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Estimations of the probability of fire occurrences in buildings

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Abstract

Fire occurrences in a particular building are really rare events. It is assumed that fires occur in accordance with the Poisson process and the number of fire occurrences in time interval can be modeled by a Poisson distribution. Using data such as the numbers of annual fire occurrences and building floor area the probability of fire occurrence in different occupancies can be estimated. In addition, the relation between the numbers of fire occurrence and the time of fire occurrence are clearly discussed. Investigations for different groups of building occupancy are illustrated in this research. Based on mean fire ignition rates and floor areas of different occupancies results show the ranking order regarding annual fire ignition per unit floor area among these occupancies. Industrial occupancy has the highest value of annual fire ignition rate, followed by residential occupancy. The rate of fire ignition only increases in public occupancy in Taiwan.

Keywords: Fire ignition rate; Fire occurrence probability; Numbers and time of fire occurrence; Poisson process

1. Introduction

Fire occurrence is really a random phenomenon affected by several uncertain factors. Various approaches have been developed to predict the occurrences due to

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fires for different types of building structures. Rutstein and Clarke [1] estimate the fire probability for different types of industrial premises. In their study, the probability of a fire is estimated by dividing the number of fires that occur each year by the number of buildings at risk. The probability of fire is clearly a non-linear function of building size. They have provided two possible reasons for describing this non-linearity. Rutstein [2] also describes a study that the fire hazard (annual fire initiation) of different occupancies can be estimated. Statistical studies reviewed by Ramachandran [3] show that the probability of a fire starting can be estimated by $F(A) = kA^{\alpha}$, where A is the total floor area of the building and k and α are constants for a particular group (risk category) of buildings. $F(A)$ is usually expressed on an annual basis and is a power function of area. Values of α are usually not close enough to 1 to be approximated as 1. Values of k and α for major building types are available for the UK [2].

Lie [4] and Burros [5] have developed methods to predict the probability of fire occurrence and to determine the probability of structural failure and monetary loss expectation due to fire. Because building fires are statistically rare events, it can be assumed that fire initiation follows a Poisson process with the mean rate of annual fire initiation (number of fires per m^2 of floor space per year) based on statistical data. Models of both Lie and Burros for estimating the probability of fire occurrence in buildings are based on this assumption.

A series of studies regarding ignition frequencies and ignition frequencies per floor area are determined for different building categories in Finland by Rahikainen, Keski-Rahkonen and Tillander [6–9]. One of these studies shows that for ignition frequencies per floor area for small buildings a strong dependence on size is observed, but it remains approximately constant for larger buildings. Additionally, periodical variations of ignition frequencies of buildings by month and week of the year, day of the week, and time of day are determined [9].

Even though various studies have moved beyond the fire occurrence or ignition such as flashover-typed and non-flashover-typed fire estimation, (conditional) probability of fire resistance failure, probability of building egress failure and fire losses (life losses and property losses) analysis [10–12], fundamental issue of fire occurrence should be discussed precisely. The objective of this paper is to generalize and modify the ideas regarding the probability of fire occurrence that Lie and Burros presented before. In addition, this study focuses on the connections between the numbers of fire occurrences and the time of fire occurrence. Investigations for different groups of occupancy in Taiwan are presented.

2. Model for estimating the probability of fire occurrence in buildings

2.1. Theory of Poisson process

Many physical problems of interest to engineers involve the possible occurrences of events at any point in time or space. For example, defects may occur on a special length of magnetic recording tape; earthquakes could strike at any time and anywhere over a seismically active region; and fires could happen at any time in any particular building. In such cases, the occurrences of the event may be appropriately modeled with a Poisson process.

On the basis of assumptions of a Poisson process [13], the number of occurrences of a fire in time t within a particular building is given by Poisson distribution; that is, if X is the number of fire occurrences in time t, then

$$P(X = x) = \frac{e^{-\lambda t}(\lambda t)^x}{x!}, \quad x = 0, 1, 2 \ldots, \text{etc.}, \tag{1}$$

where λ is the mean fire ignition rate or the average number of fire occurrence per unit time interval. Usually λ is measured in year^{-1} if t is measured in years. It follows that $E(X) = \lambda t$ and it can be shown that the variance of X is also λt.

2.2. *Derivation of fire occurrence probability*

In this section, random generations of fire occurrences for different considerations in building compartments and building purposes are presented.

2.2.1. *Building compartments are all the same*

Assume that a building is divided into a number of compartments, each of which has the same floor area, is equally equipped, and is used in the same way; that is under the case of particular compartmentation, fire ignition rate in such compartmented building may be assumed to be increased with the size of the building. This is approximately true for multi-story buildings such as offices, apartments and hotels that are subdivided into compartments of equal size [4]. However, this does not mean that the probability of fire occurrence is proportional to floor area, it should be calculated by using Eq. (2), which shows the power relation between fire probability and floor area. It is worth noting that using an ignition frequency per unit area is only a rather crude approximation. It is also recommended that all data under the situation of same compartmentation should be interpreted and applied with care, for example, careful consideration of the building size and the change of the amount of fire safety measures and service utilities while the building is enlarged.

Based on the considerations above, with the same compartments of buildings, Eq. (1) can be rewritten as

$$P(X = x) = \frac{e^{-A_F p t}(A_F p t)^x}{x!}, \quad x = 0, 1, 2, \ldots, \text{etc.}, \tag{2}$$

where A_F defines the floor area, p denotes the rate of fire ignition per year per unit floor area, and $A_F p$ denotes the mean fire ignition rate and is the same as λ in Eq. (1).

2.2.2. *Building compartments are not all the same*

It will be interesting to consider the more general case of unequal compartments (or various sectors, purposes, uses in next section). Consider a building that can be divided into n possibly unequal compartments (or various sectors, purposes and uses).

Let Y_i be the number of fire occurrences in time interval t within compartment i, $i = 1,2,\ldots n,$ and assume fire occurrences in that compartment are generated by its own Poisson process with mean ignition rate λ_i, which is independent of all other compartments. Then

$$P(Y_i = y_i) = \frac{e^{-\lambda_i t}(\lambda_i t)^{y_i}}{y_i!}, \quad y_i = 0, 1, 2, \ldots, \text{etc.}, \tag{3}$$

where λ_i is the mean fire ignition rate in compartment i. Let $Y = Y_1 + Y_2 + Y_3 + \cdots + Y_n$ denote the total number of fire occurrences within time t in the entire building. For $i = 1, 2, \ldots n$, since fire occurrence in each compartment has its own Poisson process with mean ignition rate λ_i and behaves independently. Equivalently, Y_1, Y_2, Y_3,... Y_n are all independent Poisson random variables and the distribution of $Y = Y_1 + Y_2 + Y_3 + \cdots + Y_n$ must be that Poisson distribution with parameter $\lambda_1 + \lambda_2 + \cdots + \lambda_n$. The total number of fire occurrences Y is the same as X in Eq. (1) and the sum of $\lambda_1 + \lambda_2 + \cdots + \lambda_n$ is the same as λ in Eq. (1).

2.2.3. Building is divided into sectors with different purposes

It will be more useful and practical to take into account the heterogeneity of risk within the various sectors (purposes or uses) in a building. This is a special and common issue in Taiwan, since Taiwan is a highly populated country and there are many multi-purpose or high-rise buildings over the area particularly in several commercial cities. A multi-purpose building is often used as a complex for offices, restaurants, theatres and shops in Taiwan. In this case the fire ignition rate may be considered on the basis of different sectors within the building. Based on the assumptions discussed above (similar as discussion in 2.2.2), the probability of fire occurrence of a building that is possibly divided into several different sectors can be appropriately modeled by a Poisson process. Similarly, also assume that fire occurrences in each sector constitute a Poisson process with different mean fire ignition rate and behave independently. Then, the probability of fire initiation in each sector in time t can be described by applying Poisson distribution function, and therefore so can in the entire building. Investigations of Taiwan buildings will be discussed later.

2.3. Relation between numbers of fire occurrence and time of fire occurrence

It is well known that the exponential distribution is related to the Poisson process. If fires occur according to a Poisson process, then the time till the rth occurrence of the fire is described by the gamma distribution. The gamma distribution (with integer r) is also known as the Erlang distribution. Let T_r denote the time until the rth fire occurrence in the entire building, then the mean time till the occurrence of the rth fire is $E(T_r) = r/\lambda$ which can be easily obtained by observing that distributions of the inter-arrival times in a Poisson process are independent exponential distributions. Alternatively, since the inter-arrival times are independent and identically distributed exponential random variables having mean $1/\lambda$, it follows from the properties of the exponential distribution or Poisson process that the waiting time until the rth fire has a gamma (Erlang) distribution with parameters λ and r [14]. Similarly, for a building that is subdivided into several different sectors (as discussed in 2.2.3) the first time and the rth time of fire occurrence as well as the expected time of the first and the rth fire occurrence can also be computed by applying the exponential and Erlang distributions.

3. Investigations of building fire occurrence in Taiwan

For estimating the probability of a fire starting, it is useful to analyze data for a group of buildings with similar fire risks. Moreover, it is very important to examine and compare the risks of fire ignitions by considering different types of occupancies among buildings in a country. In this section, investigations and comparisons of fire ignition characteristics regarding different types of occupancies among buildings are carefully studied.

For the purpose of comparisons among different types of buildings, fire ignition rates are usually expressed as number of fires per m^2 of floor space per year. To obtain these figures, it is necessary to collect fire statistics and data regarding the establishments in various occupancies [15]. Table 1 contains the number of fire occurrences and the total floor area of different types of building occupancies in Taiwan from 1985 to 2001. Table 1 also provides the calculations of fire ignition rates and shows ranking order of the rates among building occupancies from 1985 to 2001. From Fig. 1, it can be seen how the rate has varied over the 17 years covered by the data for the different occupancies. Generally speaking, except for public buildings, each of other building occupancies has a decreasing rate. Possibly due to more strict definitions of fire incidents (official fire identification) in Taiwan, the number of fire occurrences in shops and public buildings dropped considerably from the year 1996 to 1997 and in industry, residential and other building occupancies it dropped greatly.

In this analysis, different occupancy groups are defined as follows:

Residential buildings: Houses in which the only activity is for living or rental.

Shops: All shops which are also occupied by citizens are included in this group. For shops with only commercial activities such as department stores, restaurants, etc. are grouped in public buildings.

Industrial buildings: Buildings where the main activity is for processing, manufacture or repair, although there may also be a part of the building used for storage, offices or some other purposes.

Public: Offices, hotels, clubs, KTV, MTV, department stores, cinemas, restaurants, schools and other establishments.

Others: All buildings excluding those that are grouped as residential buildings, shops, industrial buildings and public buildings.

By summing the total fire occurrences over the 17 years, it can be calculated from Table 1 that about 60.18% ($72252/120049 = 0.6018$) of total fire ignitions is

Table 1
Floor area, number of fire occurrences and ranking order of rate for different occupancies in Taiwan for 1985–2001

Year	Residential			Shop			Industry			Public			Other		
	Floor area	Number of fire occurrences	Rate	Floor area	Number of fire occurrences	Rate	Floor area	Number of fire occurrences	Rate	Floor area	Number of fire occurrences	Rate	Floor area	Number of fire occurrences	Rate
1985	317,384	3,803	0.0120	141,937	441	0.0031	81,732	1,863	0.0228	39,530	205	0.0052	80,147	289	0.0036
1986	330,675	5,681	0.0172	145,024	567	0.0039	84,966	2,370	0.0279	41,674	273	0.0066	82,499	229	0.0028
1987	343,036	3,910	0.0114	147,859	395	0.0027	89,113	1,887	0.0212	43,354	229	0.0053	85,515	308	0.0036
1988	357,502	3,270	0.0091	151,690	360	0.0024	94,782	1,593	0.0168	44,965	206	0.0046	89,546	221	0.0025
1989	372,030	3,534	0.0095	156,454	450	0.0029	100,515	1,608	0.0160	47,201	285	0.0060	93,519	188	0.0020
1990	384,089	3,569	0.0093	164,383	450	0.0027	105,441	1,663	0.0158	49,069	283	0.0058	98,009	207	0.0021
1991	396,378	3,871	0.0098	172,484	420	0.0024	109,849	1,730	0.0157	51,569	259	0.0050	102,705	248	0.0024
1992	412,035	3,755	0.0091	181,466	403	0.0022	114,484	1,372	0.0120	54,077	279	0.0052	107,845	162	0.0015
1993	433,804	5,196	0.0120	194,110	507	0.0026	116,300	1,949	0.0168	56,448	391	0.0069	116,789	469	0.0040
1994	462,388	5,746	0.0124	209,474	599	0.0029	121,688	2,085	0.0171	58,612	425	0.0073	123,449	373	0.0030
1995	488,847	5,847	0.0120	224,815	509	0.0023	126,087	2,157	0.0171	61,514	501	0.0081	129,610	485	0.0037
1996	509,478	7,789	0.0153	235,673	691	0.0029	130,021	2,401	0.0185	64,000	630	0.0098	137,411	725	0.0053
1997	524,071	3,358	0.0064	245,310	508	0.0021	133,805	869	0.0065	66,902	561	0.0084	144,956	375	0.0026
1998	537,987	3,553	0.0066	254,526	498	0.0020	138,415	880	0.0064	71,403	567	0.0079	149,872	309	0.0021
1999	551,547	3,477	0.0063	263,462	593	0.0023	145,228	888	0.0061	76,359	738	0.0097	155,601	273	0.0018
2000	561,915	2,891	0.0051	269,670	431	0.0016	152,110	852	0.0056	81,986	750	0.0091	162,945	293	0.0018
2001	570,106	3,002	0.0053	274,045	455	0.0017	158,222	664	0.0042	85,594	686	0.0080	169,990	268	0.0016
Average	444,310	4,250	0.0096	201,905	487	0.0024	117,809	1,578	0.0134	58,486	428	0.0073	119,436	319	0.0027
Ranking order		2			5			1			3			4	

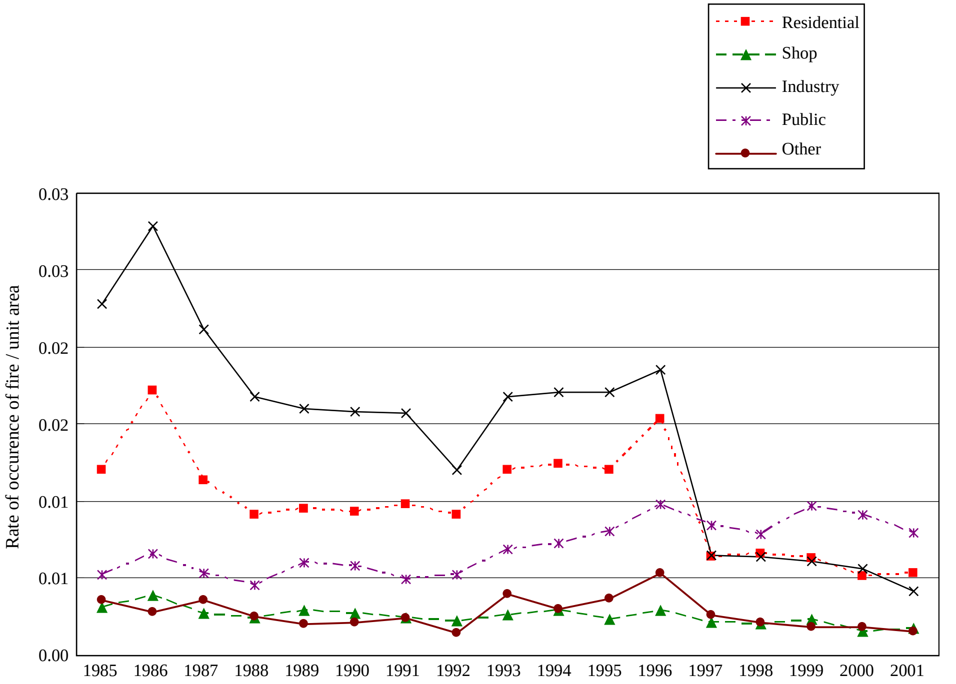

Fig. 1. Rates of occurence of fire/unit area for different occupancies in Taiwan during 1985–2001.

associated with residential buildings. Residential buildings have the highest percentage of fire occurrence, followed by industrial occupancy (about 22.35%).

Based on the data available in Table 1, annual fire ignition rate per unit floor area for different occupancies is calculated and also shown in Table 1. Taking "industry" (see definition of occupancy groups) as an example, the annual fire ignition rate per unit floor area is 1.34×10^{-2} (= 1578/117,809); that is the yearly average number of fires divided by the average floor area of industrial occupancy from 1985 to 2001. Industrial occupancy has the highest number of fires per m^2 of floor space per year, followed by residential occupancy. The rate of fire ignition only increases in public occupancy possibly with increasing public activities in Taiwan. However, this does not appear to be the case in shops (commercial activities), where annual fire ignition rate is the lowest and decreases gradually. The other occupancy is similar to shop. All these figures need to be updated and carefully interpreted if new data are added.

4. Conclusion

With the aid of building construction, data regarding building floor area and available fire statistics the probability of fire occurrence in different occupancies can be estimated by applying Poisson distribution. For a particular building that is subdivided into several sectors the probability of fire occurrence can also be predicted by applying the respective mean rate of fire ignition that is obtained from available data. Moreover, the probability distribution function of the random time of fire occurrence, the expected time of the first and the rth fire occurrence are also obtained. In addition, the relation between the numbers of fire occurrence (a discrete version) and the time of fire occurrence (a continuous version) are clearly examined.

Investigations of different building occupancies in Taiwan show that industrial occupancy has the highest value of annual fire ignition rate per unit floor area and followed by residential occupancy. It is recommended that further study be carried out by generating the joint probability of floor area, building service time or other possible attributes to accurately estimate the probability of fire occurrence for buildings. It is also recommended that the prior distribution of mean rate of fire ignition be constructed, and therefore the posterior distribution.

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