Gaussian Elimination

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What is Gaussian Elimination?

Gaussian Elimination
According to our textbook, "Gaussian Elimination is a method for solving *systems of linear equations.* Such systems are often encountered when dealing with real problems ..."

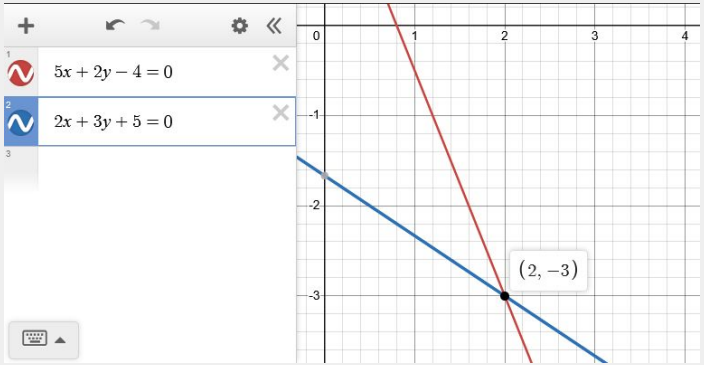
...but what does this mean?

Firstly, a system of linear equations is defined as such:

System of Linear Equations

A system of linear equations is a number of linear equations that you want to solve together so that the solution holds for each and every equation.

A good example of this is a point where two linear equations (lines) meet.



Important Note!

“Linear Equation” implies the equation is only to the 1st power.

Solving Systems of Linear Equations

Let's use a simple formula everyone knows as a basis:

$$y=mx+b_1$$

where m is the slope, and b is the intercept.

Using this equation as a basis, we run into an issue.
No matter the slope, **we cannot represent a vertical line.**

Therefore, let's convert to instead using a modified form of the equation:

$$ax + b_2y + c = 0$$

where $a = -m, b_2 = 1$ and $c = b_1$

Why go through all this trouble?

The end result, now, is we can represent
any linear equation with this formula, and
it holds true for **all points on the line.**

And thus, finally, how to solve this:
Gaussian Elimination.

Gaussian Elimination Rules

The solution to a system of equations does not change when we perform the following operations:

- Swap the order of two rows
- multiply a row with a constant $\neq 0$, or
- add a multiple of another row to a row

The proof for these properties is provided in the textbook.

Now, times for some examples!

Important Note!

The solution to a system of equations will not always be one coordinate pair*.

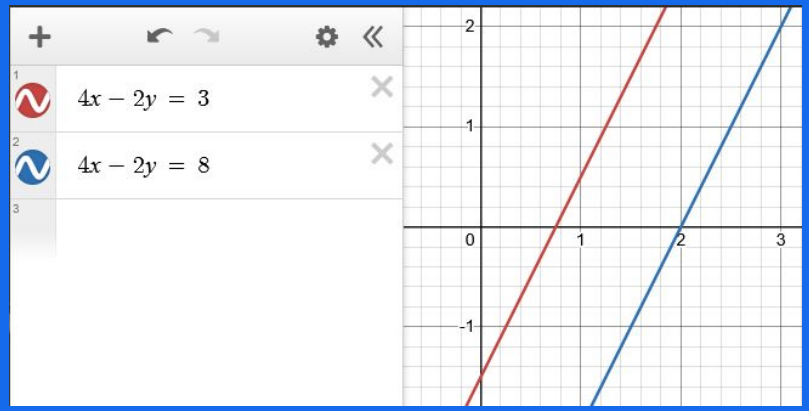
If the equations within the system are parallel, there is no solution.

Similarly, if they are the same line, there are infinite solutions.

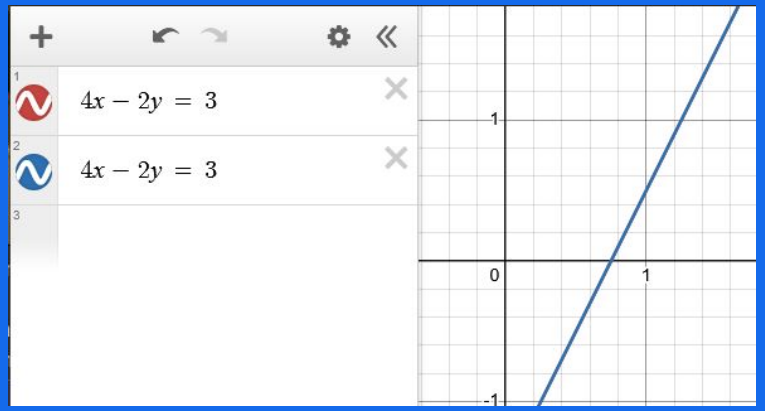
** The word pair is used here for convenience. Systems of equations can have an infinite number of dimensions.*

Examples of Other Cases

Parallel Lines (no solution)



The same line (infinite solutions)



There are more cases, but we will cover those later.

Questions?

Time for Practice!

Next Week -

Gaussian Elimination Part 2 - Electric Gaussian Boogaloo

(aka Applications of Gaussian Elimination)

Reinforcement Learning

No Project Assignment this week! Instead,

Review example problems and ensure your own understanding. Next week will deal with applications of this technique, as well as diving into some more complicated scenarios. Be prepared!