

Aerobus

v1.2

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1 Introduction	1
1.1 HOW TO	1
1.1.1 Unit Test	2
1.1.2 Benchmarks	2
1.2 Structures	2
1.2.1 Predefined discrete euclidean domains	2
1.2.2 Polynomials	3
1.2.3 Known polynomials	3
1.2.4 Conway polynomials	3
1.2.5 Taylor series	4
1.3 Operations	5
1.3.1 Field of fractions	5
1.3.2 Quotient	6
1.4 Misc	6
1.4.1 Continued Fractions	6
1.5 CUDA	6
2 Namespace Index	7
2.1 Namespace List	7
3 Concept Index	9
3.1 Concepts	9
4 Class Index	11
4.1 Class List	11
5 File Index	13
5.1 File List	13
6 Namespace Documentation	15
6.1 aerobus Namespace Reference	15
6.1.1 Detailed Description	19
6.1.2 Typedef Documentation	20
6.1.2.1 abs_t	20
6.1.2.2 add_t	20
6.1.2.3 addfractions_t	20
6.1.2.4 alternate_t	20
6.1.2.5 asin	21
6.1.2.6 asinh	21
6.1.2.7 atan	21
6.1.2.8 atanh	21
6.1.2.9 bell_t	23
6.1.2.10 bernoulli_t	23
6.1.2.11 combination_t	23

6.1.2.12 cos	23
6.1.2.13 cosh	25
6.1.2.14 div_t	25
6.1.2.15 E_fraction	25
6.1.2.16 embed_int_poly_in_fractions_t	25
6.1.2.17 exp	26
6.1.2.18 expm1	26
6.1.2.19 factorial_t	26
6.1.2.20 fpq32	26
6.1.2.21 fpq64	27
6.1.2.22 FractionField	27
6.1.2.23 gcd_t	27
6.1.2.24 geometric_sum	27
6.1.2.25 lnp1	28
6.1.2.26 make_frac_polynomial_t	28
6.1.2.27 make_int_polynomial_t	28
6.1.2.28 make_q32_t	28
6.1.2.29 make_q64_t	29
6.1.2.30 makefraction_t	29
6.1.2.31 mul_t	29
6.1.2.32 mulfractions_t	30
6.1.2.33 pi64	30
6.1.2.34 PI_fraction	30
6.1.2.35 pow_t	30
6.1.2.36 pq64	30
6.1.2.37 q32	31
6.1.2.38 q64	31
6.1.2.39 sin	31
6.1.2.40 sinh	31
6.1.2.41 SQRT2_fraction	31
6.1.2.42 SQRT3_fraction	32
6.1.2.43 stirling_1_signed_t	32
6.1.2.44 stirling_1_unsigned_t	32
6.1.2.45 stirling_2_t	32
6.1.2.46 sub_t	33
6.1.2.47 tan	33
6.1.2.48 tanh	33
6.1.2.49 taylor	33
6.1.2.50 vadd_t	34
6.1.2.51 vmul_t	34
6.1.3 Function Documentation	34
6.1.3.1 aligned_malloc()	34

6.1.3.2 field()	34
6.1.4 Variable Documentation	35
6.1.4.1 alternate_v	35
6.1.4.2 bernoulli_v	35
6.1.4.3 combination_v	35
6.1.4.4 factorial_v	36
6.2 aerobus::internal Namespace Reference	36
6.2.1 Detailed Description	39
6.2.2 Typedef Documentation	40
6.2.2.1 make_index_sequence_reverse	40
6.2.2.2 type_at_t	40
6.2.3 Function Documentation	40
6.2.3.1 index_sequence_reverse()	40
6.2.4 Variable Documentation	40
6.2.4.1 is_instantiation_of_v	40
6.3 aerobus::known_polynomials Namespace Reference	40
6.3.1 Detailed Description	40
6.3.2 Enumeration Type Documentation	40
6.3.2.1 hermite_kind	40
7 Concept Documentation	43
7.1 aerobus::IsEuclideanDomain Concept Reference	43
7.1.1 Concept definition	43
7.1.2 Detailed Description	43
7.2 aerobus::IsField Concept Reference	43
7.2.1 Concept definition	43
7.2.2 Detailed Description	44
7.3 aerobus::IsRing Concept Reference	44
7.3.1 Concept definition	44
7.3.2 Detailed Description	44
8 Class Documentation	45
8.1 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E > Struct Template Reference	45
8.2 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> > Struct Template Reference	45
8.2.1 Member Typedef Documentation	45
8.2.1.1 type	45
8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference	46
8.3.1 Member Typedef Documentation	46
8.3.1.1 type	46
8.4 aerobus::ContinuedFraction< values > Struct Template Reference	46
8.4.1 Detailed Description	46

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference	47
8.5.1 Detailed Description	47
8.5.2 Member Typedef Documentation	47
8.5.2.1 type	47
8.5.3 Member Data Documentation	48
8.5.3.1 val	48
8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference	48
8.6.1 Detailed Description	48
8.6.2 Member Typedef Documentation	49
8.6.2.1 type	49
8.6.3 Member Data Documentation	49
8.6.3.1 val	49
8.7 aerobus::ConwayPolynomial Struct Reference	49
8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference	49
8.8.1 Member Function Documentation	50
8.8.1.1 func()	50
8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference	50
8.9.1 Member Function Documentation	50
8.9.1.1 func()	50
8.10 aerobus::Embed< Small, Large, E > Struct Template Reference	51
8.10.1 Detailed Description	51
8.11 aerobus::Embed< i32, i64 > Struct Reference	51
8.11.1 Detailed Description	51
8.11.2 Member Typedef Documentation	51
8.11.2.1 type	51
8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference	52
8.12.1 Detailed Description	52
8.12.2 Member Typedef Documentation	52
8.12.2.1 type	52
8.13 aerobus::Embed< q32, q64 > Struct Reference	53
8.13.1 Detailed Description	53
8.13.2 Member Typedef Documentation	53
8.13.2.1 type	53
8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference	54
8.14.1 Detailed Description	54
8.14.2 Member Typedef Documentation	54
8.14.2.1 type	54
8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference	55
8.15.1 Detailed Description	55
8.15.2 Member Typedef Documentation	55
8.15.2.1 type	55

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference	55
8.16.1 Detailed Description	56
8.16.2 Member Typedef Documentation	56
8.16.2.1 type	56
8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference	56
8.17.1 Detailed Description	57
8.18 aerobus::i32 Struct Reference	57
8.18.1 Detailed Description	58
8.18.2 Member Typedef Documentation	59
8.18.2.1 add_t	59
8.18.2.2 div_t	59
8.18.2.3 eq_t	59
8.18.2.4 gcd_t	59
8.18.2.5 gt_t	60
8.18.2.6 inject_constant_t	60
8.18.2.7 inject_ring_t	60
8.18.2.8 inner_type	60
8.18.2.9 lt_t	60
8.18.2.10 mod_t	61
8.18.2.11 mul_t	61
8.18.2.12 one	61
8.18.2.13 pos_t	61
8.18.2.14 sub_t	62
8.18.2.15 zero	62
8.18.3 Member Data Documentation	62
8.18.3.1 eq_v	62
8.18.3.2 is_euclidean_domain	62
8.18.3.3 is_field	62
8.18.3.4 pos_v	63
8.19 aerobus::i64 Struct Reference	64
8.19.1 Detailed Description	65
8.19.2 Member Typedef Documentation	65
8.19.2.1 add_t	65
8.19.2.2 div_t	66
8.19.2.3 eq_t	66
8.19.2.4 gcd_t	66
8.19.2.5 gt_t	66
8.19.2.6 inject_constant_t	67
8.19.2.7 inject_ring_t	67
8.19.2.8 inner_type	67
8.19.2.9 lt_t	67
8.19.2.10 mod_t	68

8.19.2.11 mul_t	68
8.19.2.12 one	68
8.19.2.13 pos_t	68
8.19.2.14 sub_t	68
8.19.2.15 zero	69
8.19.3 Member Data Documentation	69
8.19.3.1 eq_v	69
8.19.3.2 gt_v	69
8.19.3.3 is_euclidean_domain	69
8.19.3.4 is_field	70
8.19.3.5 lt_v	70
8.19.3.6 pos_v	70
8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference	70
8.20.1 Member Typedef Documentation	71
8.20.1.1 type	71
8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference	71
8.21.1 Member Typedef Documentation	71
8.21.1.1 type	71
8.22 aerobus::is_prime< n > Struct Template Reference	71
8.22.1 Detailed Description	72
8.22.2 Member Data Documentation	72
8.22.2.1 value	72
8.23 aerobus::polynomial< Ring > Struct Template Reference	72
8.23.1 Detailed Description	74
8.23.2 Member Typedef Documentation	74
8.23.2.1 add_t	74
8.23.2.2 derive_t	74
8.23.2.3 div_t	75
8.23.2.4 eq_t	75
8.23.2.5 gcd_t	75
8.23.2.6 gt_t	75
8.23.2.7 inject_constant_t	76
8.23.2.8 inject_ring_t	76
8.23.2.9 lt_t	76
8.23.2.10 mod_t	76
8.23.2.11 monomial_t	77
8.23.2.12 mul_t	77
8.23.2.13 one	77
8.23.2.14 pos_t	77
8.23.2.15 simplify_t	79
8.23.2.16 sub_t	79

8.23.2.17 X	79
8.23.2.18 zero	79
8.23.3 Member Data Documentation	80
8.23.3.1 is_euclidean_domain	80
8.23.3.2 is_field	80
8.23.3.3 pos_v	80
8.24 aerobus::type_list< Ts >::pop_front Struct Reference	80
8.24.1 Detailed Description	80
8.24.2 Member Typedef Documentation	81
8.24.2.1 tail	81
8.24.2.2 type	81
8.25 aerobus::Quotient< Ring, X > Struct Template Reference	81
8.25.1 Detailed Description	82
8.25.2 Member Typedef Documentation	82
8.25.2.1 add_t	82
8.25.2.2 div_t	83
8.25.2.3 eq_t	83
8.25.2.4 inject_constant_t	83
8.25.2.5 inject_ring_t	84
8.25.2.6 mod_t	84
8.25.2.7 mul_t	84
8.25.2.8 one	84
8.25.2.9 pos_t	85
8.25.2.10 zero	85
8.25.3 Member Data Documentation	85
8.25.3.1 eq_v	85
8.25.3.2 is_euclidean_domain	85
8.25.3.3 pos_v	85
8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference	86
8.26.1 Detailed Description	86
8.26.2 Member Typedef Documentation	86
8.26.2.1 head	86
8.26.2.2 tail	86
8.27 aerobus::type_list< Ts > Struct Template Reference	87
8.27.1 Detailed Description	87
8.27.2 Member Typedef Documentation	88
8.27.2.1 at	88
8.27.2.2 concat	88
8.27.2.3 insert	88
8.27.2.4 push_back	89
8.27.2.5 push_front	89
8.27.2.6 remove	89

8.27.3 Member Data Documentation	89
8.27.3.1 length	89
8.28 aerobus::type_list<> Struct Reference	90
8.28.1 Detailed Description	90
8.28.2 Member Typedef Documentation	90
8.28.2.1 concat	90
8.28.2.2 insert	90
8.28.2.3 push_back	90
8.28.2.4 push_front	90
8.28.3 Member Data Documentation	91
8.28.3.1 length	91
8.29 aerobus::i32::val< x > Struct Template Reference	91
8.29.1 Detailed Description	91
8.29.2 Member Typedef Documentation	92
8.29.2.1 enclosing_type	92
8.29.2.2 is_zero_t	92
8.29.3 Member Function Documentation	92
8.29.3.1 get()	92
8.29.3.2 to_string()	92
8.29.4 Member Data Documentation	92
8.29.4.1 v	92
8.30 aerobus::i64::val< x > Struct Template Reference	93
8.30.1 Detailed Description	93
8.30.2 Member Typedef Documentation	94
8.30.2.1 enclosing_type	94
8.30.2.2 inner_type	94
8.30.2.3 is_zero_t	94
8.30.3 Member Function Documentation	94
8.30.3.1 get()	94
8.30.3.2 to_string()	94
8.30.4 Member Data Documentation	95
8.30.4.1 v	95
8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference	95
8.31.1 Detailed Description	96
8.31.2 Member Typedef Documentation	96
8.31.2.1 aN	96
8.31.2.2 coeff_at_t	96
8.31.2.3 enclosing_type	97
8.31.2.4 is_zero_t	97
8.31.2.5 ring_type	97
8.31.2.6 strip	97
8.31.2.7 value_at_t	97

8.31.3 Member Function Documentation	97
8.31.3.1 compensated_eval()	97
8.31.3.2 eval()	98
8.31.3.3 to_string()	98
8.31.4 Member Data Documentation	99
8.31.4.1 degree	99
8.31.4.2 is_zero_v	99
8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference	99
8.32.1 Detailed Description	99
8.32.2 Member Typedef Documentation	100
8.32.2.1 raw_t	100
8.32.2.2 type	100
8.33 aerobus::zpz< p >::val< x > Struct Template Reference	100
8.33.1 Detailed Description	100
8.33.2 Member Typedef Documentation	101
8.33.2.1 enclosing_type	101
8.33.2.2 is_zero_t	101
8.33.3 Member Function Documentation	101
8.33.3.1 get()	101
8.33.3.2 to_string()	101
8.33.4 Member Data Documentation	102
8.33.4.1 is_zero_v	102
8.33.4.2 v	102
8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference	102
8.34.1 Detailed Description	103
8.34.2 Member Typedef Documentation	103
8.34.2.1 aN	103
8.34.2.2 coeff_at_t	103
8.34.2.3 enclosing_type	103
8.34.2.4 is_zero_t	104
8.34.2.5 ring_type	104
8.34.2.6 strip	104
8.34.2.7 value_at_t	104
8.34.3 Member Function Documentation	104
8.34.3.1 compensated_eval()	104
8.34.3.2 eval()	104
8.34.3.3 to_string()	105
8.34.4 Member Data Documentation	105
8.34.4.1 degree	105
8.34.4.2 is_zero_v	105
8.35 aerobus::zpz< p > Struct Template Reference	105
8.35.1 Detailed Description	107

8.35.2 Member Typedef Documentation	107
8.35.2.1 add_t	107
8.35.2.2 div_t	107
8.35.2.3 eq_t	108
8.35.2.4 gcd_t	108
8.35.2.5 gt_t	108
8.35.2.6 inject_constant_t	109
8.35.2.7 inner_type	109
8.35.2.8 lt_t	109
8.35.2.9 mod_t	109
8.35.2.10 mul_t	110
8.35.2.11 one	110
8.35.2.12 pos_t	110
8.35.2.13 sub_t	110
8.35.2.14 zero	111
8.35.3 Member Data Documentation	111
8.35.3.1 eq_v	111
8.35.3.2 gt_v	111
8.35.3.3 is_euclidean_domain	111
8.35.3.4 is_field	111
8.35.3.5 lt_v	112
8.35.3.6 pos_v	112
9 File Documentation	113
9.1 README.md File Reference	113
9.2 src/aerobus.h File Reference	113
9.3 aerobus.h	113
9.4 src/examples.h File Reference	207
9.5 examples.h	207
10 Examples	209
10.1 examples/hermite.cpp	209
10.2 examples/custom_taylor.cpp	209
10.3 examples/fp16.cu	210
10.4 examples/continued_fractions.cpp	211
10.5 examples/modular_arithmetic.cpp	211
10.6 examples/make_polynomial.cpp	212
10.7 examples/polynomials_over_finite_field.cpp	212
10.8 examples/compensated_horner.cpp	212
Index	215

Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for **quotient ring** R/X where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	15
aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	43
aerobus::IsField	
Concept to express R is a field	43
aerobus::IsRing	
Concept to express R is a Ring	44

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >	45
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> > 45	
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	46
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	46
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	47
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	48
aerobus::ConwayPolynomial	49
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >	49
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > . .	50
aerobus::Embed< Small, Large, E >	
Embedding - struct forward declaration	51
aerobus::Embed< i32, i64 >	
Embeds i32 into i64	51
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
Embeds polynomial<Small> into polynomial<Large>	52
aerobus::Embed< q32, q64 >	
Embeds q32 into q64	53
aerobus::Embed< Quotient< Ring, X >, Ring >	
Embeds Quotient<Ring, X> into Ring	54
aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	55
aerobus::Embed< zpz< x >, i32 >	
Embeds zpz values into i32	55
aerobus::polynomial< Ring >::horner_reduction_t< P >	
Used to evaluate polynomials over a value in Ring	56
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	57
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	64
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >	70
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >	71

aerobus::is_prime< n >	71
Checks if n is prime	
aerobus::polynomial< Ring >	72
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	80
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X,	
Quotient is $\mathbb{Z}/2\mathbb{Z}$	81
aerobus::type_list< Ts >::split< index >	
Splits list at index	86
aerobus::type_list< Ts >	
Empty pure template struct to handle type list	87
aerobus::type_list<>	
Specialization for empty type list	90
aerobus::i32::val< x >	
Values in i32 , again represented as types	91
aerobus::i64::val< x >	
Values in i64	93
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	95
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	99
aerobus::zpz< p >::val< x >	
Values in zpz	100
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	102
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	105

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

src/ aerobus.h	113
src/ examples.h	207

Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - Quotient ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<typename T , typename A , typename B >
 using [gcd_t](#) = typename internal::gcd< T >::template type< A, B >
 - computes the greatest common divisor of A and B*
- template<typename... vals>
 using [vadd_t](#) = typename internal::vadd< vals... >::type
 - adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator*
- template<typename... vals>
 using [vmul_t](#) = typename internal::vmul< vals... >::type
 - multiplies multiple values (v1 * v2 + ... * vn) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator*
- template<typename val >
 using [abs_t](#) = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template [sub_t](#)< typename val::enclosing_type::zero, val > >
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
 - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add_t](#) = typename X::enclosing_type::template [add_t](#)< X, Y >
 - generic addition*
- template<typename X , typename Y >
 using [sub_t](#) = typename X::enclosing_type::template [sub_t](#)< X, Y >

- generic subtraction*
 - `template<typename X , typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
 - generic multiplication*
 - `template<typename X , typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
 - generic division*
 - `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
 - `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
 - `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
 - `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
 - `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
 - `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
 - `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
 - `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > ,`
`polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
 - `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > ,`
`i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
 - `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > ,`
`i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
 - `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
 - `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
 - `template<typename Ring , auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template`
`inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
 - `template<typename Ring , auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename`
`FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField< Ring>
 - `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type

- `template<typename T , size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T , size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T , size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T , int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T , int n, int k>`
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T , int n, int k>`
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T , int n, int k>`
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`
Stirling number of second kind – as types.
- `template<typename T , typename p , size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers , size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers , size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers , size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers , size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers , size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers , size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`
 $\cosh(x)$ *hyperbolic cosine*
- `template<typename Integers , size_t deg>`
`using cos = taylor< Integers, internal::cos_coeff, deg >`
 $\cos(x)$ *cosinus*
- `template<typename Integers , size_t deg>`
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`

- $\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$
 • template<typename Integers , size_t deg>
 using **asin** = **taylor**< Integers, internal::asin_coeff, deg >
 arcsin(x) *arc sinus*
- template<typename Integers , size_t deg>
 using **asinh** = **taylor**< Integers, internal::asinh_coeff, deg >
 arcsinh(x) *arc hyperbolic sinus*
- template<typename Integers , size_t deg>
 using **atanh** = **taylor**< Integers, internal::atanh_coeff, deg >
 arctanh(x) *arc hyperbolic tangent*
- template<typename Integers , size_t deg>
 using **tan** = **taylor**< Integers, internal::tan_coeff, deg >
 tan(x) *tangent*
- template<typename Integers , size_t deg>
 using **tanh** = **taylor**< Integers, internal::tanh_coeff, deg >
 tanh(x) *hyperbolic tangent*
- using **PI_fraction** = **ContinuedFraction**< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >
- using **E_fraction** = **ContinuedFraction**< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >
approximation of e
- using **SQRT2_fraction** = **ContinuedFraction**< 1, 2 >
approximation of $\sqrt{2}$
- using **SQRT3_fraction** = **ContinuedFraction**< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >
approximation of

Functions

- template<typename T >
 T * **aligned_malloc** (size_t count, size_t alignment)
- brief Conway polynomials tparam p characteristic of the **field** (prime number) @tparam n degree of extension
 template< int p

Variables

- template<typename T , size_t i>
 constexpr T::inner_type **factorial_v** = internal::factorial<T, i>::value
computes factorial(i) as value in T
- template<typename T , size_t k, size_t n>
 constexpr T::inner_type **combination_v** = internal::combination<T, k, n>::value
computes binomial coefficients (k among n) as value
- template<typename FloatType , typename T , size_t n>
 constexpr FloatType **bernoulli_v** = internal::bernoulli<T, n>::template value<FloatType>
 n th bernoulli number as value in FloatType
- template<typename T , size_t k>
 constexpr T::inner_type **alternate_v** = internal::alternate<T, k>::value
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	---

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh(x)` arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ `cosinus`

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of e

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

Template Parameters

<i>Ring</i>	
-------------	--

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of π as a continued fraction

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.46 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.47 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.49 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.50 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.51 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >

- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >

- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **fma_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i & 1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i & 1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i & 1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i & 1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_1_helper**
- struct **stirling_1_helper**< T, 0, 0 >
- struct **stirling_1_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, k, std::enable_if_t<(k > 0) && (n > 0)> >

- struct **stirling_2_helper**
- struct **stirling_2_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling_2_helper**< T, n, n, std::enable_if_t<(n >=0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **touchard_coeff**
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Enumerations

- enum [hermite_kind](#) { [probabilist](#) , [physicist](#) }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Enumeration Type Documentation

6.3.2.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = typename Ring::zero

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = `aN`

8.3.1 Member Typedef Documentation

8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

Examples

[examples/continued_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename q64::template inject_constant_t< a0 >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = static_cast<double>(a0)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template [add_t](#)< typename q64::template inject_constant_t< a0 >, typename q64::template [div_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.8.1 Member Function Documentation

8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.9.1 Member Function Documentation

8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- template<typename val >
using [type](#) = [i64::val](#)< static_cast< int64_t >(val::v)>
the [i64](#) representation of val

8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

8.11.2 Member Typedef Documentation

8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the polynomial<Large> representation of v

8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.12.2 Member Typedef Documentation

8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`
q64 representation of v

8.13.1 Detailed Description

embeds q32 into q64

8.13.2 Member Typedef Documentation

8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = typename val::raw_t`
Ring representation of val.

8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

8.14.2 Member Typedef Documentation

8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename FractionField< Ring >::template val< v, typename Ring::one >`
FractionField<Ring> representation of v.

8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

Template Parameters

<i>Ring</i>	an integers ring, such as i32
-------------	---

8.15.2 Member Typedef Documentation

8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i32::val< val::v >`
the i32 representation of val

8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

Template Parameters

<i>x</i>	an integer
----------	------------

8.16.2 Member Typedef Documentation

8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

Template Parameters

<i>val</i>	a value in zpz<x>
------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using [inner_type](#) = int32_t
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<auto x>
using [inject_constant_t](#) = [val](#)< static_cast< int32_t >(x)>
inject a native constant
- template<typename v >
using [inject_ring_t](#) = v
- template<typename v1 , typename v2 >
using [add_t](#) = typename add< v1, v2 >::type
addition operator yields v1 + v2

- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator yields $v1 - v2$
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
*multiplication operator yields $v1 * v2$*
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator yields $v1 / v2$
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator yields $v1 \% v2$
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator ($v1 > v2$) yields $v1 > v2$
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator ($v1 < v2$) yields $v1 < v2$
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator (boolean value)
- `template<typename v >`
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity (boolean value) yields $v > 0$ as boolean value

8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

Examples

[examples/compensated_horner.cpp](#).

8.18.2 Member Typedef Documentation

8.18.2.1 add_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields $v1 + v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.2 div_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields $v1 / v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.3 eq_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.4 gcd_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields $GCD(v1, v2)$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) yields $v1 > v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject\_constant\_t = val<static_cast<int32_t>(x)>
```

inject a native constant

Template Parameters

<i>x</i>	
----------	--

8.18.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject\_ring\_t = v
```

8.18.2.8 inner_type

```
using aerobus::i32::inner\_type = int32_t
```

8.18.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt\_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$) yields $v1 < v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields $v1 * v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.18.2.13 pos_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v</i>	a value in i32
----------	--------------------------------

8.18.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields $v1 - v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.18.3 Member Data Documentation

8.18.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.18.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.18.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.18.3.4 pos_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	a value in i32
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- using [inner_type](#) = `int64_t`
type of represented values
- template<auto x>
using [inject_constant_t](#) = `val< static_cast< int64_t >(x)>`
injects constant as an [i64](#) value
- template<typename v >
using [inject_ring_t](#) = v
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`
constant zero
- using [one](#) = `val< 1 >`
constant one
- template<typename v1 , typename v2 >
using [add_t](#) = `typename add< v1, v2 >::type`
addition operator
- template<typename v1 , typename v2 >
using [sub_t](#) = `typename sub< v1, v2 >::type`
subtraction operator
- template<typename v1 , typename v2 >
using [mul_t](#) = `typename mul< v1, v2 >::type`
multiplication operator
- template<typename v1 , typename v2 >
using [div_t](#) = `typename div< v1, v2 >::type`
division operator integer division
- template<typename v1 , typename v2 >
using [mod_t](#) = `typename remainder< v1, v2 >::type`

modulus operator

- template<typename v1 , typename v2 >
using [gt_t](#) = typename gt< v1, v2 >::type
strictly greater operator yields v1 > v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [lt_t](#) = typename lt< v1, v2 >::type
strict less operator yields v1 < v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [eq_t](#) = typename eq< v1, v2 >::type
equality operator yields v1 == v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [gcd_t](#) = [gcd_t](#)< [i64](#), v1, v2 >
greatest common divisor yields GCD(v1, v2) as instantiation of [i64::val](#)
- template<typename v >
using [pos_t](#) = typename pos< v >::type
is v positive yields v > 0 as std::true_type or std::false_type

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- template<typename v1 , typename v2 >
static constexpr bool [gt_v](#) = [gt_t](#)<v1, v2>::value
strictly greater operator yields v1 > v2 as boolean value
- template<typename v1 , typename v2 >
static constexpr bool [lt_v](#) = [lt_t](#)<v1, v2>::value
strictly smaller operator yields v1 < v2 as boolean value
- template<typename v1 , typename v2 >
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator yields v1 == v2 as boolean value
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity yields v > 0 as boolean value

8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.19.2 Member Typedef Documentation

8.19.2.1 [add_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.6 inject_constant_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

Template Parameters

<code>x</code>	
----------------	--

8.19.2.7 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)

Template Parameters

<code>v</code>	a value in i64
----------------	--------------------------------

8.19.2.8 inner_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.19.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.19.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
-----------	---

8.19.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

8.19.3 Member Data Documentation

8.19.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.19.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.19.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields $v1 < v2$ as boolean value

Template Parameters

$v1$: an element of aerobus::i64::val
$v2$: an element of aerobus::i64::val

8.19.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`

8.20.1 Member Typedef Documentation

8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = accum`

8.21.1 Member Typedef Documentation

8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.22 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.22.2 Member Data Documentation

8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [horner_reduction_t](#)
Used to evaluate polynomials over a value in Ring.
- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- using `zero` = `val`< typename Ring::zero >
constant zero
- using `one` = `val`< typename Ring::one >
constant one
- using `X` = `val`< typename Ring::one, typename Ring::zero >
generator
- template<typename P >
using `simplify_t` = typename simplify< P >::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- template<typename v1, typename v2 >
using `add_t` = typename add< v1, v2 >::type
adds two polynomials
- template<typename v1, typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
subtraction of two polynomials
- template<typename v1, typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
multiplication of two polynomials
- template<typename v1, typename v2 >
using `eq_t` = typename eq_helper< v1, v2 >::type
equality operator
- template<typename v1, typename v2 >
using `lt_t` = typename lt_helper< v1, v2 >::type
strict less operator
- template<typename v1, typename v2 >
using `gt_t` = typename gt_helper< v1, v2 >::type
strict greater operator
- template<typename v1, typename v2 >
using `div_t` = typename div< v1, v2 >::q_type
division operator
- template<typename v1, typename v2 >
using `mod_t` = typename div_helper< v1, v2, `zero`, v1 >::mod_type
modulo operator
- template<typename coeff, size_t deg>
using `monomial_t` = typename monomial< coeff, deg >::type
monomial : coeff X^deg
- template<typename v >
using `derive_t` = typename derive_helper< v >::type
derivation operator
- template<typename v >
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
checks for positivity (an > 0)
- template<typename v1, typename v2 >
using `gcd_t` = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >
greatest common divisor of two polynomials
- template<auto x>
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >
makes the constant (native type) polynomial a_0
- template<typename v >
using `inject_ring_t` = `val`< v >
makes the constant (ring type) polynomial a_0

Static Public Attributes

- static constexpr bool [is_field](#) = false
- static constexpr bool [is_euclidean_domain](#) = Ring::is_euclidean_domain
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity operator

8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

Examples

[examples/compensated_horner.cpp](#), [examples/make_polynomial.cpp](#), and [examples/modular_arithmetic.cpp](#).

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

8.23.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

Template Parameters

<i>x</i>	
----------	--

8.23.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

Template Parameters

<i>v</i>	
----------	--

8.23.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.10 mod_t

```
template<typename Ring >
```



```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.23.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.23.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($an > 0$)

Template Parameters

<i>v</i>	
----------	--

8.23.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.23.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.23.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.23.3 Member Data Documentation

8.23.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.23.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.23.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in polynomial::val
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.24 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using [tail](#) = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.24.2 Member Typedef Documentation

8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< typename Ring::zero >
zero value
- using [one](#) = [val](#)< typename Ring::one >
one
- template<typename v1 , typename v2 >
using [add_t](#) = [val](#)< typename Ring::template [add_t](#)< typename v1::type, typename v2::type > >
addition operator
- template<typename v1 , typename v2 >
using [mul_t](#) = [val](#)< typename Ring::template [mul_t](#)< typename v1::type, typename v2::type > >
subtraction operator
- template<typename v1 , typename v2 >
using [div_t](#) = [val](#)< typename Ring::template [div_t](#)< typename v1::type, typename v2::type > >
division operator
- template<typename v1 , typename v2 >
using [mod_t](#) = [val](#)< typename Ring::template [mod_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
 • `template<typename v1 , typename v2 >`
 `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
 equality operator (as type)
- `template<typename v1 >`
 `using pos_t = std::true_type`
 positivity operator always true
- `template<auto x>`
 `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
 *inject a 'constant' in quotient ring**
- `template<typename v >`
 `using inject_ring_t = val< v >`
 projects a value of Ring onto the quotient

Static Public Attributes

- `template<typename v1 , typename v2 >`
 `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
 addition operator (as boolean value)
- `template<typename v >`
 `static constexpr bool pos_v = pos_t<v>::value`
 positivity operator always true
- `static constexpr bool is_euclidean_domain = true`
 quotien rings are euclidean domain

8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.25.2 Member Typedef Documentation

8.25.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring*

Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

8.25.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

8.25.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.25.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.25.3 Member Data Documentation

8.25.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.25.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.26.2 Member Typedef Documentation

8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.27 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

8.27.2 Member Typedef Documentation

8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.27.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.27.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.3 Member Data Documentation

8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.28 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T >
- template<typename T >
using [push_back](#) = [type_list](#)< T >
- template<typename U >
using [concat](#) = U
- template<typename T , size_t index>
using [insert](#) = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t [length](#) = 0

8.28.1 Detailed Description

specialization for empty type list

8.28.2 Member Typedef Documentation

8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

8.28.2.3 push_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

8.28.2.4 push_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

8.28.3 Member Data Documentation

8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [i32](#)
Enclosing ring type.
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [DEVICE](#) valueType [get](#) ()
cast x into valueType
- static std::string [to_string](#) ()
string representation of value

Static Public Attributes

- static constexpr int32_t [v](#) = x
actual value stored in val type

8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

8.29.2 Member Typedef Documentation

8.29.2.1 `enclosing_type`

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.29.2.2 `is_zero_t`

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.29.3 Member Function Documentation

8.29.3.1 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.29.3.2 `to_string()`

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.29.4 Member Data Documentation

8.29.4.1 `v`

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```


actual value stored in val type

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using [inner_type](#) = int32_t
type of represented values
- using [enclosing_type](#) = [i64](#)
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED_DEVICE](#) valueType [get](#) ()
cast value in valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int64_t [v](#) = x
actual value

8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

x	an actual integer
-------------------	-------------------

Examples

[examples/compensated_horner.cpp](#).

8.30.2 Member Typedef Documentation

8.30.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.30.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.30.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.30.3 Member Function Documentation

8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.30.3.2 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.30.4 Member Data Documentation

8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = polynomial< Ring >
enclosing ring type
- using [aN](#) = coeffN
heavy weight coefficient (non zero)
- using [strip](#) = val< coeffs... >
remove largest coefficient
- using [is_zero_t](#) = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>
true_type if polynomial is constant zero
- template<size_t index>
using [coeff_at_t](#) = typename coeff_at< index >::type
type of coefficient at index
- template<typename x >
using [value_at_t](#) = horner_reduction_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >

Static Public Member Functions

- static std::string [to_string](#) ()
get a string representation of polynomial
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
evaluates polynomial seen as a function operating on arithmeticType
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)
Evaluate polynomial on x using compensated horner scheme.

Static Public Attributes

- static constexpr size_t [degree](#) = sizeof...(coeffs)
degree of the polynomial
- static constexpr bool [is_zero_v](#) = is_zero_t::value
true if polynomial is constant zero

8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

Examples

[examples/compensated_horner.cpp](#).

8.31.2 Member Typedef Documentation

8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.31.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.31.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>

enclosing ring type
```

8.31.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>

true_type if polynomial is constant zero
```

8.31.2.5 ring_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring

ring coefficients live in
```

8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>

remove largest coefficient
```

8.31.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

8.31.3 Member Function Documentation

8.31.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↵
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

Parameters

x	
---	--

8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

Parameters

x	value
---	-------

Returns

$P(x)$

8.31.3.3 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.31.4 Member Data Documentation

8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.31.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- using [raw_t](#) = V
- using [type](#) = [abs_t](#)< typename Ring::template [mod_t](#)< V, X > >

8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

8.32.2 Member Typedef Documentation

8.32.2.1 raw_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [zpz](#)< p >
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< [v](#)==0 >
true_type if zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED DEVICE](#) valueType [get](#) ()
get value as valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int32_t [v](#) = x % p
actual value
- static constexpr bool [is_zero_v](#) = [v](#) == 0
true if zero

8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

x	an integer
---	------------

8.33.2 Member Typedef Documentation

8.33.2.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.33.2.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true_type if zero

8.33.3 Member Function Documentation

8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

8.33.3.2 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

Returns

a string representation

8.33.4 Member Data Documentation**8.33.4.1 is_zero_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

8.33.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial< Ring >](#)
enclosing ring type
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = std::bool_constant< [aN::is_zero_t::value](#) >
- template<size_t index>
using [coeff_at_t](#) = typename [coeff_at< index >::type](#)
- template<typename x >
using [value_at_t](#) = [coeffN](#)

Static Public Member Functions

- static std::string [to_string](#) ()
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)

Static Public Attributes

- static constexpr size_t [degree](#) = 0
degree
- static constexpr bool [is_zero_v](#) = is_zero_t::value

8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.34.2 Member Typedef Documentation

8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.34.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

8.34.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.34.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

8.34.2.5 ring_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.34.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

8.34.3 Member Function Documentation

8.34.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

8.34.3.3 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.34.4 Member Data Documentation

8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.34.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in zpz

Public Types

- using `inner_type` = `int32_t`
underlying type for values
- template<auto x>
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>
injects a constant integer into mpz
- using `zero` = `val`< 0 >
zero value
- using `one` = `val`< 1 >
one value
- template<typename v1 , typename v2 >
using `add_t` = `typename add`< v1, v2 >::type
addition operator
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub`< v1, v2 >::type
subtraction operator
- template<typename v1 , typename v2 >
using `mul_t` = `typename mul`< v1, v2 >::type
multiplication operator
- template<typename v1 , typename v2 >
using `div_t` = `typename div`< v1, v2 >::type
division operator
- template<typename v1 , typename v2 >
using `mod_t` = `typename remainder`< v1, v2 >::type
modulo operator
- template<typename v1 , typename v2 >
using `gt_t` = `typename gt`< v1, v2 >::type
strictly greater operator (type)
- template<typename v1 , typename v2 >
using `lt_t` = `typename lt`< v1, v2 >::type
strictly smaller operator (type)
- template<typename v1 , typename v2 >
using `eq_t` = `typename eq`< v1, v2 >::type
equality operator (type)
- template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >
greatest common divisor
- template<typename v1 >
using `pos_t` = `typename pos`< v1 >::type
positivity operator (type)

Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value
true iff p is prime
- static constexpr bool `is_euclidean_domain` = true
always true
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
strictly greater operator (booleanvalue)

- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

8.35.1 Detailed Description

`template<int32_t p>`
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular_arithmetic.cpp](#), and [examples/polynomials_over_finite_field.cpp](#).

8.35.2 Member Typedef Documentation

8.35.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.2 div_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

Template Parameters

x	an integer
---	------------

8.35.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

8.35.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

8.35.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

8.35.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

8.35.3 Member Data Documentation**8.35.3.1 eq_v**

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

8.35.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

8.35.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt\_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos\_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 0) & 0x800000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED_DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED_DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED_DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<long double> {
00224         static constexpr INLINED_DEVICE long double eval(
00225             const long double x, const long double y, const long double z) {
00226             return x * y + z;
00227         }
00228     };
00229
00230     template<>
00231     struct fma_helper<float> {
00232         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00233             return x * y + z;
00234         }
00235     };
00236
00237     template<>
00238     struct fma_helper<int32_t> {
00239         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00240             return x * y + z;
00241         }
00242     };
00243
00244     template<>
00245     struct fma_helper<int16_t> {
00246         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00247             return x * y + z;
00248         }
00249     };
00250
00251     template<>
00252     struct fma_helper<int64_t> {
00253         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00254             return x * y + z;
00255         }
00256     };
00257
00258     #ifdef WITH_CUDA_FP16
00259     template<>
00260     struct fma_helper<__half> {
00261         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00262             #ifdef __CUDA_ARCH__
00263                 return __hfma(x, y, z);
00264             #else
00265                 return x * y + z;
00266             #endif
00267         }
00268     };
00269     template<>
00270     struct fma_helper<__half2> {
00271         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00272             #ifdef __CUDA_ARCH__
00273                 return __hfma2(x, y, z);
00274             #else
00275                 return x * y + z;
00276             #endif
00277         }
00278     };
00279     #endif
00280 } // namespace internal
00281 } // namespace aerobus
00282
00283 // compensated horner utilities
00284 namespace aerobus {
00285     namespace internal {
00286         template <typename T>
00287         struct FloatLayout;
00288
00289         #ifdef _MSC_VER
00290         template <>
00291         struct FloatLayout<long double> {
00292             static constexpr uint8_t exponent = 11;
00293             static constexpr uint8_t mantissa = 53;
00294             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00295         };

```



```

00296     #else
00297     template <>
00298     struct FloatLayout<long double> {
00299         static constexpr uint8_t exponent = 15;
00300         static constexpr uint8_t mantissa = 63;
00301         static constexpr uint8_t r = 32; // ceil(mantissa/2)
00302         static constexpr long double shift = (1LL « r) + 1;
00303     };
00304     #endif
00305
00306     template <>
00307     struct FloatLayout<double> {
00308         static constexpr uint8_t exponent = 11;
00309         static constexpr uint8_t mantissa = 53;
00310         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00311         static constexpr double shift = (1LL « r) + 1;
00312     };
00313
00314     template <>
00315     struct FloatLayout<float> {
00316         static constexpr uint8_t exponent = 8;
00317         static constexpr uint8_t mantissa = 24;
00318         static constexpr uint8_t r = 11; // ceil(mantissa/2)
00319         static constexpr float shift = (1 « r) + 1;
00320     };
00321
00322     #ifdef WITH_CUDA_FP16
00323     template <>
00324     struct FloatLayout<__half> {
00325         static constexpr uint8_t exponent = 5;
00326         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00327         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00328         static constexpr __half shift = internal::int16_convert_helper<__half, 65>::value();
00329     };
00330
00331     template <>
00332     struct FloatLayout<__half2> {
00333         static constexpr uint8_t exponent = 5;
00334         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00335         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00336         static constexpr __half2 shift = internal::int16_convert_helper<__half2, 65>::value();
00337     };
00338     #endif
00339
00340     template<typename T>
00341     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00342         T z = a * FloatLayout<T>::shift;
00343         *x = z - (z - a);
00344         *y = a - *x;
00345     }
00346
00347     template<typename T>
00348     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00349         *x = a + b;
00350         T z = *x - a;
00351         *y = (a - (*x - z)) + (b - z);
00352     }
00353
00354     template<typename T>
00355     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00356         *x = a * b;
00357         #ifdef __clang__
00358         *y = fma_helper<T>::eval(a, b, -*x);
00359         #else
00360         T ah, al, bh, bl;
00361         split(a, &ah, &al);
00362         split(b, &bh, &bl);
00363         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00364         #endif
00365     }
00366
00367     template<typename T, size_t N>
00368     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00369         T r = p1[0] + p2[0];
00370         for (int64_t i = N - 1; i >= 0; --i) {
00371             r = r * x + p1[N - i] + p2[N - i];
00372         }
00373         return r;
00374     }
00375 } // namespace internal
00376 } // namespace aerobus
00377
00378 // utilities
00379 namespace aerobus {
00380     namespace internal {
00381         template<template<typename...> typename TT, typename T>

```

```

00383     struct is_instantiation_of : std::false_type { };
00384
00385     template<template<typename...> typename TT, typename... Ts>
00386     struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00387
00388     template<template<typename...> typename TT, typename T>
00389     inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00390
00391     template<int64_t i, typename T, typename... Ts>
00392     struct type_at {
00393         static_assert(i < sizeof...(Ts) + 1, "index out of range");
00394         using type = typename type_at<i - 1, Ts...>::type;
00395     };
00396
00397     template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00398         using type = T;
00399     };
00400
00401     template<size_t i, typename... Ts>
00402     using type_at_t = typename type_at<i, Ts...>::type;
00403
00404
00405     template<size_t n, size_t i, typename E = void>
00406     struct _is_prime {};
00407
00408     template<size_t i>
00409     struct _is_prime<0, i> {
00410         static constexpr bool value = false;
00411     };
00412
00413     template<size_t i>
00414     struct _is_prime<1, i> {
00415         static constexpr bool value = false;
00416     };
00417
00418     template<size_t i>
00419     struct _is_prime<2, i> {
00420         static constexpr bool value = true;
00421     };
00422
00423     template<size_t i>
00424     struct _is_prime<3, i> {
00425         static constexpr bool value = true;
00426     };
00427
00428     template<size_t i>
00429     struct _is_prime<5, i> {
00430         static constexpr bool value = true;
00431     };
00432
00433     template<size_t i>
00434     struct _is_prime<7, i> {
00435         static constexpr bool value = true;
00436     };
00437
00438     template<size_t n, size_t i>
00439     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00440         static constexpr bool value = false;
00441     };
00442
00443     template<size_t n, size_t i>
00444     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00445         static constexpr bool value = false;
00446     };
00447
00448     template<size_t n, size_t i>
00449     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00450         static constexpr bool value = true;
00451     };
00452
00453     template<size_t n, size_t i>
00454     struct _is_prime<n, i, std::enable_if_t<(
00455         n % i == 0 &&
00456         n >= 9 &&
00457         n % 3 != 0 &&
00458         n % 2 != 0 &&
00459         i * i > n)> {
00460         static constexpr bool value = true;
00461     };
00462
00463     template<size_t n, size_t i>
00464     struct _is_prime<n, i, std::enable_if_t<(
00465         n % (i+2) == 0 &&
00466         n >= 9 &&
00467         n % 3 != 0 &&
00468         n % 2 != 0 &&
00469         i * i <= n)> {

```

```

00470         static constexpr bool value = true;
00471     };
00472
00473     template<size_t n, size_t i>
00474     struct _is_prime<n, i, std::enable_if_t<(
00475         n % (i+2) != 0 &&
00476         n % i != 0 &&
00477         n >= 9 &&
00478         n % 3 != 0 &&
00479         n % 2 != 0 &&
00480         (i * i <= n))> {
00481         static constexpr bool value = _is_prime<n, i+6>::value;
00482     };
00483 } // namespace internal
00484
00485 template<size_t n>
00486 struct is_prime {
00487     static constexpr bool value = internal::_is_prime<n, 5>::value;
00488 };
00489
00490 template<size_t n>
00491 static constexpr bool is_prime_v = is_prime<n>::value;
00492
00493 // gcd
00494 namespace internal {
00495     template <std::size_t... Is>
00496     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00497     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00498
00499     template <std::size_t N>
00500     using make_index_sequence_reverse
00501     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00502
00503     template<typename Ring, typename E = void>
00504     struct gcd;
00505
00506     template<typename Ring>
00507     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00508         template<typename A, typename B, typename E = void>
00509         struct gcd_helper {};
00510
00511         // B = 0, A > 0
00512         template<typename A, typename B>
00513         struct gcd_helper<A, B, std::enable_if_t<
00514             ((B::is_zero_t::value) &&
00515              (Ring::template gt_t<A, typename Ring::zero>::value))>> {
00516             using type = A;
00517         };
00518
00519         // B = 0, A < 0
00520         template<typename A, typename B>
00521         struct gcd_helper<A, B, std::enable_if_t<
00522             ((B::is_zero_t::value) &&
00523              !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00524             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00525         };
00526
00527         // B != 0
00528         template<typename A, typename B>
00529         struct gcd_helper<A, B, std::enable_if_t<
00530             (!B::is_zero_t::value)
00531             >> {
00532         private: // NOLINT
00533             // A / B
00534             using k = typename Ring::template div_t<A, B>;
00535             // A - (A/B)*B = A % B
00536             using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00537
00538         public:
00539             using type = typename gcd_helper<B, m>::type;
00540         };
00541
00542         template<typename A, typename B>
00543         using type = typename gcd_helper<A, B>::type;
00544     };
00545 } // namespace internal
00546
00547 // vadd and vmul
00548 namespace internal {
00549     template<typename... vals>
00550     struct vmul {};
00551
00552     template<typename v1, typename... vals>
00553     struct vmul<v1, vals...> {
00554         using type = typename v1::enclosing_type::template mul_t<v1, typename
00555         vmul<vals...>::type>;
00556     };

```

```

00567
00568     template<typename v1>
00569     struct vmul<v1> {
00570         using type = v1;
00571     };
00572
00573     template<typename... vals>
00574     struct vadd {};
00575
00576     template<typename v1, typename... vals>
00577     struct vadd<v1, vals...> {
00578         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00579     };
00580
00581     template<typename v1>
00582     struct vadd<v1> {
00583         using type = v1;
00584     };
00585 } // namespace internal
00586
00587 template<typename T, typename A, typename B>
00588 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00589
00590 template<typename... vals>
00591 using vadd_t = typename internal::vadd<vals...>::type;
00592
00593 template<typename... vals>
00594 using vmul_t = typename internal::vmul<vals...>::type;
00595
00596 template<typename val>
00597 requires IsEuclideanDomain<typename val::enclosing_type>
00598 using abs_t = std::conditional_t<
00599     val::enclosing_type::template pos_v<val>,
00600     val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00601 } // namespace aerobus
00602
00603 // embedding
00604 namespace aerobus {
00605     template<typename Small, typename Large, typename E = void>
00606     struct Embed;
00607 } // namespace aerobus
00608
00609 namespace aerobus {
00610     template<typename Ring, typename X>
00611     requires IsRing<Ring>
00612     struct Quotient {
00613         template <typename V>
00614         struct val {
00615             public:
00616                 using raw_t = V;
00617                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00618         };
00619
00620         using zero = val<typename Ring::zero>;
00621
00622         using one = val<typename Ring::one>;
00623
00624         template<typename v1, typename v2>
00625         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00626
00627         template<typename v1, typename v2>
00628         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00629
00630         template<typename v1, typename v2>
00631         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00632
00633         template<typename v1, typename v2>
00634         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00635
00636         template<typename v1, typename v2>
00637         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00638
00639         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00640
00641         template<typename v1>
00642         using pos_t = std::true_type;
00643
00644         template<typename v>
00645         static constexpr bool pos_v = pos_t<v>::value;
00646
00647         static constexpr bool is_euclidean_domain = true;
00648
00649         template<auto x>
00650         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;

```

```

00703
00704     template<typename v>
00705     using inject_ring_t = val<v>;
00706 };
00707
00708     template<typename Ring, typename X>
00709     struct Embed<Quotient<Ring, X>, Ring> {
00710         template<typename val>
00711         using type = typename val::raw_t;
00712     };
00713 } // namespace aerobus
00714
00715 // type_list
00716 namespace aerobus {
00717     template <typename... Ts>
00718     struct type_list;
00719
00720     namespace internal {
00721         template <typename T, typename... Us>
00722         struct pop_front_h {
00723             using tail = type_list<Us...>;
00724             using head = T;
00725         };
00726
00727         template <size_t index, typename L1, typename L2>
00728         struct split_h {
00729             private:
00730                 static_assert(index <= L2::length, "index ouf of bounds");
00731                 using a = typename L2::pop_front::type;
00732                 using b = typename L2::pop_front::tail;
00733                 using c = typename L1::template push_back<a>;
00734
00735             public:
00736                 using head = typename split_h<index - 1, c, b>::head;
00737                 using tail = typename split_h<index - 1, c, b>::tail;
00738         };
00739
00740         template <typename L1, typename L2>
00741         struct split_h<0, L1, L2> {
00742             using head = L1;
00743             using tail = L2;
00744         };
00745
00746         template <size_t index, typename L, typename T>
00747         struct insert_h {
00748             static_assert(index <= L::length, "index ouf of bounds");
00749             using s = typename L::template split<index>;
00750             using left = typename s::head;
00751             using right = typename s::tail;
00752             using ll = typename left::template push_back<T>;
00753             using type = typename ll::template concat<right>;
00754         };
00755
00756         template <size_t index, typename L>
00757         struct remove_h {
00758             using s = typename L::template split<index>;
00759             using left = typename s::head;
00760             using right = typename s::tail;
00761             using rr = typename right::pop_front::tail;
00762             using type = typename left::template concat<rr>;
00763         };
00764     } // namespace internal
00765
00766     template <typename... Ts>
00767     struct type_list {
00768     private:
00769         template <typename T>
00770         struct concat_h;
00771
00772         template <typename... Us>
00773         struct concat_h<type_list<Us...>> {
00774             using type = type_list<Ts..., Us...>;
00775         };
00776
00777     public:
00778         static constexpr size_t length = sizeof...(Ts);
00779
00780         template <typename T>
00781         using push_front = type_list<T, Ts...>;
00782
00783         template <size_t index>
00784         using at = internal::type_at_t<index, Ts...>;
00785
00786         struct pop_front {
00787             using type = typename internal::pop_front_h<Ts...>::head;
00788             using tail = typename internal::pop_front_h<Ts...>::tail;
00789         };
00790     };

```

```

00809
00812     template <typename T>
00813     using push_back = type_list<Ts..., T>;
00814
00817     template <typename U>
00818     using concat = typename concat_h<U>::type;
00819
00822     template <size_t index>
00823     struct split {
00824     private:
00825         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00826
00827     public:
00828         using head = typename inner::head;
00829         using tail = typename inner::tail;
00830     };
00831
00835     template <typename T, size_t index>
00836     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00837
00840     template <size_t index>
00841     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00842 };
00843
00845 template <>
00846 struct type_list<> {
00847     static constexpr size_t length = 0;
00848
00849     template <typename T>
00850     using push_front = type_list<T>;
00851
00852     template <typename T>
00853     using push_back = type_list<T>;
00854
00855     template <typename U>
00856     using concat = U;
00857
00858     // TODO(jewave): assert index == 0
00859     template <typename T, size_t index>
00860     using insert = type_list<T>;
00861 };
00862 } // namespace aerobus
00863
00864 // i16
00865 #ifdef WITH_CUDA_FP16
00866 // i16
00867 namespace aerobus {
00868     struct i16 {
00869     using inner_type = int16_t;
00870     template<int16_t x>
00871     struct val {
00872         using enclosing_type = i16;
00873         static constexpr int16_t v = x;
00874
00875         template<typename valueType>
00876         static constexpr INLINED_DEVICE valueType get() {
00877             return internal::template int16_convert_helper<valueType, x>::value();
00878         }
00879
00880         using is_zero_t = std::bool_constant<x == 0>;
00881
00882         static std::string to_string() {
00883             return std::to_string(x);
00884         }
00885     };
00886
00887     using zero = val<0>;
00888     using one = val<1>;
00889
00890     static constexpr bool is_field = false;
00891     static constexpr bool is_euclidean_domain = true;
00892     template<auto x>
00893     using inject_constant_t = val<static_cast<int16_t>(x)>;
00894
00895     template<typename v>
00896     using inject_ring_t = v;
00897
00898 private:
00899     template<typename v1, typename v2>
00900     struct add {
00901         using type = val<v1::v + v2::v>;
00902     };
00903
00904     template<typename v1, typename v2>
00905     struct sub {
00906         using type = val<v1::v - v2::v>;
00907     };
00908
00909
00910
00911
00912
00913
00914
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00918
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00920
00921
00922

```

```

00923     template<typename v1, typename v2>
00924     struct mul {
00925         using type = val<v1::v* v2::v>;
00926     };
00927
00928     template<typename v1, typename v2>
00929     struct div {
00930         using type = val<v1::v / v2::v>;
00931     };
00932
00933     template<typename v1, typename v2>
00934     struct remainder {
00935         using type = val<v1::v % v2::v>;
00936     };
00937
00938     template<typename v1, typename v2>
00939     struct gt {
00940         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00941     };
00942
00943     template<typename v1, typename v2>
00944     struct lt {
00945         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00946     };
00947
00948     template<typename v1, typename v2>
00949     struct eq {
00950         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00951     };
00952
00953     template<typename v1>
00954     struct pos {
00955         using type = std::bool_constant<(v1::v > 0)>;
00956     };
00957
00958     public:
00963     template<typename v1, typename v2>
00964     using add_t = typename add<v1, v2>::type;
00965
00970     template<typename v1, typename v2>
00971     using sub_t = typename sub<v1, v2>::type;
00972
00977     template<typename v1, typename v2>
00978     using mul_t = typename mul<v1, v2>::type;
00979
00984     template<typename v1, typename v2>
00985     using div_t = typename div<v1, v2>::type;
00986
00991     template<typename v1, typename v2>
00992     using mod_t = typename remainder<v1, v2>::type;
00993
00998     template<typename v1, typename v2>
00999     using gt_t = typename gt<v1, v2>::type;
01000
01005     template<typename v1, typename v2>
01006     using lt_t = typename lt<v1, v2>::type;
01007
01012     template<typename v1, typename v2>
01013     using eq_t = typename eq<v1, v2>::type;
01014
01018     template<typename v1, typename v2>
01019     static constexpr bool eq_v = eq_t<v1, v2>::value;
01020
01025     template<typename v1, typename v2>
01026     using gcd_t = gcd_t<i16, v1, v2>;
01027
01031     template<typename v>
01032     using pos_t = typename pos<v>::type;
01033
01037     template<typename v>
01038     static constexpr bool pos_v = pos_t<v>::value;
01039     };
01040 } // namespace aerobus
01041 #endif
01042
01043 // i32
01044 namespace aerobus {
01045     struct i32 {
01046         using inner_type = int32_t;
01047         template<int32_t x>
01048         struct val {
01049             using enclosing_type = i32;
01050             static constexpr int32_t v = x;
01051
01052             template<typename valueType>
01053             static constexpr DEVICE valueType get() {
01054                 return static_cast<valueType>(x);
01055             }
01056         };
01057     };
01058 }

```

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```

01189     template<typename v1, typename v2>
01190     using eq_t = typename eq<v1, v2>::type;
01191
01195     template<typename v1, typename v2>
01196     static constexpr bool eq_v = eq_t<v1, v2>::value;
01197
01202     template<typename v1, typename v2>
01203     using gcd_t = gcd_t<i32, v1, v2>;
01204
01208     template<typename v>
01209     using pos_t = typename pos<v>::type;
01210
01214     template<typename v>
01215     static constexpr bool pos_v = pos_t<v>::value;
01216 };
01217 } // namespace aerobus
01218
01219 // i64
01220 namespace aerobus {
01221     struct i64 {
01222         using inner_type = int64_t;
01223         template<int64_t x>
01224         struct val {
01225             using inner_type = int32_t;
01226             using enclosing_type = i64;
01227             static constexpr int64_t v = x;
01228
01229             template<typename valueType>
01230             static constexpr INLINED_DEVICE valueType get() {
01231                 return static_cast<valueType>(x);
01232             }
01233
01234             using is_zero_t = std::bool_constant<x == 0>;
01235
01236             static std::string to_string() {
01237                 return std::to_string(x);
01238             }
01239         };
01240
01241         template<auto x>
01242         using inject_constant_t = val<static_cast<int64_t>(x)>;
01243
01244         template<typename v>
01245         using inject_ring_t = v;
01246
01247         using zero = val<0>;
01248         using one = val<1>;
01249         static constexpr bool is_field = false;
01250         static constexpr bool is_euclidean_domain = true;
01251
01252     private:
01253         template<typename v1, typename v2>
01254         struct add {
01255             using type = val<v1::v + v2::v>;
01256         };
01257
01258         template<typename v1, typename v2>
01259         struct sub {
01260             using type = val<v1::v - v2::v>;
01261         };
01262
01263         template<typename v1, typename v2>
01264         struct mul {
01265             using type = val<v1::v * v2::v>;
01266         };
01267
01268         template<typename v1, typename v2>
01269         struct div {
01270             using type = val<v1::v / v2::v>;
01271         };
01272
01273         template<typename v1, typename v2>
01274         struct remainder {
01275             using type = val<v1::v % v2::v>;
01276         };
01277
01278         template<typename v1, typename v2>
01279         struct gt {
01280             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01281         };
01282
01283         template<typename v1, typename v2>
01284         struct lt {
01285             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01286         };
01287
01288         template<typename v1, typename v2>

```

```

01310     struct eq {
01311         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01312     };
01313
01314     template<typename v>
01315     struct pos {
01316         using type = std::bool_constant<(v::v > 0)>;
01317     };
01318
01319 public:
01320     template<typename v1, typename v2>
01321     using add_t = typename add<v1, v2>::type;
01322
01323     template<typename v1, typename v2>
01324     using sub_t = typename sub<v1, v2>::type;
01325
01326     template<typename v1, typename v2>
01327     using mul_t = typename mul<v1, v2>::type;
01328
01329     template<typename v1, typename v2>
01330     using div_t = typename div<v1, v2>::type;
01331
01332     template<typename v1, typename v2>
01333     using mod_t = typename remainder<v1, v2>::type;
01334
01335     template<typename v1, typename v2>
01336     using gt_t = typename gt<v1, v2>::type;
01337
01338     template<typename v1, typename v2>
01339     static constexpr bool gt_v = gt_t<v1, v2>::value;
01340
01341     template<typename v1, typename v2>
01342     using lt_t = typename lt<v1, v2>::type;
01343
01344     template<typename v1, typename v2>
01345     static constexpr bool lt_v = lt_t<v1, v2>::value;
01346
01347     template<typename v1, typename v2>
01348     using eq_t = typename eq<v1, v2>::type;
01349
01350     template<typename v1, typename v2>
01351     static constexpr bool eq_v = eq_t<v1, v2>::value;
01352
01353     template<typename v1, typename v2>
01354     using gcd_t = gcd_t<i64, v1, v2>;
01355
01356     template<typename v>
01357     using pos_t = typename pos<v>::type;
01358
01359     template<typename v>
01360     static constexpr bool pos_v = pos_t<v>::value;
01361 };
01362
01363 template<>
01364 struct Embed<i32, i64> {
01365     template<typename val>
01366     using type = i64::val<static_cast<int64_t>(val::v)>;
01367 };
01368 } // namespace aerobus
01369
01370 // z/pz
01371 namespace aerobus {
01372     template<int32_t p>
01373     struct zpz {
01374         using inner_type = int32_t;
01375
01376         template<int32_t x>
01377         struct val {
01378             using enclosing_type = zpz<p>;
01379             static constexpr int32_t v = x % p;
01380
01381             template<typename valueType>
01382             static constexpr INLINED_DEVICE valueType get() {
01383                 return static_cast<valueType>(x % p);
01384             }
01385
01386             using is_zero_t = std::bool_constant<v == 0>;
01387
01388             static constexpr bool is_zero_v = v == 0;
01389
01390             static std::string to_string() {
01391                 return std::to_string(x % p);
01392             }
01393         };
01394     };
01395
01396     template<auto x>
01397     using inject_constant_t = val<static_cast<int32_t>(x)>;

```

```

01468
01470     using zero = val<0>;
01471
01473     using one = val<1>;
01474
01476     static constexpr bool is_field = is_prime<p>::value;
01477
01479     static constexpr bool is_euclidean_domain = true;
01480
01481 private:
01482     template<typename v1, typename v2>
01483     struct add {
01484         using type = val<(v1::v + v2::v) % p>;
01485     };
01486
01487     template<typename v1, typename v2>
01488     struct sub {
01489         using type = val<(v1::v - v2::v) % p>;
01490     };
01491
01492     template<typename v1, typename v2>
01493     struct mul {
01494         using type = val<(v1::v * v2::v) % p>;
01495     };
01496
01497     template<typename v1, typename v2>
01498     struct div {
01499         using type = val<(v1::v % p) / (v2::v % p)>;
01500     };
01501
01502     template<typename v1, typename v2>
01503     struct remainder {
01504         using type = val<(v1::v % v2::v) % p>;
01505     };
01506
01507     template<typename v1, typename v2>
01508     struct gt {
01509         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01510     };
01511
01512     template<typename v1, typename v2>
01513     struct lt {
01514         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01515     };
01516
01517     template<typename v1, typename v2>
01518     struct eq {
01519         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01520     };
01521
01522     template<typename v1>
01523     struct pos {
01524         using type = std::bool_constant<(v1::v > 0)>;
01525     };
01526
01527 public:
01531     template<typename v1, typename v2>
01532     using add_t = typename add<v1, v2>::type;
01533
01537     template<typename v1, typename v2>
01538     using sub_t = typename sub<v1, v2>::type;
01539
01543     template<typename v1, typename v2>
01544     using mul_t = typename mul<v1, v2>::type;
01545
01549     template<typename v1, typename v2>
01550     using div_t = typename div<v1, v2>::type;
01551
01555     template<typename v1, typename v2>
01556     using mod_t = typename remainder<v1, v2>::type;
01557
01561     template<typename v1, typename v2>
01562     using gt_t = typename gt<v1, v2>::type;
01563
01567     template<typename v1, typename v2>
01568     static constexpr bool gt_v = gt_t<v1, v2>::value;
01569
01573     template<typename v1, typename v2>
01574     using lt_t = typename lt<v1, v2>::type;
01575
01579     template<typename v1, typename v2>
01580     static constexpr bool lt_v = lt_t<v1, v2>::value;
01581
01585     template<typename v1, typename v2>
01586     using eq_t = typename eq<v1, v2>::type;
01587
01591     template<typename v1, typename v2>

```

```

01592         static constexpr bool eq_v = eq_t<v1, v2>::value;
01593
01597         template<typename v1, typename v2>
01598         using gcd_t = gcd_t<i32, v1, v2>;
01599
01602         template<typename v1>
01603         using pos_t = typename pos<v1>::type;
01604
01607         template<typename v>
01608         static constexpr bool pos_v = pos_t<v>::value;
01609     };
01610
01613     template<int32_t x>
01614     struct Embed<zpz<x>, i32> {
01617         template <typename val>
01618         using type = i32::val<val::v>;
01619     };
01620 } // namespace aerobus
01621
01622 // polynomial
01623 namespace aerobus {
01624     // coeffN x^N + ...
01629     template<typename Ring>
01630     requires IsEuclideanDomain<Ring>
01631     struct polynomial {
01632         static constexpr bool is_field = false;
01633         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01634
01637         template<typename P>
01638         struct horner_reduction_t {
01639             template<size_t index, size_t stop>
01640             struct inner {
01641                 template<typename accum, typename x>
01642                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01643                     ::template type<
01644                         typename Ring::template add_t<
01645                             typename Ring::template mul_t<x, accum>,
01646                             typename P::template coeff_at_t<P::degree - index>
01647                             >, x>;
01648             };
01649
01650             template<size_t stop>
01651             struct inner<stop, stop> {
01652                 template<typename accum, typename x>
01653                 using type = accum;
01654             };
01655         };
01656
01660         template<typename coeffN, typename... coeffs>
01661         struct val {
01663             using ring_type = Ring;
01665             using enclosing_type = polynomial<Ring>;
01667             static constexpr size_t degree = sizeof...(coeffs);
01669             using aN = coeffN;
01671             using strip = val<coeffs...>;
01673             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01675             static constexpr bool is_zero_v = is_zero_t::value;
01676
01677         private:
01678             template<size_t index, typename E = void>
01679             struct coeff_at {};
01680
01681             template<size_t index>
01682             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01683                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01684             };
01685
01686             template<size_t index>
01687             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01688                 using type = typename Ring::zero;
01689             };
01690
01691         public:
01694             template<size_t index>
01695             using coeff_at_t = typename coeff_at<index>::type;
01696
01699             static std::string to_string() {
01700                 return string_helper<coeffN, coeffs...>::func();
01701             }
01702
01707             template<typename arithmeticType>
01708             static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01709                 #ifdef WITH_CUDA_FP16
01710                 arithmeticType start;
01711                 if constexpr (std::is_same_v<arithmeticType, __half2>) {
01712                     start = __half2(0, 0);
01713                 } else {

```

```

01714         start = static_cast<arithmeticType>(0);
01715     }
01716     #else
01717     arithmeticType start = static_cast<arithmeticType>(0);
01718     #endif
01719     return horner_evaluation<arithmeticType, val>
01720         ::template inner<0, degree + 1>
01721         ::func(start, x);
01722 }
01723
01724 template<typename arithmeticType>
01725 static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01726     return compensated_horner<arithmeticType, val>::func(x);
01727 }
01728
01729 template<typename x>
01730 using value_at_t = horner_reduction_t<val>
01731     ::template inner<0, degree + 1>
01732     ::template type<typename Ring::zero, x>;
01733
01734 };
01735
01736 template<typename coeffN>
01737 struct val<coeffN> {
01738     using ring_type = Ring;
01739     using enclosing_type = polynomial<Ring>;
01740     static constexpr size_t degree = 0;
01741     using aN = coeffN;
01742     using strip = val<coeffN>;
01743     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01744
01745     static constexpr bool is_zero_v = is_zero_t::value;
01746
01747     template<size_t index, typename E = void>
01748     struct coeff_at {};
01749
01750     template<size_t index>
01751     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01752         using type = aN;
01753     };
01754
01755     template<size_t index>
01756     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01757         using type = typename Ring::zero;
01758     };
01759
01760     template<size_t index>
01761     using coeff_at_t = typename coeff_at<index>::type;
01762
01763     static std::string to_string() {
01764         return string_helper<coeffN>::func();
01765     }
01766
01767     template<typename arithmeticType>
01768     static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01769         return coeffN::template get<arithmeticType>();
01770     }
01771
01772     template<typename arithmeticType>
01773     static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01774         return coeffN::template get<arithmeticType>();
01775     }
01776
01777     template<typename x>
01778     using value_at_t = coeffN;
01779 };
01780
01781 using zero = val<typename Ring::zero>;
01782 using one = val<typename Ring::one>;
01783 using X = val<typename Ring::one, typename Ring::zero>;
01784
01785 private:
01786     template<typename P, typename E = void>
01787     struct simplify;
01788
01789     template<typename P1, typename P2, typename I>
01790     struct add_low;
01791
01792     template<typename P1, typename P2>
01793     struct add {
01794         using type = typename simplify<typename add_low<
01795             P1,
01796             P2,
01797             internal::make_index_sequence_reverse<
01798                 std::max(P1::degree, P2::degree) + 1
01799             >::type>::type;
01800     };
01801
01802

```

```

01821     template <typename P1, typename P2, typename I>
01822     struct sub_low;
01823
01824     template <typename P1, typename P2, typename I>
01825     struct mul_low;
01826
01827     template<typename v1, typename v2>
01828     struct mul {
01829         using type = typename mul_low<
01830             v1,
01831             v2,
01832             internal::make_index_sequence_reverse<
01833                 v1::degree + v2::degree + 1
01834             >::type;
01835     };
01836
01837     template<typename coeff, size_t deg>
01838     struct monomial;
01839
01840     template<typename v, typename E = void>
01841     struct derive_helper {};
01842
01843     template<typename v>
01844     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01845         using type = zero;
01846     };
01847
01848     template<typename v>
01849     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01850         using type = typename add<
01851             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01852             typename monomial<
01853                 typename Ring::template mul_t<
01854                     typename v::aN,
01855                     typename Ring::template inject_constant_t<(v::degree)>
01856                 >,
01857                 v::degree - 1
01858             >::type
01859         >::type;
01860     };
01861
01862     template<typename v1, typename v2, typename E = void>
01863     struct eq_helper {};
01864
01865     template<typename v1, typename v2>
01866     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01867         using type = std::false_type;
01868     };
01869
01870     template<typename v1, typename v2>
01871     struct eq_helper<v1, v2, std::enable_if_t<
01872         v1::degree == v2::degree &&
01873         (v1::degree != 0 || v2::degree != 0) &&
01874         std::is_same<
01875             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01876             std::false_type
01877         >::value
01878     >
01879     > {
01880         using type = std::false_type;
01881     };
01882
01883     template<typename v1, typename v2>
01884     struct eq_helper<v1, v2, std::enable_if_t<
01885         v1::degree == v2::degree &&
01886         (v1::degree != 0 || v2::degree != 0) &&
01887         std::is_same<
01888             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01889             std::true_type
01890         >::value
01891     > {
01892         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01893     };
01894
01895     template<typename v1, typename v2>
01896     struct eq_helper<v1, v2, std::enable_if_t<
01897         v1::degree == v2::degree &&
01898         (v1::degree == 0)
01899     > {
01900         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01901     };
01902
01903     template<typename v1, typename v2, typename E = void>
01904     struct lt_helper {};
01905
01906     template<typename v1, typename v2>

```

```

01908     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01909         using type = std::true_type;
01910     };
01911
01912     template<typename v1, typename v2>
01913     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01914         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01915     };
01916
01917     template<typename v1, typename v2>
01918     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01919         using type = std::false_type;
01920     };
01921
01922     template<typename v1, typename v2, typename E = void>
01923     struct gt_helper {};
01924
01925     template<typename v1, typename v2>
01926     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01927         using type = std::true_type;
01928     };
01929
01930     template<typename v1, typename v2>
01931     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01932         using type = std::false_type;
01933     };
01934
01935     template<typename v1, typename v2>
01936     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01937         using type = std::false_type;
01938     };
01939
01940     // when high power is zero : strip
01941     template<typename P>
01942     struct simplify<P, std::enable_if_t<
01943         std::is_same<
01944             typename Ring::zero,
01945             typename P::aN
01946         >::value && (P::degree > 0)
01947     >> {
01948         using type = typename simplify<typename P::strip>::type;
01949     };
01950
01951     // otherwise : do nothing
01952     template<typename P>
01953     struct simplify<P, std::enable_if_t<
01954         !std::is_same<
01955             typename Ring::zero,
01956             typename P::aN
01957         >::value && (P::degree > 0)
01958     >> {
01959         using type = P;
01960     };
01961
01962     // do not simplify constants
01963     template<typename P>
01964     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01965         using type = P;
01966     };
01967
01968     // addition at
01969     template<typename P1, typename P2, size_t index>
01970     struct add_at {
01971         using type =
01972             typename Ring::template add_t<
01973                 typename P1::template coeff_at_t<index>,
01974                 typename P2::template coeff_at_t<index>;
01975     };
01976
01977     template<typename P1, typename P2, size_t index>
01978     using add_at_t = typename add_at<P1, P2, index>::type;
01979
01980     template<typename P1, typename P2, std::size_t... I>
01981     struct add_low<P1, P2, std::index_sequence<I...>> {
01982         using type = val<add_at_t<P1, P2, I>...>;
01983     };
01984
01985     // subtraction at
01986     template<typename P1, typename P2, size_t index>
01987     struct sub_at {
01988         using type =
01989             typename Ring::template sub_t<
01990                 typename P1::template coeff_at_t<index>,
01991                 typename P2::template coeff_at_t<index>;
01992     };
01993
01994     template<typename P1, typename P2, size_t index>

```

```

01995     using sub_at_t = typename sub_at<P1, P2, index>::type;
01996
01997     template<typename P1, typename P2, std::size_t... I>
01998     struct sub_low<P1, P2, std::index_sequence<I...> {
01999         using type = val<sub_at_t<P1, P2, I>...>;
02000     };
02001
02002     template<typename P1, typename P2>
02003     struct sub {
02004         using type = typename simplify<typename sub_low<
02005             P1,
02006             P2,
02007             internal::make_index_sequence_reverse<
02008                 std::max(P1::degree, P2::degree) + 1
02009             >::type>::type;
02010     };
02011
02012     // multiplication at
02013     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02014     struct mul_at_loop_helper {
02015         using type = typename Ring::template add_t<
02016             typename Ring::template mul_t<
02017                 typename v1::template coeff_at_t<index>,
02018                 typename v2::template coeff_at_t<k - index>
02019             >,
02020             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02021         >;
02022     };
02023
02024     template<typename v1, typename v2, size_t k, size_t stop>
02025     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02026         using type = typename Ring::template mul_t<
02027             typename v1::template coeff_at_t<stop>,
02028             typename v2::template coeff_at_t<0>;
02029     };
02030
02031     template <typename v1, typename v2, size_t k, typename E = void>
02032     struct mul_at {};
02033
02034     template<typename v1, typename v2, size_t k>
02035     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
02036         using type = typename Ring::zero;
02037     };
02038
02039     template<typename v1, typename v2, size_t k>
02040     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
02041         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02042     };
02043
02044     template<typename P1, typename P2, size_t index>
02045     using mul_at_t = typename mul_at<P1, P2, index>::type;
02046
02047     template<typename P1, typename P2, std::size_t... I>
02048     struct mul_low<P1, P2, std::index_sequence<I...> {
02049         using type = val<mul_at_t<P1, P2, I>...>;
02050     };
02051
02052     // division helper
02053     template< typename A, typename B, typename Q, typename R, typename E = void>
02054     struct div_helper {};
02055
02056     template<typename A, typename B, typename Q, typename R>
02057     struct div_helper<A, B, Q, R, std::enable_if_t<
02058         (R::degree < B::degree) ||
02059         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02060         using q_type = Q;
02061         using mod_type = R;
02062         using gcd_type = B;
02063     };
02064
02065     template<typename A, typename B, typename Q, typename R>
02066     struct div_helper<A, B, Q, R, std::enable_if_t<
02067         (R::degree >= B::degree) &&
02068         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02069     private: // NOLINT
02070         using rN = typename R::aN;
02071         using bN = typename B::aN;
02072         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02073             B::degree>::type;
02074         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02075         using qq = typename add<Q, pT>::type;
02076
02077     public:
02078         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02079         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02080         using gcd_type = rr;
02081     };

```



```

02081
02082     template<typename A, typename B>
02083     struct div {
02084         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02085         using q_type = typename div_helper<A, B, zero, A>::q_type;
02086         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02087     };
02088
02089     template<typename P>
02090     struct make_unit {
02091         using type = typename div<P, val<typename P::aN>>::q_type;
02092     };
02093
02094     template<typename coeff, size_t deg>
02095     struct monomial {
02096         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02097     };
02098
02099     template<typename coeff>
02100     struct monomial<coeff, 0> {
02101         using type = val<coeff>;
02102     };
02103
02104     template<typename arithmeticType, typename P>
02105     struct horner_evaluation {
02106         template<size_t index, size_t stop>
02107         struct inner {
02108             static constexpr DEVICE INLINED arithmeticType func(
02109                 const arithmeticType& accum, const arithmeticType& x) {
02110                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02111 stop>::func(
02112                     internal::fma_helper<arithmeticType>::eval(
02113                         x,
02114                         accum,
02115                         P::template coeff_at_t<P::degree - index>::template
02116 get<arithmeticType>()), x);
02117             }
02118         };
02119         template<size_t stop>
02120         struct inner<stop, stop> {
02121             static constexpr DEVICE INLINED arithmeticType func(
02122                 const arithmeticType& accum, const arithmeticType& x) {
02123                 return accum;
02124             }
02125         };
02126     };
02127
02128     template<typename arithmeticType, typename P>
02129     struct compensated_horner {
02130         template<int64_t index, int ghost>
02131         struct EFTHorner {
02132             static INLINED void func(
02133                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02134 *r) {
02135                 arithmeticType p;
02136                 internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02137                 constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
02138 get<arithmeticType>();
02139                 internal::two_sum<arithmeticType>(
02140                     p, coeff,
02141                     r, sigma + P::degree - index - 1);
02142                 EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02143             }
02144         };
02145         template<int ghost>
02146         struct EFTHorner<-1, ghost> {
02147             static INLINED DEVICE void func(
02148                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02149 *r) {
02150             }
02151         };
02152     };
02153
02154     static INLINED DEVICE arithmeticType func(arithmeticType x) {
02155         arithmeticType pi[P::degree], sigma[P::degree];
02156         arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02157         EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02158         arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02159         return r + c;
02160     }
02161
02162     template<typename coeff, typename... coeffs>
02163     struct string_helper {
02164         static std::string func() {
02165             std::string tail = string_helper<coeffs...>::func();

```

```

02163         std::string result = "";
02164         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02165             return tail;
02166         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02167             if (sizeof...(coeffs) == 1) {
02168                 result += "x";
02169             } else {
02170                 result += "x^" + std::to_string(sizeof...(coeffs));
02171             }
02172         } else {
02173             if (sizeof...(coeffs) == 1) {
02174                 result += coeff::to_string() + " x";
02175             } else {
02176                 result += coeff::to_string()
02177                     + " x^" + std::to_string(sizeof...(coeffs));
02178             }
02179         }
02180
02181         if (!tail.empty()) {
02182             if (tail.at(0) != '-') {
02183                 result += " + " + tail;
02184             } else {
02185                 result += " - " + tail.substr(1);
02186             }
02187         }
02188
02189         return result;
02190     }
02191 };
02192
02193 template<typename coeff>
02194 struct string_helper<coeff> {
02195     static std::string func() {
02196         if (!std::is_same<coeff, typename Ring::zero>::value) {
02197             return coeff::to_string();
02198         } else {
02199             return "";
02200         }
02201     }
02202 };
02203
02204 public:
02205     template<typename P>
02206     using simplify_t = typename simplify<P>::type;
02207
02208     template<typename v1, typename v2>
02209     using add_t = typename add<v1, v2>::type;
02210
02211     template<typename v1, typename v2>
02212     using sub_t = typename sub<v1, v2>::type;
02213
02214     template<typename v1, typename v2>
02215     using mul_t = typename mul<v1, v2>::type;
02216
02217     template<typename v1, typename v2>
02218     using eq_t = typename eq_helper<v1, v2>::type;
02219
02220     template<typename v1, typename v2>
02221     using lt_t = typename lt_helper<v1, v2>::type;
02222
02223     template<typename v1, typename v2>
02224     using gt_t = typename gt_helper<v1, v2>::type;
02225
02226     template<typename v1, typename v2>
02227     using div_t = typename div<v1, v2>::q_type;
02228
02229     template<typename v1, typename v2>
02230     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02231
02232     template<typename coeff, size_t deg>
02233     using monomial_t = typename monomial<coeff, deg>::type;
02234
02235     template<typename v>
02236     using derive_t = typename derive_helper<v>::type;
02237
02238     template<typename v>
02239     using pos_t = typename Ring::template pos_t<typename v::aN>;
02240
02241     template<typename v>
02242     static constexpr bool pos_v = pos_t<v>::value;
02243
02244     template<typename v1, typename v2>
02245     using gcd_t = std::conditional_t<
02246         Ring::is_euclidean_domain,
02247         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02248         void>;
02249 
```

```

02290     template<auto x>
02291     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02292
02293     template<typename v>
02294     using inject_ring_t = val<v>;
02295 };
02296 } // namespace aerobus
02297
02298 // fraction field
02299 namespace aerobus {
02300     namespace internal {
02301         template<typename Ring, typename E = void>
02302         requires IsEuclideanDomain<Ring>
02303         struct _FractionField {};
02304
02305         template<typename Ring>
02306         requires IsEuclideanDomain<Ring>
02307         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02308             static constexpr bool is_field = true;
02309             static constexpr bool is_euclidean_domain = true;
02310
02311         private:
02312             template<typename vall, typename val2, typename E = void>
02313             struct to_string_helper {};
02314
02315             template<typename vall, typename val2>
02316             struct to_string_helper <vall, val2,
02317                 std::enable_if_t<
02318                     Ring::template eq_t<
02319                         val2, typename Ring::one
02320                         >::value
02321                     > {
02322                 static std::string func() {
02323                     return vall::to_string();
02324                 }
02325             };
02326
02327             template<typename vall, typename val2>
02328             struct to_string_helper<vall, val2,
02329                 std::enable_if_t<
02330                     !Ring::template eq_t<
02331                         val2,
02332                         typename Ring::one
02333                         >::value
02334                     > {
02335                 static std::string func() {
02336                     return "(" + vall::to_string() + ") / (" + val2::to_string() + ")";
02337                 }
02338             };
02339
02340         public:
02341             template<typename vall, typename val2>
02342             struct val {
02343                 using x = vall;
02344                 using y = val2;
02345                 using is_zero_t = typename vall::is_zero_t;
02346                 static constexpr bool is_zero_v = vall::is_zero_t::value;
02347
02348                 using ring_type = Ring;
02349                 using enclosing_type = _FractionField<Ring>;
02350
02351                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02352
02353                 template<typename valueType, int ghost = 0>
02354                 struct get_helper {
02355                     static constexpr INLINED_DEVICE valueType get() {
02356                         return internal::staticcast<valueType, typename
02357                             ring_type::inner_type>::template func<x::v>() /
02358                             internal::staticcast<valueType, typename ring_type::inner_type>::template
02359                             func<y::v>();
02360                     }
02361                 };
02362
02363                 #ifdef WITH_CUDA_FP16
02364                 template<int ghost>
02365                 struct get_helper<__half, ghost> {
02366                     static constexpr INLINED_DEVICE __half get() {
02367                         return internal::my_float2half_rn(
02368                             internal::staticcast<float, typename ring_type::inner_type>::template
02369                             func<x::v>() /
02370                             internal::staticcast<float, typename ring_type::inner_type>::template
02371                             func<y::v>());
02372                     }
02373                 };
02374             };
02375
02376             #ifdef WITH_CUDA_FP16
02377             template<int ghost>
02378             struct get_helper<__half, ghost> {
02379                 static constexpr INLINED_DEVICE __half get() {
02380                     return internal::my_float2half_rn(
02381                         internal::staticcast<float, typename ring_type::inner_type>::template
02382                         func<x::v>() /
02383                         internal::staticcast<float, typename ring_type::inner_type>::template
02384                         func<y::v>());
02385             };

```

```

02386         template<int ghost>
02387         struct get_helper<__half2, ghost> {
02388             static constexpr INLINED_DEVICE __half2 get() {
02389                 constexpr __half tmp = internal::my_float2half_rn(
02390                     internal::staticcast<float, typename ring_type::inner_type>::template
02391 func<x::v>() /
02391                     internal::staticcast<float, typename ring_type::inner_type>::template
02392 func<y::v>());
02392                 return __half2(tmp, tmp);
02393             }
02394         };
02395     #endif
02396
02400     template<typename valueType>
02401     static constexpr INLINED_DEVICE valueType get() {
02402         return get_helper<valueType, 0>::get();
02403     }
02404
02407     static std::string to_string() {
02408         return to_string_helper<vall, val2>::func();
02409     }
02410
02415     template<typename arithmeticType>
02416     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02417         return x::eval(v) / y::eval(v);
02418     }
02419 };
02420
02422 using zero = val<typename Ring::zero, typename Ring::one>;
02424 using one = val<typename Ring::one, typename Ring::one>;
02425
02428 template<typename v>
02429 using inject_t = val<v, typename Ring::one>;
02430
02433 template<auto x>
02434 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02435
02438 template<typename v>
02439 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02440
02442 using ring_type = Ring;
02443
02444 private:
02445     template<typename v, typename E = void>
02446     struct simplify {};
02447
02448     // x = 0
02449     template<typename v>
02450     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02451         using type = typename _FractionField<Ring>::zero;
02452     };
02453
02454     // x != 0
02455     template<typename v>
02456     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02457     private:
02458         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02459         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02460         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02461
02462         using posx = std::conditional_t<
02463             !Ring::template pos_v<newx>,
02464             typename Ring::template sub_t<typename Ring::zero, newx>,
02465             newx>;
02466         using posy = std::conditional_t<
02467             !Ring::template pos_v<newy>,
02468             typename Ring::template sub_t<typename Ring::zero, newy>,
02469             newy>;
02470     public:
02471         using type = typename _FractionField<Ring>::template val<posx, posy>;
02472     };
02473
02474 public:
02477     template<typename v>
02478     using simplify_t = typename simplify<v>::type;
02479
02480 private:
02481     template<typename v1, typename v2>
02482     struct add {
02483     private:
02484         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02485         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02486         using dividend = typename Ring::template add_t<a, b>;
02487         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02488         using g = typename Ring::template gcd_t<dividend, diviser>;
02489

```

```

02490         public:
02491             using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02492     };
02493
02494     template<typename v>
02495     struct pos {
02496         using type = std::conditional_t<
02497             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02498             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02499             std::true_type,
02500             std::false_type>;
02501     };
02502
02503     template<typename v1, typename v2>
02504     struct sub {
02505     private:
02506         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02507         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02508         using dividend = typename Ring::template sub_t<a, b>;
02509         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02510         using g = typename Ring::template gcd_t<dividend, diviser>;
02511
02512     public:
02513         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02514     };
02515
02516     template<typename v1, typename v2>
02517     struct mul {
02518     private:
02519         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02520         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02521
02522     public:
02523         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02524     };
02525
02526     template<typename v1, typename v2, typename E = void>
02527     struct div {};
02528
02529     template<typename v1, typename v2>
02530     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
02531     private:
02532         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02533         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02534
02535     public:
02536         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02537     };
02538
02539     template<typename v1, typename v2>
02540     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02541         using type = one;
02542     };
02543
02544     template<typename v1, typename v2>
02545     struct eq {
02546     private:
02547         using type = std::conditional_t<
02548             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02549             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02550             std::true_type,
02551             std::false_type>;
02552     };
02553
02554     template<typename v1, typename v2, typename E = void>
02555     struct gt;
02556
02557     template<typename v1, typename v2>
02558     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
02559     > {
02560         using type = std::false_type;
02561     };
02562
02563     template<typename v1, typename v2>
02564     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
02565     (!pos<v1>::type::value) && (!pos<v2>::type::value)
02566     > {
02567         using type = typename gt<
02568             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02569             >::type;
02570     };
02571
02572     };
02573

```

```

02574     template<typename v1, typename v2>
02575     struct gt<v1, v2, std::enable_if_t<
02576         (!eq<v1, v2>::type::value) &&
02577         (pos<v1>::type::value) && (!pos<v2>::type::value)
02578         >> {
02579         using type = std::true_type;
02580     };
02581
02582     template<typename v1, typename v2>
02583     struct gt<v1, v2, std::enable_if_t<
02584         (!eq<v1, v2>::type::value) &&
02585         (!pos<v1>::type::value) && (pos<v2>::type::value)
02586         >> {
02587         using type = std::false_type;
02588     };
02589
02590     template<typename v1, typename v2>
02591     struct gt<v1, v2, std::enable_if_t<
02592         (!eq<v1, v2>::type::value) &&
02593         (pos<v1>::type::value) && (pos<v2>::type::value)
02594         >> {
02595         using type = typename Ring::template gt_t<
02596             typename Ring::template mul_t<v1::x, v2::y>,
02597             typename Ring::template mul_t<v2::y, v2::x>
02598         >;
02599     };
02600
02601     public:
02602     template<typename v1, typename v2>
02603     using add_t = typename add<v1, v2>::type;
02604
02605     template<typename v1, typename v2>
02606     using mod_t = zero;
02607
02608     template<typename v1, typename v2>
02609     using gcd_t = v1;
02610
02611     template<typename v1, typename v2>
02612     using sub_t = typename sub<v1, v2>::type;
02613
02614     template<typename v1, typename v2>
02615     using mul_t = typename mul<v1, v2>::type;
02616
02617     template<typename v1, typename v2>
02618     using div_t = typename div<v1, v2>::type;
02619
02620     template<typename v1, typename v2>
02621     using eq_t = typename eq<v1, v2>::type;
02622
02623     template<typename v1, typename v2>
02624     static constexpr bool eq_v = eq<v1, v2>::type::value;
02625
02626     template<typename v1, typename v2>
02627     using gt_t = typename gt<v1, v2>::type;
02628
02629     template<typename v1, typename v2>
02630     static constexpr bool gt_v = gt<v1, v2>::type::value;
02631
02632     template<typename v1>
02633     using pos_t = typename pos<v1>::type;
02634
02635     template<typename v>
02636     static constexpr bool pos_v = pos_t<v>::value;
02637
02638     template<typename Ring, typename E = void>
02639     requires IsEuclideanDomain<Ring>
02640     struct FractionFieldImpl {};
02641
02642     // fraction field of a field is the field itself
02643     template<typename Field>
02644     requires IsEuclideanDomain<Field>
02645     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field>> {
02646         using type = Field;
02647         template<typename v>
02648         using inject_t = v;
02649     };
02650
02651     // fraction field of a ring is the actual fraction field
02652     template<typename Ring>
02653     requires IsEuclideanDomain<Ring>
02654     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field>> {
02655         using type = _FractionField<Ring>;
02656     };
02657 } // namespace internal
02658
02659 template<typename Ring>

```

```

02699     requires IsEuclideanDomain<Ring>
02700     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02701
02704     template<typename Ring>
02705     struct Embed<Ring, FractionField<Ring> > {
02708         template<typename v>
02709         using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02710     };
02711 } // namespace aerobus
02712
02713
02714 // short names for common types
02715 namespace aerobus {
02716     template<typename X, typename Y>
02717     requires IsRing<typename X::enclosing_type> &&
02718         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02719     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02720
02721     template<typename X, typename Y>
02722     requires IsRing<typename X::enclosing_type> &&
02723         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02724     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02725
02726     template<typename X, typename Y>
02727     requires IsRing<typename X::enclosing_type> &&
02728         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02729     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02730
02731     template<typename X, typename Y>
02732     requires IsEuclideanDomain<typename X::enclosing_type> &&
02733         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02734     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02735
02736     using q32 = FractionField<i32>;
02737
02738     using fpq32 = FractionField<polynomial<q32>>;
02739
02740     using q64 = FractionField<i64>;
02741
02742     using pi64 = polynomial<i64>;
02743
02744     using pq64 = polynomial<q64>;
02745
02746     using fpq64 = FractionField<polynomial<q64>>;
02747
02748     template<typename Ring, typename v1, typename v2>
02749     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02750
02751     template<typename v>
02752     using embed_int_poly_in_fractions_t =
02753         typename Embed<
02754             polynomial<typename v::ring_type>,
02755             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02756
02757     template<int64_t p, int64_t q>
02758     using make_q64_t = typename q64::template simplify_t<
02759         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02760
02761     template<int32_t p, int32_t q>
02762     using make_q32_t = typename q32::template simplify_t<
02763         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02764
02765     #ifdef WITH_CUDA_FP16
02766     using q16 = FractionField<i16>;
02767
02768     template<int16_t p, int16_t q>
02769     using make_q16_t = typename q16::template simplify_t<
02770         typename q16::val<i16::inject_constant_t<p>, i16::inject_constant_t<q>>;
02771
02772     #endif
02773     template<typename Ring, typename v1, typename v2>
02774     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02775     template<typename Ring, typename v1, typename v2>
02776     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02777
02778     template<>
02779     struct Embed<q32, q64> {
02780         template<typename v>
02781         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02782     };
02783
02784     template<typename Small, typename Large>
02785     struct Embed<polynomial<Small>, polynomial<Large>> {
02786     private:
02787         template<typename v, typename i>
02788         struct at_low;
02789     };

```

```

02845     template<typename v, size_t i>
02846     struct at_index {
02847         using type = typename Embed<Small, Large>::template
type<typename v::template coeff_at_t<i>>;
02848     };
02849
02850     template<typename v, size_t... Is>
02851     struct at_low<v, std::index_sequence<Is...> {
02852         using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02853     };
02854
02855     public:
02856     template<typename v>
02857     using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
1>::type;
02860     };
02861
02862     template<typename Ring, auto... xs>
02863     using make_int_polynomial_t = typename polynomial<Ring>::template val<
typename Ring::template inject_constant_t<xs>...>;
02866
02867     template<typename Ring, auto... xs>
02868     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
typename FractionField<Ring>::template inject_constant_t<xs>...>;
02871 } // namespace aerobus
02872
02873 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02874 namespace aerobus {
02875     namespace internal {
02876         template<typename T, size_t x, typename E = void>
02877         struct factorial {};
02878
02879         template<typename T, size_t x>
02880         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02881             private:
02882                 template<typename, size_t, typename>
02883                 friend struct factorial;
02884             public:
02885                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02886         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02889         };
02890
02891         template<typename T>
02892         struct factorial<T, 0> {
02893             public:
02894                 using type = typename T::one;
02895         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02898         };
02899     } // namespace internal
02900
02901     template<typename T, size_t i>
02902     using factorial_t = typename internal::factorial<T, i>::type;
02905
02906     template<typename T, size_t i>
02907     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02910
02911     namespace internal {
02912         template<typename T, size_t k, size_t n, typename E = void>
02913         struct combination_helper {};
02914
02915         template<typename T, size_t k, size_t n>
02916         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02917             using type = typename FractionField<T>::template mul_t<
typename combination_helper<T, k - 1, n - 1>::type,
makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02920         };
02921
02922         template<typename T, size_t k, size_t n>
02923         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02924             using type = typename combination_helper<T, n - k, n>::type;
02927         };
02928
02929         template<typename T, size_t n>
02930         struct combination_helper<T, 0, n> {
02931             using type = typename FractionField<T>::one;
02932         };
02933
02934         template<typename T, size_t k, size_t n>
02935         struct combination {
02936             using type = typename internal::combination_helper<T, k, n>::type::x;
02937         static constexpr typename T::inner_type value =
internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02939     };

```



```

02940     } // namespace internal
02941
02942     template<typename T, size_t k, size_t n>
02943     using combination_t = typename internal::combination<T, k, n>::type;
02944
02945     template<typename T, size_t k, size_t n>
02946     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02947
02948     namespace internal {
02949         template<typename T, size_t m>
02950         struct bernoulli;
02951
02952         template<typename T, typename accum, size_t k, size_t m>
02953         struct bernoulli_helper {
02954             using type = typename bernoulli_helper<
02955                 T,
02956                 addfractions_t<T,
02957                     accum,
02958                     mulfractions_t<T,
02959                         makefraction_t<T,
02960                             combination_t<T, k, m + 1>,
02961                             typename T::one>,
02962                             typename bernoulli<T, k>::type
02963                         >,
02964                     >,
02965                     k + 1,
02966                     m>::type;
02967         };
02968
02969         template<typename T, typename accum, size_t m>
02970         struct bernoulli_helper<T, accum, m, m> {
02971             using type = accum;
02972         };
02973
02974         template<typename T, size_t m>
02975         struct bernoulli {
02976             using type = typename FractionField<T>::template mul_t<
02977                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02978                 makefraction_t<T,
02979                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02980                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02981                 >
02982             >;
02983
02984             template<typename floatType>
02985             static constexpr floatType value = type::template get<floatType>();
02986         };
02987
02988         template<typename T>
02989         struct bernoulli<T, 0> {
02990             using type = typename FractionField<T>::one;
02991
02992             template<typename floatType>
02993             static constexpr floatType value = type::template get<floatType>();
02994         };
02995     } // namespace internal
02996
02997     template<typename T, size_t n>
02998     using bernoulli_t = typename internal::bernoulli<T, n>::type;
02999
03000     template<typename FloatType, typename T, size_t n>
03001     inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
03002
03003     // bell numbers
03004     namespace internal {
03005         template<typename T, size_t n, typename E = void>
03006         struct bell_helper;
03007
03008         template<typename T, size_t n>
03009         struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
03010             template<typename accum, size_t i, size_t stop>
03011             struct sum_helper {
03012             private:
03013                 using left = typename T::template mul_t<
03014                     combination_t<T, i, n-1>,
03015                     typename bell_helper<T, i>::type>;
03016                 using new_accum = typename T::template add_t<accum, left>;
03017             public:
03018                 using type = typename sum_helper<new_accum, i+1, stop>::type;
03019             };
03020
03021             template<typename accum, size_t stop>
03022             struct sum_helper<accum, stop, stop> {
03023                 using type = accum;
03024             };
03025         };
03026     }

```

```

03040
03041     using type = typename sum_helper<typename T::zero, 0, n>::type;
03042 };
03043
03044     template<typename T>
03045     struct bell_helper<T, 0> {
03046         using type = typename T::one;
03047     };
03048
03049     template<typename T>
03050     struct bell_helper<T, 1> {
03051         using type = typename T::one;
03052     };
03053 } // namespace internal
03054
03055 template<typename T, size_t n>
03056 using bell_t = typename internal::bell_helper<T, n>::type;
03057
03058 template<typename T, size_t n>
03059 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03060
03061 namespace internal {
03062     template<typename T, int k, typename E = void>
03063     struct alternate {};
03064
03065     template<typename T, int k>
03066     struct alternate<T, k, std::enable_if_t<k % 2 == 0> > {
03067         using type = typename T::one;
03068         static constexpr typename T::inner_type value = type::template get<typename
03069 T::inner_type>();
03070     };
03071
03072     template<typename T, int k>
03073     struct alternate<T, k, std::enable_if_t<k % 2 != 0> > {
03074         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03075         static constexpr typename T::inner_type value = type::template get<typename
03076 T::inner_type>();
03077     };
03078 } // namespace internal
03079
03080 template<typename T, int k>
03081 using alternate_t = typename internal::alternate<T, k>::type;
03082
03083 template<typename T, size_t k>
03084 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03085
03086 namespace internal {
03087     template<typename T, int n, int k, typename E = void>
03088     struct stirling_l_helper {};
03089
03090     template<typename T>
03091     struct stirling_l_helper<T, 0, 0> {
03092         using type = typename T::one;
03093     };
03094
03095     template<typename T, int n>
03096     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> > {
03097         using type = typename T::zero;
03098     };
03099
03100     template<typename T, int n>
03101     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> > {
03102         using type = typename T::zero;
03103     };
03104
03105     template<typename T, int n, int k>
03106     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> > {
03107         using type = typename T::template sub_t<
03108             typename stirling_l_helper<T, n-1, k-1>::type,
03109             typename T::template mul_t<
03110                 typename T::template inject_constant_t<n-1>,
03111                 typename stirling_l_helper<T, n-1, k>::type
03112             >;
03113     };
03114 } // namespace internal
03115
03116 template<typename T, int n, int k>
03117 using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
03118
03119 template<typename T, int n, int k>
03120 using stirling_l_unsigned_t = abs_t<typename internal::stirling_l_helper<T, n, k>::type>;
03121
03122 template<typename T, int n, int k>
03123 static constexpr typename T::inner_type stirling_l_unsigned_v = stirling_l_unsigned_t<T, n, k>::v;
03124
03125 template<typename T, int n, int k>
03126 static constexpr typename T::inner_type stirling_l_signed_v = stirling_l_signed_t<T, n, k>::v;

```

```

03151
03152 namespace internal {
03153     template<typename T, int n, int k, typename E = void>
03154     struct stirling_2_helper {};
03155
03156     template<typename T, int n>
03157     struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)>> {
03158         using type = typename T::one;
03159     };
03160
03161     template<typename T, int n>
03162     struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
03163         using type = typename T::zero;
03164     };
03165
03166     template<typename T, int n>
03167     struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
03168         using type = typename T::zero;
03169     };
03170
03171     template<typename T, int n, int k>
03172     struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)>> {
03173         using type = typename T::template add_t<
03174             typename stirling_2_helper<T, n-1, k-1>::type,
03175             typename T::template mul_t<
03176                 typename T::template inject_constant_t<k>,
03177                 typename stirling_2_helper<T, n-1, k>::type
03178             >>;
03179     };
03180 } // namespace internal
03181
03182 template<typename T, int n, int k>
03183 using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03184
03185 template<typename T, int n, int k>
03186 static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03187
03188 namespace internal {
03189     template<typename T>
03190     struct pow_scalar {
03191         template<size_t p>
03192         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03193             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03194             x * func<p/2>(x) * func<p/2>(x);
03195         }
03196     };
03197
03198     template<typename T, typename p, size_t n, typename E = void>
03199     requires IsEuclideanDomain<T>
03200     struct pow;
03201
03202     template<typename T, typename p, size_t n>
03203     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
03204         using type = typename T::template mul_t<
03205             typename pow<T, p, n/2>::type,
03206             typename pow<T, p, n/2>::type
03207         >;
03208     };
03209
03210     template<typename T, typename p, size_t n>
03211     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
03212         using type = typename T::template mul_t<
03213             p,
03214             typename T::template mul_t<
03215                 typename pow<T, p, n/2>::type,
03216                 typename pow<T, p, n/2>::type
03217             >
03218         >;
03219     };
03220
03221     template<typename T, typename p, size_t n>
03222     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
03223 } // namespace internal
03224
03225 template<typename T, typename p, size_t n>
03226 using pow_t = typename internal::pow<T, p, n>::type;
03227
03228 template<typename T, typename p, size_t n>
03229 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03230
03231 template<typename T, size_t p>
03232 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03233     internal::pow_scalar<T>::template func<p>(x); }
03234
03235 namespace internal {
03236     template<typename, template<typename, size_t> typename, class>
03237     struct make_taylor_impl;

```

```

03253
03254     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03255     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03256         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
03257     };
03258 }
03259
03260 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03261 using taylor = typename internal::make_taylor_impl<
03262     T,
03263     coeff_at,
03264     internal::make_index_sequence_reverse<deg + 1>::type>;
03265
03266 namespace internal {
03267     template<typename T, size_t i>
03268     struct exp_coeff {
03269         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03270     };
03271
03272     template<typename T, size_t i, typename E = void>
03273     struct sin_coeff_helper {};
03274
03275     template<typename T, size_t i>
03276     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03277         using type = typename FractionField<T>::zero;
03278     };
03279
03280     template<typename T, size_t i>
03281     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03282         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03283     };
03284
03285     template<typename T, size_t i>
03286     struct sin_coeff {
03287         using type = typename sin_coeff_helper<T, i>::type;
03288     };
03289
03290     template<typename T, size_t i, typename E = void>
03291     struct sh_coeff_helper {};
03292
03293     template<typename T, size_t i>
03294     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03295         using type = typename FractionField<T>::zero;
03296     };
03297
03298     template<typename T, size_t i>
03299     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03300         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03301     };
03302
03303     template<typename T, size_t i>
03304     struct sh_coeff {
03305         using type = typename sh_coeff_helper<T, i>::type;
03306     };
03307
03308     template<typename T, size_t i, typename E = void>
03309     struct cos_coeff_helper {};
03310
03311     template<typename T, size_t i>
03312     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03313         using type = typename FractionField<T>::zero;
03314     };
03315
03316     template<typename T, size_t i>
03317     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03318         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03319     };
03320
03321     template<typename T, size_t i>
03322     struct cos_coeff {
03323         using type = typename cos_coeff_helper<T, i>::type;
03324     };
03325
03326     template<typename T, size_t i, typename E = void>
03327     struct cosh_coeff_helper {};
03328
03329     template<typename T, size_t i>
03330     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03331         using type = typename FractionField<T>::zero;
03332     };
03333
03334     template<typename T, size_t i>
03335     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03336         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03337     };
03338
03339     template<typename T, size_t i, typename E = void>
03340     struct sh_cosh_coeff_helper {};
03341
03342

```

```

03343     template<typename T, size_t i>
03344     struct cosh_coeff {
03345         using type = typename cosh_coeff_helper<T, i>::type;
03346     };
03347
03348     template<typename T, size_t i>
03349     struct geom_coeff { using type = typename FractionField<T>::one; };
03350
03351
03352     template<typename T, size_t i, typename E = void>
03353     struct atan_coeff_helper;
03354
03355     template<typename T, size_t i>
03356     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03357         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03358     };
03359
03360     template<typename T, size_t i>
03361     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03362         using type = typename FractionField<T>::zero;
03363     };
03364
03365     template<typename T, size_t i>
03366     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03367
03368     template<typename T, size_t i, typename E = void>
03369     struct asin_coeff_helper;
03370
03371     template<typename T, size_t i>
03372     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03373         using type = makefraction_t<T,
03374             factorial_t<T, i - 1>,
03375             typename T::template mul_t<
03376                 typename T::template val<i>,
03377                 T::template mul_t<
03378                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03379                     pow_t<T, factorial_t<T, i / 2>, 2
03380                 >,
03381                 >
03382             >;
03383     };
03384
03385     template<typename T, size_t i>
03386     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03387         using type = typename FractionField<T>::zero;
03388     };
03389
03390     template<typename T, size_t i>
03391     struct asin_coeff {
03392         using type = typename asin_coeff_helper<T, i>::type;
03393     };
03394
03395     template<typename T, size_t i>
03396     struct lnpl_coeff {
03397         using type = makefraction_t<T,
03398             alternate_t<T, i + 1>,
03399             typename T::template val<i>;
03400     };
03401
03402     template<typename T>
03403     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03404
03405     template<typename T, size_t i, typename E = void>
03406     struct asinh_coeff_helper;
03407
03408     template<typename T, size_t i>
03409     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03410         using type = makefraction_t<T,
03411             typename T::template mul_t<
03412                 alternate_t<T, i / 2>,
03413                 factorial_t<T, i - 1>
03414             >,
03415             typename T::template mul_t<
03416                 typename T::template mul_t<
03417                     typename T::template val<i>,
03418                     pow_t<T, factorial_t<T, i / 2>, 2>
03419                 >,
03420                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03421             >
03422         >;
03423     };
03424
03425     template<typename T, size_t i>
03426     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03427         using type = typename FractionField<T>::zero;
03428     };
03429

```

```

03430     template<typename T, size_t i>
03431     struct asinh_coeff {
03432         using type = typename asinh_coeff_helper<T, i>::type;
03433     };
03434
03435     template<typename T, size_t i, typename E = void>
03436     struct atanh_coeff_helper;
03437
03438     template<typename T, size_t i>
03439     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03440         // 1/i
03441         using type = typename FractionField<T>::template val<
03442             typename T::one,
03443             typename T::template inject_constant_t<i>;
03444     };
03445
03446     template<typename T, size_t i>
03447     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03448         using type = typename FractionField<T>::zero;
03449     };
03450
03451     template<typename T, size_t i>
03452     struct atanh_coeff {
03453         using type = typename atanh_coeff_helper<T, i>::type;
03454     };
03455
03456     template<typename T, size_t i, typename E = void>
03457     struct tan_coeff_helper;
03458
03459     template<typename T, size_t i>
03460     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03461         using type = typename FractionField<T>::zero;
03462     };
03463
03464     template<typename T, size_t i>
03465     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03466     private:
03467         // 4^((i+1)/2)
03468         using _4p = typename FractionField<T>::template inject_t<
03469             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03470         // 4^((i+1)/2) - 1
03471         using _4pml = typename FractionField<T>::template
03472             sub_t<_4p, typename FractionField<T>::one>;
03473         // (-1)^((i-1)/2)
03474         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03475         using dividend = typename FractionField<T>::template mul_t<
03476             altp,
03477             FractionField<T>::template mul_t<
03478                 _4p,
03479                 FractionField<T>::template mul_t<
03480                     _4pml,
03481                     bernoulli_t<T, (i + 1)>
03482                 >
03483             >;
03484     public:
03485         using type = typename FractionField<T>::template div_t<dividend,
03486             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03487     };
03488
03489     template<typename T, size_t i>
03490     struct tan_coeff {
03491         using type = typename tan_coeff_helper<T, i>::type;
03492     };
03493
03494     template<typename T, size_t i, typename E = void>
03495     struct tanh_coeff_helper;
03496
03497     template<typename T, size_t i>
03498     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03499         using type = typename FractionField<T>::zero;
03500     };
03501
03502     template<typename T, size_t i>
03503     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03504     private:
03505         using _4p = typename FractionField<T>::template inject_t<
03506             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03507         using _4pml = typename FractionField<T>::template
03508             sub_t<_4p, typename FractionField<T>::one>;
03509         using dividend =
03510             typename FractionField<T>::template mul_t<
03511                 _4p,
03512                 typename FractionField<T>::template mul_t<
03513                     _4pml,
03514                     bernoulli_t<T, (i + 1)>>::type;
03515     public:

```

```

03515         using type = typename FractionField<T>::template div_t<dividend,
03516             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03517     };
03518
03519     template<typename T, size_t i>
03520     struct tanh_coeff {
03521         using type = typename tanh_coeff_helper<T, i>::type;
03522     };
03523 } // namespace internal
03524
03525 template<typename Integers, size_t deg>
03526 using exp = taylor<Integers, internal::exp_coeff, deg>;
03527
03528 template<typename Integers, size_t deg>
03529 using expm1 = typename polynomial<FractionField<Integers>>::template sub_t<
03530     exp<Integers, deg>,
03531     typename polynomial<FractionField<Integers>>::one>;
03532
03533 template<typename Integers, size_t deg>
03534 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03535
03536 template<typename Integers, size_t deg>
03537 using atan = taylor<Integers, internal::atan_coeff, deg>;
03538
03539 template<typename Integers, size_t deg>
03540 using sin = taylor<Integers, internal::sin_coeff, deg>;
03541
03542 template<typename Integers, size_t deg>
03543 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03544
03545 template<typename Integers, size_t deg>
03546 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03547
03548 template<typename Integers, size_t deg>
03549 using cos = taylor<Integers, internal::cos_coeff, deg>;
03550
03551 template<typename Integers, size_t deg>
03552 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03553
03554 template<typename Integers, size_t deg>
03555 using asin = taylor<Integers, internal::asin_coeff, deg>;
03556
03557 template<typename Integers, size_t deg>
03558 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03559
03560 template<typename Integers, size_t deg>
03561 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03562
03563 template<typename Integers, size_t deg>
03564 using tan = taylor<Integers, internal::tan_coeff, deg>;
03565
03566 template<typename Integers, size_t deg>
03567 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03568 } // namespace aerobus
03569
03570 // continued fractions
03571 namespace aerobus {
03572     template<int64_t... values>
03573     struct ContinuedFraction {};
03574
03575     template<int64_t a0>
03576     struct ContinuedFraction<a0> {
03577         using type = typename q64::template inject_constant_t<a0>;
03578         static constexpr double val = static_cast<double>(a0);
03579     };
03580
03581     template<int64_t a0, int64_t... rest>
03582     struct ContinuedFraction<a0, rest...> {
03583         using type = q64::template add_t<
03584             typename q64::template inject_constant_t<a0>,
03585             typename q64::template div_t<
03586                 typename q64::one,
03587                 typename ContinuedFraction<rest...>::type
03588             >>;
03589         static constexpr double val = type::template get<double>();
03590     };
03591
03592     using PI_fraction =
03593         ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03594     using E_fraction =
03595         ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03596     using SQRT2_fraction =
03597         ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03598     using SQRT3_fraction =
03599         ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03600 } // NOLINT

```

```

03664 } // namespace aerobus
03665
03666 // known polynomials
03667 namespace aerobus {
03668     // CChebyshev
03669     namespace internal {
03670         template<int kind, size_t deg, typename I>
03671         struct chebyshev_helper {
03672             using type = typename polynomial<I>::template sub_t<
03673                 typename polynomial<I>::template mul_t<
03674                     typename polynomial<I>::template mul_t<
03675                         typename polynomial<I>::template inject_constant_t<2>,
03676                         typename polynomial<I>::X>,
03677                         typename chebyshev_helper<kind, deg - 1, I>::type
03678                     >,
03679                     typename chebyshev_helper<kind, deg - 2, I>::type
03680                 >;
03681         };
03682
03683         template<typename I>
03684         struct chebyshev_helper<1, 0, I> {
03685             using type = typename polynomial<I>::one;
03686         };
03687
03688         template<typename I>
03689         struct chebyshev_helper<1, 1, I> {
03690             using type = typename polynomial<I>::X;
03691         };
03692
03693         template<typename I>
03694         struct chebyshev_helper<2, 0, I> {
03695             using type = typename polynomial<I>::one;
03696         };
03697
03698         template<typename I>
03699         struct chebyshev_helper<2, 1, I> {
03700             using type = typename polynomial<I>::template mul_t<
03701                 typename polynomial<I>::template inject_constant_t<2>,
03702                 typename polynomial<I>::X>;
03703         };
03704     } // namespace internal
03705
03706     // Laguerre
03707     namespace internal {
03708         template<size_t deg, typename I>
03709         struct laguerre_helper {
03710             using Q = FractionField<I>;
03711             using PQ = polynomial<Q>;
03712
03713             private:
03714                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)lkm2)
03715                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03716                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03717                 // -x + 2k-1
03718                 using p = typename PQ::template val<
03719                     typename Q::template inject_constant_t<-1>,
03720                     typename Q::template inject_constant_t<2 * deg - 1>;
03721                 // 1/n
03722                 using factor = typename PQ::template inject_ring_t<
03723                     typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>;
03724
03725             public:
03726                 using type = typename PQ::template mul_t <
03727                     factor,
03728                     typename PQ::template sub_t<
03729                         typename PQ::template mul_t<
03730                             p,
03731                             lnm1
03732                         >,
03733                         typename PQ::template mul_t<
03734                             typename PQ::template inject_constant_t<deg-1>,
03735                             lnm2
03736                         >
03737                     >
03738                 >;
03739         };
03740
03741         template<typename I>
03742         struct laguerre_helper<0, I> {
03743             using type = typename polynomial<FractionField<I>::one;
03744         };
03745
03746         template<typename I>
03747         struct laguerre_helper<1, I> {
03748             private:
03749                 using PQ = polynomial<FractionField<I>;

```



```

03750     public:
03751         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03752     };
03753 } // namespace internal
03754
03755 // Bernstein
03756 namespace internal {
03757     template<size_t i, size_t m, typename I, typename E = void>
03758     struct bernstein_helper {};
03759
03760     template<typename I>
03761     struct bernstein_helper<0, 0, I> {
03762         using type = typename polynomial<I>::one;
03763     };
03764
03765     template<size_t i, size_t m, typename I>
03766     struct bernstein_helper<i, m, I, std::enable_if_t<
03767         (m > 0) && (i == 0)> {
03768     private:
03769         using P = polynomial<I>;
03770     public:
03771         using type = typename P::template mul_t<
03772             typename P::template sub_t<typename P::one, typename P::X>,
03773             typename bernstein_helper<i, m-1, I>::type>;
03774     };
03775
03776     template<size_t i, size_t m, typename I>
03777     struct bernstein_helper<i, m, I, std::enable_if_t<
03778         (m > 0) && (i == m)> {
03779     private:
03780         using P = polynomial<I>;
03781     public:
03782         using type = typename P::template mul_t<
03783             typename P::X,
03784             typename bernstein_helper<i-1, m-1, I>::type>;
03785     };
03786
03787     template<size_t i, size_t m, typename I>
03788     struct bernstein_helper<i, m, I, std::enable_if_t<
03789         (m > 0) && (i > 0) && (i < m)> {
03790     private:
03791         using P = polynomial<I>;
03792     public:
03793         using type = typename P::template add_t<
03794             typename P::template mul_t<
03795                 typename P::template sub_t<typename P::one, typename P::X>,
03796                 typename bernstein_helper<i, m-1, I>::type>,
03797             typename P::template mul_t<
03798                 typename P::X,
03799                 typename bernstein_helper<i-1, m-1, I>::type>;
03800     };
03801 } // namespace internal
03802
03803 // AllOne polynomials
03804 namespace internal {
03805     template<size_t deg, typename I>
03806     struct AllOneHelper {
03807         using type = aerobus::add_t<
03808             typename polynomial<I>::one,
03809             aerobus::mul_t<
03810                 typename polynomial<I>::X,
03811                 AllOneHelper<deg-1, I>::type
03812             >;
03813     };
03814
03815     template<typename I>
03816     struct AllOneHelper<0, I> {
03817         using type = typename polynomial<I>::one;
03818     };
03819 } // namespace internal
03820
03821 // Bessel polynomials
03822 namespace internal {
03823     template<size_t deg, typename I>
03824     struct BesselHelper {
03825     private:
03826         using P = polynomial<I>;
03827         using factor = typename P::template monomial_t<
03828             typename I::template inject_constant_t<(2*deg - 1)>,
03829             1>;
03830     public:
03831         using type = typename P::template add_t<
03832             typename P::template mul_t<
03833                 factor,
03834                 typename BesselHelper<deg-1, I>::type
03835             >,
03836             typename BesselHelper<deg-2, I>::type

```

```

03837         >;
03838     };
03839
03840     template<typename I>
03841     struct BesselHelper<0, I> {
03842         using type = typename polynomial<I>::one;
03843     };
03844
03845     template<typename I>
03846     struct BesselHelper<1, I> {
03847     private:
03848         using P = polynomial<I>;
03849     public:
03850         using type = typename P::template add_t<
03851             typename P::one,
03852             typename P::X
03853         >;
03854     };
03855 } // namespace internal
03856
03857 namespace known_polynomials {
03858     enum hermite_kind {
03859         probabilist,
03860         physicist
03861     };
03862 }
03863
03864 // hermite
03865 namespace internal {
03866     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03867     struct hermite_helper {};
03868
03869     template<size_t deg, typename I>
03870     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03871     private:
03872         using hnm1 = typename hermite_helper<deg - 1,
03873             known_polynomials::hermite_kind::probabilist, I>::type;
03874         using hnm2 = typename hermite_helper<deg - 2,
03875             known_polynomials::hermite_kind::probabilist, I>::type;
03876
03877     public:
03878         using type = typename polynomial<I>::template sub_t<
03879             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03880             typename polynomial<I>::template mul_t<
03881                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03882                 hnm2
03883             >
03884         >;
03885     };
03886
03887     template<size_t deg, typename I>
03888     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03889     private:
03890         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03891             I>::type;
03892         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03893             I>::type;
03894
03895     public:
03896         using type = typename polynomial<I>::template sub_t<
03897             // 2X Hn-1
03898             typename polynomial<I>::template mul_t<
03899                 typename pi64::val<typename I::template inject_constant_t<2>>,
03900                 typename I::zero>, hnm1>,
03901             typename polynomial<I>::template mul_t<
03902                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>>,
03903                 hnm2
03904             >
03905         >;
03906     };
03907
03908     template<typename I>
03909     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03910         using type = typename polynomial<I>::one;
03911     };
03912
03913     template<typename I>
03914     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03915         using type = typename polynomial<I>::X;
03916     };
03917
03918     template<typename I>
03919     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03920         using type = typename pi64::one;
03921     };
03922

```

```

03923     template<typename I>
03924     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03925         // 2X
03926         using type = typename polynomial<I>::template val<
03927             typename I::template inject_constant_t<2>,
03928             typename I::zero>;
03929     };
03930 } // namespace internal
03931
03932 // legendre
03933 namespace internal {
03934     template<size_t n, typename I>
03935     struct legendre_helper {
03936     private:
03937         using Q = FractionField<I>;
03938         using PQ = polynomial<Q>;
03939         // 1/n constant
03940         // (2n-1)/n X
03941         using fact_left = typename PQ::template monomial_t<
03942             makefraction_t<I,
03943                 typename I::template inject_constant_t<2*n-1>,
03944                 typename I::template inject_constant_t<n>
03945             >,
03946             1>;
03947         // (n-1) / n
03948         using fact_right = typename PQ::template val<
03949             makefraction_t<I,
03950                 typename I::template inject_constant_t<n-1>,
03951                 typename I::template inject_constant_t<n>>;
03952
03953     public:
03954         using type = PQ::template sub_t<
03955             typename PQ::template mul_t<
03956                 fact_left,
03957                 typename legendre_helper<n-1, I>::type
03958             >,
03959             typename PQ::template mul_t<
03960                 fact_right,
03961                 typename legendre_helper<n-2, I>::type
03962             >
03963         >;
03964     };
03965
03966     template<typename I>
03967     struct legendre_helper<0, I> {
03968         using type = typename polynomial<FractionField<I>::one>;
03969     };
03970
03971     template<typename I>
03972     struct legendre_helper<1, I> {
03973         using type = typename polynomial<FractionField<I>::X>;
03974     };
03975 } // namespace internal
03976
03977 // bernoulli polynomials
03978 namespace internal {
03979     template<size_t n>
03980     struct bernoulli_coeff {
03981         template<typename T, size_t i>
03982         struct inner {
03983         private:
03984             using F = FractionField<T>;
03985         public:
03986             using type = typename F::template mul_t<
03987                 typename F::template inject_ring_t<combination_t<T, i, n>,
03988                 bernoulli_t<T, n-i>
03989             >;
03990         };
03991     };
03992 } // namespace internal
03993
03994 namespace internal {
03995     template<size_t n>
03996     struct touchard_coeff {
03997         template<typename T, size_t i>
03998         struct inner {
03999             using type = stirling_2_t<T, n, i>;
04000         };
04001     };
04002 } // namespace internal
04003
04004 namespace internal {
04005     template<typename I = aerobus::i64>
04006     struct AbelHelper {
04007     private:
04008         using P = aerobus::polynomial<I>;
04009 
```

```

04010     public:
04011         // to keep recursion working, we need to operate on a*n and not just a
04012         template<size_t deg, I::inner_type an>
04013         struct Inner {
04014             // abel(n, a) = (x-an) * abel(n-1, a)
04015             using type = typename aerobus::mul_t<
04016                 typename Inner<deg-1, an>::type,
04017                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
04018             >;
04019         };
04020
04021         // abel(0, a) = 1
04022         template<I::inner_type an>
04023         struct Inner<0, an> {
04024             using type = P::one;
04025         };
04026
04027         // abel(1, a) = X
04028         template<I::inner_type an>
04029         struct Inner<1, an> {
04030             using type = P::X;
04031         };
04032     };
04033 } // namespace internal
04034
04035 namespace known_polynomials {
04036
04037     template<size_t n, auto a, typename I = aerobus::i64>
04038     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
04039
04040     template<size_t deg, typename I = aerobus::i64>
04041     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
04042
04043     template<size_t deg, typename I = aerobus::i64>
04044     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
04045
04046     template<size_t deg, typename I = aerobus::i64>
04047     using laguerre = typename internal::laguerre_helper<deg, I>::type;
04048
04049     template<size_t deg, typename I = aerobus::i64>
04050     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
04051         I>::type;
04052
04053     template<size_t deg, typename I = aerobus::i64>
04054     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04055
04056     template<size_t i, size_t m, typename I = aerobus::i64>
04057     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04058
04059     template<size_t deg, typename I = aerobus::i64>
04060     using legendre = typename internal::legendre_helper<deg, I>::type;
04061
04062     template<size_t deg, typename I = aerobus::i64>
04063     using bernoulli = Taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
04064
04065     template<size_t deg, typename I = aerobus::i64>
04066     using allone = typename internal::AllOneHelper<deg, I>::type;
04067
04068     template<size_t deg, typename I = aerobus::i64>
04069     using bessel = typename internal::BesselHelper<deg, I>::type;
04070
04071     template<size_t deg, typename I = aerobus::i64>
04072     using touchard = Taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
04073 } // namespace known_polynomials
04074 } // namespace aerobus
04075
04076 #ifdef AEROBUS_CONWAY_IMPORTS
04077 // conway polynomials
04078 namespace aerobus {
04079     template<int p, int n>
04080     struct ConwayPolynomial {};
04081
04082 #ifndef DO_NOT_DOCUMENT
04083     #define ZPZV ZPZ::template val
04084     #define POLYV aerobus::polynomial<ZPZ>::template val
04085     template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>; }; // NOLINT
04086     template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04087     template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04088     template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04089     template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT

```

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9.4 src/examples.h File Reference

9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_
```


Chapter 10

Examples

10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

10.2 examples/custom_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
#include <cstdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

/*
change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
to see how good is the generated assembly compared to what nvcc generates for 16 bits
*/
using int_type = aerobus::i16;
using float_type = __half2;

constexpr size_t N = 1 << 24;

template<typename T>
struct ExpmlDegree;

template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};

template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};

template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};

double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

template<typename T>
struct GetRandT;

template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};

template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};

template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};

using EXPM1 = aerobus::expml<int_type, ExpmlDegree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}
```

```

}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        out[i] = f(f(f(f(f(f(in[i]))))))) ;
    }
}

int main() {
    float_type *d_in, *d_out;
    cudaMalloc<float_type>(&d_in, N * sizeof(float_type));
    cudaMalloc<float_type>(&d_out, N * sizeof(float_type));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice);

    run<<128, 512>>>(N, d_in, d_out);

    cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost);

    cudaFree(d_in);
    cudaFree(d_out);
}

```

10.4 examples/continued_fractions.cpp

How to use `aerobus::ContinuedFraction` to get approximations of known numbers

[illegible]

10.5 examples/modular_arithmetic.cpp

How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

//  $x^3 + 2x^2 + 1$ , with coefficients in  $\mathbb{Z}/2\mathbb{Z}$ , actually  $x^3 + 1$ 
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
//  $x^3 + 5x^2 + 7x + 11$  with coefficients in  $\mathbb{Z}/17\mathbb{Z}$ , meaning actually  $x^3 + x^2 + 1$ 
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

10.6 examples/make_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

10.7 examples/polynomials_over_finite_field.cpp

How to build a known polynomial (here aerobus::known_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```

10.8 examples/compensated_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```
// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;

        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}
```


Index

abs_t
 aerobus, 20

add_t
 aerobus, 20
 aerobus::i32, 59
 aerobus::i64, 65
 aerobus::polynomial< Ring >, 74
 aerobus::Quotient< Ring, X >, 82
 aerobus::zpz< p >, 107

addfractions_t
 aerobus, 20

aerobus, 15
 abs_t, 20
 add_t, 20
 addfractions_t, 20
 aligned_malloc, 34
 alternate_t, 20
 alternate_v, 35
 asin, 21
 asinh, 21
 atan, 21
 atanh, 21
 bell_t, 23
 bernoulli_t, 23
 bernoulli_v, 35
 combination_t, 23
 combination_v, 35
 cos, 23
 cosh, 25
 div_t, 25
 E_fraction, 25
 embed_int_poly_in_fractions_t, 25
 exp, 26
 expm1, 26
 factorial_t, 26
 factorial_v, 35
 field, 34
 fpq32, 26
 fpq64, 27
 FractionField, 27
 gcd_t, 27
 geometric_sum, 27
 lnp1, 27
 make_frac_polynomial_t, 28
 make_int_polynomial_t, 28
 make_q32_t, 28
 make_q64_t, 29
 makefraction_t, 29
 mul_t, 29
 mulfractions_t, 29
 pi64, 30
 PI_fraction, 30
 pow_t, 30
 pq64, 30
 q32, 30
 q64, 31
 sin, 31
 sinh, 31
 SQRT2_fraction, 31
 SQRT3_fraction, 31
 stirling_1_signed_t, 32
 stirling_1_unsigned_t, 32
 stirling_2_t, 32
 sub_t, 33
 tan, 33
 tanh, 33
 taylor, 33
 vadd_t, 34
 vmul_t, 34

aerobus::ContinuedFraction< a0 >, 47
 type, 47
 val, 48

aerobus::ContinuedFraction< a0, rest... >, 48
 type, 49
 val, 49

aerobus::ContinuedFraction< values >, 46

aerobus::ConwayPolynomial, 49

aerobus::Embed< i32, i64 >, 51
 type, 51

aerobus::Embed< polynomial< Small >, polynomial< Large > >, 52
 type, 52

aerobus::Embed< q32, q64 >, 53
 type, 53

aerobus::Embed< Quotient< Ring, X >, Ring >, 54
 type, 54

aerobus::Embed< Ring, FractionField< Ring > >, 55
 type, 55

aerobus::Embed< Small, Large, E >, 51

aerobus::Embed< zpz< x >, i32 >, 55
 type, 56

aerobus::i32, 57
 add_t, 59
 div_t, 59
 eq_t, 59
 eq_v, 62
 gcd_t, 59
 gt_t, 60

- inject_constant_t, 60
- inject_ring_t, 60
- inner_type, 60
- is_euclidean_domain, 62
- is_field, 62
- lt_t, 60
- mod_t, 61
- mul_t, 61
- one, 61
- pos_t, 61
- pos_v, 62
- sub_t, 62
- zero, 62
- aerobus::i32::val< x >, 91
 - enclosing_type, 92
 - get, 92
 - is_zero_t, 92
 - to_string, 92
 - v, 92
- aerobus::i64, 64
 - add_t, 65
 - div_t, 66
 - eq_t, 66
 - eq_v, 69
 - gcd_t, 66
 - gt_t, 66
 - gt_v, 69
 - inject_constant_t, 67
 - inject_ring_t, 67
 - inner_type, 67
 - is_euclidean_domain, 69
 - is_field, 69
 - lt_t, 67
 - lt_v, 70
 - mod_t, 68
 - mul_t, 68
 - one, 68
 - pos_t, 68
 - pos_v, 70
 - sub_t, 68
 - zero, 69
- aerobus::i64::val< x >, 93
 - enclosing_type, 94
 - get, 94
 - inner_type, 94
 - is_zero_t, 94
 - to_string, 94
 - v, 95
- aerobus::internal, 36
 - index_sequence_reverse, 40
 - is_instantiation_of_v, 40
 - make_index_sequence_reverse, 40
 - type_at_t, 40
- aerobus::is_prime< n >, 71
 - value, 72
- aerobus::IsEuclideanDomain, 43
- aerobus::IsField, 43
- aerobus::IsRing, 44
- aerobus::known_polynomials, 40
 - hermite_kind, 40
 - physicist, 41
 - probabilist, 41
- aerobus::polynomial< Ring >, 72
 - add_t, 74
 - derive_t, 74
 - div_t, 74
 - eq_t, 75
 - gcd_t, 75
 - gt_t, 75
 - inject_constant_t, 76
 - inject_ring_t, 76
 - is_euclidean_domain, 80
 - is_field, 80
 - lt_t, 76
 - mod_t, 76
 - monomial_t, 77
 - mul_t, 77
 - one, 77
 - pos_t, 77
 - pos_v, 80
 - simplify_t, 79
 - sub_t, 79
 - X, 79
 - zero, 79
- aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >, 49
 - func, 50
- aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
 - func, 50
- aerobus::polynomial< Ring >::horner_reduction_t< P >, 56
- aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >, 70
 - type, 71
- aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >, 71
 - type, 71
- aerobus::polynomial< Ring >::val< coeffN >, 102
 - aN, 103
 - coeff_at_t, 103
 - compensated_eval, 104
 - degree, 105
 - enclosing_type, 103
 - eval, 104
 - is_zero_t, 103
 - is_zero_v, 105
 - ring_type, 104
 - strip, 104
 - to_string, 104
 - value_at_t, 104
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at<

- index, std::enable_if_t<(index < 0 | index > 0)>, 45
- type, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index == 0)>>, 46
- type, 46
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 95
- aN, 96
- coeff_at_t, 96
- compensated_eval, 97
- degree, 99
- enclosing_type, 96
- eval, 98
- is_zero_t, 97
- is_zero_v, 99
- ring_type, 97
- strip, 97
- to_string, 98
- value_at_t, 97
- aerobus::Quotient< Ring, X >, 81
- add_t, 82
- div_t, 83
- eq_t, 83
- eq_v, 85
- inject_constant_t, 83
- inject_ring_t, 83
- is_euclidean_domain, 85
- mod_t, 84
- mul_t, 84
- one, 84
- pos_t, 84
- pos_v, 85
- zero, 85
- aerobus::Quotient< Ring, X >::val< V >, 99
- raw_t, 100
- type, 100
- aerobus::type_list< Ts >, 87
- at, 88
- concat, 88
- insert, 88
- length, 89
- push_back, 88
- push_front, 89
- remove, 89
- aerobus::type_list< Ts >::pop_front, 80
- tail, 81
- type, 81
- aerobus::type_list< Ts >::split< index >, 86
- head, 86
- tail, 86
- aerobus::type_list<>, 90
- concat, 90
- insert, 90
- length, 91
- push_back, 90
- push_front, 90
- aerobus::zpz< p >, 105
- add_t, 107
- div_t, 107
- eq_t, 108
- eq_v, 111
- gcd_t, 108
- gt_t, 108
- gt_v, 111
- inject_constant_t, 109
- inner_type, 109
- is_euclidean_domain, 111
- is_field, 111
- lt_t, 109
- lt_v, 111
- mod_t, 109
- mul_t, 109
- one, 110
- pos_t, 110
- pos_v, 112
- sub_t, 110
- zero, 110
- aerobus::zpz< p >::val< x >, 100
- enclosing_type, 101
- get, 101
- is_zero_t, 101
- is_zero_v, 102
- to_string, 101
- v, 102
- aligned_malloc
- aerobus, 34
- alternate_t
- aerobus, 20
- alternate_v
- aerobus, 35
- aN
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
- asin
- aerobus, 21
- asinh
- aerobus, 21
- at
- aerobus::type_list< Ts >, 88
- atan
- aerobus, 21
- atanh
- aerobus, 21
- bell_t
- aerobus, 23
- bernoulli_t
- aerobus, 23
- bernoulli_v
- aerobus, 35
- coeff_at_t
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96

- combination_t
 - aerobus, 23
- combination_v
 - aerobus, 35
- compensated_eval
 - aerobus::polynomial< Ring >::val< coeffN >, 104
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
- concat
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- cos
 - aerobus, 23
- cosh
 - aerobus, 25
- degree
 - aerobus::polynomial< Ring >::val< coeffN >, 105
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
- derive_t
 - aerobus::polynomial< Ring >, 74
- div_t
 - aerobus, 25
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 74
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 107
- E_fraction
 - aerobus, 25
- embed_int_poly_in_fractions_t
 - aerobus, 25
- enclosing_type
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::polynomial< Ring >::val< coeffN >, 103
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
 - aerobus::zpz< p >::val< x >, 101
- eq_t
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 108
- eq_v
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 111
- eval
 - aerobus::polynomial< Ring >::val< coeffN >, 104
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 98
- exp
 - aerobus, 26
- expm1
 - aerobus, 26
- factorial_t
 - aerobus, 26
- factorial_v
 - aerobus, 35
- field
 - aerobus, 34
- fpq32
 - aerobus, 26
- fpq64
 - aerobus, 27
- FractionField
 - aerobus, 27
- func
 - aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >, 50
 - aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
- gcd_t
 - aerobus, 27
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::zpz< p >, 108
- geometric_sum
 - aerobus, 27
- get
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::zpz< p >::val< x >, 101
- gt_t
 - aerobus::i32, 60
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::zpz< p >, 108
- gt_v
 - aerobus::i64, 69
 - aerobus::zpz< p >, 111
- head
 - aerobus::type_list< Ts >::split< index >, 86
- hermite_kind
 - aerobus::known_polynomials, 40
- index_sequence_reverse
 - aerobus::internal, 40
- inject_constant_t
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::polynomial< Ring >, 76
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 109
- inject_ring_t
 - aerobus::i32, 60
 - aerobus::i64, 67

- aerobus::polynomial< Ring >, 76
- aerobus::Quotient< Ring, X >, 83
- inner_type
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::i64::val< x >, 94
 - aerobus::zpz< p >, 109
- insert
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- Introduction, 1
- is_euclidean_domain
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::polynomial< Ring >, 80
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 111
- is_field
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::polynomial< Ring >, 80
 - aerobus::zpz< p >, 111
- is_instantiation_of_v
 - aerobus::internal, 40
- is_zero_t
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::polynomial< Ring >::val< coeffN >, 103
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
 - aerobus::zpz< p >::val< x >, 101
- is_zero_v
 - aerobus::polynomial< Ring >::val< coeffN >, 105
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
 - aerobus::zpz< p >::val< x >, 102
- length
 - aerobus::type_list< Ts >, 89
 - aerobus::type_list<>, 91
- Inp1
 - aerobus, 27
- lt_t
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::polynomial< Ring >, 76
 - aerobus::zpz< p >, 109
- lt_v
 - aerobus::i64, 70
 - aerobus::zpz< p >, 111
- make_frac_polynomial_t
 - aerobus, 28
- make_index_sequence_reverse
 - aerobus::internal, 40
- make_int_polynomial_t
 - aerobus, 28
- make_q32_t
 - aerobus, 28
- make_q64_t
 - aerobus, 29
- makefraction_t
 - aerobus, 29
- mod_t
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 76
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 109
- monomial_t
 - aerobus::polynomial< Ring >, 77
- mul_t
 - aerobus, 29
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 109
- mulfractions_t
 - aerobus, 29
- one
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 110
- physicist
 - aerobus::known_polynomials, 41
- pi64
 - aerobus, 30
- PI_fraction
 - aerobus, 30
- pos_t
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 110
- pos_v
 - aerobus::i32, 62
 - aerobus::i64, 70
 - aerobus::polynomial< Ring >, 80
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 112
- pow_t
 - aerobus, 30
- pq64
 - aerobus, 30
- probabilist
 - aerobus::known_polynomials, 41
- push_back
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- push_front
 - aerobus::type_list< Ts >, 89
 - aerobus::type_list<>, 90

- q32
 - aerobus, [30](#)
- q64
 - aerobus, [31](#)
- raw_t
 - aerobus::Quotient< Ring, X >::val< V >, [100](#)
- README.md, [113](#)
- remove
 - aerobus::type_list< Ts >, [89](#)
- ring_type
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- simplify_t
 - aerobus::polynomial< Ring >, [79](#)
- sin
 - aerobus, [31](#)
- sinh
 - aerobus, [31](#)
- SQRT2_fraction
 - aerobus, [31](#)
- SQRT3_fraction
 - aerobus, [31](#)
- src/aerobus.h, [113](#)
- src/examples.h, [207](#)
- stirling_1_signed_t
 - aerobus, [32](#)
- stirling_1_unsigned_t
 - aerobus, [32](#)
- stirling_2_t
 - aerobus, [32](#)
- strip
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- sub_t
 - aerobus, [33](#)
 - aerobus::i32, [62](#)
 - aerobus::i64, [68](#)
 - aerobus::polynomial< Ring >, [79](#)
 - aerobus::zpz< p >, [110](#)
- tail
 - aerobus::type_list< Ts >::pop_front, [81](#)
 - aerobus::type_list< Ts >::split< index >, [86](#)
- tan
 - aerobus, [33](#)
- tanh
 - aerobus, [33](#)
- taylor
 - aerobus, [33](#)
- to_string
 - aerobus::i32::val< x >, [92](#)
 - aerobus::i64::val< x >, [94](#)
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [98](#)
- aerobus::zpz< p >::val< x >, [101](#)
- type
 - aerobus::ContinuedFraction< a0 >, [47](#)
 - aerobus::ContinuedFraction< a0, rest... >, [49](#)
 - aerobus::Embed< i32, i64 >, [51](#)
 - aerobus::Embed< polynomial< Small >, polynomial< Large > >, [52](#)
 - aerobus::Embed< q32, q64 >, [53](#)
 - aerobus::Embed< Quotient< Ring, X >, Ring >, [54](#)
 - aerobus::Embed< Ring, FractionField< Ring > >, [55](#)
 - aerobus::Embed< zpz< x >, i32 >, [56](#)
 - aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >, [71](#)
 - aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >, [71](#)
 - aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 || index > 0)> >, [45](#)
 - aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >, [46](#)
 - aerobus::Quotient< Ring, X >::val< V >, [100](#)
 - aerobus::type_list< Ts >::pop_front, [81](#)
- type_at_t
 - aerobus::internal, [40](#)
- v
 - aerobus::i32::val< x >, [92](#)
 - aerobus::i64::val< x >, [95](#)
 - aerobus::zpz< p >::val< x >, [102](#)
- vadd_t
 - aerobus, [34](#)
- val
 - aerobus::ContinuedFraction< a0 >, [48](#)
 - aerobus::ContinuedFraction< a0, rest... >, [49](#)
- value
 - aerobus::is_prime< n >, [72](#)
- value_at_t
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- vmul_t
 - aerobus, [34](#)
- X
 - aerobus::polynomial< Ring >, [79](#)
- zero
 - aerobus::i32, [62](#)
 - aerobus::i64, [69](#)
 - aerobus::polynomial< Ring >, [79](#)
 - aerobus::Quotient< Ring, X >, [85](#)
 - aerobus::zpz< p >, [110](#)