

Free Fall in Two Dimensions

Lab: 04

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1 Introduction

It is often necessary to calculate the distance a projectile can travel under specific circumstances. Consider a projectile that is not affected by air resistance, that is, the moment the object becomes a projectile, it can be assumed that it is now in free-fall where the only force acting on it is gravity. Theoretically, the projectile should land in the same position every time, but when we attempt to measure the distance travelled, we may measure slightly different results every time.

In this lab, we will discuss methods of calculating the horizontal range of a projectile in free fall and the margin of error associated with it's measurement. Furthermore, we will attempt to calculate a precise angle, based on a quadratic regression of our data, that will yield the greatest horizontal range of the projectile.

2 Range Of Free-Fall Projectiles

The range of a projectile can of course be measured in the traditional sense, perhaps by taking a very long tape measure starting at the initial position of the projectile and measuring to the point where the projectile reached the ground again. If we were to ignore air resistance, there would be a good argument for that projectile landing at that position every time. However when we attempt to measure the distance with our tape measure, we yeild a slightly different distance every time. This is because there exists some margin of error when utilizing the tape measure, as the limitations of physical instruments must be taken into account.

Below we have put into a table, the data collected from a projectile being launched at different angles with the same initial velocity. It also displays our measured range R_{exp} and the theoretical range R_{theory} calculated by the following equation:

$$R_{\text{theory}} = \frac{V_0^2 \sin(2\theta_0)}{2g} \left[1 + \sqrt{\frac{2gh}{V_0^2 \sin^2(2\theta_0)}} \right]$$

Table 1: The data of a free fall projectile

V_0 (m/s)	θ_0 (degree)	R_{exp} (m)	R_{theory} (m)	% error
20	20	26.3	26.234	0.243
20	30	35.4	35.348	0.147
20	40	40.5	40.196	0.756
20	45	41.1	40.816	0.695
20	50	40.4	40.196	0.507
20	60	35.4	35.348	0.147
20	70	26.3	26.236	0.243

In this case, our measured range of the projectile maintains a slight margin of error with respect to the theoretical range. This indicates there is some consistent level of inaccuracy associated with the device used to measure the range. This is known as the margin of error, or "percent error" and is useful in determining the potential inaccuracy of the measurement.

In each case, I measured the percent error relative to the theoretical range and found there to be a larger discrepancy in the percent error as the angle of the projectile got closer to the angle which produces the greatest range. That is, there was a larger percent error between the experimental range R_{exp} and theoretical range R_{theory} as the angle got closer to θ_{max} , the angle that produces the max range. This allows us to conclude that the angle θ_{max} for this projectile must lie between 40° and 50° . We can prove this by creating a quadratic regression from the data. In this case, the equation of the regression line is:

$$y = -0.024 \pm 0.000 \text{ m}/\theta^2 + 2.1219 \pm 0.029 \text{ m}/\theta - 6.7842 \pm 0.610 \text{ m}$$

From this, we can extrapolate the maximum range of the projectile by calculating the relative maxima of the derivative of the function. In this case, we get the maximum range of the projectile to be 40.911 m and the angle θ_{max} to be 42.30° .

Earlier we said that the angle θ_{max} must lie between 40° and 50° which is consistent with our results from the quadratic regression.

We can experiment with this idea further and consider a scenario where the projectile is launched 5 meters above the landing position. The following table depicts this projectile launched from varying angles:

Table 2: The data of a free fall projectile where $h_0 = 5\text{m}$				
V_0 (m/s)	θ_0 (degree)	R_{exp} (m)	R_{theory} (m)	% error
18	30	35.8	35.598	0.569
18	40	37.8	37.705	0.253
18	45	37.6	37.473	0.340
18	50	36.4	36.320	0.220
18	60	31.4	31.275	0.401

In this case, the percent error of the experimental range relative to the theoretical range **decreased** as the angle of the projectile got further from θ_{max} . This contrasts the results found in the first case where the percent error increased as the projectile angle got closer to θ_{max} .

We can apply a quadratic regression to get a better understanding of how the range of the projectile changes as its angle changes. From a quadratic regression, we get the following equation:

$$y = -0.018 \pm 0.000 \text{ m}/\theta^2 + 1.440 \pm 0.019 \text{ m}/\theta + 8.453 \pm 0.405 \text{ m}$$

From this, we can apply the same method of retrieving the maximum range of the projectile as we did in the first case. A preliminary analysis of Table 2 indicates again that something is happening between the angles of 40° and 50° . But this time the percent error is decreasing as it enters this range rather than increasing like it was in Table 1. After extrapolating the projected maximum range of the projectile, we get the maximum range of the projectile to be 37.896 m and the angle which produces this range to be 42.82° .

It appears that increasing the initial height of the projectile caused the percent error to decrease as the angle θ reached θ_{\max} . The angle θ_{\max} for Table 2 was again found to be between 40° and 50° .

3 Conclusion

It appears that there becomes a higher degree of uncertainty when the speed at which the range of the projectile goes from increasing to decreasing changes. For example, if the range goes from increasing to decreasing very quickly, then the percent error increases quickly. This type of change in the range occurs when the angle reaches θ_{\max} . In addition, When a large change occurred in the range between two relatively close angles, the percent error increased dramatically.

Furthermore, despite the projectiles in both tables starting at different initial positions, they both reached a very similar θ_{\max} . If the projectile in Table 2 was launched at the same velocity as the projectile in Table 1, perhaps this type of comparison would have been more useful. In addition, in the case where the projectile started at a higher position, the change in the range between the recorded angles was significantly smaller. This indicates that the projectile in case 2 achieved a maximum range a lot sooner than the projectile in case 1 where it started at the same height as its landing height.