

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

Here is a list of all documented concepts with brief descriptions:

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Class Index

2.1 Class List

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are aerobus::i64
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
-----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using **type** = `q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`

Static Public Attributes

- static constexpr double **val** = `type::template get<double>()`

5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
<i>...rest</i>	integers (aerobus::i64)

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in [i32](#), again represented as types

Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity (type)(v > 0)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Data Documentation

5.7.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.7.2.2 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<i>v</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in i64

Public Types

- [using inner_type = int64_t](#)
type for actual values
- [template<auto x>](#)
[using inject_constant_t = val< static_cast< int64_t >\(x\)>](#)
- [template<typename v >](#)
[using inject_ring_t = v](#)
- [using zero = val< 0 >](#)
constant zero
- [using one = val< 1 >](#)
constant one
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
addition operator
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction operator
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication operator
- [template<typename v1 , typename v2 >](#)
[using div_t = typename div< v1, v2 >::type](#)
division operator
- [template<typename v1 , typename v2 >](#)
[using mod_t = typename remainder< v1, v2 >::type](#)
modulus operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt< v1, v2 >::type](#)
strictly greater operator (v1 > v2) - type
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt< v1, v2 >::type](#)
strict less operator (v1 < v2)
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq< v1, v2 >::type](#)
equality operator (type)
- [template<typename v1 , typename v2 >](#)
[using gcd_t = gcd_t< i64, v1, v2 >](#)
greatest common divisor
- [template<typename v >](#)
[using pos_t = typename pos< v >::type](#)
is v positive (type)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator ($v1 > v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator ($v1 < v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Typedef Documentation

5.8.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) - type

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.6 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.7 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.8 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.9 pos_t

```
template<typename v >  
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive (type)

Template Parameters

v1	: an element of aerobus::i64::val
----	---

5.8.2.10 sub_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3 Member Data Documentation

5.8.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator ($v1 > v2$) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.3 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator ($v1 < v2$) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<code>v</code>	: an element of aerobus::i64::val
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.9 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< index, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.10 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< stop, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.11 aerobus::is_prime< n > Struct Template Reference

checks if `n` is prime

```
#include <aerobus.h>
```

Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`
true iff `n` is prime

5.11.1 Detailed Description

```
template<int32_t n>
struct aerobus::is_prime< n >
```

checks if `n` is prime

Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.12 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero](#) = [val< typename Ring::zero >](#)
constant zero
- [using one](#) = [val< typename Ring::one >](#)
constant one
- [using X](#) = [val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t](#) = [typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t](#) = [typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t](#) = [typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1 , typename v2 >](#)
[using mul_t](#) = [typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1 , typename v2 >](#)
[using eq_t](#) = [typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1 , typename v2 >](#)
[using lt_t](#) = [typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1 , typename v2 >](#)
[using gt_t](#) = [typename gt_helper< v1, v2 >::type](#)
strict greater operator

- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::q_type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`
modulo operator
- `template<typename coeff , size_t deg>`
`using monomial_t = typename monomial< coeff, deg >::type`
monomial : coeff X^deg
- `template<typename v >`
`using derive_t = typename derive_helper< v >::type`
derivation operator
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity (an > 0)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

5.12.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.12.2 Member Typedef Documentation

5.12.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.12.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.5 gcd_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.7 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.8 mod_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.9 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^{deg}

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.12.2.10 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.11 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.12.2.12 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.12.2.13 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.13 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.13.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.14 aerobus::Quotient< Ring, X > Struct Template Reference

Classes

- struct [val](#)

Public Types

- `using zero = val< typename Ring::zero >`
- `using one = val< typename Ring::one >`
- `template<typename v1, typename v2 >`
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
- `template<typename v1 >`
`using pos_t = std::true_type`
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `template<typename v1, typename v2 >`
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
- `static constexpr bool is_euclidean_domain = true`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.15 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- `using head = typename inner::head`
- `using tail = typename inner::tail`

5.15.1 Detailed Description

```
template<typename... Ts>  
template<size_t index>  
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

index	
-----------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.16 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- `template<typename T >`
`using push_front = type_list< T, Ts... >`
Adds T to front of the list.
- `template<size_t index>`
`using at = internal::type_at_t< index, Ts... >`
returns type at index
- `template<typename T >`
`using push_back = type_list< Ts..., T >`
pushes T at the tail of the list
- `template<typename U >`
`using concat = typename concat_h< U >::type`
concatenates two list into one
- `template<typename T , size_t index>`
`using insert = typename internal::insert_h< index, type_list< Ts... >, T >::type`
inserts type at index
- `template<size_t index>`
`using remove = typename internal::remove_h< index, type_list< Ts... > >::type`
removes type at index

Static Public Attributes

- `static constexpr size_t length = sizeof...(Ts)`
length of list

5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	
--------------	--

5.16.2 Member Typedef Documentation

5.16.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.16.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.16.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.16.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.16.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.16.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.17 aerobus::type_list<> Struct Reference

Public Types

- template<typename T >
using **push_front** = type_list< T >

- `template<typename T >`
using **push_back** = `type_list< T >`
- `template<typename U >`
using **concat** = U
- `template<typename T , size_t index>`
using **insert** = `type_list< T >`

Static Public Attributes

- static constexpr size_t **length** = 0

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.18 aerobus::i32::val< x > Struct Template Reference

values in `i32`, again represented as types

```
#include <aerobus.h>
```

Public Types

- using **ring_type** = `i32`
Enclosing ring type.
- using **is_zero_t** = `std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast x into valueType
- `static std::string to_string ()`
string representation of value
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast x into valueRing

Static Public Attributes

- static constexpr `int32_t` **v** = x
actual value stored in val type

5.18.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.18.2 Member Function Documentation

5.18.2.1 `eval()`

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.18.2.2 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<code>valueType</code>	double for example
------------------------	--------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.19 `aerobus::i64::val< x >` Struct Template Reference

values in `i64`

```
#include <aerobus.h>
```

Public Types

- `using ring_type = i64`
enclosing ring type
- `using is_zero_t = std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast value in valueType
- `static std::string to_string ()`
string representation
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast value in valueRing

Static Public Attributes

- `static constexpr int64_t v = x`
actual value

5.19.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.19.2 Member Function Documentation

5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

valueType	(double for example)
-----------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.20 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

5.20.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

5.20.2 Member Typedef Documentation

5.20.2.1 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.20.3 Member Function Documentation

5.20.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.20.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

Public Types

- `using type = std::conditional_t< Ring::template pos_v< tmp >, tmp, typename Ring::template sub_t< typename Ring::zero, tmp > >`

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- `using ring_type = zpz< p >`
enclosing ring type
- `using is_zero_t = std::bool_constant< x% p==0 >`

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.23 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct `coeff_at`
- struct `coeff_at< index, std::enable_if_t<(index< 0||index > 0)>> >`
- struct `coeff_at< index, std::enable_if_t<(index==0)>> >`

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
- `using strip = val< coeffN >`
- `using is_zero_t = std::bool_constant< aN::is_zero_t::value >`
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`

Static Public Member Functions

- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`

Static Public Attributes

- `static constexpr size_t degree = 0`
degree
- `static constexpr bool is_zero_v = is_zero_t::value`

5.23.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<code>coeffN</code>	
---------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.24 `aerobus::zpz< p >` Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- `using inner_type = int32_t`
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `using zero = val< 0 >`
- `using one = val< 1 >`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulo operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (type)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strictly smaller operator (type)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v1 >`
`using pos_t = typename pos< v1 >::type`
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.24.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

Chapter 6

File Documentation

6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035 } // namespace aerobus
00036
00037 // concepts
00038 namespace aerobus {
00039     template <typename R>
00040     concept IsRing = requires {
00041         typename R::one;
00042         typename R::zero;
00043         typename R::template add_t<typename R::one, typename R::one>;
00044         typename R::template sub_t<typename R::one, typename R::one>;
00045         typename R::template mul_t<typename R::one, typename R::one>;
00046     };
00047
00048     template <typename R>
00049     concept IsEuclideanDomain = IsRing<R> && requires {
00050         typename R::template div_t<typename R::one, typename R::one>;
00051         typename R::template mod_t<typename R::one, typename R::one>;
00052         typename R::template gcd_t<typename R::one, typename R::one>;
00053         typename R::template eq_t<typename R::one, typename R::one>;
00054         typename R::template pos_t<typename R::one>;
00055     };
00056
00057     R::template pos_v<typename R::one> == true;
00058     // typename R::template gt_t<typename R::one, typename R::zero>;
00059     R::is_euclidean_domain == true;
```

```

00067     };
00068
00070     template<typename R>
00071     concept IsField = IsEuclideanDomain<R> && requires {
00072         R::is_field == true;
00073     };
00074 } // namespace aerobus
00075
00076 // utilities
00077 namespace aerobus {
00078     namespace internal {
00079         template<template<typename...> typename TT, typename T>
00080         struct is_instantiation_of : std::false_type { };
00081
00082         template<template<typename...> typename TT, typename... Ts>
00083         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00084
00085         template<template<typename...> typename TT, typename T>
00086         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00087
00088         template<int64_t i, typename T, typename... Ts>
00089         struct type_at {
00090             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00091             using type = typename type_at<i - 1, Ts...>::type;
00092         };
00093
00094         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00095             using type = T;
00096         };
00097
00098         template<size_t i, typename... Ts>
00099         using type_at_t = typename type_at<i, Ts...>::type;
00100
00101
00102         template<int32_t n, int32_t i, typename E = void>
00103         struct _is_prime {};
00104
00105         template<int32_t i>
00106         struct _is_prime<1, i> {
00107             static constexpr bool value = false;
00108         };
00109
00110         template<int32_t i>
00111         struct _is_prime<2, i> {
00112             static constexpr bool value = true;
00113         };
00114
00115         template<int32_t i>
00116         struct _is_prime<3, i> {
00117             static constexpr bool value = true;
00118         };
00119
00120         template<int32_t i>
00121         struct _is_prime<5, i> {
00122             static constexpr bool value = true;
00123         };
00124
00125         template<int32_t i>
00126         struct _is_prime<7, i> {
00127             static constexpr bool value = true;
00128         };
00129
00130         template<int32_t n, int32_t i>
00131         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00132             static constexpr bool value = false;
00133         };
00134
00135         template<int32_t n, int32_t i>
00136         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00137             static constexpr bool value = false;
00138         };
00139
00140         template<int32_t n, int32_t i>
00141         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00142             static constexpr bool value = true;
00143         };
00144
00145         template<int32_t n, int32_t i>
00146         struct _is_prime<n, i, std::enable_if_t<(
00147             n % i == 0 &&
00148             n >= 9 &&
00149             n % 3 != 0 &&
00150             n % 2 != 0 &&
00151             i * i > n)> {
00152             static constexpr bool value = true;
00153         };
00154

```



```

00155     template<int32_t n, int32_t i>
00156     struct _is_prime<n, i, std::enable_if_t<(
00157         n % (i+2) == 0 &&
00158         n >= 9 &&
00159         n % 3 != 0 &&
00160         n % 2 != 0 &&
00161         i * i <= n)>> {
00162         static constexpr bool value = true;
00163     };
00164
00165     template<int32_t n, int32_t i>
00166     struct _is_prime<n, i, std::enable_if_t<(
00167         n % (i+2) != 0 &&
00168         n % i != 0 &&
00169         n >= 9 &&
00170         n % 3 != 0 &&
00171         n % 2 != 0 &&
00172         (i * i <= n))>> {
00173         static constexpr bool value = _is_prime<n, i+6>::value;
00174     };
00175
00176 } // namespace internal
00177
00180 template<int32_t n>
00181 struct is_prime {
00182     static constexpr bool value = internal::_is_prime<n, 5>::value;
00183 };
00184
00185 template<int32_t n>
00186 static constexpr bool is_prime_v = is_prime<n>::value;
00187
00188 namespace internal {
00189     template <std::size_t... Is>
00190     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00191     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00192
00193     template <std::size_t N>
00194     using make_index_sequence_reverse
00195     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00196
00197     template<typename Ring, typename E = void>
00198     struct gcd;
00199
00200     template<typename Ring>
00201     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00202         template<typename A, typename B, typename E = void>
00203         struct gcd_helper {};
00204
00205         // B = 0, A > 0
00206         template<typename A, typename B>
00207         struct gcd_helper<A, B, std::enable_if_t<
00208             (B::is_zero_t::value) &&
00209             (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00210             using type = A;
00211         };
00212
00213         // B = 0, A < 0
00214         template<typename A, typename B>
00215         struct gcd_helper<A, B, std::enable_if_t<
00216             ((B::is_zero_t::value) &&
00217             !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00218             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00219         };
00220
00221         // B != 0
00222         template<typename A, typename B>
00223         struct gcd_helper<A, B, std::enable_if_t<
00224             (!B::is_zero_t::value)
00225             >> {
00226             private: // NOLINT
00227                 // A / B
00228                 using k = typename Ring::template div_t<A, B>;
00229                 // A - (A/B)*B = A % B
00230                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00231
00232             public:
00233                 using type = typename gcd_helper<B, m>::type;
00234         };
00235
00236         template<typename A, typename B>
00237         using type = typename gcd_helper<A, B>::type;
00238     };
00239 } // namespace internal
00240
00241 template<typename T, typename A, typename B>
00242 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00243
00244

```

```

00252     template<typename val>
00253     requires IsEuclideanDomain<typename val::ring_type>
00254     using abs_t = std::conditional_t<
00255         val::ring_type::template pos_v<val>,
00256         val, typename val::ring_type::template sub_t<typename val::ring_type::zero, val>;
00257 } // namespace aerobus
00258
00259 // quotient ring by the principal ideal generated by X
00260 namespace aerobus {
00261     template<typename Ring, typename X>
00262     requires IsRing<Ring>
00263     struct Quotient {
00264         template <typename V>
00265         struct val {
00266             private: // NOLINT
00267                 using tmp = typename Ring::template mod_t<V, X>;
00268
00269             public:
00270                 using type = std::conditional_t<
00271                     Ring::template pos_v<tmp>,
00272                     tmp,
00273                     typename Ring::template sub_t<typename Ring::zero, tmp>
00274                 >;
00275         };
00276
00277         using zero = val<typename Ring::zero>;
00278         using one = val<typename Ring::one>;
00279
00280         template<typename v1, typename v2>
00281         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00282         template<typename v1, typename v2>
00283         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00284         template<typename v1, typename v2>
00285         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00286         template<typename v1, typename v2>
00287         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00288         template<typename v1, typename v2>
00289         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00290         template<typename v1, typename v2>
00291         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00292         template<typename v1>
00293         using pos_t = std::true_type;
00294
00295         template<typename v>
00296         static constexpr bool pos_v = pos_t<v>::value;
00297
00298         static constexpr bool is_euclidean_domain = true;
00299
00300         template<auto x>
00301         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00302
00303         template<typename v>
00304         using inject_ring_t = val<v>;
00305     };
00306 } // namespace aerobus
00307
00308 // type_list
00309 namespace aerobus {
00310     template <typename... Ts>
00311     struct type_list;
00312
00313     namespace internal {
00314         template <typename T, typename... Us>
00315         struct pop_front_h {
00316             using tail = type_list<Us...>;
00317             using head = T;
00318         };
00319     };
00320
00321     template <size_t index, typename L1, typename L2>
00322     struct split_h {
00323     private:
00324         static_assert(index <= L2::length, "index out of bounds");
00325         using a = typename L2::pop_front::type;
00326         using b = typename L2::pop_front::tail;
00327         using c = typename L1::template push_back<a>;
00328
00329     public:
00330         using head = typename split_h<index - 1, c, b>::head;
00331         using tail = typename split_h<index - 1, c, b>::tail;
00332     };
00333
00334     template <typename L1, typename L2>
00335     struct split_h<0, L1, L2> {
00336         using head = L1;
00337         using tail = L2;
00338     };
00339 }

```

```

00340     template <size_t index, typename L, typename T>
00341     struct insert_h {
00342         static_assert(index <= L::length, "index ouf of bounds");
00343         using s = typename L::template split<index>;
00344         using left = typename s::head;
00345         using right = typename s::tail;
00346         using ll = typename left::template push_back<T>;
00347         using type = typename ll::template concat<right>;
00348     };
00349
00350     template <size_t index, typename L>
00351     struct remove_h {
00352         using s = typename L::template split<index>;
00353         using left = typename s::head;
00354         using right = typename s::tail;
00355         using rr = typename right::pop_front::tail;
00356         using type = typename left::template concat<rr>;
00357     };
00358 } // namespace internal
00359
00360
00361 template <typename... Ts>
00362 struct type_list {
00363 private:
00364     template <typename T>
00365     struct concat_h;
00366
00367     template <typename... Us>
00368     struct concat_h<type_list<Us...> {
00369         using type = type_list<Ts..., Us...>;
00370     };
00371
00372 public:
00373     static constexpr size_t length = sizeof...(Ts);
00374
00375     template <typename T>
00376     using push_front = type_list<T, Ts...>;
00377
00378     template <size_t index>
00379     using at = internal::type_at_t<index, Ts...>;
00380
00381     struct pop_front {
00382         using type = typename internal::pop_front_h<Ts...>::head;
00383         using tail = typename internal::pop_front_h<Ts...>::tail;
00384     };
00385
00386     template <typename T>
00387     using push_back = type_list<Ts..., T>;
00388
00389     template <typename U>
00390     using concat = typename concat_h<U>::type;
00391
00392     template <size_t index>
00393     struct split {
00394 private:
00395         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00396
00397     public:
00398         using head = typename inner::head;
00399         using tail = typename inner::tail;
00400     };
00401
00402     template <typename T, size_t index>
00403     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00404
00405     template <size_t index>
00406     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00407 };
00408
00409 template <>
00410 struct type_list<> {
00411     static constexpr size_t length = 0;
00412
00413     template <typename T>
00414     using push_front = type_list<T>;
00415
00416     template <typename T>
00417     using push_back = type_list<T>;
00418
00419     template <typename U>
00420     using concat = U;
00421
00422     // TODO(jewave): assert index == 0
00423     template <typename T, size_t index>
00424     using insert = type_list<T>;
00425 };
00426 } // namespace aerobus

```

```

00448
00449 // i32
00450 namespace aerobus {
00451     struct i32 {
00452         using inner_type = int32_t;
00453         template<int32_t x>
00454         struct val {
00455             using ring_type = i32;
00456             static constexpr int32_t v = x;
00457
00458             template<typename valueType>
00459             static constexpr valueType get() { return static_cast<valueType>(x); }
00460
00461             using is_zero_t = std::bool_constant<x == 0>;
00462
00463             static std::string to_string() {
00464                 return std::to_string(x);
00465             }
00466
00467             template<typename valueRing>
00468             static constexpr valueRing eval(const valueRing& v) {
00469                 return static_cast<valueRing>(x);
00470             }
00471         };
00472
00473         using zero = val<0>;
00474         using one = val<1>;
00475         static constexpr bool is_field = false;
00476         static constexpr bool is_euclidean_domain = true;
00477         template<auto x>
00478         using inject_constant_t = val<static_cast<int32_t>(x)>;
00479
00480         template<typename v>
00481         using inject_ring_t = v;
00482
00483     private:
00484         template<typename v1, typename v2>
00485         struct add {
00486             using type = val<v1::v + v2::v>;
00487         };
00488
00489         template<typename v1, typename v2>
00490         struct sub {
00491             using type = val<v1::v - v2::v>;
00492         };
00493
00494         template<typename v1, typename v2>
00495         struct mul {
00496             using type = val<v1::v * v2::v>;
00497         };
00498
00499         template<typename v1, typename v2>
00500         struct div {
00501             using type = val<v1::v / v2::v>;
00502         };
00503
00504         template<typename v1, typename v2>
00505         struct remainder {
00506             using type = val<v1::v % v2::v>;
00507         };
00508
00509         template<typename v1, typename v2>
00510         struct gt {
00511             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00512         };
00513
00514         template<typename v1, typename v2>
00515         struct lt {
00516             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00517         };
00518
00519         template<typename v1, typename v2>
00520         struct eq {
00521             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00522         };
00523
00524         template<typename v1>
00525         struct pos {
00526             using type = std::bool_constant<(v1::v > 0)>;
00527         };
00528
00529     public:
00530         template<typename v1, typename v2>
00531         using add_t = typename add<v1, v2>::type;
00532
00533         template<typename v1, typename v2>
00534         using sub_t = typename sub<v1, v2>::type;
00535
00536         template<typename v1, typename v2>
00537         using mul_t = typename mul<v1, v2>::type;
00538
00539         template<typename v1, typename v2>
00540         using div_t = typename div<v1, v2>::type;
00541
00542         template<typename v1, typename v2>
00543         using remainder_t = typename remainder<v1, v2>::type;
00544
00545         template<typename v1, typename v2>
00546         using gt_t = typename gt<v1, v2>::type;
00547
00548         template<typename v1, typename v2>
00549         using lt_t = typename lt<v1, v2>::type;
00550
00551         template<typename v1, typename v2>
00552         using eq_t = typename eq<v1, v2>::type;
00553
00554         template<typename v1>
00555         using pos_t = typename pos<v1>::type;

```

```

00555
00557     template<typename v1, typename v2>
00558     using mul_t = typename mul<v1, v2>::type;
00559
00561     template<typename v1, typename v2>
00562     using div_t = typename div<v1, v2>::type;
00563
00565     template<typename v1, typename v2>
00566     using mod_t = typename remainder<v1, v2>::type;
00567
00569     template<typename v1, typename v2>
00570     using gt_t = typename gt<v1, v2>::type;
00571
00573     template<typename v1, typename v2>
00574     using lt_t = typename lt<v1, v2>::type;
00575
00577     template<typename v1, typename v2>
00578     using eq_t = typename eq<v1, v2>::type;
00579
00583     template<typename v1, typename v2>
00584     static constexpr bool eq_v = eq_t<v1, v2>::value;
00585
00587     template<typename v1, typename v2>
00588     using gcd_t = gcd_t<i32, v1, v2>;
00589
00591     template<typename v>
00592     using pos_t = typename pos<v>::type;
00593
00596     template<typename v>
00597     static constexpr bool pos_v = pos_t<v>::value;
00598 };
00599 } // namespace aerobus
00600
00601 // i64
00602 namespace aerobus {
00603     struct i64 {
00604         using inner_type = int64_t;
00605         template<int64_t x>
00606         struct val {
00607             using ring_type = i64;
00608             static constexpr int64_t v = x;
00609
00610             template<typename valueType>
00611             static constexpr valueType get() { return static_cast<valueType>(x); }
00612
00613             using is_zero_t = std::bool_constant<x == 0>;
00614
00615             static std::string to_string() {
00616                 return std::to_string(x);
00617             }
00618
00619             template<typename valueRing>
00620             static constexpr valueRing eval(const valueRing& v) {
00621                 return static_cast<valueRing>(x);
00622             }
00623         };
00624
00625         template<auto x>
00626         using inject_constant_t = val<static_cast<int64_t>(x)>;
00627
00628         template<typename v>
00629         using inject_ring_t = v;
00630
00631         using zero = val<0>;
00632         using one = val<1>;
00633         static constexpr bool is_field = false;
00634         static constexpr bool is_euclidean_domain = true;
00635
00636     private:
00637         template<typename v1, typename v2>
00638         struct add {
00639             using type = val<v1::v + v2::v>;
00640         };
00641
00642         template<typename v1, typename v2>
00643         struct sub {
00644             using type = val<v1::v - v2::v>;
00645         };
00646
00647         template<typename v1, typename v2>
00648         struct mul {
00649             using type = val<v1::v * v2::v>;
00650         };
00651
00652         template<typename v1, typename v2>
00653         struct div {
00654             using type = val<v1::v / v2::v>;
00655         };

```

```

00674     };
00675
00676     template<typename v1, typename v2>
00677     struct remainder {
00678         using type = val<v1::v% v2::v>;
00679     };
00680
00681     template<typename v1, typename v2>
00682     struct gt {
00683         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00684     };
00685
00686     template<typename v1, typename v2>
00687     struct lt {
00688         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00689     };
00690
00691     template<typename v1, typename v2>
00692     struct eq {
00693         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00694     };
00695
00696     template<typename v>
00697     struct pos {
00698         using type = std::bool_constant<(v::v > 0)>;
00699     };
00700
00701     public:
00702     template<typename v1, typename v2>
00703     using add_t = typename add<v1, v2>::type;
00704
00705     template<typename v1, typename v2>
00706     using sub_t = typename sub<v1, v2>::type;
00707
00708     template<typename v1, typename v2>
00709     using mul_t = typename mul<v1, v2>::type;
00710
00711     template<typename v1, typename v2>
00712     using div_t = typename div<v1, v2>::type;
00713
00714     template<typename v1, typename v2>
00715     using mod_t = typename remainder<v1, v2>::type;
00716
00717     template<typename v1, typename v2>
00718     using gt_t = typename gt<v1, v2>::type;
00719
00720     template<typename v1, typename v2>
00721     static constexpr bool gt_v = gt_t<v1, v2>::value;
00722
00723     template<typename v1, typename v2>
00724     using lt_t = typename lt<v1, v2>::type;
00725
00726     template<typename v1, typename v2>
00727     static constexpr bool lt_v = lt_t<v1, v2>::value;
00728
00729     template<typename v1, typename v2>
00730     using eq_t = typename eq<v1, v2>::type;
00731
00732     template<typename v1, typename v2>
00733     static constexpr bool eq_v = eq_t<v1, v2>::value;
00734
00735     template<typename v1, typename v2>
00736     using gcd_t = gcd_t<i64, v1, v2>;
00737
00738     template<typename v>
00739     using pos_t = typename pos<v>::type;
00740
00741     template<typename v>
00742     static constexpr bool pos_v = pos_t<v>::value;
00743     };
00744 } // namespace aerobus
00745
00746 // z/pz
00747 namespace aerobus {
00748     template<int32_t p>
00749     struct zpz {
00750         using inner_type = int32_t;
00751         template<int32_t x>
00752         struct val {
00753             using ring_type = zpz<p>;
00754             static constexpr int32_t v = x % p;
00755
00756             template<typename valueType>
00757             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00758
00759             using is_zero_t = std::bool_constant<x% p == 0>;
00760             static std::string to_string() {

```

```

00807         return std::to_string(x % p);
00808     }
00809
00810     template<typename valueRing>
00811     static constexpr valueRing eval(const valueRing& v) {
00812         return static_cast<valueRing>(x % p);
00813     }
00814 };
00815
00816 template<auto x>
00817 using inject_constant_t = val<static_cast<int32_t>(x)>;
00818
00819 using zero = val<0>;
00820 using one = val<1>;
00821 static constexpr bool is_field = is_prime<p>::value;
00822 static constexpr bool is_euclidean_domain = true;
00823
00824 private:
00825     template<typename v1, typename v2>
00826     struct add {
00827         using type = val<(v1::v + v2::v) % p>;
00828     };
00829
00830     template<typename v1, typename v2>
00831     struct sub {
00832         using type = val<(v1::v - v2::v) % p>;
00833     };
00834
00835     template<typename v1, typename v2>
00836     struct mul {
00837         using type = val<(v1::v * v2::v) % p>;
00838     };
00839
00840     template<typename v1, typename v2>
00841     struct div {
00842         using type = val<(v1::v % p) / (v2::v % p)>;
00843     };
00844
00845     template<typename v1, typename v2>
00846     struct remainder {
00847         using type = val<(v1::v % v2::v) % p>;
00848     };
00849
00850     template<typename v1, typename v2>
00851     struct gt {
00852         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00853     };
00854
00855     template<typename v1, typename v2>
00856     struct lt {
00857         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00858     };
00859
00860     template<typename v1, typename v2>
00861     struct eq {
00862         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00863     };
00864
00865     template<typename v1>
00866     struct pos {
00867         using type = std::bool_constant<(v1::v > 0)>;
00868     };
00869
00870 public:
00871     template<typename v1, typename v2>
00872     using add_t = typename add<v1, v2>::type;
00873
00874     template<typename v1, typename v2>
00875     using sub_t = typename sub<v1, v2>::type;
00876
00877     template<typename v1, typename v2>
00878     using mul_t = typename mul<v1, v2>::type;
00879
00880     template<typename v1, typename v2>
00881     using div_t = typename div<v1, v2>::type;
00882
00883     template<typename v1, typename v2>
00884     using mod_t = typename remainder<v1, v2>::type;
00885
00886     template<typename v1, typename v2>
00887     using gt_t = typename gt<v1, v2>::type;
00888
00889     template<typename v1, typename v2>
00890     static constexpr bool gt_v = gt_t<v1, v2>::value;
00891
00892     template<typename v1, typename v2>
00893     using lt_t = typename lt<v1, v2>::type;
00894
00895     template<typename v1, typename v2>
00896     using eq_t = typename eq<v1, v2>::type;
00897
00898     template<typename v1, typename v2>
00899     using pos_t = typename pos<v1>::type;
00900
00901     template<typename v1, typename v2>
00902     using neg_t = typename neg<v1>::type;

```



```

01020         template<size_t index>
01021         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01022             using type = typename Ring::zero;
01023         };
01024
01025         template<size_t index>
01026         using coeff_at_t = typename coeff_at<index>::type;
01027
01028         static std::string to_string() {
01029             return string_helper<coeffN>::func();
01030         }
01031
01032         template<typename valueRing>
01033         static constexpr valueRing eval(const valueRing& x) {
01034             return static_cast<valueRing>(aN::template get<valueRing>());
01035         }
01036     };
01037
01038     using zero = val<typename Ring::zero>;
01039     using one = val<typename Ring::one>;
01040     using X = val<typename Ring::one, typename Ring::zero>;
01041
01042 private:
01043     template<typename P, typename E = void>
01044     struct simplify;
01045
01046     template <typename P1, typename P2, typename I>
01047     struct add_low;
01048
01049     template<typename P1, typename P2>
01050     struct add {
01051         using type = typename simplify<typename add_low<
01052             P1,
01053             P2,
01054             internal::make_index_sequence_reverse<
01055                 std::max(P1::degree, P2::degree) + 1
01056             >::type>::type;
01057     };
01058
01059     template <typename P1, typename P2, typename I>
01060     struct sub_low;
01061
01062     template <typename P1, typename P2, typename I>
01063     struct mul_low;
01064
01065     template<typename v1, typename v2>
01066     struct mul {
01067         using type = typename mul_low<
01068             v1,
01069             v2,
01070             internal::make_index_sequence_reverse<
01071                 v1::degree + v2::degree + 1
01072             >::type;
01073     };
01074
01075     template<typename coeff, size_t deg>
01076     struct monomial;
01077
01078     template<typename v, typename E = void>
01079     struct derive_helper {};
01080
01081     template<typename v>
01082     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01083         using type = zero;
01084     };
01085
01086     template<typename v>
01087     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01088         using type = typename add<
01089             typename derive_helper<typename simplify<typename v::strip>::type,
01090                 typename monomial<
01091                     typename Ring::template mul_t<
01092                         typename v::aN,
01093                         typename Ring::template inject_constant_t<(v::degree)>
01094                     >,
01095                     v::degree - 1
01096                 >::type
01097             >::type;
01098     };
01099
01100     template<typename v1, typename v2, typename E = void>
01101     struct eq_helper {};
01102
01103     template<typename v1, typename v2>
01104     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01105         using type = std::false_type;
01106     };
01107
01108     template<typename v1, typename v2>
01109     struct eq_helper<v1, v2, std::enable_if_t<v1::degree == v2::degree> {
01110         using type = std::true_type;
01111     };

```

```

01110
01111
01112     template<typename v1, typename v2>
01113     struct eq_helper<v1, v2, std::enable_if_t<
01114         v1::degree == v2::degree &&
01115         (v1::degree != 0 || v2::degree != 0) &&
01116         std::is_same<
01117             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01118             std::false_type
01119         >::value
01120     >
01121     > {
01122         using type = std::false_type;
01123     };
01124
01125     template<typename v1, typename v2>
01126     struct eq_helper<v1, v2, std::enable_if_t<
01127         v1::degree == v2::degree &&
01128         (v1::degree != 0 || v2::degree != 0) &&
01129         std::is_same<
01130             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01131             std::true_type
01132         >::value
01133     >> {
01134         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01135     };
01136
01137     template<typename v1, typename v2>
01138     struct eq_helper<v1, v2, std::enable_if_t<
01139         v1::degree == v2::degree &&
01140         (v1::degree == 0)
01141     >> {
01142         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01143     };
01144
01145     template<typename v1, typename v2, typename E = void>
01146     struct lt_helper {};
01147
01148     template<typename v1, typename v2>
01149     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01150         using type = std::true_type;
01151     };
01152
01153     template<typename v1, typename v2>
01154     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01155         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01156     };
01157
01158     template<typename v1, typename v2>
01159     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01160         using type = std::false_type;
01161     };
01162
01163     template<typename v1, typename v2, typename E = void>
01164     struct gt_helper {};
01165
01166     template<typename v1, typename v2>
01167     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01168         using type = std::true_type;
01169     };
01170
01171     template<typename v1, typename v2>
01172     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01173         using type = std::false_type;
01174     };
01175
01176     template<typename v1, typename v2>
01177     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01178         using type = std::false_type;
01179     };
01180
01181     // when high power is zero : strip
01182     template<typename P>
01183     struct simplify<P, std::enable_if_t<
01184         std::is_same<
01185             typename Ring::zero,
01186             typename P::aN
01187         >::value && (P::degree > 0)
01188     >> {
01189         using type = typename simplify<typename P::strip>::type;
01190     };
01191
01192     // otherwise : do nothing
01193     template<typename P>
01194     struct simplify<P, std::enable_if_t<
01195         !std::is_same<
01196             typename Ring::zero,

```

```

01197         typename P::aN
01198         >::value && (P::degree > 0)
01199     » {
01200         using type = P;
01201     };
01202
01203     // do not simplify constants
01204     template<typename P>
01205     struct simplify<P, std::enable_if_t<P::degree == 0>» {
01206         using type = P;
01207     };
01208
01209     // addition at
01210     template<typename P1, typename P2, size_t index>
01211     struct add_at {
01212         using type =
01213             typename Ring::template add_t<
01214                 typename P1::template coeff_at_t<index>,
01215                 typename P2::template coeff_at_t<index>>;
01216     };
01217
01218     template<typename P1, typename P2, size_t index>
01219     using add_at_t = typename add_at<P1, P2, index>::type;
01220
01221     template<typename P1, typename P2, std::size_t... I>
01222     struct add_low<P1, P2, std::index_sequence<I...>» {
01223         using type = val<add_at_t<P1, P2, I>...>;
01224     };
01225
01226     // subtraction at
01227     template<typename P1, typename P2, size_t index>
01228     struct sub_at {
01229         using type =
01230             typename Ring::template sub_t<
01231                 typename P1::template coeff_at_t<index>,
01232                 typename P2::template coeff_at_t<index>>;
01233     };
01234
01235     template<typename P1, typename P2, size_t index>
01236     using sub_at_t = typename sub_at<P1, P2, index>::type;
01237
01238     template<typename P1, typename P2, std::size_t... I>
01239     struct sub_low<P1, P2, std::index_sequence<I...>» {
01240         using type = val<sub_at_t<P1, P2, I>...>;
01241     };
01242
01243     template<typename P1, typename P2>
01244     struct sub {
01245         using type = typename simplify<typename sub_low<
01246             P1,
01247             P2,
01248             internal::make_index_sequence_reverse<
01249                 std::max(P1::degree, P2::degree) + 1
01250             >::type>::type;
01251     };
01252
01253     // multiplication at
01254     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01255     struct mul_at_loop_helper {
01256         using type = typename Ring::template add_t<
01257             typename Ring::template mul_t<
01258                 typename v1::template coeff_at_t<index>,
01259                 typename v2::template coeff_at_t<k - index>
01260             >,
01261             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01262         >;
01263     };
01264
01265     template<typename v1, typename v2, size_t k, size_t stop>
01266     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01267         using type = typename Ring::template mul_t<
01268             typename v1::template coeff_at_t<stop>,
01269             typename v2::template coeff_at_t<0>>;
01270     };
01271
01272     template<typename v1, typename v2, size_t k, typename E = void>
01273     struct mul_at {};
01274
01275     template<typename v1, typename v2, size_t k>
01276     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>» {
01277         using type = typename Ring::zero;
01278     };
01279
01280     template<typename v1, typename v2, size_t k>
01281     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>» {
01282         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01283     };

```

```

01284
01285     template<typename P1, typename P2, size_t index>
01286     using mul_at_t = typename mul_at<P1, P2, index>::type;
01287
01288     template<typename P1, typename P2, std::size_t... I>
01289     struct mul_low<P1, P2, std::index_sequence<I...> {
01290         using type = val<mul_at_t<P1, P2, I>...>;
01291     };
01292
01293     // division helper
01294     template< typename A, typename B, typename Q, typename R, typename E = void>
01295     struct div_helper {};
01296
01297     template<typename A, typename B, typename Q, typename R>
01298     struct div_helper<A, B, Q, R, std::enable_if_t<
01299         (R::degree < B::degree) ||
01300         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01301         using q_type = Q;
01302         using mod_type = R;
01303         using gcd_type = B;
01304     };
01305
01306     template<typename A, typename B, typename Q, typename R>
01307     struct div_helper<A, B, Q, R, std::enable_if_t<
01308         (R::degree >= B::degree) &&
01309         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01310     private: // NOLINT
01311         using rN = typename R::aN;
01312         using bN = typename B::aN;
01313         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01314         B::degree>::type;
01315         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01316         using qq = typename add<Q, pT>::type;
01317     public:
01318         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01319         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01320         using gcd_type = rr;
01321     };
01322
01323     template<typename A, typename B>
01324     struct div {
01325         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01326         using q_type = typename div_helper<A, B, zero, A>::q_type;
01327         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01328     };
01329
01330     template<typename P>
01331     struct make_unit {
01332         using type = typename div<P, val<typename P::aN>::q_type>;
01333     };
01334
01335     template<typename coeff, size_t deg>
01336     struct monomial {
01337         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01338     };
01339
01340     template<typename coeff>
01341     struct monomial<coeff, 0> {
01342         using type = val<coeff>;
01343     };
01344
01345     template<typename valueRing, typename P>
01346     struct horner_evaluation {
01347         template<size_t index, size_t stop>
01348         struct inner {
01349             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01350                 constexpr valueRing coeff =
01351                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01352                     get<valueRing>());
01353                 return horner_evaluation<valueRing, P::template inner<index + 1, stop>::func(x *
01354                     accum + coeff, x);
01355             }
01356         };
01357         template<size_t stop>
01358         struct inner<stop, stop> {
01359             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01360                 return accum;
01361             }
01362         };
01363     };
01364
01365     template<typename coeff, typename... coeffs>
01366     struct string_helper {
01367         static std::string func() {
01368             std::string tail = string_helper<coeffs...>::func();

```

```

01369         std::string result = "";
01370         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01371             return tail;
01372         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01373             if (sizeof...(coeffs) == 1) {
01374                 result += "x";
01375             } else {
01376                 result += "x^" + std::to_string(sizeof...(coeffs));
01377             }
01378         } else {
01379             if (sizeof...(coeffs) == 1) {
01380                 result += coeff::to_string() + " x";
01381             } else {
01382                 result += coeff::to_string()
01383                     + " x^" + std::to_string(sizeof...(coeffs));
01384             }
01385         }
01386     }
01387     if (!tail.empty()) {
01388         result += " + " + tail;
01389     }
01390     return result;
01391 }
01392 };
01393
01394 template<typename coeff>
01395 struct string_helper<coeff> {
01396     static std::string func() {
01397         if (!std::is_same<coeff, typename Ring::zero>::value) {
01398             return coeff::to_string();
01399         } else {
01400             return "";
01401         }
01402     }
01403 };
01404 };
01405
01406 public:
01407     template<typename P>
01408     using simplify_t = typename simplify<P>::type;
01409
01410     template<typename v1, typename v2>
01411     using add_t = typename add<v1, v2>::type;
01412
01413     template<typename v1, typename v2>
01414     using sub_t = typename sub<v1, v2>::type;
01415
01416     template<typename v1, typename v2>
01417     using mul_t = typename mul<v1, v2>::type;
01418
01419     template<typename v1, typename v2>
01420     using eq_t = typename eq_helper<v1, v2>::type;
01421
01422     template<typename v1, typename v2>
01423     using lt_t = typename lt_helper<v1, v2>::type;
01424
01425     template<typename v1, typename v2>
01426     using gt_t = typename gt_helper<v1, v2>::type;
01427
01428     template<typename v1, typename v2>
01429     using div_t = typename div<v1, v2>::q_type;
01430
01431     template<typename v1, typename v2>
01432     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01433
01434     template<typename coeff, size_t deg>
01435     using monomial_t = typename monomial<coeff, deg>::type;
01436
01437     template<typename v>
01438     using derive_t = typename derive_helper<v>::type;
01439
01440     template<typename v>
01441     using pos_t = typename Ring::template pos_t<typename v::aN>;
01442
01443     template<typename v>
01444     static constexpr bool pos_v = pos_t<v>::value;
01445
01446     template<typename v1, typename v2>
01447     using gcd_t = std::conditional_t<
01448         Ring::is_euclidean_domain,
01449         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01450         void>;
01451
01452     template<auto x>
01453     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01454
01455     template<typename v>

```

```

01498     using inject_ring_t = val<v>;
01499 };
01500 } // namespace aerobus
01501
01502 // fraction field
01503 namespace aerobus {
01504     namespace internal {
01505         template<typename Ring, typename E = void>
01506         requires IsEuclideanDomain<Ring>
01507         struct _FractionField {};
01508
01509         template<typename Ring>
01510         requires IsEuclideanDomain<Ring>
01511         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
01512             static constexpr bool is_field = true;
01513             static constexpr bool is_euclidean_domain = true;
01514
01515         private:
01516             template<typename val1, typename val2, typename E = void>
01517             struct to_string_helper {};
01518
01519             template<typename val1, typename val2>
01520             struct to_string_helper<val1, val2,
01521                 std::enable_if_t<
01522                     Ring::template eq_t<
01523                         val2, typename Ring::one
01524                     >::value
01525                 >
01526             > {
01527                 static std::string func() {
01528                     return val1::to_string();
01529                 }
01530             };
01531
01532             template<typename val1, typename val2>
01533             struct to_string_helper<val1, val2,
01534                 std::enable_if_t<
01535                     !Ring::template eq_t<
01536                         val2,
01537                         typename Ring::one
01538                     >::value
01539                 >
01540             > {
01541                 static std::string func() {
01542                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01543                 }
01544             };
01545
01546         public:
01547             template<typename val1, typename val2>
01548             struct val {
01549                 using x = val1;
01550                 using y = val2;
01551                 using is_zero_t = typename val1::is_zero_t;
01552                 static constexpr bool is_zero_v = val1::is_zero_t::value;
01553
01554                 using ring_type = Ring;
01555                 using field_type = _FractionField<Ring>;
01556
01557                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01558
01559                 template<typename valueType>
01560                 static constexpr valueType get() { return static_cast<valueType>(x::v) /
01561                     static_cast<valueType>(y::v); }
01562
01563                 static std::string to_string() {
01564                     return to_string_helper<val1, val2>::func();
01565                 }
01566
01567                 template<typename valueRing>
01568                 static constexpr valueRing eval(const valueRing& v) {
01569                     return x::eval(v) / y::eval(v);
01570                 }
01571             };
01572
01573             using zero = val<typename Ring::zero, typename Ring::one>;
01574             using one = val<typename Ring::one, typename Ring::one>;
01575
01576             template<typename v>
01577             using inject_t = val<v, typename Ring::one>;
01578
01579             template<auto x>
01580             using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01581                 Ring::one>;
01582
01583             template<typename v>
01584             using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01585
01586         };
01587     };
01588 }

```

```

01611
01612     using ring_type = Ring;
01613
01614 private:
01615     template<typename v, typename E = void>
01616     struct simplify {};
01617
01618     // x = 0
01619     template<typename v>
01620     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01621         using type = typename _FractionField<Ring>::zero;
01622     };
01623
01624     // x != 0
01625     template<typename v>
01626     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01627     private:
01628         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01629         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01630         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01631
01632         using posx = std::conditional_t<
01633             !Ring::template pos_v<newx>,
01634             typename Ring::template sub_t<typename Ring::zero, newx>,
01635             newx>;
01636         using posy = std::conditional_t<
01637             !Ring::template pos_v<newy>,
01638             typename Ring::template sub_t<typename Ring::zero, newy>,
01639             newy>;
01640     public:
01641         using type = typename _FractionField<Ring>::template val<posx, posy>;
01642     };
01643
01644 public:
01645     template<typename v>
01646     using simplify_t = typename simplify<v>::type;
01647
01648 private:
01649     template<typename v1, typename v2>
01650     struct add {
01651     private:
01652         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01653         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01654         using dividend = typename Ring::template add_t<a, b>;
01655         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01656         using g = typename Ring::template gcd_t<dividend, diviser>;
01657
01658     public:
01659         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01660             diviser>;
01661     };
01662
01663     template<typename v>
01664     struct pos {
01665     private:
01666         using type = std::conditional_t<
01667             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01668             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01669             std::true_type,
01670             std::false_type>;
01671     };
01672
01673     template<typename v1, typename v2>
01674     struct sub {
01675     private:
01676         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01677         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01678         using dividend = typename Ring::template sub_t<a, b>;
01679         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01680         using g = typename Ring::template gcd_t<dividend, diviser>;
01681
01682     public:
01683         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01684             diviser>;
01685     };
01686
01687     template<typename v1, typename v2>
01688     struct mul {
01689     private:
01690         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01691         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01692
01693     public:
01694         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01695     };
01696
01697     template<typename v1, typename v2, typename E = void>
01698     struct div {};

```

```

01699
01700     template<typename v1, typename v2>
01701     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01702     private:
01703         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01704         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01705
01706     public:
01707         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01708     };
01709
01710     template<typename v1, typename v2>
01711     struct div<v1, v2, std::enable_if_t<
01712         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01713         using type = one;
01714     };
01715
01716     template<typename v1, typename v2>
01717     struct eq {
01718         using type = std::conditional_t<
01719             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01720             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01721             std::true_type,
01722             std::false_type>;
01723     };
01724
01725     template<typename TL, typename E = void>
01726     struct vadd {};
01727
01728     template<typename TL>
01729     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01730         using head = typename TL::pop_front::type;
01731         using tail = typename TL::pop_front::tail;
01732         using type = typename add<head, typename vadd<tail>::type>::type;
01733     };
01734
01735     template<typename TL>
01736     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01737         using type = typename TL::template at<0>;
01738     };
01739
01740     template<typename... vals>
01741     struct vmul {};
01742
01743     template<typename v1, typename... vals>
01744     struct vmul<v1, vals...> {
01745         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01746     };
01747
01748     template<typename v1>
01749     struct vmul<v1> {
01750         using type = v1;
01751     };
01752
01753     template<typename v1, typename v2, typename E = void>
01754     struct gt;
01755
01756     template<typename v1, typename v2>
01757     struct gt<v1, v2, std::enable_if_t<
01758         (eq<v1, v2>::type::value)
01759         >> {
01760         using type = std::false_type;
01761     };
01762
01763     template<typename v1, typename v2>
01764     struct gt<v1, v2, std::enable_if_t<
01765         (!eq<v1, v2>::type::value) &&
01766         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01767         >> {
01768         using type = typename gt<
01769             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01770             >::type;
01771     };
01772
01773     template<typename v1, typename v2>
01774     struct gt<v1, v2, std::enable_if_t<
01775         (!eq<v1, v2>::type::value) &&
01776         (pos<v1>::type::value) && (!pos<v2>::type::value)
01777         >> {
01778         using type = std::true_type;
01779     };
01780
01781     template<typename v1, typename v2>
01782     struct gt<v1, v2, std::enable_if_t<
01783         (!eq<v1, v2>::type::value) &&

```



```

01785         (!pos<v1>::type::value) && (pos<v2>::type::value)
01786     >> {
01787         using type = std::false_type;
01788     };
01789
01790     template<typename v1, typename v2>
01791     struct gt<v1, v2, std::enable_if_t<
01792         (!eq<v1, v2>::type::value) &&
01793         (pos<v1>::type::value) && (pos<v2>::type::value)
01794     >> {
01795         using type = typename Ring::template gt_t<
01796             typename Ring::template mul_t<v1::x, v2::y>,
01797             typename Ring::template mul_t<v2::y, v2::x>
01798         >;
01799     };
01800
01801     public:
01802         template<typename v1, typename v2>
01803         using add_t = typename add<v1, v2>::type;
01804         template<typename v1, typename v2>
01805         using mod_t = zero;
01806         template<typename v1, typename v2>
01807         using gcd_t = v1;
01808         template<typename... vs>
01809         using vadd_t = typename vadd<vs...>::type;
01810         template<typename... vs>
01811         using vmul_t = typename vmul<vs...>::type;
01812         template<typename v1, typename v2>
01813         using sub_t = typename sub<v1, v2>::type;
01814         template<typename v1, typename v2>
01815         using mul_t = typename mul<v1, v2>::type;
01816         template<typename v1, typename v2>
01817         using div_t = typename div<v1, v2>::type;
01818         template<typename v1, typename v2>
01819         using eq_t = typename eq<v1, v2>::type;
01820         template<typename v1, typename v2>
01821         static constexpr bool eq_v = eq<v1, v2>::type::value;
01822         template<typename v1, typename v2>
01823         using gt_t = typename gt<v1, v2>::type;
01824         template<typename v1, typename v2>
01825         static constexpr bool gt_v = gt<v1, v2>::type::value;
01826         template<typename v1>
01827         using pos_t = typename pos<v1>::type;
01828         template<typename v>
01829         static constexpr bool pos_v = pos<t<v>::value;
01830     };
01831
01832     template<typename Ring, typename E = void>
01833     requires IsEuclideanDomain<Ring>
01834     struct FractionFieldImpl {};
01835
01836     // fraction field of a field is the field itself
01837     template<typename Field>
01838     requires IsEuclideanDomain<Field>
01839     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01840         using type = Field;
01841         template<typename v>
01842         using inject_t = v;
01843     };
01844
01845     // fraction field of a ring is the actual fraction field
01846     template<typename Ring>
01847     requires IsEuclideanDomain<Ring>
01848     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01849         using type = _FractionField<Ring>;
01850     };
01851 } // namespace internal
01852
01853     template<typename Ring>
01854     requires IsEuclideanDomain<Ring>
01855     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01856 } // namespace aerobus
01857
01858 // short names for common types
01859 namespace aerobus {
01860     using q32 = FractionField<i32>;
01861     using fpq32 = FractionField<polynomial<q32>>;
01862     using q64 = FractionField<i64>;
01863     using pi64 = polynomial<i64>;
01864     using pq64 = polynomial<q64>;
01865     using fpq64 = FractionField<polynomial<q64>>;
01866     template<typename Ring, typename v1, typename v2>
01867     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01868
01869     template<typename Ring, typename v1, typename v2>
01870     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
01871     template<typename Ring, typename v1, typename v2>

```

```

01908     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01909 } // namespace aerobus
01910
01911 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
01912 namespace aerobus {
01913     namespace internal {
01914         template<typename T, size_t x, typename E = void>
01915         struct factorial {};
01916
01917         template<typename T, size_t x>
01918         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
01919             private:
01920                 template<typename, size_t, typename>
01921                 friend struct factorial;
01922             public:
01923                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
01924                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01925             };
01926
01927         template<typename T>
01928         struct factorial<T, 0> {
01929             public:
01930                 using type = typename T::one;
01931                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01932             };
01933     } // namespace internal
01934
01935     template<typename T, size_t i>
01936     using factorial_t = typename internal::factorial<T, i>::type;
01937
01938     template<typename T, size_t i>
01939     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01940
01941     namespace internal {
01942         template<typename T, size_t k, size_t n, typename E = void>
01943         struct combination_helper {};
01944
01945         template<typename T, size_t k, size_t n>
01946         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
01947             using type = typename FractionField<T>::template mul_t<
typename combination_helper<T, k - 1, n - 1>::type,
01948                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
01949             };
01950
01951         template<typename T, size_t k, size_t n>
01952         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
01953             using type = typename combination_helper<T, n - k, n>::type;
01954             };
01955
01956         template<typename T, size_t n>
01957         struct combination_helper<T, 0, n> {
01958             using type = typename FractionField<T>::one;
01959             };
01960
01961         template<typename T, size_t k, size_t n>
01962         struct combination {
01963             using type = typename internal::combination_helper<T, k, n>::type::x;
01964             static constexpr typename T::inner_type value =
01965                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
01966             };
01967     } // namespace internal
01968
01969     template<typename T, size_t k, size_t n>
01970     using combination_t = typename internal::combination<T, k, n>::type;
01971
01972     template<typename T, size_t k, size_t n>
01973     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
01974
01975     namespace internal {
01976         template<typename T, size_t m>
01977         struct bernouilli;
01978
01979         template<typename T, typename accum, size_t k, size_t m>
01980         struct bernouilli_helper {
01981             using type = typename bernouilli_helper<
01982                 T,
01983                 addfractions_t<T,
01984                     accum,
01985                     mulfractions_t<T,
01986                         makefraction_t<T,
01987                             combination_t<T, k, m + 1>,
01988                             typename T::one>,
01989                             typename bernouilli<T, k>::type

```

```

02003         >
02004         >,
02005         k + 1,
02006         m>::type;
02007     };
02008
02009     template<typename T, typename accum, size_t m>
02010     struct bernouilli_helper<T, accum, m, m> {
02011         using type = accum;
02012     };
02013
02014
02015
02016     template<typename T, size_t m>
02017     struct bernouilli {
02018         using type = typename FractionField<T>::template mul_t<
02019             typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02020             makefraction_t<T,
02021             typename T::template val<static_cast<typename T::inner_type>(-1)>,
02022             typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02023             >
02024         >;
02025
02026         template<typename floatType>
02027         static constexpr floatType value = type::template get<floatType>();
02028     };
02029
02030     template<typename T>
02031     struct bernouilli<T, 0> {
02032         using type = typename FractionField<T>::one;
02033
02034         template<typename floatType>
02035         static constexpr floatType value = type::template get<floatType>();
02036     };
02037 } // namespace internal
02038
02042     template<typename T, size_t n>
02043     using bernouilli_t = typename internal::bernouilli<T, n>::type;
02044
02049     template<typename FloatType, typename T, size_t n>
02050     inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02051
02052     namespace internal {
02053         template<typename T, int k, typename E = void>
02054         struct alternate {};
02055
02056         template<typename T, int k>
02057         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02058             using type = typename T::one;
02059             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02060         };
02061
02062         template<typename T, int k>
02063         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02064             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02065             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02066         };
02067     } // namespace internal
02068
02071     template<typename T, int k>
02072     using alternate_t = typename internal::alternate<T, k>::type;
02073
02074     namespace internal {
02075         template<typename T, int n, int k, typename E = void>
02076         struct stirling_helper {};
02077
02078         template<typename T>
02079         struct stirling_helper<T, 0, 0> {
02080             using type = typename T::one;
02081         };
02082
02083         template<typename T, int n>
02084         struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02085             using type = typename T::zero;
02086         };
02087
02088         template<typename T, int n>
02089         struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02090             using type = typename T::zero;
02091         };
02092
02093         template<typename T, int n, int k>
02094         struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02095             using type = typename T::template sub_t<
02096                 typename stirling_helper<T, n-1, k-1>::type,

```

```

02097         typename T::template mul_t<
02098             typename T::template inject_constant_t<n-1>,
02099             typename stirling_helper<T, n-1, k>::type
02100         >>;
02101     };
02102 } // namespace internal
02103
02104 template<typename T, int n, int k>
02105 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02106
02107 template<typename T, int n, int k>
02108 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02109
02110 template<typename T, int n, int k>
02111 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02112
02113 template<typename T, int n, int k>
02114 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02115
02116 template<typename T, size_t k>
02117 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02118
02119 namespace internal {
02120     template<typename T, auto p, auto n, typename E = void>
02121     struct pow {};
02122
02123     template<typename T, auto p, auto n>
02124     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
02125         using type = typename T::template mul_t<
02126             typename pow<T, p, n/2>::type,
02127             typename pow<T, p, n/2>::type
02128         >;
02129     };
02130
02131     template<typename T, auto p, auto n>
02132     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
02133         using type = typename T::template mul_t<
02134             typename T::template inject_constant_t<p>,
02135             typename T::template mul_t<
02136                 typename pow<T, p, n/2>::type,
02137                 typename pow<T, p, n/2>::type
02138             >
02139         >;
02140     };
02141
02142     template<typename T, auto p>
02143     struct pow<T, p, 0> { using type = typename T::one; };
02144 } // namespace internal
02145
02146 template<typename T, auto p, auto n>
02147 using pow_t = typename internal::pow<T, p, n>::type;
02148
02149 template<typename T, auto p, auto n>
02150 static constexpr T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02151
02152 namespace internal {
02153     template<typename, template<typename, size_t> typename, class>
02154     struct make_taylor_impl;
02155
02156     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02157     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...>> {
02158         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
02159             Is>::type...>;
02160     };
02161 }
02162
02163 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02164 using taylor = typename internal::make_taylor_impl<
02165     T,
02166     coeff_at,
02167     internal::make_index_sequence_reverse<deg + 1>::type>;
02168
02169 namespace internal {
02170     template<typename T, size_t i>
02171     struct exp_coeff {
02172         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02173     };
02174
02175     template<typename T, size_t i, typename E = void>
02176     struct sin_coeff_helper {};
02177
02178     template<typename T, size_t i>
02179     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0>> {
02180         using type = typename FractionField<T>::zero;
02181     };
02182 }

```

```

02213     template<typename T, size_t i>
02214     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02215         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02216     };
02217
02218     template<typename T, size_t i>
02219     struct sin_coeff {
02220         using type = typename sin_coeff_helper<T, i>::type;
02221     };
02222
02223     template<typename T, size_t i, typename E = void>
02224     struct sh_coeff_helper {};
02225
02226     template<typename T, size_t i>
02227     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02228         using type = typename FractionField<T>::zero;
02229     };
02230
02231     template<typename T, size_t i>
02232     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02233         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02234     };
02235
02236     template<typename T, size_t i>
02237     struct sh_coeff {
02238         using type = typename sh_coeff_helper<T, i>::type;
02239     };
02240
02241     template<typename T, size_t i, typename E = void>
02242     struct cos_coeff_helper {};
02243
02244     template<typename T, size_t i>
02245     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02246         using type = typename FractionField<T>::zero;
02247     };
02248
02249     template<typename T, size_t i>
02250     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02251         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02252     };
02253
02254     template<typename T, size_t i>
02255     struct cos_coeff {
02256         using type = typename cos_coeff_helper<T, i>::type;
02257     };
02258
02259     template<typename T, size_t i, typename E = void>
02260     struct cosh_coeff_helper {};
02261
02262     template<typename T, size_t i>
02263     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02264         using type = typename FractionField<T>::zero;
02265     };
02266
02267     template<typename T, size_t i>
02268     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02269         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02270     };
02271
02272     template<typename T, size_t i>
02273     struct cosh_coeff {
02274         using type = typename cosh_coeff_helper<T, i>::type;
02275     };
02276
02277     template<typename T, size_t i>
02278     struct geom_coeff { using type = typename FractionField<T>::one; };
02279
02280
02281     template<typename T, size_t i, typename E = void>
02282     struct atan_coeff_helper;
02283
02284     template<typename T, size_t i>
02285     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02286         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02287     };
02288
02289     template<typename T, size_t i>
02290     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02291         using type = typename FractionField<T>::zero;
02292     };
02293
02294     template<typename T, size_t i>
02295     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02296
02297     template<typename T, size_t i, typename E = void>
02298     struct asin_coeff_helper;
02299

```

```

02300     template<typename T, size_t i>
02301     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02302         using type = makefraction_t<T,
02303             factorial_t<T, i - 1>,
02304             typename T::template mul_t<
02305                 typename T::template val<i>,
02306                 T::template mul_t<
02307                     pow_t<T, 4, i / 2>,
02308                     pow<T, factorial<T, i / 2>::value, 2
02309                 >
02310             >
02311         >>;
02312     };
02313
02314     template<typename T, size_t i>
02315     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02316         using type = typename FractionField<T>::zero;
02317     };
02318
02319     template<typename T, size_t i>
02320     struct asin_coeff {
02321         using type = typename asin_coeff_helper<T, i>::type;
02322     };
02323
02324     template<typename T, size_t i>
02325     struct lnpl_coeff {
02326         using type = makefraction_t<T,
02327             alternate_t<T, i + 1>,
02328             typename T::template val<i>;
02329     };
02330
02331     template<typename T>
02332     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02333
02334     template<typename T, size_t i, typename E = void>
02335     struct asinh_coeff_helper;
02336
02337     template<typename T, size_t i>
02338     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02339         using type = makefraction_t<T,
02340             typename T::template mul_t<
02341                 alternate_t<T, i / 2>,
02342                 factorial_t<T, i - 1>
02343             >,
02344             typename T::template mul_t<
02345                 T::template mul_t<
02346                     typename T::template val<i>,
02347                     pow_t<T, (factorial<T, i / 2>::value), 2>
02348                 >,
02349                 pow_t<T, 4, i / 2>
02350             >
02351         >>;
02352     };
02353
02354     template<typename T, size_t i>
02355     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02356         using type = typename FractionField<T>::zero;
02357     };
02358
02359     template<typename T, size_t i>
02360     struct asinh_coeff {
02361         using type = typename asinh_coeff_helper<T, i>::type;
02362     };
02363
02364     template<typename T, size_t i, typename E = void>
02365     struct atanh_coeff_helper;
02366
02367     template<typename T, size_t i>
02368     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02369         // 1/i
02370         using type = typename FractionField<T>::template val<
02371             typename T::one,
02372             typename T::template val<static_cast<typename T::inner_type>(i)>>;
02373     };
02374
02375     template<typename T, size_t i>
02376     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02377         using type = typename FractionField<T>::zero;
02378     };
02379
02380     template<typename T, size_t i>
02381     struct atanh_coeff {
02382         using type = typename asinh_coeff_helper<T, i>::type;
02383     };
02384
02385     template<typename T, size_t i, typename E = void>
02386     struct tan_coeff_helper;

```

```

02387
02388     template<typename T, size_t i>
02389     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02390         using type = typename FractionField<T>::zero;
02391     };
02392
02393     template<typename T, size_t i>
02394     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02395     private:
02396         // 4^((i+1)/2)
02397         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02398         // 4^((i+1)/2) - 1
02399         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02400         // (-1)^((i-1)/2)
02401         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02402         using dividend = typename FractionField<T>::template mul_t<
02403             altp,
02404             FractionField<T>::template mul_t<
02405                 _4p,
02406                 FractionField<T>::template mul_t<
02407                     _4pml,
02408                     bernouilli_t<T, (i + 1)>
02409                 >
02410             >
02411         >;
02412     public:
02413         using type = typename FractionField<T>::template div_t<dividend,
02414             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02415     };
02416
02417     template<typename T, size_t i>
02418     struct tan_coeff {
02419         using type = typename tan_coeff_helper<T, i>::type;
02420     };
02421
02422     template<typename T, size_t i, typename E = void>
02423     struct tanh_coeff_helper;
02424
02425     template<typename T, size_t i>
02426     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02427         using type = typename FractionField<T>::zero;
02428     };
02429
02430     template<typename T, size_t i>
02431     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02432     private:
02433         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02434         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02435         using dividend =
02436             typename FractionField<T>::template mul_t<
02437                 _4p,
02438                 typename FractionField<T>::template mul_t<
02439                     _4pml,
02440                     bernouilli_t<T, (i + 1)>
02441                 >
02442             >::type;
02443     public:
02444         using type = typename FractionField<T>::template div_t<dividend,
02445             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02446     };
02447
02448     template<typename T, size_t i>
02449     struct tanh_coeff {
02450         using type = typename tanh_coeff_helper<T, i>::type;
02451     };
02452 } // namespace internal
02453
02457     template<typename T, size_t deg>
02458     using exp = taylor<T, internal::exp_coeff, deg>;
02459
02463     template<typename T, size_t deg>
02464     using expml = typename polynomial<FractionField<T>::template sub_t<
02465         exp<T, deg>,
02466         typename polynomial<FractionField<T>::one>;
02467
02471     template<typename T, size_t deg>
02472     using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02473
02477     template<typename T, size_t deg>
02478     using atan = taylor<T, internal::atan_coeff, deg>;
02479
02483     template<typename T, size_t deg>
02484     using sin = taylor<T, internal::sin_coeff, deg>;
02485
02489     template<typename T, size_t deg>

```

```

02490     using sinh = taylor<T, internal::sh_coeff, deg>;
02491
02495     template<typename T, size_t deg>
02496     using cosh = taylor<T, internal::cosh_coeff, deg>;
02497
02501     template<typename T, size_t deg>
02502     using cos = taylor<T, internal::cos_coeff, deg>;
02503
02507     template<typename T, size_t deg>
02508     using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02509
02513     template<typename T, size_t deg>
02514     using asin = taylor<T, internal::asin_coeff, deg>;
02515
02519     template<typename T, size_t deg>
02520     using asinh = taylor<T, internal::asinh_coeff, deg>;
02521
02525     template<typename T, size_t deg>
02526     using atanh = taylor<T, internal::atanh_coeff, deg>;
02527
02531     template<typename T, size_t deg>
02532     using tan = taylor<T, internal::tan_coeff, deg>;
02533
02537     template<typename T, size_t deg>
02538     using tanh = taylor<T, internal::tanh_coeff, deg>;
02539 } // namespace aerobus
02540
02541 // continued fractions
02542 namespace aerobus {
02543     template<int64_t... values>
02544     struct ContinuedFraction {};
02545
02549     template<int64_t a0>
02550     struct ContinuedFraction<a0> {
02551         using type = typename q64::template inject_constant_t<a0>;
02552         static constexpr double val = type::template get<double>();
02553     };
02554
02559     template<int64_t a0, int64_t... rest>
02560     struct ContinuedFraction<a0, rest...> {
02561         using type = q64::template add_t<
02562             typename q64::template inject_constant_t<a0>,
02563             typename q64::template div_t<
02564                 typename q64::one,
02565                 typename ContinuedFraction<rest...>::type
02566             >;
02567         static constexpr double val = type::template get<double>();
02568     };
02569
02574     using PI_fraction =
02575     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02576
02577     using E_fraction =
02578     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02579
02580     using SQRT2_fraction =
02581     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02582
02583     using SQRT3_fraction =
02584     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02585 // NOLINT
02586 } // namespace aerobus
02587
02588 // known polynomials
02589 namespace aerobus {
02590     // CChebyshev
02591     namespace internal {
02592         template<int kind, int deg>
02593         struct chebyshev_helper {
02594             using type = typename pi64::template sub_t<
02595                 typename pi64::template mul_t<
02596                     typename pi64::template mul_t<
02597                         pi64::inject_constant_t<2>,
02598                         typename pi64::X
02599                     >,
02600                     typename chebyshev_helper<kind, deg - 1>::type
02601                 >,
02602                 typename chebyshev_helper<kind, deg - 2>::type
02603             >;
02604         };
02605
02606         template<>
02607         struct chebyshev_helper<1, 0> {
02608             using type = typename pi64::one;
02609         };
02610
02611         template<>
02612         struct chebyshev_helper<1, 1> {
02613             using type = typename pi64::X;
02614         };
02615     }

```



```

02611
02612     template<>
02613     struct chebyshev_helper<2, 0> {
02614         using type = typename pi64::one;
02615     };
02616
02617     template<>
02618     struct chebyshev_helper<2, 1> {
02619         using type = typename pi64::template mul_t<
02620             typename pi64::inject_constant_t<2>,
02621             typename pi64::X>;
02622     };
02623 } // namespace internal
02624
02625 // Laguerre
02626 namespace internal {
02627     template<size_t deg>
02628     struct laguerre_helper {
02629     private:
02630         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
02631         using lnm2 = typename laguerre_helper<deg - 2>::type;
02632         using lnm1 = typename laguerre_helper<deg - 1>::type;
02633         // -x + 2k-1
02634         using p = typename pq64::template val<
02635             typename q64::template inject_constant_t<-1>,
02636             typename q64::template inject_constant_t<2 * deg - 1>;
02637         // 1/n
02638         using factor = typename pq64::template inject_ring_t<
02639             q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02640
02641     public:
02642         using type = typename pq64::template mul_t <
02643             factor,
02644             typename pq64::template sub_t<
02645                 typename pq64::template mul_t<
02646                     p,
02647                     lnm1
02648                 >,
02649                 typename pq64::template mul_t<
02650                     typename pq64::template inject_constant_t<deg-1>,
02651                     lnm2
02652                 >
02653             >
02654         >;
02655     };
02656
02657     template<>
02658     struct laguerre_helper<0> {
02659         using type = typename pq64::one;
02660     };
02661
02662     template<>
02663     struct laguerre_helper<1> {
02664         using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02665     };
02666 } // namespace internal
02667
02668 namespace known_polynomials {
02669     enum hermite_kind {
02670         probabilist,
02671         physicist
02672     };
02673 }
02674
02675 namespace internal {
02676     template<size_t deg, known_polynomials::hermite_kind kind>
02677     struct hermite_helper {};
02678
02679     template<size_t deg>
02680     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02681     private:
02682         using hnm1 = typename hermite_helper<deg - 1,
02683             known_polynomials::hermite_kind::probabilist>::type;
02684         using hnm2 = typename hermite_helper<deg - 2,
02685             known_polynomials::hermite_kind::probabilist>::type;
02686
02687     public:
02688         using type = typename pi64::template sub_t<
02689             typename pi64::template mul_t<typename pi64::X, hnm1>,
02690             typename pi64::template mul_t<
02691                 typename pi64::template inject_constant_t<deg - 1>,
02692                 hnm2
02693             >
02694         >;
02695     };
02696

```

```

02697     template<size_t deg>
02698     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02699     private:
02700         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02701         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02702     public:
02703         using type = typename pi64::template sub_t<
02704             // 2X Hn-1
02705             typename pi64::template mul_t<
02706                 typename pi64::val<typename i64::template inject_constant_t<2>,
02707                     typename i64::zero>, hnm1>,
02708                 typename pi64::template mul_t<
02709                     typename pi64::template inject_constant_t<2*(deg - 1)>,
02710                         hnm2
02711                     >
02712                 >;
02713     };
02714 };
02715
02716 template<>
02717 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02718     using type = typename pi64::one;
02719 };
02720
02721 template<>
02722 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02723     using type = typename pi64::X;
02724 };
02725
02726 template<>
02727 struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02728     using type = typename pi64::one;
02729 };
02730
02731 template<>
02732 struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02733     // 2X
02734     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
02735         typename i64::zero>;
02736 };
02737 } // namespace internal
02738
02739 namespace known_polynomials {
02740     template <size_t deg>
02741     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02742
02743     template <size_t deg>
02744     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02745
02746     template <size_t deg>
02747     using laguerre = typename internal::laguerre_helper<deg>::type;
02748
02749     template <size_t deg>
02750     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02751
02752     template <size_t deg>
02753     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
02754 } // namespace known_polynomials
02755 } // namespace aerobus
02756
02757 #ifndef AEROBUS_CONWAY_IMPORTS
02758 template<int p, int n>
02759 struct ConwayPolynomial;
02760
02761 #define ZPV ZPV::template val
02762 #define POLYV aerobus::polynomial<ZPV>::template val
02763 template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<1>; }; // NOLINT
02764 template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<1>, ZPV<1>; }; // NOLINT
02765 template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02766 template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02767 template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
02768 template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02769 template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02770 template<> struct ConwayPolynomial<2, 8> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
02771 template<> struct ConwayPolynomial<2, 9> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,

```

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```
04716  
04717 #endif // __INC_AEROBUS__ // NOLINT
```


Chapter 7

Examples

7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

7.3 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.4 PI_fraction::val

representation of PI as a continued fraction -> 3.14...

7.5 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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