

Aerobus

v1.2

Generated by Doxygen 1.9.8

1 Introduction	1
1.1 HOW TO	1
1.1.1 Unit Test	2
1.1.2 Benchmarks	2
1.2 Structures	2
1.2.1 Predefined discrete euclidean domains	2
1.2.2 Polynomials	3
1.2.3 Known polynomials	3
1.2.4 Conway polynomials	3
1.2.5 Taylor series	4
1.3 Operations	5
1.3.1 Field of fractions	5
1.3.2 Quotient	6
1.4 Misc	6
1.4.1 Continued Fractions	6
1.5 CUDA	6
2 Namespace Index	7
2.1 Namespace List	7
3 Concept Index	9
3.1 Concepts	9
4 Class Index	11
4.1 Class List	11
5 File Index	13
5.1 File List	13
6 Namespace Documentation	15
6.1 aerobus Namespace Reference	15
6.1.1 Detailed Description	19
6.1.2 Typedef Documentation	20
6.1.2.1 abs_t	20
6.1.2.2 add_t	20
6.1.2.3 addfractions_t	20
6.1.2.4 alternate_t	20
6.1.2.5 asin	21
6.1.2.6 asinh	21
6.1.2.7 atan	21
6.1.2.8 atanh	21
6.1.2.9 bell_t	23
6.1.2.10 bernoulli_t	23
6.1.2.11 combination_t	23

6.1.2.12 cos	23
6.1.2.13 cosh	25
6.1.2.14 div_t	25
6.1.2.15 E_fraction	25
6.1.2.16 embed_int_poly_in_fractions_t	25
6.1.2.17 exp	26
6.1.2.18 expm1	26
6.1.2.19 factorial_t	26
6.1.2.20 fpq32	26
6.1.2.21 fpq64	27
6.1.2.22 FractionField	27
6.1.2.23 gcd_t	27
6.1.2.24 geometric_sum	27
6.1.2.25 lnp1	28
6.1.2.26 make_frac_polynomial_t	28
6.1.2.27 make_int_polynomial_t	28
6.1.2.28 make_q32_t	28
6.1.2.29 make_q64_t	29
6.1.2.30 makefraction_t	29
6.1.2.31 mul_t	29
6.1.2.32 mulfractions_t	30
6.1.2.33 pi64	30
6.1.2.34 PI_fraction	30
6.1.2.35 pow_t	30
6.1.2.36 pq64	30
6.1.2.37 q32	31
6.1.2.38 q64	31
6.1.2.39 sin	31
6.1.2.40 sinh	31
6.1.2.41 SQRT2_fraction	31
6.1.2.42 SQRT3_fraction	32
6.1.2.43 stirling_1_signed_t	32
6.1.2.44 stirling_1_unsigned_t	32
6.1.2.45 stirling_2_t	32
6.1.2.46 sub_t	33
6.1.2.47 tan	33
6.1.2.48 tanh	33
6.1.2.49 taylor	33
6.1.2.50 vadd_t	34
6.1.2.51 vmul_t	34
6.1.3 Function Documentation	34
6.1.3.1 aligned_malloc()	34

6.1.3.2 field()	34
6.1.4 Variable Documentation	35
6.1.4.1 alternate_v	35
6.1.4.2 bernoulli_v	35
6.1.4.3 combination_v	35
6.1.4.4 factorial_v	36
6.2 aerobus::internal Namespace Reference	36
6.2.1 Detailed Description	39
6.2.2 Typedef Documentation	40
6.2.2.1 make_index_sequence_reverse	40
6.2.2.2 type_at_t	40
6.2.3 Function Documentation	40
6.2.3.1 index_sequence_reverse()	40
6.2.4 Variable Documentation	40
6.2.4.1 is_instantiation_of_v	40
6.3 aerobus::known_polynomials Namespace Reference	40
6.3.1 Detailed Description	40
6.3.2 Enumeration Type Documentation	40
6.3.2.1 hermite_kind	40
7 Concept Documentation	43
7.1 aerobus::IsEuclideanDomain Concept Reference	43
7.1.1 Concept definition	43
7.1.2 Detailed Description	43
7.2 aerobus::IsField Concept Reference	43
7.2.1 Concept definition	43
7.2.2 Detailed Description	44
7.3 aerobus::IsRing Concept Reference	44
7.3.1 Concept definition	44
7.3.2 Detailed Description	44
8 Class Documentation	45
8.1 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E > Struct Template Reference	45
8.2 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> > Struct Template Reference	45
8.2.1 Member Typedef Documentation	45
8.2.1.1 type	45
8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference	46
8.3.1 Member Typedef Documentation	46
8.3.1.1 type	46
8.4 aerobus::ContinuedFraction< values > Struct Template Reference	46
8.4.1 Detailed Description	46

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference	47
8.5.1 Detailed Description	47
8.5.2 Member Typedef Documentation	47
8.5.2.1 type	47
8.5.3 Member Data Documentation	48
8.5.3.1 val	48
8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference	48
8.6.1 Detailed Description	48
8.6.2 Member Typedef Documentation	49
8.6.2.1 type	49
8.6.3 Member Data Documentation	49
8.6.3.1 val	49
8.7 aerobus::ConwayPolynomial Struct Reference	49
8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference	49
8.8.1 Member Function Documentation	50
8.8.1.1 func()	50
8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference	50
8.9.1 Member Function Documentation	50
8.9.1.1 func()	50
8.10 aerobus::Embed< Small, Large, E > Struct Template Reference	51
8.10.1 Detailed Description	51
8.11 aerobus::Embed< i32, i64 > Struct Reference	51
8.11.1 Detailed Description	51
8.11.2 Member Typedef Documentation	51
8.11.2.1 type	51
8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference	52
8.12.1 Detailed Description	52
8.12.2 Member Typedef Documentation	52
8.12.2.1 type	52
8.13 aerobus::Embed< q32, q64 > Struct Reference	53
8.13.1 Detailed Description	53
8.13.2 Member Typedef Documentation	53
8.13.2.1 type	53
8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference	54
8.14.1 Detailed Description	54
8.14.2 Member Typedef Documentation	54
8.14.2.1 type	54
8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference	55
8.15.1 Detailed Description	55
8.15.2 Member Typedef Documentation	55
8.15.2.1 type	55

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference	55
8.16.1 Detailed Description	56
8.16.2 Member Typedef Documentation	56
8.16.2.1 type	56
8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference	56
8.17.1 Detailed Description	57
8.18 aerobus::i32 Struct Reference	57
8.18.1 Detailed Description	58
8.18.2 Member Typedef Documentation	59
8.18.2.1 add_t	59
8.18.2.2 div_t	59
8.18.2.3 eq_t	59
8.18.2.4 gcd_t	59
8.18.2.5 gt_t	60
8.18.2.6 inject_constant_t	60
8.18.2.7 inject_ring_t	60
8.18.2.8 inner_type	60
8.18.2.9 lt_t	60
8.18.2.10 mod_t	61
8.18.2.11 mul_t	61
8.18.2.12 one	61
8.18.2.13 pos_t	61
8.18.2.14 sub_t	62
8.18.2.15 zero	62
8.18.3 Member Data Documentation	62
8.18.3.1 eq_v	62
8.18.3.2 is_euclidean_domain	62
8.18.3.3 is_field	62
8.18.3.4 pos_v	63
8.19 aerobus::i64 Struct Reference	64
8.19.1 Detailed Description	65
8.19.2 Member Typedef Documentation	65
8.19.2.1 add_t	65
8.19.2.2 div_t	66
8.19.2.3 eq_t	66
8.19.2.4 gcd_t	66
8.19.2.5 gt_t	66
8.19.2.6 inject_constant_t	67
8.19.2.7 inject_ring_t	67
8.19.2.8 inner_type	67
8.19.2.9 lt_t	67
8.19.2.10 mod_t	68

8.19.2.11 mul_t	68
8.19.2.12 one	68
8.19.2.13 pos_t	68
8.19.2.14 sub_t	68
8.19.2.15 zero	69
8.19.3 Member Data Documentation	69
8.19.3.1 eq_v	69
8.19.3.2 gt_v	69
8.19.3.3 is_euclidean_domain	69
8.19.3.4 is_field	70
8.19.3.5 lt_v	70
8.19.3.6 pos_v	70
8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference	70
8.20.1 Member Typedef Documentation	71
8.20.1.1 type	71
8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference	71
8.21.1 Member Typedef Documentation	71
8.21.1.1 type	71
8.22 aerobus::is_prime< n > Struct Template Reference	71
8.22.1 Detailed Description	72
8.22.2 Member Data Documentation	72
8.22.2.1 value	72
8.23 aerobus::polynomial< Ring > Struct Template Reference	72
8.23.1 Detailed Description	74
8.23.2 Member Typedef Documentation	74
8.23.2.1 add_t	74
8.23.2.2 derive_t	74
8.23.2.3 div_t	75
8.23.2.4 eq_t	75
8.23.2.5 gcd_t	75
8.23.2.6 gt_t	75
8.23.2.7 inject_constant_t	76
8.23.2.8 inject_ring_t	76
8.23.2.9 lt_t	76
8.23.2.10 mod_t	76
8.23.2.11 monomial_t	77
8.23.2.12 mul_t	77
8.23.2.13 one	77
8.23.2.14 pos_t	77
8.23.2.15 simplify_t	79
8.23.2.16 sub_t	79

8.23.2.17 X	79
8.23.2.18 zero	79
8.23.3 Member Data Documentation	80
8.23.3.1 is_euclidean_domain	80
8.23.3.2 is_field	80
8.23.3.3 pos_v	80
8.24 aerobus::type_list< Ts >::pop_front Struct Reference	80
8.24.1 Detailed Description	80
8.24.2 Member Typedef Documentation	81
8.24.2.1 tail	81
8.24.2.2 type	81
8.25 aerobus::Quotient< Ring, X > Struct Template Reference	81
8.25.1 Detailed Description	82
8.25.2 Member Typedef Documentation	82
8.25.2.1 add_t	82
8.25.2.2 div_t	83
8.25.2.3 eq_t	83
8.25.2.4 inject_constant_t	83
8.25.2.5 inject_ring_t	84
8.25.2.6 mod_t	84
8.25.2.7 mul_t	84
8.25.2.8 one	84
8.25.2.9 pos_t	85
8.25.2.10 zero	85
8.25.3 Member Data Documentation	85
8.25.3.1 eq_v	85
8.25.3.2 is_euclidean_domain	85
8.25.3.3 pos_v	85
8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference	86
8.26.1 Detailed Description	86
8.26.2 Member Typedef Documentation	86
8.26.2.1 head	86
8.26.2.2 tail	86
8.27 aerobus::type_list< Ts > Struct Template Reference	87
8.27.1 Detailed Description	87
8.27.2 Member Typedef Documentation	88
8.27.2.1 at	88
8.27.2.2 concat	88
8.27.2.3 insert	88
8.27.2.4 push_back	89
8.27.2.5 push_front	89
8.27.2.6 remove	89

8.27.3 Member Data Documentation	89
8.27.3.1 length	89
8.28 aerobus::type_list<> Struct Reference	90
8.28.1 Detailed Description	90
8.28.2 Member Typedef Documentation	90
8.28.2.1 concat	90
8.28.2.2 insert	90
8.28.2.3 push_back	90
8.28.2.4 push_front	90
8.28.3 Member Data Documentation	91
8.28.3.1 length	91
8.29 aerobus::i32::val< x > Struct Template Reference	91
8.29.1 Detailed Description	91
8.29.2 Member Typedef Documentation	92
8.29.2.1 enclosing_type	92
8.29.2.2 is_zero_t	92
8.29.3 Member Function Documentation	92
8.29.3.1 get()	92
8.29.3.2 to_string()	92
8.29.4 Member Data Documentation	92
8.29.4.1 v	92
8.30 aerobus::i64::val< x > Struct Template Reference	93
8.30.1 Detailed Description	93
8.30.2 Member Typedef Documentation	94
8.30.2.1 enclosing_type	94
8.30.2.2 inner_type	94
8.30.2.3 is_zero_t	94
8.30.3 Member Function Documentation	94
8.30.3.1 get()	94
8.30.3.2 to_string()	94
8.30.4 Member Data Documentation	95
8.30.4.1 v	95
8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference	95
8.31.1 Detailed Description	96
8.31.2 Member Typedef Documentation	96
8.31.2.1 aN	96
8.31.2.2 coeff_at_t	96
8.31.2.3 enclosing_type	97
8.31.2.4 is_zero_t	97
8.31.2.5 ring_type	97
8.31.2.6 strip	97
8.31.2.7 value_at_t	97

8.31.3 Member Function Documentation	97
8.31.3.1 compensated_eval()	97
8.31.3.2 eval()	98
8.31.3.3 to_string()	98
8.31.4 Member Data Documentation	99
8.31.4.1 degree	99
8.31.4.2 is_zero_v	99
8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference	99
8.32.1 Detailed Description	99
8.32.2 Member Typedef Documentation	100
8.32.2.1 raw_t	100
8.32.2.2 type	100
8.33 aerobus::zpz< p >::val< x > Struct Template Reference	100
8.33.1 Detailed Description	100
8.33.2 Member Typedef Documentation	101
8.33.2.1 enclosing_type	101
8.33.2.2 is_zero_t	101
8.33.3 Member Function Documentation	101
8.33.3.1 get()	101
8.33.3.2 to_string()	101
8.33.4 Member Data Documentation	102
8.33.4.1 is_zero_v	102
8.33.4.2 v	102
8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference	102
8.34.1 Detailed Description	103
8.34.2 Member Typedef Documentation	103
8.34.2.1 aN	103
8.34.2.2 coeff_at_t	103
8.34.2.3 enclosing_type	103
8.34.2.4 is_zero_t	104
8.34.2.5 ring_type	104
8.34.2.6 strip	104
8.34.2.7 value_at_t	104
8.34.3 Member Function Documentation	104
8.34.3.1 compensated_eval()	104
8.34.3.2 eval()	104
8.34.3.3 to_string()	105
8.34.4 Member Data Documentation	105
8.34.4.1 degree	105
8.34.4.2 is_zero_v	105
8.35 aerobus::zpz< p > Struct Template Reference	105
8.35.1 Detailed Description	107

8.35.2 Member Typedef Documentation	107
8.35.2.1 add_t	107
8.35.2.2 div_t	107
8.35.2.3 eq_t	108
8.35.2.4 gcd_t	108
8.35.2.5 gt_t	108
8.35.2.6 inject_constant_t	109
8.35.2.7 inner_type	109
8.35.2.8 lt_t	109
8.35.2.9 mod_t	109
8.35.2.10 mul_t	110
8.35.2.11 one	110
8.35.2.12 pos_t	110
8.35.2.13 sub_t	110
8.35.2.14 zero	111
8.35.3 Member Data Documentation	111
8.35.3.1 eq_v	111
8.35.3.2 gt_v	111
8.35.3.3 is_euclidean_domain	111
8.35.3.4 is_field	111
8.35.3.5 lt_v	112
8.35.3.6 pos_v	112
9 File Documentation	113
9.1 README.md File Reference	113
9.2 src/aerobus.h File Reference	113
9.3 aerobus.h	113
9.4 src/examples.h File Reference	206
9.5 examples.h	206
10 Examples	207
10.1 examples/hermite.cpp	207
10.2 examples/custom_taylor.cpp	207
10.3 examples/fp16.cu	208
10.4 examples/continued_fractions.cpp	209
10.5 examples/modular_arithmetic.cpp	209
10.6 examples/make_polynomial.cpp	210
10.7 examples/polynomials_over_finite_field.cpp	210
10.8 examples/compensated_horner.cpp	210
Index	213

Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for **quotient ring** R/X where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	15
aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	43
aerobus::IsField	
Concept to express R is a field	43
aerobus::IsRing	
Concept to express R is a Ring	44

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >	45
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> > 45	
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	46
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	46
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	47
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	48
aerobus::ConwayPolynomial	49
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >	49
aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > . .	50
aerobus::Embed< Small, Large, E >	
Embedding - struct forward declaration	51
aerobus::Embed< i32, i64 >	
Embeds i32 into i64	51
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
Embeds polynomial<Small> into polynomial<Large>	52
aerobus::Embed< q32, q64 >	
Embeds q32 into q64	53
aerobus::Embed< Quotient< Ring, X >, Ring >	
Embeds Quotient<Ring, X> into Ring	54
aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	55
aerobus::Embed< zpz< x >, i32 >	
Embeds zpz values into i32	55
aerobus::polynomial< Ring >::horner_reduction_t< P >	
Used to evaluate polynomials over a value in Ring	56
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	57
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	64
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >	70
aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >	71

aerobus::is_prime< n >	71
Checks if n is prime	
aerobus::polynomial< Ring >	72
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	80
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X,	
Quotient is $\mathbb{Z}/2\mathbb{Z}$	81
aerobus::type_list< Ts >::split< index >	
Splits list at index	86
aerobus::type_list< Ts >	
Empty pure template struct to handle type list	87
aerobus::type_list<>	
Specialization for empty type list	90
aerobus::i32::val< x >	
Values in i32 , again represented as types	91
aerobus::i64::val< x >	
Values in i64	93
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	95
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	99
aerobus::zpz< p >::val< x >	
Values in zpz	100
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	102
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	105

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

src/ aerobus.h	113
src/ examples.h	206

Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<typename T , typename A , typename B >
 using [gcd_t](#) = typename internal::gcd< T >::template type< A, B >
 - computes the greatest common divisor or A and B*
- template<typename... vals>
 using [vadd_t](#) = typename internal::vadd< vals... >::type
 - adds multiple values ($v1 + v2 + \dots + vn$) vals must have same "enclosing_type" and "enclosing_type" must have an [add_t](#) binary operator*
- template<typename... vals>
 using [vmul_t](#) = typename internal::vmul< vals... >::type
 - multiplies multiple values ($v1 + v2 + \dots + vn$) vals must have same "enclosing_type" and "enclosing_type" must have an [mul_t](#) binary operator*
- template<typename val >
 using [abs_t](#) = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template [sub_t](#)< typename val::enclosing_type::zero, val > >
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
 - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add_t](#) = typename X::enclosing_type::template [add_t](#)< X, Y >
 - generic addition*
- template<typename X , typename Y >
 using [sub_t](#) = typename X::enclosing_type::template [sub_t](#)< X, Y >

- generic subtraction*
- `template<typename X , typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
- generic multiplication*
- `template<typename X , typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
- generic division*
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
- `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename Ring , auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
- `template<typename Ring , auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField< Ring>
- `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type

- `template<typename T , size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T , size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T , size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T , int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T , int n, int k>`
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T , int n, int k>`
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T , int n, int k>`
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`
Stirling number of second kind – as types.
- `template<typename T , typename p , size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers , size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers , size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers , size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers , size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers , size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers , size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`
 $\cosh(x)$ *hyperbolic cosine*
- `template<typename Integers , size_t deg>`
`using cos = taylor< Integers, internal::cos_coeff, deg >`
 $\cos(x)$ *cosinus*
- `template<typename Integers , size_t deg>`
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`

- $\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$
 • template<typename Integers , size_t deg>
 using [asin](#) = [taylor](#)< Integers, internal::asin_coeff, deg >
 arcsin(x) *arc sinus*
- template<typename Integers , size_t deg>
 using [asinh](#) = [taylor](#)< Integers, internal::asinh_coeff, deg >
 arcsinh(x) *arc hyperbolic sinus*
- template<typename Integers , size_t deg>
 using [atanh](#) = [taylor](#)< Integers, internal::atanh_coeff, deg >
 arctanh(x) *arc hyperbolic tangent*
- template<typename Integers , size_t deg>
 using [tan](#) = [taylor](#)< Integers, internal::tan_coeff, deg >
 tan(x) *tangent*
- template<typename Integers , size_t deg>
 using [tanh](#) = [taylor](#)< Integers, internal::tanh_coeff, deg >
 tanh(x) *hyperbolic tangent*
- using [PI_fraction](#) = [ContinuedFraction](#)< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >
- using [E_fraction](#) = [ContinuedFraction](#)< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >
 approximation of e
- using [SQRT2_fraction](#) = [ContinuedFraction](#)< 1, 2 >
 approximation of $\sqrt{2}$
- using [SQRT3_fraction](#) = [ContinuedFraction](#)< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >
 approximation of

Functions

- template<typename T >
 T * [aligned_malloc](#) (size_t count, size_t alignment)
- brief Conway polynomials tparam p characteristic of the [field](#) (prime number) @tparam n degree of extension
 template< int p

Variables

- template<typename T , size_t i>
 constexpr T::inner_type [factorial_v](#) = internal::factorial<T, i>::value
 computes factorial(i) as value in T
- template<typename T , size_t k, size_t n>
 constexpr T::inner_type [combination_v](#) = internal::combination<T, k, n>::value
 computes binomial coefficients (k among n) as value
- template<typename FloatType , typename T , size_t n>
 constexpr FloatType [bernoulli_v](#) = internal::bernoulli<T, n>::template value<FloatType>
 nth bernoulli number as value in FloatType
- template<typename T , size_t k>
 constexpr T::inner_type [alternate_v](#) = internal::alternate<T, k>::value
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	---

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh(x)` arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ `cosinus`

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 div_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of e

6.1.2.16 embed_int_poly_in_fractions_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

Template Parameters

<i>Ring</i>	
-------------	--

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of π as a continued fraction

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of $\sqrt{2}$

6.1.2.42 SQRT3_fraction

```
using aerobus::SQRT3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1,  
2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

6.1.2.43 `stirling_1_signed_t`

```
template<typename T , int n, int k>
using aerobus::stirling_1_signed_t = typedef typename internal::stirling_1_helper<T, n, k>::
::type
```

Stirling number of first kind (signed) – as types.

Template Parameters

T	(ring type, such as <code>aerobus::i64</code>)
n	(integer)
k	(integer)

6.1.2.44 `stirling_1_unsigned_t`

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T,
n, k>::type>
```

Stirling number of first kind (unsigned) – as types.

Template Parameters

T	(ring type, such as <code>aerobus::i64</code>)
n	(integer)
k	(integer)

6.1.2.45 stirling_2_t

```
template<typename T , int n, int k>
using aerobus::stirling_2_t = typedef typename internal::stirling_2_helper<T, n, k>::type
```

Stirling number of second kind – as types.

Template Parameters

T	(ring type, such as <code>aerobus::i64</code>)
n	(integer)
k	(integer)

6.1.2.46 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.47 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.49 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.50 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.51 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<Float↵
Type> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >

- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int16_t >

- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **fma_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling_1_helper**
- struct **stirling_1_helper**< T, 0, 0 >
- struct **stirling_1_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_1_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0)> >

- struct **stirling_2_helper**
- struct **stirling_2_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_2_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling_2_helper**< T, n, n, std::enable_if_t<(n >=0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **touchard_coeff**
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- template<template< typename... > typename TT, typename T >
constexpr bool **is_instantiation_of_v** = is_instantiation_of<TT, T>::value

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Enumerations

- enum [hermite_kind](#) { [probabilist](#) , [physicist](#) }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Enumeration Type Documentation

6.3.2.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = typename Ring::zero

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using `type` = `aN`

8.3.1 Member Typedef Documentation

8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

Examples

[examples/continued_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename q64::template inject_constant_t< a0 >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = static_cast<double>(a0)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template [add_t](#)< typename q64::template inject_constant_t< a0 >, typename q64::template [div_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.8.1 Member Function Documentation

8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 **aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference**

```
#include <aerobus.h>
```

Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType *r)

8.9.1 Member Function Documentation

8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- template<typename val >
using [type](#) = [i64::val](#)< static_cast< int64_t >(val::v)>
the [i64](#) representation of val

8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

8.11.2 Member Typedef Documentation

8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = `typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the polynomial<Large> representation of v

8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.12.2 Member Typedef Documentation

8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`
q64 representation of v

8.13.1 Detailed Description

embeds q32 into q64

8.13.2 Member Typedef Documentation

8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = typename val::raw_t`
Ring representation of val.

8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

8.14.2 Member Typedef Documentation

8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
using `type` = typename `FractionField< Ring >::template val< v, typename Ring::one >`
FractionField<Ring> representation of v.

8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

Template Parameters

<i>Ring</i>	an integers ring, such as i32
-------------	---

8.15.2 Member Typedef Documentation

8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i32::val< val::v >`
the i32 representation of val

8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

Template Parameters

<code>x</code>	an integer
----------------	------------

8.16.2 Member Typedef Documentation

8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

Template Parameters

<code>val</code>	a value in zpz<x>
------------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using [inner_type](#) = int32_t
- using [zero](#) = [val](#)< 0 >
constant zero
- using [one](#) = [val](#)< 1 >
constant one
- template<auto x>
using [inject_constant_t](#) = [val](#)< static_cast< int32_t >(x)>
inject a native constant
- template<typename v >
using [inject_ring_t](#) = v
- template<typename v1 , typename v2 >
using [add_t](#) = typename add< v1, v2 >::type
addition operator yields v1 + v2

- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator yields $v1 - v2$
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
*multiplication operator yields $v1 * v2$*
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator yields $v1 / v2$
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator yields $v1 \% v2$
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator ($v1 > v2$) yields $v1 > v2$
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator ($v1 < v2$) yields $v1 < v2$
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor yields $GCD(v1, v2)$
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator (boolean value)
- `template<typename v >`
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity (boolean value) yields $v > 0$ as boolean value

8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

Examples

[examples/compensated_horner.cpp](#).

8.18.2 Member Typedef Documentation

8.18.2.1 add_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields $v1 + v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.2 div_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields $v1 / v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.3 eq_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.4 gcd_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields $GCD(v1, v2)$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) yields $v1 > v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject\_constant\_t = val<static_cast<int32_t>(x)>
```

inject a native constant

Template Parameters

<i>x</i>	
----------	--

8.18.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject\_ring\_t = v
```

8.18.2.8 inner_type

```
using aerobus::i32::inner\_type = int32_t
```

8.18.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt\_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$) yields $v1 < v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields $v1 * v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.18.2.13 pos_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<i>v</i>	a value in i32
----------	--------------------------------

8.18.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields $v1 - v2$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.18.3 Member Data Documentation

8.18.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.18.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.18.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.18.3.4 pos_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	a value in i32
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- using [inner_type](#) = `int64_t`
type of represented values
- template<auto x>
using [inject_constant_t](#) = `val< static_cast< int64_t >(x)>`
injects constant as an [i64](#) value
- template<typename v >
using [inject_ring_t](#) = v
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`
constant zero
- using [one](#) = `val< 1 >`
constant one
- template<typename v1 , typename v2 >
using [add_t](#) = `typename add< v1, v2 >::type`
addition operator
- template<typename v1 , typename v2 >
using [sub_t](#) = `typename sub< v1, v2 >::type`
subtraction operator
- template<typename v1 , typename v2 >
using [mul_t](#) = `typename mul< v1, v2 >::type`
multiplication operator
- template<typename v1 , typename v2 >
using [div_t](#) = `typename div< v1, v2 >::type`
division operator integer division
- template<typename v1 , typename v2 >
using [mod_t](#) = `typename remainder< v1, v2 >::type`

modulus operator

- template<typename v1 , typename v2 >
using [gt_t](#) = typename gt< v1, v2 >::type
strictly greater operator yields v1 > v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [lt_t](#) = typename lt< v1, v2 >::type
strict less operator yields v1 < v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [eq_t](#) = typename eq< v1, v2 >::type
equality operator yields v1 == v2 as std::true_type or std::false_type
- template<typename v1 , typename v2 >
using [gcd_t](#) = [gcd_t](#)< [i64](#), v1, v2 >
greatest common divisor yields GCD(v1, v2) as instantiation of [i64::val](#)
- template<typename v >
using [pos_t](#) = typename pos< v >::type
is v positive yields v > 0 as std::true_type or std::false_type

Static Public Attributes

- static constexpr bool [is_field](#) = false
integers are not a field
- static constexpr bool [is_euclidean_domain](#) = true
integers are an euclidean domain
- template<typename v1 , typename v2 >
static constexpr bool [gt_v](#) = [gt_t](#)<v1, v2>::value
strictly greater operator yields v1 > v2 as boolean value
- template<typename v1 , typename v2 >
static constexpr bool [lt_v](#) = [lt_t](#)<v1, v2>::value
strictly smaller operator yields v1 < v2 as boolean value
- template<typename v1 , typename v2 >
static constexpr bool [eq_v](#) = [eq_t](#)<v1, v2>::value
equality operator yields v1 == v2 as boolean value
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity yields v > 0 as boolean value

8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.19.2 Member Typedef Documentation

8.19.2.1 [add_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div\_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq\_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd\_t = gcd\_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.6 inject_constant_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

Template Parameters

<code>x</code>	
----------------	--

8.19.2.7 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)

Template Parameters

<code>v</code>	a value in i64
----------------	--------------------------------

8.19.2.8 inner_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.19.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.19.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
-----------------	---

8.19.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

8.19.3 Member Data Documentation

8.19.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

8.19.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.19.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.19.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields $v1 < v2$ as boolean value

Template Parameters

$v1$: an element of aerobus::i64::val
$v2$: an element of aerobus::i64::val

8.19.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`

8.20.1 Member Typedef Documentation

8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `template<typename accum , typename x >`
`using type = accum`

8.21.1 Member Typedef Documentation

8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.22 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.22.2 Member Data Documentation

8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [horner_reduction_t](#)
Used to evaluate polynomials over a value in Ring.
- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- using `zero` = `val`< typename Ring::zero >
constant zero
- using `one` = `val`< typename Ring::one >
constant one
- using `X` = `val`< typename Ring::one, typename Ring::zero >
generator
- template<typename P >
using `simplify_t` = typename simplify< P >::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- template<typename v1, typename v2 >
using `add_t` = typename add< v1, v2 >::type
adds two polynomials
- template<typename v1, typename v2 >
using `sub_t` = typename sub< v1, v2 >::type
subtraction of two polynomials
- template<typename v1, typename v2 >
using `mul_t` = typename mul< v1, v2 >::type
multiplication of two polynomials
- template<typename v1, typename v2 >
using `eq_t` = typename eq_helper< v1, v2 >::type
equality operator
- template<typename v1, typename v2 >
using `lt_t` = typename lt_helper< v1, v2 >::type
strict less operator
- template<typename v1, typename v2 >
using `gt_t` = typename gt_helper< v1, v2 >::type
strict greater operator
- template<typename v1, typename v2 >
using `div_t` = typename div< v1, v2 >::q_type
division operator
- template<typename v1, typename v2 >
using `mod_t` = typename div_helper< v1, v2, `zero`, v1 >::mod_type
modulo operator
- template<typename coeff, size_t deg>
using `monomial_t` = typename monomial< coeff, deg >::type
monomial : coeff X^deg
- template<typename v >
using `derive_t` = typename derive_helper< v >::type
derivation operator
- template<typename v >
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
checks for positivity (an > 0)
- template<typename v1, typename v2 >
using `gcd_t` = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >
greatest common divisor of two polynomials
- template<auto x>
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >
makes the constant (native type) polynomial a_0
- template<typename v >
using `inject_ring_t` = `val`< v >
makes the constant (ring type) polynomial a_0

Static Public Attributes

- static constexpr bool [is_field](#) = false
- static constexpr bool [is_euclidean_domain](#) = Ring::is_euclidean_domain
- template<typename v >
static constexpr bool [pos_v](#) = [pos_t](#)<v>::value
positivity operator

8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

Examples

[examples/compensated_horner.cpp](#), [examples/make_polynomial.cpp](#), and [examples/modular_arithmetic.cpp](#).

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

8.23.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

Template Parameters

<i>x</i>	
----------	--

8.23.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

Template Parameters

<i>v</i>	
----------	--

8.23.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.10 mod_t

```
template<typename Ring >
```



```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.23.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.23.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($an > 0$)

Template Parameters

<i>v</i>	
----------	--

8.23.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.23.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.23.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.23.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.23.3 Member Data Documentation

8.23.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.23.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.23.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in polynomial::val
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.24 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using [tail](#) = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.24.2 Member Typedef Documentation

8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< typename Ring::zero >
zero value
- using [one](#) = [val](#)< typename Ring::one >
one
- template<typename v1 , typename v2 >
using [add_t](#) = [val](#)< typename Ring::template [add_t](#)< typename v1::type, typename v2::type > >
addition operator
- template<typename v1 , typename v2 >
using [mul_t](#) = [val](#)< typename Ring::template [mul_t](#)< typename v1::type, typename v2::type > >
subtraction operator
- template<typename v1 , typename v2 >
using [div_t](#) = [val](#)< typename Ring::template [div_t](#)< typename v1::type, typename v2::type > >
division operator
- template<typename v1 , typename v2 >
using [mod_t](#) = [val](#)< typename Ring::template [mod_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
 • `template<typename v1 , typename v2 >`
 `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
 equality operator (as type)
- `template<typename v1 >`
 `using pos_t = std::true_type`
 positivity operator always true
- `template<auto x>`
 `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
 *inject a 'constant' in quotient ring**
- `template<typename v >`
 `using inject_ring_t = val< v >`
 projects a value of Ring onto the quotient

Static Public Attributes

- `template<typename v1 , typename v2 >`
 `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
 addition operator (as boolean value)
- `template<typename v >`
 `static constexpr bool pos_v = pos_t<v>::value`
 positivity operator always true
- `static constexpr bool is_euclidean_domain = true`
 quotien rings are euclidean domain

8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.25.2 Member Typedef Documentation

8.25.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring*

Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

8.25.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

8.25.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.25.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.25.3 Member Data Documentation

8.25.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.25.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.25.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.26.2 Member Typedef Documentation

8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.27 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

8.27.2 Member Typedef Documentation

8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.27.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.27.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.27.3 Member Data Documentation

8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.28 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T >
- template<typename T >
using [push_back](#) = [type_list](#)< T >
- template<typename U >
using [concat](#) = U
- template<typename T , size_t index>
using [insert](#) = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t [length](#) = 0

8.28.1 Detailed Description

specialization for empty type list

8.28.2 Member Typedef Documentation

8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

8.28.2.3 push_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

8.28.2.4 push_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

8.28.3 Member Data Documentation

8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [i32](#)
Enclosing ring type.
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [DEVICE](#) valueType [get](#) ()
cast x into valueType
- static std::string [to_string](#) ()
string representation of value

Static Public Attributes

- static constexpr int32_t [v](#) = x
actual value stored in val type

8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

8.29.2 Member Typedef Documentation

8.29.2.1 enclosing_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.29.2.2 is_zero_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.29.3 Member Function Documentation

8.29.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.29.3.2 to_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.29.4 Member Data Documentation

8.29.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```


actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using [inner_type](#) = int32_t
type of represented values
- using [enclosing_type](#) = [i64](#)
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED_DEVICE](#) valueType [get](#) ()
cast value in valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int64_t [v](#) = x
actual value

8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

x	an actual integer
-------------------	-------------------

Examples

[examples/compensated_horner.cpp](#).

8.30.2 Member Typedef Documentation

8.30.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.30.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.30.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.30.3 Member Function Documentation

8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.30.3.2 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.30.4 Member Data Documentation

8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial](#)< Ring >
enclosing ring type
- using [aN](#) = coeffN
heavy weight coefficient (non zero)
- using [strip](#) = [val](#)< coeffs... >
remove largest coefficient
- using [is_zero_t](#) = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>
true_type if polynomial is constant zero
- template<size_t index>
using [coeff_at_t](#) = typename coeff_at< index >::type
type of coefficient at index
- template<typename x >
using [value_at_t](#) = [horner_reduction_t](#)< [val](#) > ::template inner< 0, degree+1 > ::template type< typename Ring::zero, x >

Static Public Member Functions

- static std::string [to_string](#) ()
get a string representation of polynomial
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
evaluates polynomial seen as a function operating on arithmeticType
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)
Evaluate polynomial on x using compensated horner scheme.

Static Public Attributes

- static constexpr size_t [degree](#) = sizeof...(coeffs)
degree of the polynomial
- static constexpr bool [is_zero_v](#) = is_zero_t::value
true if polynomial is constant zero

8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

Examples

[examples/compensated_horner.cpp](#).

8.31.2 Member Typedef Documentation

8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.31.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.31.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>

enclosing ring type
```

8.31.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>

true_type if polynomial is constant zero
```

8.31.2.5 ring_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring

ring coefficients live in
```

8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>

remove largest coefficient
```

8.31.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

8.31.3 Member Function Documentation

8.31.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

Parameters

x	
---	--

8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

Parameters

x	value
---	-------

Returns

$P(x)$

8.31.3.3 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.31.4 Member Data Documentation

8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.31.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- using [raw_t](#) = V
- using [type](#) = [abs_t](#)< typename Ring::template [mod_t](#)< V, X > >

8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

8.32.2 Member Typedef Documentation

8.32.2.1 raw_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

Public Types

- using [enclosing_type](#) = [zpz](#)< p >
enclosing ring type
- using [is_zero_t](#) = std::bool_constant< [v](#)==0 >
true_type if zero

Static Public Member Functions

- template<typename valueType >
static constexpr [INLINED DEVICE](#) valueType [get](#) ()
get value as valueType
- static std::string [to_string](#) ()
string representation

Static Public Attributes

- static constexpr int32_t [v](#) = x % p
actual value
- static constexpr bool [is_zero_v](#) = [v](#) == 0
true if zero

8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

x	an integer
---	------------

8.33.2 Member Typedef Documentation

8.33.2.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.33.2.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true_type if zero

8.33.3 Member Function Documentation

8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

8.33.3.2 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

Returns

a string representation

8.33.4 Member Data Documentation**8.33.4.1 is_zero_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

8.33.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- using [ring_type](#) = Ring
ring coefficients live in
- using [enclosing_type](#) = [polynomial< Ring >](#)
enclosing ring type
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is_zero_t](#) = std::bool_constant< [aN::is_zero_t::value](#) >
- template<size_t index>
using [coeff_at_t](#) = typename [coeff_at< index >::type](#)
- template<typename x >
using [value_at_t](#) = [coeffN](#)

Static Public Member Functions

- static std::string [to_string](#) ()
- template<typename arithmeticType >
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >
static [DEVICE INLINED](#) arithmeticType [compensated_eval](#) (const arithmeticType &x)

Static Public Attributes

- static constexpr size_t [degree](#) = 0
degree
- static constexpr bool [is_zero_v](#) = is_zero_t::value

8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.34.2 Member Typedef Documentation

8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.34.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

8.34.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.34.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

8.34.2.5 ring_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.34.2.7 value_at_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

8.34.3 Member Function Documentation

8.34.3.1 compensated_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

8.34.3.3 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.34.4 Member Data Documentation

8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.34.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in zpz

Public Types

- using `inner_type` = `int32_t`
underlying type for values
- template<auto x>
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>
injects a constant integer into mpz
- using `zero` = `val`< 0 >
zero value
- using `one` = `val`< 1 >
one value
- template<typename v1 , typename v2 >
using `add_t` = `typename add`< v1, v2 >::type
addition operator
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub`< v1, v2 >::type
subtraction operator
- template<typename v1 , typename v2 >
using `mul_t` = `typename mul`< v1, v2 >::type
multiplication operator
- template<typename v1 , typename v2 >
using `div_t` = `typename div`< v1, v2 >::type
division operator
- template<typename v1 , typename v2 >
using `mod_t` = `typename remainder`< v1, v2 >::type
modulo operator
- template<typename v1 , typename v2 >
using `gt_t` = `typename gt`< v1, v2 >::type
strictly greater operator (type)
- template<typename v1 , typename v2 >
using `lt_t` = `typename lt`< v1, v2 >::type
strictly smaller operator (type)
- template<typename v1 , typename v2 >
using `eq_t` = `typename eq`< v1, v2 >::type
equality operator (type)
- template<typename v1 , typename v2 >
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >
greatest common divisor
- template<typename v1 >
using `pos_t` = `typename pos`< v1 >::type
positivity operator (type)

Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value
true iff p is prime
- static constexpr bool `is_euclidean_domain` = true
always true
- template<typename v1 , typename v2 >
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value
strictly greater operator (booleanvalue)

- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

8.35.1 Detailed Description

`template<int32_t p>`
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular_arithmetic.cpp](#), and [examples/polynomials_over_finite_field.cpp](#).

8.35.2 Member Typedef Documentation

8.35.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.2 div_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

Template Parameters

x	an integer
---	------------

8.35.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

8.35.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

8.35.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

8.35.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

8.35.3 Member Data Documentation

8.35.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.35.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

8.35.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

8.35.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.35.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 16U) & 0x8000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477fefffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED_DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED_DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED_DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<long double> {
00224         static constexpr INLINED_DEVICE long double eval(
00225             const long double x, const long double y, const long double z) {
00226             return x * y + z;
00227         }
00228     };
00229
00230     template<>
00231     struct fma_helper<float> {
00232         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00233             return x * y + z;
00234         }
00235     };
00236
00237     template<>
00238     struct fma_helper<int32_t> {
00239         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00240             return x * y + z;
00241         }
00242     };
00243
00244     template<>
00245     struct fma_helper<int16_t> {
00246         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00247             return x * y + z;
00248         }
00249     };
00250
00251     template<>
00252     struct fma_helper<int64_t> {
00253         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00254             return x * y + z;
00255         }
00256     };
00257
00258     #ifdef WITH_CUDA_FP16
00259     template<>
00260     struct fma_helper<__half> {
00261         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00262             #ifdef __CUDA_ARCH__
00263                 return __hfma(x, y, z);
00264             #else
00265                 return x * y + z;
00266             #endif
00267         }
00268     };
00269     template<>
00270     struct fma_helper<__half2> {
00271         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00272             #ifdef __CUDA_ARCH__
00273                 return __hfma2(x, y, z);
00274             #else
00275                 return x * y + z;
00276             #endif
00277         }
00278     };
00279     #endif
00280 } // namespace internal
00281 } // namespace aerobus
00282
00283 // compensated horner utilities
00284 namespace aerobus {
00285     namespace internal {
00286         template <typename T>
00287         struct FloatLayout;
00288
00289         #ifdef _MSC_VER
00290         template <>
00291         struct FloatLayout<long double> {
00292             static constexpr uint8_t exponent = 11;
00293             static constexpr uint8_t mantissa = 53;
00294             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00295         };

```



```

00296     #else
00297     template <>
00298     struct FloatLayout<long double> {
00299         static constexpr uint8_t exponent = 15;
00300         static constexpr uint8_t mantissa = 63;
00301         static constexpr uint8_t r = 32; // ceil(mantissa/2)
00302     };
00303     #endif
00304
00305     template <>
00306     struct FloatLayout<double> {
00307         static constexpr uint8_t exponent = 11;
00308         static constexpr uint8_t mantissa = 53;
00309         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00310     };
00311
00312     template <>
00313     struct FloatLayout<float> {
00314         static constexpr uint8_t exponent = 8;
00315         static constexpr uint8_t mantissa = 24;
00316         static constexpr uint8_t r = 11; // ceil(mantissa/2)
00317     };
00318
00319     #ifdef WITH_CUDA_FP16
00320     template <>
00321     struct FloatLayout<__half> {
00322         static constexpr uint8_t exponent = 5;
00323         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00324         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00325     };
00326     #endif
00327
00328     template<typename T>
00329     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00330         T z = a * ((1 « FloatLayout<T>::r) + 1);
00331         *x = z - (z - a);
00332         *y = a - *x;
00333     }
00334
00335     template<typename T>
00336     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00337         *x = a + b;
00338         T z = *x - a;
00339         *y = (a - (*x - z)) + (b - z);
00340     }
00341
00342     template<typename T>
00343     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00344         *x = a * b;
00345         #ifdef __clang__
00346         *y = fma_helper<T>::eval(a, b, -*x);
00347         #else
00348         T ah, al, bh, bl;
00349         split(a, &ah, &al);
00350         split(b, &bh, &bl);
00351         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00352         #endif
00353     }
00354
00355     template<typename T, size_t N>
00356     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00357         T r = p1[0] + p2[0];
00358         for (int64_t i = N - 1; i >= 0; --i) {
00359             r = r * x + p1[N - i] + p2[N - i];
00360         }
00361         return r;
00362     }
00363 } // namespace internal
00364 } // namespace aerobus
00365
00366 // utilities
00367 namespace aerobus {
00368     namespace internal {
00369         template<template<typename...> typename TT, typename T>
00370         struct is_instantiation_of : std::false_type { };
00371
00372         template<template<typename...> typename TT, typename... Ts>
00373         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00374
00375         template<template<typename...> typename TT, typename T>
00376         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00377
00378         template<int64_t i, typename T, typename... Ts>
00379         struct type_at {
00380             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00381             using type = typename type_at<i - 1, Ts...>::type;
00382         };
00383     }
00384 }

```

```

00383     };
00384
00385     template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00386         using type = T;
00387     };
00388
00389     template <size_t i, typename... Ts>
00390     using type_at_t = typename type_at<i, Ts...>::type;
00391
00392
00393     template<size_t n, size_t i, typename E = void>
00394     struct _is_prime {};
00395
00396     template<size_t i>
00397     struct _is_prime<0, i> {
00398         static constexpr bool value = false;
00399     };
00400
00401     template<size_t i>
00402     struct _is_prime<1, i> {
00403         static constexpr bool value = false;
00404     };
00405
00406     template<size_t i>
00407     struct _is_prime<2, i> {
00408         static constexpr bool value = true;
00409     };
00410
00411     template<size_t i>
00412     struct _is_prime<3, i> {
00413         static constexpr bool value = true;
00414     };
00415
00416     template<size_t i>
00417     struct _is_prime<5, i> {
00418         static constexpr bool value = true;
00419     };
00420
00421     template<size_t i>
00422     struct _is_prime<7, i> {
00423         static constexpr bool value = true;
00424     };
00425
00426     template<size_t n, size_t i>
00427     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00428         static constexpr bool value = false;
00429     };
00430
00431     template<size_t n, size_t i>
00432     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00433         static constexpr bool value = false;
00434     };
00435
00436     template<size_t n, size_t i>
00437     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00438         static constexpr bool value = true;
00439     };
00440
00441     template<size_t n, size_t i>
00442     struct _is_prime<n, i, std::enable_if_t<(
00443         n % i == 0 &&
00444         n >= 9 &&
00445         n % 3 != 0 &&
00446         n % 2 != 0 &&
00447         i * i > n)> {
00448         static constexpr bool value = true;
00449     };
00450
00451     template<size_t n, size_t i>
00452     struct _is_prime<n, i, std::enable_if_t<(
00453         n % (i+2) == 0 &&
00454         n >= 9 &&
00455         n % 3 != 0 &&
00456         n % 2 != 0 &&
00457         i * i <= n)> {
00458         static constexpr bool value = true;
00459     };
00460
00461     template<size_t n, size_t i>
00462     struct _is_prime<n, i, std::enable_if_t<(
00463         n % (i+2) != 0 &&
00464         n % i != 0 &&
00465         n >= 9 &&
00466         n % 3 != 0 &&
00467         n % 2 != 0 &&
00468         (i * i <= n))> {
00469         static constexpr bool value = _is_prime<n, i+6>::value;

```

```

00470     };
00471 } // namespace internal
00472
00473 template<size_t n>
00474 struct is_prime {
00475     static constexpr bool value = internal::_is_prime<n, 5>::value;
00476 };
00477
00478 template<size_t n>
00479 static constexpr bool is_prime_v = is_prime<n>::value;
00480
00481 // gcd
00482 namespace internal {
00483     template <std::size_t... Is>
00484     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00485         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00486
00487     template <std::size_t N>
00488     using make_index_sequence_reverse
00489         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00490
00491     template<typename Ring, typename E = void>
00492     struct gcd;
00493
00494     template<typename Ring>
00495     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00496         template<typename A, typename B, typename E = void>
00497         struct gcd_helper {};
00498
00499         // B = 0, A > 0
00500         template<typename A, typename B>
00501         struct gcd_helper<A, B, std::enable_if_t<
00502             ((B::is_zero_t::value) &&
00503              (Ring::template gt_t<A, typename Ring::zero>::value))> {
00504             using type = A;
00505         };
00506
00507         // B = 0, A < 0
00508         template<typename A, typename B>
00509         struct gcd_helper<A, B, std::enable_if_t<
00510             ((B::is_zero_t::value) &&
00511              !(Ring::template gt_t<A, typename Ring::zero>::value))> {
00512             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00513         };
00514
00515         // B != 0
00516         template<typename A, typename B>
00517         struct gcd_helper<A, B, std::enable_if_t<
00518             (!B::is_zero_t::value)
00519             > {
00520             private: // NOLINT
00521                 // A / B
00522                 using k = typename Ring::template div_t<A, B>;
00523                 // A - (A/B)*B = A % B
00524                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00525
00526             public:
00527                 using type = typename gcd_helper<B, m>::type;
00528         };
00529
00530         template<typename A, typename B>
00531         using type = typename gcd_helper<A, B>::type;
00532     };
00533 } // namespace internal
00534
00535 // vadd and vmul
00536 namespace internal {
00537     template<typename... vals>
00538     struct vmul {};
00539
00540     template<typename v1, typename... vals>
00541     struct vmul<v1, vals...> {
00542         using type = typename v1::enclosing_type::template mul_t<v1, typename
00543             vmul<vals...>::type>;
00544     };
00545
00546     template<typename v1>
00547     struct vmul<v1> {
00548         using type = v1;
00549     };
00550
00551     template<typename... vals>
00552     struct vadd {};
00553
00554     template<typename v1, typename... vals>
00555     struct vadd<v1, vals...> {
00556         using type = typename v1::enclosing_type::template add_t<v1, typename

```

```

        vadd<vals...>::type>;
00567     };
00568
00569     template<typename v1>
00570     struct vadd<v1> {
00571         using type = v1;
00572     };
00573 } // namespace internal
00574
00575 template<typename T, typename A, typename B>
00576 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00577
00578 template<typename... vals>
00579 using vadd_t = typename internal::vadd<vals...>::type;
00580
00581 template<typename... vals>
00582 using vmul_t = typename internal::vmul<vals...>::type;
00583
00584 template<typename val>
00585 requires IsEuclideanDomain<typename val::enclosing_type>
00586 using abs_t = std::conditional_t<
00587     val::enclosing_type::template pos_v<val>,
00588     val, typename val::enclosing_type::template
00589     sub_t<typename val::enclosing_type::zero, val>>;
00590 } // namespace aerobus
00591
00592 // embedding
00593 namespace aerobus {
00594     template<typename Small, typename Large, typename E = void>
00595     struct Embed;
00596 } // namespace aerobus
00597
00598 namespace aerobus {
00599     template<typename Ring, typename X>
00600     requires IsRing<Ring>
00601     struct Quotient {
00602         template <typename V>
00603         struct val {
00604             public:
00605                 using raw_t = V;
00606                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00607         };
00608
00609         using zero = val<typename Ring::zero>;
00610
00611         using one = val<typename Ring::one>;
00612
00613         template<typename v1, typename v2>
00614         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00615
00616         template<typename v1, typename v2>
00617         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00618
00619         template<typename v1, typename v2>
00620         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00621
00622         template<typename v1, typename v2>
00623         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00624
00625         template<typename v1, typename v2>
00626         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00627
00628         template<typename v1, typename v2>
00629         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00630
00631         template<typename v1>
00632         using pos_t = std::true_type;
00633
00634         template<typename v>
00635         static constexpr bool pos_v = pos_t<v>::value;
00636
00637         static constexpr bool is_euclidean_domain = true;
00638
00639         template<auto x>
00640         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00641
00642         template<typename v>
00643         using inject_ring_t = val<v>;
00644     };
00645
00646     template<typename Ring, typename X>
00647     struct Embed<Quotient<Ring, X>, Ring> {
00648         template<typename val>
00649         using type = typename val::raw_t;
00650     };
00651 } // namespace aerobus
00652
00653

```

```

00711 // type_list
00712 namespace aerobus {
00713     template <typename... Ts>
00714     struct type_list;
00715
00716     namespace internal {
00717         template <typename T, typename... Us>
00718         struct pop_front_h {
00719             using tail = type_list<Us...>;
00720             using head = T;
00721         };
00722
00723         template <size_t index, typename L1, typename L2>
00724         struct split_h {
00725             private:
00726                 static_assert(index <= L2::length, "index out of bounds");
00727                 using a = typename L2::pop_front::type;
00728                 using b = typename L2::pop_front::tail;
00729                 using c = typename L1::template push_back<a>;
00730
00731             public:
00732                 using head = typename split_h<index - 1, c, b>::head;
00733                 using tail = typename split_h<index - 1, c, b>::tail;
00734             };
00735
00736         template <typename L1, typename L2>
00737         struct split_h<0, L1, L2> {
00738             using head = L1;
00739             using tail = L2;
00740         };
00741
00742         template <size_t index, typename L, typename T>
00743         struct insert_h {
00744             static_assert(index <= L::length, "index out of bounds");
00745             using s = typename L::template split<index>;
00746             using left = typename s::head;
00747             using right = typename s::tail;
00748             using ll = typename left::template push_back<T>;
00749             using type = typename ll::template concat<right>;
00750         };
00751
00752         template <size_t index, typename L>
00753         struct remove_h {
00754             using s = typename L::template split<index>;
00755             using left = typename s::head;
00756             using right = typename s::tail;
00757             using rr = typename right::pop_front::tail;
00758             using type = typename left::template concat<rr>;
00759         };
00760     } // namespace internal
00761
00762     template <typename... Ts>
00763     struct type_list {
00764     private:
00765         template <typename T>
00766         struct concat_h;
00767
00768         template <typename... Us>
00769         struct concat_h<type_list<Us...> {
00770             using type = type_list<Ts..., Us...>;
00771         };
00772
00773     public:
00774         static constexpr size_t length = sizeof...(Ts);
00775
00776         template <typename T>
00777         using push_front = type_list<T, Ts...>;
00778
00779         template <size_t index>
00780         using at = internal::type_at_t<index, Ts...>;
00781
00782         struct pop_front {
00783             using type = typename internal::pop_front_h<Ts...>::head;
00784             using tail = typename internal::pop_front_h<Ts...>::tail;
00785         };
00786
00787         template <typename T>
00788         using push_back = type_list<Ts..., T>;
00789
00790         template <typename U>
00791         using concat = typename concat_h<U>::type;
00792
00793         template <size_t index>
00794         struct split {
00795             private:
00796                 using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00797             };
00798     };
00799
00800     template <typename T>
00801     using push_front = type_list<T, Ts...>;
00802
00803     template <typename U>
00804     using concat = typename concat_h<U>::type;
00805
00806     template <size_t index>
00807     using at = internal::type_at_t<index, Ts...>;
00808
00809     struct pop_front {
00810         using type = typename internal::pop_front_h<Ts...>::head;
00811         using tail = typename internal::pop_front_h<Ts...>::tail;
00812     };
00813
00814     template <typename T>
00815     using push_back = type_list<Ts..., T>;
00816
00817     template <typename U>
00818     using concat = typename concat_h<U>::type;
00819
00820     template <size_t index>
00821     struct split {
00822         private:
00823             using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00824         };
00825     };

```

```

00815         public:
00816             using head = typename inner::head;
00817             using tail = typename inner::tail;
00818         };
00819
00823         template <typename T, size_t index>
00824         using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00825
00828         template <size_t index>
00829         using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00830     };
00831
00833     template <>
00834     struct type_list<> {
00835         static constexpr size_t length = 0;
00836
00837         template <typename T>
00838         using push_front = type_list<T>;
00839
00840         template <typename T>
00841         using push_back = type_list<T>;
00842
00843         template <typename U>
00844         using concat = U;
00845
00846         // TODO(jewave): assert index == 0
00847         template <typename T, size_t index>
00848         using insert = type_list<T>;
00849     };
00850 } // namespace aerobus
00851
00852 // i16
00853 #ifdef WITH_CUDA_FP16
00854 // i16
00855 namespace aerobus {
00856     struct i16 {
00857         using inner_type = int16_t;
00858         template<int16_t x>
00859         struct val {
00860             using enclosing_type = i16;
00861             static constexpr int16_t v = x;
00862
00863             template<typename valueType>
00864             static constexpr INLINED_DEVICE valueType get() {
00865                 return internal::template int16_convert_helper<valueType, x>::value();
00866             }
00867
00876             using is_zero_t = std::bool_constant<x == 0>;
00877
00879             static std::string to_string() {
00880                 return std::to_string(x);
00881             }
00882         };
00883
00885         using zero = val<0>;
00887         using one = val<1>;
00889         static constexpr bool is_field = false;
00891         static constexpr bool is_euclidean_domain = true;
00894         template<auto x>
00895         using inject_constant_t = val<static_cast<int16_t>(x)>;
00896
00897         template<typename v>
00898         using inject_ring_t = v;
00899
00900     private:
00901         template<typename v1, typename v2>
00902         struct add {
00903             using type = val<v1::v + v2::v>;
00904         };
00905
00906         template<typename v1, typename v2>
00907         struct sub {
00908             using type = val<v1::v - v2::v>;
00909         };
00910
00911         template<typename v1, typename v2>
00912         struct mul {
00913             using type = val<v1::v * v2::v>;
00914         };
00915
00916         template<typename v1, typename v2>
00917         struct div {
00918             using type = val<v1::v / v2::v>;
00919         };
00920
00921         template<typename v1, typename v2>
00922         struct remainder {

```

```

00923         using type = val<v1::v % v2::v>;
00924     };
00925
00926     template<typename v1, typename v2>
00927     struct gt {
00928         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00929     };
00930
00931     template<typename v1, typename v2>
00932     struct lt {
00933         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00934     };
00935
00936     template<typename v1, typename v2>
00937     struct eq {
00938         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00939     };
00940
00941     template<typename v1>
00942     struct pos {
00943         using type = std::bool_constant<(v1::v > 0)>;
00944     };
00945
00946     public:
00951     template<typename v1, typename v2>
00952     using add_t = typename add<v1, v2>::type;
00953
00954     template<typename v1, typename v2>
00955     using sub_t = typename sub<v1, v2>::type;
00956
00957     template<typename v1, typename v2>
00958     using mul_t = typename mul<v1, v2>::type;
00959
00960     template<typename v1, typename v2>
00961     using div_t = typename div<v1, v2>::type;
00962
00963     template<typename v1, typename v2>
00964     using mod_t = typename remainder<v1, v2>::type;
00965
00966     template<typename v1, typename v2>
00967     using gt_t = typename gt<v1, v2>::type;
00968
00969     template<typename v1, typename v2>
00970     using lt_t = typename lt<v1, v2>::type;
00971
00972     template<typename v1, typename v2>
00973     using eq_t = typename eq<v1, v2>::type;
00974
00975     template<typename v1, typename v2>
00976     static constexpr bool eq_v = eq_t<v1, v2>::value;
00977
00978     template<typename v1, typename v2>
00979     using gcd_t = gcd_t<i16, v1, v2>;
00980
00981     template<typename v>
00982     using pos_t = typename pos<v>::type;
00983
00984     template<typename v>
00985     static constexpr bool pos_v = pos_t<v>::value;
00986 };
00987 // namespace aerobus
00988 #endif
00989
00990 // i32
00991 namespace aerobus {
00992     struct i32 {
00993         using inner_type = int32_t;
00994         template<int32_t x>
00995         struct val {
00996             using enclosing_type = i32;
00997             static constexpr int32_t v = x;
00998
00999             template<typename valueType>
01000             static constexpr DEVICE valueType get() {
01001                 return static_cast<valueType>(x);
01002             }
01003
01004             using is_zero_t = std::bool_constant<x == 0>;
01005
01006             static std::string to_string() {
01007                 return std::to_string(x);
01008             }
01009         };
01010
01011         using zero = val<0>;
01012         using one = val<1>;
01013         static constexpr bool is_field = false;
01014     };

```

```

01068     static constexpr bool is_euclidean_domain = true;
01071     template<auto x>
01072     using inject_constant_t = val<static_cast<int32_t>(x)>;
01073
01074     template<typename v>
01075     using inject_ring_t = v;
01076
01077 private:
01078     template<typename v1, typename v2>
01079     struct add {
01080         using type = val<v1::v + v2::v>;
01081     };
01082
01083     template<typename v1, typename v2>
01084     struct sub {
01085         using type = val<v1::v - v2::v>;
01086     };
01087
01088     template<typename v1, typename v2>
01089     struct mul {
01090         using type = val<v1::v * v2::v>;
01091     };
01092
01093     template<typename v1, typename v2>
01094     struct div {
01095         using type = val<v1::v / v2::v>;
01096     };
01097
01098     template<typename v1, typename v2>
01099     struct remainder {
01100         using type = val<v1::v % v2::v>;
01101     };
01102
01103     template<typename v1, typename v2>
01104     struct gt {
01105         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01106     };
01107
01108     template<typename v1, typename v2>
01109     struct lt {
01110         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01111     };
01112
01113     template<typename v1, typename v2>
01114     struct eq {
01115         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01116     };
01117
01118     template<typename v1>
01119     struct pos {
01120         using type = std::bool_constant<(v1::v > 0)>;
01121     };
01122
01123 public:
01124     template<typename v1, typename v2>
01125     using add_t = typename add<v1, v2>::type;
01126
01127     template<typename v1, typename v2>
01128     using sub_t = typename sub<v1, v2>::type;
01129
01130     template<typename v1, typename v2>
01131     using mul_t = typename mul<v1, v2>::type;
01132
01133     template<typename v1, typename v2>
01134     using div_t = typename div<v1, v2>::type;
01135
01136     template<typename v1, typename v2>
01137     using mod_t = typename remainder<v1, v2>::type;
01138
01139     template<typename v1, typename v2>
01140     using gt_t = typename gt<v1, v2>::type;
01141
01142     template<typename v1, typename v2>
01143     using lt_t = typename lt<v1, v2>::type;
01144
01145     template<typename v1, typename v2>
01146     using eq_t = typename eq<v1, v2>::type;
01147
01148     template<typename v1, typename v2>
01149     static constexpr bool eq_v = eq_t<v1, v2>::value;
01150
01151     template<typename v1, typename v2>
01152     using gcd_t = gcd_t<i32, v1, v2>;
01153
01154     template<typename v>
01155     using pos_t = typename pos<v>::type;
01156
01157
01158
01159
01160
01161
01162
01163
01164
01165
01166
01167
01168
01169
01170
01171
01172
01173
01174
01175
01176
01177
01178
01179
01180
01181
01182
01183
01184
01185
01186
01187
01188
01189
01190
01191
01192
01193
01194
01195
01196
01197
01198

```



```

01202     template<typename v>
01203     static constexpr bool pos_v = pos_t<v>::value;
01204 };
01205 } // namespace aerobus
01206
01207 // i64
01208 namespace aerobus {
01210     struct i64 {
01212         using inner_type = int64_t;
01215         template<int64_t x>
01216         struct val {
01218             using inner_type = int32_t;
01220             using enclosing_type = i64;
01222             static constexpr int64_t v = x;
01223
01226             template<typename valueType>
01227             static constexpr INLINED_DEVICE valueType get() {
01228                 return static_cast<valueType>(x);
01229             }
01230
01232             using is_zero_t = std::bool_constant<x == 0>;
01233
01235             static std::string to_string() {
01236                 return std::to_string(x);
01237             }
01238         };
01239
01242         template<auto x>
01243         using inject_constant_t = val<static_cast<int64_t>(x)>;
01244
01249         template<typename v>
01250         using inject_ring_t = v;
01251
01253         using zero = val<0>;
01255         using one = val<1>;
01257         static constexpr bool is_field = false;
01259         static constexpr bool is_euclidean_domain = true;
01260
01261     private:
01262         template<typename v1, typename v2>
01263         struct add {
01264             using type = val<v1::v + v2::v>;
01265         };
01266
01267         template<typename v1, typename v2>
01268         struct sub {
01269             using type = val<v1::v - v2::v>;
01270         };
01271
01272         template<typename v1, typename v2>
01273         struct mul {
01274             using type = val<v1::v * v2::v>;
01275         };
01276
01277         template<typename v1, typename v2>
01278         struct div {
01279             using type = val<v1::v / v2::v>;
01280         };
01281
01282         template<typename v1, typename v2>
01283         struct remainder {
01284             using type = val<v1::v % v2::v>;
01285         };
01286
01287         template<typename v1, typename v2>
01288         struct gt {
01289             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01290         };
01291
01292         template<typename v1, typename v2>
01293         struct lt {
01294             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01295         };
01296
01297         template<typename v1, typename v2>
01298         struct eq {
01299             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01300         };
01301
01302         template<typename v>
01303         struct pos {
01304             using type = std::bool_constant<(v::v > 0)>;
01305         };
01306
01307     public:
01311         template<typename v1, typename v2>
01312         using add_t = typename add<v1, v2>::type;

```

```

01313
01317     template<typename v1, typename v2>
01318     using sub_t = typename sub<v1, v2>::type;
01319
01323     template<typename v1, typename v2>
01324     using mul_t = typename mul<v1, v2>::type;
01325
01330     template<typename v1, typename v2>
01331     using div_t = typename div<v1, v2>::type;
01332
01336     template<typename v1, typename v2>
01337     using mod_t = typename remainder<v1, v2>::type;
01338
01343     template<typename v1, typename v2>
01344     using gt_t = typename gt<v1, v2>::type;
01345
01350     template<typename v1, typename v2>
01351     static constexpr bool gt_v = gt_t<v1, v2>::value;
01352
01357     template<typename v1, typename v2>
01358     using lt_t = typename lt<v1, v2>::type;
01359
01364     template<typename v1, typename v2>
01365     static constexpr bool lt_v = lt_t<v1, v2>::value;
01366
01371     template<typename v1, typename v2>
01372     using eq_t = typename eq<v1, v2>::type;
01373
01378     template<typename v1, typename v2>
01379     static constexpr bool eq_v = eq_t<v1, v2>::value;
01380
01385     template<typename v1, typename v2>
01386     using gcd_t = gcd_t<i64, v1, v2>;
01387
01391     template<typename v>
01392     using pos_t = typename pos<v>::type;
01393
01397     template<typename v>
01398     static constexpr bool pos_v = pos_t<v>::value;
01399 };
01400
01402 template<>
01403 struct Embed<i32, i64> {
01406     template<typename val>
01407     using type = i64::val<static_cast<int64_t>(val::v)>;
01408 };
01409 } // namespace aerobus
01410
01411 // z/pz
01412 namespace aerobus {
01418     template<int32_t p>
01419     struct zpz {
01421         using inner_type = int32_t;
01422
01425         template<int32_t x>
01426         struct val {
01428             using enclosing_type = zpz<p>;
01430             static constexpr int32_t v = x % p;
01431
01434             template<typename valueType>
01435             static constexpr INLINED_DEVICE valueType get() {
01436                 return static_cast<valueType>(x % p);
01437             }
01438
01440             using is_zero_t = std::bool_constant<v == 0>;
01441
01443             static constexpr bool is_zero_v = v == 0;
01444
01447             static std::string to_string() {
01448                 return std::to_string(x % p);
01449             }
01450         };
01451
01454         template<auto x>
01455         using inject_constant_t = val<static_cast<int32_t>(x)>;
01456
01458         using zero = val<0>;
01459
01461         using one = val<1>;
01462
01464         static constexpr bool is_field = is_prime<p>::value;
01465
01467         static constexpr bool is_euclidean_domain = true;
01468
01469     private:
01470         template<typename v1, typename v2>
01471         struct add {

```

```

01472         using type = val<(v1::v + v2::v) % p>;
01473     };
01474
01475     template<typename v1, typename v2>
01476     struct sub {
01477         using type = val<(v1::v - v2::v) % p>;
01478     };
01479
01480     template<typename v1, typename v2>
01481     struct mul {
01482         using type = val<(v1::v * v2::v) % p>;
01483     };
01484
01485     template<typename v1, typename v2>
01486     struct div {
01487         using type = val<(v1::v % p) / (v2::v % p)>;
01488     };
01489
01490     template<typename v1, typename v2>
01491     struct remainder {
01492         using type = val<(v1::v % v2::v) % p>;
01493     };
01494
01495     template<typename v1, typename v2>
01496     struct gt {
01497         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01498     };
01499
01500     template<typename v1, typename v2>
01501     struct lt {
01502         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01503     };
01504
01505     template<typename v1, typename v2>
01506     struct eq {
01507         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01508     };
01509
01510     template<typename v1>
01511     struct pos {
01512         using type = std::bool_constant<(v1::v > 0)>;
01513     };
01514
01515     public:
01519     template<typename v1, typename v2>
01520     using add_t = typename add<v1, v2>::type;
01521
01525     template<typename v1, typename v2>
01526     using sub_t = typename sub<v1, v2>::type;
01527
01531     template<typename v1, typename v2>
01532     using mul_t = typename mul<v1, v2>::type;
01533
01537     template<typename v1, typename v2>
01538     using div_t = typename div<v1, v2>::type;
01539
01543     template<typename v1, typename v2>
01544     using mod_t = typename remainder<v1, v2>::type;
01545
01549     template<typename v1, typename v2>
01550     using gt_t = typename gt<v1, v2>::type;
01551
01555     template<typename v1, typename v2>
01556     static constexpr bool gt_v = gt_t<v1, v2>::value;
01557
01561     template<typename v1, typename v2>
01562     using lt_t = typename lt<v1, v2>::type;
01563
01567     template<typename v1, typename v2>
01568     static constexpr bool lt_v = lt_t<v1, v2>::value;
01569
01573     template<typename v1, typename v2>
01574     using eq_t = typename eq<v1, v2>::type;
01575
01579     template<typename v1, typename v2>
01580     static constexpr bool eq_v = eq_t<v1, v2>::value;
01581
01585     template<typename v1, typename v2>
01586     using gcd_t = gcd_t<i32, v1, v2>;
01587
01590     template<typename v1>
01591     using pos_t = typename pos<v1>::type;
01592
01595     template<typename v>
01596     static constexpr bool pos_v = pos_t<v>::value;
01597 };
01598

```

```

01601     template<int32_t x>
01602     struct Embed<zp<x>, i32> {
01603         template <typename val>
01604         using type = i32::val<val::v>;
01605     };
01606 } // namespace aerobus
01607
01608 // polynomial
01609 namespace aerobus {
01610     // coeffN x^N + ...
01611     template<typename Ring>
01612     requires IsEuclideanDomain<Ring>
01613     struct polynomial {
01614         static constexpr bool is_field = false;
01615         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01616
01617         template<typename P>
01618         struct horner_reduction_t {
01619             template<size_t index, size_t stop>
01620             struct inner {
01621                 template<typename accum, typename x>
01622                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01623                 ::template type<
01624                     typename Ring::template add_t<
01625                         typename Ring::template mul_t<x, accum>,
01626                         typename P::template coeff_at_t<P::degree - index>
01627                     >, x>;
01628             };
01629
01630             template<size_t stop>
01631             struct inner<stop, stop> {
01632                 template<typename accum, typename x>
01633                 using type = accum;
01634             };
01635         };
01636
01637         template<typename coeffN, typename... coeffs>
01638         struct val {
01639             using ring_type = Ring;
01640             using enclosing_type = polynomial<Ring>;
01641             static constexpr size_t degree = sizeof...(coeffs);
01642             using aN = coeffN;
01643             using strip = val<coeffs...>;
01644             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01645             static constexpr bool is_zero_v = is_zero_t::value;
01646
01647         private:
01648             template<size_t index, typename E = void>
01649             struct coeff_at {};
01650
01651             template<size_t index>
01652             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01653                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01654             };
01655
01656             template<size_t index>
01657             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01658                 using type = typename Ring::zero;
01659             };
01660
01661         public:
01662             template<size_t index>
01663             using coeff_at_t = typename coeff_at<index>::type;
01664
01665             static std::string to_string() {
01666                 return string_helper<coeffN, coeffs...>::func();
01667             }
01668
01669             template<typename arithmeticType>
01670             static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01671                 #ifdef WITH_CUDA_FP16
01672                 arithmeticType start;
01673                 if constexpr (std::is_same_v<arithmeticType, __half2>) {
01674                     start = __half2(0, 0);
01675                 } else {
01676                     start = static_cast<arithmeticType>(0);
01677                 }
01678                 #else
01679                 arithmeticType start = static_cast<arithmeticType>(0);
01680                 #endif
01681                 return horner_evaluation<arithmeticType, val>
01682                     ::template inner<0, degree + 1>
01683                     ::func(start, x);
01684             }
01685
01686             template<typename arithmeticType>
01687             static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {

```

```

01726         return compensated_horner<arithmeticType, val>::func(x);
01727     }
01728
01729     template<typename x>
01730     using value_at_t = horner_reduction_t<val>
01731         ::template inner<0, degree + 1>
01732         ::template type<typename Ring::zero, x>;
01733 };
01734
01735 template<typename coeffN>
01736 struct val<coeffN> {
01737     using ring_type = Ring;
01738     using enclosing_type = polynomial<Ring>;
01739     static constexpr size_t degree = 0;
01740     using aN = coeffN;
01741     using strip = val<coeffN>;
01742     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01743
01744     static constexpr bool is_zero_v = is_zero_t::value;
01745
01746     template<size_t index, typename E = void>
01747     struct coeff_at {};
01748
01749     template<size_t index>
01750     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01751         using type = aN;
01752     };
01753
01754     template<size_t index>
01755     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01756         using type = typename Ring::zero;
01757     };
01758
01759     template<size_t index>
01760     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01761         using type = typename Ring::zero;
01762     };
01763
01764     template<size_t index>
01765     using coeff_at_t = typename coeff_at<index>::type;
01766
01767     static std::string to_string() {
01768         return string_helper<coeffN>::func();
01769     }
01770
01771     template<typename arithmeticType>
01772     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01773         return coeffN::template get<arithmeticType>();
01774     }
01775
01776     template<typename arithmeticType>
01777     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01778         return coeffN::template get<arithmeticType>();
01779     }
01780
01781     template<typename x>
01782     using value_at_t = coeffN;
01783 };
01784
01785 using zero = val<typename Ring::zero>;
01786 using one = val<typename Ring::one>;
01787 using X = val<typename Ring::one, typename Ring::zero>;
01788
01789 private:
01790     template<typename P, typename E = void>
01791     struct simplify;
01792
01793     template<typename P1, typename P2, typename I>
01794     struct add_low;
01795
01796     template<typename P1, typename P2>
01797     struct add {
01798         using type = typename simplify<typename add_low<
01799             P1,
01800             P2,
01801             internal::make_index_sequence_reverse<
01802                 std::max(P1::degree, P2::degree) + 1
01803             >::type>::type;
01804     };
01805
01806     template<typename P1, typename P2, typename I>
01807     struct sub_low;
01808
01809     template<typename P1, typename P2, typename I>
01810     struct mul_low;
01811
01812     template<typename v1, typename v2>
01813     struct mul {
01814         using type = typename mul_low<
01815             v1,
01816             v2,
01817             internal::make_index_sequence_reverse<

```

```

01821         v1::degree + v2::degree + 1
01822         »::type;
01823     };
01824
01825     template<typename coeff, size_t deg>
01826     struct monomial;
01827
01828     template<typename v, typename E = void>
01829     struct derive_helper {};
01830
01831     template<typename v>
01832     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01833         using type = zero;
01834     };
01835
01836     template<typename v>
01837     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01838         using type = typename add<
01839             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01840             typename monomial<
01841                 typename Ring::template mul_t<
01842                     typename v::aN,
01843                     typename Ring::template inject_constant_t<(v::degree)>
01844                 >,
01845                 v::degree - 1
01846             >::type
01847         >::type;
01848     };
01849
01850     template<typename v1, typename v2, typename E = void>
01851     struct eq_helper {};
01852
01853     template<typename v1, typename v2>
01854     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01855         using type = std::false_type;
01856     };
01857
01858     template<typename v1, typename v2>
01859     struct eq_helper<v1, v2, std::enable_if_t<
01860         v1::degree == v2::degree &&
01861         (v1::degree != 0 || v2::degree != 0) &&
01862         std::is_same<
01863             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01864             std::false_type
01865         >::value
01866     > {
01867     > {
01868         using type = std::false_type;
01869     };
01870
01871     template<typename v1, typename v2>
01872     struct eq_helper<v1, v2, std::enable_if_t<
01873         v1::degree == v2::degree &&
01874         (v1::degree != 0 || v2::degree != 0) &&
01875         std::is_same<
01876             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01877             std::true_type
01878         >::value
01879     > {
01880     » {
01881         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01882     };
01883
01884     template<typename v1, typename v2>
01885     struct eq_helper<v1, v2, std::enable_if_t<
01886         v1::degree == v2::degree &&
01887         (v1::degree == 0)
01888     > {
01889         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01890     };
01891
01892     template<typename v1, typename v2, typename E = void>
01893     struct lt_helper {};
01894
01895     template<typename v1, typename v2>
01896     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01897         using type = std::true_type;
01898     };
01899
01900     template<typename v1, typename v2>
01901     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01902         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01903     };
01904
01905     template<typename v1, typename v2>
01906     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01907         using type = std::false_type;

```

```

01908     };
01909
01910     template<typename v1, typename v2, typename E = void>
01911     struct gt_helper {};
01912
01913     template<typename v1, typename v2>
01914     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01915         using type = std::true_type;
01916     };
01917
01918     template<typename v1, typename v2>
01919     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01920         using type = std::false_type;
01921     };
01922
01923     template<typename v1, typename v2>
01924     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01925         using type = std::false_type;
01926     };
01927
01928     // when high power is zero : strip
01929     template<typename P>
01930     struct simplify<P, std::enable_if_t<
01931         std::is_same<
01932             typename Ring::zero,
01933             typename P::aN
01934         >::value && (P::degree > 0)
01935     >> {
01936         using type = typename simplify<typename P::strip>::type;
01937     };
01938
01939     // otherwise : do nothing
01940     template<typename P>
01941     struct simplify<P, std::enable_if_t<
01942         !std::is_same<
01943             typename Ring::zero,
01944             typename P::aN
01945         >::value && (P::degree > 0)
01946     >> {
01947         using type = P;
01948     };
01949
01950     // do not simplify constants
01951     template<typename P>
01952     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01953         using type = P;
01954     };
01955
01956     // addition at
01957     template<typename P1, typename P2, size_t index>
01958     struct add_at {
01959         using type =
01960             typename Ring::template add_t<
01961                 typename P1::template coeff_at_t<index>,
01962                 typename P2::template coeff_at_t<index>;
01963     };
01964
01965     template<typename P1, typename P2, size_t index>
01966     using add_at_t = typename add_at<P1, P2, index>::type;
01967
01968     template<typename P1, typename P2, std::size_t... I>
01969     struct add_low<P1, P2, std::index_sequence<I...>> {
01970         using type = val<add_at_t<P1, P2, I>...>;
01971     };
01972
01973     // subtraction at
01974     template<typename P1, typename P2, size_t index>
01975     struct sub_at {
01976         using type =
01977             typename Ring::template sub_t<
01978                 typename P1::template coeff_at_t<index>,
01979                 typename P2::template coeff_at_t<index>;
01980     };
01981
01982     template<typename P1, typename P2, size_t index>
01983     using sub_at_t = typename sub_at<P1, P2, index>::type;
01984
01985     template<typename P1, typename P2, std::size_t... I>
01986     struct sub_low<P1, P2, std::index_sequence<I...>> {
01987         using type = val<sub_at_t<P1, P2, I>...>;
01988     };
01989
01990     template<typename P1, typename P2>
01991     struct sub {
01992         using type = typename simplify<typename sub_low<
01993             P1,
01994             P2,

```

```

01995         internal::make_index_sequence_reverse<
01996         std::max(P1::degree, P2::degree) + 1
01997         >::type>::type;
01998     };
01999
02000     // multiplication at
02001     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02002     struct mul_at_loop_helper {
02003         using type = typename Ring::template add_t<
02004             typename Ring::template mul_t<
02005                 typename v1::template coeff_at_t<index>,
02006                 typename v2::template coeff_at_t<k - index>
02007             >,
02008             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02009         >;
02010     };
02011
02012     template<typename v1, typename v2, size_t k, size_t stop>
02013     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02014         using type = typename Ring::template mul_t<
02015             typename v1::template coeff_at_t<stop>,
02016             typename v2::template coeff_at_t<0>;
02017     };
02018
02019     template <typename v1, typename v2, size_t k, typename E = void>
02020     struct mul_at {};
02021
02022     template<typename v1, typename v2, size_t k>
02023     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
02024         using type = typename Ring::zero;
02025     };
02026
02027     template<typename v1, typename v2, size_t k>
02028     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
02029         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02030     };
02031
02032     template<typename P1, typename P2, size_t index>
02033     using mul_at_t = typename mul_at<P1, P2, index>::type;
02034
02035     template<typename P1, typename P2, std::size_t... I>
02036     struct mul_low<P1, P2, std::index_sequence<I...> {
02037         using type = val<mul_at_t<P1, P2, I>...>;
02038     };
02039
02040     // division helper
02041     template< typename A, typename B, typename Q, typename R, typename E = void>
02042     struct div_helper {};
02043
02044     template<typename A, typename B, typename Q, typename R>
02045     struct div_helper<A, B, Q, R, std::enable_if_t<
02046         (R::degree < B::degree) ||
02047         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02048         using q_type = Q;
02049         using mod_type = R;
02050         using gcd_type = B;
02051     };
02052
02053     template<typename A, typename B, typename Q, typename R>
02054     struct div_helper<A, B, Q, R, std::enable_if_t<
02055         (R::degree >= B::degree) &&
02056         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02057     private: // NOLINT
02058         using rN = typename R::aN;
02059         using bN = typename B::aN;
02060         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02061             B::degree>::type;
02062         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02063         using qq = typename add<Q, pT>::type;
02064
02065     public:
02066         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02067         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02068         using gcd_type = rr;
02069     };
02070
02071     template<typename A, typename B>
02072     struct div {
02073         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02074         using q_type = typename div_helper<A, B, zero, A>::q_type;
02075         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02076     };
02077
02078     template<typename P>
02079     struct make_unit {
02080         using type = typename div<P, val<typename P::aN>::q_type>;
02081     };

```



```

02081
02082     template<typename coeff, size_t deg>
02083     struct monomial {
02084         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02085     };
02086
02087     template<typename coeff>
02088     struct monomial<coeff, 0> {
02089         using type = val<coeff>;
02090     };
02091
02092     template<typename arithmeticType, typename P>
02093     struct horner_evaluation {
02094         template<size_t index, size_t stop>
02095         struct inner {
02096             static constexpr DEVICE INLINED arithmeticType func(
02097                 const arithmeticType& accum, const arithmeticType& x) {
02098                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
stop>::func(
02099                     internal::fma_helper<arithmeticType>::eval(
02100                         x,
02101                         accum,
02102                         P::template coeff_at_t<P::degree - index>::template
get<arithmeticType>()), x);
02103             }
02104         };
02105
02106         template<size_t stop>
02107         struct inner<stop, stop> {
02108             static constexpr DEVICE INLINED arithmeticType func(
02109                 const arithmeticType& accum, const arithmeticType& x) {
02110                 return accum;
02111             }
02112         };
02113     };
02114
02115     template<typename arithmeticType, typename P>
02116     struct compensated_horner {
02117         template<int64_t index, int ghost>
02118         struct EFTHorner {
02119             static INLINED void func(
02120                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02121                 arithmeticType p;
02122                 internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02123                 constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
get<arithmeticType>();
02124                 internal::two_sum<arithmeticType>(
02125                     p, coeff,
02126                     r, sigma + P::degree - index - 1);
02127                 EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02128             }
02129         };
02130
02131         template<int ghost>
02132         struct EFTHorner<-1, ghost> {
02133             static INLINED DEVICE void func(
02134                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02135             }
02136         };
02137
02138         static INLINED DEVICE arithmeticType func(arithmeticType x) {
02139             arithmeticType pi[P::degree], sigma[P::degree];
02140             arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02141             EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02142             arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02143             return r + c;
02144         }
02145     };
02146
02147     template<typename coeff, typename... coeffs>
02148     struct string_helper {
02149         static std::string func() {
02150             std::string tail = string_helper<coeffs...>::func();
02151             std::string result = "";
02152             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02153                 return tail;
02154             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02155                 if (sizeof...(coeffs) == 1) {
02156                     result += "x";
02157                 } else {
02158                     result += "x^" + std::to_string(sizeof...(coeffs));
02159                 }
02160             } else {
02161                 if (sizeof...(coeffs) == 1) {
02162                     result += coeff::to_string() + " x";

```

```

02163         } else {
02164             result += coeff::to_string()
02165                 + " x^" + std::to_string(sizeof...(coeffs));
02166         }
02167     }
02168
02169     if (!tail.empty()) {
02170         if (tail.at(0) != '-') {
02171             result += " + " + tail;
02172         } else {
02173             result += " - " + tail.substr(1);
02174         }
02175     }
02176
02177     return result;
02178 }
02179 };
02180
02181 template<typename coeff>
02182 struct string_helper<coeff> {
02183     static std::string func() {
02184         if (!std::is_same<coeff, typename Ring::zero>::value) {
02185             return coeff::to_string();
02186         } else {
02187             return "";
02188         }
02189     }
02190 };
02191
02192 public:
02193     template<typename P>
02194     using simplify_t = typename simplify<P>::type;
02195
02196     template<typename v1, typename v2>
02197     using add_t = typename add<v1, v2>::type;
02198
02199     template<typename v1, typename v2>
02200     using sub_t = typename sub<v1, v2>::type;
02201
02202     template<typename v1, typename v2>
02203     using mul_t = typename mul<v1, v2>::type;
02204
02205     template<typename v1, typename v2>
02206     using eq_t = typename eq_helper<v1, v2>::type;
02207
02208     template<typename v1, typename v2>
02209     using lt_t = typename lt_helper<v1, v2>::type;
02210
02211     template<typename v1, typename v2>
02212     using gt_t = typename gt_helper<v1, v2>::type;
02213
02214     template<typename v1, typename v2>
02215     using div_t = typename div<v1, v2>::q_type;
02216
02217     template<typename v1, typename v2>
02218     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02219
02220     template<typename coeff, size_t deg>
02221     using monomial_t = typename monomial<coeff, deg>::type;
02222
02223     template<typename v>
02224     using derive_t = typename derive_helper<v>::type;
02225
02226     template<typename v>
02227     using pos_t = typename Ring::template pos_t<typename v::aN>;
02228
02229     template<typename v>
02230     static constexpr bool pos_v = pos_t<v>::value;
02231
02232     template<typename v1, typename v2>
02233     using gcd_t = std::conditional_t<
02234         Ring::is_euclidean_domain,
02235         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02236         void>;
02237
02238     template<auto x>
02239     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02240
02241     template<typename v>
02242     using inject_ring_t = val<v>;
02243 };
02244 } // namespace aerobus
02245
02246 // fraction field
02247 namespace aerobus {
02248     namespace internal {
02249         template<typename Ring, typename E = void>

```

```

02292     requires IsEuclideanDomain<Ring>
02293     struct _FractionField {};
02294
02295     template<typename Ring>
02296     requires IsEuclideanDomain<Ring>
02297     struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02298         static constexpr bool is_field = true;
02299         static constexpr bool is_euclidean_domain = true;
02300
02301     private:
02302         template<typename val1, typename val2, typename E = void>
02303         struct to_string_helper {};
02304
02305         template<typename val1, typename val2>
02306         struct to_string_helper<val1, val2,
02307             std::enable_if_t<
02308                 Ring::template eq_t<
02309                     val2, typename Ring::one
02310                 >::value
02311             >
02312         > {
02313             static std::string func() {
02314                 return val1::to_string();
02315             }
02316         };
02317
02318         template<typename val1, typename val2>
02319         struct to_string_helper<val1, val2,
02320             std::enable_if_t<
02321                 !Ring::template eq_t<
02322                     val2,
02323                     typename Ring::one
02324                 >::value
02325             >
02326         > {
02327             static std::string func() {
02328                 return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02329             }
02330         };
02331     };
02332
02333     public:
02334         template<typename val1, typename val2>
02335         struct val {
02336             using x = val1;
02337             using y = val2;
02338             using is_zero_t = typename val1::is_zero_t;
02339             static constexpr bool is_zero_v = val1::is_zero_t::value;
02340
02341             using ring_type = Ring;
02342             using enclosing_type = _FractionField<Ring>;
02343
02344             static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02345
02346             template<typename valueType>
02347             static constexpr INLINED_DEVICE valueType get() {
02348                 return internal::staticcast<valueType, typename ring_type::inner_type>::template
02349                     func<x::v>() /
02350                     internal::staticcast<valueType, typename ring_type::inner_type>::template
02351                     func<y::v>();
02352             }
02353
02354             static std::string to_string() {
02355                 return to_string_helper<val1, val2>::func();
02356             }
02357
02358             template<typename arithmeticType>
02359             static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02360                 return x::eval(v) / y::eval(v);
02361             }
02362         };
02363
02364         using zero = val<typename Ring::zero, typename Ring::one>;
02365         using one = val<typename Ring::one, typename Ring::one>;
02366
02367         template<typename v>
02368         using inject_t = val<v, typename Ring::one>;
02369
02370         template<auto x>
02371         using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
02372             Ring::one>;
02373
02374         template<typename v>
02375         using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02376
02377         using ring_type = Ring;
02378     private:

```

```

02405     template<typename v, typename E = void>
02406     struct simplify {};
02407
02408     // x = 0
02409     template<typename v>
02410     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02411         using type = typename _FractionField<Ring>::zero;
02412     };
02413
02414     // x != 0
02415     template<typename v>
02416     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02417     private:
02418         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02419         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02420         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02421
02422         using posx = std::conditional_t<
02423             !Ring::template pos_v<newx>,
02424             typename Ring::template sub_t<typename Ring::zero, newx>,
02425             newx>;
02426         using posy = std::conditional_t<
02427             !Ring::template pos_v<newy>,
02428             typename Ring::template sub_t<typename Ring::zero, newy>,
02429             newy>;
02430     public:
02431         using type = typename _FractionField<Ring>::template val<posx, posy>;
02432     };
02433
02434     public:
02435     template<typename v>
02436     using simplify_t = typename simplify<v>::type;
02437
02438     private:
02439     template<typename v1, typename v2>
02440     struct add {
02441     private:
02442         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02443         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02444         using dividend = typename Ring::template add_t<a, b>;
02445         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02446         using g = typename Ring::template gcd_t<dividend, diviser>;
02447
02448     public:
02449         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02450             diviser>;
02451     };
02452
02453     template<typename v>
02454     struct pos {
02455     using type = std::conditional_t<
02456         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02457         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02458         std::true_type,
02459         std::false_type>;
02460     };
02461
02462     template<typename v1, typename v2>
02463     struct sub {
02464     private:
02465         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02466         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02467         using dividend = typename Ring::template sub_t<a, b>;
02468         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02469         using g = typename Ring::template gcd_t<dividend, diviser>;
02470
02471     public:
02472         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02473             diviser>;
02474     };
02475
02476     template<typename v1, typename v2>
02477     struct mul {
02478     private:
02479         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02480         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02481
02482     public:
02483         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02484     };
02485
02486     template<typename v1, typename v2, typename E = void>
02487     struct div {};
02488
02489     template<typename v1, typename v2>
02490     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
    _FractionField<Ring>::zero>::value> {

```

```

02491     private:
02492         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02493         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02494
02495     public:
02496         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02497 };
02498
02499 template<typename v1, typename v2>
02500 struct div<v1, v2, std::enable_if_t<
02501     std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02502     using type = one;
02503 };
02504
02505 template<typename v1, typename v2>
02506 struct eq {
02507     using type = std::conditional_t<
02508         std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02509         std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02510         std::true_type,
02511         std::false_type>;
02512 };
02513
02514 template<typename v1, typename v2, typename E = void>
02515 struct gt;
02516
02517 template<typename v1, typename v2>
02518 struct gt<v1, v2, std::enable_if_t<
02519     (eq<v1, v2>::type::value)
02520     > {
02521     using type = std::false_type;
02522 };
02523
02524 template<typename v1, typename v2>
02525 struct gt<v1, v2, std::enable_if_t<
02526     (!eq<v1, v2>::type::value) &&
02527     (!pos<v1>::type::value) && (!pos<v2>::type::value)
02528     > {
02529     using type = typename gt<
02530         typename sub<zero, v1>::type, typename sub<zero, v2>::type
02531     >::type;
02532 };
02533
02534 template<typename v1, typename v2>
02535 struct gt<v1, v2, std::enable_if_t<
02536     (!eq<v1, v2>::type::value) &&
02537     (pos<v1>::type::value) && (!pos<v2>::type::value)
02538     > {
02539     using type = std::true_type;
02540 };
02541
02542 template<typename v1, typename v2>
02543 struct gt<v1, v2, std::enable_if_t<
02544     (!eq<v1, v2>::type::value) &&
02545     (!pos<v1>::type::value) && (pos<v2>::type::value)
02546     > {
02547     using type = std::false_type;
02548 };
02549
02550 template<typename v1, typename v2>
02551 struct gt<v1, v2, std::enable_if_t<
02552     (!eq<v1, v2>::type::value) &&
02553     (pos<v1>::type::value) && (pos<v2>::type::value)
02554     > {
02555     using type = typename Ring::template gt_t<
02556         typename Ring::template mul_t<v1::x, v2::y>,
02557         typename Ring::template mul_t<v2::y, v2::x>
02558     >;
02559 };
02560
02561 public:
02562     template<typename v1, typename v2>
02563     using add_t = typename add<v1, v2>::type;
02564
02565     template<typename v1, typename v2>
02566     using mod_t = zero;
02567
02568     template<typename v1, typename v2>
02569     using gcd_t = v1;
02570
02571     template<typename v1, typename v2>
02572     using sub_t = typename sub<v1, v2>::type;
02573
02574     template<typename v1, typename v2>
02575     using mul_t = typename mul<v1, v2>::type;
02576
02577     template<typename v1, typename v2>

```

```

02598         using div_t = typename div<v1, v2>::type;
02599
02600     template<typename v1, typename v2>
02601     using eq_t = typename eq<v1, v2>::type;
02602
02603     template<typename v1, typename v2>
02604     static constexpr bool eq_v = eq<v1, v2>::type::value;
02605
02606     template<typename v1, typename v2>
02607     using gt_t = typename gt<v1, v2>::type;
02608
02609     template<typename v1, typename v2>
02610     static constexpr bool gt_v = gt<v1, v2>::type::value;
02611
02612     template<typename v1>
02613     using pos_t = typename pos<v1>::type;
02614
02615     template<typename v>
02616     static constexpr bool pos_v = pos_t<v>::value;
02617 };
02618
02619 template<typename Ring, typename E = void>
02620 requires IsEuclideanDomain<Ring>
02621 struct FractionFieldImpl {};
02622
02623 // fraction field of a field is the field itself
02624 template<typename Field>
02625 requires IsEuclideanDomain<Field>
02626 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02627     using type = Field;
02628     template<typename v>
02629     using inject_t = v;
02630 };
02631
02632 // fraction field of a ring is the actual fraction field
02633 template<typename Ring>
02634 requires IsEuclideanDomain<Ring>
02635 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02636     using type = _FractionField<Ring>;
02637 };
02638 } // namespace internal
02639
02640 template<typename Ring>
02641 requires IsEuclideanDomain<Ring>
02642 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02643
02644 template<typename Ring>
02645 struct Embed<Ring, FractionField<Ring> {
02646     template<typename v>
02647     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02648 };
02649 } // namespace aerobus
02650
02651 // short names for common types
02652 namespace aerobus {
02653     template<typename X, typename Y>
02654     requires IsRing<typename X::enclosing_type> &&
02655     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02656     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02657
02658     template<typename X, typename Y>
02659     requires IsRing<typename X::enclosing_type> &&
02660     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02661     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02662
02663     template<typename X, typename Y>
02664     requires IsRing<typename X::enclosing_type> &&
02665     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02666     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02667
02668     template<typename X, typename Y>
02669     requires IsEuclideanDomain<typename X::enclosing_type> &&
02670     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02671     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02672
02673     using q32 = FractionField<i32>;
02674
02675     using fpq32 = FractionField<polynomial<q32>>;
02676
02677     using q64 = FractionField<i64>;
02678
02679     using pi64 = polynomial<i64>;
02680
02681     using pq64 = polynomial<q64>;
02682
02683     using fpq64 = FractionField<polynomial<q64>>;

```

```

02728
02733     template<typename Ring, typename v1, typename v2>
02734     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02735
02742     template<typename v>
02743     using embed_int_poly_in_fractions_t =
02744         typename Embed<
02745             polynomial<typename v::ring_type>,
02746             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02747
02751     template<int64_t p, int64_t q>
02752     using make_q64_t = typename q64::template simplify_t<
02753         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02754
02758     template<int32_t p, int32_t q>
02759     using make_q32_t = typename q32::template simplify_t<
02760         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02761
02766     template<typename Ring, typename v1, typename v2>
02767     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02772     template<typename Ring, typename v1, typename v2>
02773     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02774
02776     template<>
02777     struct Embed<q32, q64> {
02780         template<typename v>
02781         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02782     };
02783
02787     template<typename Small, typename Large>
02788     struct Embed<polynomial<Small>, polynomial<Large>> {
02789     private:
02790         template<typename v, typename i>
02791         struct at_low;
02792
02793         template<typename v, size_t i>
02794         struct at_index {
02795             using type = typename Embed<Small, Large>::template
02796             type<typename v::template coeff_at_t<i>>;
02797         };
02798
02799         template<typename v, size_t... Is>
02800         struct at_low<v, std::index_sequence<Is...> {
02801             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02802         };
02803
02804     public:
02805         template<typename v>
02806         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02807             1>::type>;
02808     };
02809
02813     template<typename Ring, auto... xs>
02814     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02815         typename Ring::template inject_constant_t<xs>...>;
02816
02820     template<typename Ring, auto... xs>
02821     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02822         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02823 } // namespace aerobus
02824
02825 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02826 namespace aerobus {
02827     namespace internal {
02828         template<typename T, size_t x, typename E = void>
02829         struct factorial {};
02830
02831         template<typename T, size_t x>
02832         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02833         private:
02834             template<typename, size_t, typename>
02835             friend struct factorial;
02836
02837         public:
02838             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02839                 x - 1>::type>;
02840             static constexpr typename T::inner_type value = type::template get<typename
02841                 T::inner_type>();
02842         };
02843
02844         template<typename T>
02845         struct factorial<T, 0> {
02846         public:
02847             using type = typename T::one;
02848             static constexpr typename T::inner_type value = type::template get<typename
02849                 T::inner_type>();
02850         };
02851     } // namespace internal

```

```

02848
02852     template<typename T, size_t i>
02853     using factorial_t = typename internal::factorial<T, i>::type;
02854
02858     template<typename T, size_t i>
02859     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02860
02861     namespace internal {
02862         template<typename T, size_t k, size_t n, typename E = void>
02863         struct combination_helper {};
02864
02865         template<typename T, size_t k, size_t n>
02866         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02867             using type = typename FractionField<T>::template mul_t<
02868                 typename combination_helper<T, k - 1, n - 1>::type,
02869                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02870         };
02871
02872         template<typename T, size_t k, size_t n>
02873         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02874             using type = typename combination_helper<T, n - k, n>::type;
02875         };
02876
02877         template<typename T, size_t n>
02878         struct combination_helper<T, 0, n> {
02879             using type = typename FractionField<T>::one;
02880         };
02881
02882         template<typename T, size_t k, size_t n>
02883         struct combination {
02884             using type = typename internal::combination_helper<T, k, n>::type::x;
02885             static constexpr typename T::inner_type value =
02886                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>());
02887         };
02888     } // namespace internal
02889
02892     template<typename T, size_t k, size_t n>
02893     using combination_t = typename internal::combination<T, k, n>::type;
02894
02899     template<typename T, size_t k, size_t n>
02900     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02901
02902     namespace internal {
02903         template<typename T, size_t m>
02904         struct bernoulli;
02905
02906         template<typename T, typename accum, size_t k, size_t m>
02907         struct bernoulli_helper {
02908             using type = typename bernoulli_helper<
02909                 T,
02910                 addfractions_t<T,
02911                     accum,
02912                     mulfractions_t<T,
02913                         makefraction_t<T,
02914                             combination_t<T, k, m + 1>,
02915                             typename T::one>,
02916                             typename bernoulli<T, k>::type
02917                         >,
02918                     >,
02919                     k + 1,
02920                     m>::type;
02921         };
02922
02923         template<typename T, typename accum, size_t m>
02924         struct bernoulli_helper<T, accum, m, m> {
02925             using type = accum;
02926         };
02927
02928
02929
02930         template<typename T, size_t m>
02931         struct bernoulli {
02932             using type = typename FractionField<T>::template mul_t<
02933                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02934                 makefraction_t<T,
02935                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02936                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02937                 >
02938             >;
02939
02940             template<typename floatType>
02941             static constexpr floatType value = type::template get<floatType>();
02942         };
02943
02944         template<typename T>
02945         struct bernoulli<T, 0> {

```



```

02946         using type = typename FractionField<T>::one;
02947
02948         template<typename floatType>
02949         static constexpr floatType value = type::template get<floatType>();
02950     };
02951 } // namespace internal
02952
02953 template<typename T, size_t n>
02954 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02955
02956 template<typename FloatType, typename T, size_t n>
02957 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02958
02959 // bell numbers
02960 namespace internal {
02961     template<typename T, size_t n, typename E = void>
02962     struct bell_helper;
02963
02964     template<typename T, size_t n>
02965     struct bell_helper<T, n, std::enable_if_t<(n > 1)> {
02966         template<typename accum, size_t i, size_t stop>
02967         struct sum_helper {
02968             private:
02969                 using left = typename T::template mul_t<
02970                     combination_t<T, i, n-1>,
02971                     typename bell_helper<T, i>::type>;
02972                 using new_accum = typename T::template add_t<accum, left>;
02973             public:
02974                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02975         };
02976
02977         template<typename accum, size_t stop>
02978         struct sum_helper<accum, stop, stop> {
02979             using type = accum;
02980         };
02981
02982         using type = typename sum_helper<typename T::zero, 0, n>::type;
02983     };
02984
02985     template<typename T>
02986     struct bell_helper<T, 0> {
02987         using type = typename T::one;
02988     };
02989
02990     template<typename T>
02991     struct bell_helper<T, 1> {
02992         using type = typename T::one;
02993     };
02994 } // namespace internal
02995
02996 template<typename T, size_t n>
02997 using bell_t = typename internal::bell_helper<T, n>::type;
02998
02999 template<typename T, size_t n>
03000 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03001
03002 namespace internal {
03003     template<typename T, int k, typename E = void>
03004     struct alternate {};
03005
03006     template<typename T, int k>
03007     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
03008         using type = typename T::one;
03009         static constexpr typename T::inner_type value = type::template get<typename
03010 T::inner_type>();
03011     };
03012
03013     template<typename T, int k>
03014     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
03015         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03016         static constexpr typename T::inner_type value = type::template get<typename
03017 T::inner_type>();
03018     };
03019 } // namespace internal
03020
03021 template<typename T, int k>
03022 using alternate_t = typename internal::alternate<T, k>::type;
03023
03024 template<typename T, size_t k>
03025 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03026
03027 namespace internal {
03028     template<typename T, int n, int k, typename E = void>
03029     struct stirling_1_helper {};
03030
03031     template<typename T>
03032     struct stirling_1_helper<T, 0, 0> {

```

```

03048         using type = typename T::one;
03049     };
03050
03051     template<typename T, int n>
03052     struct stirling_1_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03053         using type = typename T::zero;
03054     };
03055
03056     template<typename T, int n>
03057     struct stirling_1_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03058         using type = typename T::zero;
03059     };
03060
03061     template<typename T, int n, int k>
03062     struct stirling_1_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
03063         using type = typename T::template sub_t<
03064             typename stirling_1_helper<T, n-1, k-1>::type,
03065             typename T::template mul_t<
03066                 typename T::template inject_constant_t<n-1>,
03067                 typename stirling_1_helper<T, n-1, k>::type
03068             >;
03069     };
03070 } // namespace internal
03071
03072 template<typename T, int n, int k>
03073 using stirling_1_signed_t = typename internal::stirling_1_helper<T, n, k>::type;
03074
03075 template<typename T, int n, int k>
03076 using stirling_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03077
03078 template<typename T, int n, int k>
03079 static constexpr typename T::inner_type stirling_1_unsigned_v = stirling_1_unsigned_t<T, n, k>::v;
03080
03081 template<typename T, int n, int k>
03082 static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03083
03084 namespace internal {
03085     template<typename T, int n, int k, typename E = void>
03086     struct stirling_2_helper {};
03087
03088     template<typename T, int n>
03089     struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
03090         using type = typename T::one;
03091     };
03092
03093     template<typename T, int n>
03094     struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03095         using type = typename T::zero;
03096     };
03097
03098     template<typename T, int n>
03099     struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03100         using type = typename T::zero;
03101     };
03102
03103     template<typename T, int n, int k>
03104     struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03105         using type = typename T::template add_t<
03106             typename stirling_2_helper<T, n-1, k-1>::type,
03107             typename T::template mul_t<
03108                 typename T::template inject_constant_t<k>,
03109                 typename stirling_2_helper<T, n-1, k>::type
03110             >;
03111     };
03112 } // namespace internal
03113
03114 template<typename T, int n, int k>
03115 using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03116
03117 template<typename T, int n, int k>
03118 static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03119
03120 namespace internal {
03121     template<typename T>
03122     struct pow_scalar {
03123         template<size_t p>
03124         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03125             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03126             x * func<p/2>(x) * func<p/2>(x);
03127         }
03128     };
03129
03130     template<typename T, typename p, size_t n, typename E = void>
03131     requires IsEuclideanDomain<T>
03132     struct pow;
03133
03134     template<typename T, typename p, size_t n>

```

```

03159     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03160         using type = typename T::template mul_t<
03161             typename pow<T, p, n/2>::type,
03162             typename pow<T, p, n/2>::type
03163         >;
03164     };
03165
03166     template<typename T, typename p, size_t n>
03167     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03168         using type = typename T::template mul_t<
03169             p,
03170             typename T::template mul_t<
03171                 typename pow<T, p, n/2>::type,
03172                 typename pow<T, p, n/2>::type
03173             >
03174         >;
03175     };
03176
03177     template<typename T, typename p, size_t n>
03178     struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03179 } // namespace internal
03180
03181 template<typename T, typename p, size_t n>
03182 using pow_t = typename internal::pow<T, p, n>::type;
03183
03184 template<typename T, typename p, size_t n>
03185 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03186
03187 template<typename T, size_t p>
03188 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
internal::pow_scalar<T>::template func<p>(x); }
03197
03198 namespace internal {
03199     template<typename, template<typename, size_t> typename, class>
03200     struct make_taylor_impl;
03201
03202     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03203     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03204         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is::type...>;
03205     };
03206 }
03207
03212 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03213 using taylor = typename internal::make_taylor_impl<
03214     T,
03215     coeff_at,
03216     internal::make_index_sequence_reverse<deg + 1>::type;
03217
03218 namespace internal {
03219     template<typename T, size_t i>
03220     struct exp_coeff {
03221         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03222     };
03223
03224     template<typename T, size_t i, typename E = void>
03225     struct sin_coeff_helper {};
03226
03227     template<typename T, size_t i>
03228     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03229         using type = typename FractionField<T>::zero;
03230     };
03231
03232     template<typename T, size_t i>
03233     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03234         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03235     };
03236
03237     template<typename T, size_t i>
03238     struct sin_coeff {
03239         using type = typename sin_coeff_helper<T, i>::type;
03240     };
03241
03242     template<typename T, size_t i, typename E = void>
03243     struct sh_coeff_helper {};
03244
03245     template<typename T, size_t i>
03246     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03247         using type = typename FractionField<T>::zero;
03248     };
03249
03250     template<typename T, size_t i>
03251     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03252         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03253     };
03254
03255     template<typename T, size_t i>

```

```

03256     struct sh_coeff {
03257         using type = typename sh_coeff_helper<T, i>::type;
03258     };
03259
03260     template<typename T, size_t i, typename E = void>
03261     struct cos_coeff_helper {};
03262
03263     template<typename T, size_t i>
03264     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03265         using type = typename FractionField<T>::zero;
03266     };
03267
03268     template<typename T, size_t i>
03269     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03270         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
03271     };
03272
03273     template<typename T, size_t i>
03274     struct cos_coeff {
03275         using type = typename cos_coeff_helper<T, i>::type;
03276     };
03277
03278     template<typename T, size_t i, typename E = void>
03279     struct cosh_coeff_helper {};
03280
03281     template<typename T, size_t i>
03282     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03283         using type = typename FractionField<T>::zero;
03284     };
03285
03286     template<typename T, size_t i>
03287     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03288         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
03289     };
03290
03291     template<typename T, size_t i>
03292     struct cosh_coeff {
03293         using type = typename cosh_coeff_helper<T, i>::type;
03294     };
03295
03296     template<typename T, size_t i>
03297     struct geom_coeff { using type = typename FractionField<T>::one; };
03298
03299
03300     template<typename T, size_t i, typename E = void>
03301     struct atan_coeff_helper;
03302
03303     template<typename T, size_t i>
03304     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03305         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03306     };
03307
03308     template<typename T, size_t i>
03309     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03310         using type = typename FractionField<T>::zero;
03311     };
03312
03313     template<typename T, size_t i>
03314     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03315
03316     template<typename T, size_t i, typename E = void>
03317     struct asin_coeff_helper;
03318
03319     template<typename T, size_t i>
03320     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03321         using type = makefraction_t<T,
03322             factorial_t<T, i - 1>,
03323             typename T::template mul_t<
03324                 typename T::template val<i>,
03325                 T::template mul_t<
03326                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03327                     pow<T, factorial_t<T, i / 2>, 2
03328                 >
03329             >
03330         >>;
03331     };
03332
03333     template<typename T, size_t i>
03334     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03335         using type = typename FractionField<T>::zero;
03336     };
03337
03338     template<typename T, size_t i>
03339     struct asin_coeff {
03340         using type = typename asin_coeff_helper<T, i>::type;
03341     };
03342

```

```

03343     template<typename T, size_t i>
03344     struct lnpl_coeff {
03345         using type = makefraction_t<T,
03346             alternate_t<T, i + 1>,
03347             typename T::template val<i>;
03348     };
03349
03350     template<typename T>
03351     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03352
03353     template<typename T, size_t i, typename E = void>
03354     struct asinh_coeff_helper;
03355
03356     template<typename T, size_t i>
03357     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03358         using type = makefraction_t<T,
03359             typename T::template mul_t<
03360                 alternate_t<T, i / 2>,
03361                 factorial_t<T, i - 1>
03362             >,
03363             typename T::template mul_t<
03364                 typename T::template mul_t<
03365                     typename T::template val<i>,
03366                     pow_t<T, factorial_t<T, i / 2>, 2>
03367                 >,
03368                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03369             >
03370         >;
03371     };
03372
03373     template<typename T, size_t i>
03374     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03375         using type = typename FractionField<T>::zero;
03376     };
03377
03378     template<typename T, size_t i>
03379     struct asinh_coeff {
03380         using type = typename asinh_coeff_helper<T, i>::type;
03381     };
03382
03383     template<typename T, size_t i, typename E = void>
03384     struct atanh_coeff_helper;
03385
03386     template<typename T, size_t i>
03387     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03388         // 1/i
03389         using type = typename FractionField<T>::template val<
03390             typename T::one,
03391             typename T::template inject_constant_t<i>;
03392     };
03393
03394     template<typename T, size_t i>
03395     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03396         using type = typename FractionField<T>::zero;
03397     };
03398
03399     template<typename T, size_t i>
03400     struct atanh_coeff {
03401         using type = typename atanh_coeff_helper<T, i>::type;
03402     };
03403
03404     template<typename T, size_t i, typename E = void>
03405     struct tan_coeff_helper;
03406
03407     template<typename T, size_t i>
03408     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03409         using type = typename FractionField<T>::zero;
03410     };
03411
03412     template<typename T, size_t i>
03413     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03414     private:
03415         // 4^((i+1)/2)
03416         using _4p = typename FractionField<T>::template inject_t<
03417             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03418         // 4^((i+1)/2) - 1
03419         using _4pml = typename FractionField<T>::template
03420 sub_t<_4p, typename FractionField<T>::one>;
03421         // (-1)^((i-1)/2)
03422         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03423         using dividend = typename FractionField<T>::template mul_t<
03424             altp,
03425             FractionField<T>::template mul_t<
03426                 _4p,
03427                 FractionField<T>::template mul_t<
03428                     _4pml,
03429                     bernoulli_t<T, (i + 1)>

```

```

03429         >
03430         >
03431     >;
03432     public:
03433         using type = typename FractionField<T>::template div_t<dividend,
03434             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03435     };
03436
03437     template<typename T, size_t i>
03438     struct tan_coeff {
03439         using type = typename tan_coeff_helper<T, i>::type;
03440     };
03441
03442     template<typename T, size_t i, typename E = void>
03443     struct tanh_coeff_helper;
03444
03445     template<typename T, size_t i>
03446     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03447         using type = typename FractionField<T>::zero;
03448     };
03449
03450     template<typename T, size_t i>
03451     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03452     private:
03453         using _4p = typename FractionField<T>::template inject_t<
03454             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>>,
03455             typename FractionField<T>::template
03456 sub_t<_4p, typename FractionField<T>::one>;
03457         using dividend =
03458             typename FractionField<T>::template mul_t<
03459                 _4p,
03460                 typename FractionField<T>::template mul_t<
03461                     _4pml,
03462                     bernoulli_t<T, (i + 1)>>::type;
03463     public:
03464         using type = typename FractionField<T>::template div_t<dividend,
03465             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03466     };
03467
03468     template<typename T, size_t i>
03469     struct tanh_coeff {
03470         using type = typename tanh_coeff_helper<T, i>::type;
03471     };
03472 } // namespace internal
03473
03474 template<typename Integers, size_t deg>
03475 using exp = taylor<Integers, internal::exp_coeff, deg>;
03476
03477 template<typename Integers, size_t deg>
03478 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03479     exp<Integers, deg>,
03480     typename polynomial<FractionField<Integers>>::one>;
03481
03482 template<typename Integers, size_t deg>
03483 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03484
03485 template<typename Integers, size_t deg>
03486 using atan = taylor<Integers, internal::atan_coeff, deg>;
03487
03488 template<typename Integers, size_t deg>
03489 using sin = taylor<Integers, internal::sin_coeff, deg>;
03490
03491 template<typename Integers, size_t deg>
03492 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03493
03494 template<typename Integers, size_t deg>
03495 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03496
03497 template<typename Integers, size_t deg>
03498 using cos = taylor<Integers, internal::cos_coeff, deg>;
03499
03500 template<typename Integers, size_t deg>
03501 using geom_sum = taylor<Integers, internal::geom_coeff, deg>;
03502
03503 template<typename Integers, size_t deg>
03504 using asin = taylor<Integers, internal::asin_coeff, deg>;
03505
03506 template<typename Integers, size_t deg>
03507 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03508
03509 template<typename Integers, size_t deg>
03510 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03511
03512 template<typename Integers, size_t deg>
03513 using tan = taylor<Integers, internal::tan_coeff, deg>;
03514
03515 template<typename Integers, size_t deg>

```

```

03565     using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03566 } // namespace aerobus
03567
03568 // continued fractions
03569 namespace aerobus {
03572     template<int64_t... values>
03573     struct ContinuedFraction {};
03574
03577     template<int64_t a0>
03578     struct ContinuedFraction<a0> {
03580         using type = typename q64::template inject_constant_t<a0>;
03582         static constexpr double val = static_cast<double>(a0);
03583     };
03584
03588     template<int64_t a0, int64_t... rest>
03589     struct ContinuedFraction<a0, rest...> {
03591         using type = q64::template add_t<
03592             typename q64::template inject_constant_t<a0>,
03593             typename q64::template div_t<
03594                 typename q64::one,
03595                 typename ContinuedFraction<rest...>::type
03596             >>;
03597
03599         static constexpr double val = type::template get<double>();
03600     };
03601
03605     using PI_fraction =
03606     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03607     using E_fraction =
03608     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03609     using SQRT2_fraction =
03610     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03611     using SQRT3_fraction =
03612     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03612 } // namespace aerobus
03613
03614 // known polynomials
03615 namespace aerobus {
03616     // CChebyshev
03617     namespace internal {
03618         template<int kind, size_t deg, typename I>
03619         struct chebyshev_helper {
03620             using type = typename polynomial<I>::template sub_t<
03621                 typename polynomial<I>::template mul_t<
03622                     typename polynomial<I>::template mul_t<
03623                         typename polynomial<I>::template inject_constant_t<2>,
03624                         typename polynomial<I>::X>,
03625                         typename chebyshev_helper<kind, deg - 1, I>::type
03626                     >,
03627                     typename chebyshev_helper<kind, deg - 2, I>::type
03628                 >;
03629         };
03630
03631         template<typename I>
03632         struct chebyshev_helper<1, 0, I> {
03633             using type = typename polynomial<I>::one;
03634         };
03635
03636         template<typename I>
03637         struct chebyshev_helper<1, 1, I> {
03638             using type = typename polynomial<I>::X;
03639         };
03640
03641         template<typename I>
03642         struct chebyshev_helper<2, 0, I> {
03643             using type = typename polynomial<I>::one;
03644         };
03645
03646         template<typename I>
03647         struct chebyshev_helper<2, 1, I> {
03648             using type = typename polynomial<I>::template mul_t<
03649                 typename polynomial<I>::template inject_constant_t<2>,
03650                 typename polynomial<I>::X>;
03651         };
03652     } // namespace internal
03653
03654     // Laguerre
03655     namespace internal {
03656         template<size_t deg, typename I>
03657         struct laguerre_helper {
03658             using Q = FractionField<I>;
03659             using PQ = polynomial<Q>;
03660
03661         private:
03662             // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)Lkm2)
03663             using lnm2 = typename laguerre_helper<deg - 2, I>::type;

```

```

03664         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03665         // -x + 2k-1
03666         using p = typename PQ::template val<
03667             typename Q::template inject_constant_t<-1>,
03668             typename Q::template inject_constant_t<2 * deg - 1>;
03669         // 1/n
03670         using factor = typename PQ::template inject_ring_t<
03671             typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03672
03673     public:
03674         using type = typename PQ::template mul_t <
03675             factor,
03676             typename PQ::template sub_t<
03677                 typename PQ::template mul_t<
03678                     p,
03679                     lnm1
03680                 >,
03681                 typename PQ::template mul_t<
03682                     typename PQ::template inject_constant_t<deg-1>,
03683                     lnm2
03684                 >
03685             >
03686         >;
03687     };
03688
03689     template<typename I>
03690     struct laguerre_helper<0, I> {
03691         using type = typename polynomial<FractionField<I>::one;
03692     };
03693
03694     template<typename I>
03695     struct laguerre_helper<1, I> {
03696     private:
03697         using PQ = polynomial<FractionField<I>;
03698     public:
03699         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03700     };
03701 } // namespace internal
03702
03703 // Bernstein
03704 namespace internal {
03705     template<size_t i, size_t m, typename I, typename E = void>
03706     struct bernstein_helper {};
03707
03708     template<typename I>
03709     struct bernstein_helper<0, 0, I> {
03710         using type = typename polynomial<I>::one;
03711     };
03712
03713     template<size_t i, size_t m, typename I>
03714     struct bernstein_helper<i, m, I, std::enable_if_t<
03715         (m > 0) && (i == 0)>> {
03716     private:
03717         using P = polynomial<I>;
03718     public:
03719         using type = typename P::template mul_t<
03720             typename P::template sub_t<typename P::one, typename P::X>,
03721             typename bernstein_helper<i, m-1, I>::type>;
03722     };
03723
03724     template<size_t i, size_t m, typename I>
03725     struct bernstein_helper<i, m, I, std::enable_if_t<
03726         (m > 0) && (i == m)>> {
03727     private:
03728         using P = polynomial<I>;
03729     public:
03730         using type = typename P::template mul_t<
03731             typename P::X,
03732             typename bernstein_helper<i-1, m-1, I>::type>;
03733     };
03734
03735     template<size_t i, size_t m, typename I>
03736     struct bernstein_helper<i, m, I, std::enable_if_t<
03737         (m > 0) && (i > 0) && (i < m)>> {
03738     private:
03739         using P = polynomial<I>;
03740     public:
03741         using type = typename P::template add_t<
03742             typename P::template mul_t<
03743                 typename P::template sub_t<typename P::one, typename P::X>,
03744                 typename bernstein_helper<i, m-1, I>::type>,
03745                 typename P::template mul_t<
03746                     typename P::X,
03747                     typename bernstein_helper<i-1, m-1, I>::type>;
03748             >;
03749     };
03749 } // namespace internal

```



```

03750
03751 // AllOne polynomials
03752 namespace internal {
03753     template<size_t deg, typename I>
03754     struct AllOneHelper {
03755         using type = aerobus::add_t<
03756             typename polynomial<I>::one,
03757             typename aerobus::mul_t<
03758                 typename polynomial<I>::X,
03759                 typename AllOneHelper<deg-1, I>::type
03760             >>;
03761     };
03762
03763     template<typename I>
03764     struct AllOneHelper<0, I> {
03765         using type = typename polynomial<I>::one;
03766     };
03767 } // namespace internal
03768
03769 // Bessel polynomials
03770 namespace internal {
03771     template<size_t deg, typename I>
03772     struct BesselHelper {
03773     private:
03774         using P = polynomial<I>;
03775         using factor = typename P::template monomial_t<
03776             typename I::template inject_constant_t<(2*deg - 1)>,
03777             1>;
03778     public:
03779         using type = typename P::template add_t<
03780             typename P::template mul_t<
03781                 factor,
03782                 typename BesselHelper<deg-1, I>::type
03783             >,
03784             typename BesselHelper<deg-2, I>::type
03785         >;
03786     };
03787
03788     template<typename I>
03789     struct BesselHelper<0, I> {
03790         using type = typename polynomial<I>::one;
03791     };
03792
03793     template<typename I>
03794     struct BesselHelper<1, I> {
03795     private:
03796         using P = polynomial<I>;
03797     public:
03798         using type = typename P::template add_t<
03799             typename P::one,
03800             typename P::X
03801         >;
03802     };
03803 } // namespace internal
03804
03805 namespace known_polynomials {
03806     enum hermite_kind {
03807         probabilist,
03808         physicist
03809     };
03810 }
03811
03812 // hermite
03813 namespace internal {
03814     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03815     struct hermite_helper {};
03816
03817     template<size_t deg, typename I>
03818     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03819     private:
03820         using hnm1 = typename hermite_helper<deg - 1,
03821             known_polynomials::hermite_kind::probabilist, I>::type;
03822         using hnm2 = typename hermite_helper<deg - 2,
03823             known_polynomials::hermite_kind::probabilist, I>::type;
03824     public:
03825         using type = typename polynomial<I>::template sub_t<
03826             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03827             typename polynomial<I>::template mul_t<
03828                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03829                 hnm2
03830             >
03831         >;
03832     };
03833
03834     template<size_t deg, typename I>
03835     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {

```

```

03838     private:
03839     using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
I>::type;
03840     using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
I>::type;
03841
03842     public:
03843     using type = typename polynomial<I>::template sub_t<
03844         // 2X Hn-1
03845         typename polynomial<I>::template mul_t<
03846             typename pi64::val<typename I::template inject_constant_t<2>,
03847                 typename I::zero>, hnm1>,
03848
03849             typename polynomial<I>::template mul_t<
03850                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03851                 hnm2
03852             >
03853     >;
03854 };
03855
03856 template<typename I>
03857 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03858     using type = typename polynomial<I>::one;
03859 };
03860
03861 template<typename I>
03862 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03863     using type = typename polynomial<I>::X;
03864 };
03865
03866 template<typename I>
03867 struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03868     using type = typename pi64::one;
03869 };
03870
03871 template<typename I>
03872 struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03873     // 2X
03874     using type = typename polynomial<I>::template val<
03875         typename I::template inject_constant_t<2>,
03876         typename I::zero>;
03877 };
03878 } // namespace internal
03879
03880 // legendre
03881 namespace internal {
03882     template<size_t n, typename I>
03883     struct legendre_helper {
03884     private:
03885         using Q = FractionField<I>;
03886         using PQ = polynomial<Q>;
03887         // 1/n constant
03888         // (2n-1)/n X
03889         using fact_left = typename PQ::template monomial_t<
03890             makefraction_t<I,
03891                 typename I::template inject_constant_t<2*n-1>,
03892                 typename I::template inject_constant_t<n>
03893             >,
03894             1>;
03895         // (n-1) / n
03896         using fact_right = typename PQ::template val<
03897             makefraction_t<I,
03898                 typename I::template inject_constant_t<n-1>,
03899                 typename I::template inject_constant_t<n>>;
03900
03901     public:
03902         using type = PQ::template sub_t<
03903             typename PQ::template mul_t<
03904                 fact_left,
03905                 typename legendre_helper<n-1, I>::type
03906             >,
03907             typename PQ::template mul_t<
03908                 fact_right,
03909                 typename legendre_helper<n-2, I>::type
03910             >
03911         >;
03912 };
03913
03914 template<typename I>
03915 struct legendre_helper<0, I> {
03916     using type = typename polynomial<FractionField<I>::one;
03917 };
03918
03919 template<typename I>
03920 struct legendre_helper<1, I> {
03921     using type = typename polynomial<FractionField<I>::X;
03922 };

```

```

03923     } // namespace internal
03924
03925     // bernoulli polynomials
03926     namespace internal {
03927         template<size_t n>
03928         struct bernoulli_coeff {
03929             template<typename T, size_t i>
03930             struct inner {
03931                 private:
03932                     using F = FractionField<T>;
03933                 public:
03934                     using type = typename F::template mul_t<
03935                         typename F::template inject_ring_t<combination_t<T, i, n>,
03936                         bernoulli_t<T, n-i>
03937                     >;
03938             };
03939         };
03940     } // namespace internal
03941
03942     namespace internal {
03943         template<size_t n>
03944         struct touchard_coeff {
03945             template<typename T, size_t i>
03946             struct inner {
03947                 using type = stirling_2_t<T, n, i>;
03948             };
03949         };
03950     } // namespace internal
03951
03952     namespace internal {
03953         template<typename I = aerobus::i64>
03954         struct AbelHelper {
03955             private:
03956                 using P = aerobus::polynomial<I>;
03957             public:
03958                 // to keep recursion working, we need to operate on a*n and not just a
03959                 template<size_t deg, I::inner_type an>
03960                 struct Inner {
03961                     // abel(n, a) = (x-an) * abel(n-1, a)
03962                     using type = typename aerobus::mul_t<
03963                         typename Inner<deg-1, an>::type,
03964                         typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
03965                     >;
03966                 };
03967
03968                 // abel(0, a) = 1
03969                 template<I::inner_type an>
03970                 struct Inner<0, an> {
03971                     using type = P::one;
03972                 };
03973
03974                 // abel(1, a) = X
03975                 template<I::inner_type an>
03976                 struct Inner<1, an> {
03977                     using type = P::X;
03978                 };
03979             };
03980     };
03981 } // namespace internal
03982
03983 namespace known_polynomials {
03984     template<size_t n, auto a, typename I = aerobus::i64>
03985     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
03986
03987     template<size_t deg, typename I = aerobus::i64>
03988     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03989
03990     template<size_t deg, typename I = aerobus::i64>
03991     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03992
03993     template<size_t deg, typename I = aerobus::i64>
03994     using laguerre = typename internal::laguerre_helper<deg, I>::type;
03995
03996     template<size_t deg, typename I = aerobus::i64>
03997     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03998         I>::type;
03999
04000     template<size_t deg, typename I = aerobus::i64>
04001     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04002
04003     template<size_t i, size_t m, typename I = aerobus::i64>
04004     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04005
04006     template<size_t deg, typename I = aerobus::i64>
04007     using legendre = typename internal::legendre_helper<deg, I>::type;
04008
04009 }

```


Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

[illegible]

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

[illegible]

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

Generated by Doxygen

```

POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
06063     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
06064     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
06065     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
06066     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
06067     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
06068     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
06069     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
06070     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
06071     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
06072 #endif // DO_NOT_DOCUMENT
06073 } // namespace aerobus
06074 #endif // AEROBUS_CONWAY_IMPORTS
06075
06076 #endif // __INC_AEROBUS__ // NOLINT

```

9.4 src/examples.h File Reference

9.5 examples.h

[Go to the documentation of this file.](#)

```

00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_

```

Chapter 10

Examples

10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

10.2 examples/custom_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
#include <cstdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

/*
change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
to see how good is the generated assembly compared to what nvcc generates for 16 bits
*/
using int_type = aerobus::i16;
using float_type = __half2;

constexpr size_t N = 1 << 24;

template<typename T>
struct ExpmlDegree;

template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};

template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};

template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};

double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

template<typename T>
struct GetRandT;

template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};

template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};

template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};

using EXPM1 = aerobus::expml<int_type, ExpmlDegree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}
```

```

}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        out[i] = f(f(f(f(f(f(in[i]))))))) ;
    }
}

int main() {
    float_type *d_in, *d_out;
    cudaMalloc<float_type>(&d_in, N * sizeof(float_type));
    cudaMalloc<float_type>(&d_out, N * sizeof(float_type));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice);

    run<<128, 512>>>(N, d_in, d_out);

    cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost);

    cudaFree(d_in);
    cudaFree(d_out);
}

```

10.4 examples/continued_fractions.cpp

How to use `aerobus::ContinuedFraction` to get approximations of known numbers

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

static constexpr double PHI = aerobus::ContinuedFraction<
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1>::val;

static const double phi = (std::sqrt(5.0) + 1.0)/2.0;

int main() {
    std::cout << std::setprecision(15) << "Aerobus PHI : " << PHI << std::endl;
    std::cout << std::setprecision(15) << "Computed PHI : " << phi << std::endl;
    return 0;
}
```

10.5 examples/modular_arithmetic.cpp

How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

//  $x^3 + 2x^2 + 1$ , with coefficients in  $\mathbb{Z}/2\mathbb{Z}$ , actually  $x^3 + 1$ 
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
//  $x^3 + 5x^2 + 7x + 11$  with coefficients in  $\mathbb{Z}/17\mathbb{Z}$ , meaning actually  $x^3 + x^2 + 1$ 
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

10.6 examples/make_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

10.7 examples/polynomials_over_finite_field.cpp

How to build a known polynomial (here aerobus::known_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```

10.8 examples/compensated_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

publication

```
// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;
        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}
```


Index

abs_t
 aerobus, 20
add_t
 aerobus, 20
 aerobus::i32, 59
 aerobus::i64, 65
 aerobus::polynomial< Ring >, 74
 aerobus::Quotient< Ring, X >, 82
 aerobus::zpz< p >, 107
addfractions_t
 aerobus, 20
aerobus, 15
 abs_t, 20
 add_t, 20
 addfractions_t, 20
 aligned_malloc, 34
 alternate_t, 20
 alternate_v, 35
 asin, 21
 asinh, 21
 atan, 21
 atanh, 21
 bell_t, 23
 bernoulli_t, 23
 bernoulli_v, 35
 combination_t, 23
 combination_v, 35
 cos, 23
 cosh, 25
 div_t, 25
 E_fraction, 25
 embed_int_poly_in_fractions_t, 25
 exp, 26
 expm1, 26
 factorial_t, 26
 factorial_v, 35
 field, 34
 fpq32, 26
 fpq64, 27
 FractionField, 27
 gcd_t, 27
 geometric_sum, 27
 lnp1, 27
 make_frac_polynomial_t, 28
 make_int_polynomial_t, 28
 make_q32_t, 28
 make_q64_t, 29
 makefraction_t, 29
 mul_t, 29
 mulfractions_t, 29
 pi64, 30
 PI_fraction, 30
 pow_t, 30
 pq64, 30
 q32, 30
 q64, 31
 sin, 31
 sinh, 31
 SQRT2_fraction, 31
 SQRT3_fraction, 31
 stirling_1_signed_t, 32
 stirling_1_unsigned_t, 32
 stirling_2_t, 32
 sub_t, 33
 tan, 33
 tanh, 33
 taylor, 33
 vadd_t, 34
 vmul_t, 34
aerobus::ContinuedFraction< a0 >, 47
 type, 47
 val, 48
aerobus::ContinuedFraction< a0, rest... >, 48
 type, 49
 val, 49
aerobus::ContinuedFraction< values >, 46
aerobus::ConwayPolynomial, 49
aerobus::Embed< i32, i64 >, 51
 type, 51
aerobus::Embed< polynomial< Small >, polynomial< Large > >, 52
 type, 52
aerobus::Embed< q32, q64 >, 53
 type, 53
aerobus::Embed< Quotient< Ring, X >, Ring >, 54
 type, 54
aerobus::Embed< Ring, FractionField< Ring > >, 55
 type, 55
aerobus::Embed< Small, Large, E >, 51
aerobus::Embed< zpz< x >, i32 >, 55
 type, 56
aerobus::i32, 57
 add_t, 59
 div_t, 59
 eq_t, 59
 eq_v, 62
 gcd_t, 59
 gt_t, 60

- inject_constant_t, 60
- inject_ring_t, 60
- inner_type, 60
- is_euclidean_domain, 62
- is_field, 62
- lt_t, 60
- mod_t, 61
- mul_t, 61
- one, 61
- pos_t, 61
- pos_v, 62
- sub_t, 62
- zero, 62
- aerobus::i32::val< x >, 91
 - enclosing_type, 92
 - get, 92
 - is_zero_t, 92
 - to_string, 92
 - v, 92
- aerobus::i64, 64
 - add_t, 65
 - div_t, 66
 - eq_t, 66
 - eq_v, 69
 - gcd_t, 66
 - gt_t, 66
 - gt_v, 69
 - inject_constant_t, 67
 - inject_ring_t, 67
 - inner_type, 67
 - is_euclidean_domain, 69
 - is_field, 69
 - lt_t, 67
 - lt_v, 70
 - mod_t, 68
 - mul_t, 68
 - one, 68
 - pos_t, 68
 - pos_v, 70
 - sub_t, 68
 - zero, 69
- aerobus::i64::val< x >, 93
 - enclosing_type, 94
 - get, 94
 - inner_type, 94
 - is_zero_t, 94
 - to_string, 94
 - v, 95
- aerobus::internal, 36
 - index_sequence_reverse, 40
 - is_instantiation_of_v, 40
 - make_index_sequence_reverse, 40
 - type_at_t, 40
- aerobus::is_prime< n >, 71
 - value, 72
- aerobus::IsEuclideanDomain, 43
- aerobus::IsField, 43
- aerobus::IsRing, 44
- aerobus::known_polynomials, 40
 - hermite_kind, 40
 - physicist, 41
 - probabilist, 41
- aerobus::polynomial< Ring >, 72
 - add_t, 74
 - derive_t, 74
 - div_t, 74
 - eq_t, 75
 - gcd_t, 75
 - gt_t, 75
 - inject_constant_t, 76
 - inject_ring_t, 76
 - is_euclidean_domain, 80
 - is_field, 80
 - lt_t, 76
 - mod_t, 76
 - monomial_t, 77
 - mul_t, 77
 - one, 77
 - pos_t, 77
 - pos_v, 80
 - simplify_t, 79
 - sub_t, 79
 - X, 79
 - zero, 79
- aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >, 49
 - func, 50
- aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
 - func, 50
- aerobus::polynomial< Ring >::horner_reduction_t< P >, 56
- aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >, 70
 - type, 71
- aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >, 71
 - type, 71
- aerobus::polynomial< Ring >::val< coeffN >, 102
 - aN, 103
 - coeff_at_t, 103
 - compensated_eval, 104
 - degree, 105
 - enclosing_type, 103
 - eval, 104
 - is_zero_t, 103
 - is_zero_v, 105
 - ring_type, 104
 - strip, 104
 - to_string, 104
 - value_at_t, 104
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at<

- index, std::enable_if_t<(index < 0 || index > 0)>, 45
- type, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index == 0)>>, 46
- type, 46
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 95
- aN, 96
- coeff_at_t, 96
- compensated_eval, 97
- degree, 99
- enclosing_type, 96
- eval, 98
- is_zero_t, 97
- is_zero_v, 99
- ring_type, 97
- strip, 97
- to_string, 98
- value_at_t, 97
- aerobus::Quotient< Ring, X >, 81
- add_t, 82
- div_t, 83
- eq_t, 83
- eq_v, 85
- inject_constant_t, 83
- inject_ring_t, 83
- is_euclidean_domain, 85
- mod_t, 84
- mul_t, 84
- one, 84
- pos_t, 84
- pos_v, 85
- zero, 85
- aerobus::Quotient< Ring, X >::val< V >, 99
- raw_t, 100
- type, 100
- aerobus::type_list< Ts >, 87
- at, 88
- concat, 88
- insert, 88
- length, 89
- push_back, 88
- push_front, 89
- remove, 89
- aerobus::type_list< Ts >::pop_front, 80
- tail, 81
- type, 81
- aerobus::type_list< Ts >::split< index >, 86
- head, 86
- tail, 86
- aerobus::type_list<>, 90
- concat, 90
- insert, 90
- length, 91
- push_back, 90
- push_front, 90
- aerobus::zpz< p >, 105
- add_t, 107
- div_t, 107
- eq_t, 108
- eq_v, 111
- gcd_t, 108
- gt_t, 108
- gt_v, 111
- inject_constant_t, 109
- inner_type, 109
- is_euclidean_domain, 111
- is_field, 111
- lt_t, 109
- lt_v, 111
- mod_t, 109
- mul_t, 109
- one, 110
- pos_t, 110
- pos_v, 112
- sub_t, 110
- zero, 110
- aerobus::zpz< p >::val< x >, 100
- enclosing_type, 101
- get, 101
- is_zero_t, 101
- is_zero_v, 102
- to_string, 101
- v, 102
- aligned_malloc
- aerobus, 34
- alternate_t
- aerobus, 20
- alternate_v
- aerobus, 35
- aN
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
- asin
- aerobus, 21
- asinh
- aerobus, 21
- at
- aerobus::type_list< Ts >, 88
- atan
- aerobus, 21
- atanh
- aerobus, 21
- bell_t
- aerobus, 23
- bernoulli_t
- aerobus, 23
- bernoulli_v
- aerobus, 35
- coeff_at_t
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96

- combination_t
 - aerobus, 23
- combination_v
 - aerobus, 35
- compensated_eval
 - aerobus::polynomial< Ring >::val< coeffN >, 104
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
- concat
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- cos
 - aerobus, 23
- cosh
 - aerobus, 25
- degree
 - aerobus::polynomial< Ring >::val< coeffN >, 105
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
- derive_t
 - aerobus::polynomial< Ring >, 74
- div_t
 - aerobus, 25
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 74
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 107
- E_fraction
 - aerobus, 25
- embed_int_poly_in_fractions_t
 - aerobus, 25
- enclosing_type
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::polynomial< Ring >::val< coeffN >, 103
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
 - aerobus::zpz< p >::val< x >, 101
- eq_t
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 108
- eq_v
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 111
- eval
 - aerobus::polynomial< Ring >::val< coeffN >, 104
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 98
- exp
 - aerobus, 26
- expm1
 - aerobus, 26
- factorial_t
 - aerobus, 26
- factorial_v
 - aerobus, 35
- field
 - aerobus, 34
- fpq32
 - aerobus, 26
- fpq64
 - aerobus, 27
- FractionField
 - aerobus, 27
- func
 - aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >, 50
 - aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
- gcd_t
 - aerobus, 27
 - aerobus::i32, 59
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::zpz< p >, 108
- geometric_sum
 - aerobus, 27
- get
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::zpz< p >::val< x >, 101
- gt_t
 - aerobus::i32, 60
 - aerobus::i64, 66
 - aerobus::polynomial< Ring >, 75
 - aerobus::zpz< p >, 108
- gt_v
 - aerobus::i64, 69
 - aerobus::zpz< p >, 111
- head
 - aerobus::type_list< Ts >::split< index >, 86
- hermite_kind
 - aerobus::known_polynomials, 40
- index_sequence_reverse
 - aerobus::internal, 40
- inject_constant_t
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::polynomial< Ring >, 76
 - aerobus::Quotient< Ring, X >, 83
 - aerobus::zpz< p >, 109
- inject_ring_t
 - aerobus::i32, 60
 - aerobus::i64, 67

- aerobus::polynomial< Ring >, 76
- aerobus::Quotient< Ring, X >, 83
- inner_type
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::i64::val< x >, 94
 - aerobus::zpz< p >, 109
- insert
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- Introduction, 1
- is_euclidean_domain
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::polynomial< Ring >, 80
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 111
- is_field
 - aerobus::i32, 62
 - aerobus::i64, 69
 - aerobus::polynomial< Ring >, 80
 - aerobus::zpz< p >, 111
- is_instantiation_of_v
 - aerobus::internal, 40
- is_zero_t
 - aerobus::i32::val< x >, 92
 - aerobus::i64::val< x >, 94
 - aerobus::polynomial< Ring >::val< coeffN >, 103
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
 - aerobus::zpz< p >::val< x >, 101
- is_zero_v
 - aerobus::polynomial< Ring >::val< coeffN >, 105
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
 - aerobus::zpz< p >::val< x >, 102
- length
 - aerobus::type_list< Ts >, 89
 - aerobus::type_list<>, 91
- Inp1
 - aerobus, 27
- lt_t
 - aerobus::i32, 60
 - aerobus::i64, 67
 - aerobus::polynomial< Ring >, 76
 - aerobus::zpz< p >, 109
- lt_v
 - aerobus::i64, 70
 - aerobus::zpz< p >, 111
- make_frac_polynomial_t
 - aerobus, 28
- make_index_sequence_reverse
 - aerobus::internal, 40
- make_int_polynomial_t
 - aerobus, 28
- make_q32_t
 - aerobus, 28
- make_q64_t
 - aerobus, 29
- makefraction_t
 - aerobus, 29
- mod_t
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 76
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 109
- monomial_t
 - aerobus::polynomial< Ring >, 77
- mul_t
 - aerobus, 29
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 109
- mulfractions_t
 - aerobus, 29
- one
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 110
- physicist
 - aerobus::known_polynomials, 41
- pi64
 - aerobus, 30
- PI_fraction
 - aerobus, 30
- pos_t
 - aerobus::i32, 61
 - aerobus::i64, 68
 - aerobus::polynomial< Ring >, 77
 - aerobus::Quotient< Ring, X >, 84
 - aerobus::zpz< p >, 110
- pos_v
 - aerobus::i32, 62
 - aerobus::i64, 70
 - aerobus::polynomial< Ring >, 80
 - aerobus::Quotient< Ring, X >, 85
 - aerobus::zpz< p >, 112
- pow_t
 - aerobus, 30
- pq64
 - aerobus, 30
- probabilist
 - aerobus::known_polynomials, 41
- push_back
 - aerobus::type_list< Ts >, 88
 - aerobus::type_list<>, 90
- push_front
 - aerobus::type_list< Ts >, 89
 - aerobus::type_list<>, 90

- q32
 - aerobus, [30](#)
- q64
 - aerobus, [31](#)
- raw_t
 - aerobus::Quotient< Ring, X >::val< V >, [100](#)
- README.md, [113](#)
- remove
 - aerobus::type_list< Ts >, [89](#)
- ring_type
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- simplify_t
 - aerobus::polynomial< Ring >, [79](#)
- sin
 - aerobus, [31](#)
- sinh
 - aerobus, [31](#)
- SQRT2_fraction
 - aerobus, [31](#)
- SQRT3_fraction
 - aerobus, [31](#)
- src/aerobus.h, [113](#)
- src/examples.h, [206](#)
- stirling_1_signed_t
 - aerobus, [32](#)
- stirling_1_unsigned_t
 - aerobus, [32](#)
- stirling_2_t
 - aerobus, [32](#)
- strip
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- sub_t
 - aerobus, [33](#)
 - aerobus::i32, [62](#)
 - aerobus::i64, [68](#)
 - aerobus::polynomial< Ring >, [79](#)
 - aerobus::zpz< p >, [110](#)
- tail
 - aerobus::type_list< Ts >::pop_front, [81](#)
 - aerobus::type_list< Ts >::split< index >, [86](#)
- tan
 - aerobus, [33](#)
- tanh
 - aerobus, [33](#)
- taylor
 - aerobus, [33](#)
- to_string
 - aerobus::i32::val< x >, [92](#)
 - aerobus::i64::val< x >, [94](#)
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [98](#)
- aerobus::zpz< p >::val< x >, [101](#)
- type
 - aerobus::ContinuedFraction< a0 >, [47](#)
 - aerobus::ContinuedFraction< a0, rest... >, [49](#)
 - aerobus::Embed< i32, i64 >, [51](#)
 - aerobus::Embed< polynomial< Small >, polynomial< Large > >, [52](#)
 - aerobus::Embed< q32, q64 >, [53](#)
 - aerobus::Embed< Quotient< Ring, X >, Ring >, [54](#)
 - aerobus::Embed< Ring, FractionField< Ring > >, [55](#)
 - aerobus::Embed< zpz< x >, i32 >, [56](#)
 - aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >, [71](#)
 - aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >, [71](#)
 - aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 || index > 0)> >, [45](#)
 - aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >, [46](#)
 - aerobus::Quotient< Ring, X >::val< V >, [100](#)
 - aerobus::type_list< Ts >::pop_front, [81](#)
- type_at_t
 - aerobus::internal, [40](#)
- v
 - aerobus::i32::val< x >, [92](#)
 - aerobus::i64::val< x >, [95](#)
 - aerobus::zpz< p >::val< x >, [102](#)
- vadd_t
 - aerobus, [34](#)
- val
 - aerobus::ContinuedFraction< a0 >, [48](#)
 - aerobus::ContinuedFraction< a0, rest... >, [49](#)
- value
 - aerobus::is_prime< n >, [72](#)
- value_at_t
 - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
 - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- vmul_t
 - aerobus, [34](#)
- X
 - aerobus::polynomial< Ring >, [79](#)
- zero
 - aerobus::i32, [62](#)
 - aerobus::i64, [69](#)
 - aerobus::polynomial< Ring >, [79](#)
 - aerobus::Quotient< Ring, X >, [85](#)
 - aerobus::zpz< p >, [110](#)