Oklahoma and Oregon Redistricting Project Code revA

December 12, 2023

1 Final Project: Redistricing Project of Oklahoma and Oregon.

- 1.0.1 Class: IE 5318 Principles of Operations Reasearch
- 1.0.2 Team Members: Natalie Ventura, Jessica Dedeaux, Damilola Fasheru
- 1.0.3 Date: Fall 2023; 12-12-2023

```
[1]: | # GitHub Project file: https://qithub.com/Jdedeaux/Districts-Counties/tree/main
     # Credit to:
           Dr. Hamidreza Validi transportation, short path codes, and assigment ⊔
      →codes: IE 5318 Principles of Operations Reasearch at Texas Tech University
           Dr. Austin Buchanan districting example starter code: https://github.com/
      \hookrightarrow AustinLBuchanan/Districting-Examples/tree/main
           Gurobi Optimization Tutorial via the primary website: https://www.qurobi.
      -com/resources/lp-chapter-6-modeling-and-solving-linear-programming-problems/
           Dr. Gabrielle Taylor: https://github.com/gabrielletay96
           shapefiles: https://www.youtube.com/watch?v=xxq4Vm-Xq9q
     #
           for loops: https://www.youtube.com/watch?v=KWqYhaOclzw&t=172s
           Data: http://people.csail.mit.edu/ddeford/dual_graphs
           Data: https://lykhovyd.com/files/public/districting/2020/
           nested for loops: https://www.youtube.com/watch?v=APWy6Pc83qE
           Networknx Package: https://networkx.org/documentation/stable/
      →auto_examples/drawing/plot_labels_and_colors.html
```

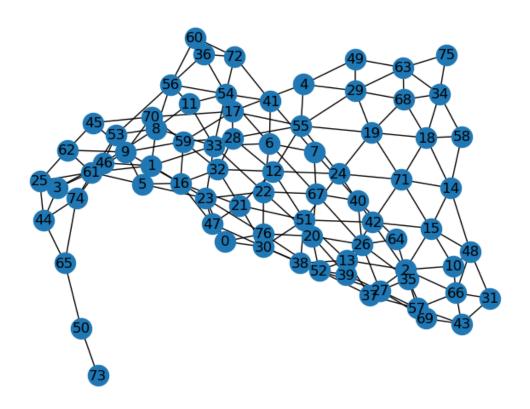
1.1 OKLAHOMA: Code Verification

```
[2]: # Import the following packages to run various cells
import gurobipy as gp
from gurobipy import GRB
import networkx as nx
import geopandas as gpd
import math
from gerrychain import Graph
import pandas as pd
import matplotlib.pyplot as plt
```

1.1.1 Step 1: Define the edges and nodes of the model

```
[3]: # Create the intial node and edges model using the edges list for Oklahoma
         # Remove the non-integers from the text file and ensure it is a plain text file.
        filepath = '/Users/jessi/Gurobi-Python Playground/OK.graph'
        # reading from the data set
        G = nx.read_edgelist(filepath,nodetype=int)
        print(type(G))
        # drawing the graph
        nx.draw(G, with_labels=True)
        # Current model description
        print("The Oklahoma graph has",G.number_of_nodes(),"nodes")
        print("The Oklahoma graph has",G.number_of_edges(),"edges")
        print("Oklahoma graph has nodes",G.nodes )
        print("Oklahoma graph has edges", G. edges )
       <class 'networkx.classes.graph.Graph'>
       The Oklahoma graph has 77 nodes
       The Oklahoma graph has 195 edges
       Oklahoma graph has nodes [0, 23, 30, 47, 76, 1, 5, 8, 33, 46, 61, 2, 10, 13, 15,
       37, 42, 57, 66, 3, 25, 44, 74, 4, 17, 29, 49, 55, 9, 16, 6, 7, 22, 41, 54, 67,
       40, 11, 53, 56, 45, 59, 62, 70, 48, 12, 21, 24, 28, 32, 51, 38, 52, 14, 18, 58,
       71, 19, 34, 68, 20, 26, 39, 27, 35, 64, 69, 63, 31, 43, 75, 36, 60, 72, 65, 50,
       Oklahoma graph has edges [(0, 23), (0, 30), (0, 47), (0, 76), (23, 1), (23, 5),
       (23, 22), (23, 33), (23, 47), (23, 76), (30, 21), (30, 38), (30, 47), (30, 51),
       (30, 52), (30, 76), (47, 16), (47, 21), (76, 20), (76, 22), (76, 52), (1, 5),
       (1, 8), (1, 33), (1, 46), (1, 61), (5, 9), (5, 16), (5, 61), (8, 11), (8, 33),
       (8, 46), (8, 53), (8, 56), (33, 6), (33, 11), (33, 22), (33, 54), (46, 3), (46, 6)
       53), (46, 61), (46, 74), (61, 3), (61, 9), (61, 25), (61, 62), (2, 10), (2, 13),
       (2, 15), (2, 37), (2, 42), (2, 57), (2, 66), (10, 15), (10, 48), (10, 66), (13, 15), (10, 15), (10, 10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 10), (10, 
       37), (13, 38), (13, 42), (13, 51), (13, 52), (15, 14), (15, 42), (15, 48), (15,
       71), (37, 27), (37, 52), (37, 57), (42, 24), (42, 51), (42, 71), (57, 27), (57,
       43), (57, 66), (57, 69), (66, 31), (66, 43), (66, 48), (3, 25), (3, 44), (3,
       74), (25, 44), (25, 62), (44, 65), (44, 74), (74, 53), (74, 65), (4, 17), (4,
       29), (4, 49), (4, 55), (17, 28), (17, 55), (17, 59), (17, 70), (29, 19), (29,
       49), (29, 55), (29, 63), (29, 68), (49, 63), (55, 12), (55, 19), (55, 24), (55,
       28), (9, 16), (9, 45), (9, 59), (9, 62), (9, 70), (16, 21), (16, 32), (16, 59),
       (6, 7), (6, 22), (6, 41), (6, 54), (6, 67), (7, 40), (7, 41), (7, 67), (22, 20),
       (22, 67), (41, 54), (41, 72), (54, 11), (54, 36), (54, 56), (54, 72), (67, 20),
       (67, 26), (67, 40), (40, 26), (40, 64), (11, 56), (53, 56), (56, 36), (56, 60),
       (45, 62), (45, 70), (59, 28), (59, 32), (59, 70), (48, 14), (48, 31), (12, 21),
       (12, 24), (12, 28), (12, 32), (12, 51), (21, 32), (21, 51), (24, 19), (24, 51),
       (24, 71), (28, 32), (51, 38), (38, 52), (52, 20), (52, 27), (52, 39), (14, 18),
```

```
(14, 58), (14, 71), (18, 19), (18, 34), (18, 58), (18, 68), (18, 71), (58, 34), (71, 19), (19, 68), (34, 63), (34, 68), (34, 75), (68, 63), (20, 26), (20, 39), (26, 27), (26, 35), (26, 39), (26, 64), (39, 27), (27, 35), (27, 69), (35, 64), (35, 69), (69, 43), (63, 75), (31, 43), (36, 60), (36, 72), (60, 72), (65, 50), (50, 73)]
```



1.1.2 Step 2: List the county populations

```
[4]: # Integrate the populations by county to the model

# create population list
population = list()

# read text file into software system using files on local drive
filepath = "/Users/jessi/Gurobi-Python Playground/OK-2.population"
file = open( filepath, "r")

# Create List for each county population
Listpop = []
for each in file:
```

```
#split the list so the populations in each range
each = each.split(' ')
    #print((each[1])
    Listpop.append(int(each[1]))

# displays the list of integers
print("Oklahoma's county populations = ", Listpop)

# the sums can be taken by list position
total_population = sum(Listpop)

#print the total population of OK
print("The total_population of Oklahoma = ", total_population)
population = Listpop #label for code clarity
n =len(population)

#print the county population as a sum districts of OK
print("Oklahoma's number of counties = ",n)
print("Population data is", type(population))
```

```
Oklahoma's county populations = [255755, 11943, 45837, 20081, 50976, 15034, 124098, 6193, 27469, 60580, 11154, 11629, 69967, 14003, 42391, 12769, 41848, 47472, 46987, 73085, 27576, 34273, 52431, 115541, 40069, 8878, 47557, 10957, 16577, 86905, 69442, 33151, 77350, 29600, 41487, 15840, 6239, 5925, 25482, 13488, 6472, 7992, 20252, 15205, 3685, 4527, 4810, 718633, 50384, 10536, 20640, 12191, 37492, 3647, 9446, 603403, 22119, 14182, 22683, 11561, 2922, 7527, 5642, 15029, 9423, 5636, 11572, 45048, 41259, 42416, 46562, 70990, 26446, 2475, 4151, 31848, 34506]
The total_population of Oklahoma = 3751351
Oklahoma's number of counties = 77
Population data is <class 'list'>
```

1.1.3 Step 3: Population Deviation Model

```
[5]: # If model is feasible, use the population deviation operations research model.
# create model
m = gp.Model()
#number of districts per the 2020 census for OK
k = 5

# create variables adapted from districting starter models
x = m.addVars(n,k, vtype=GRB.BINARY) # x_ij equals one when county i is_u
assigned to district j
y = m.addVar() # the population of least-populated district
z = m.addVar() # the population of most-populated district
# objective is to minimize absolute population deviation
```

Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 87 rows, 387 columns and 1165 nonzeros

Model fingerprint: 0x235259e8

Variable types: 2 continuous, 385 integer (385 binary)

Coefficient statistics:

Matrix range [1e+00, 7e+05]
Objective range [1e+00, 1e+00]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 1e+00]

Found heuristic solution: objective 1149841.0000

Presolve time: 0.00s

Presolved: 87 rows, 387 columns, 1165 nonzeros

Variable types: 0 continuous, 387 integer (385 binary)

Root relaxation: objective 0.000000e+00, 125 iterations, 0.00 seconds (0.00 work units)

Nodes		Current Node Obj Depth IntInf		Object	ive Bounds	Work			
Expl Unexpl				Inf Incumbent	${\tt BestBd}$	Gap	It/Node	Time	
	•	•		•	T		4.0.00/		•
	0	0	0.00000	0	7 1149841.00	0.00000	100%	_	0s
H	0	0			339690.00000	0.00000	100%	-	0s
H	0	0			190691.00000	0.00000	100%	-	0s
H	0	0			29846.000000	0.00000	100%	-	0s
H	0	0			29061.000000	0.00000	100%	_	0s
	0	0	0.00000	0	12 29061.0000	0.00000	100%	_	0s
Н	0	0			18448.000000	0.00000	100%	-	0s

0	0 0.000	00 0	12 18448.0000	0.00000	100%	_	0s
Н О	0		13568.000000	0.00000	100%	_	0s
0	0 0.000	00 0	9 13568.0000	0.00000	100%	_	0s
Н О	0		13270.000000	0.00000	100%	_	0s
0	0 0.000	00 0	11 13270.0000	0.00000	100%	_	0s
Н О	0		9740.0000000	0.00000	100%	_	0s
Н О	0		7343.0000000	0.00000	100%	_	0s
0	0 0.000	00 0	7 7343.00000	0.00000	100%	_	0s
Н О	0		4835.0000000	0.0000	100%	_	0s
0	0 0.000	00 0	7 4835.00000	0.00000	100%	-	0s
Н О	0		2400.0000000	0.00000	100%	_	0s
0	0 0.000	00 0	9 2400.00000	0.00000	100%	-	0s
0	0 0.000	00 0	9 2400.00000	0.00000	100%	-	0s
Н О	0		1655.0000000	0.00000	100%	-	0s
Н О	0		1349.0000000	0.00000	100%	-	0s
Н О	0		1059.0000000	0.0000	100%	-	0s
0	2 1.000	00 0	7 1059.00000	1.00000	100%	_	0s
Н 8	7		963.0000000	1.00000	100%	9.0	0s
H 10	7		828.0000000	1.00000	100%	9.9	0s
H 32	21		265.0000000	1.00000	100%	9.9	0s
H 84	70		207.0000000	1.00000	100%	6.4	0s
H 151	296		133.0000000	1.00000	99.2%	5.0	0s
Н 2618	1940		68.0000000	1.00000	98.5%	4.1	0s
Н 2698	1905		48.0000000	1.00000	97.9%	4.8	0s
*14689	7127	95	46.0000000	1.00000	97.8%	4.9	2s
H14723	6759		40.0000000	1.00000	97.5%	4.9	3s
H14736	6428		33.0000000	1.00000	97.0%	4.9	3s
H14772	6137		25.0000000	1.00000	96.0%	5.0	3s
H16044	6517		24.0000000	1.00000	95.8%	5.1	3s
22064	7847 infeasib	le 130	24.00000	1.00000	95.8%	5.4	5s
*27388	7146	116	14.0000000	1.00000	92.9%	5.6	5s
H29273	6390		12.0000000	1.00000	91.7%	5.7	5s
43956	6339 infeasib	le 112	12.00000	1.00000	91.7%	5.7	10s
H50017	6328		10.0000000	1.00000	90.0%	5.7	10s
H70833	6330		9.0000000	1.00000	88.9%	5.9	12s
*73086	5777	107	8.0000000	1.00000	87.5%	5.9	13s
83148	5995 infeasib		8.00000	1.00000	87.5%	5.9	15s
*83297	5907	104	5.0000000	1.00000	80.0%	5.9	15s
H111759			3.0000000	1.00000	66.7%	5.7	18s
H120568			2.0000000	1.00000	50.0%	5.6	19s
120814			2.00000	1.00000	50.0%	5.5	20s
159469		000 143	5 2.00000	1.00000	50.0%	5.5	25s
H186669	959		1.0000000	1.00000	0.00%	5.4	28s

Cutting planes:

Gomory: 1 Cover: 113 MIR: 144

```
StrongCG: 150
Flow cover: 121
Inf proof: 2
Zero half: 1
RLT: 3

Explored 188295 nodes (1027011 simplex iterations) in 28.94 seconds (16.15 work units)
Thread count was 8 (of 8 available processors)

Solution count 10: 1 2 3 ... 24

Optimal solution found (tolerance 1.00e-04)
Best objective 1.00000000000000e+00, best bound 1.000000000000e+00, gap 0.0000%
```

1.1.4 Step 4: Current model status for population

```
[6]: # print the absolute population deviation
print("The absolute population deviation is",m.objval,"person(s).")
print("Oklahoma has districts =",k)
```

The absolute population deviation is 1.0 person(s). Oklahoma has districts = 5

1.1.5 Step 5: create model to access the minimization of cut edges using the county to district populations

```
[7]: filepath = '/Users/jessi/Gurobi-Python Playground/OK_county.json'
     G = Graph.from_json(filepath)
     print(G.nodes)
     m = gp.Model()
     #check the list of nodes and matches are align with the mapping data
     print(G.nodes)
     print(G.edges)
     for node in G.nodes:
         # using the columns, located the population
         population = G.nodes[node]['TOTPOP']
     total_population = sum(G.nodes[node]['TOTPOP'] for node in G.nodes)
     print(total_population)
     # Impose a 3% population deviation (+/- 0.5%) for the Operations Research model
     deviation = 0.03
     #number of counties
     # number of districts
```

```
k
# Lower and Upper bound populations to ensure the populations of the districts ___
 ⇔do not exceed the bounds
L = math.ceil((1-deviation/2)*total_population/k)
U = math.floor((1+deviation/2)*total_population/k)
# create variables adapted from redistricitng starter:
x = m.addVars(G.nodes, k, vtype=GRB.BINARY) # x[i,j] equals one when county i_{l}
 ⇔is assigned to district j
y = m.addVars(G.edges, vtype=GRB.BINARY) # y[u,v] equals one when edge {u,v}_{\square}
 ⇒is cut
# objective is to minimize cut edges
m.setObjective(gp.quicksum(y[u,v] for u,v in G.edges), GRB.MINIMIZE)
# add constraints saying that each county i is assigned to one district
m.addConstrs( gp.quicksum(x[i,j] for j in range(k)) == 1 for i in G.nodes)
# add constraints saying that each district has population at least L and at \sqcup
 ⇔most U
m.addConstrs(gp.quicksum(G.nodes[i]['TOTPOP'] * x[i,j] for i in G.nodes) >= <math>L_{L}
 →for j in range(k) )
m.addConstrs(gp.quicksum(G.nodes[i]['TOTPOP'] * x[i,j] for i in G.nodes) <= U__

¬for j in range(k) )
# add constraints saying that edge \{i,j\} is cut if i is assigned to district v_{\sqcup}
 \rightarrowbut j is not.
m.addConstrs(x[i,v] - x[j,v] \le y[i,j]  for i,j in G.edges for v in range(k))
m.update()
# solve IP model
m.optimize()
# print the optimal minization of cut edges
print("Oklahoma: The number of cut edges is",m.objval)
print("Oklahoma: Using Lower Bound of Population =",L,"and Upper Bound of □
 →Population =",U,"and districts =",k)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76]
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
42, 43, 44, 45, 46, 47, 48, 49, 50, 51, 52, 53, 54, 55, 56, 57, 58, 59, 60, 61,
```

```
62, 63, 64, 65, 66, 67, 68, 69, 70, 71, 72, 73, 74, 75, 76]
[(0, 66), (0, 57), (0, 61), (0, 73), (0, 27), (1, 51), (1, 22), (1, 45), (1, 27), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), (1, 10), 
56), (1, 68), (1, 53), (1, 46), (2, 26), (2, 63), (2, 36), (2, 31), (2, 43), (3,
32), (3, 37), (3, 71), (3, 72), (4, 40), (4, 28), (4, 22), (4, 24), (5, 72), (5,
25), (5, 20), (5, 47), (5, 58), (6, 20), (6, 58), (6, 75), (6, 19), (7, 71), (7,
72), (7, 65), (7, 20), (8, 32), (8, 52), (8, 71), (8, 35), (9, 52), (9, 40), (9,
51), (9, 22), (10, 47), (10, 58), (10, 60), (10, 19), (10, 39), (10, 69), (11,
37), (11, 76), (11, 25), (11, 34), (12, 42), (12, 48), (12, 70), (12, 33), (13,
54), (13, 30), (13, 38), (14, 53), (14, 21), (14, 46), (14, 42), (14, 29), (14,
48), (15, 67), (15, 75), (15, 68), (15, 66), (15, 61), (16, 76), (16, 34), (16,
55), (16, 60), (16, 59), (17, 74), (17, 49), (17, 31), (18, 41), (18, 33), (19,
58), (19, 75), (19, 69), (19, 66), (19, 57), (20, 72), (20, 65), (20, 58), (20,
44), (20, 75), (21, 24), (21, 53), (21, 42), (22, 40), (22, 51), (22, 24), (22,
56), (23, 69), (23, 57), (23, 64), (23, 74), (23, 73), (23, 62), (23, 49), (24,
28), (24, 56), (24, 53), (25, 37), (25, 72), (25, 34), (25, 47), (26, 59), (26,
63), (27, 61), (27, 73), (27, 54), (27, 62), (27, 38), (28, 40), (29, 46), (29,
50), (29, 54), (29, 48), (29, 70), (29, 30), (30, 54), (30, 70), (31, 36), (31,
74), (31, 43), (32, 71), (33, 42), (34, 76), (34, 55), (34, 47), (35, 52), (35,
71), (35, 51), (35, 65), (35, 45), (36, 63), (36, 64), (36, 74), (37, 72), (38, 64)
54), (38, 62), (39, 60), (39, 59), (39, 69), (39, 63), (39, 64), (42, 48), (44,
65), (44, 45), (44, 67), (44, 75), (44, 68), (45, 51), (45, 65), (45, 68), (46,
68), (46, 53), (46, 50), (47, 55), (47, 58), (47, 60), (48, 70), (49, 74), (49,
62), (50, 68), (50, 61), (50, 54), (51, 52), (53, 56), (54, 61), (55, 60), (57,
69), (57, 66), (57, 73), (59, 60), (59, 63), (61, 66), (62, 73), (63, 64), (64,
69), (64, 74), (65, 71), (66, 75), (67, 75), (67, 68), (71, 72)]
3751351
Gurobi Optimizer version 10.0.3 build v10.0.3rc0 (mac64[arm])
CPU model: Apple M2
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 1062 rows, 580 columns and 4080 nonzeros
Model fingerprint: 0x7b871e34
Variable types: 0 continuous, 580 integer (580 binary)
Coefficient statistics:
                                 [1e+00, 7e+05]
   Matrix range
   Objective range
                                 [1e+00, 1e+00]
                                 [1e+00, 1e+00]
   Bounds range
   RHS range
                                 [1e+00, 8e+05]
Presolve time: 0.01s
Presolved: 1062 rows, 580 columns, 4080 nonzeros
Variable types: 0 continuous, 580 integer (580 binary)
Found heuristic solution: objective 102.0000000
Root relaxation: objective 0.000000e+00, 683 iterations, 0.06 seconds (0.03 work
units)
```

Objective Bounds

1

Work

Т

Nodes

Current Node

F	Expl	Unexpl	1 0	bj De	pth	In	tInf	Incumbent	;	BestBd	Gap	It/Node	Time
	C) 0	0	.00000		0	385	102.00000		0.00000	100%	_	0s
Н	C							100.0000000		0.00000	100%	_	0s
	C		3	.54115		0	394	100.00000		3.54115	96.5%	_	0s
	C	0		.75216		0	322	100.00000		9.75216	90.2%	_	0s
Н	C	0						94.0000000		9.75216	89.6%	_	0s
	C	0	9	.77816		0	322	94.00000		9.77816	89.6%	-	0s
Η	C	0						80.0000000		9.77816	87.8%	_	0s
	C	0	10	.25155		0	325	80.00000	1	0.25155	87.2%	_	0s
	C	0	10	.31184		0	310	80.00000	1	0.31184	87.1%	_	1s
	C	0	10	.32740		0	310	80.00000	1	0.32740	87.1%	-	1s
	C	0	10	.32740		0	310	80.00000	1	0.32740	87.1%	_	1s
	C	0	10	.48686		0	373	80.00000	1	0.48686	86.9%	_	1s
	C	0	10	.57637		0	316	80.00000	1	.0.57637	86.8%	-	1s
	C			.72201		0	309	80.00000		.0.72201	86.6%	-	1s
	C			.73880		0	362	80.00000		.0.73880	86.6%	-	1s
	C			.74733		0	377	80.00000		.0.74733	86.6%	-	1s
	C			.74776		0	381	80.00000		.0.74776	86.6%	-	1s
	C			.81927		0	333	80.00000		.0.81927	86.5%	-	1s
	C			.83029		0	312	80.00000		0.83029	86.5%	-	1s
	C			.83029		0	304	80.00000		.0.83029	86.5%	-	1s
	C			.83029		0	304	80.00000		.0.83029	86.5%	_	1s
	C			.83029		0	304	80.00000		0.83029	86.5%	_	1s
	C		10	.83029		0	304	80.00000		.0.83029	86.5%	_	1s
Η	36							78.0000000		7.12159	78.0%	352	2s
Н	64							74.0000000		7.12159	76.9%	303	2s
Η	66							73.0000000		7.12159	76.5%	298	2s
Н	70							61.0000000		7.12159	71.9%	295	2s
H	103							59.0000000		7.12159	71.0%	257	3s
H	104							57.0000000		7.12159	70.0%	256	3s
H	108							55.0000000		7.12159	68.9%	251	3s
H	149							51.0000000		7.12159	66.4%	225	3s
Н	194							47.0000000		7.12159		219	3s
H	196							46.0000000		7.12159		219	3s
Н	196							45.0000000		7.12159	62.0%	219	3s
Н	277		200	00007		20	405	42.0000000		7.12159	59.2%	202	3s
	509		39	.92087		23	135			7.12159	59.2%	182	5s
Н	597		20	CE 100		4 -	070	39.0000000		8.06854	53.7%	189	6s
	1273			.65139		15	272	39.00000		9.83573	49.1%		10s
	1493			.63256		15	281	39.00000		21.63256	44.5%		15s
	1937			.08872		25	376	39.00000		21.63256	44.5%		20s
	2582			cutoff		37 12		39.00000		24.30628	37.7%		25s
	3347			cutoff		43	220	39.00000		25.82863	33.8%		30s
	4361			.36045		23	332	39.00000		27.14976	30.4%		35s
	5563			.46161		27	315	39.00000		28.00574	28.2%		40s
	6376			2.66073		21	317	39.00000		28.39526	27.2%		45s
	7899	2871	30	.59452	- 2	28	361	39.00000	- 2	29.04111	25.5%	173	50s

9302	3242	32.01120	24	341	39.00000	29.49253	24.4%	170	56s
10454	3613	36.11452	26	269	39.00000	29.75403	23.7%	166	60s
11861	3965	34.24716	32	359	39.00000	30.12514	22.8%	164	65s
13467	4408	32.81638	25	284	39.00000	30.51311	21.8%	161	71s
14148	4592	36.98568	31	299	39.00000	30.67587	21.3%	160	76s
15541	4844	37.58558	32	249	39.00000	31.00584	20.5%	157	80s
17461	5413	33.46229	26	295	39.00000	31.32097	19.7%	154	86s
18819	5662	infeasible	27		39.00000	31.55234	19.1%	152	90s
20258	5973	cutoff	36		39.00000	31.72164	18.7%	151	95s
21936	6252	cutoff	31		39.00000	31.98454	18.0%	148	100s
24431	6653	34.75042	26	303	39.00000	32.31155	17.1%	146	107s
25412	6793	35.99915	26	326	39.00000	32.42132	16.9%	144	110s
27094	7021	33.79341	26	179	39.00000	32.59847	16.4%	143	115s
29648	7358	36.94286	26	309	39.00000	32.86213	15.7%	141	143s
30728	7391	37.44637	30	212	39.00000	32.97760	15.4%	140	145s
32025	7418	36.74339	28	286	39.00000	33.13178	15.0%	140	150s
33604	7480	37.67916	33	311	39.00000	33.28150	14.7%	140	155s
35171	7500	37.46878	47	200	39.00000	33.45606	14.2%	140	160s
36719	7455	35.48846	27	395	39.00000	33.63329	13.8%	139	165s
38214	7413	36.83750	33	160	39.00000	33.78725	13.4%	140	170s
39377	7407	37.01480	31	147	39.00000	33.87441	13.1%	139	177s
40074	7373	cutoff	30		39.00000	33.94368	13.0%	140	180s
41718	7249	37.27453	30	231	39.00000	34.12849	12.5%	139	185s
43176	7085	37.88204	30	194	39.00000	34.31174	12.0%	140	191s
44925	6953	37.45237	26	364	39.00000	34.48929	11.6%	140	196s
*45981	5723		52		38.0000000	34.57873	9.00%	139	199s
46669	5433	cutoff	32		38.00000	34.69002	8.71%	139	202s
H46678	2871				37.0000000	34.69002	6.24%	139	202s
47256	2457	cutoff	31		37.00000	34.79256	5.97%	139	205s
49344	384	cutoff	45		37.00000	35.63649	3.69%	138	210s

Cutting planes:

Gomory: 10 Cover: 3

Implied bound: 30

MIR: 2

Flow cover: 12 Zero half: 4 RLT: 159

Explored 50328 nodes (6849996 simplex iterations) in 210.94 seconds (148.30 work units)

Thread count was 8 (of 8 available processors)

Solution count 10: 37 38 39 \dots 57

Optimal solution found (tolerance 1.00e-04)
Best objective 3.700000000000e+01, best bound 3.70000000000e+01, gap 0.0000%

```
Oklahoma: The number of cut edges is 37.0 Oklahoma: Using Lower Bound of Population = 739017 and Upper Bound of Population = 761524 and districts = 5
```

1.1.6 Step 6: Retrieve the districts and their populations

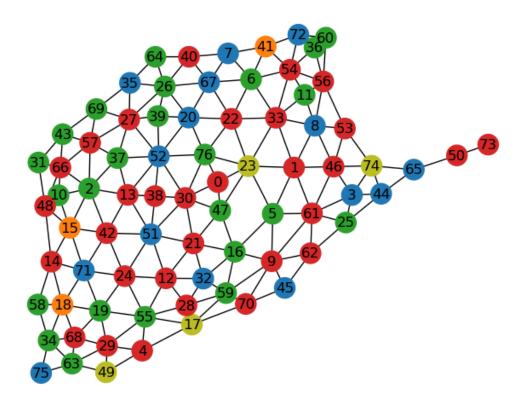
12, 13, 14, 21, 22, 24, 27, 28, 29, 30, 33, 38, 40, 42, 46, 48, 50, 53, 54, 56, 57, 61, 62, 66, 68, 70, 73]
Oklahoma: District 1 has population 741748 and contains counties [15, 18, 41]
Oklahoma: District 2 has population 751820 and contains counties [17, 23, 49, 74]
Oklahoma: District 3 has population 740351 and contains counties [2, 5, 6, 10, 11, 16, 19, 25, 26, 31, 34, 36, 37, 39, 43, 47, 55, 58, 59, 60, 63, 64, 69, 76]
Oklahoma: District 4 has population 756278 and contains counties [3, 7, 8, 20, 32, 35, 44, 45, 51, 52, 65, 67, 71, 72, 75]

1.1.7 Step 7: Node Mapping the counties and districts

```
[9]: filepath = '/Users/jessi/Gurobi-Python Playground/OK.graph'
     G = nx.read edgelist(filepath, nodetype=int)
     print(type(G))
     # List of districts
     districts = ["District 0", "District 1", "District 2", "District 3", "District 4"]
     # County to District Dictionary
     counties_district_dict = {
     "District 0": [0, 1, 4, 9, 12, 13, 14, 21, 22, 24, 27, 28, 29, 30, 33, 38, 40, u
      →42, 46, 48, 50, 53, 54, 56, 57, 61, 62, 66, 68, 70, 73],
     "District 1": [15, 18, 41],
     "District 2": [17, 23, 49, 74],
     "District 3": [2, 5, 6, 10, 11, 16, 19, 25, 26, 31, 34, 36, 37, 39, 43, 47, __
      →55, 58, 59, 60, 63, 64, 69, 76],
     "District 4": [3, 7, 8, 20, 32, 35, 44, 45, 51, 52, 65, 67, 71, 72, 75]
     }
     # Color dictionary
```

```
counties_district_colors_dict = {
   "District 0": "tab:red",
    "District 1": "tab:orange",
    "District 2": "tab:olive",
   "District 3": "tab:green",
   "District 4": "tab:blue"
}
# Defining the position
pos = nx.spring_layout(G)
# Assigning the colors
color list = []
for node in G.nodes: # iteration over nodes
    if node in counties_district_dict['District 0']:
        color_list.append(counties_district_colors_dict['District 0'])
   elif node in counties_district_dict['District 1']:
        color_list.append(counties_district_colors_dict['District 1'])
   elif node in counties_district_dict['District 2']:
        color_list.append(counties_district_colors_dict['District 2'])
   elif node in counties_district_dict['District 3']:
        color_list.append(counties_district_colors_dict['District 3'])
   else:
        color_list.append(counties_district_colors_dict['District 4'])
nx.draw(G, pos, node_color=color_list,with_labels=True)
plt.show()
```

<class 'networkx.classes.graph.Graph'>



1.1.8 Step 8: State shape Mapping the counties

```
[10]: # Read Oklahoma county shapefile and distance files
    file1 = '/Users/jessi/Gurobi-Python Playground/OK_distances.csv'
    file2 = '/Users/jessi/Gurobi-Python Playground/OK_counties.shx'
    file = '/Users/jessi/Gurobi-Python Playground/OK_counties.shp'
    #create the data frame
    df = gpd.read_file(file)

#checker
    print((df))
    print(type(df))

#assign the districts to the counties
    districts = [[i for i in range(n) if x[i,j].x > 0.5] for j in range(k)]
    print("Oklahoma's counties are in the following districts", districts)

# Mapping
    fig,ax = plt.subplots(figsize=(10,10))
    df.plot(ax=ax, column = "geometry", cmap = 'Blues')
```

```
ax.axis("off")
print("Oklahoma's counties are shown by varied blues in the diagram")
```

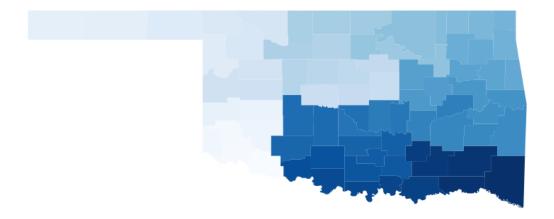
```
geometry
    POLYGON ((-99.36446 35.30577, -99.36446 35.306...
0
1
   POLYGON ((-99.42320 34.72502, -99.42314 34.725...
2
   POLYGON ((-98.53366 36.39787, -98.53369 36.404...
   POLYGON ((-95.00543 36.40142, -95.00543 36.405...
3
4
   POLYGON ((-99.14619 35.81240, -99.14280 35.812...
72 POLYGON ((-96.62036 35.95931, -96.62051 35.963...
73 POLYGON ((-97.14206 35.14521, -97.14208 35.146...
74 POLYGON ((-99.18527 35.11660, -99.18520 35.116...
75 POLYGON ((-97.24410 33.90427, -97.24505 33.903...
76 POLYGON ((-97.46132 36.43373, -97.46132 36.434...
```

[77 rows x 1 columns]

<class 'geopandas.geodataframe.GeoDataFrame'>

Oklahoma's counties are in the following districts [[0, 1, 4, 9, 12, 13, 14, 21, 22, 24, 27, 28, 29, 30, 33, 38, 40, 42, 46, 48, 50, 53, 54, 56, 57, 61, 62, 66, 68, 70, 73], [15, 18, 41], [17, 23, 49, 74], [2, 5, 6, 10, 11, 16, 19, 25, 26, 31, 34, 36, 37, 39, 43, 47, 55, 58, 59, 60, 63, 64, 69, 76], [3, 7, 8, 20, 32, 35, 44, 45, 51, 52, 65, 67, 71, 72, 75]]

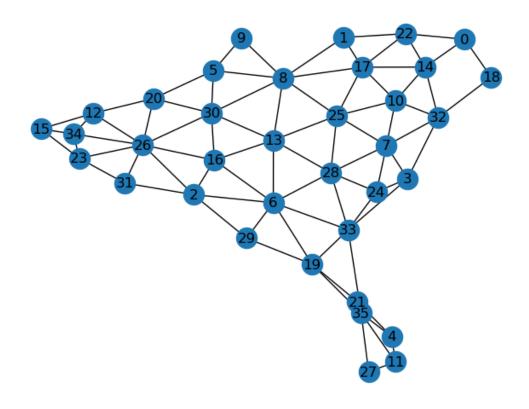
Oklahoma's counties are shown by varied blues in the diagram



1.2 OREGON

1.2.1 Step 1: Define the edges and nodes of the model

```
Class 'networkx.classes.graph.Graph'>
The Oregon graph has 36 nodes
The Oregon graph has 81 edges
Oregon graph has nodes [0, 14, 18, 22, 1, 8, 17, 2, 6, 16, 26, 29, 31, 3, 7, 24, 32, 33, 4, 11, 21, 35, 5, 9, 20, 30, 13, 19, 28, 10, 25, 27, 12, 15, 34, 23]
Oregon graph has edges [(0, 14), (0, 18), (0, 22), (14, 10), (14, 17), (14, 22), (14, 32), (18, 32), (22, 1), (22, 17), (1, 8), (1, 17), (8, 5), (8, 9), (8, 13), (8, 17), (8, 25), (8, 30), (17, 10), (17, 25), (2, 6), (2, 16), (2, 26), (2, 29), (2, 31), (6, 13), (6, 16), (6, 19), (6, 28), (6, 29), (6, 33), (16, 13), (16, 26), (16, 30), (26, 12), (26, 20), (26, 23), (26, 30), (26, 31), (26, 34), (29, 19), (31, 23), (3, 7), (3, 24), (3, 32), (3, 33), (7, 10), (7, 24), (7, 25), (7, 28), (7, 32), (24, 28), (24, 33), (32, 10), (33, 19), (33, 28), (33, 35), (4, 11), (4, 21), (4, 35), (11, 27), (11, 35), (21, 19), (21, 35), (35, 19), (35, 27), (5, 9), (5, 20), (5, 30), (20, 12), (20, 30), (30, 13), (13, 25), (13, 28), (28, 25), (10, 25), (12, 15), (12, 34), (15, 23), (15, 34), (34, 23)]
```



1.2.2 Step 2: List the county populations

```
[12]: # Integrate the populations by county to the model

# create population list
population = list()

# read text file into software system using files on local drive
filepath = "/Users/jessi/Gurobi-Python Playground/OR.population"
file = open( filepath, "r")

#remove the header to only read the integers
next(file)

# Create List for each county population
Listpop = []
for each in file:
    #split the list so the populations in each range
    each = each.split(' ')
    #print((each[1])
```

```
Listpop.append(int(each[1]))

# displays the list of integers
print("Oregon's county populations = ", Listpop)

# the sums can be taken by list position
total_population = sum(Listpop)

#print the total population of OR
print("The total_population of Oregon = ", total_population)
population = Listpop #label for code clarity
n =len(population)

#print the county population as a sum districts of OR
print("Oregon's number of counties = ",n)
print("Population data is", type(population))

districts = [ [i for i in G.nodes if x[i,j].x > 0.5] for j in range(k)]
```

Oregon's county populations = [52589, 23977, 7495, 50395, 88090, 1995, 198253, 87433, 26670, 1870, 107722, 23446, 80075, 24502, 600372, 7391, 24738, 421401, 41072, 69413, 12186, 223259, 815428, 16668, 95184, 345920, 7233, 64929, 128610, 8160, 1451, 31571, 27390, 382971, 26196, 111201]
The total_population of Oregon = 4237256
Oregon's number of counties = 36
Population data is <class 'list'>

1.2.3 Step 3: Population Deviation Model

```
[13]: # If model is feasible, use the population deviation operations research model.
      # create model
      m = gp.Model()
      #number of districts per the 2020 census for OR
      k = 6
      # create variables adapted from districting starter models
      x = m.addVars(n,k, vtype=GRB.BINARY) # x ij equals one when county i is_{1}
      ⇔assigned to district j
      v = m.addVar()
                                          # the population of least-populated district
      z = m.addVar()
                                          # the population of most-populated district
      # objective is to minimize absolute population deviation
      m.setObjective(z-y, GRB.MINIMIZE)
      # add constraints saying that each county i is assigned to one district
      m.addConstrs( sum(x[i,j] for j in range(k)) == 1 for i in range(n) )
```

Gurobi Optimizer version 10.0.3 build v10.0.3rc0 (mac64[arm])

CPU model: Apple M2

Thread count: 8 physical cores, 8 logical processors, using up to 8 threads

Optimize a model with 48 rows, 218 columns and 660 nonzeros

Model fingerprint: 0x00001389

Variable types: 2 continuous, 216 integer (216 binary)

Coefficient statistics:

Matrix range [1e+00, 8e+05]
Objective range [1e+00, 1e+00]
Bounds range [1e+00, 1e+00]
RHS range [1e+00, 1e+00]

Found heuristic solution: objective 957825.00000

Presolve time: 0.00s

Presolved: 48 rows, 218 columns, 660 nonzeros

Variable types: 0 continuous, 218 integer (216 binary)

Root relaxation: objective 0.000000e+00, 52 iterations, 0.00 seconds (0.00 work units)

Nodes			Current	Current Node			ctive Bounds		Work		
E	xpl Une	expl	Obj Dept	h Int	Inf	Incumbent	BestBd	Gap	It/Node	Time	
	_	_						_			
	0	0	0.00000	0	9 9	57825.000	0.00000	100%	-	0s	
Н	0	0			611	097.00000	0.00000	100%	_	0s	
Н	0	0			447	7504.00000	0.00000	100%	-	0s	
	0	0	0.33333	0	14 4	47504.000	0.33333	100%	-	0s	
Н	0	0			213	3397.00000	0.33333	100%	_	0s	
Н	0	0			198	3044.00000	61593.0000	68.9%	_	0s	
	0	0	61593.0000	0	13 1	98044.000	61593.0000	68.9%	_	0s	
Н	0	0			196	344.00000	61593.0000	68.6%	_	0s	
Н	0	0			139	288.00000	61593.0000	55.8%	_	0s	
Н	0	0			132	2428.00000	61593.0000	53.5%	_	0s	
	0	0	61593.0000	0	10 1	32428.000	61593.0000	53.5%	-	0s	
	0	0	109219.000	0	13 1	32428.000	109219.000	17.5%	_	0s	

```
0
           0 109219.000
                               16 132428.000 109219.000 17.5%
                                                                        0s
                                132183.00000 109219.000 17.4%
Η
     0
           0
                                                                        0s
     0
           0 109219.000
                               22 132183.000 109219.000 17.4%
                                                                        0s
Η
     0
                                132140.00000 109219.000 17.3%
                                                                        0s
                               24 132140.000 109219.000 17.3%
     0
           0 109219.000
                                                                        0s
     0
           0 109219.000
                               12 132140.000 109219.000 17.3%
                                                                        0s
     0
           0 109219.000
                               12 132140.000 109219.000 17.3%
                                                                        0s
Η
     0
                                132029.00000 109219.000 17.3%
                                                                        0s
     0
          0 109219.000
                               17 132029.000 109219.000 17.3%
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                                131960.00000 109219.000 17.2%
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                               10 131960.000 109219.000 17.2%
                                                                        0s
   31
          19
                                131825.00000 131063.000
                                                         0.58%
Η
                                                                10.1
                                                                        0s
   78
          85
                                131525.00000 131063.000
                                                         0.35%
                                                                 7.2
Η
                                                                        0s
                                                         0.22%
                                                                 4.7
Η
  183
         236
                                131347.00000 131063.000
                                                                        0s
H 604
        757
                                131323.00000 131063.000 0.20%
                                                                 3.7
                                                                        0s
H 742
        757
                                131175.00000 131063.000
                                                         0.09%
                                                                 3.7
                                                                        0s
* 9935
       3740
                          70
                                131116.00000 131063.000
                                                         0.04%
                                                                 1.2
                                                                        1s
*15801 2908
                          74
                                131080.00000 131063.000 0.01%
                                                                 1.5
                                                                        1s
*20150 2872
                          80
                                131069.00000 131063.000 0.00%
                                                                 1.6
                                                                        1s
```

Cutting planes:

Cover: 2

Flow cover: 3

Explored 20804 nodes (56687 simplex iterations) in 1.54 seconds (0.46 work units)

Thread count was 8 (of 8 available processors)

Solution count 10: 131069 131080 131116 ... 132029

Optimal solution found (tolerance 1.00e-04)

Best objective 1.310690000000e+05, best bound 1.310630000000e+05, gap 0.0046%

1.2.4 Step 4: Current model status for population

```
The absolute population deviation is 131069.0 person(s).

Oregon has districts = 6

Oregon: District 0 and contains counties [0, 16, 7, 32, 33, 20, 30, 19, 34]

Oregon: District 1 and contains counties [22]

Oregon: District 2 and contains counties [14, 26, 3, 9, 13]

Oregon: District 3 and contains counties [17, 31, 11, 35, 12, 23]

Oregon: District 4 and contains counties [8, 2, 29, 24, 28, 25, 27, 15]

Oregon: District 5 and contains counties [18, 1, 6, 4, 21, 5, 10]
```

1.2.5 Step 5: Create model to access the minimization of cut edges using the county to district populations

```
[]: # data from the county 2010 Census
     filepath = '/Users/jessi/Gurobi-Python Playground/OR_county.json'
     G = Graph.from_json(filepath)
     print(G.nodes)
     m = gp.Model()
     #check the list of nodes and matches are align with the mapping data
     print(G.nodes)
     print(G.edges)
     for node in G.nodes:
         # using the columns, located the population
         population = G.nodes[node]['TOTPOP']
     total_population = sum(G.nodes[node]['TOTPOP'] for node in G.nodes)
     print(total_population)
     # Impose a 3\% population deviation (+/- 0.5\%) for the Operations Research model
     deviation = 0.03
     #number of counties
     n
     # number of districts
     # Lower and Upper bound populations to ensure the populations of the districts __
      ⇔do not exceed the bounds
     L = math.ceil((1-deviation/2)*total_population/k)
     U = math.floor((1+deviation/2)*total_population/k)
     # create variables adapted from redistricitng starter:
     x = m.addVars(G.nodes, k, vtype=GRB.BINARY) # x[i,j] equals one when county i_{l}
      ⇔is assigned to district j
     y = m.addVars(G.edges, vtype=GRB.BINARY) # y[u,v] equals one when edge {u,v}_{\square}
      \hookrightarrow is cut
```

```
# objective is to minimize cut edges
m.setObjective(gp.quicksum(y[u,v] for u,v in G.edges), GRB.MINIMIZE)
# add constraints saying that each county i is assigned to one district
m.addConstrs(gp.quicksum(x[i,j] for j in range(k)) == 1 for i in G.nodes)
# add constraints saying that each district has population at least L and at \Box
  \hookrightarrow most U
m.addConstrs(gp.quicksum(G.nodes[i]['TOTPOP'] * x[i,j] for i in G.nodes) >= L_{\cup}
  →for j in range(k) )
m.addConstrs(gp.quicksum(G.nodes[i]['TOTPOP'] * x[i,j] for i in G.nodes) <= U_U
 →for j in range(k) )
# add constraints saying that edge \{i,j\} is cut if i is assigned to district v_{\square}
 ⇒but j is not.
m.addConstrs(x[i,v] - x[j,v] \le y[i,j]  for i,j in G.edges for v in range(k))
m.update()
# solve IP model
m.optimize()
# print the optimal minization of cut edges
print("Oregon: The number of cut edges is",m.objval)
[0, 1, 2, 3, 4, 5, 6, 7, 8, 9, 10, 11, 12, 13, 14, 15, 16, 17, 18, 19, 20, 21,
22, 23, 24, 25, 26, 27, 28, 29, 30, 31, 32, 33, 34, 35, 36, 37, 38, 39, 40, 41,
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3831074
Gurobi Optimizer version 10.0.3 build v10.0.3rc0 (mac64[arm])
CPU model: Apple M2
Thread count: 8 physical cores, 8 logical processors, using up to 8 threads
Optimize a model with 3572 rows, 1830 columns and 13860 nonzeros
Model fingerprint: 0xb7d1b581
Variable types: 0 continuous, 1830 integer (1830 binary)
Coefficient statistics:
   Matrix range
                                [1e+00, 5e+05]
                                [1e+00, 1e+00]
   Objective range
   Bounds range
                                [1e+00, 1e+00]
                                [1e+00, 6e+05]
   RHS range
Presolve time: 0.03s
```

Presolved: 3572 rows, 1830 columns, 13860 nonzeros

Variable types: 0 continuous, 1830 integer (1830 binary)

Root relaxation: objective 0.000000e+00, 2181 iterations, 0.38 seconds (0.21 work units)

Nodes		Current	Noc	ie	Object	cive Bounds	1	Wor	:k	
Expl Unexpl		Obj Dept	h Ir	ntInf	Incumbent		Gap	It/Node		
	0	0	0.00000	0	1272	_	0.00000	_	-	1s
Η	0	0			4	0000000.80	0.00000	100%	-	1s
Н	0	0			3	78.000000	0.00000	100%	-	1s
	0	0	2.53618	0	1278	378.00000	2.53618	99.3%	-	3s
Н	0	0			3	73.0000000	2.53618	99.3%	-	3s
	0	0	5.01059	0	1275	373.00000	5.01059	98.7%	-	4s
	0	0	5.10652	0	1275	373.00000	5.10652	98.6%	-	5s
H	0	0			3	70.000000	5.10652	98.6%	-	5s
H	0	0			3	64.000000	5.10652	98.6%	-	5s
	0	0	5.10933	0	1275	364.00000	5.10933	98.6%	_	5s
	0	0	5.30570	0	1276	364.00000	5.30570	98.5%	_	6s
Н	0	0			3	32.0000000	5.30570	98.4%	_	6s
	0	0	5.33643	0	1273	332.00000	5.33643	98.4%	_	6s
	0	0	5.38135	0	1274	332.00000	5.38135	98.4%	_	7s
Н	0	0			2	13.000000	5.38135	97.5%	-	8s
	0	0	5.61900	0	1273	213.00000	5.61900	97.4%	-	8s
	0	0	5.61900	0	1274	213.00000	5.61900	97.4%	-	8s
	0	0	5.61900	0	1274	213.00000	5.61900	97.4%	-	9s
	0	0	5.64039	0	1274	213.00000	5.64039	97.4%	_	9s
	0	0	5.64962	0	1275	213.00000	5.64962	97.3%	_	9s
	0	0	5.64962	0	1275	213.00000	5.64962	97.3%	_	9s
	0	0	5.66493	0	1273	213.00000	5.66493	97.3%	_	10s
H	0	0			1	98.000000	5.66493	97.1%	-	10s
	0	0	5.66493	0	1275	198.00000	5.66493	97.1%	_	10s
	0	0	5.66493	0	1275	198.00000	5.66493	97.1%	_	10s
Н	0	0			1	91.000000	5.66493	97.0%	_	10s
	0	0	5.66493	0	1275	191.00000	5.66493	97.0%	_	11s
H	0	0			1	90.000000	5.66493	97.0%	-	11s
	0	0	5.66493	0	1276	190.00000	5.66493	97.0%	-	11s
	0	0	5.66493	0	1275	190.00000	5.66493	97.0%	-	12s
H	0	0			1	73.0000000	5.66493	96.7%	-	12s
	0	0	5.66493	0	1276	173.00000	5.66493	96.7%	-	12s
	0	0	5.66493	0	1274	173.00000	5.66493	96.7%	_	13s
	0	0	5.66493	0	1274	173.00000	5.66493	96.7%	_	13s
Н	0	0			1	72.0000000	5.66493	96.7%	_	14s
	0	2	5.66493	0	1274	172.00000	5.66493	96.7%	_	14s
	1	4	5.70991	1	1273	172.00000	5.70991	96.7%	377	15s
	19	22	12.19095	5	1292	172.00000	10.97884	93.6%	958	20s
Н	27	30			1	70.000000	10.98100	93.5%	1315	22s

Η	28	30				164.0000000	10.98100	93.3%	1325	22s
Η	30	30				160.0000000	10.98100	93.1%	1340	22s
	53	54	17.17788	8	1084	160.00000	10.98100	93.1%	1459	25s
Η	61	62				153.0000000	10.98100	92.8%	1401	26s
Η	67	62				147.0000000	10.98100	92.5%	1431	26s
Η	100	99				142.0000000	10.98100	92.3%	1358	30s
Η	103	99				131.0000000	10.98100	91.6%	1361	30s
Η	139	133				123.0000000	10.98100	91.1%	1285	34s
Η	143	133				119.0000000	10.98100	90.8%	1260	34s
	146	144	26.20348	13	964	119.00000	10.98100	90.8%	1258	35s
Η	148	144				118.0000000	10.98100	90.7%	1253	35s
Η	150	144				114.0000000	10.98100	90.4%	1264	35s
Η	175	178				112.0000000	10.98100	90.2%	1174	37s
Η	180	178				110.0000000	10.98100	90.0%	1163	37s
Η	199	191				109.0000000	10.98100	89.9%	1115	38s
	226	227	26.19656	15	967	109.00000	10.98100	89.9%	1058	47s
Η	227	227				105.0000000	10.98100	89.5%	1053	47s
Η	238	227				102.0000000	10.98100	89.2%	1031	47s
Η	269	285			:	101.0000000	10.98100	89.1%	987	50s
Η	288	285				100.0000000	10.98100	89.0%	957	50s
Η	346	347				99.0000000	10.98100	88.9%	880	52s
	406	401	31.46402	20	875	99.00000	10.98100	88.9%	811	55s
Η	421	401				95.0000000	10.98100	88.4%	805	55s
Η	431	401				89.0000000	10.98100	87.7%	802	55s
Η	486	466				88.0000000	10.98100	87.5%	767	57s
Η	501	466				87.0000000	10.98100	87.4%	756	57s
	544	495	41.08958	25	917	87.00000	10.98100	87.4%	753	62s
Η	549	484				84.0000000	10.98100	86.9%	755	62s
Η	607	525				78.0000000	10.98100	85.9%	739	67s
	674	555	51.28433	30	1027	78.00000	10.98100	85.9%	725	71s
	755	652	58.50274	39	986	78.00000	10.98100	85.9%	735	76s
	885	760	71.85941	55	884	78.00000	11.65128	85.1%	711	81s
	984	813	17.74359	6	1286	78.00000	11.65128	85.1%	698	86s
	1060	888	22.45636	10	1271	78.00000	11.65128	85.1%	698	90s
	1208	1041	27.67607	14	1085	78.00000	11.65128	85.1%	690	97s
Η	1210	1035				77.0000000	11.65128	84.9%	690	97s
	1289	1095	26.36601	15	1014	77.00000	11.65128	84.9%	674	100s
	1460	1171	60.67746	51	1266	77.00000	11.65128	84.9%	649	106s
	1463	1173	28.42284	11	1273	77.00000	11.65128	84.9%	648	110s
	1468	1177	35.70242	25	1271	77.00000	11.65128	84.9%	646	116s
	1474	1181	55.56659	20	1290	77.00000	11.65128	84.9%	643	120s
	1478	1186	12.00619	14	1287	77.00000	11.65128	84.9%	655	125s
	1484	1196	24.68984	16	1098	77.00000	11.65128		664	131s
	1500	1207	28.70150		1078		12.03636		682	135s
	1524	1223	29.15261		1244		12.34477		703	140s
	1562	1251	32.46566		1275	77.00000	12.34477		719	145s
	1597	1278	38.21335	23			12.34477			150s
	1687	1348	44.79227	26	833	77.00000	12.34477	84.0%	723	155s

```
1767 1375
               48.10591
                          29 807
                                    77.00000
                                               12.34477
                                                         84.0%
                                                                 709
                                                                      162s
H 1778 1311
                                  75.0000000
                                               12.34477
                                                         83.5%
                                                                 707
                                                                      162s
H 1815
       1290
                                  74.0000000
                                               12.34477
                                                         83.3%
                                                                 701
                                                                      164s
  1837
       1329
               49.16924
                              589
                                    74.00000
                                               12.34477
                                                         83.3%
                                                                 698
                                                                      165s
                          33
H 1904
       1268
                                                         83.1%
                                                                      168s
                                  73.0000000
                                               12.34477
                                                                 686
  1957
        1335
               60.86902
                          39
                              587
                                    73.00000
                                               12.34477
                                                         83.1%
                                                                 676
                                                                      171s
  2116
       1389
               67.62673
                          49
                              769
                                    73.00000
                                               12.34477
                                                         83.1%
                                                                 650
                                                                      175s
  2214
       1416
               28.25583
                          19 1066
                                    73.00000
                                               18.39214 74.8%
                                                                 646
                                                                      180s
  2324 1473
               45.97738
                          26 1069
                                    73.00000
                                               18.39214 74.8%
                                                                      185s
                                                                 639
  2402 1494
                          32
               60.54922
                              998
                                    73.00000
                                               18.39214 74.8%
                                                                 636
                                                                      191s
H 2403 1424
                                  72.0000000
                                               18.39214 74.5%
                                                                 635
                                                                      191s
  2446 1473
               70.37679
                                    72.00000
                                               19.26548 73.2%
                                                                 638
                                                                      196s
                          36
                              887
  2542
       1547
                          22 1110
                                    72.00000
                                               19.26548 73.2%
               36.44970
                                                                 634
                                                                      200s
  2603
       1541
               41.29385
                              992
                                    72.00000
                                               19.26548 73.2%
                                                                 629
                                                                      214s
                          25
       1569
                                               19.26548 73.2%
  2624
               42.14809
                          26
                              923
                                    72.00000
                                                                 627
                                                                      217s
  2686
       1591
               51.04369
                             899
                                    72.00000
                                               19.26548 73.2%
                                                                 625
                                                                      220s
                          32
  2792 1666
               67.33191
                          54
                              858
                                    72.00000
                                               19.26548 73.2%
                                                                 623
                                                                      225s
  2921
       1682
               27.27647
                          22 1183
                                    72.00000
                                               21.30443 70.4%
                                                                 618
                                                                      235s
  3005 1750
               38.44172
                          29 1043
                                    72.00000
                                               21.30443 70.4%
                                                                 619
                                                                      242s
  3053 1804
               43.74192
                          34 972
                                    72.00000
                                               21.30443 70.4%
                                                                 617
                                                                      246s
                                               21.50653 70.1%
  3220
       1899
               60.37663
                          62 793
                                    72.00000
                                                                 609
                                                                      253s
  3320 1939
                          22 1118
                                    72.00000
                                               21.50653 70.1%
                                                                      256s
               30.38479
                                                                 603
  3418
       1913
               36.95148
                          28 1076
                                    72.00000
                                               21.50653 70.1%
                                                                 599
                                                                      289s
  3428 1938
               37.86978
                          29 1154
                                    72.00000
                                               21.50653 70.1%
                                                                 599
                                                                      296s
```

[24]: # Population Status print("Oregon: Using Lower Bound of Population =",L,"and Upper Bound of →Population =",U,"and districts =",k) print("Oregon'population deviation is greater than 100; therefore, the model is →assumed to be infeasible. This redistricting of OR is a more complex tasking →due to counties being larger than others. The model proves the infeasibility. →")

Oregon: Using Lower Bound of Population = 628935 and Upper Bound of Population = 648090 and districts = 6

Oregon'population deviation is greater than 100; therefore, the model is assumed to be infeasible. This redistricting of OR is a more complex tasking due to counties being larger than others. The model proves the infeasibility.

1.2.6 Step 6: Retrieve the districts and their populations

```
[25]: districts = [ [i for i in G.nodes if x[i,j].x > 0.5] for j in range(k)]
district_populations = [ sum(G.nodes[i]["TOTPOP"] for i in districts[j]) for j

in range(k) ]

# print district status
for j in range(k):
```

```
print("Oregon: District",j,"has population",district_populations[j],"and

contains counties",districts[j])
```

```
AttributeError
                                          Traceback (most recent call last)
Cell In[25], line 1
---> 1 districts = [ [i for i in G.nodes if x[i,j].x > 0.5] for j in range(k)]
      2 district_populations = [ sum(G.nodes[i]["TOTPOP"] for i in districts[j]

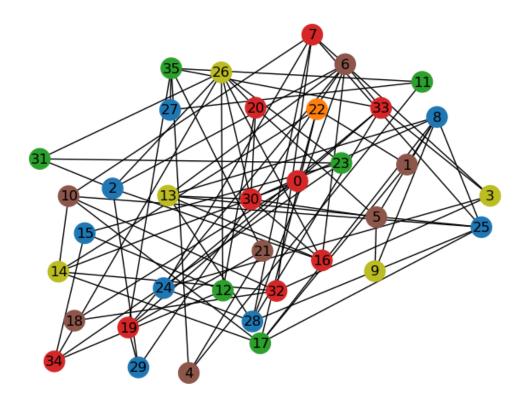
→for j in range(k) ]
      4 # print district status
Cell In[25], line 1, in stcomp>(.0)
---> 1 districts = [ [i for i in G.nodes if x[i,j].x > 0.5] for j in range(k)]
      2 district_populations = [ sum(G.nodes[i]["TOTPOP"] for i in districts[j]]

→for j in range(k) ]
      4 # print district status
Cell In[25], line 1, in stcomp>(.0)
----> 1 districts = [ [i for i in G.nodes if x[i,j].x > 0.5] for j in range(k)]
      2 district_populations = [ sum(G.nodes[i]["TOTPOP"] for i in districts[j] __
 →for j in range(k) ]
      4 # print district status
File src/gurobipy/var.pxi:125, in gurobipy.Var.__getattr__()
File src/gurobipy/var.pxi:153, in gurobipy.Var.getAttr()
File src/gurobipy/attrutil.pxi:100, in gurobipy. getattr()
AttributeError: Unable to retrieve attribute 'x'
```

1.2.7 Step 7: Node Mapping the counties and districts

```
"District 3": [17, 31, 11, 35, 12, 23],
"District 4": [8, 2, 29, 24, 28, 25, 27, 15],
"District 5": [18, 1, 6, 4, 21, 5, 10]
}
# Assigning a color to each district
counties_district_colors_dict = {
   "District 0": "tab:red",
   "District 1": "tab:orange",
   "District 2": "tab:olive",
    "District 3": "tab:green",
   "District 4": "tab:blue",
   "District 5": "tab:brown",
}
# Assigning the colors
color_list = []
for node in G.nodes: # iteration over nodes
    if node in counties_district_dict['District 0']:
        color_list.append(counties_district_colors_dict['District 0'])
   elif node in counties district dict['District 1']:
        color_list.append(counties_district_colors_dict['District 1'])
   elif node in counties_district_dict['District 2']:
        color_list.append(counties_district_colors_dict['District 2'])
   elif node in counties_district_dict['District 3']:
        color_list.append(counties_district_colors_dict['District 3'])
   elif node in counties_district_dict['District 4']:
        color_list.append(counties_district_colors_dict['District 4'])
   else:
        color_list.append(counties_district_colors_dict['District 5'])
# Map output
nx.draw(G, pos, node_color=color_list, with_labels=True)
plt.show()
```

<class 'networkx.classes.graph.Graph'>



1.2.8 Step 8: State shape Mapping the counties

```
[17]: # Read Oregon county shapefile and distance files
    file1 = '/Users/jessi/Gurobi-Python Playground/OR_distances.csv'
    file2 = '/Users/jessi/Gurobi-Python Playground/OR_counties.shx'
    file = '/Users/jessi/Gurobi-Python Playground/OR_counties.shp'
    #create the data frame
    df = gpd.read_file(file)

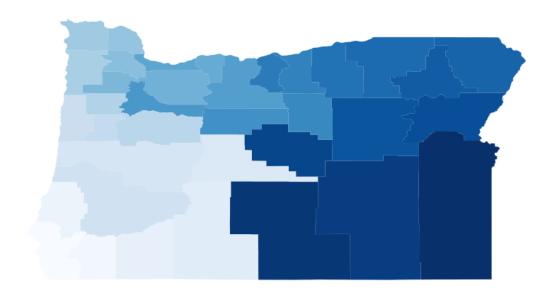
#checker
print((df))
print(type(df))

# Mapping
fig,ax = plt.subplots(figsize=(10,10))
df.plot(ax=ax, column = "geometry", cmap ='Blues')
ax.axis("off")
print("Oregon's counties are shown by varied blues in the diagram")
```

geometry

```
0
    POLYGON ((-123.35949 46.01121, -123.35959 46.0...
    POLYGON ((-121.63812 45.70477, -121.63132 45.7...
1
2
    POLYGON ((-119.89506 43.56813, -119.89506 43.5...
3
    POLYGON ((-124.14651 44.77862, -124.14609 44.7...
    POLYGON ((-124.00393 42.45154, -124.00387 42.4...
4
5
    POLYGON ((-120.36083 45.50569, -120.36080 45.5...
6
    POLYGON ((-121.40550 44.39315, -121.40256 44.3...
    POLYGON ((-123.72466 44.98427, -123.72462 44.9...
7
   POLYGON ((-121.71160 45.15282, -121.71222 45.1...
   POLYGON ((-120.82708 45.51531, -120.82814 45.5...
9
10 POLYGON ((-123.46310 45.28019, -123.46310 45.2...
11 POLYGON ((-124.54413 42.68382, -124.55310 42.6...
12 POLYGON ((-119.14139 45.93100, -119.13504 45.9...
13 POLYGON ((-121.14964 44.39065, -121.15085 44.3...
14 POLYGON ((-123.20927 45.43371, -123.20976 45.4...
15 POLYGON ((-117.74746 45.83324, -117.74745 45.8...
16 POLYGON ((-121.01452 44.39032, -121.01311 44.3...
17 POLYGON ((-122.74374 45.33207, -122.74376 45.3...
18 POLYGON ((-124.00807 46.01053, -124.00791 46.0...
19 POLYGON ((-121.31649 41.99706, -121.31761 41.9...
20 POLYGON ((-119.99817 45.68439, -119.99815 45.6...
21 POLYGON ((-122.95119 42.76756, -122.95120 42.7...
22 POLYGON ((-122.86749 45.60623, -122.86830 45.6...
23 POLYGON ((-117.94624 45.00670, -117.94605 45.0...
24 POLYGON ((-123.18202 44.72008, -123.17725 44.7...
25 POLYGON ((-122.88251 45.25531, -122.88224 45.2...
26 POLYGON ((-119.65318 44.76270, -119.65319 44.7...
27 POLYGON ((-124.46746 43.19648, -124.46524 43.2...
28 POLYGON ((-123.25099 44.55198, -123.25240 44.5...
29 POLYGON ((-120.88050 42.42859, -120.88049 42.4...
30 POLYGON ((-120.38569 44.76118, -120.38572 44.7...
31 POLYGON ((-118.23029 43.77921, -118.22811 43.7...
32 POLYGON ((-124.01620 45.64610, -124.01619 45.6...
33 POLYGON ((-123.73473 43.94406, -123.73927 43.9...
34 POLYGON ((-118.11635 45.49884, -118.11636 45.4...
35 POLYGON ((-123.81715 43.45959, -123.81710 43.4...
<class 'geopandas.geodataframe.GeoDataFrame'>
Oregon's counties are shown by varied blues in the diagram
```

31



```
[]: #Trial and Error
     # #for loop examples and set-up
     \# \# \# x is the place holder that looks in the range
     # for x in range(1, 11):
          print(x)
     # print("Range")
     # #for loop counting backwards by two which is the step function
     # for x in reversed (range(1, 11, 2)):
     # print(x)
     # print("Backward")
     # #printing within a sequence
     # ## x holds current position
     # Number = "1234-1976-3467"
     # for x in Number:
         print(x)
     # print("Number sequence")
     # #skipping a number and looping
     # for x in range(1,21):
         if \ x == 13:
              continue
           else:
              print(x)
     # #skipping a number and looping
```

```
# for x in range(1,21):
     if x == 13:
          break
      else:
          print(x)
# # Nested for loop examples
# # outer and inner loop, make sure the varible/placeholder/ counters are
 \hookrightarrow different
# #call and response
# rows =int(input("Enter the # of rows: "))
# columns =int(input("Enter the # of columns: "))
# symbol = input("Enter a symbol to use: ")
# for x in range(rows): # means 3 iterations, outer loop
# # everytime you print it ends with a new character and proceeds to the next_{\sqcup}
\hookrightarrow line.
# # so if we use a string then it will be next to each other
     for y in range(columns):
         print (symbol, end =" ")
      print() #exit inner loop
# # number string
# for x in range(3): # means 3 iterations, outer loop
# # everytime you print it ends with a new character and proceeds to the next_{\sqcup}
 ⇔line.
# # so if we use a string then it will be next to each other
    for y in range(1,10):
          print (y, end ="")
      print() #exit inner loop
####################
# If using the unmodified .dimacs file, it will need to be manipulated to only \Box
→utilize integers. Removing column 1, row 1, last row
# filepath = "/Users/jessi/Gurobi-Python Playground/MA.dimacs"
# file = open(filepath)
# #remove the header and tail to over mixing int and str
# next(file)
# last_line = None
# for line in file:
      if last_line:
          for line in file:
#
              if line.strip():
#
                   file.write("\t".join(line.split()[1:]) + "\n")
#
          print (last_line)
#
          last\_line = line
```

```
# # for line in file:
# #
            try:
               r = int(line)
# #
               if r > 999 or line.strip() == '-0':
# #
                   #filtering numbers >999 and strings with '-0'
# #
                   continue
# #
                file.append(r)
           except ValueError:
# #
# #
               pass
# # Create List of integers
# # nodes_edges = []
# # for each in file:
# #
      each = each.strip()
# #
      #split the list so the populations in each range
# #
      each = each.split(' ')
      print((each[1], each[2]))
# #
       nodes_edges.append(int(each[1], each[2]))
# #
#####################
#shifts = range(1,9000000)
# for district in districts:
      c_counties = counties_district_dict[district]
      c_color = counties_district_dict[district]
```