

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , Aerobus provides automatic implementation for **quotient ring** R/X where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

Aerobus gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, Aerobus provides some support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough constexpr in its `cuda_fp16.h` header, so we had to implement our own constexpr `static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not easily possible to make it work for `__half2` because of [another bug](#).

A workaround is to modify `cuda_fp16.h` and add a constexpr modifier to line 5039. This works but only tested on Linux with CUDA 16.1.

Once done, `nvcc` generates splendid assembly, same as for double or float:

```
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R7, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R7, R5, R7, 0.008331298828125, 0.008331298828125 ;
HFMA2 R7, R5, R7, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R7, R5, R7, 0.1666259765625, 0.1666259765625 ;
HFMA2 R7, R5, R7, 0.5, 0.5 ;
HFMA2.MMA R7, R5, R7, 1, 1 ;
HFMA2 R7, R5, R7, RZ.H0_H0 ;
```

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	17
aerobus::internal	Internal implementations, subject to breaking changes without notice	38
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	42
aerobus::libm	Holds mathematical functions (such as cosine or sin), correct to epsilon	43

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	45
aerobus::IsField	
Concept to express R is a field	45
aerobus::IsRing	
Concept to express R is a Ring	46

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >	47
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> > 47	
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	48
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	48
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	49
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	50
aerobus::ConwayPolynomial	51
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >	51
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > . .	52
aerobus::Embed< Small, Large, E >	
Embedding - struct forward declaration	53
aerobus::Embed< i32, i64 >	
Embeds i32 into i64	53
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
Embeds polynomial<Small> into polynomial<Large>	54
aerobus::Embed< q32, q64 >	
Embeds q32 into q64	55
aerobus::Embed< Quotient< Ring, X >, Ring >	
Embeds Quotient<Ring, X> into Ring	56
aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	57
aerobus::Embed< zpz< x >, i32 >	
Embeds zpz values into i32	57
aerobus::polynomial< Ring >::horner_reduction_t< P >	
Used to evaluate polynomials over a value in Ring	58
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	59
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	66
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >	72
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >	73

aerobus::is_prime< n >	
Checks if n is prime	73
aerobus::polynomial< Ring >	74
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	82
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X,	
Quotient is $\mathbb{Z}/2\mathbb{Z}$	83
aerobus::type_list< Ts >::split< index >	
Splits list at index	88
aerobus::type_list< Ts >	
Empty pure template struct to handle type list	89
aerobus::type_list<>	
Specialization for empty type list	92
aerobus::i32::val< x >	
Values in i32 , again represented as types	93
aerobus::i64::val< x >	
Values in i64	95
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	97
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	101
aerobus::zpz< p >::val< x >	
Values in zpz	102
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	104
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	107

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

src/ aerobus.h	115
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Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein
- namespace [libm](#)
holds mathematical functions (such as cosine or sin), correct to epsilon

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)

- embeds values from `Ring` to its field of fractions*
- struct `Embed< zpz< x >, i32 >`
 - embeds `zpz` values into `i32`*
- struct `i32`
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct `i64`
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct `is_prime`
 - checks if `n` is prime*
- struct `polynomial`
- struct `Quotient`
 - `Quotient` ring by the principal ideal generated by 'X' With `i32` as `Ring` and `i32::val<2>` as `X`, `Quotient` is $\mathbb{Z}/2\mathbb{Z}$.*
- struct `type_list`
 - Empty pure template struct to handle type list.*
- struct `type_list<>`
 - specialization for empty type list*
- struct `zpz`
 - congruence classes of integers modulo `p` (32 bits)*

Concepts

- concept `IsRing`
 - Concept to express `R` is a `Ring`.*
- concept `IsEuclideanDomain`
 - Concept to express `R` is an euclidean domain.*
- concept `IsField`
 - Concept to express `R` is a field.*

Typedefs

- template<typename `T` , typename `A` , typename `B` >
 using `gcd_t` = typename internal::gcd< `T` >::template type< `A`, `B` >
 - computes the greatest common divisor or `A` and `B`*
- template<typename... `vals`>
 using `vadd_t` = typename internal::vadd< `vals`... >::type
 - adds multiple values (`v1` + `v2` + ... + `vn`) `vals` must have same "enclosing_type" and "enclosing_type" must have an `add_t` binary operator*
- template<typename... `vals`>
 using `vmul_t` = typename internal::vmul< `vals`... >::type
 - multiplies multiple values (`v1` * `v2` + ... + `vn`) `vals` must have same "enclosing_type" and "enclosing_type" must have an `mul_t` binary operator*
- template<typename `val` >
 using `abs_t` = std::conditional_t< `val`::enclosing_type::template pos_v< `val` >, `val`, typename `val`::enclosing_type::template `sub_t`< typename `val`::enclosing_type::zero, `val` > >
 - computes absolute value of 'val' `val` must be a 'value' in a `Ring` satisfying 'IsEuclideanDomain' concept*
- template<typename `Ring` >
 using `FractionField` = typename internal::FractionFieldImpl< `Ring` >::type
 - Fraction field of an euclidean domain, such as \mathbb{Q} for \mathbb{Z} .*
- template<typename `X` , typename `Y` >
 using `add_t` = typename `X`::enclosing_type::template `add_t`< `X`, `Y` >
 - generic addition*

- `template<typename X , typename Y >`
`using sub_t = typename X::enclosing_type::template sub_t< X, Y >`
generic subtraction
- `template<typename X , typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
generic multiplication
- `template<typename X , typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
generic division
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
- `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename Ring , auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
- `template<typename Ring , auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField<Ring>
- `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`

- computes factorial(i), as type*

 - `template<typename T, size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T, size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T, size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T, int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T, int n, int k>`
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T, int n, int k>`
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T, int n, int k>`
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`
Stirling number of second kind – as types.
- `template<typename T, typename p, size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers, size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers, size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers, size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers, size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers, size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers, size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers, size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`
 $\cosh(x)$ *hyperbolic cosine*
- `template<typename Integers, size_t deg>`
`using cos = taylor< Integers, internal::cos_coeff, deg >`
 $\cos(x)$ *cosinus*

- [illegible]

Functions

- `template<typename T >`
`T * aligned_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the [field](#) (prime number) @tparam n degree of extension
`template< int p`

Variables

- `template<typename T , size_t i>`
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`
computes factorial(i) as value in T
- `template<typename T , size_t k, size_t n>`
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`
computes binomial coefficients (k among n) as value
- `template<typename FloatType , typename T , size_t n>`
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`
nth bernoulli number as value in FloatType
- `template<typename T , size_t k>`
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	---

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh(x)` arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ `cosinus`

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of e

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```


rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

Template Parameters

<i>Ring</i>	
-------------	--

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of π as a continued fraction

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.46 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.47 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.49 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.50 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.51 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```



```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >

- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >

- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **fma_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **Split**
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_1_helper**
- struct **stirling_1_helper**< T, 0, 0 >
- struct **stirling_1_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, 0, std::enable_if_t<(n > 0)> >

- struct **stirling_1_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0)> >
- struct **stirling_2_helper**
- struct **stirling_2_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling_2_helper**< T, n, n, std::enable_if_t<(n >= 0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) != 0> >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) == 0> >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) != 0> >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) == 0> >
- struct **touchard_coeff**
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Enumerations

- enum [hermite_kind](#) { [probabilist](#) , [physicist](#) }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Enumeration Type Documentation

6.3.2.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

6.4 aerobus::libm Namespace Reference

holds mathematical functions (such as cosine or sin), correct to epsilon

6.4.1 Detailed Description

holds mathematical functions (such as cosine or sin), correct to epsilon

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = typename Ring::zero

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = `aN`

8.3.1 Member Typedef Documentation

8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

Examples

[examples/continued_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename q64::template inject_constant_t< a0 >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = static_cast<double>(a0)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template [add_t](#)< typename q64::template inject_constant_t< a0 >, typename q64::template [div_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmetic↔
Type *r)

8.8.1 Member Function Documentation

8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P
>::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.9.1 Member Function Documentation

8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P
>::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- template<typename val >
using [type](#) = [i64::val](#)< static_cast< int64_t >(val::v)>
the [i64](#) representation of val

8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

8.11.2 Member Typedef Documentation

8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the polynomial<Large> representation of v

8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.12.2 Member Typedef Documentation

8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`
q64 representation of v

8.13.1 Detailed Description

embeds q32 into q64

8.13.2 Member Typedef Documentation

8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = typename val::raw_t`
Ring representation of val.

8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

8.14.2 Member Typedef Documentation

8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename FractionField< Ring >::template val< v, typename Ring::one >`
FractionField<Ring> representation of v.

8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

Template Parameters

<i>Ring</i>	an integers ring, such as i32
-------------	---

8.15.2 Member Typedef Documentation

8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i32::val< val::v >`
the i32 representation of val

8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into `i32`

Template Parameters

<code>x</code>	an integer
----------------	------------

8.16.2 Member Typedef Documentation

8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the `i32` representation of val

Template Parameters

<code>val</code>	a value in <code>zpz<x></code>
------------------	--------------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using [inner_type](#) = int32_t
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<auto x>
using [inject_constant_t](#) = [val](#)< static_cast< int32_t >(x)>
inject a native constant
- template<typename v >
using [inject_ring_t](#) = v
- template<typename v1 , typename v2 >
using [add_t](#) = typename add< v1, v2 >::type
addition operator yields v1 + v2

- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator yields $v1 - v2$
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
*multiplication operator yields $v1 * v2$*
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator yields $v1 / v2$
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator yields $v1 \% v2$
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator ($v1 > v2$) yields $v1 > v2$
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator ($v1 < v2$) yields $v1 < v2$
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator (boolean value)
- `template<typename v >`
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity (boolean value) yields $v > 0$ as boolean value

8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

Examples

[examples/compensated_horner.cpp](#).

8.18.2 Member Typedef Documentation

8.18.2.1 add_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields $v1 + v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.2 div_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields $v1 / v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.3 eq_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.4 gcd_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields $GCD(v1, v2)$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) yields $v1 > v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject\_constant\_t = val<static_cast<int32_t>(x)>
```

inject a native constant

Template Parameters

<i>x</i>	
----------	--

8.18.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject\_ring\_t = v
```

8.18.2.8 inner_type

```
using aerobus::i32::inner\_type = int32_t
```

8.18.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt\_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$) yields $v1 < v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields $v1 * v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.18.2.13 pos_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v</i>	a value in i32
----------	--------------------------------

8.18.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields $v1 - v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.18.3 Member Data Documentation

8.18.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.18.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.18.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.18.3.4 pos_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	a value in i32
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- using [inner_type](#) = `int64_t`
type of represented values
- template<auto x>
using [inject_constant_t](#) = `val< static_cast< int64_t >(x)>`
injects constant as an [i64](#) value
- template<typename v >
using [inject_ring_t](#) = v
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`
constant zero
- using [one](#) = `val< 1 >`
constant one
- template<typename v1 , typename v2 >
using [add_t](#) = `typename add< v1, v2 >::type`
addition operator
- template<typename v1 , typename v2 >
using [sub_t](#) = `typename sub< v1, v2 >::type`
subtraction operator
- template<typename v1 , typename v2 >
using [mul_t](#) = `typename mul< v1, v2 >::type`
multiplication operator
- template<typename v1 , typename v2 >
using [div_t](#) = `typename div< v1, v2 >::type`
division operator integer division
- template<typename v1 , typename v2 >
using [mod_t](#) = `typename remainder< v1, v2 >::type`

modulus operator

- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i64, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$ as instantiation of [i64::val](#)
- `template<typename v >`
`using pos_t = typename pos< v >::type`
is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator yields $v1 > v2$ as boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator yields $v1 < v2$ as boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator yields $v1 == v2$ as boolean value
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity yields $v > 0$ as boolean value

8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.19.2 Member Typedef Documentation

8.19.2.1 [add_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div\_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq\_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd\_t = gcd\_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt\_t = typename gt<v1, v2>::type
```


strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.6 inject_constant_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

Template Parameters

<code>x</code>	
----------------	--

8.19.2.7 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)

Template Parameters

<code>v</code>	a value in i64
----------------	--------------------------------

8.19.2.8 inner_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.19.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

8.19.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.19.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
-----------	---

8.19.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

8.19.3 Member Data Documentation

8.19.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.19.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.19.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields $v_1 < v_2$ as boolean value

Template Parameters

v_1	: an element of aerobus::i64::val
v_2	: an element of aerobus::i64::val

8.19.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`

8.20.1 Member Typedef Documentation

8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = accum`

8.21.1 Member Typedef Documentation

8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.22 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.22.2 Member Data Documentation

8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [horner_reduction_t](#)
Used to evaluate polynomials over a value in Ring.
- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- using `zero` = `val`< typename Ring::zero >
constant zero
- using `one` = `val`< typename Ring::one >
constant one
- using `X` = `val`< typename Ring::one, typename Ring::zero >
generator
- template<typename P >
using `simplify_t` = typename simplify< P >::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- template<typename v1, typename v2 >
using `add_t` = typename add< v1, v2 >::type
adds two polynomials
- template<typename v1, typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
subtraction of two polynomials
- template<typename v1, typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
multiplication of two polynomials
- template<typename v1, typename v2 >
using `eq_t` = typename eq_helper< v1, v2 >::type
equality operator
- template<typename v1, typename v2 >
using `lt_t` = typename lt_helper< v1, v2 >::type
strict less operator
- template<typename v1, typename v2 >
using `gt_t` = typename gt_helper< v1, v2 >::type
strict greater operator
- template<typename v1, typename v2 >
using `div_t` = typename div< v1, v2 >::q_type
division operator
- template<typename v1, typename v2 >
using `mod_t` = typename div_helper< v1, v2, `zero`, v1 >::mod_type
modulo operator
- template<typename coeff, size_t deg>
using `monomial_t` = typename monomial< coeff, deg >::type
monomial : coeff X^deg
- template<typename v >
using `derive_t` = typename derive_helper< v >::type
derivation operator
- template<typename v >
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
checks for positivity (an > 0)
- template<typename v1, typename v2 >
using `gcd_t` = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >
greatest common divisor of two polynomials
- template<auto x>
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >
makes the constant (native type) polynomial a_0
- template<typename v >
using `inject_ring_t` = `val`< v >
makes the constant (ring type) polynomial a_0

Static Public Attributes

- static constexpr bool `is_field` = false
- static constexpr bool `is_euclidean_domain` = `Ring::is_euclidean_domain`
- template<typename v >
static constexpr bool `pos_v` = `pos_t<v>::value`
positivity operator

8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

Examples

[examples/compensated_horner.cpp](#), [examples/make_polynomial.cpp](#), and [examples/modular_arithmetic.cpp](#).

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<code>v1</code>	
<code>v2</code>	

8.23.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<code>v</code>	
----------------	--

8.23.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

Template Parameters

<i>x</i>	
----------	--

8.23.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

Template Parameters

<i>v</i>	
----------	--

8.23.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.10 mod_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.23.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.23.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($an > 0$)

Template Parameters

<i>v</i>	
----------	--

8.23.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.23.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.23.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.23.3 Member Data Documentation

8.23.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.23.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.23.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in polynomial::val
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.24 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using [tail](#) = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.24.2 Member Typedef Documentation

8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< typename Ring::zero >
zero value
- using [one](#) = [val](#)< typename Ring::one >
one
- template<typename v1 , typename v2 >
using [add_t](#) = [val](#)< typename Ring::template [add_t](#)< typename v1::type, typename v2::type > >
addition operator
- template<typename v1 , typename v2 >
using [mul_t](#) = [val](#)< typename Ring::template [mul_t](#)< typename v1::type, typename v2::type > >
subtraction operator
- template<typename v1 , typename v2 >
using [div_t](#) = [val](#)< typename Ring::template [div_t](#)< typename v1::type, typename v2::type > >
division operator
- template<typename v1 , typename v2 >
using [mod_t](#) = [val](#)< typename Ring::template [mod_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
 • `template<typename v1 , typename v2 >`
 `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
 equality operator (as type)
- `template<typename v1 >`
 `using pos_t = std::true_type`
 positivity operator always true
- `template<auto x>`
 `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
 *inject a 'constant' in quotient ring**
- `template<typename v >`
 `using inject_ring_t = val< v >`
 projects a value of Ring onto the quotient

Static Public Attributes

- `template<typename v1 , typename v2 >`
 `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
 addition operator (as boolean value)
- `template<typename v >`
 `static constexpr bool pos_v = pos_t<v>::value`
 positivity operator always true
- `static constexpr bool is_euclidean_domain = true`
 quotien rings are euclidean domain

8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.25.2 Member Typedef Documentation

8.25.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring*

Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

8.25.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

8.25.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.25.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.25.3 Member Data Documentation

8.25.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.25.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.26.2 Member Typedef Documentation

8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.27 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

8.27.2 Member Typedef Documentation

8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.27.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.27.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.3 Member Data Documentation

8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.28 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T >
- template<typename T >
using [push_back](#) = [type_list](#)< T >
- template<typename U >
using [concat](#) = U
- template<typename T , size_t index>
using [insert](#) = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t [length](#) = 0

8.28.1 Detailed Description

specialization for empty type list

8.28.2 Member Typedef Documentation

8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

8.28.2.3 push_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

8.28.2.4 push_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```


8.28.3 Member Data Documentation

8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [i32](#)
Enclosing ring type.
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [DEVICE](#) valueType [get](#) ()
cast x into valueType
- static std::string [to_string](#) ()
string representation of value

Static Public Attributes

- static constexpr int32_t [v](#) = x
actual value stored in val type

8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

8.29.2 Member Typedef Documentation

8.29.2.1 `enclosing_type`

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.29.2.2 `is_zero_t`

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.29.3 Member Function Documentation

8.29.3.1 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.29.3.2 `to_string()`

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.29.4 Member Data Documentation

8.29.4.1 `v`

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using [inner_type](#) = int32_t
type of represented values
- using [enclosing_type](#) = [i64](#)
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED_DEVICE](#) valueType [get](#) ()
cast value in valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int64_t [v](#) = x
actual value

8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

x	an actual integer
-------------------	-------------------

Examples

[examples/compensated_horner.cpp](#).

8.30.2 Member Typedef Documentation

8.30.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.30.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.30.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.30.3 Member Function Documentation

8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.30.3.2 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.30.4 Member Data Documentation

8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = polynomial< Ring >
enclosing ring type
- using [aN](#) = coeffN
heavy weight coefficient (non zero)
- using [strip](#) = val< coeffs... >
remove largest coefficient
- using [is_zero_t](#) = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>
true_type if polynomial is constant zero
- template<size_t index>
using [coeff_at_t](#) = typename coeff_at< index >::type
type of coefficient at index
- template<typename x >
using [value_at_t](#) = horner_reduction_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >

Static Public Member Functions

- static std::string [to_string](#) ()
get a string representation of polynomial
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
evaluates polynomial seen as a function operating on arithmeticType
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)
Evaluate polynomial on x using compensated horner scheme.

Static Public Attributes

- static constexpr size_t [degree](#) = sizeof...(coeffs)
degree of the polynomial
- static constexpr bool [is_zero_v](#) = is_zero_t::value
true if polynomial is constant zero

8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

Examples

[examples/compensated_horner.cpp](#).

8.31.2 Member Typedef Documentation

8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.31.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.31.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>

enclosing ring type
```

8.31.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>

true_type if polynomial is constant zero
```

8.31.2.5 ring_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring

ring coefficients live in
```

8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>

remove largest coefficient
```

8.31.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

8.31.3 Member Function Documentation

8.31.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

Parameters

x	
---	--

8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

Parameters

x	value
---	-------

Returns

$P(x)$

8.31.3.3 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.31.4 Member Data Documentation

8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.31.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- using [raw_t](#) = V
- using [type](#) = [abs_t](#)< typename Ring::template [mod_t](#)< V, X > >

8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

8.32.2 Member Typedef Documentation

8.32.2.1 raw_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [zpz](#)< p >
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< [v](#)==0 >
true_type if zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED DEVICE](#) valueType [get](#) ()
get value as valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int32_t [v](#) = x % p
actual value
- static constexpr bool [is_zero_v](#) = [v](#) == 0
true if zero

8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

x	an integer
---	------------

8.33.2 Member Typedef Documentation

8.33.2.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.33.2.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true_type if zero

8.33.3 Member Function Documentation

8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

8.33.3.2 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

Returns

a string representation

8.33.4 Member Data Documentation**8.33.4.1 is_zero_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

8.33.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial< Ring >](#)
enclosing ring type
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = std::bool_constant< [aN::is_zero_t::value](#) >
- template<size_t index>
using [coeff_at_t](#) = typename [coeff_at< index >::type](#)
- template<typename x >
using [value_at_t](#) = [coeffN](#)

Static Public Member Functions

- static std::string [to_string](#) ()
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)

Static Public Attributes

- static constexpr size_t [degree](#) = 0
degree
- static constexpr bool [is_zero_v](#) = is_zero_t::value

8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.34.2 Member Typedef Documentation

8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.34.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

8.34.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.34.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

8.34.2.5 ring_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.34.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

8.34.3 Member Function Documentation

8.34.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

8.34.3.3 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.34.4 Member Data Documentation

8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.34.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in zpz

Public Types

- using `inner_type` = `int32_t`
underlying type for values
- template<auto x>
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>
injects a constant integer into mpz
- using `zero` = `val`< 0 >
zero value
- using `one` = `val`< 1 >
one value
- template<typename v1 , typename v2 >
using `add_t` = `typename add`< v1, v2 >::type
addition operator
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub`< v1, v2 >::type
subtraction operator
- template<typename v1 , typename v2 >
using `mul_t` = `typename mul`< v1, v2 >::type
multiplication operator
- template<typename v1 , typename v2 >
using `div_t` = `typename div`< v1, v2 >::type
division operator
- template<typename v1 , typename v2 >
using `mod_t` = `typename remainder`< v1, v2 >::type
modulo operator
- template<typename v1 , typename v2 >
using `gt_t` = `typename gt`< v1, v2 >::type
strictly greater operator (type)
- template<typename v1 , typename v2 >
using `lt_t` = `typename lt`< v1, v2 >::type
strictly smaller operator (type)
- template<typename v1 , typename v2 >
using `eq_t` = `typename eq`< v1, v2 >::type
equality operator (type)
- template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >
greatest common divisor
- template<typename v1 >
using `pos_t` = `typename pos`< v1 >::type
positivity operator (type)

Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value
true iff p is prime
- static constexpr bool `is_euclidean_domain` = true
always true
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
strictly greater operator (booleanvalue)

- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

8.35.1 Detailed Description

`template<int32_t p>`
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular_arithmetic.cpp](#), and [examples/polynomials_over_finite_field.cpp](#).

8.35.2 Member Typedef Documentation

8.35.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.2 div_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

Template Parameters

x	an integer
---	------------

8.35.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

8.35.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

8.35.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

8.35.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

8.35.3 Member Data Documentation

8.35.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

8.35.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

8.35.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00044
00045 // aligned allocation
00046 namespace aerobus {
00053     template<typename T>
00054     T* aligned_malloc(size_t count, size_t alignment) {
00055         #ifdef _MSC_VER
00056             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00057         #else
00058             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00059         #endif
00060     }
00061 } // namespace aerobus
00062
00063 // concepts
00064 namespace aerobus {
00066     template <typename R>
00067     concept IsRing = requires {
00068         typename R::one;
00069         typename R::zero;
00070         typename R::template add_t<typename R::one, typename R::one>;
00071         typename R::template sub_t<typename R::one, typename R::one>;
00072         typename R::template mul_t<typename R::one, typename R::one>;
00073     };
00074
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00090     template<typename R>
00091     concept IsField = IsEuclideanDomain<R> && requires {
00092         R::is_field == true;
00093     };
00094 } // namespace aerobus
00095
00096 #ifdef WITH_CUDA_FP16
00097 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00098 namespace aerobus {
00099     namespace internal {
00100         static constexpr DEVICE uint16_t my_internal_float2half(
00101             const float f, uint32_t &sign, uint32_t &remainder) {
00102             uint32_t x;
00103             uint32_t u;
00104             uint32_t result;
00105             x = std::bit_cast<int32_t>(f);
00106             u = (x & 0x7fffffffU);
00107             sign = ((x > 16U) & 0x8000U);
00108             // NaN/+Inf/-Inf
00109             if (u >= 0x7f800000U) {
00110                 remainder = 0U;
00111                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00112             } else if (u > 0x477fefffU) { // Overflows
00113                 remainder = 0x80000000U;
00114                 result = (sign | 0x7bfffU);
00115             } else if (u >= 0x38800000U) { // Normal numbers
00116                 remainder = u << 19U;
00117                 u -= 0x38000000U;
00118                 result = (sign | (u >> 13U));
00119             } else if (u < 0x33000001U) { // +0/-0
00120                 remainder = u;
00121                 result = sign;
00122             } else { // Denormal numbers
00123                 const uint32_t exponent = u >> 23U;
00124                 const uint32_t shift = 0x7eU - exponent;
00125                 uint32_t mantissa = (u & 0x7fffffU);
00126                 mantissa |= 0x800000U;
00127                 remainder = mantissa << (32U - shift);
00128                 result = (sign | (mantissa >> shift));

```



```

00129         result &= 0x0000FFFFU;
00130     }
00131     return static_cast<uint16_t>(result);
00132 }
00133
00134 static constexpr DEVICE __half my_float2half_rn(const float a) {
00135     __half val;
00136     __half_raw r;
00137     uint32_t sign = 0U;
00138     uint32_t remainder = 0U;
00139     r.x = my_internal_float2half(a, sign, remainder);
00140     if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00141         r.x++;
00142     }
00143
00144     val = std::bit_cast<__half>(r);
00145     return val;
00146 }
00147
00148 template<int16_t i>
00149 static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00150
00151
00152 template<typename Out, int16_t x, typename E = void>
00153 struct int16_convert_helper;
00154
00155 template<typename Out, int16_t x>
00156 struct int16_convert_helper<Out, x,
00157     std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00158     static constexpr Out value() {
00159         return static_cast<Out>(x);
00160     }
00161 };
00162
00163 template<int16_t x>
00164 struct int16_convert_helper<__half, x> {
00165     static constexpr __half value() {
00166         return convert_int16_to_half<x>;
00167     }
00168 };
00169
00170 template<int16_t x>
00171 struct int16_convert_helper<__half2, x> {
00172     static constexpr __half2 value() {
00173         return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00174     }
00175 };
00176
00177 } // namespace internal
00178 } // namespace aerobus
00179 #endif
00180
00181 // cast
00182 namespace aerobus {
00183     namespace internal {
00184         template<typename Out, typename In>
00185         struct staticcast {
00186             template<auto x>
00187             static constexpr INLINED_DEVICE Out func() {
00188                 return static_cast<Out>(x);
00189             }
00190         };
00191
00192         #ifdef WITH_CUDA_FP16
00193         template<>
00194         struct staticcast<__half, int16_t> {
00195             template<int16_t x>
00196             static constexpr INLINED_DEVICE __half func() {
00197                 return int16_convert_helper<__half, x>::value();
00198             }
00199         };
00200
00201         template<>
00202         struct staticcast<__half2, int16_t> {
00203             template<int16_t x>
00204             static constexpr INLINED_DEVICE __half2 func() {
00205                 return int16_convert_helper<__half2, x>::value();
00206             }
00207         };
00208         #endif
00209     } // namespace internal
00210 } // namespace aerobus
00211
00212 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00213 namespace aerobus {
00214     namespace internal {
00215         template<typename T>

```

```

00216     struct fma_helper;
00217
00218     template<>
00219     struct fma_helper<double> {
00220         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
00221 z) {
00221         return x * y + z;
00222     }
00223 };
00224
00225     template<>
00226     struct fma_helper<long double> {
00227         static constexpr INLINED_DEVICE long double eval(
00228             const long double x, const long double y, const long double z) {
00229             return x * y + z;
00230         }
00231     };
00232
00233     template<>
00234     struct fma_helper<float> {
00235         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00236             return x * y + z;
00237         }
00238     };
00239
00240     template<>
00241     struct fma_helper<int32_t> {
00242         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
00243 int16_t z) {
00243         return x * y + z;
00244     }
00245 };
00246
00247     template<>
00248     struct fma_helper<int16_t> {
00249         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
00250 int32_t z) {
00250         return x * y + z;
00251     }
00252 };
00253
00254     template<>
00255     struct fma_helper<int64_t> {
00256         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
00257 int64_t z) {
00257         return x * y + z;
00258     }
00259 };
00260
00261     #ifdef WITH_CUDA_FP16
00262     template<>
00263     struct fma_helper<__half> {
00264         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
00265 z) {
00265         #ifdef __CUDA_ARCH__
00266         return __hfma(x, y, z);
00267         #else
00268         return x * y + z;
00269         #endif
00270     }
00271 };
00272
00273     template<>
00274     struct fma_helper<__half2> {
00275         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
00276 __half2 z) {
00276         #ifdef __CUDA_ARCH__
00277         return __hfma2(x, y, z);
00278         #else
00279         return x * y + z;
00280         #endif
00281     }
00282     #endif
00283 } // namespace internal
00284 } // namespace aerobus
00285
00286 // compensated horner utilities
00287 namespace aerobus {
00288     namespace internal {
00289         template <typename T>
00290         struct FloatLayout;
00291
00292         #ifdef _MSC_VER
00293         template <>
00294         struct FloatLayout<long double> {
00295             static constexpr uint8_t exponent = 11;
00296             static constexpr uint8_t mantissa = 53;

```

```

00297         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00298     };
00299     #else
00300     template <>
00301     struct FloatLayout<long double> {
00302         static constexpr uint8_t exponent = 15;
00303         static constexpr uint8_t mantissa = 63;
00304         static constexpr uint8_t r = 32; // ceil(mantissa/2)
00305         static constexpr long double shift = (1LL « r) + 1;
00306     };
00307     #endif
00308
00309     template <>
00310     struct FloatLayout<double> {
00311         static constexpr uint8_t exponent = 11;
00312         static constexpr uint8_t mantissa = 53;
00313         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00314         static constexpr double shift = (1LL « r) + 1;
00315     };
00316
00317     template <>
00318     struct FloatLayout<float> {
00319         static constexpr uint8_t exponent = 8;
00320         static constexpr uint8_t mantissa = 24;
00321         static constexpr uint8_t r = 11; // ceil(mantissa/2)
00322         static constexpr float shift = (1 « r) + 1;
00323     };
00324
00325     template<typename T>
00326     struct Split {
00327         static constexpr INLINED_DEVICE void func(T a, T *x, T *y) {
00328             T z = a * FloatLayout<T>::shift;
00329             *x = z - (z - a);
00330             *y = a - *x;
00331         }
00332     };
00333
00334     #ifdef WITH_CUDA_FP16
00335     template<>
00336     struct Split<__half> {
00337         static constexpr INLINED_DEVICE void func(__half a, __half *x, __half *y) {
00338             __half z = a * __half_raw(0x5280); // TODO(JeWaVe): check this value
00339             *x = z - (z - a);
00340             *y = a - *x;
00341         }
00342     };
00343
00344     template<>
00345     struct Split<__half2> {
00346         static constexpr INLINED_DEVICE void func(__half2 a, __half2 *x, __half2 *y) {
00347             __half2 z = a * __half2(__half_raw(0x5280), __half_raw(0x5280)); // TODO(JeWaVe):
00348             check this value
00349             *x = z - (z - a);
00350             *y = a - *x;
00351         }
00352     };
00353     #endif
00354
00355     template<typename T>
00356     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00357         *x = a + b;
00358         T z = *x - a;
00359         *y = (a - (*x - z)) + (b - z);
00360     }
00361
00362     template<typename T>
00363     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00364         *x = a * b;
00365         #ifdef __clang__
00366         *y = fma_helper<T>::eval(a, b, -*x);
00367         #else
00368         T ah, al, bh, bl;
00369         Split<T>::func(a, &ah, &al);
00370         Split<T>::func(b, &bh, &bl);
00371         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00372     };
00373     #endif
00374
00375     template<typename T, size_t N>
00376     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00377         T r = p1[0] + p2[0];
00378         for (int64_t i = N - 1; i >= 0; --i) {
00379             r = r * x + p1[N - i] + p2[N - i];
00380         }
00381         return r;
00382     }

```

```

00383     } // namespace internal
00384 } // namespace aerobus
00385
00386 // utilities
00387 namespace aerobus {
00388     namespace internal {
00389         template<template<typename...> typename TT, typename T>
00390             struct is_instantiation_of : std::false_type { };
00391
00392         template<template<typename...> typename TT, typename... Ts>
00393             struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00394
00395         template<template<typename...> typename TT, typename T>
00396             inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00397
00398         template<int64_t i, typename T, typename... Ts>
00399         struct type_at {
00400             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00401             using type = typename type_at<i - 1, Ts...>::type;
00402         };
00403
00404         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00405             using type = T;
00406         };
00407
00408         template<size_t i, typename... Ts>
00409         using type_at_t = typename type_at<i, Ts...>::type;
00410
00411
00412         template<size_t n, size_t i, typename E = void>
00413         struct _is_prime {};
00414
00415         template<size_t i>
00416         struct _is_prime<0, i> {
00417             static constexpr bool value = false;
00418         };
00419
00420         template<size_t i>
00421         struct _is_prime<1, i> {
00422             static constexpr bool value = false;
00423         };
00424
00425         template<size_t i>
00426         struct _is_prime<2, i> {
00427             static constexpr bool value = true;
00428         };
00429
00430         template<size_t i>
00431         struct _is_prime<3, i> {
00432             static constexpr bool value = true;
00433         };
00434
00435         template<size_t i>
00436         struct _is_prime<5, i> {
00437             static constexpr bool value = true;
00438         };
00439
00440         template<size_t i>
00441         struct _is_prime<7, i> {
00442             static constexpr bool value = true;
00443         };
00444
00445         template<size_t n, size_t i>
00446         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00447             static constexpr bool value = false;
00448         };
00449
00450         template<size_t n, size_t i>
00451         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00452             static constexpr bool value = false;
00453         };
00454
00455         template<size_t n, size_t i>
00456         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00457             static constexpr bool value = true;
00458         };
00459
00460         template<size_t n, size_t i>
00461         struct _is_prime<n, i, std::enable_if_t<(
00462             n % i == 0 &&
00463             n >= 9 &&
00464             n % 3 != 0 &&
00465             n % 2 != 0 &&
00466             i * i > n)> {
00467             static constexpr bool value = true;
00468         };
00469

```

```

00470     template<size_t n, size_t i>
00471     struct _is_prime<n, i, std::enable_if_t<(
00472         n % (i+2) == 0 &&
00473         n >= 9 &&
00474         n % 3 != 0 &&
00475         n % 2 != 0 &&
00476         i * i <= n)>> {
00477         static constexpr bool value = true;
00478     };
00479
00480     template<size_t n, size_t i>
00481     struct _is_prime<n, i, std::enable_if_t<(
00482         n % (i+2) != 0 &&
00483         n % i != 0 &&
00484         n >= 9 &&
00485         n % 3 != 0 &&
00486         n % 2 != 0 &&
00487         (i * i <= n))>> {
00488         static constexpr bool value = _is_prime<n, i+6>::value;
00489     };
00490 } // namespace internal
00491
00492 template<size_t n>
00493 struct is_prime {
00494     static constexpr bool value = internal::_is_prime<n, 5>::value;
00495 };
00496
00497 template<size_t n>
00498 static constexpr bool is_prime_v = is_prime<n>::value;
00499
00500 // gcd
00501 namespace internal {
00502     template <std::size_t... Is>
00503     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00504     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00505
00506     template <std::size_t N>
00507     using make_index_sequence_reverse
00508     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00509
00510     template<typename Ring, typename E = void>
00511     struct gcd;
00512
00513     template<typename Ring>
00514     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00515         template<typename A, typename B, typename E = void>
00516         struct gcd_helper {};
00517
00518         // B = 0, A > 0
00519         template<typename A, typename B>
00520         struct gcd_helper<A, B, std::enable_if_t<
00521             (B::is_zero_t::value) &&
00522             (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00523             using type = A;
00524         };
00525
00526         // B = 0, A < 0
00527         template<typename A, typename B>
00528         struct gcd_helper<A, B, std::enable_if_t<
00529             ((B::is_zero_t::value) &&
00530             !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00531             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00532         };
00533
00534         // B != 0
00535         template<typename A, typename B>
00536         struct gcd_helper<A, B, std::enable_if_t<
00537             (!B::is_zero_t::value)
00538             >> {
00539             private: // NOLINT
00540                 // A / B
00541                 using k = typename Ring::template div_t<A, B>;
00542                 // A - (A/B)*B = A % B
00543                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00544
00545             public:
00546                 using type = typename gcd_helper<B, m>::type;
00547         };
00548
00549         template<typename A, typename B>
00550         using type = typename gcd_helper<A, B>::type;
00551     };
00552 } // namespace internal
00553
00554 // vadd and vmul
00555 namespace internal {
00556     template<typename... vals>

```

```

00568     struct vmul {};
00569
00570     template<typename v1, typename... vals>
00571     struct vmul<v1, vals...> {
00572         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00573     };
00574
00575     template<typename v1>
00576     struct vmul<v1> {
00577         using type = v1;
00578     };
00579
00580     template<typename... vals>
00581     struct vadd {};
00582
00583     template<typename v1, typename... vals>
00584     struct vadd<v1, vals...> {
00585         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00586     };
00587
00588     template<typename v1>
00589     struct vadd<v1> {
00590         using type = v1;
00591     };
00592 } // namespace internal
00593
00594 template<typename T, typename A, typename B>
00595 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00596
00597 template<typename... vals>
00598 using vadd_t = typename internal::vadd<vals...>::type;
00599
00600 template<typename... vals>
00601 using vmul_t = typename internal::vmul<vals...>::type;
00602
00603 template<typename val>
00604 requires IsEuclideanDomain<typename val::enclosing_type>
00605 using abs_t = std::conditional_t<
    val::enclosing_type::template pos_v<val>,
    val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00606 } // namespace aerobus
00607
00608 // embedding
00609 namespace aerobus {
00610     template<typename Small, typename Large, typename E = void>
00611     struct Embed;
00612 } // namespace aerobus
00613
00614 namespace aerobus {
00615     template<typename Ring, typename X>
00616     requires IsRing<Ring>
00617     struct Quotient {
00618         template<typename V>
00619         struct val {
00620             public:
00621                 using raw_t = V;
00622                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00623         };
00624
00625         using zero = val<typename Ring::zero>;
00626
00627         using one = val<typename Ring::one>;
00628
00629         template<typename v1, typename v2>
00630         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00631
00632         template<typename v1, typename v2>
00633         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00634
00635         template<typename v1, typename v2>
00636         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00637
00638         template<typename v1, typename v2>
00639         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00640
00641         template<typename v1, typename v2>
00642         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00643
00644         template<typename v1, typename v2>
00645         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00646
00647         template<typename v1>
00648         using pos_t = std::true_type;
00649     }
00650 }

```

```

00699     template<typename v>
00700     static constexpr bool pos_v = pos_t<v>::value;
00701
00703     static constexpr bool is_euclidean_domain = true;
00704
00708     template<auto x>
00709     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00710
00714     template<typename v>
00715     using inject_ring_t = val<v>;
00716 };
00717
00721     template<typename Ring, typename X>
00722     struct Embed<Quotient<Ring, X>, Ring> {
00723         template<typename val>
00724         using type = typename val::raw_t;
00725     };
00726 } // namespace aerobus
00727
00729 // type_list
00730 namespace aerobus {
00731     template <typename... Ts>
00732     struct type_list;
00733
00735     namespace internal {
00736         template <typename T, typename... Us>
00737         struct pop_front_h {
00738             using tail = type_list<Us...>;
00739             using head = T;
00740         };
00741
00743         template <size_t index, typename L1, typename L2>
00744         struct split_h {
00745             private:
00746                 static_assert(index <= L2::length, "index out of bounds");
00747                 using a = typename L2::pop_front::type;
00748                 using b = typename L2::pop_front::tail;
00749                 using c = typename L1::template push_back<a>;
00750
00751             public:
00752                 using head = typename split_h<index - 1, c, b>::head;
00753                 using tail = typename split_h<index - 1, c, b>::tail;
00754         };
00755
00756         template <typename L1, typename L2>
00757         struct split_h<0, L1, L2> {
00758             using head = L1;
00759             using tail = L2;
00760         };
00761
00762         template <size_t index, typename L, typename T>
00763         struct insert_h {
00764             static_assert(index <= L::length, "index out of bounds");
00765             using s = typename L::template split<index>;
00766             using left = typename s::head;
00767             using right = typename s::tail;
00768             using ll = typename left::template push_back<T>;
00769             using type = typename ll::template concat<right>;
00770         };
00771
00772         template <size_t index, typename L>
00773         struct remove_h {
00774             using s = typename L::template split<index>;
00775             using left = typename s::head;
00776             using right = typename s::tail;
00777             using rr = typename right::pop_front::tail;
00778             using type = typename left::template concat<rr>;
00779         };
00780     } // namespace internal
00781
00784     template <typename... Ts>
00785     struct type_list {
00786     private:
00787         template <typename T>
00788         struct concat_h;
00789
00790         template <typename... Us>
00791         struct concat_h<type_list<Us...>> {
00792             using type = type_list<Ts..., Us...>;
00793         };
00794
00795     public:
00796         static constexpr size_t length = sizeof...(Ts);
00797
00801         template <typename T>
00802         using push_front = type_list<T, Ts...>;
00803

```

```

00806     template <size_t index>
00807     using at = internal::type_at_t<index, Ts...>;
00808
00810     struct pop_front {
00812         using type = typename internal::pop_front_h<Ts...>::head;
00814         using tail = typename internal::pop_front_h<Ts...>::tail;
00815     };
00816
00819     template <typename T>
00820     using push_back = type_list<Ts..., T>;
00821
00824     template <typename U>
00825     using concat = typename concat_h<U>::type;
00826
00829     template <size_t index>
00830     struct split {
00831     private:
00832         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00833
00834     public:
00835         using head = typename inner::head;
00836         using tail = typename inner::tail;
00837     };
00838
00842     template <typename T, size_t index>
00843     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00844
00847     template <size_t index>
00848     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00849 };
00850
00852 template <>
00853 struct type_list<> {
00854     static constexpr size_t length = 0;
00855
00856     template <typename T>
00857     using push_front = type_list<T>;
00858
00859     template <typename T>
00860     using push_back = type_list<T>;
00861
00862     template <typename U>
00863     using concat = U;
00864
00865     // TODO(jewave): assert index == 0
00866     template <typename T, size_t index>
00867     using insert = type_list<T>;
00868 };
00869 } // namespace aerobus
00870
00871 // i16
00872 #ifdef WITH_CUDA_FP16
00873 // i16
00874 namespace aerobus {
00875     struct i16 {
00877         using inner_type = int16_t;
00878         template<int16_t x>
00881         struct val {
00883             using enclosing_type = i16;
00885             static constexpr int16_t v = x;
00886
00889             template<typename valueType>
00890             static constexpr INLINED_DEVICE valueType get() {
00891                 return internal::template int16_convert_helper<valueType, x>::value();
00892             }
00893
00895             using is_zero_t = std::bool_constant<x == 0>;
00896
00898             static std::string to_string() {
00899                 return std::to_string(x);
00900             }
00901         };
00902
00904         using zero = val<0>;
00906         using one = val<1>;
00908         static constexpr bool is_field = false;
00910         static constexpr bool is_euclidean_domain = true;
00913         template<auto x>
00914         using inject_constant_t = val<static_cast<int16_t>(x)>;
00915
00916         template<typename v>
00917         using inject_ring_t = v;
00918
00919     private:
00920         template<typename v1, typename v2>
00921         struct add {
00922             using type = val<v1::v + v2::v>;

```



```

00923     };
00924
00925     template<typename v1, typename v2>
00926     struct sub {
00927         using type = val<v1::v - v2::v>;
00928     };
00929
00930     template<typename v1, typename v2>
00931     struct mul {
00932         using type = val<v1::v * v2::v>;
00933     };
00934
00935     template<typename v1, typename v2>
00936     struct div {
00937         using type = val<v1::v / v2::v>;
00938     };
00939
00940     template<typename v1, typename v2>
00941     struct remainder {
00942         using type = val<v1::v % v2::v>;
00943     };
00944
00945     template<typename v1, typename v2>
00946     struct gt {
00947         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00948     };
00949
00950     template<typename v1, typename v2>
00951     struct lt {
00952         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00953     };
00954
00955     template<typename v1, typename v2>
00956     struct eq {
00957         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00958     };
00959
00960     template<typename v1>
00961     struct pos {
00962         using type = std::bool_constant<(v1::v > 0)>;
00963     };
00964
00965     public:
00970     template<typename v1, typename v2>
00971     using add_t = typename add<v1, v2>::type;
00972
00977     template<typename v1, typename v2>
00978     using sub_t = typename sub<v1, v2>::type;
00979
00984     template<typename v1, typename v2>
00985     using mul_t = typename mul<v1, v2>::type;
00986
00991     template<typename v1, typename v2>
00992     using div_t = typename div<v1, v2>::type;
00993
00998     template<typename v1, typename v2>
00999     using mod_t = typename remainder<v1, v2>::type;
01000
01005     template<typename v1, typename v2>
01006     using gt_t = typename gt<v1, v2>::type;
01007
01012     template<typename v1, typename v2>
01013     using lt_t = typename lt<v1, v2>::type;
01014
01019     template<typename v1, typename v2>
01020     using eq_t = typename eq<v1, v2>::type;
01021
01025     template<typename v1, typename v2>
01026     static constexpr bool eq_v = eq_t<v1, v2>::value;
01027
01032     template<typename v1, typename v2>
01033     using gcd_t = gcd_t<i16, v1, v2>;
01034
01038     template<typename v>
01039     using pos_t = typename pos<v>::type;
01040
01044     template<typename v>
01045     static constexpr bool pos_v = pos_t<v>::value;
01046     };
01047 } // namespace aerobus
01048 #endif
01049
01050 // i32
01051 namespace aerobus {
01053     struct i32 {
01054         using inner_type = int32_t;
01055         template<int32_t x>

```

```

01058     struct val {
01060         using enclosing_type = i32;
01062         static constexpr int32_t v = x;
01063
01066         template<typename valueType>
01067         static constexpr DEVICE valueType get() {
01068             return static_cast<valueType>(x);
01069         }
01070
01072         using is_zero_t = std::bool_constant<x == 0>;
01073
01075         static std::string to_string() {
01076             return std::to_string(x);
01077         }
01078     };
01079
01081     using zero = val<0>;
01083     using one = val<1>;
01085     static constexpr bool is_field = false;
01087     static constexpr bool is_euclidean_domain = true;
01090     template<auto x>
01091     using inject_constant_t = val<static_cast<int32_t>(x)>;
01092
01093     template<typename v>
01094     using inject_ring_t = v;
01095
01096 private:
01097     template<typename v1, typename v2>
01098     struct add {
01099         using type = val<v1::v + v2::v>;
01100     };
01101
01102     template<typename v1, typename v2>
01103     struct sub {
01104         using type = val<v1::v - v2::v>;
01105     };
01106
01107     template<typename v1, typename v2>
01108     struct mul {
01109         using type = val<v1::v * v2::v>;
01110     };
01111
01112     template<typename v1, typename v2>
01113     struct div {
01114         using type = val<v1::v / v2::v>;
01115     };
01116
01117     template<typename v1, typename v2>
01118     struct remainder {
01119         using type = val<v1::v % v2::v>;
01120     };
01121
01122     template<typename v1, typename v2>
01123     struct gt {
01124         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01125     };
01126
01127     template<typename v1, typename v2>
01128     struct lt {
01129         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01130     };
01131
01132     template<typename v1, typename v2>
01133     struct eq {
01134         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01135     };
01136
01137     template<typename v1>
01138     struct pos {
01139         using type = std::bool_constant<(v1::v > 0)>;
01140     };
01141
01142 public:
01147     template<typename v1, typename v2>
01148     using add_t = typename add<v1, v2>::type;
01149
01154     template<typename v1, typename v2>
01155     using sub_t = typename sub<v1, v2>::type;
01156
01161     template<typename v1, typename v2>
01162     using mul_t = typename mul<v1, v2>::type;
01163
01168     template<typename v1, typename v2>
01169     using div_t = typename div<v1, v2>::type;
01170
01175     template<typename v1, typename v2>
01176     using mod_t = typename remainder<v1, v2>::type;

```

```

01177
01182     template<typename v1, typename v2>
01183     using gt_t = typename gt<v1, v2>::type;
01184
01189     template<typename v1, typename v2>
01190     using lt_t = typename lt<v1, v2>::type;
01191
01196     template<typename v1, typename v2>
01197     using eq_t = typename eq<v1, v2>::type;
01198
01202     template<typename v1, typename v2>
01203     static constexpr bool eq_v = eq_t<v1, v2>::value;
01204
01209     template<typename v1, typename v2>
01210     using gcd_t = gcd_t<i32, v1, v2>;
01211
01215     template<typename v>
01216     using pos_t = typename pos<v>::type;
01217
01221     template<typename v>
01222     static constexpr bool pos_v = pos_t<v>::value;
01223 };
01224 } // namespace aerobus
01225
01226 // i64
01227 namespace aerobus {
01228     struct i64 {
01231         using inner_type = int64_t;
01232         template<int64_t x>
01233         struct val {
01237             using inner_type = int32_t;
01238             using enclosing_type = i64;
01239             static constexpr int64_t v = x;
01240
01245             template<typename valueType>
01246             static constexpr INLINED_DEVICE valueType get() {
01247                 return static_cast<valueType>(x);
01248             }
01249
01251             using is_zero_t = std::bool_constant<x == 0>;
01252
01254             static std::string to_string() {
01255                 return std::to_string(x);
01256             }
01257         };
01258
01261         template<auto x>
01262         using inject_constant_t = val<static_cast<int64_t>(x)>;
01263
01268         template<typename v>
01269         using inject_ring_t = v;
01270
01272         using zero = val<0>;
01273         using one = val<1>;
01274         static constexpr bool is_field = false;
01275         static constexpr bool is_euclidean_domain = true;
01276
01280     private:
01281         template<typename v1, typename v2>
01282         struct add {
01283             using type = val<v1::v + v2::v>;
01284         };
01285
01286         template<typename v1, typename v2>
01287         struct sub {
01288             using type = val<v1::v - v2::v>;
01289         };
01290
01291         template<typename v1, typename v2>
01292         struct mul {
01293             using type = val<v1::v * v2::v>;
01294         };
01295
01296         template<typename v1, typename v2>
01297         struct div {
01298             using type = val<v1::v / v2::v>;
01299         };
01300
01301         template<typename v1, typename v2>
01302         struct remainder {
01303             using type = val<v1::v % v2::v>;
01304         };
01305
01306         template<typename v1, typename v2>
01307         struct gt {
01308             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01309         };

```

```

01310
01311     template<typename v1, typename v2>
01312     struct lt {
01313         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01314     };
01315
01316     template<typename v1, typename v2>
01317     struct eq {
01318         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01319     };
01320
01321     template<typename v>
01322     struct pos {
01323         using type = std::bool_constant<(v::v > 0)>;
01324     };
01325
01326 public:
01327     template<typename v1, typename v2>
01328     using add_t = typename add<v1, v2>::type;
01329
01330     template<typename v1, typename v2>
01331     using sub_t = typename sub<v1, v2>::type;
01332
01333     template<typename v1, typename v2>
01334     using mul_t = typename mul<v1, v2>::type;
01335
01336     template<typename v1, typename v2>
01337     using div_t = typename div<v1, v2>::type;
01338
01339     template<typename v1, typename v2>
01340     using mod_t = typename remainder<v1, v2>::type;
01341
01342     template<typename v1, typename v2>
01343     using gt_t = typename gt<v1, v2>::type;
01344
01345     template<typename v1, typename v2>
01346     static constexpr bool gt_v = gt_t<v1, v2>::value;
01347
01348     template<typename v1, typename v2>
01349     using lt_t = typename lt<v1, v2>::type;
01350
01351     template<typename v1, typename v2>
01352     static constexpr bool lt_v = lt_t<v1, v2>::value;
01353
01354     template<typename v1, typename v2>
01355     using eq_t = typename eq<v1, v2>::type;
01356
01357     template<typename v1, typename v2>
01358     static constexpr bool eq_v = eq_t<v1, v2>::value;
01359
01360     template<typename v1, typename v2>
01361     using gcd_t = gcd_t<i64, v1, v2>;
01362
01363     template<typename v>
01364     using pos_t = typename pos<v>::type;
01365
01366     template<typename v>
01367     static constexpr bool pos_v = pos_t<v>::value;
01368 };
01369
01370 template<>
01371 struct Embed<i32, i64> {
01372     template<typename val>
01373     using type = i64::val<static_cast<int64_t>(val::v)>;
01374 };
01375 } // namespace aerobus
01376
01377 // z/pz
01378 namespace aerobus {
01379     template<int32_t p>
01380     struct zp {
01381         using inner_type = int32_t;
01382
01383         template<int32_t x>
01384         struct val {
01385             using enclosing_type = zp<p>;
01386             static constexpr int32_t v = x % p;
01387
01388             template<typename valueType>
01389             static constexpr INLINED_DEVICE valueType get() {
01390                 return static_cast<valueType>(x % p);
01391             }
01392
01393             using is_zero_t = std::bool_constant<v == 0>;
01394
01395             static constexpr bool is_zero_v = v == 0;
01396         };
01397     };
01398 }

```

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```

01586     template<typename v1, typename v2>
01587     static constexpr bool lt_v = lt_t<v1, v2>::value;
01588
01592     template<typename v1, typename v2>
01593     using eq_t = typename eq<v1, v2>::type;
01594
01598     template<typename v1, typename v2>
01599     static constexpr bool eq_v = eq_t<v1, v2>::value;
01600
01604     template<typename v1, typename v2>
01605     using gcd_t = gcd_t<i32, v1, v2>;
01606
01609     template<typename v1>
01610     using pos_t = typename pos<v1>::type;
01611
01614     template<typename v>
01615     static constexpr bool pos_v = pos_t<v>::value;
01616 };
01617
01620 template<int32_t x>
01621 struct Embed<zp<x>, i32> {
01624     template <typename val>
01625     using type = i32::val<val::v>;
01626 };
01627 } // namespace aerobus
01628
01629 // polynomial
01630 namespace aerobus {
01631     // coeffN x^N + ...
01632     template<typename Ring>
01633     requires IsEuclideanDomain<Ring>
01634     struct polynomial {
01639         static constexpr bool is_field = false;
01640         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01641
01644         template<typename P>
01645         struct horner_reduction_t {
01646             template<size_t index, size_t stop>
01647             struct inner {
01648                 template<typename accum, typename x>
01649                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01650                     ::template type<
01651                         typename Ring::template add_t<
01652                             typename Ring::template mul_t<x, accum>,
01653                             typename P::template coeff_at_t<P::degree - index>
01654                             >, x>;
01655             };
01656
01657             template<size_t stop>
01658             struct inner<stop, stop> {
01659                 template<typename accum, typename x>
01660                 using type = accum;
01661             };
01662         };
01663
01667         template<typename coeffN, typename... coeffs>
01668         struct val {
01670             using ring_type = Ring;
01671             using enclosing_type = polynomial<Ring>;
01672             static constexpr size_t degree = sizeof...(coeffs);
01673             using aN = coeffN;
01674             using strip = val<coeffs...>;
01675             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01676             static constexpr bool is_zero_v = is_zero_t::value;
01677
01684         private:
01685             template<size_t index, typename E = void>
01686             struct coeff_at {};
01687
01688             template<size_t index>
01689             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01690                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01691             };
01692
01693             template<size_t index>
01694             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01695                 using type = typename Ring::zero;
01696             };
01697
01698         public:
01699             template<size_t index>
01700             using coeff_at_t = typename coeff_at<index>::type;
01701
01702             static std::string to_string() {
01703                 return string_helper<coeffN, coeffs...>::func();
01704             }
01705         };
01706     };

```

```

01714     template<typename arithmeticType>
01715     static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01716         #ifdef WITH_CUDA_FP16
01717             arithmeticType start;
01718             if constexpr (std::is_same_v<arithmeticType, __half2>) {
01719                 start = __half2(0, 0);
01720             } else {
01721                 start = static_cast<arithmeticType>(0);
01722             }
01723             #else
01724             arithmeticType start = static_cast<arithmeticType>(0);
01725             #endif
01726             return horner_evaluation<arithmeticType, val>
01727                 ::template inner<0, degree + 1>
01728                 ::func(start, x);
01729     }
01730
01731     template<typename arithmeticType>
01732     static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01733         return compensated_horner<arithmeticType, val>::func(x);
01734     }
01735
01736     template<typename x>
01737     using value_at_t = horner_reduction_t<val>
01738         ::template inner<0, degree + 1>
01739         ::template type<typename Ring::zero, x>;
01740 };
01741
01742 template<typename coeffN>
01743 struct val<coeffN> {
01744     using ring_type = Ring;
01745     using enclosing_type = polynomial<Ring>;
01746     static constexpr size_t degree = 0;
01747     using aN = coeffN;
01748     using strip = val<coeffN>;
01749     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01750
01751     static constexpr bool is_zero_v = is_zero_t::value;
01752
01753     template<size_t index, typename E = void>
01754     struct coeff_at {};
01755
01756     template<size_t index>
01757     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01758         using type = aN;
01759     };
01760
01761     template<size_t index>
01762     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01763         using type = typename Ring::zero;
01764     };
01765
01766     template<size_t index>
01767     using coeff_at_t = typename coeff_at<index>::type;
01768
01769     static std::string to_string() {
01770         return string_helper<coeffN>::func();
01771     }
01772
01773     template<typename arithmeticType>
01774     static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01775         return coeffN::template get<arithmeticType>();
01776     }
01777
01778     template<typename arithmeticType>
01779     static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01780         return coeffN::template get<arithmeticType>();
01781     }
01782
01783     template<typename x>
01784     using value_at_t = coeffN;
01785 };
01786
01787 using zero = val<typename Ring::zero>;
01788 using one = val<typename Ring::one>;
01789 using X = val<typename Ring::one, typename Ring::zero>;
01790
01791 private:
01792     template<typename P, typename E = void>
01793     struct simplify;
01794
01795     template<typename P1, typename P2, typename I>
01796     struct add_low;
01797
01798     template<typename P1, typename P2>
01799     struct add {
01800         using type = typename simplify<typename add_low<

```

```

01821         P1,
01822         P2,
01823         internal::make_index_sequence_reverse<
01824         std::max(P1::degree, P2::degree) + 1
01825         >::type>::type;
01826     };
01827
01828     template <typename P1, typename P2, typename I>
01829     struct sub_low;
01830
01831     template <typename P1, typename P2, typename I>
01832     struct mul_low;
01833
01834     template<typename v1, typename v2>
01835     struct mul {
01836         using type = typename mul_low<
01837             v1,
01838             v2,
01839             internal::make_index_sequence_reverse<
01840             v1::degree + v2::degree + 1
01841             >::type>;
01842     };
01843
01844     template<typename coeff, size_t deg>
01845     struct monomial;
01846
01847     template<typename v, typename E = void>
01848     struct derive_helper {};
01849
01850     template<typename v>
01851     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01852         using type = zero;
01853     };
01854
01855     template<typename v>
01856     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01857         using type = typename add<
01858             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01859             typename monomial<
01860                 typename Ring::template mul_t<
01861                     typename v::aN,
01862                     typename Ring::template inject_constant_t<(v::degree)>
01863                 >,
01864                 v::degree - 1
01865             >::type
01866             >::type;
01867     };
01868
01869     template<typename v1, typename v2, typename E = void>
01870     struct eq_helper {};
01871
01872     template<typename v1, typename v2>
01873     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01874         using type = std::false_type;
01875     };
01876
01877
01878     template<typename v1, typename v2>
01879     struct eq_helper<v1, v2, std::enable_if_t<
01880         v1::degree == v2::degree &&
01881         (v1::degree != 0 || v2::degree != 0) &&
01882         std::is_same<
01883             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01884             std::false_type
01885         >::value
01886     > {
01887     > {
01888         using type = std::false_type;
01889     };
01890
01891     template<typename v1, typename v2>
01892     struct eq_helper<v1, v2, std::enable_if_t<
01893         v1::degree == v2::degree &&
01894         (v1::degree != 0 || v2::degree != 0) &&
01895         std::is_same<
01896             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01897             std::true_type
01898         >::value
01899     > {
01900         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01901     };
01902
01903     template<typename v1, typename v2>
01904     struct eq_helper<v1, v2, std::enable_if_t<
01905         v1::degree == v2::degree &&
01906         (v1::degree == 0)
01907     > {

```



```

01908         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01909     };
01910
01911     template<typename v1, typename v2, typename E = void>
01912     struct lt_helper {};
01913
01914     template<typename v1, typename v2>
01915     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01916         using type = std::true_type;
01917     };
01918
01919     template<typename v1, typename v2>
01920     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01921         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01922     };
01923
01924     template<typename v1, typename v2>
01925     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01926         using type = std::false_type;
01927     };
01928
01929     template<typename v1, typename v2, typename E = void>
01930     struct gt_helper {};
01931
01932     template<typename v1, typename v2>
01933     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01934         using type = std::true_type;
01935     };
01936
01937     template<typename v1, typename v2>
01938     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01939         using type = std::false_type;
01940     };
01941
01942     template<typename v1, typename v2>
01943     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01944         using type = std::false_type;
01945     };
01946
01947     // when high power is zero : strip
01948     template<typename P>
01949     struct simplify<P, std::enable_if_t<
01950         std::is_same<
01951             typename Ring::zero,
01952             typename P::aN
01953         >::value && (P::degree > 0)
01954     >> {
01955         using type = typename simplify<typename P::strip>::type;
01956     };
01957
01958     // otherwise : do nothing
01959     template<typename P>
01960     struct simplify<P, std::enable_if_t<
01961         !std::is_same<
01962             typename Ring::zero,
01963             typename P::aN
01964         >::value && (P::degree > 0)
01965     >> {
01966         using type = P;
01967     };
01968
01969     // do not simplify constants
01970     template<typename P>
01971     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01972         using type = P;
01973     };
01974
01975     // addition at
01976     template<typename P1, typename P2, size_t index>
01977     struct add_at {
01978         using type =
01979             typename Ring::template add_t<
01980                 typename P1::template coeff_at_t<index>,
01981                 typename P2::template coeff_at_t<index>;
01982     };
01983
01984     template<typename P1, typename P2, size_t index>
01985     using add_at_t = typename add_at<P1, P2, index>::type;
01986
01987     template<typename P1, typename P2, std::size_t... I>
01988     struct add_low<P1, P2, std::index_sequence<I...>> {
01989         using type = val<add_at_t<P1, P2, I>...>;
01990     };
01991
01992     // subtraction at
01993     template<typename P1, typename P2, size_t index>
01994     struct sub_at {

```

```

01995         using type =
01996             typename Ring::template sub_t<
01997                 typename P1::template coeff_at_t<index>,
01998                 typename P2::template coeff_at_t<index>;
01999     };
02000
02001     template<typename P1, typename P2, size_t index>
02002     using sub_at_t = typename sub_at<P1, P2, index>::type;
02003
02004     template<typename P1, typename P2, std::size_t... I>
02005     struct sub_low<P1, P2, std::index_sequence<I...> {
02006         using type = val<sub_at_t<P1, P2, I>...>;
02007     };
02008
02009     template<typename P1, typename P2>
02010     struct sub {
02011         using type = typename simplify<typename sub_low<
02012             P1,
02013             P2,
02014             internal::make_index_sequence_reverse<
02015                 std::max(P1::degree, P2::degree) + 1
02016             >::type>::type;
02017     };
02018
02019     // multiplication at
02020     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02021     struct mul_at_loop_helper {
02022         using type = typename Ring::template add_t<
02023             typename Ring::template mul_t<
02024                 typename v1::template coeff_at_t<index>,
02025                 typename v2::template coeff_at_t<k - index>
02026             >,
02027             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02028         >;
02029     };
02030
02031     template<typename v1, typename v2, size_t k, size_t stop>
02032     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02033         using type = typename Ring::template mul_t<
02034             typename v1::template coeff_at_t<stop>,
02035             typename v2::template coeff_at_t<0>;
02036     };
02037
02038     template<typename v1, typename v2, size_t k, typename E = void>
02039     struct mul_at {};
02040
02041     template<typename v1, typename v2, size_t k>
02042     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
02043         using type = typename Ring::zero;
02044     };
02045
02046     template<typename v1, typename v2, size_t k>
02047     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
02048         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02049     };
02050
02051     template<typename P1, typename P2, size_t index>
02052     using mul_at_t = typename mul_at<P1, P2, index>::type;
02053
02054     template<typename P1, typename P2, std::size_t... I>
02055     struct mul_low<P1, P2, std::index_sequence<I...> {
02056         using type = val<mul_at_t<P1, P2, I>...>;
02057     };
02058
02059     // division helper
02060     template<typename A, typename B, typename Q, typename R, typename E = void>
02061     struct div_helper {};
02062
02063     template<typename A, typename B, typename Q, typename R>
02064     struct div_helper<A, B, Q, R, std::enable_if_t<
02065         (R::degree < B::degree) ||
02066         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02067         using q_type = Q;
02068         using mod_type = R;
02069         using gcd_type = B;
02070     };
02071
02072     template<typename A, typename B, typename Q, typename R>
02073     struct div_helper<A, B, Q, R, std::enable_if_t<
02074         (R::degree >= B::degree) &&
02075         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02076     private: // NOLINT
02077         using rN = typename R::aN;
02078         using bN = typename B::aN;
02079         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02080             B::degree>::type;
02081         using rr = typename sub<R, typename mul<pT, B>::type>::type;

```

```

02081         using qq = typename add<Q, pT>::type;
02082
02083     public:
02084         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02085         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02086         using gcd_type = rr;
02087     };
02088
02089     template<typename A, typename B>
02090     struct div {
02091         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02092         using q_type = typename div_helper<A, B, zero, A>::q_type;
02093         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02094     };
02095
02096     template<typename P>
02097     struct make_unit {
02098         using type = typename div<P, val<typename P::aN>::q_type;
02099     };
02100
02101     template<typename coeff, size_t deg>
02102     struct monomial {
02103         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02104     };
02105
02106     template<typename coeff>
02107     struct monomial<coeff, 0> {
02108         using type = val<coeff>;
02109     };
02110
02111     template<typename arithmeticType, typename P>
02112     struct horner_evaluation {
02113         template<size_t index, size_t stop>
02114         struct inner {
02115             static constexpr DEVICE INLINED arithmeticType func(
02116                 const arithmeticType& accum, const arithmeticType& x) {
02117                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
stop>::func(
02118                     internal::fma_helper<arithmeticType>::eval(
02119                         x,
02120                         accum,
02121                         P::template coeff_at_t<P::degree - index>::template
get<arithmeticType>(), x);
02122                     );
02123             };
02124
02125             template<size_t stop>
02126             struct inner<stop, stop> {
02127                 static constexpr DEVICE INLINED arithmeticType func(
02128                     const arithmeticType& accum, const arithmeticType& x) {
02129                     return accum;
02130                 }
02131             };
02132         };
02133
02134         template<typename arithmeticType, typename P>
02135         struct compensated_horner {
02136             template<int64_t index, int ghost>
02137             struct EFTHorner {
02138                 static INLINED DEVICE void func(
02139                     arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02140                     arithmeticType p;
02141                     internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02142                     constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
get<arithmeticType>();
02143                     internal::two_sum<arithmeticType>(
02144                         p, coeff,
02145                         r, sigma + P::degree - index - 1);
02146                     EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02147                 }
02148             };
02149
02150             template<int ghost>
02151             struct EFTHorner<-1, ghost> {
02152                 static INLINED DEVICE void func(
02153                     arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02154                     }
02155             };
02156
02157             static INLINED DEVICE arithmeticType func(arithmeticType x) {
02158                 arithmeticType pi[P::degree], sigma[P::degree];
02159                 arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02160                 EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02161                 arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02162                 return r + c;

```

```

02163     }
02164 };
02165
02166 template<typename coeff, typename... coeffs>
02167 struct string_helper {
02168     static std::string func() {
02169         std::string tail = string_helper<coeffs...>::func();
02170         std::string result = "";
02171         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02172             return tail;
02173         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02174             if (sizeof...(coeffs) == 1) {
02175                 result += "x";
02176             } else {
02177                 result += "x^" + std::to_string(sizeof...(coeffs));
02178             }
02179         } else {
02180             if (sizeof...(coeffs) == 1) {
02181                 result += coeff::to_string() + " x";
02182             } else {
02183                 result += coeff::to_string()
02184                     + " x^" + std::to_string(sizeof...(coeffs));
02185             }
02186         }
02187
02188         if (!tail.empty()) {
02189             if (tail.at(0) != '-') {
02190                 result += " + " + tail;
02191             } else {
02192                 result += " - " + tail.substr(1);
02193             }
02194         }
02195
02196         return result;
02197     }
02198 };
02199
02200 template<typename coeff>
02201 struct string_helper<coeff> {
02202     static std::string func() {
02203         if (!std::is_same<coeff, typename Ring::zero>::value) {
02204             return coeff::to_string();
02205         } else {
02206             return "";
02207         }
02208     }
02209 };
02210
02211 public:
02212     template<typename P>
02213     using simplify_t = typename simplify<P>::type;
02214
02215     template<typename v1, typename v2>
02216     using add_t = typename add<v1, v2>::type;
02217
02218     template<typename v1, typename v2>
02219     using sub_t = typename sub<v1, v2>::type;
02220
02221     template<typename v1, typename v2>
02222     using mul_t = typename mul<v1, v2>::type;
02223
02224     template<typename v1, typename v2>
02225     using eq_t = typename eq_helper<v1, v2>::type;
02226
02227     template<typename v1, typename v2>
02228     using lt_t = typename lt_helper<v1, v2>::type;
02229
02230     template<typename v1, typename v2>
02231     using gt_t = typename gt_helper<v1, v2>::type;
02232
02233     template<typename v1, typename v2>
02234     using div_t = typename div<v1, v2>::q_type;
02235
02236     template<typename v1, typename v2>
02237     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02238
02239     template<typename coeff, size_t deg>
02240     using monomial_t = typename monomial<coeff, deg>::type;
02241
02242     template<typename v>
02243     using derive_t = typename derive_helper<v>::type;
02244
02245     template<typename v>
02246     using pos_t = typename Ring::template pos_t<typename v::aN>;
02247
02248     template<typename v>
02249     static constexpr bool pos_v = pos_t<v>::value;

```

```

02285
02289     template<typename v1, typename v2>
02290     using gcd_t = std::conditional_t<
02291         Ring::is_euclidean_domain,
02292         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02293         void>;
02294
02297     template<auto x>
02298     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02299
02302     template<typename v>
02303     using inject_ring_t = val<v>;
02304 };
02305 } // namespace aerobus
02306
02307 // fraction field
02308 namespace aerobus {
02309     namespace internal {
02310         template<typename Ring, typename E = void>
02311         requires IsEuclideanDomain<Ring>
02312         struct _FractionField {};
02313
02314         template<typename Ring>
02315         requires IsEuclideanDomain<Ring>
02316         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02317             static constexpr bool is_field = true;
02318             static constexpr bool is_euclidean_domain = true;
02319
02320         private:
02321             template<typename val1, typename val2, typename E = void>
02322             struct to_string_helper {};
02323
02324             template<typename val1, typename val2>
02325             struct to_string_helper <val1, val2,
02326                 std::enable_if_t<
02327                     Ring::template eq_t<
02328                         val2, typename Ring::one
02329                         >::value
02330                     >
02331                 > {
02332                 static std::string func() {
02333                     return val1::to_string();
02334                 }
02335             };
02336
02337             template<typename val1, typename val2>
02338             struct to_string_helper<val1, val2,
02339                 std::enable_if_t<
02340                     !Ring::template eq_t<
02341                         val2,
02342                         typename Ring::one
02343                         >::value
02344                     >
02345                 > {
02346                 static std::string func() {
02347                     return "(" + val1::to_string() + ")" / "(" + val2::to_string() + ")";
02348                 }
02349             };
02350         };
02351
02352     public:
02353         template<typename val1, typename val2>
02354         struct val {
02355             using x = val1;
02356             using y = val2;
02357             using is_zero_t = typename val1::is_zero_t;
02358             static constexpr bool is_zero_v = val1::is_zero_t::value;
02359
02360             using ring_type = Ring;
02361             using enclosing_type = _FractionField<Ring>;
02362
02363             static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02364
02365             template<typename valueType, int ghost = 0>
02366             struct get_helper {
02367                 static constexpr INLINED_DEVICE valueType get() {
02368                     return internal::staticcast<valueType, typename
02369 ring_type::inner_type>::template func<x::v>() /
02370 internal::staticcast<valueType, typename ring_type::inner_type>::template
02371 func<y::v>();
02372                 }
02373             };
02374
02375             #ifdef WITH_CUDA_FP16
02376             template<int ghost>
02377             struct get_helper<__half, ghost> {
02378                 static constexpr INLINED_DEVICE __half get() {
02379                     return internal::my_float2half_rn(

```

```

02388             internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /
02389             internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02390     }
02391 };
02392
02393     template<int ghost>
02394     struct get_helper<__half2, ghost> {
02395         static constexpr INLINED_DEVICE __half2 get() {
02396             constexpr __half tmp = internal::my_float2half_rn(
02397                 internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /
02398                 internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02399             return __half2(tmp, tmp);
02400         }
02401     };
02402 #endif
02403
02404     template<typename valueType>
02405     static constexpr INLINED_DEVICE valueType get() {
02406         return get_helper<valueType, 0>::get();
02407     }
02408
02409     static std::string to_string() {
02410         return to_string_helper<val1, val2>::func();
02411     }
02412
02413     template<typename arithmeticType>
02414     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02415         return x::eval(v) / y::eval(v);
02416     }
02417 };
02418
02419 using zero = val<typename Ring::zero, typename Ring::one>;
02420 using one = val<typename Ring::one, typename Ring::one>;
02421
02422 template<typename v>
02423 using inject_t = val<v, typename Ring::one>;
02424
02425 template<auto x>
02426 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02427
02428 template<typename v>
02429 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02430
02431 using ring_type = Ring;
02432
02433 private:
02434     template<typename v, typename E = void>
02435     struct simplify {};
02436
02437     // x = 0
02438     template<typename v>
02439     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02440         using type = typename _FractionField<Ring>::zero;
02441     };
02442
02443     // x != 0
02444     template<typename v>
02445     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02446     private:
02447         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02448         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02449         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02450
02451         using posx = std::conditional_t<
02452             !Ring::template pos_v<newy>,
02453             typename Ring::template sub_t<typename Ring::zero, newx>,
02454             newx>;
02455         using posy = std::conditional_t<
02456             !Ring::template pos_v<newy>,
02457             typename Ring::template sub_t<typename Ring::zero, newy>,
02458             newy>;
02459     public:
02460         using type = typename _FractionField<Ring>::template val<posx, posy>;
02461     };
02462
02463     public:
02464         template<typename v>
02465         using simplify_t = typename simplify<v>::type;
02466
02467     private:
02468         template<typename v1, typename v2>
02469         struct add {

```

```

02490     private:
02491         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02492         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02493         using dividend = typename Ring::template add_t<a, b>;
02494         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02495         using g = typename Ring::template gcd_t<dividend, divider>;
02496
02497     public:
02498         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
02499     };
02500
02501     template<typename v>
02502     struct pos {
02503         using type = std::conditional_t<
02504             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02505             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02506             std::true_type,
02507             std::false_type>;
02508     };
02509
02510     template<typename v1, typename v2>
02511     struct sub {
02512     private:
02513         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02514         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02515         using dividend = typename Ring::template sub_t<a, b>;
02516         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02517         using g = typename Ring::template gcd_t<dividend, divider>;
02518
02519     public:
02520         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
02521     };
02522
02523     template<typename v1, typename v2>
02524     struct mul {
02525     private:
02526         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02527         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02528
02529     public:
02530         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02531     };
02532
02533     template<typename v1, typename v2, typename E = void>
02534     struct div {};
02535
02536     template<typename v1, typename v2>
02537     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value>> {
02538     private:
02539         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02540         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02541
02542     public:
02543         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02544     };
02545
02546     template<typename v1, typename v2>
02547     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value>> {
02548         using type = one;
02549     };
02550
02551
02552     template<typename v1, typename v2>
02553     struct eq {
02554         using type = std::conditional_t<
02555             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02556             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02557             std::true_type,
02558             std::false_type>;
02559     };
02560
02561     template<typename v1, typename v2, typename E = void>
02562     struct gt;
02563
02564     template<typename v1, typename v2>
02565     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
02566         >> {
02567         using type = std::false_type;
02568     };
02569
02570
02571     template<typename v1, typename v2>
02572     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
02573 
```

```

02574         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02575     » {
02576         using type = typename gt<
02577             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02578             >::type;
02579     };
02580
02581     template<typename v1, typename v2>
02582     struct gt<v1, v2, std::enable_if_t<
02583         (!eq<v1, v2>::type::value) &&
02584         (pos<v1>::type::value) && (!pos<v2>::type::value)
02585         > {
02586         using type = std::true_type;
02587     };
02588
02589     template<typename v1, typename v2>
02590     struct gt<v1, v2, std::enable_if_t<
02591         (!eq<v1, v2>::type::value) &&
02592         (!pos<v1>::type::value) && (pos<v2>::type::value)
02593         > {
02594         using type = std::false_type;
02595     };
02596
02597     template<typename v1, typename v2>
02598     struct gt<v1, v2, std::enable_if_t<
02599         (!eq<v1, v2>::type::value) &&
02600         (pos<v1>::type::value) && (pos<v2>::type::value)
02601         > {
02602         using type = typename Ring::template gt_t<
02603             typename Ring::template mul_t<v1::x, v2::y>,
02604             typename Ring::template mul_t<v2::y, v2::x>
02605             >;
02606     };
02607
02608     public:
02612         template<typename v1, typename v2>
02613         using add_t = typename add<v1, v2>::type;
02614
02619         template<typename v1, typename v2>
02620         using mod_t = zero;
02621
02626         template<typename v1, typename v2>
02627         using gcd_t = v1;
02628
02632         template<typename v1, typename v2>
02633         using sub_t = typename sub<v1, v2>::type;
02634
02638         template<typename v1, typename v2>
02639         using mul_t = typename mul<v1, v2>::type;
02640
02644         template<typename v1, typename v2>
02645         using div_t = typename div<v1, v2>::type;
02646
02650         template<typename v1, typename v2>
02651         using eq_t = typename eq<v1, v2>::type;
02652
02656         template<typename v1, typename v2>
02657         static constexpr bool eq_v = eq<v1, v2>::type::value;
02658
02662         template<typename v1, typename v2>
02663         using gt_t = typename gt<v1, v2>::type;
02664
02668         template<typename v1, typename v2>
02669         static constexpr bool gt_v = gt<v1, v2>::type::value;
02670
02673         template<typename v1>
02674         using pos_t = typename pos<v1>::type;
02675
02678         template<typename v>
02679         static constexpr bool pos_v = pos_t<v>::value;
02680     };
02681
02682     template<typename Ring, typename E = void>
02683     requires IsEuclideanDomain<Ring>
02684     struct FractionFieldImpl {};
02685
02686     // fraction field of a field is the field itself
02687     template<typename Field>
02688     requires IsEuclideanDomain<Field>
02689     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02690         using type = Field;
02691         template<typename v>
02692         using inject_t = v;
02693     };
02694
02695     // fraction field of a ring is the actual fraction field
02696     template<typename Ring>

```



```

02697     requires IsEuclideanDomain<Ring>
02698     struct FractionFieldImpl<Ring, std::enable_if_t!Ring::is_field> {
02699         using type = _FractionField<Ring>;
02700     };
02701 } // namespace internal
02702
02703 template<typename Ring>
02704 requires IsEuclideanDomain<Ring>
02705 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02706
02707 template<typename Ring>
02708 struct Embed<Ring, FractionField<Ring>> {
02709     template<typename v>
02710     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02711 };
02712 } // namespace aerobus
02713
02714 // short names for common types
02715 namespace aerobus {
02716     template<typename X, typename Y>
02717     requires IsRing<typename X::enclosing_type> &&
02718         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02719     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02720
02721     template<typename X, typename Y>
02722     requires IsRing<typename X::enclosing_type> &&
02723         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02724     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02725
02726     template<typename X, typename Y>
02727     requires IsRing<typename X::enclosing_type> &&
02728         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02729     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02730
02731     template<typename X, typename Y>
02732     requires IsEuclideanDomain<typename X::enclosing_type> &&
02733         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02734     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02735
02736     using q32 = FractionField<i32>;
02737
02738     using fpq32 = FractionField<polynomial<q32>>;
02739
02740     using q64 = FractionField<i64>;
02741
02742     using pi64 = polynomial<i64>;
02743
02744     using pq64 = polynomial<q64>;
02745
02746     using fpq64 = FractionField<polynomial<q64>>;
02747
02748     template<typename Ring, typename v1, typename v2>
02749     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02750
02751     template<typename v>
02752     using embed_int_poly_in_fractions_t =
02753         typename Embed<
02754             polynomial<typename v::ring_type>,
02755             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02756
02757     template<int64_t p, int64_t q>
02758     using make_q64_t = typename q64::template simplify_t<
02759         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02760
02761     template<int32_t p, int32_t q>
02762     using make_q32_t = typename q32::template simplify_t<
02763         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02764
02765     #ifdef WITH_CUDA_FP16
02766     using q16 = FractionField<i16>;
02767
02768     template<int16_t p, int16_t q>
02769     using make_q16_t = typename q16::template simplify_t<
02770         typename q16::val<i16::inject_constant_t<p>, i16::inject_constant_t<q>>;
02771
02772     #endif
02773
02774     template<typename Ring, typename v1, typename v2>
02775     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02776     template<typename Ring, typename v1, typename v2>
02777     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02778
02779     template<>
02780     struct Embed<q32, q64> {
02781         template<typename v>
02782         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02783     };

```

```

02842
02846     template<typename Small, typename Large>
02847     struct Embed<polynomial<Small>, polynomial<Large> {
02848     private:
02849         template<typename v, typename i>
02850         struct at_low;
02851
02852         template<typename v, size_t i>
02853         struct at_index {
02854             using type = typename Embed<Small, Large>::template
02855             type<typename v::template coeff_at_t<i>>;
02856         };
02857
02858         template<typename v, size_t... Is>
02859         struct at_low<v, std::index_sequence<Is...> {
02860             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02861         };
02862     public:
02863         template<typename v>
02864         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02865         1>::type;
02866     };
02867
02868     template<typename Ring, auto... xs>
02869     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02870     typename Ring::template inject_constant_t<xs>...>;
02871
02872     template<typename Ring, auto... xs>
02873     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02874     typename FractionField<Ring>::template inject_constant_t<xs>...>;
02875 } // namespace aerobus
02876
02877 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02878 namespace aerobus {
02879     namespace internal {
02880         template<typename T, size_t x, typename E = void>
02881         struct factorial {};
02882
02883         template<typename T, size_t x>
02884         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02885         private:
02886             template<typename, size_t, typename>
02887             friend struct factorial;
02888         public:
02889             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02890             x - 1>::type>;
02891             static constexpr typename T::inner_type value = type::template get<typename
02892             T::inner_type>();
02893         };
02894
02895         template<typename T>
02896         struct factorial<T, 0> {
02897         public:
02898             using type = typename T::one;
02899             static constexpr typename T::inner_type value = type::template get<typename
02900             T::inner_type>();
02901         };
02902     } // namespace internal
02903
02904     template<typename T, size_t i>
02905     using factorial_t = typename internal::factorial<T, i>::type;
02906
02907     template<typename T, size_t i>
02908     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02909
02910     namespace internal {
02911         template<typename T, size_t k, size_t n, typename E = void>
02912         struct combination_helper {};
02913
02914         template<typename T, size_t k, size_t n>
02915         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02916             using type = typename FractionField<T>::template mul_t<
02917             typename combination_helper<T, k - 1, n - 1>::type,
02918             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02919         };
02920
02921         template<typename T, size_t k, size_t n>
02922         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02923             using type = typename combination_helper<T, n - k, n>::type;
02924         };
02925
02926         template<typename T, size_t n>
02927         struct combination_helper<T, 0, n> {
02928             using type = typename FractionField<T>::one;
02929         };
02930     }
02931
02932     template<typename T, size_t n>
02933     struct combination_helper<T, 0, n> {
02934     public:
02935         using type = typename FractionField<T>::one;
02936     };
02937 }
02938

```

```

02941     template<typename T, size_t k, size_t n>
02942     struct combination {
02943         using type = typename internal::combination_helper<T, k, n>::type::x;
02944         static constexpr typename T::inner_type value =
02945             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02946     };
02947 } // namespace internal
02948
02951 template<typename T, size_t k, size_t n>
02952 using combination_t = typename internal::combination<T, k, n>::type;
02953
02958 template<typename T, size_t k, size_t n>
02959 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02960
02961 namespace internal {
02962     template<typename T, size_t m>
02963     struct bernoulli;
02964
02965     template<typename T, typename accum, size_t k, size_t m>
02966     struct bernoulli_helper {
02967         using type = typename bernoulli_helper<
02968             T,
02969             addfractions_t<T,
02970                 accum,
02971                 mulfractions_t<T,
02972                     makefraction_t<T,
02973                         combination_t<T, k, m + 1>,
02974                         typename T::one>,
02975                         typename bernoulli<T, k>::type
02976                     >,
02977                     >,
02978                     k + 1,
02979                     m>::type;
02980     };
02981
02982     template<typename T, typename accum, size_t m>
02983     struct bernoulli_helper<T, accum, m, m> {
02984         using type = accum;
02985     };
02986
02987
02988
02989     template<typename T, size_t m>
02990     struct bernoulli {
02991         using type = typename FractionField<T>::template mul_t<
02992             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02993             makefraction_t<T,
02994                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02995                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02996             >
02997         >;
02998
02999         template<typename floatType>
03000         static constexpr floatType value = type::template get<floatType>();
03001     };
03002
03003     template<typename T>
03004     struct bernoulli<T, 0> {
03005         using type = typename FractionField<T>::one;
03006
03007         template<typename floatType>
03008         static constexpr floatType value = type::template get<floatType>();
03009     };
03010 } // namespace internal
03011
03015 template<typename T, size_t n>
03016 using bernoulli_t = typename internal::bernoulli<T, n>::type;
03017
03022 template<typename FloatType, typename T, size_t n>
03023 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
03024
03025 // bell numbers
03026 namespace internal {
03027     template<typename T, size_t n, typename E = void>
03028     struct bell_helper;
03029
03030     template<typename T, size_t n>
03031     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
03032         template<typename accum, size_t i, size_t stop>
03033         struct sum_helper {
03034             private:
03035                 using left = typename T::template mul_t<
03036                     combination_t<T, i, n-1>,
03037                     typename bell_helper<T, i>::type>;
03038                 using new_accum = typename T::template add_t<accum, left>;
03039             public:

```

```

03040         using type = typename sum_helper<new_accum, i+1, stop>::type;
03041     };
03042
03043     template<typename accum, size_t stop>
03044     struct sum_helper<accum, stop, stop> {
03045         using type = accum;
03046     };
03047
03048     using type = typename sum_helper<typename T::zero, 0, n>::type;
03049 };
03050
03051 template<typename T>
03052 struct bell_helper<T, 0> {
03053     using type = typename T::one;
03054 };
03055
03056 template<typename T>
03057 struct bell_helper<T, 1> {
03058     using type = typename T::one;
03059 };
03060 } // namespace internal
03061
03062 template<typename T, size_t n>
03063 using bell_t = typename internal::bell_helper<T, n>::type;
03064
03065 template<typename T, size_t n>
03066 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03067
03068 namespace internal {
03069     template<typename T, int k, typename E = void>
03070     struct alternate {};
03071
03072     template<typename T, int k>
03073     struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
03074         using type = typename T::one;
03075         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
03076     };
03077
03078     template<typename T, int k>
03079     struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
03080         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03081         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
03082     };
03083 } // namespace internal
03084
03085 template<typename T, int k>
03086 using alternate_t = typename internal::alternate<T, k>::type;
03087
03088 template<typename T, size_t k>
03089 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03090
03091 namespace internal {
03092     template<typename T, int n, int k, typename E = void>
03093     struct stirling_l_helper {};
03094
03095     template<typename T>
03096     struct stirling_l_helper<T, 0, 0> {
03097         using type = typename T::one;
03098     };
03099
03100     template<typename T, int n>
03101     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
03102         using type = typename T::zero;
03103     };
03104
03105     template<typename T, int n>
03106     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
03107         using type = typename T::zero;
03108     };
03109
03110     template<typename T, int n, int k>
03111     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)>> {
03112         using type = typename T::template sub_t<
03113             typename stirling_l_helper<T, n-1, k-1>::type,
03114             typename T::template mul_t<
03115                 typename T::template inject_constant_t<n-1>,
03116                 typename stirling_l_helper<T, n-1, k>::type
03117             >>;
03118     };
03119 } // namespace internal
03120
03121 template<typename T, int n, int k>
03122 using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
03123
03124 template<typename T, int n, int k>

```

```

03143     using stirring_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03144
03149     template<typename T, int n, int k>
03150     static constexpr typename T::inner_type stirring_1_unsigned_v = stirring_1_unsigned_t<T, n, k>::v;
03151
03156     template<typename T, int n, int k>
03157     static constexpr typename T::inner_type stirring_1_signed_v = stirring_1_signed_t<T, n, k>::v;
03158
03159     namespace internal {
03160         template<typename T, int n, int k, typename E = void>
03161         struct stirring_2_helper {};
03162
03163         template<typename T, int n>
03164         struct stirring_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
03165             using type = typename T::one;
03166         };
03167
03168         template<typename T, int n>
03169         struct stirring_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03170             using type = typename T::zero;
03171         };
03172
03173         template<typename T, int n>
03174         struct stirring_2_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03175             using type = typename T::zero;
03176         };
03177
03178         template<typename T, int n, int k>
03179         struct stirring_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03180             using type = typename T::template add_t<
03181                 typename stirring_2_helper<T, n-1, k-1>::type,
03182                 typename T::template mul_t<
03183                     typename T::template inject_constant_t<k>,
03184                     typename stirring_2_helper<T, n-1, k>::type
03185                 >;
03186         };
03187     } // namespace internal
03188
03193     template<typename T, int n, int k>
03194     using stirring_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03195
03200     template<typename T, int n, int k>
03201     static constexpr typename T::inner_type stirring_2_v = stirring_2_t<T, n, k>::v;
03202
03203     namespace internal {
03204         template<typename T>
03205         struct pow_scalar {
03206             template<size_t p>
03207             static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03208                 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03209                 x * func<p/2>(x) * func<p/2>(x);
03210             }
03211         };
03212
03213         template<typename T, typename p, size_t n, typename E = void>
03214         requires IsEuclideanDomain<T>
03215         struct pow;
03216
03217         template<typename T, typename p, size_t n>
03218         struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03219             using type = typename T::template mul_t<
03220                 typename pow<T, p, n/2>::type,
03221                 typename pow<T, p, n/2>::type
03222             >;
03223         };
03224
03225         template<typename T, typename p, size_t n>
03226         struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03227             using type = typename T::template mul_t<
03228                 p,
03229                 typename T::template mul_t<
03230                     typename pow<T, p, n/2>::type,
03231                     typename pow<T, p, n/2>::type
03232                 >
03233             >;
03234         };
03235
03236         template<typename T, typename p, size_t n>
03237         struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03238     } // namespace internal
03239
03244     template<typename T, typename p, size_t n>
03245     using pow_t = typename internal::pow<T, p, n>::type;
03246
03251     template<typename T, typename p, size_t n>
03252     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03253

```

```

03254     template<typename T, size_t p>
03255     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
internal::pow_scalar<T>::template func<p>(x); }
03256
03257     namespace internal {
03258         template<typename, template<typename, size_t> typename, class>
03259         struct make_taylor_impl;
03260
03261         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03262         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03263             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
03264         };
03265     }
03266
03271     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03272     using taylor = typename internal::make_taylor_impl<
03273         T,
03274         coeff_at,
03275         internal::make_index_sequence_reverse<deg + 1>::type;
03276
03277     namespace internal {
03278         template<typename T, size_t i>
03279         struct exp_coeff {
03280             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03281         };
03282
03283         template<typename T, size_t i, typename E = void>
03284         struct sin_coeff_helper {};
03285
03286         template<typename T, size_t i>
03287         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03288             using type = typename FractionField<T>::zero;
03289         };
03290
03291         template<typename T, size_t i>
03292         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03293             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03294         };
03295
03296         template<typename T, size_t i>
03297         struct sin_coeff {
03298             using type = typename sin_coeff_helper<T, i>::type;
03299         };
03300
03301         template<typename T, size_t i, typename E = void>
03302         struct sh_coeff_helper {};
03303
03304         template<typename T, size_t i>
03305         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03306             using type = typename FractionField<T>::zero;
03307         };
03308
03309         template<typename T, size_t i>
03310         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03311             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03312         };
03313
03314         template<typename T, size_t i>
03315         struct sh_coeff {
03316             using type = typename sh_coeff_helper<T, i>::type;
03317         };
03318
03319         template<typename T, size_t i, typename E = void>
03320         struct cos_coeff_helper {};
03321
03322         template<typename T, size_t i>
03323         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03324             using type = typename FractionField<T>::zero;
03325         };
03326
03327         template<typename T, size_t i>
03328         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03329             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03330         };
03331
03332         template<typename T, size_t i>
03333         struct cos_coeff {
03334             using type = typename cos_coeff_helper<T, i>::type;
03335         };
03336
03337         template<typename T, size_t i, typename E = void>
03338         struct cosh_coeff_helper {};
03339
03340         template<typename T, size_t i>
03341         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03342             using type = typename FractionField<T>::zero;

```

```

03343     };
03344
03345     template<typename T, size_t i>
03346     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03347         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03348     };
03349
03350     template<typename T, size_t i>
03351     struct cosh_coeff {
03352         using type = typename cosh_coeff_helper<T, i>::type;
03353     };
03354
03355     template<typename T, size_t i>
03356     struct geom_coeff { using type = typename FractionField<T>::one; };
03357
03358
03359     template<typename T, size_t i, typename E = void>
03360     struct atan_coeff_helper;
03361
03362     template<typename T, size_t i>
03363     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03364         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03365     };
03366
03367     template<typename T, size_t i>
03368     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03369         using type = typename FractionField<T>::zero;
03370     };
03371
03372     template<typename T, size_t i>
03373     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03374
03375     template<typename T, size_t i, typename E = void>
03376     struct asin_coeff_helper;
03377
03378     template<typename T, size_t i>
03379     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03380         using type = makefraction_t<T,
03381             factorial_t<T, i - 1>,
03382             typename T::template mul_t<
03383                 typename T::template val<i>,
03384                 T::template mul_t<
03385                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03386                     pow<T, factorial_t<T, i / 2>, 2
03387                 >
03388             >
03389         >>;
03390     };
03391
03392     template<typename T, size_t i>
03393     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03394         using type = typename FractionField<T>::zero;
03395     };
03396
03397     template<typename T, size_t i>
03398     struct asin_coeff {
03399         using type = typename asin_coeff_helper<T, i>::type;
03400     };
03401
03402     template<typename T, size_t i>
03403     struct lnpl_coeff {
03404         using type = makefraction_t<T,
03405             alternate_t<T, i + 1>,
03406             typename T::template val<i>;
03407     };
03408
03409     template<typename T>
03410     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03411
03412     template<typename T, size_t i, typename E = void>
03413     struct asinh_coeff_helper;
03414
03415     template<typename T, size_t i>
03416     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03417         using type = makefraction_t<T,
03418             typename T::template mul_t<
03419                 alternate_t<T, i / 2>,
03420                 factorial_t<T, i - 1>
03421             >,
03422             typename T::template mul_t<
03423                 typename T::template mul_t<
03424                     typename T::template val<i>,
03425                     pow_t<T, factorial_t<T, i / 2>, 2>
03426                 >,
03427                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03428             >
03429         >>;

```

```

03430     };
03431
03432     template<typename T, size_t i>
03433     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03434         using type = typename FractionField<T>::zero;
03435     };
03436
03437     template<typename T, size_t i>
03438     struct asinh_coeff {
03439         using type = typename asinh_coeff_helper<T, i>::type;
03440     };
03441
03442     template<typename T, size_t i, typename E = void>
03443     struct atanh_coeff_helper;
03444
03445     template<typename T, size_t i>
03446     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03447         // 1/i
03448         using type = typename FractionField<T>::template val<
03449             typename T::one,
03450             typename T::template inject_constant_t<i>;
03451     };
03452
03453     template<typename T, size_t i>
03454     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03455         using type = typename FractionField<T>::zero;
03456     };
03457
03458     template<typename T, size_t i>
03459     struct atanh_coeff {
03460         using type = typename atanh_coeff_helper<T, i>::type;
03461     };
03462
03463     template<typename T, size_t i, typename E = void>
03464     struct tan_coeff_helper;
03465
03466     template<typename T, size_t i>
03467     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03468         using type = typename FractionField<T>::zero;
03469     };
03470
03471     template<typename T, size_t i>
03472     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03473     private:
03474         // 4^((i+1)/2)
03475         using _4p = typename FractionField<T>::template inject_t<
03476             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03477         // 4^((i+1)/2) - 1
03478         using _4pml = typename FractionField<T>::template
03479             sub_t<_4p, typename FractionField<T>::one>;
03479         // (-1)^((i-1)/2)
03480         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03481         using dividend = typename FractionField<T>::template mul_t<
03482             altp,
03483             FractionField<T>::template mul_t<
03484                 _4p,
03485                 FractionField<T>::template mul_t<
03486                     _4pml,
03487                     bernoulli_t<T, (i + 1)>
03488                 >
03489             >
03490         >;
03491     public:
03492         using type = typename FractionField<T>::template div_t<dividend,
03493             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03494     };
03495
03496     template<typename T, size_t i>
03497     struct tan_coeff {
03498         using type = typename tan_coeff_helper<T, i>::type;
03499     };
03500
03501     template<typename T, size_t i, typename E = void>
03502     struct tanh_coeff_helper;
03503
03504     template<typename T, size_t i>
03505     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03506         using type = typename FractionField<T>::zero;
03507     };
03508
03509     template<typename T, size_t i>
03510     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03511     private:
03512         using _4p = typename FractionField<T>::template inject_t<
03513             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03514         using _4pml = typename FractionField<T>::template
03515             sub_t<_4p, typename FractionField<T>::one>;

```



```

03515         using dividend =
03516             typename FractionField<T>::template mul_t<
03517                 _4p,
03518                 typename FractionField<T>::template mul_t<
03519                     _4pml,
03520                     bernoulli_t<T, (i + 1)>>::type;
03521     public:
03522         using type = typename FractionField<T>::template div_t<dividend,
03523             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03524     };
03525
03526     template<typename T, size_t i>
03527     struct tanh_coeff {
03528         using type = typename tanh_coeff_helper<T, i>::type;
03529     };
03530 } // namespace internal
03531
03532 template<typename Integers, size_t deg>
03533 using exp = taylor<Integers, internal::exp_coeff, deg>;
03534
03535 template<typename Integers, size_t deg>
03536 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03537     exp<Integers, deg>,
03538     typename polynomial<FractionField<Integers>>::one>;
03539
03540 template<typename Integers, size_t deg>
03541 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03542
03543 template<typename Integers, size_t deg>
03544 using atan = taylor<Integers, internal::atan_coeff, deg>;
03545
03546 template<typename Integers, size_t deg>
03547 using sin = taylor<Integers, internal::sin_coeff, deg>;
03548
03549 template<typename Integers, size_t deg>
03550 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03551
03552 template<typename Integers, size_t deg>
03553 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03554
03555 template<typename Integers, size_t deg>
03556 using cos = taylor<Integers, internal::cos_coeff, deg>;
03557
03558 template<typename Integers, size_t deg>
03559 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03560
03561 template<typename Integers, size_t deg>
03562 using asin = taylor<Integers, internal::asin_coeff, deg>;
03563
03564 template<typename Integers, size_t deg>
03565 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03566
03567 template<typename Integers, size_t deg>
03568 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03569
03570 template<typename Integers, size_t deg>
03571 using tan = taylor<Integers, internal::tan_coeff, deg>;
03572
03573 template<typename Integers, size_t deg>
03574 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03575 } // namespace aerobus
03576
03577 // continued fractions
03578 namespace aerobus {
03579     template<int64_t... values>
03580     struct ContinuedFraction {};
03581
03582     template<int64_t a0>
03583     struct ContinuedFraction<a0> {
03584         using type = typename q64::template inject_constant_t<a0>;
03585         static constexpr double val = static_cast<double>(a0);
03586     };
03587
03588     template<int64_t a0, int64_t... rest>
03589     struct ContinuedFraction<a0, rest...> {
03590         using type = q64::template add_t<
03591             typename q64::template inject_constant_t<a0>,
03592             typename q64::template div_t<
03593                 typename q64::one,
03594                 typename ContinuedFraction<rest...>::type
03595             >>;
03596         static constexpr double val = type::template get<double>();
03597     };
03598
03599     using PI_fraction =
03600         ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 1>;

```

```

03666     using E_fraction =
ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03668     using Sqrt2_fraction =
ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03670     using Sqrt3_fraction =
ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
// NOLINT
03671 } // namespace aerobus
03672
03673 // known polynomials
03674 namespace aerobus {
03675     // CChebyshev
03676     namespace internal {
03677         template<int kind, size_t deg, typename I>
03678         struct chebyshev_helper {
03679             using type = typename polynomial<I>::template sub_t<
03680                 typename polynomial<I>::template mul_t<
03681                     typename polynomial<I>::template mul_t<
03682                         typename polynomial<I>::template inject_constant_t<2>,
03683                         typename polynomial<I>::X>,
03684                         typename chebyshev_helper<kind, deg - 1, I>::type
03685                     >,
03686                     typename chebyshev_helper<kind, deg - 2, I>::type
03687                 >;
03688         };
03689
03690         template<typename I>
03691         struct chebyshev_helper<1, 0, I> {
03692             using type = typename polynomial<I>::one;
03693         };
03694
03695         template<typename I>
03696         struct chebyshev_helper<1, 1, I> {
03697             using type = typename polynomial<I>::X;
03698         };
03699
03700         template<typename I>
03701         struct chebyshev_helper<2, 0, I> {
03702             using type = typename polynomial<I>::one;
03703         };
03704
03705         template<typename I>
03706         struct chebyshev_helper<2, 1, I> {
03707             using type = typename polynomial<I>::template mul_t<
03708                 typename polynomial<I>::template inject_constant_t<2>,
03709                 typename polynomial<I>::X>;
03710         };
03711     } // namespace internal
03712
03713     // Laguerre
03714     namespace internal {
03715         template<size_t deg, typename I>
03716         struct laguerre_helper {
03717             using Q = FractionField<I>;
03718             using PQ = polynomial<Q>;
03719
03720             private:
03721                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03722                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03723                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03724                 // -x + 2k-1
03725                 using p = typename PQ::template val<
03726                     typename Q::template inject_constant_t<-1>,
03727                     typename Q::template inject_constant_t<2 * deg - 1>;
03728                 // 1/n
03729                 using factor = typename PQ::template inject_ring_t<
03730                     typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03731
03732             public:
03733                 using type = typename PQ::template mul_t <
03734                     factor,
03735                     typename PQ::template sub_t<
03736                         typename PQ::template mul_t<
03737                             p,
03738                             lnm1
03739                         >,
03740                     typename PQ::template mul_t<
03741                         typename PQ::template inject_constant_t<deg-1>,
03742                         lnm2
03743                     >
03744                 >
03745             >;
03746         };
03747
03748         template<typename I>
03749         struct laguerre_helper<0, I> {

```

```

03750         using type = typename polynomial<FractionField<I>::one;
03751     };
03752
03753     template<typename I>
03754     struct laguerre_helper<1, I> {
03755     private:
03756         using PQ = polynomial<FractionField<I>;
03757     public:
03758         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03759     };
03760 } // namespace internal
03761
03762 // Bernstein
03763 namespace internal {
03764     template<size_t i, size_t m, typename I, typename E = void>
03765     struct bernstein_helper {};
03766
03767     template<typename I>
03768     struct bernstein_helper<0, 0, I> {
03769     public:
03770         using type = typename polynomial<I>::one;
03771     };
03772
03773     template<size_t i, size_t m, typename I>
03774     struct bernstein_helper<i, m, I, std::enable_if_t<
03775         (m > 0) && (i == 0)> {
03776     private:
03777         using P = polynomial<I>;
03778     public:
03779         using type = typename P::template mul_t<
03780             typename P::template sub_t<typename P::one, typename P::X>,
03781             typename bernstein_helper<i, m-1, I>::type>;
03782     };
03783
03784     template<size_t i, size_t m, typename I>
03785     struct bernstein_helper<i, m, I, std::enable_if_t<
03786         (m > 0) && (i == m)> {
03787     private:
03788         using P = polynomial<I>;
03789     public:
03790         using type = typename P::template mul_t<
03791             typename P::X,
03792             typename bernstein_helper<i-1, m-1, I>::type>;
03793     };
03794
03795     template<size_t i, size_t m, typename I>
03796     struct bernstein_helper<i, m, I, std::enable_if_t<
03797         (m > 0) && (i > 0) && (i < m)> {
03798     private:
03799         using P = polynomial<I>;
03800     public:
03801         using type = typename P::template add_t<
03802             typename P::template mul_t<
03803                 typename P::template sub_t<typename P::one, typename P::X>,
03804                 typename bernstein_helper<i, m-1, I>::type>,
03805             typename P::template mul_t<
03806                 typename P::X,
03807                 typename bernstein_helper<i-1, m-1, I>::type>;
03808     };
03809 } // namespace internal
03810
03811 // AllOne polynomials
03812 namespace internal {
03813     template<size_t deg, typename I>
03814     struct AllOneHelper {
03815     private:
03816         using type = aerobus::add_t<
03817             typename polynomial<I>::one,
03818             typename aerobus::mul_t<
03819                 typename polynomial<I>::X,
03820                 typename AllOneHelper<deg-1, I>::type>;
03821     };
03822
03823     template<typename I>
03824     struct AllOneHelper<0, I> {
03825     public:
03826         using type = typename polynomial<I>::one;
03827     };
03828 } // namespace internal
03829
03830 // Bessel polynomials
03831 namespace internal {
03832     template<size_t deg, typename I>
03833     struct BesselHelper {
03834     private:
03835         using P = polynomial<I>;
03836         using factor = typename P::template monomial_t<
03837             typename I::template inject_constant_t<(2*deg - 1)>,
03838             1>;

```

```

03837     public:
03838         using type = typename P::template add_t<
03839             typename P::template mul_t<
03840                 factor,
03841                 typename BesselHelper<deg-1, I>::type
03842             >,
03843             typename BesselHelper<deg-2, I>::type
03844         >;
03845     };
03846
03847     template<typename I>
03848     struct BesselHelper<0, I> {
03849         using type = typename polynomial<I>::one;
03850     };
03851
03852     template<typename I>
03853     struct BesselHelper<1, I> {
03854     private:
03855         using P = polynomial<I>;
03856     public:
03857         using type = typename P::template add_t<
03858             typename P::one,
03859             typename P::X
03860         >;
03861     };
03862 } // namespace internal
03863
03864 namespace known_polynomials {
03865     enum hermite_kind {
03866         probabilist,
03867         physicist
03868     };
03869 }
03870
03871 // hermite
03872 namespace internal {
03873     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03874     struct hermite_helper {};
03875
03876     template<size_t deg, typename I>
03877     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03878     private:
03879         using hnm1 = typename hermite_helper<deg - 1,
03880 known_polynomials::hermite_kind::probabilist, I>::type;
03881         using hnm2 = typename hermite_helper<deg - 2,
03882 known_polynomials::hermite_kind::probabilist, I>::type;
03883
03884     public:
03885         using type = typename polynomial<I>::template sub_t<
03886             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03887             typename polynomial<I>::template mul_t<
03888                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03889                 hnm2
03890             >
03891         >;
03892     };
03893
03894     template<size_t deg, typename I>
03895     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03896     private:
03897         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03898 I>::type;
03899         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03900 I>::type;
03901
03902     public:
03903         using type = typename polynomial<I>::template sub_t<
03904             // 2X Hn-1
03905             typename polynomial<I>::template mul_t<
03906                 typename pi64::val<typename I::template inject_constant_t<2>>,
03907                 typename I::zero>, hnm1>,
03908             typename polynomial<I>::template mul_t<
03909                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03910                 hnm2
03911             >
03912         >;
03913     };
03914
03915     template<typename I>
03916     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03917         using type = typename polynomial<I>::one;
03918     };
03919
03920     template<typename I>
03921     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03922         using type = typename polynomial<I>::X;

```

```

03923     };
03924
03925     template<typename I>
03926     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03927         using type = typename pi64::one;
03928     };
03929
03930     template<typename I>
03931     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03932         // 2X
03933         using type = typename polynomial<I>::template val<
03934             typename I::template inject_constant_t<2>,
03935             typename I::zero>;
03936     };
03937 } // namespace internal
03938
03939 // legendre
03940 namespace internal {
03941     template<size_t n, typename I>
03942     struct legendre_helper {
03943     private:
03944         using Q = FractionField<I>;
03945         using PQ = polynomial<Q>;
03946         // 1/n constant
03947         // (2n-1)/n X
03948         using fact_left = typename PQ::template monomial_t<
03949             makefraction_t<I,
03950                 typename I::template inject_constant_t<2*n-1>,
03951                 typename I::template inject_constant_t<n>
03952             >,
03953             1>;
03954         // (n-1) / n
03955         using fact_right = typename PQ::template val<
03956             makefraction_t<I,
03957                 typename I::template inject_constant_t<n-1>,
03958                 typename I::template inject_constant_t<n>>;
03959     public:
03960         using type = PQ::template sub_t<
03961             typename PQ::template mul_t<
03962                 fact_left,
03963                 typename legendre_helper<n-1, I>::type
03964             >,
03965             typename PQ::template mul_t<
03966                 fact_right,
03967                 typename legendre_helper<n-2, I>::type
03968             >
03969         >;
03970     };
03971 }
03972
03973     template<typename I>
03974     struct legendre_helper<0, I> {
03975         using type = typename polynomial<FractionField<I>::one>;
03976     };
03977
03978     template<typename I>
03979     struct legendre_helper<1, I> {
03980         using type = typename polynomial<FractionField<I>::X>;
03981     };
03982 } // namespace internal
03983
03984 // bernoulli polynomials
03985 namespace internal {
03986     template<size_t n>
03987     struct bernoulli_coeff {
03988         template<typename T, size_t i>
03989         struct inner {
03990         private:
03991             using F = FractionField<T>;
03992         public:
03993             using type = typename F::template mul_t<
03994                 typename F::template inject_ring_t<combination_t<T, i, n>,
03995                 bernoulli_t<T, n-i>
03996             >;
03997         };
03998     };
03999 } // namespace internal
04000
04001 namespace internal {
04002     template<size_t n>
04003     struct touchard_coeff {
04004         template<typename T, size_t i>
04005         struct inner {
04006             using type = stirling_2_t<T, n, i>;
04007         };
04008     };
04009 } // namespace internal

```

```

04010
04011 namespace internal {
04012     template<typename I = aerobus::i64>
04013     struct AbelHelper {
04014     private:
04015         using P = aerobus::polynomial<I>;
04016
04017     public:
04018         // to keep recursion working, we need to operate on a*n and not just a
04019         template<size_t deg, I::inner_type an>
04020         struct Inner {
04021             // abel(n, a) = (x-an) * abel(n-1, a)
04022             using type = typename aerobus::mul_t<
04023                 typename Inner<deg-1, an>::type,
04024                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
04025             >;
04026         };
04027
04028         // abel(0, a) = 1
04029         template<I::inner_type an>
04030         struct Inner<0, an> {
04031             using type = P::one;
04032         };
04033
04034         // abel(1, a) = X
04035         template<I::inner_type an>
04036         struct Inner<1, an> {
04037             using type = P::X;
04038         };
04039     };
04040 } // namespace internal
04041
04042 namespace known_polynomials {
04043
04044     template<size_t n, auto a, typename I = aerobus::i64>
04045     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
04046
04047     template<size_t deg, typename I = aerobus::i64>
04048     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
04049
04050     template<size_t deg, typename I = aerobus::i64>
04051     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
04052
04053     template<size_t deg, typename I = aerobus::i64>
04054     using laguerre = typename internal::laguerre_helper<deg, I>::type;
04055
04056     template<size_t deg, typename I = aerobus::i64>
04057     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
04058 I>::type;
04059
04060     template<size_t deg, typename I = aerobus::i64>
04061     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04062
04063     template<size_t i, size_t m, typename I = aerobus::i64>
04064     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04065
04066     template<size_t deg, typename I = aerobus::i64>
04067     using legendre = typename internal::legendre_helper<deg, I>::type;
04068
04069     template<size_t deg, typename I = aerobus::i64>
04070     using bernoulli = Taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
04071
04072     template<size_t deg, typename I = aerobus::i64>
04073     using allone = typename internal::AllOneHelper<deg, I>::type;
04074
04075     template<size_t deg, typename I = aerobus::i64>
04076     using bessel = typename internal::BesselHelper<deg, I>::type;
04077
04078     template<size_t deg, typename I = aerobus::i64>
04079     using touchard = Taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
04080 } // namespace known_polynomials
04081 } // namespace aerobus
04082
04083 #ifdef AEROBUS_CONWAY_IMPORTS
04084
04085 // Conway polynomials
04086 namespace aerobus {
04087     template<int p, int n>
04088     struct ConwayPolynomial {};
04089
04090 #ifndef DO_NOT_DOCUMENT
04091     #define ZPV ZPZ::template val
04092     #define POLYV aerobus::polynomial<ZPV>::template val
04093     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
04094     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type =

```

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9.4 src/examples.h File Reference

9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_
```


Chapter 10

Examples

10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

10.2 examples/custom_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

// TO compile with `nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu`
 // Beforehand, you need to modify `cuda_fp16.h` by adding `__CUDA_FP16_CONSTEXPR__` to line 5039 (version 12.6)
 #include <stdio>

```
#define WITH_CUDA_FP16
#include "../src/aerobus.h"
```

```
/*
You may want to change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
*/
using int_type = aerobus::i16;
using float_type = __half2;
```

```
constexpr size_t N = 1 << 26;
```

```
template<typename T>
struct ExpmlDegree;
```

```
template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};
```

```
template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};
```

```
template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};
```

```
template<>
struct ExpmlDegree<__half> {
    static constexpr size_t val = 6;
};
```

```
double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}
```

```
template<typename T>
struct GetRandT;
```

```
template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};
```

```
template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};
```

```
template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};
```

```
template<>
```



```

struct GetRandT<__half>
{
    static __half func(double min, double max) {
        return __float2half((float)rand(min, max));
    }
};

using EXPM1 = aerobus::expm1<int_type, Expm1Degree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        // fp16 FMA pipeline is quite wide so we need to feed it with a LOT of computations
        out[i] = f(f(f(f(f(f(f(f(f(f(f(f(f(f(f(in[i])))))))))))))));
    }
}

#define cudaErrorCheck(ans) { gpuAssert((ans), __FILE__, __LINE__); }
inline void gpuAssert(cudaError_t code, const char *file, int line, bool abort=true)
{
    if (code != cudaSuccess)
    {
        fprintf(stderr, "GPUassert: %s %s %d\n", cudaGetErrorString(code), file, line);
        if (abort) exit(code);
    }
}

int main() {
    // configure CUDA devices
    int deviceCount;
    int device = -1;
    int maxProcCount = 0;
    cudaErrorCheck(cudaGetDeviceCount(&deviceCount));
    for(int i = 0; i < deviceCount; ++i) {
        cudaDeviceProp prop;
        cudaErrorCheck(cudaGetDeviceProperties(&prop, i));
        int procCount = prop.multiProcessorCount;
        if(procCount > maxProcCount) {
            maxProcCount = procCount;
            device = i;
        }
    }

    if(device == -1) {
        ::printf("CANNOT FIND CUDA CAPABLE DEVICE -- aborting\n");
        ::abort();
    }

    cudaErrorCheck(cudaSetDevice(device));
    int blockSize; // The launch configurator returned block size
    int minGridSize; // The minimum grid size needed to achieve the
                    // maximum occupancy for a full device launch

    cudaErrorCheck(cudaOccupancyMaxPotentialBlockSize(&minGridSize, &blockSize, &run, 0, 0));

    ::printf("configure launch bounds to %d-%d\n", minGridSize, blockSize);

    // allocate and populate memory
    float_type *d_in, *d_out;
    cudaErrorCheck(cudaMalloc<float_type>(&d_in, N * sizeof(float_type)));
    cudaErrorCheck(cudaMalloc<float_type>(&d_out, N * sizeof(float_type)));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaErrorCheck(cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice));

    // execute kernel and get memory back from device
    run<<minGridSize, blockSize>>>(N, d_in, d_out);
    cudaErrorCheck(cudaPeekAtLastError());
    cudaErrorCheck(cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost));

    cudaErrorCheck(cudaFree(d_in));
    cudaErrorCheck(cudaFree(d_out));
}

// Example of generated SASS :

/*
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;

```

```

HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R7, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R7, R5, R7, 0.008331298828125, 0.008331298828125 ;
HFMA2 R7, R5, R7, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R7, R5, R7, 0.1666259765625, 0.1666259765625 ;
HFMA2 R7, R5, R7, 0.5, 0.5 ;
HFMA2.MMA R7, R5, R7, 1, 1 ;
HFMA2 R7, R5, R7, RZ.H0_H0 ;
HFMA2.MMA R5, R7, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R7, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R7, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R7, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R7, R5, 0.5, 0.5 ;
HFMA2 R5, R7, R5, 1, 1 ;
HFMA2.MMA R5, R7, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
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HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R6, R6, R5, RZ ;
HFMA2 R5, R6, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2 R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2 R5, R6, R5, 0.5, 0.5 ;
HFMA2.MMA R7, R6, R5, 1, 1 ;
IADD3.X R5, R8, UR11, RZ, P0, !PT ;
IADD3 R3, P0, R2, R3, RZ ;
IADD3.X R0, RZ, R0, RZ, P0, !PT ;
ISETP.GE.U32.AND P0, PT, R3, UR8, PT ;
HFMA2 R7, R6, R7, RZ.H0_H0 ;

```

```

ISETP.GE.U32.AND.EX P0, PT, R0, UR9, PT, P0 ;
STG.E desc[UR6][R4.64], R7 ;
*/

```

10.4 examples/continued_fractions.cpp

How to use `aerobus::ContinuedFraction` to get approximations of known numbers

[illegible]

10.5 examples/modular_arithmetic.cpp

How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

// x^3 + 2x^2 + 1, with coefficients in Z/2Z, actually x^3 + 1
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
// x^3 + 5x^2 + 7x + 11 with coefficients in Z/17Z, meaning actually x^3 + x^2 + 1
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

10.6 examples/make_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
```

```

        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}

```

10.7 examples/polynomials_over_finite_field.cpp

How to build a known polynomial (here `aerobus::known_polynomials::allone`) with coefficients in a finite field (here `aerobus::zpz<2>`) and get its value when evaluated at a value in this field (here 1).

```

#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}

```

10.8 examples/compensated_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```

// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {

```

```

    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;

        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}

```


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