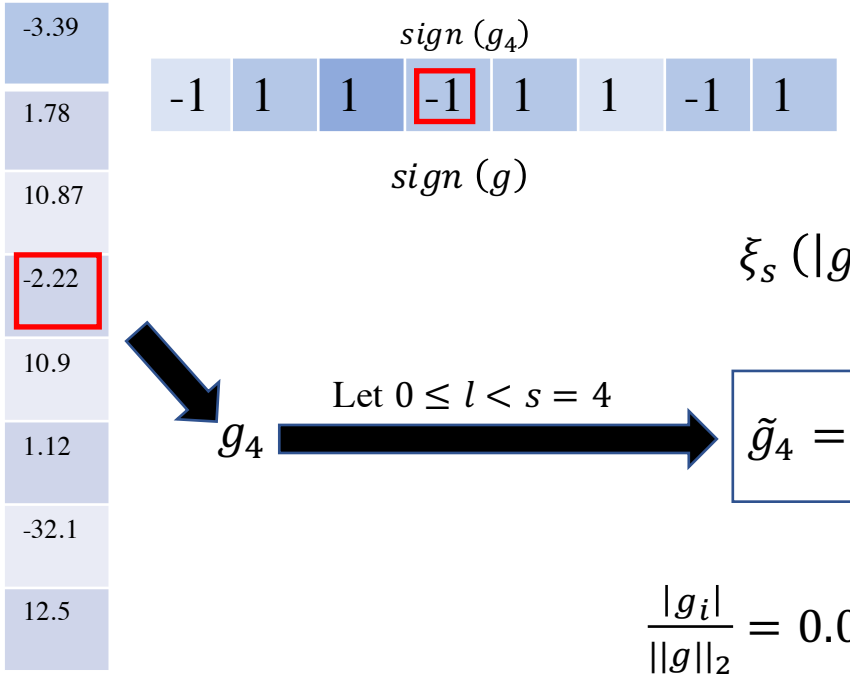
Let $0 \le l < s, s \in N$, $\tilde{g}_i = Q_s(g_i) = sign\,(g_i) * ||g|| * \xi_s\,(|g_i|, ||g||)$, s denotes the quantization level.

Sign — Gradient norm — Quantization Level



$$\xi_s\,(|g_i|, ||g||) = \begin{cases} \frac{l+1}{s} & \text{with probability } p_i = \left(\frac{s|g_i|}{||g||} - l\right), \\ \frac{l}{s} & \text{with probability } 1 - p_i. \end{cases}$$

$$\tilde{g}_4 = Q_4(g_4) = -38.0062 * \xi_4\,(|g_4|, ||g||_2) = 0$$

$$\frac{|g_i|}{||g||_2} = 0.0584, \quad \xi_4\,(|g_4|, ||g||_2) = \begin{cases} 0 \text{ with probability } p_i = 0.7664, \\ \frac{1}{4} \text{ with probability } 1 - p_i = 0.2336. \end{cases}$$