Exemple méthode POT

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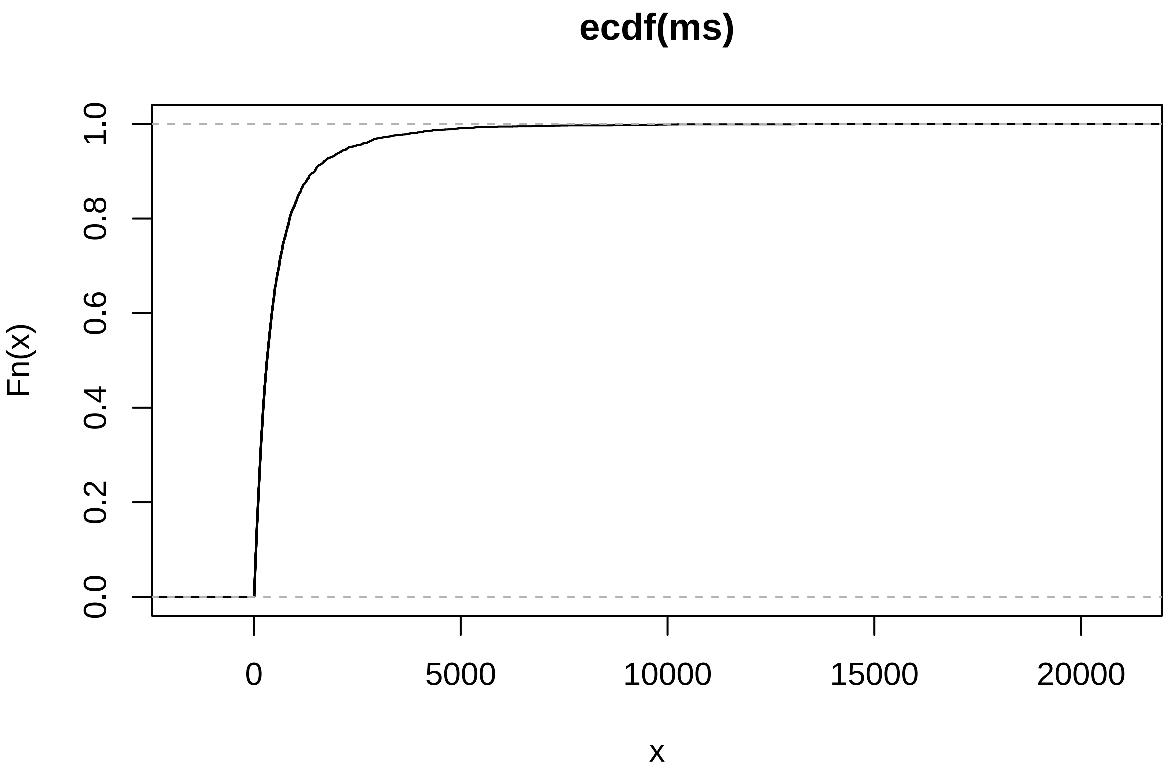
Adaptation de l'exemple fait par Étienne Marceau sur excel

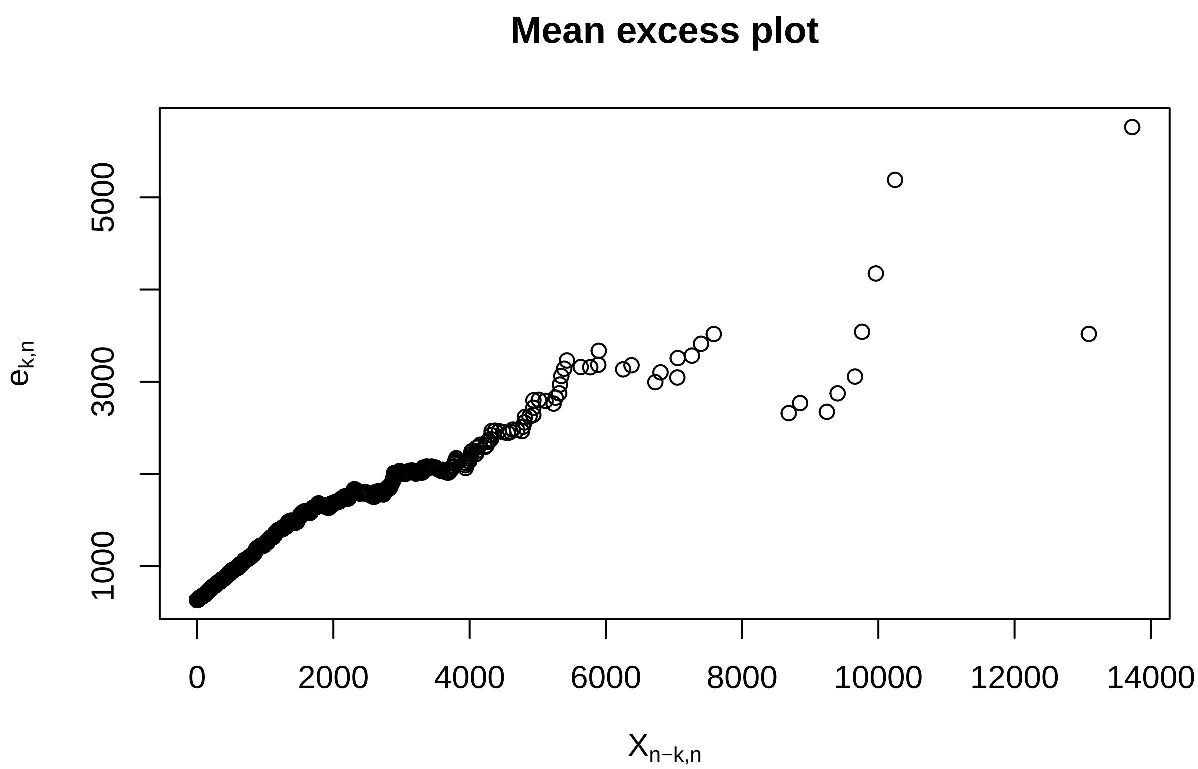
Fonction de répartition empirique

```r
Fn <- ecdf(ms)
plot(Fn)
```



Fonction d'exces moyen

```r
MeanExcess(ms)
```



Maximum de vraisemblance (Pareto)

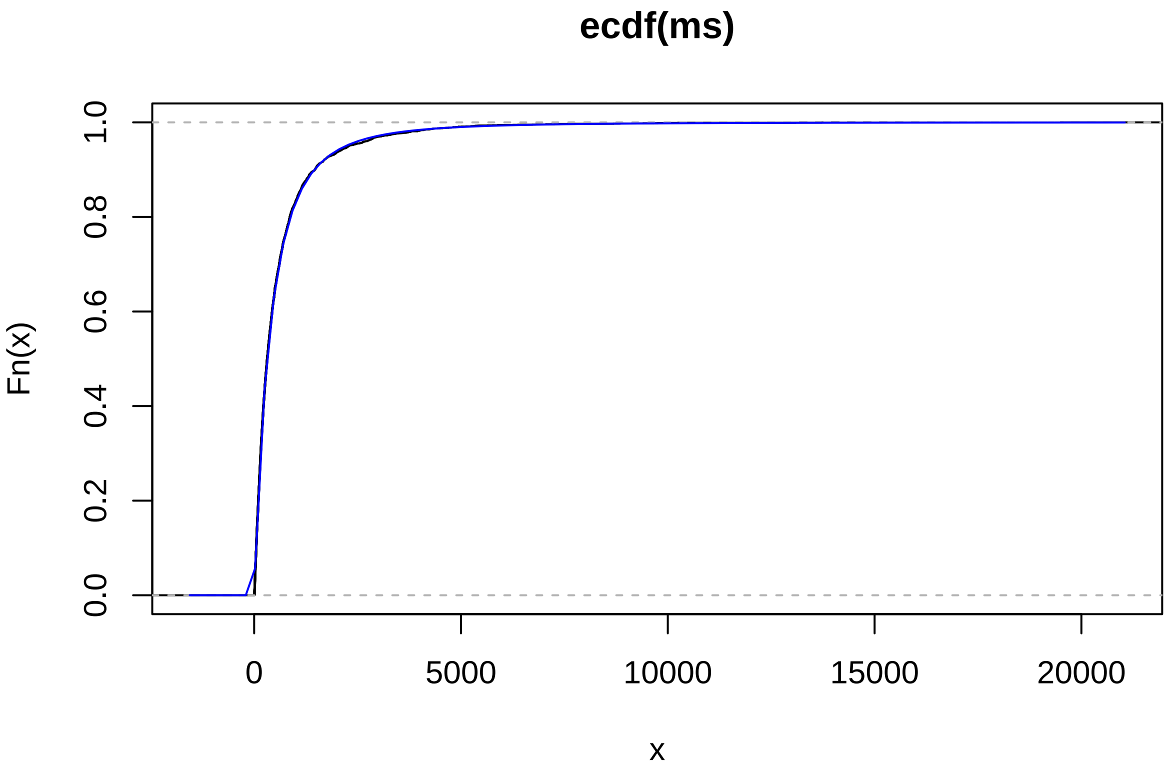
```
logvrais <- function(par){
  -sum(log(actuar::dpareto(ms, par[1], par[2])))
}
par <- constrOptim(c(1.5, 1000), logvrais, grad = NULL, ui = diag(2), ci = c(0, 0))
a <- par$par[1]
lam <- par$par[2]
model <- cbind(a, lam)
```

On trouve les paramêtres suivants :

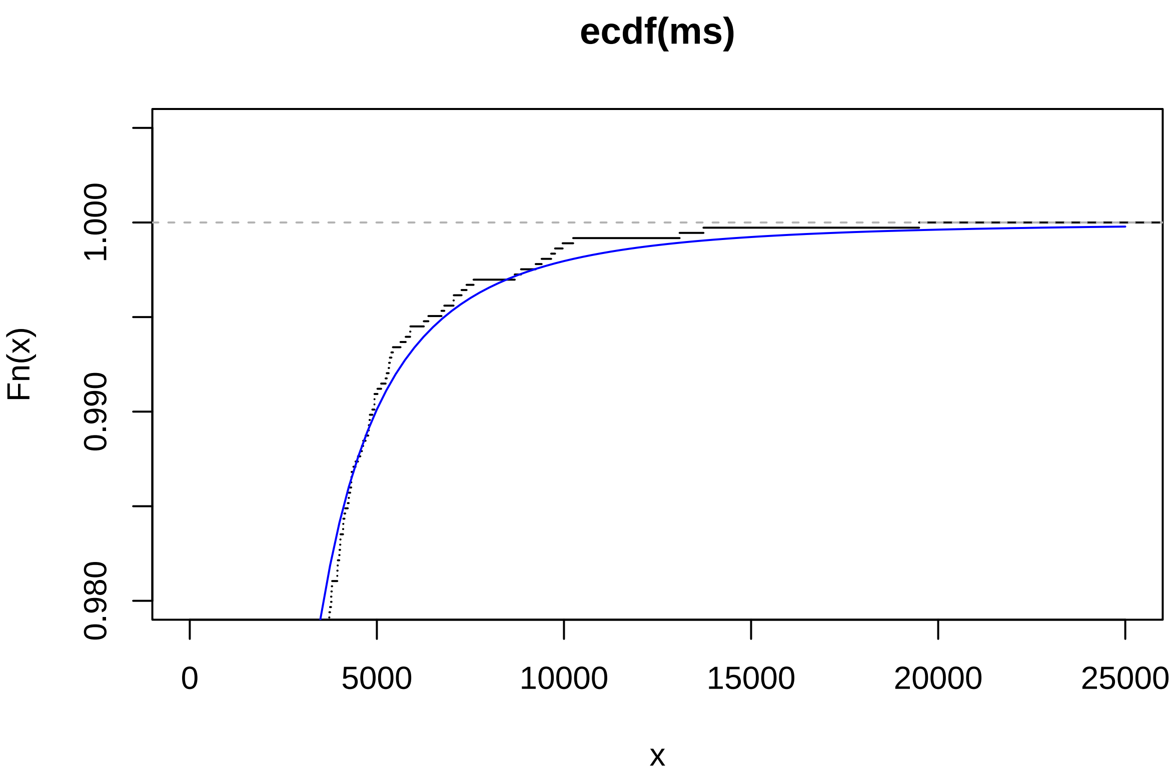
α	λ
2.61	1029.73

Comapraison courbe emprique avec Pareto

```
plot(Fn)
curve(actuar::ppareto(x, a, lam), add=T, col = "blue")
```




```
plot(Fn, ylim=c(0.98, 1.005), xlim=c(0, 25000))
curve(actuar::ppareto(x, a, lam), add=T, col = "blue")
```



Maximum de vraisemblance données élevées

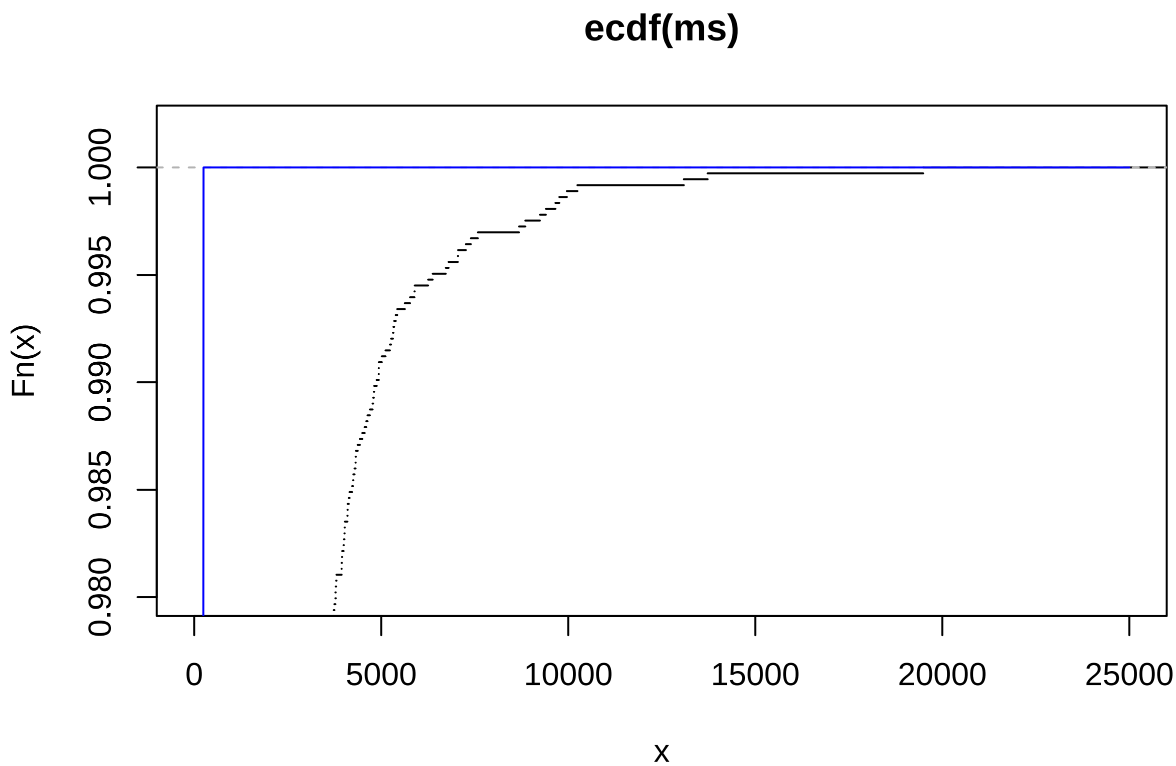
```
logvrais2 <- function(par){
  u <- 4000
  -sum(log(actuar::dpareto(ms, par[1], par[2])/actuar::ppareto(u, par[1], par[2], low=F)) * I(ms >= u))
}

par2 <- constrOptim(c(2, 0.0001), logvrais2, grad = NULL, ui = diag(2), ci = c(0, 0))
a2 <- par2$par[1]
lam2 <- par2$par[2]
model2 <- cbind(a2, lam2);model2
```

```
##            a2       lam2
## [1,] 2.796997 0.08442982
```

Comapraison avec valeur élevée

```
plot(Fn, ylim=c(0.98, 1.002), xlim=c(0, 25000))
curve(actuar::ppareto(x, a2, lam2), add=T, col = "blue")
```



Méthode POT

```
logvraisPOT <- function(par){
  u <- 4000
  -sum(log(1/par[2] * (1 + par[1]/par[2] * pmax(ms - u,0))^(-1/par[1] - 1)) * I(ms >= u))
}
par3 <- constrOptim(c(1, 1000), logvraisPOT, grad = NULL, ui =diag(2), ci = c(0, 0))
zeta <- par3$par[1]
sigma <- par3$par[2]
```

```r
model3 <- cbind(zeta, sigma);model3
```

```
##           zeta    sigma
## [1,] 0.3474587 1444.382
```

```r
# Autre façon
# logvraisPOT2 <- function(par){
# u <- 4000
#  -sum(ifelse(ms >= 4000, log(dgpd(ms, par[1], u, par[2])), 0))
#}

#constrOptim(c(1, 1000), logvraisPOT2, grad = NULL, ui =diag(2), ci = c(0, 0))
```

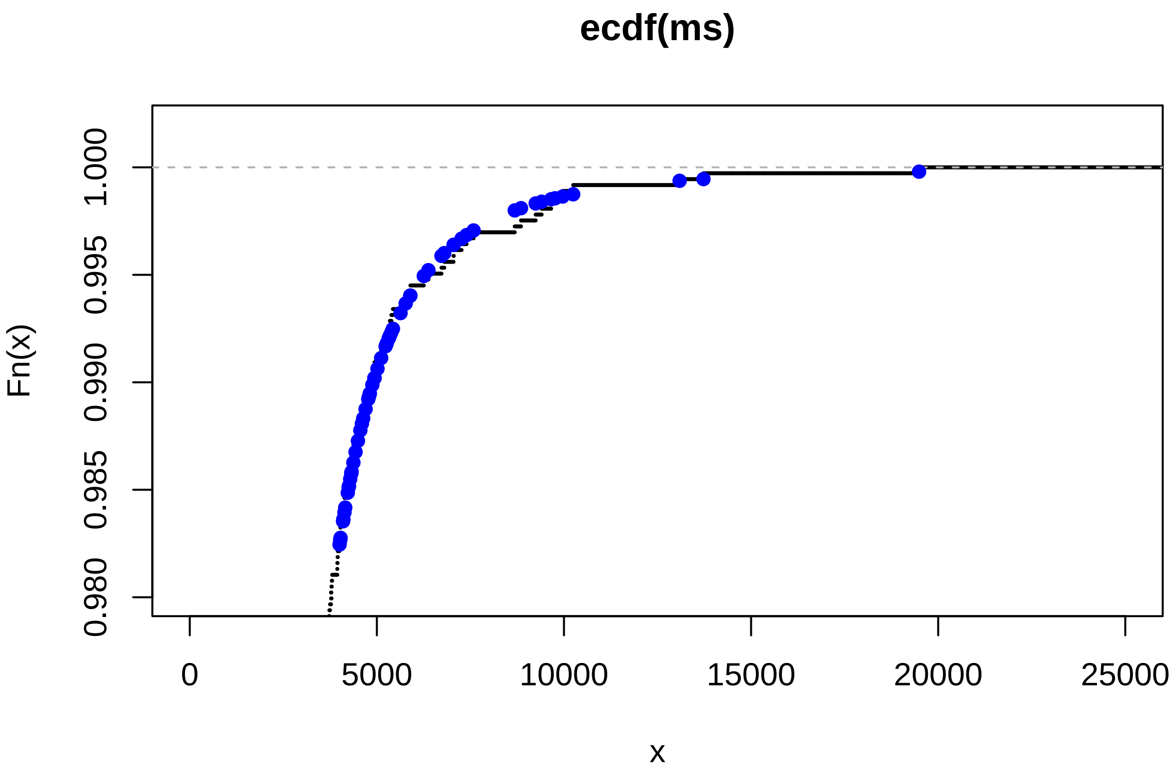
On trouve les paramètres suivants :

γ	σ
0.35	1444.38

Graphique

```r
u <- 4000
Fx.PG <- Fn(u) + (1-Fn(u)) * (1 - (1/(1 + (ms[ms >= u] - u) * zeta/sigma))^(1/zeta))

plot(Fn, ylim=c(0.98, 1.002), xlim=c(0, 25000), lwd = 2)
matplot(ms[ms >= u], Fx.PG, pch = 16, add = T, col = "blue")
```

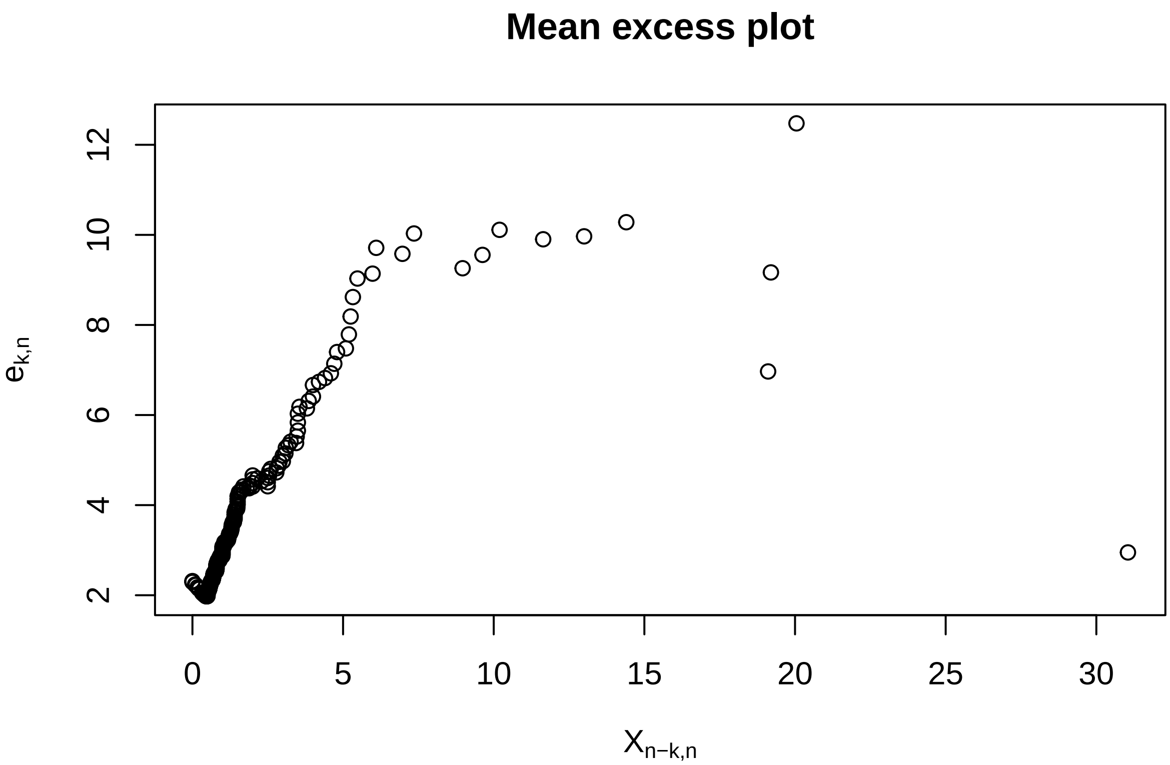


Swedish fire data

Vraisemblance avec loi Pareto

```r
fire <- as.numeric(read.table("Data Fire Swedish 1982.txt")$V1)
Fm <- ecdf(fire)

MeanExcess(fire)
```




```r
quantile(fire, 0.90)
```

```
##    90%
## 4.6289
```

```r
fire[fire > 1]
```

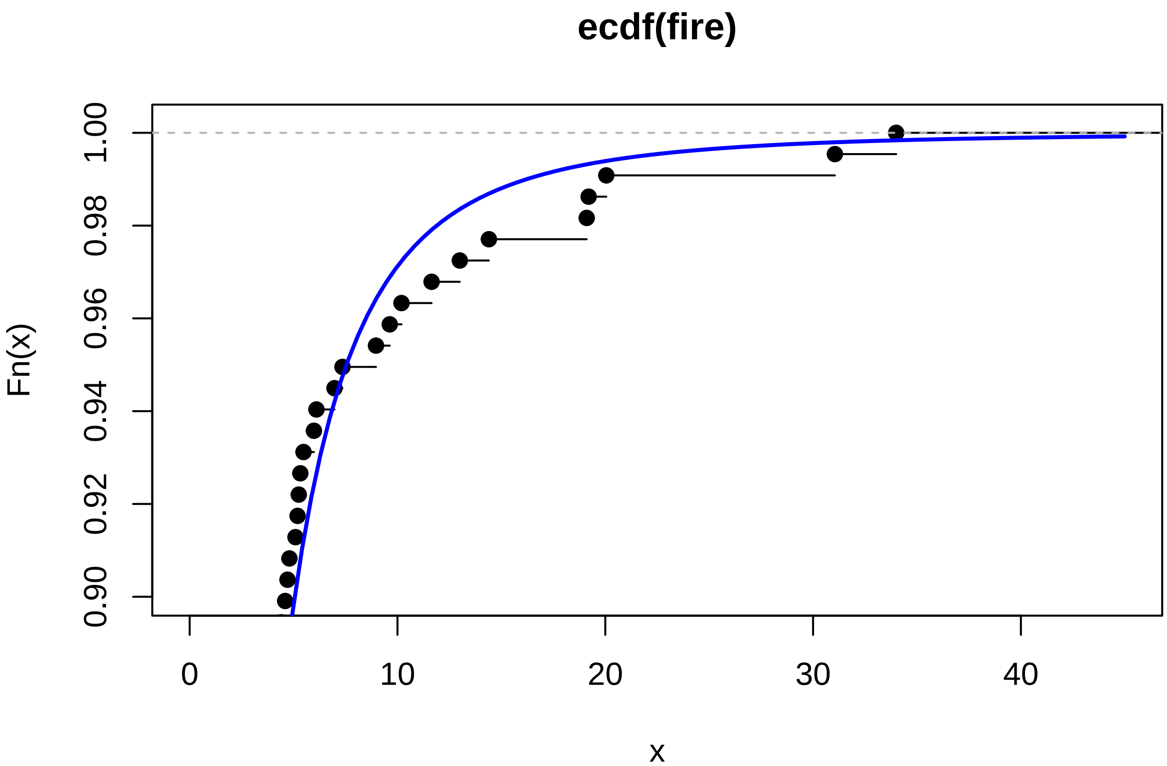
```
##   [1]  1.335  5.193  1.486  2.800  3.860 10.194  8.967  6.970  4.000  4.595
##  [11]  3.085  5.250  1.220  4.000  4.800  1.500  3.185  1.188  3.000  1.280
##  [21]  1.500  4.400  2.544  1.035  3.440  1.535  3.460  1.401  1.997  1.265
##  [31]  2.890  1.500  1.300 11.641 31.050  1.057 19.200  1.400  1.305  3.500
##  [41]  6.100  2.500  1.600  2.858  1.377  3.000  9.627  5.478  2.000  2.784
##  [51]  2.550  4.200  2.600  3.500  5.325  1.250  1.060 14.400  3.100  1.300
##  [61]  3.553  1.125  1.430  7.354  2.151  1.544  1.400  1.688 19.107  1.200
##  [71]  2.000  1.382  2.000  5.979  3.258  1.435  1.050  1.180  1.100  4.708
##  [81]  1.326  1.050 20.049  2.500  1.127  2.500  1.493  1.289  3.500  1.800
##  [91] 13.000  5.093  1.625  1.500  1.692  2.300  1.880  1.192  1.215  3.800
## [101]  1.900  1.400  1.146 34.000
```

```r
logvrais <- function(par){
  -sum(log(actuar::dpareto(fire, par[1], par[2])))
}
```

```r
param <- constrOptim(c(5, 10), logvrais, grad = NULL, ui=diag(2), ci = c(0, 0))
alpha <- param$par[1]
lambda <- param$par[2]
```

Graphique

```r
plot(Fm, ylim=c(0.90, 1.002), xlim=c(0, 45))
curve(actuar::ppareto(x, alpha, lambda), add = T, col="blue", lwd = 2)
```



Méthode POT

```r
u <- 1.268
logvraisPOT <- function(par){
  -sum(ifelse(fire >= u, log(dgpd(fire, par[1], u, par[2])), 0))
}
param <- constrOptim(c(2, 3), logvraisPOT, grad = NULL, ui=diag(2), ci = c(0, 0))
zeta <- param$par[1]
sigma <- param$par[2]
model <- cbind(zeta, sigma);model
```

```
##           zeta    sigma
## [1,] 0.6972818 1.401489
```

Analyse automatique

Graphique (fit)

```r
Fx.PG <- Fm(u) + (1-Fm(u)) * pgpd(fire[fire >= u], zeta, u, sigma)
```

```r
plot(Fm, ylim=c(0.90, 1.002), xlim=c(0, 50), lwd = 2)
matplot(sort(fire[fire >= u]), sort(Fx.PG), type = "l", lwd = 2, add = T, col = "blue")
```