

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for **quotient ring** R/X where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	15
aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	41
aerobus::IsField	
Concept to express R is a field	41
aerobus::IsRing	
Concept to express R is a Ring	42

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> >	43
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	44
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	44
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	45
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	46
aerobus::ConwayPolynomial	47
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >	47
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >	48
aerobus::Embed< Small, Large, E >	
Embedding - struct forward declaration	49
aerobus::Embed< i32, i64 >	
Embeds i32 into i64	49
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
Embeds polynomial<Small> into polynomial<Large>	50
aerobus::Embed< q32, q64 >	
Embeds q32 into q64	51
aerobus::Embed< Quotient< Ring, X >, Ring >	
Embeds Quotient<Ring, X> into Ring	52
aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	53
aerobus::Embed< zpz< x >, i32 >	
Embeds zpz values into i32	53
aerobus::polynomial< Ring >::horner_reduction_t< P >	
Used to evaluate polynomials over a value in Ring	54
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	55
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	62
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >	68
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >	69

aerobus::is_prime< n >	
Checks if n is prime	69
aerobus::polynomial< Ring >	70
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	78
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	79
aerobus::type_list< Ts >::split< index >	
Splits list at index	84
aerobus::type_list< Ts >	
Empty pure template struct to handle type list	85
aerobus::type_list<>	
Specialization for empty type list	88
aerobus::i32::val< x >	
Values in i32 , again represented as types	89
aerobus::i64::val< x >	
Values in i64	91
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	93
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	97
aerobus::zpz< p >::val< x >	
Values in zpz	98
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	100
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	103

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

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Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<typename T , typename A , typename B >
 using [gcd_t](#) = typename internal::gcd< T >::template type< A, B >
 - computes the greatest common divisor or A and B*
- template<typename... vals>
 using [vadd_t](#) = typename internal::vadd< vals... >::type
 - adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator*
- template<typename... vals>
 using [vmul_t](#) = typename internal::vmul< vals... >::type
 - multiplies multiple values (v1 * v2 + ... * vn) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator*
- template<typename val >
 using [abs_t](#) = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template [sub_t](#)< typename val::enclosing_type::zero, val > >
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
 - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add_t](#) = typename X::enclosing_type::template [add_t](#)< X, Y >
 - generic addition*
- template<typename X , typename Y >
 using [sub_t](#) = typename X::enclosing_type::template [sub_t](#)< X, Y >

- generic subtraction*
- `template<typename X , typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
- generic multiplication*
- `template<typename X , typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
- generic division*
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
- `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename Ring , auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
- `template<typename Ring , auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField< Ring>
- `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type

- `template<typename T , size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T , size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T , size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T , int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T , int n, int k>`
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T , int n, int k>`
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T , int n, int k>`
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`
Stirling number of second kind – as types.
- `template<typename T , typename p , size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<`
`deg+1 > >::type`
- `template<typename Integers , size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers , size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg`
`>, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers , size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers , size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers , size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers , size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`
 $\cosh(x)$ *hyperbolic cosine*
- `template<typename Integers , size_t deg>`
`using cos = taylor< Integers, internal::cos_coeff, deg >`
 $\cos(x)$ *cosinus*
- `template<typename Integers , size_t deg>`
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`

- $\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$
 • template<typename Integers , size_t deg>
 using **asin** = **taylor**< Integers, internal::asin_coeff, deg >
 arcsin(x) *arc sinus*
- template<typename Integers , size_t deg>
 using **asinh** = **taylor**< Integers, internal::asinh_coeff, deg >
 arcsinh(x) *arc hyperbolic sinus*
- template<typename Integers , size_t deg>
 using **atanh** = **taylor**< Integers, internal::atanh_coeff, deg >
 arctanh(x) *arc hyperbolic tangent*
- template<typename Integers , size_t deg>
 using **tan** = **taylor**< Integers, internal::tan_coeff, deg >
 tan(x) *tangent*
- template<typename Integers , size_t deg>
 using **tanh** = **taylor**< Integers, internal::tanh_coeff, deg >
 tanh(x) *hyperbolic tangent*
- using **PI_fraction** = **ContinuedFraction**< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >
- using **E_fraction** = **ContinuedFraction**< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >
 approximation of e
- using **SQRT2_fraction** = **ContinuedFraction**< 1, 2 >
 approximation of $\sqrt{2}$
- using **SQRT3_fraction** = **ContinuedFraction**< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >
 approximation of

Functions

- template<typename T >
 T * **aligned_malloc** (size_t count, size_t alignment)
- brief Conway polynomials tparam p characteristic of the **field** (prime number) @tparam n degree of extension
 template< int p

Variables

- template<typename T , size_t i>
 constexpr T::inner_type **factorial_v** = internal::factorial<T, i>::value
 computes factorial(i) as value in T
- template<typename T , size_t k, size_t n>
 constexpr T::inner_type **combination_v** = internal::combination<T, k, n>::value
 computes binomial coefficients (k among n) as value
- template<typename FloatType , typename T , size_t n>
 constexpr FloatType **bernoulli_v** = internal::bernoulli<T, n>::template value<FloatType>
 nth bernoulli number as value in FloatType
- template<typename T , size_t k>
 constexpr T::inner_type **alternate_v** = internal::alternate<T, k>::value
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a Rlng, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	---

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh(x)` arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ `cosinus`

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of e

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

Template Parameters

<i>Ring</i>	
-------------	--

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of π as a continued fraction

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.46 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.47 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.49 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.50 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.51 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >

- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >
- struct **fma_helper**< int32_t >

- struct **fma_helper**< int64_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_1_helper**
- struct **stirling_1_helper**< T, 0, 0 >
- struct **stirling_1_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, k, std::enable_if_t<(k > 0) && (n > 0)> >
- struct **stirling_2_helper**
- struct **stirling_2_helper**< T, 0, n, std::enable_if_t<(n > 0)> >

- struct **stirling_2_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling_2_helper**< T, n, n, std::enable_if_t<(n >=0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **touchard_coeff**
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N>{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Enumerations

- enum [hermite_kind](#) { [probabilist](#) , [physicist](#) }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Enumeration Type Documentation

6.3.2.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = typename Ring::zero

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = `aN`

8.3.1 Member Typedef Documentation

8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

Examples

[examples/continued_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename q64::template inject_constant_t< a0 >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = static_cast<double>(a0)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template [add_t](#)< typename q64::template inject_constant_t< a0 >, typename q64::template [div_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.8.1 Member Function Documentation

8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.9.1 Member Function Documentation

8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- template<typename val >
using [type](#) = [i64::val](#)< static_cast< int64_t >(val::v)>
the [i64](#) representation of val

8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

8.11.2 Member Typedef Documentation

8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the polynomial<Large> representation of v

8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.12.2 Member Typedef Documentation

8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`
q64 representation of v

8.13.1 Detailed Description

embeds q32 into q64

8.13.2 Member Typedef Documentation

8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = typename val::raw_t`
Ring representation of val.

8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

8.14.2 Member Typedef Documentation

8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename FractionField< Ring >::template val< v, typename Ring::one >`
FractionField<Ring> representation of v.

8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

Template Parameters

<i>Ring</i>	an integers ring, such as i32
-------------	---

8.15.2 Member Typedef Documentation

8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i32::val< val::v >`
the i32 representation of val

8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

Template Parameters

<code>x</code>	an integer
----------------	------------

8.16.2 Member Typedef Documentation

8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

Template Parameters

<code>val</code>	a value in zpz<x>
------------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using [inner_type](#) = int32_t
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<auto x>
using [inject_constant_t](#) = [val](#)< static_cast< int32_t >(x)>
inject a native constant
- template<typename v >
using [inject_ring_t](#) = v
- template<typename v1 , typename v2 >
using [add_t](#) = typename add< v1, v2 >::type
addition operator yields v1 + v2

- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator yields $v1 - v2$
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
*multiplication operator yields $v1 * v2$*
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator yields $v1 / v2$
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator yields $v1 \% v2$
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator ($v1 > v2$) yields $v1 > v2$
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator ($v1 < v2$) yields $v1 < v2$
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator (boolean value)
- `template<typename v >`
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity (boolean value) yields $v > 0$ as boolean value

8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

Examples

[examples/compensated_horner.cpp](#).

8.18.2 Member Typedef Documentation

8.18.2.1 add_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields $v1 + v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.2 div_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields $v1 / v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.3 eq_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.4 gcd_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields $GCD(v1, v2)$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) yields $v1 > v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

inject a native constant

Template Parameters

<i>x</i>	
----------	--

8.18.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

8.18.2.8 inner_type

```
using aerobus::i32::inner_type = int32_t
```

8.18.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$) yields $v1 < v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.10 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.11 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields $v1 * v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.18.2.13 pos_t

```
template<typename v >  
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v</i>	a value in i32
----------	--------------------------------

8.18.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields $v1 - v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.18.3 Member Data Documentation

8.18.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.18.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.18.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.18.3.4 pos_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	a value in i32
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- using [inner_type](#) = `int64_t`
type of represented values
- template<auto x>
using [inject_constant_t](#) = `val< static_cast< int64_t >(x)>`
injects constant as an [i64](#) value
- template<typename v >
using [inject_ring_t](#) = v
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`
constant zero
- using [one](#) = `val< 1 >`
constant one
- template<typename v1 , typename v2 >
using [add_t](#) = `typename add< v1, v2 >::type`
addition operator
- template<typename v1 , typename v2 >
using [sub_t](#) = `typename sub< v1, v2 >::type`
subtraction operator
- template<typename v1 , typename v2 >
using [mul_t](#) = `typename mul< v1, v2 >::type`
multiplication operator
- template<typename v1 , typename v2 >
using [div_t](#) = `typename div< v1, v2 >::type`
division operator integer division
- template<typename v1 , typename v2 >
using [mod_t](#) = `typename remainder< v1, v2 >::type`

modulus operator

- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i64, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$ as instantiation of [i64::val](#)
- `template<typename v >`
`using pos_t = typename pos< v >::type`
is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator yields $v1 > v2$ as boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator yields $v1 < v2$ as boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator yields $v1 == v2$ as boolean value
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity yields $v > 0$ as boolean value

8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.19.2 Member Typedef Documentation

8.19.2.1 [add_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.6 inject_constant_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

Template Parameters

<code>x</code>	
----------------	--

8.19.2.7 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
 \rightarrow [i64::val<1>](#)

Template Parameters

<code>v</code>	a value in i64
----------------	--------------------------------

8.19.2.8 inner_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.19.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.19.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
-----------	---

8.19.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

8.19.3 Member Data Documentation

8.19.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.19.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.19.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields $v_1 < v_2$ as boolean value

Template Parameters

v_1	: an element of aerobus::i64::val
v_2	: an element of aerobus::i64::val

8.19.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`

8.20.1 Member Typedef Documentation

8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = accum`

8.21.1 Member Typedef Documentation

8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.22 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.22.2 Member Data Documentation

8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [horner_reduction_t](#)
Used to evaluate polynomials over a value in Ring.
- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- using `zero` = `val`< typename Ring::zero >
constant zero
- using `one` = `val`< typename Ring::one >
constant one
- using `X` = `val`< typename Ring::one, typename Ring::zero >
generator
- template<typename P >
using `simplify_t` = typename simplify< P >::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- template<typename v1, typename v2 >
using `add_t` = typename add< v1, v2 >::type
adds two polynomials
- template<typename v1, typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
subtraction of two polynomials
- template<typename v1, typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
multiplication of two polynomials
- template<typename v1, typename v2 >
using `eq_t` = typename eq_helper< v1, v2 >::type
equality operator
- template<typename v1, typename v2 >
using `lt_t` = typename lt_helper< v1, v2 >::type
strict less operator
- template<typename v1, typename v2 >
using `gt_t` = typename gt_helper< v1, v2 >::type
strict greater operator
- template<typename v1, typename v2 >
using `div_t` = typename div< v1, v2 >::q_type
division operator
- template<typename v1, typename v2 >
using `mod_t` = typename div_helper< v1, v2, `zero`, v1 >::mod_type
modulo operator
- template<typename coeff, size_t deg>
using `monomial_t` = typename monomial< coeff, deg >::type
monomial : coeff X^deg
- template<typename v >
using `derive_t` = typename derive_helper< v >::type
derivation operator
- template<typename v >
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
checks for positivity (an > 0)
- template<typename v1, typename v2 >
using `gcd_t` = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >
greatest common divisor of two polynomials
- template<auto x>
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >
makes the constant (native type) polynomial a_0
- template<typename v >
using `inject_ring_t` = `val`< v >
makes the constant (ring type) polynomial a_0

Static Public Attributes

- static constexpr bool [is_field](#) = false
- static constexpr bool [is_euclidean_domain](#) = Ring::is_euclidean_domain
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity operator

8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

Examples

[examples/compensated_horner.cpp](#), [examples/make_polynomial.cpp](#), and [examples/modular_arithmetic.cpp](#).

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

8.23.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

Template Parameters

<i>x</i>	
----------	--

8.23.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

Template Parameters

<i>v</i>	
----------	--

8.23.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.10 mod_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.23.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.23.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($an > 0$)

Template Parameters

<i>v</i>	
----------	--

8.23.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.23.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.23.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.23.3 Member Data Documentation

8.23.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.23.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.23.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in <code>polynomial::val</code>
----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.24 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using `type` = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using `tail` = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.24.2 Member Typedef Documentation

8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< typename Ring::zero >
zero value
- using [one](#) = [val](#)< typename Ring::one >
one
- template<typename v1 , typename v2 >
using [add_t](#) = [val](#)< typename Ring::template [add_t](#)< typename v1::type, typename v2::type > >
addition operator
- template<typename v1 , typename v2 >
using [mul_t](#) = [val](#)< typename Ring::template [mul_t](#)< typename v1::type, typename v2::type > >
subtraction operator
- template<typename v1 , typename v2 >
using [div_t](#) = [val](#)< typename Ring::template [div_t](#)< typename v1::type, typename v2::type > >
division operator
- template<typename v1 , typename v2 >
using [mod_t](#) = [val](#)< typename Ring::template [mod_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
 • `template<typename v1 , typename v2 >`
 `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
 equality operator (as type)
- `template<typename v1 >`
 `using pos_t = std::true_type`
 positivity operator always true
- `template<auto x>`
 `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
 *inject a 'constant' in quotient ring**
- `template<typename v >`
 `using inject_ring_t = val< v >`
 projects a value of Ring onto the quotient

Static Public Attributes

- `template<typename v1 , typename v2 >`
 `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
 addition operator (as boolean value)
- `template<typename v >`
 `static constexpr bool pos_v = pos_t<v>::value`
 positivity operator always true
- `static constexpr bool is_euclidean_domain = true`
 quotien rings are euclidean domain

8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.25.2 Member Typedef Documentation

8.25.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring*

Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

8.25.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

8.25.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.25.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.25.3 Member Data Documentation

8.25.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.25.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.26.2 Member Typedef Documentation

8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```


The documentation for this struct was generated from the following file:

- src/aerobus.h

8.27 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

8.27.2 Member Typedef Documentation

8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.27.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.27.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.3 Member Data Documentation

8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.28 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T >
- template<typename T >
using [push_back](#) = [type_list](#)< T >
- template<typename U >
using [concat](#) = U
- template<typename T , size_t index>
using [insert](#) = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t [length](#) = 0

8.28.1 Detailed Description

specialization for empty type list

8.28.2 Member Typedef Documentation

8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

8.28.2.3 push_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

8.28.2.4 push_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

8.28.3 Member Data Documentation

8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [i32](#)
Enclosing ring type.
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [DEVICE](#) valueType [get](#) ()
cast x into valueType
- static std::string [to_string](#) ()
string representation of value

Static Public Attributes

- static constexpr int32_t [v](#) = x
actual value stored in val type

8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

8.29.2 Member Typedef Documentation

8.29.2.1 enclosing_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.29.2.2 is_zero_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.29.3 Member Function Documentation

8.29.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.29.3.2 to_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.29.4 Member Data Documentation

8.29.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using [inner_type](#) = int32_t
type of represented values
- using [enclosing_type](#) = [i64](#)
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED_DEVICE](#) valueType [get](#) ()
cast value in valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int64_t [v](#) = x
actual value

8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

x	an actual integer
-------------------	-------------------

Examples

[examples/compensated_horner.cpp](#).

8.30.2 Member Typedef Documentation

8.30.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.30.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.30.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.30.3 Member Function Documentation

8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.30.3.2 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.30.4 Member Data Documentation

8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- using `ring_type` = Ring
ring coefficients live in
- using `enclosing_type` = `polynomial< Ring >`
enclosing ring type
- using `aN` = `coeffN`
heavy weight coefficient (non zero)
- using `strip` = `val< coeffs... >`
remove largest coefficient
- using `is_zero_t` = `std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- template<size_t index>
using `coeff_at_t` = `typename coeff_at< index >::type`
type of coefficient at index
- template<typename x >
using `value_at_t` = `horner_reduction_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >`

Static Public Member Functions

- static `std::string to_string ()`
get a string representation of polynomial
- template<typename arithmeticType >
static constexpr `DEVICE INLINED` arithmeticType `eval` (const arithmeticType &x)
evaluates polynomial seen as a function operating on arithmeticType
- template<typename arithmeticType >
static `DEVICE INLINED` arithmeticType `compensated_eval` (const arithmeticType &x)
Evaluate polynomial on x using compensated horner scheme This is twice as accurate as simple eval (horner) but cannot be constexpr Please not this makes no sense on integer types as arithmetic on integers is exact in IEEE.

Static Public Attributes

- static constexpr size_t [degree](#) = sizeof...(coeffs)
degree of the polynomial
- static constexpr bool [is_zero_v](#) = is_zero_t::value
true if polynomial is constant zero

8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

Examples

[examples/compensated_horner.cpp](#).

8.31.2 Member Typedef Documentation

8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.31.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.31.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.31.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true_type if polynomial is constant zero

8.31.2.5 ring_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

8.31.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

8.31.3 Member Function Documentation

8.31.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme This is twice as accurate as simple eval (horner) but cannot be constexpr Please not this makes no sense on integer types as arithmetic on integers is exact in IEEE.

Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

Parameters

x	
---	--

8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

Parameters

x	value
---	-------

Returns

$P(x)$

8.31.3.3 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.31.4 Member Data Documentation

8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.31.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- using [raw_t](#) = V
- using [type](#) = [abs_t](#)< typename Ring::template [mod_t](#)< V, X > >

8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

8.32.2 Member Typedef Documentation

8.32.2.1 raw_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [zpz](#)< p >
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< [v](#)==0 >
true_type if zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED DEVICE](#) valueType [get](#) ()
get value as valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int32_t [v](#) = x % p
actual value
- static constexpr bool [is_zero_v](#) = [v](#) == 0
true if zero

8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

x	an integer
---	------------

8.33.2 Member Typedef Documentation

8.33.2.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.33.2.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true_type if zero

8.33.3 Member Function Documentation

8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

8.33.3.2 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

Returns

a string representation

8.33.4 Member Data Documentation**8.33.4.1 is_zero_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

8.33.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial< Ring >](#)
enclosing ring type
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = std::bool_constant< [aN::is_zero_t::value](#) >
- template<size_t index>
using [coeff_at_t](#) = typename [coeff_at< index >::type](#)
- template<typename x >
using [value_at_t](#) = [coeffN](#)

Static Public Member Functions

- static std::string [to_string](#) ()
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)

Static Public Attributes

- static constexpr size_t [degree](#) = 0
degree
- static constexpr bool [is_zero_v](#) = is_zero_t::value

8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.34.2 Member Typedef Documentation

8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.34.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

8.34.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.34.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

8.34.2.5 ring_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.34.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

8.34.3 Member Function Documentation

8.34.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

8.34.3.3 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.34.4 Member Data Documentation

8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.34.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in zpz

Public Types

- using `inner_type` = `int32_t`
underlying type for values
- template<auto x>
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>
injects a constant integer into mpz
- using `zero` = `val`< 0 >
zero value
- using `one` = `val`< 1 >
one value
- template<typename v1 , typename v2 >
using `add_t` = `typename add`< v1, v2 >::type
addition operator
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub`< v1, v2 >::type
subtraction operator
- template<typename v1 , typename v2 >
using `mul_t` = `typename mul`< v1, v2 >::type
multiplication operator
- template<typename v1 , typename v2 >
using `div_t` = `typename div`< v1, v2 >::type
division operator
- template<typename v1 , typename v2 >
using `mod_t` = `typename remainder`< v1, v2 >::type
modulo operator
- template<typename v1 , typename v2 >
using `gt_t` = `typename gt`< v1, v2 >::type
strictly greater operator (type)
- template<typename v1 , typename v2 >
using `lt_t` = `typename lt`< v1, v2 >::type
strictly smaller operator (type)
- template<typename v1 , typename v2 >
using `eq_t` = `typename eq`< v1, v2 >::type
equality operator (type)
- template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >
greatest common divisor
- template<typename v1 >
using `pos_t` = `typename pos`< v1 >::type
positivity operator (type)

Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value
true iff p is prime
- static constexpr bool `is_euclidean_domain` = true
always true
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
strictly greater operator (booleanvalue)

- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

8.35.1 Detailed Description

`template<int32_t p>`
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular_arithmetic.cpp](#), and [examples/polynomials_over_finite_field.cpp](#).

8.35.2 Member Typedef Documentation

8.35.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.2 div_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

Template Parameters

x	an integer
---	------------

8.35.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

8.35.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

8.35.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

8.35.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

8.35.3 Member Data Documentation

8.35.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

8.35.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

8.35.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 0) & 0x800000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED_DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED_DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED_DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<float> {
00224         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00225             return x * y + z;
00226         }
00227     };
00228
00229     template<>
00230     struct fma_helper<int32_t> {
00231         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00232             return x * y + z;
00233         }
00234     };
00235
00236     template<>
00237     struct fma_helper<int16_t> {
00238         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00239             return x * y + z;
00240         }
00241     };
00242
00243     template<>
00244     struct fma_helper<int64_t> {
00245         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00246             return x * y + z;
00247         }
00248     };
00249
00250     #ifdef WITH_CUDA_FP16
00251     template<>
00252     struct fma_helper<__half> {
00253         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00254             #ifdef __CUDA_ARCH__
00255                 return __hfma(x, y, z);
00256             #else
00257                 return x * y + z;
00258             #endif
00259         }
00260     };
00261     template<>
00262     struct fma_helper<__half2> {
00263         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00264             #ifdef __CUDA_ARCH__
00265                 return __hfma2(x, y, z);
00266             #else
00267                 return x * y + z;
00268             #endif
00269         }
00270     };
00271     #endif
00272 } // namespace internal
00273 } // namespace aerobus
00274
00275 // compensated horner utilities
00276 namespace aerobus {
00277     namespace internal {
00278         template <typename T>
00279         struct FloatLayout;
00280
00281         template <>
00282         struct FloatLayout<double> {
00283             static constexpr uint8_t exponent = 11;
00284             static constexpr uint8_t mantissa = 53;
00285             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00286         };
00287
00288         template <>
00289         struct FloatLayout<float> {
00290             static constexpr uint8_t exponent = 8;
00291             static constexpr uint8_t mantissa = 24;
00292             static constexpr uint8_t r = 11; // ceil(mantissa/2)
00293         };
00294
00295         #ifdef WITH_CUDA_FP16

```

```

00296     template <>
00297     struct FloatLayout<__half> {
00298         static constexpr uint8_t exponent = 5;
00299         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00300         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00301     };
00302 #endif
00303
00304     template<typename T>
00305     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00306         T z = a * ((1 < FloatLayout<T>::r) + 1);
00307         *x = z - (z - a);
00308         *y = a - *x;
00309     }
00310
00311     template<typename T>
00312     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00313         *x = a + b;
00314         T z = *x - a;
00315         *y = (a - (*x - z)) + (b - z);
00316     }
00317
00318     template<typename T>
00319     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00320         *x = a * b;
00321         T ah, al, bh, bl;
00322         split(a, &ah, &al);
00323         split(b, &bh, &bl);
00324         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00325     }
00326
00327
00328     template<typename T, size_t N>
00329     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00330         T r = p1[0] + p2[0];
00331         for (int64_t i = N - 1; i >= 0; --i) {
00332             r = r * x + p1[N - i] + p2[N - i];
00333         }
00334
00335         return r;
00336     }
00337 } // namespace internal
00338 } // namespace aerobus
00339
00340 // utilities
00341 namespace aerobus {
00342     namespace internal {
00343         template<template<typename...> typename TT, typename T>
00344         struct is_instantiation_of : std::false_type { };
00345
00346         template<template<typename...> typename TT, typename... Ts>
00347         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00348
00349         template<template<typename...> typename TT, typename T>
00350         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00351
00352         template <int64_t i, typename T, typename... Ts>
00353         struct type_at {
00354             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00355             using type = typename type_at<i - 1, Ts...>::type;
00356         };
00357
00358         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00359             using type = T;
00360         };
00361
00362         template <size_t i, typename... Ts>
00363         using type_at_t = typename type_at<i, Ts...>::type;
00364
00365
00366         template<size_t n, size_t i, typename E = void>
00367         struct _is_prime {};
00368
00369         template<size_t i>
00370         struct _is_prime<0, i> {
00371             static constexpr bool value = false;
00372         };
00373
00374         template<size_t i>
00375         struct _is_prime<1, i> {
00376             static constexpr bool value = false;
00377         };
00378
00379         template<size_t i>
00380         struct _is_prime<2, i> {
00381             static constexpr bool value = true;
00382         };

```

```

00383
00384     template<size_t i>
00385     struct _is_prime<3, i> {
00386         static constexpr bool value = true;
00387     };
00388
00389     template<size_t i>
00390     struct _is_prime<5, i> {
00391         static constexpr bool value = true;
00392     };
00393
00394     template<size_t i>
00395     struct _is_prime<7, i> {
00396         static constexpr bool value = true;
00397     };
00398
00399     template<size_t n, size_t i>
00400     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00401         static constexpr bool value = false;
00402     };
00403
00404     template<size_t n, size_t i>
00405     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00406         static constexpr bool value = false;
00407     };
00408
00409     template<size_t n, size_t i>
00410     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00411         static constexpr bool value = true;
00412     };
00413
00414     template<size_t n, size_t i>
00415     struct _is_prime<n, i, std::enable_if_t<(
00416         n % i == 0 &&
00417         n >= 9 &&
00418         n % 3 != 0 &&
00419         n % 2 != 0 &&
00420         i * i > n)>> {
00421         static constexpr bool value = true;
00422     };
00423
00424     template<size_t n, size_t i>
00425     struct _is_prime<n, i, std::enable_if_t<(
00426         n % (i+2) == 0 &&
00427         n >= 9 &&
00428         n % 3 != 0 &&
00429         n % 2 != 0 &&
00430         i * i <= n)>> {
00431         static constexpr bool value = true;
00432     };
00433
00434     template<size_t n, size_t i>
00435     struct _is_prime<n, i, std::enable_if_t<(
00436         n % (i+2) != 0 &&
00437         n % i != 0 &&
00438         n >= 9 &&
00439         n % 3 != 0 &&
00440         n % 2 != 0 &&
00441         (i * i <= n))>> {
00442         static constexpr bool value = _is_prime<n, i+6>::value;
00443     };
00444 } // namespace internal
00445
00446 template<size_t n>
00447 struct is_prime {
00448     static constexpr bool value = internal::_is_prime<n, 5>::value;
00449 };
00450
00451 template<size_t n>
00452 static constexpr bool is_prime_v = is_prime<n>::value;
00453
00454 // gcd
00455 namespace internal {
00456     template <std::size_t... Is>
00457     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00458     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00459
00460     template <std::size_t N>
00461     using make_index_sequence_reverse
00462     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00463
00464     template<typename Ring, typename E = void>
00465     struct gcd;
00466
00467     template<typename Ring>
00468     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00469         template<typename A, typename B, typename E = void>

```



```

00481     struct gcd_helper {};
00482
00483     // B = 0, A > 0
00484     template<typename A, typename B>
00485     struct gcd_helper<A, B, std::enable_if_t<
00486         ((B::is_zero_t::value) &&
00487         (Ring::template gt_t<A, typename Ring::zero>::value))> {
00488         using type = A;
00489     };
00490
00491     // B = 0, A < 0
00492     template<typename A, typename B>
00493     struct gcd_helper<A, B, std::enable_if_t<
00494         ((B::is_zero_t::value) &&
00495         !(Ring::template gt_t<A, typename Ring::zero>::value))> {
00496         using type = typename Ring::template sub_t<typename Ring::zero, A>;
00497     };
00498
00499     // B != 0
00500     template<typename A, typename B>
00501     struct gcd_helper<A, B, std::enable_if_t<
00502         (!B::is_zero_t::value)
00503         >> {
00504     private: // NOLINT
00505         // A / B
00506         using k = typename Ring::template div_t<A, B>;
00507         // A - (A/B)*B = A % B
00508         using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00509
00510     public:
00511         using type = typename gcd_helper<B, m>::type;
00512     };
00513
00514     template<typename A, typename B>
00515     using type = typename gcd_helper<A, B>::type;
00516 };
00517 } // namespace internal
00518
00519 // vadd and vmul
00520 namespace internal {
00521     template<typename... vals>
00522     struct vmul {};
00523
00524     template<typename v1, typename... vals>
00525     struct vmul<v1, vals...> {
00526         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00527     };
00528
00529     template<typename v1>
00530     struct vmul<v1> {
00531         using type = v1;
00532     };
00533
00534     template<typename... vals>
00535     struct vadd {};
00536
00537     template<typename v1, typename... vals>
00538     struct vadd<v1, vals...> {
00539         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00540     };
00541
00542     template<typename v1>
00543     struct vadd<v1> {
00544         using type = v1;
00545     };
00546 } // namespace internal
00547
00550     template<typename T, typename A, typename B>
00551     using gcd_t = typename internal::gcd<T>::template type<A, B>;
00552
00556     template<typename... vals>
00557     using vadd_t = typename internal::vadd<vals...>::type;
00558
00562     template<typename... vals>
00563     using vmul_t = typename internal::vmul<vals...>::type;
00564
00568     template<typename val>
00569     requires IsEuclideanDomain<typename val::enclosing_type>
00570     using abs_t = std::conditional_t<
00571         val::enclosing_type::template pos_v<val>,
00572         val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00573 } // namespace aerobus
00574
00575 // embedding

```

```

00576 namespace aerobus {
00581     template<typename Small, typename Large, typename E = void>
00582     struct Embed;
00583 } // namespace aerobus
00584
00585 namespace aerobus {
00590     template<typename Ring, typename X>
00591     requires IsRing<Ring>
00592     struct Quotient {
00595         template <typename V>
00596         struct val {
00597             public:
00598                 using raw_t = V;
00599                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00600         };
00601
00603         using zero = val<typename Ring::zero>;
00604
00606         using one = val<typename Ring::one>;
00607
00611         template<typename v1, typename v2>
00612         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00613
00617         template<typename v1, typename v2>
00618         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00619
00623         template<typename v1, typename v2>
00624         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00625
00629         template<typename v1, typename v2>
00630         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00631
00635         template<typename v1, typename v2>
00636         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00637
00641         template<typename v1, typename v2>
00642         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00643
00647         template<typename v1>
00648         using pos_t = std::true_type;
00649
00653         template<typename v>
00654         static constexpr bool pos_v = pos_t<v>::value;
00655
00657         static constexpr bool is_euclidean_domain = true;
00658
00662         template<auto x>
00663         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00664
00668         template<typename v>
00669         using inject_ring_t = val<v>;
00670     };
00671
00675     template<typename Ring, typename X>
00676     struct Embed<Quotient<Ring, X>, Ring> {
00679         template<typename val>
00680         using type = typename val::raw_t;
00681     };
00682 } // namespace aerobus
00683
00684 // type_list
00685 namespace aerobus {
00687     template <typename... Ts>
00688     struct type_list;
00689
00690     namespace internal {
00691         template <typename T, typename... Us>
00692         struct pop_front_h {
00693             using tail = type_list<Us...>;
00694             using head = T;
00695         };
00696
00697         template <size_t index, typename L1, typename L2>
00698         struct split_h {
00699             private:
00700                 static_assert(index <= L2::length, "index out of bounds");
00701                 using a = typename L2::pop_front::type;
00702                 using b = typename L2::pop_front::tail;
00703                 using c = typename L1::template push_back<a>;
00704
00705             public:
00706                 using head = typename split_h<index - 1, c, b>::head;
00707                 using tail = typename split_h<index - 1, c, b>::tail;
00708         };
00709
00710         template <typename L1, typename L2>
00711         struct split_h<0, L1, L2> {

```

```

00712         using head = L1;
00713         using tail = L2;
00714     };
00715
00716     template <size_t index, typename L, typename T>
00717     struct insert_h {
00718         static_assert(index <= L::length, "index out of bounds");
00719         using s = typename L::template split<index>;
00720         using left = typename s::head;
00721         using right = typename s::tail;
00722         using ll = typename left::template push_back<T>;
00723         using type = typename ll::template concat<right>;
00724     };
00725
00726     template <size_t index, typename L>
00727     struct remove_h {
00728         using s = typename L::template split<index>;
00729         using left = typename s::head;
00730         using right = typename s::tail;
00731         using rr = typename right::pop_front::tail;
00732         using type = typename left::template concat<rr>;
00733     };
00734 } // namespace internal
00735
00736 template <typename... Ts>
00737 struct type_list {
00738 private:
00739     template <typename T>
00740     struct concat_h;
00741
00742     template <typename... Us>
00743     struct concat_h<type_list<Us...> {
00744         using type = type_list<Ts..., Us...>;
00745     };
00746
00747 public:
00748     static constexpr size_t length = sizeof...(Ts);
00749
00750     template <typename T>
00751     using push_front = type_list<T, Ts...>;
00752
00753     template <size_t index>
00754     using at = internal::type_at_t<index, Ts...>;
00755
00756     struct pop_front {
00757         using type = typename internal::pop_front_h<Ts...>::head;
00758         using tail = typename internal::pop_front_h<Ts...>::tail;
00759     };
00760
00761     template <typename T>
00762     using push_back = type_list<Ts..., T>;
00763
00764     template <typename U>
00765     using concat = typename concat_h<U>::type;
00766
00767     template <size_t index>
00768     struct split {
00769     private:
00770         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00771
00772     public:
00773         using head = typename inner::head;
00774         using tail = typename inner::tail;
00775     };
00776
00777     template <typename T, size_t index>
00778     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00779
00780     template <size_t index>
00781     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00782 };
00783
00784 template <>
00785 struct type_list<> {
00786     static constexpr size_t length = 0;
00787
00788     template <typename T>
00789     using push_front = type_list<T>;
00790
00791     template <typename T>
00792     using push_back = type_list<T>;
00793
00794     template <typename U>
00795     using concat = U;
00796
00797     // TODO(jewave): assert index == 0
00798     template <typename T, size_t index>

```

```

00821         using insert = type_list<T>;
00822     };
00823 } // namespace aerobus
00824
00825 // i16
00826 #ifndef WITH_CUDA_FP16
00827 // i16
00828 namespace aerobus {
00829     struct i16 {
00830         using inner_type = int16_t;
00831         template<int16_t x>
00832         struct val {
00833             using enclosing_type = i16;
00834             static constexpr int16_t v = x;
00835
00836             template<typename valueType>
00837             static constexpr INLINED_DEVICE valueType get() {
00838                 return internal::template int16_convert_helper<valueType, x>::value();
00839             }
00840
00841             using is_zero_t = std::bool_constant<x == 0>;
00842
00843             static std::string to_string() {
00844                 return std::to_string(x);
00845             }
00846         };
00847     };
00848
00849     using zero = val<0>;
00850     using one = val<1>;
00851     static constexpr bool is_field = false;
00852     static constexpr bool is_euclidean_domain = true;
00853     template<auto x>
00854     using inject_constant_t = val<static_cast<int16_t>(x)>;
00855
00856     template<typename v>
00857     using inject_ring_t = v;
00858
00859 private:
00860     template<typename v1, typename v2>
00861     struct add {
00862         using type = val<v1::v + v2::v>;
00863     };
00864
00865     template<typename v1, typename v2>
00866     struct sub {
00867         using type = val<v1::v - v2::v>;
00868     };
00869
00870     template<typename v1, typename v2>
00871     struct mul {
00872         using type = val<v1::v * v2::v>;
00873     };
00874
00875     template<typename v1, typename v2>
00876     struct div {
00877         using type = val<v1::v / v2::v>;
00878     };
00879
00880     template<typename v1, typename v2>
00881     struct remainder {
00882         using type = val<v1::v % v2::v>;
00883     };
00884
00885     template<typename v1, typename v2>
00886     struct gt {
00887         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00888     };
00889
00890     template<typename v1, typename v2>
00891     struct lt {
00892         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00893     };
00894
00895     template<typename v1, typename v2>
00896     struct eq {
00897         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00898     };
00899
00900     template<typename v1>
00901     struct pos {
00902         using type = std::bool_constant<(v1::v > 0)>;
00903     };
00904
00905 public:
00906     template<typename v1, typename v2>
00907     using add_t = typename add<v1, v2>::type;
00908
00909     template<typename v1, typename v2>
00910     using sub_t = typename sub<v1, v2>::type;
00911
00912     template<typename v1, typename v2>
00913     using mul_t = typename mul<v1, v2>::type;
00914
00915     template<typename v1, typename v2>
00916     using div_t = typename div<v1, v2>::type;
00917
00918     template<typename v1, typename v2>
00919     using remainder_t = typename remainder<v1, v2>::type;
00920
00921     template<typename v1, typename v2>
00922     using gt_t = typename gt<v1, v2>::type;
00923
00924     template<typename v1, typename v2>
00925     using lt_t = typename lt<v1, v2>::type;
00926
00927     template<typename v1>
00928     using pos_t = typename pos<v1>::type;

```

```

00931     template<typename v1, typename v2>
00932     using sub_t = typename sub<v1, v2>::type;
00933
00938     template<typename v1, typename v2>
00939     using mul_t = typename mul<v1, v2>::type;
00940
00945     template<typename v1, typename v2>
00946     using div_t = typename div<v1, v2>::type;
00947
00952     template<typename v1, typename v2>
00953     using mod_t = typename remainder<v1, v2>::type;
00954
00959     template<typename v1, typename v2>
00960     using gt_t = typename gt<v1, v2>::type;
00961
00966     template<typename v1, typename v2>
00967     using lt_t = typename lt<v1, v2>::type;
00968
00973     template<typename v1, typename v2>
00974     using eq_t = typename eq<v1, v2>::type;
00975
00979     template<typename v1, typename v2>
00980     static constexpr bool eq_v = eq_t<v1, v2>::value;
00981
00986     template<typename v1, typename v2>
00987     using gcd_t = gcd_t<i16, v1, v2>;
00988
00992     template<typename v>
00993     using pos_t = typename pos<v>::type;
00994
00998     template<typename v>
00999     static constexpr bool pos_v = pos_t<v>::value;
01000 };
01001 } // namespace aerobus
01002 #endif
01003
01004 // i32
01005 namespace aerobus {
01006     struct i32 {
01007         using inner_type = int32_t;
01008         template<int32_t x>
01009         struct val {
01010             using enclosing_type = i32;
01011             static constexpr int32_t v = x;
01012
01013             template<typename valueType>
01014             static constexpr DEVICE valueType get() {
01015                 return static_cast<valueType>(x);
01016             }
01017
01018             using is_zero_t = std::bool_constant<x == 0>;
01019
01020             static std::string to_string() {
01021                 return std::to_string(x);
01022             }
01023         };
01024
01025         using zero = val<0>;
01026         using one = val<1>;
01027         static constexpr bool is_field = false;
01028         static constexpr bool is_euclidean_domain = true;
01029         template<auto x>
01030         using inject_constant_t = val<static_cast<int32_t>(x)>;
01031
01032         template<typename v>
01033         using inject_ring_t = v;
01034
01035     private:
01036         template<typename v1, typename v2>
01037         struct add {
01038             using type = val<v1::v + v2::v>;
01039         };
01040
01041         template<typename v1, typename v2>
01042         struct sub {
01043             using type = val<v1::v - v2::v>;
01044         };
01045
01046         template<typename v1, typename v2>
01047         struct mul {
01048             using type = val<v1::v * v2::v>;
01049         };
01050
01051         template<typename v1, typename v2>
01052         struct div {
01053             using type = val<v1::v / v2::v>;
01054         };
01055     };

```

```

01070
01071     template<typename v1, typename v2>
01072     struct remainder {
01073         using type = val<v1::v % v2::v>;
01074     };
01075
01076     template<typename v1, typename v2>
01077     struct gt {
01078         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01079     };
01080
01081     template<typename v1, typename v2>
01082     struct lt {
01083         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01084     };
01085
01086     template<typename v1, typename v2>
01087     struct eq {
01088         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01089     };
01090
01091     template<typename v1>
01092     struct pos {
01093         using type = std::bool_constant<(v1::v > 0)>;
01094     };
01095
01096     public:
01101     template<typename v1, typename v2>
01102     using add_t = typename add<v1, v2>::type;
01103
01108     template<typename v1, typename v2>
01109     using sub_t = typename sub<v1, v2>::type;
01110
01115     template<typename v1, typename v2>
01116     using mul_t = typename mul<v1, v2>::type;
01117
01122     template<typename v1, typename v2>
01123     using div_t = typename div<v1, v2>::type;
01124
01129     template<typename v1, typename v2>
01130     using mod_t = typename remainder<v1, v2>::type;
01131
01136     template<typename v1, typename v2>
01137     using gt_t = typename gt<v1, v2>::type;
01138
01143     template<typename v1, typename v2>
01144     using lt_t = typename lt<v1, v2>::type;
01145
01150     template<typename v1, typename v2>
01151     using eq_t = typename eq<v1, v2>::type;
01152
01156     template<typename v1, typename v2>
01157     static constexpr bool eq_v = eq_t<v1, v2>::value;
01158
01163     template<typename v1, typename v2>
01164     using gcd_t = gcd_t<i32, v1, v2>;
01165
01169     template<typename v>
01170     using pos_t = typename pos<v>::type;
01171
01175     template<typename v>
01176     static constexpr bool pos_v = pos_t<v>::value;
01177 };
01178 } // namespace aerobus
01179
01180 // i64
01181 namespace aerobus {
01182     struct i64 {
01185         using inner_type = int64_t;
01186         template<int64_t x>
01187         struct val {
01191             using inner_type = int32_t;
01193             using enclosing_type = i64;
01195             static constexpr int64_t v = x;
01196
01199             template<typename valueType>
01200             static constexpr INLINED_DEVICE valueType get() {
01201                 return static_cast<valueType>(x);
01202             }
01203
01205             using is_zero_t = std::bool_constant<x == 0>;
01206
01208             static std::string to_string() {
01209                 return std::to_string(x);
01210             }
01211         };
01212     };

```

```

01215     template<auto x>
01216     using inject_constant_t = val<static_cast<int64_t>(x)>;
01217
01222     template<typename v>
01223     using inject_ring_t = v;
01224
01226     using zero = val<0>;
01228     using one = val<1>;
01230     static constexpr bool is_field = false;
01232     static constexpr bool is_euclidean_domain = true;
01233
01234 private:
01235     template<typename v1, typename v2>
01236     struct add {
01237         using type = val<v1::v + v2::v>;
01238     };
01239
01240     template<typename v1, typename v2>
01241     struct sub {
01242         using type = val<v1::v - v2::v>;
01243     };
01244
01245     template<typename v1, typename v2>
01246     struct mul {
01247         using type = val<v1::v * v2::v>;
01248     };
01249
01250     template<typename v1, typename v2>
01251     struct div {
01252         using type = val<v1::v / v2::v>;
01253     };
01254
01255     template<typename v1, typename v2>
01256     struct remainder {
01257         using type = val<v1::v % v2::v>;
01258     };
01259
01260     template<typename v1, typename v2>
01261     struct gt {
01262         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01263     };
01264
01265     template<typename v1, typename v2>
01266     struct lt {
01267         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01268     };
01269
01270     template<typename v1, typename v2>
01271     struct eq {
01272         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01273     };
01274
01275     template<typename v>
01276     struct pos {
01277         using type = std::bool_constant<(v::v > 0)>;
01278     };
01279
01280 public:
01281     template<typename v1, typename v2>
01282     using add_t = typename add<v1, v2>::type;
01283
01284     template<typename v1, typename v2>
01285     using sub_t = typename sub<v1, v2>::type;
01286
01287     template<typename v1, typename v2>
01288     using mul_t = typename mul<v1, v2>::type;
01289
01290     template<typename v1, typename v2>
01291     using div_t = typename div<v1, v2>::type;
01292
01293     template<typename v1, typename v2>
01294     using mod_t = typename remainder<v1, v2>::type;
01295
01296     template<typename v1, typename v2>
01297     using gt_t = typename gt<v1, v2>::type;
01298
01299     template<typename v1, typename v2>
01300     static constexpr bool gt_v = gt_t<v1, v2>::value;
01301
01302     template<typename v1, typename v2>
01303     using lt_t = typename lt<v1, v2>::type;
01304
01305     template<typename v1, typename v2>
01306     static constexpr bool lt_v = lt_t<v1, v2>::value;
01307
01308     template<typename v1, typename v2>
01309     using eq_t = typename eq<v1, v2>::type;
01310
01311     template<typename v1, typename v2>
01312     static constexpr bool eq_v = eq_t<v1, v2>::value;
01313
01314     template<typename v1, typename v2>
01315     using pos_t = typename pos<v1, v2>::type;
01316
01317     template<typename v1, typename v2>
01318     static constexpr bool pos_v = pos_t<v1, v2>::value;
01319
01320     template<typename v1, typename v2>
01321     using is_field_t = bool;
01322
01323     template<typename v1, typename v2>
01324     static constexpr bool is_field_v = is_field;
01325
01326     template<typename v1, typename v2>
01327     using is_euclidean_domain_t = bool;
01328
01329     template<typename v1, typename v2>
01330     static constexpr bool is_euclidean_domain_v = is_euclidean_domain;
01331
01332     template<typename v1, typename v2>
01333     using inject_constant_t = val<static_cast<int64_t>(x)>;
01334
01335     template<typename v>
01336     using inject_ring_t = v;
01337
01338     using zero = val<0>;
01340     using one = val<1>;
01342     static constexpr bool is_field = false;
01344     static constexpr bool is_euclidean_domain = true;

```

```

01346
01351     template<typename v1, typename v2>
01352     static constexpr bool eq_v = eq_t<v1, v2>::value;
01353
01358     template<typename v1, typename v2>
01359     using gcd_t = gcd_t<i64, v1, v2>;
01360
01364     template<typename v>
01365     using pos_t = typename pos<v>::type;
01366
01370     template<typename v>
01371     static constexpr bool pos_v = pos_t<v>::value;
01372 };
01373
01375 template<>
01376 struct Embed<i32, i64> {
01377     template<typename val>
01380     using type = i64::val<static_cast<int64_t>(val::v)>;
01381 };
01382 } // namespace aerobus
01383
01384 // z/pz
01385 namespace aerobus {
01391     template<int32_t p>
01392     struct zpz {
01394         using inner_type = int32_t;
01395
01398         template<int32_t x>
01399         struct val {
01401             using enclosing_type = zpz<p>;
01403             static constexpr int32_t v = x % p;
01404
01407             template<typename valueType>
01408             static constexpr INLINED_DEVICE valueType get() {
01409                 return static_cast<valueType>(x % p);
01410             }
01411
01413             using is_zero_t = std::bool_constant<v == 0>;
01414
01416             static constexpr bool is_zero_v = v == 0;
01417
01420             static std::string to_string() {
01421                 return std::to_string(x % p);
01422             }
01423         };
01424
01427         template<auto x>
01428         using inject_constant_t = val<static_cast<int32_t>(x)>;
01429
01431         using zero = val<0>;
01432
01434         using one = val<1>;
01435
01437         static constexpr bool is_field = is_prime<p>::value;
01438
01440         static constexpr bool is_euclidean_domain = true;
01441
01442     private:
01443         template<typename v1, typename v2>
01444         struct add {
01445             using type = val<(v1::v + v2::v) % p>;
01446         };
01447
01448         template<typename v1, typename v2>
01449         struct sub {
01450             using type = val<(v1::v - v2::v) % p>;
01451         };
01452
01453         template<typename v1, typename v2>
01454         struct mul {
01455             using type = val<(v1::v * v2::v) % p>;
01456         };
01457
01458         template<typename v1, typename v2>
01459         struct div {
01460             using type = val<(v1::v % p) / (v2::v % p)>;
01461         };
01462
01463         template<typename v1, typename v2>
01464         struct remainder {
01465             using type = val<(v1::v % v2::v) % p>;
01466         };
01467
01468         template<typename v1, typename v2>
01469         struct gt {
01470             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01471         };

```



```

01472
01473     template<typename v1, typename v2>
01474     struct lt {
01475         using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01476     };
01477
01478     template<typename v1, typename v2>
01479     struct eq {
01480         using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01481     };
01482
01483     template<typename v1>
01484     struct pos {
01485         using type = std::bool_constant<(v1::v > 0)>;
01486     };
01487
01488     public:
01492     template<typename v1, typename v2>
01493     using add_t = typename add<v1, v2>::type;
01494
01498     template<typename v1, typename v2>
01499     using sub_t = typename sub<v1, v2>::type;
01500
01504     template<typename v1, typename v2>
01505     using mul_t = typename mul<v1, v2>::type;
01506
01510     template<typename v1, typename v2>
01511     using div_t = typename div<v1, v2>::type;
01512
01516     template<typename v1, typename v2>
01517     using mod_t = typename remainder<v1, v2>::type;
01518
01522     template<typename v1, typename v2>
01523     using gt_t = typename gt<v1, v2>::type;
01524
01528     template<typename v1, typename v2>
01529     static constexpr bool gt_v = gt_t<v1, v2>::value;
01530
01534     template<typename v1, typename v2>
01535     using lt_t = typename lt<v1, v2>::type;
01536
01540     template<typename v1, typename v2>
01541     static constexpr bool lt_v = lt_t<v1, v2>::value;
01542
01546     template<typename v1, typename v2>
01547     using eq_t = typename eq<v1, v2>::type;
01548
01552     template<typename v1, typename v2>
01553     static constexpr bool eq_v = eq_t<v1, v2>::value;
01554
01558     template<typename v1, typename v2>
01559     using gcd_t = gcd_t<i32, v1, v2>;
01560
01563     template<typename v1>
01564     using pos_t = typename pos<v1>::type;
01565
01568     template<typename v>
01569     static constexpr bool pos_v = pos_t<v>::value;
01570 };
01571
01574     template<int32_t x>
01575     struct Embed<zp<x>, i32> {
01576         template <typename val>
01577         using type = i32::val<val::v>;
01578     };
01581 } // namespace aerobus
01582
01583 // polynomial
01584 namespace aerobus {
01585     // coeffN x^N + ...
01586     template<typename Ring>
01587     requires IsEuclideanDomain<Ring>
01588     struct polynomial {
01593         static constexpr bool is_field = false;
01594         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01595
01598         template<typename P>
01599         struct horner_reduction_t {
01600             template<size_t index, size_t stop>
01601             struct inner {
01602                 template<typename accum, typename x>
01603                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01604                     ::template type<
01605                         typename Ring::template add_t<
01606                             typename Ring::template mul_t<x, accum>,
01607                             typename P::template coeff_at_t<P::degree - index>
01608                             >, x>;

```

```

01609         };
01610
01611         template<size_t stop>
01612         struct inner<stop, stop> {
01613             template<typename accum, typename x>
01614             using type = accum;
01615         };
01616     };
01617
01621     template<typename coeffN, typename... coeffs>
01622     struct val {
01623         using ring_type = Ring;
01624         using enclosing_type = polynomial<Ring>;
01625         static constexpr size_t degree = sizeof...(coeffs);
01626         using aN = coeffN;
01627         using strip = val<coeffs...>;
01628         using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01629         static constexpr bool is_zero_v = is_zero_t::value;
01630
01631     private:
01632         template<size_t index, typename E = void>
01633         struct coeff_at {};
01634
01635         template<size_t index>
01636         struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01637             using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01638         };
01639
01640         template<size_t index>
01641         struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01642             using type = typename Ring::zero;
01643         };
01644
01645     public:
01646         template<size_t index>
01647         using coeff_at_t = typename coeff_at<index>::type;
01648
01649         static std::string to_string() {
01650             return string_helper<coeffN, coeffs...>::func();
01651         }
01652
01653         template<typename arithmeticType>
01654         static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01655             #ifdef WITH_CUDA_FP16
01656             arithmeticType start;
01657             if constexpr (std::is_same_v<arithmeticType, __half2>) {
01658                 start = __half2(0, 0);
01659             } else {
01660                 start = static_cast<arithmeticType>(0);
01661             }
01662             #else
01663             arithmeticType start = static_cast<arithmeticType>(0);
01664             #endif
01665             return horner_evaluation<arithmeticType, val>
01666                 ::template inner<0, degree + 1>
01667                 ::func(start, x);
01668         }
01669
01670         template<typename arithmeticType>
01671         static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01672             return compensated_horner<arithmeticType, val>::func(x);
01673         }
01674
01675         template<typename x>
01676         using value_at_t = horner_reduction_t<val>
01677             ::template inner<0, degree + 1>
01678             ::template type<typename Ring::zero, x>;
01679     };
01680
01681     template<typename coeffN>
01682     struct val<coeffN> {
01683         using ring_type = Ring;
01684         using enclosing_type = polynomial<Ring>;
01685         static constexpr size_t degree = 0;
01686         using aN = coeffN;
01687         using strip = val<coeffN>;
01688         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01689         static constexpr bool is_zero_v = is_zero_t::value;
01690
01691         template<size_t index, typename E = void>
01692         struct coeff_at {};
01693
01694         template<size_t index>
01695         struct coeff_at<index, std::enable_if_t<(index == 0)> {
01696             using type = aN;
01697         };
01698     };

```

```

01725
01726     template<size_t index>
01727     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01728         using type = typename Ring::zero;
01729     };
01730
01731     template<size_t index>
01732     using coeff_at_t = typename coeff_at<index>::type;
01733
01734     static std::string to_string() {
01735         return string_helper<coeffN>::func();
01736     }
01737
01738     template<typename arithmeticType>
01739     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01740         return coeffN::template get<arithmeticType>();
01741     }
01742
01743     template<typename arithmeticType>
01744     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01745         return coeffN::template get<arithmeticType>();
01746     }
01747
01748     template<typename x>
01749     using value_at_t = coeffN;
01750 };
01751
01752 using zero = val<typename Ring::zero>;
01753 using one = val<typename Ring::one>;
01754 using X = val<typename Ring::one, typename Ring::zero>;
01755
01756 private:
01757     template<typename P, typename E = void>
01758     struct simplify;
01759
01760     template <typename P1, typename P2, typename I>
01761     struct add_low;
01762
01763     template<typename P1, typename P2>
01764     struct add {
01765         using type = typename simplify<typename add_low<
01766             P1,
01767             P2,
01768             internal::make_index_sequence_reverse<
01769                 std::max(P1::degree, P2::degree) + 1
01770             >::type>::type;
01771     };
01772
01773     template <typename P1, typename P2, typename I>
01774     struct sub_low;
01775
01776     template <typename P1, typename P2, typename I>
01777     struct mul_low;
01778
01779     template<typename v1, typename v2>
01780     struct mul {
01781         using type = typename mul_low<
01782             v1,
01783             v2,
01784             internal::make_index_sequence_reverse<
01785                 v1::degree + v2::degree + 1
01786             >::type;
01787     };
01788
01789     template<typename coeff, size_t deg>
01790     struct monomial;
01791
01792     template<typename v, typename E = void>
01793     struct derive_helper {};
01794
01795     template<typename v>
01796     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01797         using type = zero;
01798     };
01799
01800     template<typename v>
01801     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01802         using type = typename add<
01803             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01804             typename monomial<
01805                 typename Ring::template mul_t<
01806                     typename v::aN,
01807                     typename Ring::template inject_constant_t<(v::degree)>
01808                 >,
01809                 v::degree - 1
01810             >::type
01811         >::type;
01812     };
01813
01814

```

```

01815     };
01816
01817     template<typename v1, typename v2, typename E = void>
01818     struct eq_helper {};
01819
01820     template<typename v1, typename v2>
01821     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01822         using type = std::false_type;
01823     };
01824
01825
01826     template<typename v1, typename v2>
01827     struct eq_helper<v1, v2, std::enable_if_t<
01828         v1::degree == v2::degree &&
01829         (v1::degree != 0 || v2::degree != 0) &&
01830         std::is_same<
01831             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01832             std::false_type
01833         >::value
01834     > {
01835     > {
01836         using type = std::false_type;
01837     };
01838
01839     template<typename v1, typename v2>
01840     struct eq_helper<v1, v2, std::enable_if_t<
01841         v1::degree == v2::degree &&
01842         (v1::degree != 0 || v2::degree != 0) &&
01843         std::is_same<
01844             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01845             std::true_type
01846         >::value
01847     > {
01847     > {
01848         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01849     };
01850
01851     template<typename v1, typename v2>
01852     struct eq_helper<v1, v2, std::enable_if_t<
01853         v1::degree == v2::degree &&
01854         (v1::degree == 0)
01855     > {
01856         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01857     };
01858
01859     template<typename v1, typename v2, typename E = void>
01860     struct lt_helper {};
01861
01862     template<typename v1, typename v2>
01863     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01864         using type = std::true_type;
01865     };
01866
01867     template<typename v1, typename v2>
01868     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01869         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01870     };
01871
01872     template<typename v1, typename v2>
01873     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01874         using type = std::false_type;
01875     };
01876
01877     template<typename v1, typename v2, typename E = void>
01878     struct gt_helper {};
01879
01880     template<typename v1, typename v2>
01881     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01882         using type = std::true_type;
01883     };
01884
01885     template<typename v1, typename v2>
01886     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01887         using type = std::false_type;
01888     };
01889
01890     template<typename v1, typename v2>
01891     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01892         using type = std::false_type;
01893     };
01894
01895     // when high power is zero : strip
01896     template<typename P>
01897     struct simplify<P, std::enable_if_t<
01898         std::is_same<
01899             typename Ring::zero,
01900             typename P::aN
01901         >::value && (P::degree > 0)

```

```

01902     » {
01903         using type = typename simplify<typename P::strip>::type;
01904     };
01905
01906     // otherwise : do nothing
01907     template<typename P>
01908     struct simplify<P, std::enable_if_t<
01909         !std::is_same<
01910             typename Ring::zero,
01911             typename P::aN
01912         >::value && (P::degree > 0)
01913     > {
01914         using type = P;
01915     };
01916
01917     // do not simplify constants
01918     template<typename P>
01919     struct simplify<P, std::enable_if_t<P::degree == 0> {
01920         using type = P;
01921     };
01922
01923     // addition at
01924     template<typename P1, typename P2, size_t index>
01925     struct add_at {
01926         using type =
01927             typename Ring::template add_t<
01928                 typename P1::template coeff_at_t<index>,
01929                 typename P2::template coeff_at_t<index>;
01930     };
01931
01932     template<typename P1, typename P2, size_t index>
01933     using add_at_t = typename add_at<P1, P2, index>::type;
01934
01935     template<typename P1, typename P2, std::size_t... I>
01936     struct add_low<P1, P2, std::index_sequence<I...> {
01937         using type = val<add_at_t<P1, P2, I>...>;
01938     };
01939
01940     // subtraction at
01941     template<typename P1, typename P2, size_t index>
01942     struct sub_at {
01943         using type =
01944             typename Ring::template sub_t<
01945                 typename P1::template coeff_at_t<index>,
01946                 typename P2::template coeff_at_t<index>;
01947     };
01948
01949     template<typename P1, typename P2, size_t index>
01950     using sub_at_t = typename sub_at<P1, P2, index>::type;
01951
01952     template<typename P1, typename P2, std::size_t... I>
01953     struct sub_low<P1, P2, std::index_sequence<I...> {
01954         using type = val<sub_at_t<P1, P2, I>...>;
01955     };
01956
01957     template<typename P1, typename P2>
01958     struct sub {
01959         using type = typename simplify<typename sub_low<
01960             P1,
01961             P2,
01962             internal::make_index_sequence_reverse<
01963                 std::max(P1::degree, P2::degree) + 1
01964             >::type>::type;
01965     };
01966
01967     // multiplication at
01968     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01969     struct mul_at_loop_helper {
01970         using type = typename Ring::template add_t<
01971             typename Ring::template mul_t<
01972                 typename v1::template coeff_at_t<index>,
01973                 typename v2::template coeff_at_t<k - index>
01974             >,
01975             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01976         >;
01977     };
01978
01979     template<typename v1, typename v2, size_t k, size_t stop>
01980     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01981         using type = typename Ring::template mul_t<
01982             typename v1::template coeff_at_t<stop>,
01983             typename v2::template coeff_at_t<0>;
01984     };
01985
01986     template<typename v1, typename v2, size_t k, typename E = void>
01987     struct mul_at {};
01988

```

```

01989     template<typename v1, typename v2, size_t k>
01990     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01991         using type = typename Ring::zero;
01992     };
01993
01994     template<typename v1, typename v2, size_t k>
01995     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01996         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01997     };
01998
01999     template<typename P1, typename P2, size_t index>
02000     using mul_at_t = typename mul_at<P1, P2, index>::type;
02001
02002     template<typename P1, typename P2, std::size_t... I>
02003     struct mul_low<P1, P2, std::index_sequence<I...> {
02004         using type = val<mul_at_t<P1, P2, I>...>;
02005     };
02006
02007     // division helper
02008     template< typename A, typename B, typename Q, typename R, typename E = void>
02009     struct div_helper {};
02010
02011     template<typename A, typename B, typename Q, typename R>
02012     struct div_helper<A, B, Q, R, std::enable_if_t<
02013         (R::degree < B::degree) ||
02014         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02015         using q_type = Q;
02016         using mod_type = R;
02017         using gcd_type = B;
02018     };
02019
02020     template<typename A, typename B, typename Q, typename R>
02021     struct div_helper<A, B, Q, R, std::enable_if_t<
02022         (R::degree >= B::degree) &&
02023         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02024     private: // NOLINT
02025         using rN = typename R::aN;
02026         using bN = typename B::aN;
02027         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02028         B::degree>::type;
02029         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02030         using qq = typename add<Q, pT>::type;
02031     public:
02032         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02033         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02034         using gcd_type = rr;
02035     };
02036
02037     template<typename A, typename B>
02038     struct div {
02039         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02040         using q_type = typename div_helper<A, B, zero, A>::q_type;
02041         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02042     };
02043
02044     template<typename P>
02045     struct make_unit {
02046         using type = typename div<P, val<typename P::aN>::q_type>;
02047     };
02048
02049     template<typename coeff, size_t deg>
02050     struct monomial {
02051         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02052     };
02053
02054     template<typename coeff>
02055     struct monomial<coeff, 0> {
02056         using type = val<coeff>;
02057     };
02058
02059     template<typename arithmeticType, typename P>
02060     struct horner_evaluation {
02061         template<size_t index, size_t stop>
02062         struct inner {
02063             static constexpr DEVICE INLINED arithmeticType func(
02064                 const arithmeticType& accum, const arithmeticType& x) {
02065                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02066 stop>::func(
02067                     internal::fma_helper<arithmeticType>::eval(
02068                         x,
02069                         accum,
02070                         P::template coeff_at_t<P::degree - index>::template
02071 get<arithmeticType>()), x);
02072             };
02073         };
02074     };

```

```

02073     template<size_t stop>
02074     struct inner<stop, stop> {
02075         static constexpr DEVICE INLINED arithmeticType func(
02076             const arithmeticType& accum, const arithmeticType& x) {
02077             return accum;
02078         }
02079     };
02080 };
02081
02082 template<typename arithmeticType, typename P>
02083 struct compensated_horner {
02084     template<int64_t index, int ghost>
02085     struct EFTHorner {
02086         static INLINED void func(
02087             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02088             *r) {
02089             arithmeticType p;
02089             internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02090             constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
02091             get<arithmeticType>();
02091             internal::two_sum<arithmeticType>(
02092                 p, coeff,
02093                 r, sigma + P::degree - index - 1);
02094             EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02095         }
02096     };
02097
02098     template<int ghost>
02099     struct EFTHorner<-1, ghost> {
02100         static INLINED DEVICE void func(
02101             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02102             *r) {
02103         }
02104     };
02105
02106     static INLINED DEVICE arithmeticType func(arithmeticType x) {
02107         arithmeticType pi[P::degree], sigma[P::degree];
02108         arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02109         EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02110         arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02111         return r + c;
02112     }
02113 };
02114
02115 template<typename coeff, typename... coeffs>
02116 struct string_helper {
02117     static std::string func() {
02118         std::string tail = string_helper<coeffs...>::func();
02119         std::string result = "";
02120         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02121             return tail;
02122         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02123             if (sizeof...(coeffs) == 1) {
02124                 result += "x";
02125             } else {
02126                 result += "x^" + std::to_string(sizeof...(coeffs));
02127             }
02128         } else {
02129             if (sizeof...(coeffs) == 1) {
02130                 result += coeff::to_string() + " x";
02131             } else {
02132                 result += coeff::to_string()
02133                     + " x^" + std::to_string(sizeof...(coeffs));
02134             }
02135         }
02136         if (!tail.empty()) {
02137             if (tail.at(0) != '-') {
02138                 result += " + " + tail;
02139             } else {
02140                 result += " - " + tail.substr(1);
02141             }
02142         }
02143         return result;
02144     }
02145 };
02146
02147 template<typename coeff>
02148 struct string_helper<coeff> {
02149     static std::string func() {
02150         if (!std::is_same<coeff, typename Ring::zero>::value) {
02151             return coeff::to_string();
02152         } else {
02153             return "";
02154         }
02155     }
02156 };

```

```

02157     };
02158
02159     public:
02160         template<typename P>
02161         using simplify_t = typename simplify<P>::type;
02162
02163         template<typename v1, typename v2>
02164         using add_t = typename add<v1, v2>::type;
02165
02166         template<typename v1, typename v2>
02167         using sub_t = typename sub<v1, v2>::type;
02168
02169         template<typename v1, typename v2>
02170         using mul_t = typename mul<v1, v2>::type;
02171
02172         template<typename v1, typename v2>
02173         using eq_t = typename eq_helper<v1, v2>::type;
02174
02175         template<typename v1, typename v2>
02176         using lt_t = typename lt_helper<v1, v2>::type;
02177
02178         template<typename v1, typename v2>
02179         using gt_t = typename gt_helper<v1, v2>::type;
02180
02181         template<typename v1, typename v2>
02182         using div_t = typename div<v1, v2>::q_type;
02183
02184         template<typename v1, typename v2>
02185         using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02186
02187         template<typename coeff, size_t deg>
02188         using monomial_t = typename monomial<coeff, deg>::type;
02189
02190         template<typename v>
02191         using derive_t = typename derive_helper<v>::type;
02192
02193         template<typename v>
02194         using pos_t = typename Ring::template pos_t<typename v::aN>;
02195
02196         template<typename v>
02197         static constexpr bool pos_v = pos_t<v>::value;
02198
02199         template<typename v1, typename v2>
02200         using gcd_t = std::conditional_t<
02201             Ring::is_euclidean_domain,
02202             typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02203             void>;
02204
02205         template<auto x>
02206         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02207
02208         template<typename v>
02209         using inject_ring_t = val<v>;
02210     };
02211 } // namespace aerobus
02212
02213 // fraction field
02214 namespace aerobus {
02215     namespace internal {
02216         template<typename Ring, typename E = void>
02217         requires IsEuclideanDomain<Ring>
02218         struct _FractionField {};
02219
02220         template<typename Ring>
02221         requires IsEuclideanDomain<Ring>
02222         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02223             static constexpr bool is_field = true;
02224             static constexpr bool is_euclidean_domain = true;
02225
02226         private:
02227             template<typename val1, typename val2, typename E = void>
02228             struct to_string_helper {};
02229
02230             template<typename val1, typename val2>
02231             struct to_string_helper <val1, val2,
02232                 std::enable_if_t<
02233                     Ring::template eq_t<
02234                         val2, typename Ring::one
02235                         >::value
02236                     >
02237                 > {
02238                 static std::string func() {
02239                     return val1::to_string();
02240                 }
02241             };
02242
02243             template<typename val1, typename val2>

```



```

02287     struct to_string_helper<val1, val2,
02288         std::enable_if_t<
02289             !Ring::template eq_t<
02290                 val2,
02291                 typename Ring::one
02292             >::value
02293         >
02294     > {
02295         static std::string func() {
02296             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02297         }
02298     };
02299
02300 public:
02301     template<typename val1, typename val2>
02302     struct val {
02303         using x = val1;
02304         using y = val2;
02305         using is_zero_t = typename val1::is_zero_t;
02306         static constexpr bool is_zero_v = val1::is_zero_t::value;
02307
02308         using ring_type = Ring;
02309         using enclosing_type = _FractionField<Ring>;
02310
02311         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02312
02313         template<typename valueType>
02314         static constexpr INLINED_DEVICE valueType get() {
02315             return internal::staticcast<valueType, typename ring_type::inner_type>::template
02316 func<x::v>() /
02317             internal::staticcast<valueType, typename ring_type::inner_type>::template
02318 func<y::v>();
02319         }
02320
02321         static std::string to_string() {
02322             return to_string_helper<val1, val2>::func();
02323         }
02324
02325         template<typename arithmeticType>
02326         static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02327             return x::eval(v) / y::eval(v);
02328         }
02329     };
02330
02331     using zero = val<typename Ring::zero, typename Ring::one>;
02332     using one = val<typename Ring::one, typename Ring::one>;
02333
02334     template<typename v>
02335     using inject_t = val<v, typename Ring::one>;
02336
02337     template<auto x>
02338     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
02339 Ring::one>;
02340
02341     template<typename v>
02342     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02343
02344     using ring_type = Ring;
02345
02346 private:
02347     template<typename v, typename E = void>
02348     struct simplify {};
02349
02350     // x = 0
02351     template<typename v>
02352     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> > {
02353         using type = typename _FractionField<Ring>::zero;
02354     };
02355
02356     // x != 0
02357     template<typename v>
02358     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> > {
02359     private:
02360         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02361         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02362         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02363
02364         using posx = std::conditional_t<
02365             !Ring::template pos_v<newx>,
02366             typename Ring::template sub_t<typename Ring::zero, newx>,
02367             newx>;
02368         using posy = std::conditional_t<
02369             !Ring::template pos_v<newy>,
02370             typename Ring::template sub_t<typename Ring::zero, newy>,
02371             newy>;
02372     public:
02373         using type = typename _FractionField<Ring>::template val<posx, posy>;
02374     };

```

```

02399         };
02400
02401     public:
02402         template<typename v>
02403         using simplify_t = typename simplify<v>::type;
02404
02405     private:
02406         template<typename v1, typename v2>
02407         struct add {
02408             private:
02409                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02410                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02411                 using dividend = typename Ring::template add_t<a, b>;
02412                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02413                 using g = typename Ring::template gcd_t<dividend, diviser>;
02414
02415             public:
02416                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02417 diviser>>;
02418         };
02419
02420         template<typename v>
02421         struct pos {
02422             using type = std::conditional_t<
02423                 (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02424                 (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02425                 std::true_type,
02426                 std::false_type>;
02427         };
02428
02429         template<typename v1, typename v2>
02430         struct sub {
02431             private:
02432                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02433                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02434                 using dividend = typename Ring::template sub_t<a, b>;
02435                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02436                 using g = typename Ring::template gcd_t<dividend, diviser>;
02437
02438             public:
02439                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02440 diviser>>;
02441         };
02442
02443         template<typename v1, typename v2>
02444         struct mul {
02445             private:
02446                 using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02447                 using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02448
02449             public:
02450                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02451         };
02452
02453         template<typename v1, typename v2, typename E = void>
02454         struct div {};
02455
02456         template<typename v1, typename v2>
02457         struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value>> {
02458             private:
02459                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02460                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02461
02462             public:
02463                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02464         };
02465
02466         template<typename v1, typename v2>
02467         struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value>> {
02468             using type = one;
02469         };
02470
02471         template<typename v1, typename v2>
02472         struct eq {
02473             using type = std::conditional_t<
02474                 std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02475                 std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02476                 std::true_type,
02477                 std::false_type>;
02478         };
02479
02480         template<typename v1, typename v2, typename E = void>
02481         struct gt;
02482
02483         template<typename v1, typename v2>

```

```

02485     struct gt<v1, v2, std::enable_if_t<
02486         (eq<v1, v2>::type::value)
02487         > {
02488         using type = std::false_type;
02489     };
02490
02491     template<typename v1, typename v2>
02492     struct gt<v1, v2, std::enable_if_t<
02493         (!eq<v1, v2>::type::value) &&
02494         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02495         > {
02496         using type = typename gt<
02497             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02498             >::type;
02499     };
02500
02501     template<typename v1, typename v2>
02502     struct gt<v1, v2, std::enable_if_t<
02503         (!eq<v1, v2>::type::value) &&
02504         (pos<v1>::type::value) && (!pos<v2>::type::value)
02505         > {
02506         using type = std::true_type;
02507     };
02508
02509     template<typename v1, typename v2>
02510     struct gt<v1, v2, std::enable_if_t<
02511         (!eq<v1, v2>::type::value) &&
02512         (!pos<v1>::type::value) && (pos<v2>::type::value)
02513         > {
02514         using type = std::false_type;
02515     };
02516
02517     template<typename v1, typename v2>
02518     struct gt<v1, v2, std::enable_if_t<
02519         (!eq<v1, v2>::type::value) &&
02520         (pos<v1>::type::value) && (pos<v2>::type::value)
02521         > {
02522         using type = typename Ring::template gt_t<
02523             typename Ring::template mul_t<v1::x, v2::y>,
02524             typename Ring::template mul_t<v2::y, v2::x>
02525             >;
02526     };
02527
02528 public:
02529     template<typename v1, typename v2>
02530     using add_t = typename add<v1, v2>::type;
02531
02532     template<typename v1, typename v2>
02533     using mod_t = zero;
02534
02535     template<typename v1, typename v2>
02536     using gcd_t = v1;
02537
02538     template<typename v1, typename v2>
02539     using sub_t = typename sub<v1, v2>::type;
02540
02541     template<typename v1, typename v2>
02542     using mul_t = typename mul<v1, v2>::type;
02543
02544     template<typename v1, typename v2>
02545     using div_t = typename div<v1, v2>::type;
02546
02547     template<typename v1, typename v2>
02548     using eq_t = typename eq<v1, v2>::type;
02549
02550     template<typename v1, typename v2>
02551     static constexpr bool eq_v = eq<v1, v2>::type::value;
02552
02553     template<typename v1, typename v2>
02554     using gt_t = typename gt<v1, v2>::type;
02555
02556     template<typename v1, typename v2>
02557     static constexpr bool gt_v = gt<v1, v2>::type::value;
02558
02559     template<typename v1>
02560     using pos_t = typename pos<v1>::type;
02561
02562     template<typename v>
02563     static constexpr bool pos_v = pos_t<v>::value;
02564 };
02565
02566 template<typename Ring, typename E = void>
02567 requires IsEuclideanDomain<Ring>
02568 struct FractionFieldImpl {};
02569
02570 // fraction field of a field is the field itself
02571 template<typename Field>

```

```

02608     requires IsEuclideanDomain<Field>
02609     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02610         using type = Field;
02611         template<typename v>
02612         using inject_t = v;
02613     };
02614
02615     // fraction field of a ring is the actual fraction field
02616     template<typename Ring>
02617     requires IsEuclideanDomain<Ring>
02618     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02619         using type = _FractionField<Ring>;
02620     };
02621 } // namespace internal
02622
02623 template<typename Ring>
02624 requires IsEuclideanDomain<Ring>
02625 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02626
02627 template<typename Ring>
02628 struct Embed<Ring, FractionField<Ring> {
02629     template<typename v>
02630     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02631 };
02632 } // namespace aerobus
02633
02634 // short names for common types
02635 namespace aerobus {
02636     template<typename X, typename Y>
02637     requires IsRing<typename X::enclosing_type> &&
02638         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02639     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02640
02641     template<typename X, typename Y>
02642     requires IsRing<typename X::enclosing_type> &&
02643         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02644     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02645
02646     template<typename X, typename Y>
02647     requires IsRing<typename X::enclosing_type> &&
02648         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02649     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02650
02651     template<typename X, typename Y>
02652     requires IsEuclideanDomain<typename X::enclosing_type> &&
02653         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02654     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02655
02656     using q32 = FractionField<i32>;
02657
02658     using fpq32 = FractionField<polynomial<q32>>;
02659
02660     using q64 = FractionField<i64>;
02661
02662     using pi64 = polynomial<i64>;
02663
02664     using pq64 = polynomial<q64>;
02665
02666     using fpq64 = FractionField<polynomial<q64>>;
02667
02668     template<typename Ring, typename v1, typename v2>
02669     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02670
02671     template<typename v>
02672     using embed_int_poly_in_fractions_t =
02673         typename Embed<
02674             polynomial<typename v::ring_type>,
02675             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02676
02677     template<int64_t p, int64_t q>
02678     using make_q64_t = typename q64::template simplify_t<
02679         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02680
02681     template<int32_t p, int32_t q>
02682     using make_q32_t = typename q32::template simplify_t<
02683         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02684
02685     template<typename Ring, typename v1, typename v2>
02686     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02687     template<typename Ring, typename v1, typename v2>
02688     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02689
02690     template<>
02691     struct Embed<q32, q64> {
02692         template<typename v>
02693         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02694     };

```

```

02749     };
02750
02751     template<typename Small, typename Large>
02752     struct Embed<polynomial<Small>, polynomial<Large> > {
02753     private:
02754         template<typename v, typename i>
02755         struct at_low;
02756
02757         template<typename v, size_t i>
02758         struct at_index {
02759             using type = typename Embed<Small, Large>::template
02760             type<typename v::template coeff_at_t<i>>;
02761         };
02762
02763         template<typename v, size_t... Is>
02764         struct at_low<v, std::index_sequence<Is...> > {
02765             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02766         };
02767
02768     public:
02769         template<typename v>
02770         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02771         1>::type>;
02772     };
02773
02774     template<typename Ring, auto... xs>
02775     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02776     typename Ring::template inject_constant_t<xs>...>;
02777
02778     template<typename Ring, auto... xs>
02779     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02780     typename FractionField<Ring>::template inject_constant_t<xs>...>;
02781 } // namespace aerobus
02782
02783 // Taylor series and common integers (factorial, bernoulli...) appearing in Taylor coefficients
02784 namespace aerobus {
02785     namespace internal {
02786         template<typename T, size_t x, typename E = void>
02787         struct factorial {};
02788
02789         template<typename T, size_t x>
02790         struct factorial<T, x, std::enable_if_t<(x > 0)> > {
02791         private:
02792             template<typename, size_t, typename>
02793             friend struct factorial;
02794         public:
02795             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02796             x - 1>::type>;
02797             static constexpr typename T::inner_type value = type::template get<typename
02798             T::inner_type>();
02799         };
02800
02801         template<typename T>
02802         struct factorial<T, 0> {
02803         public:
02804             using type = typename T::one;
02805             static constexpr typename T::inner_type value = type::template get<typename
02806             T::inner_type>();
02807         };
02808     } // namespace internal
02809
02810     template<typename T, size_t i>
02811     using factorial_t = typename internal::factorial<T, i>::type;
02812
02813     template<typename T, size_t i>
02814     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02815
02816     namespace internal {
02817         template<typename T, size_t k, size_t n, typename E = void>
02818         struct combination_helper {};
02819
02820         template<typename T, size_t k, size_t n>
02821         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> > {
02822             using type = typename FractionField<T>::template mul_t<
02823             typename combination_helper<T, k - 1, n - 1>::type,
02824             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02825         };
02826
02827         template<typename T, size_t k, size_t n>
02828         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> > {
02829             using type = typename combination_helper<T, n - k, n>::type;
02830         };
02831
02832         template<typename T, size_t n>
02833         struct combination_helper<T, 0, n> {
02834             using type = typename FractionField<T>::one;
02835         };
02836     }
02837 }

```

```

02848
02849     template<typename T, size_t k, size_t n>
02850     struct combination {
02851         using type = typename internal::combination_helper<T, k, n>::type::x;
02852         static constexpr typename T::inner_type value =
02853             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>());
02854     };
02855 } // namespace internal
02856
02857 template<typename T, size_t k, size_t n>
02858 using combination_t = typename internal::combination<T, k, n>::type;
02859
02860 template<typename T, size_t k, size_t n>
02861 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02862
02863 namespace internal {
02864     template<typename T, size_t m>
02865     struct bernoulli;
02866
02867     template<typename T, typename accum, size_t k, size_t m>
02868     struct bernoulli_helper {
02869         using type = typename bernoulli_helper<
02870             T,
02871             addfractions_t<T,
02872                 accum,
02873                 mulfractions_t<T,
02874                     makefraction_t<T,
02875                         combination_t<T, k, m + 1>,
02876                         typename T::one>,
02877                         typename bernoulli<T, k>::type
02878                     >,
02879                     k + 1,
02880                     m>::type;
02881     };
02882
02883     template<typename T, typename accum, size_t m>
02884     struct bernoulli_helper<T, accum, m, m> {
02885         using type = accum;
02886     };
02887
02888     template<typename T, size_t m>
02889     struct bernoulli {
02890         using type = typename FractionField<T>::template mul_t<
02891             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02892             makefraction_t<T,
02893                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02894                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02895             >
02896         >;
02897
02898         template<typename floatType>
02899         static constexpr floatType value = type::template get<floatType>();
02900     };
02901
02902     template<typename T>
02903     struct bernoulli<T, 0> {
02904         using type = typename FractionField<T>::one;
02905
02906         template<typename floatType>
02907         static constexpr floatType value = type::template get<floatType>();
02908     };
02909 } // namespace internal
02910
02911 template<typename T, size_t n>
02912 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02913
02914 template<typename FloatType, typename T, size_t n>
02915 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02916
02917 // bell numbers
02918 namespace internal {
02919     template<typename T, size_t n, typename E = void>
02920     struct bell_helper;
02921
02922     template<typename T, size_t n>
02923     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02924         template<typename accum, size_t i, size_t stop>
02925         struct sum_helper {
02926             private:
02927                 using left = typename T::template mul_t<
02928                     combination_t<T, i, n-1>,
02929                     typename bell_helper<T, i>::type>;
02930                 using new_accum = typename T::template add_t<accum, left>;
02931             public:
02932                 using type = new_accum;
02933         };
02934     };
02935
02936     template<typename T, size_t n, typename E = void>
02937     struct bell_helper<T, n, E> {
02938         using type = E;
02939     };
02940 }
02941
02942 template<typename T, size_t n>
02943 using bell_t = typename internal::bell_helper<T, n>::type;
02944
02945 template<typename FloatType, typename T, size_t n>
02946 inline constexpr FloatType bell_v = internal::bell<T, n>::template value<FloatType>;

```

```

02947         public:
02948             using type = typename sum_helper<new_accum, i+1, stop>::type;
02949     };
02950
02951     template<typename accum, size_t stop>
02952     struct sum_helper<accum, stop, stop> {
02953         using type = accum;
02954     };
02955
02956     using type = typename sum_helper<typename T::zero, 0, n>::type;
02957 };
02958
02959 template<typename T>
02960 struct bell_helper<T, 0> {
02961     using type = typename T::one;
02962 };
02963
02964 template<typename T>
02965 struct bell_helper<T, 1> {
02966     using type = typename T::one;
02967 };
02968 } // namespace internal
02969
02970 template<typename T, size_t n>
02971 using bell_t = typename internal::bell_helper<T, n>::type;
02972
02973 template<typename T, size_t n>
02974 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02975
02976 namespace internal {
02977     template<typename T, int k, typename E = void>
02978     struct alternate {};
02979
02980     template<typename T, int k>
02981     struct alternate<T, k, std::enable_if_t<k % 2 == 0> > {
02982         using type = typename T::one;
02983         static constexpr typename T::inner_type value = type::template get<typename
02984 T::inner_type>();
02985     };
02986
02987     template<typename T, int k>
02988     struct alternate<T, k, std::enable_if_t<k % 2 != 0> > {
02989         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02990         static constexpr typename T::inner_type value = type::template get<typename
02991 T::inner_type>();
02992     };
02993 } // namespace internal
02994
02995 template<typename T, int k>
02996 using alternate_t = typename internal::alternate<T, k>::type;
02997
02998 template<typename T, size_t k>
02999 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03000
03001 namespace internal {
03002     template<typename T, int n, int k, typename E = void>
03003     struct stirling_l_helper {};
03004
03005     template<typename T>
03006     struct stirling_l_helper<T, 0, 0> {
03007         using type = typename T::one;
03008     };
03009
03010     template<typename T, int n>
03011     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> > {
03012         using type = typename T::zero;
03013     };
03014
03015     template<typename T, int n>
03016     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> > {
03017         using type = typename T::zero;
03018     };
03019
03020     template<typename T, int n, int k>
03021     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> > {
03022         using type = typename T::template sub_t<
03023             typename T::stirling_l_helper<T, n-1, k-1>::type,
03024             typename T::template mul_t<
03025                 typename T::template inject_constant_t<n-1>,
03026                 typename stirling_l_helper<T, n-1, k>::type
03027             >;
03028     };
03029 } // namespace internal
03030
03031 template<typename T, int n, int k>
03032 using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
03033
03034
03035

```

```

03050     template<typename T, int n, int k>
03051     using stirling_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03052
03053     template<typename T, int n, int k>
03054     static constexpr typename T::inner_type stirling_1_unsigned_v = stirling_1_unsigned_t<T, n, k>::v;
03055
03056     template<typename T, int n, int k>
03057     static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03058
03059     namespace internal {
03060         template<typename T, int n, int k, typename E = void>
03061         struct stirling_2_helper {};
03062
03063         template<typename T, int n>
03064         struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
03065             using type = typename T::one;
03066         };
03067
03068         template<typename T, int n>
03069         struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03070             using type = typename T::zero;
03071         };
03072
03073         template<typename T, int n>
03074         struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03075             using type = typename T::zero;
03076         };
03077
03078         template<typename T, int n, int k>
03079         struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03080             using type = typename T::template add_t<
03081                 typename stirling_2_helper<T, n-1, k-1>::type,
03082                 typename T::template mul_t<
03083                     typename T::template inject_constant_t<k>,
03084                     typename stirling_2_helper<T, n-1, k>::type
03085             >;
03086         };
03087     } // namespace internal
03088
03089     template<typename T, int n, int k>
03090     using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03091
03092     template<typename T, int n, int k>
03093     static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03094
03095     namespace internal {
03096         template<typename T>
03097         struct pow_scalar {
03098             template<size_t p>
03099             static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03100                 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03101                 x * func<p/2>(x) * func<p/2>(x);
03102         };
03103
03104         template<typename T, typename p, size_t n, typename E = void>
03105         requires IsEuclideanDomain<T>
03106         struct pow;
03107
03108         template<typename T, typename p, size_t n>
03109         struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03110             using type = typename T::template mul_t<
03111                 typename pow<T, p, n/2>::type,
03112                 typename pow<T, p, n/2>::type
03113             >;
03114         };
03115
03116         template<typename T, typename p, size_t n>
03117         struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03118             using type = typename T::template mul_t<
03119                 p,
03120                 typename T::template mul_t<
03121                     typename pow<T, p, n/2>::type,
03122                     typename pow<T, p, n/2>::type
03123                 >
03124             >;
03125         };
03126
03127         template<typename T, typename p, size_t n>
03128         struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03129     } // namespace internal
03130
03131     template<typename T, typename p, size_t n>
03132     using pow_t = typename internal::pow<T, p, n>::type;
03133
03134     template<typename T, typename p, size_t n>
03135     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;

```



```

03161
03162     template<typename T, size_t p>
03163     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
internal::pow_scalar<T>::template func<p>(x); }
03164
03165     namespace internal {
03166         template<typename, template<typename, size_t> typename, class>
03167         struct make_taylor_impl;
03168
03169         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03170         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03171             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
03172         };
03173     }
03174
03175     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03176     using taylor = typename internal::make_taylor_impl<
03177         T,
03178         coeff_at,
03179         internal::make_index_sequence_reverse<deg + 1>::type;
03180
03181     namespace internal {
03182         template<typename T, size_t i>
03183         struct exp_coeff {
03184             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03185         };
03186
03187         template<typename T, size_t i, typename E = void>
03188         struct sin_coeff_helper {};
03189
03190         template<typename T, size_t i>
03191         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03192             using type = typename FractionField<T>::zero;
03193         };
03194
03195         template<typename T, size_t i>
03196         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03197             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03198         };
03199
03200         template<typename T, size_t i>
03201         struct sin_coeff {
03202             using type = typename sin_coeff_helper<T, i>::type;
03203         };
03204
03205         template<typename T, size_t i, typename E = void>
03206         struct sh_coeff_helper {};
03207
03208         template<typename T, size_t i>
03209         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03210             using type = typename FractionField<T>::zero;
03211         };
03212
03213         template<typename T, size_t i>
03214         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03215             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03216         };
03217
03218         template<typename T, size_t i>
03219         struct sh_coeff {
03220             using type = typename sh_coeff_helper<T, i>::type;
03221         };
03222
03223         template<typename T, size_t i, typename E = void>
03224         struct cos_coeff_helper {};
03225
03226         template<typename T, size_t i>
03227         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03228             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03229         };
03230
03231         template<typename T, size_t i>
03232         struct cos_coeff {
03233             using type = typename cos_coeff_helper<T, i>::type;
03234         };
03235
03236         template<typename T, size_t i, typename E = void>
03237         struct cosh_coeff_helper {};
03238
03239         template<typename T, size_t i>
03240         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {

```

```

03250         using type = typename FractionField<T>::zero;
03251     };
03252
03253     template<typename T, size_t i>
03254     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03255         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03256     };
03257
03258     template<typename T, size_t i>
03259     struct cosh_coeff {
03260         using type = typename cosh_coeff_helper<T, i>::type;
03261     };
03262
03263     template<typename T, size_t i>
03264     struct geom_coeff { using type = typename FractionField<T>::one; };
03265
03266
03267     template<typename T, size_t i, typename E = void>
03268     struct atan_coeff_helper;
03269
03270     template<typename T, size_t i>
03271     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03272         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03273     };
03274
03275     template<typename T, size_t i>
03276     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03277         using type = typename FractionField<T>::zero;
03278     };
03279
03280     template<typename T, size_t i>
03281     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03282
03283     template<typename T, size_t i, typename E = void>
03284     struct asin_coeff_helper;
03285
03286     template<typename T, size_t i>
03287     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03288         using type = makefraction_t<T,
03289             factorial_t<T, i - 1>,
03290             typename T::template mul_t<
03291                 typename T::template val<i>,
03292                 T::template mul_t<
03293                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03294                     pow<T, factorial_t<T, i / 2>, 2>
03295                 >
03296             >
03297         >>;
03298     };
03299
03300     template<typename T, size_t i>
03301     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03302         using type = typename FractionField<T>::zero;
03303     };
03304
03305     template<typename T, size_t i>
03306     struct asin_coeff {
03307         using type = typename asin_coeff_helper<T, i>::type;
03308     };
03309
03310     template<typename T, size_t i>
03311     struct lnpl_coeff {
03312         using type = makefraction_t<T,
03313             alternate_t<T, i + 1>,
03314             typename T::template val<i>;
03315     };
03316
03317     template<typename T>
03318     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03319
03320     template<typename T, size_t i, typename E = void>
03321     struct asinh_coeff_helper;
03322
03323     template<typename T, size_t i>
03324     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03325         using type = makefraction_t<T,
03326             typename T::template mul_t<
03327                 alternate_t<T, i / 2>,
03328                 factorial_t<T, i - 1>
03329             >,
03330             typename T::template mul_t<
03331                 typename T::template mul_t<
03332                     typename T::template val<i>,
03333                     pow_t<T, factorial_t<T, i / 2>, 2>
03334                 >,
03335                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03336             >

```

```

03337         >;
03338     };
03339
03340     template<typename T, size_t i>
03341     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03342         using type = typename FractionField<T>::zero;
03343     };
03344
03345     template<typename T, size_t i>
03346     struct asinh_coeff {
03347         using type = typename asinh_coeff_helper<T, i>::type;
03348     };
03349
03350     template<typename T, size_t i, typename E = void>
03351     struct atanh_coeff_helper;
03352
03353     template<typename T, size_t i>
03354     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03355         // 1/i
03356         using type = typename FractionField<T>::template val<
03357             typename T::one,
03358             typename T::template inject_constant_t<i>;
03359     };
03360
03361     template<typename T, size_t i>
03362     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03363         using type = typename FractionField<T>::zero;
03364     };
03365
03366     template<typename T, size_t i>
03367     struct atanh_coeff {
03368         using type = typename atanh_coeff_helper<T, i>::type;
03369     };
03370
03371     template<typename T, size_t i, typename E = void>
03372     struct tan_coeff_helper;
03373
03374     template<typename T, size_t i>
03375     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03376         using type = typename FractionField<T>::zero;
03377     };
03378
03379     template<typename T, size_t i>
03380     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03381     private:
03382         // 4^((i+1)/2)
03383         using _4p = typename FractionField<T>::template inject_t<
03384             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03385         // 4^((i+1)/2) - 1
03386         using _4pml = typename FractionField<T>::template
03387             sub_t<_4p, typename FractionField<T>::one>;
03387         // (-1)^((i-1)/2)
03388         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03389         using dividend = typename FractionField<T>::template mul_t<
03390             altp,
03391             FractionField<T>::template mul_t<
03392                 _4p,
03393                 FractionField<T>::template mul_t<
03394                     _4pml,
03395                     bernoulli_t<T, (i + 1)>
03396                 >
03397             >
03398     >;
03399     public:
03400         using type = typename FractionField<T>::template div_t<dividend,
03401             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03402     };
03403
03404     template<typename T, size_t i>
03405     struct tan_coeff {
03406         using type = typename tan_coeff_helper<T, i>::type;
03407     };
03408
03409     template<typename T, size_t i, typename E = void>
03410     struct tanh_coeff_helper;
03411
03412     template<typename T, size_t i>
03413     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03414         using type = typename FractionField<T>::zero;
03415     };
03416
03417     template<typename T, size_t i>
03418     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03419     private:
03420         using _4p = typename FractionField<T>::template inject_t<
03421             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03422         using _4pml = typename FractionField<T>::template

```

```

sub_t<_4p, typename FractionField<T>::one>;
03423     using dividend =
03424         typename FractionField<T>::template mul_t<
03425             _4p,
03426             typename FractionField<T>::template mul_t<
03427                 _4pml,
03428                 bernoulli_t<T, (i + 1)>>::type;
03429     public:
03430         using type = typename FractionField<T>::template div_t<dividend,
03431             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03432     };
03433
03434     template<typename T, size_t i>
03435     struct tanh_coeff {
03436         using type = typename tanh_coeff_helper<T, i>::type;
03437     };
03438 } // namespace internal
03439
03440 template<typename Integers, size_t deg>
03441 using exp = taylor<Integers, internal::exp_coeff, deg>;
03442
03443 template<typename Integers, size_t deg>
03444 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03445     exp<Integers, deg>,
03446     typename polynomial<FractionField<Integers>>::one>;
03447
03448 template<typename Integers, size_t deg>
03449 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03450
03451 template<typename Integers, size_t deg>
03452 using atan = taylor<Integers, internal::atan_coeff, deg>;
03453
03454 template<typename Integers, size_t deg>
03455 using sin = taylor<Integers, internal::sin_coeff, deg>;
03456
03457 template<typename Integers, size_t deg>
03458 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03459
03460 template<typename Integers, size_t deg>
03461 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03462
03463 template<typename Integers, size_t deg>
03464 using cos = taylor<Integers, internal::cos_coeff, deg>;
03465
03466 template<typename Integers, size_t deg>
03467 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03468
03469 template<typename Integers, size_t deg>
03470 using asin = taylor<Integers, internal::asin_coeff, deg>;
03471
03472 template<typename Integers, size_t deg>
03473 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03474
03475 template<typename Integers, size_t deg>
03476 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03477
03478 template<typename Integers, size_t deg>
03479 using tan = taylor<Integers, internal::tan_coeff, deg>;
03480
03481 template<typename Integers, size_t deg>
03482 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03483 } // namespace aerobus
03484
03485 // continued fractions
03486 namespace aerobus {
03487     template<int64_t... values>
03488     struct ContinuedFraction {};
03489
03490     template<int64_t a0>
03491     struct ContinuedFraction<a0> {
03492         using type = typename q64::template inject_constant_t<a0>;
03493         static constexpr double val = static_cast<double>(a0);
03494     };
03495
03496     template<int64_t a0, int64_t... rest>
03497     struct ContinuedFraction<a0, rest...> {
03498         using type = q64::template add_t<
03499             typename q64::template inject_constant_t<a0>,
03500             typename q64::template div_t<
03501                 typename q64::one,
03502                 typename ContinuedFraction<rest...>::type
03503             >>;
03504
03505         static constexpr double val = type::template get<double>();
03506     };
03507
03508     using PI_fraction =

```

```

ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03574     using E_fraction =
ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03576     using SQRT2_fraction =
ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03578     using SQRT3_fraction =
ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
// NOLINT
03579 } // namespace aerobus
03580
03581 // known polynomials
03582 namespace aerobus {
03583     // CChebyshev
03584     namespace internal {
03585         template<int kind, size_t deg, typename I>
03586         struct chebyshev_helper {
03587             using type = typename polynomial<I>::template sub_t<
03588                 typename polynomial<I>::template mul_t<
03589                     typename polynomial<I>::template mul_t<
03590                         typename polynomial<I>::template inject_constant_t<2>,
03591                         typename polynomial<I>::X>,
03592                         typename chebyshev_helper<kind, deg - 1, I>::type
03593                     >,
03594                     typename chebyshev_helper<kind, deg - 2, I>::type
03595                 >;
03596         };
03597
03598         template<typename I>
03599         struct chebyshev_helper<1, 0, I> {
03600             using type = typename polynomial<I>::one;
03601         };
03602
03603         template<typename I>
03604         struct chebyshev_helper<1, 1, I> {
03605             using type = typename polynomial<I>::X;
03606         };
03607
03608         template<typename I>
03609         struct chebyshev_helper<2, 0, I> {
03610             using type = typename polynomial<I>::one;
03611         };
03612
03613         template<typename I>
03614         struct chebyshev_helper<2, 1, I> {
03615             using type = typename polynomial<I>::template mul_t<
03616                 typename polynomial<I>::template inject_constant_t<2>,
03617                 typename polynomial<I>::X>;
03618         };
03619     } // namespace internal
03620
03621     // Laguerre
03622     namespace internal {
03623         template<size_t deg, typename I>
03624         struct laguerre_helper {
03625             using Q = FractionField<I>;
03626             using PQ = polynomial<Q>;
03627
03628             private:
03629                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03630                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03631                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03632                 // -x + 2k-1
03633                 using p = typename PQ::template val<
03634                     typename Q::template inject_constant_t<-1>,
03635                     typename Q::template inject_constant_t<2 * deg - 1>;
03636                 // 1/n
03637                 using factor = typename PQ::template inject_ring_t<
03638                     typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03639
03640             public:
03641                 using type = typename PQ::template mul_t <
03642                     factor,
03643                     typename PQ::template sub_t<
03644                         typename PQ::template mul_t<
03645                             p,
03646                             lnm1
03647                         >,
03648                         typename PQ::template mul_t<
03649                             typename PQ::template inject_constant_t<deg-1>,
03650                             lnm2
03651                         >
03652                     >
03653                 >;
03654         };
03655
03656         template<typename I>

```

```

03657     struct laguerre_helper<0, I> {
03658         using type = typename polynomial<FractionField<I>::one>;
03659     };
03660
03661     template<typename I>
03662     struct laguerre_helper<1, I> {
03663     private:
03664         using PQ = polynomial<FractionField<I>;
03665     public:
03666         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03667     };
03668 } // namespace internal
03669
03670 // Bernstein
03671 namespace internal {
03672     template<size_t i, size_t m, typename I, typename E = void>
03673     struct bernstein_helper {};
03674
03675     template<typename I>
03676     struct bernstein_helper<0, 0, I> {
03677     private:
03678         using type = typename polynomial<I>::one;
03679     };
03680
03681     template<size_t i, size_t m, typename I>
03682     struct bernstein_helper<i, m, I, std::enable_if_t<
03683         (m > 0) && (i == 0)>> {
03684     private:
03685         using P = polynomial<I>;
03686     public:
03687         using type = typename P::template mul_t<
03688             typename P::template sub_t<typename P::one, typename P::X>,
03689             typename bernstein_helper<i, m-1, I>::type>;
03690     };
03691
03692     template<size_t i, size_t m, typename I>
03693     struct bernstein_helper<i, m, I, std::enable_if_t<
03694         (m > 0) && (i == m)>> {
03695     private:
03696         using P = polynomial<I>;
03697     public:
03698         using type = typename P::template mul_t<
03699             typename P::X,
03700             typename bernstein_helper<i-1, m-1, I>::type>;
03701     };
03702
03703     template<size_t i, size_t m, typename I>
03704     struct bernstein_helper<i, m, I, std::enable_if_t<
03705         (m > 0) && (i > 0) && (i < m)>> {
03706     private:
03707         using P = polynomial<I>;
03708     public:
03709         using type = typename P::template add_t<
03710             typename P::template mul_t<
03711                 typename P::template sub_t<typename P::one, typename P::X>,
03712                 typename bernstein_helper<i, m-1, I>::type>,
03713             typename P::template mul_t<
03714                 typename P::X,
03715                 typename bernstein_helper<i-1, m-1, I>::type>;
03716     };
03717 } // namespace internal
03718
03719 // AllOne polynomials
03720 namespace internal {
03721     template<size_t deg, typename I>
03722     struct AllOneHelper {
03723     private:
03724         using type = aerobus::add_t<
03725             typename polynomial<I>::one,
03726             typename aerobus::mul_t<
03727                 typename polynomial<I>::X,
03728                 typename AllOneHelper<deg-1, I>::type
03729             >>;
03730     };
03731
03732     template<typename I>
03733     struct AllOneHelper<0, I> {
03734     private:
03735         using type = typename polynomial<I>::one;
03736     };
03737 } // namespace internal
03738
03739 // Bessel polynomials
03740 namespace internal {
03741     template<size_t deg, typename I>
03742     struct BesselHelper {
03743     private:
03744         using P = polynomial<I>;
03745         using factor = typename P::template monomial_t<
03746             typename I::template inject_constant_t<(2*deg - 1)>,

```

```

03744         1>;
03745     public:
03746         using type = typename P::template add_t<
03747             typename P::template mul_t<
03748                 factor,
03749                 typename BesselHelper<deg-1, I>::type
03750             >,
03751             typename BesselHelper<deg-2, I>::type
03752         >;
03753 };
03754
03755 template<typename I>
03756 struct BesselHelper<0, I> {
03757     using type = typename polynomial<I>::one;
03758 };
03759
03760 template<typename I>
03761 struct BesselHelper<1, I> {
03762     private:
03763         using P = polynomial<I>;
03764     public:
03765         using type = typename P::template add_t<
03766             typename P::one,
03767             typename P::X
03768         >;
03769 };
03770 } // namespace internal
03771
03772 namespace known_polynomials {
03773     enum hermite_kind {
03774         probabilist,
03775         physicist
03776     };
03777 }
03778
03779 // hermite
03780 namespace internal {
03781     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03782     struct hermite_helper {};
03783
03784     template<size_t deg, typename I>
03785     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03786     private:
03787         using hnm1 = typename hermite_helper<deg - 1,
03788             known_polynomials::hermite_kind::probabilist, I>::type;
03789         using hnm2 = typename hermite_helper<deg - 2,
03790             known_polynomials::hermite_kind::probabilist, I>::type;
03791     public:
03792         using type = typename polynomial<I>::template sub_t<
03793             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03794             typename polynomial<I>::template mul_t<
03795                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03796                 hnm2
03797             >
03798         >;
03799 };
03800
03801 template<size_t deg, typename I>
03802 struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03803     private:
03804         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03805             I>::type;
03806         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03807             I>::type;
03808     public:
03809         using type = typename polynomial<I>::template sub_t<
03810             // 2X Hn-1
03811             typename polynomial<I>::template mul_t<
03812                 typename pi64::val<typename I::template inject_constant_t<2>,
03813                 typename I::zero>, hnm1>,
03814                 typename polynomial<I>::template mul_t<
03815                     typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03816                     hnm2
03817                 >
03818             >
03819         >;
03820 };
03821
03822 template<typename I>
03823 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03824     using type = typename polynomial<I>::one;
03825 };
03826
03827 template<typename I>
03828 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {

```

```

03830         using type = typename polynomial<I>::X;
03831     };
03832
03833     template<typename I>
03834     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03835         using type = typename pi64::one;
03836     };
03837
03838     template<typename I>
03839     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03840         // 2X
03841         using type = typename polynomial<I>::template val<
03842             typename I::template inject_constant_t<2>,
03843             typename I::zero>;
03844     };
03845 } // namespace internal
03846
03847 // legendre
03848 namespace internal {
03849     template<size_t n, typename I>
03850     struct legendre_helper {
03851     private:
03852         using Q = FractionField<I>;
03853         using PQ = polynomial<Q>;
03854         // 1/n constant
03855         // (2n-1)/n X
03856         using fact_left = typename PQ::template monomial_t<
03857             makefraction_t<I,
03858                 typename I::template inject_constant_t<2*n-1>,
03859                 typename I::template inject_constant_t<n>
03860             >,
03861             1>;
03862         // (n-1) / n
03863         using fact_right = typename PQ::template val<
03864             makefraction_t<I,
03865                 typename I::template inject_constant_t<n-1>,
03866                 typename I::template inject_constant_t<n>>;
03867
03868     public:
03869         using type = PQ::template sub_t<
03870             typename PQ::template mul_t<
03871                 fact_left,
03872                 typename legendre_helper<n-1, I>::type
03873             >,
03874             typename PQ::template mul_t<
03875                 fact_right,
03876                 typename legendre_helper<n-2, I>::type
03877             >
03878         >;
03879     };
03880
03881     template<typename I>
03882     struct legendre_helper<0, I> {
03883         using type = typename polynomial<FractionField<I>::one>;
03884     };
03885
03886     template<typename I>
03887     struct legendre_helper<1, I> {
03888         using type = typename polynomial<FractionField<I>::X>;
03889     };
03890 } // namespace internal
03891
03892 // bernoulli polynomials
03893 namespace internal {
03894     template<size_t n>
03895     struct bernoulli_coeff {
03896         template<typename T, size_t i>
03897         struct inner {
03898         private:
03899             using F = FractionField<T>;
03900         public:
03901             using type = typename F::template mul_t<
03902                 typename F::template inject_ring_t<combination_t<T, i, n>,
03903                 bernoulli_t<T, n-i>
03904             >;
03905         };
03906     };
03907 } // namespace internal
03908
03909 namespace internal {
03910     template<size_t n>
03911     struct touchard_coeff {
03912         template<typename T, size_t i>
03913         struct inner {
03914             using type = stirling_2<T, n, i>;
03915         };
03916     };

```



```

03917     } // namespace internal
03918
03919     namespace internal {
03920         template<typename I = aerobus::i64>
03921         struct AbelHelper {
03922             private:
03923                 using P = aerobus::polynomial<I>;
03924
03925             public:
03926                 // to keep recursion working, we need to operate on a*n and not just a
03927                 template<size_t deg, I::inner_type an>
03928                 struct Inner {
03929                     // abel(n, a) = (x-an) * abel(n-1, a)
03930                     using type = typename aerobus::mul_t<
03931                         typename Inner<deg-1, an>::type,
03932                         typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
03933                     >;
03934                 };
03935
03936                 // abel(0, a) = 1
03937                 template<I::inner_type an>
03938                 struct Inner<0, an> {
03939                     using type = P::one;
03940                 };
03941
03942                 // abel(1, a) = X
03943                 template<I::inner_type an>
03944                 struct Inner<1, an> {
03945                     using type = P::X;
03946                 };
03947             };
03948         } // namespace internal
03949
03950         namespace known_polynomials {
03951
03952             template<size_t n, auto a, typename I = aerobus::i64>
03953             using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
03954
03955             template <size_t deg, typename I = aerobus::i64>
03956             using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03957
03958             template <size_t deg, typename I = aerobus::i64>
03959             using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03960
03961             template <size_t deg, typename I = aerobus::i64>
03962             using laguerre = typename internal::laguerre_helper<deg, I>::type;
03963
03964             template <size_t deg, typename I = aerobus::i64>
03965             using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03966 I::type>;
03967
03968             template <size_t deg, typename I = aerobus::i64>
03969             using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03970
03971             template<size_t i, size_t m, typename I = aerobus::i64>
03972             using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03973
03974             template<size_t deg, typename I = aerobus::i64>
03975             using legendre = typename internal::legendre_helper<deg, I>::type;
03976
03977             template<size_t deg, typename I = aerobus::i64>
03978             using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03979
03980             template<size_t deg, typename I = aerobus::i64>
03981             using allone = typename internal::AllOneHelper<deg, I>::type;
03982
03983             template<size_t deg, typename I = aerobus::i64>
03984             using bessel = typename internal::BesselHelper<deg, I>::type;
03985
03986             template<size_t deg, typename I = aerobus::i64>
03987             using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
03988         } // namespace known_polynomials
03989     } // namespace aerobus
03990
03991     #ifndef AEROBUS_CONWAY_IMPORTS
03992
03993     // conway polynomials
03994     namespace aerobus {
03995         template<int p, int n>
03996         struct ConwayPolynomial {};
03997
03998     #ifndef DO_NOT_DOCUMENT
03999         #define ZPV ZPV::template val
04000         #define POLYV aerobus::polynomial<ZPV>::template val
04001         template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<1>; }; // NOLINT

```


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9.4 src/examples.h File Reference

9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_
```


Chapter 10

Examples

10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

10.2 examples/custom_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
#include <cstdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

constexpr size_t N = 1 << 24;

/*
change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
to see how good is the generated assembly compared to what nvcc generates for 16 bits
*/
using int_type = aerobus::i16;
using float_type = __half2;

float rand(float min, float max) {
    float range = (max - min);
    float div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

using EXPM1 = aerobus::expm1<int_type, 6>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        out[i] = f(f(f(f(f(in[i]))))));
    }
}

int main() {
    __half2 *d_in, *d_out;
    cudaMalloc<__half2>(&d_in, N * sizeof(__half2));
    cudaMalloc<__half2>(&d_out, N * sizeof(__half2));

    __half2 *in = reinterpret_cast<__half2*>(malloc(N * sizeof(__half2)));
    __half2 *out = reinterpret_cast<__half2*>(malloc(N * sizeof(__half2)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = __half2(__float2half(rand(-0.01, 0.01)), __float2half(rand(-0.01, 0.01)));
    }

    cudaMemcpy(d_in, in, N * sizeof(__half2), cudaMemcpyHostToDevice);

    run<<128, 512>>(N, d_in, d_out);

    cudaMemcpy(out, d_out, N * sizeof(__half2), cudaMemcpyDeviceToHost);

    cudaFree(d_in);
    cudaFree(d_out);
}
```

10.4 examples/continued_fractions.cpp

How to use `aerobus::ContinuedFraction` to get approximations of known numbers

```
#include <cmath>
#include <iostream>
```

```
#include <iomanip>
#include "../src/aerobus.h"

static constexpr double PHI = aerobus::ContinuedFraction<
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1>::val;

static const double phi = (std::sqrt(5.0) + 1.0)/2.0;

int main() {
    std::cout << std::setprecision(15) << "Aerobus PHI : " << PHI << std::endl;
    std::cout << std::setprecision(15) << "Computed PHI : " << phi << std::endl;
    return 0;
}
```

10.5 examples/modular_arithmetic.cpp

How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

// x^3 + 2x^2 + 1, with coefficients in Z/2Z, actually x^3 + 1
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
// x^3 + 5x^2 + 7x + 11 with coefficients in Z/17Z, meaning actually x^3 + x^2 + 1
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

10.6 examples/make_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```

// how to build your own sequence of known polynomials, here Abel polynomials
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // for recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {

```

```

        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}

```

10.7 examples/polynomials_over_finite_field.cpp

How to build a known polynomial (here `aerobus::known_polynomials::allone`) with coefficients in a finite field (here `aerobus::zpz<2>`) and get its value when evaluated at a value in this field (here 1).

```

#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}

```

10.8 examples/compensated_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```

// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE INLINED T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE INLINED T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

```

```

}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;

        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}

```


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