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# Combining engineering and data-driven approaches: Calibration of a generic fire risk model with data



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#### ABSTRACT

Two general approaches may be followed for the development of a fire risk model: statistical models based on observed fire losses can support simple cost-benefit studies but are usually not detailed enough for engineering decision-making. Engineering models, on the other hand, require many assumptions that may result in a biased risk assessment. In two related papers we show how engineering and data-driven modelling can be combined by developing generic risk models that are calibrated to statistical data on observed fire events. The focus of the present paper is on the calibration procedure. A framework is developed that is able to deal with data collection in non-homogeneous portfolios of buildings. Also incomplete data sets containing only little information on each fire event can be used for model calibration. To illustrate the capabilities of the proposed framework, it is applied to the calibration of a generic fire risk model for single family houses to Swiss insurance data. The example demonstrates that the bias in the risk estimation can be strongly reduced by model calibration.

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#### 1. Introduction

Decisions regarding investments into fire safety generally have to be made under uncertainty. This stems both from the inherent randomness of building fire events and from the fact that we are not able to fully understand and model the underlying phenomena. Probabilistic approaches for fire risk assessment allow the consistent consideration of both types of uncertainties. The overall goal of quantitative fire risk assessment is to support decisions on risk reduction measures by estimating their impact on the expected consequences (e.g. financial losses or human fatalities) of all possible fire scenarios. A basic requirement for a risk model to be used for decision-making is that the risk has to be assessed as a function of the safety measures installed; the model has to include the decision variables. Another important requirement is that the risk-relevant characteristics of the building or group of buildings to be modelled are accounted for. Finally, the model should assess the risk as accurately as possible.

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#### 1.1. Engineering and data-based fire risk assessment

Fire risk models can be based on two sources of information: statistical data and engineering models. *Empirical models* as described e.g. by Ramachandran [1] or Tillander [2] use simple parametric functions to model fire occurrence and the probability distribution of financial or human consequences given a fire event. The models are fitted to observed data and therefore may be expected to provide a fairly unbiased estimate of the observed fire risk. However, the approach can only provide average risk estimates, as the data must be collected for a more or less homogeneous group of buildings to obtain a sample size that is large enough for statistical analysis. Another drawback is that the use of data-based risk models for decision-making will always be restricted by the information content of the data available to the modeller; information on the relevant decision variables is often missing.

Engineering risk models, on the other hand, are based on an understanding of the physical processes leading to loss of property and life. For the purpose of this paper, an *engineering model* is defined very broadly as any approach that breaks down the problem of fire risk assessment into several components which are addressed by a number of interacting submodels that represent physical phenomena, such as e.g. fire spread to different rooms,

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Nomenclature General notation		P(A)	Probability of an event A
		Variable	Variable definitions
X, x X, x $\hat{x}$ E[.] Var[.] Cov[.] $f_X(x)$ $f_X(x)$ $f_{X Y}(x y)$ $p_X(x)$	Random variable, realisation Vector of RV, realisations Data set with observations of X Expectation operator Variance operator Covariance operator Probability density function for X Cumulative distribution function Conditional distribution of X given Y Discrete probability density function	X,x, \$\frac{x}{y},y, \$\frac{y}{y}\$ \$\frac{\text{\$\text{\$\frac{\text{\$\general}{2}}{\text{\$\general}}\$, \$\text{\$\text{\$\text{\$\general}{2}}\$}}} \right. \$\frac{\text{\$\text{\$\general}{2}}{\text{\$\general}{2}} \right. \$\frac{\text{\$\general}{2}}{\text{\$\general}{2}} \right. \$	Model input risk indicators Model output risk indicators Risk indicators contained in the data set (different from model input) Model calibration parameters Likelihood function, log-likelihood Maximum Likelihood parameters Covariance matrix for $\Theta$ Fisher information matrix

fire brigade response or occupant egress. Introductions to probabilistic fire risk assessment have been provided e.g. by Hasofer et al. [3], Yung [4], Magnusson et al. [5] or Ramachandran and Charters [6], to mention just a few. The methods have been applied for the development of comprehensive risk models with different focus, e.g. CESARE-RISK [7], FIRECAM [8], CRISP II [9], CUrisk [10] and B-Risk [11].

By establishing the relationship between fire risk and clearly defined physical variables or phenomena, engineering models offer a high potential for decision-making, e.g. during the design of buildings for fire safety. The methods do however always include a certain bias, i.e. a systematic error due to assumptions made in the probabilistic modelling, e.g. the probability distribution functions of basic input variables and simplified methods used to model the risk.

When comparing different fire safety designs (e.g. for demonstrating code equivalency, Beck [7] or He and Grubits [12]), fire safety engineers often use so-called "conservative" assumptions leading to a presumably safe, but unpredictable bias in the final outcome of the model. This is already problematic for a relative risk assessment, as the risk comparison will only be meaningful if the bias is the same for all options that shall be regarded. A comparison between the uncertain benefits of a safety measure and its (usually certain) costs does, however, require an absolute risk assessment. In this case, the model clearly has to assess the expected loss of property or life with as little bias as possible.

The bias, or systematic error, of a risk model may be understood as the difference between the estimated risk measure (e.g. expected consequences, exceedance probabilities for large losses) and its true value, which is generally unknown but may be approximated by statistical analysis if the data sample is large enough. This implies that the bias can be reduced by calibrating a fire risk model to statistical data.

Model calibration deals with an optimal choice of model parameters in order to represent the observations as best as possible. Ideally, a calibration approach should not only provide a point estimate for the "best-fit" parameters, but also some information on the uncertainty of the calibrated parameters. This may be achieved by using statistical methods such as the method of Maximum Likelihood (e.g. Rychlik and Rydén [13]) or a Bayesian approach to parameter estimation (e.g. Gelman et al. [14]).

If the parameters are associated with physical quantities, model calibration is also known as inverse modelling. It has recently been applied to estimate the most likely model input of fire models (e.g. heat release rate or fire growth rate) from measured output quantities such as e.g. temperature development or heat flux values. This approach can be applied either after a fire has occurred (e.g. for fire investigation, Overholt et al. [15]) or for real-time decision-making during the course of a fire event (Koo et al. [16],

Jahn et al. [17]).

Model calibration with fire loss data collected for a whole group (or *portfolio*) of buildings by e.g. fire brigades or insurance companies so far has been limited to simple statistical models like the data-based fire risk models mentioned above. Using observed loss data for the calibration of engineering fire risk models can be expected to provide valuable input for an improved prediction before a fire occurs, e.g. for evaluating the effect of different fire safety measures. The aim of the present paper and a companion paper by De Sanctis et al. [18] is to show how this may be realized in practise.

#### 1.2. Outline of the calibration problem

The general idea of the approach followed in the two related papers is illustrated in Fig. 1. First we develop a risk model estimating the random model output Y (e.g. the financial loss due to a fire) as a function of some model input X. The model can be adjusted to observations of X and Y made in real fire events by fitting a set of calibration parameters  $\Theta$  to statistical data. The development of such a fire risk model, i.e. a model that may be calibrated, is discussed in De Sanctis et al. [18]. The modelling strategy chosen is based on the principles of generic risk assessment described in JCSS [19]. The consequences of an exposure event (e.g. fire ignition) are modelled using a hierarchical approach, with a vulnerability model estimating the direct effects of the exposure and a robustness model assessing the indirect consequences, see Fig. 1.

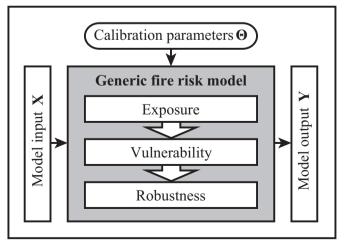


Fig. 1. Calibration of a generic fire risk model to data.

The focus of the present paper is on the calibration of a generic fire risk model to statistical data from real fire events; the model itself is treated mainly as a "black box" estimating the risk for each building as a function of the model input and the calibration parameters. The calibration of the model parameters  $\Theta$  requires a data set with sufficient sample size containing information on both the model input X and the model output Y; in this setting the parameters  $\Theta$  are the only unknowns. Another important requirement is that the data has not been extensively used during the development of the risk model, i.e. that the data is an independent sample providing new information.

Possible data sources for the calibration of building fire risk models are fire brigade reports or data collected by fire insurance companies. Two main problems have to be solved when using such data for model calibration:

*Data collection at portfolio level*: the data is typically collected in a non-homogeneous portfolio (group) of buildings, with a variety of building-specific factors influencing the risk.

*Incomplete data sets*: the data does not necessarily contain all information relevant to an engineering approach for fire risk assessment.

The first problem is well-known to anybody analysing fire loss data with research questions regarding e.g. the effectiveness of fire safety measures (Thomas [20]). In contrast to lab experiments where it is possible to vary only one factor at once, the analysis of data from real fire events is more complex due to the interaction of different variables. When calibrating an engineering model to realworld data, this problem can be reduced by explicitly modelling the effect of different building characteristics on the fire risk. This is achieved by following the generic modelling approach described in JCSS [19]. A generic fire risk model estimates the risk based on a set of risk indicators describing the characteristics of the building (e.g. floor area, number of rooms) and the fire event (e.g. room of fire ignition, fire brigade intervention time). The advantage of this approach is that the same model can be applied to all buildings within a specific class the model has been developed for, which allows for an estimation of the model calibration parameters based on data collected at portfolio level. Also the effect of fire safety measures can be assessed for a portfolio of buildings to support decisions that have to be made at portfolio level (e.g. code-making decisions that generally affect a whole class of buildings).

Often, the data available for calibration is incomplete, which leads to the second problem mentioned above: the risk indicators used by the engineering model may not be recorded in the data set. As a short-term solution to this problem, the required risk indicators can be estimated based on the data and other sources of information. However, such an approach introduces additional uncertainties into the calibration procedure, as the model input and/or output (X and Y in Fig. 1) are not known with certainty. A long-term strategy should therefore be to improve the data base by collecting the information required for an engineering modelling approach. By identifying observable indicators relevant for fire risk assessment, the development of generic fire risk models helps to formulate the requirements for future data collection. With an improved data base, the models can be further updated and developed.

Data on observed fire events may also be used to assess the quality of an engineering model. Here, the goal is to judge whether the model can be used to predict the behaviour in real fires after its calibration to statistical data. The process of model calibration and model validation can help to find inconsistencies in the model structure, for example if the model is not able to represent the observed losses in different groups of buildings, e.g. small and large buildings. While this may give helpful hints for improving the model, it is clear that a statistical approach like the one described in this paper always has to be coupled with a thorough

understanding of the underlying physical processes.

In the present paper we show how a calibration procedure may be formulated that is able to deal with the two problems of data collection on portfolio level and incomplete data sets discussed above. The general framework is presented in Section 2. The calibration procedure is then applied to a generic model for fire risk assessment in single family houses described in the companion paper by De Sanctis et al. [18]. The idea is to start with a group of buildings where large amounts of data are available for exploring the possibilities of calibrating a fire risk model to real-world data. Section 3 contains a short introduction to the model and the data used for calibration, a discussion of the calibration results and some remarks on model validation. The paper ends with a short summary and conclusion in Section 4.

### 2. Framework for the calibration of an engineering fire risk model to statistical data

## 2.1. Calibration with data from non-homogeneous building portfolios

In a single building, fire is a rare event, but information on a large number of fire events in building portfolios is provided by e.g. fire insurance statistics or fire brigade reports. When using such data for model calibration, one has to bear in mind that different building characteristics may have an influence on the outcome of the fire. In the following, it will be discussed how a generic fire risk model can be calibrated to statistical data on fire events collected in a non-homogeneous portfolio of buildings.

#### 2.1.1. Generic fire risk modelling and calibration

The aim of generic fire risk modelling is to estimate the risk based on a set of indicators describing the system (e.g. the building or a building fire event). A risk indicator may be understood as any measurable or observable characteristic of a system or its components containing information on the risk (JCSS [19]). In the case of building fires, risk indicators may provide information on the characteristics of the building (e.g. floor area, number of rooms) and/or the fire event (e.g. room of fire ignition, fire brigade intervention time), see De Sanctis et al. [18]. The model output furthermore depends on a set of model parameters, which in contrast to the risk indicators are not observable. Model parameters can be defined based on engineering knowledge or estimated from statistical data during the process of model calibration. A simple example of a model parameter is the fire occurrence rate which cannot be directly observed but only estimated from statistical data.

The principles of generic fire risk modelling are described more in detail in De Sanctis et al. [18], [21]. For the calibration procedure discussed in this paper, it is sufficient to look at a generic model as a black box: for a given set of calibration parameters and a certain model input (e.g. a set of building-specific risk indicators), the model provides the probability distribution of the model output (e.g. the fire loss). With data containing evidence on both model input and model output in a non-homogeneous portfolio of buildings, it is possible to calibrate the model at portfolio level. In doing so, the engineering knowledge used to build the generic fire risk model is combined with information from observed fire events and the bias introduced by assumptions made during the modelling process is minimised.

#### 2.1.2. Calibration based on the Maximum Likelihood Method

As a starting point for formulating the calibration procedure we assume that there exists a generic probabilistic model assessing the distribution of the random model output Y conditional on a set

of risk indicators X=x (the model input) and the calibration parameters  $\Theta=\theta$  (bold letters denote vectors, upper case for random variables and lower case for realisations of random variables). Treating the model as a black box, it can be expressed as a conditional probability density function  $f_{Y|X,\Theta}(y|x,\theta)$ . We further assume that the data set used for calibration contains complete information on the model output and all risk indicators in n independent fire events and that it has not been used during model development. The observations are stored in a matrix  $\hat{\mathbf{x}} = \begin{bmatrix} \hat{\mathbf{x}}_1, \dots, \hat{\mathbf{x}}_n \end{bmatrix}^T$  for the model input and a matrix  $\mathbf{\hat{y}} = \begin{bmatrix} \hat{\mathbf{y}}_1, \dots, \hat{\mathbf{y}}_n \end{bmatrix}^T$  for the model output. During the calibration of the model to statistical data, the goal is to find the parameters  $\mathbf{\theta}^*$  leading to the best representation of the observations stored in  $\hat{\mathbf{y}}$  and  $\hat{\mathbf{x}}$  by the model.

When applying a generic model at portfolio level, the model output for each building will depend on the building-specific risk indicators (the model input) and on the calibration parameters, which are assumed to be the same for all buildings. A simple calibration procedure that is able to deal with data from nonhomogeneous portfolios can be formulated based on the Maximum Likelihood Method (see e.g. Rychlik and Rydén [13] for an introduction to this method). The idea of this method is to find the parameters of a probabilistic model that maximise the "likelihood" of the observations as evaluated by the model. Statistical data on observed fire events typically contain only one observation per building. Therefore, the likelihood has to be evaluated at portfolio level. When following a generic modelling strategy, this is not problematic, because the same model can be applied to a variety of buildings or fire events; the differences between the individual observations are captured by the risk indicators  $\hat{x}_i$ .

The likelihood L and log-likelihood l are defined as follows:

$$L(\boldsymbol{\theta}|\hat{\mathbf{y}}, \hat{\mathbf{x}}) = \prod_{i=1}^{n} f_{\mathbf{y}_{i}|\mathbf{x}_{i},\boldsymbol{\Theta}}(\hat{\mathbf{y}}_{i}|\hat{\mathbf{x}}_{i},\boldsymbol{\theta})$$

$$L(\boldsymbol{\theta}|\hat{\mathbf{y}}, \hat{\mathbf{x}}) = \sum_{i=1}^{n} \ln(f_{\mathbf{y}_{i}|\mathbf{x}_{i},\boldsymbol{\Theta}}(\hat{\mathbf{y}}_{i}|\hat{\mathbf{x}}_{i},\boldsymbol{\theta}))$$
(1)

The Maximum Likelihood parameters  $\theta^*$  are determined by maximising the likelihood L or, equivalently, by minimising the negative log-likelihood -l:

$$\theta^* = \min_{\theta} \left( -l(\theta | \hat{\mathbf{y}}, \hat{\mathbf{x}}) \right) \tag{2}$$

A nice property of the Maximum Likelihood approach is that it provides not only a point estimate for the calibration parameters but also their statistical uncertainty. If the data set used for calibration is sufficiently large, it may be assumed that the uncertain parameters  $\Theta$  are asymptotically normally distributed. Their expected value is the Maximum Likelihood estimate, i.e.  $E[\Theta] = \Theta^*$ . The covariance matrix  $C_{\Theta}$  of the parameters is determined as the inverse of the (observed) Fisher information matrix, which is defined as the negative Hessian matrix H of the log-likelihood function evaluated at the Maximum Likelihood estimate. For the example of two calibration parameters, this is expressed as follows:

$$C_{\Theta} = \begin{bmatrix} \operatorname{Var}[\theta_{1}] & \operatorname{Cov}[\theta_{1}, \theta_{2}] \\ \operatorname{Cov}[\theta_{1}, \theta_{2}] & \operatorname{Var}[\theta_{2}] \end{bmatrix} = \left\{ -H \Big|_{\theta=\theta^{*}} \right\}^{-1} = \left\{ -\begin{bmatrix} \frac{\partial^{2}l}{\partial^{2}\theta_{1}} & \frac{\partial^{2}l}{\partial^{2}\theta_{1}} \\ \frac{\partial^{2}l}{\partial^{2}\theta_{1}} & \frac{\partial^{2}l}{\partial^{2}\theta_{2}} & \frac{\partial^{2}l}{\partial^{2}\theta_{2}} \\ \frac{\partial^{2}l}{\partial^{2}\theta_{1}} & \frac{\partial^{2}l}{\partial^{2}\theta_{2}} & \frac{\partial^{2}l}{\partial^{2}\theta_{2}} \end{bmatrix}_{\theta_{1}=\theta_{1}^{*}}^{-1} \right\}$$

$$(3)$$

For practical applications, the Maximum Likelihood estimation

can be performed using a numerical routine to solve Eq. (2). The Hessian matrix is then typically determined as a by-product of the optimisation.

It should be noted that there is one implicit assumption of the Maximum Likelihood calibration which leads to a small inconsistency in the quantification of statistical uncertainty: it is assumed that the (unknown) true values of the model parameters  $\Theta$ are the same for all buildings or fire events. As a result, also the statistical uncertainty is assumed to be the same (and to have the same realisation) for all individual objects. This is not necessarily realistic when applying the model to a non-homogeneous portfolio of buildings. However, the simplification only affects the variance, not the mean value of the model output. Therefore, the framework is already applicable to many practical situations where a risk model shall be used only to assess expected values, e.g. the expected loss in a building or a portfolio of buildings. It should be noted that the abovementioned assumption only relates to the variability of the calibration parameters  $\Theta$ . A large share of the population variability is explicitly accounted for by modelling the fire risk for each building as a function of the building-specific risk indicators X.

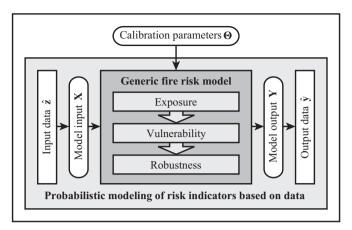
#### 2.2. Calibration with incomplete fire loss data

In the previous section, it was assumed that the data used for calibration contains complete information on a set of risk indicators describing the input X and output Y of the engineering model. With real data sets, the situation is often less favourable. In the following, we discuss how a calibration can be performed with a data set containing only little information on the buildings and/or fire events. Also the situation where information is available only for "large" fires is shortly discussed.

2.2.1. Missing information on the risk indicators used by the model Model calibration becomes very difficult or even impossible if the data contains no information at all on the specific conditions under which the observed fire losses occurred. However, in practise fire loss data typically provide some basic information on the buildings and/or the fire events, although not necessarily on the risk indicators used as a model input. Such information allows at least for a rough estimation of the risk indicators needed during the calibration. To give an example, fire insurance data may not contain information on the building's floor area, which is a basic input variable for most engineering approaches to fire risk assessment. However, for a given occupancy class, the floor area is generally correlated with the building's monetary (or insured) value. By making use of such correlations, the calibration procedure described in Section 2.1 can still be applied, but the uncertainty in the estimation of the risk indicators (e.g. floor area) from the information contained in the data (e.g. insured value) has to be quantified.

Fig. 2 illustrates the calibration procedure for data sets with limited information content. The risk indicators used as model input (vector X) are estimated from the available information (input data, vector  $\hat{\mathbf{Z}}$ ) using probabilistic or, if possible, deterministic assumptions. If necessary, a similar approach can be followed on the model output side (Here it is assumed that the data set  $\hat{\mathbf{Y}}$  contains the same variable as Y).

Instead of evidence from the data, an uncertain estimate of the model input now enters the calibration. Accordingly, the observed risk indicators  $\hat{X}_i$  in the likelihood formulation (Eq. (1)) are replaced by a random vector  $X_i$ . The distribution of  $X_i$  is specified conditional on a vector  $\hat{Z}_i$  containing the information provided in the data set. The assumptions on the model input side are expressed by a conditional probability density function  $f_{XZ}(x|z)$ . In



**Fig. 2.** Calibration with data containing only limited information on the risk indicators used by the model.

the example mentioned above, the input data  $\hat{z}_i$  would be the known insured value of the building i, the model input  $X_i$  would be its floor area and the probability distribution  $f_{X|Z}(x|z)$  could be constructed e.g. based on some knowledge on the range of typical prices per square metre floor area.

The log-likelihood can be reformulated based on the total probability theorem:

$$l\left(\boldsymbol{\theta}\middle|\hat{\mathbf{y}},\hat{\mathbf{z}}\right) = \sum_{i=1}^{n} \ln \left\{ \int_{D\mathbf{X}} f_{\mathbf{Y}_{i}\mid\mathbf{X}_{i},\boldsymbol{\Theta}}\left(\hat{\mathbf{y}}_{i}\mid\mathbf{x}_{i},\boldsymbol{\theta}\right) \cdot f_{\mathbf{X}_{i}\mid\mathbf{Z}_{i}}\left(\mathbf{x}_{i}\mid\hat{\mathbf{z}}_{i}\right) d\mathbf{x}_{i} \right\} \tag{4}$$

Where  $D_X$  is the domain of X. From a computational point of view, the likelihood formulation in Eq. (4) is highly inconvenient: due to the uncertainty in X, the number of model evaluations per entry in the data set is, at least in theory, infinite. For practical applications, the probability density function  $f_{X|Z}(x|z)$  can however be discretized and limited to a reasonable range. The level of discretization should reflect the uncertainty inherent in the distribution of the risk indicators, a rough discretization being appropriate for variables with a high degree of uncertainty. With a discrete probability mass function  $p_{X|Z}(x|z)$ , the log-likelihood is expressed as:

$$l\left(\boldsymbol{\theta}\middle|\hat{\mathbf{y}},\hat{\mathbf{z}}\right) = \sum_{i=1}^{n} \ln\left\{\sum_{j=1}^{k_{i}} f_{\mathbf{x}_{i}}|_{\mathbf{x}_{i},\boldsymbol{\Theta}}\left(\hat{\mathbf{y}}_{i}\middle|\mathbf{x}_{ij},\boldsymbol{\theta}\right) \cdot p_{\mathbf{x}_{i}}|_{\mathbf{Z}_{i}}\left(\mathbf{x}_{ij}\middle|\hat{\mathbf{z}}_{i}\right)\right\}$$
(5)

Here,  $k_i$  refers to the number of different model input combinations (possible realisations of the vector  $\mathbf{X}_i$ ) encompassed by the discrete probability mass function  $p_{\mathbf{X}|\mathbf{Z}}\!\left(\mathbf{x}_i \left| \mathbf{\hat{z}}_i \right.\right)$  for the data set entry

 $\hat{Z}_i$ . It depends on the number of variables in X and on the level of discretization used to define  $p_{X|Z}(X|z)$ .

The capabilities of the calibration approach for incomplete data sets are obviously limited by the information content of the data. Nevertheless, the calibration of a risk model can still be valuable even if the data base is very poor. Also a rough calibration may help to discover inconsistencies in the engineering model, e.g. if it is not able to reproduce the observed fire and loss characteristics in different groups of buildings. Finally, the lessons learnt during the calibration of a fire risk model can help to formulate the requirements for future data collection.

#### 2.2.2. Calibration with data sets limited to large fire events

The discussion above was focussed on the problem of limited information content of the data used for calibration. Another problem typical for fire loss data is that information on small fire events is missing, e.g. on those fires that are not reported to the fire brigade or insurance company. This situation can, however, easily be handled by a conditional Maximum Likelihood approach. The calibration is then still based on the likelihood formulation in Eqs. (1) or (5), but the (unconditional) distribution of the model output,  $f_{Y|X,\Theta}(y|x,\theta)$ , is replaced by a distribution conditional on the event determining whether the fire is included in the data base. A simple example is the case of insurance data containing only losses larger than a certain excess (deductible)  $y_0$ . The conditional distribution  $f_{Y|X,\Theta,Y>y_0}(y|x,\theta,Y>y_0)$  is in this case derived from  $f_{Y|X,\Theta}(y|x,\theta)$  by truncation at  $y_0$ .

#### 3. Calibration of a fire risk model for single family houses to Swiss insurance data

#### 3.1. Short introduction to the engineering risk model

In the following, the calibration procedure is applied for the calibration of a simple fire risk model to data from real fire events. The model used for testing our approach is a generic fire risk model for single family houses. This example was chosen because it deals with a well-defined group of buildings where large amounts of data are available for calibration. The model estimates the probability distribution of the financial loss due to a fire for a given set of building-specific risk indicators. It may be used e.g. in an insurance context or for the economic evaluation of codemaking decisions from a societal point of view (Fischer [22]). Loss of life and injuries are not within the scope of the model. The monetary losses refer to damages at the building structure only. Loss of contents and consequential losses are excluded for consistency with the data set used for calibration (see Section 3.2).

The model is described in detail in a companion paper focusing on the development of a generic fire risk model which facilitates calibration, see De Sanctis et al. [18]. Herein, only a short introduction to the model is provided for the convenience of the reader.

In the generic risk model, each house is described by a set of building-specific risk indicators listed in Table 1. The table also contains the definition of some fire-specific risk indicators and the model calibration parameters. For a complete list of all indicators used by the model see De Sanctis et al. [18].

An overview on the model structure can be found in Fig. 3. The model consists of several sub-models, each of which will be shortly described in the following.

#### 3.1.1. Ignition model (Exposure)

The focus of the present paper is on the calibration of a model for the fire risk conditional on a fire. If the goal is to assess yearly risk, one has to multiply with the yearly rate of fire occurrence. Both models (fire occurrence and consequences given fire) have to use the same definition of a fire event. The ignition model we use determines the rate of fire occurrence based on a "power law" function of the insured value in CHF. The parameters were fitted to Swiss insurance data for residential buildings with an insured value below 1Mio. CHF, see Fischer et al. [23]. "Fire occurrence" implies that the fire has been reported to the insurance company. The same data is used to calibrate the loss model, see Section 3.2.

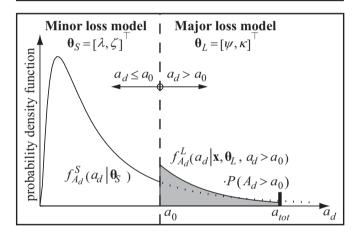
#### 3.1.2. Minor loss model (Vulnerability)

The analysis of fire insurance statistics shows that the sum of fire losses is dominated by large losses, see e.g. Fontana et al. [24]. For the single family house model this means that small losses, e.g. those resulting from fires confined to the room of fire origin, are of

**Table 1**Risk indicators used as input for the fire risk model and definition of the calibration parameters (upper case for random variables, lower case for realisations).

Building-specific risk indicators (model input)	Х	х
Total building floor area [m²]	A <sub>tot</sub>	a <sub>tot</sub>
Floor area of largest room [m <sup>2</sup> ]	Amax	a <sub>max</sub>
Number of rooms [dimensionless]	N <sub>R</sub>	$n_R$
Number of connections between rooms [dimentionless]	N-	nc
Insured value [CHF]	V	ν
Fire-specific risk indicators		
Floor area of room of fire ignition [m²]	$A_0$	$a_0$
Area of fire spread [m <sup>2</sup> ]	$A_d$	$a_d$
Model output		У
Financial loss (building structure) [CHF]	С	c
Calibration parameters		θ
Minor loss model parameters $\Theta_S = [\Lambda, Z]^T$	$\Theta_{S}$	$\theta_S$
Mean of log-log fire spread area Ad	Λ	λ
Std. dev. of log-log fire spread area $A_d$	Z	ζ
Major loss model parameters $\Theta_L = [\Psi, K]^T$		$\Theta_L$
"Fire spread coefficient"	Ψ	Ψ
"Control time exponent"	K	κ

#### **Ignition model**



#### Financial loss model

**Fig. 3.** Overview on the components of the engineering model described in De Sanctis et al. [18].

minor importance for the expected loss. Therefore, these "minor losses" are modelled based on a simplified statistical approach, with engineering modelling focusing on the tail of the loss distribution, see Fig. 3. For the minor loss model  $f_{Al}^S$  ( $a_d | \theta_S$ ) we assume a (shifted) log-lognormal distribution for the fire spread area  $A_l$ . The distribution is independent of the building-specific risk indicators and used only for small losses confined to the room of fire origin ( $a_l \leq a_0$ ). The distribution parameters  $\theta_S = [\lambda, \zeta]^T$  (mean and standard deviation of the log-log fire spread area) are estimated from the data, while the shift is fixed and introduced only to ensure positive values in the logarithm. The probability mass in the tail of the loss distribution is redistributed according to an engineering model, the "major loss model". The minor loss model here only provides the probability of fire spread beyond the room of fire origin:

$$P(A_d > a_0) = 1 - F_{Ad}^{S} \left( a_0 | \boldsymbol{\theta}_S \right) \tag{6}$$

#### 3.1.3. Major loss model (Robustness)

The major loss model becomes relevant if the fire has spread beyond the room of fire origin  $(a_d > a_0)$ . The conditional density function  $f_{\mathcal{A}_l}^L\left(a_d \big| \mathbf{x}, \pmb{\theta}_L, a_d > a_0\right)$  is derived from an engineering model that is composed of a fire spread model and a fire brigade model. The calibration parameters  $\pmb{\theta}_L = [\psi, \kappa]^{\mathsf{T}}$  are related to these two sub-models.

The fire spread model describes the development of the area damaged  $A_d$  as a function of time conditional on the risk indicators  $[a_{tot}, a_{max}, n_R, n_C]^T$  (see Table 1). The model is modified by the "fire spread coefficient"  $\psi$ . This parameter describes how well the reference case ( $\psi = 1$ ) used during model development represents reality. For higher  $\psi$ , the fire develops faster and vice versa.

The fire brigade model is based on a simple time-line approach: first, the starting time of fire brigade actions is defined as a random variable. Next, the average time needed to extinguish or confine the fire ("control time") is modelled as a "power law" function of the area damaged at the starting time, which is estimated using the fire spread model mentioned above. The exponent defining the shape of this control time model is the second calibration parameter  $\kappa$ , with  $\kappa=1.0$  denoting a linear relationship between the area damaged and the control. An initial estimate for this parameter,  $\kappa=0.52$ , was derived based on foreign fire brigade statistics. Finally, the area damaged  $A_d$  at the end of the fire brigade actions is again determined based on the fire spread model.

An effect of the engineering approach developed for the major loss model is that the upper tail of the damage size distribution is not explained by the log-lognormal distribution chosen for the minor loss model anymore. In addition, it should be noted that in contrast to the simplified minor loss model, the major loss model for the simplified minor loss model, the major loss model for the building-specific risk indicators summarised in x (see Table 1) and thus specific to each building evaluated by the risk model.

#### 3.1.4. Financial loss model

Both the minor and the major loss model are defined in terms of the "fire spread area"  $A_d$ , which should be understood mainly as a proxy for the monetary fire loss, see De Sanctis et al. [18]. For the conversion to financial losses it is assumed that the ratio between the monetary loss C and the insured value V is the same as the ratio between the area damaged  $A_d$  and the total floor area  $A_{tot}$ , i.e.  $C = V \cdot A_d / A_{tot}$ .

#### 3.2. Description of the data set used for calibration

For the calibration of the risk model described in Section 3.1 we use fire loss data provided by AGV, the public building insurance company of Aargau (a canton/state of Switzerland). The loss data includes also small losses, as no excess (deductible) is borne by the policy holders. Only the building structure is insured by AGV; losses to contents and consequential losses are insured on the private market. The data provides information on all claims due to fires in single family houses (detached, semi-detached and row houses) submitted to AGV from 1999 to 2008. The resulting data set contains the following information on n = 1996 fire events: the building's insured value, its year of construction, its volume in  $m^3$  and the fire loss amount.

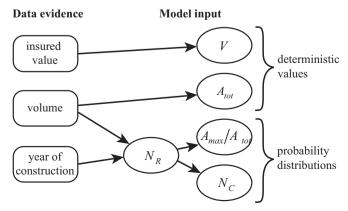
A comparison with the risk indicators used by the model (Table 1) reveals that only for the insured value V and the financial loss C information is readily available. The missing building-specific risk indicators were estimated based on the evidence provided by the data. This was done partly using deterministic assumptions as in the case of the total floor area  $A_{tot}$  which was calculated from the building's volume assuming a room height of 2.7 m. For the remaining indicators  $N_R$ ,  $N_C$  and  $A_{max}$ , we derived

probability distribution functions using a two-step procedure: first we estimated the probability distribution function of the number of rooms  $N_R$  based on the building's volume and the year of construction. For this task, we used statistical information published by the Swiss Federal Statistical Office [25]. For the number of connections between rooms,  $N_C$ , and the size of the largest room relative to the total floor area,  $A_{max}/A_{tot}$ , we assumed probability distribution functions conditional on the number of rooms  $N_R$ . No statistical information could be found on these two risk indicators. In order to come to reasonable estimates, we conducted a survey of typical single family house layouts found on the online real estate portal homegate (www.homegate.ch). These estimates can be expected to be very uncertain. An illustration of the procedure used to estimate the building-specific risk indicators is found in Fig. 4.

The limitations of the data set used for calibration are obvious: almost all building-specific risk indicators in Table 1 had to be estimated based on assumptions and information on fire development, fire brigade actions and risk reduction measures is missing completely. Another factor is the small number of large losses (only 10 out of 1996 claims are larger than 500,000 CHF), especially for the calibration of model components that mainly influence the tail of the loss size probability distribution. The amount of data can be increased e.g. by extending the data set to a longer time period. Improving the quality of the data requires more effort, but could be aimed at in the future: all risk indicators listed in Table 1 are, at least in principle, quantifiable.

The companion paper by De Sanctis et al. [18] also discusses a number of fire specific risk indicators, which are also observable in the case of a fire event, e.g. the floor area of the room of fire origin or the fire brigade intervention time. For model calibration with the data set described above, these risk indicators have been modelled as random variables within the fire risk model and are therefore not part of the model input X. Note that the same approach is required for risk assessment prior to the occurrence of a fire, where information on the fire-specific risk indicators is also not available.

A full list of all risk indicators used by the model is provided in Table 1 of De Sanctis et al. [18]. This list can be used as a starting point for defining future data collection requirements. As a minimum, the data should contain full information on the five building-specific risk indicators defined as model input for the present paper. In addition, also information on the fire event should be collected, including e.g. the floor area of the room of fire origin, the fire brigade intervention time and the final fire spread area or the number of rooms affected by the fire.



**Fig. 4.** Illustration of the assumptions made to estimate the building-specific risk indicators (model input) from the evidence provided by the data.

#### 3.3. Application of the calibration procedure

In the following, it is shown how the single family house model was calibrated to the observed loss data. As discussed in Section 3.1, the model is composed of a simple minor loss model for fires confined to the room of fire origin (small losses) and a more complex engineering model for major losses. The respective calibration parameters are  $\theta_S = [\lambda, \zeta]^T$  for the minor loss model and  $\theta_l = [\psi, \kappa]^T$  for the major loss model. The characteristics of each building i are described by the random vector  $X_i = [A_{tot,i}, A_{max,i}, N_{R,i}, N_{C,i}, V_i]^T$  (see Table 1 for variable definition). As the data set contains no evidence on most of these buildingspecific risk indicators (see Section 3.2), the model input is generated by a discrete probability distribution  $p_{X|Z}(x|z)$  for the random vector X<sub>i</sub> conditional on the information available in the data set. The data input is represented by a vector  $\hat{z}_i$  containing the insured value, the volume and the year of construction for each individual building. The model output is defined as the financial loss in case of fire, i.e.  $Y_i = G$ .

The incomplete data set requires estimating the likelihood based on Eq. (5), which is computationally expensive. The number of model evaluations  $k_i$  per observation could be reduced by assuming that combinations of risk indicators with  $p_{X_i|Z_i}\left(x_{ij}\left|\hat{Z}_i\right.\right)<10^{-6}$  are negligible. Further reductions in computation time were achieved by estimating the calibration parameters  $\theta_S$  of the minor loss model separately from the parameters  $\theta_L$  of the major loss model: first the minor loss model is fitted to the whole data set and then the calibration of the engineering model is performed with fixed minor loss model parameters:

$$\mathbf{\theta}_{L}^{*} = \min_{\mathbf{\theta}_{L}} \left( -l \left( \mathbf{\theta}_{L} \middle| \hat{\mathbf{y}}, \, \hat{\mathbf{z}}, \, \mathbf{\theta}_{S}^{*} \right) \right) \tag{7}$$

Eq. (7) was solved using the interior-point algorithm implemented in the MATLAB<sup>®</sup> routine *fmincon*.

#### 3.3.1. Calibration results and comparison with the data

Using the procedure described above, the calibration parameters are estimated as:

$$\theta^* = [\lambda, \zeta, \psi, \kappa]^{\mathsf{T}} = [-0.13, 0.91, 1.73, 1.44]^{\mathsf{T}}$$
 (8)

Based on the Maximum Likelihood parameters  $\theta^*$  and the building-specific information  $\hat{z}_i$  contained in the data set, the loss size probability distribution function for each building (with index i) can now be determined based on the total probability theorem:

$$f_{f_{i}\mid\mathbf{z}_{i},\boldsymbol{\Theta}}\left(y_{i}\mid\mathbf{\hat{z}}_{i},\boldsymbol{\Theta}^{*}\right) = \sum_{j=1}^{k_{i}}f_{f_{i}\mid\mathbf{x}_{i},\boldsymbol{\Theta}}\left(y_{i}\mid\mathbf{x}_{ij},\boldsymbol{\Theta}^{*}\right)\cdot p_{X_{i}\mid\mathbf{z}_{i}}\left(\mathbf{x}_{ij}\mid\mathbf{\hat{z}}_{i}\right)$$

$$\tag{9}$$

Here,  $f_{k_i|X_i,\Theta}$  is the probability density function for the model output  $Y_i$  (financial loss in building i) conditional on the model input, i.e. the building-specific risk indicators  $x_i$ . The model has to be evaluated  $k_i$  times to account for the uncertainty in the unknown risk indicators  $X_i$  as expressed by the discretized probability mass function  $p_{X_i|Z_i}$  (see also Section 2.2.1). It should be noted that, even though the same model is used for all buildings, the probability density functions  $f_{X_i|X_i,\Theta}$  and  $f_{X_i|Z_i,\Theta}$  are specific to the building i due to the conditioning on the risk-indicators  $x_i = \left[a_{tot,i}, a_{max,i}, n_{R,i}, n_{C,i}, v_i\right]^\mathsf{T}$  and the information  $\hat{z}_i$  contained in the data set, i.e. the insured value, the volume and the year of construction.

For comparison with the data, the loss size distribution has to be aggregated at portfolio level:

$$f_{Y,Portfolio}(y) = \frac{1}{n} \sum_{i=1}^{n} f_{fi}|_{Z_{i},\Theta} \left(y_{i} \middle| \hat{Z}_{i}, \Theta^{*} \right)$$

$$= \frac{1}{n} \sum_{i=1}^{n} \sum_{j=1}^{k_{i}} f_{fi}|_{X_{i},\Theta} \left(y \middle| X_{ij}, \Theta^{*} \right) \cdot p_{X_{i} \middle| Z_{i}} \left(X_{ij} \middle| \hat{Z}_{i} \right)$$

$$(10)$$

Where n denotes the number of buildings in the considered portfolio, or the sample size of the data set. The probability density function  $f_{Y,Portfolio}$  may be interpreted as the loss size distribution for an arbitrary building, which is averaged over all individual buildings in the portfolio. Fig. 5 shows a comparison of the portfolio loss size distribution with the data. The cumulative distribution function  $F_{Y,Portfolio}(y)$  is illustrated in Fig. 5a, which allows judging the overall fit of the model in the whole range of the observed losses. Plotting the complementary cumulative probability distribution function  $1 - F_{Y,Portfolio}(y)$  on a logarithmic scale, as in Fig. 5b, puts emphasis on the important upper tail of the probability distribution. The expected value E[c] assessed on the basis of the modelled probability distribution function shows a 7% deviation from the sample mean.

A new engineering risk model cannot be expected to perfectly represent the observations made in real fires right from the beginning; typically a few iterations are required to improve the model. In the case of our single family house model, the results of the first calibration trials revealed problems with parts of the model that could not be calibrated due to the sparse information contained in the data set. An advantage of our approach is that engineering knowledge can fill the gap when information is lacking in the observed data and vice versa. The choice of calibration parameters was therefore guided by the availability of quantitative engineering knowledge: parameters with a clear physical meaning, like the time of fire spread beyond the room of fire origin, can be defined based on physical models or expert judgement. Processes that are more difficult to quantify are captured by the calibration parameters.

3.3.2. Effect of the calibration on the portfolio loss size distribution Fig. 6 shows a comparison of the single family house model before and after calibration with the observed loss size distribution at portfolio level. All model curves are based on the Maximum Likelihood parameters for the minor loss model,  $\theta_s^* = \left[\lambda^*, \zeta^*\right]^T = [-0.13, 0.91]^T$ . Therefore, the lower part of the loss size distributions is the same for both models.

The loss size distribution before calibration (dashed line) is based on an initial estimate for the major loss model parameters  $\theta_L = [\psi, \kappa]^T$ , see De Sanctis et al. [18] for details. The bias of this model may be evaluated e.g. by comparing the expected fire loss E[c] to the sample mean of the loss data. It is seen that the model underestimates the observed fire risk by more than 20%. This bias can be reduced by using the Maximum Likelihood parameters  $\theta^*$  (solid line), which leads to an overestimation of the expected loss E[c], but only by about 7%. The positive effect of calibration is even more pronounced when comparing exceedance probabilities for large losses. To give an example, the probability of a loss larger than 500,000 CHF is between 0.004 and 0.005 both in the observed loss size distribution and in the calibrated model. The estimate provided by the model before calibration, 1.6·10<sup>-4</sup>, is more than an order of magnitude smaller.

For illustrative purposes, also the shifted log-lognormal distribution used for the minor loss model is shown in Fig. 6. Note that this distribution is not truncated at the building's insured value, as this is a feature of the building-specific major loss model. Therefore, the probability of large losses is strongly overestimated by the minor loss model.

The individual effect of the two calibration parameters for the

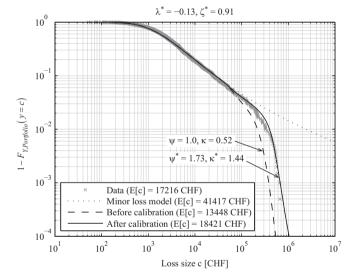


Fig. 6. Comparison of the model before and after calibration.

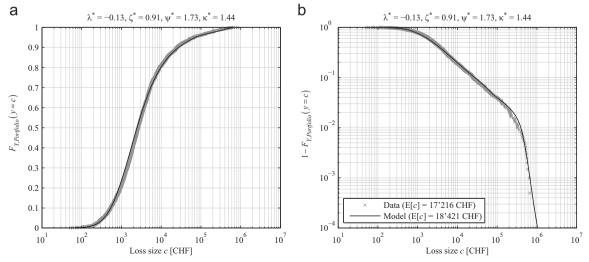


Fig. 5. Comparison of the calibrated model with the data.

major loss model,  $\theta_L = [\psi, \kappa]^T$ , is illustrated in Fig. 7. The solid lines in both graphs are based on the Maximum Likelihood parameters  $\theta^*$ . In Fig. 7a only the fire spread coefficient  $\psi$  is varied. Choosing a higher value (e.g.  $\psi^* + 0.5$ , as illustrated with the dotted line) implies increasing the velocity of the fire development in the model and therefore increases the probability of large loss events, and vice versa.

Fig. 7b shows the effect of varying the control time exponent  $\kappa$ . It is seen that this parameter has an effect mainly on the shape of the loss size probability distribution. When comparing the expected losses E[c] in Fig. 7b, one may argue that the data can be represented better when choosing a larger value than the Maximum Likelihood estimate  $\kappa^* = 1.44$ . However, our aim is not to model the expected loss of an average building at portfolio level but to best represent the loss size probability distribution at object level, as a function of the building-specific risk indicators. Whether this goal was achieved is discussed in the following.

#### 3.3.3. Results for different sub-portfolios

Fig. 8 shows a comparison of the model with the observed losses for different groups of buildings. The effect of the building characteristics is illustrated by dividing the data into two

equal-sized groups according to different risk indicators. For Fig. 8a, the data set has been separated according to the building's insured value V. The distribution of the loss size in both subsets is estimated using the Maximum Likelihood parameters  $\theta^*$ ; only the risk indicators describing the individual buildings differ. The comparison with the observed loss size distributions shows that the model is able to describe the differences between the two groups of buildings fairly well. It should be noted that the statistical uncertainty in the tail of the observed loss distribution is higher than in Figs. 5 and 7 because of the reduced sample size.

Fig. 8b shows a similar analysis: again the data set is divided in two equal-sized groups, but this time according to the volume of the buildings. Also here the model is at least qualitatively able to describe the different characteristics of the two groups, but not as well as in Fig. 8a. This may be explained by the fact that the building's volume is not directly used as a model input. However, most of the risk indicators needed by the engineering model are directly or indirectly derived from it, see Fig. 4. This indirect estimation of the model input introduces large uncertainties into the calibration procedure. With a data set containing evidence on all model input risk indicators, it should be possible to produce better results.

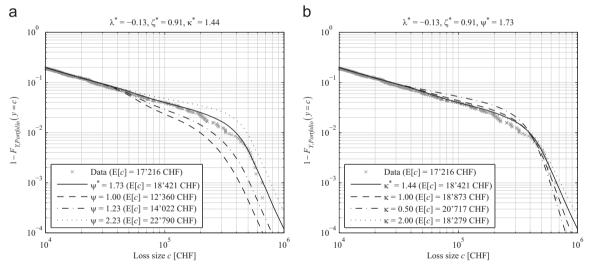


Fig. 7. Effect of the calibration parameters on the loss distribution at portfolio level.

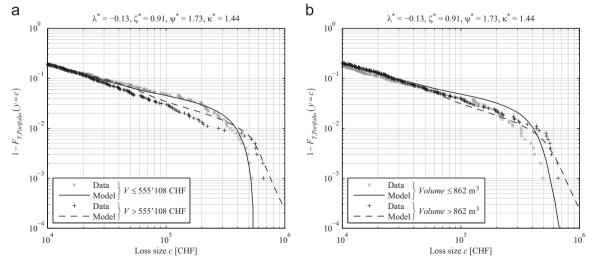


Fig. 8. Comparison of the model with data for different types of buildings. Model validation and discussion of the calibration results.

In engineering decision-making, the goal is not simply to describe or explain the observations made in the past; instead, the models shall be used to predict future outcomes of decisions. The aim of model validation in this context is to judge whether it can be reasonably assumed that the model is able to estimate the fire risk in a new context after its calibration to a limited data set. Only an informal discussion of this requirement is presented in the following, as the focus of the present paper is on model calibration rather than validation; a more rigorous treatment can be found e.g. in Hastie et al. [26].

A simple approach for model validation is to calibrate the model only to a subset of the observations before applying it to the remaining data. The two subsets should be comparable in terms of the range of relevant building characteristics to avoid problems introduced by the effect of different risk indicators. For Fig. 9 we randomly selected 60% of the data to be used as a "training set", i.e. for calibrating the model. The resulting Maximum Likelihood parameters are similar, but not equal to the parameters in Eq. (8) that were derived from the calibration with the whole data set. The model is then applied to the remaining 40% of the data; this "test set" is used to compare the prediction with observations. The same procedure is applied ten times with different random training sets; Fig. 9 shows only two examples. The modelled loss size probability distributions of the two subsets (solid and dashed line) differ only slightly because the probability distribution of the building-specific risk indicators is similar. Therefore, the best results are achieved with random subsets where also the observed loss distributions were similar. Fig. 9a shows an example with only small differences in the tail of the probability distributions that can easily be explained by statistical uncertainty. The worst outcome out of the ten validation trials is shown in Fig. 9b.

No trend could be observed that might explain the differences between the model calibrated to a random training set and the observations in the corresponding test set. Instead, outcomes like the one in Fig. 9b seem to occur completely at random. The differences can be attributed to the large variation of the loss size, of which only a small fraction is explained by the building-specific risk indicators.

Accounting for fire specific risk indicators such as e.g. the ignition source, room of fire ignition or presence of fire brigades would improve the situation. Note, however, that this information is not available when estimating fire risk before a fire has occurred. This lack of information leads to considerable statistical uncertainty especially in the important upper tail of the loss size

probability distribution, which explains the large variation in observed frequencies of large losses when using small data samples to estimate  $F_{Y,Portfolio}(y)$  (e.g. when comparing the training and test data in Fig. 9b). The effect of statistical uncertainty on the results of model calibration can be reduced by increasing the sample size, e.g. by using the whole data set for calibration as in Section 3.3. However, even with a very large data set, model calibration can only reduce the bias, or systematic error, of the model and not the inherent uncertainties of building fire events.

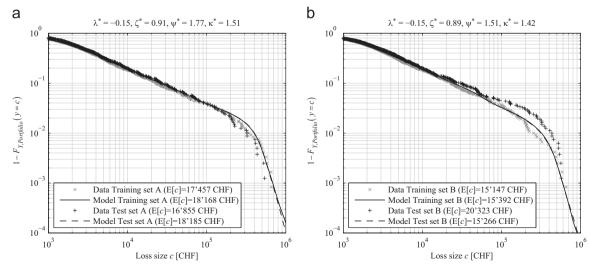
Our goal was to develop a risk model that after calibration with fire loss data can be applied for evaluating the efficiency of risk reduction measures in a non-homogeneous building portfolio. Based on this goal we can derive three different requirements that have to be fulfilled by the model:

Physical modelling approach: the model assumptions and the final performance of the model have to be consistent with the physical processes underlying the problem. This property of the model is important for evaluating the effect of fire safety investments, especially if no information on the risk reduction measures is contained in the data.

Influence of building-specific risk indicators: the behaviour of the model for buildings with different risk indicators has to be consistent with the observed losses in different groups of buildings. A good fit to the data in relation to different building characteristics is also an indicator for an appropriate physical modelling.

*Overall fit to the data*: after aggregation at portfolio level, the model has to be able to represent the observed loss size probability distribution. This property is important for modelling the risk in absolute terms (expected loss in CHF) with as little bias as possible.

The Maximum Likelihood calibration improves both the overall fit to the data and an appropriate dependence of the building-specific risk indicators. The calibration does, however, not guarantee that the model is consistent with the physics underlying the problem. Judging the plausibility of the risk assessment results from an engineering point of view thus remains an important task during the process of model development. Adjusting the model assumptions to achieve an optimal fit to the data while disregarding physical understanding of the fire problem is clearly not a valid approach. A good engineering model will not require much trade-off between the three requirements discussed above: the fit



**Fig. 9.** Validation results for two different random training sets (60% of the data) and corresponding test sets (remaining 40% of the data). (a) shows a typical example with only small differences between the two samples, (b) the worst outcome out of ten validation trials.

to the observed loss data will generally be good if the model is able to represent the physics and characteristics of real fire events.

#### 4. Summary and conclusion

In the present paper it is shown how an engineering model for fire risk assessment can be calibrated to statistical data on real fire events, e.g. fire insurance statistics or fire brigade reports. The data is usually collected in a non-homogeneous portfolio of buildings with different risk-relevant characteristics. In a companion paper by De Sanctis et al. [18] it has been shown how the differences between the individual buildings can be accounted for by developing generic models for fire risk assessment. The framework presented in this paper allows to calibrate such a model to statistical data. The framework is applicable also if the data does not contain full information on the model input or if information on small fire events is missing.

As an example, we apply the calibration procedure to a generic fire risk model for single family houses that was described in De Sanctis et al. [18]. The calibration is conducted using Swiss building fire insurance data. Additional assumptions for several model input variables were necessary because the data set does not contain full information on all risk indicators required by the model. Nevertheless, the calibration can reduce the bias in the engineering model considerably. We also show how the model can be validated by using only one part of the data for calibration.

The procedure described in this paper provides a consistent way of combining a physical modelling approach with statistical information from real fire events. The calibration with real-world data helps to reduce the bias (systematic error) introduced by simplified modelling assumptions. This is important especially in the context of cost-benefit studies where the risk reduction due to a fire safety measure shall be compared to its costs.

Even with data containing only very little information on the problem at hand, the performance of the model can be improved with the aid of model calibration. Besides the direct bias-reducing effect, the calibration and validation of an engineering model with statistical data can reveal inconsistencies in the model structure and foster an improved physical understanding of the problem at hand. Finally, the development of generic fire risk models helps to provide the requirements for future data collection by defining observable risk indicators that contain information on the risk and by highlighting the deficiencies of current data collection efforts. Based on this feedback loop, both the physical models and the data collection can be improved in the long run.

The calibration of engineering models is obviously limited by the data available for calibration. This holds especially for rare events like structural collapse or multiple death fires, where the data base will always remain small. But also the quality of the physical models can be a limiting factor: calibrating a model that is not able to capture the behaviour of real fires at least qualitatively will not be successful. The strength of our approach is the combination of engineering knowledge with statistical data: observations from real fire events are most helpful in areas where the uncertainties are high and the understanding of the physical processes is poor. Engineering models, on the other hand, can be used to fill the gaps in the available data.

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