A STATISTICAL STUDY OF LARGE FIRE LOSSES WITH APPLICATION TO A PROBLEM IN CATASTROPHE INSURANCE

BY

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This study was undertaken in 1950 as part of an investigation into the problem of rating catastrophe fire insurance. It was subsequently put together as a paper and submitted, together with the paper I presented to the Society last year, as a thesis for Part III and IV of the Fellowship Examinations. Fortunately I managed to pass the examinations and did not need to rely on a thesis; but, since the study has been put together as a paper, it may prove of interest to members and encourage others to make investigations into the statistical aspects of catastrophe insurance, not only in the fire field but in all lines of insurance.

Certain large organizations have been for many years self insurers of their fire risks. Such an organization needs an insurance policy to cover it against fire losses of catastrophic proportions, the chances of which are so remote that no credible statistics are available to assess the cost of the cover. The most usual form of contract covers losses by fire up to a specified amount in excess of a fixed sum. Thus a policy might cover a loss up to $1,000,000. in excess of $200,000. In the case of a fire loss of less than $200,000. no payment would be made under the policy, but in the case of a fire loss of $500,000., a payment of $300,000. would be made. Under no circumstances would a payment in excess of $1,000,000. be made, however large the actual loss. The sum of $1,000,000. in the above example is referred to as the sum insured and the sum of $200,000. the attachment point. For simplicity, it is usual to employ the notation "$1,000,000/$200,000" to designate this cover. By definition catastrophe insurance requires the attachment point to be fixed sufficiently high so that the probability of a claim is remote. Catastrophe policies may cover other lines such as explosion and wind but this study is limited to the fire risk.

The assessing of a suitable premium for a catastrophe policy involves especial difficulties because the usual rating methods cannot, by the nature of the contract, apply. If sufficient data were available to determine the premium directly, the risk covered would not be a catastrophe, and hence the premium must be determined on a judgment basis. The premium required will not, as in ordinary insurance, vary in proportion with the sum insured and it is not possible therefore to develop a rate of premium.

The premium will depend not only upon the nature of the risk, details of the individual exposures, etc., but also the sum insured and the attachment point. The underwriter is presented with a difficult task in trying to take all these aspects into account. If it were possible to determine some approximate law concerning the distribution of catastrophe fire losses, it might be possible to simplify the underwriting by removing some of the special difficulties introduced by the non-linear relation between the sum insured and the premium and the effect of varying the attachment point. Clearly the distribution of large fire losses must be a function of the distribution of exposures and hence no exact law can exist. At the same time by studying a sufficiently large body of data an empirical law may be found which will have some practical application.

The most suitable body of data for this purpose appears to be the analysis of large fire losses in the United States and Canada issued each year by the National Fire Protection Association. The figures are not perfect in that they are estimates and some of them include business interruption and rent insurance, but they do provide a body of statistics which is sufficiently accurate for this investigation and is free from the distortion due to under-insurance which makes ordinary insurance statistics difficult to handle.

It seems unwise to include in this investigation the experience of the war years, so that the data available are limited to the four years 1946–1949 (figures for 1950 were not available when this paper was drafted). It has been considered desirable to exclude from the data certain classes of fire losses which are not normally covered by policies issued by the fire department of an insurance company. These classes are:—

(1) Property of the Armed Forces,
(2) Forest fires,
(3) Those fires listed under the heading Transportation, which consist mainly of aviation and ship fire losses.

With these classes excluded, we are left with 759 fires, each with an estimated loss of $250,000 or more. This should provide a sufficiently large body of data for our purpose.

To simplify the study of these losses, they were grouped according to the size of loss. In view of the very large concentration of losses at the lower values, a logarithm scale was used, the lower limit of each group being fixed at 1.189 (the fourth root of 2) times the lower limit of the preceding group. The following table shows the distribution of losses.

TABLE 1

Distribution of large losses, 1946–1949, according to size

Size of Loss Group $	*No. of Losses*	*Size of Loss Group* $	*No. of Losses*
250,000—	248	1,420,000	10
297,000—	144	1,680,000	5
354,000—	77	2,000,000	9
420,000—	57	2,380,000	3
500,000—	85	2,830,000	3
595,000—	22	3,360,000	2
707,000—	33	4,000,000	1
841,000—	19	4,760,000	1
1,000,000—	28	5,660,000	1
1,190,000—	11	6,730,000	0

If we exclude the humps at $500,000, $1,000,000, and $2,000,000, which are obviously due to rounding in estimating the amount of loss, these figures reveal a fairly regular pattern which encourages further investigation.

Considering the figures for the four individual years, it is found that, allowing for the smallness of the data, each year follows the same distribution. It is found also that the total losses vary from year to year no more than would be expected as a result of chance variation, except for the first group, $250,000 to $296,999, which has higher figures for the last two years than the first two years. This may be due to difficulty in dealing with borderline cases around the $250,000 loss size.

For the period under consideration, values were fairly steady and it was not considered necessary to allow for inflationary trends.

In the following table, the expected has been taken as one quarter the four-year total.

TABLE 2

Number of losses in individual years by size
(A = Actual, E = Expected)

Size of Loss Group	*Expected Losses*	*1946* A	*1946* A-E	*1946* √E	*1947* A	*1947* A-E	*1947* √E	*1948* A	*1948* A-E	*1948* √E	*1949* A	*1949* A-E	*1949* √E
$													
250,000—	62	42	−20	8	47	−15	8	92	+30	8	67	+5	8
297,000—	36	28	− 8	6	36	0	6	43	+ 7	6	37	+1	6
354,000—	19	10	− 9	4	25	+ 6	4	25	+ 6	4	17	−2	4
420,000—	14	9	− 5	4	12	− 2	4	20	+ 6	4	16	+2	4
500,000—	40	41	+ 1	6	38	− 2	6	39	− 1	6	41	+1	6
1,000,000—	14	15	+ 1	4	9	− 5	4	21	+ 7	4	9	−5	4
2,000,000—	5	7	+ 2	2	5	0	2	4	− 1	2	4	−1	2

Instead of studying the distribution of these large losses in the form shown in Tables 1 and 2, it was decided to convert them into an excess of loss form. This procedure has four advantages:

1. Since we wish to use the results in relation to catastrophe insurance problems, results in this form will be easier to handle;

2. The financial affect of any graduation of the figures will be immediately apparent;

3. Full account can be taken of the distribution of the losses within the groups;

4. No weight will be given to the losses of exactly $250,000, and very little weight will be given to the other losses in the $250,000—$296,999 group. This is the only group which reveals unsatisfactory year to year variation.

The procedure adopted was to calculate the cost of the excess of loss over $250,000, $500,000, $750,000, and so on in steps of $250,000 for all the 759 losses studied. The calculation of those excess of loss costs is set out in Table 3, and the results summarized in Table 4.

TABLE 3

Calculation of Excess of Loss Costs for the 1946–49 Large Losses
(All figures in millions of dollars)

Size of Loss Group $	*No. of Losses*	*Total Amt. of Losses*	*First $250,000 of each loss*	*Total Excess of $250,000*	*Total Excess of loss over $250,000 in $250,000 steps* 1st	2nd	3rd	4th	5th	6th	7th	8th	9th	10th	11th	Bal.
250,000—	248	64.4	62.0	2.4	2.4											
297,000—	144	45.9	36.0	9.9	9.9											
354,000—	77	29.8	19.2	10.6	10.6											
420,000—	57	25.8	14.3	11.5	11.5											
500,000—	85	44.2	21.2	23.0	21.3	1.7										
595,000—	22	14.1	5.5	8.6	5.5	3.1										
707,000—	33	25.2	8.3	16.9	8.2	8.3	0.4									
841,000—	19	17.3	4.7	12.6	4.8	4.7	3.1									
1,000,000—	28	30.3	7.0	23.3	7.0	7.0	7.0	2.3								
1,190,000—	11	14.1	2.8	11.3	2.7	2.8	2.7	2.8	0.3							
1,420,000—	10	15.2	2.5	12.7	2.5	2.5	2.5	2.5	2.5	0.2						
1,680,000—	5	8.8	1.2	7.6	1.3	1.2	1.3	1.2	1.3	1.2	0.1					
2,000,000—	9	18.7	2.3	16.4	2.2	2.3	2.2	2.3	2.2	2.3	2.2	.7				
2,380,000—	3	8.1	.7	7.4	.8	.7	.8	.7	.8	.7	.8	.7	.8	.6		
2,830,000—	3	9.0	.8	8.2	.7	.8	.7	.8	.7	.8	.7	.8	.7	.8	.7	
3,360,000—	2	7.3	.5	6.8	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	.5	1.3
4,000,000—	1	4.0	.2	3.8	.3	.2	.3	.2	.3	.2	.3	.2	.3	.2	.3	1.0
4,760,000—	1	5.0	.3	4.7	.2	.3	.2	.3	.2	.3	.2	.3	.2	.3	.2	2.0
5,660,000—	1	6.0	.2	5.8	.3	.2	.3	.2	.3	.2	.3	.2	.3	.2	.3	3.0
6,730,000—	—	—	—													
TOTALS	759	393.2	189.7	203.5	92.7	36.3	22.0	13.8	9.1	6.4	5.1	3.4	2.8	2.6	2.0	7.3

TABLE 4

Excess of loss costs of the 1946–49 large losses

Attachment Point	*Cost of $250,000 cover over attachment point*	*Cost of unlimited cover over attachment point*
$	$	$
250,000	92,700,000	203,500,000
500,000	36,300,000	110,800,000
750,000	22,000,000	74,500,000
1,000,000	13,800,000	52,500,000
1,250,000	9,100,000	38,700,000
1,500,000	6,400,000	29,600,000
1,750,000	5,100,000	23,200,000
2,000,000	3,400,000	18,100,000
2,250,000	2,800,000	14,700,000
2,500,000	2,600,000	11,900,000
2,750,000	2,000,000	9,300,000
3,000,000	.	7,300,000

It is to the last column of Table 4 that we shall turn our attention. The logarithms of the amounts shown decrease steadily and their first differences decrease rapidly at first and then become practically constant. As will be seen from the following table, a very satisfactory fit can be obtained by assuming that the second difference of the logarithms decrease in geometric progression.

TABLE 5

Graduation of Excess of Loss Costs

		Ungraduated				*Graduated*		
Attachment Point $	x	*Cost in millions of dollars of unlimited cover over, attachment point.* λ'_x	$log\lambda'_x$	$\Delta log\lambda'_x$	$\Delta^2 log\lambda_x$	$\Delta log\lambda_x$	$log\lambda_x$	λ_x
250,000	1	203.5	2.3086	−.264	.0800	−.2623	2.3086	203.5
500,000	2	110.8	2.0444	−.172	.0400	−.1823	2.0463	111.3
750,000	3	74.5	1.8722	−.152	.0200	−.1423	1.8640	73.11
1,000,000	4	52.5	1.7202	−.133	.0100	−.1223	1.7217	52.69
1,250,000	5	38.7	1.5877	−.116	.0050	−.1123	1.5994	39.76
1,500,000	6	29.6	1.4713	−.106	.0025	−.1073	1.4871	30.70
1,750,000	7	23.2	1.3655	−.108	.0012	−.1048	1.3798	23.97
2,000,000	8	18.1	1.2577	−.090	.0006	−.1036	1.2750	18.84
2,250,000	9	14.7	1.1673	−.092	.0003	−.1030	1.1714	14.84
2,500,000	10	11.9	1.0755	−.107	.0002	−.1027	1.0684	11.70
2,750,000	11	9.3	0.9685	−.105	.0001	−.1025	.9659	9.24
3,000,000	12	7.3	0.8633			−.1024	.8635	7.30

The graduated values of λ_x follow a law which is in form the same as Makeham's famous law of mortality

$$\lambda_x = ks^x g^{c^x}.$$

Where $c = .5$, $\log g = .6400$, $\log s = -.1023$.
The similarity is, however, in form only.

The following table shows the closeness of fit of the graduation.

TABLE 6

Comparison of Graduated and Ungraduated Values of Excess of Loss Cost

Attachment Point	x	*Cost of Unlimited Cover over Attachment Point (in millions of dollars)*			*Cost of $250,000 Cover over Attachment Point (in millions of dollars)*		
		Ungraduated	*Graduated*	*Difference*	*Ungraduated*	*Graduated*	*Difference*
$							
250,000	1	203.5	203.5	—	92.7	92.2	−0.5
500,000	2	110.8	111.3	+0.5	36.3	38.19	+1.9
750,000	3	74.5	73.11	−1.4	22.0	20.42	−1.6
1,000,000	4	52.5	52.69	+0.2	13.8	12.93	−1.1
1,250,000	5	38.7	39.76	+1.1	9.1	9.06	—
1,500,000	6	29.6	30.70	+1.1	6.4	6.73	+0.3
1,750,000	7	23.2	23.97	+0.8	5.1	5.13	—
2,000,000	8	18.1	18.84	+0.7	3.4	4.00	+0.6
2,250,000	9	14.7	14.84	+0.1	2.8	3.14	+0.3
2,500,000	10	11.9	11.70	−0.2	2.6	2.46	−0.1
2,750,000	11	9.3	9.24	−0.1	2.0	1.94	−0.1
3,000,000	12	7.3	7.30			1.53	
3,250,000	13		5.77			1.22	
3,500,000	14		4.55			.95	
3,750,000	15		3.60			.76	
4,000,000	16		2.84			.59	
4,250,000	17		2.25			.48	
4,500,000	18		1.77			.35	
4,750,000	19		1.42			.31	

It is necessary to investigate how the actual number of losses recorded in Table 1 compare with the losses expected by the foregoing graduation formula.

The total number of losses expected for the amount of $250,000 and over is

$$-4\frac{d\lambda_x}{dx} = 4\,\lambda_x\,\mu_x\,,$$

where $\mu_x = -\log_e s - \log_e g \,.\, \log_e c \,.\, c^x$
and the factor of 4 is introduced because λ_x is in units of $1,000,000 and the interval used for x is $250,000. The results are shown in Table 7.

TABLE 7

Comparison of actual and expected number of losses

Size of Loss Group $	*Actual Losses*	*Expected Losses*	*A-E*	$\Sigma(A\text{-}E)$	*Size of Loss Group* $	*Actual Losses*	*Expected Losses*	*A-E*	$\Sigma(A\text{-}E)$
250,000—	248	125.5			1,420,000—	10	9.2	+ .8	+ .6
297,000—	144	108.8	+35.2	+14.1	1,680,000—	5	7.1	−2.1	− .2
354,000—	77	91.3	−14.3	−21.1	2,000,000—	9	5.7	+3.3	+1.9
420,000—	57	74.4	−17.4	− 6.8	2,380,000—	3	4.3	+ .3	−1.4
500,000—	85	58.1	+26.9	+10.6	2,830,000—	3	3.2	− .2	− .1
595,000—	22	44.4	−22.4	−16.3	3,360,000—	2	2.2	− .2	+ .1
707,000—	33	32.7	+ .3	+ 6.1	4,000,000—	1	1.4	− .4	+ .3
841,000—	19	23.6	− 4.6	+ 5.8	4,760,000—	1	.7	+ .3	+ .7
1,000,000—	28	16.9	+11.1	+10.4	5,660,000—	1	.4	+ .6	+ .4
1,190,000—	11	12.3	− 1.3	− .7	6,730,000—	—	.2	− .2	− .2

The first group has negligible weight in the calculation of excess of loss costs and it is not surprising therefore that the graduation should provide an unsatisfactory estimate of the number of losses. For the number of losses over $300,000 the graduation has of course, removed the humps which occur at the $500,000, $1,000,000 and $2,000,000 points.

This distribution of large fire losses, depending as it does on the distribution of fire exposures, cannot be expected to apply to any particular group of properties for which a catastrophe cover is required but the pattern is of interest and may provide a valuable guide to underwriters in determining the correct premium for a particular cover when the premium for some other cover has been established.