Math 221 Worksheet 24 December 3, 2020 Section 5.1: Areas Between Curves

1.	Consider the curves defined by the equations $x = 3 - y^2$ and $x = -1$.
	(a) Find the points where the curves intersect. Sketch the curves.
	(b) Write down two different integrals which represent the area of the region enclosed by the curves. (One integral should be in terms of x , and the other should be in terms of y .)
	(c) Evaluate both integrals to check that they are equal.
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2.	Consider the curves defined by the equations $y = x^2$ and $y = \sqrt{x}$.

(a) Find the points where the curves intersect. Sketch the curves.

	(b)	Write down an integral in terms of x that represents the area between the two curves.
	(c)	Write down an integral in terms of y that represents the area between the two curves. (You will need to solv for x in the equations $y=x^2$ and $y=\sqrt{x}$.)
3.		$f(x)=x^2$, and let (a,a^2) be a point on the graph of f . Find the equation of the line tangent to the graph of f at (a,a^2) .
	(b)	Sketch the graph, the point, and the tangent line. Shade the region enclosed by the x -axis, the graph, and th tangent line.
	(c)	If the shaded region has area $\frac{2}{3}$, what is the value of a ?

- 4. Let $f(x) = \sin x$ and $g(x) = \cos x$ for $x \in [0, \frac{\pi}{2}]$.
 - (a) Find the points where the graphs of f and g intersect. Sketch the graphs.

(b) Find the area of the region enclosed by the graph of f, the graph of g, and the lines $x = \frac{\pi}{2}$ and x = 0.

5. Each set of equations defines some curves. Find the area of the region enclosed by the curves.

(a)
$$y = \frac{8}{x}, y = 2x, x = 4$$

(b)
$$x = 3 + y^2$$
, $x = 2 - y^2$, $y = 1$, $y = -2$

(c)
$$y = e^{1+2x}$$
, $y = e^{1-x}$, $x = -2$, $x = 1$

(d)
$$y = 2x^2 + 10$$
, $y = 4x + 16$, $x = -2$, $x = 5$

(e)
$$x = -y^2 + 10$$
, $x = (y - 2)^2$

(f)
$$y = xe^{-x^2}$$
, $y = x + 1$, $x = 2$, $x = 0$