The Matrix Pt. 2

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Matrix Transpose

Original Matrix	Transpose Operation	Transposed Matrix
$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$	A^T	$\begin{bmatrix} a & c \\ b & d \end{bmatrix}$
$v = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$	v^T	$\begin{bmatrix} x & y & z \end{bmatrix}$
$B = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix}$	B^T	$\begin{bmatrix} 1 & 4 \\ 2 & 5 \\ 3 & 6 \end{bmatrix}$

Useful Matrices

Rotation Matrix

$$\begin{bmatrix} r\cos(\phi) & -r\sin(\phi) \\ -r\sin(\phi) & \cos(\phi) \end{bmatrix}$$

where r is the radius,
and ϕ is radians rotated by (counter-clockwise)

Scaling Matrix

$$S(f_x, f_y) = \begin{bmatrix} f_x & 0 \\ 0 & f_y \end{bmatrix}$$

where f_x is the factor applied to the x dimension, and f_y is the factor applied to the y dimension.

Shearing Matrix

$$H_{xy}(\mathrm{s}) = \begin{pmatrix} 1 & s \\ 0 & 1 \end{pmatrix} \text{ or } H_{yx}(\mathrm{s}) = \begin{pmatrix} 1 & 0 \\ s & 1 \end{pmatrix}$$

Note that the first subscript of H refers to which coordinate is changed, and the second subscript refers to the coordinate that is used to scale by s and add to that first coordinate.

Matrix Properties

Theorem 6.1: Matrix Arithmetic Properties

In the following, we assume that the sizes of the matrices are such that the operations are defined.

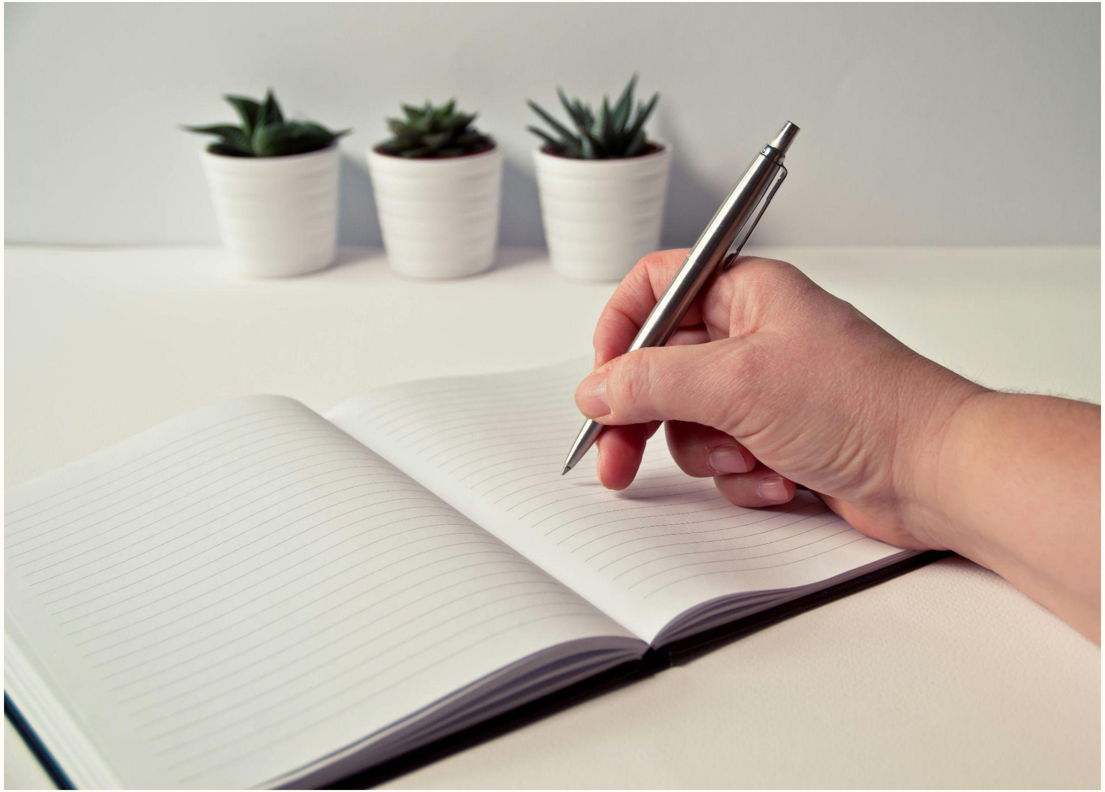
$$
\begin{array}{lll}
(i) & k(l\mathbf{A}) = (kl)\mathbf{A} & \text{(associativity)} \\
(ii) & (k+l)\mathbf{A} = k\mathbf{A} + l\mathbf{A} & \text{(distributivity)} \\
(iii) & k(\mathbf{A}+\mathbf{B}) = k\mathbf{A} + k\mathbf{B} & \text{(distributivity)} \\
(iv) & \mathbf{A}+\mathbf{B} = \mathbf{B}+\mathbf{A} & \text{(commutativity)} \\
(v) & \mathbf{A}+(\mathbf{B}+\mathbf{C}) = (\mathbf{A}+\mathbf{B})+\mathbf{C} & \text{(associativity)} \\
(vi) & \mathbf{A}+(-1)\mathbf{A} = \mathbf{O} & \text{(additive inverse)} \\
(vii) & \mathbf{A}(\mathbf{B}+\mathbf{C}) = \mathbf{A}\mathbf{B}+\mathbf{A}\mathbf{C} & \text{(distributivity)} \\
(viii) & (\mathbf{A}+\mathbf{B})\mathbf{C} = \mathbf{A}\mathbf{C}+\mathbf{B}\mathbf{C} & \text{(distributivity)} \\
(ix) & (\mathbf{A}\mathbf{B})\mathbf{C} = \mathbf{A}(\mathbf{B}\mathbf{C}) & \text{(associativity)} \\
(x) & \mathbf{I}\mathbf{A} = \mathbf{A}\mathbf{I} = \mathbf{A} & \text{(multiplicative one)} \\
(xi) & (k\mathbf{A})^{\mathrm{T}} = k\mathbf{A}^{\mathrm{T}} & \text{(transpose rule 1)} \\
(xii) & (\mathbf{A}+\mathbf{B})^{\mathrm{T}} = \mathbf{A}^{\mathrm{T}}+\mathbf{B}^{\mathrm{T}} & \text{(transpose rule 2)} \\
(xiii) & (\mathbf{A}^{\mathrm{T}})^{\mathrm{T}} = \mathbf{A} & \text{(transpose rule 3)} \\
(xiv) & (\mathbf{A}\mathbf{B})^{\mathrm{T}} = \mathbf{B}^{\mathrm{T}}\mathbf{A}^{\mathrm{T}} & \text{(transpose rule 4)}
\end{array}
\tag{6.48}
$$

In addition, we have the following trivial set of rules: $1\mathbf{A} = \mathbf{A}$, $0\mathbf{A} = \mathbf{O}$, $k\mathbf{O} = \mathbf{O}$, and $\mathbf{A} + \mathbf{O} = \mathbf{A}$.

https://immersivemath.com/ila/ch06_matrices/ch06.html

Questions?

Let’s do some practice!

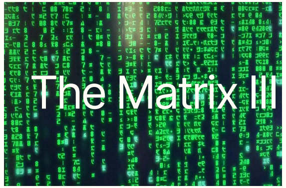


https://images.pexels.com/photos/796603/pexels-photo-796603.jpeg?cs=srgb&dl=hand-desk-notebook-796603.jpg&fm=jpg

Onward to practicality

https://github.com/fastai/numerical-linear-algebra/blob/master/nbs/2.%20Topic%20Modeling%20with%20NMF%20and%20SVD.ipynb

Next Week -



Reinforcement Learning

Goal: Write a script to perform Gaussian Elimination automatically when provided a system of linear equations and notebook exercises.