Satellite Geodesy

Exercise3-Double differences solution

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To receivers at the approximate WGS84 positions

$$x_{1,0} = (4.159404458308991 \cdot 10^6, 672972.065, 4.77245255894603 \cdot 10^6)^T$$

and

$$x_{2,0} = (4.1551687403802983 \cdot 10^6, 672949.963, 4.776113966971923 \cdot 10^6)^T$$

observe phases on the frequency f_1 to all visible satellites. The observations are stored in the files Phase1.dat and Phase2.dat. The WGS84 positions of all visible satellites are stored in the file VisibleSatellites.dat.

1) Compute the double differences solution.

Single difference step only eliminate the clock errors. An adjustment process is performed to find out the ambiguity vector. The adjustment model this denotes as:

$$\mathbf{Y} = \mathbf{A}\boldsymbol{\beta} + \boldsymbol{\varepsilon}$$

The estimation vector is:

$$\boldsymbol{\beta} = (\Delta\mathbf{b}^T, \nabla N^1, \cdots, \nabla N^{k-1})^T$$

The phase observation model is given by (In this exercise, $k = 11$):

$$\Phi_i^j = r_i^j + N_i^j\lambda + c\left(dt_i + dt^j\right) + I^j + T^j + \varepsilon_i^j, i = 1,2,\cdots \quad j = 1,\cdots,k$$

where the upper index refers to the satellite and the lower index refers to the receiver.

For the observation from both receivers k single differences are calculated by:

$$\Delta\Phi^j = \Phi_1^j - \Phi_2^j = \Delta r^j + \lambda\Delta N^j + c\Delta t + \varepsilon^j$$

The k-1 double differences are the subtraction of the single differences from the $1,\cdots,k-1$ satellites to the k satellite:

$$\nabla\Phi^j = \Delta\Phi^j - \Delta\Phi^k = \nabla r^j + \lambda\Delta N^j + \varepsilon^j, \qquad j = 1,\cdots,k-1$$

The computational process of one epoch can be done by:

$$\begin{bmatrix} \nabla\Phi^1 \\ \nabla\Phi^2 \\ \vdots \\ \nabla\Phi^{k-1} \end{bmatrix} = \begin{bmatrix} 1 & 0 & \dots & 0 & -1 \\ 0 & 1 & \dots & 0 & -1 \\ & & \ddots & & \\ 0 & 0 & \cdots & 1 & -1 \end{bmatrix} \begin{bmatrix} \Delta\Phi^1 \\ \Delta\Phi^2 \\ \vdots \\ \Delta\Phi^k \end{bmatrix}$$

Linearize this function we get:

$$\nabla\Phi^{j} - \nabla\Phi_0^{j} = -\left(e^{j} - e^{k}\right)^{T}\Delta b + \lambda\Delta N^{j} + \nabla\varepsilon^{j}, \qquad j = 1,\cdots,k-1$$

For the adjustment purpose, the observation vector is set up as (n is the number of epochs, which equals 25 in this exercise):

$$\mathbf{Y} = \begin{bmatrix} \nabla\Phi^{1}(t_1) - \nabla\Phi_0^{1}(t_1) \\ \nabla\Phi^{2}(t_1) - \nabla\Phi_0^{1}(t_1) \\ \vdots \\ \nabla\Phi^{k-1}(t_1) - \nabla\Phi_0^{k-1}(t_1) \\ \vdots \\ \vdots \\ \nabla\Phi^{1}(t_n) - \nabla\Phi_0^{1}(t_n) \\ \nabla\Phi^{2}(t_n) - \nabla\Phi_0^{1}(t_n) \\ \vdots \\ \nabla\Phi^{k-1}(t_n) - \nabla\Phi_0^{k-1}(t_n) \end{bmatrix}$$

whereas the initial vector $\nabla\Phi_0$ is computed by the double differences of the approximate distance (since the positions of satellites and the approximate positions of the receivers are known) between the satellites and the observer, with similar procedure of the computation of the phase double difference.

The design matrix is built up by:

$$\mathbf{A} = \begin{bmatrix} -(\boldsymbol{e}^{1}(t_1) - \boldsymbol{e}^{k}(t_1))^{T} & \lambda & 0 & \cdots & 0 \\ -(\boldsymbol{e}^{2}(t_1) - \boldsymbol{e}^{k}(t_1))^{T} & 0 & \lambda & \cdots & 0 \\ & & \ddots & & \\ -(\boldsymbol{e}^{k-1}(t_1) - \boldsymbol{e}^{k}(t_1))^{T} & 0 & 0 & \cdots & \lambda \\ & & \vdots & & \\ & & \vdots & & \\ -(\boldsymbol{e}^{1}(t_n) - \boldsymbol{e}^{k}(t_n))^{T} & \lambda & 0 & \cdots & 0 \\ -(\boldsymbol{e}^{2}(t_n) - \boldsymbol{e}^{k}(t_n))^{T} & 0 & \lambda & \cdots & 0 \\ & & \ddots & & \\ -(\boldsymbol{e}^{k-1}(t_n) - \boldsymbol{e}^{k}(t_n))^{T} & 0 & 0 & \cdots & \lambda \end{bmatrix}$$

where $\boldsymbol{e}^{k}(t_n)$ shows the normal vectors of relative vector between satellite k and the receiver in epoch n. And λ is the wavelength of f_1 frequency.

$$\boldsymbol{e}(t) = \frac{SatPos - RecPos}{||SatPos - RecPos||}$$

$$f_1 = 1575.42$$

$$\lambda = \frac{c}{f}$$

with $c = 299792458\ \mathrm{m/s}$

The weight matrix of one epoch is

$$\mathbf{P} = \frac{1}{k}\begin{bmatrix} k-1 & -1 & -1 & \cdots & -1 & -1 \\ -1 & k-1 & -1 & \cdots & -1 & -1 \\ & & & \ddots & & \\ -1 & -1 & -1 & \dots & -1 & k-1 \end{bmatrix}$$

The multi-epoch weight matrix is thus derived by a block diagonal matrix

$$\overline{\boldsymbol{P}} = \begin{bmatrix} \mathbf{P} & 0 & \dots & 0 \\ 0 & \mathbf{P} & \dots & 0 \\ & & \ddots & \\ 0 & 0 & \cdots & \mathbf{P} \end{bmatrix}$$

The least squares estimation results in that:

$$\widehat{\boldsymbol{\beta}} = (\mathbf{A}^{\boldsymbol{T}}\overline{\boldsymbol{P}}\mathbf{A})^{-\mathbf{1}}\mathbf{A}^{\boldsymbol{T}}\overline{\boldsymbol{P}}\boldsymbol{Y}$$

The numerical result is (with the unit meter):

$$\widehat{\Delta\mathbf{b}} = [-2.8826, -81.5066, -18.0775]^T$$

$$\widehat{\nabla \boldsymbol{N}} = [-17.9909, -76.0157, -60.9937, -294.0005, -244.0008, -197.0038, -46.0045, 57.0052, -266.0077, 37.0044]^T$$

2) Approximate the matrix $\boldsymbol{Q_{22}}$ by its main diagonal.

Split the estimation parameters into two parts. One part is the baseline correction estimation and another part is the estimated ambiguity vector.

$$\boldsymbol{\beta} = [\boldsymbol{\beta}_1^{\mathrm{T}}, \boldsymbol{\beta}_2^{\mathrm{T}}]^{\mathrm{T}}$$

Accordingly, the design matrix is separated into two parts.

$$\mathbf{Y} = [\mathbf{A_1} \quad \mathbf{A_2}]\begin{bmatrix} \boldsymbol{\beta}_1 \\ \boldsymbol{\beta}_2 \end{bmatrix}$$

Hence the normal equation contains four poarts:

$$\mathbf{N} = \begin{bmatrix} \boldsymbol{A_1^T}\overline{\boldsymbol{P}}\boldsymbol{A_1} & \boldsymbol{A_1^T}\overline{\boldsymbol{P}}\boldsymbol{A_2} \\ \boldsymbol{A_2^T}\overline{\boldsymbol{P}}\boldsymbol{A_1} & \boldsymbol{A_2^T}\overline{\boldsymbol{P}}\boldsymbol{A_2} \end{bmatrix}$$

The inverse of the norm equation is also divided into four blocks

$$\mathbf{Q} = \begin{bmatrix} \mathbf{Q_{11}} & \mathbf{Q_{12}} \\ \mathbf{Q_{21}} & \mathbf{Q_{22}} \end{bmatrix}$$

The diagonal elements of result are finally:

$$\mathbf{Q_{22}} = \begin{bmatrix} 3.8561 \\ 3.8037 \\ 3.3662 \\ 3.1613 \\ 2.9795 \\ 2.3883 \\ 2.7156 \\ 2.4506 \\ 2.4311 \\ 2.2183 \end{bmatrix}$$

3) Compute the confidence sphere for the ambiguity double differences.
Use the value $\boldsymbol{\chi_{247,0.05} = 284.66}$

The precision of the previous estimation process is given by:

$$\sigma^2 = \frac{(\mathbf{Y} - \mathbf{A}\widehat{\boldsymbol{\beta}})^T \overline{\boldsymbol{P}} (\mathbf{Y} - \mathbf{A}\widehat{\boldsymbol{\beta}})}{m - u} = 9.4519 \cdot 10^{-6}$$

with $\mathrm{m} = 250, \mathrm{u} = 13$

The longest major axis is computed by ($\lambda_{max} = 3.8561$)

$$\sqrt{\sigma^2 \lambda_{max} \chi^2{}_{247,0.05}} = 0.1019$$

which indicates that there is only one candidate of the ambiguity fixing.
Round the float the estimated ambiguity vector and plug in into the probability test:

$$\left(\beta_2 - \hat{\beta}_2\right)^T \mathbf{Q_{22}} \left(\beta_2 - \hat{\beta}_2\right) = 1.5596 \cdot 10^{-4}$$

and

$$\chi^2{}_{247,0.05} \sigma^2 = 0.0027$$

$$\left(\beta_2 - \hat{\beta}_2\right)^T \mathbf{Q_{22}} \left(\beta_2 - \hat{\beta}_2\right) \leq \chi^2{}_{247,0.05} \sigma^2$$

Hence the candidate is checked.

4) Resolve the ambiguity double differences.

The ambiguity double difference is:

$$\boldsymbol{\beta}_2 = \begin{bmatrix} -18 \\ -76 \\ -61 \\ -294 \\ -244 \\ -197 \\ -46 \\ 57 \\ -266 \\ 37 \end{bmatrix}$$