Vectors

John Hohman & Micah Hanevich

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What is a Vector?

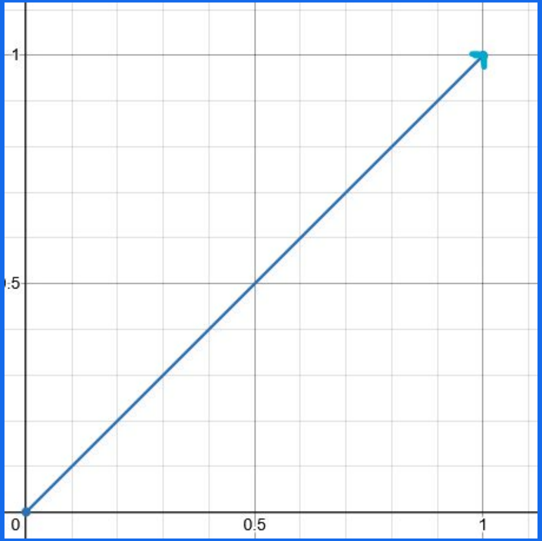
A vector is completely defined by its

1. Direction, and
2. Length

By extension, then, a Vector is effectively "momentum".

Example 1.1 shows a simple vector from (0,0) to (1,1).

Example 1.1



How to refer to Vectors

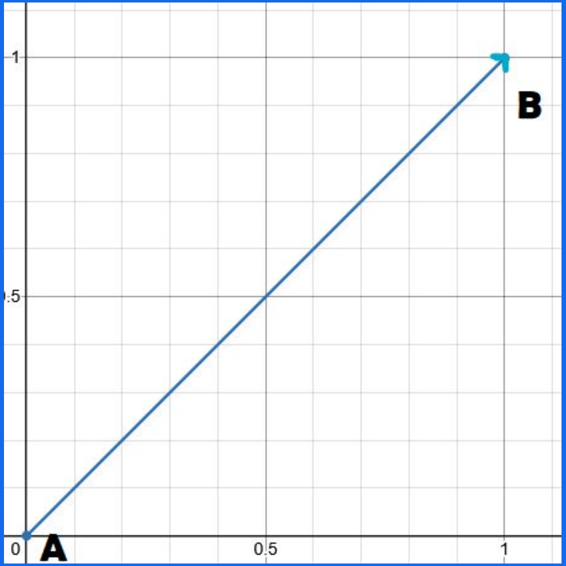
The common form for referring to a Vector mathematically is as follows:

Let A and B be two points. A directed line segment from A to B is denoted by:

$$\overrightarrow{AB}$$

This directed line segment **constitutes a Vector**.

Example 1.2



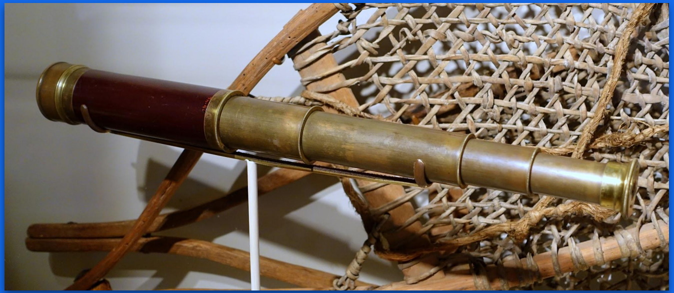
Did you notice?

For a moment, let's return back to the "What is a Vector" slide. Specifically, the section stating how a Vector is "completely defined".

Notice that something is missing from the definition of a Vector: (x,y) **points**; references to locations on a Cartesian Coordinate System.

What are some potential uses you can imagine using the lack of points in a Vector's definition?

Note: Mathematically referring to a Vector asks us to use the "starting point" and "ending point" to refer to them. These are for simplicity, and are not necessary to define a Vector.



Using the Vector's Definition

Since a Vector is only defined by

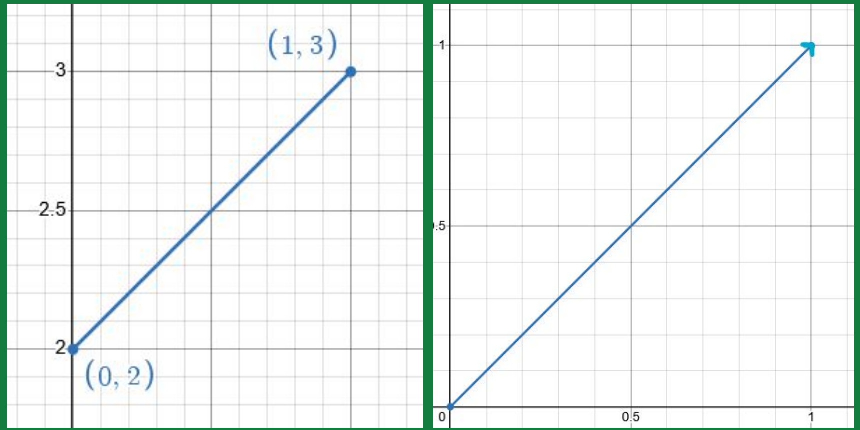
1. Direction, and
2. Length,

Vectors can be applied to **any point**, function, array, etc. and used to represent movement.

Additionally, using this definition of vectors, the same vector can be used in multiple places.

Using the information above, Examples 1.3a and 1.3b are showing the ***same vector****.*

Examples 1.3a (left) & 1.3b (right)



Important Things

- When referring to a Vector mathematically, order of points is paramount.

$$\overrightarrow{AB}! = \overrightarrow{BA}$$

- The Zero Vector (a vector with no direction or length) is referred to using the notation to the right.

$$\vec{0} \; or \; \mathbf{0}$$

- Vectors are not limited to two dimensions.
- The notation to the right is used to refer to the **length** of the Vector.

$$||\overrightarrow{AB}||$$

Take it away, John!

Vector Operations

Operation	Syntax	Example
Addition	$\vec{u}+\vec{v}=\vec{w}$	$(1,1)+(2,3)=(3,4)$
Subtraction	$\vec{u}-\vec{v}=\vec{w}$	$(1,1)-(2,3)=(-1,-2)$
Scalar Vector Multiplication	$c\vec{v}=\vec{w}$	$5\cdot(1,1)=(5,5)$

Vector Properties

Property	Form
Vector Commutativity	$\vec{u}+\vec{v}=\vec{v}+\vec{u}$
Vector Addition Associativity	$(\vec{u}+\vec{v})+\vec{w}=\vec{v}+(\vec{u}+\vec{w})$
Zero Existence	$\vec{v}+\vec{0}=\vec{v}$
Negative Vector Existence	$\vec{v}+(-\vec{v})=\vec{0}$
Scalar Multiplication Associativity	$k(l\vec{v})=(kl)\vec{v}$
Multiplication by One	$1\vec{v}=\vec{v}$
Multiplication by Zero	$0\vec{v}=\vec{0}$
Multiplication by Zero Vector	$k\vec{0}=\vec{0}$
Distributive Property 1	$k(\vec{u}+\vec{v})=k\vec{u}+k\vec{v}$
Distributive Property 2	$(k+l)\vec{v}=k\vec{v}+l\vec{v}$

Basis Vectors

Dimension	Canonical Basis Vectors
1D	$e_1 = (1)$
2D	$e_1 = (1, 0)$, $e_2 = (0, 1)$
3D	$e_1 = (1, 0, 0)$, $e_2 = (0, 1, 0)$, $e_3 = (0, 0, 1)$

Canonical Basis in $\mathbb{R}^n$	
The set of all basis vectors in $\mathbb{R}^n$ is given by:	
e_1	$(1, 0, \ldots, 0)$
e_2	$(0, 1, \ldots, 0)$
$\vdots$	$\vdots$
e_n	$(0, 0, \ldots, 1)$

Let's do some practice!

https://images.pexels.com/photos/796603/pexels-photo-796603.jpeg?cs=srgb&dl=hand-desk-notebook-796603.jpg&fm=jpg

Questions?

Next Week -

The Vector Dot Product

Reinforcement Learning

`Goal: Use Vectors to have an object bounce around the screen (Ex. VHS Logo)`