The Vector Product

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What is a Vector Product?

Definition: The cross product of two vectors $\mathbf{A}$ and $\mathbf{B}$ is given by:

$$\mathbf{A} \times \mathbf{B} = |\mathbf{A}||\mathbf{B}| \sin\theta \, \mathbf{n}$$

where θ is the angle between $\mathbf{A}$ and $\mathbf{B}$, and $\mathbf{n}$ is a unit vector perpendicular to both $\mathbf{A}$ and $\mathbf{B}$, following the right-hand rule.

Properties of the Vector Product

Properties:

1. $\mathbf{A} \times \mathbf{B}$ is orthogonal to both $\mathbf{A}$ and $\mathbf{B}$.
2. The magnitude of $\mathbf{A} \times \mathbf{B}$ is given by:

$$|\mathbf{A} \times \mathbf{B}| = |\mathbf{A}||\mathbf{B}| \sin\theta$$

3. The vectors $\mathbf{A}, \mathbf{B}$, and $\mathbf{A} \times \mathbf{B}$ form a right-handed system.
4. If $\mathbf{A}$ or $\mathbf{B}$ is the zero vector, then $\mathbf{A} \times \mathbf{B} = \mathbf{0}$.
5. If $\mathbf{A}$ and $\mathbf{B}$ are parallel (i.e., $\theta = 0°$ or $180°$), then $\mathbf{A} \times \mathbf{B} = \mathbf{0}$.
6. The cross product is anti-commutative:

$$\mathbf{A} \times \mathbf{B} = -(\mathbf{B} \times \mathbf{A})$$

The Determinant

Determinant Formula:

The cross product can be computed using the determinant:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix} = (A_2 B_3 - A_3 B_2)\mathbf{i} - (A_1 B_3 - A_3 B_1)\mathbf{j} + (A_1 B_2 - A_2 B_1)\mathbf{k}$$

where $\mathbf{i}, \mathbf{j}, \mathbf{k}$ are unit vectors along the x, y, and z-axes, respectively.

Vector Product in a Orthonormal Basis

Theorem: For three-dimensional vectors **A** and **B**, and for a positively oriented and orthonormal basis $\{\mathbf{i}, \mathbf{j}, \mathbf{k}\}$, the vector product is given by:

$$\mathbf{A} \times \mathbf{B} = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ A_1 & A_2 & A_3 \\ B_1 & B_2 & B_3 \end{vmatrix}$$

Given that:

$$\mathbf{A} = A_1\mathbf{i} + A_2\mathbf{j} + A_3\mathbf{k}, \quad \mathbf{B} = B_1\mathbf{i} + B_2\mathbf{j} + B_3\mathbf{k}$$

the vector product can be computed using the properties from Theorem 4.1:

$$\mathbf{A} \times \mathbf{B} = (A_2B_3 - A_3B_2)\mathbf{i} - (A_1B_3 - A_3B_1)\mathbf{j} + (A_1B_2 - A_2B_1)\mathbf{k}$$

Let's do some practice, once more, again!

https://images.pexels.com/photos/796603/pexels-photo-796603.jpeg?cs=srgb&dl=hand-desk-notebook-796603.jpg&fm=jpg

Questions?

Next Week -

The Gaussian Elimination

Reinforcement Learning

`Goal: Use normalization to have an object move consistently on screen along diagonals`

`Goal: Use OpenMP to parallelize a Dot Product`