

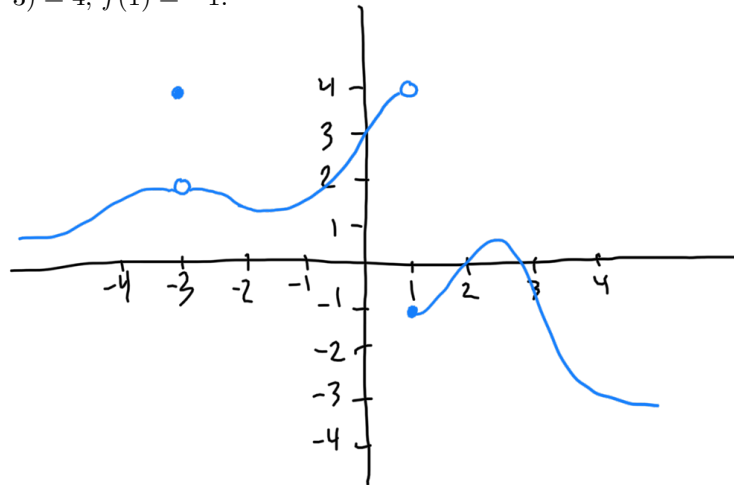
1. Guess the value of the following limits:

(a) $\lim_{s \rightarrow 5} s - 3$ 2, because $5 - 3 = 2$

(b) $\lim_{u \rightarrow -2} u^2 - \cos(\pi u)$ 3, because $(-2)^2 - \cos(\pi(-2)) = 4 - 1 = 3$

(c) $\lim_{v \rightarrow 4} \frac{v+3}{4v-2}$ $\frac{1}{2}$, because $\frac{4+3}{4 \cdot 4 - 2} = \frac{7}{14} = \frac{1}{2}$
(here it's important that the denom. is $\neq 0$)

2. Sketch the graph of a function f that satisfies all of the following: $\lim_{x \rightarrow -3^-} f(x) = 2$, $\lim_{x \rightarrow -3^+} f(x) = 2$, $\lim_{x \rightarrow 1^-} f(x) = 4$, $\lim_{x \rightarrow 1^+} f(x) = -1$, $f(-3) = 4$, $f(1) = -1$.



3. Determine the following infinite limits:

(a) $\lim_{s \rightarrow 1^-} \frac{s^2 - 4}{s - 1}$ $+\infty$, b/c $\frac{s^2 - 4}{s - 1} \approx \frac{-3}{\text{something small \& neg.}}$
when s is close to 1 but less than 1

(b) $\lim_{u \rightarrow 3^+} \frac{u^2 - 2u - 8}{u^2 - 6u + 9}$ $-\infty$, b/c $\frac{u^2 - 2u - 8}{u^2 - 6u + 9} = \frac{u^2 - 2u - 8}{(u - 3)^2}$
 $\approx \frac{-5}{\text{something small \& pos.}}$

(c) $\lim_{t \rightarrow 9^-} \frac{\sqrt{t}}{(t - 9)^3}$ $+\infty$ (similar reasoning to (b)) for u close to 3

(d) $\lim_{\theta \rightarrow \pi^+} \frac{\theta - 4}{\sin(\theta)}$ $+\infty$ (similar reasoning to (a))

4. Consider the function $f(x) = \frac{2x-3}{(x-2)(x+4)}$.

(a) Find all the vertical asymptotes of f .

$$\lim_{x \rightarrow 2^+} f(x) = \infty, \quad \lim_{x \rightarrow -4^+} f(x) = \infty \quad (\text{check this}),$$

while $f(x)$ exists everywhere except $x=2, -4$.

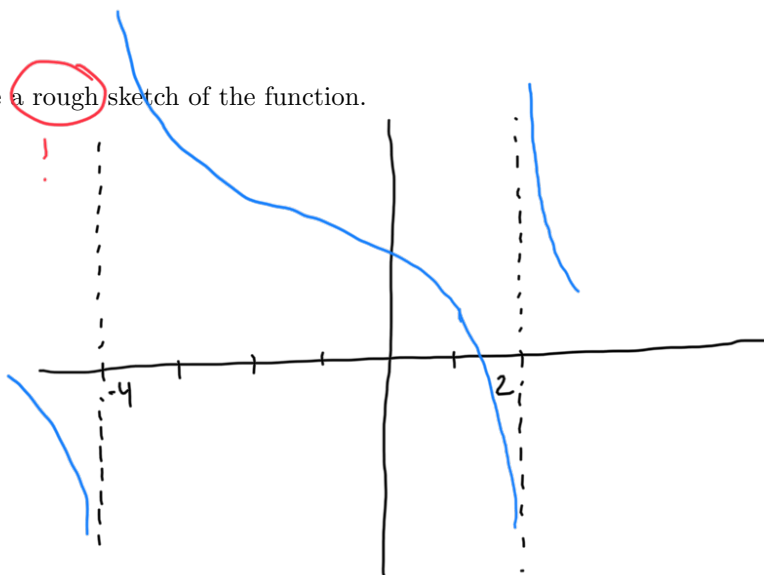
So the (only) vertical asymptotes are $x=2$ and $x=-4$.

(b) Compute $\lim_{x \rightarrow 2^+} f(x)$, $\lim_{x \rightarrow 2^-} f(x)$, $\lim_{x \rightarrow -4^+} f(x)$, and $\lim_{x \rightarrow -4^-} f(x)$.

$$\lim_{x \rightarrow 2^+} f(x) = \infty, \quad \lim_{x \rightarrow 2^-} f(x) = -\infty$$

$$\lim_{x \rightarrow -4^+} f(x) = \infty, \quad \lim_{x \rightarrow -4^-} f(x) = -\infty$$

(c) Make a rough sketch of the function.



5. Consider the function $f(x) = \tan\left(\frac{1}{x}\right)$.

(a) Show that $f(x) = 0$ for $x = \frac{1}{\pi}, \frac{1}{2\pi}, \frac{1}{3\pi}, \dots$

If k is an integer, then $\sin(k\pi) = 0$ and $\cos(k\pi) = \pm 1$.

$$\text{So } \tan\left(\frac{1}{\frac{1}{k\pi}}\right) = \tan(k\pi) = \frac{\sin(k\pi)}{\cos(k\pi)} = 0.$$

(b) Show that $f(x) = 1$ for $x = \frac{4}{\pi}, \frac{4}{5\pi}, \frac{4}{9\pi}, \dots$

If k is an integer, then $\sin\left(\frac{\pi}{4} + k\pi\right) = \cos\left(\frac{\pi}{4} + k\pi\right)$, so

$$\tan\left(\frac{1}{\frac{4}{(4k+1)\pi}}\right) = \tan\left(\frac{(4k+1)\pi}{4}\right) = \tan\left(\frac{\pi}{4} + k\pi\right) = 1.$$

(c) What can you conclude about $\lim_{x \rightarrow 0^+} \tan\left(\frac{1}{x}\right)$?

The limit does not exist.

$\tan\left(\frac{1}{x}\right)$ "hits" both 0 and 1 infinitely many times as $x \rightarrow 0^+$.