Satellite Geodesy

Exercise1-Astronomical position determination

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The exact position of the observer on earth can be obtained by observing the zenith-distances to two stars with approximate position. The computational steps are given below.

1) The observation data is in ICRS coordinate system. It is necessary to derive the unit vectors of the observations after transforming the parameters into radius form.

$$\overrightarrow{\mathrm{xICRS1}} = (\cos(\delta 1) * \cos(\alpha 1), \cos(\delta 1) * \sin(\alpha 1), \sin(\delta 1))$$
$$\overrightarrow{\mathrm{xICRS2}} = (\cos(\delta 2) * \cos(\alpha 2), \cos(\delta 2) * \sin(\alpha 2), \sin(\delta 2))$$

With the observation data:

$$\alpha 1 = 17h\ 46\min 1.94s,\ \delta 1 = 56.73^{\circ}, z_1 = 52.9823^{\circ}$$
$$\alpha 2 = 18h\ 24\min 6.44s,\ \delta 2 = 37.90^{\circ}, z_2 = 59.3379^{\circ}$$

The results:

$$\overrightarrow{\mathrm{xICRS1}} = [\,-0.0334\ \ -0.5476\ \ 0.8360]$$
$$\overrightarrow{\mathrm{xICRS2}} = [\,0.0828\ \ -0.7847\ 0.6142]$$

2) Precession correction for each star

a. Compute Julia date of 2014/11/27/19/0/0 as well as the date difference.

$$\mathrm{T} = \frac{JD - G2JD(date)}{36525}$$

b. Precession angles are calculated for correction process.

$$\xi\mathrm{A} = (2306''.2181 + 1''.39656T - 0''.000139T^2)T + (0''.30188 - 0''.000344T)T^2 + 0''.017998T^3$$
$$\mathrm{zA} = (2306''.2181 + 1''.39656T - 0''.000139T^2)T + (1''.09468 - 0''.000066T)T^2 + 0''.018203T^3$$
$$\theta\mathrm{A} = (2306''.2181 + 1''.39656T - 0''.000139T^2)T + (0''.30188 - 0''.000344T)T^2 + 0''.041833T^3$$

c. Both vectors are corrected and transformed into CEP system(Celestial Ephemeris Pole)

$$\overrightarrow{\mathrm{xCEP1}} = R3(-zA)R2(\theta A)R3(\xi A)\ \overrightarrow{\mathrm{xICRS1}}$$
$$\overrightarrow{\mathrm{xCEP2}} = R3(-zA)R2(\theta A)R3(\xi A)\ \overrightarrow{\mathrm{xICRS2}}$$

The results:

$$\overrightarrow{\mathrm{xCEP1}} = [\ -0.0328\ \ \ -0.5477\ \ 0.8360]$$
$$\overrightarrow{\mathrm{xCEP2}} = [\,0.0845\ \ \ -0.7844\ \ 0.6144]$$

3) Earth rotation correction

a. Calculate the GMST with a certain algorithm

b. Transform the unit vector into earth-fixed system

$$\overrightarrow{\mathrm{xITRS1}} = R3(\theta A)\overrightarrow{\mathrm{xCEP1}}$$
$$\overrightarrow{\mathrm{xITRS2}} = R3(\theta A)\overrightarrow{\mathrm{xCEP2}}$$

The results:

$$\overrightarrow{\mathrm{xITRS1}} = [0.0474\ -0.5466\ \ 0.8360]$$
$$\overrightarrow{\mathrm{xITRS2}} = [\,0.1981\ -0.7637\ 0.6144]$$

4) To obtain the position with vector geometry. The zenith angle is known. The unit vector across the observation point can be expressed as

$$\overrightarrow{\mathrm{xP}} = (cosBcosL, cosBsinL, sinB)$$

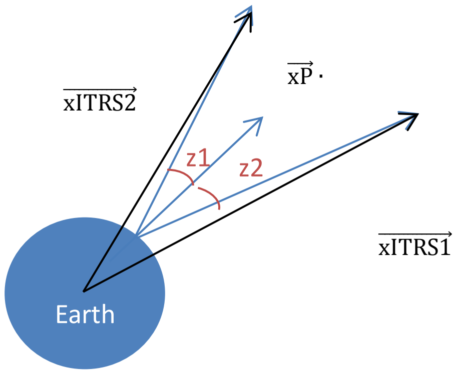
Let

$$\overrightarrow{\mathrm{xITRS1}} = [x1, y1, z1]$$
$$\overrightarrow{\mathrm{xITRS2}} = [x2, y2, z2]$$

The vector geometry rule is as

$$\overrightarrow{\mathrm{xITRS1}} \cdot \overrightarrow{\mathrm{xP}} = \cos(z1)$$
$$\overrightarrow{\mathrm{xITRS2}} \cdot \overrightarrow{\mathrm{xP}} = \cos(z2)$$

Since the stars are far enough from the observer, the two unit vectors across the observer are considered approximately the unit vectors $\overrightarrow{\mathrm{xITRS1}}$ and $\overrightarrow{\mathrm{xITRS2}}$.



In matrix notation

$$\begin{bmatrix} \cos(z1) \\ \cos(z2) \end{bmatrix} = \begin{bmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \end{bmatrix} \begin{bmatrix} cosBcosL \\ cosBsinL \\ sinB \end{bmatrix}$$

With two equations, the two unknowns L and B can be solved uniquely. But the equations are not linear. Therefore, the equation should be linearized first with respect to both unknowns. The approximate position L0 and B0 are used as the initial values.

Let

$$A = \begin{bmatrix} x1 & y1 & z1 \\ x2 & y2 & z2 \end{bmatrix}$$

The linearized equation is given below with the higher order terms (hot)

$$\begin{bmatrix} \cos(z1) \\ \cos(z2) \end{bmatrix} - \mathrm{A} \begin{bmatrix} cosB0cosL0 \\ cosB0sinL0 \\ sinB0 \end{bmatrix} = \mathrm{A} \left. \begin{bmatrix} -sinBcosL \\ -sinBsinL \\ cosB \end{bmatrix} \right|_{TP} (\mathrm{B}\text{-}\mathrm{B0}) + \mathrm{A} \left. \begin{bmatrix} -cosBsinL \\ cosBcosL \\ 0 \end{bmatrix} \right|_{TP} (L - L0) + hot$$

with initial values

$$\mathrm{L0} = \frac{9.16\pi}{180}$$

$$\mathrm{B0} = \frac{48.75\pi}{180}$$

Note

$$\Delta \mathrm{y} = \begin{bmatrix} \cos(z1) \\ \cos(z2) \end{bmatrix} - \mathrm{A} \begin{bmatrix} cosB0cosL0 \\ cosB0sinL0 \\ sinB0 \end{bmatrix} + hot$$

$$dB = \mathrm{B} - \mathrm{B0}$$
$$dL = L - L0$$

the observation equation is rewritten as

$$\Delta \mathrm{y} = \mathrm{A}[C \quad D] \begin{bmatrix} dB \\ dL \end{bmatrix} + hot$$

Let

$$\mathrm{Anew} = \mathrm{A}[C \quad D]$$
$$X = \begin{bmatrix} dB \\ dL \end{bmatrix}$$

Hence

$$\Delta \mathrm{y} = \mathrm{Anew} \cdot X + \mathrm{hot}$$
$$X = (Anew^T Anew)^{-1} Anew^T \Delta \mathrm{y}$$

dB and dL are estimated iteratively while until dB<0.0000001. While the condition is not satisfied, the Taylor points are substituted by B=B0+dB and L=L0+dL.

5) Results discussion

The estimated position is

$$\mathrm{L} = 9^{\circ}.3002$$
$$\mathrm{B} = 48^{\circ}.8349$$

6) Finally the results are transformed into Cartesian coordinates in GRS80 ellipsoid.

$$\begin{bmatrix} xG \\ yG \\ zG \end{bmatrix} = \begin{bmatrix} (N+H)cosBcosL \\ (N+H)cosBsinL \\ (N(1-e^2)+H)sinB \end{bmatrix}$$

with

$$\mathrm{N} = \frac{a}{\sqrt{1 - e^2 sin^2 \varphi}}$$
$$\mathrm{e}^2 = \frac{a^2 - b^2}{a^2}$$
$$\mathrm{f} = \frac{a - b}{a}$$

The results:

$$\begin{bmatrix} xG \\ yG \\ zG \end{bmatrix} = \begin{bmatrix} 4150981.2065 \\ 679767.1042 \\ 4778494.8910 \end{bmatrix}$$

With the given Helmert Transform parameters the corresponding coordinates in Rauenberg datum are calculated.

$$\begin{bmatrix} xR \\ yR \\ zR \end{bmatrix} = \begin{bmatrix} tx \\ ty \\ tz \end{bmatrix} + (1 + \delta m) \cdot R1(\alpha) \cdot R2(\beta) \cdot R3(\theta) \begin{bmatrix} xG \\ yG \\ zG \end{bmatrix}$$

The results:

$$\begin{bmatrix} xR \\ yR \\ zR \end{bmatrix} = \begin{bmatrix} 4150445.1192 \\ 679576.4222 \\ 4778184.4560 \end{bmatrix}$$