Math 221 Worksheet 22
November 17, 2020
Section 6.3: Logarithmic Functions; Section 6.4: Derivatives of Logarithmic Functions

1. Compute the derivatives of the following functions.

   (a) $f(x) = \ln(3x^2 - 5x)$

   (b) $g(u) = \dfrac{u + \ln(5u)}{\sin(u)}$

   (c) $f(s) = \ln\left(\sqrt{\frac{2s+1}{4s}}\right)$

   (d) $h(u) = e^{4u} \ln(ue^u)$

   (e) $y = x \log_4(\sin(x))$

   (f) $y = \log_2(x \log_5 x)$

2. Find the equation of the tangent line to the curve $y = \ln(x^2)$ at the point $(e, 2)$.

3. Sketch the graph of $f(x) = x + e^x$ using the curve sketching techniques you learned in Chapter 3.

4. Find $y'$ if $2e^y + \ln(xy) = 2x^2y + 4$.

5. Find a formula for the $n$th derivative of $g(s) = e^{4s}$.

6. Compute the following integrals.

(a) $\displaystyle\int_0^{\frac{e-1}{2}} \frac{5}{1+2x}\,dx$

(b) $\displaystyle\int \frac{\sin(\ln x)}{x}\,dx$

(c) $\displaystyle\int_1^{e} \frac{(\ln t)^4}{t}\,dt$

(d) $\displaystyle\int_0^{\ln(1+\pi)} e^x \cos(1-e^x)\,dx$

(e) $\int \frac{\log_{10} x}{x} dx$

7. Determine the values of $x$ that satisfy the inequality $1 < e^{4x-2} < 2$.

8. Solve the following equations:

   (a) $e^{4x-6} = 8$.

   (b) $e - e^{-4x} = 4$.

   (c) $\ln(x) + \ln(x - 1) = 1$.

9. Differentiate the following functions:

   (a) $G(x) = 4^{C/x}$, where $C$ is a constant

(b) $y = x^x$

(c) $y = (\sin x)^{\ln x}$

(d) $y = (3x^2 + 5)^{\frac{1}{x}}$

10. Find $y'$ if $x^y = y^x$.

11. A computer is programmed to inscribe a series of rectangles in the first quadrant under the curve of $y = e^{-x}$. What is the area of the largest rectangle that can be inscribed?

12. Let $a \neq -1$ be a constant. Calculate $\int \left( \frac{x}{a} + \frac{a}{x} + x^a + a^x + ax \right) \, dx$.

13. Sketch the graph of $f(x) = \ln(1 + x^2)$ using the curve sketching techniques you learned in Chapter 3.

14.

(a) $\displaystyle\int_0^{\sqrt{3}/4} \frac{dx}{1+16x^2}$

(b) $\displaystyle\int \frac{1+x}{1+x^2}\,dx$

(c) $\displaystyle\int_0^{\pi/2} \frac{\sin x}{1+\cos^2 x}\,dx$

(d) $\displaystyle\int \frac{dx}{\sqrt{1-x^2}\,\sin^{-1} x}$

(e) $\displaystyle\int_{1/\sqrt{3}}^{\sqrt{3}} \frac{8}{1+x^2}\,dx$