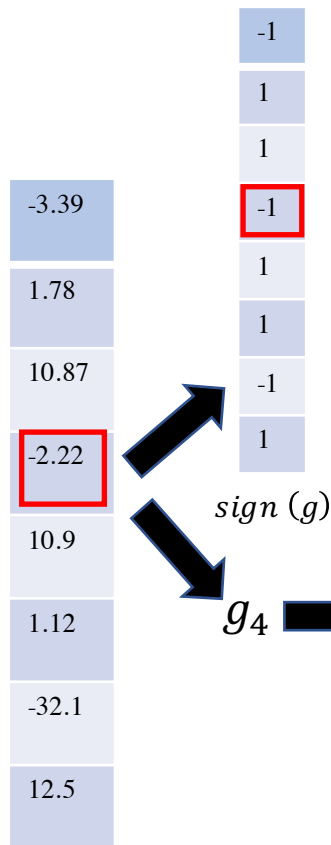


Let  $0 \leq l < s, s \in N$ ,

$$\tilde{g}_i = Q_s(g_i) = \underbrace{\text{sign}(g_i)}_{\text{Sign}} * \underbrace{\|g\|}_{\text{Gradient norm}} * \underbrace{\xi_s(|g_i|, \|g\|)}_{\text{Quantization Level}}, \text{ s denotes the quantization level.}$$

$$\xi_s(|g_i|, \|g\|) = \begin{cases} \frac{l+1}{s} & \text{with probability } p_i = \left(\frac{s|g_i|}{\|g\|} - l\right), \\ \frac{l}{s} & \text{with probability } 1 - p_i. \end{cases}$$



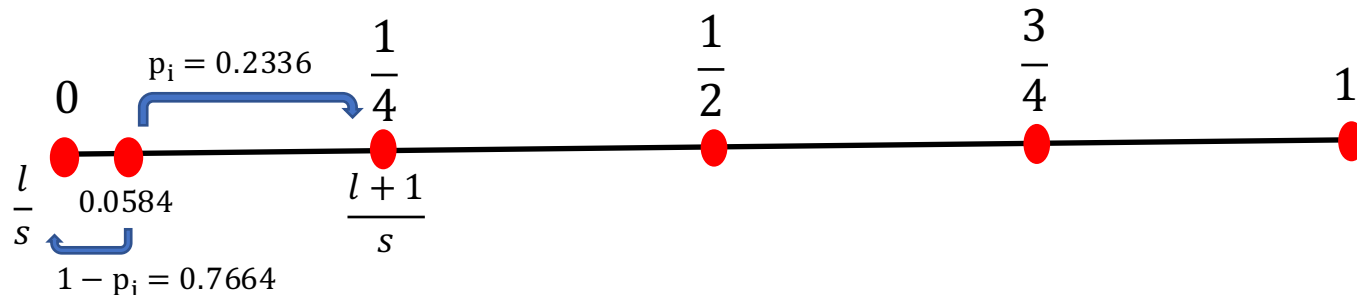
$\text{sign}(g)$

$g_4$

Let  $0 \leq l < s = 4$

$$\tilde{g}_4 = Q_4(g_4) = -38.0062 * \xi_4(|g_4|, \|g\|_2) = 0$$

$$\frac{|g_i|}{\|g\|_2} = 0.0584, \quad \xi_4(|g_4|, \|g\|_2) = \begin{cases} 0 & \text{with probability } p_i = 0.7664, \\ \frac{1}{4} & \text{with probability } 1 - p_i = 0.2336. \end{cases}$$



Original  
Gradient