Exemple méthode POT

Jérémie Barde

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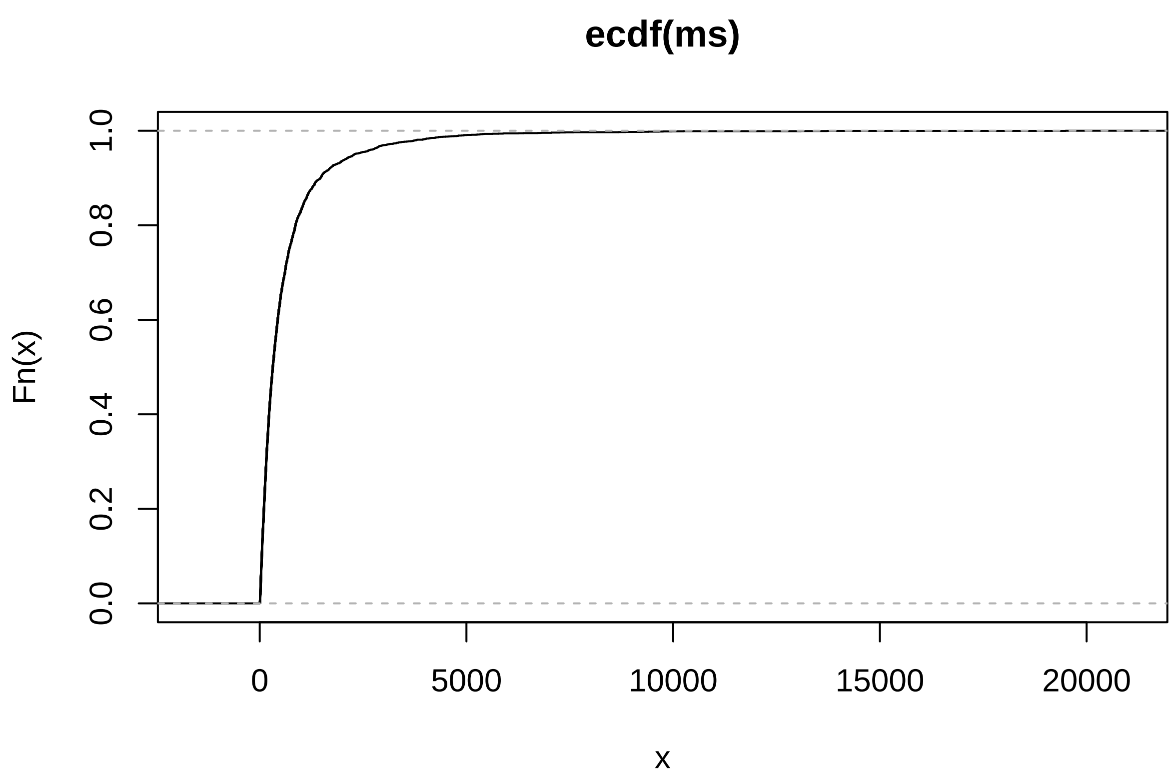
Exemple fait par Étienne Marceau

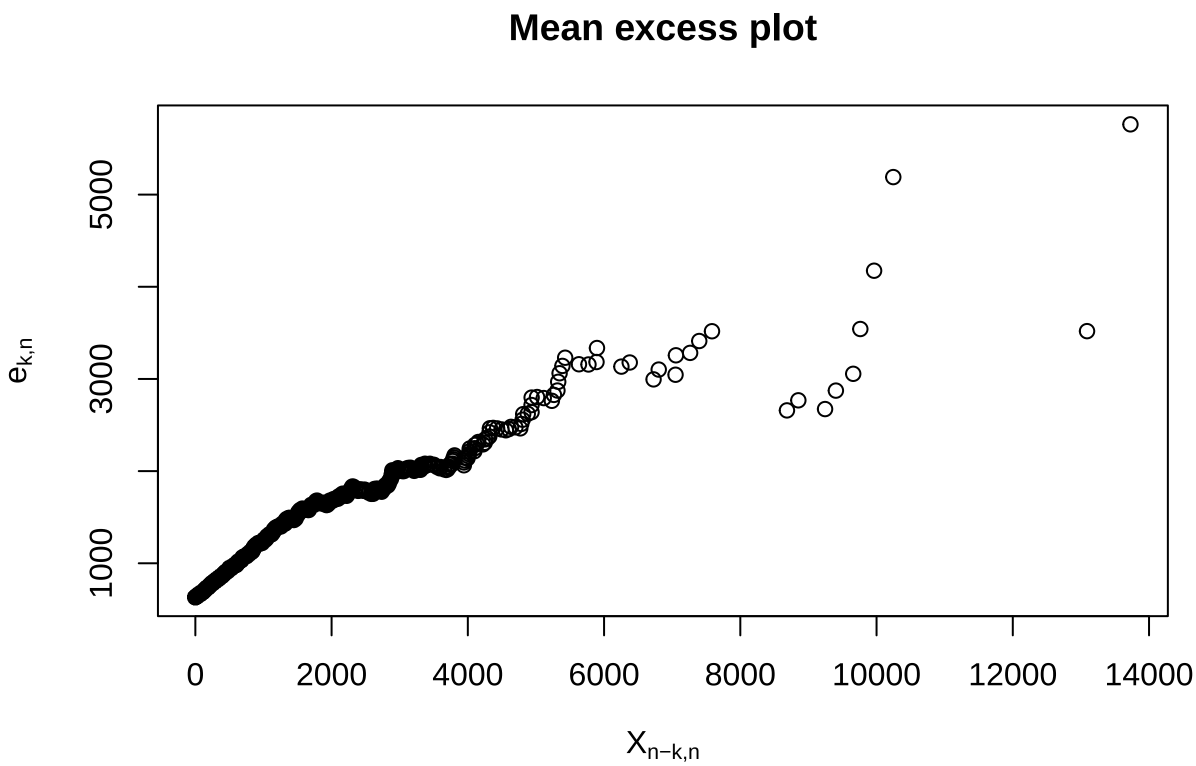
Fonction de répartition empirique

```
Fn <- ecdf(ms)
plot(Fn)
```



Fonction d'exces moyen

```
MeanExcess(ms)
```



Maximum de vraisemblance (Pareto)

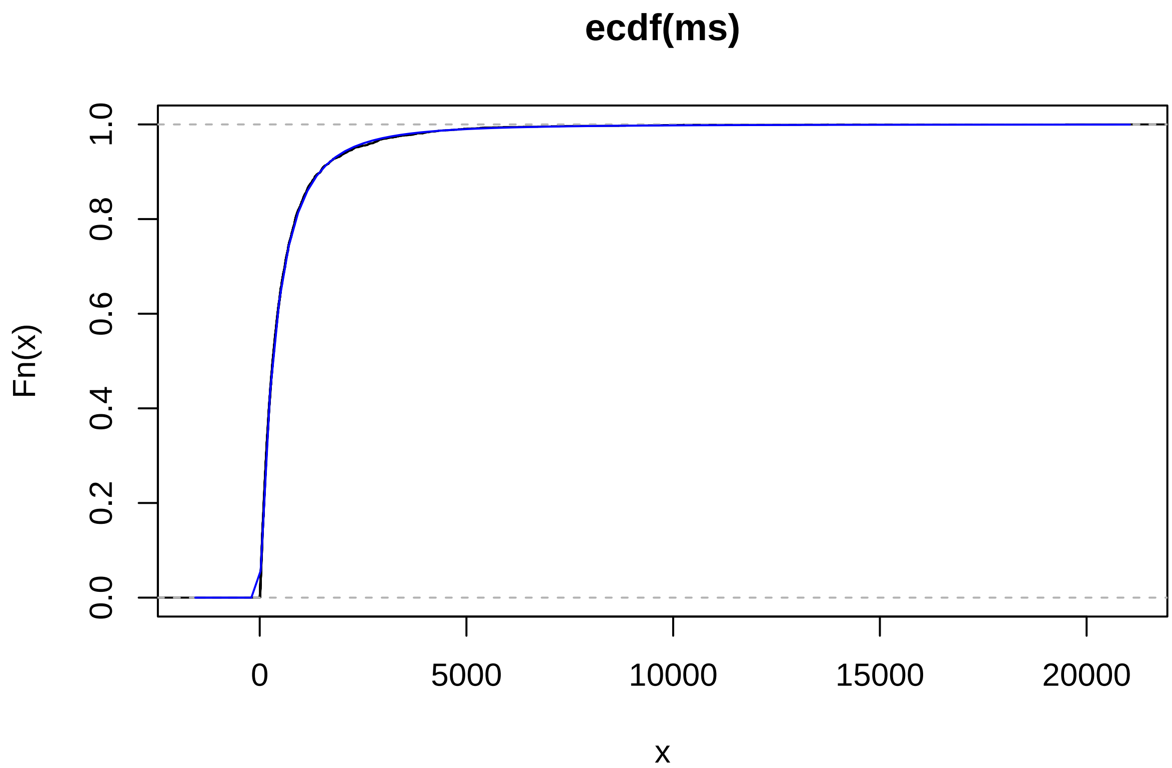
```
logvrais <- function(par){
  -sum(log(actuar::dpareto(ms, par[1], par[2])))
}
par <- constrOptim(c(1.5, 1000), logvrais, grad = NULL, ui = diag(2), ci = c(0, 0))
a <- par$par[1]
lam <- par$par[2]
model <- cbind(a, lam)
```

On trouve les paramêtres suivants :

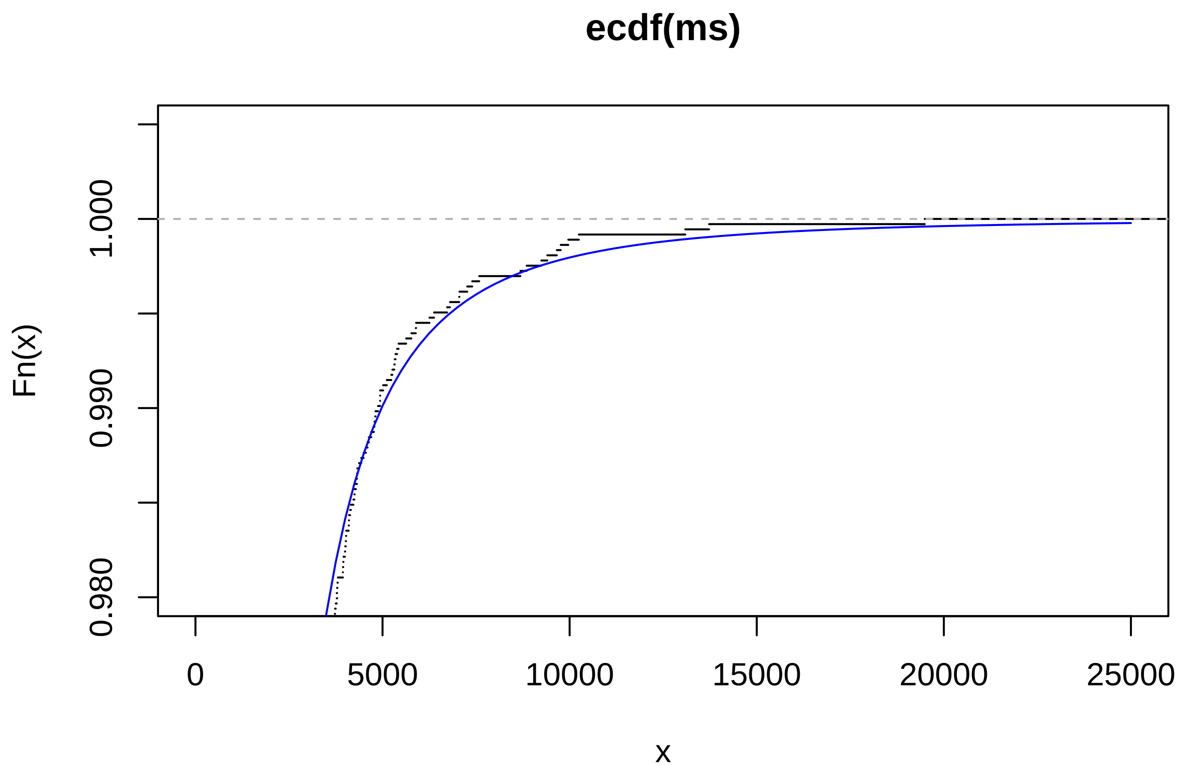
α	λ
2.61	1029.73

Comapraison courbe emprique avec Pareto

```
plot(Fn)
curve(actuar::ppareto(x, a, lam), add=T, col = "blue")
```




```
plot(Fn, ylim=c(0.98, 1.005), xlim=c(0, 25000))
curve(actuar::ppareto(x, a, lam), add=T, col = "blue")
```



Maximum de vraisemblance données élevées

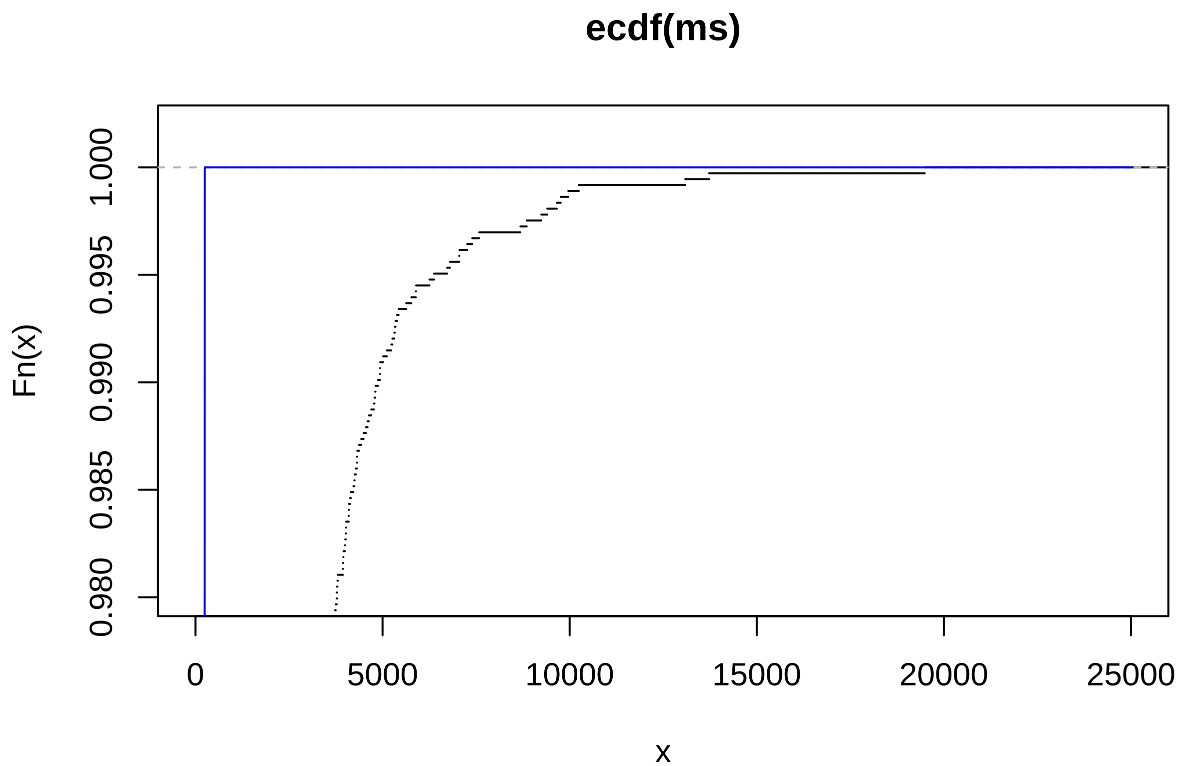
```
logvrais2 <- function(par){
  u <- 4000
  -sum(log(actuar::dpareto(ms, par[1], par[2])/actuar::ppareto(u, par[1], par[2], low=F)) * I(ms >= u))
}

par2 <- constrOptim(c(2, 0.0001), logvrais2, grad = NULL, ui = diag(2), ci = c(0, 0))
a2 <- par2$par[1]
lam2 <- par2$par[2]
model2 <- cbind(a2, lam2);model2
```

```
##             a2       lam2
## [1,] 2.796997 0.08442982
```

Comapraison avec valeur élevée

```
plot(Fn, ylim=c(0.98, 1.002), xlim=c(0, 25000))
curve(actuar::ppareto(x, a2, lam2), add=T, col = "blue")
```



Méthode POT

```
logvraisPOT <- function(par){
  u <- 4000
  -sum(log(1/par[2] * (1 + par[1]/par[2] * pmax(ms - u,0))^(-1/par[1] - 1)) * I(ms >= u))
}
par3 <- constrOptim(c(1, 1000), logvraisPOT, grad = NULL, ui =diag(2), ci = c(0, 0))
zeta <- par3$par[1]
sigma <- par3$par[2]
model3 <- cbind(zeta, sigma);model3
```

```
##           zeta    sigma
## [1,] 0.3474587 1444.382
```

```
# Autre façon
# logvraisPOT2 <- function(par){
# u <- 4000
#  -sum(ifelse(ms >= 4000, log(dgpd(ms, par[1], u, par[2])), 0))
#}

#constrOptim(c(1, 1000), logvraisPOT2, grad = NULL, ui =diag(2), ci = c(0, 0))
```

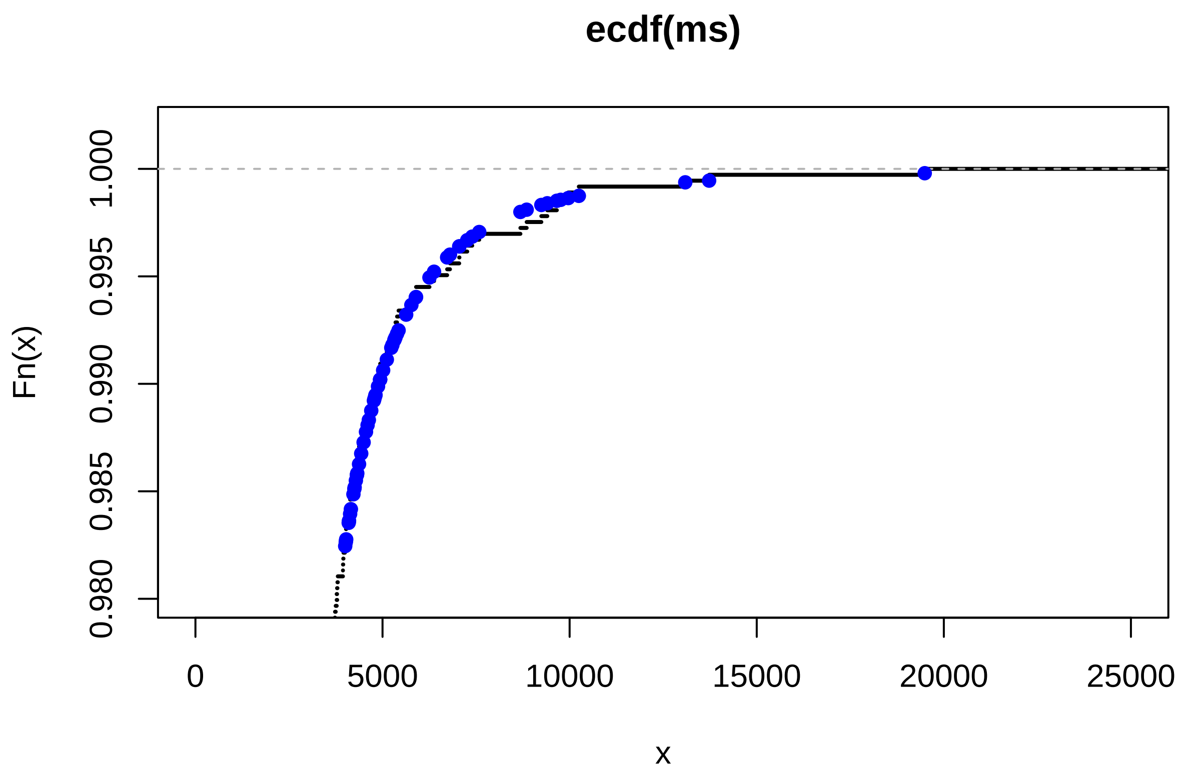
On trouve les paramètres suivants :

γ	σ
0.35	1444.38

Graphique

```
u <- 4000
Fx.PG <- Fn(u) + (1-Fn(u)) * (1 - (1/(1 + (ms[ms >= u] - u) * zeta/sigma))^(1/zeta))

plot(Fn, ylim=c(0.98, 1.002), xlim=c(0, 25000), lwd = 2)
matplot(ms[ms >= u], Fx.PG, pch = 16, add = T, col = "blue")
```

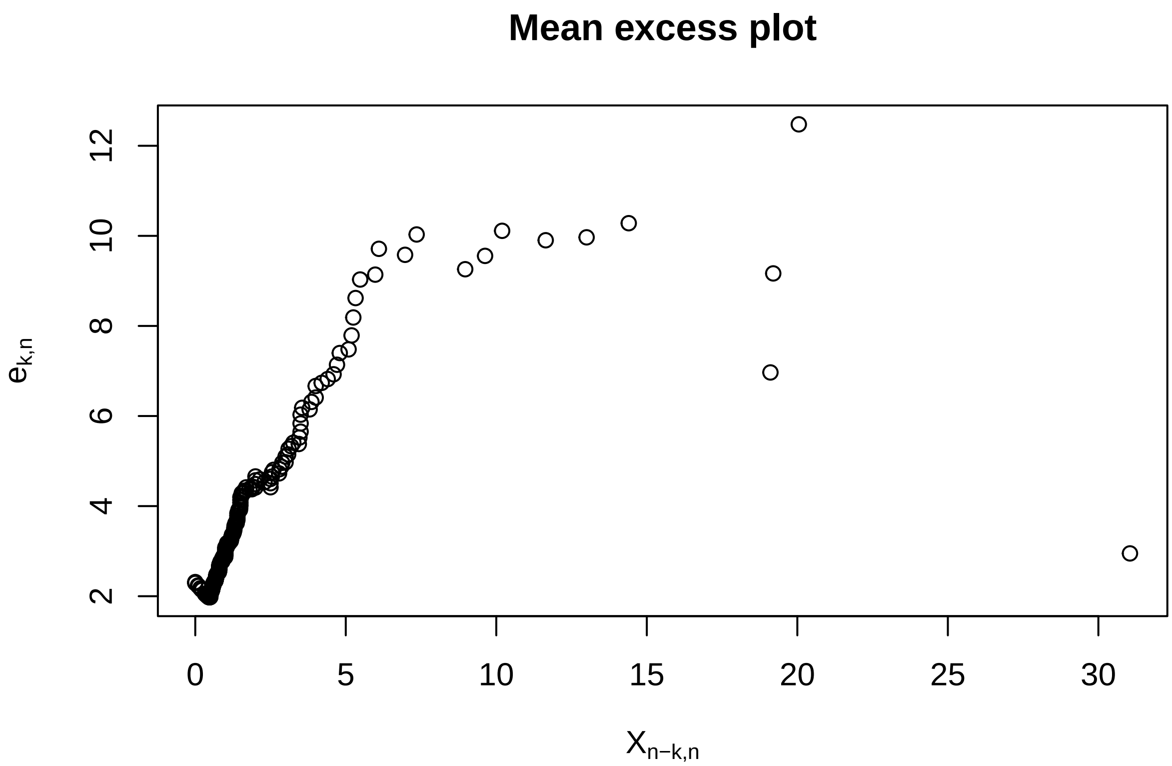


Swedish fire data

Vraisemblance avec loi Pareto

```
fire <- as.numeric(read.table("data/Data Fire Swedish 1982.txt")$V1)
Fm <- ecdf(fire)

MeanExcess(fire)
```




```
quantile(fire, 0.90)
```

```
##    90%
## 4.6289
```

```
fire[fire > 5]
```

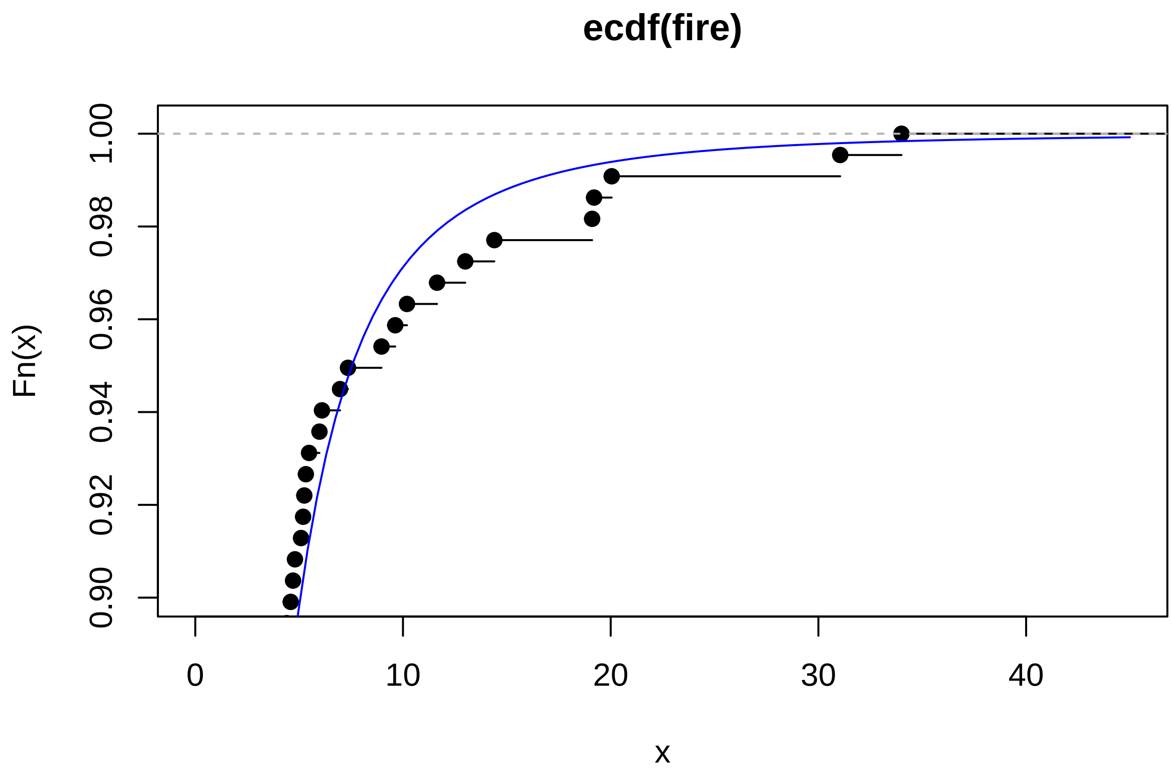
```
##  [1]  5.193 10.194  8.967  6.970  5.250 11.641 31.050 19.200  6.100  9.627
## [11]  5.478  5.325 14.400  7.354 19.107  5.979 20.049 13.000  5.093 34.000
```

```
logvrais <- function(par){
  -sum(log(actuar::dpareto(fire, par[1], par[2])))
}

param <- constrOptim(c(5, 10), logvrais, grad = NULL, ui=diag(2), ci = c(0, 0))
alpha <- param$par[1]
lambda <- param$par[2]
```

Graphique

```
plot(Fm, ylim=c(0.90, 1.002), xlim=c(0, 45))
curve(actuar::ppareto(x, alpha, lambda), add = T, col="blue")
```



Méthode POT

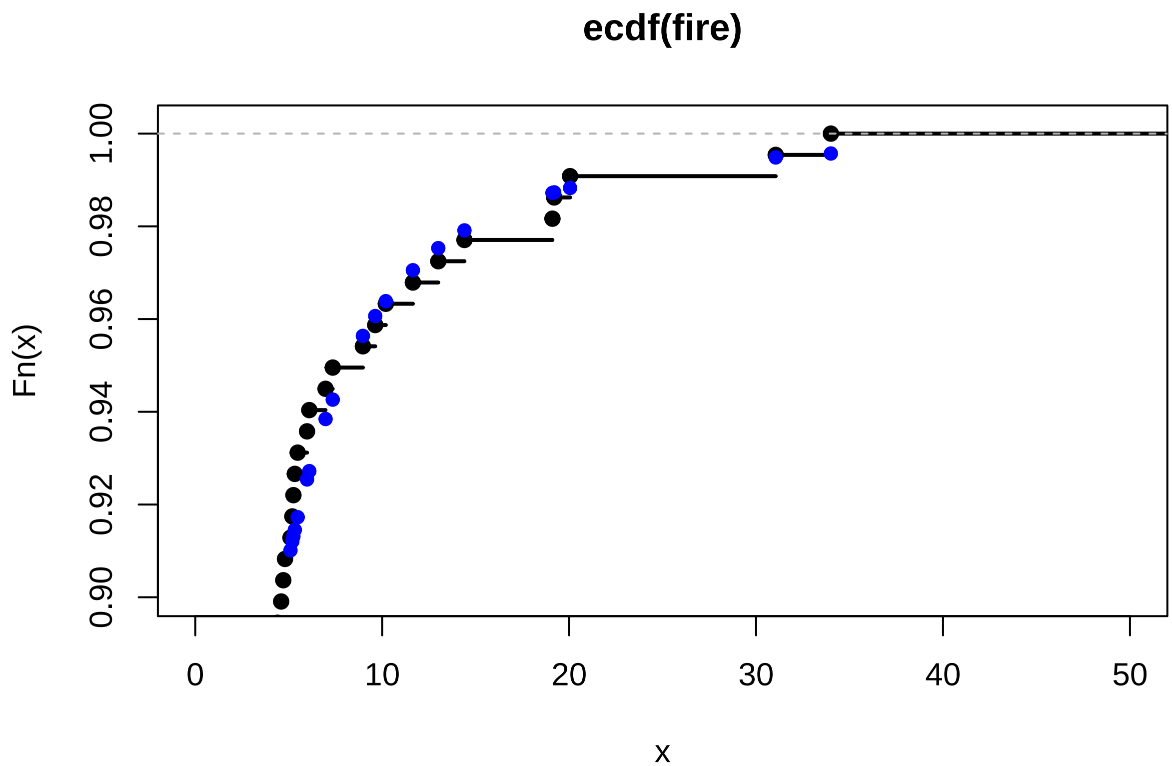
```
u <- 5
logvraisPOT <- function(par){
  -sum(ifelse(fire >= u, log(dgpd(fire, par[1], u, par[2])), 0))
}
param <- constrOptim(c(2, 3), logvraisPOT, grad = NULL, ui=diag(2), ci = c(0, 0))
zeta <- param$par[1]
sigma <- param$par[2]
model <- cbind(zeta, sigma);model
```

```
##           zeta    sigma
## [1,] 0.4354472 4.520378
```

Graphique

```
u <- 5
Fx.PG <- Fm(u) + (1-Fm(u)) * pgpd(fire[fire >= u], zeta, u, sigma)

plot(Fm, ylim=c(0.90, 1.002), xlim=c(0, 50), lwd = 2)
matplot(fire[fire >= u], Fx.PG, pch = 16, add = T, col = "blue")
```


ecdf(fire)
Fn(x)
0.90
0.92
0.94
0.96
0.98
1.00
0
10
20
30
40
50
x