Math 221 Sec 003 Quiz 5 Solution

1. Consider a hot air balloon rising vertically from a launch site located on the ground. A person is initially standing 21 m from the launch site and begins running towards the site at a rate of 8 m/s at the moment of launch. If the hot air balloon is rising at a constant rate of 6 m/s, how fast is the distance between the person and the balloon changing 2 seconds after the person starts running? (You may express your answer in fraction. Don't forget the unit.)

Solution:

Let x be the distance from person to launch site, and let y be the height of the balloon.

$$\frac{dx}{dt} = -8, \quad \frac{dy}{dt} = 6$$

(correct values +1)

Let z be the distance between the person and the balloon.

$$z^2 = x^2 + y^2$$

(correct formula +1)

After 2 seconds, $x = 5$, $y = 12$.

Therefore, $z = 13$.

(correct values +1)

$$2z\frac{dz}{dt} = 2x\frac{dx}{dt} + 2y\frac{dy}{dt}$$

(derivative +1)

$$26\frac{dz}{dt} = 10(-8) + 24(6)$$
$$\frac{dz}{dt} = \frac{32}{13}$$

(correct answer +1)

Hence, the person and the balloon are parting at $\frac{32}{13}$m/s.

2. Find all global extrema of the following function

$$f(x) = \begin{cases} x - 5 & 0 \le x < 1 \\ 2x^2 - 8x & 1 \le x \le 4 \end{cases}$$

and identify their nature (maxima, minima).

Solution:

Endpoints: $x = 0, 4$

Point not differentiable: $x = 1$

Critical point: $f'(x) = 0$

None for $0 \le x < 1$ as $f'(x) = 1 \ne 0$.

For $1 \le x \le 4$, $f'(x) = 4x - 8$. Zero at $x = 2$.

(correctly identifying all the points +2,
-1 for each extra point or missing point)

$f(0) = -5$

$f(1) = -6$

$f(2) = -8$ (min)

$f(4) = 0$ (max)

(correct values after substitution +2,
-1 for each wrong value)

(Correct nature +1)