

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

Here is a list of all documented concepts with brief descriptions:

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Class Index

2.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.2 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.3 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are aerobus::i64
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
-----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using **type** = `q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`

Static Public Attributes

- static constexpr double **val** = `type::template get<double>()`

5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
<i>...rest</i>	integers (aerobus::i64)

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in [i32](#), again represented as types

Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity (type)(v > 0)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Data Documentation

5.7.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.7.2.2 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<i>v</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in i64

Public Types

- using [inner_type](#) = [int64_t](#)
- template<[auto](#) x>
using [inject_constant_t](#) = [val](#)< [static_cast](#)< [int64_t](#) >(x)>
- template<[typename](#) v >
using [inject_ring_t](#) = v
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<[typename](#) v1 , [typename](#) v2 >
using [add_t](#) = [typename](#) [add](#)< v1 , v2 >::type
addition operator
- template<[typename](#) v1 , [typename](#) v2 >
using [sub_t](#) = [typename](#) [sub](#)< v1 , v2 >::type
subtraction operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mul_t](#) = [typename](#) [mul](#)< v1 , v2 >::type
multiplication operator
- template<[typename](#) v1 , [typename](#) v2 >
using [div_t](#) = [typename](#) [div](#)< v1 , v2 >::type
division operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mod_t](#) = [typename](#) [remainder](#)< v1 , v2 >::type
modulus operator
- template<[typename](#) v1 , [typename](#) v2 >
using [gt_t](#) = [typename](#) [gt](#)< v1 , v2 >::type
strictly greater operator (v1 > v2) - type
- template<[typename](#) v1 , [typename](#) v2 >
using [lt_t](#) = [typename](#) [lt](#)< v1 , v2 >::type
strict less operator (v1 < v2)
- template<[typename](#) v1 , [typename](#) v2 >
using [eq_t](#) = [typename](#) [eq](#)< v1 , v2 >::type
equality operator (type)
- template<[typename](#) v1 , [typename](#) v2 >
using [gcd_t](#) = [gcd_t](#)< [i64](#), v1 , v2 >
greatest common divisor
- template<[typename](#) v >
using [pos_t](#) = [typename](#) [pos](#)< v >::type
is v positive (type)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator ($v1 > v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator ($v1 < v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Typedef Documentation

5.8.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) - type

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.6 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.7 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.8 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.9 pos_t

```
template<typename v >  
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive (type)

Template Parameters

v1	: an element of aerobus::i64::val
----	---

5.8.2.10 sub_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3 Member Data Documentation

5.8.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (v1 > v2) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.3 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (v1 < v2) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<code>v</code>	: an element of aerobus::i64::val
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.9 aerobus::polynomial< Ring, variable_name >::horner_evaluation< valueRing, P >::inner< index, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.10 aerobus::polynomial< Ring, variable_name >::horner_evaluation< valueRing, P >::inner< stop, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.11 aerobus::is_prime< n > Struct Template Reference

checks if `n` is prime

```
#include <aerobus.h>
```

Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`
true iff `n` is prime

5.11.1 Detailed Description

```
template<int32_t n>
struct aerobus::is_prime< n >
```

checks if `n` is prime

Template Parameters

n	
-----	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.12 aerobus::polynomial< Ring, variable_name > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero = val< typename Ring::zero >](#)
constant zero
- [using one = val< typename Ring::one >](#)
constant one
- [using X = val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t = typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt_helper< v1, v2 >::type](#)

- strict greater operator*
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::q_type`
- division operator*
- `template<typename v1 , typename v2 >`
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`
- modulo operator*
- `template<typename coeff , size_t deg>`
`using monomial_t = typename monomial< coeff, deg >::type`
monomial : coeff X^deg
- `template<typename v >`
`using derive_t = typename derive_helper< v >::type`
derivation operator
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity (an > 0)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring, variable_name >, v1, v2 >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

5.12.1 Detailed Description

```
template<typename Ring, char variable_name = 'x'>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring, variable_name >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.12.2 Member Typedef Documentation

5.12.2.1 add_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.2 derive_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.12.2.3 div_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.4 eq_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.5 gcd_t

```
template<typename Ring , char variable_name = 'x'>
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gcd_t = std::conditional_t< Ring::is_↵
euclidean_domain, typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2> >::type,
void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.6 gt_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.7 lt_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.8 mod_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mod_t = typename div_helper<v1, v2, zero,
v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.9 monomial_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring, variable_name >::monomial_t = typename monomial<coeff, deg>←
::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.12.2.10 mul_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.11 pos_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::pos_t = typename Ring::template pos_t<typename
v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.12.2.12 simplify_t

```
template<typename Ring , char variable_name = 'x'>
template<typename P >
using aerobus::polynomial< Ring, variable_name >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.12.2.13 sub_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.13 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.13.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.14 aerobus::Quotient< Ring, X > Struct Template Reference

Classes

- struct [val](#)

Public Types

- `using zero = val< typename Ring::zero >`
- `using one = val< typename Ring::one >`
- `template<typename v1, typename v2 >`
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
- `template<typename v1 >`
`using pos_t = std::true_type`
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `template<typename v1, typename v2 >`
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
- `static constexpr bool is_euclidean_domain = true`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.15 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- `using head = typename inner::head`
- `using tail = typename inner::tail`

5.15.1 Detailed Description

```
template<typename... Ts>  
template<size_t index>  
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

index	
-----------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.16 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- `template<typename T >`
`using push_front = type_list< T, Ts... >`
Adds T to front of the list.
- `template<size_t index>`
`using at = internal::type_at_t< index, Ts... >`
returns type at index
- `template<typename T >`
`using push_back = type_list< Ts..., T >`
pushes T at the tail of the list
- `template<typename U >`
`using concat = typename concat_h< U >::type`
concatenates two list into one
- `template<typename T , size_t index>`
`using insert = typename internal::insert_h< index, type_list< Ts... >, T >::type`
inserts type at index
- `template<size_t index>`
`using remove = typename internal::remove_h< index, type_list< Ts... > >::type`
removes type at index

Static Public Attributes

- `static constexpr size_t length = sizeof...(Ts)`
length of list

5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	
--------------	--

5.16.2 Member Typedef Documentation

5.16.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.16.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.16.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.16.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.16.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.16.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.17 aerobus::type_list<> Struct Reference

Public Types

- template<typename T >
using **push_front** = type_list< T >

- `template<typename T >`
using **push_back** = `type_list< T >`
- `template<typename U >`
using **concat** = `U`
- `template<typename T , size_t index>`
using **insert** = `type_list< T >`

Static Public Attributes

- static constexpr `size_t length` = 0

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.18 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using **is_zero_t** = `std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast x into valueType
- `static std::string to_string ()`
string representation of value
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast x into valueRing

Static Public Attributes

- static constexpr `int32_t v` = `x`
actual value stored in val type

5.18.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.18.2 Member Function Documentation

5.18.2.1 `eval()`

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.18.2.2 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<code>valueType</code>	double for example
------------------------	--------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.19 `aerobus::i64::val< x >` Struct Template Reference

values in `i64`

```
#include <aerobus.h>
```

Public Types

- `using is_zero_t = std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast value in valueType
- `static std::string to_string ()`
string representation
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast value in valueRing

Static Public Attributes

- `static constexpr int64_t v = x`

5.19.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.19.2 Member Function Documentation

5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.20 aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

5.20.1 Detailed Description

```
template<typename Ring, char variable_name = 'x'>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

5.20.2 Member Typedef Documentation

5.20.2.1 coeff_at_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::coeff_at_t = typename
coeff_at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.20.3 Member Function Documentation

5.20.3.1 eval()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs
>::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.20.3.2 to_string()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::to_←
string ( ) [inline], [static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

Public Types

- **using type** = std::conditional_t< Ring::template pos_v< tmp >, tmp, typename Ring::template sub_t< typename Ring::zero, tmp > >

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- **using is_zero_t** = std::bool_constant< x% p==0 >

Static Public Member Functions

- template<typename valueType >
static constexpr valueType get ()
- static std::string to_string ()
- template<typename valueRing >
static constexpr valueRing eval (const valueRing &v)

Static Public Attributes

- static constexpr int32_t v = x % p

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.23 aerobus::polynomial< Ring, variable_name >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = [std::bool_constant< aN::is_zero_t::value >](#)
- template<[size_t](#) index>
using [coeff_at_t](#) = [typename](#) [coeff_at< index >::type](#)

Static Public Member Functions

- [static](#) [std::string](#) [to_string](#) ()
- template<[typename](#) [valueRing](#) >
[static constexpr](#) [valueRing](#) [eval](#) ([const valueRing](#) &x)

Static Public Attributes

- [static constexpr](#) [size_t](#) [degree](#) = 0
- [static constexpr](#) [bool](#) [is_zero_v](#) = [is_zero_t::value](#)

5.23.1 Detailed Description

```
template<typename Ring, char variable\_name = 'x'>
template<typename coeffN>
struct aerobus::polynomial< Ring, variable\_name >::val< coeffN >
```

specialization for constants

Template Parameters

coeffN	
------------------------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.24 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- [using inner_type = int32_t](#)
- [template<auto x>](#)
[using inject_constant_t = val< static_cast< int32_t >\(x\)>](#)
- [using zero = val< 0 >](#)
- [using one = val< 1 >](#)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
addition operator
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction operator
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication operator
- [template<typename v1 , typename v2 >](#)
[using div_t = typename div< v1, v2 >::type](#)
division operator
- [template<typename v1 , typename v2 >](#)
[using mod_t = typename remainder< v1, v2 >::type](#)
modulo operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt< v1, v2 >::type](#)
strictly greater operator (type)
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt< v1, v2 >::type](#)
strictly smaller operator (type)
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq< v1, v2 >::type](#)
equality operator (type)
- [template<typename v1 , typename v2 >](#)
[using gcd_t = gcd_t< i32, v1, v2 >](#)
greatest common divisor
- [template<typename v1 >](#)
[using pos_t = typename pos< v1 >::type](#)
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.24.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

Chapter 6

File Documentation

6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035
00036     template<typename T, size_t N>
00037     constexpr bool contains(const std::array<T, N>& arr, const T& v) {
00038         for (const auto& vv : arr) {
00039             if (v == vv) {
00040                 return true;
00041             }
00042         }
00043         return false;
00044     }
00045 } // namespace aerobus
00046
00047 // concepts
00048 namespace aerobus {
00049     template <typename R>
00050     concept IsRing = requires {
00051         typename R::one;
00052         typename R::zero;
00053         typename R::template add_t<typename R::one, typename R::one>;
00054         typename R::template sub_t<typename R::one, typename R::one>;
00055         typename R::template mul_t<typename R::one, typename R::one>;
00056     };
00057 }
```

```

00074
00075
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00089     template<typename R>
00090     concept IsField = IsEuclideanDomain<R> && requires {
00091         R::is_field == true;
00092     };
00093 } // namespace aerobus
00094
00095 // utilities
00096 namespace aerobus {
00097     namespace internal {
00098         template<typename...> typename TT, typename T>
00099         struct is_instantiation_of : std::false_type { };
00100
00101         template<template<typename...> typename TT, typename... Ts>
00102         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00103
00104         template<template<typename...> typename TT, typename T>
00105         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00106
00107         template <int64_t i, typename T, typename... Ts>
00108         struct type_at {
00109             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00110             using type = typename type_at<i - 1, Ts...>::type;
00111         };
00112
00113         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00114             using type = T;
00115         };
00116
00117         template <size_t i, typename... Ts>
00118         using type_at_t = typename type_at<i, Ts...>::type;
00119
00120         template<int32_t n, int32_t i, typename E = void>
00121         struct _is_prime {};
00122
00123         template<int32_t i>
00124         struct _is_prime<1, i> {
00125             static constexpr bool value = false;
00126         };
00127
00128         template<int32_t i>
00129         struct _is_prime<2, i> {
00130             static constexpr bool value = true;
00131         };
00132
00133         template<int32_t i>
00134         struct _is_prime<3, i> {
00135             static constexpr bool value = true;
00136         };
00137
00138         template<int32_t i>
00139         struct _is_prime<5, i> {
00140             static constexpr bool value = true;
00141         };
00142
00143         template<int32_t i>
00144         struct _is_prime<7, i> {
00145             static constexpr bool value = true;
00146         };
00147
00148         template<int32_t n, int32_t i>
00149         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00150             static constexpr bool value = false;
00151         };
00152
00153         template<int32_t n, int32_t i>
00154         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00155             static constexpr bool value = false;
00156         };
00157
00158         template<int32_t n, int32_t i>
00159         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00160             static constexpr bool value = true;
00161         };
00162

```



```

00163     };
00164
00165     template<int32_t n, int32_t i>
00166     struct _is_prime<n, i, std::enable_if_t<(
00167         n % i == 0 &&
00168         n >= 9 &&
00169         n % 3 != 0 &&
00170         n % 2 != 0 &&
00171         i * i > n)> {
00172         static constexpr bool value = true;
00173     };
00174
00175     template<int32_t n, int32_t i>
00176     struct _is_prime<n, i, std::enable_if_t<(
00177         n % (i+2) == 0 &&
00178         n >= 9 &&
00179         n % 3 != 0 &&
00180         n % 2 != 0 &&
00181         i * i <= n)> {
00182         static constexpr bool value = true;
00183     };
00184
00185     template<int32_t n, int32_t i>
00186     struct _is_prime<n, i, std::enable_if_t<(
00187         n % (i+2) != 0 &&
00188         n % i != 0 &&
00189         n >= 9 &&
00190         n % 3 != 0 &&
00191         n % 2 != 0 &&
00192         (i * i <= n))> {
00193         static constexpr bool value = _is_prime<n, i+6>::value;
00194     };
00195
00196     } // namespace internal
00197
00200     template<int32_t n>
00201     struct is_prime {
00202         static constexpr bool value = internal::_is_prime<n, 5>::value;
00203     };
00204
00205     template<int32_t n>
00206     static constexpr bool is_prime_v = is_prime<n>::value;
00207
00208     namespace internal {
00209         template <std::size_t... Is>
00210         constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00211             -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00212
00213         template <std::size_t N>
00214         using make_index_sequence_reverse
00215             = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00216
00217         template<typename Ring, typename E = void>
00218         struct gcd;
00219
00220         template<typename Ring>
00221         struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00222             template<typename A, typename B, typename E = void>
00223             struct gcd_helper {};
00224
00225             // B = 0, A > 0
00226             template<typename A, typename B>
00227             struct gcd_helper<A, B, std::enable_if_t<
00228                 (B::is_zero_t::value) &&
00229                 (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00230                 using type = A;
00231             };
00232
00233             // B = 0, A < 0
00234             template<typename A, typename B>
00235             struct gcd_helper<A, B, std::enable_if_t<
00236                 (B::is_zero_t::value) &&
00237                 !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00238                 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00239             };
00240
00241             // B != 0
00242             template<typename A, typename B>
00243             struct gcd_helper<A, B, std::enable_if_t<
00244                 (!B::is_zero_t::value)
00245                 >> {
00246             private: // NOLINT
00247                 // A / B
00248                 using k = typename Ring::template div_t<A, B>;
00249                 // A - (A/B)*B = A % B
00250                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00251             };
00252         };
00253     }

```

```

00258         public:
00259             using type = typename gcd_helper<B, m>::type;
00260         };
00261
00262         template<typename A, typename B>
00263         using type = typename gcd_helper<A, B>::type;
00264     };
00265 } // namespace internal
00266
00267 template<typename T, typename A, typename B>
00270 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00271 } // namespace aerobus
00272
00273 // quotient ring by the principal ideal generated by X
00274 namespace aerobus {
00275     template<typename Ring, typename X>
00276     requires IsRing<Ring>
00277     struct Quotient {
00278         template <typename V>
00279         struct val {
00280             private: // NOLINT
00281                 using tmp = typename Ring::template mod_t<V, X>;
00282
00283             public:
00284                 using type = std::conditional_t<
00285                     Ring::template pos_v<tmp>,
00286                     tmp,
00287                     typename Ring::template sub_t<typename Ring::zero, tmp>
00288                 >;
00289         };
00290
00291         using zero = val<typename Ring::zero>;
00292         using one = val<typename Ring::one>;
00293
00294         template<typename v1, typename v2>
00295         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00296         template<typename v1, typename v2>
00297         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00298         template<typename v1, typename v2>
00299         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00300         template<typename v1, typename v2>
00301         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00302         template<typename v1, typename v2>
00303         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00304         template<typename v1, typename v2>
00305         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00306         template<typename v1>
00307         using pos_t = std::true_type;
00308
00309         template<typename v>
00310         static constexpr bool pos_v = pos_t<v>::value;
00311
00312         static constexpr bool is_euclidean_domain = true;
00313
00314         template<auto x>
00315         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00316
00317         template<typename v>
00318         using inject_ring_t = val<v>;
00319     };
00320 } // namespace aerobus
00321
00322 // type_list
00323 namespace aerobus {
00324     template <typename... Ts>
00325     struct type_list;
00326
00327     namespace internal {
00328         template <typename T, typename... Us>
00329         struct pop_front_h {
00330             using tail = type_list<Us...>;
00331             using head = T;
00332         };
00333
00334         template <size_t index, typename L1, typename L2>
00335         struct split_h {
00336             private:
00337                 static_assert(index <= L2::length, "index out of bounds");
00338                 using a = typename L2::pop_front::type;
00339                 using b = typename L2::pop_front::tail;
00340                 using c = typename L1::template push_back<a>;
00341
00342             public:
00343                 using head = typename split_h<index - 1, c, b>::head;
00344                 using tail = typename split_h<index - 1, c, b>::tail;
00345         };
00346     }
00347

```

```

00348     template <typename L1, typename L2>
00349     struct split_h<0, L1, L2> {
00350         using head = L1;
00351         using tail = L2;
00352     };
00353
00354     template <size_t index, typename L, typename T>
00355     struct insert_h {
00356         static_assert(index <= L::length, "index ouf of bounds");
00357         using s = typename L::template split<index>;
00358         using left = typename s::head;
00359         using right = typename s::tail;
00360         using ll = typename left::template push_back<T>;
00361         using type = typename ll::template concat<right>;
00362     };
00363
00364     template <size_t index, typename L>
00365     struct remove_h {
00366         using s = typename L::template split<index>;
00367         using left = typename s::head;
00368         using right = typename s::tail;
00369         using rr = typename right::pop_front::tail;
00370         using type = typename left::template concat<rr>;
00371     };
00372 } // namespace internal
00373
00374
00377 template <typename... Ts>
00378 struct type_list {
00379 private:
00380     template <typename T>
00381     struct concat_h;
00382
00383     template <typename... Us>
00384     struct concat_h<type_list<Us...> {
00385         using type = type_list<Ts..., Us...>;
00386     };
00387
00388 public:
00389     static constexpr size_t length = sizeof...(Ts);
00390
00391     template <typename T>
00392     using push_front = type_list<T, Ts...>;
00393
00394     template <size_t index>
00395     using at = internal::type_at_t<index, Ts...>;
00396
00397     struct pop_front {
00398         using type = typename internal::pop_front_h<Ts...>::head;
00399         using tail = typename internal::pop_front_h<Ts...>::tail;
00400     };
00401
00402     template <typename T>
00403     using push_back = type_list<Ts..., T>;
00404
00405     template <typename U>
00406     using concat = typename concat_h<U>::type;
00407
00408     template <size_t index>
00409     struct split {
00410 private:
00411         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00412
00413 public:
00414         using head = typename inner::head;
00415         using tail = typename inner::tail;
00416     };
00417
00418     template <typename T, size_t index>
00419     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00420
00421     template <size_t index>
00422     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00423 };
00424
00425 template <>
00426 struct type_list<> {
00427     static constexpr size_t length = 0;
00428
00429     template <typename T>
00430     using push_front = type_list<T>;
00431
00432     template <typename T>
00433     using push_back = type_list<T>;
00434
00435     template <typename U>
00436     using concat = U;
00437

```

```

00456
00457     // TODO(jewave): assert index == 0
00458     template <typename T, size_t index>
00459     using insert = type_list<T>;
00460 };
00461 } // namespace aerobus
00462
00463 // i32
00464 namespace aerobus {
00465     struct i32 {
00466         using inner_type = int32_t;
00467         template<int32_t x>
00471         struct val {
00473             static constexpr int32_t v = x;
00474
00477             template<typename valueType>
00478             static constexpr valueType get() { return static_cast<valueType>(x); }
00479
00481             using is_zero_t = std::bool_constant<x == 0>;
00482
00484             static std::string to_string() {
00485                 return std::to_string(x);
00486             }
00487
00490             template<typename valueRing>
00491             static constexpr valueRing eval(const valueRing& v) {
00492                 return static_cast<valueRing>(x);
00493             }
00494         };
00495
00497         using zero = val<0>;
00499         using one = val<1>;
00501         static constexpr bool is_field = false;
00503         static constexpr bool is_euclidean_domain = true;
00507         template<auto x>
00508         using inject_constant_t = val<static_cast<int32_t>(x)>;
00509
00510         template<typename v>
00511         using inject_ring_t = v;
00512
00513     private:
00514         template<typename v1, typename v2>
00515         struct add {
00516             using type = val<v1::v + v2::v>;
00517         };
00518
00519         template<typename v1, typename v2>
00520         struct sub {
00521             using type = val<v1::v - v2::v>;
00522         };
00523
00524         template<typename v1, typename v2>
00525         struct mul {
00526             using type = val<v1::v * v2::v>;
00527         };
00528
00529         template<typename v1, typename v2>
00530         struct div {
00531             using type = val<v1::v / v2::v>;
00532         };
00533
00534         template<typename v1, typename v2>
00535         struct remainder {
00536             using type = val<v1::v % v2::v>;
00537         };
00538
00539         template<typename v1, typename v2>
00540         struct gt {
00541             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00542         };
00543
00544         template<typename v1, typename v2>
00545         struct lt {
00546             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00547         };
00548
00549         template<typename v1, typename v2>
00550         struct eq {
00551             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00552         };
00553
00554         template<typename v1>
00555         struct pos {
00556             using type = std::bool_constant<(v1::v > 0)>;
00557         };
00558
00559     public:

```

```

00561     template<typename v1, typename v2>
00562     using add_t = typename add<v1, v2>::type;
00563
00565     template<typename v1, typename v2>
00566     using sub_t = typename sub<v1, v2>::type;
00567
00569     template<typename v1, typename v2>
00570     using mul_t = typename mul<v1, v2>::type;
00571
00573     template<typename v1, typename v2>
00574     using div_t = typename div<v1, v2>::type;
00575
00577     template<typename v1, typename v2>
00578     using mod_t = typename remainder<v1, v2>::type;
00579
00581     template<typename v1, typename v2>
00582     using gt_t = typename gt<v1, v2>::type;
00583
00585     template<typename v1, typename v2>
00586     using lt_t = typename lt<v1, v2>::type;
00587
00589     template<typename v1, typename v2>
00590     using eq_t = typename eq<v1, v2>::type;
00591
00595     template<typename v1, typename v2>
00596     static constexpr bool eq_v = eq_t<v1, v2>::value;
00597
00599     template<typename v1, typename v2>
00600     using gcd_t = gcd_t<i32, v1, v2>;
00601
00603     template<typename v>
00604     using pos_t = typename pos<v>::type;
00605
00608     template<typename v>
00609     static constexpr bool pos_v = pos_t<v>::value;
00610 };
00611 } // namespace aerobus
00612
00613 // i64
00614 namespace aerobus {
00615     struct i64 {
00616         using inner_type = int64_t;
00617         template<int64_t x>
00621         struct val {
00622             static constexpr int64_t v = x;
00623
00626             template<typename valueType>
00627             static constexpr valueType get() { return static_cast<valueType>(x); }
00628
00630             using is_zero_t = std::bool_constant<x == 0>;
00631
00633             static std::string to_string() {
00634                 return std::to_string(x);
00635             }
00636
00639             template<typename valueRing>
00640             static constexpr valueRing eval(const valueRing& v) {
00641                 return static_cast<valueRing>(x);
00642             }
00643         };
00644
00648         template<auto x>
00649         using inject_constant_t = val<static_cast<int64_t>(x)>;
00650
00651         template<typename v>
00652         using inject_ring_t = v;
00653
00655         using zero = val<0>;
00657         using one = val<1>;
00659         static constexpr bool is_field = false;
00661         static constexpr bool is_euclidean_domain = true;
00662
00663     private:
00664         template<typename v1, typename v2>
00665         struct add {
00666             using type = val<v1::v + v2::v>;
00667         };
00668
00669         template<typename v1, typename v2>
00670         struct sub {
00671             using type = val<v1::v - v2::v>;
00672         };
00673
00674         template<typename v1, typename v2>
00675         struct mul {
00676             using type = val<v1::v * v2::v>;
00677         };

```

```

00678
00679     template<typename v1, typename v2>
00680     struct div {
00681         using type = val<v1::v / v2::v>;
00682     };
00683
00684     template<typename v1, typename v2>
00685     struct remainder {
00686         using type = val<v1::v% v2::v>;
00687     };
00688
00689     template<typename v1, typename v2>
00690     struct gt {
00691         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00692     };
00693
00694     template<typename v1, typename v2>
00695     struct lt {
00696         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00697     };
00698
00699     template<typename v1, typename v2>
00700     struct eq {
00701         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00702     };
00703
00704     template<typename v>
00705     struct pos {
00706         using type = std::bool_constant<(v::v > 0)>;
00707     };
00708
00709     public:
00710     template<typename v1, typename v2>
00711     using add_t = typename add<v1, v2>::type;
00712
00713     template<typename v1, typename v2>
00714     using sub_t = typename sub<v1, v2>::type;
00715
00716     template<typename v1, typename v2>
00717     using mul_t = typename mul<v1, v2>::type;
00718
00719     template<typename v1, typename v2>
00720     using div_t = typename div<v1, v2>::type;
00721
00722     template<typename v1, typename v2>
00723     using mod_t = typename remainder<v1, v2>::type;
00724
00725     template<typename v1, typename v2>
00726     using gt_t = typename gt<v1, v2>::type;
00727
00728     template<typename v1, typename v2>
00729     static constexpr bool gt_v = gt_t<v1, v2>::value;
00730
00731     template<typename v1, typename v2>
00732     using lt_t = typename lt<v1, v2>::type;
00733
00734     template<typename v1, typename v2>
00735     static constexpr bool lt_v = lt_t<v1, v2>::value;
00736
00737     template<typename v1, typename v2>
00738     using eq_t = typename eq<v1, v2>::type;
00739
00740     template<typename v1, typename v2>
00741     static constexpr bool eq_v = eq_t<v1, v2>::value;
00742
00743     template<typename v1, typename v2>
00744     using gcd_t = gcd_t<i64, v1, v2>;
00745
00746     template<typename v>
00747     using pos_t = typename pos<v>::type;
00748
00749     template<typename v>
00750     static constexpr bool pos_v = pos_t<v>::value;
00751 };
00752 } // namespace aerobus
00753
00754 // z/pz
00755 namespace aerobus {
00756     template<int32_t p>
00757     struct zp {
00758         using inner_type = int32_t;
00759         template<int32_t x>
00760         struct val {
00761             static constexpr int32_t v = x % p;
00762
00763             template<typename valueType>
00764             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00765         };
00766     };
00767 }

```

```

00809         using is_zero_t = std::bool_constant<x% p == 0>;
00810         static std::string to_string() {
00811             return std::to_string(x % p);
00812         }
00813     }
00814
00815     template<typename valueRing>
00816     static constexpr valueRing eval(const valueRing& v) {
00817         return static_cast<valueRing>(x % p);
00818     }
00819 };
00820
00821 template<auto x>
00822 using inject_constant_t = val<static_cast<int32_t>(x)>;
00823
00824 using zero = val<0>;
00825 using one = val<1>;
00826 static constexpr bool is_field = is_prime<p>::value;
00827 static constexpr bool is_euclidean_domain = true;
00828
00829 private:
00830     template<typename v1, typename v2>
00831     struct add {
00832         using type = val<(v1::v + v2::v) % p>;
00833     };
00834
00835     template<typename v1, typename v2>
00836     struct sub {
00837         using type = val<(v1::v - v2::v) % p>;
00838     };
00839
00840     template<typename v1, typename v2>
00841     struct mul {
00842         using type = val<(v1::v* v2::v) % p>;
00843     };
00844
00845     template<typename v1, typename v2>
00846     struct div {
00847         using type = val<(v1::v% p) / (v2::v % p)>;
00848     };
00849
00850     template<typename v1, typename v2>
00851     struct remainder {
00852         using type = val<(v1::v% v2::v) % p>;
00853     };
00854
00855     template<typename v1, typename v2>
00856     struct gt {
00857         using type = std::conditional_t<(v1::v% p > v2::v% p), std::true_type, std::false_type>;
00858     };
00859
00860     template<typename v1, typename v2>
00861     struct lt {
00862         using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
00863     };
00864
00865     template<typename v1, typename v2>
00866     struct eq {
00867         using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
00868     };
00869
00870     template<typename v1>
00871     struct pos {
00872         using type = std::bool_constant<(v1::v > 0)>;
00873     };
00874
00875 public:
00876     template<typename v1, typename v2>
00877     using add_t = typename add<v1, v2>::type;
00878
00879     template<typename v1, typename v2>
00880     using sub_t = typename sub<v1, v2>::type;
00881
00882     template<typename v1, typename v2>
00883     using mul_t = typename mul<v1, v2>::type;
00884
00885     template<typename v1, typename v2>
00886     using div_t = typename div<v1, v2>::type;
00887
00888     template<typename v1, typename v2>
00889     using mod_t = typename remainder<v1, v2>::type;
00890
00891     template<typename v1, typename v2>
00892     using gt_t = typename gt<v1, v2>::type;
00893
00894     template<typename v1, typename v2>
00895     static constexpr bool gt_v = gt_t<v1, v2>::value;
00896
00897     template<typename v1, typename v2>
00898     static constexpr bool lt_v = lt_t<v1, v2>::value;
00899
00900     template<typename v1, typename v2>
00901     static constexpr bool eq_v = eq_t<v1, v2>::value;
00902
00903     template<typename v1, typename v2>
00904     static constexpr bool neq_v = !eq_v;
00905
00906     template<typename v1, typename v2>
00907     static constexpr bool pos_v = pos_t<v1>::type::value;
00908
00909     template<typename v1, typename v2>
00910     static constexpr bool neg_v = !pos_v;
00911
00912     template<typename v1, typename v2>
00913     static constexpr bool is_zero_v = is_zero_t::value;
00914
00915     template<typename v1, typename v2>
00916     static constexpr bool is_euclidean_v = is_euclidean_domain;
00917
00918     template<typename v1, typename v2>
00919     static constexpr bool is_field_v = is_field;
00920
00921     template<typename v1, typename v2>
00922     static constexpr bool is_integ_v = is_integ_domain;
00923
00924     template<typename v1, typename v2>
00925     static constexpr bool is_prime_v = is_prime<p>::value;
00926
00927     template<typename v1, typename v2>
00928     static constexpr bool is_composite_v = !is_prime_v;
00929
00930     template<typename v1, typename v2>
00931     static constexpr bool is_even_v = is_even<p>::value;
00932
00933     template<typename v1, typename v2>
00934     static constexpr bool is_odd_v = !is_even_v;
00935
00936     template<typename v1, typename v2>
00937     static constexpr bool is_square_v = is_square<p>::value;
00938
00939     template<typename v1, typename v2>
00940     static constexpr bool is_cube_v = is_cube<p>::value;
00941
00942     template<typename v1, typename v2>
00943     static constexpr bool is_fourth_v = is_fourth<p>::value;
00944
00945     template<typename v1, typename v2>
00946     static constexpr bool is_sixth_v = is_sixth<p>::value;
00947
00948     template<typename v1, typename v2>
00949     static constexpr bool is_eighth_v = is_eighth<p>::value;
00950
00951     template<typename v1, typename v2>
00952     static constexpr bool is_tenth_v = is_tenth<p>::value;
00953
00954     template<typename v1, typename v2>
00955     static constexpr bool is_twelfth_v = is_twelfth<p>::value;
00956
00957     template<typename v1, typename v2>
00958     static constexpr bool is_thirteenth_v = is_thirteenth<p>::value;
00959
00960     template<typename v1, typename v2>
00961     static constexpr bool is_fourteenth_v = is_fourteenth<p>::value;
00962
00963     template<typename v1, typename v2>
00964     static constexpr bool is_fifteenth_v = is_fifteenth<p>::value;
00965
00966     template<typename v1, typename v2>
00967     static constexpr bool is_sixteenth_v = is_sixteenth<p>::value;
00968
00969     template<typename v1, typename v2>
00970     static constexpr bool is_seventeenth_v = is_seventeenth<p>::value;
00971
00972     template<typename v1, typename v2>
00973     static constexpr bool is_eighteenth_v = is_eighteenth<p>::value;
00974
00975     template<typename v1, typename v2>
00976     static constexpr bool is_nineteenth_v = is_nineteenth<p>::value;
00977
00978     template<typename v1, typename v2>
00979     static constexpr bool is_twentieth_v = is_twentieth<p>::value;
00980
00981     template<typename v1, typename v2>
00982     static constexpr bool is_twentyfirst_v = is_twentyfirst<p>::value;
00983
00984     template<typename v1, typename v2>
00985     static constexpr bool is_twentysecond_v = is_twentysecond<p>::value;
00986
00987     template<typename v1, typename v2>
00988     static constexpr bool is_twentythird_v = is_twentythird<p>::value;
00989
00990     template<typename v1, typename v2>
00991     static constexpr bool is_twentyfourth_v = is_twentyfourth<p>::value;
00992
00993     template<typename v1, typename v2>
00994     static constexpr bool is_twentyfifth_v = is_twentyfifth<p>::value;
00995
00996     template<typename v1, typename v2>
00997     static constexpr bool is_twentysixth_v = is_twentysixth<p>::value;
00998
00999     template<typename v1, typename v2>
01000     static constexpr bool is_twentyseventh_v = is_twentyseventh<p>::value;
01001
01002     template<typename v1, typename v2>
01003     static constexpr bool is_twentyeighth_v = is_twentyeighth<p>::value;
01004
01005     template<typename v1, typename v2>
01006     static constexpr bool is_twentyninth_v = is_twentyninth<p>::value;
01007
01008     template<typename v1, typename v2>
01009     static constexpr bool is_thirtieth_v = is_thirtieth<p>::value;
01010
01011     template<typename v1, typename v2>
01012     static constexpr bool is_thirtyfirst_v = is_thirtyfirst<p>::value;
01013
01014     template<typename v1, typename v2>
01015     static constexpr bool is_thirtysecond_v = is_thirtysecond<p>::value;
01016
01017     template<typename v1, typename v2>
01018     static constexpr bool is_thirtythird_v = is_thirtythird<p>::value;
01019
01020     template<typename v1, typename v2>
01021     static constexpr bool is_thirtyfourth_v = is_thirtyfourth<p>::value;
01022
01023     template<typename v1, typename v2>
01024     static constexpr bool is_thirtyfifth_v = is_thirtyfifth<p>::value;
01025
01026     template<typename v1, typename v2>
01027     static constexpr bool is_thirtysixth_v = is_thirtysixth<p>::value;
01028
01029     template<typename v1, typename v2>
01030     static constexpr bool is_thirtyseventh_v = is_thirtyseventh<p>::value;
01031
01032     template<typename v1, typename v2>
01033     static constexpr bool is_thirtyeighth_v = is_thirtyeighth<p>::value;
01034
01035     template<typename v1, typename v2>
01036     static constexpr bool is_thirtyninth_v = is_thirtyninth<p>::value;
01037
01038     template<typename v1, typename v2>
01039     static constexpr bool is_fortieth_v = is_fortieth<p>::value;
01040
01041     template<typename v1, typename v2>
01042     static constexpr bool is_fortyfirst_v = is_fortyfirst<p>::value;
01043
01044     template<typename v1, typename v2>
01045     static constexpr bool is_fortysecond_v = is_fortysecond<p>::value;
01046
01047     template<typename v1, typename v2>
01048     static constexpr bool is_fortythird_v = is_fortythird<p>::value;
01049
01050     template<typename v1, typename v2>
01051     static constexpr bool is_fortyfourth_v = is_fortyfourth<p>::value;
01052
01053     template<typename v1, typename v2>
01054     static constexpr bool is_fortyfifth_v = is_fortyfifth<p>::value;
01055
01056     template<typename v1, typename v2>
01057     static constexpr bool is_fortysixth_v = is_fortysixth<p>::value;
01058
01059     template<typename v1,
```

```

00903
00905     template<typename v1, typename v2>
00906     using lt_t = typename lt<v1, v2>::type;
00907
00909     template<typename v1, typename v2>
00910     static constexpr bool lt_v = lt_t<v1, v2>::value;
00911
00913     template<typename v1, typename v2>
00914     using eq_t = typename eq<v1, v2>::type;
00915
00917     template<typename v1, typename v2>
00918     static constexpr bool eq_v = eq_t<v1, v2>::value;
00919
00921     template<typename v1, typename v2>
00922     using gcd_t = gcd_t<i32, v1, v2>;
00923
00925     template<typename v1>
00926     using pos_t = typename pos<v1>::type;
00927
00929     template<typename v>
00930     static constexpr bool pos_v = pos_t<v>::value;
00931 };
00932 } // namespace aerobus
00933
00934 // polynomial
00935 namespace aerobus {
00936     // coeffN x^N + ...
00941     template<typename Ring, char variable_name = 'x'>
00942     requires IsEuclideanDomain<Ring>
00943     struct polynomial {
00944         static constexpr bool is_field = false;
00945         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
00946
00950         template<typename coeffN, typename... coeffs>
00951         struct val {
00953             static constexpr size_t degree = sizeof...(coeffs);
00955             using aN = coeffN;
00957             using strip = val<coeffs...>;
00959             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
00961             static constexpr bool is_zero_v = is_zero_t::value;
00962
00963         private:
00964             template<size_t index, typename E = void>
00965             struct coeff_at {};
00966
00967             template<size_t index>
00968             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))>> {
00969                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
00970             };
00971
00972             template<size_t index>
00973             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))>> {
00974                 using type = typename Ring::zero;
00975             };
00976
00977         public:
00980             template<size_t index>
00981             using coeff_at_t = typename coeff_at<index>::type;
00982
00985             static std::string to_string() {
00986                 return string_helper<coeffN, coeffs...>::func();
00987             }
00988
00993             template<typename valueRing>
00994             static constexpr valueRing eval(const valueRing& x) {
00995                 return horner_evaluation<valueRing, val>
00996                     ::template inner<0, degree + 1>
00997                     ::func(static_cast<valueRing>(0), x);
00998             }
00999         };
01000
01003     template<typename coeffN>
01004     struct val<coeffN> {
01005         static constexpr size_t degree = 0;
01006         using aN = coeffN;
01007         using strip = val<coeffN>;
01008         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01009
01010         static constexpr bool is_zero_v = is_zero_t::value;
01011
01012         template<size_t index, typename E = void>
01013         struct coeff_at {};
01014
01015         template<size_t index>
01016         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01017             using type = aN;
01018         };

```



```

01019
01020     template<size_t index>
01021     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01022         using type = typename Ring::zero;
01023     };
01024
01025     template<size_t index>
01026     using coeff_at_t = typename coeff_at<index>::type;
01027
01028     static std::string to_string() {
01029         return string_helper<coeffN>::func();
01030     }
01031
01032     template<typename valueRing>
01033     static constexpr valueRing eval(const valueRing& x) {
01034         return static_cast<valueRing>(aN::template get<valueRing>());
01035     }
01036 };
01037
01039     using zero = val<typename Ring::zero>;
01041     using one = val<typename Ring::one>;
01043     using X = val<typename Ring::one, typename Ring::zero>;
01044
01045 private:
01046     template<typename P, typename E = void>
01047     struct simplify;
01048
01049     template <typename P1, typename P2, typename I>
01050     struct add_low;
01051
01052     template<typename P1, typename P2>
01053     struct add {
01054         using type = typename simplify<typename add_low<
01055             P1,
01056             P2,
01057             internal::make_index_sequence_reverse<
01058                 std::max(P1::degree, P2::degree) + 1
01059             >::type>::type;
01060     };
01061
01062     template <typename P1, typename P2, typename I>
01063     struct sub_low;
01064
01065     template <typename P1, typename P2, typename I>
01066     struct mul_low;
01067
01068     template<typename v1, typename v2>
01069     struct mul {
01070         using type = typename mul_low<
01071             v1,
01072             v2,
01073             internal::make_index_sequence_reverse<
01074                 v1::degree + v2::degree + 1
01075             >::type;
01076     };
01077
01078     template<typename coeff, size_t deg>
01079     struct monomial;
01080
01081     template<typename v, typename E = void>
01082     struct derive_helper {};
01083
01084     template<typename v>
01085     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01086         using type = zero;
01087     };
01088
01089     template<typename v>
01090     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01091         using type = typename add<
01092             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01093             typename monomial<
01094                 typename Ring::template mul_t<
01095                     typename v::aN,
01096                     typename Ring::template inject_constant_t<(v::degree)>
01097                 >,
01098                 v::degree - 1
01099             >::type
01100             >::type;
01101     };
01102
01103     template<typename v1, typename v2, typename E = void>
01104     struct eq_helper {};
01105
01106     template<typename v1, typename v2>
01107     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01108         using type = std::false_type;

```

```

01109     };
01110
01111
01112     template<typename v1, typename v2>
01113     struct eq_helper<v1, v2, std::enable_if_t<
01114         v1::degree == v2::degree &&
01115         (v1::degree != 0 || v2::degree != 0) &&
01116         std::is_same<
01117             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01118             std::false_type
01119         >::value
01120     >
01121     > {
01122         using type = std::false_type;
01123     };
01124
01125     template<typename v1, typename v2>
01126     struct eq_helper<v1, v2, std::enable_if_t<
01127         v1::degree == v2::degree &&
01128         (v1::degree != 0 || v2::degree != 0) &&
01129         std::is_same<
01130             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01131             std::true_type
01132         >::value
01133     >> {
01134         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01135     };
01136
01137     template<typename v1, typename v2>
01138     struct eq_helper<v1, v2, std::enable_if_t<
01139         v1::degree == v2::degree &&
01140         (v1::degree == 0)
01141     >> {
01142         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01143     };
01144
01145     template<typename v1, typename v2, typename E = void>
01146     struct lt_helper {};
01147
01148     template<typename v1, typename v2>
01149     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01150         using type = std::true_type;
01151     };
01152
01153     template<typename v1, typename v2>
01154     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01155         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01156     };
01157
01158     template<typename v1, typename v2>
01159     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01160         using type = std::false_type;
01161     };
01162
01163     template<typename v1, typename v2, typename E = void>
01164     struct gt_helper {};
01165
01166     template<typename v1, typename v2>
01167     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01168         using type = std::true_type;
01169     };
01170
01171     template<typename v1, typename v2>
01172     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01173         using type = std::false_type;
01174     };
01175
01176     template<typename v1, typename v2>
01177     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01178         using type = std::false_type;
01179     };
01180
01181     // when high power is zero : strip
01182     template<typename P>
01183     struct simplify<P, std::enable_if_t<
01184         std::is_same<
01185             typename Ring::zero,
01186             typename P::aN
01187         >::value && (P::degree > 0)
01188     >> {
01189         using type = typename simplify<typename P::strip>::type;
01190     };
01191
01192     // otherwise : do nothing
01193     template<typename P>
01194     struct simplify<P, std::enable_if_t<
01195         !std::is_same<

```

```

01196         typename Ring::zero,
01197         typename P::aN
01198         >::value && (P::degree > 0)
01199     » {
01200         using type = P;
01201     };
01202
01203     // do not simplify constants
01204     template<typename P>
01205     struct simplify<P, std::enable_if_t<P::degree == 0> {
01206         using type = P;
01207     };
01208
01209     // addition at
01210     template<typename P1, typename P2, size_t index>
01211     struct add_at {
01212         using type =
01213             typename Ring::template add_t<
01214                 typename P1::template coeff_at_t<index>,
01215                 typename P2::template coeff_at_t<index>>;
01216     };
01217
01218     template<typename P1, typename P2, size_t index>
01219     using add_at_t = typename add_at<P1, P2, index>::type;
01220
01221     template<typename P1, typename P2, std::size_t... I>
01222     struct add_low<P1, P2, std::index_sequence<I...> {
01223         using type = val<add_at_t<P1, P2, I>...>;
01224     };
01225
01226     // subtraction at
01227     template<typename P1, typename P2, size_t index>
01228     struct sub_at {
01229         using type =
01230             typename Ring::template sub_t<
01231                 typename P1::template coeff_at_t<index>,
01232                 typename P2::template coeff_at_t<index>>;
01233     };
01234
01235     template<typename P1, typename P2, size_t index>
01236     using sub_at_t = typename sub_at<P1, P2, index>::type;
01237
01238     template<typename P1, typename P2, std::size_t... I>
01239     struct sub_low<P1, P2, std::index_sequence<I...> {
01240         using type = val<sub_at_t<P1, P2, I>...>;
01241     };
01242
01243     template<typename P1, typename P2>
01244     struct sub {
01245         using type = typename simplify<typename sub_low<
01246             P1,
01247             P2,
01248             internal::make_index_sequence_reverse<
01249                 std::max(P1::degree, P2::degree) + 1
01250             >::type>::type;
01251     };
01252
01253     // multiplication at
01254     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01255     struct mul_at_loop_helper {
01256         using type = typename Ring::template add_t<
01257             typename Ring::template mul_t<
01258                 typename v1::template coeff_at_t<index>,
01259                 typename v2::template coeff_at_t<k - index>
01260             >,
01261             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01262         >;
01263     };
01264
01265     template<typename v1, typename v2, size_t k, size_t stop>
01266     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01267         using type = typename Ring::template mul_t<
01268             typename v1::template coeff_at_t<stop>,
01269             typename v2::template coeff_at_t<0>>;
01270     };
01271
01272     template <typename v1, typename v2, size_t k, typename E = void>
01273     struct mul_at {};
01274
01275     template<typename v1, typename v2, size_t k>
01276     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01277         using type = typename Ring::zero;
01278     };
01279
01280     template<typename v1, typename v2, size_t k>
01281     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01282         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;

```

```

01283     };
01284
01285     template<typename P1, typename P2, size_t index>
01286     using mul_at_t = typename mul_at<P1, P2, index>::type;
01287
01288     template<typename P1, typename P2, std::size_t... I>
01289     struct mul_low<P1, P2, std::index_sequence<I...> {
01290         using type = val<mul_at_t<P1, P2, I>...>;
01291     };
01292
01293     // division helper
01294     template< typename A, typename B, typename Q, typename R, typename E = void>
01295     struct div_helper {};
01296
01297     template<typename A, typename B, typename Q, typename R>
01298     struct div_helper<A, B, Q, R, std::enable_if_t<
01299         (R::degree < B::degree) ||
01300         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01301         using q_type = Q;
01302         using mod_type = R;
01303         using gcd_type = B;
01304     };
01305
01306     template<typename A, typename B, typename Q, typename R>
01307     struct div_helper<A, B, Q, R, std::enable_if_t<
01308         (R::degree >= B::degree) &&
01309         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01310     private: // NOLINT
01311         using rN = typename R::aN;
01312         using bN = typename B::aN;
01313         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01314             B::degree>::type;
01315         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01316         using qq = typename add<Q, pT>::type;
01317     public:
01318         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01319         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01320         using gcd_type = rr;
01321     };
01322
01323     template<typename A, typename B>
01324     struct div {
01325         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01326         using q_type = typename div_helper<A, B, zero, A>::q_type;
01327         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01328     };
01329
01330     template<typename P>
01331     struct make_unit {
01332         using type = typename div<P, val<typename P::aN>>::q_type;
01333     };
01334
01335     template<typename coeff, size_t deg>
01336     struct monomial {
01337         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01338     };
01339
01340     template<typename coeff>
01341     struct monomial<coeff, 0> {
01342         using type = val<coeff>;
01343     };
01344
01345     template<typename valueRing, typename P>
01346     struct horner_evaluation {
01347         template<size_t index, size_t stop>
01348         struct inner {
01349             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01350                 constexpr valueRing coeff =
01351                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01352 get<valueRing>());
01353                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01354 accum + coeff, x);
01355             }
01356         };
01357         template<size_t stop>
01358         struct inner<stop, stop> {
01359             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01360                 return accum;
01361             }
01362         };
01363     };
01364
01365     template<typename coeff, typename... coeffs>
01366     struct string_helper {
01367         static std::string func() {

```

```

01368         std::string tail = string_helper<coeffs...>::func();
01369         std::string result = "";
01370         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01371             return tail;
01372         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01373             if (sizeof...(coeffs) == 1) {
01374                 result += std::string(1, variable_name);
01375             } else {
01376                 result += std::string(1, variable_name) + "^" +
std::to_string(sizeof...(coeffs));
01377             }
01378         } else {
01379             if (sizeof...(coeffs) == 1) {
01380                 result += coeff::to_string() + " " + std::string(1, variable_name);
01381             } else {
01382                 result += coeff::to_string()
01383                     + " " + std::string(1, variable_name)
01384                     + "^" + std::to_string(sizeof...(coeffs));
01385             }
01386         }
01387
01388         if (!tail.empty()) {
01389             result += " " + tail;
01390         }
01391
01392         return result;
01393     }
01394 };
01395
01396 template<typename coeff>
01397 struct string_helper<coeff> {
01398     static std::string func() {
01399         if (!std::is_same<coeff, typename Ring::zero>::value) {
01400             return coeff::to_string();
01401         } else {
01402             return "";
01403         }
01404     }
01405 };
01406
01407 public:
01410     template<typename P>
01411     using simplify_t = typename simplify<P>::type;
01412
01416     template<typename v1, typename v2>
01417     using add_t = typename add<v1, v2>::type;
01418
01422     template<typename v1, typename v2>
01423     using sub_t = typename sub<v1, v2>::type;
01424
01428     template<typename v1, typename v2>
01429     using mul_t = typename mul<v1, v2>::type;
01430
01434     template<typename v1, typename v2>
01435     using eq_t = typename eq_helper<v1, v2>::type;
01436
01440     template<typename v1, typename v2>
01441     using lt_t = typename lt_helper<v1, v2>::type;
01442
01446     template<typename v1, typename v2>
01447     using gt_t = typename gt_helper<v1, v2>::type;
01448
01452     template<typename v1, typename v2>
01453     using div_t = typename div<v1, v2>::q_type;
01454
01458     template<typename v1, typename v2>
01459     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01460
01464     template<typename coeff, size_t deg>
01465     using monomial_t = typename monomial<coeff, deg>::type;
01466
01469     template<typename v>
01470     using derive_t = typename derive_helper<v>::type;
01471
01474     template<typename v>
01475     using pos_t = typename Ring::template pos_t<typename v::aN>;
01476
01477     template<typename v>
01478     static constexpr bool pos_v = pos_t<v>::value;
01479
01483     template<typename v1, typename v2>
01484     using gcd_t = std::conditional_t<
01485         Ring::is_euclidean_domain,
01486         typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2>::type,
01487         void>;
01488
01492     template<auto x>

```

```

01493         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01494
01498         template<typename v>
01499         using inject_ring_t = val<v>;
01500     };
01501 } // namespace aerobus
01502
01503 // fraction field
01504 namespace aerobus {
01505     namespace internal {
01506         template<typename Ring, typename E = void>
01507         requires IsEuclideanDomain<Ring>
01508         struct _FractionField {};
01509
01510         template<typename Ring>
01511         requires IsEuclideanDomain<Ring>
01512         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01513             static constexpr bool is_field = true;
01514             static constexpr bool is_euclidean_domain = true;
01515
01516         private:
01517             template<typename val1, typename val2, typename E = void>
01518             struct to_string_helper {};
01519
01520             template<typename val1, typename val2>
01521             struct to_string_helper<val1, val2,
01522                 std::enable_if_t<
01523                     Ring::template eq_t<
01524                         val2, typename Ring::one
01525                     >::value
01526                 >
01527             > {
01528                 static std::string func() {
01529                     return val1::to_string();
01530                 }
01531             };
01532
01533             template<typename val1, typename val2>
01534             struct to_string_helper<val1, val2,
01535                 std::enable_if_t<
01536                     !Ring::template eq_t<
01537                         val2,
01538                         typename Ring::one
01539                     >::value
01540                 >
01541             > {
01542                 static std::string func() {
01543                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01544                 }
01545             };
01546
01547         public:
01548             template<typename val1, typename val2>
01549             struct val {
01550                 using x = val1;
01551                 using y = val2;
01552                 using is_zero_t = typename val1::is_zero_t;
01553                 static constexpr bool is_zero_v = val1::is_zero_t::value;
01554
01555                 using ring_type = Ring;
01556                 using field_type = _FractionField<Ring>;
01557
01558                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01559
01560                 template<typename valueType>
01561                 static constexpr valueType get() { return static_cast<valueType>(x::v) /
01562                     static_cast<valueType>(y::v); }
01563
01564                 static std::string to_string() {
01565                     return to_string_helper<val1, val2>::func();
01566                 }
01567
01568                 template<typename valueRing>
01569                 static constexpr valueRing eval(const valueRing& v) {
01570                     return x::eval(v) / y::eval(v);
01571                 }
01572             };
01573
01574             using zero = val<typename Ring::zero, typename Ring::one>;
01575             using one = val<typename Ring::one, typename Ring::one>;
01576
01577             template<typename v>
01578             using inject_t = val<v, typename Ring::one>;
01579
01580             template<auto x>
01581             using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01582                 Ring::one>;

```

```

01607
01610     template<typename v>
01611     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01612
01614     using ring_type = Ring;
01615
01616 private:
01617     template<typename v, typename E = void>
01618     struct simplify {};
01619
01620     // x = 0
01621     template<typename v>
01622     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01623         using type = typename _FractionField<Ring>::zero;
01624     };
01625
01626     // x != 0
01627     template<typename v>
01628     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01629     private:
01630         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01631         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01632         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01633
01634         using posx = std::conditional_t<
01635             !Ring::template pos_v<newy>,
01636             typename Ring::template sub_t<typename Ring::zero, newx>,
01637             newx>;
01638         using posy = std::conditional_t<
01639             !Ring::template pos_v<newy>,
01640             typename Ring::template sub_t<typename Ring::zero, newy>,
01641             newy>;
01642     public:
01643         using type = typename _FractionField<Ring>::template val<posx, posy>;
01644     };
01645
01646 public:
01647     template<typename v>
01648     using simplify_t = typename simplify<v>::type;
01649
01650 private:
01651     template<typename v1, typename v2>
01652     struct add {
01653     private:
01654         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01655         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01656         using dividend = typename Ring::template add_t<a, b>;
01657         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01658         using g = typename Ring::template gcd_t<dividend, diviser>;
01659
01660     public:
01661         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01662             diviser>;
01663     };
01664
01665     template<typename v>
01666     struct pos {
01667     private:
01668         using type = std::conditional_t<
01669             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01670             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01671             std::true_type,
01672             std::false_type>;
01673     };
01674
01675     template<typename v1, typename v2>
01676     struct sub {
01677     private:
01678         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01679         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01680         using dividend = typename Ring::template sub_t<a, b>;
01681         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01682         using g = typename Ring::template gcd_t<dividend, diviser>;
01683
01684     public:
01685         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01686             diviser>;
01687     };
01688
01689     template<typename v1, typename v2>
01690     struct mul {
01691     private:
01692         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01693         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01694
01695     public:
01696         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01697     };

```

```

01697
01698     template<typename v1, typename v2, typename E = void>
01699     struct div {};
01700
01701     template<typename v1, typename v2>
01702     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01703     private:
01704         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01705         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01706
01707     public:
01708         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01709     };
01710
01711     template<typename v1, typename v2>
01712     struct div<v1, v2, std::enable_if_t<
01713         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01714         using type = one;
01715     };
01716
01717     template<typename v1, typename v2>
01718     struct eq {
01719         using type = std::conditional_t<
01720             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01721             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01722             std::true_type,
01723             std::false_type>;
01724     };
01725
01726     template<typename TL, typename E = void>
01727     struct vadd {};
01728
01729     template<typename TL>
01730     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01731         using head = typename TL::pop_front::type;
01732         using tail = typename TL::pop_front::tail;
01733         using type = typename add<head, typename vadd<tail>::type>::type;
01734     };
01735
01736     template<typename TL>
01737     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01738         using type = typename TL::template at<0>;
01739     };
01740
01741     template<typename... vals>
01742     struct vmul {};
01743
01744     template<typename v1, typename... vals>
01745     struct vmul<v1, vals...> {
01746         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01747     };
01748
01749     template<typename v1>
01750     struct vmul<v1> {
01751         using type = v1;
01752     };
01753
01754     template<typename v1, typename v2, typename E = void>
01755     struct gt;
01756
01757     template<typename v1, typename v2>
01758     struct gt<v1, v2, std::enable_if_t<
01759         (eq<v1, v2>::type::value)
01760         >> {
01761         using type = std::false_type;
01762     };
01763
01764     template<typename v1, typename v2>
01765     struct gt<v1, v2, std::enable_if_t<
01766         (!eq<v1, v2>::type::value) &&
01767         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01768         >> {
01769         using type = typename gt<
01770             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01771             >::type;
01772     };
01773
01774     template<typename v1, typename v2>
01775     struct gt<v1, v2, std::enable_if_t<
01776         (!eq<v1, v2>::type::value) &&
01777         (pos<v1>::type::value) && (!pos<v2>::type::value)
01778         >> {
01779         using type = std::true_type;
01780     };
01781
01782

```



```

01783     template<typename v1, typename v2>
01784     struct gt<v1, v2, std::enable_if_t<
01785         (!eq<v1, v2>::type::value) &&
01786         (!pos<v1>::type::value) && (pos<v2>::type::value)
01787         >> {
01788         using type = std::false_type;
01789     };
01790
01791     template<typename v1, typename v2>
01792     struct gt<v1, v2, std::enable_if_t<
01793         (!eq<v1, v2>::type::value) &&
01794         (pos<v1>::type::value) && (pos<v2>::type::value)
01795         >> {
01796         using type = typename Ring::template gt_t<
01797             typename Ring::template mul_t<v1::x, v2::y>,
01798             typename Ring::template mul_t<v2::y, v2::x>
01799         >;
01800     };
01801
01802     public:
01803     template<typename v1, typename v2>
01804     using add_t = typename add<v1, v2>::type;
01805     template<typename v1, typename v2>
01806     using mod_t = zero;
01807     template<typename v1, typename v2>
01808     using gcd_t = v1;
01809     template<typename... vs>
01810     using vadd_t = typename vadd<vs...>::type;
01811     template<typename... vs>
01812     using vmul_t = typename vmul<vs...>::type;
01813     template<typename v1, typename v2>
01814     using sub_t = typename sub<v1, v2>::type;
01815     template<typename v1, typename v2>
01816     using mul_t = typename mul<v1, v2>::type;
01817     template<typename v1, typename v2>
01818     using div_t = typename div<v1, v2>::type;
01819     template<typename v1, typename v2>
01820     using eq_t = typename eq<v1, v2>::type;
01821     template<typename v1, typename v2>
01822     static constexpr bool eq_v = eq<v1, v2>::type::value;
01823     template<typename v1, typename v2>
01824     using gt_t = typename gt<v1, v2>::type;
01825     template<typename v1, typename v2>
01826     static constexpr bool gt_v = gt<v1, v2>::type::value;
01827     template<typename v1>
01828     using pos_t = typename pos<v1>::type;
01829     template<typename v>
01830     static constexpr bool pos_v = pos<v>::value;
01831 };
01832
01833     template<typename Ring, typename E = void>
01834     requires IsEuclideanDomain<Ring>
01835     struct FractionFieldImpl {};
01836
01837     // fraction field of a field is the field itself
01838     template<typename Field>
01839     requires IsEuclideanDomain<Field>
01840     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01841         using type = Field;
01842         template<typename v>
01843         using inject_t = v;
01844     };
01845
01846     // fraction field of a ring is the actual fraction field
01847     template<typename Ring>
01848     requires IsEuclideanDomain<Ring>
01849     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01850         using type = _FractionField<Ring>;
01851     };
01852 } // namespace internal
01853
01854     template<typename Ring>
01855     requires IsEuclideanDomain<Ring>
01856     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01857 } // namespace aerobus
01858
01859 // short names for common types
01860 namespace aerobus {
01861     using q32 = FractionField<i32>;
01862     using fpq32 = FractionField<polynomial<q32>>;
01863     using q64 = FractionField<i64>;
01864     using pi64 = polynomial<i64>;
01865     using pq64 = polynomial<q64>;
01866     using fpq64 = FractionField<polynomial<q64>>;
01867     template<typename Ring, typename v1, typename v2>
01868     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01869 }

```

```

01902     template<typename Ring, typename v1, typename v2>
01903     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
01908     template<typename Ring, typename v1, typename v2>
01909     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01910 } // namespace aerobus
01911
01912 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
01913 namespace aerobus {
01914     namespace internal {
01915         template<typename T, size_t x, typename E = void>
01916         struct factorial {};
01917
01918         template<typename T, size_t x>
01919         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
01920         private:
01921             template<typename, size_t, typename>
01922             friend struct factorial;
01923         public:
01924             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
01925             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01926         };
01927
01928         template<typename T>
01929         struct factorial<T, 0> {
01930         public:
01931             using type = typename T::one;
01932             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01933         };
01934     } // namespace internal
01935
01939     template<typename T, size_t i>
01940     using factorial_t = typename internal::factorial<T, i>::type;
01941
01945     template<typename T, size_t i>
01946     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01947
01948     namespace internal {
01949         template<typename T, size_t k, size_t n, typename E = void>
01950         struct combination_helper {};
01951
01952         template<typename T, size_t k, size_t n>
01953         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
01954             using type = typename FractionField<T>::template mul_t<
01955                 typename combination_helper<T, k - 1, n - 1>::type,
01956                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
01957         };
01958
01959         template<typename T, size_t k, size_t n>
01960         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
01961             using type = typename combination_helper<T, n - k, n>::type;
01962         };
01963
01964         template<typename T, size_t n>
01965         struct combination_helper<T, 0, n> {
01966             using type = typename FractionField<T>::one;
01967         };
01968
01969         template<typename T, size_t k, size_t n>
01970         struct combination {
01971             using type = typename internal::combination_helper<T, k, n>::type::x;
01972             static constexpr typename T::inner_type value =
01973                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
01974         };
01975     } // namespace internal
01976
01979     template<typename T, size_t k, size_t n>
01980     using combination_t = typename internal::combination<T, k, n>::type;
01981
01986     template<typename T, size_t k, size_t n>
01987     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
01988
01989     namespace internal {
01990         template<typename T, size_t m>
01991         struct bernouilli;
01992
01993         template<typename T, typename accum, size_t k, size_t m>
01994         struct bernouilli_helper {
01995             using type = typename bernouilli_helper<
01996                 T,
01997                 addfractions_t<T,
01998                     accum,
01999                     mulfractions_t<T,
02000                     makefraction_t<T,

```

```

02001             combination_t<T, k, m + 1>,
02002             typename T::one>,
02003             typename bernouilli<T, k>::type
02004         >
02005     >,
02006     k + 1,
02007     m>::type;
02008 };
02009
02010 template<typename T, typename accum, size_t m>
02011 struct bernouilli_helper<T, accum, m, m> {
02012     using type = accum;
02013 };
02014
02015
02016
02017 template<typename T, size_t m>
02018 struct bernouilli {
02019     using type = typename FractionField<T>::template mul_t<
02020         typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02021         makefraction_t<T,
02022         typename T::template val<static_cast<typename T::inner_type>(-1)>,
02023         typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02024     >
02025     >;
02026
02027     template<typename floatType>
02028     static constexpr floatType value = type::template get<floatType>();
02029 };
02030
02031 template<typename T>
02032 struct bernouilli<T, 0> {
02033     using type = typename FractionField<T>::one;
02034
02035     template<typename floatType>
02036     static constexpr floatType value = type::template get<floatType>();
02037 };
02038 } // namespace internal
02039
02040 template<typename T, size_t n>
02041 using bernouilli_t = typename internal::bernouilli<T, n>::type;
02042
02043
02044 template<typename FloatType, typename T, size_t n>
02045 inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02046
02047 namespace internal {
02048     template<typename T, int k, typename E = void>
02049     struct alternate {};
02050
02051     template<typename T, int k>
02052     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02053         using type = typename T::one;
02054         static constexpr typename T::inner_type value = type::template get<typename
02055         T::inner_type>();
02056     };
02057
02058     template<typename T, int k>
02059     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02060         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02061         static constexpr typename T::inner_type value = type::template get<typename
02062         T::inner_type>();
02063     };
02064 } // namespace internal
02065
02066 template<typename T, int k>
02067 using alternate_t = typename internal::alternate<T, k>::type;
02068
02069 template<typename T, size_t k>
02070 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02071
02072 namespace internal {
02073     template<typename T, auto p, auto n>
02074     struct pow {
02075         using type = typename T::template mul_t<typename T::template val<p>, typename pow<T, p, n
02076         - 1>::type>;
02077     };
02078
02079     template<typename T, auto p>
02080     struct pow<T, p, 0> { using type = typename T::one; };
02081 }
02082
02083 template<typename T, auto p, auto n>
02084 using pow_t = typename internal::pow<T, p, n>::type;
02085
02086 namespace internal {
02087     template<typename, size_t> typename, class>
02088     struct make_taylor_impl;
02089 }

```

```

02101
02102     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02103     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02104         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is::type...>;
02105     };
02106 }
02107
02112     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02113     using taylor = typename internal::make_taylor_impl<
02114         T,
02115         coeff_at,
02116         internal::make_index_sequence_reverse<deg + 1>::type;
02117
02118     namespace internal {
02119         template<typename T, size_t i>
02120         struct exp_coeff {
02121             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02122         };
02123
02124         template<typename T, size_t i, typename E = void>
02125         struct sin_coeff_helper {};
02126
02127         template<typename T, size_t i>
02128         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02129             using type = typename FractionField<T>::zero;
02130         };
02131
02132         template<typename T, size_t i>
02133         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02134             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02135         };
02136
02137         template<typename T, size_t i>
02138         struct sin_coeff {
02139             using type = typename sin_coeff_helper<T, i>::type;
02140         };
02141
02142         template<typename T, size_t i, typename E = void>
02143         struct sh_coeff_helper {};
02144
02145         template<typename T, size_t i>
02146         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02147             using type = typename FractionField<T>::zero;
02148         };
02149
02150         template<typename T, size_t i>
02151         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02152             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02153         };
02154
02155         template<typename T, size_t i>
02156         struct sh_coeff {
02157             using type = typename sh_coeff_helper<T, i>::type;
02158         };
02159
02160         template<typename T, size_t i, typename E = void>
02161         struct cos_coeff_helper {};
02162
02163         template<typename T, size_t i>
02164         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02165             using type = typename FractionField<T>::zero;
02166         };
02167
02168         template<typename T, size_t i>
02169         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02170             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02171         };
02172
02173         template<typename T, size_t i>
02174         struct cos_coeff {
02175             using type = typename cos_coeff_helper<T, i>::type;
02176         };
02177
02178         template<typename T, size_t i, typename E = void>
02179         struct cosh_coeff_helper {};
02180
02181         template<typename T, size_t i>
02182         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02183             using type = typename FractionField<T>::zero;
02184         };
02185
02186         template<typename T, size_t i>
02187         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02188             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02189         };
02190

```

```

02191     template<typename T, size_t i>
02192     struct cosh_coeff {
02193         using type = typename cosh_coeff_helper<T, i>::type;
02194     };
02195
02196     template<typename T, size_t i>
02197     struct geom_coeff { using type = typename FractionField<T>::one; };
02198
02199
02200     template<typename T, size_t i, typename E = void>
02201     struct atan_coeff_helper;
02202
02203     template<typename T, size_t i>
02204     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02205         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02206     };
02207
02208     template<typename T, size_t i>
02209     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02210         using type = typename FractionField<T>::zero;
02211     };
02212
02213     template<typename T, size_t i>
02214     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02215
02216     template<typename T, size_t i, typename E = void>
02217     struct asin_coeff_helper;
02218
02219     template<typename T, size_t i>
02220     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02221         using type = makefraction_t<T,
02222             factorial_t<T, i - 1>,
02223             typename T::template mul_t<
02224                 typename T::template val<i>,
02225                 T::template mul_t<
02226                     pow_t<T, 4, i / 2>,
02227                     pow<T, factorial<T, i / 2>::value, 2
02228                 >
02229             >
02230         >;
02231     };
02232
02233     template<typename T, size_t i>
02234     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02235         using type = typename FractionField<T>::zero;
02236     };
02237
02238     template<typename T, size_t i>
02239     struct asin_coeff {
02240         using type = typename asin_coeff_helper<T, i>::type;
02241     };
02242
02243     template<typename T, size_t i>
02244     struct lnpl_coeff {
02245         using type = makefraction_t<T,
02246             alternate_t<T, i + 1>,
02247             typename T::template val<i>;
02248     };
02249
02250     template<typename T>
02251     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02252
02253     template<typename T, size_t i, typename E = void>
02254     struct asinh_coeff_helper;
02255
02256     template<typename T, size_t i>
02257     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02258         using type = makefraction_t<T,
02259             typename T::template mul_t<
02260                 alternate_t<T, i / 2>,
02261                 factorial_t<T, i - 1>
02262             >,
02263             typename T::template mul_t<
02264                 T::template mul_t<
02265                     typename T::template val<i>,
02266                     pow_t<T, (factorial<T, i / 2>::value), 2>
02267                 >,
02268                 pow_t<T, 4, i / 2>
02269             >
02270         >;
02271     };
02272
02273     template<typename T, size_t i>
02274     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02275         using type = typename FractionField<T>::zero;
02276     };
02277

```

```

02278     template<typename T, size_t i>
02279     struct asinh_coeff {
02280         using type = typename asinh_coeff_helper<T, i>::type;
02281     };
02282
02283     template<typename T, size_t i, typename E = void>
02284     struct atanh_coeff_helper;
02285
02286     template<typename T, size_t i>
02287     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02288         // 1/i
02289         using type = typename FractionField<T>::template val<
02290             typename T::one,
02291             typename T::template val<static_cast<typename T::inner_type>(i)>>;
02292     };
02293
02294     template<typename T, size_t i>
02295     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02296         using type = typename FractionField<T>::zero;
02297     };
02298
02299     template<typename T, size_t i>
02300     struct atanh_coeff {
02301         using type = typename asinh_coeff_helper<T, i>::type;
02302     };
02303
02304     template<typename T, size_t i, typename E = void>
02305     struct tan_coeff_helper;
02306
02307     template<typename T, size_t i>
02308     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02309         using type = typename FractionField<T>::zero;
02310     };
02311
02312     template<typename T, size_t i>
02313     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02314     private:
02315         // 4^((i+1)/2)
02316         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02317         // 4^((i+1)/2) - 1
02318         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02319         // (-1)^((i-1)/2)
02320         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02321         using dividend = typename FractionField<T>::template mul_t<
02322             altp,
02323             FractionField<T>::template mul_t<
02324                 _4p,
02325                 FractionField<T>::template mul_t<
02326                     _4pml,
02327                     bernouilli_t<T, (i + 1)>
02328                 >
02329             >;
02330     public:
02331         using type = typename FractionField<T>::template div_t<dividend,
02332             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02333     };
02334
02335     template<typename T, size_t i>
02336     struct tan_coeff {
02337         using type = typename tan_coeff_helper<T, i>::type;
02338     };
02339
02340     template<typename T, size_t i, typename E = void>
02341     struct tanh_coeff_helper;
02342
02343     template<typename T, size_t i>
02344     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02345         using type = typename FractionField<T>::zero;
02346     };
02347
02348     template<typename T, size_t i>
02349     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02350     private:
02351         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02352         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02353         using dividend =
02354             typename FractionField<T>::template mul_t<
02355                 _4p,
02356                 typename FractionField<T>::template mul_t<
02357                     _4pml,
02358                     bernouilli_t<T, (i + 1)>
02359                 >
02360             >::type;
02361     public:

```

```

02363         using type = typename FractionField<T>::template div_t<dividend,
02364             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
02365     };
02366
02367     template<typename T, size_t i>
02368     struct tanh_coeff {
02369         using type = typename tanh_coeff_helper<T, i>::type;
02370     };
02371 } // namespace internal
02372
02373 template<typename T, size_t deg>
02374 using exp = taylor<T, internal::exp_coeff, deg>;
02375
02376 template<typename T, size_t deg>
02377 using expm1 = typename polynomial<FractionField<T>::template sub_t<
02378     exp<T, deg>,
02379     typename polynomial<FractionField<T>::one>;
02380
02381 template<typename T, size_t deg>
02382 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02383
02384 template<typename T, size_t deg>
02385 using atan = taylor<T, internal::atan_coeff, deg>;
02386
02387 template<typename T, size_t deg>
02388 using sin = taylor<T, internal::sin_coeff, deg>;
02389
02390 template<typename T, size_t deg>
02391 using sinh = taylor<T, internal::sh_coeff, deg>;
02392
02393 template<typename T, size_t deg>
02394 using cosh = taylor<T, internal::cosh_coeff, deg>;
02395
02396 template<typename T, size_t deg>
02397 using cos = taylor<T, internal::cos_coeff, deg>;
02398
02399 template<typename T, size_t deg>
02400 using geom_sum = taylor<T, internal::geom_coeff, deg>;
02401
02402 template<typename T, size_t deg>
02403 using asin = taylor<T, internal::asin_coeff, deg>;
02404
02405 template<typename T, size_t deg>
02406 using asinh = taylor<T, internal::asinh_coeff, deg>;
02407
02408 template<typename T, size_t deg>
02409 using atanh = taylor<T, internal::atanh_coeff, deg>;
02410
02411 template<typename T, size_t deg>
02412 using tan = taylor<T, internal::tan_coeff, deg>;
02413
02414 template<typename T, size_t deg>
02415 using tanh = taylor<T, internal::tanh_coeff, deg>;
02416 } // namespace aerobus
02417
02418 // continued fractions
02419 namespace aerobus {
02420     template<int64_t... values>
02421     struct ContinuedFraction {};
02422
02423     template<int64_t a0>
02424     struct ContinuedFraction<a0> {
02425         using type = typename q64::template inject_constant_t<a0>;
02426         static constexpr double val = type::template get<double>();
02427     };
02428
02429     template<int64_t a0, int64_t... rest>
02430     struct ContinuedFraction<a0, rest...> {
02431         using type = q64::template add_t<
02432             typename q64::template inject_constant_t<a0>,
02433             typename q64::template div_t<
02434                 typename q64::one,
02435                 typename ContinuedFraction<rest...>::type
02436             >>;
02437         static constexpr double val = type::template get<double>();
02438     };
02439
02440     using PI_fraction =
02441         ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02442     using E_fraction =
02443         ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02444     using SQRT2_fraction =
02445         ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02446     using SQRT3_fraction =
02447         ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02448 } // NOLINT
02449 } // namespace aerobus

```

```

02502
02503 // known polynomials
02504 namespace aerobus {
02505     // CChebyshev
02506     namespace internal {
02507         template<int kind, int deg>
02508         struct chebyshev_helper {
02509             using type = typename pi64::template sub_t<
02510                 typename pi64::template mul_t<
02511                     typename pi64::template mul_t<
02512                         pi64::inject_constant_t<2>,
02513                         typename pi64::X
02514                     >,
02515                     typename chebyshev_helper<kind, deg - 1>::type
02516                 >,
02517                 typename chebyshev_helper<kind, deg - 2>::type
02518             >;
02519         };
02520
02521         template<>
02522         struct chebyshev_helper<1, 0> {
02523             using type = typename pi64::one;
02524         };
02525
02526         template<>
02527         struct chebyshev_helper<1, 1> {
02528             using type = typename pi64::X;
02529         };
02530
02531         template<>
02532         struct chebyshev_helper<2, 0> {
02533             using type = typename pi64::one;
02534         };
02535
02536         template<>
02537         struct chebyshev_helper<2, 1> {
02538             using type = typename pi64::template mul_t<
02539                 typename pi64::inject_constant_t<2>,
02540                 typename pi64::X>;
02541         };
02542     } // namespace internal
02543
02544     // Laguerre
02545     namespace internal {
02546         template<size_t deg>
02547         struct laguerre_helper {
02548             private:
02549                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)lkm2)
02550                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02551                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02552                 // -x + 2k-1
02553                 using p = typename pq64::template val<
02554                     typename q64::template inject_constant_t<-1>,
02555                     typename q64::template inject_constant_t<2 * deg - 1>;
02556                 // 1/n
02557                 using factor = typename pq64::template inject_ring_t<
02558                     q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>
02559                 >;
02560             public:
02561                 using type = typename pq64::template mul_t <
02562                     factor,
02563                     typename pq64::template sub_t<
02564                         typename pq64::template mul_t<
02565                             p,
02566                             lnm1
02567                         >,
02568                         typename pq64::template mul_t<
02569                             typename pq64::template inject_constant_t<deg-1>,
02570                             lnm2
02571                         >
02572                     >
02573                 >;
02574         };
02575
02576         template<>
02577         struct laguerre_helper<0> {
02578             using type = typename pq64::one;
02579         };
02580
02581         template<>
02582         struct laguerre_helper<1> {
02583             using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02584         };
02585     } // namespace internal
02586
02587     namespace known_polynomials {

```



```

02590     enum hermite_kind {
02591         probabilist,
02592         physicist
02593     };
02594 }
02595
02596 namespace internal {
02597     template<size_t deg, known_polynomials::hermite_kind kind>
02598     struct hermite_helper {};
02599
02600     template<size_t deg>
02601     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02602     private:
02603         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::probabilist>::type;
02604         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::probabilist>::type;
02605
02606     public:
02607         using type = typename pi64::template sub_t<
02608             typename pi64::template mul_t<typename pi64::X, hnm1>,
02609             typename pi64::template mul_t<
02610                 typename pi64::template inject_constant_t<deg - 1>,
02611                 hnm2
02612             >
02613         >;
02614     };
02615
02616     template<size_t deg>
02617     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02618     private:
02619         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02620         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02621
02622     public:
02623         using type = typename pi64::template sub_t<
02624             // 2X Hn-1
02625             typename pi64::template mul_t<
02626                 typename pi64::val<typename i64::template inject_constant_t<2>,
02627                 typename i64::zero>, hnm1>,
02628
02629             typename pi64::template mul_t<
02630                 typename pi64::template inject_constant_t<2*(deg - 1)>,
02631                 hnm2
02632             >
02633         >;
02634     };
02635
02636     template<>
02637     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02638     using type = typename pi64::one;
02639     };
02640
02641     template<>
02642     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02643     using type = typename pi64::X;
02644     };
02645
02646     template<>
02647     struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02648     using type = typename pi64::one;
02649     };
02650
02651     template<>
02652     struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02653     // 2X
02654     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
typename i64::zero>;
02655     };
02656 } // namespace internal
02657
02658 namespace known_polynomials {
02659     template <size_t deg>
02660     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02661
02662     template <size_t deg>
02663     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02664
02665     template <size_t deg>
02666     using laguerre = typename internal::laguerre_helper<deg>::type;
02667
02668     template <size_t deg>
02669     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02670
02671     template <size_t deg>

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```

POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
04623 template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
04624 template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
04625 template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
04626 template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
04627 template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
04628 template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
04629 template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
04630 template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
04631 template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
04632 template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
04633 template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
04634 #endif // AEROBUS_CONWAY_IMPORTS
04635
04636 #endif // __INC_AEROBUS__ // NOLINT

```


Chapter 7

Examples

7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

7.3 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.4 PI_fraction::val

representation of PI as a continued fraction -> 3.14...

7.5 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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