

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for **quotient ring** R/X where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	15
aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	41
aerobus::IsField	
Concept to express R is a field	41
aerobus::IsRing	
Concept to express R is a Ring	42

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> >	43
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	44
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	44
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	45
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	46
aerobus::ConwayPolynomial	47
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Embedding - struct forward declaration	47
aerobus::Embed< i32, i64 >	
Embeds i32 into i64	48
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
Embeds polynomial<Small> into polynomial<Large>	49
aerobus::Embed< q32, q64 >	
Embeds q32 into q64	50
aerobus::Embed< Quotient< Ring, X >, Ring >	
Embeds Quotient<Ring, X> into Ring	50
aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	51
aerobus::Embed< zpz< x >, i32 >	
Embeds zpz values into i32	52
aerobus::polynomial< Ring >::horner_reduction_t< P >	
Used to evaluate polynomials over a value in Ring	53
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	54
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	59
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >	65
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >	66
aerobus::is_prime< n >	
Checks if n is prime	67

aerobus::polynomial< Ring >	67
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	75
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	76
aerobus::type_list< Ts >::split< index >	
Splits list at index	81
aerobus::type_list< Ts >	
Empty pure template struct to handle type list	82
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aerobus::i32::val< x >	
Values in i32 , again represented as types	86
aerobus::i64::val< x >	
Values in i64	88
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	90
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	94
aerobus::zpz< p >::val< x >	
Values in zpz	94
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	97
aerobus::zpz< p >	
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Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

src/ aerobus.h	109
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Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - Quotient ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<typename T , typename A , typename B >
 using [gcd_t](#) = typename internal::gcd< T >::template type< A, B >
 - computes the greatest common divisor of A and B*
- template<typename... vals>
 using [vadd_t](#) = typename internal::vadd< vals... >::type
 - adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator*
- template<typename... vals>
 using [vmul_t](#) = typename internal::vmul< vals... >::type
 - multiplies multiple values (v1 * v2 + ... * vn) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator*
- template<typename val >
 using [abs_t](#) = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template [sub_t](#)< typename val::enclosing_type::zero, val > >
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
 - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add_t](#) = typename X::enclosing_type::template [add_t](#)< X, Y >
 - generic addition*
- template<typename X , typename Y >
 using [sub_t](#) = typename X::enclosing_type::template [sub_t](#)< X, Y >

- generic subtraction*
 - `template<typename X , typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
 - generic multiplication*
 - `template<typename X , typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
 - generic division*
 - `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
 - `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
 - `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
 - `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
 - `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
 - `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
 - `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
 - `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > ,`
`polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
 - `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > ,`
`i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
 - `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > ,`
`i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
 - `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
 - `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
 - `template<typename Ring , auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template`
`inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
 - `template<typename Ring , auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename`
`FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField< Ring>
 - `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type

- `template<typename T , size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T , size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T , size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T , int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T , int n, int k>`
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T , int n, int k>`
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T , int n, int k>`
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`
Stirling number of second kind – as types.
- `template<typename T , typename p , size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers , size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers , size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers , size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers , size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers , size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers , size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`
 $\cosh(x)$ *hyperbolic cosine*
- `template<typename Integers , size_t deg>`
`using cos = taylor< Integers, internal::cos_coeff, deg >`
 $\cos(x)$ *cosinus*
- `template<typename Integers , size_t deg>`
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	---

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh(x)` arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ `cosinus`

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of e

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

Template Parameters

<i>Ring</i>	
-------------	--

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of π as a continued fraction

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.42 Sqrt3_fraction

```
using aerobus::Sqrt3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

6.1.2.43 stirling_1_signed_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_signed_t = typedef typename internal::stirling_1_helper<T, n, k>::type
```

Stirling number of first king (signed) – as types.

Template Parameters

<i>T</i>	(ring type, such as aerobus::i64)
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.44 stirling_1_unsigned_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T, n, k>::type>
```

Stirling number of first king (unsigned) – as types.

Template Parameters

<i>T</i>	(ring type, such as aerobus::i64)
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.45 stirling_2_t

```
template<typename T , int n, int k>
using aerobus::stirling_2_t = typedef typename internal::stirling_2_helper<T, n, k>::type
```

Stirling number of second king – as types.

Template Parameters

<i>T</i>	(ring type, such as aerobus::i64)
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.46 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.47 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.49 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.50 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.51 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >

- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >
- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >

- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_1_helper**
- struct **stirling_1_helper**< T, 0, 0 >
- struct **stirling_1_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0)> >
- struct **stirling_2_helper**
- struct **stirling_2_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling_2_helper**< T, n, n, std::enable_if_t<(n >=0)> >

- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **touchard_coeff**
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Enumerations

- enum [hermite_kind](#) { [probabilist](#) , [physicist](#) }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Enumeration Type Documentation

6.3.2.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = typename Ring::zero

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = `aN`

8.3.1 Member Typedef Documentation

8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

Examples

[examples/continued_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename q64::template inject_constant_t< a0 >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = static_cast<double>(a0)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template [add_t](#)< typename q64::template inject_constant_t< a0 >, typename q64::template [div_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.8.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.9 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
using `type = i64::val< static_cast< int64_t >(val::v)>`
the i64 representation of val

8.9.1 Detailed Description

embeds [i32](#) into [i64](#)

8.9.2 Member Typedef Documentation

8.9.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.10 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

Public Types

- template<typename v >
using [type](#) = typename at_low< v, typename [internal::make_index_sequence_reverse](#)< v::degree+1 > >::type
the polynomial<Large> representation of v

8.10.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.10.2 Member Typedef Documentation

8.10.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- template<typename v >
using **type** = [make_q64_t](#)< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>
q64 representation of v

8.11.1 Detailed Description

embeds q32 into q64

8.11.2 Member Typedef Documentation

8.11.2.1 type

```
template<typename v >  
using aerobus::Embed< q32, q64 >::type = make\_q64\_t<static_cast<int64_t>(v::x::v), static_↵  
cast<int64_t>(v::y::v)>
```

q64 representation of v

Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.12 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

Public Types

- template<typename val >
using **type** = typename val::raw_t
Ring representation of val.

8.12.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

8.12.2 Member Typedef Documentation

8.12.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.13 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

Public Types

- template<typename v >
using [type](#) = typename [FractionField](#)< Ring >::template val< v, typename Ring::one >
FractionField<Ring> representation of v.

8.13.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

Template Parameters

<i>Ring</i>	an integers ring, such as i32
-------------	---

8.13.2 Member Typedef Documentation

8.13.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

[FractionField](#)<Ring> representation of v.

Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 [aerobus::Embed](#)< [zpz](#)< [x](#) >, [i32](#) > Struct Template Reference

embeds [zpz](#) values into [i32](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i32::val< val::v >`
the [i32](#) representation of val

8.14.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds [zpz](#) values into [i32](#)

Template Parameters

<i>x</i>	an integer
----------	------------

8.14.2 Member Typedef Documentation

8.14.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the `i32` representation of `val`

Template Parameters

<i>val</i>	a value in <code>zpz<x></code>
------------	--------------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.15 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference

Used to evaluate polynomials over a value in `Ring`.

```
#include <aerobus.h>
```

Classes

- struct `inner`
- struct `inner< stop, stop >`

8.15.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in `Ring`.

Template Parameters

<i>P</i>	a value in <code>polynomial<Ring></code>
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.16 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using [inner_type](#) = int32_t
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<auto x>
using [inject_constant_t](#) = [val](#)< static_cast< int32_t >(x)>
inject a native constant
- template<typename v >
using [inject_ring_t](#) = v
- template<typename v1 , typename v2 >
using [add_t](#) = typename add< v1, v2 >::type
addition operator yields v1 + v2
- template<typename v1 , typename v2 >
using [sub_t](#) = typename sub< v1, v2 >::type
subtraction operator yields v1 - v2
- template<typename v1 , typename v2 >
using [mul_t](#) = typename mul< v1, v2 >::type
*multiplication operator yields v1 * v2*
- template<typename v1 , typename v2 >
using [div_t](#) = typename div< v1, v2 >::type
division operator yields v1 / v2
- template<typename v1 , typename v2 >
using [mod_t](#) = typename remainder< v1, v2 >::type
modulus operator yields v1 % v2
- template<typename v1 , typename v2 >
using [gt_t](#) = typename gt< v1, v2 >::type
strictly greater operator (v1 > v2) yields v1 > v2
- template<typename v1 , typename v2 >
using [lt_t](#) = typename lt< v1, v2 >::type
strict less operator (v1 < v2) yields v1 < v2
- template<typename v1 , typename v2 >
using [eq_t](#) = typename eq< v1, v2 >::type
equality operator (type) yields v1 == v2 as std::integral_constant<bool>
- template<typename v1 , typename v2 >
using [gcd_t](#) = [gcd_t](#)< [i32](#), v1, v2 >
greatest common divisor yields GCD(v1, v2)
- template<typename v >
using [pos_t](#) = typename pos< v >::type
positivity operator yields v > 0 as std::true_type or std::false_type

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- template<typename v1 , typename v2 >
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator (boolean value)
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity (boolean value) yields $v > 0$ as boolean value

8.16.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

8.16.2 Member Typedef Documentation

8.16.2.1 [add_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::add\_t = typename add<v1, v2>::type
```

addition operator yields $v1 + v2$

Template Parameters

v1	a value in i32
v2	a value in i32

8.16.2.2 [div_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::div\_t = typename div<v1, v2>::type
```

division operator yields $v1 / v2$

Template Parameters

v1	a value in i32
v2	a value in i32

8.16.2.3 [eq_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::eq\_t = typename eq<v1, v2>::type
```

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields $GCD(v1, v2)$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) yields $v1 > v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

inject a native constant

Template Parameters

<i>x</i>	
----------	--

8.16.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

8.16.2.8 inner_type

```
using aerobus::i32::inner_type = int32_t
```

8.16.2.9 lt_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$) yields $v1 < v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.10 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.11 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields $v1 * v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.16.2.13 pos_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v</i>	a value in i32
----------	--------------------------------

8.16.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields $v1 - v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.16.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.16.3 Member Data Documentation

8.16.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.16.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.16.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.16.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	a value in i32
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.17 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- using [inner_type](#) = `int64_t`
type of represented values
- template<auto x>
using [inject_constant_t](#) = `val< static_cast< int64_t >(x)>`
injects constant as an [i64](#) value
- template<typename v >
using [inject_ring_t](#) = v
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`

- constant zero*
 - using `one` = `val`< 1 >
- constant one*
 - template<typename v1 , typename v2 >
using `add_t` = typename add< v1, v2 >::type
- addition operator*
 - template<typename v1 , typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
- subtraction operator*
 - template<typename v1 , typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
- multiplication operator*
 - template<typename v1 , typename v2 >
using `div_t` = typename div< v1, v2 >::type
- division operator integer division*
 - template<typename v1 , typename v2 >
using `mod_t` = typename remainder< v1, v2 >::type
- modulus operator*
 - template<typename v1 , typename v2 >
using `gt_t` = typename gt< v1, v2 >::type
- strictly greater operator yields v1 > v2 as std::true_type or std::false_type*
 - template<typename v1 , typename v2 >
using `lt_t` = typename lt< v1, v2 >::type
- strict less operator yields v1 < v2 as std::true_type or std::false_type*
 - template<typename v1 , typename v2 >
using `eq_t` = typename eq< v1, v2 >::type
- equality operator yields v1 == v2 as std::true_type or std::false_type*
 - template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t`< i64, v1, v2 >
- greatest common divisor yields GCD(v1, v2) as instantiation of i64::val*
 - template<typename v >
using `pos_t` = typename pos< v >::type
- is v positive yields v > 0 as std::true_type or std::false_type*

Static Public Attributes

- static constexpr bool `is_field` = false
- integers are not a field*
- static constexpr bool `is_euclidean_domain` = true
- integers are an euclidean domain*
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
- strictly greater operator yields v1 > v2 as boolean value*
- template<typename v1 , typename v2 >
static constexpr bool `lt_v` = `lt_t`<v1, v2>::value
- strictly smaller operator yields v1 < v2 as boolean value*
- template<typename v1 , typename v2 >
static constexpr bool `eq_v` = `eq_t`<v1, v2>::value
- equality operator yields v1 == v2 as boolean value*
- template<typename v >
static constexpr bool `pos_v` = `pos_t`<v>::value
- positivity yields v > 0 as boolean value*

8.17.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.17.2 Member Typedef Documentation

8.17.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.5 `gt_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields `v1 > v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.6 `inject_constant_t`

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

Template Parameters

<code>x</code>	
----------------	--

8.17.2.7 `inject_ring_t`

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
`-> i64::val<1>`

Template Parameters

<code>v</code>	a value in i64
----------------	--------------------------------

8.17.2.8 `inner_type`

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.17.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.17.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
-----------------	---

8.17.2.14 `sub_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::sub\_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.2.15 `zero`

```
using aerobus::i64::zero = val<0>
```

constant zero

8.17.3 Member Data Documentation

8.17.3.1 `eq_v`

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq\_v = eq\_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.3.2 `gt_v`

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt\_v = gt\_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.17.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.17.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.17.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields $v_1 < v_2$ as boolean value

Template Parameters

v_1	: an element of aerobus::i64::val
v_2	: an element of aerobus::i64::val

8.17.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.18 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = typename horner_reduction_t< P >::template inner< index+1, stop > ::template type< type-`
`name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↵`
`at_t< P::degree - index > >, x >`

8.18.1 Member Typedef Documentation

8.18.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_↵
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.19 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = accum`

8.19.1 Member Typedef Documentation

8.19.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.20 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.20.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.20.2 Member Data Documentation

8.20.2.1 value

```
template<size_t n>
constexpr bool aerobus::is\_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.21 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [horner_reduction_t](#)
Used to evaluate polynomials over a value in Ring.
- struct [val](#)
values (seen as types) in polynomial ring
- struct [val](#)< [coeffN](#) >
specialization for constants

Public Types

- using `zero` = `val`< typename Ring::zero >
constant zero
- using `one` = `val`< typename Ring::one >
constant one
- using `X` = `val`< typename Ring::one, typename Ring::zero >
generator
- template<typename P >
using `simplify_t` = typename simplify< P >::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- template<typename v1, typename v2 >
using `add_t` = typename add< v1, v2 >::type
adds two polynomials
- template<typename v1, typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
subtraction of two polynomials
- template<typename v1, typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
multiplication of two polynomials
- template<typename v1, typename v2 >
using `eq_t` = typename eq_helper< v1, v2 >::type
equality operator
- template<typename v1, typename v2 >
using `lt_t` = typename lt_helper< v1, v2 >::type
strict less operator
- template<typename v1, typename v2 >
using `gt_t` = typename gt_helper< v1, v2 >::type
strict greater operator
- template<typename v1, typename v2 >
using `div_t` = typename div< v1, v2 >::q_type
division operator
- template<typename v1, typename v2 >
using `mod_t` = typename div_helper< v1, v2, `zero`, v1 >::mod_type
modulo operator
- template<typename coeff, size_t deg>
using `monomial_t` = typename monomial< coeff, deg >::type
monomial : coeff X^{deg}
- template<typename v >
using `derive_t` = typename derive_helper< v >::type
derivation operator
- template<typename v >
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
checks for positivity (an > 0)
- template<typename v1, typename v2 >
using `gcd_t` = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >
greatest common divisor of two polynomials
- template<auto x>
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >
makes the constant (native type) polynomial a_0
- template<typename v >
using `inject_ring_t` = `val`< v >
makes the constant (ring type) polynomial a_0

Static Public Attributes

- static constexpr bool [is_field](#) = false
- static constexpr bool [is_euclidean_domain](#) = Ring::is_euclidean_domain
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity operator

8.21.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

Examples

[examples/make_polynomial.cpp](#), and [examples/modular_arithmetic.cpp](#).

8.21.2 Member Typedef Documentation

8.21.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

8.21.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial a_0

Template Parameters

<i>x</i>	
----------	--

8.21.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial a_0

Template Parameters

<i>v</i>	
----------	--

8.21.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.10 mod_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.21.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.21.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($a_n > 0$)

Template Parameters

<i>v</i>	
----------	--

8.21.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.21.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.21.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.21.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.21.3 Member Data Documentation

8.21.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.21.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.21.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in polynomial::val
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.22 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using [tail](#) = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.22.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.22.2 Member Typedef Documentation

8.22.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.22.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.23 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< typename Ring::zero >
zero value
- using [one](#) = [val](#)< typename Ring::one >
one
- template<typename v1 , typename v2 >
using [add_t](#) = [val](#)< typename Ring::template [add_t](#)< typename v1::type, typename v2::type > >
addition operator
- template<typename v1 , typename v2 >
using [mul_t](#) = [val](#)< typename Ring::template [mul_t](#)< typename v1::type, typename v2::type > >
subtraction operator
- template<typename v1 , typename v2 >
using [div_t](#) = [val](#)< typename Ring::template [div_t](#)< typename v1::type, typename v2::type > >
division operator
- template<typename v1 , typename v2 >
using [mod_t](#) = [val](#)< typename Ring::template [mod_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
- template<typename v1 , typename v2 >
using `eq_t` = typename Ring::template `eq_t`< typename v1::type, typename v2::type >
equality operator (as type)
- template<typename v1 >
using `pos_t` = std::true_type
positivity operator always true
- template<auto x>
using `inject_constant_t` = val< typename Ring::template `inject_constant_t`< x > >
*inject a 'constant' in quotient ring**
- template<typename v >
using `inject_ring_t` = val< v >
projects a value of Ring onto the quotient

Static Public Attributes

- template<typename v1 , typename v2 >
static constexpr bool `eq_v` = Ring::template `eq_t`<typename v1::type, typename v2::type>::value
addition operator (as boolean value)
- template<typename v >
static constexpr bool `pos_v` = `pos_t`<v>::value
positivity operator always true
- static constexpr bool `is_euclidean_domain` = true
quotien rings are euclidean domain

8.23.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' <code>i32</code> ', must satisfy the <code>IsRing</code> concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1↔
::type, typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring*

Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

8.23.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

8.23.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.23.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.23.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.23.3 Member Data Documentation

8.23.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.23.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.23.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.24 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.24.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.24.2 Member Typedef Documentation

8.24.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.24.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.25 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.25.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

8.25.2 Member Typedef Documentation

8.25.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.25.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.25.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.25.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.25.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.25.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.25.3 Member Data Documentation

8.25.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.26 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T >
- template<typename T >
using [push_back](#) = [type_list](#)< T >
- template<typename U >
using [concat](#) = U
- template<typename T , size_t index>
using [insert](#) = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t [length](#) = 0

8.26.1 Detailed Description

specialization for empty type list

8.26.2 Member Typedef Documentation

8.26.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

8.26.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

8.26.2.3 push_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

8.26.2.4 push_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

8.26.3 Member Data Documentation

8.26.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.27 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [i32](#)
Enclosing ring type.
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [DEVICE](#) valueType [get](#) ()
cast x into valueType
- static std::string [to_string](#) ()
string representation of value

Static Public Attributes

- static constexpr int32_t [v](#) = x
actual value stored in val type

8.27.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

x	an actual integer
---	-------------------

8.27.2 Member Typedef Documentation

8.27.2.1 enclosing_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.27.2.2 is_zero_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.27.3 Member Function Documentation

8.27.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.27.3.2 to_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.27.4 Member Data Documentation

8.27.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.28 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using [inner_type](#) = int32_t
type of represented values
- using [enclosing_type](#) = [i64](#)
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED_DEVICE](#) valueType [get](#) ()
cast value in valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int64_t [v](#) = x
actual value

8.28.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

x	an actual integer
-------------------	-------------------

8.28.2 Member Typedef Documentation

8.28.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.28.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.28.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.28.3 Member Function Documentation

8.28.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.28.3.2 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.28.4 Member Data Documentation

8.28.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.29 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- using `ring_type` = `Ring`
ring coefficients live in
- using `enclosing_type` = `polynomial< Ring >`
enclosing ring type
- using `aN` = `coeffN`
heavy weight coefficient (non zero)
- using `strip` = `val< coeffs... >`
remove largest coefficient
- using `is_zero_t` = `std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- template<size_t index>
using `coeff_at_t` = `typename coeff_at< index >::type`
type of coefficient at index
- template<typename x >
using `value_at_t` = `horner_reduction_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >`

Static Public Member Functions

- static `std::string to_string ()`
get a string representation of polynomial
- template<typename arithmeticType >
static constexpr `DEVICE INLINED arithmeticType eval (const arithmeticType &x)`
evaluates polynomial seen as a function operating on arithmeticType

Static Public Attributes

- static constexpr size_t [degree](#) = sizeof...(coeffs)
degree of the polynomial
- static constexpr bool [is_zero_v](#) = is_zero_t::value
true if polynomial is constant zero

8.29.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

8.29.2 Member Typedef Documentation

8.29.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.29.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.29.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.29.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true_type if polynomial is constant zero

8.29.2.5 ring_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

8.29.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

8.29.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

8.29.3 Member Function Documentation

8.29.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

8.29.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.29.4 Member Data Documentation

8.29.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.29.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t←
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.30 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- using [raw_t](#) = V
- using [type](#) = [abs_t](#)< typename Ring::template [mod_t](#)< V, X > >

8.30.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

8.30.2 Member Typedef Documentation

8.30.2.1 raw_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

8.30.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs\_t<typename Ring::template mod\_t<V, X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.31 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

Public Types

- using `enclosing_type` = `zpz< p >`
enclosing ring type
- using `is_zero_t` = `std::bool_constant< v==0 >`
true_type if zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr INLINED_DEVICE valueType get ()`
get value as valueType
- `static std::string to_string ()`
string representation

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value
- `static constexpr bool is_zero_v = v == 0`
true if zero

8.31.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

<code>x</code>	an integer
----------------	------------

8.31.2 Member Typedef Documentation

8.31.2.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.31.2.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true_type if zero

8.31.3 Member Function Documentation

8.31.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

8.31.3.2 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

Returns

a string representation

8.31.4 Member Data Documentation

8.31.4.1 is_zero_v

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

8.31.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.32 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial< Ring >](#)
enclosing ring type
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = [std::bool_constant< aN::is_zero_t::value >](#)
- template<size_t index>
using [coeff_at_t](#) = [typename coeff_at< index >::type](#)
- template<typename x >
using [value_at_t](#) = [coeffN](#)

Static Public Member Functions

- static [std::string to_string \(\)](#)
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)

Static Public Attributes

- static constexpr size_t [degree](#) = 0
degree
- static constexpr bool [is_zero_v](#) = [is_zero_t::value](#)

8.32.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.32.2 Member Typedef Documentation

8.32.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.32.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

8.32.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.32.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

8.32.2.5 ring_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

8.32.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.32.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

8.32.3 Member Function Documentation

8.32.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >↵
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

8.32.3.2 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.32.4 Member Data Documentation

8.32.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.32.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.33 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

Classes

- struct `val`
values in zpz

Public Types

- using `inner_type` = `int32_t`
underlying type for values
- template<auto x>
using `inject_constant_t` = `val< static_cast< int32_t >(x)>`
injects a constant integer into zpz
- using `zero` = `val< 0 >`
zero value
- using `one` = `val< 1 >`
one value
- template<typename v1 , typename v2 >
using `add_t` = `typename add< v1, v2 >::type`
addition operator
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub< v1, v2 >::type`
subtraction operator
- template<typename v1 , typename v2 >
using `mul_t` = `typename mul< v1, v2 >::type`
multiplication operator
- template<typename v1 , typename v2 >
using `div_t` = `typename div< v1, v2 >::type`
division operator
- template<typename v1 , typename v2 >
using `mod_t` = `typename remainder< v1, v2 >::type`
modulo operator
- template<typename v1 , typename v2 >
using `gt_t` = `typename gt< v1, v2 >::type`
strictly greater operator (type)
- template<typename v1 , typename v2 >
using `lt_t` = `typename lt< v1, v2 >::type`
strictly smaller operator (type)
- template<typename v1 , typename v2 >
using `eq_t` = `typename eq< v1, v2 >::type`
equality operator (type)
- template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t< i32, v1, v2 >`
greatest common divisor
- template<typename v1 >
using `pos_t` = `typename pos< v1 >::type`
positivity operator (type)

Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value
true iff p is prime
- static constexpr bool `is_euclidean_domain` = true
always true
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
strictly greater operator (booleanvalue)
- template<typename v1 , typename v2 >
static constexpr bool `lt_v` = `lt_t`<v1, v2>::value
strictly smaller operator (booleanvalue)
- template<typename v1 , typename v2 >
static constexpr bool `eq_v` = `eq_t`<v1, v2>::value
equality operator (booleanvalue)
- template<typename v >
static constexpr bool `pos_v` = `pos_t`<v>::value
positivity operator (boolean value)

8.33.1 Detailed Description

template<int32_t p>
struct aerobus::zpz< p >

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular_arithmetic.cpp](#), and [examples/polynomials_over_finite_field.cpp](#).

8.33.2 Member Typedef Documentation

8.33.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.2 div_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

Template Parameters

<i>x</i>	an integer
----------	------------

8.33.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

8.33.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.33.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.33.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

8.33.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

v1	a value in zpz::val
----	-------------------------------------

8.33.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

8.33.3 Member Data Documentation

8.33.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.33.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

8.33.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

8.33.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.33.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

v1	a value in zpz::val
----	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 0) & 0x800000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<float> {
00224         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00225             return x * y + z;
00226         }
00227     };
00228
00229     template<>
00230     struct fma_helper<int32_t> {
00231         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00232             return x * y + z;
00233         }
00234     };
00235
00236     template<>
00237     struct fma_helper<int16_t> {
00238         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00239             return x * y + z;
00240         }
00241     };
00242
00243     template<>
00244     struct fma_helper<int64_t> {
00245         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00246             return x * y + z;
00247         }
00248     };
00249
00250     #ifdef WITH_CUDA_FP16
00251     template<>
00252     struct fma_helper<__half> {
00253         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00254             #ifdef __CUDA_ARCH__
00255                 return __hfma(x, y, z);
00256             #else
00257                 return x * y + z;
00258             #endif
00259         }
00260     };
00261     template<>
00262     struct fma_helper<__half2> {
00263         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00264             #ifdef __CUDA_ARCH__
00265                 return __hfma2(x, y, z);
00266             #else
00267                 return x * y + z;
00268             #endif
00269         }
00270     };
00271     #endif
00272 } // namespace internal
00273 } // namespace aerobus
00274
00275 // utilities
00276 namespace aerobus {
00277     namespace internal {
00278         template<template<typename...> typename TT, typename T>
00279         struct is_instantiation_of : std::false_type { };
00280
00281         template<template<typename...> typename TT, typename... Ts>
00282         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00283
00284         template<template<typename...> typename TT, typename T>
00285         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00286
00287         template<int64_t i, typename T, typename... Ts>
00288         struct type_at {
00289             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00290             using type = typename type_at<i - 1, Ts...>::type;
00291         };
00292
00293         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00294             using type = T;
00295         };

```

```

00296
00297     template <size_t i, typename... Ts>
00298     using type_at_t = typename type_at<i, Ts...>::type;
00299
00300
00301     template<size_t n, size_t i, typename E = void>
00302     struct _is_prime {};
00303
00304     template<size_t i>
00305     struct _is_prime<0, i> {
00306         static constexpr bool value = false;
00307     };
00308
00309     template<size_t i>
00310     struct _is_prime<1, i> {
00311         static constexpr bool value = false;
00312     };
00313
00314     template<size_t i>
00315     struct _is_prime<2, i> {
00316         static constexpr bool value = true;
00317     };
00318
00319     template<size_t i>
00320     struct _is_prime<3, i> {
00321         static constexpr bool value = true;
00322     };
00323
00324     template<size_t i>
00325     struct _is_prime<5, i> {
00326         static constexpr bool value = true;
00327     };
00328
00329     template<size_t i>
00330     struct _is_prime<7, i> {
00331         static constexpr bool value = true;
00332     };
00333
00334     template<size_t n, size_t i>
00335     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00336         static constexpr bool value = false;
00337     };
00338
00339     template<size_t n, size_t i>
00340     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00341         static constexpr bool value = false;
00342     };
00343
00344     template<size_t n, size_t i>
00345     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00346         static constexpr bool value = true;
00347     };
00348
00349     template<size_t n, size_t i>
00350     struct _is_prime<n, i, std::enable_if_t<(
00351         n % i == 0 &&
00352         n >= 9 &&
00353         n % 3 != 0 &&
00354         n % 2 != 0 &&
00355         i * i > n)>> {
00356         static constexpr bool value = true;
00357     };
00358
00359     template<size_t n, size_t i>
00360     struct _is_prime<n, i, std::enable_if_t<(
00361         n % (i+2) == 0 &&
00362         n >= 9 &&
00363         n % 3 != 0 &&
00364         n % 2 != 0 &&
00365         i * i <= n)>> {
00366         static constexpr bool value = true;
00367     };
00368
00369     template<size_t n, size_t i>
00370     struct _is_prime<n, i, std::enable_if_t<(
00371         n % (i+2) != 0 &&
00372         n % i != 0 &&
00373         n >= 9 &&
00374         n % 3 != 0 &&
00375         n % 2 != 0 &&
00376         (i * i <= n))>> {
00377         static constexpr bool value = _is_prime<n, i+6>::value;
00378     };
00379
00380 } // namespace internal
00381
00382 template<size_t n>

```

```

00385     struct is_prime {
00387         static constexpr bool value = internal::_is_prime<n, 5>::value;
00388     };
00389
00393     template<size_t n>
00394     static constexpr bool is_prime_v = is_prime<n>::value;
00395
00396     // gcd
00397     namespace internal {
00398         template <std::size_t... Is>
00399         constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00400             -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00401
00402         template <std::size_t N>
00403         using make_index_sequence_reverse
00404             = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00405
00411         template<typename Ring, typename E = void>
00412         struct gcd;
00413
00414         template<typename Ring>
00415         struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00416             template<typename A, typename B, typename E = void>
00417             struct gcd_helper {};
00418
00419             // B = 0, A > 0
00420             template<typename A, typename B>
00421             struct gcd_helper<A, B, std::enable_if_t<
00422                 (B::is_zero_t::value) &&
00423                 (Ring::template gt_t<A, typename Ring::zero>::value)> {
00424                 using type = A;
00425             };
00426
00427             // B = 0, A < 0
00428             template<typename A, typename B>
00429             struct gcd_helper<A, B, std::enable_if_t<
00430                 (B::is_zero_t::value) &&
00431                 !(Ring::template gt_t<A, typename Ring::zero>::value)> {
00432                 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00433             };
00434
00435             // B != 0
00436             template<typename A, typename B>
00437             struct gcd_helper<A, B, std::enable_if_t<
00438                 (!B::is_zero_t::value)
00439                 > {
00440             private: // NOLINT
00441                 // A / B
00442                 using k = typename Ring::template div_t<A, B>;
00443                 // A - (A/B)*B = A % B
00444                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00445
00446             public:
00447                 using type = typename gcd_helper<B, m>::type;
00448             };
00449
00450             template<typename A, typename B>
00451             using type = typename gcd_helper<A, B>::type;
00452         };
00453     } // namespace internal
00454
00455     // vadd and vmul
00456     namespace internal {
00457         template<typename... vals>
00458         struct vmul {};
00459
00460         template<typename v1, typename... vals>
00461         struct vmul<v1, vals...> {
00462             using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00463         };
00464
00465         template<typename v1>
00466         struct vmul<v1> {
00467             using type = v1;
00468         };
00469
00470         template<typename... vals>
00471         struct vadd {};
00472
00473         template<typename v1, typename... vals>
00474         struct vadd<v1, vals...> {
00475             using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00476         };
00477
00478         template<typename v1>

```

```

00479     struct vadd<v1> {
00480         using type = v1;
00481     };
00482 } // namespace internal
00483
00486 template<typename T, typename A, typename B>
00487 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00488
00492 template<typename... vals>
00493 using vadd_t = typename internal::vadd<vals...>::type;
00494
00498 template<typename... vals>
00499 using vmul_t = typename internal::vmul<vals...>::type;
00500
00504 template<typename val>
00505 requires IsEuclideanDomain<typename val::enclosing_type>
00506 using abs_t = std::conditional_t<
00507     val::enclosing_type::template pos_v<val>,
00508     val, typename val::enclosing_type::template
00509     sub_t<typename val::enclosing_type::zero, val>>;
00510 } // namespace aerobus
00511
00511 // embedding
00512 namespace aerobus {
00513     template<typename Small, typename Large, typename E = void>
00514     struct Embed;
00515 } // namespace aerobus
00520
00521 namespace aerobus {
00522     template<typename Ring, typename X>
00523     requires IsRing<Ring>
00524     struct Quotient {
00525         template <typename V>
00526         struct val {
00527             public:
00528                 using raw_t = V;
00529                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00530         };
00531
00532         using zero = val<typename Ring::zero>;
00533
00534         using one = val<typename Ring::one>;
00535
00536         template<typename v1, typename v2>
00537         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00538
00539         template<typename v1, typename v2>
00540         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00541
00542         template<typename v1, typename v2>
00543         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00544
00545         template<typename v1, typename v2>
00546         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00547
00548         template<typename v1, typename v2>
00549         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00550
00551         template<typename v1, typename v2>
00552         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00553
00554         template<typename v1>
00555         using pos_t = std::true_type;
00556
00557         template<typename v>
00558         static constexpr bool pos_v = pos_t<v>::value;
00559
00560         static constexpr bool is_euclidean_domain = true;
00561
00562         template<auto x>
00563         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00564
00565         template<typename v>
00566         using inject_ring_t = val<v>;
00567     };
00568
00569     template<typename Ring, typename X>
00570     struct Embed<Quotient<Ring, X>, Ring> {
00571         template<typename val>
00572         using type = typename val::raw_t;
00573     };
00574 } // namespace aerobus
00575
00576 // type_list
00577 namespace aerobus {
00578     template <typename... Ts>
00579     struct type_list;
00580 }

```

```

00625
00626 namespace internal {
00627     template <typename T, typename... Us>
00628     struct pop_front_h {
00629         using tail = type_list<Us...>;
00630         using head = T;
00631     };
00632
00633     template <size_t index, typename L1, typename L2>
00634     struct split_h {
00635     private:
00636         static_assert(index <= L2::length, "index ouf of bounds");
00637         using a = typename L2::pop_front::type;
00638         using b = typename L2::pop_front::tail;
00639         using c = typename L1::template push_back<a>;
00640
00641     public:
00642         using head = typename split_h<index - 1, c, b>::head;
00643         using tail = typename split_h<index - 1, c, b>::tail;
00644     };
00645
00646     template <typename L1, typename L2>
00647     struct split_h<0, L1, L2> {
00648         using head = L1;
00649         using tail = L2;
00650     };
00651
00652     template <size_t index, typename L, typename T>
00653     struct insert_h {
00654         static_assert(index <= L::length, "index ouf of bounds");
00655         using s = typename L::template split<index>;
00656         using left = typename s::head;
00657         using right = typename s::tail;
00658         using ll = typename left::template push_back<T>;
00659         using type = typename ll::template concat<right>;
00660     };
00661
00662     template <size_t index, typename L>
00663     struct remove_h {
00664         using s = typename L::template split<index>;
00665         using left = typename s::head;
00666         using right = typename s::tail;
00667         using rr = typename right::pop_front::tail;
00668         using type = typename left::template concat<rr>;
00669     };
00670 } // namespace internal
00671
00672 template <typename... Ts>
00673 struct type_list {
00674 private:
00675     template <typename T>
00676     struct concat_h;
00677
00678     template <typename... Us>
00679     struct concat_h<type_list<Us...> {
00680         using type = type_list<Ts..., Us...>;
00681     };
00682
00683 public:
00684     static constexpr size_t length = sizeof...(Ts);
00685
00686     template <typename T>
00687     using push_front = type_list<T, Ts...>;
00688
00689     template <size_t index>
00690     using at = internal::type_at_t<index, Ts...>;
00691
00692     struct pop_front {
00693         using type = typename internal::pop_front_h<Ts...>::head;
00694         using tail = typename internal::pop_front_h<Ts...>::tail;
00695     };
00696
00697     template <typename T>
00698     using push_back = type_list<Ts..., T>;
00699
00700     template <typename U>
00701     using concat = typename concat_h<U>::type;
00702
00703     template <size_t index>
00704     struct split {
00705     private:
00706         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
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00708     public:
00709         using head = typename inner::head;
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01419
01420     public:
01421         using type = typename inner::type;
01422     };
01423
01424     template <size_t index>
01425     struct concat {
01426     private:
01427         using inner = internal::concat_h<index, type_list<>>;
01428
01429     public:
01430         using type = typename inner::type;
01431     };
01432
01433     template <size_t index>
01434     struct split {
01435     private:
01436         using inner = internal::split_h<index, type_list<>>;
01437
01438     public:
01439         using head = typename inner::head;
01440         using tail = typename inner::tail;
01441     };
01442
01443     template <size_t index>
01444     struct insert {
01445     private:
01446         using inner = internal::insert_h<index, type_list<>>;
01447
01448     public:
01449         using type = typename inner::type;
01450     };
01451
01452     template <size_t index>
01453     struct remove {
01454     private:
01455         using inner = internal::remove_h<index, type_list<>>;
01456
01457     public:
01458         using type = typename inner::type;
01459     };
01460
01461     template <size_t index>
01462     struct pop_back {
01463     private:
01464         using inner = internal::pop_back_h<index, type_list<>>;
01465
01466     public:
01467         using type = typename inner::type;
01468     };
01469
01470     template <size_t index>
01471     struct push_back {
01472     private:
01473         using inner = internal::push_back_h<index, type_list<>>;
01474
01475     public:
01476         using type = typename inner::type;
01477     };
01478
01479     template <size_t index>
01480     struct pop_front {
01481     private:
01482         using inner = internal::pop_front_h<index, type_list<>>;
01483
01484     public:
01485         using type = typename inner::type;
01486     };
01487
01488     template <size_t index>
01489     struct push_front {
01490     private:
01491         using inner = internal::push_front_h<index, type_list<>>;
01492
01493     public:
01494         using type = typename inner::type;
01495     };
01496
01497     template <size_t index>
01498     struct concat {
01499     private:
01500         using inner = internal::concat_h<index, type_list<>>;
01501
01502     public:
01503         using type = typename inner::type;
01504     };
01505
01506     template <size_t index>
01507     struct split {
01508     private:
01509         using inner = internal::split_h<index, type_list<>>;
01510
01511     public:
01512         using head = typename inner::head;
01513         using tail = typename inner::tail;
01514     };
01515
01516     template <size_t index>
01517     struct insert {
01518     private:
01519         using inner = internal::insert_h<index, type_list<>>;
01520
01521     public:
01522         using type = typename inner::type;
01523     };
01524
01525     template <size_t index>
01526     struct remove {
01527     private:
01528         using inner = internal::remove_h<index, type_list<>>;
01529
01530     public:
01531         using type = typename inner::type;
01532     };
01533
01534     template <size_t index>
01535     struct pop_back {
01536     private:
01537         using inner = internal::pop_back_h<index, type_list<>>;
01538
01539     public:
01540         using type = typename inner::type;
01541     };
01542
01543     template <size_t index>
01544     struct push_back {
01545     private:
01546         using inner = internal::push_back_h<index, type_list<>>;
01547
01548     public:
01549         using type = typename inner::type;
01550     };
01551
01552     template <size_t index>
01553     struct pop_front {
01554     private:
01555         using inner = internal::pop_front_h<index, type_list<>>;
01556
01557     public:
01558         using type = typename inner::type;
01559     };
01560
01561     template <size_t index>
01562     struct push_front {
01563     private:
01564         using inner = internal::push_front_h<index, type_list<>>;
01565
01566     public:
01567         using type = typename inner::type;
01568     };
01569
01570     template <size_t index>
01571     struct concat {
01572     private:
01573         using inner = internal::concat_h<index, type_list<>>;
01574
01575     public:
01576         using type = typename inner::type;
01577     };
01578
01579     template <size_t index>
01580     struct split {
01581     private:
01582         using inner = internal::split_h<index, type_list<>>;
01583
01584     public:
01585         using head = typename inner::head;
01586         using tail = typename inner::tail;
01587     };
01588
01589     template <size_t index>
01590     struct insert {
01591     private:
01592         using inner = internal::insert_h<index, type_list<>>;
01593
01594     public:
01595         using type = typename inner::type;
01596     };
01597
01598     template <size_t index>
01599     struct remove {
01600     private:
01601         using inner = internal::remove_h<index, type_list<>>;
01602
01603     public:
01604         using type = typename inner::type;
01605     };
01606
01607     template <size_t index>
01608     struct pop_back {
01609     private:
01610         using inner = internal::pop_back_h<index, type_list<>>;
01611
01612     public:
01613         using type = typename inner::type;
01614     };
01615
01616     template <size_t index>
01617     struct push_back {
01618     private:
01619         using inner = internal::push_back_h<index, type_list<>>;
01620
01621     public:
01622         using type = typename inner::type;
01623     };
01624
01625     template <size_t index>
01626     struct pop_front {
01627     private:
01628         using inner = internal::pop_front_h<index, type_list<>>;
01629
01630     public:
01631         using type = typename inner::type;
01632     };
01633
01634     template <size_t index>
01635     struct push_front {
01636     private:
01637         using inner = internal::push_front_h<index, type_list<>>;
01638
01639     public:
01640         using type = typename inner::type;
01641     };
01642
01643     template <size_t index>
01644     struct concat {
01645     private:
01646         using inner = internal::concat_h
```



```

00728
00732     template <typename T, size_t index>
00733     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00734
00737     template <size_t index>
00738     using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00739 };
00740
00742 template <>
00743 struct type_list<> {
00744     static constexpr size_t length = 0;
00745
00746     template <typename T>
00747     using push_front = type_list<T>;
00748
00749     template <typename T>
00750     using push_back = type_list<T>;
00751
00752     template <typename U>
00753     using concat = U;
00754
00755     // TODO(jewave): assert index == 0
00756     template <typename T, size_t index>
00757     using insert = type_list<T>;
00758 };
00759 } // namespace aerobus
00760
00761 // i16
00762 #ifdef WITH_CUDA_FP16
00763 // i16
00764 namespace aerobus {
00765     struct i16 {
00766         using inner_type = int16_t;
00767         template<int16_t x>
00770         struct val {
00771             using enclosing_type = i16;
00772             static constexpr int16_t v = x;
00773
00774             template<typename valueType>
00775             static constexpr INLINED_DEVICE valueType get() {
00776                 return internal::template int16_convert_helper<valueType, x>::value();
00777             }
00778
00779             using is_zero_t = std::bool_constant<x == 0>;
00780
00781             static std::string to_string() {
00782                 return std::to_string(x);
00783             }
00784         };
00785
00786         using zero = val<0>;
00787         using one = val<1>;
00788         static constexpr bool is_field = false;
00789         static constexpr bool is_euclidean_domain = true;
00790         template<auto x>
00791         using inject_constant_t = val<static_cast<int16_t>(x)>;
00792
00793         template<typename v>
00794         using inject_ring_t = v;
00795
00796     private:
00797         template<typename v1, typename v2>
00798         struct add {
00799             using type = val<v1::v + v2::v>;
00800         };
00801
00802         template<typename v1, typename v2>
00803         struct sub {
00804             using type = val<v1::v - v2::v>;
00805         };
00806
00807         template<typename v1, typename v2>
00808         struct mul {
00809             using type = val<v1::v * v2::v>;
00810         };
00811
00812         template<typename v1, typename v2>
00813         struct div {
00814             using type = val<v1::v / v2::v>;
00815         };
00816
00817         template<typename v1, typename v2>
00818         struct remainder {
00819             using type = val<v1::v % v2::v>;
00820         };
00821
00822         template<typename v1, typename v2>
00823

```

```

00836     struct gt {
00837         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00838     };
00839
00840     template<typename v1, typename v2>
00841     struct lt {
00842         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00843     };
00844
00845     template<typename v1, typename v2>
00846     struct eq {
00847         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00848     };
00849
00850     template<typename v1>
00851     struct pos {
00852         using type = std::bool_constant<(v1::v > 0)>;
00853     };
00854
00855     public:
00856     template<typename v1, typename v2>
00857     using add_t = typename add<v1, v2>::type;
00858
00859     template<typename v1, typename v2>
00860     using sub_t = typename sub<v1, v2>::type;
00861
00862     template<typename v1, typename v2>
00863     using mul_t = typename mul<v1, v2>::type;
00864
00865     template<typename v1, typename v2>
00866     using div_t = typename div<v1, v2>::type;
00867
00868     template<typename v1, typename v2>
00869     using mod_t = typename remainder<v1, v2>::type;
00870
00871     template<typename v1, typename v2>
00872     using gt_t = typename gt<v1, v2>::type;
00873
00874     template<typename v1, typename v2>
00875     using lt_t = typename lt<v1, v2>::type;
00876
00877     template<typename v1, typename v2>
00878     using eq_t = typename eq<v1, v2>::type;
00879
00880     template<typename v1, typename v2>
00881     static constexpr bool eq_v = eq_t<v1, v2>::value;
00882
00883     template<typename v1, typename v2>
00884     using gcd_t = gcd_t<i16, v1, v2>;
00885
00886     template<typename v>
00887     using pos_t = typename pos<v>::type;
00888
00889     template<typename v>
00890     static constexpr bool pos_v = pos_t<v>::value;
00891 };
00892 } // namespace aerobus
00893 #endif
00894
00895 // i32
00896 namespace aerobus {
00897     struct i32 {
00898         using inner_type = int32_t;
00899         template<int32_t x>
00900         struct val {
00901             using enclosing_type = i32;
00902             static constexpr int32_t v = x;
00903
00904             template<typename valueType>
00905             static constexpr DEVICE valueType get() {
00906                 return static_cast<valueType>(x);
00907             }
00908
00909             using is_zero_t = std::bool_constant<x == 0>;
00910
00911             static std::string to_string() {
00912                 return std::to_string(x);
00913             }
00914         };
00915     };
00916
00917     using zero = val<0>;
00918     using one = val<1>;
00919     static constexpr bool is_field = false;
00920     static constexpr bool is_euclidean_domain = true;
00921     template<auto x>
00922     using inject_constant_t = val<static_cast<int32_t>(x)>;
00923

```

```

00983     template<typename v>
00984     using inject_ring_t = v;
00985
00986 private:
00987     template<typename v1, typename v2>
00988     struct add {
00989         using type = val<v1::v + v2::v>;
00990     };
00991
00992     template<typename v1, typename v2>
00993     struct sub {
00994         using type = val<v1::v - v2::v>;
00995     };
00996
00997     template<typename v1, typename v2>
00998     struct mul {
00999         using type = val<v1::v * v2::v>;
01000     };
01001
01002     template<typename v1, typename v2>
01003     struct div {
01004         using type = val<v1::v / v2::v>;
01005     };
01006
01007     template<typename v1, typename v2>
01008     struct remainder {
01009         using type = val<v1::v % v2::v>;
01010     };
01011
01012     template<typename v1, typename v2>
01013     struct gt {
01014         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01015     };
01016
01017     template<typename v1, typename v2>
01018     struct lt {
01019         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01020     };
01021
01022     template<typename v1, typename v2>
01023     struct eq {
01024         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01025     };
01026
01027     template<typename v1>
01028     struct pos {
01029         using type = std::bool_constant<(v1::v > 0)>;
01030     };
01031
01032 public:
01033     template<typename v1, typename v2>
01034     using add_t = typename add<v1, v2>::type;
01035
01036     template<typename v1, typename v2>
01037     using sub_t = typename sub<v1, v2>::type;
01038
01039     template<typename v1, typename v2>
01040     using mul_t = typename mul<v1, v2>::type;
01041
01042     template<typename v1, typename v2>
01043     using div_t = typename div<v1, v2>::type;
01044
01045     template<typename v1, typename v2>
01046     using mod_t = typename remainder<v1, v2>::type;
01047
01048     template<typename v1, typename v2>
01049     using gt_t = typename gt<v1, v2>::type;
01050
01051     template<typename v1, typename v2>
01052     using lt_t = typename lt<v1, v2>::type;
01053
01054     template<typename v1, typename v2>
01055     using eq_t = typename eq<v1, v2>::type;
01056
01057     template<typename v1, typename v2>
01058     static constexpr bool eq_v = eq_t<v1, v2>::value;
01059
01060     template<typename v1, typename v2>
01061     using gcd_t = gcd_t<i32, v1, v2>;
01062
01063     template<typename v>
01064     using pos_t = typename pos<v>::type;
01065
01066     template<typename v>
01067     static constexpr bool pos_v = pos_t<v>::value;
01068 };
01069 } // namespace aerobus

```

```

01115
01116 // i64
01117 namespace aerobus {
01118     struct i64 {
01119         using inner_type = int64_t;
01120         template<int64_t x>
01121         struct val {
01122             using inner_type = int32_t;
01123             using enclosing_type = i64;
01124             static constexpr int64_t v = x;
01125
01126             template<typename valueType>
01127             static constexpr INLINED_DEVICE valueType get() {
01128                 return static_cast<valueType>(x);
01129             }
01130
01131             using is_zero_t = std::bool_constant<x == 0>;
01132
01133             static std::string to_string() {
01134                 return std::to_string(x);
01135             }
01136         };
01137
01138         template<auto x>
01139         using inject_constant_t = val<static_cast<int64_t>(x)>;
01140
01141         template<typename v>
01142         using inject_ring_t = v;
01143
01144         using zero = val<0>;
01145         using one = val<1>;
01146         static constexpr bool is_field = false;
01147         static constexpr bool is_euclidean_domain = true;
01148
01149     private:
01150         template<typename v1, typename v2>
01151         struct add {
01152             using type = val<v1::v + v2::v>;
01153         };
01154
01155         template<typename v1, typename v2>
01156         struct sub {
01157             using type = val<v1::v - v2::v>;
01158         };
01159
01160         template<typename v1, typename v2>
01161         struct mul {
01162             using type = val<v1::v * v2::v>;
01163         };
01164
01165         template<typename v1, typename v2>
01166         struct div {
01167             using type = val<v1::v / v2::v>;
01168         };
01169
01170         template<typename v1, typename v2>
01171         struct remainder {
01172             using type = val<v1::v % v2::v>;
01173         };
01174
01175         template<typename v1, typename v2>
01176         struct gt {
01177             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01178         };
01179
01180         template<typename v1, typename v2>
01181         struct lt {
01182             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01183         };
01184
01185         template<typename v1, typename v2>
01186         struct eq {
01187             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01188         };
01189
01190         template<typename v>
01191         struct pos {
01192             using type = std::bool_constant<(v::v > 0)>;
01193         };
01194
01195     public:
01196         template<typename v1, typename v2>
01197         using add_t = typename add<v1, v2>::type;
01198
01199         template<typename v1, typename v2>
01200         using sub_t = typename sub<v1, v2>::type;
01201
01202         template<typename v1, typename v2>
01203         using mul_t = typename mul<v1, v2>::type;
01204
01205         template<typename v1, typename v2>
01206         using div_t = typename div<v1, v2>::type;
01207
01208         template<typename v1, typename v2>
01209         using remainder_t = typename remainder<v1, v2>::type;
01210
01211         template<typename v1, typename v2>
01212         using gt_t = typename gt<v1, v2>::type;
01213
01214         template<typename v1, typename v2>
01215         using lt_t = typename lt<v1, v2>::type;
01216
01217         template<typename v1, typename v2>
01218         using eq_t = typename eq<v1, v2>::type;
01219
01220         template<typename v>
01221         using pos_t = typename pos<v>::type;
01222
01223         template<typename v1, typename v2>
01224         using is_zero_t = is_zero_t;
01225
01226         template<typename v1, typename v2>
01227         using is_euclidean_domain_t = is_euclidean_domain;
01228
01229     };
01230
01231 }
01232
01233 
```

```

01232     template<typename v1, typename v2>
01233     using mul_t = typename mul<v1, v2>::type;
01234
01239     template<typename v1, typename v2>
01240     using div_t = typename div<v1, v2>::type;
01241
01245     template<typename v1, typename v2>
01246     using mod_t = typename remainder<v1, v2>::type;
01247
01252     template<typename v1, typename v2>
01253     using gt_t = typename gt<v1, v2>::type;
01254
01259     template<typename v1, typename v2>
01260     static constexpr bool gt_v = gt_t<v1, v2>::value;
01261
01266     template<typename v1, typename v2>
01267     using lt_t = typename lt<v1, v2>::type;
01268
01273     template<typename v1, typename v2>
01274     static constexpr bool lt_v = lt_t<v1, v2>::value;
01275
01280     template<typename v1, typename v2>
01281     using eq_t = typename eq<v1, v2>::type;
01282
01287     template<typename v1, typename v2>
01288     static constexpr bool eq_v = eq_t<v1, v2>::value;
01289
01294     template<typename v1, typename v2>
01295     using gcd_t = gcd_t<i64, v1, v2>;
01296
01300     template<typename v>
01301     using pos_t = typename pos<v>::type;
01302
01306     template<typename v>
01307     static constexpr bool pos_v = pos_t<v>::value;
01308 };
01309
01311 template<>
01312 struct Embed<i32, i64> {
01313     template<typename val>
01314     using type = i64::val<static_cast<int64_t>(val::v)>;
01315 };
01316 } // namespace aerobus
01317
01319 // z/pz
01320 namespace aerobus {
01321     template<int32_t p>
01322     struct zpz {
01323         using inner_type = int32_t;
01324
01325         template<int32_t x>
01326         struct val {
01327             using enclosing_type = zpz<p>;
01328             static constexpr int32_t v = x % p;
01329
01330             template<typename valueType>
01331             static constexpr INLINED_DEVICE valueType get() {
01332                 return static_cast<valueType>(x % p);
01333             }
01334
01335             using is_zero_t = std::bool_constant<v == 0>;
01336
01337             static constexpr bool is_zero_v = v == 0;
01338
01339             static std::string to_string() {
01340                 return std::to_string(x % p);
01341             }
01342         };
01343
01344         template<auto x>
01345         using inject_constant_t = val<static_cast<int32_t>(x)>;
01346
01347         using zero = val<0>;
01348
01349         using one = val<1>;
01350
01351         static constexpr bool is_field = is_prime<p>::value;
01352
01353         static constexpr bool is_euclidean_domain = true;
01354
01355     private:
01356         template<typename v1, typename v2>
01357         struct add {
01358             using type = val<(v1::v + v2::v) % p>;
01359         };
01360
01361     template<typename v1, typename v2>

```

```

01385     struct sub {
01386         using type = val<(v1::v - v2::v) % p>;
01387     };
01388
01389     template<typename v1, typename v2>
01390     struct mul {
01391         using type = val<(v1::v* v2::v) % p>;
01392     };
01393
01394     template<typename v1, typename v2>
01395     struct div {
01396         using type = val<(v1::v% p) / (v2::v % p)>;
01397     };
01398
01399     template<typename v1, typename v2>
01400     struct remainder {
01401         using type = val<(v1::v% v2::v) % p>;
01402     };
01403
01404     template<typename v1, typename v2>
01405     struct gt {
01406         using type = std::conditional_t<(v1::v% p > v2::v% p), std::true_type, std::false_type>;
01407     };
01408
01409     template<typename v1, typename v2>
01410     struct lt {
01411         using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01412     };
01413
01414     template<typename v1, typename v2>
01415     struct eq {
01416         using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01417     };
01418
01419     template<typename v1>
01420     struct pos {
01421         using type = std::bool_constant<(v1::v > 0)>;
01422     };
01423
01424 public:
01425     template<typename v1, typename v2>
01426     using add_t = typename add<v1, v2>::type;
01427
01428     template<typename v1, typename v2>
01429     using sub_t = typename sub<v1, v2>::type;
01430
01431     template<typename v1, typename v2>
01432     using mul_t = typename mul<v1, v2>::type;
01433
01434     template<typename v1, typename v2>
01435     using div_t = typename div<v1, v2>::type;
01436
01437     template<typename v1, typename v2>
01438     using mod_t = typename remainder<v1, v2>::type;
01439
01440     template<typename v1, typename v2>
01441     using gt_t = typename gt<v1, v2>::type;
01442
01443     template<typename v1, typename v2>
01444     static constexpr bool gt_v = gt_t<v1, v2>::value;
01445
01446     template<typename v1, typename v2>
01447     using lt_t = typename lt<v1, v2>::type;
01448
01449     template<typename v1, typename v2>
01450     static constexpr bool lt_v = lt_t<v1, v2>::value;
01451
01452     template<typename v1, typename v2>
01453     using eq_t = typename eq<v1, v2>::type;
01454
01455     template<typename v1, typename v2>
01456     static constexpr bool eq_v = eq_t<v1, v2>::value;
01457
01458     template<typename v1, typename v2>
01459     using gcd_t = gcd_t<i32, v1, v2>;
01460
01461     template<typename v1>
01462     using pos_t = typename pos<v1>::type;
01463
01464     template<typename v>
01465     static constexpr bool pos_v = pos_t<v>::value;
01466 };
01467
01468 template<int32_t x>
01469 struct Embed<zp<x>, i32> {
01470     template<typename val>
01471     using type = i32::val<val::v>;
01472 };

```

```

01516     };
01517 } // namespace aerobus
01518
01519 // polynomial
01520 namespace aerobus {
01521     // coeffN x^N + ...
01522     template<typename Ring>
01523     requires IsEuclideanDomain<Ring>
01524     struct polynomial {
01525         static constexpr bool is_field = false;
01526         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01527
01528         template<typename P>
01529         struct horner_reduction_t {
01530             template<size_t index, size_t stop>
01531             struct inner {
01532                 template<typename accum, typename x>
01533                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01534                     ::template type<
01535                         typename Ring::template add_t<
01536                             typename Ring::template mul_t<x, accum>,
01537                             typename P::template coeff_at_t<P::degree - index>
01538                             >, x>;
01539             };
01540
01541             template<size_t stop>
01542             struct inner<stop, stop> {
01543                 template<typename accum, typename x>
01544                 using type = accum;
01545             };
01546
01547             template<typename coeffN, typename... coeffs>
01548             struct val {
01549                 using ring_type = Ring;
01550                 using enclosing_type = polynomial<Ring>;
01551                 static constexpr size_t degree = sizeof...(coeffs);
01552                 using aN = coeffN;
01553                 using strip = val<coeffs...>;
01554                 using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01555                 static constexpr bool is_zero_v = is_zero_t::value;
01556
01557             private:
01558                 template<size_t index, typename E = void>
01559                 struct coeff_at {};
01560
01561                 template<size_t index>
01562                 struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01563                     using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01564                 };
01565
01566                 template<size_t index>
01567                 struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01568                     using type = typename Ring::zero;
01569                 };
01570
01571             public:
01572                 template<size_t index>
01573                 using coeff_at_t = typename coeff_at<index>::type;
01574
01575                 static std::string to_string() {
01576                     return string_helper<coeffN, coeffs...>::func();
01577                 }
01578
01579                 template<typename arithmeticType>
01580                 static constexpr DEVICE INLINEED arithmeticType eval(const arithmeticType& x) {
01581                     #ifdef WITH_CUDA_FP16
01582                     arithmeticType start;
01583                     if constexpr (std::is_same_v<arithmeticType, __half2>) {
01584                         start = __half2(0, 0);
01585                     } else {
01586                         start = static_cast<arithmeticType>(0);
01587                     }
01588                     #else
01589                     arithmeticType start = static_cast<arithmeticType>(0);
01590                     #endif
01591                     return horner_evaluation<arithmeticType, val>
01592                         ::template inner<0, degree + 1>
01593                         ::func(start, x);
01594                 }
01595
01596                 template<typename x>
01597                 using value_at_t = horner_reduction_t<val>
01598                     ::template inner<0, degree + 1>
01599                     ::template type<typename Ring::zero, x>;
01600             };
01601         };
01602     };
01603 }

```

```

01629     template<typename coeffN>
01630     struct val<coeffN> {
01631         using ring_type = Ring;
01632         using enclosing_type = polynomial<Ring>;
01633         static constexpr size_t degree = 0;
01634         using aN = coeffN;
01635         using strip = val<coeffN>;
01636         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01637
01638         static constexpr bool is_zero_v = is_zero_t::value;
01639
01640         template<size_t index, typename E = void>
01641         struct coeff_at {};
01642
01643         template<size_t index>
01644         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01645             using type = aN;
01646         };
01647
01648         template<size_t index>
01649         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01650             using type = typename Ring::zero;
01651         };
01652
01653         template<size_t index>
01654         using coeff_at_t = typename coeff_at<index>::type;
01655
01656         static std::string to_string() {
01657             return string_helper<coeffN>::func();
01658         }
01659
01660         template<typename arithmeticType>
01661         static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01662             return coeffN::template get<arithmeticType>();
01663         }
01664
01665         template<typename x>
01666         using value_at_t = coeffN;
01667     };
01668
01669     using zero = val<typename Ring::zero>;
01670     using one = val<typename Ring::one>;
01671     using X = val<typename Ring::one, typename Ring::zero>;
01672
01673 private:
01674     template<typename P, typename E = void>
01675     struct simplify;
01676
01677     template <typename P1, typename P2, typename I>
01678     struct add_low;
01679
01680     template<typename P1, typename P2>
01681     struct add {
01682         using type = typename simplify<typename add_low<
01683             P1,
01684             P2,
01685             internal::make_index_sequence_reverse<
01686                 std::max(P1::degree, P2::degree) + 1
01687             >::type>::type;
01688     };
01689
01690     template <typename P1, typename P2, typename I>
01691     struct sub_low;
01692
01693     template <typename P1, typename P2, typename I>
01694     struct mul_low;
01695
01696     template<typename v1, typename v2>
01697     struct mul {
01698         using type = typename mul_low<
01699             v1,
01700             v2,
01701             internal::make_index_sequence_reverse<
01702                 v1::degree + v2::degree + 1
01703             >::type;
01704     };
01705
01706     template<typename coeff, size_t deg>
01707     struct monomial;
01708
01709     template<typename v, typename E = void>
01710     struct derive_helper {};
01711
01712     template<typename v>
01713     struct derive_helper<v, std::enable_if_t<v::degree == 0>> {
01714         using type = zero;
01715     };
01716

```



```

01722
01723     template<typename v>
01724     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01725         using type = typename add<
01726             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01727             typename monomial<
01728                 typename Ring::template mul_t<
01729                     typename v::aN,
01730                     typename Ring::template inject_constant_t<(v::degree)>
01731                 >,
01732                 v::degree - 1
01733             >::type
01734         >::type;
01735     };
01736
01737     template<typename v1, typename v2, typename E = void>
01738     struct eq_helper {};
01739
01740     template<typename v1, typename v2>
01741     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01742         using type = std::false_type;
01743     };
01744
01745     template<typename v1, typename v2>
01746     struct eq_helper<v1, v2, std::enable_if_t<
01747         v1::degree == v2::degree &&
01748         (v1::degree != 0 || v2::degree != 0) &&
01749         std::is_same<
01750             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01751             std::false_type
01752         >::value
01753     >
01754     > {
01755     > {
01756         using type = std::false_type;
01757     };
01758
01759     template<typename v1, typename v2>
01760     struct eq_helper<v1, v2, std::enable_if_t<
01761         v1::degree == v2::degree &&
01762         (v1::degree != 0 || v2::degree != 0) &&
01763         std::is_same<
01764             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01765             std::true_type
01766         >::value
01767     > {
01768         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01769     };
01770
01771     template<typename v1, typename v2>
01772     struct eq_helper<v1, v2, std::enable_if_t<
01773         v1::degree == v2::degree &&
01774         (v1::degree == 0)
01775     > {
01776         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01777     };
01778
01779     template<typename v1, typename v2, typename E = void>
01780     struct lt_helper {};
01781
01782     template<typename v1, typename v2>
01783     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01784         using type = std::true_type;
01785     };
01786
01787     template<typename v1, typename v2>
01788     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01789         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01790     };
01791
01792     template<typename v1, typename v2>
01793     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01794         using type = std::false_type;
01795     };
01796
01797     template<typename v1, typename v2, typename E = void>
01798     struct gt_helper {};
01799
01800     template<typename v1, typename v2>
01801     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01802         using type = std::true_type;
01803     };
01804
01805     template<typename v1, typename v2>
01806     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01807         using type = std::false_type;
01808     };

```

```

01809
01810     template<typename v1, typename v2>
01811     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01812         using type = std::false_type;
01813     };
01814
01815     // when high power is zero : strip
01816     template<typename P>
01817     struct simplify<P, std::enable_if_t<
01818         std::is_same<
01819             typename Ring::zero,
01820             typename P::aN
01821         >::value && (P::degree > 0)
01822     > {
01823         using type = typename simplify<typename P::strip>::type;
01824     };
01825
01826     // otherwise : do nothing
01827     template<typename P>
01828     struct simplify<P, std::enable_if_t<
01829         !std::is_same<
01830             typename Ring::zero,
01831             typename P::aN
01832         >::value && (P::degree > 0)
01833     > {
01834         using type = P;
01835     };
01836
01837     // do not simplify constants
01838     template<typename P>
01839     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01840         using type = P;
01841     };
01842
01843     // addition at
01844     template<typename P1, typename P2, size_t index>
01845     struct add_at {
01846         using type =
01847             typename Ring::template add_t<
01848                 typename P1::template coeff_at_t<index>,
01849                 typename P2::template coeff_at_t<index>;
01850     };
01851
01852     template<typename P1, typename P2, size_t index>
01853     using add_at_t = typename add_at<P1, P2, index>::type;
01854
01855     template<typename P1, typename P2, std::size_t... I>
01856     struct add_low<P1, P2, std::index_sequence<I...> {
01857         using type = val<add_at_t<P1, P2, I>...>;
01858     };
01859
01860     // subtraction at
01861     template<typename P1, typename P2, size_t index>
01862     struct sub_at {
01863         using type =
01864             typename Ring::template sub_t<
01865                 typename P1::template coeff_at_t<index>,
01866                 typename P2::template coeff_at_t<index>;
01867     };
01868
01869     template<typename P1, typename P2, size_t index>
01870     using sub_at_t = typename sub_at<P1, P2, index>::type;
01871
01872     template<typename P1, typename P2, std::size_t... I>
01873     struct sub_low<P1, P2, std::index_sequence<I...> {
01874         using type = val<sub_at_t<P1, P2, I>...>;
01875     };
01876
01877     template<typename P1, typename P2>
01878     struct sub {
01879         using type = typename simplify<typename sub_low<
01880             P1,
01881             P2,
01882             internal::make_index_sequence_reverse<
01883                 std::max(P1::degree, P2::degree) + 1
01884             >::type>::type;
01885     };
01886
01887     // multiplication at
01888     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01889     struct mul_at_loop_helper {
01890         using type = typename Ring::template add_t<
01891             typename Ring::template mul_t<
01892                 typename v1::template coeff_at_t<index>,
01893                 typename v2::template coeff_at_t<k - index>
01894             >,
01895             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type

```

```

01896         >;
01897     };
01898
01899     template<typename v1, typename v2, size_t k, size_t stop>
01900     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01901         using type = typename Ring::template mul_t<
01902             typename v1::template coeff_at_t<stop>,
01903             typename v2::template coeff_at_t<0>;
01904     };
01905
01906     template<typename v1, typename v2, size_t k, typename E = void>
01907     struct mul_at {};
01908
01909     template<typename v1, typename v2, size_t k>
01910     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01911         using type = typename Ring::zero;
01912     };
01913
01914     template<typename v1, typename v2, size_t k>
01915     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01916         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01917     };
01918
01919     template<typename P1, typename P2, size_t index>
01920     using mul_at_t = typename mul_at<P1, P2, index>::type;
01921
01922     template<typename P1, typename P2, std::size_t... I>
01923     struct mul_low<P1, P2, std::index_sequence<I...> {
01924         using type = val<mul_at_t<P1, P2, I>...>;
01925     };
01926
01927     // division helper
01928     template<typename A, typename B, typename Q, typename R, typename E = void>
01929     struct div_helper {};
01930
01931     template<typename A, typename B, typename Q, typename R>
01932     struct div_helper<A, B, Q, R, std::enable_if_t<
01933         (R::degree < B::degree) ||
01934         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01935         using q_type = Q;
01936         using mod_type = R;
01937         using gcd_type = B;
01938     };
01939
01940     template<typename A, typename B, typename Q, typename R>
01941     struct div_helper<A, B, Q, R, std::enable_if_t<
01942         (R::degree >= B::degree) &&
01943         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01944     private: // NOLINT
01945         using rN = typename R::aN;
01946         using bN = typename B::aN;
01947         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01948             B::degree>::type;
01949         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01950         using qq = typename add<Q, pT>::type;
01951     public:
01952         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01953         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01954         using gcd_type = rr;
01955     };
01956
01957     template<typename A, typename B>
01958     struct div {
01959         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01960         using q_type = typename div_helper<A, B, zero, A>::q_type;
01961         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01962     };
01963
01964     template<typename P>
01965     struct make_unit {
01966         using type = typename div<P, val<typename P::aN>::q_type>;
01967     };
01968
01969     template<typename coeff, size_t deg>
01970     struct monomial {
01971         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01972     };
01973
01974     template<typename coeff>
01975     struct monomial<coeff, 0> {
01976         using type = val<coeff>;
01977     };
01978
01979     template<typename arithmeticType, typename P>
01980     struct horner_evaluation {
01981         template<size_t index, size_t stop>

```

```

01982         struct inner {
01983             static constexpr DEVICE INLINED arithmeticType func(
01984                 const arithmeticType& accum, const arithmeticType& x) {
01985                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
stop>::func(
01986                     internal::fma_helper<arithmeticType>::eval(
01987                         x,
01988                         accum,
01989                         P::template coeff_at_t<P::degree - index>::template
get<arithmeticType>(), x);
01990                     }
01991             };
01992
01993             template<size_t stop>
01994             struct inner<stop, stop> {
01995                 static constexpr DEVICE INLINED arithmeticType func(
01996                     const arithmeticType& accum, const arithmeticType& x) {
01997                     return accum;
01998                 }
01999             };
02000         };
02001
02002         template<typename coeff, typename... coeffs>
02003         struct string_helper {
02004             static std::string func() {
02005                 std::string tail = string_helper<coeffs...>::func();
02006                 std::string result = "";
02007                 if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02008                     return tail;
02009                 } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02010                     if (sizeof...(coeffs) == 1) {
02011                         result += "x";
02012                     } else {
02013                         result += "x^" + std::to_string(sizeof...(coeffs));
02014                     }
02015                 } else {
02016                     if (sizeof...(coeffs) == 1) {
02017                         result += coeff::to_string() + " x";
02018                     } else {
02019                         result += coeff::to_string()
02020                             + " x^" + std::to_string(sizeof...(coeffs));
02021                     }
02022                 }
02023
02024                 if (!tail.empty()) {
02025                     if (tail.at(0) != '-') {
02026                         result += " + " + tail;
02027                     } else {
02028                         result += " - " + tail.substr(1);
02029                     }
02030                 }
02031
02032                 return result;
02033             }
02034         };
02035
02036         template<typename coeff>
02037         struct string_helper<coeff> {
02038             static std::string func() {
02039                 if (!std::is_same<coeff, typename Ring::zero>::value) {
02040                     return coeff::to_string();
02041                 } else {
02042                     return "";
02043                 }
02044             }
02045         };
02046
02047     public:
02048         template<typename P>
02049         using simplify_t = typename simplify<P>::type;
02050
02051         template<typename v1, typename v2>
02052         using add_t = typename add<v1, v2>::type;
02053
02054         template<typename v1, typename v2>
02055         using sub_t = typename sub<v1, v2>::type;
02056
02057         template<typename v1, typename v2>
02058         using mul_t = typename mul<v1, v2>::type;
02059
02060         template<typename v1, typename v2>
02061         using eq_t = typename eq_helper<v1, v2>::type;
02062
02063         template<typename v1, typename v2>
02064         using lt_t = typename lt_helper<v1, v2>::type;
02065
02066         template<typename v1, typename v2>

```

```

02087     using gt_t = typename gt_helper<v1, v2>::type;
02088
02092     template<typename v1, typename v2>
02093     using div_t = typename div<v1, v2>::q_type;
02094
02098     template<typename v1, typename v2>
02099     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02100
02104     template<typename coeff, size_t deg>
02105     using monomial_t = typename monomial<coeff, deg>::type;
02106
02109     template<typename v>
02110     using derive_t = typename derive_helper<v>::type;
02111
02114     template<typename v>
02115     using pos_t = typename Ring::template pos_t<typename v::aN>;
02116
02119     template<typename v>
02120     static constexpr bool pos_v = pos_t<v>::value;
02121
02125     template<typename v1, typename v2>
02126     using gcd_t = std::conditional_t<
02127         Ring::is_euclidean_domain,
02128         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02129         void>;
02130
02133     template<auto x>
02134     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02135
02138     template<typename v>
02139     using inject_ring_t = val<v>;
02140 };
02141 } // namespace aerobus
02142
02143 // fraction field
02144 namespace aerobus {
02145     namespace internal {
02146         template<typename Ring, typename E = void>
02147         requires IsEuclideanDomain<Ring>
02148         struct _FractionField {};
02149
02150         template<typename Ring>
02151         requires IsEuclideanDomain<Ring>
02152         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
02153             static constexpr bool is_field = true;
02154             static constexpr bool is_euclidean_domain = true;
02155
02156         private:
02157             template<typename val1, typename val2, typename E = void>
02158             struct to_string_helper {};
02159
02160             template<typename val1, typename val2>
02161             struct to_string_helper<val1, val2,
02162                 std::enable_if_t<
02163                     Ring::template eq_t<
02164                         val2, typename Ring::one
02165                         >::value
02166                     >
02167             > {
02168                 static std::string func() {
02169                     return val1::to_string();
02170                 }
02171             };
02172
02173             template<typename val1, typename val2>
02174             struct to_string_helper<val1, val2,
02175                 std::enable_if_t<
02176                     !Ring::template eq_t<
02177                         val2,
02178                         typename Ring::one
02179                         >::value
02180                     >
02181             > {
02182                 static std::string func() {
02183                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02184                 }
02185             };
02186
02187         public:
02188             template<typename val1, typename val2>
02189             struct val {
02190                 using x = val1;
02191                 using y = val2;
02192                 using is_zero_t = typename val1::is_zero_t;
02193                 static constexpr bool is_zero_v = val1::is_zero_t::value;
02194
02195                 using ring_type = Ring;

```

```

02205         using enclosing_type = _FractionField<Ring>;
02206
02209         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02210
02214         template<typename valueType>
02215         static constexpr INLINED_DEVICE valueType get() {
02216             return internal::staticcast<valueType, typename ring_type::inner_type>::template
func<x::v>() /
02217             internal::staticcast<valueType, typename ring_type::inner_type>::template
func<y::v>();
02218     }
02219
02222     static std::string to_string() {
02223         return to_string_helper<val1, val2>::func();
02224     }
02225
02230     template<typename arithmeticType>
02231     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02232         return x::eval(v) / y::eval(v);
02233     }
02234 };
02235
02237 using zero = val<typename Ring::zero, typename Ring::one>;
02239 using one = val<typename Ring::one, typename Ring::one>;
02240
02243 template<typename v>
02244 using inject_t = val<v, typename Ring::one>;
02245
02248 template<auto x>
02249 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02250
02253 template<typename v>
02254 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02255
02257 using ring_type = Ring;
02258
02259 private:
02260     template<typename v, typename E = void>
02261     struct simplify {};
02262
02263     // x = 0
02264     template<typename v>
02265     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02266         using type = typename _FractionField<Ring>::zero;
02267     };
02268
02269     // x != 0
02270     template<typename v>
02271     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02272     private:
02273         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02274         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02275         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02276
02277         using posx = std::conditional_t<
02278             !Ring::template pos_v<newy>,
02279             typename Ring::template sub_t<typename Ring::zero, newx>,
02280             newx>;
02281         using posy = std::conditional_t<
02282             !Ring::template pos_v<newy>,
02283             typename Ring::template sub_t<typename Ring::zero, newy>,
02284             newy>;
02285     public:
02286         using type = typename _FractionField<Ring>::template val<posx, posy>;
02287     };
02288
02289     public:
02292     template<typename v>
02293     using simplify_t = typename simplify<v>::type;
02294
02295     private:
02296     template<typename v1, typename v2>
02297     struct add {
02298     private:
02299         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02300         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02301         using dividend = typename Ring::template add_t<a, b>;
02302         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02303         using g = typename Ring::template gcd_t<dividend, divider>;
02304
02305     public:
02306         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02307     };
02308
02309     template<typename v>

```

```

02310 struct pos {
02311     using type = std::conditional_t<
02312         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02313         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02314         std::true_type,
02315         std::false_type>;
02316 };
02317
02318 template<typename v1, typename v2>
02319 struct sub {
02320     private:
02321         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02322         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02323         using dividend = typename Ring::template sub_t<a, b>;
02324         using divisor = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02325         using g = typename Ring::template gcd_t<dividend, divisor>;
02326
02327     public:
02328         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02329     };
02330
02331 template<typename v1, typename v2>
02332 struct mul {
02333     private:
02334         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02335         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02336
02337     public:
02338         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02339     };
02340
02341 template<typename v1, typename v2, typename E = void>
02342 struct div {};
02343
02344 template<typename v1, typename v2>
02345 struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
02346     private:
02347         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02348         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02349
02350     public:
02351         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02352     };
02353
02354 template<typename v1, typename v2>
02355 struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02356     using type = one;
02357 };
02358
02359 template<typename v1, typename v2>
02360 struct eq {
02361     using type = std::conditional_t<
02362         std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02363         std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02364         std::true_type,
02365         std::false_type>;
02366 };
02367
02368 template<typename v1, typename v2, typename E = void>
02369 struct gt;
02370
02371 template<typename v1, typename v2>
02372 struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
    >> {
02373     using type = std::false_type;
02374 };
02375
02376 template<typename v1, typename v2>
02377 struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(!pos<v1>::type::value) && (!pos<v2>::type::value)
    >> {
02378     using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
    >::type;
02379 };
02380
02381 template<typename v1, typename v2>
02382 struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
    >> {
02383     using type = std::true_type;
02384 };
02385
02386 template<typename v1, typename v2>
02387 struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
    >> {
02388     using type = std::true_type;
02389 };
02390
02391 template<typename v1, typename v2>
02392 struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
    >> {
02393     using type = std::true_type;
02394 };

```

```

02395         };
02396
02397     template<typename v1, typename v2>
02398     struct gt<v1, v2, std::enable_if_t<
02399         (!eq<v1, v2>::type::value) &&
02400         (!pos<v1>::type::value) && (pos<v2>::type::value)
02401         >> {
02402         using type = std::false_type;
02403     };
02404
02405     template<typename v1, typename v2>
02406     struct gt<v1, v2, std::enable_if_t<
02407         (!eq<v1, v2>::type::value) &&
02408         (pos<v1>::type::value) && (pos<v2>::type::value)
02409         >> {
02410         using type = typename Ring::template gt_t<
02411             typename Ring::template mul_t<v1::x, v2::y>,
02412             typename Ring::template mul_t<v2::y, v2::x>
02413         >;
02414     };
02415
02416     public:
02420     template<typename v1, typename v2>
02421     using add_t = typename add<v1, v2>::type;
02422
02427     template<typename v1, typename v2>
02428     using mod_t = zero;
02429
02434     template<typename v1, typename v2>
02435     using gcd_t = v1;
02436
02440     template<typename v1, typename v2>
02441     using sub_t = typename sub<v1, v2>::type;
02442
02446     template<typename v1, typename v2>
02447     using mul_t = typename mul<v1, v2>::type;
02448
02452     template<typename v1, typename v2>
02453     using div_t = typename div<v1, v2>::type;
02454
02458     template<typename v1, typename v2>
02459     using eq_t = typename eq<v1, v2>::type;
02460
02464     template<typename v1, typename v2>
02465     static constexpr bool eq_v = eq<v1, v2>::type::value;
02466
02470     template<typename v1, typename v2>
02471     using gt_t = typename gt<v1, v2>::type;
02472
02476     template<typename v1, typename v2>
02477     static constexpr bool gt_v = gt<v1, v2>::type::value;
02478
02481     template<typename v1>
02482     using pos_t = typename pos<v1>::type;
02483
02486     template<typename v>
02487     static constexpr bool pos_v = pos<t<v>>::value;
02488 };
02489
02490 template<typename Ring, typename E = void>
02491 requires IsEuclideanDomain<Ring>
02492 struct FractionFieldImpl {};
02493
02494 // fraction field of a field is the field itself
02495 template<typename Field>
02496 requires IsEuclideanDomain<Field>
02497 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02498     using type = Field;
02499     template<typename v>
02500     using inject_t = v;
02501 };
02502
02503 // fraction field of a ring is the actual fraction field
02504 template<typename Ring>
02505 requires IsEuclideanDomain<Ring>
02506 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02507     using type = _FractionField<Ring>;
02508 };
02509 } // namespace internal
02510
02513 template<typename Ring>
02514 requires IsEuclideanDomain<Ring>
02515 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02516
02519 template<typename Ring>
02520 struct Embed<Ring, FractionField<Ring> {
02523     template<typename v>

```



```

02524         using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02525     };
02526 } // namespace aerobus
02527
02528
02529 // short names for common types
02530 namespace aerobus {
02531     template<typename X, typename Y>
02532     requires IsRing<typename X::enclosing_type> &&
02533     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02534     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02535
02536     template<typename X, typename Y>
02537     requires IsRing<typename X::enclosing_type> &&
02538     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02539     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02540
02541     template<typename X, typename Y>
02542     requires IsRing<typename X::enclosing_type> &&
02543     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02544     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02545
02546     template<typename X, typename Y>
02547     requires IsRing<typename X::enclosing_type> &&
02548     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02549     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02550
02551     using q32 = FractionField<i32>;
02552
02553     using fpq32 = FractionField<polynomial<q32>>;
02554
02555     using q64 = FractionField<i64>;
02556
02557     using pi64 = polynomial<i64>;
02558
02559     using pq64 = polynomial<q64>;
02560
02561     using fpq64 = FractionField<polynomial<q64>>;
02562
02563     template<typename Ring, typename v1, typename v2>
02564     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02565
02566     template<typename v>
02567     using embed_int_poly_in_fractions_t =
02568         typename Embed<
02569             polynomial<typename v::ring_type>,
02570             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02571
02572     template<int64_t p, int64_t q>
02573     using make_q64_t = typename q64::template simplify_t<
02574         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02575
02576     template<int32_t p, int32_t q>
02577     using make_q32_t = typename q32::template simplify_t<
02578         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02579
02580     template<typename Ring, typename v1, typename v2>
02581     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02582     template<typename Ring, typename v1, typename v2>
02583     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02584
02585     template<>
02586     struct Embed<q32, q64> {
02587         template<typename v>
02588         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02589     };
02590
02591     template<typename Small, typename Large>
02592     struct Embed<polynomial<Small>, polynomial<Large>> {
02593     private:
02594         template<typename v, typename i>
02595         struct at_low;
02596
02597         template<typename v, size_t i>
02598         struct at_index {
02599             using type = typename Embed<Small, Large>::template
02600                 type<typename v::template coeff_at_t<i>>;
02601         };
02602
02603         template<typename v, size_t... Is>
02604         struct at_low<v, std::index_sequence<Is...>> {
02605             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02606         };
02607
02608     public:
02609         template<typename v>
02610         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +

```

```

1>::type;
02663     };
02664
02665     template<typename Ring, auto... xs>
02666     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02670         typename Ring::template inject_constant_t<xs>...>;
02671
02672     template<typename Ring, auto... xs>
02673     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02674         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02675 } // namespace aerobus
02676
02677 // Taylor series and common integers (factorial, bernoulli...) appearing in Taylor coefficients
02678 namespace aerobus {
02679     namespace internal {
02680         template<typename T, size_t x, typename E = void>
02681         struct factorial {};
02682
02683         template<typename T, size_t x>
02684         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02685         private:
02686             template<typename, size_t, typename>
02687             friend struct factorial;
02688         public:
02689             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02690 x - 1>::type>;
02691         static constexpr typename T::inner_type value = type::template get<typename
02692 T::inner_type>();
02693     };
02694
02695     template<typename T>
02696     struct factorial<T, 0> {
02697     public:
02698         using type = typename T::one;
02699         static constexpr typename T::inner_type value = type::template get<typename
02700 T::inner_type>();
02701     };
02702 } // namespace internal
02703
02704 template<typename T, size_t i>
02705 using factorial_t = typename internal::factorial<T, i>::type;
02706
02707 template<typename T, size_t i>
02708 inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02709
02710 namespace internal {
02711     template<typename T, size_t k, size_t n, typename E = void>
02712     struct combination_helper {};
02713
02714     template<typename T, size_t k, size_t n>
02715     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02716     using type = typename FractionField<T>::template mul_t<
02717         typename combination_helper<T, k - 1, n - 1>::type,
02718         makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02719     };
02720
02721     template<typename T, size_t k, size_t n>
02722     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02723     using type = typename combination_helper<T, n - k, n>::type;
02724     };
02725
02726     template<typename T, size_t n>
02727     struct combination_helper<T, 0, n> {
02728     using type = typename FractionField<T>::one;
02729     };
02730
02731     template<typename T, size_t k, size_t n>
02732     struct combination {
02733     using type = typename internal::combination_helper<T, k, n>::type::x;
02734     static constexpr typename T::inner_type value =
02735         internal::combination_helper<T, k, n>::type::template get<typename
02736 T::inner_type>();
02737     };
02738 } // namespace internal
02739
02740 template<typename T, size_t k, size_t n>
02741 using combination_t = typename internal::combination<T, k, n>::type;
02742
02743 template<typename T, size_t k, size_t n>
02744 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02745
02746 namespace internal {
02747     template<typename T, size_t m>
02748     struct bernoulli;
02749
02750     template<typename T, typename accum, size_t k, size_t m>
02751     struct bernoulli_helper {

```

```

02763         using type = typename bernoulli_helper<
02764             T,
02765             addfractions_t<T,
02766                 accum,
02767                 mulfractions_t<T,
02768                     makefraction_t<T,
02769                         combination_t<T, k, m + 1>,
02770                         typename T::one>,
02771                         typename bernoulli<T, k>::type
02772                     >,
02773                     >,
02774                     k + 1,
02775                     m>::type;
02776     };
02777
02778     template<typename T, typename accum, size_t m>
02779     struct bernoulli_helper<T, accum, m, m> {
02780         using type = accum;
02781     };
02782
02783
02784
02785     template<typename T, size_t m>
02786     struct bernoulli {
02787         using type = typename FractionField<T>::template mul_t<
02788             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02789             makefraction_t<T,
02790                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02791                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02792             >
02793         >;
02794
02795         template<typename floatType>
02796         static constexpr floatType value = type::template get<floatType>();
02797     };
02798
02799     template<typename T>
02800     struct bernoulli<T, 0> {
02801         using type = typename FractionField<T>::one;
02802
02803         template<typename floatType>
02804         static constexpr floatType value = type::template get<floatType>();
02805     };
02806 } // namespace internal
02807
02811 template<typename T, size_t n>
02812 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02813
02818 template<typename FloatType, typename T, size_t n>
02819 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02820
02821 // bell numbers
02822 namespace internal {
02823     template<typename T, size_t n, typename E = void>
02824     struct bell_helper;
02825
02826     template<typename T, size_t n>
02827     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02828         template<typename accum, size_t i, size_t stop>
02829         struct sum_helper {
02830             private:
02831                 using left = typename T::template mul_t<
02832                     combination_t<T, i, n-1>,
02833                     typename bell_helper<T, i>::type>;
02834                 using new_accum = typename T::template add_t<accum, left>;
02835             public:
02836                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02837         };
02838
02839         template<typename accum, size_t stop>
02840         struct sum_helper<accum, stop, stop> {
02841             using type = accum;
02842         };
02843
02844         using type = typename sum_helper<typename T::zero, 0, n>::type;
02845     };
02846
02847     template<typename T>
02848     struct bell_helper<T, 0> {
02849         using type = typename T::one;
02850     };
02851
02852     template<typename T>
02853     struct bell_helper<T, 1> {
02854         using type = typename T::one;
02855     };
02856 } // namespace internal

```

```

02857
02861     template<typename T, size_t n>
02862     using bell_t = typename internal::bell_helper<T, n>::type;
02863
02867     template<typename T, size_t n>
02868     static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02869
02870     namespace internal {
02871         template<typename T, int k, typename E = void>
02872         struct alternate {};
02873
02874         template<typename T, int k>
02875         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02876             using type = typename T::one;
02877             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02878         };
02879
02880         template<typename T, int k>
02881         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02882             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02883             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02884         };
02885     } // namespace internal
02886
02889     template<typename T, int k>
02890     using alternate_t = typename internal::alternate<T, k>::type;
02891
02894     template<typename T, size_t k>
02895     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02896
02897     namespace internal {
02898         template<typename T, int n, int k, typename E = void>
02899         struct stirling_l_helper {};
02900
02901         template<typename T>
02902         struct stirling_l_helper<T, 0, 0> {
02903             using type = typename T::one;
02904         };
02905
02906         template<typename T, int n>
02907         struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02908             using type = typename T::zero;
02909         };
02910
02911         template<typename T, int n>
02912         struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02913             using type = typename T::zero;
02914         };
02915
02916         template<typename T, int n, int k>
02917         struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02918             using type = typename T::template sub_t<
02919                 typename stirling_l_helper<T, n-1, k-1>::type,
02920                 typename T::template mul_t<
02921                     typename T::template inject_constant_t<n-1>,
02922                     typename stirling_l_helper<T, n-1, k>::type
02923                 >;
02924         };
02925     } // namespace internal
02926
02931     template<typename T, int n, int k>
02932     using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
02933
02938     template<typename T, int n, int k>
02939     using stirling_l_unsigned_t = abs_t<typename internal::stirling_l_helper<T, n, k>::type>;
02940
02945     template<typename T, int n, int k>
02946     static constexpr typename T::inner_type stirling_l_unsigned_v = stirling_l_unsigned_t<T, n, k>::v;
02947
02952     template<typename T, int n, int k>
02953     static constexpr typename T::inner_type stirling_l_signed_v = stirling_l_signed_t<T, n, k>::v;
02954
02955     namespace internal {
02956         template<typename T, int n, int k, typename E = void>
02957         struct stirling_2_helper {};
02958
02959         template<typename T, int n>
02960         struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
02961             using type = typename T::one;
02962         };
02963
02964         template<typename T, int n>
02965         struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02966             using type = typename T::zero;
02967         };

```

```

02968
02969     template<typename T, int n>
02970     struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
02971         using type = typename T::zero;
02972     };
02973
02974     template<typename T, int n, int k>
02975     struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)>> {
02976         using type = typename T::template add_t<
02977             typename stirling_2_helper<T, n-1, k-1>::type,
02978             typename T::template mul_t<
02979                 typename T::template inject_constant_t<k>,
02980                 typename stirling_2_helper<T, n-1, k>::type
02981             >;
02982     };
02983 } // namespace internal
02984
02985 template<typename T, int n, int k>
02986 using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
02987
02988 template<typename T, int n, int k>
02989 static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
02990
02991 namespace internal {
02992     template<typename T>
02993     struct pow_scalar {
02994         template<size_t p>
02995         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02996             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02997             x * func<p/2>(x) * func<p/2>(x);
02998         }
02999     };
03000
03001     template<typename T, typename p, size_t n, typename E = void>
03002     requires IsEuclideanDomain<T>
03003     struct pow;
03004
03005     template<typename T, typename p, size_t n>
03006     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
03007         using type = typename T::template mul_t<
03008             typename pow<T, p, n/2>::type,
03009             typename pow<T, p, n/2>::type
03010         >;
03011     };
03012
03013     template<typename T, typename p, size_t n>
03014     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
03015         using type = typename T::template mul_t<
03016             p,
03017             typename T::template mul_t<
03018                 typename pow<T, p, n/2>::type,
03019                 typename pow<T, p, n/2>::type
03020             >
03021         >;
03022     };
03023
03024     template<typename T, typename p, size_t n>
03025     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
03026 } // namespace internal
03027
03028 template<typename T, typename p, size_t n>
03029 using pow_t = typename internal::pow<T, p, n>::type;
03030
03031 template<typename T, typename p, size_t n>
03032 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03033
03034 template<typename T, size_t p>
03035 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03036     internal::pow_scalar<T>::template func<p>(x); }
03037
03038 namespace internal {
03039     template<typename, template<typename, size_t> typename, class>
03040     struct make_taylor_impl;
03041
03042     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03043     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...>> {
03044         using type = typename polynomial<FractionField<T>>::template val<typename coeff_at<T,
03045             Is>::type...>;
03046     };
03047 }
03048
03049 template<typename T, template<typename, size_t> index> typename coeff_at, size_t deg>
03050 using taylor = typename internal::make_taylor_impl<
03051     T,
03052     coeff_at,
03053     internal::make_index_sequence_reverse<deg + 1>::type;
03054

```

```

03073 namespace internal {
03074     template<typename T, size_t i>
03075     struct exp_coeff {
03076         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03077     };
03078
03079     template<typename T, size_t i, typename E = void>
03080     struct sin_coeff_helper {};
03081
03082     template<typename T, size_t i>
03083     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03084         using type = typename FractionField<T>::zero;
03085     };
03086
03087     template<typename T, size_t i>
03088     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03089         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03090     };
03091
03092     template<typename T, size_t i>
03093     struct sin_coeff {
03094         using type = typename sin_coeff_helper<T, i>::type;
03095     };
03096
03097     template<typename T, size_t i, typename E = void>
03098     struct sh_coeff_helper {};
03099
03100     template<typename T, size_t i>
03101     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03102         using type = typename FractionField<T>::zero;
03103     };
03104
03105     template<typename T, size_t i>
03106     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03107         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03108     };
03109
03110     template<typename T, size_t i>
03111     struct sh_coeff {
03112         using type = typename sh_coeff_helper<T, i>::type;
03113     };
03114
03115     template<typename T, size_t i, typename E = void>
03116     struct cos_coeff_helper {};
03117
03118     template<typename T, size_t i>
03119     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03120         using type = typename FractionField<T>::zero;
03121     };
03122
03123     template<typename T, size_t i>
03124     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03125         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03126     };
03127
03128     template<typename T, size_t i>
03129     struct cos_coeff {
03130         using type = typename cos_coeff_helper<T, i>::type;
03131     };
03132
03133     template<typename T, size_t i, typename E = void>
03134     struct cosh_coeff_helper {};
03135
03136     template<typename T, size_t i>
03137     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03138         using type = typename FractionField<T>::zero;
03139     };
03140
03141     template<typename T, size_t i>
03142     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03143         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03144     };
03145
03146     template<typename T, size_t i>
03147     struct cosh_coeff {
03148         using type = typename cosh_coeff_helper<T, i>::type;
03149     };
03150
03151     template<typename T, size_t i>
03152     struct geom_coeff { using type = typename FractionField<T>::one; };
03153
03154     template<typename T, size_t i, typename E = void>
03155     struct atan_coeff_helper;
03156
03157     template<typename T, size_t i>
03158     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03159

```

```

03160         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03161     };
03162
03163     template<typename T, size_t i>
03164     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03165         using type = typename FractionField<T>::zero;
03166     };
03167
03168     template<typename T, size_t i>
03169     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03170
03171     template<typename T, size_t i, typename E = void>
03172     struct asin_coeff_helper;
03173
03174     template<typename T, size_t i>
03175     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03176         using type = makefraction_t<T,
03177             factorial_t<T, i - 1>,
03178             typename T::template mul_t<
03179                 typename T::template val<i>,
03180                 T::template mul_t<
03181                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03182                     pow<T, factorial_t<T, i / 2>, 2
03183                 >
03184             >
03185         >;
03186     };
03187
03188     template<typename T, size_t i>
03189     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03190         using type = typename FractionField<T>::zero;
03191     };
03192
03193     template<typename T, size_t i>
03194     struct asin_coeff {
03195         using type = typename asin_coeff_helper<T, i>::type;
03196     };
03197
03198     template<typename T, size_t i>
03199     struct lnpl_coeff {
03200         using type = makefraction_t<T,
03201             alternate_t<T, i + 1>,
03202             typename T::template val<i>;
03203     };
03204
03205     template<typename T>
03206     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03207
03208     template<typename T, size_t i, typename E = void>
03209     struct asinh_coeff_helper;
03210
03211     template<typename T, size_t i>
03212     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03213         using type = makefraction_t<T,
03214             typename T::template mul_t<
03215                 alternate_t<T, i / 2>,
03216                 factorial_t<T, i - 1>
03217             >,
03218             typename T::template mul_t<
03219                 typename T::template mul_t<
03220                     typename T::template val<i>,
03221                     pow_t<T, factorial_t<T, i / 2>, 2>
03222                 >,
03223                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03224             >
03225         >;
03226     };
03227
03228     template<typename T, size_t i>
03229     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03230         using type = typename FractionField<T>::zero;
03231     };
03232
03233     template<typename T, size_t i>
03234     struct asinh_coeff {
03235         using type = typename asinh_coeff_helper<T, i>::type;
03236     };
03237
03238     template<typename T, size_t i, typename E = void>
03239     struct atanh_coeff_helper;
03240
03241     template<typename T, size_t i>
03242     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03243         // 1/i
03244         using type = typename FractionField<T>::template val<
03245             typename T::one,
03246             typename T::template inject_constant_t<i>;

```

```

03247     };
03248
03249     template<typename T, size_t i>
03250     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03251         using type = typename FractionField<T>::zero;
03252     };
03253
03254     template<typename T, size_t i>
03255     struct atanh_coeff {
03256         using type = typename atanh_coeff_helper<T, i>::type;
03257     };
03258
03259     template<typename T, size_t i, typename E = void>
03260     struct tan_coeff_helper;
03261
03262     template<typename T, size_t i>
03263     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03264         using type = typename FractionField<T>::zero;
03265     };
03266
03267     template<typename T, size_t i>
03268     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03269     private:
03270         //  $4^{(i+1)/2}$ 
03271         using _4p = typename FractionField<T>::template inject_t<
03272             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03273         //  $4^{(i+1)/2} - 1$ 
03274         using _4pml = typename FractionField<T>::template
03275             sub_t<_4p, typename FractionField<T>::one>;
03276         //  $(-1)^{(i-1)/2}$ 
03277         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03278         using dividend = typename FractionField<T>::template mul_t<
03279             altp,
03280             FractionField<T>::template mul_t<
03281                 _4p,
03282                 FractionField<T>::template mul_t<
03283                     _4pml,
03284                     bernoulli_t<T, (i + 1)>
03285                 >
03286             >;
03287     public:
03288         using type = typename FractionField<T>::template div_t<dividend,
03289             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03290     };
03291
03292     template<typename T, size_t i>
03293     struct tan_coeff {
03294         using type = typename tan_coeff_helper<T, i>::type;
03295     };
03296
03297     template<typename T, size_t i, typename E = void>
03298     struct tanh_coeff_helper;
03299
03300     template<typename T, size_t i>
03301     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03302         using type = typename FractionField<T>::zero;
03303     };
03304
03305     template<typename T, size_t i>
03306     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03307     private:
03308         using _4p = typename FractionField<T>::template inject_t<
03309             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03310         using _4pml = typename FractionField<T>::template
03311             sub_t<_4p, typename FractionField<T>::one>;
03312         using dividend =
03313             typename FractionField<T>::template mul_t<
03314                 _4p,
03315                 typename FractionField<T>::template mul_t<
03316                     _4pml,
03317                     bernoulli_t<T, (i + 1)>>::type;
03318     public:
03319         using type = typename FractionField<T>::template div_t<dividend,
03320             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03321     };
03322
03323     template<typename T, size_t i>
03324     struct tanh_coeff {
03325         using type = typename tanh_coeff_helper<T, i>::type;
03326     };
03327 } // namespace internal
03328
03329 template<typename Integers, size_t deg>
03330 using exp = taylor<Integers, internal::exp_coeff, deg>;
03331
03332 template<typename Integers, size_t deg>

```



```

03338     using expm1 = typename polynomial<FractionField<Integers>>::template sub_t<
03339         exp<Integers, deg>,
03340         typename polynomial<FractionField<Integers>>::one>;
03341
03342     template<typename Integers, size_t deg>
03343     using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03344
03345     template<typename Integers, size_t deg>
03346     using atan = taylor<Integers, internal::atan_coeff, deg>;
03347
03348     template<typename Integers, size_t deg>
03349     using sin = taylor<Integers, internal::sin_coeff, deg>;
03350
03351     template<typename Integers, size_t deg>
03352     using sinh = taylor<Integers, internal::sh_coeff, deg>;
03353
03354     template<typename Integers, size_t deg>
03355     using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03356
03357     template<typename Integers, size_t deg>
03358     using cos = taylor<Integers, internal::cos_coeff, deg>;
03359
03360     template<typename Integers, size_t deg>
03361     using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03362
03363     template<typename Integers, size_t deg>
03364     using asin = taylor<Integers, internal::asin_coeff, deg>;
03365
03366     template<typename Integers, size_t deg>
03367     using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03368
03369     template<typename Integers, size_t deg>
03370     using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03371
03372     template<typename Integers, size_t deg>
03373     using tan = taylor<Integers, internal::tan_coeff, deg>;
03374
03375     template<typename Integers, size_t deg>
03376     using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03377 } // namespace aerobus
03378
03379 // continued fractions
03380 namespace aerobus {
03381     template<int64_t... values>
03382     struct ContinuedFraction {};
03383
03384     template<int64_t a0>
03385     struct ContinuedFraction<a0> {
03386         using type = typename q64::template inject_constant_t<a0>;
03387         static constexpr double val = static_cast<double>(a0);
03388     };
03389
03390     template<int64_t a0, int64_t... rest>
03391     struct ContinuedFraction<a0, rest...> {
03392         using type = q64::template add_t<
03393             typename q64::template inject_constant_t<a0>,
03394             typename q64::template div_t<
03395                 typename q64::one,
03396                 typename ContinuedFraction<rest...>::type
03397             >;
03398         static constexpr double val = type::template get<double>();
03399     };
03400
03401     using PI_fraction =
03402     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03403     using E_fraction =
03404     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03405     using SQRT2_fraction =
03406     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03407     using SQRT3_fraction =
03408     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03409 // NOLINT
03410 } // namespace aerobus
03411
03412 // known polynomials
03413 namespace aerobus {
03414     // CChebyshev
03415     namespace internal {
03416         template<int kind, size_t deg, typename I>
03417         struct chebyshev_helper {
03418             using type = typename polynomial<I>::template sub_t<
03419                 typename polynomial<I>::template mul_t<
03420                     typename polynomial<I>::template mul_t<
03421                         typename polynomial<I>::template inject_constant_t<2>,
03422                         typename polynomial<I>::X>,
03423                     typename chebyshev_helper<kind, deg - 1, I>::type
03424                 >
03425             >
03426         };
03427     }
03428 }

```

```

03481         >,
03482         typename chebyshev_helper<kind, deg - 2, I>::type
03483     >;
03484 };
03485
03486 template<typename I>
03487 struct chebyshev_helper<1, 0, I> {
03488     using type = typename polynomial<I>::one;
03489 };
03490
03491 template<typename I>
03492 struct chebyshev_helper<1, 1, I> {
03493     using type = typename polynomial<I>::X;
03494 };
03495
03496 template<typename I>
03497 struct chebyshev_helper<2, 0, I> {
03498     using type = typename polynomial<I>::one;
03499 };
03500
03501 template<typename I>
03502 struct chebyshev_helper<2, 1, I> {
03503     using type = typename polynomial<I>::template mul_t<
03504         typename polynomial<I>::template inject_constant_t<2>,
03505         typename polynomial<I>::X>;
03506 };
03507 } // namespace internal
03508
03509 // Laguerre
03510 namespace internal {
03511     template<size_t deg, typename I>
03512     struct laguerre_helper {
03513         using Q = FractionField<I>;
03514         using PQ = polynomial<Q>;
03515
03516     private:
03517         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)Lkm2)
03518         using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03519         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03520         // -x + 2k-1
03521         using p = typename PQ::template val<
03522             typename Q::template inject_constant_t<-1>,
03523             typename Q::template inject_constant_t<2 * deg - 1>;
03524         // 1/n
03525         using factor = typename PQ::template inject_ring_t<
03526             typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03527
03528     public:
03529         using type = typename PQ::template mul_t<
03530             factor,
03531             typename PQ::template sub_t<
03532                 typename PQ::template mul_t<
03533                     p,
03534                     lnm1
03535                 >,
03536                 typename PQ::template mul_t<
03537                     typename PQ::template inject_constant_t<deg-1>,
03538                     lnm2
03539                 >
03540             >
03541         >;
03542     };
03543
03544 template<typename I>
03545 struct laguerre_helper<0, I> {
03546     using type = typename polynomial<FractionField<I>::one;
03547 };
03548
03549 template<typename I>
03550 struct laguerre_helper<1, I> {
03551     private:
03552         using PQ = polynomial<FractionField<I>;
03553     public:
03554         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03555     };
03556 } // namespace internal
03557
03558 // Bernstein
03559 namespace internal {
03560     template<size_t i, size_t m, typename I, typename E = void>
03561     struct bernstein_helper {};
03562
03563 template<typename I>
03564 struct bernstein_helper<0, 0, I> {
03565     using type = typename polynomial<I>::one;
03566 };

```

```

03567
03568     template<size_t i, size_t m, typename I>
03569     struct bernstein_helper<i, m, I, std::enable_if_t<
03570         (m > 0) && (i == 0)>> {
03571     private:
03572         using P = polynomial<I>;
03573     public:
03574         using type = typename P::template mul_t<
03575             typename P::template sub_t<typename P::one, typename P::X>,
03576             typename bernstein_helper<i, m-1, I>::type>;
03577     };
03578
03579     template<size_t i, size_t m, typename I>
03580     struct bernstein_helper<i, m, I, std::enable_if_t<
03581         (m > 0) && (i == m)>> {
03582     private:
03583         using P = polynomial<I>;
03584     public:
03585         using type = typename P::template mul_t<
03586             typename P::X,
03587             typename bernstein_helper<i-1, m-1, I>::type>;
03588     };
03589
03590     template<size_t i, size_t m, typename I>
03591     struct bernstein_helper<i, m, I, std::enable_if_t<
03592         (m > 0) && (i > 0) && (i < m)>> {
03593     private:
03594         using P = polynomial<I>;
03595     public:
03596         using type = typename P::template add_t<
03597             typename P::template mul_t<
03598                 typename P::template sub_t<typename P::one, typename P::X>,
03599                 typename bernstein_helper<i, m-1, I>::type>,
03600             typename P::template mul_t<
03601                 typename P::X,
03602                 typename bernstein_helper<i-1, m-1, I>::type>;
03603     };
03604 } // namespace internal
03605
03606 // AllOne polynomials
03607 namespace internal {
03608     template<size_t deg, typename I>
03609     struct AllOneHelper {
03610     private:
03611         using type = aerobus::add_t<
03612             typename polynomial<I>::one,
03613             typename aerobus::mul_t<
03614                 typename polynomial<I>::X,
03615                 typename AllOneHelper<deg-1, I>::type
03616             >>;
03617     };
03618
03619     template<typename I>
03620     struct AllOneHelper<0, I> {
03621     private:
03622         using type = typename polynomial<I>::one;
03623     };
03624 } // namespace internal
03625
03626 // Bessel polynomials
03627 namespace internal {
03628     template<size_t deg, typename I>
03629     struct BesselHelper {
03630     private:
03631         using P = polynomial<I>;
03632         using factor = typename P::template monomial_t<
03633             typename I::template inject_constant_t<(2*deg - 1)>,
03634             1>;
03635     public:
03636         using type = typename P::template add_t<
03637             typename P::template mul_t<
03638                 factor,
03639                 typename BesselHelper<deg-1, I>::type
03640             >,
03641             typename BesselHelper<deg-2, I>::type
03642         >;
03643     };
03644
03645     template<typename I>
03646     struct BesselHelper<0, I> {
03647     private:
03648         using type = typename polynomial<I>::one;
03649     };
03650
03651     template<typename I>
03652     struct BesselHelper<1, I> {
03653     private:
03654         using P = polynomial<I>;
03655     public:
03656         using type = typename P::template add_t<

```

```

03654         typename P::one,
03655         typename P::X
03656     >;
03657 };
03658 } // namespace internal
03659
03660 namespace known_polynomials {
03661     enum hermite_kind {
03662         probabilist,
03663         physicist
03664     };
03665 }
03666
03667 // hermite
03668 namespace internal {
03669     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03670     struct hermite_helper {};
03671
03672     template<size_t deg, typename I>
03673     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03674     private:
03675         using hnm1 = typename hermite_helper<deg - 1,
03676         known_polynomials::hermite_kind::probabilist, I>::type;
03677         using hnm2 = typename hermite_helper<deg - 2,
03678         known_polynomials::hermite_kind::probabilist, I>::type;
03679
03680     public:
03681         using type = typename polynomial<I>::template sub_t<
03682             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03683             typename polynomial<I>::template mul_t<
03684                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03685                 hnm2
03686             >
03687         >;
03688     };
03689
03690     template<size_t deg, typename I>
03691     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03692     private:
03693         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03694         I>::type;
03695         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03696         I>::type;
03697
03698     public:
03699         using type = typename polynomial<I>::template sub_t<
03700             // 2X Hn-1
03701             typename polynomial<I>::template mul_t<
03702                 typename pi64::val<typename I::template inject_constant_t<2>,
03703                 typename I::zero>, hnm1>,
03704             typename polynomial<I>::template mul_t<
03705                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03706                 hnm2
03707             >
03708         >;
03709     };
03710
03711     template<typename I>
03712     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03713         using type = typename polynomial<I>::one;
03714     };
03715
03716     template<typename I>
03717     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03718         using type = typename polynomial<I>::X;
03719     };
03720
03721     template<typename I>
03722     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03723         using type = typename pi64::one;
03724     };
03725
03726     template<typename I>
03727     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03728         // 2X
03729         using type = typename polynomial<I>::template val<
03730             typename I::template inject_constant_t<2>,
03731             typename I::zero>;
03732     };
03733 } // namespace internal
03734
03735 // legendre
03736 namespace internal {
03737     template<size_t n, typename I>
03738     struct legendre_helper {
03739     private:

```

```

03740         using Q = FractionField<I>;
03741         using PQ = polynomial<Q>;
03742         // 1/n constant
03743         // (2n-1)/n X
03744         using fact_left = typename PQ::template monomial_t<
03745             makefraction_t<I,
03746                 typename I::template inject_constant_t<2*n-1>,
03747                 typename I::template inject_constant_t<n>
03748             >,
03749             1>;
03750         // (n-1) / n
03751         using fact_right = typename PQ::template val<
03752             makefraction_t<I,
03753                 typename I::template inject_constant_t<n-1>,
03754                 typename I::template inject_constant_t<n>>>;
03755
03756     public:
03757         using type = PQ::template sub_t<
03758             typename PQ::template mul_t<
03759                 fact_left,
03760                 typename legendre_helper<n-1, I>::type
03761             >,
03762             typename PQ::template mul_t<
03763                 fact_right,
03764                 typename legendre_helper<n-2, I>::type
03765             >
03766         >;
03767     };
03768
03769     template<typename I>
03770     struct legendre_helper<0, I> {
03771         using type = typename polynomial<FractionField<I>::one>;
03772     };
03773
03774     template<typename I>
03775     struct legendre_helper<1, I> {
03776         using type = typename polynomial<FractionField<I>::X>;
03777     };
03778 } // namespace internal
03779
03780 // bernoulli polynomials
03781 namespace internal {
03782     template<size_t n>
03783     struct bernoulli_coeff {
03784         template<typename T, size_t i>
03785         struct inner {
03786             private:
03787                 using F = FractionField<T>;
03788             public:
03789                 using type = typename F::template mul_t<
03790                     typename F::template inject_ring_t<combination_t<T, i, n>,
03791                     bernoulli_t<T, n-i>
03792                 >;
03793         };
03794     };
03795 } // namespace internal
03796
03797 namespace internal {
03798     template<size_t n>
03799     struct touchard_coeff {
03800         template<typename T, size_t i>
03801         struct inner {
03802             using type = stirling_2_t<T, n, i>;
03803         };
03804     };
03805 } // namespace internal
03806
03807 namespace internal {
03808     template<typename I = aerobus::i64>
03809     struct AbelHelper {
03810     private:
03811         using P = aerobus::polynomial<I>;
03812
03813     public:
03814         // to keep recursion working, we need to operate on a*n and not just a
03815         template<size_t deg, I::inner_type an>
03816         struct Inner {
03817             // abel(n, a) = (x-an) * abel(n-1, a)
03818             using type = typename aerobus::mul_t<
03819                 typename Inner<deg-1, an>::type,
03820                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
03821             >;
03822         };
03823
03824         // abel(0, a) = 1
03825         template<I::inner_type an>
03826         struct Inner<0, an> {

```

```

03827         using type = P::one;
03828     };
03829
03830     // abel(1, a) = X
03831     template<I::inner_type an>
03832     struct Inner<1, an> {
03833         using type = P::X;
03834     };
03835 };
03836 } // namespace internal
03837
03838 namespace known_polynomials {
03839
03840     template<size_t n, auto a, typename I = aerobus::i64>
03841     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
03842
03843     template<size_t deg, typename I = aerobus::i64>
03844     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03845
03846     template<size_t deg, typename I = aerobus::i64>
03847     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03848
03849     template<size_t deg, typename I = aerobus::i64>
03850     using laguerre = typename internal::laguerre_helper<deg, I>::type;
03851
03852     template<size_t deg, typename I = aerobus::i64>
03853     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03854 I>::type;
03855
03856     template<size_t deg, typename I = aerobus::i64>
03857     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03858
03859     template<size_t i, size_t m, typename I = aerobus::i64>
03860     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03861
03862     template<size_t deg, typename I = aerobus::i64>
03863     using legendre = typename internal::legendre_helper<deg, I>::type;
03864
03865     template<size_t deg, typename I = aerobus::i64>
03866     using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03867
03868     template<size_t deg, typename I = aerobus::i64>
03869     using allone = typename internal::AllOneHelper<deg, I>::type;
03870
03871     template<size_t deg, typename I = aerobus::i64>
03872     using bessel = typename internal::BesselHelper<deg, I>::type;
03873
03874     template<size_t deg, typename I = aerobus::i64>
03875     using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
03876 } // namespace known_polynomials
03877 } // namespace aerobus
03878
03879 #ifdef AEROBUS_CONWAY_IMPORTS
03880
03881 // conway polynomials
03882 namespace aerobus {
03883     template<int p, int n>
03884     struct ConwayPolynomial {};
03885
03886 #ifndef DO_NOT_DOCUMENT
03887     #define ZPV ZPZ::template val
03888     #define POLYV aerobus::polynomial<ZPV>::template val
03889     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
03890     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03891     template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03892     template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03893     template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03894     template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03895     template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03896     template<> struct ConwayPolynomial<2, 8> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03897     template<> struct ConwayPolynomial<2, 9> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>; }; //
NOLINT
03898     template<> struct ConwayPolynomial<2, 10> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>,
ZPV<1>; }; // NOLINT
03899     template<> struct ConwayPolynomial<2, 11> { using ZPV = aerobus::zpz<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>,
ZPV<1>,

```

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```

05900     template<> struct ConwayPolynomial<983, 1> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<978>; }; // NOLINT
05901     template<> struct ConwayPolynomial<983, 2> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<981>, ZPZV<5>; }; // NOLINT
05902     template<> struct ConwayPolynomial<983, 3> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<978>; }; // NOLINT
05903     template<> struct ConwayPolynomial<983, 4> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<5>, ZPZV<567>, ZPZV<5>; }; // NOLINT
05904     template<> struct ConwayPolynomial<983, 5> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<8>, ZPZV<978>; }; // NOLINT
05905     template<> struct ConwayPolynomial<983, 6> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<849>, ZPZV<296>, ZPZV<228>, ZPZV<5>; }; // NOLINT
05906     template<> struct ConwayPolynomial<983, 7> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<978>; }; // NOLINT
05907     template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<738>, ZPZV<276>, ZPZV<530>, ZPZV<5>; }; //
NOLINT
05908     template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<858>, ZPZV<87>, ZPZV<978>;
}; // NOLINT
05909     template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<985>; }; // NOLINT
05910     template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<989>, ZPZV<6>; }; // NOLINT
05911     template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<985>; }; // NOLINT
05912     template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<10>, ZPZV<794>, ZPZV<6>; }; // NOLINT
05913     template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<985>; }; // NOLINT
05914     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
05915     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
05916     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
05917     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
05918     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
05919     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
05920     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
05921     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
05922     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
05923     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
05924     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
05925     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
05926     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
05927 #endif // DO_NOT_DOCUMENT
05928 } // namespace aerobus
05929 #endif // AEROBUS_CONWAY_IMPORTS
05930
05931 #endif // __INC_AEROBUS__ // NOLINT

```

9.4 src/examples.h File Reference

9.5 examples.h

[Go to the documentation of this file.](#)

```

00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00042 #endif // SRC_EXAMPLES_H_

```

Chapter 10

Examples

10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

10.2 examples/custom_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```



```
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

// x^3 + 2x^2 + 1, with coefficients in Z/2Z, actually x^3 + 1
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
// x^3 + 5x^2 + 7x + 11 with coefficients in Z/17Z, meaning actually x^3 + x^2 + 1
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value = " << v << std::endl;
    return 0;
}
```

10.6 examples/make_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

10.7 examples/polynomials_over_finite_field.cpp

How to build a known polynomial (here aerobus::known_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
```

```
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1 in  $\mathbb{Z}/2\mathbb{Z}$  is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```


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