Теорема о неявном отображении

$m,n\in\mathbb{N}$, $\underset{\text{откр.}}{O} \subseteq \mathbb{R}^{m+n}$; $x=(x_1,\dots,x_n)$, $y=(y_1,\dots,y_m)$, $r\in\mathbb{N}$

$\exists$ $F\in C^r(O\to\mathbb{R}^m)$; $\exists$ $x^0=(x^0_1,\dots,x^0_n)$, $y^0=(y^0_1,\dots,y^0_m)$:

i). $F(x^0,y^0)=0$

ii) $F'_y(x^0,y^0)\neq 0$, $\left(F'_y=\frac{\partial F}{\partial y}=\begin{pmatrix} F'_{1y_1} & \dots & F'_{1y_m} \\ \dots & \dots & \dots \\ F'_{my_1} & \dots & F'_{my_m}\end{pmatrix}\right)$

Тогда $\exists$ откр-m $U(x^0)$, $U(y^0)$ и $f: U(x^0)\to U(y^0)$:

1). $F(x,y)=0 \iff y=f(x)$ в $U(x^0)\times U(y^0)$

2). $f\in C^r(U(x^0)\to U(y^0))$

3). $f'(x)=-\left(F'_y(x,f(x))\right)^{-1}\cdot F'_x(x,f(x))$

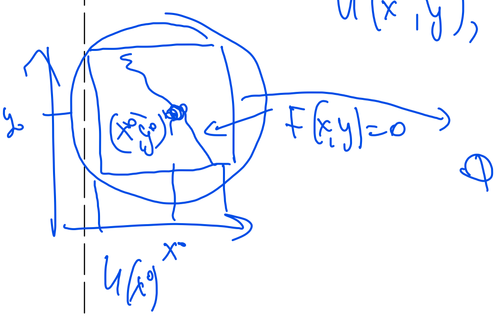
$F'=\left(\underbrace{F'_x}_{n} \;\middle|\; \underbrace{F'_y}_{m}\right)\Big\updownarrow m$

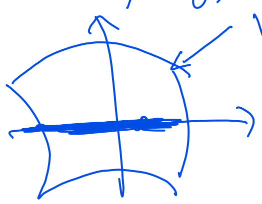
$\Phi(x,y)=(\underbrace{x}_{n\text{-вектор}}, \underbrace{F(x,y)}_{m\text{ вектор}}) : O \to \mathbb{R}^{n+m}$, $O\subseteq\mathbb{R}^{n+m}$

$$\Phi'=\left(\begin{array}{c|c} E_n = \begin{matrix}1 & 0 & \dots\\ 0 & 1 & \dots\\ \dots & & \ddots\end{matrix} & 0 \\ \hline F'_x & F'_y\end{array}\right)$$

$\det\Phi' = \underbrace{\det E_n}_{1}\cdot\underbrace{\det F'_y}_{\neq 0}\neq 0$ вблизи в O.

По предыдущ. теор. в окрестности точки (x^0,y^0) Φ обратимо, $\exists$ окр. $U(x^0)$, $U(y^0)$: $U(x^0)\times U(y^0)\subseteq U(x^0,y^0)$, ($\Phi\big|_{U(x^0)\times U(y^0)}$ — обратимо)





$V=\Phi(U(x^0)\times U(y^0))$

$\forall\; x\in U(x^0))$