Electric Circuits 2

Lab: 02

Jaden Moore

Orange Coast College
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1 Introduction

In this lab, we attach a resistance R_L across two end terminals of a circuit. We then analyze the behavior of the various electrical quantities across the resistor as the resistance increases. We then prove Thévenin's theorem by creating Thévenin's equivalent circuit and comparing the electrical quantities with the original circuit quantities. Throughout the lab, we utilize the web based circuit builder CircuitLab to generate and analyze the circuit.

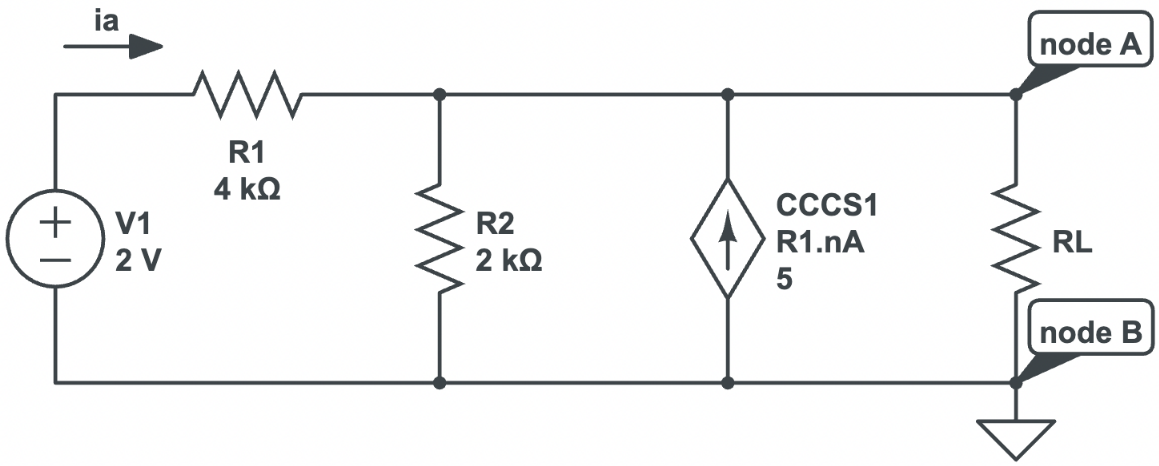

Figure 1: Electric circuit with resistor R_L across terminals A-B

Consider the circuit presented in Figure (1). We place increasing resistances across R_L and measure the current i_L, the voltage, v_L, and power p_L across the resistor and record the data in the table below.

2 Data

Table 1: Data collected from CircuitLab for different resistances R_L

R_L [Ω]	i_L [A]	v_L [V]	p_L [W]	$p_{L-theory}$ [W]
10	2.941E-03	029.41E-03	086.5E-06	086.505E-06
56	2.698E-03	151.08E-03	407.6E-06	407.588E-06
110	2.459E-03	270.49E-03	665.1E-06	665.144E-06
180	2.206E-03	397.06E-03	875.9E-06	875.865E-06
200	2.143E-03	428.57E-03	918.4E-06	918.367E-06
270	1.948E-03	525.97E-03	1.025E-03	001.025E-03
360	1.744E-03	627.91E-03	1.095E-03	001.095E-03
430	1.613E-03	693.55E-03	1.119E-03	001.119E-03
470	1.546E-03	726.80E-03	1.124E-03	001.124E-03
560	1.415E-03	792.45E-03	1.121E-03	001.121E-03
750	1.200E-03	900.00E-03	1.080E-03	001.080E-03
1000	1.000E-03	1.000	1.000E-03	001.000E-03
1800	652.2E-06	1.174	765.6E-06	765.595E-06
2700	468.8E-06	1.266	593.3E-06	593.262E-06
3600	365.9E-06	1.317	481.9E-06	481.856E-06
5600	245.9E-06	1.377	338.6E-06	338.619E-06
27000	54.55E-06	1.473	80.33E-06	080.331E-06
110000	13.57E-06	1.493	20.27E-06	020.270E-06
220000	6.803E-06	1.497	10.18E-06	010.181E-06
750000	1.999E-06	1.499	2.996E-06	002.996E-06
1100000	1.363E-06	1.499	2.044E-06	002.044E-06
2400000	624.9E-09	1.500	937.1E-09	937.109E-09
4700000	319.1E-09	1.500	478.6E-09	478.622E-09
6200000	241.9E-09	1.500	362.8E-09	362.845E-09
8200000	182.9E-09	1.500	274.4E-09	274.357E-09
10000000	150.0E-09	1.500	225.0E-09	224.978E-09

Using the table above, we plot the electrical quantities over the resistance R_L to further identify and analyze the trends.

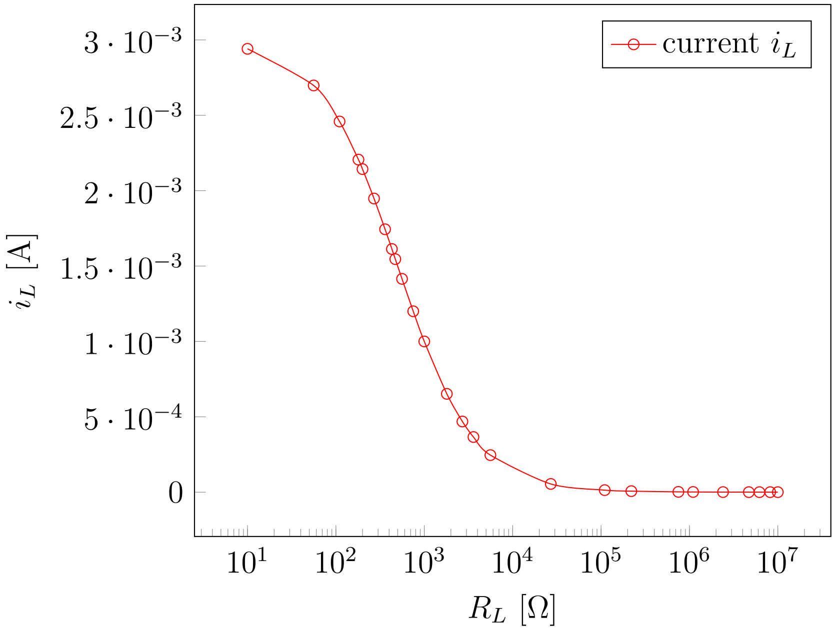

Figure 2: Current through load resistance R_L

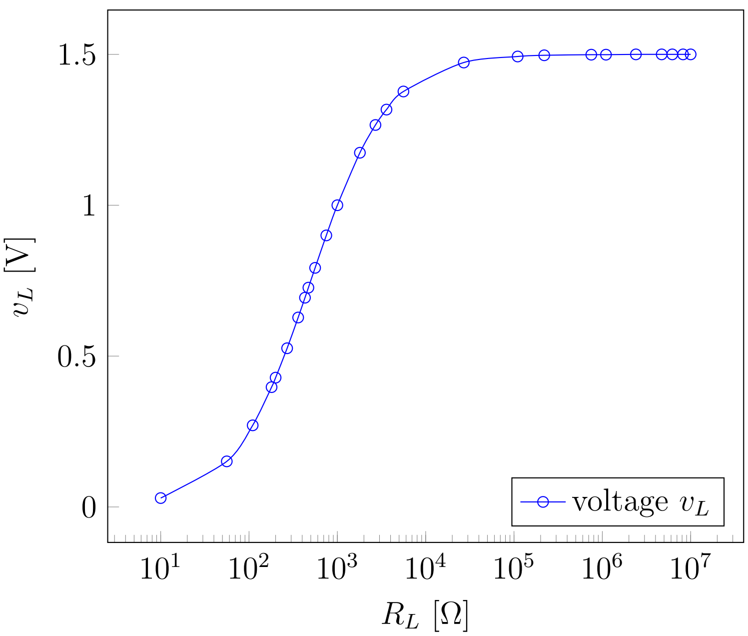

Figure 3: Voltage through load resistance R_L

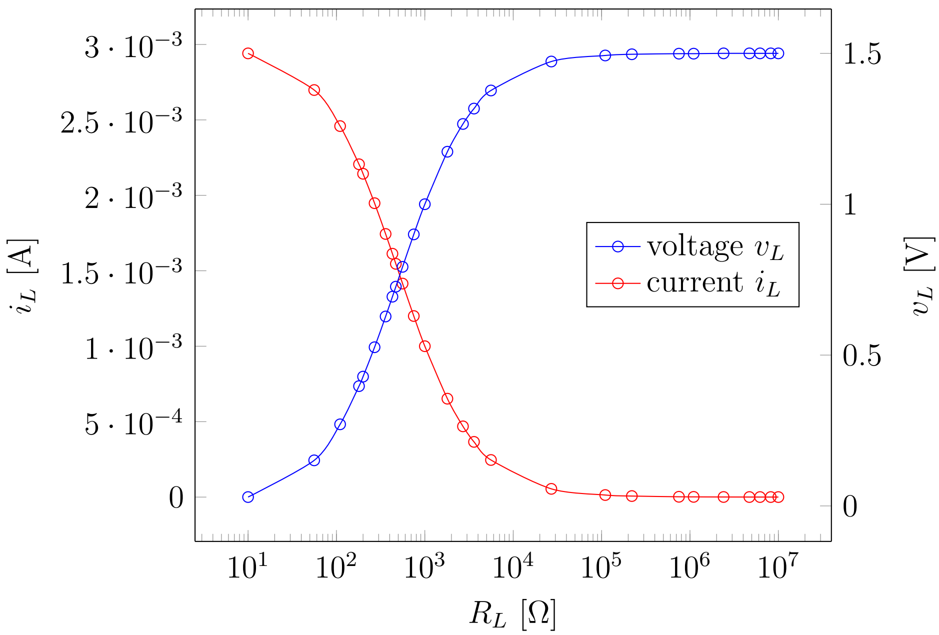

Figure 4: Current and voltage through load resistance R_L

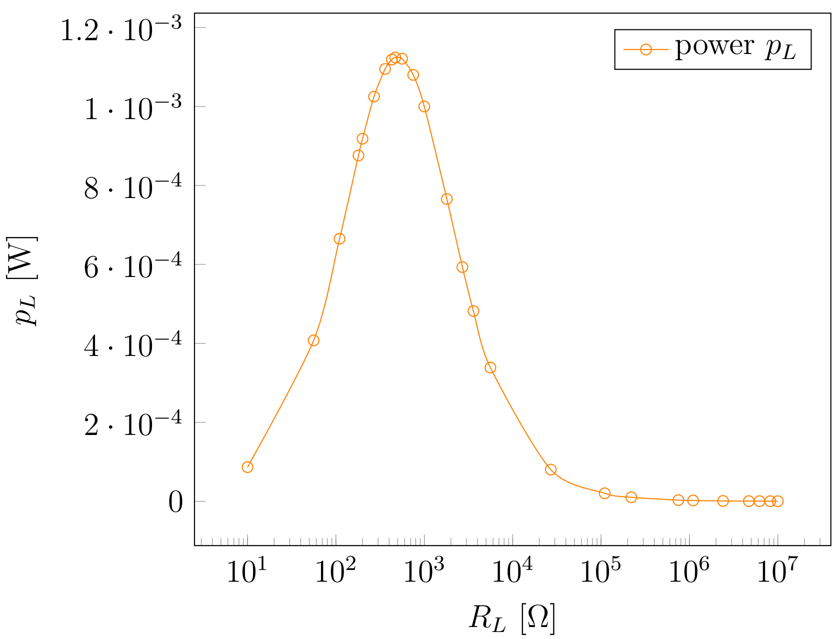

Figure 5: Power p_L through load resistance R_L

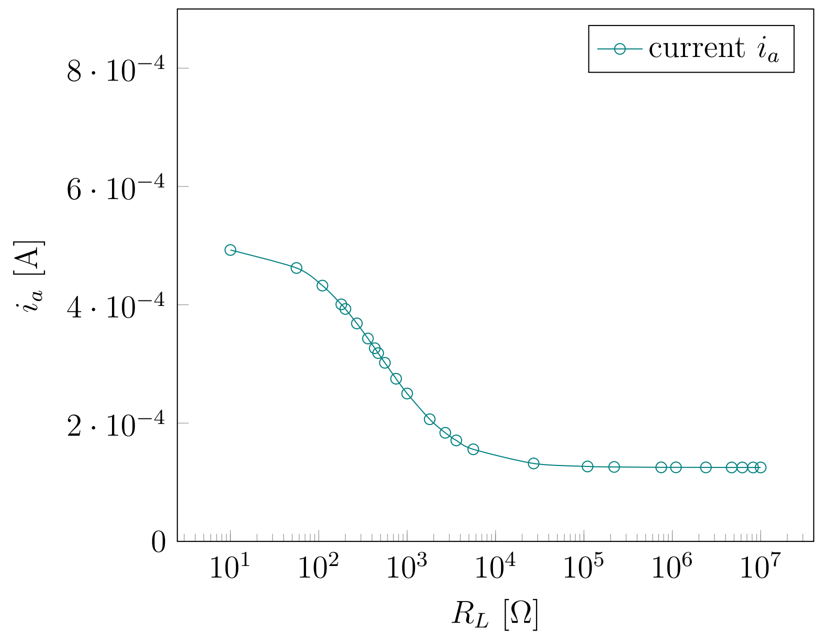

Figure 6: Current i_a through voltage source v_1 over resistance R_L

To preface the analysis, we realize that at points in the figures where there are fewer data points, the graphs appear to be more of a straight line. However, this is a limitation in the plotting software. If we introduced more data points, the graph would gain back its curvature in those regions.

3 Analysis and Discussion

From Figure (2), we see that the current through the load resistor R_L decreases as resistance increases. Inversely, from Figure (3), the voltage through R_L increases as the resistance increases. In both cases, the graphs appear to scale exponentially, and then arrive asymptotically at a specific value as the resistance grows increasingly large. From theory, we know that as the resistance of a resistor approaches infinity, the element begins to act like an open circuit, which explains why the current approaches zero as the resistance increases as seen in Figure (2). That is, no current begins to flow through a resistor as the resistance approaches very large values. Furthermore, when the resistor begins to act like an open circuit, the potential difference across its terminals are at a maximum. From this, we see that the potential difference between nodes A and B in Figure (1) is 1.5 V because the voltage approaches 1.5 V in Figure (2) as R_L increases. In this case, since node B is at ground, we get that $v_A = 1.5\,\text{V}$ when R_L approaches infinity. Additionally, we realize that when the resistance is very small, the resistor begins to act like a closed circuit and the current approaches a maximum while the voltage across approaches zero.

In the analysis of Figure (4), we see the comparison of Figure (2) and (3) in greater detail. The graphs appear to be inverses of each other which agrees with Ohm's Law. Furthermore, we realize that at the point of intersection of the voltage and current in Figure (4), the power P_L across the resistor is at a maximum as seen in Figure (5). However, after the intersection point, the power decreases exponentially and approaches zero. This makes intuitive sense because the current approaches zero as resistance grows very large and thus causes the power to approach zero.

The data tabulated in Table (1) was gathered from circuit simulation software CircuitLab, which produces a percent error when rounding the electrical quantities. We can obtain a percent error between the software and theory such that

$$\% \text{ error} = \left(\frac{|p_{exp} - p_{theory}|}{p_{theory}} \right) 100 \tag{1}$$

The column $p_{L-theory}$ in Table (1) contains the power as calculated from theory using $p = i_L v_L$. For example, using $R_L = 56\Omega$, we can obtain a percent error of the power between the software and theory using Equation (1) such that

$$\% \text{ error} = \left(\frac{|407.6 \,\text{x}10^{-6} - 407.588 \,\text{x}10^{-6}|}{407.588 \,\text{x}10^{-6}} \right) 100 = 0.00294\%$$

In this case, the software produces a very small percent error with respect to the theoretical values, which indicate that the software is accurate. Furthermore, analyzing the power gathered in Table (1), we see that for all resistances, the percent error between the software and theory is very small, thus we can conclude that the software is very accurate to at least 0.01% error in all cases.

In figure (5), we show the current through the voltage source as resistance R_L increases. From the graph, we see that as R_L increases the current i_s decreases toward 125 µA. In comparison, we see that the current i_L through the load decreases toward zero. This makes intuitive sense because as R_L gets very large, it begins to act like an open circuit, which causes the current to only flow through R_1 and R_2. Inversely, as R_L approaches zero, it begins to act like a short circuit, which would cause no current to flow through R_2 and instead only travel through R_1 which becomes the only resistance in the circuit.

4 Thévenin's Equivalent Circuit Analysis

Furthermore, we can validate Thévenin's Theorem by converting Figure (1) into Thévenin's equivalent circuit and analyzing various resistances and electrical quantities to determine whether the circuit is truly equivalent. Below, we reintroduce Figure (1) for reference.

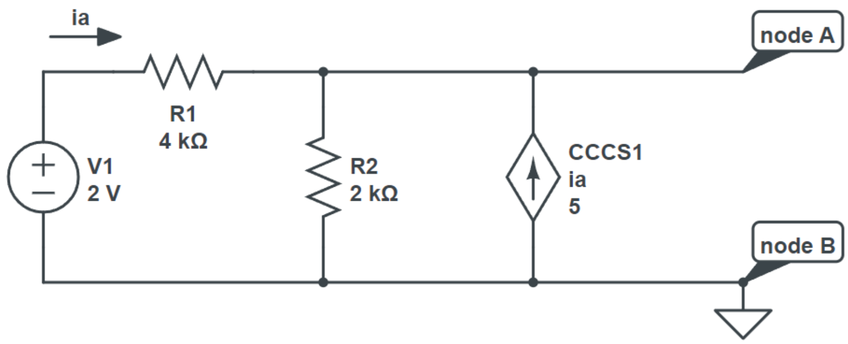

Figure 7: Electric circuit with terminals A-B

Using CircuitLab, we first measure the voltage at node A to obtain $v_{oc} = 1.5\,\text{V}$. We then attach a wire across terminals A-B to obtain $i_{sc} = 3\,\text{mA}$. We can then calculate R_{th} such that

$$R_{th} = \frac{V_{oc}}{i_{sc}} = \frac{1.5\,\text{V}}{3\,\text{mA}} = 500\,\Omega \tag{2}$$

From this, we build Thévenin's Circuit such that

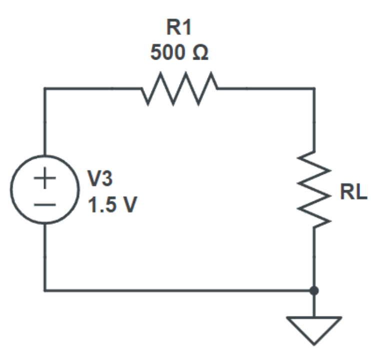

Figure 8: Thévenin's Circuit with load resistor R_L

To validate Thévenin's Theorem, we test various resistances across R_L and compare them to the values obtained in Table (1) using the original circuit shown in Figure (1).

For example, testing $R_L = 56\Omega$ for Thévenin's circuit, we can use Ohm's Law to get that

$$\begin{aligned} i_s &= \frac{v_{oc}}{R_{eq}} = \frac{1.5}{500+56} = 2.6978\,\mathrm{mA} \\ v_L &= \left(\frac{R_L}{R_L+500}\right) v_{oc} = \left(\frac{56}{56+500}\right)(1.5) = 151.079\,\mathrm{mV} \end{aligned} \tag{3}$$

From Table (1) for $R_L = 56\Omega$, we see that Thévenin's equivalent circuit produces the same electrical quantities as the original circuit which proves Thévenin's Theorem experimentally. One interesting difference would be that the current through the voltage source would act differently than the current through the voltage source in the original circuit. This is because as R_L becomes very large, and begins to act like an open circuit, no current would flow in Thévenin's circuit. However, in the original circuit, there are other branches for the current to flow through, thus current never approaches zero. In any case, this does not affect the validity of Thévenin's theorem. We can conclude that the graphs and data across the resistor R_L obtained from the original circuit would apply to Thévenin's circuit load R_L. Lastly, in analyzing the power data and graph, we note that when $R_L = 500\Omega$, the power is at a maximum which is the same as R_{th} as calculated in Equation (2). This proves the maximum power transfer theorem, which states that the power across the resistor R_L is at a maximum when $R_L = R_{th}$. Thus, the experimental data agrees with theory.

5 Conclusion

From this lab, we gain a better understanding of the behavior of various electrical quantities across a resistor as the resistance changes. Furthermore, we confirm that values obtained from software are within a very small percent error with respect to the theoretical values which indicates that the software is sufficiently accurate. Additionally, we confirm the validity of Thévenin's Theorem by comparing the values produced by the original circuit and Thévenin's equivalent circuit.

6 CircuitLab Reference

Below are the links to the circuits created for this lab in CircuitLab.

- Original Circuit: `https://www.circuitlab.com/editor/#?id=gsme39a23q99`
- Thévenin's Circuit: `https://www.circuitlab.com/editor/#?id=z6wa4dha4een`