

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.



For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of  $ZmZ$ .

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , Aerobus provides automatic implementation for **quotient ring**  $R/X$  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

Aerobus gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

## 1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, Aerobus provides some support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

<<<<<< HEAD More, it's (at this time), not easily possible to make it work for \_\_half2 because of another bug.

A workaround is to modify `cuda_fp16.h` and add a `constexpr` modifier to line 5039. This works but only tested on Linux with CUDA 16.1.

Once done, `nvcc` generates splendid assembly, same as for `double` or `float`:

HFMA2.MMA	R5,	R6,	RZ,	0.0013885498046875,	0.0013885498046875 ;
HFMA2	R5,	R6,	R5,	0.008331298828125,	0.008331298828125 ;
HFMA2.MMA	R5,	R6,	R5,	0.041656494140625,	0.041656494140625 ;
HFMA2	R5,	R6,	R5,	0.1666259765625,	0.1666259765625 ;
HFMA2.MMA	R5,	R6,	R5,	0.5,	0.5 ;
HFMA2	R5,	R6,	R5,	1,	1 ;
HFMA2.MMA	R5,	R6,	R5,	RZ ;	
HFMA2	R7,	R5,	RZ.H0_H0,	0.0013885498046875,	0.0013885498046875 ;
HFMA2.MMA	R7,	R5,	R7,	0.008331298828125,	0.008331298828125 ;
HFMA2	R7,	R5,	R7,	0.041656494140625,	0.041656494140625 ;
HFMA2.MMA	R7,	R5,	R7,	0.1666259765625,	0.1666259765625 ;
HFMA2	R7,	R5,	R7,	0.5,	0.5 ;
HFMA2.MMA	R7,	R5,	R7,	1,	1 ;
HFMA2	R7,	R5,	RZ.H0_H0 ;		

More, it's (at this time), not easy to make it work for `__half2` because of [another bug](#).

One workaround is to add `constexpr` modifier on line 5039 of file `cuda_fp16.h`. Once done, ``examples/fp16.cu"` compiles and generates proper assembly.

<blockquote><blockquote><blockquote><blockquote><blockquote>  
</blockquote></blockquote></blockquote></blockquote></blockquote></blockquote></blockquote>

Please push to make these bug fixed by NVIDIA. <https://zenodo.org/badge/latestdoi/499577459>  
DOI



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

<a href="#">aerobus</a>	Main namespace for all publicly exposed types or functions . . . . .	17
<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	33
<a href="#">aerobus::known_polynomials</a>	Families of well known polynomials such as Hermite or Bernstein . . . . .	37



## Chapter 3

# Concept Index

### 3.1 Concepts

Here is a list of all concepts with brief descriptions:

<a href="#">aerobus::IsEuclideanDomain</a>	
Concept to express R is an euclidean domain . . . . .	39
<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	39
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	39





# Chapter 4

## Class Index

### 4.1 Class List

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<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt;</a>	41
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt;</a>	41
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<a href="#">aerobus::ContinuedFraction&lt; a0 &gt;</a>	
Specialization for only one coefficient, technically just 'a0'	42
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<a href="#">aerobus::Embed&lt; Quotient&lt; Ring, X &gt;, Ring &gt;</a>	
Embeds Quotient<Ring, X> into Ring	47
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# Chapter 5

## File Index

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# Chapter 6

## Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial< Small > into polynomial< Large >*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient< Ring, X > into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*
- struct [Embed< zpz< x >, i32 >](#)  
*embeds zpz values into i32*
- struct [i32](#)  
*32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)  
*64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)

- *checks if  $n$  is prime*
- struct [polynomial](#)
- struct [Quotient](#)  
*[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)  
*Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)  
*specialization for empty type list*
- struct [zpz](#)  
*congruence classes of integers modulo  $p$  (32 bits)*

## Concepts

- concept [IsRing](#)  
*Concept to express  $R$  is a Ring.*
- concept [IsEuclideanDomain](#)  
*Concept to express  $R$  is an euclidean domain.*
- concept [IsField](#)  
*Concept to express  $R$  is a field.*

## Typedefs

- template<typename T , typename A , typename B >  
using [gcd\\_t](#) = typename internal::gcd< T >::template type< A, B >  
*computes the greatest common divisor of A and B*
- template<typename... vals>  
using [vadd\\_t](#) = typename internal::vadd< vals... >::type  
*adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an [add\\_t](#) binary operator*
- template<typename... vals>  
using [vmul\\_t](#) = typename internal::vmul< vals... >::type  
*multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an [mul\\_t](#) binary operator*
- template<typename val >  
using [abs\\_t](#) = std::conditional\_t< val::enclosing\_type::template pos\_v< val >, val, typename val::enclosing\_type::template [sub\\_t](#)< typename val::enclosing\_type::zero, val > >  
*computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >  
using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type  
*Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .*
- template<typename X , typename Y >  
using [add\\_t](#) = typename X::enclosing\_type::template [add\\_t](#)< X, Y >  
*generic addition*
- template<typename X , typename Y >  
using [sub\\_t](#) = typename X::enclosing\_type::template [sub\\_t](#)< X, Y >  
*generic subtraction*
- template<typename X , typename Y >  
using [mul\\_t](#) = typename X::enclosing\_type::template [mul\\_t](#)< X, Y >  
*generic multiplication*
- template<typename X , typename Y >  
using [div\\_t](#) = typename X::enclosing\_type::template [div\\_t](#)< X, Y >  
*generic division*
- using [q32](#) = [FractionField](#)< [i32](#) >  
*32 bits rationals rationals with 32 bits numerator and denominator*

- using `fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- using `q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
- using `pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
- using `pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
- using `fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
- template<typename Ring , typename v1 , typename v2 >  
using `makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
- template<typename v >  
using `embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
- template<int64\_t p, int64\_t q>  
using `make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- template<int32\_t p, int32\_t q>  
using `make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- template<typename Ring , typename v1 , typename v2 >  
using `addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- template<typename Ring , typename v1 , typename v2 >  
using `mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- template<typename Ring , auto... xs>  
using `make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
- template<typename Ring , auto... xs>  
using `make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField< Ring>*
- template<typename T , size\_t i>  
using `factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*
- template<typename T , size\_t k, size\_t n>  
using `combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- template<typename T , size\_t n>  
using `bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- template<typename T , size\_t n>  
using `bell_t = typename internal::bell_helper< T, n >::type`  
*Bell numbers.*

- `template<typename T , int k>`  
using `alternate_t` = `typename internal::alternate< T, k >::type`  
 $(-1)^k$  as type in  $T$
- `template<typename T , int n, int k>`  
using `stirling_1_signed_t` = `typename internal::stirling_1_helper< T, n, k >::type`  
*Stirling number of first king (signed) – as types.*
- `template<typename T , int n, int k>`  
using `stirling_1_unsigned_t` = `abs_t< typename internal::stirling_1_helper< T, n, k >::type >`  
*Stirling number of first king (unsigned) – as types.*
- `template<typename T , int n, int k>`  
using `stirling_2_t` = `typename internal::stirling_2_helper< T, n, k >::type`  
*Stirling number of second king – as types.*
- `template<typename T , typename p , size_t n>`  
using `pow_t` = `typename internal::pow< T, p, n >::type`  
 $p^n$  (as 'val' type in  $T$ )
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`  
using `taylor` = `typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers , size_t deg>`  
using `exp` = `taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers , size_t deg>`  
using `expm1` = `typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers , size_t deg>`  
using `lnp1` = `taylor< Integers, internal::lnp1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`  
using `atan` = `taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers , size_t deg>`  
using `sin` = `taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers , size_t deg>`  
using `sinh` = `taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers , size_t deg>`  
using `cosh` = `taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers , size_t deg>`  
using `cos` = `taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*
- `template<typename Integers , size_t deg>`  
using `geometric_sum` = `taylor< Integers, internal::geom_coeff, deg >`  
 $\frac{1}{1-x}$  *zero development of  $\frac{1}{1-x}$*
- `template<typename Integers , size_t deg>`  
using `asin` = `taylor< Integers, internal::asin_coeff, deg >`  
 $\arcsin(x)$  *arc sinus*
- `template<typename Integers , size_t deg>`  
using `asinh` = `taylor< Integers, internal::asinh_coeff, deg >`  
 $\operatorname{arcsinh}(x)$  *arc hyperbolic sinus*
- `template<typename Integers , size_t deg>`  
using `atanh` = `taylor< Integers, internal::atanh_coeff, deg >`



- `arctanh(x)` *arc hyperbolic tangent*
- `template<typename Integers , size_t deg>`  
using `tan = Taylor< Integers, internal::tan_coeff, deg >`  
`tan(x)` *tangent*
- `template<typename Integers , size_t deg>`  
using `tanh = Taylor< Integers, internal::tanh_coeff, deg >`  
`tanh(x)` *hyperbolic tangent*
- using `PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 1 >`
- using `E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`  
*approximation of e*
- using `SQRT2_fraction = ContinuedFraction< 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2 >`  
*approximation of  $\sqrt{2}$*
- using `SQRT3_fraction = ContinuedFraction< 1, 1, 2 >`  
*approximation of  $\sqrt{3}$*

## Functions

- `template<typename T >`  
`T * aligned\_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the [field](#) (prime number) @tparam n degree of extension  
`template< int p`

## Variables

- `template<typename T , size_t i>`  
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`  
*computes factorial(i) as value in T*
- `template<typename T , size_t k, size_t n>`  
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`  
*computes binomial coefficients (k among n) as value*
- `template<typename FloatType , typename T , size_t n>`  
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`  
*nth bernoulli number as value in FloatType*
- `template<typename T , size_t k>`  
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`  
 *$(-1)^k$  as value from T*

### 6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

### 6.1.2 Typedef Documentation

#### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept
```

## Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
generic addition
```

#### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
helper type : adds two fractions
```

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
(-1)^k as type in T
```

#### Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example
----------	-------------------------------------

### 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
arcsin( $x$ ) arc sinus
```

#### Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

### 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
arcsinh( $x$ ) arc hyperbolic sinus
```

#### Template Parameters

<i>Integers</i>	Ring type (for example i64)
-----------------	-----------------------------

## Template Parameters

<i>deg</i>	taylor approximation degree
------------	-----------------------------

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
arctan( $x$ )
```

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
arctanh( $x$ ) arc hyperbolic tangent
```

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
Bell numbers.
```

## Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

## 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
nth bernoulli number as type in T
```

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

## 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
```

```
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
computes binomial coefficient (k among n) as type
```

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

#### 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
cos( $x$ ) cosinus
```

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
cosh( $x$ ) hyperbolic cosine
```

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
generic division
```

#### Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

#### 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
approximation of  $e$ 
```

#### 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
embed a polynomial with integers coefficients into rational coefficients polynomials
Lives in polynomial<FractionField<Ring>>
```

## Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

## 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
 $e^x$ 
```

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub\_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
 $e^x - 1$ 
```

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
computes factorial(i), as type
```

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
```

## 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
polynomial with 64 bits rational coefficients
```

## 6.1.2.22 FractionField

```
template<typename Ring >
```

using `aerobus::FractionField` = typedef typename internal::FractionFieldImpl<Ring>::type  
 Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	
-------------	--

#### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
computes the greatest common divisor of A and B
```

#### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

#### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
 $\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$ 
```

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.25 lnp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
 $\ln(1+x)$ 
```

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >↔
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
make a polynomial with coefficients in FractionField<Ring>
```

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
make a polynomial with coefficients in Ring
```

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
helper type : make a fraction from numerator and denominator
```

#### Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

### 6.1.2.29 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
helper type : make a fraction from numerator and denominator
```

#### Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

### 6.1.2.30 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
helper type : the rational V1/V2 in the field of fractions of Ring
```

#### Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

### 6.1.2.31 mul\_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
generic multiplication
```

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

## 6.1.2.32 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
helper type : multiplies two fractions
```

## Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

## 6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
polynomial with 64 bits integers coefficients
```

## 6.1.2.34 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
representation of  $\pi$  as a continued fraction
```

## 6.1.2.35 pow\_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
p^n (as 'val' type in T)
```

## Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

## 6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
polynomial with 64 bits rationals coefficients
```

## 6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
32 bits rationals rationals with 32 bits numerator and denominator
```

## 6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
64 bits rationals rationals with 64 bits numerator and denominator
```



**6.1.2.39 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
sin( $x$ )
```

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.40 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
sinh( $x$ )
```

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.41 SQRT2\_fraction**

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
approximation of  $\sqrt{2}$ 
```

**6.1.2.42 SQRT3\_fraction**

```
using aerobus::SQRT3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1,
2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
approximation of
```

**6.1.2.43 stirling\_1\_signed\_t**

```
template<typename T , int n, int k>
using aerobus::stirling_1_signed_t = typedef typename internal::stirling_1_helper<T, n, k>←
::type
Stirling number of first kind (signed) – as types.
```

**Template Parameters**

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

**6.1.2.44 stirling\_1\_unsigned\_t**

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T,
n, k>::type>
Stirling number of first kind (unsigned) – as types.
```

## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.45 `stirling_2_t`

```
template<typename T , int n, int k>
using aerobus::stirling\_2\_t = typedef typename internal::stirling_2_helper<T, n, k>::type
Stirling number of second kind – as types.
```

## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.46 `sub_t`

```
template<typename X , typename Y >
using aerobus::sub\_t = typedef typename X::enclosing_type::template sub\_t<X, Y>
generic subtraction
```

## Template Parameters

<i>X</i>	a value in a ring providing <code>sub_t</code> operator
<i>Y</i>	a value in same ring

6.1.2.47 `tan`

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
tan(x) tangent
```

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

6.1.2.48 `tanh`

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
tanh(x) hyperbolic tangent
```

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.49 `taylor`

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

#### Template Parameters

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff_at</i>	- implementation giving the 'value' (seen as type in <code>FractionField&lt;T&gt;</code> )
<i>deg</i>	

### 6.1.2.50 `vadd_t`

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have an
add_t binary operator
```

#### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.2.51 `vmul_t`

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
multiplies multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have
an mul_t binary operator
```

#### Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

### 6.1.3.1 `aligned_malloc()`

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
'portable' aligned allocation of count elements of type T
```

#### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

#### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

### 6.1.3.2 field()

brief Conway polynomials tparam p characteristic of the aerobus::field (  
prime number )

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
(-1)^k as value from T
```

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<Float↵
Type> [inline], [constexpr]
nth bernoulli number as value in FloatType
```

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
computes binomial coefficients (k among n) as value
```

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
computes factorial(i) as value in T
```

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i64</a> for example)
----------	---

## Template Parameters

<i>i</i>	
----------	--

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, l >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >

- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i == 0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i == m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k > (n/2) && k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k <= (n/2) && k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >
- struct **fma\_helper**
- struct **fma\_helper**< double >
- struct **fma\_helper**< float >
- struct **fma\_helper**< int16\_t >
- struct **fma\_helper**< int32\_t >
- struct **fma\_helper**< int64\_t >
- struct **fma\_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t<Field::is\_field> >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field> >
- struct **gcd**
  - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable\_if\_t<Ring::is\_euclidean\_domain> >
- struct **geom\_coeff**

- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **Split**
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling\_1\_helper**
- struct **stirling\_1\_helper**< T, 0, 0 >
- struct **stirling\_1\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0)> >
- struct **stirling\_2\_helper**
- struct **stirling\_2\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling\_2\_helper**< T, n, n, std::enable\_if\_t<(n >=0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**

- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **touchard\_coeff**
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N >{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

## 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

## 6.2.2 Typedef Documentation

### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{}) [constexpr]
```



## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Enumeration Type Documentation

#### 6.3.2.1 hermite\_kind

```
enum aerobus::known\_polynomials::hermite\_kind
```

#### Enumerator

<a href="#">probabilist</a>	
<a href="#">physicist</a>	



# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

#### 7.2.2 Detailed Description

Concept to express R is a field.

### 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename Ring::zero

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = `aN`

### 8.3.1 Member Typedef Documentation

#### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

#### Template Parameters

<i>...values</i>	are int64_t
------------------	----------------

#### Examples

[examples/continued\\_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

#### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
*represented value as [aerobus::q64](#)*

#### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
*represented value as double*

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

## Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

## 8.5.2 Member Typedef Documentation

### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
represented value as double
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using **type** = q64::template **add\_t**< typename q64::template inject\_constant\_t< a0 >, typename q64::template **div\_t**< typename q64::one, typename **ContinuedFraction**< rest... >::type > >  
represented value as aerobus::q64

### Static Public Attributes

- static constexpr double **val** = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

## Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
represented value as aerobus::q64
```

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
represented value as double
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

### 8.8.1 Member Function Documentation

#### 8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P
>::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## 8.9 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

### 8.9.1 Member Function Documentation

#### 8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

#### Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = i64::val< static_cast< int64_t >(val::v)>`  
*the i64 representation of val*

### 8.11.1 Detailed Description

embeds i32 into i64

### 8.11.2 Member Typedef Documentation

#### 8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
the i64 representation of val
```

#### Template Parameters

<i>val</i>	a value in i32
------------	----------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>  
`#include <aerobus.h>`

## Public Types

- `template<typename v >`  
`using type = typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`  
*the polynomial<Large> representation of v*

### 8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

#### Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

### 8.12.2 Member Typedef Documentation

#### 8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
```

```
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
the polynomial<Large> representation of v
```

#### Template Parameters

<code>v</code>	a value in polynomial<Small>
----------------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

#### Public Types

- `template<typename v >`  
using `type` = `make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`  
*q64 representation of v*

### 8.13.1 Detailed Description

embeds q32 into q64

### 8.13.2 Member Typedef Documentation

#### 8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_↵
cast<int64_t>(v::y::v)>
q64 representation of v
```

#### Template Parameters

<code>v</code>	a value in q32
----------------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

#### Public Types

- `template<typename val >`  
using `type` = `typename val::raw_t`  
*Ring representation of val.*

### 8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
Ring representation of val.
```

#### Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions  
`#include <aerobus.h>`

### Public Types

- template<typename v >  
using [type](#) = typename [FractionField](#)< Ring >::template val< v, typename Ring::one >  
*FractionField<Ring> representation of v.*

### 8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

<i>Ring</i>	an integers ring, such as <a href="#">i32</a>
-------------	---

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↵
::template val<v, typename Ring::one>
FractionField<Ring> representation of v.
```

#### Template Parameters

<code>v</code>	a Ring value
----------------	--------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)  
`#include <aerobus.h>`

### Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the [i32](#) representation of val*

### 8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into [i32](#)

#### Template Parameters

<code>x</code>	an integer
----------------	------------

## 8.16.2 Member Typedef Documentation

### 8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
the i32 representation of val
```

#### Template Parameters

<code>val</code>	a value in zpz<x>
------------------	-------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.17 aerobus::polynomial< Ring >::horner\_reduction\_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

### Classes

- struct [inner](#)
- struct [inner< stop, stop >](#)

### 8.17.1 Detailed Description

```
template<typename Ring>
```

```
template<typename P>
```

```
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

#### Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in i32, again represented as types*

### Public Types

- using [inner\\_type](#) = int32\_t
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*inject a native constant*
- template<typename v >  
using [inject\\_ring\\_t](#) = v
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator yields v1 + v2*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction operator yields v1 - v2*

- `template<typename v1 , typename v2 >`  
`using mul\_t = typename mul< v1, v2 >::type`  
*multiplication operator yields  $v1 * v2$*
- `template<typename v1 , typename v2 >`  
`using div\_t = typename div< v1, v2 >::type`  
*division operator yields  $v1 / v2$*
- `template<typename v1 , typename v2 >`  
`using mod\_t = typename remainder< v1, v2 >::type`  
*modulus operator yields  $v1 \% v2$*
- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator ( $v1 < v2$ ) yields  $v1 < v2$*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i32, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator (boolean value)*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity (boolean value) yields  $v > 0$  as boolean value*

### 8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

#### Examples

[examples/compensated\\_horner.cpp](#).

### 8.18.2 Member Typedef Documentation

#### 8.18.2.1 [add\\_t](#)

`template<typename v1 , typename v2 >`  
`using aerobus::i32::add\_t = typename add<v1, v2>::type`  
 addition operator yields  $v1 + v2$

#### Template Parameters

<a href="#">v1</a>	a value in <a href="#">i32</a>
<a href="#">v2</a>	a value in <a href="#">i32</a>

### 8.18.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
division operator yields v1 / v2
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
equality operator (type) yields v1 == v2 as std::integral_constant<bool>
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
greatest common divisor yields GCD(v1, v2)
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
strictly greater operator (v1 > v2) yields v1 > v2
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
inject a native constant
```

#### Template Parameters

<i>x</i>	
----------	--



### 8.18.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

### 8.18.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

### 8.18.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
strict less operator (v1 < v2) yields v1 < v2
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
modulus operator yields v1 % v2
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
multiplication operator yields v1 * v2
```

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.12 one

```
using aerobus::i32::one = val<1>
constant one
```

### 8.18.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
positivity operator yields v > 0 as std::true_type or std::false_type
```

#### Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

#### 8.18.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
subtraction operator yields v1 - v2
```

##### Template Parameters

v1	a value in <a href="#">i32</a>
v2	a value in <a href="#">i32</a>

#### 8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
constant zero
```

### 8.18.3 Member Data Documentation

#### 8.18.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
equality operator (boolean value)
```

##### Template Parameters

v1	
v2	

#### 8.18.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
integers are an euclidean domain
```

#### 8.18.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
integers are not a field
```

#### 8.18.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
positivity (boolean value) yields v > 0 as boolean value
```

##### Template Parameters

v	a value in <a href="#">i32</a>
---	--------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations  

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i64](#)*

## Public Types

- using [inner\\_type](#) = int64\_t  
*type of represented values*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int64\_t >(x)>  
*injects constant as an [i64](#) value*
- template<typename v >  
using [inject\\_ring\\_t](#) = v  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> [i64::val<1>](#)*
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = typename mul< v1, v2 >::type  
*multiplication operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = typename div< v1, v2 >::type  
*division operator integer division*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = typename remainder< v1, v2 >::type  
*modulus operator*
- template<typename v1 , typename v2 >  
using [gt\\_t](#) = typename gt< v1, v2 >::type  
*strictly greater operator yields v1 > v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [lt\\_t](#) = typename lt< v1, v2 >::type  
*strict less operator yields v1 < v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename eq< v1, v2 >::type  
*equality operator yields v1 == v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [gcd\\_t](#) = [gcd\\_t](#)< [i64](#), v1, v2 >  
*greatest common divisor yields GCD(v1, v2) as instantiation of [i64::val](#)*
- template<typename v >  
using [pos\\_t](#) = typename pos< v >::type  
*is v positive yields v > 0 as std::true\_type or std::false\_type*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- template<typename v1 , typename v2 >  
static constexpr bool [gt\\_v](#) = [gt\\_t](#)<v1, v2>::value  
*strictly greater operator yields v1 > v2 as boolean value*
- template<typename v1 , typename v2 >  
static constexpr bool [lt\\_v](#) = [lt\\_t](#)<v1, v2>::value  
*strictly smaller operator yields v1 < v2 as boolean value*
- template<typename v1 , typename v2 >  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator yields v1 == v2 as boolean value*
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity yields v > 0 as boolean value*

### 8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
addition operator
```

#### Template Parameters

<a href="#">v1</a>	: an element of <a href="#">aerobus::i64::val</a>
<a href="#">v2</a>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.2 [div\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::div\_t = typename div<v1, v2>::type
division operator integer division
```

#### Template Parameters

<a href="#">v1</a>	: an element of <a href="#">aerobus::i64::val</a>
<a href="#">v2</a>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.3 [eq\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::eq\_t = typename eq<v1, v2>::type
equality operator yields v1 == v2 as std::true_type or std::false_type
```

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
greatest common divisor yields GCD(v1, v2) as instantiation of i64::val
```

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
strictly greater operator yields v1 > v2 as std::true_type or std::false_type
```

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
injects constant as an i64 value
```

## Template Parameters

<code>x</code>	
----------------	--

## 8.19.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
injects a value used for internal consistency and quotient rings implementations for example i64::inject\_ring\_t<i64::val<1>>
-> i64::val<1>
```

## Template Parameters

<code>v</code>	a value in <a href="#">i64</a>
----------------	--------------------------------

## 8.19.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
type of represented values
```

### 8.19.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
strict less operator yields v1 < v2 as std::true_type or std::false_type
```

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
modulus operator
```

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
multiplication operator
```

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.12 one

```
using aerobus::i64::one = val<1>
constant one
```

### 8.19.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
is v positive yields v > 0 as std::true_type or std::false_type
```

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
-----------------	---

### 8.19.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
subtraction operator
```

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.15 zero**

using [aerobus::i64::zero](#) = `val<0>`  
 constant zero

**8.19.3 Member Data Documentation****8.19.3.1 eq\_v**

template<typename v1 , typename v2 >  
 constexpr bool aerobus::i64::eq\_v = [eq\\_t](#)<v1, v2>::value [static], [constexpr]  
 equality operator yields `v1 == v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.3.2 gt\_v**

template<typename v1 , typename v2 >  
 constexpr bool aerobus::i64::gt\_v = [gt\\_t](#)<v1, v2>::value [static], [constexpr]  
 strictly greater operator yields `v1 > v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.3.3 is\_euclidean\_domain**

constexpr bool aerobus::i64::is\_euclidean\_domain = true [static], [constexpr]  
 integers are an euclidean domain

**8.19.3.4 is\_field**

constexpr bool aerobus::i64::is\_field = false [static], [constexpr]  
 integers are not a field

**8.19.3.5 lt\_v**

template<typename v1 , typename v2 >  
 constexpr bool aerobus::i64::lt\_v = [lt\\_t](#)<v1, v2>::value [static], [constexpr]  
 strictly smaller operator yields `v1 < v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
positivity yields v > 0 as boolean value
```

#### Template Parameters

<code>v</code>	: an element of <code>aerobus::i64::val</code>
----------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.20 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
`using type = typename horner_reduction_t< P >::template inner< index+1, stop > ::template type< type-`  
`name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↵`  
`at_t< P::degree - index > >, x >`

### 8.20.1 Member Typedef Documentation

#### 8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_↵
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.21 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
`using type = accum`

### 8.21.1 Member Typedef Documentation

#### 8.21.1.1 type

```
template<typename Ring >
template<typename P >
```



```
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.22 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool [value](#) = internal::\_is\_prime<n, 5>::value  
*true iff n is prime*

### 8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

<i>n</i>	
----------	--

### 8.22.2 Member Data Documentation

#### 8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
true iff n is prime
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [horner\\_reduction\\_t](#)  
*Used to evaluate polynomials over a value in Ring.*
- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

## Public Types

- using `zero` = `val`< typename Ring::zero >  
*constant zero*
- using `one` = `val`< typename Ring::one >  
*constant one*
- using `X` = `val`< typename Ring::one, typename Ring::zero >  
*generator*
- template<typename P >  
using `simplify_t` = typename simplify< P >::type  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- template<typename v1, typename v2 >  
using `add_t` = typename add< v1, v2 >::type  
*adds two polynomials*
- template<typename v1, typename v2 >  
using `sub_t` = typename sub< v1, v2 >::type  
*subtraction of two polynomials*
- template<typename v1, typename v2 >  
using `mul_t` = typename mul< v1, v2 >::type  
*multiplication of two polynomials*
- template<typename v1, typename v2 >  
using `eq_t` = typename eq\_helper< v1, v2 >::type  
*equality operator*
- template<typename v1, typename v2 >  
using `lt_t` = typename lt\_helper< v1, v2 >::type  
*strict less operator*
- template<typename v1, typename v2 >  
using `gt_t` = typename gt\_helper< v1, v2 >::type  
*strict greater operator*
- template<typename v1, typename v2 >  
using `div_t` = typename div< v1, v2 >::q\_type  
*division operator*
- template<typename v1, typename v2 >  
using `mod_t` = typename div\_helper< v1, v2, `zero`, v1 >::mod\_type  
*modulo operator*
- template<typename coeff, size\_t deg>  
using `monomial_t` = typename monomial< coeff, deg >::type  
*monomial : coeff X<sup>deg</sup>*
- template<typename v >  
using `derive_t` = typename derive\_helper< v >::type  
*derivation operator*
- template<typename v >  
using `pos_t` = typename Ring::template pos\_t< typename v::aN >  
*checks for positivity (an > 0)*
- template<typename v1, typename v2 >  
using `gcd_t` = std::conditional\_t< Ring::is\_euclidean\_domain, typename make\_unit< gcd\_t< polynomial< Ring >, v1, v2 > >::type, void >  
*greatest common divisor of two polynomials*
- template<auto x>  
using `inject_constant_t` = `val`< typename Ring::template inject\_constant\_t< x > >  
*makes the constant (native type) polynomial a\_0*
- template<typename v >  
using `inject_ring_t` = `val`< v >  
*makes the constant (ring type) polynomial a\_0*

### Static Public Attributes

- static constexpr bool [is\\_field](#) = false
- static constexpr bool [is\\_euclidean\\_domain](#) = Ring::is\_euclidean\_domain
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator*

## 8.23.1 Detailed Description

**template**<typename Ring>  
**requires** IsEuclideanDomain<Ring>  
**struct** aerobus::polynomial< Ring >

polynomial with coefficients in Ring Ring must be an integral domain

### Examples

[examples/compensated\\_horner.cpp](#), [examples/make\\_polynomial.cpp](#), and [examples/modular\\_arithmetic.cpp](#).

## 8.23.2 Member Typedef Documentation

### 8.23.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
adds two polynomials
```

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
derivation operator
```

#### Template Parameters

<i>v</i>	
----------	--

### 8.23.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
division operator
```

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

**8.23.2.4 eq\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
equality operator
```

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.5 gcd\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
greatest common divisor of two polynomials
```

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.6 gt\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
strict greater operator
```

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.7 inject\_constant\_t**

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
makes the constant (native type) polynomial a_0
```

**Template Parameters**

<i>x</i>	
----------	--

**8.23.2.8 inject\_ring\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

#### Template Parameters

<i>v</i>	
----------	--

#### 8.23.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
strict less operator
```

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.10 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
modulo operator
```

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
monomial : coeff X^deg
```

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

#### 8.23.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
multiplication of two polynomials
```

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

**8.23.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
constant one
```

**8.23.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
checks for positivity (an > 0)
```

**Template Parameters**

<i>v</i>	
----------	--

**8.23.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
```

**Template Parameters**

<i>P</i>	
----------	--

**8.23.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
subtraction of two polynomials
```

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
generator
```

**8.23.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
constant zero
```

### 8.23.3 Member Data Documentation

#### 8.23.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.23.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.23.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
positivity operator
```

#### Template Parameters

<i>v</i>	a value in <code>polynomial::val</code>
----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.24 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- using `type` = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using `tail` = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

### 8.24.1 Detailed Description

```
template<typename... Ts>
```

```
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

### 8.24.2 Member Typedef Documentation

#### 8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
remaining types in parent list when front is removed
```

#### 8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*zero value*
- using [one](#) = [val](#)< typename Ring::one >  
*one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = [val](#)< typename Ring::template [add\\_t](#)< typename v1::type, typename v2::type > >  
*addition operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = [val](#)< typename Ring::template [mul\\_t](#)< typename v1::type, typename v2::type > >  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = [val](#)< typename Ring::template [div\\_t](#)< typename v1::type, typename v2::type > >  
*division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = [val](#)< typename Ring::template [mod\\_t](#)< typename v1::type, typename v2::type > >  
*modulus operator*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename Ring::template [eq\\_t](#)< typename v1::type, typename v2::type >  
*equality operator (as type)*
- template<typename v1 >  
using [pos\\_t](#) = std::true\_type  
*positivity operator always true*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< typename Ring::template [inject\\_constant\\_t](#)< x > >  
*inject a 'constant' in quotient ring\**
- template<typename v >  
using [inject\\_ring\\_t](#) = [val](#)< v >  
*projects a value of Ring onto the quotient*

### Static Public Attributes

- template<typename v1 , typename v2 >  
static constexpr bool [eq\\_v](#) = Ring::template [eq\\_t](#)<typename v1::type, typename v2::type>::value  
*addition operator (as boolean value)*
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator always true*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*quotien rings are euclidean domain*



### 8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as ' <a href="#">i32</a> ', must satisfy the <code>IsRing</code> concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 `add_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1↵
::type, typename v2::type> >
addition operator
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 8.25.2.2 `div_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↵
::type, typename v2::type> >
division operator
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 8.25.2.3 `eq_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
equality operator (as type)
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 8.25.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
inject a 'constant' in quotient ring*
```

##### Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

#### 8.25.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
projects a value of Ring onto the quotient
```

##### Template Parameters

v	a value in Ring
---	-----------------

#### 8.25.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
modulus operator
```

##### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

#### 8.25.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
substraction operator
```

##### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

#### 8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.25.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
positivity operator always true
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
zero value
```

## 8.25.3 Member Data Documentation

### 8.25.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
addition operator (as boolean value)
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
quotien rings are euclidean domain
```

### 8.25.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
positivity operator always true
```

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

### Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.26.1 Detailed Description

```
template<typename... Ts>
```

```
template<size_t index>
```

```
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.26.2 Member Typedef Documentation

#### 8.26.2.1 head

```
template<typename... Ts>
```

```
template<size_t index>
```

```
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.26.2.2 tail

```
template<typename... Ts>
```

```
template<size_t index>
```

```
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.27 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*

- `template<size_t index>`  
`using at = internal::type_at_t< index, Ts... >`  
*returns type at index*
- `template<typename T >`  
`using push_back = type_list< Ts..., T >`  
*pushes T at the tail of the list*
- `template<typename U >`  
`using concat = typename concat_h< U >::type`  
*concatenates two list into one*
- `template<typename T , size_t index>`  
`using insert = typename internal::insert_h< index, type_list< Ts... >, T >::type`  
*inserts type at index*
- `template<size_t index>`  
`using remove = typename internal::remove_h< index, type_list< Ts... > >::type`  
*removes type at index*

### Static Public Attributes

- `static constexpr size_t length = sizeof...(Ts)`  
*length of list*

## 8.27.1 Detailed Description

**template<typename... Ts>**  
**struct aerobus::type\_list< Ts >**

Empty pure template struct to handle type list.  
A list of types.

### Template Parameters

<code>...Ts</code>	types to store and manipulate at compile time
--------------------	---

## 8.27.2 Member Typedef Documentation

### 8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
returns type at index
```

### Template Parameters

<code>index</code>	
--------------------	--

### 8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
concatenates two list into one
```

### Template Parameters

<code>U</code>	
----------------	--

### 8.27.2.3 insert

```
template<typename... Ts>
template<typename T, size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
inserts type at index
```

#### Template Parameters

<i>index</i>	
<i>T</i>	

### 8.27.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
pushes T at the tail of the list
```

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.5 push\_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
Adds T to front of the list.
```

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
removes type at index
```

#### Template Parameters

<i>index</i>	
--------------	--

## 8.27.3 Member Data Documentation

### 8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
length of list
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.28 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t [length](#) = 0

### 8.28.1 Detailed Description

specialization for empty type list

### 8.28.2 Member Typedef Documentation

#### 8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

#### 8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

#### 8.28.2.3 push\_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

#### 8.28.2.4 push\_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

### 8.28.3 Member Data Documentation

#### 8.28.3.1 length

```
constexpr size_t aerobus::type\_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [i32](#)  
*Enclosing ring type.*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [DEVICE](#) valueType [get](#) ()  
*cast x into valueType*
- static std::string [to\\_string](#) ()  
*string representation of value*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x  
*actual value stored in val type*

### 8.29.1 Detailed Description

```
template<int32_t x>
```

```
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

#### Template Parameters

<i>x</i>	an actual integer
----------	-------------------

### 8.29.2 Member Typedef Documentation

#### 8.29.2.1 enclosing\_type

```
template<int32_t x>
```

```
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

#### 8.29.2.2 is\_zero\_t

```
template<int32_t x>
```

```
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.29.3 Member Function Documentation

#### 8.29.3.1 get()

```
template<int32_t x>
```

```
template<typename valueType >
```

```
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```



cast x into valueType

## Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

## 8.29.3.2 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
string representation of value
```

## 8.29.4 Member Data Documentation

## 8.29.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
actual value stored in val type
The documentation for this struct was generated from the following file:
```

- src/[aerobus.h](#)

## 8.30 aerobus::i64::val&lt; x &gt; Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

## Public Types

- using [inner\\_type](#) = int32\_t  
*type of represented values*
- using [enclosing\\_type](#) = [i64](#)  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

## Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED DEVICE](#) valueType [get](#) ()  
*cast value in valueType*
- static std::string [to\\_string](#) ()  
*string representation*

## Static Public Attributes

- static constexpr int64\_t [v](#) = x  
*actual value*

## 8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.30.2 Member Typedef Documentation

### 8.30.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
enclosing ring type
```

### 8.30.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
type of represented values
```

### 8.30.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
is value zero
```

## 8.30.3 Member Function Documentation

### 8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
cast value in valueType
```

## Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.30.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
string representation
```

## 8.30.4 Member Data Documentation

### 8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
actual value
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

### Public Types

- using `ring_type` = Ring  
*ring coefficients live in*
- using `enclosing_type` = polynomial< Ring >  
*enclosing ring type*
- using `aN` = coeffN  
*heavy weight coefficient (non zero)*
- using `strip` = val< coeffs... >  
*remove largest coefficient*
- using `is_zero_t` = std::bool\_constant<(degree==0) &&(aN::is\_zero\_t::value)>  
*true\_type if polynomial is constant zero*
- template<size\_t index>  
using `coeff_at_t` = typename coeff\_at< index >::type  
*type of coefficient at index*
- template<typename x >  
using `value_at_t` = horner\_reduction\_t< val > ::template inner< 0, degree+1 > ::template type< typename Ring::zero, x >

### Static Public Member Functions

- static std::string `to_string` ()  
*get a string representation of polynomial*
- template<typename arithmeticType >  
static constexpr **DEVICE INLINED** arithmeticType `eval` (const arithmeticType &x)  
*evaluates polynomial seen as a function operating on arithmeticType*
- template<typename arithmeticType >  
static **DEVICE INLINED** arithmeticType `compensated_eval` (const arithmeticType &x)  
*Evaluate polynomial on x using compensated horner scheme.*

### Static Public Attributes

- static constexpr size\_t `degree` = sizeof...(coeffs)  
*degree of the polynomial*
- static constexpr bool `is_zero_v` = is\_zero\_t::value  
*true if polynomial is constant zero*

#### 8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.31.2 Member Typedef Documentation

### 8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
heavy weight coefficient (non zero)
```

### 8.31.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
type of coefficient at index
```

#### Template Parameters

<i>index</i>	
--------------	--

### 8.31.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
enclosing ring type
```

### 8.31.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
true_type if polynomial is constant zero
```

### 8.31.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
ring coefficients live in
```

### 8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
remove largest coefficient
```

### 8.31.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

### 8.31.3 Member Function Documentation

#### 8.31.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

##### Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

##### Parameters

<i>x</i>	
----------	--

#### 8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

##### Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

##### Parameters

<i>x</i>	value
----------	-------

##### Returns

$P(x)$

#### 8.31.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

### 8.31.4 Member Data Documentation

#### 8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
degree of the polynomial
```

#### 8.31.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
true if polynomial is constant zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

#### Public Types

- using [raw\\_t](#) = V
- using [type](#) = [abs\\_t](#)< typename Ring::template [mod\\_t](#)< V, X > >

#### 8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

<a href="#">V</a>	a value from 'Ring'
-------------------	---------------------

### 8.32.2 Member Typedef Documentation

#### 8.32.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [zpz](#)< p >  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< [v](#)==0 >  
*true\_type if zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED DEVICE](#) valueType [get](#) ()  
*get value as valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x % p  
*actual value*
- static constexpr bool [is\\_zero\\_v](#) = [v](#) == 0  
*true if zero*

### 8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

#### Template Parameters

<a href="#">x</a>	an integer
-------------------	------------

## 8.33.2 Member Typedef Documentation

### 8.33.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
enclosing ring type
```



### 8.33.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
true_type if zero
```

## 8.33.3 Member Function Documentation

### 8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
get value as valueType
```

#### Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

### 8.33.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
string representation
```

#### Returns

a string representation

## 8.33.4 Member Data Documentation

### 8.33.4.1 is\_zero\_v

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
true if zero
```

### 8.33.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
actual value
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants  
#include <aerobus.h>

## Classes

- struct `coeff_at`
- struct `coeff_at< index, std::enable_if_t<(index< 0||index > 0)>> >`
- struct `coeff_at< index, std::enable_if_t<(index==0)>> >`

## Public Types

- using `ring_type` = Ring  
*ring coefficients live in*
- using `enclosing_type` = `polynomial< Ring >`  
*enclosing ring type*
- using `aN` = `coeffN`
- using `strip` = `val< coeffN >`
- using `is_zero_t` = `std::bool_constant< aN::is_zero_t::value >`
- template<size\_t index>  
using `coeff_at_t` = `typename coeff_at< index >::type`
- template<typename x >  
using `value_at_t` = `coeffN`

## Static Public Member Functions

- static `std::string to_string ()`
- template<typename arithmeticType >  
static constexpr `DEVICE INLINED` arithmeticType `eval` (const arithmeticType &x)
- template<typename arithmeticType >  
static `DEVICE INLINED` arithmeticType `compensated_eval` (const arithmeticType &x)

## Static Public Attributes

- static constexpr size\_t `degree` = 0  
*degree*
- static constexpr bool `is_zero_v` = `is_zero_t::value`

### 8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

#### Template Parameters

<code>coeffN</code>	
---------------------	--

### 8.34.2 Member Typedef Documentation

#### 8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.34.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.34.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
enclosing ring type
```

### 8.34.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

### 8.34.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
ring coefficients live in
```

### 8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

### 8.34.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

## 8.34.3 Member Function Documentation

### 8.34.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated↵
_eval (
    const arithmeticType & x ) [inline], [static]
```

### 8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
```

```
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >↵
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

### 8.34.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

## 8.34.4 Member Data Documentation

### 8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
degree
```

### 8.34.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in zpz*

### Public Types

- using [inner\\_type](#) = int32\_t  
*underlying type for values*
- template<auto x>  
using [inject\\_constant\\_t](#) = val< static\_cast< int32\_t >(x)>  
*injects a constant integer into zpz*
- using [zero](#) = val< 0 >  
*zero value*
- using [one](#) = val< 1 >  
*one value*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction operator*

- `template<typename v1 , typename v2 >`  
`using mul\_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
`using div\_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod\_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos\_t = typename pos< v1 >::type`  
*positivity operator (type)*

### Static Public Attributes

- `static constexpr bool is\_field = is\_prime<p>::value`  
*true iff p is prime*
- `static constexpr bool is\_euclidean\_domain = true`  
*always true*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt\_v = gt\_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt\_v = lt\_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity operator (boolean value)*

### 8.35.1 Detailed Description

`template<int32_t p>`

`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

### Template Parameters

<i>p</i>	a integer
----------	-----------

### Examples

[examples/modular\\_arithmetic.cpp](#), and [examples/polynomials\\_over\\_finite\\_field.cpp](#).

## 8.35.2 Member Typedef Documentation

### 8.35.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
addition operator
```

### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
division operator
```

### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
equality operator (type)
```

### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
greatest common divisor
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.35.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
strictly greater operator (type)
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.35.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
injects a constant integer into zpz
```

## Template Parameters

<i>x</i>	an integer
----------	------------

## 8.35.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
underlying type for values
```

## 8.35.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
strictly smaller operator (type)
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.35.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
modulo operator
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.35.2.10 mul\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
multiplication operator
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.35.2.11 one**

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
one value
```

**8.35.2.12 pos\_t**

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
positivity operator (type)
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

**8.35.2.13 sub\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
subtraction operator
```

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.35.2.14 zero**

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
zero value
```



### 8.35.3 Member Data Documentation

#### 8.35.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
equality operator (booleanvalue)
```

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.35.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
strictly greater operator (booleanvalue)
```

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.35.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
always true
```

#### 8.35.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
true iff p is prime
```

#### 8.35.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
strictly smaller operator (booleanvalue)
```

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.35.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <stdint>
#include <stddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <stdint>
00006 #include <stddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
```

```

00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static consteval DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x >> 16U) & 0x8000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477fefffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7fffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;
00128             }
00129             return static_cast<uint16_t>(result);
00130         }
00131
00132         static consteval DEVICE __half my_float2half_rn(const float a) {
00133             __half val;
00134             __half_raw r;
00135             uint32_t sign = 0U;
00136             uint32_t remainder = 0U;
00137             r.x = my_internal_float2half(a, sign, remainder);

```

```

00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template <int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template <typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template <typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template <int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template <int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174
00175     } // namespace internal
00176 } // namespace aerobus
00177 #endif
00178
00179 // cast
00180 namespace aerobus {
00181     namespace internal {
00182         template<typename Out, typename In>
00183         struct staticcast {
00184             template<auto x>
00185             static constexpr INLINED_DEVICE Out func() {
00186                 return static_cast<Out>(x);
00187             }
00188         };
00189
00190         #ifdef WITH_CUDA_FP16
00191         template<>
00192         struct staticcast<__half, int16_t> {
00193             template<int16_t x>
00194             static constexpr INLINED_DEVICE __half func() {
00195                 return int16_convert_helper<__half, x>::value();
00196             }
00197         };
00198
00199         template<>
00200         struct staticcast<__half2, int16_t> {
00201             template<int16_t x>
00202             static constexpr INLINED_DEVICE __half2 func() {
00203                 return int16_convert_helper<__half2, x>::value();
00204             }
00205         };
00206         #endif
00207     } // namespace internal
00208 } // namespace aerobus
00209
00210 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00211 namespace aerobus {
00212     namespace internal {
00213         template<typename T>
00214         struct fma_helper;
00215
00216         template<>
00217         struct fma_helper<double> {
00218             static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
00219 z) {
00219                 return x * y + z;
00220             }
00221         };
00222
00223     template<>

```

```

00224     struct fma_helper<long double> {
00225         static constexpr INLINED_DEVICE long double eval(
00226             const long double x, const long double y, const long double z) {
00227             return x * y + z;
00228         }
00229     };
00230
00231     template<>
00232     struct fma_helper<float> {
00233         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00234             return x * y + z;
00235         }
00236     };
00237
00238     template<>
00239     struct fma_helper<int32_t> {
00240         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
00241             int16_t z) {
00242             return x * y + z;
00243         }
00244     };
00245     template<>
00246     struct fma_helper<int16_t> {
00247         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
00248             int32_t z) {
00249             return x * y + z;
00250         }
00251     };
00252     template<>
00253     struct fma_helper<int64_t> {
00254         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
00255             int64_t z) {
00256             return x * y + z;
00257         }
00258     };
00259     #ifdef WITH_CUDA_FP16
00260     template<>
00261     struct fma_helper<__half> {
00262         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
00263             z) {
00264             #ifdef __CUDA_ARCH__
00265                 return __hfma(x, y, z);
00266             #else
00267                 return x * y + z;
00268             #endif
00269         };
00270     template<>
00271     struct fma_helper<__half2> {
00272         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
00273             __half2 z) {
00274             #ifdef __CUDA_ARCH__
00275                 return __hfma2(x, y, z);
00276             #else
00277                 return x * y + z;
00278             #endif
00279         };
00280     #endif
00281     } // namespace internal
00282 } // namespace aerobus
00283
00284 // compensated horner utilities
00285 namespace aerobus {
00286     namespace internal {
00287         template <typename T>
00288         struct FloatLayout;
00289
00290         #ifdef _MSC_VER
00291         template <>
00292         struct FloatLayout<long double> {
00293             static constexpr uint8_t exponent = 11;
00294             static constexpr uint8_t mantissa = 53;
00295             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00296         };
00297         #else
00298         template <>
00299         struct FloatLayout<long double> {
00300             static constexpr uint8_t exponent = 15;
00301             static constexpr uint8_t mantissa = 63;
00302             static constexpr uint8_t r = 32; // ceil(mantissa/2)
00303             static constexpr long double shift = (1LL « r) + 1;
00304         };
00305         #endif

```

```

00306
00307     template <>
00308     struct FloatLayout<double> {
00309         static constexpr uint8_t exponent = 11;
00310         static constexpr uint8_t mantissa = 53;
00311         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00312         static constexpr double shift = (1LL « r) + 1;
00313     };
00314
00315     template <>
00316     struct FloatLayout<float> {
00317         static constexpr uint8_t exponent = 8;
00318         static constexpr uint8_t mantissa = 24;
00319         static constexpr uint8_t r = 11; // ceil(mantissa/2)
00320         static constexpr float shift = (1 « r) + 1;
00321     };
00322
00323     template<typename T>
00324     struct Split {
00325         static constexpr INLINED_DEVICE void func(T a, T *x, T *y) {
00326             T z = a * FloatLayout<T>::shift;
00327             *x = z - (z - a);
00328             *y = a - *x;
00329         }
00330     };
00331
00332     #ifdef WITH_CUDA_FP16
00333     template<>
00334     struct Split<__half> {
00335         static constexpr INLINED_DEVICE void func(__half a, __half *x, __half *y) {
00336             __half z = a * __half_raw(0x5280); // TODO(JeWaVe): check this value
00337             *x = z - (z - a);
00338             *y = a - *x;
00339         }
00340     };
00341
00342     template<>
00343     struct Split<__half2> {
00344         static constexpr INLINED_DEVICE void func(__half2 a, __half2 *x, __half2 *y) {
00345             __half2 z = a * __half2(__half_raw(0x5280), __half_raw(0x5280)); // TODO(JeWaVe):
00346             check this value
00347             *x = z - (z - a);
00348             *y = a - *x;
00349         }
00350     };
00351     #endif
00352
00353     template<typename T>
00354     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00355         *x = a + b;
00356         T z = *x - a;
00357         *y = (a - (*x - z)) + (b - z);
00358     }
00359
00360     template<typename T>
00361     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00362         *x = a * b;
00363         #ifdef __clang__
00364             *y = fma_helper<T>::eval(a, b, -*x);
00365         #else
00366             T ah, al, bh, bl;
00367             Split<T>::func(a, &ah, &al);
00368             Split<T>::func(b, &bh, &bl);
00369             *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00370         #endif
00371     }
00372
00373     template<typename T, size_t N>
00374     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00375         T r = p1[0] + p2[0];
00376         for (int64_t i = N - 1; i >= 0; --i) {
00377             r = r * x + p1[N - i] + p2[N - i];
00378         }
00379         return r;
00380     }
00381 } // namespace internal
00382 } // namespace aerobus
00383
00384 // utilities
00385 namespace aerobus {
00386     namespace internal {
00387         template<typename...> typename TT, typename T>
00388         struct is_instantiation_of : std::false_type { };
00389
00390         template<template<typename...> typename TT, typename... Ts>
00391         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };

```

```

00392
00393     template<template<typename...> typename TT, typename T>
00394     inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00395
00396     template<int64_t i, typename T, typename... Ts>
00397     struct type_at {
00398         static_assert(i < sizeof...(Ts) + 1, "index out of range");
00399         using type = typename type_at<i - 1, Ts...>::type;
00400     };
00401
00402     template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00403         using type = T;
00404     };
00405
00406     template<size_t i, typename... Ts>
00407     using type_at_t = typename type_at<i, Ts...>::type;
00408
00409
00410     template<size_t n, size_t i, typename E = void>
00411     struct _is_prime {};
00412
00413     template<size_t i>
00414     struct _is_prime<0, i> {
00415         static constexpr bool value = false;
00416     };
00417
00418     template<size_t i>
00419     struct _is_prime<1, i> {
00420         static constexpr bool value = false;
00421     };
00422
00423     template<size_t i>
00424     struct _is_prime<2, i> {
00425         static constexpr bool value = true;
00426     };
00427
00428     template<size_t i>
00429     struct _is_prime<3, i> {
00430         static constexpr bool value = true;
00431     };
00432
00433     template<size_t i>
00434     struct _is_prime<5, i> {
00435         static constexpr bool value = true;
00436     };
00437
00438     template<size_t i>
00439     struct _is_prime<7, i> {
00440         static constexpr bool value = true;
00441     };
00442
00443     template<size_t n, size_t i>
00444     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00445         static constexpr bool value = false;
00446     };
00447
00448     template<size_t n, size_t i>
00449     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00450         static constexpr bool value = false;
00451     };
00452
00453     template<size_t n, size_t i>
00454     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00455         static constexpr bool value = true;
00456     };
00457
00458     template<size_t n, size_t i>
00459     struct _is_prime<n, i, std::enable_if_t<(
00460         n % i == 0 &&
00461         n >= 9 &&
00462         n % 3 != 0 &&
00463         n % 2 != 0 &&
00464         i * i > n)> {
00465         static constexpr bool value = true;
00466     };
00467
00468     template<size_t n, size_t i>
00469     struct _is_prime<n, i, std::enable_if_t<(
00470         n % (i+2) == 0 &&
00471         n >= 9 &&
00472         n % 3 != 0 &&
00473         n % 2 != 0 &&
00474         i * i <= n)> {
00475         static constexpr bool value = true;
00476     };
00477
00478     template<size_t n, size_t i>

```



```

00479     struct _is_prime<n, i, std::enable_if_t<(
00480         n % (i+2) != 0 &&
00481         n % i != 0 &&
00482         n >= 9 &&
00483         n % 3 != 0 &&
00484         n % 2 != 0 &&
00485         (i * i <= n))> {
00486         static constexpr bool value = _is_prime<n, i+6>::value;
00487     };
00488 } // namespace internal
00489
00492 template<size_t n>
00493 struct is_prime {
00494     static constexpr bool value = internal::_is_prime<n, 5>::value;
00495 };
00496
00497 template<size_t n>
00500 static constexpr bool is_prime_v = is_prime<n>::value;
00501
00502 // gcd
00503 namespace internal {
00504     template<std::size_t... Is>
00505     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00506         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00507
00508     template<std::size_t N>
00509     using make_index_sequence_reverse
00510         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00511
00512     template<typename Ring, typename E = void>
00513     struct gcd;
00514
00515     template<typename Ring>
00516     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00517         template<typename A, typename B, typename E = void>
00518         struct gcd_helper {};
00519
00520         // B = 0, A > 0
00521         template<typename A, typename B>
00522         struct gcd_helper<A, B, std::enable_if_t<
00523             ((B::is_zero_t::value) &&
00524              (Ring::template gt_t<A, typename Ring::zero>::value))>> {
00525             using type = A;
00526         };
00527
00528         // B = 0, A < 0
00529         template<typename A, typename B>
00530         struct gcd_helper<A, B, std::enable_if_t<
00531             ((B::is_zero_t::value) &&
00532              !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00533             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00534         };
00535
00536         // B != 0
00537         template<typename A, typename B>
00538         struct gcd_helper<A, B, std::enable_if_t<
00539             (!B::is_zero_t::value)
00540             >> {
00541             private: // NOLINT
00542                 // A / B
00543                 using k = typename Ring::template div_t<A, B>;
00544                 // A - (A/B)*B = A % B
00545                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00546
00547             public:
00548                 using type = typename gcd_helper<B, m>::type;
00549         };
00550
00551         template<typename A, typename B>
00552         using type = typename gcd_helper<A, B>::type;
00553     };
00554 } // namespace internal
00555
00556 // vadd and vmul
00557 namespace internal {
00558     template<typename... vals>
00559     struct vmul {};
00560
00561     template<typename v1, typename... vals>
00562     struct vmul<v1, vals...> {
00563         using type = typename v1::enclosing_type::template mul_t<v1, typename
00564             vmul<vals...>::type>;
00565     };
00566
00567     template<typename v1>
00568     struct vmul<v1> {
00569         using type = v1;
00570     };

```

```

00576     };
00577
00578     template<typename... vals>
00579     struct vadd {};
00580
00581     template<typename v1, typename... vals>
00582     struct vadd<v1, vals...> {
00583         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00584     };
00585
00586     template<typename v1>
00587     struct vadd<v1> {
00588         using type = v1;
00589     };
00590 } // namespace internal
00591
00592 template<typename T, typename A, typename B>
00593 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00594
00595 template<typename... vals>
00596 using vadd_t = typename internal::vadd<vals...>::type;
00597
00598 template<typename... vals>
00599 using vmul_t = typename internal::vmul<vals...>::type;
00600
00601 template<typename val>
00602 requires IsEuclideanDomain<typename val::enclosing_type>
00603 using abs_t = std::conditional_t<
00604     val::enclosing_type::template pos_v<val>,
00605     val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00606 } // namespace aerobus
00607
00608 // embedding
00609 namespace aerobus {
00610     template<typename Small, typename Large, typename E = void>
00611     struct Embed;
00612 } // namespace aerobus
00613
00614 namespace aerobus {
00615     template<typename Ring, typename X>
00616     requires IsRing<Ring>
00617     struct Quotient {
00618         template <typename V>
00619         struct val {
00620             public:
00621                 using raw_t = V;
00622                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00623         };
00624
00625         using zero = val<typename Ring::zero>;
00626
00627         using one = val<typename Ring::one>;
00628
00629         template<typename v1, typename v2>
00630         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00631
00632         template<typename v1, typename v2>
00633         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00634
00635         template<typename v1, typename v2>
00636         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00637
00638         template<typename v1, typename v2>
00639         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00640
00641         template<typename v1, typename v2>
00642         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00643
00644         template<typename v1, typename v2>
00645         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00646
00647         template<typename v1>
00648         using pos_t = std::true_type;
00649
00650         template<typename v>
00651         static constexpr bool pos_v = pos_t<v>::value;
00652
00653         static constexpr bool is_euclidean_domain = true;
00654
00655         template<auto x>
00656         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00657
00658         template<typename v>
00659         using inject_ring_t = val<v>;
00660     };

```

```

00715
00719     template<typename Ring, typename X>
00720     struct Embed<Quotient<Ring, X>, Ring> {
00723         template<typename val>
00724         using type = typename val::raw_t;
00725     };
00726 } // namespace aerobus
00727
00728 // type_list
00729 namespace aerobus {
00731     template <typename... Ts>
00732     struct type_list;
00733
00734     namespace internal {
00735         template <typename T, typename... Us>
00736         struct pop_front_h {
00737             using tail = type_list<Us...>;
00738             using head = T;
00739         };
00740
00741         template <size_t index, typename L1, typename L2>
00742         struct split_h {
00743             private:
00744                 static_assert(index <= L2::length, "index ouf of bounds");
00745                 using a = typename L2::pop_front::type;
00746                 using b = typename L2::pop_front::tail;
00747                 using c = typename L1::template push_back<a>;
00748
00749             public:
00750                 using head = typename split_h<index - 1, c, b>::head;
00751                 using tail = typename split_h<index - 1, c, b>::tail;
00752         };
00753
00754         template <typename L1, typename L2>
00755         struct split_h<0, L1, L2> {
00756             using head = L1;
00757             using tail = L2;
00758         };
00759
00760         template <size_t index, typename L, typename T>
00761         struct insert_h {
00762             static_assert(index <= L::length, "index ouf of bounds");
00763             using s = typename L::template split<index>;
00764             using left = typename s::head;
00765             using right = typename s::tail;
00766             using ll = typename left::template push_back<T>;
00767             using type = typename ll::template concat<right>;
00768         };
00769
00770         template <size_t index, typename L>
00771         struct remove_h {
00772             using s = typename L::template split<index>;
00773             using left = typename s::head;
00774             using right = typename s::tail;
00775             using rr = typename right::pop_front::tail;
00776             using type = typename left::template concat<rr>;
00777         };
00778     } // namespace internal
00779
00782     template <typename... Ts>
00783     struct type_list {
00784     private:
00785         template <typename T>
00786         struct concat_h;
00787
00788         template <typename... Us>
00789         struct concat_h<type_list<Us...>> {
00790             using type = type_list<Ts..., Us...>;
00791         };
00792
00793     public:
00795         static constexpr size_t length = sizeof...(Ts);
00796
00799         template <typename T>
00800         using push_front = type_list<T, Ts...>;
00801
00804         template <size_t index>
00805         using at = internal::type_at_t<index, Ts...>;
00806
00808         struct pop_front {
00810             using type = typename internal::pop_front_h<Ts...>::head;
00812             using tail = typename internal::pop_front_h<Ts...>::tail;
00813         };
00814
00817         template <typename T>
00818         using push_back = type_list<Ts..., T>;
00819

```

```

00822     template <typename U>
00823     using concat = typename concat_h<U>::type;
00824
00827     template <size_t index>
00828     struct split {
00829     private:
00830         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00831
00832     public:
00833         using head = typename inner::head;
00834         using tail = typename inner::tail;
00835     };
00836
00840     template <typename T, size_t index>
00841     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00842
00845     template <size_t index>
00846     using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00847 };
00848
00850 template <>
00851 struct type_list<> {
00852     static constexpr size_t length = 0;
00853
00854     template <typename T>
00855     using push_front = type_list<T>;
00856
00857     template <typename T>
00858     using push_back = type_list<T>;
00859
00860     template <typename U>
00861     using concat = U;
00862
00863     // TODO(jewave): assert index == 0
00864     template <typename T, size_t index>
00865     using insert = type_list<T>;
00866 };
00867 } // namespace aerobus
00868
00869 // i16
00870 #ifdef WITH_CUDA_FP16
00871 // i16
00872 namespace aerobus {
00873     struct i16 {
00874         using inner_type = int16_t;
00875         template<int16_t x>
00876         struct val {
00877             using enclosing_type = i16;
00878             static constexpr int16_t v = x;
00879
00880             template<typename valueType>
00881             static constexpr INLINED_DEVICE valueType get() {
00882                 return internal::template int16_convert_helper<valueType, x>::value();
00883             }
00884
00885             using is_zero_t = std::bool_constant<x == 0>;
00886
00887             static std::string to_string() {
00888                 return std::to_string(x);
00889             }
00890         };
00891
00892         using is_zero_t = std::bool_constant<x == 0>;
00893
00894         static std::string to_string() {
00895             return std::to_string(x);
00896         }
00897     };
00898
00899     using zero = val<0>;
00900     using one = val<1>;
00901     static constexpr bool is_field = false;
00902     static constexpr bool is_euclidean_domain = true;
00903     template<auto x>
00904     using inject_constant_t = val<static_cast<int16_t>(x)>;
00905
00906     template<typename v>
00907     using inject_ring_t = v;
00908
00909 private:
00910     template<typename v1, typename v2>
00911     struct add {
00912         using type = val<v1::v + v2::v>;
00913     };
00914
00915     template<typename v1, typename v2>
00916     struct sub {
00917         using type = val<v1::v - v2::v>;
00918     };
00919
00920     template<typename v1, typename v2>
00921     struct mul {
00922         using type = val<v1::v * v2::v>;
00923     };
00924 }

```

```

00932
00933     template<typename v1, typename v2>
00934     struct div {
00935         using type = val<v1::v / v2::v>;
00936     };
00937
00938     template<typename v1, typename v2>
00939     struct remainder {
00940         using type = val<v1::v % v2::v>;
00941     };
00942
00943     template<typename v1, typename v2>
00944     struct gt {
00945         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00946     };
00947
00948     template<typename v1, typename v2>
00949     struct lt {
00950         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00951     };
00952
00953     template<typename v1, typename v2>
00954     struct eq {
00955         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00956     };
00957
00958     template<typename v1>
00959     struct pos {
00960         using type = std::bool_constant<(v1::v > 0)>;
00961     };
00962
00963     public:
00964         template<typename v1, typename v2>
00965         using add_t = typename add<v1, v2>::type;
00966
00967         template<typename v1, typename v2>
00968         using sub_t = typename sub<v1, v2>::type;
00969
00970         template<typename v1, typename v2>
00971         using mul_t = typename mul<v1, v2>::type;
00972
00973         template<typename v1, typename v2>
00974         using div_t = typename div<v1, v2>::type;
00975
00976         template<typename v1, typename v2>
00977         using mod_t = typename remainder<v1, v2>::type;
00978
00979         template<typename v1, typename v2>
00980         using gt_t = typename gt<v1, v2>::type;
00981
00982         template<typename v1, typename v2>
00983         using lt_t = typename lt<v1, v2>::type;
00984
00985         template<typename v1, typename v2>
00986         using eq_t = typename eq<v1, v2>::type;
00987
00988         template<typename v1, typename v2>
00989         static constexpr bool eq_v = eq_t<v1, v2>::value;
00990
00991         template<typename v1, typename v2>
00992         using gcd_t = gcd_t<i16, v1, v2>;
00993
00994         template<typename v>
00995         using pos_t = typename pos<v>::type;
00996
00997         template<typename v>
00998         static constexpr bool pos_v = pos_t<v>::value;
00999     };
01000 } // namespace aerobus
01001 #endif
01002
01003 // i32
01004 namespace aerobus {
01005     struct i32 {
01006         using inner_type = int32_t;
01007         template<int32_t x>
01008         struct val {
01009             using enclosing_type = i32;
01010             static constexpr int32_t v = x;
01011
01012             template<typename valueType>
01013             static constexpr DEVICE valueType get() {
01014                 return static_cast<valueType>(x);
01015             }
01016
01017             using is_zero_t = std::bool_constant<x == 0>;
01018         };
01019     };
01020 }

```

```

01073         static std::string to_string() {
01074             return std::to_string(x);
01075         }
01076     };
01077
01078     using zero = val<0>;
01079     using one = val<1>;
01080     static constexpr bool is_field = false;
01081     static constexpr bool is_euclidean_domain = true;
01082     template<auto x>
01083     using inject_constant_t = val<static_cast<int32_t>(x)>;
01084
01085     template<typename v>
01086     using inject_ring_t = v;
01087
01088 private:
01089     template<typename v1, typename v2>
01090     struct add {
01091         using type = val<v1::v + v2::v>;
01092     };
01093
01094     template<typename v1, typename v2>
01095     struct sub {
01096         using type = val<v1::v - v2::v>;
01097     };
01098
01099     template<typename v1, typename v2>
01100     struct mul {
01101         using type = val<v1::v * v2::v>;
01102     };
01103
01104     template<typename v1, typename v2>
01105     struct div {
01106         using type = val<v1::v / v2::v>;
01107     };
01108
01109     template<typename v1, typename v2>
01110     struct remainder {
01111         using type = val<v1::v % v2::v>;
01112     };
01113
01114     template<typename v1, typename v2>
01115     struct gt {
01116         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01117     };
01118
01119     template<typename v1, typename v2>
01120     struct lt {
01121         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01122     };
01123
01124     template<typename v1, typename v2>
01125     struct eq {
01126         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01127     };
01128
01129     template<typename v1>
01130     struct pos {
01131         using type = std::bool_constant<(v1::v > 0)>;
01132     };
01133
01134 public:
01135     template<typename v1, typename v2>
01136     using add_t = typename add<v1, v2>::type;
01137
01138     template<typename v1, typename v2>
01139     using sub_t = typename sub<v1, v2>::type;
01140
01141     template<typename v1, typename v2>
01142     using mul_t = typename mul<v1, v2>::type;
01143
01144     template<typename v1, typename v2>
01145     using div_t = typename div<v1, v2>::type;
01146
01147     template<typename v1, typename v2>
01148     using mod_t = typename remainder<v1, v2>::type;
01149
01150     template<typename v1, typename v2>
01151     using gt_t = typename gt<v1, v2>::type;
01152
01153     template<typename v1, typename v2>
01154     using lt_t = typename lt<v1, v2>::type;
01155
01156     template<typename v1, typename v2>
01157     using eq_t = typename eq<v1, v2>::type;
01158
01159     template<typename v1, typename v2>

```

```

01201         static constexpr bool eq_v = eq_t<v1, v2>::value;
01202
01207         template<typename v1, typename v2>
01208         using gcd_t = gcd_t<i32, v1, v2>;
01209
01213         template<typename v>
01214         using pos_t = typename pos<v>::type;
01215
01219         template<typename v>
01220         static constexpr bool pos_v = pos_t<v>::value;
01221     };
01222 } // namespace aerobus
01223
01224 // i64
01225 namespace aerobus {
01226     struct i64 {
01229         using inner_type = int64_t;
01232         template<int64_t x>
01233         struct val {
01235             using inner_type = int32_t;
01237             using enclosing_type = i64;
01239             static constexpr int64_t v = x;
01240
01243             template<typename valueType>
01244             static constexpr INLINED_DEVICE valueType get() {
01245                 return static_cast<valueType>(x);
01246             }
01247
01249             using is_zero_t = std::bool_constant<x == 0>;
01250
01252             static std::string to_string() {
01253                 return std::to_string(x);
01254             }
01255         };
01256
01259         template<auto x>
01260         using inject_constant_t = val<static_cast<int64_t>(x)>;
01261
01266         template<typename v>
01267         using inject_ring_t = v;
01268
01270         using zero = val<0>;
01272         using one = val<1>;
01274         static constexpr bool is_field = false;
01276         static constexpr bool is_euclidean_domain = true;
01277
01278     private:
01279         template<typename v1, typename v2>
01280         struct add {
01281             using type = val<v1::v + v2::v>;
01282         };
01283
01284         template<typename v1, typename v2>
01285         struct sub {
01286             using type = val<v1::v - v2::v>;
01287         };
01288
01289         template<typename v1, typename v2>
01290         struct mul {
01291             using type = val<v1::v * v2::v>;
01292         };
01293
01294         template<typename v1, typename v2>
01295         struct div {
01296             using type = val<v1::v / v2::v>;
01297         };
01298
01299         template<typename v1, typename v2>
01300         struct remainder {
01301             using type = val<v1::v % v2::v>;
01302         };
01303
01304         template<typename v1, typename v2>
01305         struct gt {
01306             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01307         };
01308
01309         template<typename v1, typename v2>
01310         struct lt {
01311             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01312         };
01313
01314         template<typename v1, typename v2>
01315         struct eq {
01316             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01317         };
01318

```

```

01319     template<typename v>
01320     struct pos {
01321         using type = std::bool_constant<(v::v > 0)>;
01322     };
01323
01324     public:
01325     template<typename v1, typename v2>
01326     using add_t = typename add<v1, v2>::type;
01327
01328     template<typename v1, typename v2>
01329     using sub_t = typename sub<v1, v2>::type;
01330
01331     template<typename v1, typename v2>
01332     using mul_t = typename mul<v1, v2>::type;
01333
01334     template<typename v1, typename v2>
01335     using div_t = typename div<v1, v2>::type;
01336
01337     template<typename v1, typename v2>
01338     using mod_t = typename remainder<v1, v2>::type;
01339
01340     template<typename v1, typename v2>
01341     using gt_t = typename gt<v1, v2>::type;
01342
01343     template<typename v1, typename v2>
01344     static constexpr bool gt_v = gt_t<v1, v2>::value;
01345
01346     template<typename v1, typename v2>
01347     using lt_t = typename lt<v1, v2>::type;
01348
01349     template<typename v1, typename v2>
01350     static constexpr bool lt_v = lt_t<v1, v2>::value;
01351
01352     template<typename v1, typename v2>
01353     using eq_t = typename eq<v1, v2>::type;
01354
01355     template<typename v1, typename v2>
01356     static constexpr bool eq_v = eq_t<v1, v2>::value;
01357
01358     template<typename v1, typename v2>
01359     using gcd_t = gcd_t<i64, v1, v2>;
01360
01361     template<typename v>
01362     using pos_t = typename pos<v>::type;
01363
01364     template<typename v>
01365     static constexpr bool pos_v = pos_t<v>::value;
01366 };
01367
01368 template<>
01369 struct Embed<i32, i64> {
01370     template<typename val>
01371     using type = i64::val<static_cast<int64_t>(val::v)>;
01372 };
01373 } // namespace aerobus
01374
01375 // z/pz
01376 namespace aerobus {
01377     template<int32_t p>
01378     struct zp {
01379         using inner_type = int32_t;
01380
01381         template<int32_t x>
01382         struct val {
01383             using enclosing_type = zp<p>;
01384             static constexpr int32_t v = x % p;
01385
01386             template<typename valueType>
01387             static constexpr INLINED_DEVICE valueType get() {
01388                 return static_cast<valueType>(x % p);
01389             }
01390
01391             using is_zero_t = std::bool_constant<v == 0>;
01392
01393             static constexpr bool is_zero_v = v == 0;
01394
01395             static std::string to_string() {
01396                 return std::to_string(x % p);
01397             }
01398         };
01399     };
01400
01401     template<auto x>
01402     using inject_constant_t = val<static_cast<int32_t>(x)>;
01403
01404     using zero = val<0>;
01405
01406     using one = val<1>;

```



```

01479
01481     static constexpr bool is_prime<p>::value;
01482
01484     static constexpr bool is_euclidean_domain = true;
01485
01486 private:
01487     template<typename v1, typename v2>
01488     struct add {
01489         using type = val<(v1::v + v2::v) % p>;
01490     };
01491
01492     template<typename v1, typename v2>
01493     struct sub {
01494         using type = val<(v1::v - v2::v) % p>;
01495     };
01496
01497     template<typename v1, typename v2>
01498     struct mul {
01499         using type = val<(v1::v * v2::v) % p>;
01500     };
01501
01502     template<typename v1, typename v2>
01503     struct div {
01504         using type = val<(v1::v % p) / (v2::v % p)>;
01505     };
01506
01507     template<typename v1, typename v2>
01508     struct remainder {
01509         using type = val<(v1::v % v2::v) % p>;
01510     };
01511
01512     template<typename v1, typename v2>
01513     struct gt {
01514         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01515     };
01516
01517     template<typename v1, typename v2>
01518     struct lt {
01519         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01520     };
01521
01522     template<typename v1, typename v2>
01523     struct eq {
01524         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01525     };
01526
01527     template<typename v1>
01528     struct pos {
01529         using type = std::bool_constant<(v1::v > 0)>;
01530     };
01531
01532 public:
01533     template<typename v1, typename v2>
01534     using add_t = typename add<v1, v2>::type;
01535
01536     template<typename v1, typename v2>
01537     using sub_t = typename sub<v1, v2>::type;
01538
01539     template<typename v1, typename v2>
01540     using mul_t = typename mul<v1, v2>::type;
01541
01542     template<typename v1, typename v2>
01543     using div_t = typename div<v1, v2>::type;
01544
01545     template<typename v1, typename v2>
01546     using mod_t = typename remainder<v1, v2>::type;
01547
01548     template<typename v1, typename v2>
01549     using gt_t = typename gt<v1, v2>::type;
01550
01551     template<typename v1, typename v2>
01552     static constexpr bool gt_v = gt_t<v1, v2>::value;
01553
01554     template<typename v1, typename v2>
01555     using lt_t = typename lt<v1, v2>::type;
01556
01557     template<typename v1, typename v2>
01558     static constexpr bool lt_v = lt_t<v1, v2>::value;
01559
01560     template<typename v1, typename v2>
01561     using eq_t = typename eq<v1, v2>::type;
01562
01563     template<typename v1, typename v2>
01564     static constexpr bool eq_v = eq_t<v1, v2>::value;
01565
01566     template<typename v1, typename v2>
01567     using gcd_t = gcd_t<i32, v1, v2>;
01568
01569     template<typename v1, typename v2>
01570     static constexpr bool gcd_v = gcd_t<i32, v1, v2>::value;

```

```

01604
01607     template<typename v1>
01608     using pos_t = typename pos<v1>::type;
01609
01612     template<typename v>
01613     static constexpr bool pos_v = pos_t<v>::value;
01614 };
01615
01618     template<int32_t x>
01619     struct Embed<zp<x>, i32> {
01622         template <typename val>
01623         using type = i32::val<val::v>;
01624     };
01625 } // namespace aerobus
01626
01627 // polynomial
01628 namespace aerobus {
01629     // coeffN x^N + ...
01634     template<typename Ring>
01635     requires IsEuclideanDomain<Ring>
01636     struct polynomial {
01637         static constexpr bool is_field = false;
01638         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01639
01642         template<typename P>
01643         struct horner_reduction_t {
01644             template<size_t index, size_t stop>
01645             struct inner {
01646                 template<typename accum, typename x>
01647                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01648                 ::template type<
01649                     typename Ring::template add_t<
01650                         typename Ring::template mul_t<x, accum>,
01651                         typename P::template coeff_at_t<P::degree - index>
01652                     >, x>;
01653             };
01654
01655             template<size_t stop>
01656             struct inner<stop, stop> {
01657                 template<typename accum, typename x>
01658                 using type = accum;
01659             };
01660         };
01661
01665         template<typename coeffN, typename... coeffs>
01666         struct val {
01668             using ring_type = Ring;
01670             using enclosing_type = polynomial<Ring>;
01672             static constexpr size_t degree = sizeof...(coeffs);
01674             using aN = coeffN;
01676             using strip = val<coeffs...>;
01678             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01680             static constexpr bool is_zero_v = is_zero_t::value;
01681
01682         private:
01683             template<size_t index, typename E = void>
01684             struct coeff_at {};
01685
01686             template<size_t index>
01687             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01688                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01689             };
01690
01691             template<size_t index>
01692             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01693                 using type = typename Ring::zero;
01694             };
01695
01696         public:
01699             template<size_t index>
01700             using coeff_at_t = typename coeff_at<index>::type;
01701
01704             static std::string to_string() {
01705                 return string_helper<coeffN, coeffs...>::func();
01706             }
01707
01712             template<typename arithmeticType>
01713             static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01714                 #ifdef WITH_CUDA_FP16
01715                 arithmeticType start;
01716                 if constexpr (std::is_same_v<arithmeticType, __half2>) {
01717                     start = __half2(0, 0);
01718                 } else {
01719                     start = static_cast<arithmeticType>(0);
01720                 }
01721                 #else
01722                 arithmeticType start = static_cast<arithmeticType>(0);

```

```

01723         #endif
01724         return horner_evaluation<arithmeticType, val>
01725             ::template inner<0, degree + 1>
01726             ::func(start, x);
01727     }
01728
01741     template<typename arithmeticType>
01742     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01743         return compensated_horner<arithmeticType, val>::func(x);
01744     }
01745
01746     template<typename x>
01747     using value_at_t = horner_reduction_t<val>
01748         ::template inner<0, degree + 1>
01749         ::template type<typename Ring::zero, x>;
01750 };
01751
01754 template<typename coeffN>
01755 struct val<coeffN> {
01756     using ring_type = Ring;
01757     using enclosing_type = polynomial<Ring>;
01758     static constexpr size_t degree = 0;
01759     using aN = coeffN;
01760     using strip = val<coeffN>;
01761     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01762
01763     static constexpr bool is_zero_v = is_zero_t::value;
01764
01765     template<size_t index, typename E = void>
01766     struct coeff_at {};
01767
01768     template<size_t index>
01769     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01770         using type = aN;
01771     };
01772
01773     template<size_t index>
01774     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01775         using type = typename Ring::zero;
01776     };
01777
01778     template<size_t index>
01779     using coeff_at_t = typename coeff_at<index>::type;
01780
01781     static std::string to_string() {
01782         return string_helper<coeffN>::func();
01783     }
01784
01785     template<typename arithmeticType>
01786     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01787         return coeffN::template get<arithmeticType>();
01788     }
01789
01790     template<typename arithmeticType>
01791     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01792         return coeffN::template get<arithmeticType>();
01793     }
01794
01795     template<typename x>
01796     using value_at_t = coeffN;
01797 };
01798
01800 using zero = val<typename Ring::zero>;
01801 using one = val<typename Ring::one>;
01802 using X = val<typename Ring::one, typename Ring::zero>;
01803
01804 private:
01805     template<typename P, typename E = void>
01806     struct simplify;
01807
01808     template<typename P1, typename P2, typename I>
01809     struct add_low;
01810
01811     template<typename P1, typename P2>
01812     struct add {
01813         using type = typename simplify<typename add_low<
01814             P1,
01815             P2,
01816             internal::make_index_sequence_reverse<
01817                 std::max(P1::degree, P2::degree) + 1
01818             >::type>::type;
01819     };
01820
01821     template<typename P1, typename P2, typename I>
01822     struct sub_low;
01823
01824     template<typename P1, typename P2, typename I>

```

```

01830     struct mul_low;
01831
01832     template<typename v1, typename v2>
01833     struct mul {
01834         using type = typename mul_low<
01835             v1,
01836             v2,
01837             internal::make_index_sequence_reverse<
01838                 v1::degree + v2::degree + 1
01839             >::type;
01840     };
01841
01842     template<typename coeff, size_t deg>
01843     struct monomial;
01844
01845     template<typename v, typename E = void>
01846     struct derive_helper {};
01847
01848     template<typename v>
01849     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01850         using type = zero;
01851     };
01852
01853     template<typename v>
01854     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01855         using type = typename add<
01856             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01857             typename monomial<
01858                 typename Ring::template mul_t<
01859                     typename v::aN,
01860                     typename Ring::template inject_constant_t<(v::degree)>
01861                 >,
01862                 v::degree - 1
01863             >::type
01864         >::type;
01865     };
01866
01867     template<typename v1, typename v2, typename E = void>
01868     struct eq_helper {};
01869
01870     template<typename v1, typename v2>
01871     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01872         using type = std::false_type;
01873     };
01874
01875
01876     template<typename v1, typename v2>
01877     struct eq_helper<v1, v2, std::enable_if_t<
01878         v1::degree == v2::degree &&
01879         (v1::degree != 0 || v2::degree != 0) &&
01880         std::is_same<
01881             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01882             std::false_type
01883         >::value
01884     >
01885     > {
01886         using type = std::false_type;
01887     };
01888
01889     template<typename v1, typename v2>
01890     struct eq_helper<v1, v2, std::enable_if_t<
01891         v1::degree == v2::degree &&
01892         (v1::degree != 0 || v2::degree != 0) &&
01893         std::is_same<
01894             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01895             std::true_type
01896         >::value
01897     > {
01898         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01899     };
01900
01901     template<typename v1, typename v2>
01902     struct eq_helper<v1, v2, std::enable_if_t<
01903         v1::degree == v2::degree &&
01904         (v1::degree == 0)
01905     > {
01906         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01907     };
01908
01909     template<typename v1, typename v2, typename E = void>
01910     struct lt_helper {};
01911
01912     template<typename v1, typename v2>
01913     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01914         using type = std::true_type;
01915     };
01916

```

```

01917     template<typename v1, typename v2>
01918     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01919         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01920     };
01921
01922     template<typename v1, typename v2>
01923     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01924         using type = std::false_type;
01925     };
01926
01927     template<typename v1, typename v2, typename E = void>
01928     struct gt_helper {};
01929
01930     template<typename v1, typename v2>
01931     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01932         using type = std::true_type;
01933     };
01934
01935     template<typename v1, typename v2>
01936     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01937         using type = std::false_type;
01938     };
01939
01940     template<typename v1, typename v2>
01941     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01942         using type = std::false_type;
01943     };
01944
01945     // when high power is zero : strip
01946     template<typename P>
01947     struct simplify<P, std::enable_if_t<
01948         std::is_same<
01949             typename Ring::zero,
01950             typename P::aN
01951         >::value && (P::degree > 0)
01952     >> {
01953         using type = typename simplify<typename P::strip>::type;
01954     };
01955
01956     // otherwise : do nothing
01957     template<typename P>
01958     struct simplify<P, std::enable_if_t<
01959         !std::is_same<
01960             typename Ring::zero,
01961             typename P::aN
01962         >::value && (P::degree > 0)
01963     >> {
01964         using type = P;
01965     };
01966
01967     // do not simplify constants
01968     template<typename P>
01969     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01970         using type = P;
01971     };
01972
01973     // addition at
01974     template<typename P1, typename P2, size_t index>
01975     struct add_at {
01976         using type =
01977             typename Ring::template add_t<
01978                 typename P1::template coeff_at_t<index>,
01979                 typename P2::template coeff_at_t<index>;
01980     };
01981
01982     template<typename P1, typename P2, size_t index>
01983     using add_at_t = typename add_at<P1, P2, index>::type;
01984
01985     template<typename P1, typename P2, std::size_t... I>
01986     struct add_low<P1, P2, std::index_sequence<I...>> {
01987         using type = val<add_at_t<P1, P2, I>...>;
01988     };
01989
01990     // subtraction at
01991     template<typename P1, typename P2, size_t index>
01992     struct sub_at {
01993         using type =
01994             typename Ring::template sub_t<
01995                 typename P1::template coeff_at_t<index>,
01996                 typename P2::template coeff_at_t<index>;
01997     };
01998
01999     template<typename P1, typename P2, size_t index>
02000     using sub_at_t = typename sub_at<P1, P2, index>::type;
02001
02002     template<typename P1, typename P2, std::size_t... I>
02003     struct sub_low<P1, P2, std::index_sequence<I...>> {

```

```

02004         using type = val<sub_at_t<P1, P2, I>...>;
02005     };
02006
02007     template<typename P1, typename P2>
02008     struct sub {
02009         using type = typename simplify<typename sub_low<
02010             P1,
02011             P2,
02012             internal::make_index_sequence_reverse<
02013                 std::max(P1::degree, P2::degree) + 1
02014             >::type>::type;
02015     };
02016
02017     // multiplication at
02018     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02019     struct mul_at_loop_helper {
02020         using type = typename Ring::template add_t<
02021             typename Ring::template mul_t<
02022                 typename v1::template coeff_at_t<index>,
02023                 typename v2::template coeff_at_t<k - index>
02024             >,
02025             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02026         >;
02027     };
02028
02029     template<typename v1, typename v2, size_t k, size_t stop>
02030     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02031         using type = typename Ring::template mul_t<
02032             typename v1::template coeff_at_t<stop>,
02033             typename v2::template coeff_at_t<0>;
02034     };
02035
02036     template <typename v1, typename v2, size_t k, typename E = void>
02037     struct mul_at {};
02038
02039     template<typename v1, typename v2, size_t k>
02040     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
02041         using type = typename Ring::zero;
02042     };
02043
02044     template<typename v1, typename v2, size_t k>
02045     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
02046         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02047     };
02048
02049     template<typename P1, typename P2, size_t index>
02050     using mul_at_t = typename mul_at<P1, P2, index>::type;
02051
02052     template<typename P1, typename P2, std::size_t... I>
02053     struct mul_low<P1, P2, std::index_sequence<I...> {
02054         using type = val<mul_at_t<P1, P2, I>...>;
02055     };
02056
02057     // division helper
02058     template< typename A, typename B, typename Q, typename R, typename E = void>
02059     struct div_helper {};
02060
02061     template<typename A, typename B, typename Q, typename R>
02062     struct div_helper<A, B, Q, R, std::enable_if_t<
02063         (R::degree < B::degree) ||
02064         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02065         using q_type = Q;
02066         using mod_type = R;
02067         using gcd_type = B;
02068     };
02069
02070     template<typename A, typename B, typename Q, typename R>
02071     struct div_helper<A, B, Q, R, std::enable_if_t<
02072         (R::degree >= B::degree) &&
02073         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02074     private: // NOLINT
02075         using rN = typename R::aN;
02076         using bN = typename B::aN;
02077         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02078             B::degree>::type;
02079         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02080         using qq = typename add<Q, pT>::type;
02081     public:
02082         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02083         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02084         using gcd_type = rr;
02085     };
02086
02087     template<typename A, typename B>
02088     struct div {
02089         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");

```

```

02090         using q_type = typename div_helper<A, B, zero, A>::q_type;
02091         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02092     };
02093
02094     template<typename P>
02095     struct make_unit {
02096         using type = typename div<P, val<typename P::aN>::q_type;
02097     };
02098
02099     template<typename coeff, size_t deg>
02100     struct monomial {
02101         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02102     };
02103
02104     template<typename coeff>
02105     struct monomial<coeff, 0> {
02106         using type = val<coeff>;
02107     };
02108
02109     template<typename arithmeticType, typename P>
02110     struct horner_evaluation {
02111         template<size_t index, size_t stop>
02112         struct inner {
02113             static constexpr DEVICE INLINED arithmeticType func(
02114                 const arithmeticType& accum, const arithmeticType& x) {
02115                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
stop>::func(
02116                     internal::fma_helper<arithmeticType>::eval(
02117                         x,
02118                         accum,
02119                         P::template coeff_at_t<P::degree - index>::template
get<arithmeticType>()), x);
02120             }
02121         };
02122
02123         template<size_t stop>
02124         struct inner<stop, stop> {
02125             static constexpr DEVICE INLINED arithmeticType func(
02126                 const arithmeticType& accum, const arithmeticType& x) {
02127                 return accum;
02128             }
02129         };
02130     };
02131
02132     template<typename arithmeticType, typename P>
02133     struct compensated_horner {
02134         template<int64_t index, int ghost>
02135         struct EFTHorner {
02136             static INLINED DEVICE void func(
02137                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02138                 arithmeticType p;
02139                 internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02140                 constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
get<arithmeticType>();
02141                 internal::two_sum<arithmeticType>(
02142                     p, coeff,
02143                     r, sigma + P::degree - index - 1);
02144                 EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02145             }
02146         };
02147
02148         template<int ghost>
02149         struct EFTHorner<-1, ghost> {
02150             static INLINED DEVICE void func(
02151                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
*r) {
02152             }
02153         };
02154
02155         static INLINED DEVICE arithmeticType func(arithmeticType x) {
02156             arithmeticType pi[P::degree], sigma[P::degree];
02157             arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02158             EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02159             arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02160             return r + c;
02161         }
02162     };
02163
02164     template<typename coeff, typename... coeffs>
02165     struct string_helper {
02166         static std::string func() {
02167             std::string tail = string_helper<coeffs...>::func();
02168             std::string result = "";
02169             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02170                 return tail;
02171             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {

```

```

02172         if (sizeof...(coeffs) == 1) {
02173             result += "x";
02174         } else {
02175             result += "x^" + std::to_string(sizeof...(coeffs));
02176         }
02177     } else {
02178         if (sizeof...(coeffs) == 1) {
02179             result += coeff::to_string() + " x";
02180         } else {
02181             result += coeff::to_string()
02182                 + " x^" + std::to_string(sizeof...(coeffs));
02183         }
02184     }
02185
02186     if (!tail.empty()) {
02187         if (tail.at(0) != '-') {
02188             result += " + " + tail;
02189         } else {
02190             result += " - " + tail.substr(1);
02191         }
02192     }
02193
02194     return result;
02195 }
02196 };
02197
02198 template<typename coeff>
02199 struct string_helper<coeff> {
02200     static std::string func() {
02201         if (!std::is_same<coeff, typename Ring::zero>::value) {
02202             return coeff::to_string();
02203         } else {
02204             return "";
02205         }
02206     }
02207 };
02208
02209 public:
02210     template<typename P>
02211     using simplify_t = typename simplify<P>::type;
02212
02213     template<typename v1, typename v2>
02214     using add_t = typename add<v1, v2>::type;
02215
02216     template<typename v1, typename v2>
02217     using sub_t = typename sub<v1, v2>::type;
02218
02219     template<typename v1, typename v2>
02220     using mul_t = typename mul<v1, v2>::type;
02221
02222     template<typename v1, typename v2>
02223     using eq_t = typename eq_helper<v1, v2>::type;
02224
02225     template<typename v1, typename v2>
02226     using lt_t = typename lt_helper<v1, v2>::type;
02227
02228     template<typename v1, typename v2>
02229     using gt_t = typename gt_helper<v1, v2>::type;
02230
02231     template<typename v1, typename v2>
02232     using div_t = typename div<v1, v2>::q_type;
02233
02234     template<typename v1, typename v2>
02235     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02236
02237     template<typename coeff, size_t deg>
02238     using monomial_t = typename monomial<coeff, deg>::type;
02239
02240     template<typename v>
02241     using derive_t = typename derive_helper<v>::type;
02242
02243     template<typename v>
02244     using pos_t = typename Ring::template pos_t<typename v::aN>;
02245
02246     template<typename v>
02247     static constexpr bool pos_v = pos_t<v>::value;
02248
02249     template<typename v1, typename v2>
02250     using gcd_t = std::conditional_t<
02251         Ring::is_euclidean_domain,
02252         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02253         void>;
02254
02255     template<auto x>
02256     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02257
02258     template<typename v>

```



```

02301         using inject_ring_t = val<v>;
02302     };
02303 } // namespace aerobus
02304
02305 // fraction field
02306 namespace aerobus {
02307     namespace internal {
02308         template<typename Ring, typename E = void>
02309         requires IsEuclideanDomain<Ring>
02310         struct _FractionField {};
02311
02312         template<typename Ring>
02313         requires IsEuclideanDomain<Ring>
02314         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02315             static constexpr bool is_field = true;
02316             static constexpr bool is_euclidean_domain = true;
02317
02318         private:
02319             template<typename val1, typename val2, typename E = void>
02320             struct to_string_helper {};
02321
02322             template<typename val1, typename val2>
02323             struct to_string_helper<val1, val2,
02324                 std::enable_if_t<
02325                     Ring::template eq_t<
02326                         val2, typename Ring::one
02327                         >::value
02328                     >
02329             > {
02330                 static std::string func() {
02331                     return val1::to_string();
02332                 }
02333             };
02334
02335             template<typename val1, typename val2>
02336             struct to_string_helper<val1, val2,
02337                 std::enable_if_t<
02338                     !Ring::template eq_t<
02339                         val2,
02340                         typename Ring::one
02341                         >::value
02342                     >
02343             > {
02344                 static std::string func() {
02345                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02346                 }
02347             };
02348         };
02349
02350     public:
02351         template<typename val1, typename val2>
02352         struct val {
02353             using x = val1;
02354             using y = val2;
02355             using is_zero_t = typename val1::is_zero_t;
02356             static constexpr bool is_zero_v = val1::is_zero_t::value;
02357
02358             using ring_type = Ring;
02359             using enclosing_type = _FractionField<Ring>;
02360
02361             static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02362
02363             template<typename valueType, int ghost = 0>
02364             struct get_helper {
02365                 static constexpr INLINED_DEVICE valueType get() {
02366                     return internal::staticcast<valueType, typename
02367 ring_type::inner_type>::template func<x::v>() /
02368                 internal::staticcast<valueType, typename ring_type::inner_type>::template
02369 func<y::v>());
02370             };
02371
02372             #ifdef WITH_CUDA_FP16
02373             template<int ghost>
02374             struct get_helper<__half, ghost> {
02375                 static constexpr INLINED_DEVICE __half get() {
02376                     return internal::my_float2half_rn(
02377                 internal::staticcast<float, typename ring_type::inner_type>::template
02378 func<x::v>() /
02379                 internal::staticcast<float, typename ring_type::inner_type>::template
02380 func<y::v>());
02381             };
02382
02383             template<int ghost>
02384             struct get_helper<__half2, ghost> {
02385                 static constexpr INLINED_DEVICE __half2 get() {
02386                     constexpr __half tmp = internal::my_float2half_rn(

```

```

02395                                     internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /                                     internal::staticcast<float, typename ring_type::inner_type>::template
02396                                     internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02397                                     return __half2(tmp, tmp);
02398                                     }
02399                                     };
02400                                     #endif
02401
02405                                     template<typename valueType>
02406                                     static constexpr INLINED_DEVICE valueType get() {
02407                                     return get_helper<valueType, 0>::get();
02408                                     }
02409
02412                                     static std::string to_string() {
02413                                     return to_string_helper<val1, val2>::func();
02414                                     }
02415
02420                                     template<typename arithmeticType>
02421                                     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02422                                     return x::eval(v) / y::eval(v);
02423                                     }
02424                                     };
02425
02427                                     using zero = val<typename Ring::zero, typename Ring::one>;
02429                                     using one = val<typename Ring::one, typename Ring::one>;
02430
02433                                     template<typename v>
02434                                     using inject_t = val<v, typename Ring::one>;
02435
02438                                     template<auto x>
02439                                     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02440
02443                                     template<typename v>
02444                                     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02445
02447                                     using ring_type = Ring;
02448
02449                                     private:
02450                                     template<typename v, typename E = void>
02451                                     struct simplify {};
02452
02453                                     // x = 0
02454                                     template<typename v>
02455                                     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02456                                     using type = typename _FractionField<Ring>::zero;
02457                                     };
02458
02459                                     // x != 0
02460                                     template<typename v>
02461                                     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02462                                     private:
02463                                     using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02464                                     using newx = typename Ring::template div_t<typename v::x, _gcd>;
02465                                     using newy = typename Ring::template div_t<typename v::y, _gcd>;
02466
02467                                     using posx = std::conditional_t<
02468                                     !Ring::template pos_v<newy>,
02469                                     typename Ring::template sub_t<typename Ring::zero, newx>,
02470                                     newx>;
02471                                     using posy = std::conditional_t<
02472                                     !Ring::template pos_v<newy>,
02473                                     typename Ring::template sub_t<typename Ring::zero, newy>,
02474                                     newy>;
02475                                     public:
02476                                     using type = typename _FractionField<Ring>::template val<posx, posy>;
02477                                     };
02478
02479                                     public:
02482                                     template<typename v>
02483                                     using simplify_t = typename simplify<v>::type;
02484
02485                                     private:
02486                                     template<typename v1, typename v2>
02487                                     struct add {
02488                                     private:
02489                                     using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02490                                     using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02491                                     using dividend = typename Ring::template add_t<a, b>;
02492                                     using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02493                                     using g = typename Ring::template gcd_t<dividend, divider>;
02494
02495                                     public:
02496                                     using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02497                                     };

```

```

02498
02499     template<typename v>
02500     struct pos {
02501         using type = std::conditional_t<
02502             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02503             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02504             std::true_type,
02505             std::false_type>;
02506     };
02507
02508     template<typename v1, typename v2>
02509     struct sub {
02510     private:
02511         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02512         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02513         using dividend = typename Ring::template sub_t<a, b>;
02514         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02515         using g = typename Ring::template gcd_t<dividend, diviser>;
02516
02517     public:
02518         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02519     };
02520
02521     template<typename v1, typename v2>
02522     struct mul {
02523     private:
02524         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02525         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02526
02527     public:
02528         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02529     };
02530
02531     template<typename v1, typename v2, typename E = void>
02532     struct div {};
02533
02534     template<typename v1, typename v2>
02535     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
02536     private:
02537         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02538         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02539
02540     public:
02541         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02542     };
02543
02544     template<typename v1, typename v2>
02545     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02546         using type = one;
02547     };
02548
02549
02550     template<typename v1, typename v2>
02551     struct eq {
02552     private:
02553         using type = std::conditional_t<
std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
std::true_type,
std::false_type>;
02554
02555     };
02556
02557     template<typename v1, typename v2, typename E = void>
02558     struct gt;
02559
02560     template<typename v1, typename v2>
02561     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
> {
02562         using type = std::false_type;
02563     };
02564
02565     template<typename v1, typename v2>
02566     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(!pos<v1>::type::value) && (!pos<v2>::type::value)
> {
02567         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02568     };
02569
02570     template<typename v1, typename v2>
02571     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
> {
02572         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02573     };
02574
02575     template<typename v1, typename v2>
02576     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
> {
02577         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02578     };
02579
02580     template<typename v1, typename v2>
02581     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
> {
02582         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02583     };

```

```

02583         » {
02584             using type = std::true_type;
02585         };
02586
02587     template<typename v1, typename v2>
02588     struct gt<v1, v2, std::enable_if_t<
02589         (!eq<v1, v2>::type::value) &&
02590         (!pos<v1>::type::value) && (pos<v2>::type::value)
02591         > {
02592         using type = std::false_type;
02593     };
02594
02595     template<typename v1, typename v2>
02596     struct gt<v1, v2, std::enable_if_t<
02597         (!eq<v1, v2>::type::value) &&
02598         (pos<v1>::type::value) && (pos<v2>::type::value)
02599         > {
02600         using type = typename Ring::template gt_t<
02601             typename Ring::template mul_t<v1::x, v2::y>,
02602             typename Ring::template mul_t<v2::y, v2::x>
02603         >;
02604     };
02605
02606 public:
02610     template<typename v1, typename v2>
02611     using add_t = typename add<v1, v2>::type;
02612
02617     template<typename v1, typename v2>
02618     using mod_t = zero;
02619
02624     template<typename v1, typename v2>
02625     using gcd_t = v1;
02626
02630     template<typename v1, typename v2>
02631     using sub_t = typename sub<v1, v2>::type;
02632
02636     template<typename v1, typename v2>
02637     using mul_t = typename mul<v1, v2>::type;
02638
02642     template<typename v1, typename v2>
02643     using div_t = typename div<v1, v2>::type;
02644
02648     template<typename v1, typename v2>
02649     using eq_t = typename eq<v1, v2>::type;
02650
02654     template<typename v1, typename v2>
02655     static constexpr bool eq_v = eq<v1, v2>::type::value;
02656
02660     template<typename v1, typename v2>
02661     using gt_t = typename gt<v1, v2>::type;
02662
02666     template<typename v1, typename v2>
02667     static constexpr bool gt_v = gt<v1, v2>::type::value;
02668
02671     template<typename v1>
02672     using pos_t = typename pos<v1>::type;
02673
02676     template<typename v>
02677     static constexpr bool pos_v = pos_t<v>::value;
02678 };
02679
02680 template<typename Ring, typename E = void>
02681 requires IsEuclideanDomain<Ring>
02682 struct FractionFieldImpl {};
02683
02684 // fraction field of a field is the field itself
02685 template<typename Field>
02686 requires IsEuclideanDomain<Field>
02687 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02688     using type = Field;
02689     template<typename v>
02690     using inject_t = v;
02691 };
02692
02693 // fraction field of a ring is the actual fraction field
02694 template<typename Ring>
02695 requires IsEuclideanDomain<Ring>
02696 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02697     using type = _FractionField<Ring>;
02698 };
02699 } // namespace internal
02700
02703 template<typename Ring>
02704 requires IsEuclideanDomain<Ring>
02705 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02706
02709 template<typename Ring>

```

```

02710     struct Embed<Ring, FractionField<Ring>> {
02713         template<typename v>
02714             using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02715     };
02716 } // namespace aerobus
02717
02718 // short names for common types
02719 namespace aerobus {
02720     template<typename X, typename Y>
02721     requires IsRing<typename X::enclosing_type> &&
02722         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02723     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02724
02725     template<typename X, typename Y>
02726     requires IsRing<typename X::enclosing_type> &&
02727         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02728     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02729
02730     template<typename X, typename Y>
02731     requires IsRing<typename X::enclosing_type> &&
02732         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02733     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02734
02735     template<typename X, typename Y>
02736     requires IsEuclideanDomain<typename X::enclosing_type> &&
02737         (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02738     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02739
02740     using q32 = FractionField<i32>;
02741
02742     using fpq32 = FractionField<polynomial<q32>>;
02743
02744     using q64 = FractionField<i64>;
02745
02746     using pi64 = polynomial<i64>;
02747
02748     using pq64 = polynomial<q64>;
02749
02750     using fpq64 = FractionField<polynomial<q64>>;
02751
02752     template<typename Ring, typename v1, typename v2>
02753     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02754
02755     template<typename v>
02756     using embed_int_poly_in_fractions_t =
02757         typename Embed<
02758             polynomial<typename v::ring_type>,
02759             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02760         >;
02761
02762     template<int64_t p, int64_t q>
02763     using make_q64_t = typename q64::template simplify_t<
02764         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02765     >;
02766
02767     template<int32_t p, int32_t q>
02768     using make_q32_t = typename q32::template simplify_t<
02769         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02770     >;
02771
02772     #ifdef WITH_CUDA_FP16
02773     using q16 = FractionField<i16>;
02774
02775     template<int16_t p, int16_t q>
02776     using make_q16_t = typename q16::template simplify_t<
02777         typename q16::val<i16::inject_constant_t<p>, i16::inject_constant_t<q>>;
02778     >;
02779
02780     #endif
02781
02782     template<typename Ring, typename v1, typename v2>
02783     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02784
02785     template<typename Ring, typename v1, typename v2>
02786     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02787
02788     template<>
02789     struct Embed<q32, q64> {
02790         template<typename v>
02791         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02792     };
02793
02794     template<typename Small, typename Large>
02795     struct Embed<polynomial<Small>, polynomial<Large>> {
02796     private:
02797         template<typename v, typename i>
02798         struct at_low;
02799
02800         template<typename v, size_t i>
02801         struct at_index {
02802             using type = typename Embed<Small, Large>::template
02803                 type<typename v::template coeff_at_t<i>>;
02804         };
02805     };

```



```

02951
02956 template<typename T, size_t k, size_t n>
02957 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02958
02959 namespace internal {
02960     template<typename T, size_t m>
02961     struct bernoulli;
02962
02963     template<typename T, typename accum, size_t k, size_t m>
02964     struct bernoulli_helper {
02965         using type = typename bernoulli_helper<
02966             T,
02967             addfractions_t<T,
02968                 accum,
02969                 mulfractions_t<T,
02970                     makefraction_t<T,
02971                         combination_t<T, k, m + 1>,
02972                         typename T::one>,
02973                         typename bernoulli<T, k>::type
02974                     >
02975             >,
02976             k + 1,
02977             m>::type;
02978     };
02979
02980     template<typename T, typename accum, size_t m>
02981     struct bernoulli_helper<T, accum, m, m> {
02982         using type = accum;
02983     };
02984
02985
02986
02987     template<typename T, size_t m>
02988     struct bernoulli {
02989         using type = typename FractionField<T>::template mul_t<
02990             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02991             makefraction_t<T,
02992                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02993                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02994             >
02995         >;
02996
02997         template<typename floatType>
02998         static constexpr floatType value = type::template get<floatType>();
02999     };
03000
03001     template<typename T>
03002     struct bernoulli<T, 0> {
03003         using type = typename FractionField<T>::one;
03004
03005         template<typename floatType>
03006         static constexpr floatType value = type::template get<floatType>();
03007     };
03008 } // namespace internal
03009
03010 template<typename T, size_t n>
03011 using bernoulli_t = typename internal::bernoulli<T, n>::type;
03012
03013 template<typename FloatType, typename T, size_t n>
03014 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
03015
03016 // bell numbers
03017 namespace internal {
03018     template<typename T, size_t n, typename E = void>
03019     struct bell_helper;
03020
03021     template<typename T, size_t n>
03022     struct bell_helper<T, n, std::enable_if_t<(n > 1)> {
03023         template<typename accum, size_t i, size_t stop>
03024         struct sum_helper {
03025             private:
03026                 using left = typename T::template mul_t<
03027                     combination_t<T, i, n-1>,
03028                     typename bell_helper<T, i>::type>;
03029                 using new_accum = typename T::template add_t<accum, left>;
03030             public:
03031                 using type = typename sum_helper<new_accum, i+1, stop>::type;
03032         };
03033
03034         template<typename accum, size_t stop>
03035         struct sum_helper<accum, stop, stop> {
03036             using type = accum;
03037         };
03038
03039         using type = typename sum_helper<typename T::zero, 0, n>::type;
03040     };
03041 }
03042
03043
03044
03045
03046
03047
03048

```

```

03049     template<typename T>
03050     struct bell_helper<T, 0> {
03051         using type = typename T::one;
03052     };
03053
03054     template<typename T>
03055     struct bell_helper<T, 1> {
03056         using type = typename T::one;
03057     };
03058 } // namespace internal
03059
03063 template<typename T, size_t n>
03064 using bell_t = typename internal::bell_helper<T, n>::type;
03065
03069 template<typename T, size_t n>
03070 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03071
03072 namespace internal {
03073     template<typename T, int k, typename E = void>
03074     struct alternate {};
03075
03076     template<typename T, int k>
03077     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
03078         using type = typename T::one;
03079         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
03080     };
03081
03082     template<typename T, int k>
03083     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
03084         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03085         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
03086     };
03087 } // namespace internal
03088
03091 template<typename T, int k>
03092 using alternate_t = typename internal::alternate<T, k>::type;
03093
03096 template<typename T, size_t k>
03097 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03098
03099 namespace internal {
03100     template<typename T, int n, int k, typename E = void>
03101     struct stirling_l_helper {};
03102
03103     template<typename T>
03104     struct stirling_l_helper<T, 0, 0> {
03105         using type = typename T::one;
03106     };
03107
03108     template<typename T, int n>
03109     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03110         using type = typename T::zero;
03111     };
03112
03113     template<typename T, int n>
03114     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03115         using type = typename T::zero;
03116     };
03117
03118     template<typename T, int n, int k>
03119     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
03120         using type = typename T::template sub_t<
03121             typename T::template stirling_l_helper<T, n-1, k-1>::type,
03122             typename T::template mul_t<
03123                 typename T::template inject_constant_t<n-1>,
03124                 typename stirling_l_helper<T, n-1, k>::type
03125             >;
03126     };
03127 } // namespace internal
03128
03133 template<typename T, int n, int k>
03134 using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
03135
03140 template<typename T, int n, int k>
03141 using stirling_l_unsigned_t = abs_t<typename internal::stirling_l_helper<T, n, k>::type>;
03142
03147 template<typename T, int n, int k>
03148 static constexpr typename T::inner_type stirling_l_unsigned_v = stirling_l_unsigned_t<T, n, k>::v;
03149
03154 template<typename T, int n, int k>
03155 static constexpr typename T::inner_type stirling_l_signed_v = stirling_l_signed_t<T, n, k>::v;
03156
03157 namespace internal {
03158     template<typename T, int n, int k, typename E = void>
03159     struct stirling_2_helper {};

```



```

03160
03161     template<typename T, int n>
03162     struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)>> {
03163         using type = typename T::one;
03164     };
03165
03166     template<typename T, int n>
03167     struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
03168         using type = typename T::zero;
03169     };
03170
03171     template<typename T, int n>
03172     struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
03173         using type = typename T::zero;
03174     };
03175
03176     template<typename T, int n, int k>
03177     struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)>> {
03178         using type = typename T::template add_t<
03179             typename stirling_2_helper<T, n-1, k-1>::type,
03180             typename T::template mul_t<
03181                 typename T::template inject_constant_t<k>,
03182                 typename stirling_2_helper<T, n-1, k>::type
03183             >>;
03184     };
03185 } // namespace internal
03186
03191 template<typename T, int n, int k>
03192 using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03193
03198 template<typename T, int n, int k>
03199 static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03200
03201 namespace internal {
03202     template<typename T>
03203     struct pow_scalar {
03204         template<size_t p>
03205         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03206             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03207             x * func<p/2>(x) * func<p/2>(x);
03208         }
03209     };
03210
03211     template<typename T, typename p, size_t n, typename E = void>
03212     requires IsEuclideanDomain<T>
03213     struct pow;
03214
03215     template<typename T, typename p, size_t n>
03216     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
03217         using type = typename T::template mul_t<
03218             typename pow<T, p, n/2>::type,
03219             typename pow<T, p, n/2>::type
03220         >;
03221     };
03222
03223     template<typename T, typename p, size_t n>
03224     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
03225         using type = typename T::template mul_t<
03226             p,
03227             typename T::template mul_t<
03228                 typename pow<T, p, n/2>::type,
03229                 typename pow<T, p, n/2>::type
03230             >
03231         >;
03232     };
03233
03234     template<typename T, typename p, size_t n>
03235     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
03236 } // namespace internal
03237
03242 template<typename T, typename p, size_t n>
03243 using pow_t = typename internal::pow<T, p, n>::type;
03244
03249 template<typename T, typename p, size_t n>
03250 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03251
03252     template<typename T, size_t p>
03253     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03254         internal::pow_scalar<T>::template func<p>(x); }
03255
03256     namespace internal {
03257         template<typename, template<typename, size_t> typename, class>
03258         struct make_taylor_impl;
03259
03260         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03261         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...>> {
03262             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,

```

```

Is::type...>;
03262     };
03263     };
03264
03265     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03270     using taylor = typename internal::make_taylor_impl<
03271         T,
03272         coeff_at,
03273         internal::make_index_sequence_reverse<deg + 1>::type;
03274
03275     namespace internal {
03276         template<typename T, size_t i>
03277         struct exp_coeff {
03278             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03279         };
03280
03281         template<typename T, size_t i, typename E = void>
03282         struct sin_coeff_helper {};
03283
03284         template<typename T, size_t i>
03285         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03286             using type = typename FractionField<T>::zero;
03287         };
03288
03289         template<typename T, size_t i>
03290         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03291             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03292         };
03293
03294         template<typename T, size_t i>
03295         struct sin_coeff {
03296             using type = typename sin_coeff_helper<T, i>::type;
03297         };
03298
03299         template<typename T, size_t i, typename E = void>
03300         struct sh_coeff_helper {};
03301
03302         template<typename T, size_t i>
03303         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03304             using type = typename FractionField<T>::zero;
03305         };
03306
03307         template<typename T, size_t i>
03308         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03309             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03310         };
03311
03312         template<typename T, size_t i>
03313         struct sh_coeff {
03314             using type = typename sh_coeff_helper<T, i>::type;
03315         };
03316
03317         template<typename T, size_t i, typename E = void>
03318         struct cos_coeff_helper {};
03319
03320         template<typename T, size_t i>
03321         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03322             using type = typename FractionField<T>::zero;
03323         };
03324
03325         template<typename T, size_t i>
03326         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03327             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03328         };
03329
03330         template<typename T, size_t i>
03331         struct cos_coeff {
03332             using type = typename cos_coeff_helper<T, i>::type;
03333         };
03334
03335         template<typename T, size_t i, typename E = void>
03336         struct cosh_coeff_helper {};
03337
03338         template<typename T, size_t i>
03339         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03340             using type = typename FractionField<T>::zero;
03341         };
03342
03343         template<typename T, size_t i>
03344         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03345             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03346         };
03347
03348         template<typename T, size_t i>
03349         struct cosh_coeff {
03350             using type = typename cosh_coeff_helper<T, i>::type;
03351         };

```

```

03352
03353     template<typename T, size_t i>
03354     struct geom_coeff { using type = typename FractionField<T>::one; };
03355
03356
03357     template<typename T, size_t i, typename E = void>
03358     struct atan_coeff_helper;
03359
03360     template<typename T, size_t i>
03361     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03362         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03363     };
03364
03365     template<typename T, size_t i>
03366     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03367         using type = typename FractionField<T>::zero;
03368     };
03369
03370     template<typename T, size_t i>
03371     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03372
03373     template<typename T, size_t i, typename E = void>
03374     struct asin_coeff_helper;
03375
03376     template<typename T, size_t i>
03377     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03378         using type = makefraction_t<T,
03379             factorial_t<T, i - 1>,
03380             typename T::template mul_t<
03381                 typename T::template val<i>,
03382                 T::template mul_t<
03383                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03384                     pow_t<T, factorial_t<T, i / 2>, 2
03385                 >,
03386                 >
03387             >>;
03388     };
03389
03390     template<typename T, size_t i>
03391     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03392         using type = typename FractionField<T>::zero;
03393     };
03394
03395     template<typename T, size_t i>
03396     struct asin_coeff {
03397         using type = typename asin_coeff_helper<T, i>::type;
03398     };
03399
03400     template<typename T, size_t i>
03401     struct lnpl_coeff {
03402         using type = makefraction_t<T,
03403             alternate_t<T, i + 1>,
03404             typename T::template val<i>;
03405     };
03406
03407     template<typename T>
03408     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03409
03410     template<typename T, size_t i, typename E = void>
03411     struct asinh_coeff_helper;
03412
03413     template<typename T, size_t i>
03414     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03415         using type = makefraction_t<T,
03416             typename T::template mul_t<
03417                 alternate_t<T, i / 2>,
03418                 factorial_t<T, i - 1>
03419             >,
03420             typename T::template mul_t<
03421                 typename T::template mul_t<
03422                     typename T::template val<i>,
03423                     pow_t<T, factorial_t<T, i / 2>, 2>
03424                 >,
03425                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03426             >
03427         >;
03428     };
03429
03430     template<typename T, size_t i>
03431     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03432         using type = typename FractionField<T>::zero;
03433     };
03434
03435     template<typename T, size_t i>
03436     struct asinh_coeff {
03437         using type = typename asinh_coeff_helper<T, i>::type;
03438     };

```

```

03439
03440     template<typename T, size_t i, typename E = void>
03441     struct atanh_coeff_helper;
03442
03443     template<typename T, size_t i>
03444     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03445         // 1/i
03446         using type = typename FractionField<T>::template val<
03447             typename T::one,
03448             typename T::template inject_constant_t<i>;
03449     };
03450
03451     template<typename T, size_t i>
03452     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03453         using type = typename FractionField<T>::zero;
03454     };
03455
03456     template<typename T, size_t i>
03457     struct atanh_coeff {
03458         using type = typename atanh_coeff_helper<T, i>::type;
03459     };
03460
03461     template<typename T, size_t i, typename E = void>
03462     struct tan_coeff_helper;
03463
03464     template<typename T, size_t i>
03465     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03466         using type = typename FractionField<T>::zero;
03467     };
03468
03469     template<typename T, size_t i>
03470     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03471     private:
03472         // 4^((i+1)/2)
03473         using _4p = typename FractionField<T>::template inject_t<
03474             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03475         // 4^((i+1)/2) - 1
03476         using _4pml = typename FractionField<T>::template
03477             sub_t<_4p, typename FractionField<T>::one>;
03478         // (-1)^((i-1)/2)
03479         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03480         using dividend = typename FractionField<T>::template mul_t<
03481             altp,
03482             FractionField<T>::template mul_t<
03483                 _4p,
03484                 FractionField<T>::template mul_t<
03485                     _4pml,
03486                     bernoulli_t<T, (i + 1)>
03487                 >
03488             >;
03489     public:
03490         using type = typename FractionField<T>::template div_t<dividend,
03491             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03492     };
03493
03494     template<typename T, size_t i>
03495     struct tan_coeff {
03496         using type = typename tan_coeff_helper<T, i>::type;
03497     };
03498
03499     template<typename T, size_t i, typename E = void>
03500     struct tanh_coeff_helper;
03501
03502     template<typename T, size_t i>
03503     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03504         using type = typename FractionField<T>::zero;
03505     };
03506
03507     template<typename T, size_t i>
03508     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03509     private:
03510         using _4p = typename FractionField<T>::template inject_t<
03511             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03512         using _4pml = typename FractionField<T>::template
03513             sub_t<_4p, typename FractionField<T>::one>;
03514         using dividend =
03515             typename FractionField<T>::template mul_t<
03516                 _4p,
03517                 typename FractionField<T>::template mul_t<
03518                     _4pml,
03519                     bernoulli_t<T, (i + 1)>>::type;
03520     public:
03521         using type = typename FractionField<T>::template div_t<dividend,
03522             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03523     };

```

```

03524     template<typename T, size_t i>
03525     struct tanh_coeff {
03526         using type = typename tanh_coeff_helper<T, i>::type;
03527     };
03528 } // namespace internal
03529
03533 template<typename Integers, size_t deg>
03534 using exp = taylor<Integers, internal::exp_coeff, deg>;
03535
03539 template<typename Integers, size_t deg>
03540 using expm1 = typename polynomial<FractionField<Integers>>::template sub_t<
03541     exp<Integers, deg>,
03542     typename polynomial<FractionField<Integers>>::one>;
03543
03547 template<typename Integers, size_t deg>
03548 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03549
03553 template<typename Integers, size_t deg>
03554 using atan = taylor<Integers, internal::atan_coeff, deg>;
03555
03559 template<typename Integers, size_t deg>
03560 using sin = taylor<Integers, internal::sin_coeff, deg>;
03561
03565 template<typename Integers, size_t deg>
03566 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03567
03572 template<typename Integers, size_t deg>
03573 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03574
03579 template<typename Integers, size_t deg>
03580 using cos = taylor<Integers, internal::cos_coeff, deg>;
03581
03586 template<typename Integers, size_t deg>
03587 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03588
03593 template<typename Integers, size_t deg>
03594 using asin = taylor<Integers, internal::asin_coeff, deg>;
03595
03600 template<typename Integers, size_t deg>
03601 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03602
03607 template<typename Integers, size_t deg>
03608 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03609
03614 template<typename Integers, size_t deg>
03615 using tan = taylor<Integers, internal::tan_coeff, deg>;
03616
03621 template<typename Integers, size_t deg>
03622 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03623 } // namespace aerobus
03624
03625 // continued fractions
03626 namespace aerobus {
03629     template<int64_t... values>
03630     struct ContinuedFraction {};
03631
03634     template<int64_t a0>
03635     struct ContinuedFraction<a0> {
03637         using type = typename q64::template inject_constant_t<a0>;
03639         static constexpr double val = static_cast<double>(a0);
03640     };
03641
03645     template<int64_t a0, int64_t... rest>
03646     struct ContinuedFraction<a0, rest...> {
03648         using type = q64::template add_t<
03649             typename q64::template inject_constant_t<a0>,
03650             typename q64::template div_t<
03651                 typename q64::one,
03652                 typename ContinuedFraction<rest...>::type
03653             >;
03654
03656         static constexpr double val = type::template get<double>();
03657     };
03658
03662     using PI_fraction =
03663     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03664     using E_fraction =
03665     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03666     using SQRT2_fraction =
03667     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03668     using SQRT3_fraction =
03669     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03669 } // namespace aerobus
03670
03671 // known polynomials
03672 namespace aerobus {

```

```

03673 // CChebyshev
03674 namespace internal {
03675     template<int kind, size_t deg, typename I>
03676     struct chebyshev_helper {
03677         using type = typename polynomial<I>::template sub_t<
03678             typename polynomial<I>::template mul_t<
03679                 typename polynomial<I>::template mul_t<
03680                     typename polynomial<I>::template inject_constant_t<2>,
03681                     typename polynomial<I>::X>,
03682                     typename chebyshev_helper<kind, deg - 1, I>::type
03683                 >,
03684                 typename chebyshev_helper<kind, deg - 2, I>::type
03685             >;
03686     };
03687
03688     template<typename I>
03689     struct chebyshev_helper<1, 0, I> {
03690         using type = typename polynomial<I>::one;
03691     };
03692
03693     template<typename I>
03694     struct chebyshev_helper<1, 1, I> {
03695         using type = typename polynomial<I>::X;
03696     };
03697
03698     template<typename I>
03699     struct chebyshev_helper<2, 0, I> {
03700         using type = typename polynomial<I>::one;
03701     };
03702
03703     template<typename I>
03704     struct chebyshev_helper<2, 1, I> {
03705         using type = typename polynomial<I>::template mul_t<
03706             typename polynomial<I>::template inject_constant_t<2>,
03707             typename polynomial<I>::X>;
03708     };
03709 } // namespace internal
03710
03711 // Laguerre
03712 namespace internal {
03713     template<size_t deg, typename I>
03714     struct laguerre_helper {
03715         using Q = FractionField<I>;
03716         using PQ = polynomial<Q>;
03717
03718     private:
03719         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03720         using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03721         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03722         // -x + 2k-1
03723         using p = typename PQ::template val<
03724             typename Q::template inject_constant_t<-1>,
03725             typename Q::template inject_constant_t<2 * deg - 1>;
03726         // 1/n
03727         using factor = typename PQ::template inject_ring_t<
03728             typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>;
03729
03730     public:
03731         using type = typename PQ::template mul_t <
03732             factor,
03733             typename PQ::template sub_t<
03734                 typename PQ::template mul_t<
03735                     p,
03736                     lnm1
03737                 >,
03738                 typename PQ::template mul_t<
03739                     typename PQ::template inject_constant_t<deg-1>,
03740                     lnm2
03741                 >
03742             >
03743         >;
03744     };
03745
03746     template<typename I>
03747     struct laguerre_helper<0, I> {
03748         using type = typename polynomial<FractionField<I>::one;
03749     };
03750
03751     template<typename I>
03752     struct laguerre_helper<1, I> {
03753     private:
03754         using PQ = polynomial<FractionField<I>;
03755     public:
03756         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03757     };
03758 } // namespace internal

```

```

03759
03760 // Bernstein
03761 namespace internal {
03762     template<size_t i, size_t m, typename I, typename E = void>
03763     struct bernstein_helper {};
03764
03765     template<typename I>
03766     struct bernstein_helper<0, 0, I> {
03767         using type = typename polynomial<I>::one;
03768     };
03769
03770     template<size_t i, size_t m, typename I>
03771     struct bernstein_helper<i, m, I, std::enable_if_t<
03772         (m > 0) && (i == 0)>> {
03773     private:
03774         using P = polynomial<I>;
03775     public:
03776         using type = typename P::template mul_t<
03777             typename P::template sub_t<typename P::one, typename P::X>,
03778             typename bernstein_helper<i, m-1, I>::type>;
03779     };
03780
03781     template<size_t i, size_t m, typename I>
03782     struct bernstein_helper<i, m, I, std::enable_if_t<
03783         (m > 0) && (i == m)>> {
03784     private:
03785         using P = polynomial<I>;
03786     public:
03787         using type = typename P::template mul_t<
03788             typename P::X,
03789             typename bernstein_helper<i-1, m-1, I>::type>;
03790     };
03791
03792     template<size_t i, size_t m, typename I>
03793     struct bernstein_helper<i, m, I, std::enable_if_t<
03794         (m > 0) && (i > 0) && (i < m)>> {
03795     private:
03796         using P = polynomial<I>;
03797     public:
03798         using type = typename P::template add_t<
03799             typename P::template mul_t<
03800                 typename P::template sub_t<typename P::one, typename P::X>,
03801                 typename bernstein_helper<i, m-1, I>::type>,
03802                 typename P::template mul_t<
03803                     typename P::X,
03804                     typename bernstein_helper<i-1, m-1, I>::type>;
03805             >;
03806     };
03807 } // namespace internal
03808
03809 // AllOne polynomials
03810 namespace internal {
03811     template<size_t deg, typename I>
03812     struct AllOneHelper {
03813     private:
03814         using type = aerobus::add_t<
03815             typename polynomial<I>::one,
03816             typename aerobus::mul_t<
03817                 typename polynomial<I>::X,
03818                 typename AllOneHelper<deg-1, I>::type
03819             >>;
03820     };
03821
03822     template<typename I>
03823     struct AllOneHelper<0, I> {
03824     private:
03825         using type = typename polynomial<I>::one;
03826     };
03827 } // namespace internal
03828
03829 // Bessel polynomials
03830 namespace internal {
03831     template<size_t deg, typename I>
03832     struct BesselHelper {
03833     private:
03834         using P = polynomial<I>;
03835         using factor = typename P::template monomial_t<
03836             typename I::template inject_constant_t<(2*deg - 1)>,
03837             1>;
03838     public:
03839         using type = typename P::template add_t<
03840             typename P::template mul_t<
03841                 factor,
03842                 typename BesselHelper<deg-1, I>::type
03843             >,
03844             typename BesselHelper<deg-2, I>::type
03845         >;
03846     };
03847
03848     template<typename I>

```

```

03846     struct BesselHelper<0, I> {
03847         using type = typename polynomial<I>::one;
03848     };
03849
03850     template<typename I>
03851     struct BesselHelper<1, I> {
03852     private:
03853         using P = polynomial<I>;
03854     public:
03855         using type = typename P::template add_t<
03856             typename P::one,
03857             typename P::X
03858         >;
03859     };
03860 } // namespace internal
03861
03862 namespace known_polynomials {
03863     enum hermite_kind {
03864         probabilist,
03865         physicist
03866     };
03867 }
03870
03871 // hermite
03872 namespace internal {
03873     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03874     struct hermite_helper {};
03875
03876     template<size_t deg, typename I>
03877     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03878     private:
03879         using hnm1 = typename hermite_helper<deg - 1,
03880 known_polynomials::hermite_kind::probabilist, I>::type;
03881         using hnm2 = typename hermite_helper<deg - 2,
03882 known_polynomials::hermite_kind::probabilist, I>::type;
03883     public:
03884         using type = typename polynomial<I>::template sub_t<
03885             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03886             typename polynomial<I>::template mul_t<
03887                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03888                 hnm2
03889             >
03890         >;
03891     };
03892
03893     template<size_t deg, typename I>
03894     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03895     private:
03896         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03897 I>::type;
03898         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03899 I>::type;
03900     public:
03901         using type = typename polynomial<I>::template sub_t<
03902             // 2X Hn-1
03903             typename polynomial<I>::template mul_t<
03904                 typename pi64::val<typename I::template inject_constant_t<2>,
03905                 typename I::zero>, hnm1>,
03906             typename polynomial<I>::template mul_t<
03907                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03908                 hnm2
03909             >
03910         >;
03911     };
03912
03913     template<typename I>
03914     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03915         using type = typename polynomial<I>::one;
03916     };
03917
03918     template<typename I>
03919     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03920         using type = typename polynomial<I>::X;
03921     };
03922
03923     template<typename I>
03924     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03925         using type = typename pi64::one;
03926     };
03927
03928     template<typename I>
03929     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03930         // 2X
03931         using type = typename polynomial<I>::template val<

```



```

03932         typename I::template inject_constant_t<2>,
03933         typename I::zero>;
03934     };
03935 } // namespace internal
03936
03937 // legendre
03938 namespace internal {
03939     template<size_t n, typename I>
03940     struct legendre_helper {
03941     private:
03942         using Q = FractionField<I>;
03943         using PQ = polynomial<Q>;
03944         // 1/n constant
03945         // (2n-1)/n X
03946         using fact_left = typename PQ::template monomial_t<
03947             makefraction_t<I,
03948                 typename I::template inject_constant_t<2*n-1>,
03949                 typename I::template inject_constant_t<n>
03950             >,
03951             1>;
03952         // (n-1) / n
03953         using fact_right = typename PQ::template val<
03954             makefraction_t<I,
03955                 typename I::template inject_constant_t<n-1>,
03956                 typename I::template inject_constant_t<n>>;
03957     public:
03958         using type = PQ::template sub_t<
03959             typename PQ::template mul_t<
03960                 fact_left,
03961                 typename legendre_helper<n-1, I>::type
03962             >,
03963             typename PQ::template mul_t<
03964                 fact_right,
03965                 typename legendre_helper<n-2, I>::type
03966             >
03967         >;
03968     };
03969 };
03970
03971 template<typename I>
03972 struct legendre_helper<0, I> {
03973     using type = typename polynomial<FractionField<I>::one>;
03974 };
03975
03976 template<typename I>
03977 struct legendre_helper<1, I> {
03978     using type = typename polynomial<FractionField<I>::X>;
03979 };
03980 } // namespace internal
03981
03982 // bernoulli polynomials
03983 namespace internal {
03984     template<size_t n>
03985     struct bernoulli_coeff {
03986     template<typename T, size_t i>
03987     struct inner {
03988     private:
03989         using F = FractionField<T>;
03990     public:
03991         using type = typename F::template mul_t<
03992             typename F::template inject_ring_t<combination_t<T, i, n>,
03993             bernoulli_t<T, n-i>
03994         >;
03995     };
03996     };
03997 } // namespace internal
03998
03999 namespace internal {
04000     template<size_t n>
04001     struct touchard_coeff {
04002     template<typename T, size_t i>
04003     struct inner {
04004         using type = stirling_2_t<T, n, i>;
04005     };
04006     };
04007 } // namespace internal
04008
04009 namespace internal {
04010     template<typename I = aerobus::i64>
04011     struct AbelHelper {
04012     private:
04013         using P = aerobus::polynomial<I>;
04014
04015     public:
04016         // to keep recursion working, we need to operate on a*n and not just a
04017         template<size_t deg, I::inner_type an>
04018         struct Inner {

```

```

04019         // Abel(n, a) = (x-an) * Abel(n-1, a)
04020         using type = typename aerobus::mul_t<
04021             typename Inner<deg-1, an>::type,
04022             typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
04023         >;
04024     };
04025
04026     // Abel(0, a) = 1
04027     template<I::inner_type an>
04028     struct Inner<0, an> {
04029         using type = P::one;
04030     };
04031
04032     // Abel(1, a) = X
04033     template<I::inner_type an>
04034     struct Inner<1, an> {
04035         using type = P::X;
04036     };
04037 };
04038 } // namespace internal
04039
04040 namespace known_polynomials {
04041
04042     template<size_t n, auto a, typename I = aerobus::i64>
04043     using Abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
04044
04045     template<size_t deg, typename I = aerobus::i64>
04046     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
04047
04048     template<size_t deg, typename I = aerobus::i64>
04049     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
04050
04051     template<size_t deg, typename I = aerobus::i64>
04052     using laguerre = typename internal::laguerre_helper<deg, I>::type;
04053
04054     template<size_t deg, typename I = aerobus::i64>
04055     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
04056         I>::type;
04057
04058     template<size_t deg, typename I = aerobus::i64>
04059     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04060
04061     template<size_t i, size_t m, typename I = aerobus::i64>
04062     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04063
04064     template<size_t deg, typename I = aerobus::i64>
04065     using legendre = typename internal::legendre_helper<deg, I>::type;
04066
04067     template<size_t deg, typename I = aerobus::i64>
04068     using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
04069
04070     template<size_t deg, typename I = aerobus::i64>
04071     using allone = typename internal::AllOneHelper<deg, I>::type;
04072
04073     template<size_t deg, typename I = aerobus::i64>
04074     using bessel = typename internal::BesselHelper<deg, I>::type;
04075
04076     template<size_t deg, typename I = aerobus::i64>
04077     using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
04078 } // namespace known_polynomials
04079 } // namespace aerobus
04080
04081 #ifndef AEROBUS_CONWAY_IMPORTS
04082
04083 // Conway polynomials
04084 namespace aerobus {
04085     template<int p, int n>
04086     struct ConwayPolynomial {};
04087
04088 #ifndef DO_NOT_DOCUMENT
04089     #define ZPV ZPV::template val
04090     #define POLYV aerobus::polynomial<ZPV>::template val
04091     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
04092     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
04093     template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
04094     template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
04095     template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>; }; // NOLINT
04096     template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
04097     template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpv<2>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT

```

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```

06098     template<> struct ConwayPolynomial<977, 6> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<729>, ZPZV<830>, ZPZV<753>, ZPZV<3>; }; // NOLINT
06099     template<> struct ConwayPolynomial<977, 7> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<974>; }; // NOLINT
06100     template<> struct ConwayPolynomial<977, 8> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<855>, ZPZV<807>, ZPZV<77>, ZPZV<3>; }; //
NOLINT
06101     template<> struct ConwayPolynomial<977, 9> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<450>, ZPZV<740>, ZPZV<974>;
}; // NOLINT
06102     template<> struct ConwayPolynomial<983, 1> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<978>; }; // NOLINT
06103     template<> struct ConwayPolynomial<983, 2> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<981>, ZPZV<5>; }; // NOLINT
06104     template<> struct ConwayPolynomial<983, 3> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<978>; }; // NOLINT
06105     template<> struct ConwayPolynomial<983, 4> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<5>, ZPZV<567>, ZPZV<5>; }; // NOLINT
06106     template<> struct ConwayPolynomial<983, 5> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<8>, ZPZV<978>; }; // NOLINT
06107     template<> struct ConwayPolynomial<983, 6> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<849>, ZPZV<296>, ZPZV<228>, ZPZV<5>; }; // NOLINT
06108     template<> struct ConwayPolynomial<983, 7> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<978>; }; // NOLINT
06109     template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<738>, ZPZV<276>, ZPZV<530>, ZPZV<5>; }; //
NOLINT
06110     template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<858>, ZPZV<87>, ZPZV<978>;
}; // NOLINT
06111     template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<985>; }; // NOLINT
06112     template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<989>, ZPZV<6>; }; // NOLINT
06113     template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<985>; }; // NOLINT
06114     template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<10>, ZPZV<794>, ZPZV<6>; }; // NOLINT
06115     template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<985>; }; // NOLINT
06116     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
06117     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
06118     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
06119     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
06120     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
06121     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
06122     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
06123     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
06124     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
06125     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
06126     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
06127     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
06128     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
06129 #endif // DO_NOT_DOCUMENT
06130 } // namespace aerobus
06131 #endif // AEROBUS_CONWAY_IMPORTS
06132
06133 #endif // __INC_AEROBUS__ // NOLINT

```

## 9.4 src/examples.h File Reference

## 9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
```



```
00002 #define SRC_EXAMPLES_H_  
00050 #endif // SRC_EXAMPLES_H_
```



# Chapter 10

## Examples

### 10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

### 10.2 examples/custom\_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

## 10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using aerobus::i16 Warning : due to an NVIDIA bug (lack of constexpr operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
<<<< HEAD
// TO GET optimal performances, modify cuda_fp16.h by adding __CUDA_FP16_CONSTEXPR__ to line 5039 (version
12.6)
=====
// Beforehand, you need to modify cuda_fp16.h by adding __CUDA_FP16_CONSTEXPR__ to line 5039 (version 12.6)
>>>> main
#include <stdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

/*
You may want to change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
*/
using int_type = aerobus::i16;
using float_type = __half2;

constexpr size_t N = 1 << 26;

template<typename T>
struct ExpmlDegree;

template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};

template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};

template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};

template<>
struct ExpmlDegree<__half> {
    static constexpr size_t val = 6;
};

double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

template<typename T>
struct GetRandT;

template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};

template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};

template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};

template<>
struct GetRandT<__half> {
    static __half func(double min, double max) {
        return __float2half((float)rand(min, max));
    }
};
```

```

},
using EXPM1 = aerobus::expml<int_type, ExpmlDegree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
<<<<< HEAD
        out[i] = f(f(f(f(f(f(f(f(f(f(in[i])))))))))));
=====
        // fp16 FMA pipeline is quite wide so we need to feed it with a LOT of computations
        out[i] = f(f(f(f(f(f(f(f(f(f(f(f(f(f(f(f(in[i])))))))))))))));
>>>>> main
    }
}

#define cudaErrorCheck(ans) { gpuAssert((ans), __FILE__, __LINE__); }
inline void gpuAssert(cudaError_t code, const char *file, int line, bool abort=true)
{
    if (code != cudaSuccess)
    {
        fprintf(stderr, "GPUAssert: %s %s %d\n", cudaGetErrorString(code), file, line);
        if (abort) exit(code);
    }
}

int main() {
    // configure CUDA devices
    int deviceCount;
    int device = -1;
    int maxProcCount = 0;
    cudaErrorCheck(cudaGetDeviceCount(&deviceCount));
    for(int i = 0; i < deviceCount; ++i) {
        cudaDeviceProp prop;
        cudaErrorCheck(cudaGetDeviceProperties(&prop, i));
        int procCount = prop.multiProcessorCount;
        if(procCount > maxProcCount) {
            maxProcCount = procCount;
            device = i;
        }
    }

    if(device == -1) {
        ::printf("CANNOT FIND CUDA CAPABLE DEVICE -- aborting\n");
        ::abort();
    }

    cudaErrorCheck(cudaSetDevice(device));
    int blockSize; // The launch configurator returned block size
    int minGridSize; // The minimum grid size needed to achieve the
                    // maximum occupancy for a full device launch

    cudaErrorCheck(cudaOccupancyMaxPotentialBlockSize( &minGridSize, &blockSize, &run, 0, 0));

    ::printf("configure launch bounds to %d-%d\n", minGridSize, blockSize);

    // allocate and populate memory
    float_type *d_in, *d_out;
    cudaErrorCheck(cudaMalloc<float_type>(&d_in, N * sizeof(float_type)));
    cudaErrorCheck(cudaMalloc<float_type>(&d_out, N * sizeof(float_type)));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaErrorCheck(cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice));

    // execute kernel and get memory back from device
    run<<minGridSize, blockSize>>(N, d_in, d_out);
    cudaErrorCheck(cudaPeekAtLastError());
    cudaErrorCheck(cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost));

    cudaErrorCheck(cudaFree(d_in));
    cudaErrorCheck(cudaFree(d_out));
}

// Example of generated SASS :

/*
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;

```

```
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R7, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R7, R5, R7, 0.008331298828125, 0.008331298828125 ;
HFMA2 R7, R5, R7, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R7, R5, R7, 0.1666259765625, 0.1666259765625 ;
HFMA2 R7, R5, R7, 0.5, 0.5 ;
HFMA2.MMA R7, R5, R7, 1, 1 ;
HFMA2 R7, R5, R7, RZ.H0_H0 ;
HFMA2.MMA R5, R7, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R7, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R7, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R7, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R7, R5, 0.5, 0.5 ;
HFMA2 R5, R7, R5, 1, 1 ;
HFMA2.MMA R5, R7, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
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HFMA2 R6, R5, R6, RZ.H0_H0 ;
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HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
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HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R6, R6, R5, RZ ;
HFMA2 R5, R6, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2 R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2 R5, R6, R5, 0.5, 0.5 ;
HFMA2.MMA R7, R6, R5, 1, 1 ;
IADD3.X R5, R8, UR11, RZ, P0, !PT ;
IADD3 R3, P0, R2, R3, RZ ;
IADD3.X R0, RZ, R0, RZ, P0, !PT ;
ISETP.GE.U32.AND P0, PT, R3, UR8, PT ;
HFMA2 R7, R6, R7, RZ.H0_H0 ;
```

```

ISETP.GE.U32.AND.EX P0, PT, R0, UR9, PT, P0 ;
STG.E desc[UR6][R4.64], R7 ;
*/

```

## 10.4 examples/continued\_fractions.cpp

## How to use `aerobus::ContinuedFraction` to get approximations of known numbers

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

static constexpr double PHI = aerobus::ContinuedFraction<
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1>::val;

static const double phi = (std::sqrt(5.0) + 1.0)/2.0;

int main() {
    std::cout << std::setprecision(15) << "Aerobus PHI : " << PHI << std::endl;
    std::cout << std::setprecision(15) << "Computed PHI : " << phi << std::endl;
    return 0;
}
```

## 10.5 examples/modular\_arithmetic.cpp

## How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

//  $x^3 + 2x^2 + 1$ , with coefficients in  $\mathbb{Z}/2\mathbb{Z}$ , actually  $x^3 + 1$ 
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
//  $x^3 + 5x^2 + 7x + 11$  with coefficients in  $\mathbb{Z}/17\mathbb{Z}$ , meaning actually  $x^3 + x^2 + 1$ 
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

## 10.6 examples/make polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
```

```

struct Inner<0, an> {
    using type = P::one;
};

// abel(1, a) = X
template<I::inner_type an>
struct Inner<1, an> {
    using type = P::X;
};

};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}

```

## 10.7 examples/polynomials\_over\_finite\_field.cpp

How to build a known polynomial (here aerobus::known\_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```

#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}

```

## 10.8 examples/compensated\_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```

// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

```



```

}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;

        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}

```



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