The Determinant pt. 2

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The Adjoint Matrix

$$\mathbf{A} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$$

$$\operatorname{adj}(\mathbf{A}) = \begin{bmatrix} a_{22} & -a_{12} \\ -a_{21} & a_{11} \end{bmatrix}$$

$$\mathbf{A}^{-1} = \frac{1}{\det(\mathbf{A})}\operatorname{adj}(\mathbf{A})$$

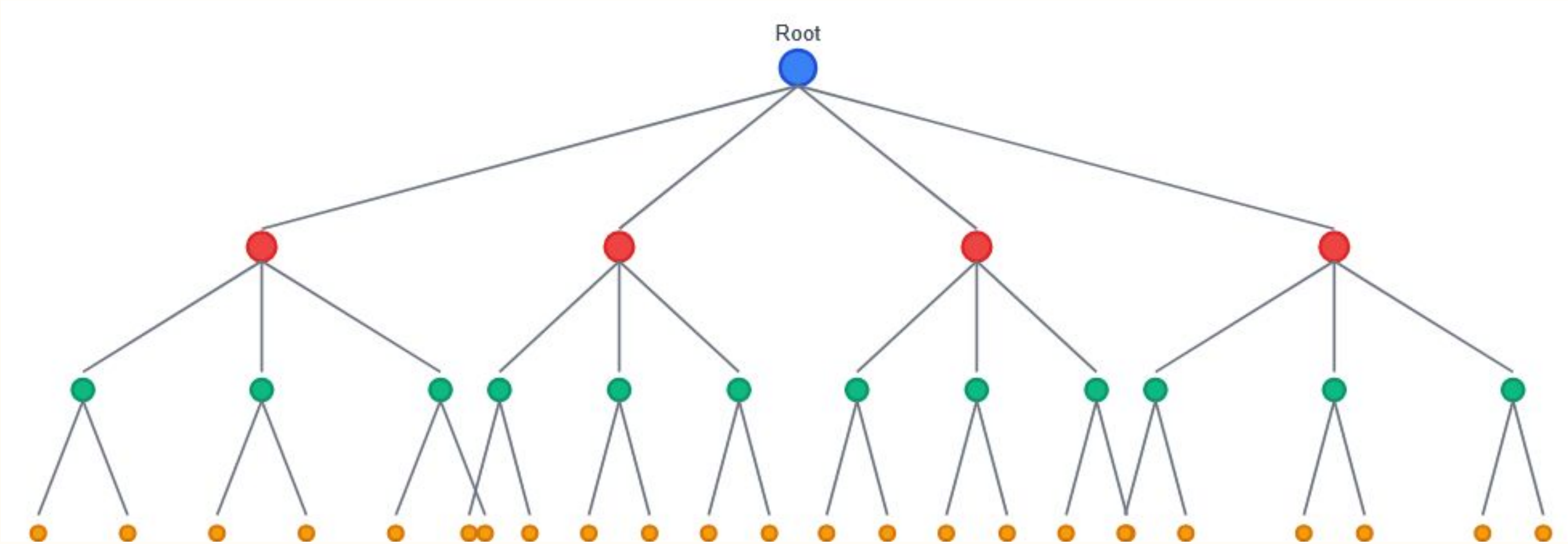
Expansion along a column

Algorithm:

```
Def det(Matrix):
    # basecase
    If Matrix.shape[0] == 2:
        Return Matrix[0][0]*Matrix[1][1] - Matrix[0][1]*Matrix[1][0]
    Multiplier = 1
    Total = 0
    For j in range(0,matrix.shape[0]):
        if(j.shape[0] % 2 == 0)
            Multiplier = -1
        minor = [row[:j] + row[j+1:] for row in Matrix[1:]]
        Total  += Multiplier * Matrix[0][j] * det(MInor)

    Return Total
```

Big O(no!)



A Better Alg

Cramer's Rule

$$\begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} y_1 \\ y_2 \end{pmatrix}$$

If det(**A**), $a_{11}a_{22} - a_{12}a_{21} \neq 0$, then the solution is *unique* and...

$$\mathrm{x}_1 = \frac{\begin{vmatrix} y_1 & a_{12} \\ y_2 & a_{22} \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{y_2 a_{11} - y_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

$$\mathrm{x}_2 = \frac{\begin{vmatrix} a_{11} & y_1 \\ a_{21} & y_2 \end{vmatrix}}{\begin{vmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{vmatrix}} = \frac{y_2 a_{11} - y_1 a_{21}}{a_{11}a_{22} - a_{12}a_{21}}$$

Square Matrices

The following equivalence holds for all square matrices <u>A</u>:

I. The column vectors of **A** is a basis
II. The row vectors of **A** is a basis
III. The matrix equation **Ax = 0** has *only one solution*, **x = 0**
IV. The matrix equation **Ax = y** has a solution for every **y**
V. The matrix **A** is invertible
VI. The determinant of **A ≠ 0**

Let solve a few!

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Questions?

Onward to practicality

https://github.com/fastai/numerical-linear-algebra/blob/master/nbs/2.%20Topic%20Modeling%20with%20NMF%20and%20SVD.ipynb

Next Week - Rank

Reinforcement Learning

Goal: Continue Notebook 3 in https://github.com/fastai/numerical-linear-algebra/blob/master/README.md