[redacted]

ASSIGNMENT 3

1) There are 3 instances $\lambda_i > 0$. These instances correspond to 3 support vectors, x_1, x_2 and x_3

$x_1 = (-0.6, -1.3)$ $x_2 = (-2, 0.5)$ $x_3 = (1, 2)$

$\lambda_1 = 0.0533$ $\lambda_2 = 0.1316$ $\lambda_3 = 0.1849$

$y_1 = -1$ $y_2 = -1$ $y_3 = 1$

Let $w = (w_1, w_2)$ and b denote the parameters of DB.

Using eq'n: $w = \sum_{i=1}^{N} \lambda_i y_i x_i$

$x_i = (x_{i1}, x_{i2}, x_{i3})$, we can solve for w_1, w_2 and w_3.

$$w_1 = \sum_{i:\lambda_i>0} \lambda_i y_i x_{ij} = \sum_{i=1}^{3} \lambda_i x_{i1} y_i$$
$$= \lambda_1 y_1 x_{11} + \lambda_2 y_2 x_{21} + \lambda_3 y_3 x_{31}$$
$$= (0.0533 \times -1 \times -0.6) + (0.1316 \times -1 \times -2) + (0.1849 \times 1 \times 1)$$
$$= 0.48008$$

$$w_2 = \sum_{i:\lambda_i>0} \lambda_i y_i x_{ij} = \sum_{i=1}^{3} \lambda_i x_{i2} y_i$$
$$= \lambda_1 y_1 x_{12} + \lambda_2 y_2 x_{22} + \lambda_3 y_3 x_{32}$$
$$= (0.0533 \times -1 \times -1.3) + (0.1316 \times -1 \times 0.5) + (0.1849 \times 1 \times 2)$$
$$= 0.37329$$

Thus $w = (0.48008, 0.37329)$

For support vectors x_i (ie $\lambda_i > 0$),

$$w = \sum_{i=1}^{N} \lambda_i y_i x_i = \sum_{i:\lambda_i>0} \lambda_i y_i x_i$$

$$w_j = \sum_{i=1}^{N} \lambda_i y_i x_{ij} = \sum_{i:\lambda_i>0} \lambda_i y_i x_{ij}$$

Bias term b can be calculated using equation $\lambda_i [y_i(w.x_i + b) - 1] = 0$ for each support vector.

Recall: for a support vector x_i ($\lambda_i > 0$) we have

$$y_i(w.x_i + b) - 1 = 0$$

$$\Leftrightarrow y_i(w.x_i + b) = 1$$

$$\Leftrightarrow b = y_i - wx_i/y_i$$

$w = (w_1, w_2) = (0.48008, 0.37329)$, $x_i = (x_{i1}, x_{i2}, x_{i3})$

$b^{(K)} = y_i - w_1 \times x_{i1} - w_2 \times x_{i2}, K = 1, 2, \ldots, m$

where m is the no. of support vectors

(m = 2) i = 1, 2, 3 in the question.

$$b^{(1)} = -1 - w.x_1$$

$$= -1 - (0.48008 \times -0.6) - (0.37329 \times -1.3)$$

$$= -0.226675$$

$$b^{(2)} = -1 - w.x_2$$

$$= -1 - (0.48008 \times -2) - (0.37329 \times 0.5)$$

$$= -0.22484$$

$$b^{(3)} = 1 - wx_3$$

$$= 1 - (0.48008 \times 1) - (0.37329 \times 2) = -0.22008$$

Averaging $b^{(1)}$, $b^{(2)}$ and $b^{(3)}$ we get

$$b = \frac{-0.226675 + (-0.22484) + (-0.22008)}{3}$$

$$= \boxed{0.223865}$$

$$b \approx 0.22$$

Decision boundary :

$$0.48008\,x_1 + 0.37329\,x_2 - 0.2239 = 0$$

With found parameters w and b of DB, a test instance Z is:

$$f(z) = \text{sign}\,(w \cdot z + b)$$
$$= \text{sign}\left[\sum_{i=1}^{N} \lambda_i y_i x_i \cdot Z + b\right]$$

If $f(z) > 0$ (or $w \cdot Z + b \gtrsim 1$) then Z is classified as positive class (class label $y = 1$)
If $f(z) < 0$ (or $w \cdot Z + b \lesssim -1$) then Z is classified as negative class (ie. class label $y = -1$)