Circuit Theory and Electronics Fundamentals - T2

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Abstract

In this report, we show two analysis of a circuit with 4 meshes and 8 nodes, containing 1 capacitor and, for $t < 0$, a constant voltage source and sinusoidal for $t > 0$. *Octave* was used for theoretical analysis and *Ngspice* for simulation analysis. The results were consistent with the exception of a small offset from 0 at the beginning of the oscillation of the forced solution in the simulation.

Resumo

Neste relatório, mostramos duas análise de um circuito com 4 malhas e 8 nós, contendo 1 condensador e, para uma fonte de tensão constante para $t < 0$ e sinusoidal $t > 0$. Foi ulizado para uma análise teórica o *Octave* e para a análise em simulação o *Ngspice*. Os resultados foram concordantes com exceção de um pequeno desvio ao 0 no início da oscilação da solução forçada na simulação.

"Уж небо осенью дышало,
Уж реже солнышко блистало,
Короче становился день,
Лесов таинственная сень
С печальным шумом обнажалась,
Ложился на поля туман,
Гусей крикливых караван
Тянулся к югу: приближалась
Довольно скучная пора;
Стоял ноябрь уж у двора."
Пушкин А.С.

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1 Introduction

The main goal of this laboratory assignment is to analyse a RC circuit with sinusoidal excitation both theoretically and computationally, by simulating the circuit using *Ngspice*. Throughout the theoretical analysis, the circuit will be essentially analysed using some methods and concepts such as the nodal analysis method and the Thévenin equivalent circuit or solving some differential equations to find the capacitor's natural response and the total solution of the system.

All the results and plots obtained in the theoretical analysis will be compared to the values and similar plots obtained by the simulation made in *Ngspice* and discussed.

The analysed circuit was a RC circuit which can be seen in Fig. 1.

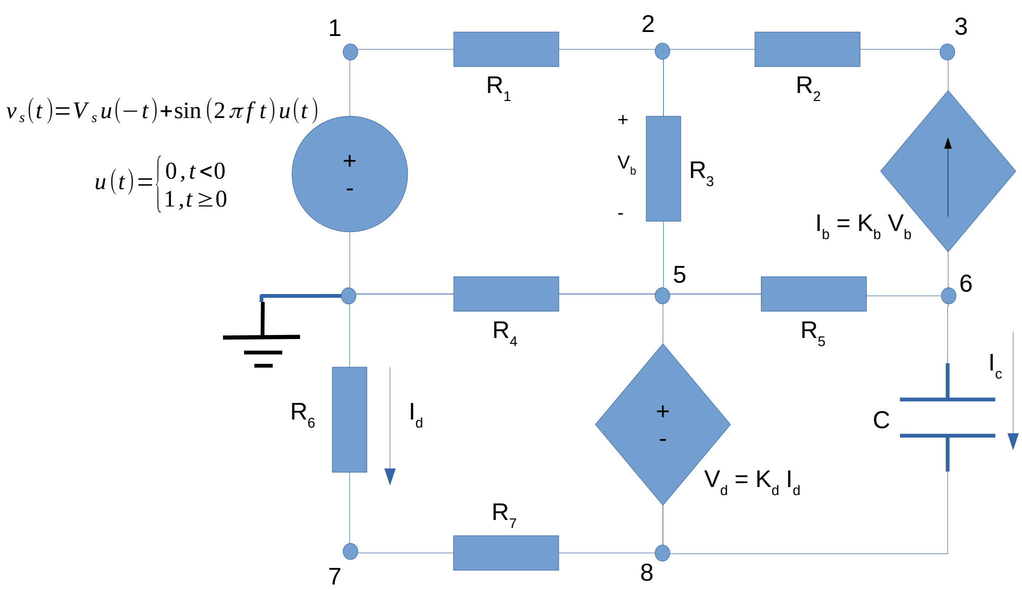


Figure 1: *Circuit analysed. It consists, essentially, in a circuit with 4 meshes, 8 nodes and 11 branches (in which we count 7 resistors, 2 voltage sources - one of them controlled by the current I_d - and 2 current sources - one independent and one dependent, controlled by the voltage V_b as it can be seen in the figure). The labels used in this figure will be used throughout this report.*

As a starting point to this laboratory assignment, some values of the symbolic variables presented in Fig. 1 were given and are presented in Table 1:

Resistors $[k\Omega]$	
R_1	1.03994439216
R_2	2.07923431764
R_3	3.06168544529
R_4	4.09516986362
R_5	3.00136467001
R_6	2.03324628446
R_7	1.02216788331

Capacitance $[\mu F]$	
C	1.01674167773
Voltages $[V]$	
V_s	5.03847501972
Dependent sources constants	
K_b	7.01505323139 mS
K_c	8.37372457746 $k\Omega$

Table 1: Summary table with all the known values at the starting point

2 Theoretical Analysis

2.1 Nodal Analysis at $t < 0$

First of all, one can start by applying the nodal analysis method at $t < 0$ to find the voltages of all nodes and, afterwards, using Ohm's law, the currents that flow through each branch of the circuit. Note that the labels used to refer to each node and branch are exactly the same as presented in Fig. 1.

So, the next step is to write linearly indepedent equations that will allow us to get the voltages at each node. Note that for $t < 0$, we will have $u(t) = 0 \implies I_c = 0$ and so, by applying the condition associated with the independent voltage source, one will have a constant value for $v_s(t)$, such that $v_s(t) = V_s$ for $t < 0$ (as it can be seen in the expression for the independent voltage source in Fig. 1.

$$\begin{cases} V_1 - V_4 = V_s \quad \text{(independent voltage source)} \\ (V_2 - V_1)G_1 + (V_2 - V_3)G_2 + (V_2 - V_5)G_3 = 0 \quad \text{(node 2)} \\ (V_3 - V_2)G_2 + (V_5 - V_2)K_b = 0 \quad \text{(node 3)} \\ V_4 = 0 \quad \text{(node 4 assigned to GND)} \\ (V_5 - V_2)G_3 + (V_5 - V_4)G_4 + (V_5 - V_6)G_5 + (V_8 - V_7)G_7 = 0 \quad \text{(supernodal equation)} \\ (V_6 - V_5)G_5 + (V_2 - V_5)K_b = 0 \quad \text{(node 6)} \\ (V_7 - V_4)G_6 + (V_7 - V_8)G_7 = 0 \quad \text{(node 7)} \\ V_5 - V_8 = (V_7 - V_4)K_dG_6 \quad \text{(restriction equation involving dependent voltage source)} \end{cases} \tag{1}$$

Note that the last equation of this system, the restriction equation involving the current-controlled voltage source, urges as the result of the combination of two equations. As shown in Fig. 1, one can write V_d as:

$$V_d = K_d I_d \tag{2}$$

and also:

$$V_d = V_5 - V_8 \tag{3}$$

Because I_d is the current which flows through the resistor R_6 (one could write an equivalent equation by using the resistor R_7, as the current flowing through it is exactly the same), one can easily write, using Ohm's Law:

$$I_d = (V_7 - V_4)G_6 \tag{4}$$

Combining these 3 equations, one can obtain the restriction equation used in the previous system:

$$V_d = V_5 - V_8 = (V_7 - V_4)G_6K_d \tag{5}$$

Converting this system of equations to its matricial form, one can obtain:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2 - K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3 & 0 & -G_4 & G_3 + G_4 + G_5 & -G_5 & -G_7 & G_7 \\ 0 & K_b & 0 & 0 & -G_5 - K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6 + G_7 & -G_7 \\ 0 & 0 & 0 & -K_dG_6 & 1 & 0 & K_dG_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} V_s \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{6}$$

The matrix system 6 is composed of a very sparse matrix, thus we also advise for using correspondingly adapted sparse matrix solvers if possible, but this is already out of the scope of this discipline and laboratory. Getting back to the point, the solution to the matrix system is,

Finally, to find the currents in the circuit, one can apply Ohm's law as one knows all the resistor values and the voltages in all of the circuit's nodes (presented in (20).

Labeling the positive and negative terminals of each component as inserted in *Ngspice*, and as it can be seen in Fig. something, one can obtain

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} 5.038475 \\ 4.758224 \\ 4.170535 \\ -0.000000 \\ 4.798515 \\ 5.646841 \\ -1.834523 \\ -2.756787 \end{bmatrix} V \tag{7}$$

Branch	Current (A)
I(R1)	−0.000269
I(R2)	0.000283
I(R3)	−0.000013
I(R4)	−0.001172
I(R5)	−0.000283
I(R6)	0.000902
I(R7)	0.000902
I(Vs)	−0.000269
I(Vd)	−0.000902
Id	0.000902
Ib	−0.000283
Ic	0.000000

Table 2: Theoretical solution for voltages for all nodes and current for all branches for $t < 0$.

2.2 R_{eq} and time constant τ

Next up, we need to determine the equivalent resistance, R_eq, as seen from the capacitor terminals to find the time constant associated with this circuit, $\tau = R_{eq}C$, so that one can analyse, afterwards, the natural and the forced solution of the circuit.

To do that, one can start by "shutting down" the independent voltage source of the circuit, V_s, by making it $V_s = 0$, i.e., replacing it by a short circuit. Afterward, as we want to calculate R_{eq} as seen by the terminals of the capacitor, one should now substitute the capacitor by a voltage source with value $V_x = V_6 - V_8$, where one can use the values obtained in section 2.1 for V_6 and V_8. This whole process is adopted because the value R_{eq} which we are seeking is nothing more than the Thévenin Resistance, R_{th}, of the circuit as seen by the terminals of the capacitor.

Rewriting the system of equations with this new constraints (knowing the value for V_x and imposing $V_s = 0$), one can write the system as:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1+G_2+G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2-K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3+K_b & 0 & -G_4 & G_3+G_4-K_b & 0 & -G_7 & G_7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6+G_7 & -G_7 \\ 0 & 0 & 0 & -K_dG_6 & 1 & 0 & K_dG_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \\ 0 \end{bmatrix} \tag{8}$$

Which take us to this result:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 8.40363 \\ 0 \\ 0 \end{bmatrix} V \tag{9}$$

Now that one obtained the values for every node voltage, and having in mind that I_x can be written as $I_x = -(I_b + I_5)$:

$$\begin{cases} I_b = K_b(V_2 - V_5) = 0mA \\ I_5 = (V_6 - V_8)G_5 \approx -2.80mA \end{cases} \implies I_x = -I_5 = 2.80mA \tag{10}$$

Now that one obtained the values of V_x and I_x, one can finally obtain the value for R_eq, which can be written as:

$$R_{eq} = \frac{V_x}{I_x} \approx 3.0014k\Omega \tag{11}$$

Finally, as for RC circuits, the time constant can be expressed as:

$$\tau = R_{eq} \tag{12}$$

one can obtain that for this specific circuit, *tau* can be written as:

$$\tau \approx 3.0516ms \tag{13}$$

2.3 Node analysis for $t \geq 0$ - natural solution for $v_6(t)$

Analysing now the circuit in $t > 0$ in order to determine the evolution of the voltage on node 6, V_6, with time, $v_6(t)$, one can start by determining its natural solution. To do that, we start by assuming the general form of this solution which can be written as:

$$v_{6n}(t) = v_{6n}(t \to \infty) + (v_{6n}(t = 0) - v_{6n}(t \to \infty))e^{-\frac{t}{\tau}} \tag{14}$$

in which τ represents the time constant for this RC circuit, i.e., $\tau = R_{eq}C$, as calculated on the previous section.

Starting by $v_6(t = 0)$, the initial condition for the solution which we are seeking is V_x, the voltage obtained in the last section for the voltage source by which one replaced the capacitor. So on, using the last section results, as $V_x = V_6 - V_8$ and $V_8 = 0$, one can infer that $V_x = V_6$, and so one can conclude that $v_6(t = 0) = V_x$.

Only $v_(t \to \infty)$ remains unknows. To find it, one can run again the nodal analysis system of equations with $V_s = 0$, as we are searching only for the natural solution by now. So, one can write the equivalent matricial equation for $t \to \infty$ as:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2 - K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3 + K_b & 0 & -G_4 & G_3 + G_4 - K_b & 0 & -G_7 & G_7 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 \\ 0 & K_b & 0 & 0 & -G_5 - K_b & G_5 & 0 & 0 \\ 0 & 0 & 0 & -K_dG_6 & 1 & 0 & K_dG_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{15}$$

which results in the following solution:

$$\begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} V \tag{16}$$

Knowing $v_{6n}(t = 0)$ and $v_{6n}(t \to \infty)$ one can now use equation (14) to find the expression for the natural solution of the voltage on node 6:

$$v_{6n}(t) \approx 8.4036e^{-\frac{t}{0.0030516}} \tag{17}$$

Plotting the function obtained on the interval $t \in [0, 20]ms$, one can get the graph shown in Fig. 2

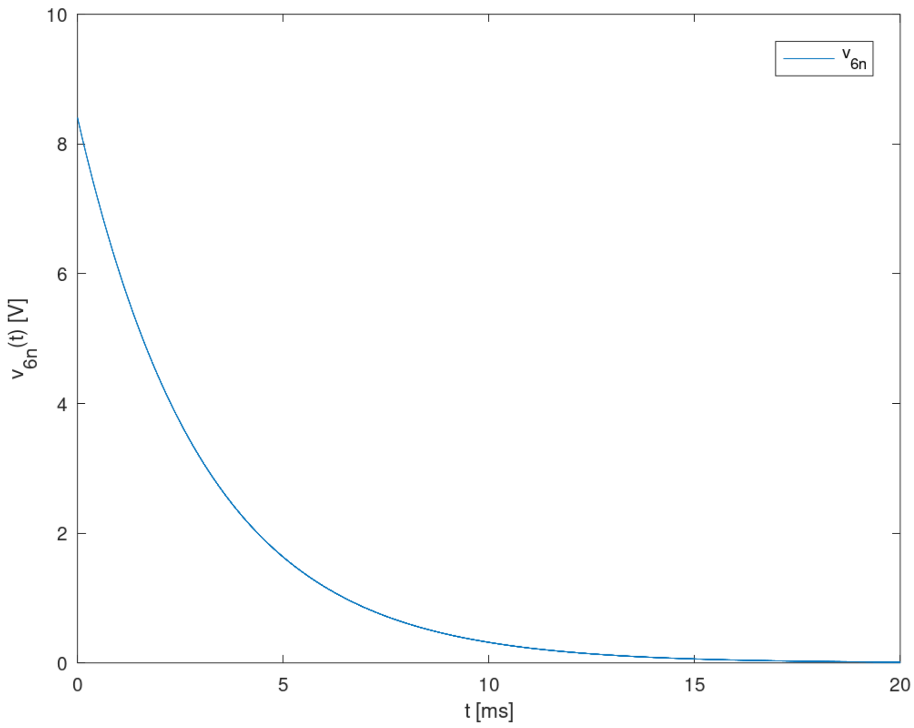


Figure 2: *Plot of the natural solution for the voltage on node 6 in the interval* $t \in [0, 20]ms$ *as shown on the plot*

2.4 Node analysis for $t \geq 0$ - forced solution $v_6(t)$

To complete the solution of the system, the forced solution needs to be determined. That was done using the nodal analysis method equivalent to 15 but adding the current flowing thought the capacitor and changing the voltage source to it's new function ($V_s(t) = sin(2\pi ft)$, $t \geq 0$) resulting in the change of only this 3 equations.

$$\begin{cases} V_1 - V_4 = -ie^{i2\pi fCt} \quad \text{(independent voltage source)} \\ (V_5 - V_2)G_3 + (V_5 - V_4)G_4 + (V_5 - V_6)G_5 + (V_8 - V_7)G_7 + (V_8 - V_6)i2\pi fC = 0 \quad \text{(supernodal equation)} \\ (V_6 - V_5)G_5 + (V_2 - V_5)K_b + (V_6 - V_8)i2\pi fC = 0 \quad \text{(node 6)} \end{cases} \tag{18}$$

Note that the the complex phasors are being used instead of the normal sine function in order to obtain the complex amplitudes. Converting this system of equations to its matricial form, one can obtain:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G_1 & G_1 + G_2 + G_3 & -G_2 & 0 & -G_3 & 0 & 0 & 0 \\ 0 & -G_2 - K_b & G_2 & 0 & K_b & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G_3 & 0 & -G_4 & G_3 + G_4 + G_5 & -G_5 - i2\pi fC & -G_7 & G_7 + i2\pi fC \\ 0 & K_b & 0 & 0 & -G_5 - K_b & G_5 + i2\pi fC & 0 & -i2\pi fC \\ 0 & 0 & 0 & -G_6 & 0 & 0 & G_6 + G_7 & -G_7 \\ 0 & 0 & 0 & -K_d G_6 & 1 & 0 & K_d G_6 & -1 \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \\ V_7 \\ V_8 \end{bmatrix} = \begin{bmatrix} -ie^{i2\pi fCt} \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix} \tag{19}$$

With the matrix defined, a program in octave was used to solve the system symbolically, to which was then substituted the frequency for $1000Hz$ and, in order to obtain the complex amplitude, time for 0, obtaining the values in Tab. 3.

The information that these complex amplitudes contain is the phase and amplitude of the oscillation, in this case forced, values that can be obtained through the complex amplitude norm and its argument, respectively, as it was done and the results are presented in Tab. 3.

Node	Complex Amplitude (v)
V(1)	-0.000000 + i(-1.000000)
V(2)	0.000000 + i(-0.944378)
V(3)	-0.000000 + i(-0.827738)
V(5)	0.000000 + i(-0.952375)
V(6)	-0.086752 + i(0.542623)
V(7)	0.000000 + i(0.364103)
V(8)	0.000000 + i(0.547147)

Node	Amplitude (V)	Phase (rad)
V(1)	1.000000	-1.570796
V(2)	0.944378	-1.570796
V(3)	0.827738	-1.570796
V(5)	0.952375	-1.570796
V(6)	0.549514	1.729330
V(7)	0.364103	1.570796
V(8)	0.547147	1.570796

Table 3: Amplitudes for the forced solution osculations

With this information the forced solution can be plotted, in specific, in Fig.3.

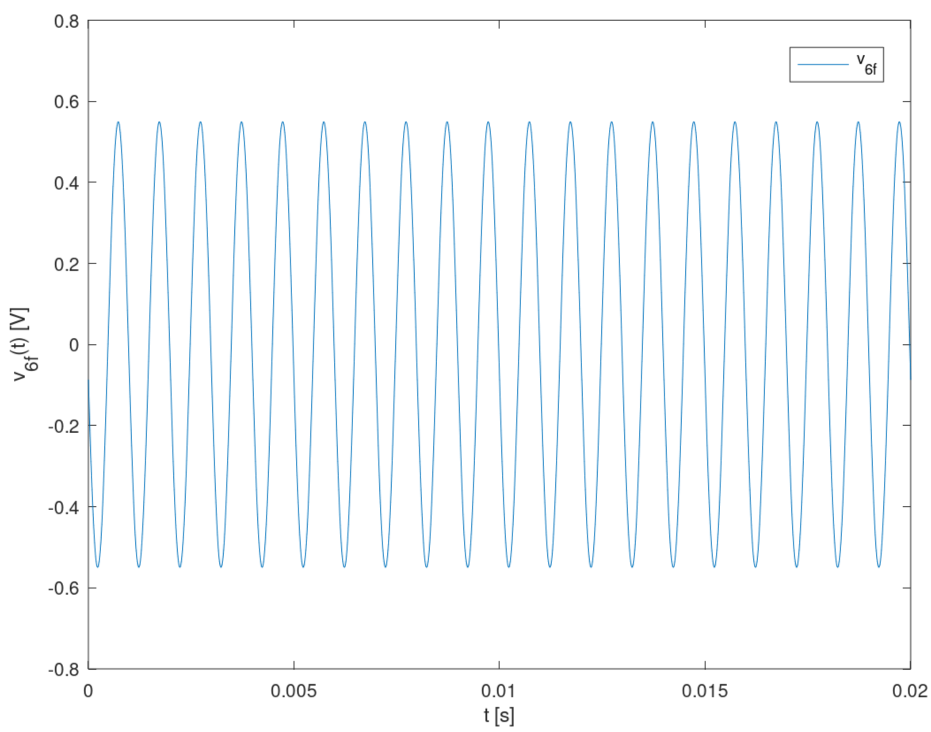


Figure 3: *Plot of the forced solution for the voltage on node 6 in the interval* $t \in [0, 20]ms$ *as shown on the plot*

With both natural, obtained in Fig. 2 and forced solutions, obtained in Fig. 3 determined it's easy to add the 2 in order to obtain the complete solution, which for $V_6(t)$ is ploted in Fig. 4

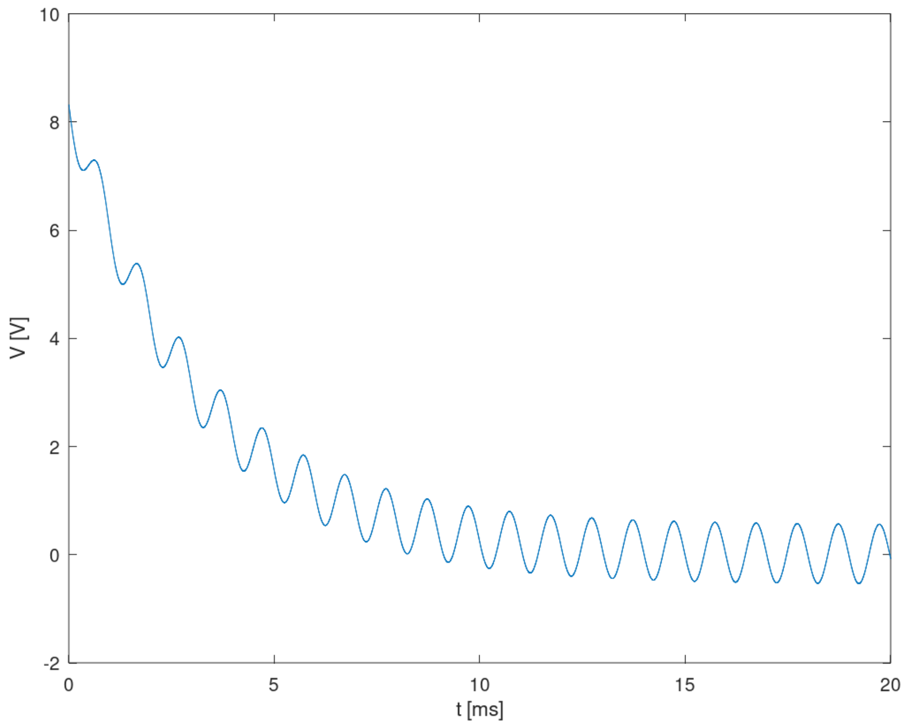


Figure 4: *Plot of the complete solution for the voltage on node 6 in the interval* $t \in [0, 20]ms$ *as shown on the plot*

For further understanding the behaving of the system, the voltage in node 6 and the input voltage can both be ploted for before and after $t = 0$, by merging the results from the previous theoretical analysis, resulting in what is seen in Fig. 5.

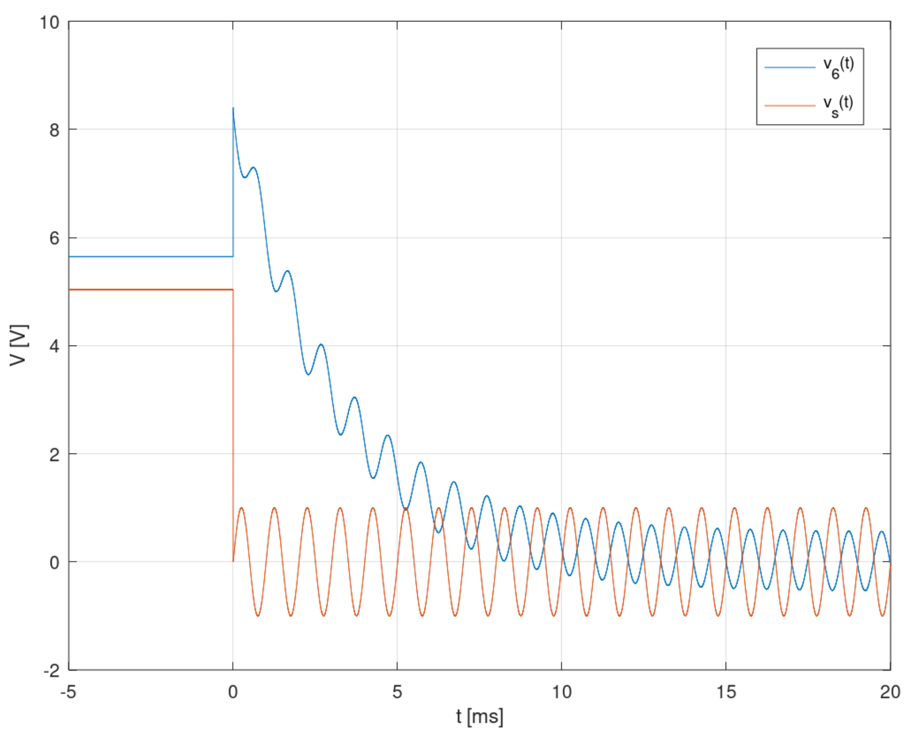


Figure 5: *Plot of the complete solution for the voltage on node 6 and the input voltage in the interval* $t \in [-5, 20]ms$ *as shown on the plot*

As expected we can see that the discontinuity in the voltage source causes a discontinuity in the voltage in node 6, just not as intense and the system takes some time to reach the equilibrium and adapt to a behavior similar to its source. It's also notable that these 2 voltages are separated in phase by half a period.

2.5 Magnitude and Phase as Functions of Frequency

Once the system was solved symbolically, the complex amplitudes in function of the frequency could be obtain symbol by keeping $t = 0$, and varying only the frequency, which in this case was done in the interval between 0.1 and 10^6 Hz. This time the goal was specifically to get the magnitude and the phase, instead of the complex amplitude, so as before were used the norm and the argument.

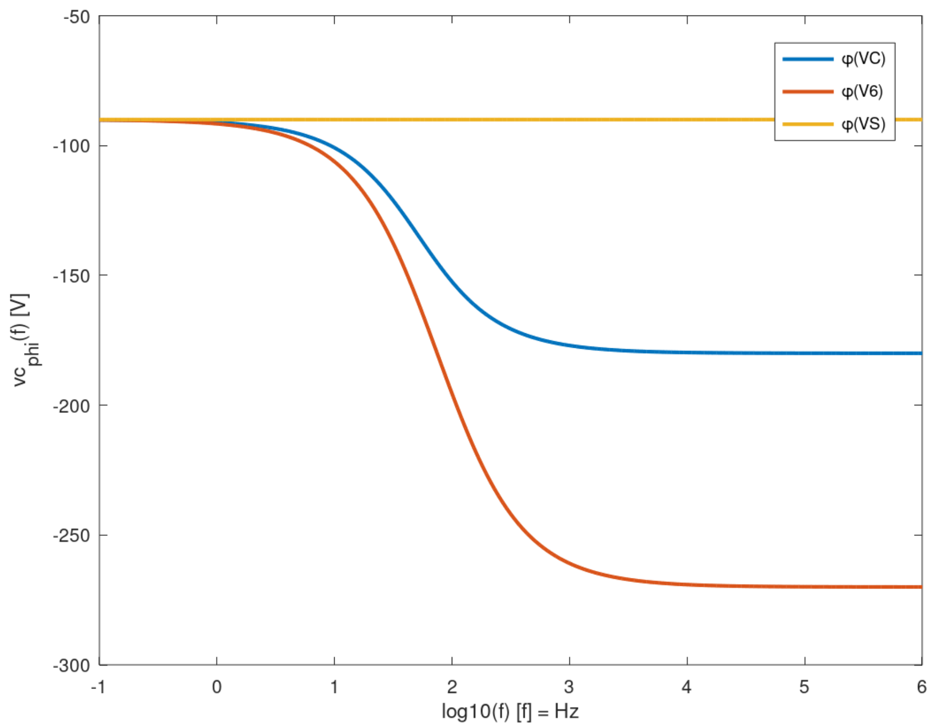


Figure 6: *Plot of the phase in function of the frequency. The frequency is presented in a logarithmic scale, contrary to the phase which is in linear scale in degrees.*

As expected the phase for the voltage source is not dependent on the frequency, which was clear in the formula used to define it. More interestingly, the phase for both V_c and V_6 seem to decrease around $50Hz$, stabilizing in a quarter and half a period, respectively, for higher frequencies.

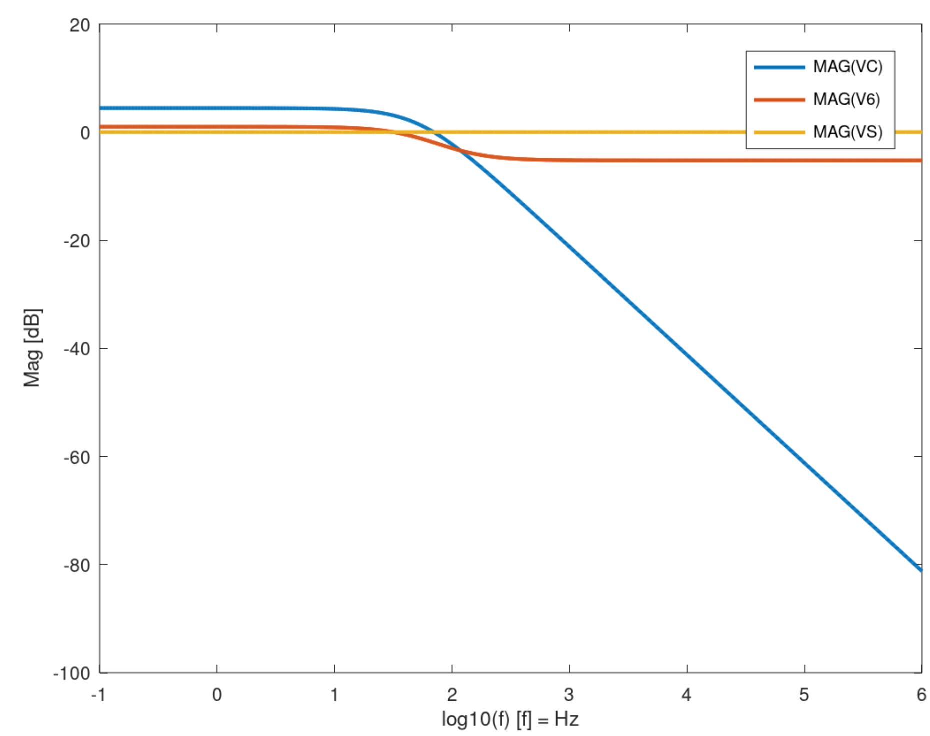


Figure 7: *Plot of the magnitude in function of the frequency. The frequency is presented in a logarithmic scale, as is the magnitude, which is in dB.*

With amplitude 1, V_s has a constant magnitude for any frequency as expected, contrary to the other amplitudes that decrease with the increase in frequency, and for really high oscillation in the source, the amplitude of the oscillation in node 6 is practically 0. The critical frequency seems to be at $80Hz$.

3 Simulation Analysis

3.1 Nodal Analysis at t < 0

For the first part, as mentioned in the theoretical analysis, because the system has stabilized, we expect that for a long period of time no current will pass through the capacitor. The values obtained for this are presented in Tab. 4.

Node/Branch	$@(i)[A]/v[V]$
@c1[i]	0.000000e+00
@gb[i]	-2.82647e-04
@r1[i]	-2.69487e-04
@r2[i]	2.826469e-04
@r3[i]	-1.31600e-05
@r4[i]	-1.17175e-03
@r5[i]	-2.82647e-04
@r6[i]	9.022631e-04
@r7[i]	9.022631e-04
v(1)	5.038475e+00
v(2)	4.758224e+00
v(3)	4.170534e+00
v(5)	4.798515e+00
v(6)	5.646842e+00
v(7)	-1.83452e+00
v(8)	-2.75679e+00
cfp1	-1.83452e+00

Table 4: Tension and current for every branch and node of the circuit for the stable solution when V_s is stable.

As we can see, the current value in the conductor is 0, which goes according to the expectations, i.e, for a long period of time, the charge accumulates there and the potential difference in the capacitor is equal to the potential difference that is coming in from the circuit, which means no current is pulled to it. The rest of the values are equal to the ones obtained in the theoretical analysis.

3.2 R_{eq} and time constant τ

The only thing that needed change from the previous subsection was the value for V_s, that changes to 0, in order to determine the R_{eq} and that way τ.

Node/Branch	$@(i)[A]/v[V]$
@gb[i]	8.369438e-18
@r1[i]	7.979758e-18
@r2[i]	-8.36944e-18
@r3[i]	3.896799e-19
@r4[i]	-1.73508e-18
@r5[i]	-2.79994e-03
@r6[i]	1.301043e-18
@r7[i]	8.876953e-19
v(1)	0.000000e+00
v(2)	8.298505e-15
v(3)	2.570053e-14
v(5)	7.105427e-15
v(6)	8.403629e+00
v(7)	-2.64534e-15
v(8)	-3.55271e-15
cfp1	-2.64534e-15

Table 5: Tension and current for every branch and node of the circuit for the stable solution when V_s is null.

As we can see in Tab. 6, the values for the currents are practically 0 for every branch, excepting the currents going through R_5, C_1 and V_d which, although are not presented in the table, one can easily see from the nodes tensions, specially for the tension in node 6. The result for node 6 in the theoretical analysis was $V_6 = 8.40363V$ while here is $V_6 = 8.403629V$, which represents but an approximation in octave. $I_5 = -2.79994 \times 10^{-3}A$, $I_b \approx 0A$, which means that $I_x = 2.79994 \times 10^{-3}A$ while in the theoretical analysis we got $I_x = 2.80mA$, thus once again is but an approximation from Octave. This means that $R_{eq} = 3.0014k\Omega$. This means that the value for τ is the same as in the theoretical analysis.

3.3 Natural solution for $v_6(t)$

To simulate that the circuit was active since infinite, we use initial conditions has suggested by the teacher, while zeroing the voltage source v_s, which results in what is shown in Fig. 8

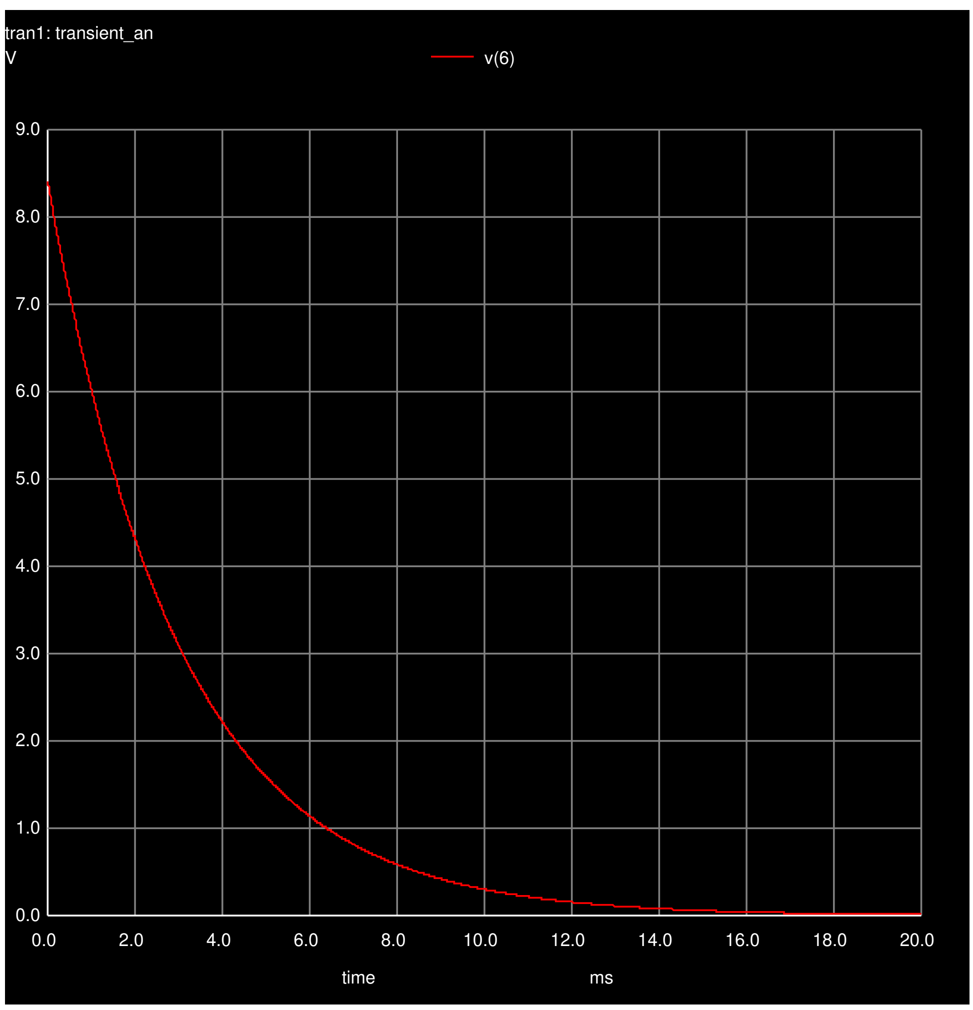


Figure 8: *Natural solution for $v_s = 0$, having been charged, the capacitor, previously, since infinite (due to initial conditions).*

This result is the same, as expected, as the one in Fig. 2.

3.4 Forced solution for $v_6(t)$

Having given no initial conditions and putting $v_s = sin(2\pi ft)$, we get the shown in Fig. 9. One should notice that this graph is not exactly the same as the one in Fig. 3, because here the system is behaving as if it were in the real world. What this means is that the voltage source starts by immediately having that sinusoidal behavior but it still is not in equilibrium with the rest of the circuit, which means that v_6 goes through a transient state first, before reaching the steady state form, in those initial milliseconds.

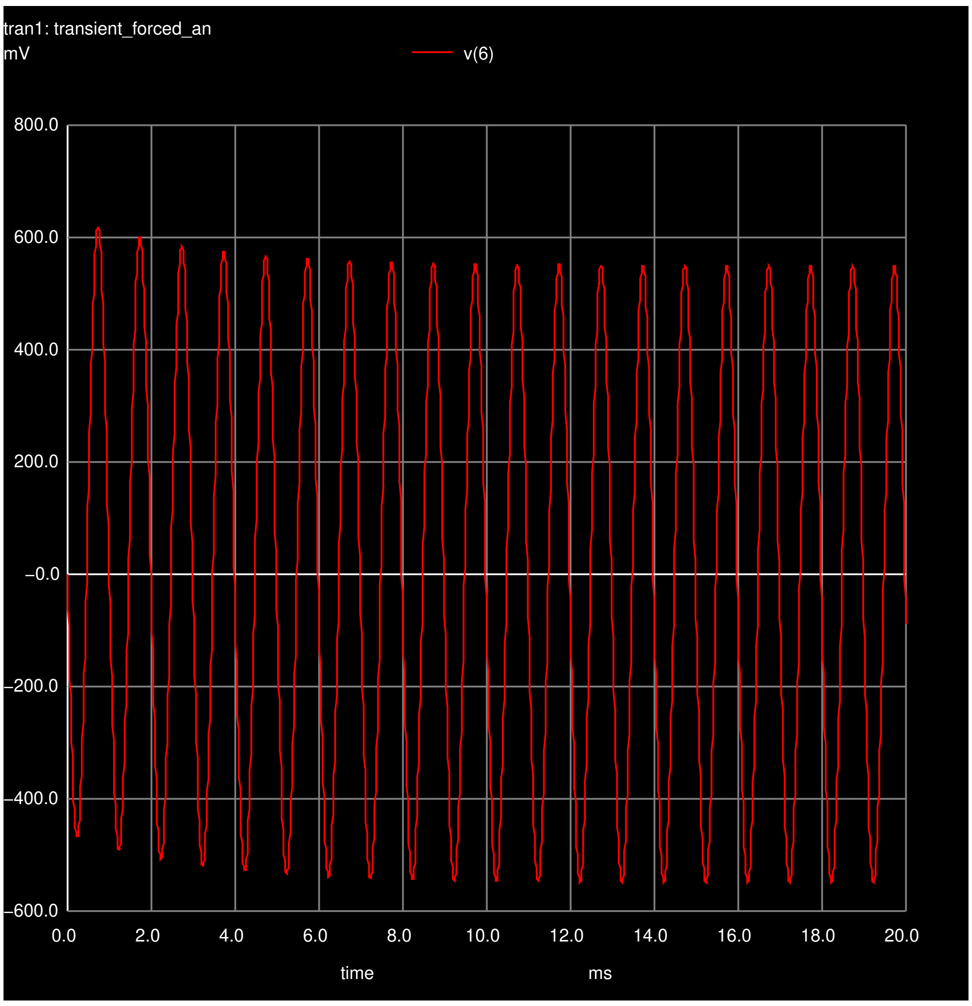


Figure 9: *Forced solution for $v_s = sin(2\pi f t)$, having zero initial conditions, i.e., capacitor totally discharged.*

If we want to add the forced and natural solution to get the real solution, we do the same for this part, but impose the initial conditions from the previous section. Thus, the result is what is shown in Fig. 10, in red. In the same figure, in blue, one can see the voltage source behavior as well. As discussed in the theoretical analysis, the voltage source and v_6 are totally out of phase, which only means that the current takes some time to be "felt" in node 6.

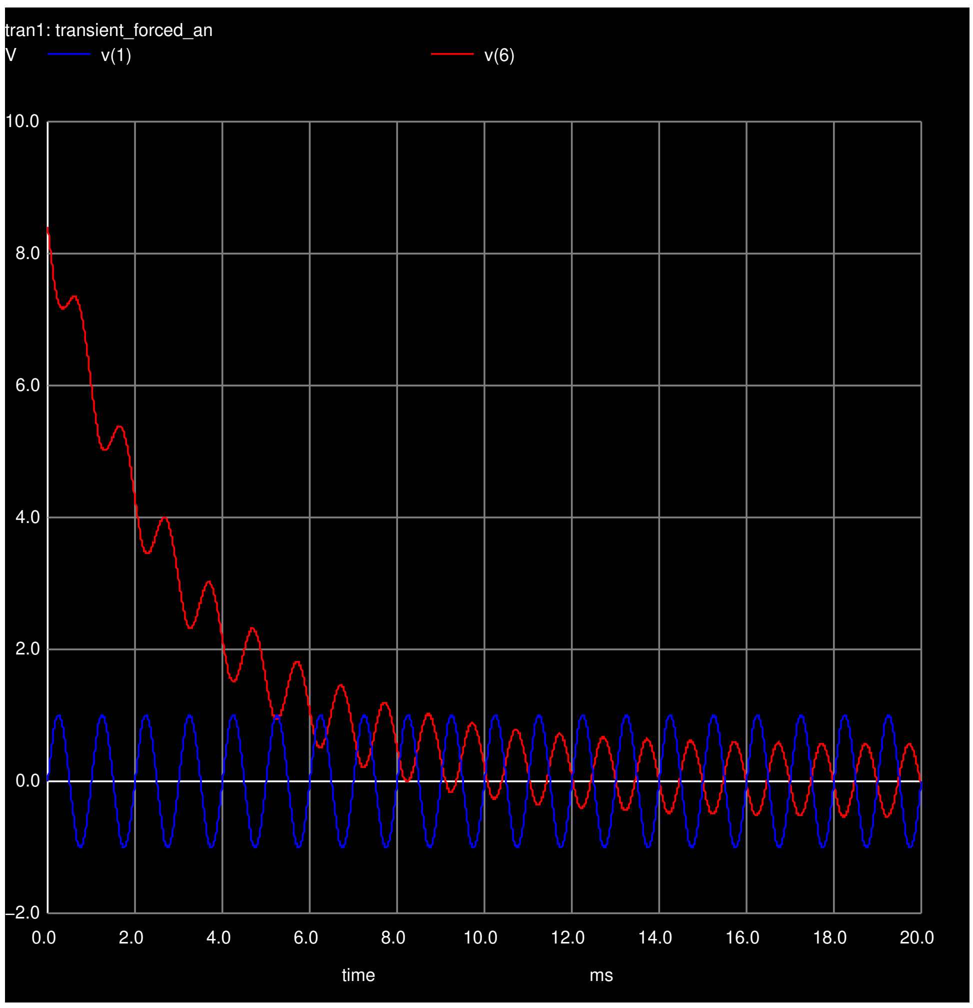


Figure 10: *Sum of forced and natural solution in red and v_s in red.*

As asked in the powerpoint, it is also shown the behaviour of tV_x in the sum of the two solutions, as shown in Fig. 11.

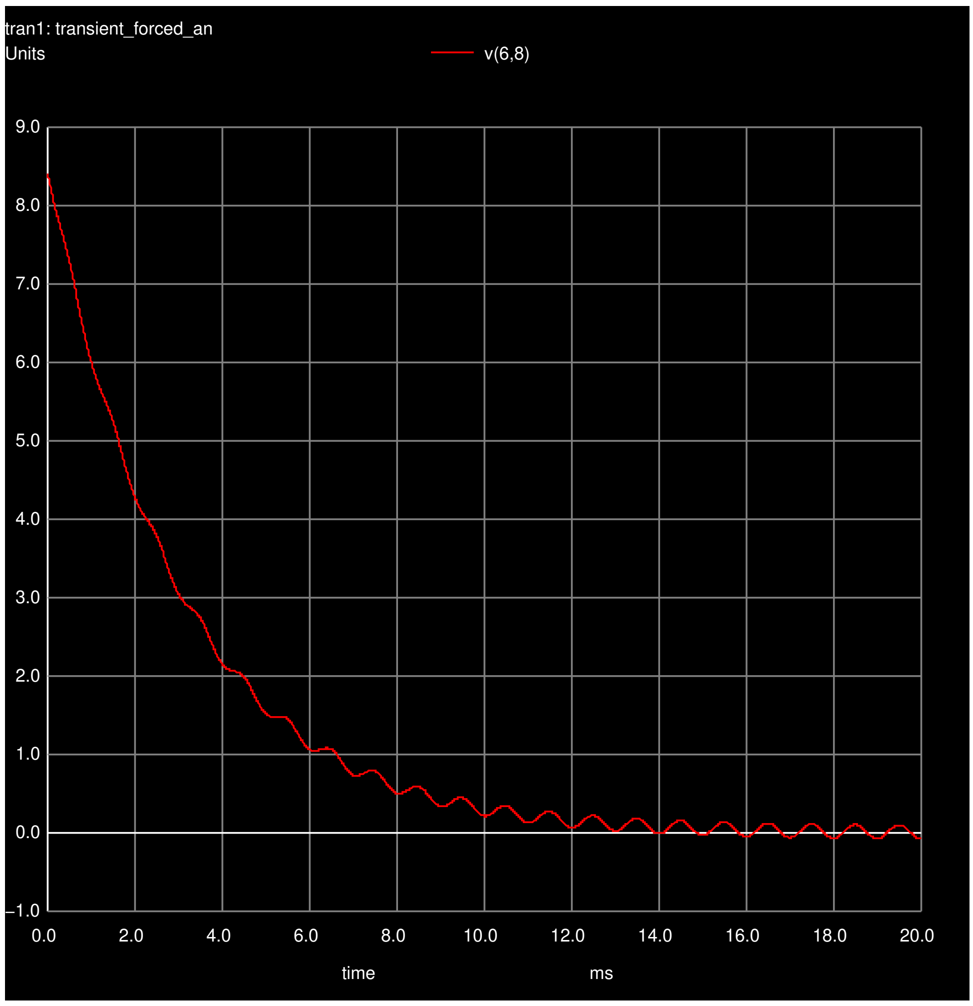


Figure 11: *Sum of forced and natural solution for* v_x.

All of the graphs are according to the theoretical analysis.

3.5 Magnitude and Phase as Functions of Frequency

The magnitude as a function of the frequency for several variables, in dB, is shown in Fig. 12.

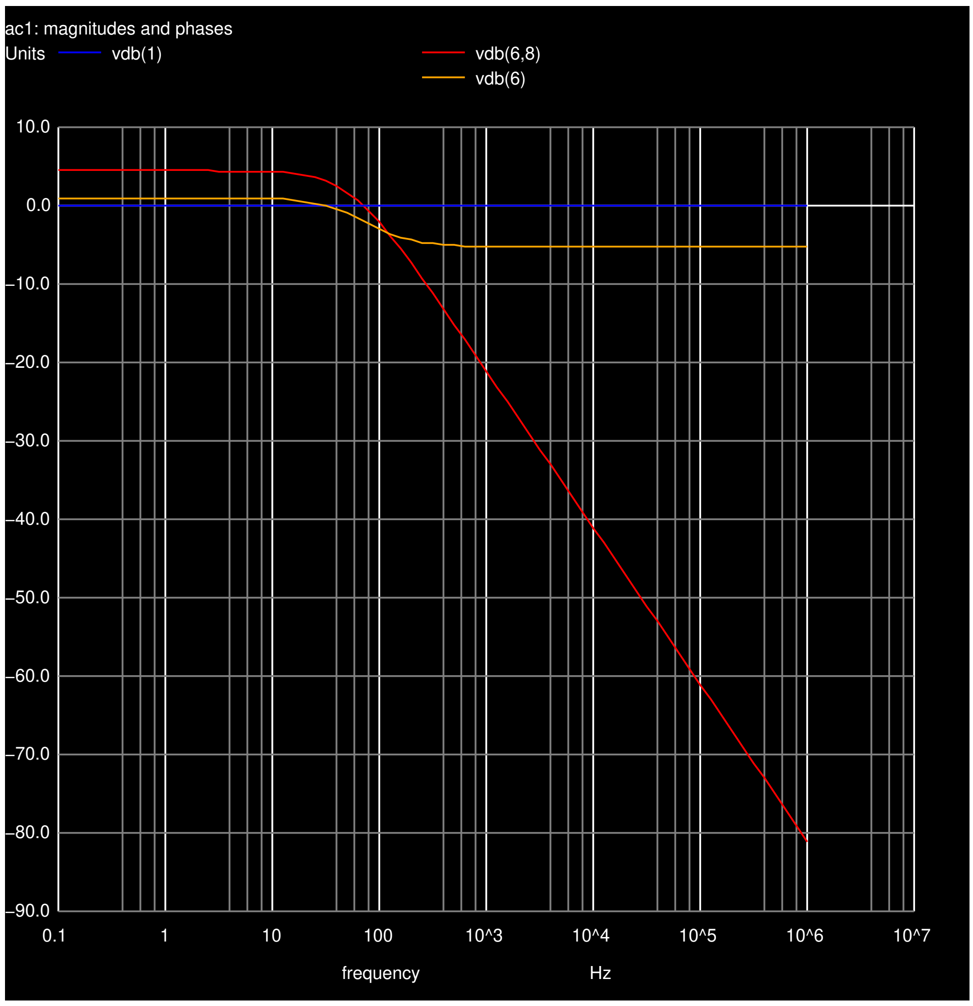


Figure 12: *Magnitudes of v_6 (orange), $v_6 - v_8$ (red) and v_s (blue) as a function of frequency.*

As expected, the magnitudes for v_6 and $v_6 - v_8$ start by accompanying the behaviour of the circuit but, for frequencies that are greater than 1000Hz, the system saturates and it no longer has the ability to go accordingly to the voltage source. Thus, due to saturation, the potential difference goes to zero, and the singular voltage for node 6 stabilizes at a certain value of saturation. One expects always that the magnitude of the source with respect to itself to be the same, thus, because it is a logarithmic scale, it is 0, as seen. For the phases response, check Fig. 13.

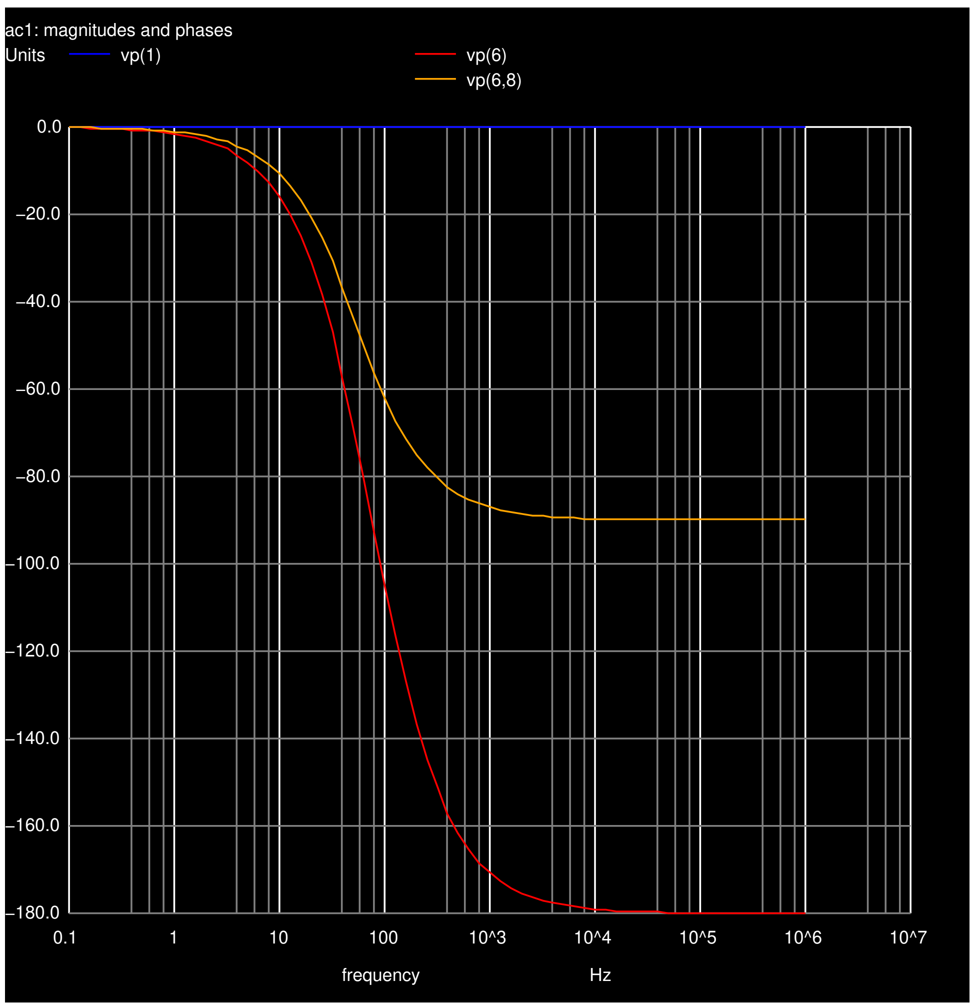


Figure 13: *Phases of v_6 (red), $v_6 - v_8$ (orange) and v_s (blue) as a function of frequency.*

The main difference in Fig. 13 to Fig. 6 is the base level, i.e., the voltage level. Octave assumes it to be $-\pi/2$, while ngspice rightly sets it to 0. However, the evolution of the phases throughout time is equivalent. This concludes our simulation analysis.

4 Side by side comparison between analysis results

Node/Branch	$@(i)[A]/v[V]$
@c1[i]	0.000000e+00
@gb[i]	-2.82647e-04
@r1[i]	-2.69487e-04
@r2[i]	2.826469e-04
@r3[i]	-1.31600e-05
@r4[i]	-1.17175e-03
@r5[i]	-2.82647e-04
@r6[i]	9.022631e-04
@r7[i]	9.022631e-04
v(1)	5.038475e+00
v(2)	4.758224e+00
v(3)	4.170534e+00
v(5)	4.798515e+00
v(6)	5.646842e+00
v(7)	-1.83452e+00
v(8)	-2.75679e+00
cfp1	-1.83452e+00

Table 6: Solution from simulation.

Branch	Current (A)
I(R1)	−0.000269
I(R2)	0.000283
I(R3)	−0.000013
I(R4)	−0.001172
I(R5)	−0.000283
I(R6)	0.000902
I(R7)	0.000902
I(Vs)	−0.000269
I(Vd)	−0.000902
Id	0.000902
Ib	−0.000283
Ic	0.000000

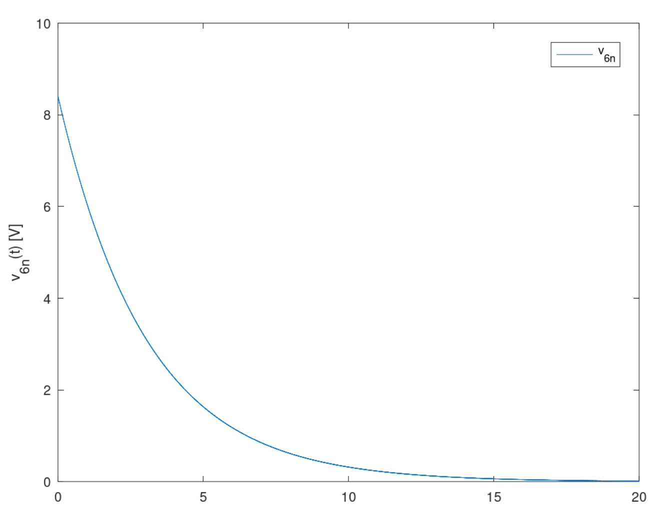
Table 7: Currents from theoretical solution

$$\begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} 5.038475 \\ 4.758224 \\ 4.170535 \\ -0.000000 \\ 4.798515 \\ 5.646841 \\ -1.834523 \\ -2.756787 \end{bmatrix} V \tag{20}$$

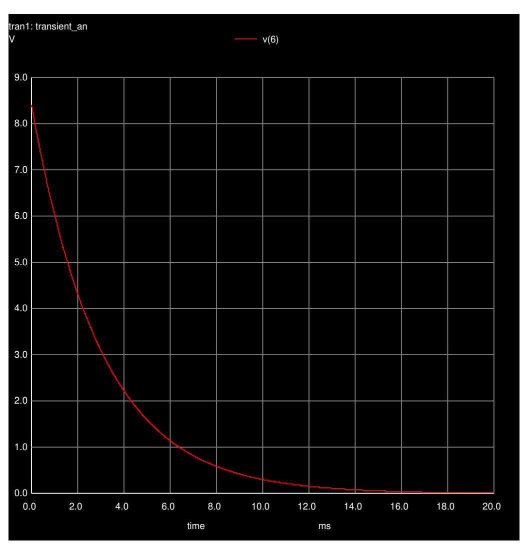
Table 8: Voltages from theoretical solution

Table 9: Solution for voltages for all nodes and current for all branches for $t < 0$.

The only difference found between the 2 solutions, for the digits presetned here are for V_3 and V_6, but only in the last digit presented, meaning a difference around $10^{-6}V$. Differences that are probably result of numerical errors, rather than real differences in solutions.



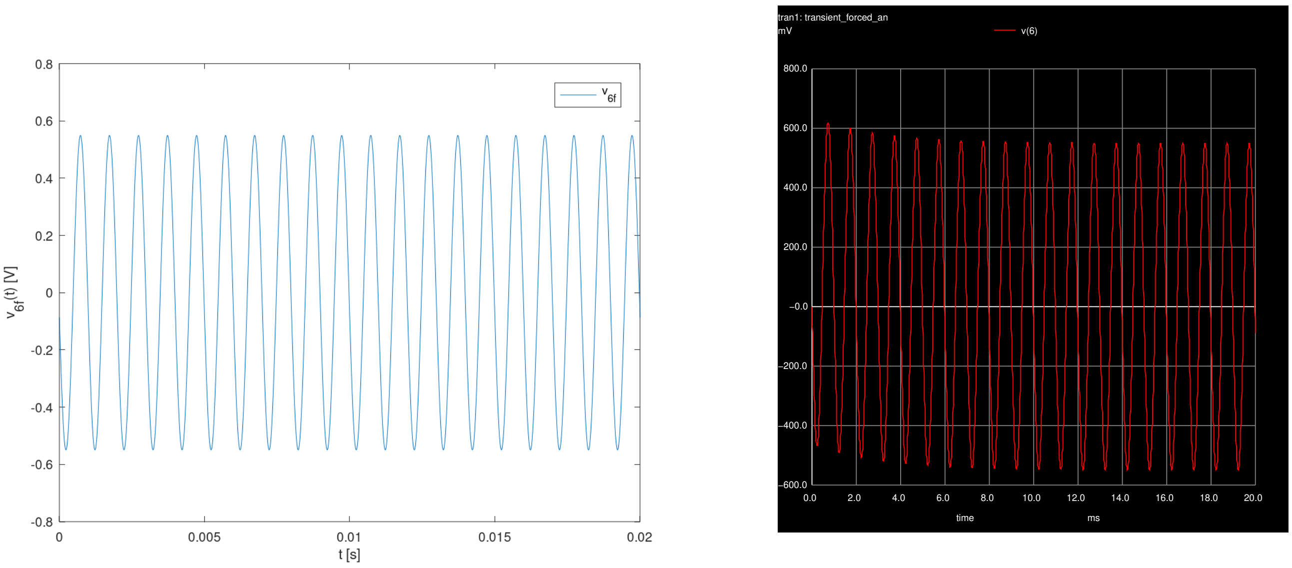
(a) Theoretical analysis



(b) Simulation analysis

Figure 14: Natural solution for voltage in node 6 for $t = [0, 20]$.

The solution for the natural solution in node 6 appears to be exactly the same.

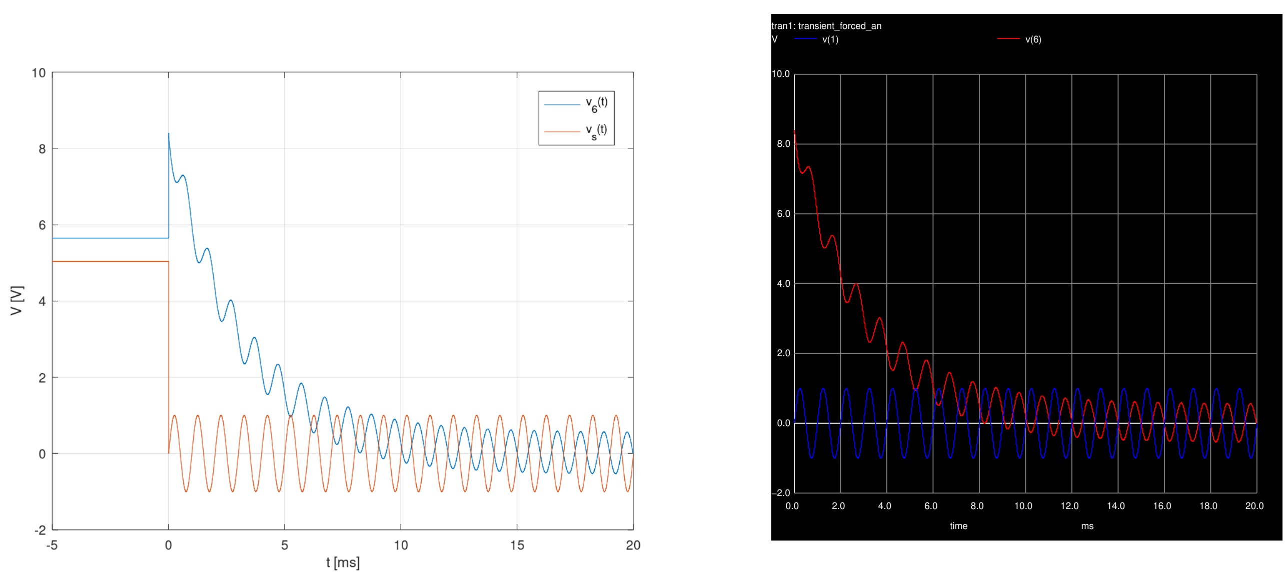


(a) Theoretical analysis

(b) Simulation analysis

Figure 15: Forced solution for voltage in node 6 for $t = [0, 20]$.

As mentioned before, the only difference found between these 2 solution is that in the forced solution a small offset seems to exist in the first moments of the oscillation, but appears to stabilize on the expected solution.

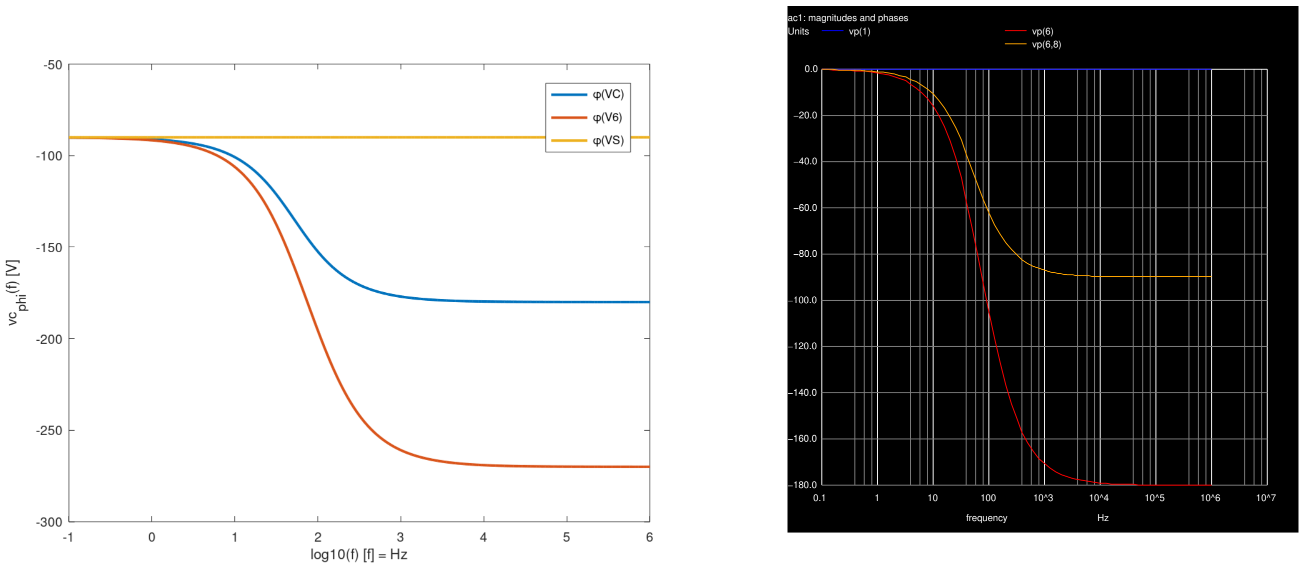


(a) Theoretical analysis

(b) Simulation analysis

Figure 16: Complete solution for voltage in node 6 for $t = [-5, 20]$ and $t = [0, 20]$.

Besides the part of the forced solution that is now impossible to notice, the plots looks identical even in close inspection.

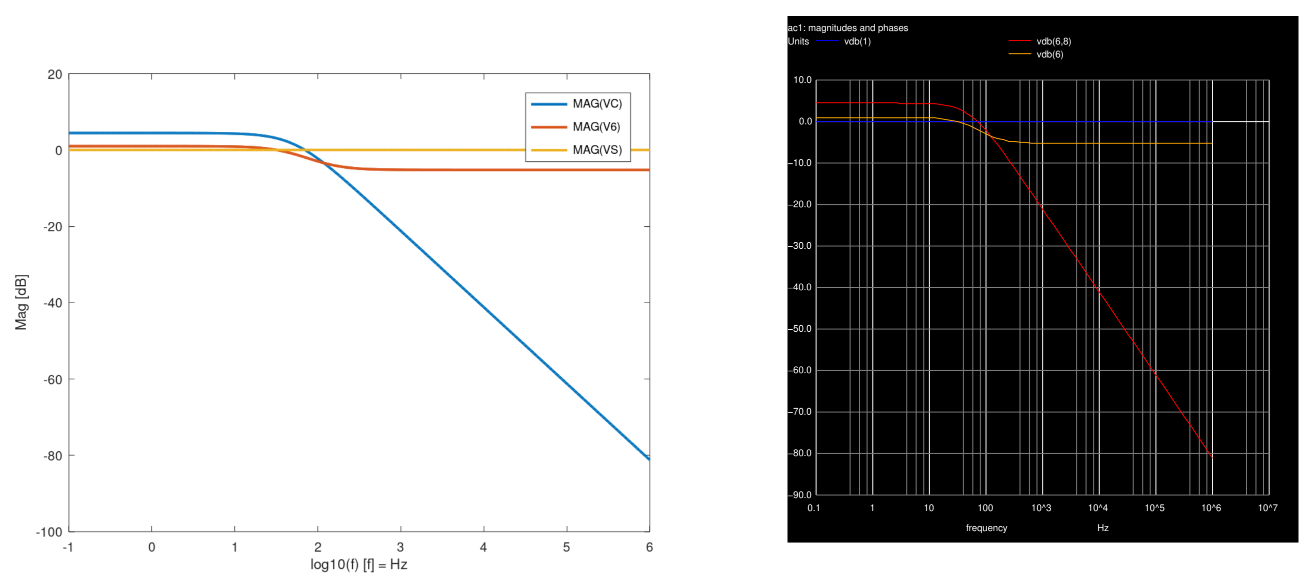


(a) Theoretical analysis

(b) Simulation analysis

Figure 17: Phase difference as a function of the frequency of the voltage source.

As mentioned before the same transition to a phase difference of a quarter and half a period is seen but *Ngspice* automatically offsets the phases in order for the source to be at 0 degrees.



(a) Theoretical analysis

(b) Simulation analysis

Figure 18: Magnitude as a function of the frequency of the voltage source.

Once again no differences can be found between the 2 plots.

5 Conclusion

The goal of this laboratory assignment was to analyse a RC circuit with sinusoidal excitation. This circuit full analysis was made both theoretically (with the computational help of *Octave*) and computationally, simulating the circuit using *Ngspice*.

The comparison of the results obtained from both methods has been made throughout the report, and it was summarized on section 4 where all the values and plots obtained were compared.

Unsurprisingly, the results obtained with both methods were pretty consistent and similar to each other. The values obtained for the nodes voltages and the branches currents in $t < 0$ by the simulation matched every digit of the values obtained theoretically (the differences seen are product of approximations done by Octave). In addition, the plots made on both methods to analyse the circuit at $t \geq 0$ were very similar, except the plots obtained to the forced solution on node 6, where the plot obtained by *Ngspice* had a small offset on the beginning of the interval plotted due to an initial transient state, as the circuit doesn't *really* start in equilibrium.

Therefore, and concluding, as all the results and *plots* obtained were consistent with each other, it can be stated that the goals for this laboratory were successfully achieved.

References

[1] Ngspice official website, *http://ngspice.sourceforge.net/*