

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

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Class Index

2.1 Class List

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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--	----

Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.2 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.3 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<i>...values</i>	
------------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

Public Types

- using **type** = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct **val**
values in i32, again represented as types

Public Types

- using **inner_type** = int32_t
- using **zero** = val< 0 >
constant zero
- using **one** = val< 1 >
constant one
- template<auto x>
using **inject_constant_t** = val< static_cast< int32_t >(x)>
- template<typename v >
using **inject_ring_t** = v
- template<typename v1 , typename v2 >
using **add_t** = typename add< v1, v2 >::type
addition operator
- template<typename v1 , typename v2 >
using **sub_t** = typename sub< v1, v2 >::type
subtraction operator
- template<typename v1 , typename v2 >
using **mul_t** = typename mul< v1, v2 >::type
multiplication operator
- template<typename v1 , typename v2 >
using **div_t** = typename div< v1, v2 >::type
division operator
- template<typename v1 , typename v2 >
using **mod_t** = typename remainder< v1, v2 >::type
modulus operator
- template<typename v1 , typename v2 >
using **gt_t** = typename gt< v1, v2 >::type
strictly greater operator (v1 > v2)

- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity (type)(v > 0)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Data Documentation

5.7.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.7.2.2 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<i>v</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- `using inner_type = int64_t`
- `template<auto x>`
`using inject_constant_t = val< static_cast< int64_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`

- strictly greater operator ($v1 > v2$) - type*
 • `template<typename v1 , typename v2 >`
 `using lt_t = typename lt< v1, v2 >::type`
 strict less operator ($v1 < v2$)
- `template<typename v1 , typename v2 >`
 `using eq_t = typename eq< v1, v2 >::type`
 equality operator (type)
- `template<typename v1 , typename v2 >`
 `using gcd_t = gcd_t< i64, v1, v2 >`
 greatest common divisor
- `template<typename v >`
 `using pos_t = typename pos< v >::type`
 is v positive (type)

Static Public Attributes

- `static constexpr bool is_field = false`
 integers are not a field
- `static constexpr bool is_euclidean_domain = true`
 integers are an euclidean domain
- `template<typename v1 , typename v2 >`
 `static constexpr bool gt_v = gt_t<v1, v2>::value`
 strictly greater operator ($v1 > v2$) - boolean value
- `template<typename v1 , typename v2 >`
 `static constexpr bool lt_v = lt_t<v1, v2>::value`
 strictly smaller operator ($v1 < v2$) - boolean value
- `template<typename v1 , typename v2 >`
 `static constexpr bool eq_v = eq_t<v1, v2>::value`
 equality operator (boolean value)
- `template<typename v >`
 `static constexpr bool pos_v = pos_t<v>::value`
 positivity (boolean value)

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Data Documentation

5.8.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.8.2.2 gt_v

```
template<typename v1 , typename v2 >  
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator ($v1 > v2$) - boolean value

Template Parameters

v1	
v2	

5.8.2.3 lt_v

```
template<typename v1 , typename v2 >  
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator ($v1 < v2$) - boolean value

Template Parameters

v1	
v2	

5.8.2.4 pos_v

```
template<typename v >  
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

v	
---	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.9 aerobus::polynomial< Ring, variable_name >::eval_helper< valueRing, P >::inner< index, stop > Struct Template Reference

Static Public Member Functions

- static constexpr valueRing func (const valueRing &accum, const valueRing &x)

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.10 aerobus::polynomial< Ring, variable_name >::eval_helper< valueRing, P >::inner< stop, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.11 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`
true iff n is prime

5.11.1 Detailed Description

```
template<int32_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.12 aerobus::polynomial< Ring, variable_name > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero](#) = [val< typename Ring::zero >](#)
constant zero
- [using one](#) = [val< typename Ring::one >](#)
constant one
- [using X](#) = [val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t](#) = [typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t](#) = [typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t](#) = [typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1 , typename v2 >](#)
[using mul_t](#) = [typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1 , typename v2 >](#)
[using eq_t](#) = [typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1 , typename v2 >](#)
[using lt_t](#) = [typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1 , typename v2 >](#)
[using gt_t](#) = [typename gt_helper< v1, v2 >::type](#)
strict greater operator
- [template<typename v1 , typename v2 >](#)
[using div_t](#) = [typename div< v1, v2 >::q_type](#)
division operator
- [template<typename v1 , typename v2 >](#)
[using mod_t](#) = [typename div_helper< v1, v2, zero, v1 >::mod_type](#)
modulo operator
- [template<typename coeff , size_t deg>](#)
[using monomial_t](#) = [typename monomial< coeff, deg >::type](#)
monomial : coeff X^{deg}
- [template<typename v >](#)
[using derive_t](#) = [typename derive_helper< v >::type](#)
derivation operator
- [template<typename v >](#)
[using pos_t](#) = [typename Ring::template pos_t< typename v::aN >](#)
checks for positivity (an > 0)

- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring, variable_name >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

5.12.1 Detailed Description

```
template<typename Ring, char variable_name = 'x'>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring, variable_name >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.12.2 Member Typedef Documentation

5.12.2.1 add_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<code>v1</code>	
<code>v2</code>	

5.12.2.2 derive_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.12.2.3 div_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.4 eq_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.5 gcd_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gcd_t = std::conditional_t< Ring::is_↵
euclidean_domain, typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2> >::type,
void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.6 gt_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.7 lt_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.8 mod_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.9 monomial_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring, variable_name >::monomial_t = typename monomial<coeff, deg>↔::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.12.2.10 mul_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.11 pos_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.12.2.12 simplify_t

```
template<typename Ring , char variable_name = 'x'>
template<typename P >
using aerobus::polynomial< Ring, variable_name >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.12.2.13 sub_t

```
template<typename Ring , char variable_name = 'x'>
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.13 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.13.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.14 aerobus::Quotient< Ring, X > Struct Template Reference

Classes

- struct [val](#)

Public Types

- using **zero** = val< typename Ring::zero >
- using **one** = val< typename Ring::one >
- template<typename v1 , typename v2 >
using **add_t** = val< typename Ring::template add_t< typename v1::type, typename v2::type > >
- template<typename v1 , typename v2 >
using **mul_t** = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >
- template<typename v1 , typename v2 >
using **div_t** = val< typename Ring::template div_t< typename v1::type, typename v2::type > >
- template<typename v1 , typename v2 >
using **mod_t** = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >
- template<typename v1 , typename v2 >
using **eq_t** = typename Ring::template eq_t< typename v1::type, typename v2::type >
- template<typename v1 >
using **pos_t** = std::true_type
- template<auto x>
using **inject_constant_t** = val< typename Ring::template inject_constant_t< x > >
- template<typename v >
using **inject_ring_t** = val< v >

Static Public Attributes

- template<typename v1 , typename v2 >
static constexpr bool **eq_v** = Ring::template eq_t<typename v1::type, typename v2::type>::value
- template<typename v >
static constexpr bool **pos_v** = pos_t<v>::value
- static constexpr bool **is_euclidean_domain** = true

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.15 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using **head** = typename inner::head
- using **tail** = typename inner::tail

5.15.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

index	
-----------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.16 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = internal::type_at_t< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename concat_h< U >::type
concatenates two list into one
- template<typename T , size_t index>
using [insert](#) = typename internal::insert_h< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename internal::remove_h< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t **length** = sizeof...(Ts)
length of list

5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	
--------------	--

5.16.2 Member Typedef Documentation

5.16.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.16.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.16.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.16.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.16.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.16.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.17 aerobus::type_list<> Struct Reference

Public Types

- template<typename T >
using **push_front** = type_list< T >

- `template<typename T >`
using **push_back** = `type_list< T >`
- `template<typename U >`
using **concat** = `U`
- `template<typename T , size_t index>`
using **insert** = `type_list< T >`

Static Public Attributes

- static constexpr `size_t length` = 0

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.18 aerobus::i32::val< x > Struct Template Reference

values in `i32`, again represented as types

```
#include <aerobus.h>
```

Public Types

- using **is_zero_t** = `std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast x into valueType
- `static std::string to_string ()`
string representation of value
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast x into valueRing

Static Public Attributes

- `static constexpr int32_t v` = `x`
actual value stored in val type

5.18.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.18.2 Member Function Documentation

5.18.2.1 `eval()`

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.18.2.2 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<code>valueType</code>	double for example
------------------------	--------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.19 `aerobus::i64::val< x >` Struct Template Reference

values in `i64`

```
#include <aerobus.h>
```

Public Types

- `using is_zero_t = std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast value in valueType
- `static std::string to_string ()`
string representation
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast value in valueRing

Static Public Attributes

- `static constexpr int64_t v = x`

5.19.1 Detailed Description

`template<int64_t x>`
`struct aerobus::i64::val< x >`

values in `i64`

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.19.2 Member Function Documentation

5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.20 aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- **using aN = coeffN**
heavy weight coefficient (non zero)
- **using strip = val< coeffs... >**
remove largest coefficient
- **using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>**
true_type if polynomial is constant zero
- **template<size_t index>**
using coeff_at_t = typename coeff_at< index >::type
type of coefficient at index

Static Public Member Functions

- **static std::string to_string ()**
get a string representation of polynomial
- **template<typename valueRing >**
static constexpr valueRing eval (const valueRing &x)
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- **static constexpr size_t degree = sizeof...(coeffs)**
degree of the polynomial
- **static constexpr bool is_zero_v = is_zero_t::value**
true if polynomial is constant zero

5.20.1 Detailed Description

```
template<typename Ring, char variable_name = 'x'>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

5.20.2 Member Typedef Documentation

5.20.2.1 coeff_at_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::coeff_at_t = typename
coeff_at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.20.3 Member Function Documentation

5.20.3.1 eval()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs
>::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.20.3.2 to_string()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::to_←
string ( ) [inline], [static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

Public Types

- **using type** = std::conditional_t< Ring::template pos_v< tmp >, tmp, typename Ring::template sub_t< typename Ring::zero, tmp > >

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- **using is_zero_t** = std::bool_constant< x% p==0 >

Static Public Member Functions

- template<typename valueType >
static constexpr valueType **get** ()
- static std::string **to_string** ()
- template<typename valueRing >
static constexpr valueRing **eval** (const valueRing &v)

Static Public Attributes

- static constexpr int32_t **v** = x % p

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.23 aerobus::polynomial< Ring, variable_name >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = [std::bool_constant< aN::is_zero_t::value >](#)
- template<[size_t](#) index>
using [coeff_at_t](#) = [typename](#) [coeff_at< index >::type](#)

Static Public Member Functions

- [static](#) [std::string](#) [to_string](#) ()
- template<[typename](#) [valueRing](#) >
[static constexpr](#) [valueRing](#) [eval](#) ([const](#) [valueRing](#) &x)

Static Public Attributes

- [static constexpr](#) [size_t](#) [degree](#) = 0
- [static constexpr](#) [bool](#) [is_zero_v](#) = [is_zero_t::value](#)

5.23.1 Detailed Description

```
template<typename Ring, char variable\_name = 'x'>
template<typename coeffN>
struct aerobus::polynomial< Ring, variable\_name >::val< coeffN >
```

specialization for constants

Template Parameters

coeffN	
------------------------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.24 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- [using inner_type = int32_t](#)
- [template<auto x>](#)
[using inject_constant_t = val< static_cast< int32_t >\(x\)>](#)
- [using zero = val< 0 >](#)
- [using one = val< 1 >](#)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
addition operator
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction operator
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication operator
- [template<typename v1 , typename v2 >](#)
[using div_t = typename div< v1, v2 >::type](#)
division operator
- [template<typename v1 , typename v2 >](#)
[using mod_t = typename remainder< v1, v2 >::type](#)
modulo operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt< v1, v2 >::type](#)
strictly greater operator (type)
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt< v1, v2 >::type](#)
strictly smaller operator (type)
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq< v1, v2 >::type](#)
equality operator (type)
- [template<typename v1 , typename v2 >](#)
[using gcd_t = gcd_t< i32, v1, v2 >](#)
greatest common divisor
- [template<typename v1 >](#)
[using pos_t = typename pos< v1 >::type](#)
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.24.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

Chapter 6

File Documentation

6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035
00036     template<typename T, size_t N>
00037     constexpr bool contains(const std::array<T, N>& arr, const T& v) {
00038         for (const auto& vv : arr) {
00039             if (v == vv) {
00040                 return true;
00041             }
00042         }
00043         return false;
00044     }
00045 } // namespace aerobus
00046
00047 // concepts
00048 namespace aerobus {
00049     template <typename R>
00050     concept IsRing = requires {
00051         typename R::one;
00052         typename R::zero;
00053         typename R::template add_t<typename R::one, typename R::one>;
00054         typename R::template sub_t<typename R::one, typename R::one>;
00055         typename R::template mul_t<typename R::one, typename R::one>;
00056     };
00057 }
```

```

00074
00075
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00089     template<typename R>
00090     concept IsField = IsEuclideanDomain<R> && requires {
00091         R::is_field == true;
00092     };
00093 } // namespace aerobus
00094
00095 // utilities
00096 namespace aerobus {
00097     namespace internal {
00098         template<typename...> typename TT, typename T>
00099         struct is_instantiation_of : std::false_type {};
00100
00101         template<template<typename...> typename TT, typename... Ts>
00102         struct is_instantiation_of<TT, TT<Ts...> : std::true_type {};
00103
00104         template<template<typename...> typename TT, typename T>
00105         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00106
00107         template <int64_t i, typename T, typename... Ts>
00108         struct type_at {
00109             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00110             using type = typename type_at<i - 1, Ts...>::type;
00111         };
00112
00113         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00114             using type = T;
00115         };
00116
00117         template <size_t i, typename... Ts>
00118         using type_at_t = typename type_at<i, Ts...>::type;
00119
00120         template<int32_t n, int32_t i, typename E = void>
00121         struct _is_prime {};
00122
00123         template<int32_t i>
00124         struct _is_prime<1, i> {
00125             static constexpr bool value = false;
00126         };
00127
00128         template<int32_t i>
00129         struct _is_prime<2, i> {
00130             static constexpr bool value = true;
00131         };
00132
00133         template<int32_t i>
00134         struct _is_prime<3, i> {
00135             static constexpr bool value = true;
00136         };
00137
00138         template<int32_t i>
00139         struct _is_prime<5, i> {
00140             static constexpr bool value = true;
00141         };
00142
00143         template<int32_t i>
00144         struct _is_prime<7, i> {
00145             static constexpr bool value = true;
00146         };
00147
00148         template<int32_t n, int32_t i>
00149         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00150             static constexpr bool value = false;
00151         };
00152
00153         template<int32_t n, int32_t i>
00154         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00155             static constexpr bool value = false;
00156         };
00157
00158         template<int32_t n, int32_t i>
00159         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00160             static constexpr bool value = true;
00161         };
00162

```

```

00163     };
00164
00165     template<int32_t n, int32_t i>
00166     struct _is_prime<n, i, std::enable_if_t<(
00167         n % i == 0 &&
00168         n >= 9 &&
00169         n % 3 != 0 &&
00170         n % 2 != 0 &&
00171         i * i > n)> {
00172         static constexpr bool value = true;
00173     };
00174
00175     template<int32_t n, int32_t i>
00176     struct _is_prime<n, i, std::enable_if_t<(
00177         n % (i+2) == 0 &&
00178         n >= 9 &&
00179         n % 3 != 0 &&
00180         n % 2 != 0 &&
00181         i * i <= n)> {
00182         static constexpr bool value = true;
00183     };
00184
00185     template<int32_t n, int32_t i>
00186     struct _is_prime<n, i, std::enable_if_t<(
00187         n % (i+2) != 0 &&
00188         n % i != 0 &&
00189         n >= 9 &&
00190         n % 3 != 0 &&
00191         n % 2 != 0 &&
00192         (i * i <= n))> {
00193         static constexpr bool value = _is_prime<n, i+6>::value;
00194     };
00195
00196     } // namespace internal
00197
00200     template<int32_t n>
00201     struct is_prime {
00202         static constexpr bool value = internal::_is_prime<n, 5>::value;
00203     };
00204
00205     template<int32_t n>
00206     static constexpr bool is_prime_v = is_prime<n>::value;
00207
00208     namespace internal {
00209         template <std::size_t... Is>
00210         constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00211             -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00212
00213         template <std::size_t N>
00214         using make_index_sequence_reverse
00215             = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00216
00217         template<typename Ring, typename E = void>
00218         struct gcd;
00219
00220         template<typename Ring>
00221         struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00222             template<typename A, typename B, typename E = void>
00223             struct gcd_helper {};
00224
00225             // B = 0, A > 0
00226             template<typename A, typename B>
00227             struct gcd_helper<A, B, std::enable_if_t<
00228                 (B::is_zero_t::value) &&
00229                 (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00230                 using type = A;
00231             };
00232
00233             // B = 0, A < 0
00234             template<typename A, typename B>
00235             struct gcd_helper<A, B, std::enable_if_t<
00236                 (B::is_zero_t::value) &&
00237                 !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00238                 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00239             };
00240
00241             // B != 0
00242             template<typename A, typename B>
00243             struct gcd_helper<A, B, std::enable_if_t<
00244                 (!B::is_zero_t::value)
00245                 >> {
00246             private: // NOLINT
00247                 // A / B
00248                 using k = typename Ring::template div_t<A, B>;
00249                 // A - (A/B)*B = A % B
00250                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00251             };
00252         };
00253     }

```

```

00258         public:
00259             using type = typename gcd_helper<B, m>::type;
00260         };
00261
00262         template<typename A, typename B>
00263         using type = typename gcd_helper<A, B>::type;
00264     };
00265 } // namespace internal
00266
00267 template<typename T, typename A, typename B>
00270 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00271 } // namespace aerobus
00272
00273 // quotient ring by the principal ideal generated by X
00274 namespace aerobus {
00275     template<typename Ring, typename X>
00276     requires IsRing<Ring>
00277     struct Quotient {
00278         template <typename V>
00279         struct val {
00280             private: // NOLINT
00281                 using tmp = typename Ring::template mod_t<V, X>;
00282
00283             public:
00284                 using type = std::conditional_t<
00285                     Ring::template pos_v<tmp>,
00286                     tmp,
00287                     typename Ring::template sub_t<typename Ring::zero, tmp>
00288                 >;
00289         };
00290
00291         using zero = val<typename Ring::zero>;
00292         using one = val<typename Ring::one>;
00293
00294         template<typename v1, typename v2>
00295         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00296         template<typename v1, typename v2>
00297         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00298         template<typename v1, typename v2>
00299         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00300         template<typename v1, typename v2>
00301         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00302         template<typename v1, typename v2>
00303         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00304         template<typename v1, typename v2>
00305         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00306         template<typename v1>
00307         using pos_t = std::true_type;
00308
00309         template<typename v>
00310         static constexpr bool pos_v = pos_t<v>::value;
00311
00312         static constexpr bool is_euclidean_domain = true;
00313
00314         template<auto x>
00315         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00316
00317         template<typename v>
00318         using inject_ring_t = val<v>;
00319     };
00320 } // namespace aerobus
00321
00322 // type_list
00323 namespace aerobus {
00324     template <typename... Ts>
00325     struct type_list;
00326
00327     namespace internal {
00328         template <typename T, typename... Us>
00329         struct pop_front_h {
00330             using tail = type_list<Us...>;
00331             using head = T;
00332         };
00333
00334         template <size_t index, typename L1, typename L2>
00335         struct split_h {
00336             private:
00337                 static_assert(index <= L2::length, "index out of bounds");
00338                 using a = typename L2::pop_front::type;
00339                 using b = typename L2::pop_front::tail;
00340                 using c = typename L1::template push_back<a>;
00341
00342             public:
00343                 using head = typename split_h<index - 1, c, b>::head;
00344                 using tail = typename split_h<index - 1, c, b>::tail;
00345         };
00346     }
00347

```

```

00348     template <typename L1, typename L2>
00349     struct split_h<0, L1, L2> {
00350         using head = L1;
00351         using tail = L2;
00352     };
00353
00354     template <size_t index, typename L, typename T>
00355     struct insert_h {
00356         static_assert(index <= L::length, "index ouf of bounds");
00357         using s = typename L::template split<index>;
00358         using left = typename s::head;
00359         using right = typename s::tail;
00360         using ll = typename left::template push_back<T>;
00361         using type = typename ll::template concat<right>;
00362     };
00363
00364     template <size_t index, typename L>
00365     struct remove_h {
00366         using s = typename L::template split<index>;
00367         using left = typename s::head;
00368         using right = typename s::tail;
00369         using rr = typename right::pop_front::tail;
00370         using type = typename left::template concat<rr>;
00371     };
00372 } // namespace internal
00373
00374
00377 template <typename... Ts>
00378 struct type_list {
00379 private:
00380     template <typename T>
00381     struct concat_h;
00382
00383     template <typename... Us>
00384     struct concat_h<type_list<Us...> {
00385         using type = type_list<Ts..., Us...>;
00386     };
00387
00388 public:
00389     static constexpr size_t length = sizeof...(Ts);
00390
00391     template <typename T>
00392     using push_front = type_list<T, Ts...>;
00393
00394     template <size_t index>
00395     using at = internal::type_at_t<index, Ts...>;
00401
00403     struct pop_front {
00404         using type = typename internal::pop_front_h<Ts...>::head;
00405         using tail = typename internal::pop_front_h<Ts...>::tail;
00406     };
00407
00408     template <typename T>
00409     using push_back = type_list<Ts..., T>;
00410
00411     template <typename U>
00412     using concat = typename concat_h<U>::type;
00413
00414     template <size_t index>
00415     struct split {
00416     private:
00417         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00418
00419     public:
00420         using head = typename inner::head;
00421         using tail = typename inner::tail;
00422     };
00423
00424     template <typename T, size_t index>
00425     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00426
00427     template <size_t index>
00428     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00429 };
00430
00431
00432 template <>
00433 struct type_list<> {
00434     static constexpr size_t length = 0;
00435
00436     template <typename T>
00437     using push_front = type_list<T>;
00438
00439     template <typename T>
00440     using push_back = type_list<T>;
00441
00442     template <typename U>
00443     using concat = U;

```

```

00456
00457     // TODO(jewave): assert index == 0
00458     template <typename T, size_t index>
00459     using insert = type_list<T>;
00460 };
00461 } // namespace aerobus
00462
00463 // i32
00464 namespace aerobus {
00465     struct i32 {
00466         using inner_type = int32_t;
00467         template<int32_t x>
00471         struct val {
00473             static constexpr int32_t v = x;
00474
00477             template<typename valueType>
00478             static constexpr valueType get() { return static_cast<valueType>(x); }
00479
00481             using is_zero_t = std::bool_constant<x == 0>;
00482
00484             static std::string to_string() {
00485                 return std::to_string(x);
00486             }
00487
00490             template<typename valueRing>
00491             static constexpr valueRing eval(const valueRing& v) {
00492                 return static_cast<valueRing>(x);
00493             }
00494         };
00495
00497         using zero = val<0>;
00499         using one = val<1>;
00501         static constexpr bool is_field = false;
00503         static constexpr bool is_euclidean_domain = true;
00507         template<auto x>
00508         using inject_constant_t = val<static_cast<int32_t>(x)>;
00509
00510         template<typename v>
00511         using inject_ring_t = v;
00512
00513     private:
00514         template<typename v1, typename v2>
00515         struct add {
00516             using type = val<v1::v + v2::v>;
00517         };
00518
00519         template<typename v1, typename v2>
00520         struct sub {
00521             using type = val<v1::v - v2::v>;
00522         };
00523
00524         template<typename v1, typename v2>
00525         struct mul {
00526             using type = val<v1::v * v2::v>;
00527         };
00528
00529         template<typename v1, typename v2>
00530         struct div {
00531             using type = val<v1::v / v2::v>;
00532         };
00533
00534         template<typename v1, typename v2>
00535         struct remainder {
00536             using type = val<v1::v % v2::v>;
00537         };
00538
00539         template<typename v1, typename v2>
00540         struct gt {
00541             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00542         };
00543
00544         template<typename v1, typename v2>
00545         struct lt {
00546             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00547         };
00548
00549         template<typename v1, typename v2>
00550         struct eq {
00551             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00552         };
00553
00554         template<typename v1>
00555         struct pos {
00556             using type = std::bool_constant<(v1::v > 0)>;
00557         };
00558
00559     public:

```



```

00561     template<typename v1, typename v2>
00562     using add_t = typename add<v1, v2>::type;
00563
00565     template<typename v1, typename v2>
00566     using sub_t = typename sub<v1, v2>::type;
00567
00569     template<typename v1, typename v2>
00570     using mul_t = typename mul<v1, v2>::type;
00571
00573     template<typename v1, typename v2>
00574     using div_t = typename div<v1, v2>::type;
00575
00577     template<typename v1, typename v2>
00578     using mod_t = typename remainder<v1, v2>::type;
00579
00581     template<typename v1, typename v2>
00582     using gt_t = typename gt<v1, v2>::type;
00583
00585     template<typename v1, typename v2>
00586     using lt_t = typename lt<v1, v2>::type;
00587
00589     template<typename v1, typename v2>
00590     using eq_t = typename eq<v1, v2>::type;
00591
00595     template<typename v1, typename v2>
00596     static constexpr bool eq_v = eq_t<v1, v2>::value;
00597
00599     template<typename v1, typename v2>
00600     using gcd_t = gcd_t<i32, v1, v2>;
00601
00603     template<typename v>
00604     using pos_t = typename pos<v>::type;
00605
00608     template<typename v>
00609     static constexpr bool pos_v = pos_t<v>::value;
00610 };
00611 } // namespace aerobus
00612
00613 // i64
00614 namespace aerobus {
00615     struct i64 {
00616         using inner_type = int64_t;
00617         template<int64_t x>
00621         struct val {
00622             static constexpr int64_t v = x;
00623
00626             template<typename valueType>
00627             static constexpr valueType get() { return static_cast<valueType>(x); }
00628
00630             using is_zero_t = std::bool_constant<x == 0>;
00631
00633             static std::string to_string() {
00634                 return std::to_string(x);
00635             }
00636
00639             template<typename valueRing>
00640             static constexpr valueRing eval(const valueRing& v) {
00641                 return static_cast<valueRing>(x);
00642             }
00643         };
00644
00648         template<auto x>
00649         using inject_constant_t = val<static_cast<int64_t>(x)>;
00650
00651         template<typename v>
00652         using inject_ring_t = v;
00653
00655         using zero = val<0>;
00657         using one = val<1>;
00659         static constexpr bool is_field = false;
00661         static constexpr bool is_euclidean_domain = true;
00662
00663     private:
00664         template<typename v1, typename v2>
00665         struct add {
00666             using type = val<v1::v + v2::v>;
00667         };
00668
00669         template<typename v1, typename v2>
00670         struct sub {
00671             using type = val<v1::v - v2::v>;
00672         };
00673
00674         template<typename v1, typename v2>
00675         struct mul {
00676             using type = val<v1::v * v2::v>;
00677         };

```

```

00678
00679     template<typename v1, typename v2>
00680     struct div {
00681         using type = val<v1::v / v2::v>;
00682     };
00683
00684     template<typename v1, typename v2>
00685     struct remainder {
00686         using type = val<v1::v% v2::v>;
00687     };
00688
00689     template<typename v1, typename v2>
00690     struct gt {
00691         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00692     };
00693
00694     template<typename v1, typename v2>
00695     struct lt {
00696         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00697     };
00698
00699     template<typename v1, typename v2>
00700     struct eq {
00701         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00702     };
00703
00704     template<typename v>
00705     struct pos {
00706         using type = std::bool_constant<(v::v > 0)>;
00707     };
00708
00709     public:
00710     template<typename v1, typename v2>
00711     using add_t = typename add<v1, v2>::type;
00712
00713     template<typename v1, typename v2>
00714     using sub_t = typename sub<v1, v2>::type;
00715
00716     template<typename v1, typename v2>
00717     using mul_t = typename mul<v1, v2>::type;
00718
00719     template<typename v1, typename v2>
00720     using div_t = typename div<v1, v2>::type;
00721
00722     template<typename v1, typename v2>
00723     using mod_t = typename remainder<v1, v2>::type;
00724
00725     template<typename v1, typename v2>
00726     using gt_t = typename gt<v1, v2>::type;
00727
00728     template<typename v1, typename v2>
00729     static constexpr bool gt_v = gt_t<v1, v2>::value;
00730
00731     template<typename v1, typename v2>
00732     using lt_t = typename lt<v1, v2>::type;
00733
00734     template<typename v1, typename v2>
00735     static constexpr bool lt_v = lt_t<v1, v2>::value;
00736
00737     template<typename v1, typename v2>
00738     using eq_t = typename eq<v1, v2>::type;
00739
00740     template<typename v1, typename v2>
00741     static constexpr bool eq_v = eq_t<v1, v2>::value;
00742
00743     template<typename v1, typename v2>
00744     using gcd_t = gcd_t<i64, v1, v2>;
00745
00746     template<typename v>
00747     using pos_t = typename pos<v>::type;
00748
00749     template<typename v>
00750     static constexpr bool pos_v = pos_t<v>::value;
00751 };
00752 } // namespace aerobus
00753
00754 // z/pz
00755 namespace aerobus {
00756     template<int32_t p>
00757     struct zp {
00758         using inner_type = int32_t;
00759         template<int32_t x>
00760         struct val {
00761             static constexpr int32_t v = x % p;
00762         };
00763
00764         template<typename valueType>
00765         static constexpr valueType get() { return static_cast<valueType>(x % p); }
00766     };
00767 }

```

```

00790
00791     using is_zero_t = std::bool_constant<x% p == 0>;
00792     static std::string to_string() {
00793         return std::to_string(x % p);
00794     }
00795
00796     template<typename valueRing>
00797     static constexpr valueRing eval(const valueRing& v) {
00798         return static_cast<valueRing>(x % p);
00799     }
00800 };
00801
00802 template<auto x>
00803 using inject_constant_t = val<static_cast<int32_t>(x)>;
00804
00805 using zero = val<0>;
00806 using one = val<1>;
00807 static constexpr bool is_field = is_prime<p>::value;
00808 static constexpr bool is_euclidean_domain = true;
00809
00810 private:
00811     template<typename v1, typename v2>
00812     struct add {
00813         using type = val<(v1::v + v2::v) % p>;
00814     };
00815
00816     template<typename v1, typename v2>
00817     struct sub {
00818         using type = val<(v1::v - v2::v) % p>;
00819     };
00820
00821     template<typename v1, typename v2>
00822     struct mul {
00823         using type = val<(v1::v * v2::v) % p>;
00824     };
00825
00826     template<typename v1, typename v2>
00827     struct div {
00828         using type = val<(v1::v % p) / (v2::v % p)>;
00829     };
00830
00831     template<typename v1, typename v2>
00832     struct remainder {
00833         using type = val<(v1::v % v2::v) % p>;
00834     };
00835
00836     template<typename v1, typename v2>
00837     struct gt {
00838         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00839     };
00840
00841     template<typename v1, typename v2>
00842     struct lt {
00843         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00844     };
00845
00846     template<typename v1, typename v2>
00847     struct eq {
00848         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00849     };
00850
00851     template<typename v1>
00852     struct pos {
00853         using type = std::bool_constant<(v1::v > 0)>;
00854     };
00855
00856 public:
00857     template<typename v1, typename v2>
00858     using add_t = typename add<v1, v2>::type;
00859
00860     template<typename v1, typename v2>
00861     using sub_t = typename sub<v1, v2>::type;
00862
00863     template<typename v1, typename v2>
00864     using mul_t = typename mul<v1, v2>::type;
00865
00866     template<typename v1, typename v2>
00867     using div_t = typename div<v1, v2>::type;
00868
00869     template<typename v1, typename v2>
00870     using mod_t = typename remainder<v1, v2>::type;
00871
00872     template<typename v1, typename v2>
00873     using gt_t = typename gt<v1, v2>::type;
00874
00875     template<typename v1, typename v2>
00876     using lt_t = typename lt<v1, v2>::type;
00877
00878     template<typename v1, typename v2>
00879     using eq_t = typename eq<v1, v2>::type;
00880
00881     static constexpr bool gt_v = gt_t<v1, v2>::value;
00882
00883

```

```

00884
00886     template<typename v1, typename v2>
00887     using lt_t = typename lt<v1, v2>::type;
00888
00890     template<typename v1, typename v2>
00891     static constexpr bool lt_v = lt_t<v1, v2>::value;
00892
00894     template<typename v1, typename v2>
00895     using eq_t = typename eq<v1, v2>::type;
00896
00898     template<typename v1, typename v2>
00899     static constexpr bool eq_v = eq_t<v1, v2>::value;
00900
00902     template<typename v1, typename v2>
00903     using gcd_t = gcd_t<i32, v1, v2>;
00904
00906     template<typename v1>
00907     using pos_t = typename pos<v1>::type;
00908
00910     template<typename v>
00911     static constexpr bool pos_v = pos_t<v>::value;
00912 };
00913 } // namespace aerobus
00914
00915 // polynomial
00916 namespace aerobus {
00917     // coeffN x^N + ...
00922     template<typename Ring, char variable_name = 'x'>
00923     requires IsEuclideanDomain<Ring>
00924     struct polynomial {
00925         static constexpr bool is_field = false;
00926         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
00927
00931         template<typename coeffN, typename... coeffs>
00932         struct val {
00933             static constexpr size_t degree = sizeof...(coeffs);
00934             using aN = coeffN;
00935             using strip = val<coeffs...>;
00936             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
00937             static constexpr bool is_zero_v = is_zero_t::value;
00938
00944         private:
00945             template<size_t index, typename E = void>
00946             struct coeff_at {};
00947
00948             template<size_t index>
00949             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
00950                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
00951             };
00952
00953             template<size_t index>
00954             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
00955                 using type = typename Ring::zero;
00956             };
00957
00958         public:
00961             template<size_t index>
00962             using coeff_at_t = typename coeff_at<index>::type;
00963
00966             static std::string to_string() {
00967                 return string_helper<coeffN, coeffs...>::func();
00968             }
00969
00974             template<typename valueRing>
00975             static constexpr valueRing eval(const valueRing& x) {
00976                 return eval_helper<valueRing, val>::template inner<0, degree +
1>::func(static_cast<valueRing>(0), x);
00977             }
00978         };
00979
00982         template<typename coeffN>
00983         struct val<coeffN> {
00984             static constexpr size_t degree = 0;
00985             using aN = coeffN;
00986             using strip = val<coeffN>;
00987             using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
00988
00989             static constexpr bool is_zero_v = is_zero_t::value;
00990
00991             template<size_t index, typename E = void>
00992             struct coeff_at {};
00993
00994             template<size_t index>
00995             struct coeff_at<index, std::enable_if_t<(index == 0)> {
00996                 using type = aN;
00997             };
00998

```

```

00999         template<size_t index>
01000         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01001             using type = typename Ring::zero;
01002         };
01003
01004         template<size_t index>
01005         using coeff_at_t = typename coeff_at<index>::type;
01006
01007         static std::string to_string() {
01008             return string_helper<coeffN>::func();
01009         }
01010
01011         template<typename valueRing>
01012         static constexpr valueRing eval(const valueRing& x) {
01013             return static_cast<valueRing>(aN::template get<valueRing>());
01014         }
01015     };
01016
01018     using zero = val<typename Ring::zero>;
01020     using one = val<typename Ring::one>;
01022     using X = val<typename Ring::one, typename Ring::zero>;
01023
01024 private:
01025     template<typename P, typename E = void>
01026     struct simplify;
01027
01028     template <typename P1, typename P2, typename I>
01029     struct add_low;
01030
01031     template<typename P1, typename P2>
01032     struct add {
01033         using type = typename simplify<typename add_low<
01034             P1,
01035             P2,
01036             internal::make_index_sequence_reverse<
01037                 std::max(P1::degree, P2::degree) + 1
01038             >::type>::type;
01039     };
01040
01041     template <typename P1, typename P2, typename I>
01042     struct sub_low;
01043
01044     template <typename P1, typename P2, typename I>
01045     struct mul_low;
01046
01047     template<typename v1, typename v2>
01048     struct mul {
01049         using type = typename mul_low<
01050             v1,
01051             v2,
01052             internal::make_index_sequence_reverse<
01053                 v1::degree + v2::degree + 1
01054             >::type;
01055     };
01056
01057     template<typename coeff, size_t deg>
01058     struct monomial;
01059
01060     template<typename v, typename E = void>
01061     struct derive_helper {};
01062
01063     template<typename v>
01064     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01065         using type = zero;
01066     };
01067
01068     template<typename v>
01069     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01070         using type = typename add<
01071             typename derive_helper<typename simplify<typename v::strip>::type,
01072                 typename monomial<
01073                     typename Ring::template mul_t<
01074                         typename v::aN,
01075                         typename Ring::template inject_constant_t<(v::degree)>
01076                     >,
01077                     v::degree - 1
01078                 >::type
01079             >::type;
01080     };
01081
01082     template<typename v1, typename v2, typename E = void>
01083     struct eq_helper {};
01084
01085     template<typename v1, typename v2>
01086     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01087         using type = std::false_type;
01088     };

```

```

01089
01090
01091     template<typename v1, typename v2>
01092     struct eq_helper<v1, v2, std::enable_if_t<
01093         v1::degree == v2::degree &&
01094         (v1::degree != 0 || v2::degree != 0) &&
01095         std::is_same<
01096             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01097             std::false_type
01098         >::value
01099     > {
01100     > {
01101         using type = std::false_type;
01102     };
01103
01104     template<typename v1, typename v2>
01105     struct eq_helper<v1, v2, std::enable_if_t<
01106         v1::degree == v2::degree &&
01107         (v1::degree != 0 || v2::degree != 0) &&
01108         std::is_same<
01109             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01110             std::true_type
01111         >::value
01112     > {
01113     > {
01114         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01115     };
01116
01117     template<typename v1, typename v2>
01118     struct eq_helper<v1, v2, std::enable_if_t<
01119         v1::degree == v2::degree &&
01120         (v1::degree == 0)
01121     > {
01122     > {
01123         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01124     };
01125
01126     template<typename v1, typename v2, typename E = void>
01127     struct lt_helper {};
01128
01129     template<typename v1, typename v2>
01130     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01131     > {
01132         using type = std::true_type;
01133     };
01134
01135     template<typename v1, typename v2>
01136     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01137     > {
01138         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01139     };
01140
01141     template<typename v1, typename v2>
01142     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01143     > {
01144         using type = std::false_type;
01145     };
01146
01147     template<typename v1, typename v2, typename E = void>
01148     struct gt_helper {};
01149
01150     template<typename v1, typename v2>
01151     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01152     > {
01153         using type = std::true_type;
01154     };
01155
01156     template<typename v1, typename v2>
01157     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01158     > {
01159         using type = std::false_type;
01160     };
01161
01162     // when high power is zero : strip
01163     template<typename P>
01164     struct simplify<P, std::enable_if_t<
01165         std::is_same<
01166             typename Ring::zero,
01167             typename P::aN
01168         >::value && (P::degree > 0)
01169     > {
01170     > {
01171         using type = typename simplify<typename P::strip>::type;
01172     };
01173
01174     // otherwise : do nothing
01175     template<typename P>
01176     struct simplify<P, std::enable_if_t<
01177         !std::is_same<
01178             typename Ring::zero,

```

```

01176         typename P::aN
01177         >::value && (P::degree > 0)
01178     » {
01179         using type = P;
01180     };
01181
01182     // do not simplify constants
01183     template<typename P>
01184     struct simplify<P, std::enable_if_t<P::degree == 0>» {
01185         using type = P;
01186     };
01187
01188     // addition at
01189     template<typename P1, typename P2, size_t index>
01190     struct add_at {
01191         using type =
01192             typename Ring::template add_t<
01193                 typename P1::template coeff_at_t<index>,
01194                 typename P2::template coeff_at_t<index>>;
01195     };
01196
01197     template<typename P1, typename P2, size_t index>
01198     using add_at_t = typename add_at<P1, P2, index>::type;
01199
01200     template<typename P1, typename P2, std::size_t... I>
01201     struct add_low<P1, P2, std::index_sequence<I...>» {
01202         using type = val<add_at_t<P1, P2, I>...>;
01203     };
01204
01205     // subtraction at
01206     template<typename P1, typename P2, size_t index>
01207     struct sub_at {
01208         using type =
01209             typename Ring::template sub_t<
01210                 typename P1::template coeff_at_t<index>,
01211                 typename P2::template coeff_at_t<index>>;
01212     };
01213
01214     template<typename P1, typename P2, size_t index>
01215     using sub_at_t = typename sub_at<P1, P2, index>::type;
01216
01217     template<typename P1, typename P2, std::size_t... I>
01218     struct sub_low<P1, P2, std::index_sequence<I...>» {
01219         using type = val<sub_at_t<P1, P2, I>...>;
01220     };
01221
01222     template<typename P1, typename P2>
01223     struct sub {
01224         using type = typename simplify<typename sub_low<
01225             P1,
01226             P2,
01227             internal::make_index_sequence_reverse<
01228                 std::max(P1::degree, P2::degree) + 1
01229             >::type>::type;
01230     };
01231
01232     // multiplication at
01233     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01234     struct mul_at_loop_helper {
01235         using type = typename Ring::template add_t<
01236             typename Ring::template mul_t<
01237                 typename v1::template coeff_at_t<index>,
01238                 typename v2::template coeff_at_t<k - index>
01239             >,
01240             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01241         >;
01242     };
01243
01244     template<typename v1, typename v2, size_t k, size_t stop>
01245     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01246         using type = typename Ring::template mul_t<
01247             typename v1::template coeff_at_t<stop>,
01248             typename v2::template coeff_at_t<0>>;
01249     };
01250
01251     template<typename v1, typename v2, size_t k, typename E = void>
01252     struct mul_at {};
01253
01254     template<typename v1, typename v2, size_t k>
01255     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>» {
01256         using type = typename Ring::zero;
01257     };
01258
01259     template<typename v1, typename v2, size_t k>
01260     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>» {
01261         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01262     };

```

```

01263
01264     template<typename P1, typename P2, size_t index>
01265     using mul_at_t = typename mul_at<P1, P2, index>::type;
01266
01267     template<typename P1, typename P2, std::size_t... I>
01268     struct mul_low<P1, P2, std::index_sequence<I...> {
01269         using type = val<mul_at_t<P1, P2, I>...>;
01270     };
01271
01272     // division helper
01273     template< typename A, typename B, typename Q, typename R, typename E = void>
01274     struct div_helper {};
01275
01276     template<typename A, typename B, typename Q, typename R>
01277     struct div_helper<A, B, Q, R, std::enable_if_t<
01278         (R::degree < B::degree) ||
01279         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01280         using q_type = Q;
01281         using mod_type = R;
01282         using gcd_type = B;
01283     };
01284
01285     template<typename A, typename B, typename Q, typename R>
01286     struct div_helper<A, B, Q, R, std::enable_if_t<
01287         (R::degree >= B::degree) &&
01288         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01289     private: // NOLINT
01290         using rN = typename R::aN;
01291         using bN = typename B::aN;
01292         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01293         B::degree>::type;
01294         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01295         using qq = typename add<Q, pT>::type;
01296
01297     public:
01298         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01299         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01300         using gcd_type = rr;
01301     };
01302
01303     template<typename A, typename B>
01304     struct div {
01305         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01306         using q_type = typename div_helper<A, B, zero, A>::q_type;
01307         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01308     };
01309
01310     template<typename P>
01311     struct make_unit {
01312         using type = typename div<P, val<typename P::aN>>::q_type;
01313     };
01314
01315     template<typename coeff, size_t deg>
01316     struct monomial {
01317         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01318     };
01319
01320     template<typename coeff>
01321     struct monomial<coeff, 0> {
01322         using type = val<coeff>;
01323     };
01324
01325     template<typename valueRing, typename P>
01326     struct eval_helper {
01327         template<size_t index, size_t stop>
01328         struct inner {
01329             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01330                 constexpr valueRing coeff =
01331                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01332                     get<valueRing>());
01333                 return eval_helper<valueRing, P>::template inner<index + 1, stop>::func(x * accum
01334                     + coeff, x);
01335             }
01336         };
01337
01338         template<size_t stop>
01339         struct inner<stop, stop> {
01340             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01341                 return accum;
01342             }
01343         };
01344     };
01345
01346     template<typename coeff, typename... coeffs>
01347     struct string_helper {
01348         static std::string func() {
01349             std::string tail = string_helper<coeffs...>::func();

```



```

01347         std::string result = "";
01348         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01349             return tail;
01350         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01351             if (sizeof...(coeffs) == 1) {
01352                 result += std::string(1, variable_name);
01353             } else {
01354                 result += std::string(1, variable_name) + "^" +
std::to_string(sizeof...(coeffs));
01355             }
01356         } else {
01357             if (sizeof...(coeffs) == 1) {
01358                 result += coeff::to_string() + " " + std::string(1, variable_name);
01359             } else {
01360                 result += coeff::to_string()
01361                     + " " + std::string(1, variable_name)
01362                     + "^" + std::to_string(sizeof...(coeffs));
01363             }
01364         }
01365
01366         if (!tail.empty()) {
01367             result += " " + tail;
01368         }
01369
01370         return result;
01371     }
01372 };
01373
01374 template<typename coeff>
01375 struct string_helper<coeff> {
01376     static std::string func() {
01377         if (!std::is_same<coeff, typename Ring::zero>::value) {
01378             return coeff::to_string();
01379         } else {
01380             return "";
01381         }
01382     }
01383 };
01384
01385 public:
01386     template<typename P>
01387     using simplify_t = typename simplify<P>::type;
01388
01389     template<typename v1, typename v2>
01390     using add_t = typename add<v1, v2>::type;
01391
01392     template<typename v1, typename v2>
01393     using sub_t = typename sub<v1, v2>::type;
01394
01395     template<typename v1, typename v2>
01396     using mul_t = typename mul<v1, v2>::type;
01397
01398     template<typename v1, typename v2>
01399     using eq_t = typename eq_helper<v1, v2>::type;
01400
01401     template<typename v1, typename v2>
01402     using lt_t = typename lt_helper<v1, v2>::type;
01403
01404     template<typename v1, typename v2>
01405     using gt_t = typename gt_helper<v1, v2>::type;
01406
01407     template<typename v1, typename v2>
01408     using div_t = typename div<v1, v2>::q_type;
01409
01410     template<typename v1, typename v2>
01411     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01412
01413     template<typename coeff, size_t deg>
01414     using monomial_t = typename monomial<coeff, deg>::type;
01415
01416     template<typename v>
01417     using derive_t = typename derive_helper<v>::type;
01418
01419     template<typename v>
01420     using pos_t = typename Ring::template pos_t<typename v::aN>;
01421
01422     template<typename v>
01423     static constexpr bool pos_v = pos_t<v>::value;
01424
01425     template<typename v1, typename v2>
01426     using gcd_t = std::conditional_t<
01427         Ring::is_euclidean_domain,
01428         typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2>::type,
01429         void>;
01430
01431     template<auto x>
01432     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;

```

```

01472
01473     template<typename v>
01474     using inject_ring_t = val<v>;
01475 };
01476 } // namespace aerobus
01477
01478 // fraction field
01479 namespace aerobus {
01480     namespace internal {
01481         template<typename Ring, typename E = void>
01482         requires IsEuclideanDomain<Ring>
01483         struct _FractionField {};
01484
01485         template<typename Ring>
01486         requires IsEuclideanDomain<Ring>
01487         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> > {
01488             static constexpr bool is_field = true;
01489             static constexpr bool is_euclidean_domain = true;
01490
01491         private:
01492             template<typename val1, typename val2, typename E = void>
01493             struct to_string_helper {};
01494
01495             template<typename val1, typename val2>
01496             struct to_string_helper <val1, val2,
01497                 std::enable_if_t<
01498                     Ring::template eq_t<
01499                         val2, typename Ring::one
01500                         >::value
01501                     >
01502                 > {
01503                 static std::string func() {
01504                     return val1::to_string();
01505                 }
01506             };
01507
01508             template<typename val1, typename val2>
01509             struct to_string_helper<val1, val2,
01510                 std::enable_if_t<
01511                     !Ring::template eq_t<
01512                         val2,
01513                         typename Ring::one
01514                         >::value
01515                     >
01516                 > {
01517                 static std::string func() {
01518                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01519                 }
01520             };
01521
01522         public:
01523             template<typename val1, typename val2>
01524             struct val {
01525                 using x = val1;
01526                 using y = val2;
01527                 using is_zero_t = typename val1::is_zero_t;
01528                 static constexpr bool is_zero_v = val1::is_zero_t::value;
01529
01530                 using ring_type = Ring;
01531                 using field_type = _FractionField<Ring>;
01532
01533                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01534
01535                 template<typename valueType>
01536                 static constexpr valueType get() { return static_cast<valueType>(x::v) /
01537                     static_cast<valueType>(y::v); }
01538
01539                 static std::string to_string() {
01540                     return to_string_helper<val1, val2>::func();
01541                 }
01542
01543                 template<typename valueRing>
01544                 static constexpr valueRing eval(const valueRing& v) {
01545                     return x::eval(v) / y::eval(v);
01546                 }
01547             };
01548
01549             using zero = val<typename Ring::zero, typename Ring::one>;
01550             using one = val<typename Ring::one, typename Ring::one>;
01551
01552             template<typename v>
01553             using inject_t = val<v, typename Ring::one>;
01554
01555             template<auto x>
01556             using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01557             Ring::one>;
01558

```

```

01588     template<typename v>
01589     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01590
01592     using ring_type = Ring;
01593
01594 private:
01595     template<typename v, typename E = void>
01596     struct simplify {};
01597
01598     // x = 0
01599     template<typename v>
01600     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01601         using type = typename _FractionField<Ring>::zero;
01602     };
01603
01604     // x != 0
01605     template<typename v>
01606     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01607     private:
01608         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01609         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01610         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01611
01612         using posx = std::conditional_t<
01613             !Ring::template pos_v<newx>,
01614             typename Ring::template sub_t<typename Ring::zero, newx>,
01615             newx>;
01616         using posy = std::conditional_t<
01617             !Ring::template pos_v<newy>,
01618             typename Ring::template sub_t<typename Ring::zero, newy>,
01619             newy>;
01620     public:
01621         using type = typename _FractionField<Ring>::template val<posx, posy>;
01622     };
01623
01624 public:
01627     template<typename v>
01628     using simplify_t = typename simplify<v>::type;
01629
01630 private:
01631     template<typename v1, typename v2>
01632     struct add {
01633     private:
01634         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01635         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01636         using dividend = typename Ring::template add_t<a, b>;
01637         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01638         using g = typename Ring::template gcd_t<dividend, diviser>;
01639
01640     public:
01641         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01642             diviser>;
01643     };
01644
01645     template<typename v>
01646     struct pos {
01647     private:
01648         using type = std::conditional_t<
01649             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01650             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01651             std::true_type,
01652             std::false_type>;
01653     };
01654
01655     template<typename v1, typename v2>
01656     struct sub {
01657     private:
01658         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01659         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01660         using dividend = typename Ring::template sub_t<a, b>;
01661         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01662         using g = typename Ring::template gcd_t<dividend, diviser>;
01663
01664     public:
01665         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01666             diviser>;
01667     };
01668
01669     template<typename v1, typename v2>
01670     struct mul {
01671     private:
01672         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01673         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01674
01675     public:
01676         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01677     };

```

```

01676     template<typename v1, typename v2, typename E = void>
01677     struct div {};
01678
01679     template<typename v1, typename v2>
01680     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01681     private:
01682         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01683         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01684
01685     public:
01686         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01687     };
01688
01689     template<typename v1, typename v2>
01690     struct div<v1, v2, std::enable_if_t<
01691         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01692         using type = one;
01693     };
01694
01695     template<typename v1, typename v2>
01696     struct eq {
01697         using type = std::conditional_t<
01698             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01699             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01700             std::true_type,
01701             std::false_type>;
01702     };
01703
01704     template<typename TL, typename E = void>
01705     struct vadd {};
01706
01707     template<typename TL>
01708     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01709         using head = typename TL::pop_front::type;
01710         using tail = typename TL::pop_front::tail;
01711         using type = typename add<head, typename vadd<tail>::type>::type;
01712     };
01713
01714     template<typename TL>
01715     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01716         using type = typename TL::template at<0>;
01717     };
01718
01719     template<typename... vals>
01720     struct vmul {};
01721
01722     template<typename v1, typename... vals>
01723     struct vmul<v1, vals...> {
01724         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01725     };
01726
01727     template<typename v1>
01728     struct vmul<v1> {
01729         using type = v1;
01730     };
01731
01732     template<typename v1, typename v2, typename E = void>
01733     struct gt;
01734
01735     template<typename v1, typename v2>
01736     struct gt<v1, v2, std::enable_if_t<
01737         (eq<v1, v2>::type::value)
01738         > {
01739         using type = std::false_type;
01740     };
01741
01742     template<typename v1, typename v2>
01743     struct gt<v1, v2, std::enable_if_t<
01744         (!eq<v1, v2>::type::value) &&
01745         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01746         > {
01747         using type = typename gt<
01748             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01749             >::type;
01750     };
01751
01752     template<typename v1, typename v2>
01753     struct gt<v1, v2, std::enable_if_t<
01754         (!eq<v1, v2>::type::value) &&
01755         (pos<v1>::type::value) && (!pos<v2>::type::value)
01756         > {
01757         using type = std::true_type;
01758     };
01759
01760     template<typename v1, typename v2>

```

```

01762     struct gt<v1, v2, std::enable_if_t<
01763         (!eq<v1, v2>::type::value) &&
01764         (!pos<v1>::type::value) && (pos<v2>::type::value)
01765         >> {
01766         using type = std::false_type;
01767     };
01768
01769     template<typename v1, typename v2>
01770     struct gt<v1, v2, std::enable_if_t<
01771         (!eq<v1, v2>::type::value) &&
01772         (pos<v1>::type::value) && (pos<v2>::type::value)
01773         >> {
01774         using type = typename Ring::template gt_t<
01775             typename Ring::template mul_t<v1::x, v2::y>,
01776             typename Ring::template mul_t<v2::y, v2::x>
01777         >;
01778     };
01779
01780     public:
01781     template<typename v1, typename v2>
01782     using add_t = typename add<v1, v2>::type;
01783     template<typename v1, typename v2>
01784     using mod_t = zero;
01785     template<typename v1, typename v2>
01786     using gcd_t = v1;
01787     template<typename... vs>
01788     using vadd_t = typename vadd<vs...>::type;
01789     template<typename... vs>
01790     using vmul_t = typename vmul<vs...>::type;
01791     template<typename v1, typename v2>
01792     using sub_t = typename sub<v1, v2>::type;
01793     template<typename v1, typename v2>
01794     using mul_t = typename mul<v1, v2>::type;
01795     template<typename v1, typename v2>
01796     using div_t = typename div<v1, v2>::type;
01797     template<typename v1, typename v2>
01798     using eq_t = typename eq<v1, v2>::type;
01799     template<typename v1, typename v2>
01800     static constexpr bool eq_v = eq<v1, v2>::type::value;
01801     template<typename v1, typename v2>
01802     using gt_t = typename gt<v1, v2>::type;
01803     template<typename v1, typename v2>
01804     static constexpr bool gt_v = gt<v1, v2>::type::value;
01805     template<typename v1>
01806     using pos_t = typename pos<v1>::type;
01807     template<typename v>
01808     static constexpr bool pos_v = pos<t<v>::value;
01809 };
01810
01811 template<typename Ring, typename E = void>
01812 requires IsEuclideanDomain<Ring>
01813 struct FractionFieldImpl {};
01814
01815 // fraction field of a field is the field itself
01816 template<typename Field>
01817 requires IsEuclideanDomain<Field>
01818 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01819     using type = Field;
01820     template<typename v>
01821     using inject_t = v;
01822 };
01823
01824 // fraction field of a ring is the actual fraction field
01825 template<typename Ring>
01826 requires IsEuclideanDomain<Ring>
01827 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01828     using type = _FractionField<Ring>;
01829 };
01830 } // namespace internal
01831
01832 template<typename Ring>
01833 requires IsEuclideanDomain<Ring>
01834 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01835 } // namespace aerobus
01836
01837 // short names for common types
01838 namespace aerobus {
01839     using q32 = FractionField<i32>;
01840     using fpq32 = FractionField<polynomial<q32>>;
01841     using q64 = FractionField<i64>;
01842     using pi64 = polynomial<i64>;
01843     using fpq64 = FractionField<polynomial<q64>>;
01844     template<typename Ring, typename v1, typename v2>
01845     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01846
01847     template<typename Ring, typename v1, typename v2>
01848     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;

```

```

01884     template<typename Ring, typename v1, typename v2>
01885     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01886 } // namespace aerobus
01887
01888 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
01889 namespace aerobus {
01890     namespace internal {
01891         template<typename T, size_t x, typename E = void>
01892         struct factorial {};
01893
01894         template<typename T, size_t x>
01895         struct factorial<T, x, std::enable_if_t<(x > 0)> {
01896         private:
01897             template<typename, size_t, typename>
01898             friend struct factorial;
01899         public:
01900             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
01901             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01902         };
01903
01904         template<typename T>
01905         struct factorial<T, 0> {
01906         public:
01907             using type = typename T::one;
01908             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01909         };
01910     } // namespace internal
01911
01912     template<typename T, size_t i>
01913     using factorial_t = typename internal::factorial<T, i>::type;
01914
01915     template<typename T, size_t i>
01916     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01917
01918     namespace internal {
01919         template<typename T, size_t k, size_t n, typename E = void>
01920         struct combination_helper {};
01921
01922         template<typename T, size_t k, size_t n>
01923         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
01924         using type = typename FractionField<T>::template mul_t<
01925             typename combination_helper<T, k - 1, n - 1>::type,
01926             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
01927         };
01928
01929         template<typename T, size_t k, size_t n>
01930         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
01931         using type = typename combination_helper<T, n - k, n>::type;
01932         };
01933
01934         template<typename T, size_t n>
01935         struct combination_helper<T, 0, n> {
01936         using type = typename FractionField<T>::one;
01937         };
01938
01939         template<typename T, size_t k, size_t n>
01940         struct combination {
01941         using type = typename internal::combination_helper<T, k, n>::type::x;
01942         static constexpr typename T::inner_type value =
01943             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
01944         };
01945     } // namespace internal
01946
01947     template<typename T, size_t k, size_t n>
01948     using combination_t = typename internal::combination<T, k, n>::type;
01949
01950     template<typename T, size_t k, size_t n>
01951     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
01952
01953     namespace internal {
01954         template<typename T, size_t m>
01955         struct bernouilli;
01956
01957         template<typename T, typename accum, size_t k, size_t m>
01958         struct bernouilli_helper {
01959         using type = typename bernouilli_helper<
01960             T,
01961             addfractions_t<T,
01962                 accum,
01963                 mulfractions_t<T,
01964                     makefraction_t<T,
01965                         combination_t<T, k, m + 1>,
01966                         typename T::one>,

```

```

01979         typename bernouilli<T, k>::type
01980     >
01981     >,
01982     k + 1,
01983     m>::type;
01984 };
01985
01986 template<typename T, typename accum, size_t m>
01987 struct bernouilli_helper<T, accum, m, m> {
01988     using type = accum;
01989 };
01990
01991
01992
01993 template<typename T, size_t m>
01994 struct bernouilli {
01995     using type = typename FractionField<T>::template mul_t<
01996         typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
01997         makefraction_t<T,
01998             typename T::template val<static_cast<typename T::inner_type>(-1)>,
01999             typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02000         >
02001     >;
02002
02003     template<typename floatType>
02004     static constexpr floatType value = type::template get<floatType>();
02005 };
02006
02007 template<typename T>
02008 struct bernouilli<T, 0> {
02009     using type = typename FractionField<T>::one;
02010
02011     template<typename floatType>
02012     static constexpr floatType value = type::template get<floatType>();
02013 };
02014 } // namespace internal
02015
02016 template<typename T, size_t n>
02017 using bernouilli_t = typename internal::bernouilli<T, n>::type;
02018
02019 template<typename FloatType, typename T, size_t n>
02020 inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02021
02022 namespace internal {
02023     template<typename T, int k, typename E = void>
02024     struct alternate {};
02025
02026     template<typename T, int k>
02027     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02028         using type = typename T::one;
02029         static constexpr typename T::inner_type value = type::template get<typename
02030 T::inner_type>();
02031     };
02032
02033     template<typename T, int k>
02034     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02035         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02036         static constexpr typename T::inner_type value = type::template get<typename
02037 T::inner_type>();
02038     };
02039
02040     template<typename T, int k>
02041     struct alternate_t = typename internal::alternate<T, k>::type;
02042
02043     template<typename T, size_t k>
02044     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02045
02046     namespace internal {
02047         template<typename T, auto p, auto n>
02048         struct pow {
02049             using type = typename T::template mul_t<typename T::template val<p>, typename pow<T, p, n
02050 - 1>::type>;
02051         };
02052
02053         template<typename T, auto p>
02054         struct pow<T, p, 0> { using type = typename T::one; };
02055     }
02056
02057     template<typename T, auto p, auto n>
02058     using pow_t = typename internal::pow<T, p, n>::type;
02059
02060     namespace internal {
02061         template<typename, template<typename, size_t> typename, class>
02062         struct make_taylor_impl;
02063
02064         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>

```

```

02079     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02080         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
02081     };
02082 }
02083
02088 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02089 using taylor = typename internal::make_taylor_impl<
02090     T,
02091     coeff_at,
02092     internal::make_index_sequence_reverse<deg + 1>::type;
02093
02094 namespace internal {
02095     template<typename T, size_t i>
02096     struct exp_coeff {
02097         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02098     };
02099
02100     template<typename T, size_t i, typename E = void>
02101     struct sin_coeff_helper {};
02102
02103     template<typename T, size_t i>
02104     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02105         using type = typename FractionField<T>::zero;
02106     };
02107
02108     template<typename T, size_t i>
02109     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02110         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02111     };
02112
02113     template<typename T, size_t i>
02114     struct sin_coeff {
02115         using type = typename sin_coeff_helper<T, i>::type;
02116     };
02117
02118     template<typename T, size_t i, typename E = void>
02119     struct sh_coeff_helper {};
02120
02121     template<typename T, size_t i>
02122     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02123         using type = typename FractionField<T>::zero;
02124     };
02125
02126     template<typename T, size_t i>
02127     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02128         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02129     };
02130
02131     template<typename T, size_t i>
02132     struct sh_coeff {
02133         using type = typename sh_coeff_helper<T, i>::type;
02134     };
02135
02136     template<typename T, size_t i, typename E = void>
02137     struct cos_coeff_helper {};
02138
02139     template<typename T, size_t i>
02140     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02141         using type = typename FractionField<T>::zero;
02142     };
02143
02144     template<typename T, size_t i>
02145     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02146         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02147     };
02148
02149     template<typename T, size_t i>
02150     struct cos_coeff {
02151         using type = typename cos_coeff_helper<T, i>::type;
02152     };
02153
02154     template<typename T, size_t i, typename E = void>
02155     struct cosh_coeff_helper {};
02156
02157     template<typename T, size_t i>
02158     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02159         using type = typename FractionField<T>::zero;
02160     };
02161
02162     template<typename T, size_t i>
02163     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02164         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02165     };
02166
02167     template<typename T, size_t i>
02168     struct cosh_coeff {

```



```

02169         using type = typename cosh_coeff_helper<T, i>::type;
02170     };
02171
02172     template<typename T, size_t i>
02173     struct geom_coeff { using type = typename FractionField<T>::one; };
02174
02175
02176     template<typename T, size_t i, typename E = void>
02177     struct atan_coeff_helper;
02178
02179     template<typename T, size_t i>
02180     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02181         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02182     };
02183
02184     template<typename T, size_t i>
02185     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02186         using type = typename FractionField<T>::zero;
02187     };
02188
02189     template<typename T, size_t i>
02190     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02191
02192     template<typename T, size_t i, typename E = void>
02193     struct asin_coeff_helper;
02194
02195     template<typename T, size_t i>
02196     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02197         using type = makefraction_t<T,
02198             factorial_t<T, i - 1>,
02199             typename T::template mul_t<
02200                 typename T::template val<i>,
02201                 T::template mul_t<
02202                     pow_t<T, 4, i / 2>,
02203                     pow<T, factorial<T, i / 2>::value, 2
02204                 >
02205             >
02206         >>;
02207     };
02208
02209     template<typename T, size_t i>
02210     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02211         using type = typename FractionField<T>::zero;
02212     };
02213
02214     template<typename T, size_t i>
02215     struct asin_coeff {
02216         using type = typename asin_coeff_helper<T, i>::type;
02217     };
02218
02219     template<typename T, size_t i>
02220     struct lnpl_coeff {
02221         using type = makefraction_t<T,
02222             alternate_t<T, i + 1>,
02223             typename T::template val<i>;
02224     };
02225
02226     template<typename T>
02227     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02228
02229     template<typename T, size_t i, typename E = void>
02230     struct asinh_coeff_helper;
02231
02232     template<typename T, size_t i>
02233     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02234         using type = makefraction_t<T,
02235             typename T::template mul_t<
02236                 alternate_t<T, i / 2>,
02237                 factorial_t<T, i - 1>
02238             >,
02239             typename T::template mul_t<
02240                 T::template mul_t<
02241                     typename T::template val<i>,
02242                     pow_t<T, (factorial<T, i / 2>::value), 2>
02243                 >,
02244                 pow_t<T, 4, i / 2>
02245             >
02246         >>;
02247     };
02248
02249     template<typename T, size_t i>
02250     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02251         using type = typename FractionField<T>::zero;
02252     };
02253
02254     template<typename T, size_t i>
02255     struct asinh_coeff {

```

```

02256         using type = typename asinh_coeff_helper<T, i>::type;
02257     };
02258
02259     template<typename T, size_t i, typename E = void>
02260     struct atanh_coeff_helper;
02261
02262     template<typename T, size_t i>
02263     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02264         // 1/i
02265         using type = typename FractionField<T>::template val<
02266             typename T::one,
02267             typename T::template val<static_cast<typename T::inner_type>(i)>;
02268     };
02269
02270     template<typename T, size_t i>
02271     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02272         using type = typename FractionField<T>::zero;
02273     };
02274
02275     template<typename T, size_t i>
02276     struct atanh_coeff {
02277         using type = typename asinh_coeff_helper<T, i>::type;
02278     };
02279
02280     template<typename T, size_t i, typename E = void>
02281     struct tan_coeff_helper;
02282
02283     template<typename T, size_t i>
02284     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02285         using type = typename FractionField<T>::zero;
02286     };
02287
02288     template<typename T, size_t i>
02289     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02290     private:
02291         // 4^((i+1)/2)
02292         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02293         // 4^((i+1)/2) - 1
02294         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02295         // (-1)^((i-1)/2)
02296         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02297         using dividend = typename FractionField<T>::template mul_t<
02298             altp,
02299             FractionField<T>::template mul_t<
02300                 _4p,
02301                 FractionField<T>::template mul_t<
02302                     _4pml,
02303                     bernouilli_t<T, (i + 1)>
02304                 >
02305             >
02306     >;
02307     public:
02308         using type = typename FractionField<T>::template div_t<dividend,
02309             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02310     };
02311
02312     template<typename T, size_t i>
02313     struct tan_coeff {
02314         using type = typename tan_coeff_helper<T, i>::type;
02315     };
02316
02317     template<typename T, size_t i, typename E = void>
02318     struct tanh_coeff_helper;
02319
02320     template<typename T, size_t i>
02321     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02322         using type = typename FractionField<T>::zero;
02323     };
02324
02325     template<typename T, size_t i>
02326     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02327     private:
02328         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02329         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02330         using dividend =
02331             typename FractionField<T>::template mul_t<
02332                 _4p,
02333                 typename FractionField<T>::template mul_t<
02334                     _4pml,
02335                     bernouilli_t<T, (i + 1)>
02336                 >
02337             >::type;
02338     public:
02339         using type = typename FractionField<T>::template div_t<dividend,
02340             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;

```

```

02341     };
02342
02343     template<typename T, size_t i>
02344     struct tanh_coeff {
02345         using type = typename tanh_coeff_helper<T, i>::type;
02346     };
02347 } // namespace internal
02348
02352 template<typename T, size_t deg>
02353 using exp = taylor<T, internal::exp_coeff, deg>;
02354
02358 template<typename T, size_t deg>
02359 using expm1 = typename polynomial<FractionField<T>::template sub_t<
02360     exp<T, deg>,
02361     typename polynomial<FractionField<T>::one>;
02362
02366 template<typename T, size_t deg>
02367 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02368
02372 template<typename T, size_t deg>
02373 using atan = taylor<T, internal::atan_coeff, deg>;
02374
02378 template<typename T, size_t deg>
02379 using sin = taylor<T, internal::sin_coeff, deg>;
02380
02384 template<typename T, size_t deg>
02385 using sinh = taylor<T, internal::sh_coeff, deg>;
02386
02390 template<typename T, size_t deg>
02391 using cosh = taylor<T, internal::cosh_coeff, deg>;
02392
02396 template<typename T, size_t deg>
02397 using cos = taylor<T, internal::cos_coeff, deg>;
02398
02402 template<typename T, size_t deg>
02403 using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02404
02408 template<typename T, size_t deg>
02409 using asin = taylor<T, internal::asin_coeff, deg>;
02410
02414 template<typename T, size_t deg>
02415 using asinh = taylor<T, internal::asinh_coeff, deg>;
02416
02420 template<typename T, size_t deg>
02421 using atanh = taylor<T, internal::atanh_coeff, deg>;
02422
02426 template<typename T, size_t deg>
02427 using tan = taylor<T, internal::tan_coeff, deg>;
02428
02432 template<typename T, size_t deg>
02433 using tanh = taylor<T, internal::tanh_coeff, deg>;
02434 } // namespace aerobus
02435
02436 // continued fractions
02437 namespace aerobus {
02440     template<int64_t... values>
02441     struct ContinuedFraction {};
02442
02443     template<int64_t a0>
02444     struct ContinuedFraction<a0> {
02445         using type = typename q64::template inject_constant_t<a0>;
02446         static constexpr double val = type::template get<double>();
02447     };
02448
02449     template<int64_t a0, int64_t... rest>
02450     struct ContinuedFraction<a0, rest...> {
02451         using type = q64::template add_t<
02452             typename q64::template inject_constant_t<a0>,
02453             typename q64::template div_t<
02454                 typename q64::one,
02455                 typename ContinuedFraction<rest...>::type
02456             >;
02457         static constexpr double val = type::template get<double>();
02458     };
02459
02464     using PI_fraction =
02465     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02467     using E_fraction =
02468     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02469     using SQRT2_fraction =
02470     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02471     using SQRT3_fraction =
02472     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02473 // NOLINT
02472 } // namespace aerobus
02473
02474 // known polynomials

```

```

02475 namespace aerobus {
02476     namespace internal {
02477         template<int kind, int deg>
02478         struct chebyshev_helper {
02479             using type = typename pi64::template sub_t<
02480                 typename pi64::template mul_t<
02481                     typename pi64::template mul_t<
02482                         pi64::inject_constant_t<2>,
02483                         typename pi64::X
02484                     >,
02485                     typename chebyshev_helper<kind, deg-1>::type
02486                 >,
02487                 typename chebyshev_helper<kind, deg-2>::type
02488             >;
02489         };
02490
02491         template<>
02492         struct chebyshev_helper<1, 0> {
02493             using type = typename pi64::one;
02494         };
02495
02496         template<>
02497         struct chebyshev_helper<1, 1> {
02498             using type = typename pi64::X;
02499         };
02500
02501         template<>
02502         struct chebyshev_helper<2, 0> {
02503             using type = typename pi64::one;
02504         };
02505
02506         template<>
02507         struct chebyshev_helper<2, 1> {
02508             using type = typename pi64::template mul_t<
02509                 typename pi64::inject_constant_t<2>,
02510                 typename pi64::X>;
02511         };
02512     } // namespace internal
02513
02514     template<size_t deg>
02515     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02516
02517     template<size_t deg>
02518     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02519 } // namespace aerobus
02520
02521 #ifdef AEROBUS_CONWAY_IMPORTS
02522 template<int p, int n>
02523 struct ConwayPolynomial;
02524
02525 #define ZPV ZPZ::template val
02526 #define POLYV aerobus::polynomial<ZPV>::template val
02527 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02528     ZPV<1>; }; // NOLINT
02529 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02530     ZPV<1>, ZPV<1>; }; // NOLINT
02531 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02532     ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02533 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02534     ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02535 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02536     ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
02537 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02538     ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
02539 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02540     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02541 template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02542     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
02543 template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02544     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
02545 template<> struct ConwayPolynomial<2, 10> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02546     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<1>; }; //
02547     NOLINT
02548 template<> struct ConwayPolynomial<2, 11> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02549     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; };
02550     // NOLINT
02551 template<> struct ConwayPolynomial<2, 12> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02552     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>,
02553     ZPV<1>; }; // NOLINT
02554 template<> struct ConwayPolynomial<2, 13> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02555     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>,
02556     ZPV<1>; }; // NOLINT
02557 template<> struct ConwayPolynomial<2, 14> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,
02558     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>,
02559     ZPV<0>, ZPV<0>, ZPV<1>; }; // NOLINT
02560 template<> struct ConwayPolynomial<2, 15> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPV<1>,

```

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Chapter 7

Examples

7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

7.3 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.4 PI_fraction::val

representation of PI as a continued fraction -> 3.14...

7.5 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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