MATH 424: Homework Chapter 9: Logistic Regression

Due on Wednesday, December 6, 2017

Kafai 11:10am

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Contents

Q 24

Flight response of geese. Offshore oil drilling near an Alaskan estuary has led to increased air traf- ficmostly large helicoptersin the area. The U.S. Fish and Wildlife Service commissioned a study to investigate the impact these helicopters have on the flocks of Pacific brant geese, which inhabit the estuary in Fall before migrating (Sta- tistical Case Studies: A Collaboration between Academe and Industry, 1998). Two large heli- copters were flown repeatedly over the estuary at different altitudes and lateral distances from the flock. The flight responses of the geese (recorded as low or high), altitude (x 1 = hundreds of meters), and lateral distance (x 2 = hundreds of meters) for each of 464 helicopter overflights were recorded and are saved in the PACGEESE file. (The data for the first 10 overflights are shown in the table, p. 503.) MINITAB was used to fit the logistic regression model = 0 + 1 x 1 + 2 x 2 , where y = 1 if high response, 0 if low response, = P(y = 1), and = ln[/(1)]. The result- ing printout is shown above.
(a) Is the overall logit model statistically useful for predicting geese flight response? Test using = .01.
(b) Conduct a test to determine if flight response of the geese depends on altitude of the heli- copter. Test using = .01.
(c) Conduct a test to determine if flight response of the geese depends on lateral distance of helicopter from the flock. Test using = .01.
(d) Predict the probability of a high flight response from the geese for a helicopter flying over the estuary at an altitude of x 1 = 6 hundred meters and at a lateral distance of x 2 = 3 hundred meters.

a

Using the model output in the text as reference:

$$H_0 = \hat{\beta}_1 = \hat{\beta}_2 = 0$$

$$H_a = \hat{\beta}_1, \hat{\beta}_2 \neq 0$$

We can compare the Chi squared values and their respective d.f.'s by invoking
'1-pchisq(model$null.deviance - model$deviance, model$df.null - model$df.residual)'
In the same motion as the F-test as we would do for a linear regression: taking the ratio of two Chi squared values and exracting a p-value from that.
We can extract a p-value = 0. p is less than $\alpha = 0.01$, therefore we can reject H_0 and state that the model is statistically useful for predicting geese flight response.

b

$$H_0 = \hat{\beta}_1 = 0$$

$$H_a = \hat{\beta}_1 \neq 0$$

by looking at the model summary, we can see that the z-score for this test is z=2.914 , with a corresponding p-value of 0.00357. We can conclude that p is less than α and therefore that the flight response does in fact depend on the altitude of the helicopter.

Q 24 continued on next page...

c

$$H_0 = \hat{\beta}_2 = 0$$

$$H_a = \hat{\beta}_2 \neq 0$$

by looking at the model summary, we can see that the z-score for this test is z=-10.625 , with a corresponding p-value of less than 2×10^{-16}. We can conclude that p is less than $\alpha = 0.01$ and therefore that the flight response does in fact depend on the lateral distance of the helicopter.

d

For

$$x_1 = 6, x_2 = 3$$

where both are measured in hundreds of meters, the predicted probability from the logistic model

$$\pi^* = \beta_0 + \beta_1 x_1 + \beta_2 x_2$$

using the R commands:
: newdata = data.frame(ALTITUDE=6, LATERAL=3)
: predict(model,newdata,type="response")
is 0.946. From this we can conclude that there is a 94.6% chance that the geese have a high flight response when the Altitude is held at a fixed 600m, and the Lateral distance is held at a fixed 300m based on the data reported.

Q 26

Groundwater contamination in wells. Many New Hampshire counties mandate the use of reformulated gasoline, leading to an increase in groundwater contamination. Refer to the Environmental Science and Technology (January 2005) study of the factors related to methyl tert-butyl ether (MTBE) contamination in public and private New Hampshire wells, Exercise 6.11 (p. 343). Data were collected for a sample of 223 wells and are saved in the MTBE file. Recall that the list of potential predictors of MTBE level include well class (public or private), aquifer (bedrock or unconsolidated), pH level (standard units), well depth (meters), amount of dissolved oxygen (mil ligrams per liter), distance from well to nearest fuel source (meters), and percentage of adjacent land allocated to industry. For this exercise, consider the dependent variable y = 1 if a detectible level of MTBE is found, 0 if the level of MTBE found is below limit. Using the independent variables indentified in Exercise 6.11, fit a logistic regres sion model for the probability of a detectible level of MTBE. Interpret the results of the logistic regression. Do you recommend using the model? Explain.

a

The model in question is defined as follows:

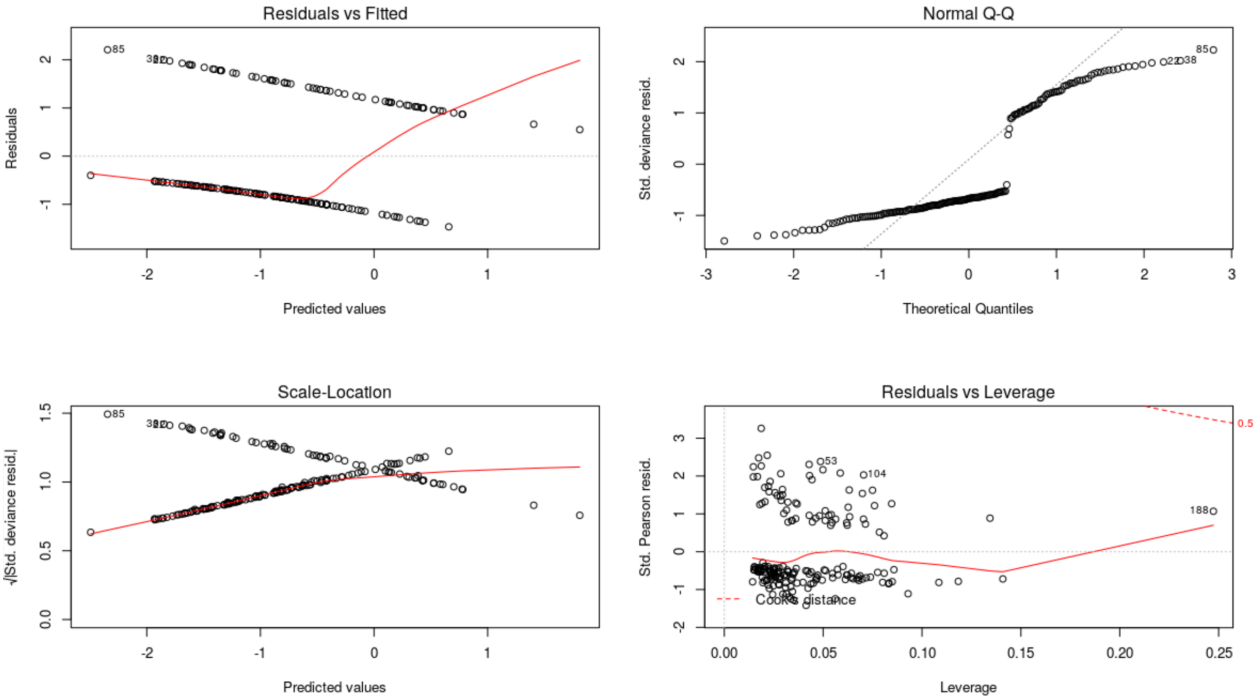
$$\pi^* = \beta_0 + \beta_1 x_1 + \beta_2 x_2 + \beta_3 x_3 + \beta_4 x_4 + \beta_5 x_5 + \beta_6 x_6 + \beta_7 x_7$$

With

$$x_1 \begin{cases} 0 : \text{Below limit} \\ 1 : \text{Detectable} \end{cases}$$

Q 26 [a] continued on next page...

```
        Call:
glm(formula = detet ~ wellclass + aquifer + ph + welldepth +
    do2 + d2f + pind, family = binomial(link = "logit"))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.4664  -0.8663  -0.6549   1.0395   2.2075

Coefficients:
                   Estimate Std. Error z value Pr(>z)
(Intercept)       1.1713423  1.7544128   0.668   0.5044
wellclassPublic   0.8066518  0.3807905   2.118   0.0341 *
aquiferUnconsoli -0.2693525  0.6760558  -0.398   0.6903
ph               -0.4098715  0.2328449  -1.760   0.0784 .
welldepth         0.0084778  0.0034056   2.489   0.0128 *
do2               0.0062126  0.0742396   0.084   0.9333
d2f              -0.0001209  0.0001642  -0.736   0.4617
pind              0.0234640  0.0317899   0.738   0.4605
---
Signif. codes:  0    ***    0.001    **    0.01    *    0.05    .    0.1         1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 242.21  on 190  degrees of freedom
Residual deviance: 221.13  on 183  degrees of freedom
  (32 observations deleted due to missingness)
AIC: 237.13

Number of Fisher Scoring iterations: 4
```



Looking at the graphs for this model from the R output, we can see that the residuals for both groups are distributed normally, with none of them breaching 3 standards deviations, or the Cooks' distance. So the weight of any single point does not overly pull the model in any direction too far.
Continuing the analysis, we test the model for overall model adequacy.

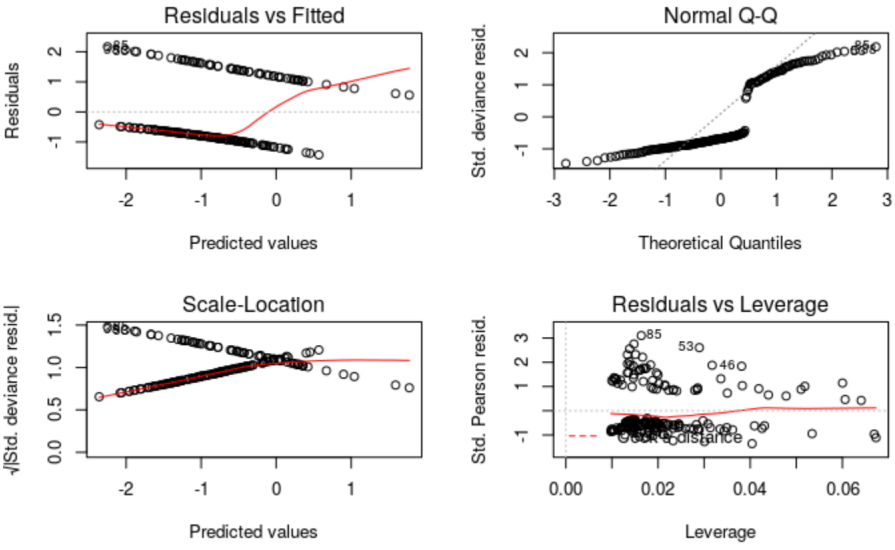
$$H_0 = \hat{\beta}_1 = \hat{\beta}_2 = \ldots = \hat{\beta}_7 = 0$$
$$H_a = \text{at least one } \beta_i (i = 0, 7) \neq 0$$

To do this we use the same method as in Question 24. From our χ^2 testing, we get a p-value = 0.00365, which: p-value is less than $\alpha = 0.5$, so we can reject the null hypothesis and state that the model is statistically useful for predicting the probability of detection in MTBE levels.
After globally validating the model, we can then move to see if each of the $\hat{\beta}$'s are statistically significant.
For i = 1 to 7 :

$$H_0 : \hat{\beta}_i = 0$$
$$H_a : \hat{\beta}_i \neq 0$$

We can refer to the R printout to see the p-values associated with the z-scores for the individual betas. After doing a backwards variable selection based on the p-values, we arrive at a model predicting the Detection status of MTBE with the WellClass, pH level, and WellDepth. The p-values for the respective predictors are 0.00575, 0.05476, 0.00187. The pH level was the only one to come close to the default $\alpha = 0.05$. Testing overall model adequacy again, we use the χ^2 testing method form Q 24, we arrive at a p-value of 0.000258. Comparing this to the previous model's p-value of 0.00365. Following the manual variable selection, we have improved the p-value by a factor of 10.

```
        Call:
glm(formula = detet ~ wellclass + ph + welldepth, family = binomial(link = "logit"))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.4267  -0.8812  -0.6763   1.0872   2.1673

Coefficients:
                 Estimate Std. Error z value Pr(>z)
(Intercept)      0.834621   1.535032   0.544  0.58664
wellclassPublic  0.958560   0.347112   2.762  0.00575 **
ph              -0.410929   0.213941  -1.921  0.05476 .
welldepth        0.009641   0.003100   3.111  0.00187 **
---
Signif. codes:
0    ***    0.001    **    0.01    *    0.05    .    0.1         1

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 242.21  on 190  degrees of freedom
Residual deviance: 223.09  on 187  degrees of freedom
  (32 observations deleted due to missingness)
AIC: 231.09

Number of Fisher Scoring iterations: 4
```



Looking at the residuals quickly, after model reformation, we see that nothing has changed in respect to the status of normality or leverage.

Depending on the tolerance of the client requesting the model, I would also reccommend removing the pH term as the p-value was just above 0.05. Depending on the desire of the company, I would reccommend either this model, or this model with the pH value removed.

Interpretation of Betas

The betas in their current states represent changes in the log-odds π^*. In order to obtain the percentage

change in odds for each unit increase, we must first transform it back to find the percentage change in odds. We calculate for all i's (1,3)

$$e^{\hat{\beta}_i} - 1$$

Interpretation of $\hat{\beta}_1$: this corresponds to the the well being public or not. If the well is public, the well has a 160% higher odds of having detectable levels of MTBE while keeping any other x's held fixed.
Interpretation of $\hat{\beta}_2$: this corresponds to the the wells' pH value. For every one unit increase in the pH value of the well, the odds of the well having detectable levels of MTBE decreases by 33.7% while keeping any other x's held fixed.
Interpretation of $\hat{\beta}_3$: this corresponds to the the wells' depth. For every one unit increase in the depth of the well in meters, the odds that the well will have detectable levels of MTBE will decrease by 0.959% while keeping any other x's held fixed.
The model proposed estimates the probability of the level of the MTBE level being low or high. With 0 being "Below Limit", and 1 being "Detect". The higher the result of the model, the more likely that the well has a detectable level of MTBE.

Q 28

A new dental bonding agent. When bonding teeth, orthodontists must maintain a dry field. A new bonding adhesive (called Smartbond) has been developed to eliminate the necessity of a dry field. However, there is concern that the new bonding adhesive may not stick to the tooth as well as the current standard, a composite adhesive (Trends in Biomaterials and Artificial Organs, Jan- uary 2003). Tests were conducted on a sample of 10 extracted teeth bonded with the new adhesive and a sample of 10 extracted teeth bonded with the composite adhesive. The Adhesive Remnant Index (ARI), which measures the residual adhe- sive of a bonded tooth on a scale of 1 to 5, was determined for each of the 20 bonded teeth after 1 hour of drying. (Note: An ARI score of 1 implies all adhesive remains on the tooth, while a score of 5 means none of the adhesive remains on the tooth.) The data are listed in the accompanying table. Fit a logistic regression model for the prob- ability of the new (Smartbond) adhesive based on the ARI value of the bonded tooth. Interpret the results.

a

Fitting the model,

$$\pi^* = \beta_0 + \beta_1 x_1$$

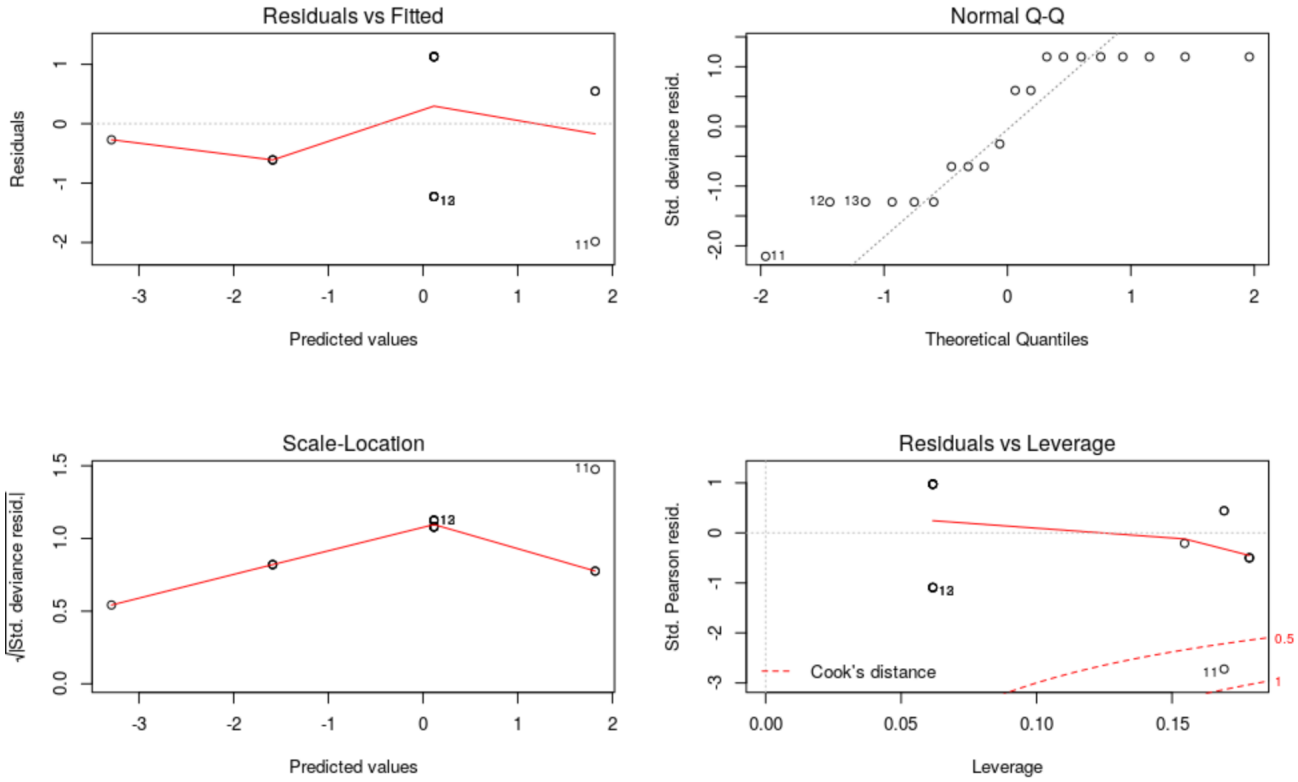
$$x_1 \begin{cases} 0 : \text{Composite} \\ 1 : \text{SmartBond} \end{cases}$$

Fitting model to the data gives us the following R output table:

```
Call:
glm(formula = BONDING$ADHESIVE ~ BONDING$ARISCORE, family = binomial(link = "logit"))

Deviance Residuals:
    Min       1Q   Median       3Q      Max
-1.9842  -1.2267   0.1391   1.1289   1.1289

Coefficients:
                 Estimate Std. Error z value Pr(>z)
```

```
(Intercept)          3.521      2.179    1.616    0.106
BONDING$ARISCORE    -1.703      1.044   -1.631    0.103

(Dispersion parameter for binomial family taken to be 1)

    Null deviance: 27.726  on 19  degrees of freedom
Residual deviance: 23.447  on 18  degrees of freedom
AIC: 27.447

Number of Fisher Scoring iterations: 4
```

Taking a look at the model residuals:



There appears to be one point (11) which is close to breaching the 3 Std Deviation threshold for leverage, but other than that, nothing seems to be extraneous considering the small sample size.
Moving to global model validation, we look at the p-score from the χ^2 testing.

$$H_0 = \beta_1 = 0$$

$$H_a = \beta_1 \neq 0$$

The result is a p-value = 0.0386. Comparing this with a default $\alpha = 0.05$ leads to the conclusion that p , α, therefore we can reject the null hypothesis and state that the model is statistically significant for predicting the category of adhesive used based on the ARISCORE value.

$$H_0 = \hat{\beta}_1 = 0$$

$$H_a = \hat{\beta}_1 \neq 0$$

Moving to individual beta analysis, we can refer to the R printout above to determine whether or not the term itself is statistically significant. The z-score found is -1.63, with an accompanying p-value of 0.103. Therefore we can reject the null hypothesis and state that the individual beta of the ARISCORE is statistically significant in predicting the probability of the type of adhesive used on a tooth.
Armed with this information, we can recommend the given model in predicting the probability of the type of adhesive used on a tooth, with 1 being representative of SmartBond, and zero being representative of a Compound adhesive.
The betas in their current states represent changes in the log-odds π^*. In order to obtain the percentage change in odds for each unit increase, we must first transform it back to find the percentage change in odds. We calculate this for $\hat{\beta}_1$:

$$e^{-1.703} - 1 = -0.818$$

To interpret $\hat{\beta}_1$, it represents the -81.8% change in odds for each unit change in ARISCORE while keeping any other variables held fixed.