The Determinant

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Introduction

The determinant det is function that outputs a scalar from an input n x n matrix A with following properties.

1. det(I) = 1
2. det(A) = 0 if 2 columns are equal
3. $|\ldots \lambda\_1\mathbf{a\_1} + \lambda\_2\mathbf{a\_2} \ldots| = \lambda\_1|\ldots \mathbf{a\_1} \ldots| + \lambda\_2|\ldots \mathbf{a\_2} \ldots|$

It has been proven only one function has these properties

Properties of the Determinant

1. $|\dots \mathbf{0} \dots| = 0$
2. $|\dots \mathbf{a\_i} \dots \mathbf{a\_j} \dots| = -|\dots \mathbf{a\_j} \dots \mathbf{a\_i} \dots|$
3. $|\dots \mathbf{a\_i} \dots \mathbf{a\_j} \dots| = -|\dots \mathbf{a\_i{+}\lambda a\_j} \dots \mathbf{a\_j} \dots|$
4. det(A)=det(A^T)
5. det(AB) = det(A)det(B)
6. det(A^-1)=(det(A))^-1

Permutations

1. A perm of elements is bijection from the set{1,2,...,n} to itself

4. N choose k ways to choose an inversion of some size

2. A transposition is where only 2 elements positions are changed

5. Inversions can be odd or even based on the number of inversions

3. The permutation σ′s inverse is the inverse function σ^-1

Determinants

Determinants with a Transposition Matrix

A matrix's determinant does not change under transposition.

det(**A**) = det(**A**^{T})

Example:

$$\det(\mathrm{A}) = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix} \quad det(A) = (1 * 4) - (2 * 3)$$

$$\det(\mathrm{A}^T) = \begin{bmatrix} 1 & 3 \\ 2 & 4 \end{bmatrix} \quad det(A^T) = (1 * 4) - (3 * 2)$$

Determinants of a Product of Matrices

A matrix product's determinant is the product of corresponding determinants.

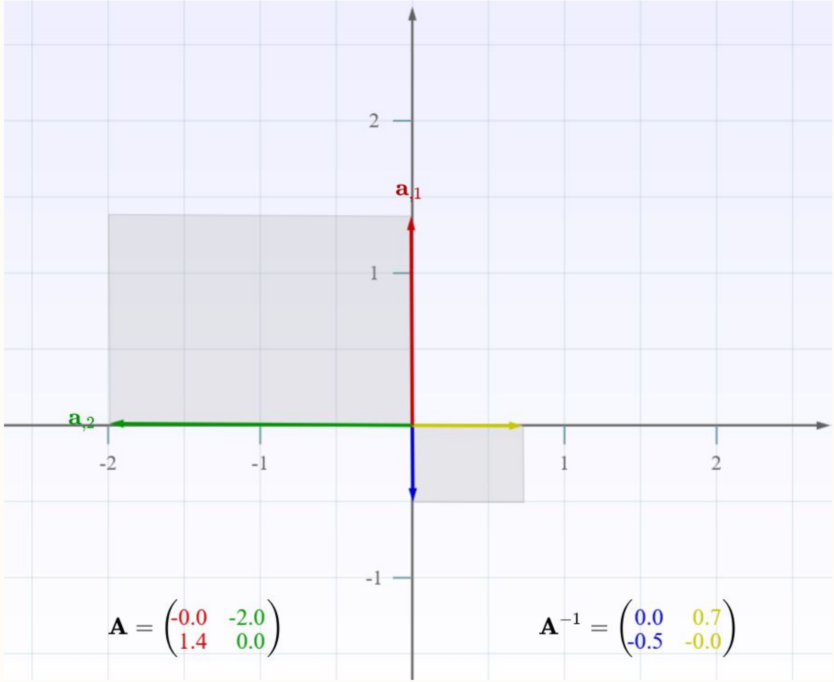
$$\det(\mathbf{AB}) = \det(\mathbf{A})\,\det(\mathbf{B})$$

Determinants of an Inverse Matrix

The determinant of the *inverse of a matrix* is the same as the inverse of the determinant

$$\det(A^{-1}) = \frac{1}{det(A)}$$

Below, the area of the parallelogram is the determinant of the matrix. The bigger det(**A**) gets, the smaller the inverse gets (and vice versa).



Matrix Inverse

Requirements for a Matrix to have an inverse

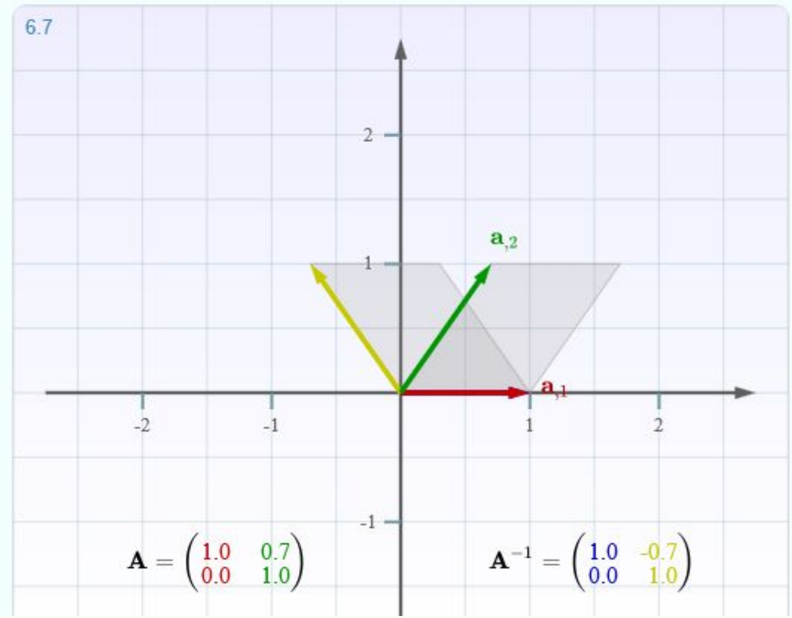
1. Is the Matrix a Square?
   a. If so, continue.

2. Is the determinant of the matrix, det(a)=0
   a. If det(a) = 0, there is no inverse
   b. If det(a) ≠ 0, then there is an inverse

$$\mathbf{Ax} = \mathbf{y} \iff \mathbf{Ax} = \mathbf{Iy}$$

$$\mathbf{A}^{-1}\mathbf{Ax} = \mathbf{A}^{-1}\mathbf{Iy} \iff \mathbf{Ix} = \mathbf{A}^{-1}\mathbf{y}$$

https://www.geeksforgeeks.org/check-if-a-matrix-is-invertible/



https://immersivemath.com/ila/ch06_matrices/ch06.html

Let solve a few!

Image by juicy_fish on Freepik

Questions?

Onward to practicality

https://github.com/fastai/numerical-linear-algebra/blob/master/nbs/2.%20Topic%20Modeling%20with%20NMF%20and%20SVD.ipynb

Next Week - Determinants pt. 2

Reinforcement Learning

Goal: Continue Notebook 3 in https://github.com/fastai/numerical-linear-algebra/blob/master/README.md