

Aerobus

v1.2

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# Chapter 1

## Concept Index

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# Class Index

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# File Index

### 3.1 File List

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## Chapter 4

# Concept Documentation

### 4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 4.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 4.2.2 Detailed Description

Concept to express R is a field.

## 4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

### 4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

## Chapter 5

# Class Documentation

### 5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

#### Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

#### Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

### 5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are <a href="#">aerobus::i64</a>
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using **type** = typename q64::template inject\_constant\_t< a0 >

### Static Public Attributes

- static constexpr double **val** = type::template get<double>()

### 5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.



## Template Parameters

<i>a0</i>	an integer ( <a href="#">aerobus::i64</a> )
-----------	---------------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

## Public Types

- using **type** = q64::template add\_t< typename q64::template inject\_constant\_t< a0 >, typename q64::template div\_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >

## Static Public Attributes

- static constexpr double **val** = type::template get<double>()

### 5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

## Template Parameters

<i>a0</i>	an integer ( <a href="#">aerobus::i64</a> )
<i>...rest</i>	integers ( <a href="#">aerobus::i64</a> )

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

## Public Types

- using **inner\_type** = int32\_t
- using **zero** = [val](#)< 0 >  
*constant zero*
- using **one** = [val](#)< 1 >  
*constant one*
- template<auto x>  
using **inject\_constant\_t** = [val](#)< static\_cast< int32\_t >(x)>
- template<typename v >  
using **inject\_ring\_t** = v
- template<typename v1, typename v2 >  
using **add\_t** = typename add< v1, v2 >::type
- template<typename v1, typename v2 >  
using **sub\_t** = typename sub< v1, v2 >::type
- template<typename v1, typename v2 >  
using **mul\_t** = typename mul< v1, v2 >::type
- template<typename v1, typename v2 >  
using **div\_t** = typename div< v1, v2 >::type
- template<typename v1, typename v2 >  
using **mod\_t** = typename remainder< v1, v2 >::type  
*modulus operator yields v1 % v2 for example : [i32::mod\\_t](#)<[i32::val](#)<7>, [i32::val](#)<2>>*
- template<typename v1, typename v2 >  
using **gt\_t** = typename gt< v1, v2 >::type
- template<typename v1, typename v2 >  
using **lt\_t** = typename lt< v1, v2 >::type
- template<typename v1, typename v2 >  
using **eq\_t** = typename eq< v1, v2 >::type
- template<typename v1, typename v2 >  
using **gcd\_t** = gcd\_t< [i32](#), v1, v2 >
- template<typename v >  
using **pos\_t** = typename pos< v >::type

## Static Public Attributes

- static constexpr bool **is\_field** = false  
*integers are not a field*
- static constexpr bool **is\_euclidean\_domain** = true  
*integers are an euclidean domain*
- template<typename v1, typename v2 >  
static constexpr bool **eq\_v** = eq\_t<v1, v2>::value
- template<typename v >  
static constexpr bool **pos\_v** = pos\_t<v>::value

### 5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 5.7.2 Member Typedef Documentation

### 5.7.2.1 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

#### Template Parameters

<code>v1</code>	a value in <code>i32</code>
<code>v2</code>	a value in <code>i32</code>

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
values in `i64`

### Public Types

- using `inner_type` = `int64_t`  
type for actual values
- template<auto x>  
using `inject_constant_t` = `val< static_cast< int64_t >(x)>`
- template<typename v >  
using `inject_ring_t` = v  
*injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>> -> i64::val<1>`*
- using `zero` = `val< 0 >`  
constant zero
- using `one` = `val< 1 >`  
constant one
- template<typename v1 , typename v2 >  
using `add_t` = `typename add< v1, v2 >::type`
- template<typename v1 , typename v2 >  
using `sub_t` = `typename sub< v1, v2 >::type`

- `template<typename v1 , typename v2 >`  
using **mul\_t** = typename mul< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **div\_t** = typename div< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **mod\_t** = typename remainder< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **gt\_t** = typename gt< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **lt\_t** = typename lt< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **eq\_t** = typename eq< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **gcd\_t** = gcd\_t< i64, v1, v2 >
- `template<typename v >`  
using **pos\_t** = typename pos< v >::type

### Static Public Attributes

- static constexpr bool **is\_field** = false  
*integers are not a field*
- static constexpr bool **is\_euclidean\_domain** = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
static constexpr bool **gt\_v** = gt\_t<v1, v2>::value  
*strictly greater operator yields v1 > v2 as boolean value*
- `template<typename v1 , typename v2 >`  
static constexpr bool **lt\_v** = lt\_t<v1, v2>::value
- `template<typename v1 , typename v2 >`  
static constexpr bool **eq\_v** = eq\_t<v1, v2>::value
- `template<typename v >`  
static constexpr bool **pos\_v** = pos\_t<v>::value

## 5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

## 5.8.2 Member Typedef Documentation

### 5.8.2.1 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64>::val<1>> -> i64::val<1>`

#### Template Parameters

<i>v</i>	a value in i64
----------	----------------

## 5.8.3 Member Data Documentation

### 5.8.3.1 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.9 aerobus::is\_prime< n > Struct Template Reference

checks if  $n$  is prime

```
#include <aerobus.h>
```

#### Static Public Attributes

- static constexpr bool **value** = internal::\_is\_prime< $n$ , 5>::value  
*true iff  $n$  is prime*

### 5.9.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if  $n$  is prime

#### Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.10 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1, typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1, typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1, typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1, typename v2 >](#)  
[using eq\\_t = typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1, typename v2 >](#)  
[using lt\\_t = typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1, typename v2 >](#)  
[using gt\\_t = typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*
- [template<typename v1, typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::q\\_type](#)  
*division operator*
- [template<typename v1, typename v2 >](#)  
[using mod\\_t = typename div\\_helper< v1, v2, zero, v1 >::mod\\_type](#)  
*modulo operator*
- [template<typename coeff, size\\_t deg>](#)  
[using monomial\\_t = typename monomial< coeff, deg >::type](#)  
*monomial : coeff X<sup>deg</sup>*
- [template<typename v >](#)  
[using derive\\_t = typename derive\\_helper< v >::type](#)

- derivation operator*
  - ```
template<typename v >
using pos_t = typename Ring::template pos_t< typename v::aN >
```

*checks for positivity (an > 0)*
  - ```
template<typename v1 , typename v2 >
using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<
Ring >, v1, v2 > >::type, void >
```

*greatest common divisor of two polynomials*
  - ```
template<auto x>
using inject_constant_t = val< typename Ring::template inject_constant_t< x > >
```
  - ```
template<typename v >
using inject_ring_t = val< v >
```

## Static Public Attributes

- ```
static constexpr bool is_field = false
```
- ```
static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain
```
- ```
template<typename v >
static constexpr bool pos_v = pos_t<v>::value
```

*positivity operator*

### 5.10.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

### 5.10.2 Member Typedef Documentation

#### 5.10.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

#### 5.10.2.2 derive\_t

```
template<typename Ring >
```

```
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

|          |  |
|----------|--|
| <i>v</i> |  |
|----------|--|

### 5.10.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.2.5 gcd\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |



### 5.10.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.2.7 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.2.8 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.2.9 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

**Template Parameters**

|              |  |
|--------------|--|
| <i>coeff</i> |  |
| <i>deg</i>   |  |

**5.10.2.10 mul\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**5.10.2.11 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

|          |  |
|----------|--|
| <i>v</i> |  |
|----------|--|

**5.10.2.12 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

|          |  |
|----------|--|
| <i>P</i> |  |
|----------|--|

**5.10.2.13 sub\_t**

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

#### Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

### 5.10.3 Member Data Documentation

#### 5.10.3.1 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

#### Template Parameters

|          |                                            |
|----------|--------------------------------------------|
| <i>v</i> | a value in <a href="#">polynomial::val</a> |
|----------|--------------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.11 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

#### Public Types

- using **type** = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using **tail** = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 5.11.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.12 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< [typename](#) Ring::zero >  
*zero value*
- using [one](#) = [val](#)< [typename](#) Ring::one >  
*one*
- template<[typename](#) v1 , [typename](#) v2 >  
using [add\\_t](#) = [val](#)< [typename](#) Ring::template [add\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*addition operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [mul\\_t](#) = [val](#)< [typename](#) Ring::template [mul\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*subtraction operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [div\\_t](#) = [val](#)< [typename](#) Ring::template [div\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*division operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [mod\\_t](#) = [val](#)< [typename](#) Ring::template [mod\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*modulus operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [eq\\_t](#) = [typename](#) Ring::template [eq\\_t](#)< [typename](#) v1::type, [typename](#) v2::type >  
*equality operator (as type)*
- template<[typename](#) v1 >  
using [pos\\_t](#) = `std::true_type`  
*positivity operator always true*
- template<`auto` x>  
using [inject\\_constant\\_t](#) = [val](#)< [typename](#) Ring::template [inject\\_constant\\_t](#)< x > >
- template<[typename](#) v >  
using [inject\\_ring\\_t](#) = [val](#)< v >

### Static Public Attributes

- template<[typename](#) v1 , [typename](#) v2 >  
[static constexpr bool](#) [eq\\_v](#) = Ring::template [eq\\_t](#)<[typename](#) v1::type, [typename](#) v2::type>::value  
*addition operator (as boolean value)*
- template<[typename](#) v >  
[static constexpr bool](#) [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator always true*
- [static constexpr bool](#) [is\\_euclidean\\_domain](#) = `true`  
*quotien rings are euclidean domain*

### 5.12.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

|             |                                                                               |
|-------------|-------------------------------------------------------------------------------|
| <i>Ring</i> | A ring type, such as ' <a href="#">i32</a> ', must satisfy the IsRing concept |
| <i>X</i>    | a value in Ring, such as <code>i32::val&lt;2&gt;</code>                       |

### 5.12.2 Member Typedef Documentation

#### 5.12.2.1 `add_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

#### 5.12.2.2 `div_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

#### 5.12.2.3 `eq_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
```

```
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

#### 5.12.2.4 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

#### 5.12.2.5 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

#### 5.12.2.6 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

## Template Parameters

|                 |                          |
|-----------------|--------------------------|
| <code>v1</code> | a value in quotient ring |
|-----------------|--------------------------|

## 5.12.3 Member Data Documentation

## 5.12.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

## Template Parameters

|                 |                          |
|-----------------|--------------------------|
| <code>v1</code> | a value in quotient ring |
| <code>v2</code> | a value in quotient ring |

## 5.12.3.2 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

|                 |                          |
|-----------------|--------------------------|
| <code>v1</code> | a value in quotient ring |
|-----------------|--------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.13 aerobus::type\_list&lt; Ts &gt;::split&lt; index &gt; Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using **head** = typename inner::head
- using **tail** = typename inner::tail

### 5.13.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.14 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = internal::type\_at\_t< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename concat\_h< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using [insert](#) = typename internal::insert\_h< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename internal::remove\_h< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*



### Static Public Attributes

- static constexpr size\_t **length** = sizeof...(Ts)  
*length of list*

## 5.14.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

## 5.14.2 Member Typedef Documentation

### 5.14.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

#### Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 5.14.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

#### Template Parameters

|          |  |
|----------|--|
| <i>U</i> |  |
|----------|--|

### 5.14.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
| <i>T</i>     |  |

**5.14.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

**5.14.2.5 push\_front**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

**Template Parameters**

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

**5.14.2.6 remove**

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.15 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using **push\_front** = [type\\_list](#)< T >
- template<typename T >  
using **push\_back** = [type\\_list](#)< T >
- template<typename U >  
using **concat** = U
- template<typename T , size\_t index>  
using **insert** = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t **length** = 0

#### 5.15.1 Detailed Description

specialization for empty type list

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.16 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using **enclosing\_type** = [i32](#)  
*Enclosing ring type.*
- using **is\_zero\_t** = std::bool\_constant< x==0 >  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`  
*cast x into valueType*
- `static std::string to\_string ()`  
*string representation of value*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`  
*cast x into valueRing*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*

### 5.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

#### Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

### 5.16.2 Member Function Documentation

#### 5.16.2.1 [eval\(\)](#)

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

#### Template Parameters

|                        |                    |
|------------------------|--------------------|
| <code>valueRing</code> | double for example |
|------------------------|--------------------|

#### 5.16.2.2 [get\(\)](#)

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

#### Template Parameters

|                  |                    |
|------------------|--------------------|
| <i>valueType</i> | double for example |
|------------------|--------------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.17 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using **enclosing\_type** = [i64](#)  
*enclosing ring type*
- using **is\_zero\_t** = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr valueType **get** ()  
*cast value in valueType*
- static std::string **to\_string** ()  
*string representation*
- template<typename valueRing >  
static constexpr valueRing **eval** (const valueRing &v)  
*cast value in valueRing*

### Static Public Attributes

- static constexpr int64\_t **v** = x  
*actual value*

### 5.17.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

## Template Parameters

|          |                   |
|----------|-------------------|
| <i>x</i> | an actual integer |
|----------|-------------------|

## 5.17.2 Member Function Documentation

### 5.17.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

## Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueRing</i> | (double for example) |
|------------------|----------------------|

### 5.17.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

## Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueType</i> | (double for example) |
|------------------|----------------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.18 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- `using enclosing_type = polynomial< Ring >`  
*enclosing ring type*
- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true\_type if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 5.18.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

|                        |                           |
|------------------------|---------------------------|
| <code>coeffN</code>    | high degree coefficient   |
| <code>...coeffs</code> | lower degree coefficients |

### 5.18.2 Member Typedef Documentation

#### 5.18.2.1 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

## 5.18.3 Member Function Documentation

### 5.18.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

|                  |                         |
|------------------|-------------------------|
| <i>valueRing</i> | usually float or double |
|------------------|-------------------------|

#### Parameters

|          |       |
|----------|-------|
| <i>x</i> | value |
|----------|-------|

#### Returns

$P(x)$

### 5.18.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## 5.19 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

### 5.19.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

|   |                     |
|---|---------------------|
| V | a value from 'Ring' |
|---|---------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.20 aerobus::zpz< p >::val< x > Struct Template Reference

### Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< x% p==0 >`

### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.21 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

### Public Types

- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is\\_zero\\_t = std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)

### Static Public Member Functions

- [static std::string to\\_string \(\)](#)
- [template<typename valueRing >](#)  
[static constexpr valueRing eval \(const valueRing &x\)](#)

### Static Public Attributes

- [static constexpr size\\_t degree = 0](#)  
*degree*
- [static constexpr bool is\\_zero\\_v = is\\_zero\\_t::value](#)

#### 5.21.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

#### Template Parameters

|                        |  |
|------------------------|--|
| <a href="#">coeffN</a> |  |
|------------------------|--|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.22 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)

### Public Types

- [using inner\\_type = int32\\_t](#)
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int32\\_t >\(x\)>](#)
- [using zero = val< 0 >](#)
- [using one = val< 1 >](#)
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*addition operator*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction operator*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication operator*
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)  
*division operator*
- [template<typename v1 , typename v2 >](#)  
[using mod\\_t = typename remainder< v1, v2 >::type](#)  
*modulo operator*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt< v1, v2 >::type](#)  
*strictly greater operator (type)*
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt< v1, v2 >::type](#)  
*strictly smaller operator (type)*
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq< v1, v2 >::type](#)  
*equality operator (type)*
- [template<typename v1 , typename v2 >](#)  
[using gcd\\_t = gcd\\_t< i32, v1, v2 >](#)  
*greatest common divisor*
- [template<typename v1 >](#)  
[using pos\\_t = typename pos< v1 >::type](#)  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

### 5.22.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

### 5.22.2 Member Typedef Documentation

#### 5.22.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

#### Template Parameters

|                 |                                  |
|-----------------|----------------------------------|
| <code>v1</code> | a value in <code>zpz::val</code> |
| <code>v2</code> | a value in <code>zpz::val</code> |

#### 5.22.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 5.22.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 5.22.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 5.22.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 5.22.2.6 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 5.22.2.7 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 5.22.2.8 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 5.22.2.9 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

## 5.22.2.10 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 5.22.3 Member Data Documentation

## 5.22.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 5.22.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 5.22.3.3 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 5.22.3.4 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## Chapter 6

# File Documentation

### 6.1 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 6.2 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00027
00028 // aligned allocation
00029 namespace aerobus {
00036     template<typename T>
00037     T* aligned_malloc(size_t count, size_t alignment) {
00038         #ifdef _MSC_VER
```

```

00039         return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00040     #else
00041     return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00042     #endif
00043 }
00044 } // namespace aerobus
00045
00046 // concepts
00047 namespace aerobus {
00048     template <typename R>
00049     concept IsRing = requires {
00050         typename R::one;
00051         typename R::zero;
00052         typename R::template add_t<typename R::one, typename R::one>;
00053         typename R::template sub_t<typename R::one, typename R::one>;
00054         typename R::template mul_t<typename R::one, typename R::one>;
00055     };
00056
00057     template <typename R>
00058     concept IsEuclideanDomain = IsRing<R> && requires {
00059         typename R::template div_t<typename R::one, typename R::one>;
00060         typename R::template mod_t<typename R::one, typename R::one>;
00061         typename R::template gcd_t<typename R::one, typename R::one>;
00062         typename R::template eq_t<typename R::one, typename R::one>;
00063         typename R::template pos_t<typename R::one>;
00064
00065         R::template pos_v<typename R::one> == true;
00066         // typename R::template gt_t<typename R::one, typename R::zero>;
00067         R::is_euclidean_domain == true;
00068     };
00069
00070     template<typename R>
00071     concept IsField = IsEuclideanDomain<R> && requires {
00072         R::is_field == true;
00073     };
00074 } // namespace aerobus
00075
00076 // utilities
00077 namespace aerobus {
00078     namespace internal {
00079         template<template<typename...> typename TT, typename T>
00080         struct is_instantiation_of : std::false_type { };
00081
00082         template<template<typename...> typename TT, typename... Ts>
00083         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00084
00085         template<template<typename...> typename TT, typename T>
00086         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00087
00088         template<int64_t i, typename T, typename... Ts>
00089         struct type_at {
00090             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00091             using type = typename type_at<i - 1, Ts...>::type;
00092         };
00093
00094         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00095             using type = T;
00096         };
00097
00098         template <size_t i, typename... Ts>
00099         using type_at_t = typename type_at<i, Ts...>::type;
00100
00101         template<size_t n, size_t i, typename E = void>
00102         struct _is_prime {};
00103
00104         template<size_t i>
00105         struct _is_prime<0, i> {
00106             static constexpr bool value = false;
00107         };
00108
00109         template<size_t i>
00110         struct _is_prime<1, i> {
00111             static constexpr bool value = false;
00112         };
00113
00114         template<size_t i>
00115         struct _is_prime<2, i> {
00116             static constexpr bool value = true;
00117         };
00118
00119         template<size_t i>
00120         struct _is_prime<3, i> {
00121             static constexpr bool value = true;
00122         };
00123
00124         template<size_t i>

```

```

00129     struct _is_prime<5, i> {
00130         static constexpr bool value = true;
00131     };
00132
00133     template<size_t i>
00134     struct _is_prime<7, i> {
00135         static constexpr bool value = true;
00136     };
00137
00138     template<size_t n, size_t i>
00139     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00140         static constexpr bool value = false;
00141     };
00142
00143     template<size_t n, size_t i>
00144     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00145         static constexpr bool value = false;
00146     };
00147
00148     template<size_t n, size_t i>
00149     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00150         static constexpr bool value = true;
00151     };
00152
00153     template<size_t n, size_t i>
00154     struct _is_prime<n, i, std::enable_if_t<(
00155         n % i == 0 &&
00156         n >= 9 &&
00157         n % 3 != 0 &&
00158         n % 2 != 0 &&
00159         i * i > n)>> {
00160         static constexpr bool value = true;
00161     };
00162
00163     template<size_t n, size_t i>
00164     struct _is_prime<n, i, std::enable_if_t<(
00165         n % (i+2) == 0 &&
00166         n >= 9 &&
00167         n % 3 != 0 &&
00168         n % 2 != 0 &&
00169         i * i <= n)>> {
00170         static constexpr bool value = true;
00171     };
00172
00173     template<size_t n, size_t i>
00174     struct _is_prime<n, i, std::enable_if_t<(
00175         n % (i+2) != 0 &&
00176         n % i != 0 &&
00177         n >= 9 &&
00178         n % 3 != 0 &&
00179         n % 2 != 0 &&
00180         (i * i <= n))>> {
00181         static constexpr bool value = _is_prime<n, i+6>::value;
00182     };
00183
00184 } // namespace internal
00185
00186 template<size_t n>
00187 struct is_prime {
00188     static constexpr bool value = internal::_is_prime<n, 5>::value;
00189 };
00190
00191 template<size_t n>
00192 static constexpr bool is_prime_v = is_prime<n>::value;
00193
00194 // gcd
00195 namespace internal {
00196     template <std::size_t... Is>
00197     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00198         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00199
00200     template <std::size_t N>
00201     using make_index_sequence_reverse
00202         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00203
00204     template<typename Ring, typename E = void>
00205     struct gcd;
00206
00207     template<typename Ring>
00208     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00209         template<typename A, typename B, typename E = void>
00210         struct gcd_helper {};
00211
00212         // B = 0, A > 0
00213         template<typename A, typename B>
00214         struct gcd_helper<A, B, std::enable_if_t<
00215             (B::is_zero_t::value) &&

```

```

00227         (Ring::template gt_t<A, typename Ring::zero>::value))» {
00228             using type = A;
00229         };
00230
00231         // B = 0, A < 0
00232         template<typename A, typename B>
00233         struct gcd_helper<A, B, std::enable_if_t<
00234             ((B::is_zero_t::value) &&
00235              !(Ring::template gt_t<A, typename Ring::zero>::value))» {
00236             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00237         };
00238
00239         // B != 0
00240         template<typename A, typename B>
00241         struct gcd_helper<A, B, std::enable_if_t<
00242             (!B::is_zero_t::value)
00243             » {
00244             private: // NOLINT
00245                 // A / B
00246                 using k = typename Ring::template div_t<A, B>;
00247                 // A - (A/B)*B = A % B
00248                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00249
00250             public:
00251                 using type = typename gcd_helper<B, m>::type;
00252             };
00253
00254         template<typename A, typename B>
00255         using type = typename gcd_helper<A, B>::type;
00256     };
00257 } // namespace internal
00258
00259 // vadd and vmul
00260 namespace internal {
00261     template<typename... vals>
00262     struct vmul {};
00263
00264     template<typename v1, typename... vals>
00265     struct vmul<v1, vals...> {
00266         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00267     };
00268
00269     template<typename v1>
00270     struct vmul<v1> {
00271         using type = v1;
00272     };
00273
00274     template<typename... vals>
00275     struct vadd {};
00276
00277     template<typename v1, typename... vals>
00278     struct vadd<v1, vals...> {
00279         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00280     };
00281
00282     template<typename v1>
00283     struct vadd<v1> {
00284         using type = v1;
00285     };
00286 } // namespace internal
00287
00288 template<typename T, typename A, typename B>
00289 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00290
00291 template<typename... vals>
00292 using vadd_t = typename internal::vadd<vals...>::type;
00293
00294 template<typename... vals>
00295 using vmul_t = typename internal::vmul<vals...>::type;
00296
00297 template<typename val>
00298 requires IsEuclideanDomain<typename val::enclosing_type>
00299 using abs_t = std::conditional_t<
00300     val::enclosing_type::template pos_v<val>,
00301     val, typename val::enclosing_type::template sub_t<typename
val::enclosing_type::zero, val>;
00302 } // namespace aerobus
00303
00304 namespace aerobus {
00305     template<typename Ring, typename X>
00306     requires IsRing<Ring>
00307     struct Quotient {
00308         template <typename V>
00309         struct val {
00310             public:

```

```

00328         using type = abs_t<typename Ring::template mod_t<V, X>>;
00329     };
00330
00332     using zero = val<typename Ring::zero>;
00333
00335     using one = val<typename Ring::one>;
00336
00340     template<typename v1, typename v2>
00341     using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00342
00346     template<typename v1, typename v2>
00347     using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00348
00352     template<typename v1, typename v2>
00353     using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00354
00358     template<typename v1, typename v2>
00359     using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00360
00364     template<typename v1, typename v2>
00365     using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00366
00370     template<typename v1, typename v2>
00371     static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00372
00376     template<typename v1>
00377     using pos_t = std::true_type;
00378
00382     template<typename v>
00383     static constexpr bool pos_v = pos_t<v>::value;
00384
00386     static constexpr bool is_euclidean_domain = true;
00387
00391     template<auto x>
00392     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00393
00397     template<typename v>
00398     using inject_ring_t = val<v>;
00399 };
00400 } // namespace aerobus
00401
00402 // type_list
00403 namespace aerobus {
00404     template <typename... Ts>
00405     struct type_list;
00406
00407     namespace internal {
00408         template <typename T, typename... Us>
00409         struct pop_front_h {
00410             using tail = type_list<Us...>;
00411             using head = T;
00412         };
00413     };
00414
00415     template <size_t index, typename L1, typename L2>
00416     struct split_h {
00417     private:
00418         static_assert(index <= L2::length, "index ouf of bounds");
00419         using a = typename L2::pop_front::type;
00420         using b = typename L2::pop_front::tail;
00421         using c = typename L1::template push_back<a>;
00422
00423     public:
00424         using head = typename split_h<index - 1, c, b>::head;
00425         using tail = typename split_h<index - 1, c, b>::tail;
00426     };
00427
00428     template <typename L1, typename L2>
00429     struct split_h<0, L1, L2> {
00430         using head = L1;
00431         using tail = L2;
00432     };
00433
00434     template <size_t index, typename L, typename T>
00435     struct insert_h {
00436         static_assert(index <= L::length, "index ouf of bounds");
00437         using s = typename L::template split<index>;
00438         using left = typename s::head;
00439         using right = typename s::tail;
00440         using ll = typename left::template push_back<T>;
00441         using type = typename ll::template concat<right>;
00442     };
00443
00444     template <size_t index, typename L>
00445     struct remove_h {
00446         using s = typename L::template split<index>;
00447         using left = typename s::head;
00448         using right = typename s::tail;

```

```

00449         using rr = typename right::pop_front::tail;
00450         using type = typename left::template concat<rr>;
00451     };
00452 } // namespace internal
00453
00454 template <typename... Ts>
00455 struct type_list {
00456 private:
00457     template <typename T>
00458     struct concat_h;
00459
00460     template <typename... Us>
00461     struct concat_h<type_list<Us...> {
00462         using type = type_list<Ts..., Us...>;
00463     };
00464
00465 public:
00466     static constexpr size_t length = sizeof...(Ts);
00467
00468     template <typename T>
00469     using push_front = type_list<T, Ts...>;
00470
00471     template <size_t index>
00472     using at = internal::type_at_t<index, Ts...>;
00473
00474     struct pop_front {
00475         using type = typename internal::pop_front_h<Ts...>::head;
00476         using tail = typename internal::pop_front_h<Ts...>::tail;
00477     };
00478
00479     template <typename T>
00480     using push_back = type_list<Ts..., T>;
00481
00482     template <typename U>
00483     using concat = typename concat_h<U>::type;
00484
00485     template <size_t index>
00486     struct split {
00487     private:
00488         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00489
00490     public:
00491         using head = typename inner::head;
00492         using tail = typename inner::tail;
00493     };
00494
00495     template <typename T, size_t index>
00496     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00497
00498     template <size_t index>
00499     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00500 };
00501
00502 template <>
00503 struct type_list<> {
00504     static constexpr size_t length = 0;
00505
00506     template <typename T>
00507     using push_front = type_list<T>;
00508
00509     template <typename T>
00510     using push_back = type_list<T>;
00511
00512     template <typename U>
00513     using concat = U;
00514
00515     // TODO(jewave): assert index == 0
00516     template <typename T, size_t index>
00517     using insert = type_list<T>;
00518 };
00519 } // namespace aerobus
00520
00521 // i32
00522 namespace aerobus {
00523     struct i32 {
00524         using inner_type = int32_t;
00525         template<int32_t x>
00526         struct val {
00527             using enclosing_type = i32;
00528             static constexpr int32_t v = x;
00529
00530             template<typename valueType>
00531             static constexpr valueType get() { return static_cast<valueType>(x); }
00532
00533             using is_zero_t = std::bool_constant<x == 0>;
00534
00535             static std::string to_string() {

```

```

00568         return std::to_string(x);
00569     }
00570
00573     template<typename valueRing>
00574     static constexpr valueRing eval(const valueRing& v) {
00575         return static_cast<valueRing>(x);
00576     }
00577 };
00578
00580 using zero = val<0>;
00582 using one = val<1>;
00584 static constexpr bool is_field = false;
00586 static constexpr bool is_euclidean_domain = true;
00590 template<auto x>
00591 using inject_constant_t = val<static_cast<int32_t>(x)>;
00592
00593 template<typename v>
00594 using inject_ring_t = v;
00595
00596 private:
00597     template<typename v1, typename v2>
00598     struct add {
00599         using type = val<v1::v + v2::v>;
00600     };
00601
00602     template<typename v1, typename v2>
00603     struct sub {
00604         using type = val<v1::v - v2::v>;
00605     };
00606
00607     template<typename v1, typename v2>
00608     struct mul {
00609         using type = val<v1::v * v2::v>;
00610     };
00611
00612     template<typename v1, typename v2>
00613     struct div {
00614         using type = val<v1::v / v2::v>;
00615     };
00616
00617     template<typename v1, typename v2>
00618     struct remainder {
00619         using type = val<v1::v % v2::v>;
00620     };
00621
00622     template<typename v1, typename v2>
00623     struct gt {
00624         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00625     };
00626
00627     template<typename v1, typename v2>
00628     struct lt {
00629         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00630     };
00631
00632     template<typename v1, typename v2>
00633     struct eq {
00634         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00635     };
00636
00637     template<typename v1>
00638     struct pos {
00639         using type = std::bool_constant<(v1::v > 0)>;
00640     };
00641
00642 public:
00643     template<typename v1, typename v2>
00644     using add_t = typename add<v1, v2>::type;
00645
00646     template<typename v1, typename v2>
00647     using sub_t = typename sub<v1, v2>::type;
00648
00649     template<typename v1, typename v2>
00650     using mul_t = typename mul<v1, v2>::type;
00651
00652     template<typename v1, typename v2>
00653     using div_t = typename div<v1, v2>::type;
00654
00655     template<typename v1, typename v2>
00656     using mod_t = typename remainder<v1, v2>::type;
00657
00658     template<typename v1, typename v2>
00659     using gt_t = typename gt<v1, v2>::type;
00660
00661     template<typename v1, typename v2>
00662     using lt_t = typename lt<v1, v2>::type;
00663
00664     template<typename v1, typename v2>
00665     using eq_t = typename eq<v1, v2>::type;
00666
00667     template<typename v1>
00668     using pos_t = typename pos<v1>::type;

```

```

00704     template<typename v1, typename v2>
00705     using eq_t = typename eq<v1, v2>::type;
00706
00711     template<typename v1, typename v2>
00712     static constexpr bool eq_v = eq_t<v1, v2>::value;
00713
00719     template<typename v1, typename v2>
00720     using gcd_t = gcd_t<i32, v1, v2>;
00721
00726     template<typename v>
00727     using pos_t = typename pos<v>::type;
00728
00733     template<typename v>
00734     static constexpr bool pos_v = pos_t<v>::value;
00735 };
00736 } // namespace aerobus
00737
00738 // i64
00739 namespace aerobus {
00741     struct i64 {
00743         using inner_type = int64_t;
00744         template<int64_t x>
00745         struct val {
00747             using enclosing_type = i64;
00748             static constexpr int64_t v = x;
00749
00755             template<typename valueType>
00756             static constexpr valueType get() { return static_cast<valueType>(x); }
00757
00759             using is_zero_t = std::bool_constant<x == 0>;
00760
00762             static std::string to_string() {
00763                 return std::to_string(x);
00764             }
00765
00768             template<typename valueRing>
00769             static constexpr valueRing eval(const valueRing& v) {
00770                 return static_cast<valueRing>(x);
00771             }
00772         };
00773
00777         template<auto x>
00778         using inject_constant_t = val<static_cast<int64_t>(x)>;
00779
00784         template<typename v>
00785         using inject_ring_t = v;
00786
00788         using zero = val<0>;
00789         using one = val<1>;
00792         static constexpr bool is_field = false;
00794         static constexpr bool is_euclidean_domain = true;
00795
00796     private:
00797         template<typename v1, typename v2>
00798         struct add {
00799             using type = val<v1::v + v2::v>;
00800         };
00801
00802         template<typename v1, typename v2>
00803         struct sub {
00804             using type = val<v1::v - v2::v>;
00805         };
00806
00807         template<typename v1, typename v2>
00808         struct mul {
00809             using type = val<v1::v * v2::v>;
00810         };
00811
00812         template<typename v1, typename v2>
00813         struct div {
00814             using type = val<v1::v / v2::v>;
00815         };
00816
00817         template<typename v1, typename v2>
00818         struct remainder {
00819             using type = val<v1::v % v2::v>;
00820         };
00821
00822         template<typename v1, typename v2>
00823         struct gt {
00824             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00825         };
00826
00827         template<typename v1, typename v2>
00828         struct lt {
00829             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00830         };

```



```

00831
00832     template<typename v1, typename v2>
00833     struct eq {
00834         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00835     };
00836
00837     template<typename v>
00838     struct pos {
00839         using type = std::bool_constant<(v::v > 0)>;
00840     };
00841
00842     public:
00843     template<typename v1, typename v2>
00844     using add_t = typename add<v1, v2>::type;
00845
00846     template<typename v1, typename v2>
00847     using sub_t = typename sub<v1, v2>::type;
00848
00849     template<typename v1, typename v2>
00850     using mul_t = typename mul<v1, v2>::type;
00851
00852     template<typename v1, typename v2>
00853     using div_t = typename div<v1, v2>::type;
00854
00855     template<typename v1, typename v2>
00856     using mod_t = typename remainder<v1, v2>::type;
00857
00858     template<typename v1, typename v2>
00859     using gt_t = typename gt<v1, v2>::type;
00860
00861     template<typename v1, typename v2>
00862     static constexpr bool gt_v = gt_t<v1, v2>::value;
00863
00864     template<typename v1, typename v2>
00865     using lt_t = typename lt<v1, v2>::type;
00866
00867     template<typename v1, typename v2>
00868     static constexpr bool lt_v = lt_t<v1, v2>::value;
00869
00870     template<typename v1, typename v2>
00871     using eq_t = typename eq<v1, v2>::type;
00872
00873     template<typename v1, typename v2>
00874     static constexpr bool eq_v = eq_t<v1, v2>::value;
00875
00876     template<typename v1, typename v2>
00877     using gcd_t = gcd_t<i64, v1, v2>;
00878
00879     template<typename v>
00880     using pos_t = typename pos<v>::type;
00881
00882     template<typename v>
00883     static constexpr bool pos_v = pos_t<v>::value;
00884 };
00885 } // namespace aerobus
00886
00887 // z/pz
00888 namespace aerobus {
00889     template<int32_t p>
00890     struct zp {
00891         using inner_type = int32_t;
00892         template<int32_t x>
00893         struct val {
00894             using enclosing_type = zp<p>;
00895             static constexpr int32_t v = x % p;
00896
00897             template<typename valueType>
00898             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00899
00900             using is_zero_t = std::bool_constant<x % p == 0>;
00901             static std::string to_string() {
00902                 return std::to_string(x % p);
00903             }
00904
00905             template<typename valueRing>
00906             static constexpr valueRing eval(const valueRing& v) {
00907                 return static_cast<valueRing>(x % p);
00908             }
00909         };
00910     };
00911
00912     template<auto x>
00913     using inject_constant_t = val<static_cast<int32_t>(x)>;
00914
00915     using zero = val<0>;
00916     using one = val<1>;
00917     static constexpr bool is_prime = is_prime<p>::value;
00918     static constexpr bool is_euclidean_domain = true;
00919 }

```

```

00987
00988     private:
00989         template<typename v1, typename v2>
00990         struct add {
00991             using type = val<(v1::v + v2::v) % p>;
00992         };
00993
00994         template<typename v1, typename v2>
00995         struct sub {
00996             using type = val<(v1::v - v2::v) % p>;
00997         };
00998
00999         template<typename v1, typename v2>
01000         struct mul {
01001             using type = val<(v1::v * v2::v) % p>;
01002         };
01003
01004         template<typename v1, typename v2>
01005         struct div {
01006             using type = val<(v1::v % p) / (v2::v % p)>;
01007         };
01008
01009         template<typename v1, typename v2>
01010         struct remainder {
01011             using type = val<(v1::v % v2::v) % p>;
01012         };
01013
01014         template<typename v1, typename v2>
01015         struct gt {
01016             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01017         };
01018
01019         template<typename v1, typename v2>
01020         struct lt {
01021             using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01022         };
01023
01024         template<typename v1, typename v2>
01025         struct eq {
01026             using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01027         };
01028
01029         template<typename v1>
01030         struct pos {
01031             using type = std::bool_constant<(v1::v > 0)>;
01032         };
01033
01034     public:
01038         template<typename v1, typename v2>
01039         using add_t = typename add<v1, v2>::type;
01040
01044         template<typename v1, typename v2>
01045         using sub_t = typename sub<v1, v2>::type;
01046
01050         template<typename v1, typename v2>
01051         using mul_t = typename mul<v1, v2>::type;
01052
01056         template<typename v1, typename v2>
01057         using div_t = typename div<v1, v2>::type;
01058
01062         template<typename v1, typename v2>
01063         using mod_t = typename remainder<v1, v2>::type;
01064
01068         template<typename v1, typename v2>
01069         using gt_t = typename gt<v1, v2>::type;
01070
01074         template<typename v1, typename v2>
01075         static constexpr bool gt_v = gt_t<v1, v2>::value;
01076
01080         template<typename v1, typename v2>
01081         using lt_t = typename lt<v1, v2>::type;
01082
01086         template<typename v1, typename v2>
01087         static constexpr bool lt_v = lt_t<v1, v2>::value;
01088
01092         template<typename v1, typename v2>
01093         using eq_t = typename eq<v1, v2>::type;
01094
01098         template<typename v1, typename v2>
01099         static constexpr bool eq_v = eq_t<v1, v2>::value;
01100
01104         template<typename v1, typename v2>
01105         using gcd_t = gcd_t<i32, v1, v2>;
01106
01109         template<typename v1>
01110         using pos_t = typename pos<v1>::type;
01111

```

```

01114         template<typename v>
01115         static constexpr bool pos_v = pos_t<v>::value;
01116     };
01117 } // namespace aerobus
01118
01119 // polynomial
01120 namespace aerobus {
01121     // coeffN x^N + ...
01122     template<typename Ring>
01123     requires IsEuclideanDomain<Ring>
01124     struct polynomial {
01125         static constexpr bool is_field = false;
01126         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01127
01128         template<typename coeffN, typename... coeffs>
01129         struct val {
01130             using enclosing_type = polynomial<Ring>;
01131             static constexpr size_t degree = sizeof...(coeffs);
01132             using aN = coeffN;
01133             using strip = val<coeffs...>;
01134             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01135             static constexpr bool is_zero_v = is_zero_t::value;
01136
01137         private:
01138             template<size_t index, typename E = void>
01139             struct coeff_at {};
01140
01141             template<size_t index>
01142             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))>> {
01143                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01144             };
01145
01146             template<size_t index>
01147             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))>> {
01148                 using type = typename Ring::zero;
01149             };
01150
01151         public:
01152             template<size_t index>
01153             using coeff_at_t = typename coeff_at<index>::type;
01154
01155             static std::string to_string() {
01156                 return string_helper<coeffN, coeffs...>::func();
01157             }
01158
01159             template<typename valueRing>
01160             static constexpr valueRing eval(const valueRing& x) {
01161                 return horner_evaluation<valueRing, val>
01162                     ::template inner<0, degree + 1>
01163                     ::func(static_cast<valueRing>(0), x);
01164             }
01165         };
01166     };
01167
01168     template<typename coeffN>
01169     struct val<coeffN> {
01170         using enclosing_type = polynomial<Ring>;
01171         static constexpr size_t degree = 0;
01172         using aN = coeffN;
01173         using strip = val<coeffN>;
01174         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01175         static constexpr bool is_zero_v = is_zero_t::value;
01176
01177         template<size_t index, typename E = void>
01178         struct coeff_at {};
01179
01180         template<size_t index>
01181         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01182             using type = aN;
01183         };
01184
01185         template<size_t index>
01186         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01187             using type = typename Ring::zero;
01188         };
01189
01190         template<size_t index>
01191         using coeff_at_t = typename coeff_at<index>::type;
01192
01193         static std::string to_string() {
01194             return string_helper<coeffN>::func();
01195         }
01196
01197         template<typename valueRing>
01198         static constexpr valueRing eval(const valueRing& x) {
01199             return static_cast<valueRing>(aN::template get<valueRing>());
01200         }
01201     };
01202 }

```

```

01226     };
01227
01229     using zero = val<typename Ring::zero>;
01231     using one = val<typename Ring::one>;
01233     using X = val<typename Ring::one, typename Ring::zero>;
01234
01235 private:
01236     template<typename P, typename E = void>
01237     struct simplify;
01238
01239     template<typename P1, typename P2, typename I>
01240     struct add_low;
01241
01242     template<typename P1, typename P2>
01243     struct add {
01244         using type = typename simplify<typename add_low<
01245             P1,
01246             P2,
01247             internal::make_index_sequence_reverse<
01248                 std::max(P1::degree, P2::degree) + 1
01249             >::type>::type;
01250     };
01251
01252     template<typename P1, typename P2, typename I>
01253     struct sub_low;
01254
01255     template<typename P1, typename P2, typename I>
01256     struct mul_low;
01257
01258     template<typename v1, typename v2>
01259     struct mul {
01260         using type = typename mul_low<
01261             v1,
01262             v2,
01263             internal::make_index_sequence_reverse<
01264                 v1::degree + v2::degree + 1
01265             >::type;
01266     };
01267
01268     template<typename coeff, size_t deg>
01269     struct monomial;
01270
01271     template<typename v, typename E = void>
01272     struct derive_helper {};
01273
01274     template<typename v>
01275     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01276         using type = zero;
01277     };
01278
01279     template<typename v>
01280     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01281         using type = typename add<
01282             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01283             typename monomial<
01284                 typename Ring::template mul_t<
01285                     typename v::aN,
01286                     typename Ring::template inject_constant_t<(v::degree)>
01287                 >,
01288                 v::degree - 1
01289             >::type
01290         >::type;
01291     };
01292
01293     template<typename v1, typename v2, typename E = void>
01294     struct eq_helper {};
01295
01296     template<typename v1, typename v2>
01297     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01298         using type = std::false_type;
01299     };
01300
01301
01302     template<typename v1, typename v2>
01303     struct eq_helper<v1, v2, std::enable_if_t<
01304         v1::degree == v2::degree &&
01305         (v1::degree != 0 || v2::degree != 0) &&
01306         std::is_same<
01307             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01308             std::false_type
01309         >::value
01310     > {
01311     > {
01312         using type = std::false_type;
01313     };
01314
01315     template<typename v1, typename v2>

```

```

01316     struct eq_helper<v1, v2, std::enable_if_t<
01317         v1::degree == v2::degree &&
01318         (v1::degree != 0 || v2::degree != 0) &&
01319         std::is_same<
01320             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01321             std::true_type
01322         >::value
01323     > {
01324         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01325     };
01326
01327     template<typename v1, typename v2>
01328     struct eq_helper<v1, v2, std::enable_if_t<
01329         v1::degree == v2::degree &&
01330         (v1::degree == 0)
01331     > {
01332         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01333     };
01334
01335     template<typename v1, typename v2, typename E = void>
01336     struct lt_helper {};
01337
01338     template<typename v1, typename v2>
01339     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01340         using type = std::true_type;
01341     };
01342
01343     template<typename v1, typename v2>
01344     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01345         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01346     };
01347
01348     template<typename v1, typename v2>
01349     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01350         using type = std::false_type;
01351     };
01352
01353     template<typename v1, typename v2, typename E = void>
01354     struct gt_helper {};
01355
01356     template<typename v1, typename v2>
01357     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01358         using type = std::true_type;
01359     };
01360
01361     template<typename v1, typename v2>
01362     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01363         using type = std::false_type;
01364     };
01365
01366     template<typename v1, typename v2>
01367     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01368         using type = std::false_type;
01369     };
01370
01371     // when high power is zero : strip
01372     template<typename P>
01373     struct simplify<P, std::enable_if_t<
01374         std::is_same<
01375             typename Ring::zero,
01376             typename P::aN
01377         >::value && (P::degree > 0)
01378     > {
01379         using type = typename simplify<typename P::strip>::type;
01380     };
01381
01382     // otherwise : do nothing
01383     template<typename P>
01384     struct simplify<P, std::enable_if_t<
01385         !std::is_same<
01386             typename Ring::zero,
01387             typename P::aN
01388         >::value && (P::degree > 0)
01389     > {
01390         using type = P;
01391     };
01392
01393     // do not simplify constants
01394     template<typename P>
01395     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01396         using type = P;
01397     };
01398
01399     // addition at
01400     template<typename P1, typename P2, size_t index>
01401     struct add_at {
01402         using type =

```

```

01403         typename Ring::template add_t<
01404             typename P1::template coeff_at_t<index>,
01405             typename P2::template coeff_at_t<index>>;
01406     };
01407
01408     template<typename P1, typename P2, size_t index>
01409     using add_at_t = typename add_at<P1, P2, index>::type;
01410
01411     template<typename P1, typename P2, std::size_t... I>
01412     struct add_low<P1, P2, std::index_sequence<I...> {
01413         using type = val<add_at_t<P1, P2, I>...>;
01414     };
01415
01416     // subtraction at
01417     template<typename P1, typename P2, size_t index>
01418     struct sub_at {
01419         using type =
01420             typename Ring::template sub_t<
01421                 typename P1::template coeff_at_t<index>,
01422                 typename P2::template coeff_at_t<index>>;
01423     };
01424
01425     template<typename P1, typename P2, size_t index>
01426     using sub_at_t = typename sub_at<P1, P2, index>::type;
01427
01428     template<typename P1, typename P2, std::size_t... I>
01429     struct sub_low<P1, P2, std::index_sequence<I...> {
01430         using type = val<sub_at_t<P1, P2, I>...>;
01431     };
01432
01433     template<typename P1, typename P2>
01434     struct sub {
01435         using type = typename simplify<typename sub_low<
01436             P1,
01437             P2,
01438             internal::make_index_sequence_reverse<
01439                 std::max(P1::degree, P2::degree) + 1
01440             >::type>::type;
01441     };
01442
01443     // multiplication at
01444     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01445     struct mul_at_loop_helper {
01446         using type = typename Ring::template add_t<
01447             typename Ring::template mul_t<
01448                 typename v1::template coeff_at_t<index>,
01449                 typename v2::template coeff_at_t<k - index>
01450             >,
01451             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01452         >;
01453     };
01454
01455     template<typename v1, typename v2, size_t k, size_t stop>
01456     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01457         using type = typename Ring::template mul_t<
01458             typename v1::template coeff_at_t<stop>,
01459             typename v2::template coeff_at_t<0>>;
01460     };
01461
01462     template<typename v1, typename v2, size_t k, typename E = void>
01463     struct mul_at {};
01464
01465     template<typename v1, typename v2, size_t k>
01466     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01467         using type = typename Ring::zero;
01468     };
01469
01470     template<typename v1, typename v2, size_t k>
01471     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01472         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01473     };
01474
01475     template<typename P1, typename P2, size_t index>
01476     using mul_at_t = typename mul_at<P1, P2, index>::type;
01477
01478     template<typename P1, typename P2, std::size_t... I>
01479     struct mul_low<P1, P2, std::index_sequence<I...> {
01480         using type = val<mul_at_t<P1, P2, I>...>;
01481     };
01482
01483     // division helper
01484     template<typename A, typename B, typename Q, typename R, typename E = void>
01485     struct div_helper {};
01486
01487     template<typename A, typename B, typename Q, typename R>
01488     struct div_helper<A, B, Q, R, std::enable_if_t<
01489         (R::degree < B::degree) ||

```

```

01490         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01491             using q_type = Q;
01492             using mod_type = R;
01493             using gcd_type = B;
01494         };
01495
01496         template<typename A, typename B, typename Q, typename R>
01497         struct div_helper<A, B, Q, R, std::enable_if_t<
01498             (R::degree >= B::degree) &&
01499             !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01500             private: // NOLINT
01501                 using rN = typename R::aN;
01502                 using bN = typename B::aN;
01503                 using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01504                     B::degree>::type;
01505                 using rr = typename sub<R, typename mul<pT, B::type>::type>::type;
01506                 using qq = typename add<Q, pT>::type;
01507             public:
01508                 using q_type = typename div_helper<A, B, qq, rr>::q_type;
01509                 using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01510                 using gcd_type = rr;
01511         };
01512
01513         template<typename A, typename B>
01514         struct div {
01515             static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01516             using q_type = typename div_helper<A, B, zero, A>::q_type;
01517             using m_type = typename div_helper<A, B, zero, A>::mod_type;
01518         };
01519
01520         template<typename P>
01521         struct make_unit {
01522             using type = typename div<P, val<typename P::aN>::q_type>;
01523         };
01524
01525         template<typename coeff, size_t deg>
01526         struct monomial {
01527             using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01528         };
01529
01530         template<typename coeff>
01531         struct monomial<coeff, 0> {
01532             using type = val<coeff>;
01533         };
01534
01535         template<typename valueRing, typename P>
01536         struct horner_evaluation {
01537             template<size_t index, size_t stop>
01538             struct inner {
01539                 static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01540                     constexpr valueRing coeff =
01541                         static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01542                             get<valueRing>());
01543                     return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01544                         accum + coeff, x);
01545                 }
01546             };
01547             template<size_t stop>
01548             struct inner<stop, stop> {
01549                 static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01550                     return accum;
01551                 }
01552             };
01553         };
01554
01555         template<typename coeff, typename... coeffs>
01556         struct string_helper {
01557             static std::string func() {
01558                 std::string tail = string_helper<coeffs...>::func();
01559                 std::string result = "";
01560                 if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01561                     return tail;
01562                 } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01563                     if (sizeof...(coeffs) == 1) {
01564                         result += "x";
01565                     } else {
01566                         result += "x^" + std::to_string(sizeof...(coeffs));
01567                     }
01568                 } else {
01569                     if (sizeof...(coeffs) == 1) {
01570                         result += coeff::to_string() + " x";
01571                     } else {
01572                         result += coeff::to_string()
01573                             + " x^" + std::to_string(sizeof...(coeffs));
01574                     }
01575                 }
01576             }
01577         };

```

```

01574         }
01575
01576         if (!tail.empty()) {
01577             result += " + " + tail;
01578         }
01579
01580         return result;
01581     }
01582 };
01583
01584 template<typename coeff>
01585 struct string_helper<coeff> {
01586     static std::string func() {
01587         if (!std::is_same<coeff, typename Ring::zero>::value) {
01588             return coeff::to_string();
01589         } else {
01590             return "";
01591         }
01592     }
01593 };
01594
01595 public:
01596     template<typename P>
01597     using simplify_t = typename simplify<P>::type;
01598
01599     template<typename v1, typename v2>
01600     using add_t = typename add<v1, v2>::type;
01601
01602     template<typename v1, typename v2>
01603     using sub_t = typename sub<v1, v2>::type;
01604
01605     template<typename v1, typename v2>
01606     using mul_t = typename mul<v1, v2>::type;
01607
01608     template<typename v1, typename v2>
01609     using eq_t = typename eq_helper<v1, v2>::type;
01610
01611     template<typename v1, typename v2>
01612     using lt_t = typename lt_helper<v1, v2>::type;
01613
01614     template<typename v1, typename v2>
01615     using gt_t = typename gt_helper<v1, v2>::type;
01616
01617     template<typename v1, typename v2>
01618     using div_t = typename div<v1, v2>::q_type;
01619
01620     template<typename v1, typename v2>
01621     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01622
01623     template<typename coeff, size_t deg>
01624     using monomial_t = typename monomial<coeff, deg>::type;
01625
01626     template<typename v>
01627     using derive_t = typename derive_helper<v>::type;
01628
01629     template<typename v>
01630     using pos_t = typename Ring::template pos_t<typename v::aN>;
01631
01632     template<typename v>
01633     static constexpr bool pos_v = pos_t<v>::value;
01634
01635     template<typename v1, typename v2>
01636     using gcd_t = std::conditional_t<
01637         Ring::is_euclidean_domain,
01638         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01639         void>;
01640
01641     template<auto x>
01642     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01643
01644     template<typename v>
01645     using inject_ring_t = val<v>;
01646 };
01647 } // namespace aerobus
01648
01649 // fraction field
01650 namespace aerobus {
01651     namespace internal {
01652         template<typename Ring, typename E = void>
01653         requires IsEuclideanDomain<Ring>
01654         struct _FractionField {};
01655
01656         template<typename Ring>
01657         requires IsEuclideanDomain<Ring>
01658         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01659             static constexpr bool is_field = true;
01660             static constexpr bool is_euclidean_domain = true;
01661         };
01662     }
01663 }

```



```

01706
01707 private:
01708     template<typename val1, typename val2, typename E = void>
01709     struct to_string_helper {};
01710
01711     template<typename val1, typename val2>
01712     struct to_string_helper <val1, val2,
01713         std::enable_if_t<
01714             Ring::template eq_t<
01715                 val2, typename Ring::one
01716             >::value
01717         >
01718     > {
01719         static std::string func() {
01720             return val1::to_string();
01721         }
01722     };
01723
01724     template<typename val1, typename val2>
01725     struct to_string_helper<val1, val2,
01726         std::enable_if_t<
01727             !Ring::template eq_t<
01728                 val2,
01729                 typename Ring::one
01730             >::value
01731         >
01732     > {
01733         static std::string func() {
01734             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01735         }
01736     };
01737
01738 public:
01742     template<typename val1, typename val2>
01743     struct val {
01744         using x = val1;
01745         using y = val2;
01746         using is_zero_t = typename val1::is_zero_t;
01747         static constexpr bool is_zero_v = val1::is_zero_t::value;
01748
01749         using ring_type = Ring;
01750         using enclosing_type = _FractionField<Ring>;
01751
01752         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01753
01754         template<typename valueType>
01755         static constexpr valueType get() { return static_cast<valueType>(x::v) /
01756             static_cast<valueType>(y::v); }
01757
01758         static std::string to_string() {
01759             return to_string_helper<val1, val2>::func();
01760         }
01761
01762         template<typename valueRing>
01763         static constexpr valueRing eval(const valueRing& v) {
01764             return x::eval(v) / y::eval(v);
01765         }
01766     };
01767
01768     using zero = val<typename Ring::zero, typename Ring::one>;
01769     using one = val<typename Ring::one, typename Ring::one>;
01770
01771     template<typename v>
01772     using inject_t = val<v, typename Ring::one>;
01773
01774     template<auto x>
01775     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01776     Ring::one>;
01777
01778     template<typename v>
01779     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01780
01781     using ring_type = Ring;
01782
01783 private:
01784     template<typename v, typename E = void>
01785     struct simplify {};
01786
01787     // x = 0
01788     template<typename v>
01789     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01790         using type = typename _FractionField<Ring>::zero;
01791     };
01792
01793     // x != 0
01794     template<typename v>
01795     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {

```

```

01819     private:
01820         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01821         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01822         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01823
01824         using posx = std::conditional_t<
01825             !Ring::template pos_v<newy>,
01826             typename Ring::template sub_t<typename Ring::zero, newx>,
01827             newx>;
01828         using posy = std::conditional_t<
01829             !Ring::template pos_v<newy>,
01830             typename Ring::template sub_t<typename Ring::zero, newy>,
01831             newy>;
01832     public:
01833         using type = typename _FractionField<Ring>::template val<posx, posy>;
01834     };
01835
01836 public:
01837     template<typename v>
01838     using simplify_t = typename simplify<v>::type;
01839
01840 private:
01841     template<typename v1, typename v2>
01842     struct add {
01843     private:
01844         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01845         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01846         using dividend = typename Ring::template add_t<a, b>;
01847         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01848         using g = typename Ring::template gcd_t<dividend, divider>;
01849
01850     public:
01851         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01852             divider>>;
01853     };
01854
01855     template<typename v>
01856     struct pos {
01857     private:
01858         using type = std::conditional_t<
01859             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01860             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01861             std::true_type,
01862             std::false_type>;
01863     };
01864
01865     template<typename v1, typename v2>
01866     struct sub {
01867     private:
01868         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01869         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01870         using dividend = typename Ring::template sub_t<a, b>;
01871         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01872         using g = typename Ring::template gcd_t<dividend, divider>;
01873
01874     public:
01875         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01876             divider>>;
01877     };
01878
01879     template<typename v1, typename v2>
01880     struct mul {
01881     private:
01882         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01883         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01884
01885     public:
01886         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
01887     };
01888
01889     template<typename v1, typename v2, typename E = void>
01890     struct div {};
01891
01892     template<typename v1, typename v2>
01893     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
01894         _FractionField<Ring>::zero>::value>> {
01895     private:
01896         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01897         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01898
01899     public:
01900         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
01901     };
01902
01903     template<typename v1, typename v2>
01904     struct div<v1, v2, std::enable_if_t<
01905         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value>> {
01906         using type = one;

```

```

01905         };
01906
01907     template<typename v1, typename v2>
01908     struct eq {
01909         using type = std::conditional_t<
01910             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01911             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01912             std::true_type,
01913             std::false_type>;
01914     };
01915
01916     template<typename v1, typename v2, typename E = void>
01917     struct gt;
01918
01919     template<typename v1, typename v2>
01920     struct gt<v1, v2, std::enable_if_t<
01921         (eq<v1, v2>::type::value)
01922         >> {
01923         using type = std::false_type;
01924     };
01925
01926     template<typename v1, typename v2>
01927     struct gt<v1, v2, std::enable_if_t<
01928         (!eq<v1, v2>::type::value) &&
01929         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01930         >> {
01931         using type = typename gt<
01932             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01933             >::type;
01934     };
01935
01936     template<typename v1, typename v2>
01937     struct gt<v1, v2, std::enable_if_t<
01938         (!eq<v1, v2>::type::value) &&
01939         (pos<v1>::type::value) && (!pos<v2>::type::value)
01940         >> {
01941         using type = std::true_type;
01942     };
01943
01944     template<typename v1, typename v2>
01945     struct gt<v1, v2, std::enable_if_t<
01946         (!eq<v1, v2>::type::value) &&
01947         (!pos<v1>::type::value) && (pos<v2>::type::value)
01948         >> {
01949         using type = std::false_type;
01950     };
01951
01952     template<typename v1, typename v2>
01953     struct gt<v1, v2, std::enable_if_t<
01954         (!eq<v1, v2>::type::value) &&
01955         (pos<v1>::type::value) && (pos<v2>::type::value)
01956         >> {
01957         using type = typename Ring::template gt_t<
01958             typename Ring::template mul_t<v1::x, v2::y>,
01959             typename Ring::template mul_t<v2::y, v2::x>
01960             >;
01961     };
01962
01963     public:
01964     template<typename v1, typename v2>
01965     using add_t = typename add<v1, v2>::type;
01966
01967     template<typename v1, typename v2>
01968     using mod_t = zero;
01969
01970     template<typename v1, typename v2>
01971     using gcd_t = v1;
01972
01973     template<typename v1, typename v2>
01974     using sub_t = typename sub<v1, v2>::type;
01975
01976     template<typename v1, typename v2>
01977     using mul_t = typename mul<v1, v2>::type;
01978
01979     template<typename v1, typename v2>
01980     using div_t = typename div<v1, v2>::type;
01981
01982     template<typename v1, typename v2>
01983     using eq_t = typename eq<v1, v2>::type;
01984
01985     template<typename v1, typename v2>
01986     static constexpr bool eq_v = eq<v1, v2>::type::value;
01987
01988     template<typename v1, typename v2>
01989     using gt_t = typename gt<v1, v2>::type;
01990
01991     template<typename v1, typename v2>

```

```

02025         static constexpr bool gt_v = gt<v1, v2>::type::value;
02026
02029         template<typename v1>
02030         using pos_t = typename pos<v1>::type;
02031
02034         template<typename v>
02035         static constexpr bool pos_v = pos_t<v>::value;
02036     };
02037
02038     template<typename Ring, typename E = void>
02039     requires IsEuclideanDomain<Ring>
02040     struct FractionFieldImpl {};
02041
02042     // fraction field of a field is the field itself
02043     template<typename Field>
02044     requires IsEuclideanDomain<Field>
02045     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02046         using type = Field;
02047         template<typename v>
02048         using inject_t = v;
02049     };
02050
02051     // fraction field of a ring is the actual fraction field
02052     template<typename Ring>
02053     requires IsEuclideanDomain<Ring>
02054     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02055         using type = _FractionField<Ring>;
02056     };
02057 } // namespace internal
02058
02062     template<typename Ring>
02063     requires IsEuclideanDomain<Ring>
02064     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02065 } // namespace aerobus
02066
02067 // short names for common types
02068 namespace aerobus {
02071     using q32 = FractionField<i32>;
02074     using fpq32 = FractionField<polynomial<q32>;
02077     using q64 = FractionField<i64>;
02079     using pi64 = polynomial<i64>;
02081     using pq64 = polynomial<q64>;
02083     using fpq64 = FractionField<polynomial<q64>;
02088     template<typename Ring, typename v1, typename v2>
02089     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02090
02095     template<typename Ring, typename v1, typename v2>
02096     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02101     template<typename Ring, typename v1, typename v2>
02102     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02103 } // namespace aerobus
02104
02105 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
02106 namespace aerobus {
02107     namespace internal {
02108         template<typename T, size_t x, typename E = void>
02109         struct factorial {};
02110
02111         template<typename T, size_t x>
02112         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02113             private:
02114                 template<typename, size_t, typename>
02115                 friend struct factorial;
02116             public:
02117                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02118 x - 1>::type>;
02119                 static constexpr typename T::inner_type value = type::template get<typename
02120 T::inner_type>();
02121             };
02122
02123         template<typename T>
02124         struct factorial<T, 0> {
02125             public:
02126                 using type = typename T::one;
02127                 static constexpr typename T::inner_type value = type::template get<typename
02128 T::inner_type>();
02129             };
02130         };
02131     } // namespace internal
02132
02133     template<typename T, size_t i>
02134     using factorial_t = typename internal::factorial<T, i>::type;
02135
02136     template<typename T, size_t i>
02137     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02138
02139     namespace internal {
02140         template<typename T, size_t k, size_t n, typename E = void>

```

```

02143     struct combination_helper {};
02144
02145     template<typename T, size_t k, size_t n>
02146     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02147         using type = typename FractionField<T>::template mul_t<
02148             typename combination_helper<T, k - 1, n - 1>::type,
02149             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02150     };
02151
02152     template<typename T, size_t k, size_t n>
02153     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02154         using type = typename combination_helper<T, n - k, n>::type;
02155     };
02156
02157     template<typename T, size_t n>
02158     struct combination_helper<T, 0, n> {
02159         using type = typename FractionField<T>::one;
02160     };
02161
02162     template<typename T, size_t k, size_t n>
02163     struct combination {
02164         using type = typename internal::combination_helper<T, k, n>::type::x;
02165         static constexpr typename T::inner_type value =
02166             T::inner_type>());
02167     };
02168     } // namespace internal
02169
02170     template<typename T, size_t k, size_t n>
02171     using combination_t = typename internal::combination<T, k, n>::type;
02172
02173     template<typename T, size_t k, size_t n>
02174     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02175
02176     namespace internal {
02177         template<typename T, size_t m>
02178         struct bernouilli;
02179
02180         template<typename T, typename accum, size_t k, size_t m>
02181         struct bernouilli_helper {
02182             using type = typename bernouilli_helper<
02183                 T,
02184                 addfractions_t<T,
02185                     accum,
02186                     mulfractions_t<T,
02187                         makefraction_t<T,
02188                             combination_t<T, k, m + 1>,
02189                             typename T::one>,
02190                             typename bernouilli<T, k>::type
02191                         >,
02192                     >,
02193                     k + 1,
02194                     m>::type;
02195         };
02196
02197         template<typename T, typename accum, size_t m>
02198         struct bernouilli_helper<T, accum, m, m> {
02199             using type = accum;
02200         };
02201
02202         template<typename T, size_t m>
02203         struct bernouilli {
02204             using type = typename FractionField<T>::template mul_t<
02205                 typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02206                 makefraction_t<T,
02207                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02208                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02209                 >,
02210                 >;
02211
02212         template<typename floatType>
02213         static constexpr floatType value = type::template get<floatType>();
02214     };
02215
02216     template<typename T>
02217     struct bernouilli<T, 0> {
02218         using type = typename FractionField<T>::one;
02219
02220         template<typename floatType>
02221         static constexpr floatType value = type::template get<floatType>();
02222     };
02223     } // namespace internal
02224
02225     template<typename T, size_t n>
02226     using bernouilli_t = typename internal::bernouilli<T, n>::type;

```

```

02238
02243     template<typename FloatType, typename T, size_t n >
02244     inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02245
02246     namespace internal {
02247         template<typename T, int k, typename E = void>
02248         struct alternate {};
02249
02250         template<typename T, int k>
02251         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02252             using type = typename T::one;
02253             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02254         };
02255
02256         template<typename T, int k>
02257         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02258             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02259             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02260         };
02261     } // namespace internal
02262
02263     template<typename T, int k>
02264     using alternate_t = typename internal::alternate<T, k>::type;
02265
02266     namespace internal {
02267         template<typename T, int n, int k, typename E = void>
02268         struct stirling_helper {};
02269
02270         template<typename T>
02271         struct stirling_helper<T, 0, 0> {
02272             using type = typename T::one;
02273         };
02274
02275         template<typename T, int n>
02276         struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02277             using type = typename T::zero;
02278         };
02279
02280         template<typename T, int n>
02281         struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02282             using type = typename T::zero;
02283         };
02284
02285         template<typename T, int n, int k>
02286         struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02287             using type = typename T::template sub_t<
02288                 typename stirling_helper<T, n-1, k-1>::type,
02289                 typename T::template mul_t<
02290                     typename T::template inject_constant_t<n-1>,
02291                     typename stirling_helper<T, n-1, k>::type
02292                 >;
02293         };
02294     } // namespace internal
02295
02296     template<typename T, int n, int k>
02297     using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02298
02299     template<typename T, int n, int k>
02300     using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02301
02302     template<typename T, int n, int k>
02303     static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02304
02305     template<typename T, int n, int k>
02306     static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02307
02308     template<typename T, size_t k>
02309     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02310
02311     namespace internal {
02312         template<typename T, auto p, auto n, typename E = void>
02313         struct pow {};
02314
02315         template<typename T, auto p, auto n>
02316         struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02317             using type = typename T::template mul_t<
02318                 typename pow<T, p, n/2>::type,
02319                 typename pow<T, p, n/2>::type
02320             >;
02321         };
02322
02323         template<typename T, auto p, auto n>
02324         struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02325             using type = typename T::template mul_t<

```

```

02347         typename T::template inject_constant_t<p>,
02348         typename T::template mul_t<
02349             typename pow<T, p, n/2>::type,
02350             typename pow<T, p, n/2>::type
02351         >
02352     >;
02353 };
02354
02355     template<typename T, auto p>
02356     struct pow<T, p, 0> { using type = typename T::one; };
02357 } // namespace internal
02358
02363     template<typename T, auto p, auto n>
02364     using pow_t = typename internal::pow<T, p, n>::type;
02365
02370     template<typename T, auto p, auto n>
02371     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02372
02373     namespace internal {
02374         template<typename, template<typename, size_t> typename, class>
02375         struct make_taylor_impl;
02376
02377         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02378         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02379             using type = typename polynomial<FractionField<T>>::template val<typename coeff_at<T,
Is>::type...>;
02380         };
02381     }
02382
02387     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02388     using taylor = typename internal::make_taylor_impl<
02389         T,
02390         coeff_at,
02391         internal::make_index_sequence_reverse<deg + 1>::type;
02392
02393     namespace internal {
02394         template<typename T, size_t i>
02395         struct exp_coeff {
02396             using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02397         };
02398
02399         template<typename T, size_t i, typename E = void>
02400         struct sin_coeff_helper {};
02401
02402         template<typename T, size_t i>
02403         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02404             using type = typename FractionField<T>::zero;
02405         };
02406
02407         template<typename T, size_t i>
02408         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02409             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02410         };
02411
02412         template<typename T, size_t i>
02413         struct sin_coeff {
02414             using type = typename sin_coeff_helper<T, i>::type;
02415         };
02416
02417         template<typename T, size_t i, typename E = void>
02418         struct sh_coeff_helper {};
02419
02420         template<typename T, size_t i>
02421         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02422             using type = typename FractionField<T>::zero;
02423         };
02424
02425         template<typename T, size_t i>
02426         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02427             using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02428         };
02429
02430         template<typename T, size_t i>
02431         struct sh_coeff {
02432             using type = typename sh_coeff_helper<T, i>::type;
02433         };
02434
02435         template<typename T, size_t i, typename E = void>
02436         struct cos_coeff_helper {};
02437
02438         template<typename T, size_t i>
02439         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02440             using type = typename FractionField<T>::zero;
02441         };
02442
02443         template<typename T, size_t i>
02444         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {

```

```

02445         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02446     };
02447
02448     template<typename T, size_t i>
02449     struct cos_coeff {
02450         using type = typename cos_coeff_helper<T, i>::type;
02451     };
02452
02453     template<typename T, size_t i, typename E = void>
02454     struct cosh_coeff_helper {};
02455
02456     template<typename T, size_t i>
02457     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02458         using type = typename FractionField<T>::zero;
02459     };
02460
02461     template<typename T, size_t i>
02462     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02463         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02464     };
02465
02466     template<typename T, size_t i>
02467     struct cosh_coeff {
02468         using type = typename cosh_coeff_helper<T, i>::type;
02469     };
02470
02471     template<typename T, size_t i>
02472     struct geom_coeff { using type = typename FractionField<T>::one; };
02473
02474
02475     template<typename T, size_t i, typename E = void>
02476     struct atan_coeff_helper;
02477
02478     template<typename T, size_t i>
02479     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02480         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02481     };
02482
02483     template<typename T, size_t i>
02484     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02485         using type = typename FractionField<T>::zero;
02486     };
02487
02488     template<typename T, size_t i>
02489     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02490
02491     template<typename T, size_t i, typename E = void>
02492     struct asin_coeff_helper;
02493
02494     template<typename T, size_t i>
02495     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02496         using type = makefraction_t<T,
02497             factorial_t<T, i - 1>,
02498             typename T::template mul_t<
02499                 typename T::template val<i>,
02500                 T::template mul_t<
02501                     pow_t<T, 4, i / 2>,
02502                     pow<T, factorial<T, i / 2>::value, 2
02503                 >
02504             >
02505         >>;
02506     };
02507
02508     template<typename T, size_t i>
02509     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02510         using type = typename FractionField<T>::zero;
02511     };
02512
02513     template<typename T, size_t i>
02514     struct asin_coeff {
02515         using type = typename asin_coeff_helper<T, i>::type;
02516     };
02517
02518     template<typename T, size_t i>
02519     struct lnpl_coeff {
02520         using type = makefraction_t<T,
02521             alternate_t<T, i + 1>,
02522             typename T::template val<i>;
02523     };
02524
02525     template<typename T>
02526     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02527
02528     template<typename T, size_t i, typename E = void>
02529     struct asinh_coeff_helper;
02530
02531     template<typename T, size_t i>

```



```

02532     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02533         using type = makefraction_t<T,
02534             typename T::template mul_t<
02535                 alternate_t<T, i / 2>,
02536                 factorial_t<T, i - 1>
02537             >,
02538             typename T::template mul_t<
02539                 T::template mul_t<
02540                     typename T::template val<i>,
02541                     pow_t<T, (factorial<T, i / 2>::value), 2>
02542                 >,
02543                 pow_t<T, 4, i / 2>
02544             >
02545         >;
02546     };
02547
02548     template<typename T, size_t i>
02549     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02550         using type = typename FractionField<T>::zero;
02551     };
02552
02553     template<typename T, size_t i>
02554     struct asinh_coeff {
02555         using type = typename asinh_coeff_helper<T, i>::type;
02556     };
02557
02558     template<typename T, size_t i, typename E = void>
02559     struct atanh_coeff_helper;
02560
02561     template<typename T, size_t i>
02562     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02563         // 1/i
02564         using type = typename FractionField<T>::template val<
02565             typename T::one,
02566             typename T::template val<static_cast<typename T::inner_type>(i)>
02567         >;
02568     };
02569
02570     template<typename T, size_t i>
02571     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02572         using type = typename FractionField<T>::zero;
02573     };
02574
02575     template<typename T, size_t i>
02576     struct atanh_coeff {
02577         using type = typename asinh_coeff_helper<T, i>::type;
02578     };
02579
02580     template<typename T, size_t i, typename E = void>
02581     struct tan_coeff_helper;
02582
02583     template<typename T, size_t i>
02584     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02585         using type = typename FractionField<T>::zero;
02586     };
02587
02588     template<typename T, size_t i>
02589     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02590     private:
02591         // 4^((i+1)/2)
02592         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02593         // 4^((i+1)/2) - 1
02594         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02595         // (-1)^((i-1)/2)
02596         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02597         using dividend = typename FractionField<T>::template mul_t<
02598             altp,
02599             FractionField<T>::template mul_t<
02600                 _4p,
02601                 FractionField<T>::template mul_t<
02602                     _4pml,
02603                     bernouilli_t<T, (i + 1)>
02604                 >
02605             >
02606         >;
02607     public:
02608         using type = typename FractionField<T>::template div_t<dividend,
02609             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>
02610         >;
02611     };
02612
02613     template<typename T, size_t i>
02614     struct tan_coeff {
02615         using type = typename tan_coeff_helper<T, i>::type;
02616     };
02617
02618     template<typename T, size_t i, typename E = void>
02619     struct tanh_coeff_helper;

```

```

02618
02619     template<typename T, size_t i>
02620     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02621         using type = typename FractionField<T>::zero;
02622     };
02623
02624     template<typename T, size_t i>
02625     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02626     private:
02627         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02628         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02629         using dividend =
02630             typename FractionField<T>::template mul_t<
02631                 _4p,
02632                 typename FractionField<T>::template mul_t<
02633                     _4pml,
02634                     bernouilli_t<T, (i + 1)>
02635                 >::type;
02636     public:
02637         using type = typename FractionField<T>::template div_t<dividend,
FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02638     };
02639
02640     template<typename T, size_t i>
02641     struct tanh_coeff {
02642         using type = typename tanh_coeff_helper<T, i>::type;
02643     };
02644 } // namespace internal
02645
02646 template<typename T, size_t deg>
02647 using exp = taylor<T, internal::exp_coeff, deg>;
02648
02649 template<typename T, size_t deg>
02650 using expml = typename polynomial<FractionField<T>>::template sub_t<
exp<T, deg>,
02651     typename polynomial<FractionField<T>>::one>;
02652
02653 template<typename T, size_t deg>
02654 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02655
02656 template<typename T, size_t deg>
02657 using atan = taylor<T, internal::atan_coeff, deg>;
02658
02659 template<typename T, size_t deg>
02660 using sin = taylor<T, internal::sin_coeff, deg>;
02661
02662 template<typename T, size_t deg>
02663 using sinh = taylor<T, internal::sh_coeff, deg>;
02664
02665 template<typename T, size_t deg>
02666 using cosh = taylor<T, internal::cosh_coeff, deg>;
02667
02668 template<typename T, size_t deg>
02669 using cos = taylor<T, internal::cos_coeff, deg>;
02670
02671 template<typename T, size_t deg>
02672 using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02673
02674 template<typename T, size_t deg>
02675 using asin = taylor<T, internal::asin_coeff, deg>;
02676
02677 template<typename T, size_t deg>
02678 using asinh = taylor<T, internal::asinh_coeff, deg>;
02679
02680 template<typename T, size_t deg>
02681 using atanh = taylor<T, internal::atanh_coeff, deg>;
02682
02683 template<typename T, size_t deg>
02684 using tan = taylor<T, internal::tan_coeff, deg>;
02685
02686 template<typename T, size_t deg>
02687 using tanh = taylor<T, internal::tanh_coeff, deg>;
02688 } // namespace aerobus
02689
02690 // continued fractions
02691 namespace aerobus {
02692     template<int64_t... values>
02693     struct ContinuedFraction {};
02694
02695     template<int64_t a0>
02696     struct ContinuedFraction<a0> {
02697         using type = typename q64::template inject_constant_t<a0>;
02698         static constexpr double val = type::template get<double>();
02699     };
02700 }

```

```

02753     template<int64_t a0, int64_t... rest>
02754     struct ContinuedFraction<a0, rest...> {
02755         using type = q64::template add_t<
02756             typename q64::template inject_constant_t<a0>,
02757             typename q64::template div_t<
02758                 typename q64::one,
02759                 typename ContinuedFraction<rest...>::type
02760             >>;
02761         static constexpr double val = type::template get<double>();
02762     };
02763
02768     using PI_fraction =
ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02771     using E_fraction =
ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02773     using SQRT2_fraction =
ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02775     using SQRT3_fraction =
ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
// NOLINT
02776 } // namespace aerobus
02777
02778 // known polynomials
02779 namespace aerobus {
02780     // CChebyshev
02781     namespace internal {
02782         template<int kind, size_t deg>
02783         struct chebyshev_helper {
02784             using type = typename pi64::template sub_t<
02785                 typename pi64::template mul_t<
02786                     typename pi64::template mul_t<
02787                         pi64::inject_constant_t<2>,
02788                         typename pi64::X>,
02789                     typename chebyshev_helper<kind, deg - 1>::type
02790                 >,
02791                 typename chebyshev_helper<kind, deg - 2>::type
02792             >;
02793         };
02794
02795         template<>
02796         struct chebyshev_helper<1, 0> {
02797             using type = typename pi64::one;
02798         };
02799
02800         template<>
02801         struct chebyshev_helper<1, 1> {
02802             using type = typename pi64::X;
02803         };
02804
02805         template<>
02806         struct chebyshev_helper<2, 0> {
02807             using type = typename pi64::one;
02808         };
02809
02810         template<>
02811         struct chebyshev_helper<2, 1> {
02812             using type = typename pi64::template mul_t<
02813                 typename pi64::inject_constant_t<2>,
02814                 typename pi64::X>;
02815         };
02816     } // namespace internal
02817
02818     // Laguerre
02819     namespace internal {
02820         template<size_t deg>
02821         struct laguerre_helper {
02822             private:
02823                 // Lk = (1 / k) * ((2 * k - 1 - x) * L_{k-1} - (k - 2) L_{k-2})
02824                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02825                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02826                 // -x + 2k-1
02827                 using p = typename pq64::template val<
02828                     typename q64::template inject_constant_t<-1>,
02829                     typename q64::template inject_constant_t<2 * deg - 1>>;
02830                 // 1/n
02831                 using factor = typename pq64::template inject_ring_t<
02832                     q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02833
02834             public:
02835                 using type = typename pq64::template mul_t <
02836                     factor,
02837                     typename pq64::template sub_t<
02838                         typename pq64::template mul_t<
02839                             p,
02840                             lnm1
02841                         >,
02842                     typename pq64::template mul_t<

```

```

02843         typename pq64::template inject_constant_t<deg-1>,
02844         lnm2
02845     >
02846 >
02847 >;
02848
02849 };
02850
02851 template<>
02852 struct laguerre_helper<0> {
02853     using type = typename pq64::one;
02854 };
02855
02856 template<>
02857 struct laguerre_helper<1> {
02858     using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02859 };
02860 } // namespace internal
02861
02862 // Bernstein
02863 namespace internal {
02864     template<size_t i, size_t m, typename E = void>
02865     struct bernstein_helper {};
02866
02867     template<>
02868     struct bernstein_helper<0, 0> {
02869         using type = typename pi64::one;
02870     };
02871
02872     template<size_t i, size_t m>
02873     struct bernstein_helper<i, m, std::enable_if_t<
02874         (m > 0) && (i == 0)>> {
02875         using type = typename pi64::mul_t<
02876             typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02877             typename bernstein_helper<i, m-1>::type>;
02878     };
02879
02880     template<size_t i, size_t m>
02881     struct bernstein_helper<i, m, std::enable_if_t<
02882         (m > 0) && (i == m)>> {
02883         using type = typename pi64::template mul_t<
02884             typename pi64::X,
02885             typename bernstein_helper<i-1, m-1>::type>;
02886     };
02887
02888     template<size_t i, size_t m>
02889     struct bernstein_helper<i, m, std::enable_if_t<
02890         (m > 0) && (i > 0) && (i < m)>> {
02891         using type = typename pi64::add_t<
02892             typename pi64::mul_t<
02893                 typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02894                 typename bernstein_helper<i, m-1>::type>,
02895             typename pi64::mul_t<
02896                 typename pi64::X,
02897                 typename bernstein_helper<i-1, m-1>::type>;
02898     };
02899 } // namespace internal
02900
02901 namespace known_polynomials {
02902     enum hermite_kind {
02903         probabilist,
02904         physicist
02905     };
02906 }
02907
02908 namespace internal {
02909     template<size_t deg, known_polynomials::hermite_kind kind>
02910     struct hermite_helper {};
02911
02912     template<size_t deg>
02913     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02914     private:
02915         using hnm1 = typename hermite_helper<deg - 1,
02916 known_polynomials::hermite_kind::probabilist>::type;
02917         using hnm2 = typename hermite_helper<deg - 2,
02918 known_polynomials::hermite_kind::probabilist>::type;
02919
02919     public:
02920         using type = typename pi64::template sub_t<
02921             typename pi64::template mul_t<typename pi64::X, hnm1>,
02922             typename pi64::template mul_t<
02923                 typename pi64::template inject_constant_t<deg - 1>,
02924                 hnm2
02925             >
02926         >;
02927     };
02928

```

```

02929     template<size_t deg>
02930     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02931     private:
02932         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02933         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02934     public:
02935         using type = typename pi64::template sub_t<
02936             // 2X Hn-1
02937             typename pi64::template mul_t<
02938                 typename pi64::val<typename i64::template inject_constant_t<2>,
02939                     typename i64::zero>, hnm1>,
02940                 typename pi64::template mul_t<
02941                     typename pi64::template inject_constant_t<2*(deg - 1)>,
02942                         hnm2
02943                     >
02944                 >;
02945     };
02946 };
02947 };
02948
02949 template<>
02950 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02951     using type = typename pi64::one;
02952 };
02953
02954 template<>
02955 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02956     using type = typename pi64::X;
02957 };
02958
02959 template<>
02960 struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02961     using type = typename pi64::one;
02962 };
02963
02964 template<>
02965 struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02966     // 2X
02967     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
typename i64::zero>;
02968 };
02969 } // namespace internal
02970
02971 namespace known_polynomials {
02972     template <size_t deg>
02973     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02974
02975     template <size_t deg>
02976     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02977
02978     template <size_t deg>
02979     using laguerre = typename internal::laguerre_helper<deg>::type;
02980
02981     template <size_t deg>
02982     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02983
02984     template <size_t deg>
02985     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
02986
02987     template<size_t i, size_t m>
02988     using bernstein = typename internal::bernstein_helper<i, m>::type;
02989 } // namespace known_polynomials
02990 } // namespace aerobus
02991
03000
03001 #ifdef AEROBUS_CONWAY_IMPORTS
03002 template<int p, int n>
03003 struct ConwayPolynomial;
03004
03005 #define ZPZV ZPZ::template val
03006 #define POLYV aerobus::polynomial<ZPZ>::template val
03007 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>; }; // NOLINT
03008 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>, ZPZV<1>; }; // NOLINT
03009 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03010 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03011 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03012 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03013 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT

```



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```
        POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;  
    }; // NOLINT  
04953 #endif // AEROBUS_CONWAY_IMPORTS  
04954  
04955 #endif // __INC_AEROBUS__ // NOLINT
```



## Chapter 7

# Examples

### 7.1 QuotientRing

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject\_constant\_t<1>

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject\_constant\_t<1>

Template Parameters

|   |                                      |
|---|--------------------------------------|
| x | a 'constant' from Ring point of view |
|---|--------------------------------------|

### 7.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

|       |                                               |
|-------|-----------------------------------------------|
| ...Ts | types to store and manipulate at compile time |
|-------|-----------------------------------------------|

### 7.3 i32::template

inject a native constant

inject a native constant

Template Parameters

|   |                                              |
|---|----------------------------------------------|
| x | inject_constant_2<2> -> i32::template val<2> |
|---|----------------------------------------------|

## 7.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 7.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 7.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 7.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 7.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  `<i32::val<7>, i32::val<2>>`

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  `<i32::val<7>, i32::val<2>>`

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 7.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` `<i32::val<2>, i32::val<2>>`

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` `<i32::val<2>, i32::val<2>>`

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 7.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

|           |                                                           |
|-----------|-----------------------------------------------------------|
| <i>v1</i> |                                                           |
| <i>v2</i> | <code>&lt;i32::val&lt;1&gt;, i32::val&lt;1&gt;&gt;</code> |

## 7.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  `<i32::val<6>, i32::val<15>>`

greatest common divisor yields  $GCD(v1, v2)$  `<i32::val<6>, i32::val<15>>`

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 7.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

|     |                |
|-----|----------------|
| $v$ | a value in i32 |
|-----|----------------|

## 7.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

|     |                                                       |
|-----|-------------------------------------------------------|
| $v$ | a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code> |
|-----|-------------------------------------------------------|

## 7.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

|     |                                         |
|-----|-----------------------------------------|
| $x$ | <code>inject_constant_t&lt;2&gt;</code> |
|-----|-----------------------------------------|

## 7.15 i64::add\_t

addition operator

addition operator

Template Parameters

|      |                                                                                                             |
|------|-------------------------------------------------------------------------------------------------------------|
| $v1$ | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| $v2$ | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |



## 7.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 7.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 7.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 7.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

|           |                                                                               |
|-----------|-------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 7.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code> |

## 7.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 7.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 7.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 7.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 7.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

### Template Parameters

|                 |                                                                                                              |
|-----------------|--------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                            |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;6&gt;, i64::val&lt;15&gt;&gt;</code> |

## 7.26 i64::pos\_t

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                          |
|-----------------|------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code> |
|-----------------|------------------------------------------------------------------------------------------|

## 7.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

|                |                                                                                          |
|----------------|------------------------------------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code> |
|----------------|------------------------------------------------------------------------------------------|

## 7.28 polynomial

makes the constant (native type) polynomial a\_0

makes the constant (native type) polynomial a\_0

Template Parameters

|   |                                      |
|---|--------------------------------------|
| x | <i32>::template inject_constant_t<2> |
|---|--------------------------------------|

## 7.29 q32::add\_t

addition operator

addition operator

Template Parameters

|    |                                                                                  |
|----|----------------------------------------------------------------------------------|
| v1 | a value                                                                          |
| v2 | a value <q32::val<i32::val<1>, i32::val<2>>, q32::val<i32::val<1>, i32::val<3>>> |

## 7.30 FractionField

Fraction field of an euclidean domain, such as Q for Z.

Fraction field of an euclidean domain, such as Q for Z

Template Parameters

|      |                                                                 |
|------|-----------------------------------------------------------------|
| Ring | <i64> is q64 (rationals with 64 bits numerator and denominator) |
|------|-----------------------------------------------------------------|

## 7.31 PI\_fraction::val

representation of PI as a continued fraction -> 3.14...

## 7.32 E\_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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