RooAbsCollection::printLatex()	65
Generate toy Monte Carlo datasets	RooAbsPdf::generate()	66
Create integrals of functions	RooAbsReal::createIntegral()	66
Automate fit studies	RooMCStudy	67
Reduce a dataset	RooAbsData::reduce()	68

# **Plotting**

# Make a plot frame - RooAbsRealLValue::frame()

Usage example: RooPlot\* frame = x.frame(...)

Create a new RooPlot on the heap with a drawing frame initialized for this object, but no plot contents. Use x.frame() as the first argument to the y.plotOn(...) method, for example. The caller is responsible for deleting the returned object.

This function supports the following optional named arguments

Range(double lo, double hi) Restrict plot frame to the specified range

Range(const char\* name) Restrict plot frame to range with the specified name

```
Bins(Int_t nbins) Set default binning for datasets to specified number of bins
                                    Choose plot range such that all points in given data set fit
  AutoRange(const RooAbsData&
     data, double margin=0.1)
                                    inside the range with given fractional margin.
AutoSymRange(const RooAbsData
                                    Choose plot range such that all points in given data set fit
     data, double margin=0.1)
                                    inside the range and such that center of range coincides
                                    with mean of distribution in given dataset.
        Name(const char* name) Give specified name to RooPlot object
     Title(const char* title) Give specified title to RooPlot object
```

#### Some examples:

```
// Create frame with name "foo" and title "bar"
x.frame(Name("foo"),Title("bar"));
// Create frame with range (-10,10) and default binning of 25 bins
x.frame(Range(-10,10),Bins(25));
// Create frame with range that fits all events in data with 10% margin that
// is centered around mean of data
x.frame(AutoSymRange(data)) ;
```

# Draw a PDF on a frame - RooAbsPdf::plotOn()

```
Usage example: RooPlot* frame = pdf.plotOn(frame,...);
```

Plots (projects) the PDF on a specified frame. If a PDF is plotted in an empty frame, it will show a unit normalized curve in the frame variable, taken at the present value of other observables defined for this PDF.

If a PDF is plotted in a frame in which a dataset has already been plotted, it will show a projected curve integrated over all variables that were present in the shown dataset except for the one on the xaxis. The normalization of the curve will also be adjusted to the event count of the plotted dataset. An informational message will be printed for each projection step that is performed

**Projection control** 

This function takes the following named arguments

observables listed in set from the projection, results in set from the projection in discrete places.
---

resulting a

Project(const RooArgSet& set) Override default projection behavior by projecting over observables given in set and complete ignoring the default projection behavior. Advanced use only.

ProjWData(const RooAbsData& d) Override default projection *technique* (integration by default). For observables present in given dataset projection of PDF is achieved by constructing a Monte-Carlo summation of the curve for all observable values in given set. Consult manual sections 'Selecting arbitrarily shaped regions for plotting' (p.42) and 'Using conditional p.d.f.s for fitting, plotting and generating' (p.50) for details ProjWData(const RooArgSet& s, As above but only consider subset 's' of observables in const RooAbsData& d) dataset 'd' for projection through data averaging ProjectionRange(const char\* rn) Override default range of projection integrals to a different range specified by given range name. This technique allows you to project a finite width slice in a real-valued observable Miscellaneous content control Normalization(Double\_t scale, Adjust normalization by given scale factor. Interpretation of ScaleType code) number depends on code: Relative: relative adjustment factor, NumEvent: scale to match given number of events. Name(const chat\* name) Give curve specified name in frame. Useful if curve is to be referenced later Asymmetry(const RooCategory& c) Show the asymmetry of the PDF in given two-state category  $(A^{+}-A^{-})/(A^{+}+A^{-})$  rather than the PDF projection. Category must have two states with indices -1 and +1 or three states with indices -1,0 and +1. Shift entire curve such that lowest visible point is at exactly ShiftToZero(Bool\_t flag) zero. Mostly useful when plotting  $-\log(L)$  or  $\chi^2$  distributions AddTo(const char\* name,Double\_t Add constructed projection to already existing curve with wgtSelf, double\_t wgtOther) given name and relative weight factors Plotting control LineStyle(Int\_t style) Select line style by ROOT line style code, default is solid LineColor(Int\_t color) Select line color by ROOT color code, default is blue LineWidth(Int\_t width) Select line with in pixels, default is 3 FillStyle(Int\_t style) Select fill style, default is not filled. If a filled style is selected, also use VLines() to add vertical downward lines at end of curve to ensure proper closure FillColor(Int\_t color) Select fill color by ROOT color code Range(const char\* name) Only draw curve in range defined by given name Range(double lo, double hi) Only draw curve in specified range

**VLines()** Add vertical lines to y=0 at end points of curve

densely spaced curve points

-- Control precision of drawn curve w.r.t to scale of plot, default is 1e-3. Higher precision will result in more and more

Precision(Double\_t eps)

## Draw parameters of a PDF on a frame - RooAbsPdf::paramOn()

Usage example: pdf.paramOn(frame,...)

Add a box with parameter values (and errors) to the specified frame

The following named arguments are supported

Parameters(const RooArgSet& Only the specified subset of parameters will be shown. By param) default all non-constant parameters are shown

ShowConstant(Bool\_t flag) Also display constant parameters

Classic parameter formatting options, provided for Format(const char\* optStr)

backward compatibility

Format(const char\* what,...) Parameter formatting options, details are given below

Label (const chat\* label) Add header line with given label to parameter box

Layout(Double\_t xmin, Specify relative position of left, right side and top of box. Double\_t xmax, Double\_t ymax) Vertical size of box is calculated automatically from number

lines in box

The Format(const\_char\* what,...) has the following structure

const char\* what Controls what is shown. "N" adds name, "E" adds error, "A" shows asymmetric error, "U" shows unit, "H" hides the value

FixedPrecision(int n) Controls precision, set fixed number of digits

AutoPrecision(int n) Controls precision. Number of shown digits is calculated from error + n specified additional digits (1 is sensible

default)

Example use: pdf.paramOn(frame,Label("fit result"),Format("NEU",AutoPrecision(1)));

# Draw data on a frame - RooAbsData::plotOn()

Usage example: data.plot0n(frame,...)

Plots the dataset on the specified frame. By default an unbinned dataset will use the default binning of the target frame. A binned dataset will by default retain its intrinsic binning.

The following optional named arguments can be used to modify the default behavior

#### Data representation options

Show the asymmetry of the data in given two-state category Asymmetry(const RooCategory& c)  $(A^{+}-A^{-}) / (A^{+}+A^{-})$ . Category must have two states with

	indices -1 and +1 or three states with indices -1, 0 and +1.	
ErrorType(RooAbsData::EType)	Select the type of error drawn: Poisson (default) draws asymmetric Poisson confidence intervals. SumW2 draws symmetric sum-of-weights error	
Binning(double xlo, double xhi, int nbins)	Use specified binning to draw dataset	
Binning(const RooAbsBinning&)	Use specified binning to draw dataset	
Binning(const char* name)	Use binning with specified name to draw dataset	
RefreshNorm(Bool_t flag)	Force refreshing for PDF normalization information in frame. If set, any subsequent PDF will normalize to this dataset, even if it is not the first one added to the frame. By default only the 1st dataset added to a frame will update the normalization information	
Histogram drawing options		
DrawOption(const char* opt)	Select ROOT draw option for resulting TGraph object	
LineStyle(Int_t style)	Select line style by ROOT line style code, default is solid	
LineColor(Int_t color)	Select line color by ROOT color code, default is black	
LineWidth(Int_t width)	Select line with in pixels, default is 3	
<pre>MarkerStyle(Int_t style)</pre>	Select the ROOT marker style, default is 21	
<pre>MarkerColor(Int_t color)</pre>	Select the ROOT marker color, default is black	
<pre>MarkerSize(Double_t size)</pre>	Select the ROOT marker size	
<pre>XErrorSize(Double_t frac)</pre>	Select size of X error bar as fraction of the bin width, default is 1	
Name (accept about a cons)	Misc. other options	
Name(const chat* name)	Give curve specified name in frame. Useful if curve is to be referenced later	
<pre>Invisble(Bool_t flag)</pre>	Add curve to frame, but do not display. Useful in combination AddTo()	

# Draw data statistics on a frame - RooAbsData::statOn()

Add constructed histogram to already existing histogram with given name and relative weight factors

Usage example: data.statOn(frame,...)

AddTo(const char\* name, Double\_t wgtSelf, Double\_t

wgtOther)

Add a box with statistics information to the specified frame. By default a box with the event count, mean and RMS of the plotted variable is added.

The following optional named arguments are accepted

What(const char\* whatstr) Controls what is printed: "N" = count, "M" is mean, "R" is RMS. Format(const char\* optStr) Classic parameter formatting options, provided for backward compatibility Format(const char\* what,...) Parameter formatting options, details given below Label (const chat\* label) Add header label to parameter box Layout(Double\_t xmin, Specify relative position of left, right side of box and top of Double\_t xmax, Double\_t ymax) box. Vertical size of the box is calculated automatically from number lines in box Cut(const char\* expression) Apply given cut expression to data when calculating statistics. CutRange(const char\* Only consider events within given range when calculating

The Format(const char\* what,...) has the following structure

rangeName)

const char\* what Controls what is shown. "N" adds name, "E" adds error, A" shows asymmetric error, "U" shows unit, "H" hides the value

statistics. Multiple CutRange() argument may be specified

FixedPrecision(int n) Controls precision, set fixed number of digits

to combine ranges

AutoPrecision(int n) Controls precision. Number of shown digits is calculated from error + n specified additional digits (1 is sensible default)

**VerbatimName(Bool\_t flag)** Put variable name in a \verb+ + clause.

# Fill a 2D or 3D root histogram from a dataset – RooAbsData::createHistogram()

Usage example: TH1\* hist = data.createHistogram(name,xvar,...)

Create and fill a ROOT histogram TH1,TH2 or TH3 with the values of this dataset.

This function accepts the following arguments

const char\* name Name of the ROOT histogram

const RooAbsRealLValue& xvar Observable to be mapped on x axis of ROOT histogram

Binning(const char\* name) Apply binning with given name to x axis of histogram

Binning(RooAbsBinning& binning) Apply specified binning to x axis of histogram

Binning(double lo, double hi, int nbins) Apply specified binning to x axis of histogram

YVar(const RooAbsRealLValue& var,...)

ZVar(const RooAbsRealLValue& Observable to be mapped on z axis of ROOT histogram var,...)

The YVar() and ZVar() arguments can be supplied with optional Binning() arguments to control the binning of the Y and Z axes, e.g.

The caller takes ownership of the returned histogram

# Fill a 2D or 3D root histogram from a PDF – RooAbsReal::createHistogram()

Usage example: TH1\* hist = pdf.createHistogram(name,xvar,...)

Create and fill a ROOT histogram TH1, TH2 or TH3 with the values of this function.

This function accepts the following arguments

const char\* name Name of the ROOT histogram const RooAbsRealLValue& xvar Observable to be mapped on x axis of ROOT histogram Binning(const char\* name) Apply binning with given name to x axis of histogram Binning(RooAbsBinning& binning) Apply specified binning to x axis of histogram Binning(double lo, double hi, Apply specified binning to x axis of histogram int nbins) ConditionalObservables(const Do not normalized PDF over following observables when RooArgSet& set) projecting PDF into histogram YVar(const RooAbsRealLValue& Observable to be mapped on y axis of ROOT histogram var,...) ZVar(const RooAbsRealLValue& Observable to be mapped on z axis of ROOT histogram var,...)

The YVar() and ZVar() arguments can be supplied with optional Binning() arguments to control the binning of the Y and Z axes, e.g.

The caller takes ownership of the returned histogram.

# Fitting and generating

# Fit a PDF to data - RooAbsPdf::fitTo()

Usage example: pdf.fitTo(data,...)

Fit PDF to given dataset. If dataset is unbinned, an unbinned maximum likelihood is performed. If the dataset is binned, a binned maximum likelihood is performed. By default the fit is executed through the MINUIT commands MIGRAD, HESSE and MINOS in succession.

The following named arguments are supported

#### Options to control construction of -log(L)

ConditionalObservables(const RooArgSet& set)	Do not normalize PDF over listed observables
<pre>Extended(Bool_t flag)</pre>	Add extended likelihood term, off by default
Range(const char* name)	Fit only data inside range with given name
<pre>Range(Double_t lo, Double_t hi)</pre>	Fit only data inside given range. A range named "fit" is created on the fly on all observables.
NumCPU(int num)	Parallelize NLL calculation on num CPUs
Optimize(Bool_t flag)	Activate constant term optimization (on by default)
SplitRange(Bool_t flag)	Use separate fit ranges in a simultaneous fit. Actual range name for each subsample is assumed to by rangeName_{indexState} where indexState is the state of the master index category of the simultaneous fit

#### Options to control flow of fit procedure

InitialHesse(Bool_t flag)	Flag controls if HESSE before MIGRAD as well, off by default
Hesse(Bool_t flag)	Flag controls if HESSE is run after MIGRAD, on by default
Minos(Bool_t flag)	Flag controls if MINOS is run after HESSE, on by default
Minos(const RooArgSet& set)	Only run MINOS on given subset of arguments
Save(Bool_t flag)	Flag controls if RooFitResult object is produced and returned, off by default
Strategy(Int_t flag)	Set MINUIT strategy (0 through 2, default is 1)

#### FitOptions(const char\* optStr)

Steer fit with classic options string (for backward compatibility). Use of this option excludes use of any of the new style steering options

#### Options to control informational output

Verbose(Bool\_t flag) Flag controls if verbose output is printed (NLL, parameter

changes during fit

Timer(Bool\_t flag) Time CPU and wall clock consumption of fit steps, off by

default

PrintLevel(Int\_t level) Set MINUIT print level (1 through 3, default is 1). At 1 all

RooFit informational messages are suppressed as well.

# Print fit results as a LaTeX table – RooAbsCollection::printLatex()

Usage example: paramList.printLatex(...) ;

Output content of collection as LaTex table. By default a table with two columns is created: the left column contains the name of each variable, the right column the value.

The following optional named arguments can be used to modify the default behavior

Columns(Int\_t ncol) Fold table into multiple columns, i.e. ncol=3 will result in 3 x

2 = 6 total columns

**Sibling(const** Define sibling list. The sibling list is assumed to have objects with the same name in the same order. If this is not

the case warnings will be printed. If a single sibling list is specified, 3 columns will be output: the (common) name, the value of this list and the value in the sibling list. Multiple sibling lists can be specified by repeating the Sibling()

command.

Format(const char\* str) Classic format string, provided for backward compatibility

Format(...) Formatting arguments, details are given below

OutputFile(const char\* fname) Send output to file with given name rather than standard

output

The Format(const char\* what,...) has the following structure

const char\* what Controls what is shown. "N" adds name, "E" adds error, "A"

shows asymmetric error, "U" shows unit, "H" hides the value

FixedPrecision(int n) Controls precision, set fixed number of digits

AutoPrecision(int n) Controls precision. Number of shown digits is calculated

from error + n specified additional digits (1 is sensible

default)

**VerbatimName(Bool\_t flag)** Put variable name in a \verb+ + clause.

Example use:

list.printLatex(Columns(2), Format("NEU",AutoPrecision(1),VerbatimName()));

# Generate toy Monte Carlo datasets - RooAbsPdf::generate()

Usage example: RooDataSet\* data = pdf.generate(x,...) ;

Generate a new dataset containing the specified variables with events sampled from our distribution. Generate the specified number of events or expectedEvents() if not specified.

Any variables of this PDF that are not in whatVars will use their current values and be treated as fixed parameters. Returns zero in case of an error. The caller takes ownership of the returned dataset.

The following named arguments are supported

Verbose(Bool\_t flag) Print informational messages during event generation

NumEvent(int nevt) Generate specified number of events

Extended() The actual number of events generated will be sampled

from a Poisson distribution with mu=nevt. For use with

extended maximum likelihood fits

ProtoData(const RooDataSet&
 data, Bool\_t randOrder)

Use specified dataset as prototype dataset. If randOrder is set to true the order of the events in the dataset will be read in a random order the order of the events in the dataset will

be read in a random order number of events in the

prototype dataset

If ProtoData() is used, the specified existing dataset as a prototype: the new dataset will contain the same number of events as the prototype (unless otherwise specified), and any prototype variables not in whatVars will be copied into the new dataset for each generated event and also used to set our PDF parameters.

The user can specify a number of events to generate that will override the default. The result is a copy of the prototype dataset with only variables in whatVars randomized. Variables in whatVars that are not in the prototype will be added as new columns to the generated dataset.

# Create integrals of functions-RooAbsReal::createIntegral()

Usage example: RooAbsReal\* intOfFunc = func.createIntegral(x,...) ;

Create an object that represents the integral of the function over one or more observables listed in iset

The actual integration calculation is only performed when the return object is evaluated. The name of the integral object is automatically constructed from the name of the input function, the variables it integrates and the range integrates over

The following named arguments are accepted

NormSet(const RooArgSet&) Specify normalization set, mostly useful when working with PDFS

NumIntConfig(const Use given configuration for any numeric integration, if RooNumIntConfig&) necessary

Range(const char\* name) Integrate only over given range. Multiple ranges may be specified by passing multiple Range() arguments

# Automate fit studies - class RooMCStudy

Usage example: RooMCStudy mgr(model,observables,...) ;

Construct Monte Carlo Study Manager. This class automates generating data from a given PDF, fitting the PDF to that data and accumulating the fit statistics.

The constructor accepts the following arguments

const RooAbsPdf& model	The PDF to be studied
const RooArgSet& observables	The variables of the PDF to be considered the observables
FitModel(const RooAbsPdf&)	The PDF for fitting, if it is different from the PDF for generating
ConditionalObservables(const RooArgSet& set)	The set of observables that the PDF should <i>not</i> be normalized over
Binned(Bool_t flag)	Bin the dataset before fitting it. Speeds up fitting of large data samples
FitOptions(const char*)	Classic fit options, provided for backward compatibility
FitOptions()	Options to be used for fitting. All named arguments inside FitOptions()are passed to RooAbsPdf::fitTo();
<pre>Verbose(Bool_t flag)</pre>	Activate informational messages in event generation phase
<pre>Extended(Bool_t flag)</pre>	Determine number of events for each sample anew from a Poisson distribution
ProtoData(const RooDataSet&, Bool_t randOrder)	Prototype data for the event generation. If the randOrder flag is set, the order of the dataset will be re-randomized for each generation cycle to protect against systematic biases if the number of generated events does not exactly match the number of events in the prototype dataset at the cost of reduced precision with mu equal to the specified number of events

The plotParam() method plots the distribution of the fitted value of the given parameter on a newly created frame. This function accepts the following optional arguments

 $\label{prop:continuous} \textbf{FrameRange(double lo, double hi)} \quad \textbf{Set range of frame to given specification}$ 

FrameBins (int bins) Set default number of bins of frame to given number

Frame(...) Pass supplied named arguments to

RooAbsRealLValue::frame() function. See frame() function for list of allowed arguments

If no frame specifications are given, the AutoRange() feature will be used to set the range. Any other named argument is passed to the RooAbsData::plot0n() call. See that function for allowed options

The plotPull() method plots the distribution of pull values for the specified parameter on a newly created frame. If asymmetric errors are calculated in the fit (by MINOS) those will be used in the pull calculation This function accepts the following optional arguments

FrameRange(double lo, double Set range of frame to given specification

FrameBins(int bins) Set default number of bins of frame to given number

Frame(...) Pass supplied named arguments to RooAbsRealLValue::frame() function. See frame() function for list of allowed arguments

FitGauss(Bool\_t flag) Add a Gaussian fit to the frame

If no frame specifications are given, the AutoSymRange() feature will be used to set the range Any other named argument is passed to the RooAbsData::plotOn() call. See that function for allowed options

# **Data manipulation**

## Reduce a dataset - RooAbsData::reduce()

Usage example: RooAbsData\* reducedData = data.reduce(...) ;

Create a reduced copy of this dataset. The caller takes ownership of the returned dataset

The following optional named arguments are accepted

SelectVars(const RooArgSet& Only retain the listed observables in the output dataset

Cut(const char\* expression) Only retain event surviving the given cut expression

Cut(const RooFormulaVar& Only retain event surviving the given cut formula expr)

CutRange(const char\* name) Only retain events inside range with given name. Multiple CutRange arguments may be given to select multiple

ranges

EventRange(int lo, int hi) Only retain events with given sequential event numbers

Name(const char\* name) Give specified name to output dataset

Title(const char\* name) Give specified title to output dataset

# Appendix B – Selected statistical issues

This section is scheduled for the next version of the manual

# Appendix C - RooFit class structure

This section is scheduled for the next version of the manual

**General philosophy** 

Generic value objects - RooAbsArg

Real valued objects

Discrete valued objects

**Datasets** 

**Collections**