Gaussian Elimination

According to our textbook,
"Gaussian Elimination is a method for solving *systems of linear equations*. Such systems are often encountered when dealing with real problems ..."

System of Linear Equations

A system of linear equations is a number of linear equations that you want to solve together so that the solution holds for each and every equation.

Let's use a simple formula everyone knows as a basis:

$$y=mx+b_1$$

where m is the slope, and b is the intercept.

Using this equation as a basis, we run into an issue.
No matter the slope, **we cannot represent a vertical line**.

Therefore, let's convert to instead using a modified form of the equation:

$$ax + b_2 y + c = 0$$

where $a = -m, b_2 = 1$ and $c = b_1$

Why go through all this trouble?

The end result, now, is we can represent **any** linear equation with this formula, and it holds true for **all points on the line**.

And thus, finally, how to solve this:
Gaussian Elimination.

The solution to a system of equations does not change
when we perform the following operations:

- Swap the order of two rows
- multiply a row with a constant $\neq 0$, or
- add a multiple of another row to a row

The proof for these properties is provided in the textbook.

Let's assume we have the following system of equations: $\begin{cases} -x + y = 8, \\ 2x + 4y = -10 \end{cases}$
To solve this, we can use the second and third rules of Gaussian Elimination to add two times the first row to the last. $\begin{cases} -x + y = 8, \\ 6y = 6 \end{cases}$
The last row can be simplified down to $y = 1$, which can then be substituted back into the first equation, giving us: $-x + 1 = 8$, simplified down to $x = -7$
Therefore, the solution to this system of equations is The coordinate pair $(-7, 1)$.

Example Problems

1.

$$\begin{cases} 2x + 3y = 5, \\ 6x + y = 7 \end{cases}$$

SOLUTION: (1,1)

2.

$$\begin{cases} 2x + 4y = 4, \\ 3x + 5y = 5 \end{cases}$$

SOLUTION: (0,1)

3.

$$\begin{cases} 135x - 256y = e, \\ 135x - 256y = e + 4 \end{cases}$$

SOLUTION: N/A
(There is no point where these parallel lines meet.)

4.

$$\begin{cases} x + 3y = 1, \\ 3x + 9y = 3 \end{cases}$$

SOLUTION: ∞
(The solution is the line itself.)

5.

$$\begin{cases} 2x + 3y + 0.5z = 2, \\ 4x + 6y + z = 4, \\ x + 1.5y + 0.25z = 1 \end{cases}$$

SOLUTION: ∞
(The solution is the **plane** itself.)