

Aerobus

v1.2

Generated by Doxygen 1.9.8



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# Chapter 1

## Concept Index

### 1.1 Concepts

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# Class Index

### 2.1 Class List

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## Chapter 3

# File Index

### 3.1 File List

Here is a list of all documented files with brief descriptions:

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# Chapter 4

## Concept Documentation

### 4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 4.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 4.2.2 Detailed Description

Concept to express R is a field.

## 4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

### 4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

## Chapter 5

# Class Documentation

### 5.1 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

### 5.2 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

#### Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

### 5.3 `aerobus::polynomial< Ring, variable_name >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

#### Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

### 5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	
------------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

### Public Types

- using **type** = typename q64::template inject\_constant\_t< a0 >

### Static Public Attributes

- static constexpr double **val** = type::template get<double>()

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

### Public Types

- using **type** = q64::template add\_t< typename q64::template inject\_constant\_t< a0 >, typename q64::template div\_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >



### Static Public Attributes

- static constexpr double **val** = type::template get<double>()

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct **val**  
*values in i32*

### Public Types

- using **inner\_type** = int32\_t
- using **zero** = val< 0 >  
*constant zero*
- using **one** = val< 1 >  
*constant one*
- template<auto x>  
using **inject\_constant\_t** = val< static\_cast< int32\_t >(x)>
- template<typename v >  
using **inject\_ring\_t** = v
- template<typename v1 , typename v2 >  
using **add\_t** = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using **sub\_t** = typename sub< v1, v2 >::type  
*subtraction operator*
- template<typename v1 , typename v2 >  
using **mul\_t** = typename mul< v1, v2 >::type  
*multiplication operator*
- template<typename v1 , typename v2 >  
using **div\_t** = typename div< v1, v2 >::type  
*division operator*
- template<typename v1 , typename v2 >  
using **mod\_t** = typename remainder< v1, v2 >::type  
*modulus operator*
- template<typename v1 , typename v2 >  
using **gt\_t** = typename gt< v1, v2 >::type  
*strictly greater operator (v1 > v2)*

- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strict less operator ( $v1 < v2$ )*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v >`  
`using pos_t = typename pos< v >::type`  
*positivity ( $v > 0$ )*

### Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in i64*

## Public Types

- `using inner_type = int64_t`
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int64_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (v1 > v2)*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strict less operator (v1 < v2)*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i64, v1, v2 >`  
*greatest common divisor*
- `template<typename v >`  
`using pos_t = typename pos< v >::type`  
*is v positive*

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.9 `aerobus::polynomial< Ring, variable_name >::eval_helper< valueRing, P >::inner< index, stop >` Struct Template Reference

### Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.10 `aerobus::polynomial< Ring, variable_name >::eval_helper< valueRing, P >::inner< stop, stop >` Struct Template Reference

### Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.11 `aerobus::is_prime< n >` Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool **value** = `internal::_is_prime<n, 5>::value`  
*true iff n is prime*

### 5.11.1 Detailed Description

```
template<int32_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

## Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.12 aerobus::polynomial< Ring, variable\_name > Struct Template Reference

```
#include <aerobus.h>
```

## Classes

- struct [val](#)
- struct [val< coeffN >](#)

## Public Types

- [using zero](#) = [val< typename Ring::zero >](#)  
*constant zero*
- [using one](#) = [val< typename Ring::one >](#)  
*constant one*
- [using X](#) = [val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t](#) = [typename simplify< P >::type](#)  
*simplifies a polynomial (deletes highest degree if null, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t](#) = [typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t](#) = [typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t](#) = [typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t](#) = [typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t](#) = [typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t](#) = [typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*

- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::q_type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial :  $\text{coeff } X^{\text{deg}}$*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity ( $an > 0$ )*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`  
`Ring, variable_name >, v1, v2 >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

## Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 5.12.1 Detailed Description

```
template<typename Ring, char variable_name = 'x'>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring, variable_name >
```

polynomial with coefficients in Ring Ring must be an integral domain

### 5.12.2 Member Typedef Documentation

#### 5.12.2.1 add\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::add_t = typename add<v1, v2>::type
```

adds two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 5.12.2.2 derive\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::derive_t = typename derive_helper<v>::type
```

derivation operator

## Template Parameters

<i>v</i>	
----------	--

## 5.12.2.3 div\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::div_t = typename div<v1, v2>::q_type
```

division operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 5.12.2.4 eq\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 5.12.2.5 gcd\_t

```
template<typename Ring , char variable_name = 'x'>
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gcd_t = std::conditional_t< Ring::is_↵
euclidean_domain, typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2> >::type,
void>
```

greatest common divisor of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 5.12.2.6 gt\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 5.12.2.7 lt\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 5.12.2.8 mod\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mod_t = typename div_helper<v1, v2, zero,
v1>::mod_type
```

modulo operator



## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 5.12.2.9 monomial\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring, variable_name >::monomial_t = typename monomial<coeff, deg>↵
::type
```

monomial : coeff X^deg

## Template Parameters

<i>coeff</i>	
<i>deg</i>	

## 5.12.2.10 mul\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 5.12.2.11 pos\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename v >
using aerobus::polynomial< Ring, variable_name >::pos_t = typename Ring::template pos_t<typename↵
v::aN>
```

checks for positivity (an > 0)

## Template Parameters

<i>v</i>	
----------	--

### 5.12.2.12 `simplify_t`

```
template<typename Ring , char variable_name = 'x'>
template<typename P >
using aerobus::polynomial< Ring, variable_name >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (deletes highest degree if null, do nothing otherwise)

#### Template Parameters

<i>P</i>	
----------	--

### 5.12.2.13 `sub_t`

```
template<typename Ring , char variable_name = 'x'>
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring, variable_name >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.13 `aerobus::type_list< Ts >::pop_front` Struct Reference

### Public Types

- using **type** = `typename internal::pop_front_h< Ts... >::head`
- using **tail** = `typename internal::pop_front_h< Ts... >::tail`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.14 `aerobus::Quotient< Ring, X >` Struct Template Reference

### Classes

- struct `val`

### Public Types

- `using zero = val< typename Ring::zero >`
- `using one = val< typename Ring::one >`
- `template<typename v1 , typename v2 >`  
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`
- `template<typename v1 , typename v2 >`  
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`
- `template<typename v1 , typename v2 >`  
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`
- `template<typename v1 , typename v2 >`  
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
- `template<typename v1 >`  
`using pos_t = std::true_type`
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

### Static Public Attributes

- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`
- `static constexpr bool is_euclidean_domain = true`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.15 aerobus::type\_list< Ts >::split< index > Struct Template Reference

### Public Types

- `using head = typename inner::head`
- `using tail = typename inner::tail`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.16 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

## Classes

- struct [pop\\_front](#)
- struct [split](#)

## Public Types

- template<typename T >  
using **push\_front** = [type\\_list](#)< T, Ts... >
- template<uint64\_t index>  
using **at** = internal::type\_at\_t< index, Ts... >
- template<typename T >  
using **push\_back** = [type\\_list](#)< Ts..., T >
- template<typename U >  
using **concat** = typename concat\_h< U >::type
- template<uint64\_t index, typename T >  
using **insert** = typename internal::insert\_h< index, [type\\_list](#)< Ts... >, T >::type
- template<uint64\_t index>  
using **remove** = typename internal::remove\_h< index, [type\\_list](#)< Ts... > >::type

## Static Public Attributes

- static constexpr size\_t **length** = sizeof...(Ts)

### 5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.17 aerobus::type\_list<> Struct Reference

### Public Types

- template<typename T >  
using **push\_front** = [type\\_list](#)< T >
- template<typename T >  
using **push\_back** = [type\\_list](#)< T >
- template<typename U >  
using **concat** = U
- template<uint64\_t index, typename T >  
using **insert** = [type\\_list](#)< T >

**Static Public Attributes**

- static constexpr size\_t **length** = 0

The documentation for this struct was generated from the following file:

- src/aerobus.h

**5.18 aerobus::i32::val< x > Struct Template Reference**

values in [i32](#)

```
#include <aerobus.h>
```

**Public Types**

- [using is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

**Static Public Member Functions**

- template<typename [valueType](#) >  
[static constexpr valueType get](#) ()  
*cast x into valueType*
- [static](#) std::string [to\\_string](#) ()  
*string representation of value*
- template<typename [valueRing](#) >  
[static constexpr valueRing eval](#) (const [valueRing](#) &v)  
*cast x into valueRing*

**Static Public Attributes**

- [static constexpr int32\\_t v](#) = x

**5.18.1 Detailed Description**

```
template<int32\_t x>
struct aerobus::i32::val< x >
```

values in [i32](#)

**Template Parameters**

<a href="#">x</a>	an actual integer
-------------------	-------------------

## 5.18.2 Member Function Documentation

### 5.18.2.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<i>valueRing</i>	double for example
------------------	--------------------

### 5.18.2.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.19 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- `using is_zero_t = std::bool_constant< x==0 >`  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`  
*cast value in valueType*
- `static std::string to_string ()`  
*string representation*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`  
*cast value in valueRing*

## Static Public Attributes

- `static constexpr int64_t v = x`

### 5.19.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

#### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

### 5.19.2 Member Function Documentation

#### 5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

#### Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

#### 5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType



## Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.20 aerobus::polynomial< Ring, variable\_name >::val< coeffN, coeffs > Struct Template Reference

### Public Types

- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

### Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

### Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*

## 5.20.1 Member Typedef Documentation

### 5.20.1.1 coeff\_at\_t

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::coeff_at_t = typename
coeff_at<index>::type
```

type of coefficient at index

## Template Parameters

<i>index</i>	
--------------	--

## 5.20.2 Member Function Documentation

### 5.20.2.1 eval()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs
>::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

## Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

## Parameters

<i>x</i>	value
----------	-------

## Returns

$P(x)$

### 5.20.2.2 to\_string()

```
template<typename Ring , char variable_name = 'x'>
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring, variable_name >::val< coeffN, coeffs >::to_↵
string ( ) [inline], [static]
```

get a string representation of polynomial

## Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

### Public Types

- using **type** = std::conditional\_t< Ring::template pos\_v< tmp >, tmp, [typename](#) Ring::template sub\_t< [typename](#) Ring::zero, tmp > >

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.22 aerobus::zpz< p >::val< x > Struct Template Reference

### Public Types

- using **is\_zero\_t** = std::bool\_constant< x% p==0 >

### Static Public Member Functions

- template<typename valueType >  
static constexpr valueType **get** ()
- static std::string **to\_string** ()
- template<typename valueRing >  
static constexpr valueRing **eval** (const valueRing &v)

### Static Public Attributes

- static constexpr int32\_t **v** = x % p

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.23 aerobus::polynomial< Ring, variable\_name >::val< coeffN > Struct Template Reference

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at](#)< index, std::enable\_if\_t<(index< 0||index > 0)> >
- struct [coeff\\_at](#)< index, std::enable\_if\_t<(index==0)> >

**Public Types**

- using **aN** = `coeffN`
- using **strip** = `val< coeffN >`
- using **is\_zero\_t** = `std::bool_constant< aN::is_zero_t::value >`
- template<size\_t index>  
using **coeff\_at\_t** = `typename coeff_at< index >::type`

**Static Public Member Functions**

- static `std::string to_string ()`
- template<typename valueRing >  
static `constexpr valueRing eval (const valueRing &x)`

**Static Public Attributes**

- static `constexpr size_t degree = 0`
- static `constexpr bool is_zero_v = is_zero_t::value`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

**5.24 aerobus::zpz< p > Struct Template Reference**

```
#include <aerobus.h>
```

**Classes**

- struct `val`

**Public Types**

- using **inner\_type** = `int32_t`
- template<auto x>  
using **inject\_constant\_t** = `val< static_cast< int32_t >(x)>`
- using **zero** = `val< 0 >`
- using **one** = `val< 1 >`
- template<typename v1, typename v2 >  
using **add\_t** = `typename add< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **sub\_t** = `typename sub< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **mul\_t** = `typename mul< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **div\_t** = `typename div< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **mod\_t** = `typename remainder< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **gt\_t** = `typename gt< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **lt\_t** = `typename lt< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **eq\_t** = `typename eq< v1, v2 >::type`
- template<typename v1, typename v2 >  
using **gcd\_t** = `gcd_t< i32, v1, v2 >`
- template<typename v1 >  
using **pos\_t** = `typename pos< v1 >::type`

### Static Public Attributes

- static constexpr bool **is\_field** = [is\\_prime](#)<p>::value
- static constexpr bool **is\_euclidean\_domain** = true
- template<typename v >  
static constexpr bool **pos\_v** = pos\_t<v>::value

#### 5.24.1 Detailed Description

```
template<int32_t p>  
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- src/aerobus.h



## Chapter 6

# File Documentation

### 6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035
00036     template<typename T, size_t N>
00037     constexpr bool contains(const std::array<T, N>& arr, const T& v) {
00038         for (const auto& vv : arr) {
00039             if (v == vv) {
00040                 return true;
00041             }
00042         }
00043         return false;
00044     }
00045 } // namespace aerobus
00046
00047 // concepts
00048 namespace aerobus {
00049     template <typename R>
00050     concept IsRing = requires {
00051         typename R::one;
00052         typename R::zero;
00053         typename R::template add_t<typename R::one, typename R::one>;
00054         typename R::template sub_t<typename R::one, typename R::one>;
00055         typename R::template mul_t<typename R::one, typename R::one>;
00056     };
00057 }
```

```

00074
00075
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00089     template<typename R>
00090     concept IsField = IsEuclideanDomain<R> && requires {
00091         R::is_field == true;
00092     };
00093 } // namespace aerobus
00094
00095 // utilities
00096 namespace aerobus {
00097     namespace internal {
00098         template<typename...> typename TT, typename T>
00099         struct is_instantiation_of : std::false_type {};
00100
00101         template<template<typename...> typename TT, typename... Ts>
00102         struct is_instantiation_of<TT, TT<Ts...> : std::true_type {};
00103
00104         template<template<typename...> typename TT, typename T>
00105         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00106
00107         template <int64_t i, typename T, typename... Ts>
00108         struct type_at {
00109             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00110             using type = typename type_at<i - 1, Ts...>::type;
00111         };
00112
00113         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00114             using type = T;
00115         };
00116
00117         template <size_t i, typename... Ts>
00118         using type_at_t = typename type_at<i, Ts...>::type;
00119
00120         template<int32_t n, int32_t i, typename E = void>
00121         struct _is_prime {};
00122
00123         template<int32_t i>
00124         struct _is_prime<1, i> {
00125             static constexpr bool value = false;
00126         };
00127
00128         template<int32_t i>
00129         struct _is_prime<2, i> {
00130             static constexpr bool value = true;
00131         };
00132
00133         template<int32_t i>
00134         struct _is_prime<3, i> {
00135             static constexpr bool value = true;
00136         };
00137
00138         template<int32_t i>
00139         struct _is_prime<5, i> {
00140             static constexpr bool value = true;
00141         };
00142
00143         template<int32_t i>
00144         struct _is_prime<7, i> {
00145             static constexpr bool value = true;
00146         };
00147
00148         template<int32_t n, int32_t i>
00149         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00150             static constexpr bool value = false;
00151         };
00152
00153         template<int32_t n, int32_t i>
00154         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00155             static constexpr bool value = false;
00156         };
00157
00158         template<int32_t n, int32_t i>
00159         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00160             static constexpr bool value = true;
00161         };
00162

```



```

00163     };
00164
00165     template<int32_t n, int32_t i>
00166     struct _is_prime<n, i, std::enable_if_t<(
00167         n % i == 0 &&
00168         n >= 9 &&
00169         n % 3 != 0 &&
00170         n % 2 != 0 &&
00171         i * i > n)> {
00172         static constexpr bool value = true;
00173     };
00174
00175     template<int32_t n, int32_t i>
00176     struct _is_prime<n, i, std::enable_if_t<(
00177         n % (i+2) == 0 &&
00178         n >= 9 &&
00179         n % 3 != 0 &&
00180         n % 2 != 0 &&
00181         i * i <= n)> {
00182         static constexpr bool value = true;
00183     };
00184
00185     template<int32_t n, int32_t i>
00186     struct _is_prime<n, i, std::enable_if_t<(
00187         n % (i+2) != 0 &&
00188         n % i != 0 &&
00189         n >= 9 &&
00190         n % 3 != 0 &&
00191         n % 2 != 0 &&
00192         (i * i <= n))> {
00193         static constexpr bool value = _is_prime<n, i+6>::value;
00194     };
00195
00196     } // namespace internal
00197
00200     template<int32_t n>
00201     struct is_prime {
00202         static constexpr bool value = internal::_is_prime<n, 5>::value;
00203     };
00204
00205     template<int32_t n>
00206     static constexpr bool is_prime_v = is_prime<n>::value;
00207
00208     namespace internal {
00209         template <std::size_t... Is>
00210         constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00211             -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00212
00213         template <std::size_t N>
00214         using make_index_sequence_reverse
00215             = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00216
00217         template<typename Ring, typename E = void>
00218         struct gcd;
00219
00220         template<typename Ring>
00221         struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00222             template<typename A, typename B, typename E = void>
00223             struct gcd_helper {};
00224
00225             // B = 0, A > 0
00226             template<typename A, typename B>
00227             struct gcd_helper<A, B, std::enable_if_t<
00228                 (B::is_zero_t::value) &&
00229                 (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00230                 using type = A;
00231             };
00232
00233             // B = 0, A < 0
00234             template<typename A, typename B>
00235             struct gcd_helper<A, B, std::enable_if_t<
00236                 (B::is_zero_t::value) &&
00237                 !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00238                 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00239             };
00240
00241             // B != 0
00242             template<typename A, typename B>
00243             struct gcd_helper<A, B, std::enable_if_t<
00244                 (!B::is_zero_t::value)
00245                 >> {
00246             private: // NOLINT
00247                 // A / B
00248                 using k = typename Ring::template div_t<A, B>;
00249                 // A - (A/B)*B = A % B
00250                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00251             };
00252         };
00253     }

```

```

00258         public:
00259             using type = typename gcd_helper<B, m>::type;
00260         };
00261
00262         template<typename A, typename B>
00263         using type = typename gcd_helper<A, B>::type;
00264     };
00265 } // namespace internal
00266
00267 template<typename T, typename A, typename B>
00270 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00271 } // namespace aerobus
00272
00273 // quotient ring by the principal ideal generated by X
00274 namespace aerobus {
00275     template<typename Ring, typename X>
00276     requires IsRing<Ring>
00277     struct Quotient {
00278         template <typename V>
00279         struct val {
00280             private: // NOLINT
00281                 using tmp = typename Ring::template mod_t<V, X>;
00282
00283             public:
00284                 using type = std::conditional_t<
00285                     Ring::template pos_v<tmp>,
00286                     tmp,
00287                     typename Ring::template sub_t<typename Ring::zero, tmp>
00288                 >;
00289         };
00290
00291         using zero = val<typename Ring::zero>;
00292         using one = val<typename Ring::one>;
00293
00294         template<typename v1, typename v2>
00295         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00296         template<typename v1, typename v2>
00297         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00298         template<typename v1, typename v2>
00299         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00300         template<typename v1, typename v2>
00301         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00302         template<typename v1, typename v2>
00303         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00304         template<typename v1, typename v2>
00305         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00306         template<typename v1>
00307         using pos_t = std::true_type;
00308
00309         template<typename v>
00310         static constexpr bool pos_v = pos_t<v>::value;
00311
00312         static constexpr bool is_euclidean_domain = true;
00313
00314         template<auto x>
00315         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00316
00317         template<typename v>
00318         using inject_ring_t = val<v>;
00319     };
00320 } // namespace aerobus
00321
00322 // type_list
00323 namespace aerobus {
00324     template <typename... Ts>
00325     struct type_list;
00326
00327     namespace internal {
00328         template <typename T, typename... Us>
00329         struct pop_front_h {
00330             using tail = type_list<Us...>;
00331             using head = T;
00332         };
00333
00334         template <uint64_t index, typename L1, typename L2>
00335         struct split_h {
00336             private:
00337                 static_assert(index <= L2::length, "index out of bounds");
00338                 using a = typename L2::pop_front::type;
00339                 using b = typename L2::pop_front::tail;
00340                 using c = typename L1::template push_back<a>;
00341
00342             public:
00343                 using head = typename split_h<index - 1, c, b>::head;
00344                 using tail = typename split_h<index - 1, c, b>::tail;
00345         };
00346     }
00347

```

```

00348     template <typename L1, typename L2>
00349     struct split_h<0, L1, L2> {
00350         using head = L1;
00351         using tail = L2;
00352     };
00353
00354     template <uint64_t index, typename L, typename T>
00355     struct insert_h {
00356         static_assert(index <= L::length, "index out of bounds");
00357         using s = typename L::template split<index>;
00358         using left = typename s::head;
00359         using right = typename s::tail;
00360         using ll = typename left::template push_back<T>;
00361         using type = typename ll::template concat<right>;
00362     };
00363
00364     template <uint64_t index, typename L>
00365     struct remove_h {
00366         using s = typename L::template split<index>;
00367         using left = typename s::head;
00368         using right = typename s::tail;
00369         using rr = typename right::pop_front::tail;
00370         using type = typename left::template concat<rr>;
00371     };
00372 } // namespace internal
00373
00374 template <typename... Ts>
00375 struct type_list {
00376 private:
00377     template <typename T>
00378     struct concat_h;
00379
00380     template <typename... Us>
00381     struct concat_h<type_list<Us...> {
00382         using type = type_list<Ts..., Us...>;
00383     };
00384
00385 public:
00386     static constexpr size_t length = sizeof...(Ts);
00387
00388     template <typename T>
00389     using push_front = type_list<T, Ts...>;
00390
00391     template <uint64_t index>
00392     using at = internal::type_at_t<index, Ts...>;
00393
00394     struct pop_front {
00395         using type = typename internal::pop_front_h<Ts...>::head;
00396         using tail = typename internal::pop_front_h<Ts...>::tail;
00397     };
00398
00399     template <typename T>
00400     using push_back = type_list<Ts..., T>;
00401
00402     template <typename U>
00403     using concat = typename concat_h<U>::type;
00404
00405     template <uint64_t index>
00406     struct split {
00407     private:
00408         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00409
00410     public:
00411         using head = typename inner::head;
00412         using tail = typename inner::tail;
00413     };
00414
00415     template <uint64_t index, typename T>
00416     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00417
00418     template <uint64_t index>
00419     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00420 };
00421
00422 template <>
00423 struct type_list<> {
00424     static constexpr size_t length = 0;
00425
00426     template <typename T>
00427     using push_front = type_list<T>;
00428
00429     template <typename T>
00430     using push_back = type_list<T>;
00431
00432     template <typename U>
00433     using concat = U;
00434

```

```

00435         // TODO(jewave): assert index == 0
00436         template <uint64_t index, typename T>
00437         using insert = type_list<T>;
00438     };
00439 } // namespace aerobus
00440
00441 // i32
00442 namespace aerobus {
00443     struct i32 {
00444         using inner_type = int32_t;
00445         template<int32_t x>
00446         struct val {
00447             static constexpr int32_t v = x;
00448
00449             template<typename valueType>
00450             static constexpr valueType get() { return static_cast<valueType>(x); }
00451
00452             using is_zero_t = std::bool_constant<x == 0>;
00453
00454             static std::string to_string() {
00455                 return std::to_string(x);
00456             }
00457
00458             template<typename valueRing>
00459             static constexpr valueRing eval(const valueRing& v) {
00460                 return static_cast<valueRing>(x);
00461             }
00462         };
00463
00464         using zero = val<0>;
00465         using one = val<1>;
00466         static constexpr bool is_field = false;
00467         static constexpr bool is_euclidean_domain = true;
00468         template<auto x>
00469         using inject_constant_t = val<static_cast<int32_t>(x)>;
00470
00471         template<typename v>
00472         using inject_ring_t = v;
00473
00474     private:
00475         template<typename v1, typename v2>
00476         struct add {
00477             using type = val<v1::v + v2::v>;
00478         };
00479
00480         template<typename v1, typename v2>
00481         struct sub {
00482             using type = val<v1::v - v2::v>;
00483         };
00484
00485         template<typename v1, typename v2>
00486         struct mul {
00487             using type = val<v1::v * v2::v>;
00488         };
00489
00490         template<typename v1, typename v2>
00491         struct div {
00492             using type = val<v1::v / v2::v>;
00493         };
00494
00495         template<typename v1, typename v2>
00496         struct remainder {
00497             using type = val<v1::v % v2::v>;
00498         };
00499
00500         template<typename v1, typename v2>
00501         struct gt {
00502             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00503         };
00504
00505         template<typename v1, typename v2>
00506         struct lt {
00507             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00508         };
00509
00510         template<typename v1, typename v2>
00511         struct eq {
00512             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00513         };
00514
00515         template<typename v1>
00516         struct pos {
00517             using type = std::bool_constant<(v1::v > 0)>;
00518         };
00519
00520     public:
00521         template<typename v1, typename v2>

```

```

00539     using add_t = typename add<v1, v2>::type;
00540
00542     template<typename v1, typename v2>
00543     using sub_t = typename sub<v1, v2>::type;
00544
00546     template<typename v1, typename v2>
00547     using mul_t = typename mul<v1, v2>::type;
00548
00550     template<typename v1, typename v2>
00551     using div_t = typename div<v1, v2>::type;
00552
00554     template<typename v1, typename v2>
00555     using mod_t = typename remainder<v1, v2>::type;
00556
00558     template<typename v1, typename v2>
00559     using gt_t = typename gt<v1, v2>::type;
00560
00562     template<typename v1, typename v2>
00563     using lt_t = typename lt<v1, v2>::type;
00564
00566     template<typename v1, typename v2>
00567     using eq_t = typename eq<v1, v2>::type;
00568
00570     template<typename v1, typename v2>
00571     using gcd_t = gcd_t<i32, v1, v2>;
00572
00574     template<typename v>
00575     using pos_t = typename pos<v>::type;
00576
00577     template<typename v>
00578     static constexpr bool pos_v = pos_t<v>::value;
00579 };
00580 } // namespace aerobus
00581
00582 // i64
00583 namespace aerobus {
00584     struct i64 {
00585         using inner_type = int64_t;
00586         template<int64_t x>
00587         struct val {
00588             static constexpr int64_t v = x;
00589
00590             template<typename valueType>
00591             static constexpr valueType get() { return static_cast<valueType>(x); }
00592
00593             using is_zero_t = std::bool_constant<x == 0>;
00594
00595             static std::string to_string() {
00596                 return std::to_string(x);
00597             }
00598
00599             template<typename valueRing>
00600             static constexpr valueRing eval(const valueRing& v) {
00601                 return static_cast<valueRing>(x);
00602             }
00603         };
00604     };
00605
00607     template<auto x>
00608     using inject_constant_t = val<static_cast<int64_t>(x)>;
00609
00611     template<typename v>
00612     using inject_ring_t = v;
00613
00615     using zero = val<0>;
00616     using one = val<1>;
00617     static constexpr bool is_field = false;
00618     static constexpr bool is_euclidean_domain = true;
00619
00621 private:
00622     template<typename v1, typename v2>
00623     struct add {
00624         using type = val<v1::v + v2::v>;
00625     };
00626
00628     template<typename v1, typename v2>
00629     struct sub {
00630         using type = val<v1::v - v2::v>;
00631     };
00632
00634     template<typename v1, typename v2>
00635     struct mul {
00636         using type = val<v1::v * v2::v>;
00637     };
00638
00640     template<typename v1, typename v2>
00641     struct div {
00642         using type = val<v1::v / v2::v>;
00643     };

```

```

00651     };
00652
00653     template<typename v1, typename v2>
00654     struct remainder {
00655         using type = val<v1::v% v2::v>;
00656     };
00657
00658     template<typename v1, typename v2>
00659     struct gt {
00660         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00661     };
00662
00663     template<typename v1, typename v2>
00664     struct lt {
00665         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00666     };
00667
00668     template<typename v1, typename v2>
00669     struct eq {
00670         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00671     };
00672
00673     template<typename v>
00674     struct pos {
00675         using type = std::bool_constant<(v::v > 0)>;
00676     };
00677
00678     public:
00679         template<typename v1, typename v2>
00680         using add_t = typename add<v1, v2>::type;
00681
00682         template<typename v1, typename v2>
00683         using sub_t = typename sub<v1, v2>::type;
00684
00685         template<typename v1, typename v2>
00686         using mul_t = typename mul<v1, v2>::type;
00687
00688         template<typename v1, typename v2>
00689         using div_t = typename div<v1, v2>::type;
00690
00691         template<typename v1, typename v2>
00692         using mod_t = typename remainder<v1, v2>::type;
00693
00694         template<typename v1, typename v2>
00695         using gt_t = typename gt<v1, v2>::type;
00696
00697         template<typename v1, typename v2>
00698         using lt_t = typename lt<v1, v2>::type;
00699
00700         template<typename v1, typename v2>
00701         using eq_t = typename eq<v1, v2>::type;
00702
00703         template<typename v1, typename v2>
00704         using gcd_t = gcd_t<i64, v1, v2>;
00705
00706         template<typename v>
00707         using pos_t = typename pos<v>::type;
00708
00709         template<typename v>
00710         static constexpr bool pos_v = pos_t<v>::value;
00711     };
00712 } // namespace aerobus
00713
00714 // z/pz
00715 namespace aerobus {
00716     template<int32_t p>
00717     struct zp {
00718         using inner_type = int32_t;
00719         template<int32_t x>
00720         struct val {
00721             static constexpr int32_t v = x % p;
00722
00723             template<typename valueType>
00724             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00725
00726             using is_zero_t = std::bool_constant<x% p == 0>;
00727             static std::string to_string() {
00728                 return std::to_string(x % p);
00729             }
00730
00731             template<typename valueRing>
00732             static constexpr valueRing eval(const valueRing& v) {
00733                 return static_cast<valueRing>(x % p);
00734             }
00735         };
00736     };
00737
00738     template<auto x>

```

```

00752         using inject_constant_t = val<static_cast<int32_t>(x)>;
00753
00754         using zero = val<0>;
00755         using one = val<1>;
00756         static constexpr bool is_field = is_prime<p>::value;
00757         static constexpr bool is_euclidean_domain = true;
00758
00759     private:
00760         template<typename v1, typename v2>
00761         struct add {
00762             using type = val<(v1::v + v2::v) % p>;
00763         };
00764
00765         template<typename v1, typename v2>
00766         struct sub {
00767             using type = val<(v1::v - v2::v) % p>;
00768         };
00769
00770         template<typename v1, typename v2>
00771         struct mul {
00772             using type = val<(v1::v * v2::v) % p>;
00773         };
00774
00775         template<typename v1, typename v2>
00776         struct div {
00777             using type = val<(v1::v % p) / (v2::v % p)>;
00778         };
00779
00780         template<typename v1, typename v2>
00781         struct remainder {
00782             using type = val<(v1::v % v2::v) % p>;
00783         };
00784
00785         template<typename v1, typename v2>
00786         struct gt {
00787             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00788         };
00789
00790         template<typename v1, typename v2>
00791         struct lt {
00792             using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00793         };
00794
00795         template<typename v1, typename v2>
00796         struct eq {
00797             using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00798         };
00799
00800         template<typename v1>
00801         struct pos {
00802             using type = std::bool_constant<(v1::v > 0)>;
00803         };
00804
00805     public:
00806         template<typename v1, typename v2>
00807         using add_t = typename add<v1, v2>::type;
00808
00809         template<typename v1, typename v2>
00810         using sub_t = typename sub<v1, v2>::type;
00811
00812         template<typename v1, typename v2>
00813         using mul_t = typename mul<v1, v2>::type;
00814
00815         template<typename v1, typename v2>
00816         using div_t = typename div<v1, v2>::type;
00817
00818         template<typename v1, typename v2>
00819         using mod_t = typename remainder<v1, v2>::type;
00820
00821         template<typename v1, typename v2>
00822         using gt_t = typename gt<v1, v2>::type;
00823
00824         template<typename v1, typename v2>
00825         using lt_t = typename lt<v1, v2>::type;
00826
00827         template<typename v1, typename v2>
00828         using eq_t = typename eq<v1, v2>::type;
00829
00830         template<typename v1, typename v2>
00831         using gcd_t = gcd_t<i32, v1, v2>;
00832
00833         template<typename v1>
00834         using pos_t = typename pos<v1>::type;
00835
00836         template<typename v>
00837         static constexpr bool pos_v = pos_t<v>::value;
00838     };

```

```

00839 } // namespace aerobus
00840
00841 // polynomial
00842 namespace aerobus {
00843     // coeffN x^N + ...
00844     template<typename Ring, char variable_name = 'x'>
00845     requires IsEuclideanDomain<Ring>
00850     struct polynomial {
00851         static constexpr bool is_field = false;
00852         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
00853
00854         template<typename coeffN, typename... coeffs>
00855         struct val {
00856             static constexpr size_t degree = sizeof...(coeffs);
00857             using aN = coeffN;
00858             using strip = val<coeffs...>;
00859             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
00860
00861         private:
00862             template<size_t index, typename E = void>
00863             struct coeff_at {};
00864
00865             template<size_t index>
00866             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))>> {
00867                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
00868             };
00869
00870             template<size_t index>
00871             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))>> {
00872                 using type = typename Ring::zero;
00873             };
00874
00875         public:
00876             template<size_t index>
00877             using coeff_at_t = typename coeff_at<index>::type;
00878
00879             static std::string to_string() {
00880                 return string_helper<coeffN, coeffs...>::func();
00881             }
00882
00883             template<typename valueRing>
00884             static constexpr valueRing eval(const valueRing& x) {
00885                 return eval_helper<valueRing, val>::template inner<0, degree +
00886                 1>::func(static_cast<valueRing>(0), x);
00887             }
00888         };
00889     };
00890
00891     // specialization for constants
00892     template<typename coeffN>
00893     struct val<coeffN> {
00894         static constexpr size_t degree = 0;
00895         using aN = coeffN;
00896         using strip = val<coeffN>;
00897         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
00898
00899         static constexpr bool is_zero_v = is_zero_t::value;
00900
00901         template<size_t index, typename E = void>
00902         struct coeff_at {};
00903
00904         template<size_t index>
00905         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
00906             using type = aN;
00907         };
00908
00909         template<size_t index>
00910         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
00911             using type = typename Ring::zero;
00912         };
00913
00914         template<size_t index>
00915         using coeff_at_t = typename coeff_at<index>::type;
00916
00917         static std::string to_string() {
00918             return string_helper<coeffN>::func();
00919         }
00920
00921         template<typename valueRing>
00922         static constexpr valueRing eval(const valueRing& x) {
00923             return static_cast<valueRing>(aN::template get<valueRing>());
00924         }
00925     };
00926
00927     using zero = val<typename Ring::zero>;
00928     using one = val<typename Ring::one>;
00929     using X = val<typename Ring::one, typename Ring::zero>;
00930
00931 }

```



```

00944     private:
00945         template<typename P, typename E = void>
00946             struct simplify;
00947
00948         template <typename P1, typename P2, typename I>
00949             struct add_low;
00950
00951         template<typename P1, typename P2>
00952             struct add {
00953                 using type = typename simplify<typename add_low<
00954                     P1,
00955                     P2,
00956                     internal::make_index_sequence_reverse<
00957                         std::max(P1::degree, P2::degree) + 1
00958                     >::type>::type;
00959             };
00960
00961         template <typename P1, typename P2, typename I>
00962             struct sub_low;
00963
00964         template <typename P1, typename P2, typename I>
00965             struct mul_low;
00966
00967         template<typename v1, typename v2>
00968             struct mul {
00969                 using type = typename mul_low<
00970                     v1,
00971                     v2,
00972                     internal::make_index_sequence_reverse<
00973                         v1::degree + v2::degree + 1
00974                     >::type;
00975             };
00976
00977         template<typename coeff, size_t deg>
00978             struct monomial;
00979
00980         template<typename v, typename E = void>
00981             struct derive_helper {};
00982
00983         template<typename v>
00984             struct derive_helper<v, std::enable_if_t<v::degree == 0> {
00985                 using type = zero;
00986             };
00987
00988         template<typename v>
00989             struct derive_helper<v, std::enable_if_t<v::degree != 0> {
00990                 using type = typename add<
00991                     typename derive_helper<typename simplify<typename v::strip>::type>::type,
00992                     typename monomial<
00993                         typename Ring::template mul_t<
00994                             typename v::aN,
00995                             typename Ring::template inject_constant_t<(v::degree)>
00996                         >,
00997                         v::degree - 1
00998                     >::type
00999                 >::type;
01000             };
01001
01002         template<typename v1, typename v2, typename E = void>
01003             struct eq_helper {};
01004
01005         template<typename v1, typename v2>
01006             struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01007                 using type = std::false_type;
01008             };
01009
01010
01011         template<typename v1, typename v2>
01012             struct eq_helper<v1, v2, std::enable_if_t<
01013                 v1::degree == v2::degree &&
01014                 (v1::degree != 0 || v2::degree != 0) &&
01015                 std::is_same<
01016                     typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01017                     std::false_type
01018                 >::value
01019             > {
01020             > {
01021                 using type = std::false_type;
01022             };
01023
01024         template<typename v1, typename v2>
01025             struct eq_helper<v1, v2, std::enable_if_t<
01026                 v1::degree == v2::degree &&
01027                 (v1::degree != 0 || v2::degree != 0) &&
01028                 std::is_same<
01029                     typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01030                     std::true_type

```

```

01031         >::value
01032     » {
01033         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01034     };
01035
01036     template<typename v1, typename v2>
01037     struct eq_helper<v1, v2, std::enable_if_t<
01038         v1::degree == v2::degree &&
01039         (v1::degree == 0)>
01040     » {
01041         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01042     };
01043
01044     template<typename v1, typename v2, typename E = void>
01045     struct lt_helper {};
01046
01047     template<typename v1, typename v2>
01048     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01049         using type = std::true_type;
01050     };
01051
01052     template<typename v1, typename v2>
01053     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01054         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01055     };
01056
01057     template<typename v1, typename v2>
01058     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01059         using type = std::false_type;
01060     };
01061
01062     template<typename v1, typename v2, typename E = void>
01063     struct gt_helper {};
01064
01065     template<typename v1, typename v2>
01066     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01067         using type = std::true_type;
01068     };
01069
01070     template<typename v1, typename v2>
01071     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01072         using type = std::false_type;
01073     };
01074
01075     template<typename v1, typename v2>
01076     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01077         using type = std::false_type;
01078     };
01079
01080     // when high power is zero : strip
01081     template<typename P>
01082     struct simplify<P, std::enable_if_t<
01083         std::is_same<
01084             typename Ring::zero,
01085             typename P::aN
01086         >::value && (P::degree > 0)>
01087     » {
01088         using type = typename simplify<typename P::strip>::type;
01089     };
01090
01091     // otherwise : do nothing
01092     template<typename P>
01093     struct simplify<P, std::enable_if_t<
01094         !std::is_same<
01095             typename Ring::zero,
01096             typename P::aN
01097         >::value && (P::degree > 0)>
01098     » {
01099         using type = P;
01100     };
01101
01102     // do not simplify constants
01103     template<typename P>
01104     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01105         using type = P;
01106     };
01107
01108     // addition at
01109     template<typename P1, typename P2, size_t index>
01110     struct add_at {
01111         using type =
01112             typename Ring::template add_t<
01113                 typename P1::template coeff_at_t<index>,
01114                 typename P2::template coeff_at_t<index>>;
01115     };
01116
01117     template<typename P1, typename P2, size_t index>

```

```

01118     using add_at_t = typename add_at<P1, P2, index>::type;
01119
01120     template<typename P1, typename P2, std::size_t... I>
01121     struct add_low<P1, P2, std::index_sequence<I...> {
01122         using type = val<add_at_t<P1, P2, I>...>;
01123     };
01124
01125     // subtraction at
01126     template<typename P1, typename P2, size_t index>
01127     struct sub_at {
01128         using type =
01129             typename Ring::template sub_t<
01130                 typename P1::template coeff_at_t<index>,
01131                 typename P2::template coeff_at_t<index>>;
01132     };
01133
01134     template<typename P1, typename P2, size_t index>
01135     using sub_at_t = typename sub_at<P1, P2, index>::type;
01136
01137     template<typename P1, typename P2, std::size_t... I>
01138     struct sub_low<P1, P2, std::index_sequence<I...> {
01139         using type = val<sub_at_t<P1, P2, I>...>;
01140     };
01141
01142     template<typename P1, typename P2>
01143     struct sub {
01144         using type = typename simplify<typename sub_low<
01145             P1,
01146             P2,
01147             internal::make_index_sequence_reverse<
01148                 std::max(P1::degree, P2::degree) + 1
01149             >::type>::type;
01150     };
01151
01152     // multiplication at
01153     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01154     struct mul_at_loop_helper {
01155         using type = typename Ring::template add_t<
01156             typename Ring::template mul_t<
01157                 typename v1::template coeff_at_t<index>,
01158                 typename v2::template coeff_at_t<k - index>
01159             >,
01160             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01161         >;
01162     };
01163
01164     template<typename v1, typename v2, size_t k, size_t stop>
01165     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01166         using type = typename Ring::template mul_t<
01167             typename v1::template coeff_at_t<stop>,
01168             typename v2::template coeff_at_t<0>>;
01169     };
01170
01171     template<typename v1, typename v2, size_t k, typename E = void>
01172     struct mul_at {};
01173
01174     template<typename v1, typename v2, size_t k>
01175     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01176         using type = typename Ring::zero;
01177     };
01178
01179     template<typename v1, typename v2, size_t k>
01180     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01181         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01182     };
01183
01184     template<typename P1, typename P2, size_t index>
01185     using mul_at_t = typename mul_at<P1, P2, index>::type;
01186
01187     template<typename P1, typename P2, std::size_t... I>
01188     struct mul_low<P1, P2, std::index_sequence<I...> {
01189         using type = val<mul_at_t<P1, P2, I>...>;
01190     };
01191
01192     // division helper
01193     template<typename A, typename B, typename Q, typename R, typename E = void>
01194     struct div_helper {};
01195
01196     template<typename A, typename B, typename Q, typename R>
01197     struct div_helper<A, B, Q, R, std::enable_if_t<
01198         (R::degree < B::degree) ||
01199         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01200         using q_type = Q;
01201         using mod_type = R;
01202         using gcd_type = B;
01203     };
01204

```

```

01205     template<typename A, typename B, typename Q, typename R>
01206     struct div_helper<A, B, Q, R, std::enable_if_t<
01207         (R::degree >= B::degree) &&
01208         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01209     private: // NOLINT
01210         using rN = typename R::aN;
01211         using bN = typename B::aN;
01212         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
B::degree>::type;
01213         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01214         using qq = typename add<Q, pT>::type;
01215
01216     public:
01217         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01218         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01219         using gcd_type = rr;
01220     };
01221
01222     template<typename A, typename B>
01223     struct div {
01224         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01225         using q_type = typename div_helper<A, B, zero, A>::q_type;
01226         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01227     };
01228
01229     template<typename P>
01230     struct make_unit {
01231         using type = typename div<P, val<typename P::aN>::q_type>;
01232     };
01233
01234     template<typename coeff, size_t deg>
01235     struct monomial {
01236         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01237     };
01238
01239     template<typename coeff>
01240     struct monomial<coeff, 0> {
01241         using type = val<coeff>;
01242     };
01243
01244     template<typename valueRing, typename P>
01245     struct eval_helper {
01246         template<size_t index, size_t stop>
01247         struct inner {
01248             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01249                 constexpr valueRing coeff =
01250                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
get<valueRing>());
01251                 return eval_helper<valueRing, P>::template inner<index + 1, stop>::func(x * accum
+ coeff, x);
01252             }
01253         };
01254
01255         template<size_t stop>
01256         struct inner<stop, stop> {
01257             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01258                 return accum;
01259             }
01260         };
01261     };
01262
01263     template<typename coeff, typename... coeffs>
01264     struct string_helper {
01265         static std::string func() {
01266             std::string tail = string_helper<coeffs...>::func();
01267             std::string result = "";
01268             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01269                 return tail;
01270             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01271                 if (sizeof...(coeffs) == 1) {
01272                     result += std::string(1, variable_name);
01273                 } else {
01274                     result += std::string(1, variable_name) + "^" +
std::to_string(sizeof...(coeffs));
01275                 }
01276             } else {
01277                 if (sizeof...(coeffs) == 1) {
01278                     result += coeff::to_string() + " " + std::string(1, variable_name);
01279                 } else {
01280                     result += coeff::to_string()
+ " " + std::string(1, variable_name)
+ "^" + std::to_string(sizeof...(coeffs));
01281                 }
01282             }
01283         }
01284     };
01285
01286     if (!tail.empty()) {
01287         result += " " + tail;

```

```

01288         }
01289
01290         return result;
01291     }
01292 };
01293
01294 template<typename coeff>
01295 struct string_helper<coeff> {
01296     static std::string func() {
01297         if (!std::is_same<coeff, typename Ring::zero>::value) {
01298             return coeff::to_string();
01299         } else {
01300             return "";
01301         }
01302     }
01303 };
01304
01305 public:
01306     template<typename P>
01307     using simplify_t = typename simplify<P>::type;
01308
01309     template<typename v1, typename v2>
01310     using add_t = typename add<v1, v2>::type;
01311
01312     template<typename v1, typename v2>
01313     using sub_t = typename sub<v1, v2>::type;
01314
01315     template<typename v1, typename v2>
01316     using mul_t = typename mul<v1, v2>::type;
01317
01318     template<typename v1, typename v2>
01319     using eq_t = typename eq_helper<v1, v2>::type;
01320
01321     template<typename v1, typename v2>
01322     using lt_t = typename lt_helper<v1, v2>::type;
01323
01324     template<typename v1, typename v2>
01325     using gt_t = typename gt_helper<v1, v2>::type;
01326
01327     template<typename v1, typename v2>
01328     using div_t = typename div<v1, v2>::q_type;
01329
01330     template<typename v1, typename v2>
01331     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01332
01333     template<typename coeff, size_t deg>
01334     using monomial_t = typename monomial<coeff, deg>::type;
01335
01336     template<typename v>
01337     using derive_t = typename derive_helper<v>::type;
01338
01339     template<typename v>
01340     using pos_t = typename Ring::template pos_t<typename v::aN>;
01341
01342     template<typename v>
01343     static constexpr bool pos_v = pos_t<v>::value;
01344
01345     template<typename v1, typename v2>
01346     using gcd_t = std::conditional_t<
01347         Ring::is_euclidean_domain,
01348         typename make_unit<gcd_t<polynomial<Ring, variable_name>, v1, v2>::type,
01349         void>;
01350
01351     template<auto x>
01352     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01353
01354     template<typename v>
01355     using inject_ring_t = val<v>;
01356 };
01357 } // namespace aerobus
01358
01359 // fraction field
01360 namespace aerobus {
01361     namespace internal {
01362         template<typename Ring, typename E = void>
01363         requires IsEuclideanDomain<Ring>
01364         struct _FractionField {};
01365
01366         template<typename Ring>
01367         requires IsEuclideanDomain<Ring>
01368         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01369             static constexpr bool is_field = true;
01370             static constexpr bool is_euclidean_domain = true;
01371
01372         private:
01373             template<typename val1, typename val2, typename E = void>
01374             struct to_string_helper {};
01375         };
01376     }
01377 }

```

```

01418
01419     template<typename val1, typename val2>
01420     struct to_string_helper <val1, val2,
01421         std::enable_if_t<
01422             Ring::template eq_t<
01423                 val2, typename Ring::one
01424             >::value
01425         >
01426     > {
01427         static std::string func() {
01428             return val1::to_string();
01429         }
01430     };
01431
01432     template<typename val1, typename val2>
01433     struct to_string_helper<val1, val2,
01434         std::enable_if_t<
01435             !Ring::template eq_t<
01436                 val2,
01437                 typename Ring::one
01438             >::value
01439         >
01440     > {
01441         static std::string func() {
01442             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01443         }
01444     };
01445
01446 public:
01447     template<typename val1, typename val2>
01448     struct val {
01449         using x = val1;
01450         using y = val2;
01451         using is_zero_t = typename val1::is_zero_t;
01452         static constexpr bool is_zero_v = val1::is_zero_t::value;
01453
01454         using ring_type = Ring;
01455         using field_type = _FractionField<Ring>;
01456
01457         static constexpr bool is_integer = std::is_same<val2, typename Ring::one>::value;
01458
01459         template<typename valueType>
01460         static constexpr valueType get() { return static_cast<valueType>(x::v) /
01461             static_cast<valueType>(y::v); }
01462
01463         static std::string to_string() {
01464             return to_string_helper<val1, val2>::func();
01465         }
01466
01467         template<typename valueRing>
01468         static constexpr valueRing eval(const valueRing& v) {
01469             return x::eval(v) / y::eval(v);
01470         }
01471     };
01472
01473     using zero = val<typename Ring::zero, typename Ring::one>;
01474     using one = val<typename Ring::one, typename Ring::one>;
01475
01476     template<typename v>
01477     using inject_t = val<v, typename Ring::one>;
01478
01479     template<auto x>
01480     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01481     Ring::one>;
01482
01483     template<typename v>
01484     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01485
01486     using ring_type = Ring;
01487
01488 private:
01489     template<typename v, typename E = void>
01490     struct simplify {};
01491
01492     // x = 0
01493     template<typename v>
01494     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> > {
01495         using type = typename _FractionField<Ring>::zero;
01496     };
01497
01498     // x != 0
01499     template<typename v>
01500     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> > {
01501     private:
01502         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01503         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01504         using newy = typename Ring::template div_t<typename v::y, _gcd>;

```

```

01526
01527         using posx = std::conditional_t<
01528             !Ring::template pos_v<newy>,
01529             typename Ring::template sub_t<typename Ring::zero, newx>,
01530             newx>;
01531         using posy = std::conditional_t<
01532             !Ring::template pos_v<newy>,
01533             typename Ring::template sub_t<typename Ring::zero, newy>,
01534             newy>;
01535     public:
01536         using type = typename _FractionField<Ring>::template val<posx, posy>;
01537     };
01538
01539     public:
01540     template<typename v>
01541     using simplify_t = typename simplify<v>::type;
01542
01543     private:
01544     template<typename v1, typename v2>
01545     struct add {
01546     private:
01547         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01548         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01549         using dividend = typename Ring::template add_t<a, b>;
01550         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01551         using g = typename Ring::template gcd_t<dividend, diviser>;
01552
01553     public:
01554         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01555             dividend>;
01556     };
01557
01558     template<typename v>
01559     struct pos {
01560     using type = std::conditional_t<
01561         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01562         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01563         std::true_type,
01564         std::false_type>;
01565     };
01566
01567     template<typename v1, typename v2>
01568     struct sub {
01569     private:
01570         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01571         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01572         using dividend = typename Ring::template sub_t<a, b>;
01573         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01574         using g = typename Ring::template gcd_t<dividend, diviser>;
01575
01576     public:
01577         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01578             dividend>;
01579     };
01580
01581     template<typename v1, typename v2>
01582     struct mul {
01583     private:
01584         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01585         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01586
01587     public:
01588         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01589     };
01590
01591     template<typename v1, typename v2, typename E = void>
01592     struct div {};
01593
01594     template<typename v1, typename v2>
01595     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
01596         _FractionField<Ring>::zero>::value> {
01597     private:
01598         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01599         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01600
01601     public:
01602         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01603     };
01604
01605     template<typename v1, typename v2>
01606     struct div<v1, v2, std::enable_if_t<
01607         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01608         using type = one;
01609     };
01610
01611     template<typename v1, typename v2>
01612     struct eq {

```

```

01612         using type = std::conditional_t<
01613             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01614             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01615             std::true_type,
01616             std::false_type>;
01617     };
01618
01619     template<typename TL, typename E = void>
01620     struct vadd {};
01621
01622     template<typename TL>
01623     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01624         using head = typename TL::pop_front::type;
01625         using tail = typename TL::pop_front::tail;
01626         using type = typename add<head, typename vadd<tail>::type>::type;
01627     };
01628
01629     template<typename TL>
01630     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01631         using type = typename TL::template at<0>;
01632     };
01633
01634     template<typename... vals>
01635     struct vmul {};
01636
01637     template<typename v1, typename... vals>
01638     struct vmul<v1, vals...> {
01639         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01640     };
01641
01642     template<typename v1>
01643     struct vmul<v1> {
01644         using type = v1;
01645     };
01646
01647
01648     template<typename v1, typename v2, typename E = void>
01649     struct gt;
01650
01651     template<typename v1, typename v2>
01652     struct gt<v1, v2, std::enable_if_t<
01653         (eq<v1, v2>::type::value)
01654         > {
01655         using type = std::false_type;
01656     };
01657
01658     template<typename v1, typename v2>
01659     struct gt<v1, v2, std::enable_if_t<
01660         (!eq<v1, v2>::type::value) &&
01661         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01662         > {
01663         using type = typename gt<
01664             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01665             >::type;
01666     };
01667
01668     template<typename v1, typename v2>
01669     struct gt<v1, v2, std::enable_if_t<
01670         (!eq<v1, v2>::type::value) &&
01671         (pos<v1>::type::value) && (!pos<v2>::type::value)
01672         > {
01673         using type = std::true_type;
01674     };
01675
01676     template<typename v1, typename v2>
01677     struct gt<v1, v2, std::enable_if_t<
01678         (!eq<v1, v2>::type::value) &&
01679         (!pos<v1>::type::value) && (pos<v2>::type::value)
01680         > {
01681         using type = std::false_type;
01682     };
01683
01684     template<typename v1, typename v2>
01685     struct gt<v1, v2, std::enable_if_t<
01686         (!eq<v1, v2>::type::value) &&
01687         (pos<v1>::type::value) && (pos<v2>::type::value)
01688         > {
01689         using type = typename Ring::template gt_t<
01690             typename Ring::template mul_t<v1::x, v2::y>,
01691             typename Ring::template mul_t<v2::y, v2::x>
01692             >;
01693     };
01694
01695 public:
01696     template<typename v1, typename v2>
01697     using add_t = typename add<v1, v2>::type;
01698     template<typename v1, typename v2>

```



```

01701         using mod_t = zero;
01705         template<typename v1, typename v2>
01706         using gcd_t = v1;
01709         template<typename... vs>
01710         using vadd_t = typename vadd<vs...>::type;
01713         template<typename... vs>
01714         using vmul_t = typename vmul<vs...>::type;
01716         template<typename v1, typename v2>
01717         using sub_t = typename sub<v1, v2>::type;
01719         template<typename v1, typename v2>
01720         using mul_t = typename mul<v1, v2>::type;
01722         template<typename v1, typename v2>
01723         using div_t = typename div<v1, v2>::type;
01725         template<typename v1, typename v2>
01726         using eq_t = typename eq<v1, v2>::type;
01728         template<typename v1, typename v2>
01729         static constexpr bool eq_v = eq<v1, v2>::type::value;
01731         template<typename v1, typename v2>
01732         using gt_t = typename gt<v1, v2>::type;
01734         template<typename v1, typename v2>
01735         static constexpr bool gt_v = gt<v1, v2>::type::value;
01737         template<typename v>
01738         using pos_t = typename pos<v>::type;
01740         template<typename v>
01741         static constexpr bool pos_v = pos<v>::value;
01742     };
01743
01744     template<typename Ring, typename E = void>
01745     requires IsEuclideanDomain<Ring>
01746     struct FractionFieldImpl {};
01747
01748     // fraction field of a field is the field itself
01749     template<typename Field>
01750     requires IsEuclideanDomain<Field>
01751     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01752         using type = Field;
01753         template<typename v>
01754         using inject_t = v;
01755     };
01756
01757     // fraction field of a ring is the actual fraction field
01758     template<typename Ring>
01759     requires IsEuclideanDomain<Ring>
01760     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01761         using type = _FractionField<Ring>;
01762     };
01763 } // namespace internal
01764
01765     template<typename Ring>
01766     requires IsEuclideanDomain<Ring>
01767     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01768 } // namespace aerobus
01769
01770 // short names for common types
01771 namespace aerobus {
01772     using q32 = FractionField<i32>;
01773     using fpq32 = FractionField<polynomial<q32>;
01774     using q64 = FractionField<i64>;
01775     using pi64 = polynomial<i64>;
01776     using fpq64 = FractionField<polynomial<q64>;
01777     template<typename Ring, typename v1, typename v2>
01778     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01779
01780     template<typename Ring, typename v1, typename v2>
01781     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
01782     template<typename Ring, typename v1, typename v2>
01783     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01784 } // namespace aerobus
01785
01786 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
01787 namespace aerobus {
01788     namespace internal {
01789         template<typename T, size_t x, typename E = void>
01790         struct factorial {};
01791
01792         template<typename T, size_t x>
01793         struct factorial<T, x, std::enable_if_t<(x > 0)> {
01794             private:
01795                 template<typename, size_t, typename>
01796                 friend struct factorial;
01797             public:
01798                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
01799 x - 1>::type>;
01800                 static constexpr typename T::inner_type value = type::template get<typename
01801 T::inner_type>();
01802             };
01803         };
01804     }
01805 }

```

```

01811     template<typename T>
01812     struct factorial<T, 0> {
01813     public:
01814         using type = typename T::one;
01815         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01816     };
01817 } // namespace internal
01818
01822 template<typename T, size_t i>
01823 using factorial_t = typename internal::factorial<T, i>::type;
01824
01825 template<typename T, size_t i>
01826 inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01827
01828 namespace internal {
01829     template<typename T, size_t k, size_t n, typename E = void>
01830     struct combination_helper {};
01831
01832     template<typename T, size_t k, size_t n>
01833     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
01834         using type = typename FractionField<T>::template mul_t<
01835             typename combination_helper<T, k - 1, n - 1>::type,
01836             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
01837     };
01838
01839     template<typename T, size_t k, size_t n>
01840     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
01841         using type = typename combination_helper<T, n - k, n>::type;
01842     };
01843
01844     template<typename T, size_t n>
01845     struct combination_helper<T, 0, n> {
01846         using type = typename FractionField<T>::one;
01847     };
01848
01849     template<typename T, size_t k, size_t n>
01850     struct combination {
01851         using type = typename internal::combination_helper<T, k, n>::type::x;
01852         static constexpr typename T::inner_type value =
01853             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
01854     };
01855 } // namespace internal
01856
01859 template<typename T, size_t k, size_t n>
01860 using combination_t = typename internal::combination<T, k, n>::type;
01861
01862 template<typename T, size_t k, size_t n>
01863 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
01864
01865 namespace internal {
01866     template<typename T, size_t m>
01867     struct bernouilli;
01868
01869     template<typename T, typename accum, size_t k, size_t m>
01870     struct bernouilli_helper {
01871         using type = typename bernouilli_helper<
01872             T,
01873             addfractions_t<T,
01874                 accum,
01875                 mulfractions_t<T,
01876                     makefraction_t<T,
01877                         combination_t<T, k, m + 1>,
01878                         typename T::one>,
01879                         typename bernouilli<T, k>::type
01880                     >,
01881                     >,
01882                     k + 1,
01883                     m>::type;
01884     };
01885
01886     template<typename T, typename accum, size_t m>
01887     struct bernouilli_helper<T, accum, m, m> {
01888         using type = accum;
01889     };
01890
01891
01892
01893     template<typename T, size_t m>
01894     struct bernouilli {
01895         using type = typename FractionField<T>::template mul_t<
01896             typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
01897             makefraction_t<T,
01898                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
01899                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
01900             >

```

```

01901         >;
01902
01903         template<typename floatType>
01904         static constexpr floatType value = type::template get<floatType>();
01905     };
01906
01907     template<typename T>
01908     struct bernouilli<T, 0> {
01909         using type = typename FractionField<T>::one;
01910
01911         template<typename floatType>
01912         static constexpr floatType value = type::template get<floatType>();
01913     };
01914 } // namespace internal
01915
01919 template<typename T, size_t n>
01920 using bernouilli_t = typename internal::bernouilli<T, n>::type;
01921
01922 template<typename FloatType, typename T, size_t n>
01923 inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
01924
01925 namespace internal {
01926     template<typename T, int k, typename E = void>
01927     struct alternate {};
01928
01929     template<typename T, int k>
01930     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
01931         using type = typename T::one;
01932         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01933     };
01934
01935     template<typename T, int k>
01936     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
01937         using type = typename T::template sub_t<typename T::zero, typename T::one>;
01938         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01939     };
01940 } // namespace internal
01941
01944 template<typename T, int k>
01945 using alternate_t = typename internal::alternate<T, k>::type;
01946
01947 template<typename T, size_t k>
01948 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
01949
01950 // pow
01951 namespace internal {
01952     template<typename T, auto p, auto n>
01953     struct pow {
01954         using type = typename T::template mul_t<typename T::template val<p>, typename pow<T, p, n
- 1>::type>;
01955     };
01956
01957     template<typename T, auto p>
01958     struct pow<T, p, 0> { using type = typename T::one; };
01959 }
01960
01961 template<typename T, auto p, auto n>
01962 using pow_t = typename internal::pow<T, p, n>::type;
01963
01964 namespace internal {
01965     template<typename, template<typename, size_t> typename, class>
01966     struct make_taylor_impl;
01967
01968     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
01969     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
01970         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
01971     };
01972 }
01973
01974 // generic taylor serie, depending on coefficients
01975 template<typename T, template<typename, size_t> typename coeff_at, size_t deg>
01976 using taylor = typename internal::make_taylor_impl<
T,
coeff_at,
internal::make_index_sequence_reverse<deg + 1>::type>;
01980
01981 namespace internal {
01982     template<typename T, size_t i>
01983     struct exp_coeff {
01984         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
01985     };
01986
01987     template<typename T, size_t i, typename E = void>
01988     struct sin_coeff_helper {};

```

```

01989
01990     template<typename T, size_t i>
01991     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
01992         using type = typename FractionField<T>::zero;
01993     };
01994
01995     template<typename T, size_t i>
01996     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
01997         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
01998     };
01999
02000     template<typename T, size_t i>
02001     struct sin_coeff {
02002         using type = typename sin_coeff_helper<T, i>::type;
02003     };
02004
02005     template<typename T, size_t i, typename E = void>
02006     struct sh_coeff_helper {};
02007
02008     template<typename T, size_t i>
02009     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02010         using type = typename FractionField<T>::zero;
02011     };
02012
02013     template<typename T, size_t i>
02014     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02015         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02016     };
02017
02018     template<typename T, size_t i>
02019     struct sh_coeff {
02020         using type = typename sh_coeff_helper<T, i>::type;
02021     };
02022
02023     template<typename T, size_t i, typename E = void>
02024     struct cos_coeff_helper {};
02025
02026     template<typename T, size_t i>
02027     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02028         using type = typename FractionField<T>::zero;
02029     };
02030
02031     template<typename T, size_t i>
02032     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02033         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02034     };
02035
02036     template<typename T, size_t i>
02037     struct cos_coeff {
02038         using type = typename cos_coeff_helper<T, i>::type;
02039     };
02040
02041     template<typename T, size_t i, typename E = void>
02042     struct cosh_coeff_helper {};
02043
02044     template<typename T, size_t i>
02045     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02046         using type = typename FractionField<T>::zero;
02047     };
02048
02049     template<typename T, size_t i>
02050     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02051         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02052     };
02053
02054     template<typename T, size_t i>
02055     struct cosh_coeff {
02056         using type = typename cosh_coeff_helper<T, i>::type;
02057     };
02058
02059     template<typename T, size_t i>
02060     struct geom_coeff { using type = typename FractionField<T>::one; };
02061
02062     template<typename T, size_t i, typename E = void>
02063     struct atan_coeff_helper;
02064
02065     template<typename T, size_t i>
02066     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02067         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02068     };
02069
02070     template<typename T, size_t i>
02071     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02072         using type = typename FractionField<T>::zero;
02073     };
02074
02075

```

```

02076     template<typename T, size_t i>
02077     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02078
02079     template<typename T, size_t i, typename E = void>
02080     struct asin_coeff_helper;
02081
02082     template<typename T, size_t i>
02083     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02084         using type = makefraction_t<T,
02085             factorial_t<T, i - 1>,
02086             typename T::template mul_t<
02087                 typename T::template val<i>,
02088                 T::template mul_t<
02089                     pow_t<T, 4, i / 2>,
02090                     pow<T, factorial<T, i / 2>::value, 2
02091                 >
02092             >
02093         >>;
02094     };
02095
02096     template<typename T, size_t i>
02097     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02098         using type = typename FractionField<T>::zero;
02099     };
02100
02101     template<typename T, size_t i>
02102     struct asin_coeff {
02103         using type = typename asin_coeff_helper<T, i>::type;
02104     };
02105
02106     template<typename T, size_t i>
02107     struct lnpl_coeff {
02108         using type = makefraction_t<T,
02109             alternate_t<T, i + 1>,
02110             typename T::template val<i>;
02111     };
02112
02113     template<typename T>
02114     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02115
02116     template<typename T, size_t i, typename E = void>
02117     struct asinh_coeff_helper;
02118
02119     template<typename T, size_t i>
02120     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02121         using type = makefraction_t<T,
02122             typename T::template mul_t<
02123                 alternate_t<T, i / 2>,
02124                 factorial_t<T, i - 1>
02125             >,
02126             typename T::template mul_t<
02127                 T::template mul_t<
02128                     typename T::template val<i>,
02129                     pow_t<T, (factorial<T, i / 2>::value), 2>
02130                 >,
02131                 pow_t<T, 4, i / 2>
02132             >
02133         >>;
02134     };
02135
02136     template<typename T, size_t i>
02137     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02138         using type = typename FractionField<T>::zero;
02139     };
02140
02141     template<typename T, size_t i>
02142     struct asinh_coeff {
02143         using type = typename asinh_coeff_helper<T, i>::type;
02144     };
02145
02146     template<typename T, size_t i, typename E = void>
02147     struct atanh_coeff_helper;
02148
02149     template<typename T, size_t i>
02150     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02151         // 1/i
02152         using type = typename FractionField<T>::template val<
02153             typename T::one,
02154             typename T::template val<static_cast<typename T::inner_type>(i)>;
02155     };
02156
02157     template<typename T, size_t i>
02158     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02159         using type = typename FractionField<T>::zero;
02160     };
02161
02162     template<typename T, size_t i>

```

```

02163     struct atanh_coeff {
02164         using type = typename asinh_coeff_helper<T, i>::type;
02165     };
02166
02167     template<typename T, size_t i, typename E = void>
02168     struct tan_coeff_helper;
02169
02170     template<typename T, size_t i>
02171     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02172         using type = typename FractionField<T>::zero;
02173     };
02174
02175     template<typename T, size_t i>
02176     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02177     private:
02178         // 4^((i+1)/2)
02179         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02180         // 4^((i+1)/2) - 1
02181         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02182         // (-1)^((i-1)/2)
02183         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02184         using dividend = typename FractionField<T>::template mul_t<
02185             altp,
02186             FractionField<T>::template mul_t<
02187                 _4p,
02188                 FractionField<T>::template mul_t<
02189                     _4pml,
02190                     bernouilli_t<T, (i + 1)>
02191                 >
02192             >
02193         >;
02194     public:
02195         using type = typename FractionField<T>::template div_t<dividend,
02196             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02197     };
02198
02199     template<typename T, size_t i>
02200     struct tanh_coeff {
02201         using type = typename tan_coeff_helper<T, i>::type;
02202     };
02203
02204     template<typename T, size_t i, typename E = void>
02205     struct tanh_coeff_helper;
02206
02207     template<typename T, size_t i>
02208     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02209         using type = typename FractionField<T>::zero;
02210     };
02211
02212     template<typename T, size_t i>
02213     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02214     private:
02215         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02216         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02217         using dividend =
02218             typename FractionField<T>::template mul_t<
02219                 _4p,
02220                 typename FractionField<T>::template mul_t<
02221                     _4pml,
02222                     bernouilli_t<T, (i + 1)>
02223                 >
02224             >::type;
02225     public:
02226         using type = typename FractionField<T>::template div_t<dividend,
02227             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02228     };
02229
02230     template<typename T, size_t i>
02231     struct tanh_coeff {
02232         using type = typename tanh_coeff_helper<T, i>::type;
02233     };
02234 } // namespace internal
02235
02236 template<typename T, size_t deg>
02237 using exp = taylor<T, internal::exp_coeff, deg>;
02238
02239 template<typename T, size_t deg>
02240 using expml = typename polynomial<FractionField<T>::template sub_t<
02241     exp<T, deg>,
02242     typename polynomial<FractionField<T>::one>;
02243
02244 template<typename T, size_t deg>
02245 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02246
02247 template<typename T, size_t deg>

```

```

02260     using atan = taylor<T, internal::atan_coeff, deg>;
02261
02265     template<typename T, size_t deg>
02266     using sin = taylor<T, internal::sin_coeff, deg>;
02267
02271     template<typename T, size_t deg>
02272     using sinh = taylor<T, internal::sh_coeff, deg>;
02273
02277     template<typename T, size_t deg>
02278     using cosh = taylor<T, internal::cosh_coeff, deg>;
02279
02283     template<typename T, size_t deg>
02284     using cos = taylor<T, internal::cos_coeff, deg>;
02285
02289     template<typename T, size_t deg>
02290     using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02291
02295     template<typename T, size_t deg>
02296     using asin = taylor<T, internal::asin_coeff, deg>;
02297
02301     template<typename T, size_t deg>
02302     using asinh = taylor<T, internal::asinh_coeff, deg>;
02303
02307     template<typename T, size_t deg>
02308     using atanh = taylor<T, internal::atanh_coeff, deg>;
02309
02313     template<typename T, size_t deg>
02314     using tan = taylor<T, internal::tan_coeff, deg>;
02315
02319     template<typename T, size_t deg>
02320     using tanh = taylor<T, internal::tanh_coeff, deg>;
02321 } // namespace aerobus
02322
02323 // continued fractions
02324 namespace aerobus {
02327     template<int64_t... values>
02328     struct ContinuedFraction {};
02329
02330     template<int64_t a0>
02331     struct ContinuedFraction<a0> {
02332         using type = typename q64::template inject_constant_t<a0>;
02333         static constexpr double val = type::template get<double>();
02334     };
02335
02336     template<int64_t a0, int64_t... rest>
02337     struct ContinuedFraction<a0, rest...> {
02338         using type = q64::template add_t<
02339             typename q64::template inject_constant_t<a0>,
02340             typename q64::template div_t<
02341                 typename q64::one,
02342                 typename ContinuedFraction<rest...>::type
02343             >;
02344         static constexpr double val = type::template get<double>();
02345     };
02346
02351     using PI_fraction =
02352     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02354     using E_fraction =
02355     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02356     using SQRT2_fraction =
02357     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02358     using SQRT3_fraction =
02359     ContinuedFraction<1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02359 } // namespace aerobus
02360
02361 // known polynomials
02362 namespace aerobus {
02363     namespace internal {
02364         template<int kind, int deg>
02365         struct chebyshev_helper {
02366             using type = typename pi64::template sub_t<
02367                 typename pi64::template mul_t<
02368                     typename pi64::template mul_t<
02369                         pi64::inject_constant_t<2>,
02370                         typename pi64::X
02371                     >,
02372                     typename chebyshev_helper<kind, deg-1>::type
02373                 >,
02374                 typename chebyshev_helper<kind, deg-2>::type
02375             >;
02376         };
02377
02378         template<>
02379         struct chebyshev_helper<1, 0> {
02380             using type = typename pi64::one;
02381         };

```

```

02382
02383     template<>
02384     struct chebyshev_helper<1, 1> {
02385         using type = typename pi64::X;
02386     };
02387
02388     template<>
02389     struct chebyshev_helper<2, 0> {
02390         using type = typename pi64::one;
02391     };
02392
02393     template<>
02394     struct chebyshev_helper<2, 1> {
02395         using type = typename pi64::template mul_t<
02396             typename pi64::inject_constant_t<2>,
02397             typename pi64::X>;
02398     };
02399 } // namespace internal
02400
02401     template<size_t deg>
02402     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02403
02404     template<size_t deg>
02405     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02406 } // namespace aerobus
02407
02408 #ifdef AEROBUS_CONWAY_IMPORTS
02409 template<int p, int n>
02410 struct ConwayPolynomial;
02411
02412 #define ZPZV ZPZ::template val
02413 #define POLYV aerobus::polynomial<ZPZ>::template val
02414 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02415     ZPZV<1>; }; // NOLINT
02416 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02417     ZPZV<1>, ZPZV<1>; }; // NOLINT
02418 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02419     ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02420 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02421     ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02422 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02423     ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
02424 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02425     ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
02426 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02427     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02428 template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02429     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02430 template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02431     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02432 template<> struct ConwayPolynomial<2, 10> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02433     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; //
02434     NOLINT
02435 template<> struct ConwayPolynomial<2, 11> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02436     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; //
02437     NOLINT
02438 template<> struct ConwayPolynomial<2, 12> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02439     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>,
02440     ZPZV<1>; }; // NOLINT
02441 template<> struct ConwayPolynomial<2, 13> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02442     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>,
02443     ZPZV<1>; }; // NOLINT
02444 template<> struct ConwayPolynomial<2, 14> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02445     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>,
02446     ZPZV<1>; }; // NOLINT
02447 template<> struct ConwayPolynomial<2, 15> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02448     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>,
02449     ZPZV<1>; }; // NOLINT
02450 template<> struct ConwayPolynomial<2, 16> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02451     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>,
02452     ZPZV<1>; }; // NOLINT
02453 template<> struct ConwayPolynomial<2, 17> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02454     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>,
02455     ZPZV<0>; }; // NOLINT
02456 template<> struct ConwayPolynomial<2, 18> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02457     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>,
02458     ZPZV<1>; }; // NOLINT
02459 template<> struct ConwayPolynomial<2, 19> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02460     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>,
02461     ZPZV<1>; }; // NOLINT
02462 template<> struct ConwayPolynomial<2, 20> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
02463     ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>,
02464     ZPZV<1>; }; // NOLINT
02465 template<> struct ConwayPolynomial<3, 1> { using ZPZ = aerobus::zpz<3>; using type = POLYV<ZPZV<1>,
02466     ZPZV<1>; }; // NOLINT
02467 template<> struct ConwayPolynomial<3, 2> { using ZPZ = aerobus::zpz<3>; using type = POLYV<ZPZV<1>,

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```

POLYV<ZPV<1>, ZPV<0>, ZPV<2>, ZPV<849>, ZPV<296>, ZPV<228>, ZPV<5>; }; // NOLINT
04339 template<> struct ConwayPolynomial<983, 7> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<3>, ZPV<978>; }; // NOLINT
04340 template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<7>, ZPV<738>, ZPV<276>, ZPV<530>, ZPV<5>; }; //
NOLINT
04341 template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<858>, ZPV<87>, ZPV<978>;
}; // NOLINT
04342 template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<985>; }; // NOLINT
04343 template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<989>, ZPV<6>; }; // NOLINT
04344 template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<4>, ZPV<985>; }; // NOLINT
04345 template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<10>, ZPV<794>, ZPV<6>; }; // NOLINT
04346 template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<3>, ZPV<985>; }; // NOLINT
04347 template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<637>, ZPV<855>, ZPV<278>, ZPV<6>; }; // NOLINT
04348 template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<985>; }; // NOLINT
04349 template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<15>, ZPV<941>, ZPV<786>, ZPV<234>, ZPV<6>; }; //
NOLINT
04350 template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<9>, ZPV<466>, ZPV<222>, ZPV<985>;
}; // NOLINT
04351 template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<990>; }; // NOLINT
04352 template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<995>, ZPV<7>; }; // NOLINT
04353 template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<2>, ZPV<990>; }; // NOLINT
04354 template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<4>, ZPV<622>, ZPV<7>; }; // NOLINT
04355 template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<10>, ZPV<990>; }; // NOLINT
04356 template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<981>, ZPV<58>, ZPV<260>, ZPV<7>; }; // NOLINT
04357 template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<990>; }; // NOLINT
04358 template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<934>, ZPV<473>, ZPV<241>, ZPV<7>; }; //
NOLINT
04359 template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<39>, ZPV<732>, ZPV<616>, ZPV<990>;
}; // NOLINT
04360 #endif // AEROBUS_CONWAY_IMPORTS
04361
04362 #endif // __INC_AEROBUS__ // NOLINT

```



# Chapter 7

## Examples

### 7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

### 7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

### 7.3 polynomial

makes the constant (native type) polynomial a\_0

makes the constant (native type) polynomial a\_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

## 7.4 PI\_fraction::val

representation of PI as a continued fraction -> 3.14...

## 7.5 E\_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...



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