

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download [aerobus.h](#)
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including [aerobus.h](#).

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

results on my laptop :

```
./benchmarks_avx512.exe
[std math] 5.358e-01 Gsin/s
[std fast math] 3.389e+00 Gsin/s
[aerobus deg 1] 1.871e+01 Gsin/s
average error (vs std) : 4.36e-02
max error (vs std) : 1.50e-01
[aerobus deg 3] 1.943e+01 Gsin/s
average error (vs std) : 1.85e-04
max error (vs std) : 8.17e-04
[aerobus deg 5] 1.335e+01 Gsin/s
average error (vs std) : 6.07e-07
max error (vs std) : 3.63e-06
[aerobus deg 7] 8.634e+00 Gsin/s
average error (vs std) : 1.27e-09
max error (vs std) : 9.75e-09
[aerobus deg 9] 6.171e+00 Gsin/s
average error (vs std) : 1.89e-12
max error (vs std) : 1.78e-11
[aerobus deg 11] 4.731e+00 Gsin/s
average error (vs std) : 2.12e-15
max error (vs std) : 2.40e-14
[aerobus deg 13] 3.862e+00 Gsin/s
average error (vs std) : 3.16e-17
max error (vs std) : 3.33e-16
[aerobus deg 15] 3.359e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
[aerobus deg 17] 2.947e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
```


1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `Fraction<Field<Integers>>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`

- cos
- tan
- sh
- cosh
- tanh
- asin
- acos
- acosh
- asinh
- atanh

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in `Integers (typename)` and `index (size_t)`;
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_serie = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_serie<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
```

```

vbroadcastsd    ymm3, QWORD PTR .LC23[rip]
vbroadcastsd    ymm2, QWORD PTR .LC25[rip]
.L3:
vmovupd ymm15, YMMWORD PTR [rsi+rax]
vmovapd ymm0, ymm15
vfmadd132pd     ymm0, ymm14, ymm1
vfmadd132pd     ymm0, ymm13, ymm15
vfmadd132pd     ymm0, ymm12, ymm15
vfmadd132pd     ymm0, ymm11, ymm15
vfmadd132pd     ymm0, ymm10, ymm15
vfmadd132pd     ymm0, ymm9, ymm15
vfmadd132pd     ymm0, ymm8, ymm15
vfmadd132pd     ymm0, ymm7, ymm15
vfmadd132pd     ymm0, ymm6, ymm15
vfmadd132pd     ymm0, ymm5, ymm15
vfmadd132pd     ymm0, ymm4, ymm15
vfmadd132pd     ymm0, ymm3, ymm15
vfmadd132pd     ymm0, ymm2, ymm15
vfmadd132pd     ymm0, ymm1, ymm15
vmovupd YMMWORD PTR [rdx+rax], ymm0
add    rax, 32
cmp    rcx, rax
jne    .L3
mov    rax, rdi
and    rax, -4
vzeroupper

```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, `Aerobus` allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```

using namespace aerobus;
using ZmZ = zp<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;

```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define `rational functions` by taking the ring of fractions of polynomials:

```

using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;

```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for `quotient ring R/X` where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```

using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;

```

As we could have using `zp<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

Aerobus gives an implementation for [continued fractions](#). It can be used this way:

```
using namespace aerobus;  
using T = ContinuedFraction<1,2,3,4>;  
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, Aerobus provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	45
aerobus::IsField	
Concept to express R is a field	45
aerobus::IsRing	
Concept to express R is a Ring	46

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	48
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	48
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	49
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Specialization for multiple coefficients (strictly more than one)	50
aerobus::ConwayPolynomial	51
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aerobus::Embed< q32, q64 >	
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aerobus::Embed< Quotient< Ring, X >, Ring >	
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aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	55
aerobus::Embed< zpz< x >, i32 >	
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aerobus::i32	
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Removes types from head of the list	72

aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	73
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aerobus::type_list< Ts >	
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Values in i32 , again represented as types	82
aerobus::i64::val< x >	
Values in i64	84
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	86
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	90
aerobus::zpz< p >::val< x >	
Values in zpz	91
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	93
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	96

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

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Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<[typename T](#) , [typename A](#) , [typename B](#) >
 - [using gcd_t = typename internal::gcd< T >::template type< A, B >](#)
 - computes the greatest common divisor of A and B*
- template<[typename... vals](#)>
 - [using vadd_t = typename internal::vadd< vals... >::type](#)
 - adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator*
- template<[typename... vals](#)>
 - [using vmul_t = typename internal::vmul< vals... >::type](#)
 - multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator*
- template<[typename val](#) >
 - [using abs_t = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template sub_t< typename val::enclosing_type::zero, val > >](#)
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<[typename Ring](#) >
 - [using FractionField = typename internal::FractionFieldImpl< Ring >::type](#)
- template<[typename X](#) , [typename Y](#) >
 - [using add_t = typename X::enclosing_type::template add_t< X, Y >](#)
 - generic addition*
- template<[typename X](#) , [typename Y](#) >
 - [using sub_t = typename X::enclosing_type::template sub_t< X, Y >](#)
 - generic subtraction*

- `template<typename X, typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
generic multiplication
- `template<typename X, typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
generic division
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring, typename v1, typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
- `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type >, polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p >, i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p >, i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<typename Ring, typename v1, typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring, typename v1, typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename Ring, auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
- `template<typename Ring, auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField<Ring>
- `template<typename T, size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type
- `template<typename T, size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`

- computes binomial coefficient (k among n) as type*

```
template<typename T, size_t n>
using bernoulli_t = typename internal::bernoulli< T, n >::type
```

nth bernoulli number as type in T
- ```
template<typename T, size_t n>
using bell_t = typename internal::bell_helper< T, n >::type
```

*Bell numbers.*
- ```
template<typename T, int k>
using alternate_t = typename internal::alternate< T, k >::type
```

$(-1)^k$ as type in T
- ```
template<typename T, int n, int k>
using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type
```

*Stirling number of first kind (signed) – as types.*
- ```
template<typename T, int n, int k>
using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >
```

Stirling number of first kind (unsigned) – as types.
- ```
template<typename T, typename p, size_t n>
using pow_t = typename internal::pow< T, p, n >::type
```

*$p^n$  (as 'val' type in T)*
- ```
template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>
using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<
deg+1 > >::type
```
- ```
template<typename Integers, size_t deg>
using exp = taylor< Integers, internal::exp_coeff, deg >
```

*$e^x$*
- ```
template<typename Integers, size_t deg>
using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg
>, typename polynomial< FractionField< Integers > >::one >
```

$e^x - 1$
- ```
template<typename Integers, size_t deg>
using ln1 = taylor< Integers, internal::ln1_coeff, deg >
```

*$\ln(1 + x)$*
- ```
template<typename Integers, size_t deg>
using atan = taylor< Integers, internal::atan_coeff, deg >
```

$\arctan(x)$
- ```
template<typename Integers, size_t deg>
using sin = taylor< Integers, internal::sin_coeff, deg >
```

*$\sin(x)$*
- ```
template<typename Integers, size_t deg>
using sinh = taylor< Integers, internal::sh_coeff, deg >
```

$\sinh(x)$
- ```
template<typename Integers, size_t deg>
using cosh = taylor< Integers, internal::cosh_coeff, deg >
```

*$\cosh(x)$  hyperbolic cosine*
- ```
template<typename Integers, size_t deg>
using cos = taylor< Integers, internal::cos_coeff, deg >
```

$\cos(x)$ cosinus
- ```
template<typename Integers, size_t deg>
using geometric_sum = taylor< Integers, internal::geom_coeff, deg >
```

*$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$*
- ```
template<typename Integers, size_t deg>
using asin = taylor< Integers, internal::asin_coeff, deg >
```

$\arcsin(x)$ arc sinus

- `template<typename Integers , size_t deg>`
`using asinh = taylor< Integers, internal::asinh_coeff, deg >`
`arcsinh(x)` *arc hyperbolic sinus*
- `template<typename Integers , size_t deg>`
`using atanh = taylor< Integers, internal::atanh_coeff, deg >`
`arctanh(x)` *arc hyperbolic tangent*
- `template<typename Integers , size_t deg>`
`using tan = taylor< Integers, internal::tan_coeff, deg >`
`tan(x)` *tangent*
- `template<typename Integers , size_t deg>`
`using tanh = taylor< Integers, internal::tanh_coeff, deg >`
`tanh(x)` *hyperbolic tangent*
- `using PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >`
- `using E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`
- `using SQRT2_fraction = ContinuedFraction< 1, 2 >`
approximation of $\sqrt{2}$
- `using SQRT3_fraction = ContinuedFraction< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >`
approximation of

Functions

- `template<typename T >`
`T * aligned_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the field (prime number) @tparam n degree of extension
`template< int p`

Variables

- `template<typename T , size_t i>`
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`
computes factorial(i) as value in T
- `template<typename T , size_t k, size_t n>`
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`
computes binomial coefficients (k among n) as value
- `template<typename FloatType , typename T , size_t n>`
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`
 n th bernoulli number as value in $FloatType$
- `template<typename T , size_t k>`
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, <code>aerobus::i64</code> for example
----------	--

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\operatorname{arctan}(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$ arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ cosinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in polynomial<FractionField<Ring>>

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

e^x

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 ln1

```
template<typename Integers , size_t deg>
using aerobus::ln1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
...xs	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

T	(some ring type, such as aerobus::i64)
p	must be an instantiation of $T::val$
n	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 Sqrt2_fraction

```
using aerobus::Sqrt2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.42 Sqrt3_fraction

```
using aerobus::Sqrt3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

6.1.2.43 stirling_signed_t

```
template<typename T , int n, int k>
using aerobus::stirling_signed_t = typedef typename internal::stirling_helper<T, n, k>::type
```

Stirling number of first kind (signed) – as types.

Template Parameters

<i>T</i>	(ring type, such as aerobus::i64)
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.44 stirling_unsigned_t

```
template<typename T , int n, int k>
using aerobus::stirling_unsigned_t = typedef abs_t<typename internal::stirling_helper<T, n, k>::type>
```

Stirling number of first kind (unsigned) – as types.

Template Parameters

<i>T</i>	(ring type, such as aerobus::i64)
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.45 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.46 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.47 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
deg + 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.49 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.50 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```



```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>
[inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >

- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >
- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t<Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**

greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain

- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >

- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_helper**
- struct **stirling_helper**< T, 0, 0 >
- struct **stirling_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, k, std::enable_if_t<(k > 0) && (n > 0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**

- struct **vadd**< **v1** >
- struct **vadd**< **v1**, **vals...** >
- struct **vmul**
- struct **vmul**< **v1** >
- struct **vmul**< **v1**, **vals...** >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Typedefs

- template<size_t deg, typename I = aerobus::i64>
using chebyshev_T = typename internal::chebyshev_helper< 1, deg, I >::type
Chebyshev polynomials of first kind.
- template<size_t deg, typename I = aerobus::i64>
using chebyshev_U = typename internal::chebyshev_helper< 2, deg, I >::type
Chebyshev polynomials of second kind.
- template<size_t deg, typename I = aerobus::i64>
using laguerre = typename internal::laguerre_helper< deg, I >::type
Laguerre polynomials.
- template<size_t deg, typename I = aerobus::i64>
using hermite_prob = typename internal::hermite_helper< deg, hermite_kind::probabilist, I >::type
Hermite polynomials - probabilist form.
- template<size_t deg, typename I = aerobus::i64>
using hermite_phys = typename internal::hermite_helper< deg, hermite_kind::physicist, I >::type
Hermite polynomials - physicist form.
- template<size_t i, size_t m, typename I = aerobus::i64>
using bernstein = typename internal::bernstein_helper< i, m, I >::type
Bernstein polynomials.
- template<size_t deg, typename I = aerobus::i64>
using legendre = typename internal::legendre_helper< deg, I >::type
Legendre polynomials.
- template<size_t deg, typename I = aerobus::i64>
using bernoulli = taylor< I, internal::bernoulli_coeff< deg >::template inner, deg >
Bernoulli polynomials.

Enumerations

- enum hermite_kind { probabilist , physicist }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Typedef Documentation

6.3.2.1 bernoulli

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::bernoulli = typedef taylor<I, internal::bernoulli_coeff<deg><←
::template inner, deg>
```

Bernoulli polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to aerobus::i64)

6.3.2.2 bernstein

```
template<size_t i, size_t m, typename I = aerobus::i64>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m, I>::type
```

Bernstein polynomials.

Lives in polynomial

See also

[See in Wikipedia](#)

Template Parameters

<i>i</i>	<i>index of polynomial (between 0 and m)</i>
<i>m</i>	<i>degree of polynomial</i>
<i>I</i>	<i>Integers ring (defaults to aerobus::i64)</i>

6.3.2.3 chebyshev_T

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg, I>::type
```

Chebyshev polynomials of first kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>integer</i>	rings (defaults to aerobus::i64)

6.3.2.4 chebyshev_U

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg, I>::type
```

Chebyshev polynomials of second kind.

Lives in polynomial

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	<i>degree of polynomial</i>
<i>integer</i>	<i>rings (defaults to aerobus::i64)</i>

6.3.2.5 hermite_phys

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist, I>::type
```

Hermite polynomials - physicist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.6 hermite_prob

```
template<size_t deg, typename I = aerobus::i64>
```



```
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist, I>::type
```

Hermite polynomials - probabilist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.7 laguerre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg,
I>::type
```

Laguerre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to aerobus::i64)

6.3.2.8 legendre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg,
I>::type
```

Legendre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers Ring (defaults to aerobus::i64)

6.3.3 Enumeration Type Documentation

6.3.3.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `using type = typename Ring::zero`

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- [using type = aN](#)

8.3.1 Member Typedef Documentation

8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<i>...values</i>	are int64_t
------------------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- `using type = typename q64::template inject_constant_t< a0 >`
represented value as `aerobus::q64`

Static Public Attributes

- `static constexpr double val = static_cast<double>(a0)`
represented value as `double`

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as `double`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- `using type = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`
represented value as `aerobus::q64`

Static Public Attributes

- `static constexpr double val = type::template get<double>()`
represented value as `double`

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<code>a0</code>	integer (int64_t)
<code>...rest</code>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename ContinuedFraction<rest...>::type > >
```

represented value as `aerobus::q64`

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.8.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i64::val< static_cast< int64_t >(val::v)>`
the [i64](#) representation of val

8.9.1 Detailed Description

embeds [i32](#) into [i64](#)

8.9.2 Member Typedef Documentation

8.9.2.1 type

```
template<typename val >  
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.10 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds `polynomial<Small>` into `polynomial<Large>`

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
`using type = typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the `polynomial<Large>` representation of v

8.10.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.10.2 Member Typedef Documentation

8.10.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
```

the polynomial<Large> reprensentation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.11 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- ```
template<typename v >
using type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>
```

*q64 representation of v*

### 8.11.1 Detailed Description

embeds q32 into q64

## 8.11.2 Member Typedef Documentation

### 8.11.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>
```

q64 representation of v

#### Template Parameters

|   |                |
|---|----------------|
| v | a value in q32 |
|---|----------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.12 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- template<typename val >  
using type = typename val::raw\_t  
*Ring representation of val.*

### 8.12.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

|             |                  |
|-------------|------------------|
| <i>Ring</i> | a Euclidean ring |
| <i>X</i>    | a value in Ring  |

## 8.12.2 Member Typedef Documentation

### 8.12.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

|            |                              |
|------------|------------------------------|
| <i>val</i> | a value in Quotient<Ring, X> |
|------------|------------------------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.13 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- template<typename v >  
using type = typename FractionField< Ring >::template val< v, typename Ring::one >  
*FractionField<Ring> representation of v.*

### 8.13.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

|             |                                               |
|-------------|-----------------------------------------------|
| <i>Ring</i> | an integers ring, such as <a href="#">i32</a> |
|-------------|-----------------------------------------------|

## 8.13.2 Member Typedef Documentation

### 8.13.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

FractionField<Ring> representation of v.

#### Template Parameters

|   |              |
|---|--------------|
| v | a Ring value |
|---|--------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.14 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the [i32](#) representation of val*

### 8.14.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into [i32](#)

#### Template Parameters

|   |            |
|---|------------|
| x | an integer |
|---|------------|

## 8.14.2 Member Typedef Documentation

### 8.14.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the `i32` representation of `val`

#### Template Parameters

|                  |                                      |
|------------------|--------------------------------------|
| <code>val</code> | a value in <code>zpz&lt;x&gt;</code> |
|------------------|--------------------------------------|

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.15 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in `i32`, again represented as types*

### Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`

- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator yields v1 % v2 for example : i32::mod\_t<i32::val<7>, i32::val<2>>*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

### Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

## 8.15.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::add_t = typename add<v1, v2>::type
```

### 8.15.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

### 8.15.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```



#### 8.15.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

#### 8.15.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

#### 8.15.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

#### 8.15.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

#### 8.15.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

#### 8.15.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

#### 8.15.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

##### Template Parameters

|                 |                             |
|-----------------|-----------------------------|
| <code>v1</code> | a value in <code>i32</code> |
| <code>v2</code> | a value in <code>i32</code> |

#### 8.15.2.11 mul\_t

```
template<typename v1 , typename v2 >
```

```
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

#### 8.15.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

#### 8.15.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

#### 8.15.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

#### 8.15.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.15.3 Member Data Documentation

#### 8.15.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

#### 8.15.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.15.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

## 8.15.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.16 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in i64*

## Public Types

- [using inner\\_type = int64\\_t](#)  
*type of represented values*
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int64\\_t >\(x\)>](#)
- [template<typename v >](#)  
[using inject\\_ring\\_t = v](#)  
*injects a value used for internal consistency and quotient rings implementations for example i64::inject\_ring\_t<i64::val<1>>*  
*-> i64::val<1>*
- [using zero = val< 0 >](#)  
*constant zero*
- [using one = val< 1 >](#)  
*constant one*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mod\\_t = typename remainder< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq< v1 , v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gcd\\_t = gcd\\_t< i64, v1, v2 >](#)
- [template<typename v >](#)  
[using pos\\_t = typename pos< v >::type](#)

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields v1 > v2 as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 8.16.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

#### 8.16.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

#### 8.16.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

#### 8.16.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

#### 8.16.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

### 8.16.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

### 8.16.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>>`  
 -> `i64::val<1>`

#### Template Parameters

|                |                             |
|----------------|-----------------------------|
| <code>v</code> | a value in <code>i64</code> |
|----------------|-----------------------------|

### 8.16.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

### 8.16.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

### 8.16.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

### 8.16.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

### 8.16.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

### 8.16.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

### 8.16.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

### 8.16.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.16.3 Member Data Documentation

### 8.16.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

|                 |                                                   |
|-----------------|---------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a> |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> |

### 8.16.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.16.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

### 8.16.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.17 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- `static constexpr bool value` = `internal::_is_prime<n, 5>::value`  
*true iff n is prime*

### 8.17.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

|          |  |
|----------|--|
| <i>n</i> |  |
|----------|--|

### 8.17.2 Member Data Documentation

#### 8.17.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.18 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::q\\_type](#)  
*division operator*



- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial : coeff X^deg*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity (an > 0)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`  
`Ring >, v1, v2 > >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

### Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

## 8.18.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 8.18.2.2 `derive_t`

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

|                |  |
|----------------|--|
| <code>v</code> |  |
|----------------|--|

### 8.18.2.3 `div_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.4 `eq_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.5 `gcd_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.7 inject\_constant\_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.18.2.8 inject\_ring\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

## 8.18.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.10 mod\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.11 monomial\_t**

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

**Template Parameters**

|              |  |
|--------------|--|
| <i>coeff</i> |  |
| <i>deg</i>   |  |

**8.18.2.12 mul\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

**8.18.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

|          |  |
|----------|--|
| <i>v</i> |  |
|----------|--|

**8.18.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

|          |  |
|----------|--|
| <i>P</i> |  |
|----------|--|

**8.18.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.18.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

### 8.18.3 Member Data Documentation

#### 8.18.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.18.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.18.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

|          |                                            |
|----------|--------------------------------------------|
| <i>v</i> | a value in <a href="#">polynomial::val</a> |
|----------|--------------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- [using type](#) = [typename](#) internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- [using tail](#) = [typename](#) internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.19.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.19.2 Member Typedef Documentation

### 8.19.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.19.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< [typename](#) [Ring](#)::zero >  
*zero value*
- using [one](#) = [val](#)< [typename](#) [Ring](#)::one >  
*one*
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >  
using [add\\_t](#) = [val](#)< [typename](#) [Ring](#)::template [add\\_t](#)< [typename](#) [v1](#)::type, [typename](#) [v2](#)::type > >  
*addition operator*
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >  
using [mul\\_t](#) = [val](#)< [typename](#) [Ring](#)::template [mul\\_t](#)< [typename](#) [v1](#)::type, [typename](#) [v2](#)::type > >  
*subtraction operator*
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >  
using [div\\_t](#) = [val](#)< [typename](#) [Ring](#)::template [div\\_t](#)< [typename](#) [v1](#)::type, [typename](#) [v2](#)::type > >  
*division operator*
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >  
using [mod\\_t](#) = [val](#)< [typename](#) [Ring](#)::template [mod\\_t](#)< [typename](#) [v1](#)::type, [typename](#) [v2](#)::type > >

- modulus operator*  
 • `template<typename v1 , typename v2 >`  
   `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
   *equality operator (as type)*
- `template<typename v1 >`  
   `using pos_t = std::true_type`  
   *positivity operator always true*
- `template<auto x>`  
   `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
   `using inject_ring_t = val< v >`

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
   `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
   *addition operator (as boolean value)*
- `template<typename v >`  
   `static constexpr bool pos_v = pos_t<v>::value`  
   *positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
   *quotien rings are euclidean domain*

## 8.20.1 Detailed Description

`template<typename Ring, typename X>`  
**requires** `IsRing<Ring>`  
**struct** `aerobus::Quotient< Ring, X >`

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

### Template Parameters

|             |                                                                                         |
|-------------|-----------------------------------------------------------------------------------------|
| <i>Ring</i> | A ring type, such as ' <code>i32</code> ', must satisfy the <code>IsRing</code> concept |
| <i>X</i>    | a value in Ring, such as <code>i32::val&lt;2&gt;</code>                                 |

## 8.20.2 Member Typedef Documentation

### 8.20.2.1 add\_t

`template<typename Ring , typename X >`  
`template<typename v1 , typename v2 >`  
`using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,`  
`typename v2::type> >`

addition operator



## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.20.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

### 8.20.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.20.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
|----|--------------------------|

### 8.20.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.20.3 Member Data Documentation

### 8.20.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

### 8.20.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.20.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
|-----------|--------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

### Public Types

- [using head](#) = [typename](#) inner::head
- [using tail](#) = [typename](#) inner::tail

### 8.21.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.21.2 Member Typedef Documentation

#### 8.21.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.21.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

## Classes

- struct `pop_front`  
*removes types from head of the list*
- struct `split`  
*splits list at index*

## Public Types

- template<typename T >  
using `push_front` = `type_list`< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using `at` = `internal::type_at_t`< index, Ts... >  
*returns type at index*
- template<typename T >  
using `push_back` = `type_list`< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using `concat` = `typename concat_h`< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using `insert` = `typename internal::insert_h`< index, `type_list`< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using `remove` = `typename internal::remove_h`< index, `type_list`< Ts... > >::type  
*removes type at index*

## Static Public Attributes

- `static constexpr size_t length` = `sizeof...(Ts)`  
*length of list*

### 8.22.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

### 8.22.2 Member Typedef Documentation

#### 8.22.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

**8.22.2.2 concat**

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

**Template Parameters**

|          |  |
|----------|--|
| <i>U</i> |  |
|----------|--|

**8.22.2.3 insert**

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
| <i>T</i>     |  |

**8.22.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

**8.22.2.5 push\_front**

```
template<typename... Ts>
```

```
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

#### 8.22.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.22.3 Member Data Documentation

#### 8.22.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.23 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

#### Public Types

- `template<typename T >`  
`using push_front = type_list< T >`
- `template<typename T >`  
`using push_back = type_list< T >`
- `template<typename U >`  
`using concat = U`
- `template<typename T , size_t index>`  
`using insert = type_list< T >`

## Static Public Attributes

- `static constexpr size_t length = 0`

### 8.23.1 Detailed Description

specialization for empty type list

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 concat

```
template<typename U >
using aerobus::type_list<>::concat = U
```

#### 8.23.2.2 insert

```
template<typename T , size_t index>
using aerobus::type_list<>::insert = type_list<T>
```

#### 8.23.2.3 push\_back

```
template<typename T >
using aerobus::type_list<>::push_back = type_list<T>
```

#### 8.23.2.4 push\_front

```
template<typename T >
using aerobus::type_list<>::push_front = type_list<T>
```

### 8.23.3 Member Data Documentation

#### 8.23.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.24 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```



## Public Types

- `using enclosing_type = i32`  
*Enclosing ring type.*
- `using is_zero_t = std::bool_constant< x==0 >`  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr DEVICE valueType get ()`  
*cast x into valueType*
- `static std::string to_string ()`  
*string representation of value*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*

### 8.24.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

#### Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

### 8.24.2 Member Typedef Documentation

#### 8.24.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

#### 8.24.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.24.3 Member Function Documentation

#### 8.24.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get () [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

|                  |                    |
|------------------|--------------------|
| <i>valueType</i> | double for example |
|------------------|--------------------|

#### 8.24.3.2 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string () [inline], [static]
```

string representation of value

### 8.24.4 Member Data Documentation

#### 8.24.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

#### Public Types

- [using inner\\_type = int32\\_t](#)  
*type of represented values*
- [using enclosing\\_type = i64](#)  
*enclosing ring type*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr INLINED_DEVICE valueType get ()`  
*cast value in valueType*
- `static std::string to_string ()`  
*string representation*

## Static Public Attributes

- `static constexpr int64_t v = x`  
*actual value*

### 8.25.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

#### 8.25.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

#### 8.25.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.25.3 Member Function Documentation

#### 8.25.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get () [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueType</i> | (double for example) |
|------------------|----------------------|

#### 8.25.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string () [inline], [static]
```

string representation

### 8.25.4 Member Data Documentation

#### 8.25.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.26 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- `using ring_type = Ring`  
*ring coefficients live in*
- `using enclosing_type = polynomial< Ring >`  
*enclosing ring type*
- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true\_type if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 8.26.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

|                        |                           |
|------------------------|---------------------------|
| <code>coeffN</code>    | high degree coefficient   |
| <code>...coeffs</code> | lower degree coefficients |

## 8.26.2 Member Typedef Documentation

### 8.26.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

### 8.26.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.26.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.26.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.26.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

### 8.26.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

## 8.26.3 Member Function Documentation

### 8.26.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs
>::eval (
 const valueRing & x) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

|                  |                         |
|------------------|-------------------------|
| <i>valueRing</i> | usually float or double |
|------------------|-------------------------|

#### Parameters

|          |       |
|----------|-------|
| <i>x</i> | value |
|----------|-------|

#### Returns

$P(x)$

### 8.26.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string () [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.26.4 Member Data Documentation

### 8.26.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

### 8.26.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t←
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.27 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- [using raw\\_t = V](#)
- [using type = abs\\_t< typename Ring::template mod\\_t< V, X > >](#)

### 8.27.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

|                   |                     |
|-------------------|---------------------|
| <a href="#">V</a> | a value from 'Ring' |
|-------------------|---------------------|



## 8.27.2 Member Typedef Documentation

### 8.27.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.27.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.28 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< v==0 >`  
*true\_type if zero*

### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr INLINED_DEVICE valueType get ()`  
*get value as valueType*
- `static std::string to_string ()`  
*string representation*

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*
- `static constexpr bool is_zero_v = v == 0`  
*true if zero*

## 8.28.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

## Template Parameters

|   |            |
|---|------------|
| x | an integer |
|---|------------|

## 8.28.2 Member Typedef Documentation

### 8.28.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.28.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.28.3 Member Function Documentation

### 8.28.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get () [inline],
[static], [constexpr]
```

get value as valueType

## Template Parameters

|                  |                                   |
|------------------|-----------------------------------|
| <i>valueType</i> | an arithmetic type, such as float |
|------------------|-----------------------------------|

### 8.28.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string () [inline], [static]
```

string representation

**Returns**

a string representation

**8.28.4 Member Data Documentation****8.28.4.1 is\_zero\_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

**8.28.4.2 v**

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

**8.29 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference**

specialization for constants

```
#include <aerobus.h>
```

**Classes**

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

**Public Types**

- [using ring\\_type = Ring](#)  
*ring coefficients live in*
- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is\\_zero\\_t = std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)

## Static Public Member Functions

- `static std::string to_string ()`

## Static Public Attributes

- `static constexpr size_t degree = 0`  
*degree*
- `static constexpr bool is_zero_v = is_zero_t::value`

### 8.29.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

|               |  |
|---------------|--|
| <i>coeffN</i> |  |
|---------------|--|

### 8.29.2 Member Typedef Documentation

#### 8.29.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

#### 8.29.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

#### 8.29.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.29.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

### 8.29.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

### 8.29.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

## 8.29.3 Member Function Documentation

### 8.29.3.1 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string () [inline], [static]
```

## 8.29.4 Member Data Documentation

### 8.29.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

### 8.29.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.30 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in zpz*

### Public Types

- `using inner_type = int32_t`  
*underlying type for values*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`  
*injects a constant integer into zpz*
- `using zero = val< 0 >`  
*zero value*
- `using one = val< 1 >`  
*one value*
- `template<typename v1, typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1, typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1, typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1, typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1, typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1, typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1, typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1, typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1, typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos_t = typename pos< v1 >::type`  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`  
*true iff p is prime*
- `static constexpr bool is_euclidean_domain = true`  
*always true*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

### 8.30.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

#### Template Parameters

|          |           |
|----------|-----------|
| <i>p</i> | a integer |
|----------|-----------|

### 8.30.2 Member Typedef Documentation

#### 8.30.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)



## Template Parameters

|                 |                                  |
|-----------------|----------------------------------|
| <code>v1</code> | a value in <code>zpz::val</code> |
| <code>v2</code> | a value in <code>zpz::val</code> |

## 8.30.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

## Template Parameters

|                |            |
|----------------|------------|
| <code>x</code> | an integer |
|----------------|------------|

## 8.30.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

## 8.30.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

## Template Parameters

|                 |                                  |
|-----------------|----------------------------------|
| <code>v1</code> | a value in <code>zpz::val</code> |
| <code>v2</code> | a value in <code>zpz::val</code> |

## 8.30.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

**8.30.2.10 mul\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

**8.30.2.11 one**

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

**8.30.2.12 pos\_t**

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

**8.30.2.13 sub\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.30.3 Member Data Documentation

## 8.30.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.30.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.30.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051 template<typename T>
00052 T* aligned_malloc(size_t count, size_t alignment) {
00053 #ifdef _MSC_VER
00054 return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055 #else
00056 return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057 #endif
00058 }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064 template <typename R>
00065 concept IsRing = requires {
00066 typename R::one;
00067 typename R::zero;
00068 typename R::template add_t<typename R::one, typename R::one>;
00069 typename R::template sub_t<typename R::one, typename R::one>;
00070 typename R::template mul_t<typename R::one, typename R::one>;
00071 };
00072
00074 template <typename R>
00075 concept IsEuclideanDomain = IsRing<R> && requires {
00076 typename R::template div_t<typename R::one, typename R::one>;
00077 typename R::template mod_t<typename R::one, typename R::one>;
00078 typename R::template gcd_t<typename R::one, typename R::one>;
00079 typename R::template eq_t<typename R::one, typename R::one>;
00080 typename R::template pos_t<typename R::one>;
00081
00082 R::template pos_v<typename R::one> == true;
00083 // typename R::template gt_t<typename R::one, typename R::zero>;
00084 R::is_euclidean_domain == true;
00085 };
00086
00088 template<typename R>
00089 concept IsField = IsEuclideanDomain<R> && requires {
00090 R::is_field == true;
00091 };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097 namespace internal {
00098 static constexpr DEVICE uint16_t my_internal_float2half(
00099 const float f, uint32_t &sign, uint32_t &remainder) {
00100 uint32_t x;
00101 uint32_t u;
00102 uint32_t result;
00103 x = std::bit_cast<int32_t>(f);
00104 u = (x & 0x7fffffffU);
00105 sign = ((x >> 16U) & 0x8000U);
00106 // NaN/+Inf/-Inf
00107 if (u >= 0x7f800000U) {
00108 remainder = 0U;
00109 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110 } else if (u > 0x477fefffU) { // Overflows
00111 remainder = 0x80000000U;
00112 result = (sign | 0x7bfffU);
00113 } else if (u >= 0x38800000U) { // Normal numbers
00114 remainder = u << 19U;
00115 u -= 0x38000000U;
00116 result = (sign | (u >> 13U));
00117 } else if (u < 0x33000001U) { // +0/-0
00118 remainder = u;
00119 result = sign;
00120 } else { // Denormal numbers
00121 const uint32_t exponent = u >> 23U;
00122 const uint32_t shift = 0x7eU - exponent;
00123 uint32_t mantissa = (u & 0x7ffffU);
00124 mantissa |= 0x800000U;
00125 remainder = mantissa << (32U - shift);
00126 result = (sign | (mantissa >> shift));
00127 result &= 0x0000ffffU;

```

```

00128 }
00129 return static_cast<uint16_t>(result);
00130 }
00131
00132 static constexpr DEVICE __half my_float2half_rn(const float a) {
00133 __half val;
00134 __half_raw r;
00135 uint32_t sign = 0U;
00136 uint32_t remainder = 0U;
00137 r.x = my_internal_float2half(a, sign, remainder);
00138 if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139 r.x++;
00140 }
00141
00142 val = std::bit_cast<__half>(r);
00143 return val;
00144 }
00145
00146 template<int16_t i>
00147 static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150 template<typename Out, int16_t x, typename E = void>
00151 struct int16_convert_helper;
00152
00153 template<typename Out, int16_t x>
00154 struct int16_convert_helper<Out, x,
00155 std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156 static constexpr Out value() {
00157 return static_cast<Out>(x);
00158 }
00159 };
00160
00161 template<int16_t x>
00162 struct int16_convert_helper<__half, x> {
00163 static constexpr __half value() {
00164 return convert_int16_to_half<x>;
00165 }
00166 };
00167
00168 template<int16_t x>
00169 struct int16_convert_helper<__half2, x> {
00170 static constexpr __half2 value() {
00171 return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172 }
00173 };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180 namespace internal {
00181 template<typename Out, typename In>
00182 struct staticcast {
00183 template<auto x>
00184 static constexpr INLINED DEVICE Out func() {
00185 return static_cast<Out>(x);
00186 }
00187 };
00188
00189 #ifdef WITH_CUDA_FP16
00190 template<>
00191 struct staticcast<__half, int16_t> {
00192 template<int16_t x>
00193 static constexpr INLINED DEVICE __half func() {
00194 return int16_convert_helper<__half, x>::value();
00195 }
00196 };
00197
00198 template<>
00199 struct staticcast<__half2, int16_t> {
00200 template<int16_t x>
00201 static constexpr INLINED DEVICE __half2 func() {
00202 return int16_convert_helper<__half2, x>::value();
00203 }
00204 };
00205 #endif
00206 } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211 namespace internal {
00212 template<typename T>
00213 struct fma_helper;
00214 }

```

```

00215 template<>
00216 struct fma_helper<double> {
00217 static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
00218 z) {
00219 return x * y + z;
00220 };
00221
00222 template<>
00223 struct fma_helper<float> {
00224 static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00225 return x * y + z;
00226 };
00227
00228 template<>
00229 struct fma_helper<int32_t> {
00230 static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
00231 int16_t z) {
00232 return x * y + z;
00233 };
00234
00235 template<>
00236 struct fma_helper<int16_t> {
00237 static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
00238 int32_t z) {
00239 return x * y + z;
00240 };
00241
00242 template<>
00243 struct fma_helper<int64_t> {
00244 static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
00245 int64_t z) {
00246 return x * y + z;
00247 };
00248
00249 #ifdef WITH_CUDA_FP16
00250 template<>
00251 struct fma_helper<__half> {
00252 static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
00253 z) {
00254 #ifdef __CUDA_ARCH__
00255 return __hfma(x, y, z);
00256 #else
00257 return x * y + z;
00258 #endif
00259 };
00260
00261 template<>
00262 struct fma_helper<__half2> {
00263 static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
00264 __half2 z) {
00265 #ifdef __CUDA_ARCH__
00266 return __hfma2(x, y, z);
00267 #else
00268 return x * y + z;
00269 #endif
00270 };
00271 #endif
00272 } // namespace internal
00273 } // namespace aerobus
00274
00275 // utilities
00276 namespace aerobus {
00277 namespace internal {
00278 template<template<typename...> typename TT, typename T>
00279 struct is_instantiation_of : std::false_type { };
00280
00281 template<template<typename...> typename TT, typename... Ts>
00282 struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00283
00284 template<template<typename...> typename TT, typename T>
00285 inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00286
00287 template<int64_t i, typename T, typename... Ts>
00288 struct type_at {
00289 static_assert(i < sizeof...(Ts) + 1, "index out of range");
00290 using type = typename type_at<i - 1, Ts...>::type;
00291 };
00292
00293 template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00294 using type = T;
00295 };

```



```

00296
00297 template <size_t i, typename... Ts>
00298 using type_at_t = typename type_at<i, Ts...>::type;
00299
00300
00301 template<size_t n, size_t i, typename E = void>
00302 struct _is_prime {};
00303
00304 template<size_t i>
00305 struct _is_prime<0, i> {
00306 static constexpr bool value = false;
00307 };
00308
00309 template<size_t i>
00310 struct _is_prime<1, i> {
00311 static constexpr bool value = false;
00312 };
00313
00314 template<size_t i>
00315 struct _is_prime<2, i> {
00316 static constexpr bool value = true;
00317 };
00318
00319 template<size_t i>
00320 struct _is_prime<3, i> {
00321 static constexpr bool value = true;
00322 };
00323
00324 template<size_t i>
00325 struct _is_prime<5, i> {
00326 static constexpr bool value = true;
00327 };
00328
00329 template<size_t i>
00330 struct _is_prime<7, i> {
00331 static constexpr bool value = true;
00332 };
00333
00334 template<size_t n, size_t i>
00335 struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00336 static constexpr bool value = false;
00337 };
00338
00339 template<size_t n, size_t i>
00340 struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00341 static constexpr bool value = false;
00342 };
00343
00344 template<size_t n, size_t i>
00345 struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00346 static constexpr bool value = true;
00347 };
00348
00349 template<size_t n, size_t i>
00350 struct _is_prime<n, i, std::enable_if_t<(
00351 n % i == 0 &&
00352 n >= 9 &&
00353 n % 3 != 0 &&
00354 n % 2 != 0 &&
00355 i * i > n)>> {
00356 static constexpr bool value = true;
00357 };
00358
00359 template<size_t n, size_t i>
00360 struct _is_prime<n, i, std::enable_if_t<(
00361 n % (i+2) == 0 &&
00362 n >= 9 &&
00363 n % 3 != 0 &&
00364 n % 2 != 0 &&
00365 i * i <= n)>> {
00366 static constexpr bool value = true;
00367 };
00368
00369 template<size_t n, size_t i>
00370 struct _is_prime<n, i, std::enable_if_t<(
00371 n % (i+2) != 0 &&
00372 n % i != 0 &&
00373 n >= 9 &&
00374 n % 3 != 0 &&
00375 n % 2 != 0 &&
00376 (i * i <= n))>> {
00377 static constexpr bool value = _is_prime<n, i+6>::value;
00378 };
00379
00380 } // namespace internal
00381
00382 template<size_t n>

```

```

00385 struct is_prime {
00387 static constexpr bool value = internal::_is_prime<n, 5>::value;
00388 };
00389
00393 template<size_t n>
00394 static constexpr bool is_prime_v = is_prime<n>::value;
00395
00396 // gcd
00397 namespace internal {
00398 template <std::size_t... Is>
00399 constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00400 -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00401
00402 template <std::size_t N>
00403 using make_index_sequence_reverse
00404 = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00405
00411 template<typename Ring, typename E = void>
00412 struct gcd;
00413
00414 template<typename Ring>
00415 struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00416 template<typename A, typename B, typename E = void>
00417 struct gcd_helper {};
00418
00419 // B = 0, A > 0
00420 template<typename A, typename B>
00421 struct gcd_helper<A, B, std::enable_if_t<
00422 (B::is_zero_t::value) &&
00423 (Ring::template gt_t<A, typename Ring::zero>::value)> {
00424 using type = A;
00425 };
00426
00427 // B = 0, A < 0
00428 template<typename A, typename B>
00429 struct gcd_helper<A, B, std::enable_if_t<
00430 (B::is_zero_t::value) &&
00431 !(Ring::template gt_t<A, typename Ring::zero>::value)> {
00432 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00433 };
00434
00435 // B != 0
00436 template<typename A, typename B>
00437 struct gcd_helper<A, B, std::enable_if_t<
00438 (!B::is_zero_t::value)
00439 > {
00440 private: // NOLINT
00441 // A / B
00442 using k = typename Ring::template div_t<A, B>;
00443 // A - (A/B)*B = A % B
00444 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00445
00446 public:
00447 using type = typename gcd_helper<B, m>::type;
00448 };
00449
00450 template<typename A, typename B>
00451 using type = typename gcd_helper<A, B>::type;
00452 };
00453 } // namespace internal
00454
00455 // vadd and vmul
00456 namespace internal {
00457 template<typename... vals>
00458 struct vmul {};
00459
00460 template<typename v1, typename... vals>
00461 struct vmul<v1, vals...> {
00462 using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00463 };
00464
00465 template<typename v1>
00466 struct vmul<v1> {
00467 using type = v1;
00468 };
00469
00470 template<typename... vals>
00471 struct vadd {};
00472
00473 template<typename v1, typename... vals>
00474 struct vadd<v1, vals...> {
00475 using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00476 };
00477
00478 template<typename v1>

```

```

00479 struct vadd<v1> {
00480 using type = v1;
00481 };
00482 } // namespace internal
00483
00486 template<typename T, typename A, typename B>
00487 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00488
00492 template<typename... vals>
00493 using vadd_t = typename internal::vadd<vals...>::type;
00494
00498 template<typename... vals>
00499 using vmul_t = typename internal::vmul<vals...>::type;
00500
00504 template<typename val>
00505 requires IsEuclideanDomain<typename val::enclosing_type>
00506 using abs_t = std::conditional_t<
00507 val::enclosing_type::template pos_v<val>,
00508 val, typename val::enclosing_type::template
00509 sub_t<typename val::enclosing_type::zero, val>>;
00510 } // namespace aerobus
00511
00511 // embedding
00512 namespace aerobus {
00513 template<typename Small, typename Large, typename E = void>
00514 struct Embed;
00515 } // namespace aerobus
00520
00521 namespace aerobus {
00522 template<typename Ring, typename X>
00523 requires IsRing<Ring>
00524 struct Quotient {
00525 template <typename V>
00526 struct val {
00527 public:
00528 using raw_t = V;
00529 using type = abs_t<typename Ring::template mod_t<V, X>>;
00530 };
00531
00532 using zero = val<typename Ring::zero>;
00533
00534 using one = val<typename Ring::one>;
00535
00536 template<typename v1, typename v2>
00537 using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00538
00539 template<typename v1, typename v2>
00540 using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00541
00542 template<typename v1, typename v2>
00543 using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00544
00545 template<typename v1, typename v2>
00546 using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00547
00548 template<typename v1, typename v2>
00549 using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00550
00551 template<typename v1, typename v2>
00552 static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00553
00554 template<typename v1>
00555 using pos_t = std::true_type;
00556
00557 template<typename v>
00558 static constexpr bool pos_v = pos_t<v>::value;
00559
00560 static constexpr bool is_euclidean_domain = true;
00561
00562 template<auto x>
00563 using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00564
00565 template<typename v>
00566 using inject_ring_t = val<v>;
00567 };
00568
00569 template<typename Ring, typename X>
00570 struct Embed<Quotient<Ring, X>, Ring> {
00571 template<typename val>
00572 using type = typename val::raw_t;
00573 };
00574 } // namespace aerobus
00575
00576 // type_list
00577 namespace aerobus {
00578 template <typename... Ts>
00579 struct type_list;
00580 }

```

```

00629
00630 namespace internal {
00631 template <typename T, typename... Us>
00632 struct pop_front_h {
00633 using tail = type_list<Us...>;
00634 using head = T;
00635 };
00636
00637 template <size_t index, typename L1, typename L2>
00638 struct split_h {
00639 private:
00640 static_assert(index <= L2::length, "index ouf of bounds");
00641 using a = typename L2::pop_front::type;
00642 using b = typename L2::pop_front::tail;
00643 using c = typename L1::template push_back<a>;
00644
00645 public:
00646 using head = typename split_h<index - 1, c, b>::head;
00647 using tail = typename split_h<index - 1, c, b>::tail;
00648 };
00649
00650 template <typename L1, typename L2>
00651 struct split_h<0, L1, L2> {
00652 using head = L1;
00653 using tail = L2;
00654 };
00655
00656 template <size_t index, typename L, typename T>
00657 struct insert_h {
00658 static_assert(index <= L::length, "index ouf of bounds");
00659 using s = typename L::template split<index>;
00660 using left = typename s::head;
00661 using right = typename s::tail;
00662 using ll = typename left::template push_back<T>;
00663 using type = typename ll::template concat<right>;
00664 };
00665
00666 template <size_t index, typename L>
00667 struct remove_h {
00668 using s = typename L::template split<index>;
00669 using left = typename s::head;
00670 using right = typename s::tail;
00671 using rr = typename right::pop_front::tail;
00672 using type = typename left::template concat<rr>;
00673 };
00674 } // namespace internal
00675
00676 template <typename... Ts>
00677 struct type_list {
00678 private:
00679 template <typename T>
00680 struct concat_h;
00681
00682 template <typename... Us>
00683 struct concat_h<type_list<Us...> {
00684 using type = type_list<Ts..., Us...>;
00685 };
00686
00687 public:
00688 static constexpr size_t length = sizeof...(Ts);
00689
00690 template <typename T>
00691 using push_front = type_list<T, Ts...>;
00692
00693 template <size_t index>
00694 using at = internal::type_at_t<index, Ts...>;
00695
00696 struct pop_front {
00697 using type = typename internal::pop_front_h<Ts...>::head;
00698 using tail = typename internal::pop_front_h<Ts...>::tail;
00699 };
00700
00701 template <typename T>
00702 using push_back = type_list<Ts..., T>;
00703
00704 template <typename U>
00705 using concat = typename concat_h<U>::type;
00706
00707 template <size_t index>
00708 struct split {
00709 private:
00710 using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00711
00712 public:
00713 using head = typename inner::head;
00714 using tail = typename inner::tail;
00715 };
00716 };

```

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```

00842 struct gt {
00843 using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00844 };
00845
00846 template<typename v1, typename v2>
00847 struct lt {
00848 using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00849 };
00850
00851 template<typename v1, typename v2>
00852 struct eq {
00853 using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00854 };
00855
00856 template<typename v1>
00857 struct pos {
00858 using type = std::bool_constant<(v1::v > 0)>;
00859 };
00860
00861 public:
00862 template<typename v1, typename v2>
00863 using add_t = typename add<v1, v2>::type;
00864
00865 template<typename v1, typename v2>
00866 using sub_t = typename sub<v1, v2>::type;
00867
00868 template<typename v1, typename v2>
00869 using mul_t = typename mul<v1, v2>::type;
00870
00871 template<typename v1, typename v2>
00872 using div_t = typename div<v1, v2>::type;
00873
00874 template<typename v1, typename v2>
00875 using mod_t = typename remainder<v1, v2>::type;
00876
00877 template<typename v1, typename v2>
00878 using gt_t = typename gt<v1, v2>::type;
00879
00880 template<typename v1, typename v2>
00881 using lt_t = typename lt<v1, v2>::type;
00882
00883 template<typename v1, typename v2>
00884 using eq_t = typename eq<v1, v2>::type;
00885
00886 template<typename v1, typename v2>
00887 static constexpr bool eq_v = eq_t<v1, v2>::value;
00888
00889 template<typename v1, typename v2>
00890 using gcd_t = gcd_t<i16, v1, v2>;
00891
00892 template<typename v>
00893 using pos_t = typename pos<v>::type;
00894
00895 template<typename v>
00896 static constexpr bool pos_v = pos_t<v>::value;
00897 };
00898 } // namespace aerobus
00899 #endif
00900
00901 // i32
00902 namespace aerobus {
00903 struct i32 {
00904 using inner_type = int32_t;
00905 template<int32_t x>
00906 struct val {
00907 using enclosing_type = i32;
00908 static constexpr int32_t v = x;
00909
00910 template<typename valueType>
00911 static constexpr DEVICE valueType get() {
00912 return static_cast<valueType>(x);
00913 }
00914
00915 using is_zero_t = std::bool_constant<x == 0>;
00916
00917 static std::string to_string() {
00918 return std::to_string(x);
00919 }
00920 };
00921 };
00922
00923 using zero = val<0>;
00924 using one = val<1>;
00925 static constexpr bool is_field = false;
00926 static constexpr bool is_euclidean_domain = true;
00927 template<auto x>
00928 using inject_constant_t = val<static_cast<int32_t>(x)>;
00929 }
00930

```

```

01002 template<typename v>
01003 using inject_ring_t = v;
01004
01005 private:
01006 template<typename v1, typename v2>
01007 struct add {
01008 using type = val<v1::v + v2::v>;
01009 };
01010
01011 template<typename v1, typename v2>
01012 struct sub {
01013 using type = val<v1::v - v2::v>;
01014 };
01015
01016 template<typename v1, typename v2>
01017 struct mul {
01018 using type = val<v1::v * v2::v>;
01019 };
01020
01021 template<typename v1, typename v2>
01022 struct div {
01023 using type = val<v1::v / v2::v>;
01024 };
01025
01026 template<typename v1, typename v2>
01027 struct remainder {
01028 using type = val<v1::v % v2::v>;
01029 };
01030
01031 template<typename v1, typename v2>
01032 struct gt {
01033 using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01034 };
01035
01036 template<typename v1, typename v2>
01037 struct lt {
01038 using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01039 };
01040
01041 template<typename v1, typename v2>
01042 struct eq {
01043 using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01044 };
01045
01046 template<typename v1>
01047 struct pos {
01048 using type = std::bool_constant<(v1::v > 0)>;
01049 };
01050
01051 public:
01052 template<typename v1, typename v2>
01053 using add_t = typename add<v1, v2>::type;
01054
01055 template<typename v1, typename v2>
01056 using sub_t = typename sub<v1, v2>::type;
01057
01058 template<typename v1, typename v2>
01059 using mul_t = typename mul<v1, v2>::type;
01060
01061 template<typename v1, typename v2>
01062 using div_t = typename div<v1, v2>::type;
01063
01064 template<typename v1, typename v2>
01065 using mod_t = typename remainder<v1, v2>::type;
01066
01067 template<typename v1, typename v2>
01068 using gt_t = typename gt<v1, v2>::type;
01069
01070 template<typename v1, typename v2>
01071 using lt_t = typename lt<v1, v2>::type;
01072
01073 template<typename v1, typename v2>
01074 using eq_t = typename eq<v1, v2>::type;
01075
01076 template<typename v1, typename v2>
01077 static constexpr bool eq_v = eq_t<v1, v2>::value;
01078
01079 template<typename v1, typename v2>
01080 using gcd_t = gcd_t<i32, v1, v2>;
01081
01082 template<typename v>
01083 using pos_t = typename pos<v>::type;
01084
01085 template<typename v>
01086 static constexpr bool pos_v = pos_t<v>::value;
01087 };
01088 // namespace aerobus

```

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```

01267 template<typename v1, typename v2>
01268 using mul_t = typename mul<v1, v2>::type;
01269
01275 template<typename v1, typename v2>
01276 using div_t = typename div<v1, v2>::type;
01277
01282 template<typename v1, typename v2>
01283 using mod_t = typename remainder<v1, v2>::type;
01284
01290 template<typename v1, typename v2>
01291 using gt_t = typename gt<v1, v2>::type;
01292
01297 template<typename v1, typename v2>
01298 static constexpr bool gt_v = gt_t<v1, v2>::value;
01299
01305 template<typename v1, typename v2>
01306 using lt_t = typename lt<v1, v2>::type;
01307
01313 template<typename v1, typename v2>
01314 static constexpr bool lt_v = lt_t<v1, v2>::value;
01315
01321 template<typename v1, typename v2>
01322 using eq_t = typename eq<v1, v2>::type;
01323
01329 template<typename v1, typename v2>
01330 static constexpr bool eq_v = eq_t<v1, v2>::value;
01331
01337 template<typename v1, typename v2>
01338 using gcd_t = gcd_t<i64, v1, v2>;
01339
01344 template<typename v>
01345 using pos_t = typename pos<v>::type;
01346
01351 template<typename v>
01352 static constexpr bool pos_v = pos_t<v>::value;
01353 };
01354
01356 template<>
01357 struct Embed<i32, i64> {
01360 template<typename val>
01361 using type = i64::val<static_cast<int64_t>(val::v)>;
01362 };
01363 } // namespace aerobus
01364
01365 // z/pz
01366 namespace aerobus {
01372 template<int32_t p>
01373 struct zpz {
01375 using inner_type = int32_t;
01376
01379 template<int32_t x>
01380 struct val {
01382 using enclosing_type = zpz<p>;
01384 static constexpr int32_t v = x % p;
01385
01388 template<typename valueType>
01389 static constexpr INLINED_DEVICE valueType get() {
01390 return static_cast<valueType>(x % p);
01391 }
01392
01394 using is_zero_t = std::bool_constant<v == 0>;
01395
01397 static constexpr bool is_zero_v = v == 0;
01398
01401 static std::string to_string() {
01402 return std::to_string(x % p);
01403 }
01404 };
01405
01408 template<auto x>
01409 using inject_constant_t = val<static_cast<int32_t>(x)>;
01410
01412 using zero = val<0>;
01413
01415 using one = val<1>;
01416
01418 static constexpr bool is_prime<p>::value;
01419
01421 static constexpr bool is_euclidean_domain = true;
01422
01423 private:
01424 template<typename v1, typename v2>
01425 struct add {
01426 using type = val<(v1::v + v2::v) % p>;
01427 };
01428
01429 template<typename v1, typename v2>

```

```

01430 struct sub {
01431 using type = val<(v1::v - v2::v) % p>;
01432 };
01433
01434 template<typename v1, typename v2>
01435 struct mul {
01436 using type = val<(v1::v* v2::v) % p>;
01437 };
01438
01439 template<typename v1, typename v2>
01440 struct div {
01441 using type = val<(v1::v% p) / (v2::v % p)>;
01442 };
01443
01444 template<typename v1, typename v2>
01445 struct remainder {
01446 using type = val<(v1::v% v2::v) % p>;
01447 };
01448
01449 template<typename v1, typename v2>
01450 struct gt {
01451 using type = std::conditional_t<(v1::v% p > v2::v% p), std::true_type, std::false_type>;
01452 };
01453
01454 template<typename v1, typename v2>
01455 struct lt {
01456 using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01457 };
01458
01459 template<typename v1, typename v2>
01460 struct eq {
01461 using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01462 };
01463
01464 template<typename v1>
01465 struct pos {
01466 using type = std::bool_constant<(v1::v > 0)>;
01467 };
01468
01469 public:
01470 template<typename v1, typename v2>
01471 using add_t = typename add<v1, v2>::type;
01472
01473 template<typename v1, typename v2>
01474 using sub_t = typename sub<v1, v2>::type;
01475
01476 template<typename v1, typename v2>
01477 using mul_t = typename mul<v1, v2>::type;
01478
01479 template<typename v1, typename v2>
01480 using div_t = typename div<v1, v2>::type;
01481
01482 template<typename v1, typename v2>
01483 using mod_t = typename remainder<v1, v2>::type;
01484
01485 template<typename v1, typename v2>
01486 using gt_t = typename gt<v1, v2>::type;
01487
01488 template<typename v1, typename v2>
01489 static constexpr bool gt_v = gt_t<v1, v2>::value;
01490
01491 template<typename v1, typename v2>
01492 using lt_t = typename lt<v1, v2>::type;
01493
01494 template<typename v1, typename v2>
01495 static constexpr bool lt_v = lt_t<v1, v2>::value;
01496
01497 template<typename v1, typename v2>
01498 using eq_t = typename eq<v1, v2>::type;
01499
01500 template<typename v1, typename v2>
01501 static constexpr bool eq_v = eq_t<v1, v2>::value;
01502
01503 template<typename v1, typename v2>
01504 using gcd_t = gcd_t<i32, v1, v2>;
01505
01506 template<typename v1>
01507 using pos_t = typename pos<v1>::type;
01508
01509 template<typename v>
01510 static constexpr bool pos_v = pos_t<v>::value;
01511
01512 };
01513
01514 template<int32_t x>
01515 struct Embed<zpz<x>, i32> {
01516 template<typename val>
01517 using type = i32::val<val::v>;
01518

```

```

01561 };
01562 } // namespace aerobus
01563
01564 // polynomial
01565 namespace aerobus {
01566 // coeffN x^N + ...
01571 template<typename Ring>
01572 requires IsEuclideanDomain<Ring>
01573 struct polynomial {
01574 static constexpr bool is_field = false;
01575 static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01576
01580 template<typename coeffN, typename... coeffs>
01581 struct val {
01583 using ring_type = Ring;
01585 using enclosing_type = polynomial<Ring>;
01587 static constexpr size_t degree = sizeof...(coeffs);
01589 using aN = coeffN;
01591 using strip = val<coeffs...>;
01593 using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01595 static constexpr bool is_zero_v = is_zero_t::value;
01596
01597 private:
01598 template<size_t index, typename E = void>
01599 struct coeff_at {};
01600
01601 template<size_t index>
01602 struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01603 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01604 };
01605
01606 template<size_t index>
01607 struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01608 using type = typename Ring::zero;
01609 };
01610
01611 public:
01614 template<size_t index>
01615 using coeff_at_t = typename coeff_at<index>::type;
01616
01619 static std::string to_string() {
01620 return string_helper<coeffN, coeffs...>::func();
01621 }
01622
01627 template<typename valueRing>
01628 static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {
01629 #ifdef WITH_CUDA_FP16
01630 valueRing start;
01631 if constexpr (std::is_same_v<valueRing, __half2>) {
01632 start = __half2(0, 0);
01633 } else {
01634 start = static_cast<valueRing>(0);
01635 }
01636 #else
01637 valueRing start = static_cast<valueRing>(0);
01638 #endif
01639 return horner_evaluation<valueRing, val>
01640 ::template inner<0, degree + 1>
01641 ::func(start, x);
01642 }
01643 };
01644
01647 template<typename coeffN>
01648 struct val<coeffN> {
01650 using ring_type = Ring;
01652 using enclosing_type = polynomial<Ring>;
01654 static constexpr size_t degree = 0;
01655 using aN = coeffN;
01656 using strip = val<coeffN>;
01657 using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01658
01659 static constexpr bool is_zero_v = is_zero_t::value;
01660
01661 template<size_t index, typename E = void>
01662 struct coeff_at {};
01663
01664 template<size_t index>
01665 struct coeff_at<index, std::enable_if_t<(index == 0)> {
01666 using type = aN;
01667 };
01668
01669 template<size_t index>
01670 struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01671 using type = typename Ring::zero;
01672 };
01673
01674 template<size_t index>

```

```

01675 using coeff_at_t = typename coeff_at<index>::type;
01676
01677 static std::string to_string() {
01678 return string_helper<coeffN>::func();
01679 }
01680 };
01681
01682 using zero = val<typename Ring::zero>;
01683 using one = val<typename Ring::one>;
01684 using X = val<typename Ring::one, typename Ring::zero>;
01685
01686 private:
01687 template<typename P, typename E = void>
01688 struct simplify;
01689
01690 template<typename P1, typename P2, typename I>
01691 struct add_low;
01692
01693 template<typename P1, typename P2>
01694 struct add {
01695 using type = typename simplify<typename add_low<
01696 P1,
01697 P2,
01698 internal::make_index_sequence_reverse<
01699 std::max(P1::degree, P2::degree) + 1
01700 >::type>::type;
01701 };
01702
01703 template<typename P1, typename P2, typename I>
01704 struct sub_low;
01705
01706 template<typename P1, typename P2, typename I>
01707 struct mul_low;
01708
01709 template<typename v1, typename v2>
01710 struct mul {
01711 using type = typename mul_low<
01712 v1,
01713 v2,
01714 internal::make_index_sequence_reverse<
01715 v1::degree + v2::degree + 1
01716 >::type;
01717 };
01718
01719 template<typename coeff, size_t deg>
01720 struct monomial;
01721
01722 template<typename v, typename E = void>
01723 struct derive_helper {};
01724
01725 template<typename v>
01726 struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01727 using type = zero;
01728 };
01729
01730 template<typename v>
01731 struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01732 using type = typename add<
01733 typename derive_helper<typename simplify<typename v::strip>::type>::type,
01734 typename monomial<
01735 typename Ring::template mul_t<
01736 typename v::aN,
01737 typename Ring::template inject_constant_t<(v::degree)>
01738 >,
01739 v::degree - 1
01740 >::type
01741 >::type;
01742 };
01743
01744 template<typename v1, typename v2, typename E = void>
01745 struct eq_helper {};
01746
01747 template<typename v1, typename v2>
01748 struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01749 using type = std::false_type;
01750 };
01751
01752 template<typename v1, typename v2>
01753 struct eq_helper<v1, v2, std::enable_if_t<
01754 v1::degree == v2::degree &&
01755 (v1::degree != 0 || v2::degree != 0) &&
01756 std::is_same<
01757 typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01758 std::false_type
01759 >::value
01760 >

```

```

01765 > {
01766 using type = std::false_type;
01767 };
01768
01769 template<typename v1, typename v2>
01770 struct eq_helper<v1, v2, std::enable_if_t<
01771 v1::degree == v2::degree &&
01772 (v1::degree != 0 || v2::degree != 0) &&
01773 std::is_same<
01774 typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01775 std::true_type
01776 >::value
01777 > {
01778 using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01779 };
01780
01781 template<typename v1, typename v2>
01782 struct eq_helper<v1, v2, std::enable_if_t<
01783 v1::degree == v2::degree &&
01784 (v1::degree == 0)
01785 > {
01786 using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01787 };
01788
01789 template<typename v1, typename v2, typename E = void>
01790 struct lt_helper {};
01791
01792 template<typename v1, typename v2>
01793 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01794 using type = std::true_type;
01795 };
01796
01797 template<typename v1, typename v2>
01798 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01799 using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01800 };
01801
01802 template<typename v1, typename v2>
01803 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01804 using type = std::false_type;
01805 };
01806
01807 template<typename v1, typename v2, typename E = void>
01808 struct gt_helper {};
01809
01810 template<typename v1, typename v2>
01811 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01812 using type = std::true_type;
01813 };
01814
01815 template<typename v1, typename v2>
01816 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01817 using type = std::false_type;
01818 };
01819
01820 template<typename v1, typename v2>
01821 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01822 using type = std::false_type;
01823 };
01824
01825 // when high power is zero : strip
01826 template<typename P>
01827 struct simplify<P, std::enable_if_t<
01828 std::is_same<
01829 typename Ring::zero,
01830 typename P::aN
01831 >::value && (P::degree > 0)
01832 > {
01833 using type = typename simplify<typename P::strip>::type;
01834 };
01835
01836 // otherwise : do nothing
01837 template<typename P>
01838 struct simplify<P, std::enable_if_t<
01839 !std::is_same<
01840 typename Ring::zero,
01841 typename P::aN
01842 >::value && (P::degree > 0)
01843 > {
01844 using type = P;
01845 };
01846
01847 // do not simplify constants
01848 template<typename P>
01849 struct simplify<P, std::enable_if_t<P::degree == 0>> {
01850 using type = P;
01851 };

```

```

01852
01853 // addition at
01854 template<typename P1, typename P2, size_t index>
01855 struct add_at {
01856 using type =
01857 typename Ring::template add_t<
01858 typename P1::template coeff_at_t<index>,
01859 typename P2::template coeff_at_t<index>>;
01860 };
01861
01862 template<typename P1, typename P2, size_t index>
01863 using add_at_t = typename add_at<P1, P2, index>::type;
01864
01865 template<typename P1, typename P2, std::size_t... I>
01866 struct add_low<P1, P2, std::index_sequence<I...> {
01867 using type = val<add_at_t<P1, P2, I>...>;
01868 };
01869
01870 // subtraction at
01871 template<typename P1, typename P2, size_t index>
01872 struct sub_at {
01873 using type =
01874 typename Ring::template sub_t<
01875 typename P1::template coeff_at_t<index>,
01876 typename P2::template coeff_at_t<index>>;
01877 };
01878
01879 template<typename P1, typename P2, size_t index>
01880 using sub_at_t = typename sub_at<P1, P2, index>::type;
01881
01882 template<typename P1, typename P2, std::size_t... I>
01883 struct sub_low<P1, P2, std::index_sequence<I...> {
01884 using type = val<sub_at_t<P1, P2, I>...>;
01885 };
01886
01887 template<typename P1, typename P2>
01888 struct sub {
01889 using type = typename simplify<typename sub_low<
01890 P1,
01891 P2,
01892 internal::make_index_sequence_reverse<
01893 std::max(P1::degree, P2::degree) + 1
01894 >::type>::type;
01895 };
01896
01897 // multiplication at
01898 template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01899 struct mul_at_loop_helper {
01900 using type = typename Ring::template add_t<
01901 typename Ring::template mul_t<
01902 typename v1::template coeff_at_t<index>,
01903 typename v2::template coeff_at_t<k - index>
01904 >,
01905 typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01906 >;
01907 };
01908
01909 template<typename v1, typename v2, size_t k, size_t stop>
01910 struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01911 using type = typename Ring::template mul_t<
01912 typename v1::template coeff_at_t<stop>,
01913 typename v2::template coeff_at_t<0>>;
01914 };
01915
01916 template <typename v1, typename v2, size_t k, typename E = void>
01917 struct mul_at {};
01918
01919 template<typename v1, typename v2, size_t k>
01920 struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01921 using type = typename Ring::zero;
01922 };
01923
01924 template<typename v1, typename v2, size_t k>
01925 struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01926 using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01927 };
01928
01929 template<typename P1, typename P2, size_t index>
01930 using mul_at_t = typename mul_at<P1, P2, index>::type;
01931
01932 template<typename P1, typename P2, std::size_t... I>
01933 struct mul_low<P1, P2, std::index_sequence<I...> {
01934 using type = val<mul_at_t<P1, P2, I>...>;
01935 };
01936
01937 // division helper
01938 template< typename A, typename B, typename Q, typename R, typename E = void>

```

```

01939 struct div_helper {};
01940
01941 template<typename A, typename B, typename Q, typename R>
01942 struct div_helper<A, B, Q, R, std::enable_if_t<
01943 (R::degree < B::degree) ||
01944 (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01945 using q_type = Q;
01946 using mod_type = R;
01947 using gcd_type = B;
01948 };
01949
01950 template<typename A, typename B, typename Q, typename R>
01951 struct div_helper<A, B, Q, R, std::enable_if_t<
01952 (R::degree >= B::degree) &&
01953 !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01954 private: // NOLINT
01955 using rN = typename R::aN;
01956 using bN = typename B::aN;
01957 using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
B::degree>::type;
01958 using rr = typename sub<R, typename mul<pT, B>::type>::type;
01959 using qq = typename add<Q, pT>::type;
01960
01961 public:
01962 using q_type = typename div_helper<A, B, qq, rr>::q_type;
01963 using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01964 using gcd_type = rr;
01965 };
01966
01967 template<typename A, typename B>
01968 struct div {
01969 static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01970 using q_type = typename div_helper<A, B, zero, A>::q_type;
01971 using m_type = typename div_helper<A, B, zero, A>::mod_type;
01972 };
01973
01974 template<typename P>
01975 struct make_unit {
01976 using type = typename div<P, val<typename P::aN>>::q_type;
01977 };
01978
01979 template<typename coeff, size_t deg>
01980 struct monomial {
01981 using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01982 };
01983
01984 template<typename coeff>
01985 struct monomial<coeff, 0> {
01986 using type = val<coeff>;
01987 };
01988
01989 template<typename valueRing, typename P>
01990 struct horner_evaluation {
01991 template<size_t index, size_t stop>
01992 struct inner {
01993 static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01994 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(
internal::fma_helper<valueRing>::eval(
01995 x,
01996 accum,
01997 P::template coeff_at_t<P::degree - index>::template get<valueRing>()), x);
01998 }
01999 };
02000 };
02001
02002 template<size_t stop>
02003 struct inner<stop, stop> {
02004 static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
02005 return accum;
02006 }
02007 };
02008
02009 template<typename coeff, typename... coeffs>
02010 struct string_helper {
02011 static std::string func() {
02012 std::string tail = string_helper<coeffs...>::func();
02013 std::string result = "";
02014 if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02015 return tail;
02016 } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02017 if (sizeof...(coeffs) == 1) {
02018 result += "x";
02019 } else {
02020 result += "x^" + std::to_string(sizeof...(coeffs));
02021 }
02022 }

```

```

02023 } else {
02024 if (sizeof...(coeffs) == 1) {
02025 result += coeff::to_string() + " x";
02026 } else {
02027 result += coeff::to_string()
02028 + " x^" + std::to_string(sizeof...(coeffs));
02029 }
02030 }
02031 }
02032 if (!tail.empty()) {
02033 result += " + " + tail;
02034 }
02035 return result;
02036 }
02037 };
02038 };
02039
02040 template<typename coeff>
02041 struct string_helper<coeff> {
02042 static std::string func() {
02043 if (!std::is_same<coeff, typename Ring::zero::value>) {
02044 return coeff::to_string();
02045 } else {
02046 return "";
02047 }
02048 }
02049 };
02050
02051 public:
02052 template<typename P>
02053 using simplify_t = typename simplify<P>::type;
02054
02055 template<typename v1, typename v2>
02056 using add_t = typename add<v1, v2>::type;
02057
02058 template<typename v1, typename v2>
02059 using sub_t = typename sub<v1, v2>::type;
02060
02061 template<typename v1, typename v2>
02062 using mul_t = typename mul<v1, v2>::type;
02063
02064 template<typename v1, typename v2>
02065 using eq_t = typename eq_helper<v1, v2>::type;
02066
02067 template<typename v1, typename v2>
02068 using lt_t = typename lt_helper<v1, v2>::type;
02069
02070 template<typename v1, typename v2>
02071 using gt_t = typename gt_helper<v1, v2>::type;
02072
02073 template<typename v1, typename v2>
02074 using div_t = typename div<v1, v2>::q_type;
02075
02076 template<typename v1, typename v2>
02077 using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02078
02079 template<typename coeff, size_t deg>
02080 using monomial_t = typename monomial<coeff, deg>::type;
02081
02082 template<typename v>
02083 using derive_t = typename derive_helper<v>::type;
02084
02085 template<typename v>
02086 using pos_t = typename Ring::template pos_t<typename v::aN>;
02087
02088 template<typename v>
02089 static constexpr bool pos_v = pos_t<v>::value;
02090
02091 template<typename v1, typename v2>
02092 using gcd_t = std::conditional_t<
02093 Ring::is_euclidean_domain,
02094 typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02095 void>;
02096
02097 template<auto x>
02098 using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02099
02100 template<typename v>
02101 using inject_ring_t = val<v>;
02102 };
02103 } // namespace aerobus
02104
02105 // fraction field
02106 namespace aerobus {
02107 namespace internal {
02108 template<typename Ring, typename E = void>
02109 requires IsEuclideanDomain<Ring>

```



```

02154 struct _FractionField {};
02155
02156 template<typename Ring>
02157 requires IsEuclideanDomain<Ring>
02158 struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02159 static constexpr bool is_field = true;
02160 static constexpr bool is_euclidean_domain = true;
02161
02162 private:
02163 template<typename val1, typename val2, typename E = void>
02164 struct to_string_helper {};
02165
02166 template<typename val1, typename val2>
02167 struct to_string_helper<val1, val2,
02168 std::enable_if_t<
02169 Ring::template eq_t<
02170 val2, typename Ring::one
02171 >::value
02172 >
02173 > {
02174 static std::string func() {
02175 return val1::to_string();
02176 }
02177 };
02178
02179 template<typename val1, typename val2>
02180 struct to_string_helper<val1, val2,
02181 std::enable_if_t<
02182 !Ring::template eq_t<
02183 val2,
02184 typename Ring::one
02185 >::value
02186 >
02187 > {
02188 static std::string func() {
02189 return "(" + val1::to_string() + ") / (" + val2::to_string() + ")";
02190 }
02191 };
02192 };
02193
02194 public:
02195 template<typename val1, typename val2>
02196 struct val {
02197 using x = val1;
02198 using y = val2;
02199 using is_zero_t = typename val1::is_zero_t;
02200 static constexpr bool is_zero_v = val1::is_zero_t::value;
02201
02202 using ring_type = Ring;
02203 using enclosing_type = _FractionField<Ring>;
02204
02205 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02206
02207 template<typename valueType>
02208 static constexpr INLINED DEVICE valueType get() {
02209 return internal::staticcast<valueType, typename ring_type::inner_type>::template
02210 func<x::v>() /
02211 internal::staticcast<valueType, typename ring_type::inner_type>::template
02212 func<y::v>();
02213 }
02214
02215 static std::string to_string() {
02216 return to_string_helper<val1, val2>::func();
02217 }
02218
02219 template<typename valueRing>
02220 static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
02221 return x::eval(v) / y::eval(v);
02222 }
02223 };
02224
02225 using zero = val<typename Ring::zero, typename Ring::one>;
02226 using one = val<typename Ring::one, typename Ring::one>;
02227
02228 template<typename v>
02229 using inject_t = val<v, typename Ring::one>;
02230
02231 template<auto x>
02232 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
02233 Ring::one>;
02234
02235 template<typename v>
02236 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02237
02238 using ring_type = Ring;
02239
02240 private:
02241 template<typename v, typename E = void>

```

```

02267 struct simplify {};
02268
02269 // x = 0
02270 template<typename v>
02271 struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02272 using type = typename _FractionField<Ring>::zero;
02273 };
02274
02275 // x != 0
02276 template<typename v>
02277 struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02278 private:
02279 using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02280 using newx = typename Ring::template div_t<typename v::x, _gcd>;
02281 using newy = typename Ring::template div_t<typename v::y, _gcd>;
02282
02283 using posx = std::conditional_t<
02284 !Ring::template pos_v<newy>,
02285 typename Ring::template sub_t<typename Ring::zero, newx>,
02286 newx>;
02287 using posy = std::conditional_t<
02288 !Ring::template pos_v<newy>,
02289 typename Ring::template sub_t<typename Ring::zero, newy>,
02290 newy>;
02291 public:
02292 using type = typename _FractionField<Ring>::template val<posx, posy>;
02293 };
02294
02295 public:
02296 template<typename v>
02297 using simplify_t = typename simplify<v>::type;
02298
02299 private:
02300 template<typename v1, typename v2>
02301 struct add {
02302 private:
02303 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02304 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02305 using dividend = typename Ring::template add_t<a, b>;
02306 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02307 using g = typename Ring::template gcd_t<dividend, diviser>;
02308
02309 public:
02310 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02311 diviser>;
02312 };
02313
02314 template<typename v>
02315 struct pos {
02316 using type = std::conditional_t<
02317 (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02318 (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02319 std::true_type,
02320 std::false_type>;
02321 };
02322
02323 template<typename v1, typename v2>
02324 struct sub {
02325 private:
02326 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02327 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02328 using dividend = typename Ring::template sub_t<a, b>;
02329 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02330 using g = typename Ring::template gcd_t<dividend, diviser>;
02331
02332 public:
02333 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02334 diviser>;
02335 };
02336
02337 template<typename v1, typename v2>
02338 struct mul {
02339 private:
02340 using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02341 using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02342
02343 public:
02344 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02345 };
02346
02347 template<typename v1, typename v2, typename E = void>
02348 struct div {};
02349
02350 template<typename v1, typename v2>
02351 struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
02352 _FractionField<Ring>::zero>::value> {
02353 private:

```

```

02353 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02354 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02355
02356 public:
02357 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02358 };
02359
02360 template<typename v1, typename v2>
02361 struct div<v1, v2, std::enable_if_t<
02362 std::is_same<zero, v1::value && std::is_same<v2, zero>::value> {
02363 using type = one;
02364 };
02365
02366 template<typename v1, typename v2>
02367 struct eq {
02368 using type = std::conditional_t<
02369 std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x::value &&
02370 std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y::value,
02371 std::true_type,
02372 std::false_type>;
02373 };
02374
02375 template<typename v1, typename v2, typename E = void>
02376 struct gt;
02377
02378 template<typename v1, typename v2>
02379 struct gt<v1, v2, std::enable_if_t<
02380 (eq<v1, v2>::type::value)
02381 > {
02382 using type = std::false_type;
02383 };
02384
02385 template<typename v1, typename v2>
02386 struct gt<v1, v2, std::enable_if_t<
02387 (!eq<v1, v2>::type::value) &&
02388 (!pos<v1>::type::value) && (!pos<v2>::type::value)
02389 > {
02390 using type = typename gt<
02391 typename sub<zero, v1>::type, typename sub<zero, v2>::type
02392 >::type;
02393 };
02394
02395 template<typename v1, typename v2>
02396 struct gt<v1, v2, std::enable_if_t<
02397 (!eq<v1, v2>::type::value) &&
02398 (pos<v1>::type::value) && (!pos<v2>::type::value)
02399 > {
02400 using type = std::true_type;
02401 };
02402
02403 template<typename v1, typename v2>
02404 struct gt<v1, v2, std::enable_if_t<
02405 (!eq<v1, v2>::type::value) &&
02406 (!pos<v1>::type::value) && (pos<v2>::type::value)
02407 > {
02408 using type = std::false_type;
02409 };
02410
02411 template<typename v1, typename v2>
02412 struct gt<v1, v2, std::enable_if_t<
02413 (!eq<v1, v2>::type::value) &&
02414 (pos<v1>::type::value) && (pos<v2>::type::value)
02415 > {
02416 using type = typename Ring::template gt_t<
02417 typename Ring::template mul_t<v1::x, v2::y>,
02418 typename Ring::template mul_t<v2::y, v2::x>
02419 >;
02420 };
02421
02422 public:
02423 template<typename v1, typename v2>
02424 using add_t = typename add<v1, v2>::type;
02425
02426 template<typename v1, typename v2>
02427 using mod_t = zero;
02428
02429 template<typename v1, typename v2>
02430 using gcd_t = v1;
02431
02432 template<typename v1, typename v2>
02433 using sub_t = typename sub<v1, v2>::type;
02434
02435 template<typename v1, typename v2>
02436 using mul_t = typename mul<v1, v2>::type;
02437
02438 template<typename v1, typename v2>
02439 using div_t = typename div<v1, v2>::type;

```

```

02461
02465 template<typename v1, typename v2>
02466 using eq_t = typename eq<v1, v2>::type;
02467
02471 template<typename v1, typename v2>
02472 static constexpr bool eq_v = eq<v1, v2>::type::value;
02473
02477 template<typename v1, typename v2>
02478 using gt_t = typename gt<v1, v2>::type;
02479
02483 template<typename v1, typename v2>
02484 static constexpr bool gt_v = gt<v1, v2>::type::value;
02485
02488 template<typename v1>
02489 using pos_t = typename pos<v1>::type;
02490
02493 template<typename v>
02494 static constexpr bool pos_v = pos<t<v>::value;
02495 };
02496
02497 template<typename Ring, typename E = void>
02498 requires IsEuclideanDomain<Ring>
02499 struct FractionFieldImpl {};
02500
02501 // fraction field of a field is the field itself
02502 template<typename Field>
02503 requires IsEuclideanDomain<Field>
02504 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02505 using type = Field;
02506 template<typename v>
02507 using inject_t = v;
02508 };
02509
02510 // fraction field of a ring is the actual fraction field
02511 template<typename Ring>
02512 requires IsEuclideanDomain<Ring>
02513 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02514 using type = _FractionField<Ring>;
02515 };
02516 } // namespace internal
02517
02521 template<typename Ring>
02522 requires IsEuclideanDomain<Ring>
02523 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02524
02527 template<typename Ring>
02528 struct Embed<Ring, FractionField<Ring> {
02529 template<typename v>
02530 using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02531 };
02532 } // namespace aerobus
02533
02534 // short names for common types
02535 namespace aerobus {
02536 template<typename X, typename Y>
02537 requires IsRing<typename X::enclosing_type> &&
02538 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02539 using add_t = typename X::enclosing_type::template add_t<X, Y>;
02540
02541 template<typename X, typename Y>
02542 requires IsRing<typename X::enclosing_type> &&
02543 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02544 using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02545
02546 template<typename X, typename Y>
02547 requires IsRing<typename X::enclosing_type> &&
02548 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02549 using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02550
02551 template<typename X, typename Y>
02552 requires IsEuclideanDomain<typename X::enclosing_type> &&
02553 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02554 using div_t = typename X::enclosing_type::template div_t<X, Y>;
02555
02556 using q32 = FractionField<i32>;
02557
02558 using fpq32 = FractionField<polynomial<q32>>;
02559
02560 using q64 = FractionField<i64>;
02561
02562 using pi64 = polynomial<i64>;
02563
02564 using pq64 = polynomial<q64>;
02565
02566 using fpq64 = FractionField<polynomial<q64>>;
02567
02568
02569
02570
02571
02572
02573
02574
02575
02576
02577
02578
02579
02580
02581
02582
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02590
02591

```

```

02596 template<typename Ring, typename v1, typename v2>
02597 using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02598
02605 template<typename v>
02606 using embed_int_poly_in_fractions_t =
02607 typename Embed<
02608 polynomial<typename v::ring_type>,
02609 polynomial<FractionField<typename v::ring_type>>::template type<v>;
02610
02614 template<int64_t p, int64_t q>
02615 using make_q64_t = typename q64::template simplify_t<
02616 typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02617
02621 template<int32_t p, int32_t q>
02622 using make_q32_t = typename q32::template simplify_t<
02623 typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02624
02629 template<typename Ring, typename v1, typename v2>
02630 using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02635 template<typename Ring, typename v1, typename v2>
02636 using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02637
02639 template<>
02640 struct Embed<q32, q64> {
02643 template<typename v>
02644 using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02645 };
02646
02650 template<typename Small, typename Large>
02651 struct Embed<polynomial<Small>, polynomial<Large>> {
02652 private:
02653 template<typename v, typename i>
02654 struct at_low;
02655
02656 template<typename v, size_t i>
02657 struct at_index {
02658 using type = typename Embed<Small, Large>::template
02659 type<typename v::template coeff_at_t<i>>;
02660 };
02661
02662 template<typename v, size_t... Is>
02663 struct at_low<v, std::index_sequence<Is...>> {
02664 using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02665 };
02666 public:
02667 template<typename v>
02668 using type = typename
02669 at_low<v, typename internal::make_index_sequence_reverse<v::degree + 1>::type>;
02671 };
02672
02676 template<typename Ring, auto... xs>
02677 using make_int_polynomial_t = typename polynomial<Ring>::template val<
02678 typename Ring::template inject_constant_t<xs>...>;
02679
02683 template<typename Ring, auto... xs>
02684 using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02685 typename FractionField<Ring>::template inject_constant_t<xs>...>;
02686 } // namespace aerobus
02687
02688 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02689 namespace aerobus {
02690 namespace internal {
02691 template<typename T, size_t x, typename E = void>
02692 struct factorial {};
02693
02694 template<typename T, size_t x>
02695 struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02696 private:
02697 template<typename, size_t, typename>
02698 friend struct factorial;
02699 public:
02700 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02701 x - 1>::type>;
02702 static constexpr typename T::inner_type value = type::template
02703 get<typename T::inner_type>();
02704 };
02705
02706 template<typename T>
02707 struct factorial<T, 0> {
02708 public:
02709 using type = typename T::one;
02710 static constexpr typename T::inner_type value = type::template
02711 get<typename T::inner_type>();
02712 };
02713 } // namespace internal
02714 }

```

```

02715 template<typename T, size_t i>
02716 using factorial_t = typename internal::factorial<T, i>::type;
02717
02721 template<typename T, size_t i>
02722 inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02723
02724 namespace internal {
02725 template<typename T, size_t k, size_t n, typename E = void>
02726 struct combination_helper {};
02727
02728 template<typename T, size_t k, size_t n>
02729 struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02730 using type = typename FractionField<T>::template mul_t<
02731 typename combination_helper<T, k - 1, n - 1>::type,
02732 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02733 };
02734
02735 template<typename T, size_t k, size_t n>
02736 struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02737 using type = typename combination_helper<T, n - k, n>::type;
02738 };
02739
02740 template<typename T, size_t n>
02741 struct combination_helper<T, 0, n> {
02742 using type = typename FractionField<T>::one;
02743 };
02744
02745 template<typename T, size_t k, size_t n>
02746 struct combination {
02747 using type = typename internal::combination_helper<T, k, n>::type::x;
02748 static constexpr typename T::inner_type value =
02749 internal::combination_helper<T, k, n>::type::template
02750 get<typename T::inner_type>();
02751 };
02752 } // namespace internal
02753
02755 template<typename T, size_t k, size_t n>
02756 using combination_t = typename internal::combination<T, k, n>::type;
02757
02762 template<typename T, size_t k, size_t n>
02763 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02764
02765 namespace internal {
02766 template<typename T, size_t m>
02767 struct bernoulli;
02768
02769 template<typename T, typename accum, size_t k, size_t m>
02770 struct bernoulli_helper {
02771 using type = typename bernoulli_helper<
02772 T,
02773 addfractions_t<T,
02774 accum,
02775 mulfractions_t<T,
02776 makefraction_t<T,
02777 combination_t<T, k, m + 1>,
02778 typename T::one>,
02779 typename bernoulli<T, k>::type
02780 >,
02781 >,
02782 k + 1,
02783 m>::type;
02784 };
02785
02786 template<typename T, typename accum, size_t m>
02787 struct bernoulli_helper<T, accum, m, m> {
02788 using type = accum;
02789 };
02790
02791
02792
02793 template<typename T, size_t m>
02794 struct bernoulli {
02795 using type = typename FractionField<T>::template mul_t<
02796 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02797 makefraction_t<T,
02798 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02799 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02800 >
02801 >;
02802
02803 template<typename floatType>
02804 static constexpr floatType value = type::template get<floatType>();
02805 };
02806
02807 template<typename T>
02808 struct bernoulli<T, 0> {
02809 using type = typename FractionField<T>::one;

```

```

02810
02811 template<typename floatType>
02812 static constexpr floatType value = type::template get<floatType>();
02813 };
02814 } // namespace internal
02815
02816 template<typename T, size_t n>
02817 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02818
02819 template<typename FloatType, typename T, size_t n>
02820 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02821
02822 // bell numbers
02823 namespace internal {
02824 template<typename T, size_t n, typename E = void>
02825 struct bell_helper;
02826
02827 template<typename T, size_t n>
02828 struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02829 template<typename accum, size_t i, size_t stop>
02830 struct sum_helper {
02831 private:
02832 using left = typename T::template mul_t<
02833 combination_t<T, i, n-1>,
02834 typename bell_helper<T, i>::type>;
02835 using new_accum = typename T::template add_t<accum, left>;
02836 public:
02837 using type = typename sum_helper<new_accum, i+1, stop>::type;
02838 };
02839
02840 template<typename accum, size_t stop>
02841 struct sum_helper<accum, stop, stop> {
02842 using type = accum;
02843 };
02844
02845 using type = typename sum_helper<typename T::zero, 0, n>::type;
02846 };
02847
02848 template<typename T>
02849 struct bell_helper<T, 0> {
02850 using type = typename T::one;
02851 };
02852
02853 template<typename T>
02854 struct bell_helper<T, 1> {
02855 using type = typename T::one;
02856 };
02857 } // namespace internal
02858
02859 template<typename T, size_t n>
02860 using bell_t = typename internal::bell_helper<T, n>::type;
02861
02862 template<typename T, size_t n>
02863 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02864
02865 namespace internal {
02866 template<typename T, int k, typename E = void>
02867 struct alternate {};
02868
02869 template<typename T, int k>
02870 struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
02871 using type = typename T::one;
02872 static constexpr typename T::inner_type value = type::template
02873 get<typename T::inner_type>();
02874 };
02875
02876 template<typename T, int k>
02877 struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
02878 using type = typename T::template sub_t<typename T::zero, typename T::one>;
02879 static constexpr typename T::inner_type value = type::template
02880 get<typename T::inner_type>();
02881 };
02882 } // namespace internal
02883
02884 template<typename T, int k>
02885 using alternate_t = typename internal::alternate<T, k>::type;
02886
02887 namespace internal {
02888 template<typename T, int n, int k, typename E = void>
02889 struct stirling_helper {};
02890
02891 template<typename T>
02892 struct stirling_helper<T, 0, 0> {
02893 using type = typename T::one;
02894 };
02895
02896 template<typename T, int n>

```

```

02910 struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02911 using type = typename T::zero;
02912 };
02913
02914 template<typename T, int n>
02915 struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02916 using type = typename T::zero;
02917 };
02918
02919 template<typename T, int n, int k>
02920 struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02921 using type = typename T::template sub_t<
02922 typename stirling_helper<T, n-1, k-1>::type,
02923 typename T::template mul_t<
02924 typename T::template inject_constant_t<n-1>,
02925 typename stirling_helper<T, n-1, k>::type
02926 >;
02927 };
02928 } // namespace internal
02929
02930 template<typename T, int n, int k>
02931 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02932
02933 template<typename T, int n, int k>
02934 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02935
02936 template<typename T, int n, int k>
02937 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02938
02939 template<typename T, int n, int k>
02940 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02941
02942 template<typename T, size_t k>
02943 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02944
02945 namespace internal {
02946 template<typename T>
02947 struct pow_scalar {
02948 template<size_t p>
02949 static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02950 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02951 x * func<p/2>(x) * func<p/2>(x);
02952 }
02953 };
02954
02955 template<typename T, typename p, size_t n, typename E = void>
02956 requires IsEuclideanDomain<T>
02957 struct pow;
02958
02959 template<typename T, typename p, size_t n>
02960 struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02961 using type = typename T::template mul_t<
02962 typename pow<T, p, n/2>::type,
02963 typename pow<T, p, n/2>::type
02964 >;
02965 };
02966
02967 template<typename T, typename p, size_t n>
02968 struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02969 using type = typename T::template mul_t<
02970 p,
02971 typename T::template mul_t<
02972 typename pow<T, p, n/2>::type,
02973 typename pow<T, p, n/2>::type
02974 >
02975 >;
02976 };
02977
02978 template<typename T, typename p, size_t n>
02979 struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
02980 } // namespace internal
02981
02982 template<typename T, typename p, size_t n>
02983 using pow_t = typename internal::pow<T, p, n>::type;
02984
02985 template<typename T, typename p, size_t n>
02986 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02987
02988 template<typename T, size_t p>
02989 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
02990 internal::pow_scalar<T>::template func<p>(x); }
02991
02992 namespace internal {
02993 template<typename, template<typename, size_t> typename, class>
02994 struct make_taylor_impl;
02995 }

```



```

03022 template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03023 struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03024 using type = typename polynomial<FractionField<T>>::template
val<typename coeff_at<T, Is>::type...>;
03025 };
03026 }
03027
03032 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03033 using taylor = typename internal::make_taylor_impl<
03034 T,
03035 coeff_at,
03036 internal::make_index_sequence_reverse<deg + 1>::type;
03037
03038 namespace internal {
03039 template<typename T, size_t i>
03040 struct exp_coeff {
03041 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03042 };
03043
03044 template<typename T, size_t i, typename E = void>
03045 struct sin_coeff_helper {};
03046
03047 template<typename T, size_t i>
03048 struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03049 using type = typename FractionField<T>::zero;
03050 };
03051
03052 template<typename T, size_t i>
03053 struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03054 using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03055 };
03056
03057 template<typename T, size_t i>
03058 struct sin_coeff {
03059 using type = typename sin_coeff_helper<T, i>::type;
03060 };
03061
03062 template<typename T, size_t i, typename E = void>
03063 struct sh_coeff_helper {};
03064
03065 template<typename T, size_t i>
03066 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03067 using type = typename FractionField<T>::zero;
03068 };
03069
03070 template<typename T, size_t i>
03071 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03072 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03073 };
03074
03075 template<typename T, size_t i>
03076 struct sh_coeff {
03077 using type = typename sh_coeff_helper<T, i>::type;
03078 };
03079
03080 template<typename T, size_t i, typename E = void>
03081 struct cos_coeff_helper {};
03082
03083 template<typename T, size_t i>
03084 struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03085 using type = typename FractionField<T>::zero;
03086 };
03087
03088 template<typename T, size_t i>
03089 struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03090 using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03091 };
03092
03093 template<typename T, size_t i>
03094 struct cos_coeff {
03095 using type = typename cos_coeff_helper<T, i>::type;
03096 };
03097
03098 template<typename T, size_t i, typename E = void>
03099 struct cosh_coeff_helper {};
03100
03101 template<typename T, size_t i>
03102 struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03103 using type = typename FractionField<T>::zero;
03104 };
03105
03106 template<typename T, size_t i>
03107 struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03108 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03109 };
03110
03111 template<typename T, size_t i>

```

```

03112 struct cosh_coeff {
03113 using type = typename cosh_coeff_helper<T, i>::type;
03114 };
03115
03116 template<typename T, size_t i>
03117 struct geom_coeff { using type = typename FractionField<T>::one; };
03118
03119
03120 template<typename T, size_t i, typename E = void>
03121 struct atan_coeff_helper;
03122
03123 template<typename T, size_t i>
03124 struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03125 using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>>;
03126 };
03127
03128 template<typename T, size_t i>
03129 struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03130 using type = typename FractionField<T>::zero;
03131 };
03132
03133 template<typename T, size_t i>
03134 struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03135
03136 template<typename T, size_t i, typename E = void>
03137 struct asin_coeff_helper;
03138
03139 template<typename T, size_t i>
03140 struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03141 using type = makefraction_t<T,
03142 factorial_t<T, i - 1>,
03143 typename T::template mul_t<
03144 typename T::template val<i>,
03145 T::template mul_t<
03146 pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03147 pow_t<T, factorial_t<T, i / 2>, 2
03148 >
03149 >
03150 >;
03151 };
03152
03153 template<typename T, size_t i>
03154 struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03155 using type = typename FractionField<T>::zero;
03156 };
03157
03158 template<typename T, size_t i>
03159 struct asin_coeff {
03160 using type = typename asin_coeff_helper<T, i>::type;
03161 };
03162
03163 template<typename T, size_t i>
03164 struct lnpl_coeff {
03165 using type = makefraction_t<T,
03166 alternate_t<T, i + 1>,
03167 typename T::template val<i>>;
03168 };
03169
03170 template<typename T>
03171 struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03172
03173 template<typename T, size_t i, typename E = void>
03174 struct asinh_coeff_helper;
03175
03176 template<typename T, size_t i>
03177 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03178 using type = makefraction_t<T,
03179 typename T::template mul_t<
03180 alternate_t<T, i / 2>,
03181 factorial_t<T, i - 1>
03182 >,
03183 typename T::template mul_t<
03184 typename T::template mul_t<
03185 typename T::template val<i>,
03186 pow_t<T, factorial_t<T, i / 2>, 2>
03187 >,
03188 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03189 >
03190 >;
03191 };
03192
03193 template<typename T, size_t i>
03194 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03195 using type = typename FractionField<T>::zero;
03196 };
03197
03198 template<typename T, size_t i>

```

```

03199 struct asinh_coeff {
03200 using type = typename asinh_coeff_helper<T, i>::type;
03201 };
03202
03203 template<typename T, size_t i, typename E = void>
03204 struct atanh_coeff_helper;
03205
03206 template<typename T, size_t i>
03207 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03208 // 1/i
03209 using type = typename FractionField<T>::template val<
03210 typename T::one,
03211 typename T::template inject_constant_t<i>>;
03212 };
03213
03214 template<typename T, size_t i>
03215 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03216 using type = typename FractionField<T>::zero;
03217 };
03218
03219 template<typename T, size_t i>
03220 struct atanh_coeff {
03221 using type = typename atanh_coeff_helper<T, i>::type;
03222 };
03223
03224 template<typename T, size_t i, typename E = void>
03225 struct tan_coeff_helper;
03226
03227 template<typename T, size_t i>
03228 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03229 using type = typename FractionField<T>::zero;
03230 };
03231
03232 template<typename T, size_t i>
03233 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03234 private:
03235 // 4^((i+1)/2)
03236 using _4p = typename FractionField<T>::template inject_t<
03237 pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03238 // 4^((i+1)/2) - 1
03239 using _4pml = typename FractionField<T>::template
03240 sub_t<_4p, typename FractionField<T>::one>;
03241 // (-1)^((i-1)/2)
03242 using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03243 using dividend = typename FractionField<T>::template mul_t<
03244 altp,
03245 _4p,
03246 FractionField<T>::template mul_t<
03247 _4pml,
03248 bernoulli_t<T, (i + 1)>
03249 >
03250 >;
03251 public:
03252 using type = typename FractionField<T>::template div_t<dividend,
03253 typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03254 };
03255
03256 template<typename T, size_t i>
03257 struct tan_coeff {
03258 using type = typename tan_coeff_helper<T, i>::type;
03259 };
03260
03261 template<typename T, size_t i, typename E = void>
03262 struct tanh_coeff_helper;
03263
03264 template<typename T, size_t i>
03265 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03266 using type = typename FractionField<T>::zero;
03267 };
03268
03269 template<typename T, size_t i>
03270 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03271 private:
03272 using _4p = typename FractionField<T>::template inject_t<
03273 pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03274 using _4pml = typename FractionField<T>::template
03275 sub_t<_4p, typename FractionField<T>::one>;
03276 using dividend =
03277 typename FractionField<T>::template mul_t<
03278 _4p,
03279 typename FractionField<T>::template mul_t<
03280 _4pml,
03281 bernoulli_t<T, (i + 1)>>::type;
03282 public:
03283 using type = typename FractionField<T>::template div_t<dividend,

```

```

03284 FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03285 };
03286
03287 template<typename T, size_t i>
03288 struct tanh_coeff {
03289 using type = typename tanh_coeff_helper<T, i>::type;
03290 };
03291 } // namespace internal
03292
03296 template<typename Integers, size_t deg>
03297 using exp = taylor<Integers, internal::exp_coeff, deg>;
03298
03302 template<typename Integers, size_t deg>
03303 using expm1 = typename polynomial<FractionField<Integers>>::template sub_t<
03304 exp<Integers, deg>,
03305 typename polynomial<FractionField<Integers>>::one>;
03306
03310 template<typename Integers, size_t deg>
03311 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03312
03316 template<typename Integers, size_t deg>
03317 using atan = taylor<Integers, internal::atan_coeff, deg>;
03318
03322 template<typename Integers, size_t deg>
03323 using sin = taylor<Integers, internal::sin_coeff, deg>;
03324
03328 template<typename Integers, size_t deg>
03329 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03330
03335 template<typename Integers, size_t deg>
03336 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03337
03342 template<typename Integers, size_t deg>
03343 using cos = taylor<Integers, internal::cos_coeff, deg>;
03344
03349 template<typename Integers, size_t deg>
03350 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03351
03356 template<typename Integers, size_t deg>
03357 using asin = taylor<Integers, internal::asin_coeff, deg>;
03358
03363 template<typename Integers, size_t deg>
03364 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03365
03370 template<typename Integers, size_t deg>
03371 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03372
03377 template<typename Integers, size_t deg>
03378 using tan = taylor<Integers, internal::tan_coeff, deg>;
03379
03384 template<typename Integers, size_t deg>
03385 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03386 } // namespace aerobus
03387
03388 // continued fractions
03389 namespace aerobus {
03392 template<int64_t... values>
03393 struct ContinuedFraction {};
03394
03397 template<int64_t a0>
03398 struct ContinuedFraction<a0> {
03400 using type = typename q64::template inject_constant_t<a0>;
03402 static constexpr double val = static_cast<double>(a0);
03403 };
03404
03408 template<int64_t a0, int64_t... rest>
03409 struct ContinuedFraction<a0, rest...> {
03411 using type = q64::template add_t<
03412 typename q64::template inject_constant_t<a0>,
03413 typename q64::template div_t<
03414 typename q64::one,
03415 typename ContinuedFraction<rest...>::type
03416 >;
03417
03419 static constexpr double val = type::template get<double>();
03420 };
03421
03426 using PI_fraction =
03427 ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03429 using E_fraction =
03430 ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03431 using SQRT2_fraction =
03432 ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03433 using SQRT3_fraction =
03434 ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03435 // NOLINT
03436 } // namespace aerobus

```

```

03435
03436 // known polynomials
03437 namespace aerobus {
03438 // CChebyshev
03439 namespace internal {
03440 template<int kind, size_t deg, typename I>
03441 struct chebyshev_helper {
03442 using type = typename polynomial<I>::template sub_t<
03443 typename polynomial<I>::template mul_t<
03444 typename polynomial<I>::template mul_t<
03445 typename polynomial<I>::template inject_constant_t<2>,
03446 typename polynomial<I>::X>,
03447 typename chebyshev_helper<kind, deg - 1, I>::type
03448 >,
03449 typename chebyshev_helper<kind, deg - 2, I>::type
03450 >;
03451 };
03452
03453 template<typename I>
03454 struct chebyshev_helper<1, 0, I> {
03455 using type = typename polynomial<I>::one;
03456 };
03457
03458 template<typename I>
03459 struct chebyshev_helper<1, 1, I> {
03460 using type = typename polynomial<I>::X;
03461 };
03462
03463 template<typename I>
03464 struct chebyshev_helper<2, 0, I> {
03465 using type = typename polynomial<I>::one;
03466 };
03467
03468 template<typename I>
03469 struct chebyshev_helper<2, 1, I> {
03470 using type = typename polynomial<I>::template mul_t<
03471 typename polynomial<I>::template inject_constant_t<2>,
03472 typename polynomial<I>::X>;
03473 };
03474 } // namespace internal
03475
03476 // Laguerre
03477 namespace internal {
03478 template<size_t deg, typename I>
03479 struct laguerre_helper {
03480 using Q = FractionField<I>;
03481 using PQ = polynomial<Q>;
03482
03483 private:
03484 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03485 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03486 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03487 // -x + 2k-1
03488 using p = typename PQ::template val<
03489 typename Q::template inject_constant_t<-1>,
03490 typename Q::template inject_constant_t<2 * deg - 1>;
03491 // 1/n
03492 using factor = typename PQ::template inject_ring_t<
03493 typename Q::template
03494 val<typename I::one, typename I::template inject_constant_t<deg>>>;
03495
03496 public:
03497 using type = typename PQ::template mul_t <
03498 factor,
03499 typename PQ::template sub_t<
03500 typename PQ::template mul_t<
03501 p,
03502 lnm1
03503 >,
03504 typename PQ::template mul_t<
03505 typename PQ::template inject_constant_t<deg-1>,
03506 lnm2
03507 >
03508 >;
03509 };
03510
03511 template<typename I>
03512 struct laguerre_helper<0, I> {
03513 using type = typename polynomial<FractionField<I>>::one;
03514 };
03515
03516 template<typename I>
03517 struct laguerre_helper<1, I> {
03518 private:
03519 using PQ = polynomial<FractionField<I>>;
03520 public:

```

```

03521 using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03522 };
03523 } // namespace internal
03524
03525 // Bernstein
03526 namespace internal {
03527 template<size_t i, size_t m, typename I, typename E = void>
03528 struct bernstein_helper {};
03529
03530 template<typename I>
03531 struct bernstein_helper<0, 0, I> {
03532 using type = typename polynomial<I>::one;
03533 };
03534
03535 template<size_t i, size_t m, typename I>
03536 struct bernstein_helper<i, m, I, std::enable_if_t<
03537 (m > 0) && (i == 0)>> {
03538 private:
03539 using P = polynomial<I>;
03540 public:
03541 using type = typename P::template mul_t<
03542 typename P::template sub_t<typename P::one, typename P::X>,
03543 typename bernstein_helper<i, m-1, I>::type>;
03544 };
03545
03546 template<size_t i, size_t m, typename I>
03547 struct bernstein_helper<i, m, I, std::enable_if_t<
03548 (m > 0) && (i == m)>> {
03549 private:
03550 using P = polynomial<I>;
03551 public:
03552 using type = typename P::template mul_t<
03553 typename P::X,
03554 typename bernstein_helper<i-1, m-1, I>::type>;
03555 };
03556
03557 template<size_t i, size_t m, typename I>
03558 struct bernstein_helper<i, m, I, std::enable_if_t<
03559 (m > 0) && (i > 0) && (i < m)>> {
03560 private:
03561 using P = polynomial<I>;
03562 public:
03563 using type = typename P::template add_t<
03564 typename P::template mul_t<
03565 typename P::template sub_t<typename P::one, typename P::X>,
03566 typename bernstein_helper<i, m-1, I>::type>,
03567 typename P::template mul_t<
03568 typename P::X,
03569 typename bernstein_helper<i-1, m-1, I>::type>>;
03570 };
03571 } // namespace internal
03572
03573 namespace known_polynomials {
03574 enum hermite_kind {
03575 probabilist,
03576 physicist
03577 };
03578 }
03579
03580 // hermite
03581 namespace internal {
03582 template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03583 struct hermite_helper {};
03584
03585 template<size_t deg, typename I>
03586 struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03587 private:
03588 using hnm1 = typename hermite_helper<deg - 1,
03589 known_polynomials::hermite_kind::probabilist, I>::type;
03590 using hnm2 = typename hermite_helper<deg - 2,
03591 known_polynomials::hermite_kind::probabilist, I>::type;
03592 public:
03593 using type = typename polynomial<I>::template sub_t<
03594 typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03595 typename polynomial<I>::template mul_t<
03596 typename polynomial<I>::template inject_constant_t<deg - 1>,
03597 hnm2
03598 >
03599 >;
03600 };
03601
03602 template<size_t deg, typename I>
03603 struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03604 private:
03605 using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03606 I>::type;

```

```

03608 using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
I>::type;
03609
03610 public:
03611 using type = typename polynomial<I>::template sub_t<
03612 // 2X Hn-1
03613 typename polynomial<I>::template mul_t<
03614 typename pi64::val<typename I::template inject_constant_t<2>,
03615 typename I::zero>, hnm1>,
03616
03617 typename polynomial<I>::template mul_t<
03618 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03619 hnm2
03620 >
03621 >;
03622 };
03623
03624 template<typename I>
03625 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03626 using type = typename polynomial<I>::one;
03627 };
03628
03629 template<typename I>
03630 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03631 using type = typename polynomial<I>::X;
03632 };
03633
03634 template<typename I>
03635 struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03636 using type = typename pi64::one;
03637 };
03638
03639 template<typename I>
03640 struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03641 // 2X
03642 using type = typename polynomial<I>::template val<
03643 typename I::template inject_constant_t<2>,
03644 typename I::zero>;
03645 };
03646 } // namespace internal
03647
03648 // legendre
03649 namespace internal {
03650 template<size_t n, typename I>
03651 struct legendre_helper {
03652 private:
03653 using Q = FractionField<I>;
03654 using PQ = polynomial<Q>;
03655 // 1/n constant
03656 // (2n-1)/n X
03657 using fact_left = typename PQ::template monomial_t<
03658 makefraction_t<I,
03659 typename I::template inject_constant_t<2*n-1>,
03660 typename I::template inject_constant_t<n>
03661 >,
03662 1>;
03663 // (n-1) / n
03664 using fact_right = typename PQ::template val<
03665 makefraction_t<I,
03666 typename I::template inject_constant_t<n-1>,
03667 typename I::template inject_constant_t<n>»>;
03668
03669 public:
03670 using type = PQ::template sub_t<
03671 typename PQ::template mul_t<
03672 fact_left,
03673 typename legendre_helper<n-1, I>::type
03674 >,
03675 typename PQ::template mul_t<
03676 fact_right,
03677 typename legendre_helper<n-2, I>::type
03678 >
03679 >;
03680 };
03681
03682 template<typename I>
03683 struct legendre_helper<0, I> {
03684 using type = typename polynomial<FractionField<I>>::one;
03685 };
03686
03687 template<typename I>
03688 struct legendre_helper<1, I> {
03689 using type = typename polynomial<FractionField<I>>::X;
03690 };
03691 } // namespace internal
03692
03693 // bernoulli polynomials

```

```

03694 namespace internal {
03695 template<size_t n>
03696 struct bernoulli_coeff {
03697 template<typename T, size_t i>
03698 struct inner {
03699 private:
03700 using F = FractionField<T>;
03701 public:
03702 using type = typename F::template mul_t<
03703 typename F::template inject_ring_t<combination_t<T, i, n>>,
03704 bernoulli_t<T, n-i>
03705 >;
03706 };
03707 };
03708 } // namespace internal
03709
03710 namespace known_polynomials {
03711 template <size_t deg, typename I = aerobus::i64>
03712 using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03713
03714 template <size_t deg, typename I = aerobus::i64>
03715 using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03716
03717 template <size_t deg, typename I = aerobus::i64>
03718 using laguerre = typename internal::laguerre_helper<deg, I>::type;
03719
03720 template <size_t deg, typename I = aerobus::i64>
03721 using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03722 I>::type;
03723
03724 template <size_t deg, typename I = aerobus::i64>
03725 using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03726
03727 template<size_t i, size_t m, typename I = aerobus::i64>
03728 using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03729
03730 template<size_t deg, typename I = aerobus::i64>
03731 using legendre = typename internal::legendre_helper<deg, I>::type;
03732
03733 template<size_t deg, typename I = aerobus::i64>
03734 using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03735 } // namespace known_polynomials
03736 } // namespace aerobus
03737
03738 #ifdef AEROBUS_CONWAY_IMPORTS
03739 // conway polynomials
03740 namespace aerobus {
03741 template<int p, int n>
03742 struct ConwayPolynomial {};
03743
03744 #ifndef DO_NOT_DOCUMENT
03745 #define ZPZV ZPZ::template val
03746 #define POLYV aerobus::polynomial<ZPZ>::template val
03747 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type =
03748 POLYV<ZPZV<1>, ZPZV<1>>; }; // NOLINT
03749 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type =
03750 POLYV<ZPZV<1>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03751 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type =
03752 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03753 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type =
03754 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>>; }; // NOLINT
03755 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type =
03756 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>>; }; // NOLINT
03757 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type =
03758 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>>; }; // NOLINT
03759 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type =
03760 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03761 template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type =
03762 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03763 template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type =
03764 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>>; };
03765 // NOLINT
03766 template<> struct ConwayPolynomial<2, 10> { using ZPZ = aerobus::zpz<2>; using type =
03767 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03768 template<> struct ConwayPolynomial<2, 11> { using ZPZ = aerobus::zpz<2>; using type =
03769 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03770 template<> struct ConwayPolynomial<2, 12> { using ZPZ = aerobus::zpz<2>; using type =
03771 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>>; }; // NOLINT
03772 template<> struct ConwayPolynomial<2, 13> { using ZPZ = aerobus::zpz<2>; using type =
03773 POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>>; }; // NOLINT
03774 template<> struct ConwayPolynomial<2, 14> { using ZPZ = aerobus::zpz<2>; using type =

```



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## Chapter 10

# Examples

### 10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject\_constant\_t<1>>

Template Parameters

|   |                                      |
|---|--------------------------------------|
| x | a 'constant' from Ring point of view |
|---|--------------------------------------|

### 10.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

|       |                                               |
|-------|-----------------------------------------------|
| ...Ts | types to store and manipulate at compile time |
|-------|-----------------------------------------------|

### 10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

|   |                                              |
|---|----------------------------------------------|
| x | inject_constant_2<2> -> i32::template val<2> |
|---|----------------------------------------------|

## 10.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |



## 10.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

|           |                            |
|-----------|----------------------------|
| <i>v1</i> |                            |
| <i>v2</i> | <i32::val<1>, i32::val<1>> |

## 10.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

|     |                |
|-----|----------------|
| $v$ | a value in i32 |
|-----|----------------|

## 10.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

|     |                                                       |
|-----|-------------------------------------------------------|
| $v$ | a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code> |
|-----|-------------------------------------------------------|

## 10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

|     |                                         |
|-----|-----------------------------------------|
| $x$ | <code>inject_constant_t&lt;2&gt;</code> |
|-----|-----------------------------------------|

## 10.15 i64::add\_t

addition operator

addition operator

Template Parameters

|      |                                                                                                          |
|------|----------------------------------------------------------------------------------------------------------|
| $v1$ | : an element of <code>aerobus::i64::val</code>                                                           |
| $v2$ | : an element of <code>aerobus::i64::val</code> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

|           |                                                                               |
|-----------|-------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 10.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code> |

## 10.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

|                 |                                                                              |
|-----------------|------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<2>, i64::val<2>> |

## 10.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)

### Template Parameters

|                 |                                                                               |
|-----------------|-------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 10.26 i64::pos\_t

is v positive yields  $v > 0$  as [std::true\\_type](#) or [std::false\\_type](#)

is v positive yields  $v > 0$  as [std::true\\_type](#) or [std::false\\_type](#)

### Template Parameters

|                |                                                                 |
|----------------|-----------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>> |
|----------------|-----------------------------------------------------------------|

## 10.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

|                |                                                                 |
|----------------|-----------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>> |
|----------------|-----------------------------------------------------------------|

## 10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

|                |                                                               |
|----------------|---------------------------------------------------------------|
| <code>x</code> | <code>&lt;i32&gt;::template inject_constant_t&lt;2&gt;</code> |
|----------------|---------------------------------------------------------------|

## 10.29 q32::add\_t

addition operator

addition operator

Template Parameters

|                 |                                                                                                                                         |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | a value                                                                                                                                 |
| <code>v2</code> | a value <code>&lt;q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;2&gt;&gt;, q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;3&gt;&gt;&gt;</code> |

## 10.30 FractionField

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$

Template Parameters

|             |                                                                                    |
|-------------|------------------------------------------------------------------------------------|
| <i>Ring</i> | <code>&lt;i64&gt;</code> is q64 (rationals with 64 bits numerator and denominator) |
|-------------|------------------------------------------------------------------------------------|

## 10.31 PI\_fraction::val

representation of  $\pi$  as a continued fraction -> 3.14...

## 10.32 E\_fraction::val

approximation of  $e$  -> 2.718...

approximation of  $e$  -> 2.718...

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