Report on
The Geometry of Drinfeld modular forms

Context and summary Let $A = \mathbb{F}_q[T]$ where $\mathbb{F}_q$ is a finite field with q elements, $K = \mathbb{F}_q(T)$, $K_\infty = \mathbb{F}_q((1/T))$ and C be the $1/T$-adic completion of an algebraic closure of K_∞. Studied since the late 1970s, Drinfeld modular forms are analogues over C of classical modular forms. Let $M(\Gamma)$ be the graded C-algebra of Drinfeld modular forms for a congruence subgroup $\Gamma \subset \mathrm{GL}_2(A)$ of any weight and type. Describing the structure of this algebra by a minimal set of generators and their relations is an important problem on modular forms. Such a description is known for the congruence subgroups $\mathrm{GL}_2(A)$ (Gekeler 1988, after Goss), $\Gamma(N)$ where $N \in A$ with $\deg N = 1$ (Cornelissen, 1997) and more recently $\Gamma_0(T)$ (Dalal-Kumar, 2023). The author proposes to follow the approach recently developed by Voight and Zureick-Brown (2022) in which they uses the log canonical ring of stacky (classical) modular curves and their geometric invariants (genus, elliptic points, cusps) to compute minimal presentations for such algebras, and to adapt it to the Drinfeld setting.

The text consists of three main results and some examples. All these results assume that q is odd. Theorem 1.1 describes $M(\Gamma)$ as sections of a log canonical bundle on the stacky Drinfeld modular curve for Γ, provided that all determinants of matrices in Γ are squares in $\mathbb{F}_q^\times$ (see also point 1 below for an additional hypothesis). Theorem 1.2 compare the algebras and subspaces of Drinfeld modular forms for Γ and $\Gamma_2 = \{\gamma \in \Gamma \mid \det\gamma \in (\mathbb{F}_q^\times)^2\}$. Theorem 1.3 generalizes this to a larger class of congruence subgroups, by replacing Γ_2 with any Γ' such that $\Gamma \cap \mathrm{SL}_2(A) \subset \Gamma' \subset \Gamma$, using a different method. Theorem 1.1 offers a new perspective to describe the algebra of Drinfeld modular forms, provided that we are able to compute the canonical ring for stacky Drinfeld modular curves. The author present examples where they compute this ring to obtain the structure of $M(\mathrm{GL}_2(A)_2)$ and $M(\Gamma_0(T)_2)$, using a technique of O'Dorney (2015) for genus-0 curves: although these structures can be recovered directly from Theorem 1.3 and previous results on $M(\mathrm{GL}_2(A))$ and $M(\Gamma_0(T))$ (see point 5 below), they serve as an illustration, in accessible cases, of what the method would be able to achieve in general.

Opinion The paper presents a new perspective of study for the structure of the algebra of Drinfeld modular forms and laid the groundwork for future exploration. I believe this type of material deserves to be published in JNT. However *the text contains inaccuracies (detailed below), including a couple of what I believe are major ones. I am not able to recommend it for publication at this point. I suggest that it be thoroughly revised before being resubmitted* and I encourage the author to complete this work.

Comments and suggestions They are gathered in several lists, organized as follows: major mathematical inaccuracies; minor mathematical inaccuracies; other mathematical comments and suggestions; typographical or grammatical comments and suggestions; bibliographical comments. I would like to apologize in advance if, in some places, my comments are not relevant or if I misunderstood what the author meant.

Major mathematical inaccuracies.

1. Page 3, Theorem 1.1 (see also Theorem 6.1) is stated for congruence subgroups $\Gamma \subset \mathrm{GL}_2(A)$ which contain all diagonal matrices in $\mathrm{GL}_2(A)$ and such that for any $\gamma \in \Gamma$, $\det \gamma$ is a square in $\mathbb{F}_q^\times$. However these two conditions seem incompatible to me: take λ (resp. λ') a square (resp. a non-square) in $\mathbb{F}_q^\times$, the matrix $\left(\begin{smallmatrix} \lambda & 0 \\ 0 & \lambda' \end{smallmatrix}\right)$ belongs to Γ but has non-square determinant $\lambda\lambda'$ in $\mathbb{F}_q^\times$. A clarification is required. I am not familiar enough with the technique of the proof to see if the assumption on diagonal matrices may be lifted.

2. Page 14, proof of Theorem 6.1 (I am not sure whether this is a major or minor inaccuracy). I could not see why the proof is complete. Starting with $f \in M_{k,l}(\Gamma)$, the author associates an element of $H^0(\mathscr{X}_\Gamma, \Omega^1_{\mathscr{X}_\Gamma}(2\Delta)^{\otimes k/2})$ but I wonder why this provides an isomorphism.

3. Page 16, line 7: I have some doubts about the assertion for the set of representatives of left cosets of $\mathbb{F}_q^\times$ in Γ_e. Since $\#\Gamma_e = n(q-1)$, there should be n representatives, instead of $n(q-1)^2$ as in the text. Take $\gamma \in \Gamma_e$ such that $\langle \gamma \rangle = \Gamma_e$. Since $\mathrm{Z}(\mathbb{F}_q) \subset \Gamma_e$, the subgroup $\mathrm{Z}(\mathbb{F}_q)$, which is the unique subgroup of order $q-1$ in the cyclic group Γ_e, is generated by γ^n. A set of representatives of $\Gamma_e/\mathbb{F}_q^\times$ would be for instance $\{\gamma^0, \dots, \gamma^{n-1}\}$. As a consequence, the equation for $\Gamma_e/\mathbb{F}_q^\times$ on line 11 should be revised, as well as the rest of the proof of Proposition 6.8.

4. Page 18, line 5: I suspect that defining f_1 and f_2 by these u-power series is not enough to ensure that they are Drinfeld modular forms for Γ_2 (note that Lemma 3.9 only ensures that *if* they are Drinfeld modular forms, they are uniquely determined by their expansion). This seems to be a major issue in the proof of Theorem 1.2.

5. Page 21.

   (a) Example 7.1: is it obvious that Theorem 6 of [O'D15] proves $R_D \simeq C[g,h] \simeq M(\mathrm{GL}_2(A)_2)$? Same on page 24 for $\Gamma_0(T)_2$. The way you apply this theorem seems to be at the core of both examples so more details would be appreciated.

   (b) This is more of a general comment on the examples. Example 7.1 proves that $M(\mathrm{GL}_2(A)_2) = C[g,h]$ using geometric invariants to compute the canonical ring. However this conclusion would follow much more directly by combining Theorem 6.12 (which gives $M(\mathrm{GL}_2(A)_2) = M(\mathrm{GL}_2(A)))$ and $M(\mathrm{GL}_2(A)) = C[g,h]$, without the whole machinery of canonical rings for stacky modular curves. If this is true, it should be stated explicitly.

Same for the last example on page 24: $M(\Gamma_0(T)_2) \simeq C[U,V,Z]/(UV - Z^2)$ (with U, V, Z corresponding respectively to Δ_T, Δ_W, E_T), which can be obtained directly from Theorem 6.12 and [DK23]. Note that you already use the result of [DK23] on $M(\Gamma_0(T))$ in your proof.

6. Page 22, table on the top of the page, third line. To compare elliptic points for $\Gamma_1(\alpha T + \beta)$ and $\Gamma_1(\alpha T + \beta)_2$, you use Proposition 6.8. However the proposition has an assumption, that the congruence subgroup should contain all diagonal matrices of $\mathrm{GL}_2(A)$, which is not satisfied by $\Gamma_1(\alpha T + \beta)$. Same when you use Proposition 6.8 just before Conjecture 7.2. Your example should be revised accordingly.

Minor mathematical inaccuracies.

7. Page 4, end of the statement of Theorem 1.3: could you say what you mean by "nontrivial l'"? Also it would be worth stating a more precise relation between those l' and the type l. Same for Theorem 6.12 on page 18.

8. Page 4, paragraph 2.1, definition of the valuation v. For $v : K^\times \to \mathbb{Z}$, both sums $\sum_1^n$ should start at index 0 instead of 1. Also it seems that you assume $a_n \neq 0$ and $b_m \neq 0$ in this definition (same for the extension of v to $K_\infty^\times$ with $a_n \neq 0$).

9. Page 5, Definition 2.1, non-square case: do you also assume here that $\gamma \in \Gamma_z$?

10. Page 7, definitions of stacky curve and coarse space morphism:

    (a) Is there any assumption on the field $\mathbb{K}$ to define a stacky curve over $\mathbb{K}$?

    (b) You state that the coarse space morphism is unique but according to [VZB22, Remark 5.3.2] this is only up to isomorphism.

    (c) According to [VZB22], the bijection is between the set of isomorphism classes of F-points on $\mathscr{X}$ on one side, and $X(F)$, the set of all F-points, on the other side (your phrasing suggests that it is the set of isomorphism class of F-points on X).

11. Page 8, definition of log divisor: your definition seems to differ from [VZB22, Def. 5.6.2] where they allow only non-stacky points.

12. Page 8, last line: the right-hand side is a divisor with coefficients in $\mathbb{Q}$ but I thought we were working with Weil divisors, i.e. with coefficients in $\mathbb{Z}$? (indeed in [VZB22, Prop. 5.5.6] the formula has coefficients in $\mathbb{Z}$)

13. Page 9, Lemma 3.3: could you check the proof? I am not sure it shows that the two exponential functions differ by a factor $\tilde{\pi}\alpha^{-1}$. Alternatively a more direct proof would be to do a change of variable in $\sum_{a \in A} \frac{1}{z+a}$, or use the fact that the exponential function attached to a lattice is a $\mathbb{F}_q$-linear power series.

14. Page 11, Definition 4.2, second •. The choice of notation in "$U \in \mathrm{Cov}(U)$" could be misleading: I suggest to use $\mathcal{U} \in \mathrm{Cov}(U)$. The following sentences should be : "the covering $\mathcal{U} \cap V \overset{def}{=} \{U' \cap V : U' \in \mathcal{U}\}$ belongs to $\mathrm{Cov}(V)$" (instead of $\mathrm{Cov}(U)$).

15. Page 14, line 6: "$f(dz)^{\otimes k/2}$ is a global section of the twist by 2Δ of sheaf of holomorphic differentials...". Do you need to assume $k \geq 2$ here?

16. Page 14, Remark 6.3: can you justify the assertions (1) and (2)? I suspect that they may derive from the subsequent Section 6.2.

17. Page 14, line -2: $\det(\gamma)^{(q-1)/2} \equiv -1 \bmod q-1$ does not make sense since $\det\gamma$ belongs to $\mathbb{F}_q^\times$ but is not an element in $\mathbb{Z}/(q-1)\mathbb{Z}$. Perhaps you meant $(\det\gamma)^{(q-1)/2} = -1$.

18. Page 15, Lemma 6.5: we get the impression that the proof is not complete since it does not provide the values of both solutions.

19. Page 17, Proposition 6.10 and its proof:

    (a) It is implicit that $k/2$ is an integer: this holds provided that q is odd. This assumption should appear in the statement of the proposition. Same for Proposition 6.11.

    (b) Proof, line 1: to have $\alpha^{k-2l} = 1$, we need that f is not identically zero (but when it is not the case, the statement of the proposition is trivial, fortunately).

    (c) The proof may be significantly shortened. By assumption on β, the matrix $\left(\begin{smallmatrix} \beta & 0 \\ 0 & 1 \end{smallmatrix}\right)$ belongs to Γ_2 therefore $f(\beta z) = \beta^{-l} f(z)$. All we have to do now is prove that $\beta^{-l} = \beta^{-k/2}$, that is $\alpha^{-k} = \alpha^{-2l}$. But this comes from $k \equiv 2l \bmod q-1$ (this is Lemma 3.10, assuming that f is not identically zero).

20. Page 18, proof of Theorem 6.12:

    (a) Line 4: is it obvious that Γ' is normal in Γ? (shouldn't that be add as an assumption in the statement of Theorem 6.12?)

    (b) Line 4: when you say that $f_{|\gamma}$ is weakly modular, precise for which congruence subgroup.

    (c) Line 7: you mention actions of the groups Γ' and Γ/Γ': precise the sets or objects they act on.

    (d) Line 8: can you quickly justify why $\Gamma/\Gamma' = \det(\Gamma)/\det(\Gamma')$? I suspect this comes from $\Gamma_1 \subset \Gamma'$. More generally the role of the assumption $\Gamma_1 \subset \Gamma'$ does not appear clearly in the proof.

21. Page 20, line 3: the valence formula is valid for any $f \in M_{k,m}(\mathrm{GL}_2(A))$ provided that f is not identically zero.

22. Page 22, Conjecture 7.2. The statement should be made more precise: do you conjecture that $\Gamma_1(\alpha T+\beta)$ and $\Gamma_0(\alpha T+\beta)$ are both "non-square" for any choice of α and β?

23. Page 22, Remark 7.3: in "for $z \in \Omega$, we have

$$\frac{az+b}{cz+d} = z \iff \begin{cases} a = d \text{ and } b = 0 = c & \text{or} \\ cz^2 + (d-a)z - b = 0 & \text{irreducible''} \end{cases}$$

I suppose you meant "for $z \in \Omega$, we have

$$\frac{az+b}{cz+d} = z \iff \begin{cases} a = d \text{ and } b = 0 = c & \text{or} \\ cx^2 + (d-a)x - b \text{ is irreducible over } K.\text{''} \end{cases}$$

However the right-hand side of this equivalence does not depend on $z \in \Omega$. Perhaps it should be read as :

$$\exists z \in \Omega,\ \frac{az+b}{cz+d} = z \iff \begin{cases} a = d \text{ and } b = 0 = c & \text{or} \\ cx^2 + (d-a)x - b \text{ is irreducible over } K. \end{cases}$$

Even then, I am not sure about the irreducibility condition: shouldn't we need it over K_∞? If so, revise the remark accordingly. Also I was unable to understand the paragraph "We see that... will remain the same" (to start with, what do you mean by "consideration of only the constants in A?").

24. Page 23, line -8: I am not sure I understand the conclusion of the example, which seems unfinished. Among the geometric invariants we need, you have computed the genus and the number of elliptic points. Do we lack the number of cusps to go further?

Other mathematical comments and suggestions.

25. ~~Page 2, line -9: "between modular forms", I suggest to add "of weight k" since this~~ quantity appears in $f \mapsto f(z)dz^{\otimes k/2}$.

26. ~~Page 3, statement of Theorem 1.1. It would be worth saying that under the assumption~~ that q is odd, we have that $k/2$ is an integer when $M_{k,l}(\Gamma) \neq 0$.

27. Page 6, line -6: "the Carlitz module is the image of the ring homomorphism...". This is a bit unconventional, as well as inconsistent with your definition of algebraic Drinfeld module on the bottom of page 5 (as a ring homorphism $\varphi : A \to C\{X^q\}$). Also with this homomorphism definition, the rank of the Carlitz can be read off directly from $\deg(TX + X^q)$ (there is no need of $\deg \varphi$). Same on page 7, Example 2.7 ("the image of a degree 2 ring homomorphism").

28. ~~Page 7, line -2. $\mathscr{X}_{\text{ét}}$ has not been introduced.~~

29. ~~Page 8, definition of a gerbe over a stacky curve: I do not see where is the stacky~~ curve in this definition.

30. ~~Page 8, paragraph 3: the notion of "μ_n-gerbe" has not been introduced.~~

31. Page 9, Remark 3.2: "so that the series expansion coefficients for modular forms at cusps are elements in A". I suggest to precise something like "for certain generating modular forms" (not for all of them).

32. Page 9, Remark 3.5 (1) "in any case when Γ has a single cusp": this is true also for the second interpretation (2). Perhaps move it to the beginning of the remark? ("there are several interpretations of the second condition when Γ as a single cusp, or more generally about holomorphy at the cusp ∞")

33. Page 9, Definition 3.7: to avoid repetition, you could put this definition of weakly modular form before Definition 3.4 (modular form).

34. ~~Page 10, Lemma 3.9: the proof is a paraphrase of [Gek88, Remark 5.8iii]. It does not~~ seem useful to write it again.

35. ~~Page 10, Example 3.12: I suggest to add "Section 8" to the reference [Gek88].~~

36. Page 11, Definition 4.3, line 3: the notion of affinoid (that is, affinoid space) should be defined earlier, for instance after affinoid algebra (Definition 4.1).

37. ~~Page 12, Example 4.13. I suspect that $\mathcal{M}_A^2$ "includes cusps" in the moduli interpreta-~~ tion but on the top of the same page, the notation is rather $\overline{\mathcal{M}_\Gamma^2}$?

38. ~~Page 12, line -1: please recall the definition of $\mathbb{P}^1(\ldots,\ldots)$.~~

39. Page 13, Theorem 5.4. Recall the notation Coh. Also check that this corresponds to [PY16, 7.4] (their statement is for $\mathrm{Coh}^{\heartsuit}$, which is a full subcategory of Coh).

40. Page 14, paragraph "By rigid analytic GAGA...": what is the integer n in $\mathbb{P}^n_C$ (possibly $n = 1$)?

41. Page 14, Remark 6.3: I am not sure if the notion of log canonical divisor has been previously defined.

42. Page 15, Lemma 6.6: the proof is a general fact on group actions, namely that stabilizers of two elements in the same orbit are conjugate subgroups, so the proof of the lemma may be shortened. Also it looks like the assumptions that q is odd and $e_1 \neq e_2$ are unnecessary here.

43. Page 15, proof of Lemma 6.7:

    (a) ~~I could not see how Lemma 6.6 is used.~~

    (b) ~~It seems that the proof if is not complete, or lacks a conclusion.~~

44. Page 16, line 7: should one read $n \mid q+1$ instead of $n \mid q-1$?

45. ~~Page 16, Corollary 6.9: as the proof explains, it is not exactly a corollary of Proposi-~~tion 6.8. This corollary might be integrated into the proposition.

46. ~~Page 17, line -1: if you use Lemma 6.5 here, it would be good to write it.~~

47. ~~Page 20, line 4: the characteristic of C is not q but the prime number of which it is a~~ power.

~~*Typographical or grammatical comments and suggestions.*~~

48. Page 2, paragraph 4, line 4: in $\prod_p (\mathbb{F}_q[T])_p$, the letter p might be replaced with P or $\mathfrak{p}$ to emphasize that it is an ideal, or a polynomial, and not the characteristic of C.

49. Page 3, Theorem 1.1, line 3: perhaps precise "the stacky modular curve $\mathscr{X}_\Gamma$".

50. Page 4, line 9: $R = R(\mathscr{X}, \mathscr{L})$. Is this notation used afterwards?

51. Page 5, Definition 2.1, "every γ [...] has $\det \gamma$ a square in $\mathbb{F}_q^\times$": perhaps it sounds more conventional to say "every γ [...] has a square determinant in $\mathbb{F}_q^\times$".

52. Page 6, Lemma 2.2. For consistency, you may use the letter X for the indeterminate instead of x (same as on pages 5 and 6). Also I suggest to add "for some $n \geq 0$ and $a_0, \ldots, a_n$ in K". Finally this may not be the optimal location for this lemma since you mentioned already $\mathbb{F}_q$-linear polynomials: it would be better on page 5, before $C\{X^q\}$.

53. Page 6, Definition 2.4: "for a family of pairs", I suppose you mean "for any family of pairs". Also "of form $\varphi_i(T)$..." should be "of the form $\varphi_i(T)$...".

54. Page 6, lines -2 and -1: I suggest to harmonize the notation ($|a|_\infty$ and $|0|$) and perhaps to move it to the notation paragraph on page 4.

55. Page 8, line 5: I suggest to add a comma between "Zariski locally" and "every stacky curve". Also I am not sure about the interpretation "in some sense, "most" stacky curve have a quotient description [...]": to me it seems this description is valid for any stacky curve but it is only of *local* nature.

56. Page 8, Definition 2.10: "invertble" should be "invertible".

57. Page 9, Remark 3.6: "for $b \in A$", I suggest to precise "for any $b \in A$".

58. Page 11, Definition 4.2 (1): I suggest to replace & with "and".

59. Page 11, lines -16 and -15: I suggest changing the word order, "The action by a congruence subgroup $\Gamma \leqslant \mathrm{GL}_2(A)$". Same on page 12, Remark 4.12. Same on page 19, line -1 with "f some Drinfeld modular form".

60. Page 11, line -13: "archimedian" should be "archimedean". Same with "achimedean" on page 13, Theorem 5.4.

61. Page 11, line -2: I suggest "the affine Drinfeld modular curves".

62. Page 12, line -4: "the moduli corresponding problem" should be "the corresponding moduli problem".

63. Page 12, line -3: I suppose one should read "a rigidification of $\mathcal{M}_A^2//\mu_{q-1}$".

64. Page 13, Definitions 5.2 and 5.3: I suppose that the field L should be C.

65. Page 13, Theorem 5.4 starts with a k-affinoid algebra where k is a non-archimedean field: the definition of C-affinoid algebra (Def. 4.1) should be extended to this case.

66. Page 13, Theorem 6.1, last line: I would be in favor of writing $f(z)(dz)^{\otimes k/2}$ instead of $f(dz)^{\otimes k/2}$ (throughout the paper).

67. Page 14, line 1: the notation $\Omega_\Omega^{\otimes k/2}$ is not optimal but I am not sure there is anything we can do about it.

68. Page 17, line -3, "we have $\beta^{-n} = \beta^{-k/2}$ or $\alpha^{-2n} = \alpha^{-k}$": "or" should be "that is".

69. Page 18, line 11: $f_1(\alpha\gamma_2 z)$ should be $f_1(\left(\begin{smallmatrix} \alpha & 0 \\ 0 & 1 \end{smallmatrix}\right)\gamma_2 z)$.

70. Page 18, proof of Theorem 6.12:

    (a) "Peterson" should be "Petersson".

    (b) on line 3 of the proof, I suggest to write $f_{|\gamma}(z)$ instead of $f_{|\gamma}$.

71. Page 19, line -6: "see how differs" should be "see how it differs".

72. Page 20, lines 8 to 10: "From [VZB22, Definition 5.6.2]... non-stacky points on X": this has already been said at the bottom of page 8, perhaps it is unnecessary to recall it. Same at the bottom of page 20 from "We recall from [VZB22]..." up to the definition of log divisor.

73. Page 20, line -16: "... in $\mathrm{GL}_2(A)$ as the group of upper triangular matrices is strictly larger than...". I suspect this should be "... in $\mathrm{GL}_2(A)$. As the group of upper triangular matrices is strictly larger than...".

74. Page 20, line -6: "Defintions" should be "Definitions".

75. Page 21, line 10: "some new example computations", perhaps you meant "some new examples of computations".

76. Page 21, line -10: for $\alpha T + \beta$, please state that $\alpha, \beta \in \mathbb{F}_q$ and $\alpha \neq 0$. Note also that throughout the text you consider the level N to be a monic polynomial (see page 4, line -4) so you may take $\alpha = 1$ on pages 21 to 24.

77. Page 21, line -9: for the algebra $M(\Gamma_0(T))_R$, it would be good to mention that these are Drinfeld modular forms with coefficients in R.

78. Page 22, Remark 7.3, line 3: "the determinant $(aN + C)d - bcN$" should be "the polynomial $(aN + C)d - bcN$" (there is no matrix at this point).

79. Page 23, line 9: "Riemann-Hurwitz" should be "Riemann-Hurwitz formula". Same in Example 7.5.

80. Page 23, line -8: "comparision" should be "comparison".

Bibliographical comments.

81. Page 2, line 10: [Gek86, Page 52] seems to be Page 53 (more precisely Theorem 5.4).

82. Page 2, paragraph 3: [BBP] deals with Drinfeld modular forms of rank $r \geq 2$ which is a much more difficult setting than rank 2, where this text concentrates on. For rank 2, an isomorphism between Drinfeld modular forms and sections of a line bundle may be found earlier, for instance in G. Böckle, *An Eichler-Shimura isomorphism over function fields between Drinfeld modular forms and cohomology classes of crystals*, Prop. 5.6. Similarly, Pink's work [Pin12] deals with higher rank. Perhaps this difference of rank with the set-up of this text should be emphasized.

83. Page 2, paragraph 5, line 5–8: the statement of [Arm08, Prop. 4.16] is actually not related to Gekeler's problem so I am not sure this reference is relevant here. Moreover this correspondence between $M^2_{2,1}(\Gamma_0(N))$ and holomorphic differentials on the Drinfeld modular curve $X_0(N)$ was known previously in the literature, for instance in [GR, Section 2.10].

84. Page 2, last paragraph, line 2: the sentence refers to a paper of "Gerritzen-van der Put cited above" but this paper has not been cited before and is not in the bibliographical references.

85. Page 6, line 5: [Pap23, Definition 3.1.4] does not seem to exist. It could be Definition 3.1.3.

86. Page 7, line -2: [Alp23, Section 2.2.9] does not exist in the current version of the paper. It rather seems to be 2.3.3.

87. Page 9, Lemma 3.3: [Gek99, Page 10] should be Page 494 in the published version.

88. Page 10, line 14: the reference to [GR96] for the rigid analytic space $\Gamma\backslash\Omega$ should rather be Section 2.2.

89. Page 14, paragraph "By rigid analytic GAGA...", line -2: instead of [PY16, Theorem 7.4], I suggest to refer to Theorem 5.4 as cited on page 13.

90. Page 20, line -12: in [Bre16], the reference should be page 312 in the published version.

91. Pages 25-26: several references have no complete bibliographical entries (journal name, etc.): [AVO07], [AOV10], [CT07], [EGH23]... Complete them if possible.