

Aerobus

v1.2

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<b>1 Introduction</b>	<b>1</b>
1.1 HOW TO	1
1.1.1 Unit Test	2
1.1.2 Benchmarks	2
1.2 Structures	2
1.2.1 Predefined discrete euclidean domains	2
1.2.2 Polynomials	3
1.2.3 Known polynomials	3
1.2.4 Conway polynomials	3
1.2.5 Taylor series	4
1.3 Operations	5
1.3.1 Field of fractions	5
1.3.2 Quotient	6
1.4 Misc	6
1.4.1 Continued Fractions	6
1.5 CUDA	6
<b>2 Namespace Index</b>	<b>7</b>
2.1 Namespace List	7
<b>3 Concept Index</b>	<b>9</b>
3.1 Concepts	9
<b>4 Class Index</b>	<b>11</b>
4.1 Class List	11
<b>5 File Index</b>	<b>13</b>
5.1 File List	13
<b>6 Namespace Documentation</b>	<b>15</b>
6.1 aerobus Namespace Reference	15
6.1.1 Detailed Description	19
6.1.2 Typedef Documentation	20
6.1.2.1 abs_t	20
6.1.2.2 add_t	20
6.1.2.3 addfractions_t	20
6.1.2.4 alternate_t	20
6.1.2.5 asin	21
6.1.2.6 asinh	21
6.1.2.7 atan	21
6.1.2.8 atanh	21
6.1.2.9 bell_t	23
6.1.2.10 bernoulli_t	23
6.1.2.11 combination_t	23

6.1.2.12 cos	23
6.1.2.13 cosh	25
6.1.2.14 div_t	25
6.1.2.15 E_fraction	25
6.1.2.16 embed_int_poly_in_fractions_t	25
6.1.2.17 exp	26
6.1.2.18 expm1	26
6.1.2.19 factorial_t	26
6.1.2.20 fpq32	26
6.1.2.21 fpq64	27
6.1.2.22 FractionField	27
6.1.2.23 gcd_t	27
6.1.2.24 geometric_sum	27
6.1.2.25 lnp1	28
6.1.2.26 make_frac_polynomial_t	28
6.1.2.27 make_int_polynomial_t	28
6.1.2.28 make_q32_t	28
6.1.2.29 make_q64_t	29
6.1.2.30 makefraction_t	29
6.1.2.31 mul_t	29
6.1.2.32 mulfractions_t	30
6.1.2.33 pi64	30
6.1.2.34 PI_fraction	30
6.1.2.35 pow_t	30
6.1.2.36 pq64	30
6.1.2.37 q32	31
6.1.2.38 q64	31
6.1.2.39 sin	31
6.1.2.40 sinh	31
6.1.2.41 SQRT2_fraction	31
6.1.2.42 SQRT3_fraction	32
6.1.2.43 stirling_1_signed_t	32
6.1.2.44 stirling_1_unsigned_t	32
6.1.2.45 stirling_2_t	32
6.1.2.46 sub_t	33
6.1.2.47 tan	33
6.1.2.48 tanh	33
6.1.2.49 taylor	33
6.1.2.50 vadd_t	34
6.1.2.51 vmul_t	34
6.1.3 Function Documentation	34
6.1.3.1 aligned_malloc()	34

6.1.3.2 field()	34
6.1.4 Variable Documentation	35
6.1.4.1 alternate_v	35
6.1.4.2 bernoulli_v	35
6.1.4.3 combination_v	35
6.1.4.4 factorial_v	36
6.2 aerobus::internal Namespace Reference	36
6.2.1 Detailed Description	39
6.2.2 Typedef Documentation	40
6.2.2.1 make_index_sequence_reverse	40
6.2.2.2 type_at_t	40
6.2.3 Function Documentation	40
6.2.3.1 index_sequence_reverse()	40
6.2.4 Variable Documentation	40
6.2.4.1 is_instantiation_of_v	40
6.3 aerobus::known_polynomials Namespace Reference	40
6.3.1 Detailed Description	40
6.3.2 Enumeration Type Documentation	40
6.3.2.1 hermite_kind	40
<b>7 Concept Documentation</b>	<b>43</b>
7.1 aerobus::IsEuclideanDomain Concept Reference	43
7.1.1 Concept definition	43
7.1.2 Detailed Description	43
7.2 aerobus::IsField Concept Reference	43
7.2.1 Concept definition	43
7.2.2 Detailed Description	44
7.3 aerobus::IsRing Concept Reference	44
7.3.1 Concept definition	44
7.3.2 Detailed Description	44
<b>8 Class Documentation</b>	<b>45</b>
8.1 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E > Struct Template Reference	45
8.2 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0  index > 0)> > Struct Template Reference	45
8.2.1 Member Typedef Documentation	45
8.2.1.1 type	45
8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference	46
8.3.1 Member Typedef Documentation	46
8.3.1.1 type	46
8.4 aerobus::ContinuedFraction< values > Struct Template Reference	46
8.4.1 Detailed Description	46

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference	47
8.5.1 Detailed Description	47
8.5.2 Member Typedef Documentation	47
8.5.2.1 type	47
8.5.3 Member Data Documentation	48
8.5.3.1 val	48
8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference	48
8.6.1 Detailed Description	48
8.6.2 Member Typedef Documentation	49
8.6.2.1 type	49
8.6.3 Member Data Documentation	49
8.6.3.1 val	49
8.7 aerobus::ConwayPolynomial Struct Reference	49
8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference	49
8.8.1 Member Function Documentation	50
8.8.1.1 func()	50
8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference	50
8.9.1 Member Function Documentation	50
8.9.1.1 func()	50
8.10 aerobus::Embed< Small, Large, E > Struct Template Reference	51
8.10.1 Detailed Description	51
8.11 aerobus::Embed< i32, i64 > Struct Reference	51
8.11.1 Detailed Description	51
8.11.2 Member Typedef Documentation	51
8.11.2.1 type	51
8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference	52
8.12.1 Detailed Description	52
8.12.2 Member Typedef Documentation	52
8.12.2.1 type	52
8.13 aerobus::Embed< q32, q64 > Struct Reference	53
8.13.1 Detailed Description	53
8.13.2 Member Typedef Documentation	53
8.13.2.1 type	53
8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference	54
8.14.1 Detailed Description	54
8.14.2 Member Typedef Documentation	54
8.14.2.1 type	54
8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference	55
8.15.1 Detailed Description	55
8.15.2 Member Typedef Documentation	55
8.15.2.1 type	55

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference	55
8.16.1 Detailed Description	56
8.16.2 Member Typedef Documentation	56
8.16.2.1 type	56
8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference	56
8.17.1 Detailed Description	57
8.18 aerobus::i32 Struct Reference	57
8.18.1 Detailed Description	58
8.18.2 Member Typedef Documentation	59
8.18.2.1 add_t	59
8.18.2.2 div_t	59
8.18.2.3 eq_t	59
8.18.2.4 gcd_t	59
8.18.2.5 gt_t	60
8.18.2.6 inject_constant_t	60
8.18.2.7 inject_ring_t	60
8.18.2.8 inner_type	60
8.18.2.9 lt_t	60
8.18.2.10 mod_t	61
8.18.2.11 mul_t	61
8.18.2.12 one	61
8.18.2.13 pos_t	61
8.18.2.14 sub_t	62
8.18.2.15 zero	62
8.18.3 Member Data Documentation	62
8.18.3.1 eq_v	62
8.18.3.2 is_euclidean_domain	62
8.18.3.3 is_field	62
8.18.3.4 pos_v	63
8.19 aerobus::i64 Struct Reference	64
8.19.1 Detailed Description	65
8.19.2 Member Typedef Documentation	65
8.19.2.1 add_t	65
8.19.2.2 div_t	66
8.19.2.3 eq_t	66
8.19.2.4 gcd_t	66
8.19.2.5 gt_t	66
8.19.2.6 inject_constant_t	67
8.19.2.7 inject_ring_t	67
8.19.2.8 inner_type	67
8.19.2.9 lt_t	67
8.19.2.10 mod_t	68

8.19.2.11 mul_t	68
8.19.2.12 one	68
8.19.2.13 pos_t	68
8.19.2.14 sub_t	68
8.19.2.15 zero	69
8.19.3 Member Data Documentation	69
8.19.3.1 eq_v	69
8.19.3.2 gt_v	69
8.19.3.3 is_euclidean_domain	69
8.19.3.4 is_field	70
8.19.3.5 lt_v	70
8.19.3.6 pos_v	70
8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference	70
8.20.1 Member Typedef Documentation	71
8.20.1.1 type	71
8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference	71
8.21.1 Member Typedef Documentation	71
8.21.1.1 type	71
8.22 aerobus::is_prime< n > Struct Template Reference	71
8.22.1 Detailed Description	72
8.22.2 Member Data Documentation	72
8.22.2.1 value	72
8.23 aerobus::polynomial< Ring > Struct Template Reference	72
8.23.1 Detailed Description	74
8.23.2 Member Typedef Documentation	74
8.23.2.1 add_t	74
8.23.2.2 derive_t	74
8.23.2.3 div_t	75
8.23.2.4 eq_t	75
8.23.2.5 gcd_t	75
8.23.2.6 gt_t	75
8.23.2.7 inject_constant_t	76
8.23.2.8 inject_ring_t	76
8.23.2.9 lt_t	76
8.23.2.10 mod_t	76
8.23.2.11 monomial_t	77
8.23.2.12 mul_t	77
8.23.2.13 one	77
8.23.2.14 pos_t	77
8.23.2.15 simplify_t	79
8.23.2.16 sub_t	79



8.23.2.17 X	79
8.23.2.18 zero	79
8.23.3 Member Data Documentation	80
8.23.3.1 is_euclidean_domain	80
8.23.3.2 is_field	80
8.23.3.3 pos_v	80
8.24 aerobus::type_list< Ts >::pop_front Struct Reference	80
8.24.1 Detailed Description	80
8.24.2 Member Typedef Documentation	81
8.24.2.1 tail	81
8.24.2.2 type	81
8.25 aerobus::Quotient< Ring, X > Struct Template Reference	81
8.25.1 Detailed Description	82
8.25.2 Member Typedef Documentation	82
8.25.2.1 add_t	82
8.25.2.2 div_t	83
8.25.2.3 eq_t	83
8.25.2.4 inject_constant_t	83
8.25.2.5 inject_ring_t	84
8.25.2.6 mod_t	84
8.25.2.7 mul_t	84
8.25.2.8 one	84
8.25.2.9 pos_t	85
8.25.2.10 zero	85
8.25.3 Member Data Documentation	85
8.25.3.1 eq_v	85
8.25.3.2 is_euclidean_domain	85
8.25.3.3 pos_v	85
8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference	86
8.26.1 Detailed Description	86
8.26.2 Member Typedef Documentation	86
8.26.2.1 head	86
8.26.2.2 tail	86
8.27 aerobus::type_list< Ts > Struct Template Reference	87
8.27.1 Detailed Description	87
8.27.2 Member Typedef Documentation	88
8.27.2.1 at	88
8.27.2.2 concat	88
8.27.2.3 insert	88
8.27.2.4 push_back	89
8.27.2.5 push_front	89
8.27.2.6 remove	89

8.27.3 Member Data Documentation	89
8.27.3.1 length	89
8.28 aerobus::type_list<> Struct Reference	90
8.28.1 Detailed Description	90
8.28.2 Member Typedef Documentation	90
8.28.2.1 concat	90
8.28.2.2 insert	90
8.28.2.3 push_back	90
8.28.2.4 push_front	90
8.28.3 Member Data Documentation	91
8.28.3.1 length	91
8.29 aerobus::i32::val< x > Struct Template Reference	91
8.29.1 Detailed Description	91
8.29.2 Member Typedef Documentation	92
8.29.2.1 enclosing_type	92
8.29.2.2 is_zero_t	92
8.29.3 Member Function Documentation	92
8.29.3.1 get()	92
8.29.3.2 to_string()	92
8.29.4 Member Data Documentation	92
8.29.4.1 v	92
8.30 aerobus::i64::val< x > Struct Template Reference	93
8.30.1 Detailed Description	93
8.30.2 Member Typedef Documentation	94
8.30.2.1 enclosing_type	94
8.30.2.2 inner_type	94
8.30.2.3 is_zero_t	94
8.30.3 Member Function Documentation	94
8.30.3.1 get()	94
8.30.3.2 to_string()	94
8.30.4 Member Data Documentation	95
8.30.4.1 v	95
8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference	95
8.31.1 Detailed Description	96
8.31.2 Member Typedef Documentation	96
8.31.2.1 aN	96
8.31.2.2 coeff_at_t	96
8.31.2.3 enclosing_type	97
8.31.2.4 is_zero_t	97
8.31.2.5 ring_type	97
8.31.2.6 strip	97
8.31.2.7 value_at_t	97

8.31.3 Member Function Documentation	97
8.31.3.1 compensated_eval()	97
8.31.3.2 eval()	98
8.31.3.3 to_string()	98
8.31.4 Member Data Documentation	99
8.31.4.1 degree	99
8.31.4.2 is_zero_v	99
8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference	99
8.32.1 Detailed Description	99
8.32.2 Member Typedef Documentation	100
8.32.2.1 raw_t	100
8.32.2.2 type	100
8.33 aerobus::zpz< p >::val< x > Struct Template Reference	100
8.33.1 Detailed Description	100
8.33.2 Member Typedef Documentation	101
8.33.2.1 enclosing_type	101
8.33.2.2 is_zero_t	101
8.33.3 Member Function Documentation	101
8.33.3.1 get()	101
8.33.3.2 to_string()	101
8.33.4 Member Data Documentation	102
8.33.4.1 is_zero_v	102
8.33.4.2 v	102
8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference	102
8.34.1 Detailed Description	103
8.34.2 Member Typedef Documentation	103
8.34.2.1 aN	103
8.34.2.2 coeff_at_t	103
8.34.2.3 enclosing_type	103
8.34.2.4 is_zero_t	104
8.34.2.5 ring_type	104
8.34.2.6 strip	104
8.34.2.7 value_at_t	104
8.34.3 Member Function Documentation	104
8.34.3.1 compensated_eval()	104
8.34.3.2 eval()	104
8.34.3.3 to_string()	105
8.34.4 Member Data Documentation	105
8.34.4.1 degree	105
8.34.4.2 is_zero_v	105
8.35 aerobus::zpz< p > Struct Template Reference	105
8.35.1 Detailed Description	107

8.35.2 Member Typedef Documentation	107
8.35.2.1 add_t	107
8.35.2.2 div_t	107
8.35.2.3 eq_t	108
8.35.2.4 gcd_t	108
8.35.2.5 gt_t	108
8.35.2.6 inject_constant_t	109
8.35.2.7 inner_type	109
8.35.2.8 lt_t	109
8.35.2.9 mod_t	109
8.35.2.10 mul_t	110
8.35.2.11 one	110
8.35.2.12 pos_t	110
8.35.2.13 sub_t	110
8.35.2.14 zero	111
8.35.3 Member Data Documentation	111
8.35.3.1 eq_v	111
8.35.3.2 gt_v	111
8.35.3.3 is_euclidean_domain	111
8.35.3.4 is_field	111
8.35.3.5 lt_v	112
8.35.3.6 pos_v	112
<b>9 File Documentation</b>	<b>113</b>
9.1 README.md File Reference	113
9.2 src/aerobus.h File Reference	113
9.3 aerobus.h	113
9.4 src/examples.h File Reference	206
9.5 examples.h	206
<b>10 Examples</b>	<b>207</b>
10.1 examples/hermite.cpp	207
10.2 examples/custom_taylor.cpp	207
10.3 examples/fp16.cu	208
10.4 examples/continued_fractions.cpp	209
10.5 examples/modular_arithmetic.cpp	209
10.6 examples/make_polynomial.cpp	210
10.7 examples/polynomials_over_finite_field.cpp	210
10.8 examples/compensated_horner.cpp	210
<b>Index</b>	<b>213</b>

# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.



For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , `Aerobus` provides automatic implementation for **quotient ring**  $R/X$  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

## 1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

<a href="#">aerobus</a>	Main namespace for all publicly exposed types or functions . . . . .	15
<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	36
<a href="#">aerobus::known_polynomials</a>	Families of well known polynomials such as Hermite or Bernstein . . . . .	40



## Chapter 3

# Concept Index

### 3.1 Concepts

Here is a list of all concepts with brief descriptions:

<a href="#">aerobus::IsEuclideanDomain</a>	
Concept to express R is an euclidean domain . . . . .	43
<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	43
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	44



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, E &gt; . . . . .</a>	<a href="#">45</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt; 45</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt; . . . . .</a>	<a href="#">46</a>
<a href="#">aerobus::ContinuedFraction&lt; values &gt;</a>	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$ . . . . .	<a href="#">46</a>
<a href="#">aerobus::ContinuedFraction&lt; a0 &gt;</a>	
Specialization for only one coefficient, technically just 'a0' . . . . .	<a href="#">47</a>
<a href="#">aerobus::ContinuedFraction&lt; a0, rest... &gt;</a>	
Specialization for multiple coefficients (strictly more than one) . . . . .	<a href="#">48</a>
<a href="#">aerobus::ConwayPolynomial</a> . . . . .	<a href="#">49</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::compensated_horner&lt; arithmeticType, P &gt;::EFTHorner&lt; index, ghost &gt;</a>	<a href="#">49</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::compensated_horner&lt; arithmeticType, P &gt;::EFTHorner&lt;-1, ghost &gt; . .</a>	<a href="#">50</a>
<a href="#">aerobus::Embed&lt; Small, Large, E &gt;</a>	
Embedding - struct forward declaration . . . . .	<a href="#">51</a>
<a href="#">aerobus::Embed&lt; i32, i64 &gt;</a>	
Embeds i32 into i64 . . . . .	<a href="#">51</a>
<a href="#">aerobus::Embed&lt; polynomial&lt; Small &gt;, polynomial&lt; Large &gt; &gt;</a>	
Embeds polynomial<Small> into polynomial<Large> . . . . .	<a href="#">52</a>
<a href="#">aerobus::Embed&lt; q32, q64 &gt;</a>	
Embeds q32 into q64 . . . . .	<a href="#">53</a>
<a href="#">aerobus::Embed&lt; Quotient&lt; Ring, X &gt;, Ring &gt;</a>	
Embeds Quotient<Ring, X> into Ring . . . . .	<a href="#">54</a>
<a href="#">aerobus::Embed&lt; Ring, FractionField&lt; Ring &gt; &gt;</a>	
Embeds values from Ring to its field of fractions . . . . .	<a href="#">55</a>
<a href="#">aerobus::Embed&lt; zpz&lt; x &gt;, i32 &gt;</a>	
Embeds zpz values into i32 . . . . .	<a href="#">55</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;</a>	
Used to evaluate polynomials over a value in Ring . . . . .	<a href="#">56</a>
<a href="#">aerobus::i32</a>	
32 bits signed integers, seen as a algebraic ring with related operations . . . . .	<a href="#">57</a>
<a href="#">aerobus::i64</a>	
64 bits signed integers, seen as a algebraic ring with related operations . . . . .	<a href="#">64</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;::inner&lt; index, stop &gt;</a>	<a href="#">70</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;::inner&lt; stop, stop &gt;</a>	<a href="#">71</a>

<a href="#">aerobus::is_prime&lt; n &gt;</a>	71
Checks if n is prime	
<a href="#">aerobus::polynomial&lt; Ring &gt;</a>	72
<a href="#">aerobus::type_list&lt; Ts &gt;::pop_front</a>	80
Removes types from head of the list	
<a href="#">aerobus::Quotient&lt; Ring, X &gt;</a>	81
Quotient ring by the principal ideal generated by 'X' With <a href="#">i32</a> as Ring and <a href="#">i32::val&lt;2&gt;</a> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	
<a href="#">aerobus::type_list&lt; Ts &gt;::split&lt; index &gt;</a>	86
Splits list at index	
<a href="#">aerobus::type_list&lt; Ts &gt;</a>	87
Empty pure template struct to handle type list	
<a href="#">aerobus::type_list&lt;&gt;</a>	90
Specialization for empty type list	
<a href="#">aerobus::i32::val&lt; x &gt;</a>	91
Values in <a href="#">i32</a> , again represented as types	
<a href="#">aerobus::i64::val&lt; x &gt;</a>	93
Values in <a href="#">i64</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN, coeffs &gt;</a>	95
Values (seen as types) in polynomial ring	
<a href="#">aerobus::Quotient&lt; Ring, X &gt;::val&lt; V &gt;</a>	99
Projection values in the quotient ring	
<a href="#">aerobus::zpz&lt; p &gt;::val&lt; x &gt;</a>	100
Values in <a href="#">zpz</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;</a>	102
Specialization for constants	
<a href="#">aerobus::zpz&lt; p &gt;</a>	105
Congruence classes of integers modulo p (32 bits)	



# Chapter 5

## File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

src/ <a href="#">aerobus.h</a> . . . . .	113
src/ <a href="#">examples.h</a> . . . . .	206



## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial<Small> into polynomial<Large>*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient<Ring, X> into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
  - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
  - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)
  - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
  - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)
  - Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)
  - specialization for empty type list*
- struct [zpz](#)
  - congruence classes of integers modulo p (32 bits)*

## Concepts

- concept [IsRing](#)
  - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
  - Concept to express R is an euclidean domain.*
- concept [IsField](#)
  - Concept to express R is a field.*

## Typedefs

- template<typename T , typename A , typename B >
 using [gcd\\_t](#) = typename internal::gcd< T >::template type< A, B >
  - computes the greatest common divisor or A and B*
- template<typename... vals>
 using [vadd\\_t](#) = typename internal::vadd< vals... >::type
  - adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator*
- template<typename... vals>
 using [vmul\\_t](#) = typename internal::vmul< vals... >::type
  - multiplies multiple values (v1 \* v2 + ... + vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator*
- template<typename val >
 using [abs\\_t](#) = std::conditional\_t< val::enclosing\_type::template pos\_v< val >, val, typename val::enclosing\_type::template [sub\\_t](#)< typename val::enclosing\_type::zero, val > >
  - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
  - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add\\_t](#) = typename X::enclosing\_type::template [add\\_t](#)< X, Y >
  - generic addition*
- template<typename X , typename Y >
 using [sub\\_t](#) = typename X::enclosing\_type::template [sub\\_t](#)< X, Y >

- generic subtraction*
  - `template<typename X , typename Y >`  
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
  - generic multiplication*
  - `template<typename X , typename Y >`  
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
  - generic division*
  - `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
  - `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
  - `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
  - `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
  - `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
  - `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
  - `template<typename v >`  
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > ,`  
`polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
  - `template<int64_t p, int64_t q>`  
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > ,`  
`i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
  - `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > ,`  
`i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
  - `template<typename Ring , auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template`  
`inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
  - `template<typename Ring , auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename`  
`FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField< Ring>*
  - `template<typename T , size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*

- `template<typename T , size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T , size_t n>`  
`using bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T , size_t n>`  
`using bell_t = typename internal::bell_helper< T, n >::type`  
*Bell numbers.*
- `template<typename T , int k>`  
`using alternate_t = typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T , int n, int k>`  
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T , int n, int k>`  
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T , int n, int k>`  
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`  
*Stirling number of second kind – as types.*
- `template<typename T , typename p , size_t n>`  
`using pow_t = typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`  
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<`  
`deg+1 > >::type`
- `template<typename Integers , size_t deg>`  
`using exp = taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers , size_t deg>`  
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg`  
`>, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers , size_t deg>`  
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`  
`using atan = taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers , size_t deg>`  
`using sin = taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers , size_t deg>`  
`using sinh = taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers , size_t deg>`  
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers , size_t deg>`  
`using cos = taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*
- `template<typename Integers , size_t deg>`  
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`



## 6.1.2 Typedef Documentation

### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

#### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T



## Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example
----------	---

## 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh( $x$ )` arc hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

## Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

## 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

## 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

## 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  `cosinus`

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

## Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

## 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of  $e$

## 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

## Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

## 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

### 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

### 6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

#### Template Parameters

<i>Ring</i>	
-------------	--

### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

#### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator



## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.29 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.30 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

## Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

## 6.1.2.31 mul\_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

### 6.1.2.32 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

### 6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

### 6.1.2.34 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of  $\pi$  as a continued fraction

### 6.1.2.35 pow\_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

#### Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

### 6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.37 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.38 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

### 6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$$\sin(x)$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$$\sinh(x)$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.41 SQRT2\_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of  $\sqrt{2}$



**6.1.2.46 sub\_t**

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

**Template Parameters**

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

**6.1.2.47 tan**

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.48 tanh**

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.49 taylor**

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

**Template Parameters**

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff<sub>↔</sub> _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

### 6.1.2.50 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.2.51 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

### 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

#### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

#### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

### 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

##### Template Parameters

<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >



- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >
- struct **fma\_helper**
- struct **fma\_helper**< double >
- struct **fma\_helper**< float >
- struct **fma\_helper**< int16\_t >

- struct **fma\_helper**< int32\_t >
- struct **fma\_helper**< int64\_t >
- struct **fma\_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**
  - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling\_1\_helper**
- struct **stirling\_1\_helper**< T, 0, 0 >
- struct **stirling\_1\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, k, std::enable\_if\_t<(k > 0) && (n > 0)> >

- struct **stirling\_2\_helper**
- struct **stirling\_2\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling\_2\_helper**< T, n, n, std::enable\_if\_t<(n >=0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **touchard\_coeff**
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N >{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

### 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

## 6.2.2 Typedef Documentation

### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Enumeration Type Documentation

#### 6.3.2.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```

## Enumerator

probabilist	
physicist	



# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.



## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename Ring::zero

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = `aN`

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

### 8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

#### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

#### Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

#### Examples

[examples/continued\\_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
represented value as double

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

#### Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

### 8.5.2 Member Typedef Documentation

#### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template [add\\_t](#)< typename q64::template inject\_constant\_t< a0 >, typename q64::template [div\\_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

#### Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.8.1 Member Function Documentation

### 8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.9.1 Member Function Documentation

### 8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

### Public Types

- template<typename val >  
using [type](#) = [i64::val](#)< static\_cast< int64\_t >(val::v)>  
*the [i64](#) representation of val*

### 8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

### 8.11.2 Member Typedef Documentation

#### 8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

### Template Parameters

<i>val</i>	a value in <a href="#">i32</a>
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = `typename at_low< v, typename internal::make\_index\_sequence\_reverse< v::degree+1 > >::type`  
*the polynomial<Large> representation of v*

### 8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

#### Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

### 8.12.2 Member Typedef Documentation

#### 8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v



## Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

## Public Types

- `template<typename v >`  
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`  
*q64 representation of v*

### 8.13.1 Detailed Description

embeds q32 into q64

### 8.13.2 Member Typedef Documentation

#### 8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

## Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = typename val::raw_t`  
*Ring representation of val.*

### 8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = typename `FractionField< Ring >::template val< v, typename Ring::one >`  
*FractionField<Ring> representation of v.*

### 8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

<i>Ring</i>	an integers ring, such as <a href="#">i32</a>
-------------	---

### 8.15.2 Member Typedef Documentation

#### 8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

#### Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the i32 representation of val*

### 8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

#### Template Parameters

<code>x</code>	an integer
----------------	------------

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

#### Template Parameters

<code>val</code>	a value in zpz<x>
------------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.17 aerobus::polynomial< Ring >::horner\_reduction\_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

## Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

### 8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

#### Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

## Public Types

- using [inner\\_type](#) = int32\_t
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*inject a native constant*
- template<typename v >  
using [inject\\_ring\\_t](#) = v
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator yields v1 + v2*

- `template<typename v1 , typename v2 >`  
`using sub\_t = typename sub< v1, v2 >::type`  
*subtraction operator yields  $v1 - v2$*
- `template<typename v1 , typename v2 >`  
`using mul\_t = typename mul< v1, v2 >::type`  
*multiplication operator yields  $v1 * v2$*
- `template<typename v1 , typename v2 >`  
`using div\_t = typename div< v1, v2 >::type`  
*division operator yields  $v1 / v2$*
- `template<typename v1 , typename v2 >`  
`using mod\_t = typename remainder< v1, v2 >::type`  
*modulus operator yields  $v1 \% v2$*
- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator ( $v1 < v2$ ) yields  $v1 < v2$*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i32, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator (boolean value)*
- `template<typename v >`  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity (boolean value) yields  $v > 0$  as boolean value*

## 8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

### Examples

[examples/compensated\\_horner.cpp](#).

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields  $v1 + v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.2 div\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields  $v1 / v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.3 eq\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.4 gcd\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields  $GCD(v1, v2)$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.5 gt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.6 inject\_constant\_t**

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

inject a native constant

**Template Parameters**

<i>x</i>	
----------	--

**8.18.2.7 inject\_ring\_t**

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

**8.18.2.8 inner\_type**

```
using aerobus::i32::inner_type = int32_t
```

**8.18.2.9 lt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

strict less operator ( $v1 < v2$ ) yields  $v1 < v2$



## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.10 mod\_t**

```
template<typename v1 , typename v2 >  
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.11 mul\_t**

```
template<typename v1 , typename v2 >  
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields  $v1 * v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.12 one**

```
using aerobus::i32::one = val<1>
```

constant one

**8.18.2.13 pos\_t**

```
template<typename v >  
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`

## Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

#### 8.18.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields  $v1 - v2$

##### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

#### 8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.18.3 Member Data Documentation

#### 8.18.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

##### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.18.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.18.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

#### 8.18.3.4 pos\_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields  $v > 0$  as boolean value

## Template Parameters

<code>v</code>	a value in <a href="#">i32</a>
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
values in [i64](#)

### Public Types

- using [inner\\_type](#) = `int64_t`  
*type of represented values*
- `template<auto x>`  
using [inject\\_constant\\_t](#) = `val< static_cast< int64_t >(x)>`  
*injects constant as an i64 value*
- `template<typename v >`  
using [inject\\_ring\\_t](#) = `v`  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`  
*constant zero*
- using [one](#) = `val< 1 >`  
*constant one*
- `template<typename v1 , typename v2 >`  
using [add\\_t](#) = `typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1 , typename v2 >`  
using [sub\\_t](#) = `typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
using [mul\\_t](#) = `typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
using [div\\_t](#) = `typename div< v1, v2 >::type`  
*division operator integer division*
- `template<typename v1 , typename v2 >`  
using [mod\\_t](#) = `typename remainder< v1, v2 >::type`

*modulus operator*

- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i64, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt\_v = gt\_t<v1, v2>::value`  
*strictly greater operator yields  $v1 > v2$  as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt\_v = lt\_t<v1, v2>::value`  
*strictly smaller operator yields  $v1 < v2$  as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator yields  $v1 == v2$  as boolean value*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity yields  $v > 0$  as boolean value*

### 8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.2 div\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator integer division

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.3 eq\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.4 gcd\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.5 gt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

#### Template Parameters

<code>x</code>	
----------------	--

#### 8.19.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> [i64::val<1>](#)

#### Template Parameters

<code>v</code>	a value in <a href="#">i64</a>
----------------	--------------------------------

#### 8.19.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

#### 8.19.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.10 mod\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.12 one**

```
using aerobus::i64::one = val<1>
```

constant one

**8.19.2.13 pos\_t**

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
-----------	---

**8.19.2.14 sub\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator



## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.19.3 Member Data Documentation

## 8.19.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.19.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

### 8.19.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields  $v1 < v2$  as boolean value

#### Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields  $v > 0$  as boolean value

#### Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a>
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`

## 8.20.1 Member Typedef Documentation

### 8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
`using type = accum`

## 8.21.1 Member Typedef Documentation

### 8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool [value](#) = internal::\_is\_prime<n, 5>::value  
*true iff n is prime*

## 8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

<i>n</i>	
----------	--

## 8.22.2 Member Data Documentation

### 8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [horner\\_reduction\\_t](#)  
*Used to evaluate polynomials over a value in Ring.*
- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

## Public Types

- using `zero` = `val`< typename Ring::zero >  
*constant zero*
- using `one` = `val`< typename Ring::one >  
*constant one*
- using `X` = `val`< typename Ring::one, typename Ring::zero >  
*generator*
- template<typename P >  
using `simplify_t` = typename simplify< P >::type  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- template<typename v1, typename v2 >  
using `add_t` = typename add< v1, v2 >::type  
*adds two polynomials*
- template<typename v1, typename v2 >  
using `sub_t` = typename sub< v1, v2 >::type  
*subtraction of two polynomials*
- template<typename v1, typename v2 >  
using `mul_t` = typename mul< v1, v2 >::type  
*multiplication of two polynomials*
- template<typename v1, typename v2 >  
using `eq_t` = typename eq\_helper< v1, v2 >::type  
*equality operator*
- template<typename v1, typename v2 >  
using `lt_t` = typename lt\_helper< v1, v2 >::type  
*strict less operator*
- template<typename v1, typename v2 >  
using `gt_t` = typename gt\_helper< v1, v2 >::type  
*strict greater operator*
- template<typename v1, typename v2 >  
using `div_t` = typename div< v1, v2 >::q\_type  
*division operator*
- template<typename v1, typename v2 >  
using `mod_t` = typename div\_helper< v1, v2, `zero`, v1 >::mod\_type  
*modulo operator*
- template<typename coeff, size\_t deg>  
using `monomial_t` = typename monomial< coeff, deg >::type  
*monomial : coeff X^deg*
- template<typename v >  
using `derive_t` = typename derive\_helper< v >::type  
*derivation operator*
- template<typename v >  
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >  
*checks for positivity (an > 0)*
- template<typename v1, typename v2 >  
using `gcd_t` = std::conditional\_t< Ring::is\_euclidean\_domain, typename make\_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >  
*greatest common divisor of two polynomials*
- template<auto x>  
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >  
*makes the constant (native type) polynomial a\_0*
- template<typename v >  
using `inject_ring_t` = `val`< v >  
*makes the constant (ring type) polynomial a\_0*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false
- static constexpr bool [is\\_euclidean\\_domain](#) = Ring::is\_euclidean\_domain
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator*

### 8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

#### Examples

[examples/compensated\\_horner.cpp](#), [examples/make\\_polynomial.cpp](#), and [examples/modular\\_arithmetic.cpp](#).

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 8.23.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.5 gcd\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.7 inject\_constant\_t**

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

**Template Parameters**

<i>x</i>	
----------	--

**8.23.2.8 inject\_ring\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

**Template Parameters**

<i>v</i>	
----------	--

**8.23.2.9 lt\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.10 mod\_t**

```
template<typename Ring >
```



```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

#### 8.23.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

#### 8.23.2.14 pos\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ( $an > 0$ )

## Template Parameters

<i>v</i>	
----------	--

**8.23.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

## Template Parameters

<i>P</i>	
----------	--

**8.23.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

**8.23.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.23.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

### 8.23.3 Member Data Documentation

#### 8.23.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.23.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.23.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in <a href="#">polynomial::val</a>
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.24 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.24.2 Member Typedef Documentation

### 8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*zero value*
- using [one](#) = [val](#)< typename Ring::one >  
*one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = [val](#)< typename Ring::template [add\\_t](#)< typename v1::type, typename v2::type > >  
*addition operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = [val](#)< typename Ring::template [mul\\_t](#)< typename v1::type, typename v2::type > >  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = [val](#)< typename Ring::template [div\\_t](#)< typename v1::type, typename v2::type > >  
*division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = [val](#)< typename Ring::template [mod\\_t](#)< typename v1::type, typename v2::type > >

- modulus operator*  
 • `template<typename v1 , typename v2 >`  
   `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
   *equality operator (as type)*
- `template<typename v1 >`  
   `using pos_t = std::true_type`  
   *positivity operator always true*
- `template<auto x>`  
   `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`  
   *inject a 'constant' in quotient ring\**
- `template<typename v >`  
   `using inject_ring_t = val< v >`  
   *projects a value of Ring onto the quotient*

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
   `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
   *addition operator (as boolean value)*
- `template<typename v >`  
   `static constexpr bool pos_v = pos_t<v>::value`  
   *positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
   *quotien rings are euclidean domain*

### 8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as ' <a href="#">i32</a> ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring\*

## Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

### 8.25.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

#### Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

### 8.25.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one



### 8.25.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.25.3 Member Data Documentation

### 8.25.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.25.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.26.2 Member Typedef Documentation

#### 8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.27 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = [internal::type\\_at\\_t](#)< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename [concat\\_h](#)< U >::type  
*concatenates two list into one*
- template<typename T, size\_t index>  
using [insert](#) = typename [internal::insert\\_h](#)< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename [internal::remove\\_h](#)< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*

### Static Public Attributes

- static constexpr size\_t [length](#) = sizeof...(Ts)  
*length of list*

### 8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

### Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

## 8.27.2 Member Typedef Documentation

### 8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

### Template Parameters

<i>index</i>	
--------------	--

### 8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

### Template Parameters

<i>U</i>	
----------	--

### 8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

### Template Parameters

<i>index</i>	
<i>T</i>	

### 8.27.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.5 push\_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

#### Template Parameters

<i>index</i>	
--------------	--

## 8.27.3 Member Data Documentation

### 8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.28 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t [length](#) = 0

### 8.28.1 Detailed Description

specialization for empty type list

## 8.28.2 Member Typedef Documentation

### 8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

### 8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

### 8.28.2.3 push\_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

### 8.28.2.4 push\_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

### 8.28.3 Member Data Documentation

#### 8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [i32](#)  
*Enclosing ring type.*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [DEVICE](#) valueType [get](#) ()  
*cast x into valueType*
- static std::string [to\\_string](#) ()  
*string representation of value*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x  
*actual value stored in val type*

### 8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

## 8.29.2 Member Typedef Documentation

### 8.29.2.1 `enclosing_type`

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

### 8.29.2.2 `is_zero_t`

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.29.3 Member Function Documentation

### 8.29.3.1 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

### Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

### 8.29.3.2 `to_string()`

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

## 8.29.4 Member Data Documentation

### 8.29.4.1 `v`

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```



actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using [inner\\_type](#) = int32\_t  
*type of represented values*
- using [enclosing\\_type](#) = [i64](#)  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED\\_DEVICE](#) valueType [get](#) ()  
*cast value in valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int64\_t [v](#) = x  
*actual value*

### 8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

<a href="#">x</a>	an actual integer
-------------------	-------------------

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.30.2 Member Typedef Documentation

### 8.30.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.30.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.30.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.30.3 Member Function Documentation

### 8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

#### Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.30.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

### 8.30.4 Member Data Documentation

#### 8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

### Public Types

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = polynomial< Ring >  
*enclosing ring type*
- using [aN](#) = coeffN  
*heavy weight coefficient (non zero)*
- using [strip](#) = val< coeffs... >  
*remove largest coefficient*
- using [is\\_zero\\_t](#) = std::bool\_constant<(degree==0) &&(aN::is\_zero\_t::value)>  
*true\_type if polynomial is constant zero*
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename coeff\_at< index >::type  
*type of coefficient at index*
- template<typename x >  
using [value\\_at\\_t](#) = horner\_reduction\_t< val > ::template inner< 0, degree+1 > ::template type< typename Ring::zero, x >

### Static Public Member Functions

- static std::string [to\\_string](#) ()  
*get a string representation of polynomial*
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)  
*evaluates polynomial seen as a function operating on arithmeticType*
- template<typename arithmeticType >  
static [DEVICE INLINED](#) arithmeticType [compensated\\_eval](#) (const arithmeticType &x)  
*Evaluate polynomial on x using compensated horner scheme.*

## Static Public Attributes

- static constexpr size\_t [degree](#) = sizeof...(coeffs)  
*degree of the polynomial*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value  
*true if polynomial is constant zero*

### 8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

#### Examples

[examples/compensated\\_horner.cpp](#).

### 8.31.2 Member Typedef Documentation

#### 8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

#### 8.31.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.31.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>

enclosing ring type
```

### 8.31.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>

true_type if polynomial is constant zero
```

### 8.31.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring

ring coefficients live in
```

### 8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>

remove largest coefficient
```

### 8.31.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

## 8.31.3 Member Function Documentation

### 8.31.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

**Template Parameters**

<i>arithmeticType</i>	float for example
-----------------------	-------------------

**Parameters**

x	
---	--

**8.31.3.2 eval()**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

**Template Parameters**

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

**Parameters**

x	value
---	-------

**Returns**

P(x)

**8.31.3.3 to\_string()**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

**Returns**

something like  $a_n X^n + \dots + a_1 X + a_0$

### 8.31.4 Member Data Documentation

#### 8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

#### 8.31.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- using [raw\\_t](#) = V
- using [type](#) = [abs\\_t](#)< typename Ring::template [mod\\_t](#)< V, X > >

### 8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

V	a value from 'Ring'
---	---------------------

## 8.32.2 Member Typedef Documentation

### 8.32.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [zpz](#)< p >  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< [v](#)==0 >  
*true\_type if zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED DEVICE](#) valueType [get](#) ()  
*get value as valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x % p  
*actual value*
- static constexpr bool [is\\_zero\\_v](#) = [v](#) == 0  
*true if zero*

### 8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz



## Template Parameters

x	an integer
---	------------

## 8.33.2 Member Typedef Documentation

### 8.33.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.33.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.33.3 Member Function Documentation

### 8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

## Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

### 8.33.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

**Returns**

a string representation

**8.33.4 Member Data Documentation****8.33.4.1 is\_zero\_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

**8.33.4.2 v**

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

**8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference**

specialization for constants

```
#include <aerobus.h>
```

**Classes**

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

**Public Types**

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = [polynomial< Ring >](#)  
*enclosing ring type*
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is\\_zero\\_t](#) = std::bool\_constant< [aN::is\\_zero\\_t::value](#) >
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename [coeff\\_at< index >::type](#)
- template<typename x >  
using [value\\_at\\_t](#) = [coeffN](#)

## Static Public Member Functions

- static std::string [to\\_string](#) ()
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >  
static [DEVICE INLINED](#) arithmeticType [compensated\\_eval](#) (const arithmeticType &x)

## Static Public Attributes

- static constexpr size\_t [degree](#) = 0  
*degree*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value

### 8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

### 8.34.2 Member Typedef Documentation

#### 8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

#### 8.34.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

#### 8.34.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

#### 8.34.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

#### 8.34.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

#### 8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

#### 8.34.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

### 8.34.3 Member Function Documentation

#### 8.34.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

#### 8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

### 8.34.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

## 8.34.4 Member Data Documentation

### 8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

### 8.34.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in zpz*

## Public Types

- using `inner_type` = `int32_t`  
*underlying type for values*
- template<auto x>  
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>  
*injects a constant integer into mpz*
- using `zero` = `val`< 0 >  
*zero value*
- using `one` = `val`< 1 >  
*one value*
- template<typename v1 , typename v2 >  
using `add_t` = `typename add`< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using `sub_t` = `typename sub`< v1, v2 >::type  
*subtraction operator*
- template<typename v1 , typename v2 >  
using `mul_t` = `typename mul`< v1, v2 >::type  
*multiplication operator*
- template<typename v1 , typename v2 >  
using `div_t` = `typename div`< v1, v2 >::type  
*division operator*
- template<typename v1 , typename v2 >  
using `mod_t` = `typename remainder`< v1, v2 >::type  
*modulo operator*
- template<typename v1 , typename v2 >  
using `gt_t` = `typename gt`< v1, v2 >::type  
*strictly greater operator (type)*
- template<typename v1 , typename v2 >  
using `lt_t` = `typename lt`< v1, v2 >::type  
*strictly smaller operator (type)*
- template<typename v1 , typename v2 >  
using `eq_t` = `typename eq`< v1, v2 >::type  
*equality operator (type)*
- template<typename v1 , typename v2 >  
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >  
*greatest common divisor*
- template<typename v1 >  
using `pos_t` = `typename pos`< v1 >::type  
*positivity operator (type)*

## Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value  
*true iff p is prime*
- static constexpr bool `is_euclidean_domain` = true  
*always true*
- template<typename v1 , typename v2 >  
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value  
*strictly greater operator (booleanvalue)*

- `template<typename v1 , typename v2 >`  
`static constexpr bool lt\_v = lt\_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity operator (boolean value)*

### 8.35.1 Detailed Description

`template<int32_t p>`  
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular\\_arithmetic.cpp](#), and [examples/polynomials\\_over\\_finite\\_field.cpp](#).

### 8.35.2 Member Typedef Documentation

#### 8.35.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

#### 8.35.2.2 div\_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >  
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.3 eq\_t

```
template<int32_t p>  
template<typename v1 , typename v2 >  
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.4 gcd\_t

```
template<int32_t p>  
template<typename v1 , typename v2 >  
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.5 gt\_t

```
template<int32_t p>  
template<typename v1 , typename v2 >  
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>



### 8.35.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

#### Template Parameters

x	an integer
---	------------

### 8.35.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

### 8.35.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

### 8.35.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

### 8.35.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.35.3 Member Data Documentation

### 8.35.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.35.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.35.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 16U) & 0x8000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED_DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED_DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED_DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<long double> {
00224         static constexpr INLINED_DEVICE long double eval(
00225             const long double x, const long double y, const long double z) {
00226             return x * y + z;
00227         }
00228     };
00229
00230     template<>
00231     struct fma_helper<float> {
00232         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00233             return x * y + z;
00234         }
00235     };
00236
00237     template<>
00238     struct fma_helper<int32_t> {
00239         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00240             return x * y + z;
00241         }
00242     };
00243
00244     template<>
00245     struct fma_helper<int16_t> {
00246         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00247             return x * y + z;
00248         }
00249     };
00250
00251     template<>
00252     struct fma_helper<int64_t> {
00253         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00254             return x * y + z;
00255         }
00256     };
00257
00258     #ifdef WITH_CUDA_FP16
00259     template<>
00260     struct fma_helper<__half> {
00261         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00262             #ifdef __CUDA_ARCH__
00263                 return __hfma(x, y, z);
00264             #else
00265                 return x * y + z;
00266             #endif
00267         }
00268     };
00269     template<>
00270     struct fma_helper<__half2> {
00271         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00272             #ifdef __CUDA_ARCH__
00273                 return __hfma2(x, y, z);
00274             #else
00275                 return x * y + z;
00276             #endif
00277         }
00278     };
00279     #endif
00280 } // namespace internal
00281 } // namespace aerobus
00282
00283 // compensated horner utilities
00284 namespace aerobus {
00285     namespace internal {
00286         template <typename T>
00287         struct FloatLayout;
00288
00289         #ifdef _MSC_VER
00290         template <>
00291         struct FloatLayout<long double> {
00292             static constexpr uint8_t exponent = 11;
00293             static constexpr uint8_t mantissa = 53;
00294             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00295         };

```



```

00296     #else
00297     template <>
00298     struct FloatLayout<long double> {
00299         static constexpr uint8_t exponent = 15;
00300         static constexpr uint8_t mantissa = 63;
00301         static constexpr uint8_t r = 32; // ceil(mantissa/2)
00302         static constexpr long double shift = (1LL « r) + 1;
00303     };
00304     #endif
00305
00306     template <>
00307     struct FloatLayout<double> {
00308         static constexpr uint8_t exponent = 11;
00309         static constexpr uint8_t mantissa = 53;
00310         static constexpr uint8_t r = 27; // ceil(mantissa/2)
00311         static constexpr double shift = (1LL « r) + 1;
00312     };
00313
00314     template <>
00315     struct FloatLayout<float> {
00316         static constexpr uint8_t exponent = 8;
00317         static constexpr uint8_t mantissa = 24;
00318         static constexpr uint8_t r = 11; // ceil(mantissa/2)
00319         static constexpr float shift = (1 « r) + 1;
00320     };
00321
00322     #ifdef WITH_CUDA_FP16
00323     template <>
00324     struct FloatLayout<__half> {
00325         static constexpr uint8_t exponent = 5;
00326         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00327         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00328         static constexpr __half shift = internal::int16_convert_helper<__half, 65>::value();
00329     };
00330
00331     template <>
00332     struct FloatLayout<__half2> {
00333         static constexpr uint8_t exponent = 5;
00334         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00335         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00336         static constexpr __half2 shift = internal::int16_convert_helper<__half2, 65>::value();
00337     };
00338     #endif
00339
00340     template<typename T>
00341     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00342         T z = a * FloatLayout<T>::shift;
00343         *x = z - (z - a);
00344         *y = a - *x;
00345     }
00346
00347     template<typename T>
00348     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00349         *x = a + b;
00350         T z = *x - a;
00351         *y = (a - (*x - z)) + (b - z);
00352     }
00353
00354     template<typename T>
00355     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00356         *x = a * b;
00357         #ifdef __clang__
00358         *y = fma_helper<T>::eval(a, b, -*x);
00359         #else
00360         T ah, al, bh, bl;
00361         split(a, &ah, &al);
00362         split(b, &bh, &bl);
00363         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00364         #endif
00365     }
00366
00367     template<typename T, size_t N>
00368     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00369         T r = p1[0] + p2[0];
00370         for (int64_t i = N - 1; i >= 0; --i) {
00371             r = r * x + p1[N - i] + p2[N - i];
00372         }
00373         return r;
00374     }
00375 } // namespace internal
00376 } // namespace aerobus
00377
00378 // utilities
00379 namespace aerobus {
00380     namespace internal {
00381         template<template<typename...> typename TT, typename T>

```

```

00383     struct is_instantiation_of : std::false_type { };
00384
00385     template<template<typename...> typename TT, typename... Ts>
00386     struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00387
00388     template<template<typename...> typename TT, typename T>
00389     inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00390
00391     template<int64_t i, typename T, typename... Ts>
00392     struct type_at {
00393         static_assert(i < sizeof...(Ts) + 1, "index out of range");
00394         using type = typename type_at<i - 1, Ts...>::type;
00395     };
00396
00397     template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00398         using type = T;
00399     };
00400
00401     template<size_t i, typename... Ts>
00402     using type_at_t = typename type_at<i, Ts...>::type;
00403
00404
00405     template<size_t n, size_t i, typename E = void>
00406     struct _is_prime {};
00407
00408     template<size_t i>
00409     struct _is_prime<0, i> {
00410         static constexpr bool value = false;
00411     };
00412
00413     template<size_t i>
00414     struct _is_prime<1, i> {
00415         static constexpr bool value = false;
00416     };
00417
00418     template<size_t i>
00419     struct _is_prime<2, i> {
00420         static constexpr bool value = true;
00421     };
00422
00423     template<size_t i>
00424     struct _is_prime<3, i> {
00425         static constexpr bool value = true;
00426     };
00427
00428     template<size_t i>
00429     struct _is_prime<5, i> {
00430         static constexpr bool value = true;
00431     };
00432
00433     template<size_t i>
00434     struct _is_prime<7, i> {
00435         static constexpr bool value = true;
00436     };
00437
00438     template<size_t n, size_t i>
00439     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00440         static constexpr bool value = false;
00441     };
00442
00443     template<size_t n, size_t i>
00444     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00445         static constexpr bool value = false;
00446     };
00447
00448     template<size_t n, size_t i>
00449     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00450         static constexpr bool value = true;
00451     };
00452
00453     template<size_t n, size_t i>
00454     struct _is_prime<n, i, std::enable_if_t<(
00455         n % i == 0 &&
00456         n >= 9 &&
00457         n % 3 != 0 &&
00458         n % 2 != 0 &&
00459         i * i > n)> {
00460         static constexpr bool value = true;
00461     };
00462
00463     template<size_t n, size_t i>
00464     struct _is_prime<n, i, std::enable_if_t<(
00465         n % (i+2) == 0 &&
00466         n >= 9 &&
00467         n % 3 != 0 &&
00468         n % 2 != 0 &&
00469         i * i <= n)> {

```

```

00470         static constexpr bool value = true;
00471     };
00472
00473     template<size_t n, size_t i>
00474     struct _is_prime<n, i, std::enable_if_t<(
00475         n % (i+2) != 0 &&
00476         n % i != 0 &&
00477         n >= 9 &&
00478         n % 3 != 0 &&
00479         n % 2 != 0 &&
00480         (i * i <= n))> {
00481         static constexpr bool value = _is_prime<n, i+6>::value;
00482     };
00483 } // namespace internal
00484
00485 template<size_t n>
00486 struct is_prime {
00487     static constexpr bool value = internal::_is_prime<n, 5>::value;
00488 };
00489
00490 template<size_t n>
00491 static constexpr bool is_prime_v = is_prime<n>::value;
00492
00493 // gcd
00494 namespace internal {
00495     template <std::size_t... Is>
00496     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00497     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00498
00499     template <std::size_t N>
00500     using make_index_sequence_reverse
00501     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00502
00503     template<typename Ring, typename E = void>
00504     struct gcd;
00505
00506     template<typename Ring>
00507     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00508         template<typename A, typename B, typename E = void>
00509         struct gcd_helper {};
00510
00511         // B = 0, A > 0
00512         template<typename A, typename B>
00513         struct gcd_helper<A, B, std::enable_if_t<
00514             ((B::is_zero_t::value) &&
00515              (Ring::template gt_t<A, typename Ring::zero>::value))>> {
00516             using type = A;
00517         };
00518
00519         // B = 0, A < 0
00520         template<typename A, typename B>
00521         struct gcd_helper<A, B, std::enable_if_t<
00522             ((B::is_zero_t::value) &&
00523              !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00524             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00525         };
00526
00527         // B != 0
00528         template<typename A, typename B>
00529         struct gcd_helper<A, B, std::enable_if_t<
00530             (!B::is_zero_t::value)
00531             >> {
00532         private: // NOLINT
00533             // A / B
00534             using k = typename Ring::template div_t<A, B>;
00535             // A - (A/B)*B = A % B
00536             using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00537
00538         public:
00539             using type = typename gcd_helper<B, m>::type;
00540         };
00541
00542         template<typename A, typename B>
00543         using type = typename gcd_helper<A, B>::type;
00544     };
00545 } // namespace internal
00546
00547 // vadd and vmul
00548 namespace internal {
00549     template<typename... vals>
00550     struct vmul {};
00551
00552     template<typename v1, typename... vals>
00553     struct vmul<v1, vals...> {
00554         using type = typename v1::enclosing_type::template mul_t<v1, typename
00555         vmul<vals...>::type>;
00556     };

```

```

00567
00568     template<typename v1>
00569     struct vmul<v1> {
00570         using type = v1;
00571     };
00572
00573     template<typename... vals>
00574     struct vadd {};
00575
00576     template<typename v1, typename... vals>
00577     struct vadd<v1, vals...> {
00578         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00579     };
00580
00581     template<typename v1>
00582     struct vadd<v1> {
00583         using type = v1;
00584     };
00585 } // namespace internal
00586
00587 template<typename T, typename A, typename B>
00588 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00589
00590 template<typename... vals>
00591 using vadd_t = typename internal::vadd<vals...>::type;
00592
00593 template<typename... vals>
00594 using vmul_t = typename internal::vmul<vals...>::type;
00595
00596 template<typename val>
00597 requires IsEuclideanDomain<typename val::enclosing_type>
00598 using abs_t = std::conditional_t<
00599     val::enclosing_type::template pos_v<val>,
00600     val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00601 } // namespace aerobus
00602
00603 // embedding
00604 namespace aerobus {
00605     template<typename Small, typename Large, typename E = void>
00606     struct Embed;
00607 } // namespace aerobus
00608
00609 namespace aerobus {
00610     template<typename Ring, typename X>
00611     requires IsRing<Ring>
00612     struct Quotient {
00613         template <typename V>
00614         struct val {
00615             public:
00616                 using raw_t = V;
00617                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00618         };
00619
00620         using zero = val<typename Ring::zero>;
00621
00622         using one = val<typename Ring::one>;
00623
00624         template<typename v1, typename v2>
00625         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00626
00627         template<typename v1, typename v2>
00628         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00629
00630         template<typename v1, typename v2>
00631         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00632
00633         template<typename v1, typename v2>
00634         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00635
00636         template<typename v1, typename v2>
00637         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00638
00639         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00640
00641         template<typename v1>
00642         using pos_t = std::true_type;
00643
00644         template<typename v>
00645         static constexpr bool pos_v = pos_t<v>::value;
00646
00647         static constexpr bool is_euclidean_domain = true;
00648
00649         template<auto x>
00650         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;

```

```

00703
00704     template<typename v>
00705     using inject_ring_t = val<v>;
00706 };
00707
00708     template<typename Ring, typename X>
00709     struct Embed<Quotient<Ring, X>, Ring> {
00710         template<typename val>
00711         using type = typename val::raw_t;
00712     };
00713 } // namespace aerobus
00714
00715 // type_list
00716 namespace aerobus {
00717     template <typename... Ts>
00718     struct type_list;
00719
00720     namespace internal {
00721         template <typename T, typename... Us>
00722         struct pop_front_h {
00723             using tail = type_list<Us...>;
00724             using head = T;
00725         };
00726
00727         template <size_t index, typename L1, typename L2>
00728         struct split_h {
00729             private:
00730                 static_assert(index <= L2::length, "index ouf of bounds");
00731                 using a = typename L2::pop_front::type;
00732                 using b = typename L2::pop_front::tail;
00733                 using c = typename L1::template push_back<a>;
00734
00735             public:
00736                 using head = typename split_h<index - 1, c, b>::head;
00737                 using tail = typename split_h<index - 1, c, b>::tail;
00738         };
00739
00740         template <typename L1, typename L2>
00741         struct split_h<0, L1, L2> {
00742             using head = L1;
00743             using tail = L2;
00744         };
00745
00746         template <size_t index, typename L, typename T>
00747         struct insert_h {
00748             static_assert(index <= L::length, "index ouf of bounds");
00749             using s = typename L::template split<index>;
00750             using left = typename s::head;
00751             using right = typename s::tail;
00752             using ll = typename left::template push_back<T>;
00753             using type = typename ll::template concat<right>;
00754         };
00755
00756         template <size_t index, typename L>
00757         struct remove_h {
00758             using s = typename L::template split<index>;
00759             using left = typename s::head;
00760             using right = typename s::tail;
00761             using rr = typename right::pop_front::tail;
00762             using type = typename left::template concat<rr>;
00763         };
00764     } // namespace internal
00765
00766     template <typename... Ts>
00767     struct type_list {
00768     private:
00769         template <typename T>
00770         struct concat_h;
00771
00772         template <typename... Us>
00773         struct concat_h<type_list<Us...>> {
00774             using type = type_list<Ts..., Us...>;
00775         };
00776
00777     public:
00778         static constexpr size_t length = sizeof...(Ts);
00779
00780         template <typename T>
00781         using push_front = type_list<T, Ts...>;
00782
00783         template <size_t index>
00784         using at = internal::type_at_t<index, Ts...>;
00785
00786         struct pop_front {
00787             using type = typename internal::pop_front_h<Ts...>::head;
00788             using tail = typename internal::pop_front_h<Ts...>::tail;
00789         };
00790     };

```

```

00809
00812     template <typename T>
00813     using push_back = type_list<Ts..., T>;
00814
00817     template <typename U>
00818     using concat = typename concat_h<U>::type;
00819
00822     template <size_t index>
00823     struct split {
00824     private:
00825         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00826
00827     public:
00828         using head = typename inner::head;
00829         using tail = typename inner::tail;
00830     };
00831
00835     template <typename T, size_t index>
00836     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00837
00840     template <size_t index>
00841     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00842 };
00843
00845 template <>
00846 struct type_list<> {
00847     static constexpr size_t length = 0;
00848
00849     template <typename T>
00850     using push_front = type_list<T>;
00851
00852     template <typename T>
00853     using push_back = type_list<T>;
00854
00855     template <typename U>
00856     using concat = U;
00857
00858     // TODO(jewave): assert index == 0
00859     template <typename T, size_t index>
00860     using insert = type_list<T>;
00861 };
00862 } // namespace aerobus
00863
00864 // i16
00865 #ifndef WITH_CUDA_FP16
00866 // i16
00867 namespace aerobus {
00868     struct i16 {
00869     using inner_type = int16_t;
00870     template<int16_t x>
00871     struct val {
00872         using enclosing_type = i16;
00873         static constexpr int16_t v = x;
00874
00875         template<typename valueType>
00876         static constexpr INLINED_DEVICE valueType get() {
00877             return internal::template int16_convert_helper<valueType, x>::value();
00878         }
00879
00880         using is_zero_t = std::bool_constant<x == 0>;
00881
00882         static std::string to_string() {
00883             return std::to_string(x);
00884         }
00885     };
00886
00887     using zero = val<0>;
00888     using one = val<1>;
00889
00890     static constexpr bool is_field = false;
00891     static constexpr bool is_euclidean_domain = true;
00892     template<auto x>
00893     using inject_constant_t = val<static_cast<int16_t>(x)>;
00894
00895     template<typename v>
00896     using inject_ring_t = v;
00897
00898 private:
00899     template<typename v1, typename v2>
00900     struct add {
00901         using type = val<v1::v + v2::v>;
00902     };
00903
00904     template<typename v1, typename v2>
00905     struct sub {
00906         using type = val<v1::v - v2::v>;
00907     };
00908
00909
00910
00911
00912
00913
00914
00915
00916
00917
00918
00919
00920
00921
00922

```

```

00923     template<typename v1, typename v2>
00924     struct mul {
00925         using type = val<v1::v* v2::v>;
00926     };
00927
00928     template<typename v1, typename v2>
00929     struct div {
00930         using type = val<v1::v / v2::v>;
00931     };
00932
00933     template<typename v1, typename v2>
00934     struct remainder {
00935         using type = val<v1::v % v2::v>;
00936     };
00937
00938     template<typename v1, typename v2>
00939     struct gt {
00940         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00941     };
00942
00943     template<typename v1, typename v2>
00944     struct lt {
00945         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00946     };
00947
00948     template<typename v1, typename v2>
00949     struct eq {
00950         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00951     };
00952
00953     template<typename v1>
00954     struct pos {
00955         using type = std::bool_constant<(v1::v > 0)>;
00956     };
00957
00958     public:
00963     template<typename v1, typename v2>
00964     using add_t = typename add<v1, v2>::type;
00965
00970     template<typename v1, typename v2>
00971     using sub_t = typename sub<v1, v2>::type;
00972
00977     template<typename v1, typename v2>
00978     using mul_t = typename mul<v1, v2>::type;
00979
00984     template<typename v1, typename v2>
00985     using div_t = typename div<v1, v2>::type;
00986
00991     template<typename v1, typename v2>
00992     using mod_t = typename remainder<v1, v2>::type;
00993
00998     template<typename v1, typename v2>
00999     using gt_t = typename gt<v1, v2>::type;
01000
01005     template<typename v1, typename v2>
01006     using lt_t = typename lt<v1, v2>::type;
01007
01012     template<typename v1, typename v2>
01013     using eq_t = typename eq<v1, v2>::type;
01014
01018     template<typename v1, typename v2>
01019     static constexpr bool eq_v = eq_t<v1, v2>::value;
01020
01025     template<typename v1, typename v2>
01026     using gcd_t = gcd_t<i16, v1, v2>;
01027
01031     template<typename v>
01032     using pos_t = typename pos<v>::type;
01033
01037     template<typename v>
01038     static constexpr bool pos_v = pos_t<v>::value;
01039     };
01040 } // namespace aerobus
01041 #endif
01042
01043 // i32
01044 namespace aerobus {
01045     struct i32 {
01046         using inner_type = int32_t;
01047         template<int32_t x>
01050         struct val {
01051             using enclosing_type = i32;
01052             static constexpr int32_t v = x;
01053
01059             template<typename valueType>
01060             static constexpr DEVICE valueType get() {
01061                 return static_cast<valueType>(x);

```

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```

01189     template<typename v1, typename v2>
01190     using eq_t = typename eq<v1, v2>::type;
01191
01195     template<typename v1, typename v2>
01196     static constexpr bool eq_v = eq_t<v1, v2>::value;
01197
01202     template<typename v1, typename v2>
01203     using gcd_t = gcd_t<i32, v1, v2>;
01204
01208     template<typename v>
01209     using pos_t = typename pos<v>::type;
01210
01214     template<typename v>
01215     static constexpr bool pos_v = pos_t<v>::value;
01216 };
01217 } // namespace aerobus
01218
01219 // i64
01220 namespace aerobus {
01221     struct i64 {
01222         using inner_type = int64_t;
01223         template<int64_t x>
01224         struct val {
01225             using inner_type = int32_t;
01226             using enclosing_type = i64;
01227             static constexpr int64_t v = x;
01228
01229             template<typename valueType>
01230             static constexpr INLINED_DEVICE valueType get() {
01231                 return static_cast<valueType>(x);
01232             }
01233
01234             using is_zero_t = std::bool_constant<x == 0>;
01235
01236             static std::string to_string() {
01237                 return std::to_string(x);
01238             }
01239         };
01240
01241         template<auto x>
01242         using inject_constant_t = val<static_cast<int64_t>(x)>;
01243
01244         template<typename v>
01245         using inject_ring_t = v;
01246
01247         using zero = val<0>;
01248         using one = val<1>;
01249         static constexpr bool is_field = false;
01250         static constexpr bool is_euclidean_domain = true;
01251
01252     private:
01253         template<typename v1, typename v2>
01254         struct add {
01255             using type = val<v1::v + v2::v>;
01256         };
01257
01258         template<typename v1, typename v2>
01259         struct sub {
01260             using type = val<v1::v - v2::v>;
01261         };
01262
01263         template<typename v1, typename v2>
01264         struct mul {
01265             using type = val<v1::v * v2::v>;
01266         };
01267
01268         template<typename v1, typename v2>
01269         struct div {
01270             using type = val<v1::v / v2::v>;
01271         };
01272
01273         template<typename v1, typename v2>
01274         struct remainder {
01275             using type = val<v1::v % v2::v>;
01276         };
01277
01278         template<typename v1, typename v2>
01279         struct gt {
01280             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01281         };
01282
01283         template<typename v1, typename v2>
01284         struct lt {
01285             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01286         };
01287
01288         template<typename v1, typename v2>

```

```

01310     struct eq {
01311         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01312     };
01313
01314     template<typename v>
01315     struct pos {
01316         using type = std::bool_constant<(v::v > 0)>;
01317     };
01318
01319 public:
01320     template<typename v1, typename v2>
01321     using add_t = typename add<v1, v2>::type;
01322
01323     template<typename v1, typename v2>
01324     using sub_t = typename sub<v1, v2>::type;
01325
01326     template<typename v1, typename v2>
01327     using mul_t = typename mul<v1, v2>::type;
01328
01329     template<typename v1, typename v2>
01330     using div_t = typename div<v1, v2>::type;
01331
01332     template<typename v1, typename v2>
01333     using mod_t = typename remainder<v1, v2>::type;
01334
01335     template<typename v1, typename v2>
01336     using gt_t = typename gt<v1, v2>::type;
01337
01338     template<typename v1, typename v2>
01339     static constexpr bool gt_v = gt_t<v1, v2>::value;
01340
01341     template<typename v1, typename v2>
01342     using lt_t = typename lt<v1, v2>::type;
01343
01344     template<typename v1, typename v2>
01345     static constexpr bool lt_v = lt_t<v1, v2>::value;
01346
01347     template<typename v1, typename v2>
01348     using eq_t = typename eq<v1, v2>::type;
01349
01350     template<typename v1, typename v2>
01351     static constexpr bool eq_v = eq_t<v1, v2>::value;
01352
01353     template<typename v1, typename v2>
01354     using gcd_t = gcd_t<i64, v1, v2>;
01355
01356     template<typename v>
01357     using pos_t = typename pos<v>::type;
01358
01359     template<typename v>
01360     static constexpr bool pos_v = pos_t<v>::value;
01361 };
01362
01363 template<>
01364 struct Embed<i32, i64> {
01365     template<typename val>
01366     using type = i64::val<static_cast<int64_t>(val::v)>;
01367 };
01368 } // namespace aerobus
01369
01370 // z/pz
01371 namespace aerobus {
01372     template<int32_t p>
01373     struct zpz {
01374         using inner_type = int32_t;
01375
01376         template<int32_t x>
01377         struct val {
01378             using enclosing_type = zpz<p>;
01379             static constexpr int32_t v = x % p;
01380
01381             template<typename valueType>
01382             static constexpr INLINED_DEVICE valueType get() {
01383                 return static_cast<valueType>(x % p);
01384             }
01385
01386             using is_zero_t = std::bool_constant<v == 0>;
01387
01388             static constexpr bool is_zero_v = v == 0;
01389
01390             static std::string to_string() {
01391                 return std::to_string(x % p);
01392             }
01393         };
01394     };
01395
01396     template<auto x>
01397     using inject_constant_t = val<static_cast<int32_t>(x)>;

```

```

01468
01470     using zero = val<0>;
01471
01473     using one = val<1>;
01474
01476     static constexpr bool is_field = is_prime<p>::value;
01477
01479     static constexpr bool is_euclidean_domain = true;
01480
01481 private:
01482     template<typename v1, typename v2>
01483     struct add {
01484         using type = val<(v1::v + v2::v) % p>;
01485     };
01486
01487     template<typename v1, typename v2>
01488     struct sub {
01489         using type = val<(v1::v - v2::v) % p>;
01490     };
01491
01492     template<typename v1, typename v2>
01493     struct mul {
01494         using type = val<(v1::v * v2::v) % p>;
01495     };
01496
01497     template<typename v1, typename v2>
01498     struct div {
01499         using type = val<(v1::v % p) / (v2::v % p)>;
01500     };
01501
01502     template<typename v1, typename v2>
01503     struct remainder {
01504         using type = val<(v1::v % v2::v) % p>;
01505     };
01506
01507     template<typename v1, typename v2>
01508     struct gt {
01509         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01510     };
01511
01512     template<typename v1, typename v2>
01513     struct lt {
01514         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01515     };
01516
01517     template<typename v1, typename v2>
01518     struct eq {
01519         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01520     };
01521
01522     template<typename v1>
01523     struct pos {
01524         using type = std::bool_constant<(v1::v > 0)>;
01525     };
01526
01527 public:
01531     template<typename v1, typename v2>
01532     using add_t = typename add<v1, v2>::type;
01533
01537     template<typename v1, typename v2>
01538     using sub_t = typename sub<v1, v2>::type;
01539
01543     template<typename v1, typename v2>
01544     using mul_t = typename mul<v1, v2>::type;
01545
01549     template<typename v1, typename v2>
01550     using div_t = typename div<v1, v2>::type;
01551
01555     template<typename v1, typename v2>
01556     using mod_t = typename remainder<v1, v2>::type;
01557
01561     template<typename v1, typename v2>
01562     using gt_t = typename gt<v1, v2>::type;
01563
01567     template<typename v1, typename v2>
01568     static constexpr bool gt_v = gt_t<v1, v2>::value;
01569
01573     template<typename v1, typename v2>
01574     using lt_t = typename lt<v1, v2>::type;
01575
01579     template<typename v1, typename v2>
01580     static constexpr bool lt_v = lt_t<v1, v2>::value;
01581
01585     template<typename v1, typename v2>
01586     using eq_t = typename eq<v1, v2>::type;
01587
01591     template<typename v1, typename v2>

```

```

01592         static constexpr bool eq_v = eq_t<v1, v2>::value;
01593
01597         template<typename v1, typename v2>
01598         using gcd_t = gcd_t<i32, v1, v2>;
01599
01602         template<typename v1>
01603         using pos_t = typename pos<v1>::type;
01604
01607         template<typename v>
01608         static constexpr bool pos_v = pos_t<v>::value;
01609     };
01610
01613     template<int32_t x>
01614     struct Embed<zpz<x>, i32> {
01617         template <typename val>
01618         using type = i32::val<val::v>;
01619     };
01620 } // namespace aerobus
01621
01622 // polynomial
01623 namespace aerobus {
01624     // coeffN x^N + ...
01629     template<typename Ring>
01630     requires IsEuclideanDomain<Ring>
01631     struct polynomial {
01632         static constexpr bool is_field = false;
01633         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01634
01637         template<typename P>
01638         struct horner_reduction_t {
01639             template<size_t index, size_t stop>
01640             struct inner {
01641                 template<typename accum, typename x>
01642                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01643                     ::template type<
01644                         typename Ring::template add_t<
01645                             typename Ring::template mul_t<x, accum>,
01646                             typename P::template coeff_at_t<P::degree - index>
01647                             >, x>;
01648             };
01649
01650             template<size_t stop>
01651             struct inner<stop, stop> {
01652                 template<typename accum, typename x>
01653                 using type = accum;
01654             };
01655         };
01656
01660         template<typename coeffN, typename... coeffs>
01661         struct val {
01663             using ring_type = Ring;
01665             using enclosing_type = polynomial<Ring>;
01667             static constexpr size_t degree = sizeof...(coeffs);
01669             using aN = coeffN;
01671             using strip = val<coeffs...>;
01673             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01675             static constexpr bool is_zero_v = is_zero_t::value;
01676
01677         private:
01678             template<size_t index, typename E = void>
01679             struct coeff_at {};
01680
01681             template<size_t index>
01682             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01683                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01684             };
01685
01686             template<size_t index>
01687             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01688                 using type = typename Ring::zero;
01689             };
01690
01691         public:
01694             template<size_t index>
01695             using coeff_at_t = typename coeff_at<index>::type;
01696
01699             static std::string to_string() {
01700                 return string_helper<coeffN, coeffs...>::func();
01701             }
01702
01707             template<typename arithmeticType>
01708             static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01709                 #ifdef WITH_CUDA_FP16
01710                 arithmeticType start;
01711                 if constexpr (std::is_same_v<arithmeticType, __half2>) {
01712                     start = __half2(0, 0);
01713                 } else {

```

```

01714         start = static_cast<arithmeticType>(0);
01715     }
01716     #else
01717     arithmeticType start = static_cast<arithmeticType>(0);
01718     #endif
01719     return horner_evaluation<arithmeticType, val>
01720         ::template inner<0, degree + 1>
01721         ::func(start, x);
01722 }
01723
01724 template<typename arithmeticType>
01725 static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01726     return compensated_horner<arithmeticType, val>::func(x);
01727 }
01728
01729 template<typename x>
01730 using value_at_t = horner_reduction_t<val>
01731     ::template inner<0, degree + 1>
01732     ::template type<typename Ring::zero, x>;
01733
01734 };
01735
01736 template<typename coeffN>
01737 struct val<coeffN> {
01738     using ring_type = Ring;
01739     using enclosing_type = polynomial<Ring>;
01740     static constexpr size_t degree = 0;
01741     using aN = coeffN;
01742     using strip = val<coeffN>;
01743     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01744
01745     static constexpr bool is_zero_v = is_zero_t::value;
01746
01747     template<size_t index, typename E = void>
01748     struct coeff_at {};
01749
01750     template<size_t index>
01751     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01752         using type = aN;
01753     };
01754
01755     template<size_t index>
01756     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01757         using type = typename Ring::zero;
01758     };
01759
01760     template<size_t index>
01761     using coeff_at_t = typename coeff_at<index>::type;
01762
01763     static std::string to_string() {
01764         return string_helper<coeffN>::func();
01765     }
01766
01767     template<typename arithmeticType>
01768     static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01769         return coeffN::template get<arithmeticType>();
01770     }
01771
01772     template<typename arithmeticType>
01773     static DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01774         return coeffN::template get<arithmeticType>();
01775     }
01776
01777     template<typename x>
01778     using value_at_t = coeffN;
01779 };
01780
01781 using zero = val<typename Ring::zero>;
01782 using one = val<typename Ring::one>;
01783 using X = val<typename Ring::one, typename Ring::zero>;
01784
01785 private:
01786     template<typename P, typename E = void>
01787     struct simplify;
01788
01789     template<typename P1, typename P2, typename I>
01790     struct add_low;
01791
01792     template<typename P1, typename P2>
01793     struct add {
01794         using type = typename simplify<typename add_low<
01795             P1,
01796             P2,
01797             internal::make_index_sequence_reverse<
01798                 std::max(P1::degree, P2::degree) + 1
01799             >::type>::type;
01800     };
01801
01802

```

```

01821     template <typename P1, typename P2, typename I>
01822     struct sub_low;
01823
01824     template <typename P1, typename P2, typename I>
01825     struct mul_low;
01826
01827     template<typename v1, typename v2>
01828     struct mul {
01829         using type = typename mul_low<
01830             v1,
01831             v2,
01832             internal::make_index_sequence_reverse<
01833                 v1::degree + v2::degree + 1
01834             >::type;
01835     };
01836
01837     template<typename coeff, size_t deg>
01838     struct monomial;
01839
01840     template<typename v, typename E = void>
01841     struct derive_helper {};
01842
01843     template<typename v>
01844     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01845         using type = zero;
01846     };
01847
01848     template<typename v>
01849     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01850         using type = typename add<
01851             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01852             typename monomial<
01853                 typename Ring::template mul_t<
01854                     typename v::aN,
01855                     typename Ring::template inject_constant_t<(v::degree)>
01856                 >,
01857                 v::degree - 1
01858             >::type
01859         >::type;
01860     };
01861
01862     template<typename v1, typename v2, typename E = void>
01863     struct eq_helper {};
01864
01865     template<typename v1, typename v2>
01866     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01867         using type = std::false_type;
01868     };
01869
01870     template<typename v1, typename v2>
01871     struct eq_helper<v1, v2, std::enable_if_t<
01872         v1::degree == v2::degree &&
01873         (v1::degree != 0 || v2::degree != 0) &&
01874         std::is_same<
01875             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01876             std::false_type
01877         >::value
01878     >
01879     > {
01880         using type = std::false_type;
01881     };
01882
01883     template<typename v1, typename v2>
01884     struct eq_helper<v1, v2, std::enable_if_t<
01885         v1::degree == v2::degree &&
01886         (v1::degree != 0 || v2::degree != 0) &&
01887         std::is_same<
01888             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01889             std::true_type
01890         >::value
01891     > {
01892         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01893     };
01894
01895     template<typename v1, typename v2>
01896     struct eq_helper<v1, v2, std::enable_if_t<
01897         v1::degree == v2::degree &&
01898         (v1::degree == 0)
01899     > {
01900         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01901     };
01902
01903     template<typename v1, typename v2, typename E = void>
01904     struct lt_helper {};
01905
01906     template<typename v1, typename v2>

```

```

01908     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01909         using type = std::true_type;
01910     };
01911
01912     template<typename v1, typename v2>
01913     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01914         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01915     };
01916
01917     template<typename v1, typename v2>
01918     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01919         using type = std::false_type;
01920     };
01921
01922     template<typename v1, typename v2, typename E = void>
01923     struct gt_helper {};
01924
01925     template<typename v1, typename v2>
01926     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01927         using type = std::true_type;
01928     };
01929
01930     template<typename v1, typename v2>
01931     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01932         using type = std::false_type;
01933     };
01934
01935     template<typename v1, typename v2>
01936     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01937         using type = std::false_type;
01938     };
01939
01940     // when high power is zero : strip
01941     template<typename P>
01942     struct simplify<P, std::enable_if_t<
01943         std::is_same<
01944             typename Ring::zero,
01945             typename P::aN
01946         >::value && (P::degree > 0)
01947     >> {
01948         using type = typename simplify<typename P::strip>::type;
01949     };
01950
01951     // otherwise : do nothing
01952     template<typename P>
01953     struct simplify<P, std::enable_if_t<
01954         !std::is_same<
01955             typename Ring::zero,
01956             typename P::aN
01957         >::value && (P::degree > 0)
01958     >> {
01959         using type = P;
01960     };
01961
01962     // do not simplify constants
01963     template<typename P>
01964     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01965         using type = P;
01966     };
01967
01968     // addition at
01969     template<typename P1, typename P2, size_t index>
01970     struct add_at {
01971         using type =
01972             typename Ring::template add_t<
01973                 typename P1::template coeff_at_t<index>,
01974                 typename P2::template coeff_at_t<index>;
01975     };
01976
01977     template<typename P1, typename P2, size_t index>
01978     using add_at_t = typename add_at<P1, P2, index>::type;
01979
01980     template<typename P1, typename P2, std::size_t... I>
01981     struct add_low<P1, P2, std::index_sequence<I...>> {
01982         using type = val<add_at_t<P1, P2, I>...>;
01983     };
01984
01985     // subtraction at
01986     template<typename P1, typename P2, size_t index>
01987     struct sub_at {
01988         using type =
01989             typename Ring::template sub_t<
01990                 typename P1::template coeff_at_t<index>,
01991                 typename P2::template coeff_at_t<index>;
01992     };
01993
01994     template<typename P1, typename P2, size_t index>

```

```

01995     using sub_at_t = typename sub_at<P1, P2, index>::type;
01996
01997     template<typename P1, typename P2, std::size_t... I>
01998     struct sub_low<P1, P2, std::index_sequence<I...> {
01999         using type = val<sub_at_t<P1, P2, I>...>;
02000     };
02001
02002     template<typename P1, typename P2>
02003     struct sub {
02004         using type = typename simplify<typename sub_low<
02005             P1,
02006             P2,
02007             internal::make_index_sequence_reverse<
02008                 std::max(P1::degree, P2::degree) + 1
02009             >::type>::type;
02010     };
02011
02012     // multiplication at
02013     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02014     struct mul_at_loop_helper {
02015         using type = typename Ring::template add_t<
02016             typename Ring::template mul_t<
02017                 typename v1::template coeff_at_t<index>,
02018                 typename v2::template coeff_at_t<k - index>
02019             >,
02020             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02021         >;
02022     };
02023
02024     template<typename v1, typename v2, size_t k, size_t stop>
02025     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02026         using type = typename Ring::template mul_t<
02027             typename v1::template coeff_at_t<stop>,
02028             typename v2::template coeff_at_t<0>;
02029     };
02030
02031     template <typename v1, typename v2, size_t k, typename E = void>
02032     struct mul_at {};
02033
02034     template<typename v1, typename v2, size_t k>
02035     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
02036         using type = typename Ring::zero;
02037     };
02038
02039     template<typename v1, typename v2, size_t k>
02040     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
02041         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02042     };
02043
02044     template<typename P1, typename P2, size_t index>
02045     using mul_at_t = typename mul_at<P1, P2, index>::type;
02046
02047     template<typename P1, typename P2, std::size_t... I>
02048     struct mul_low<P1, P2, std::index_sequence<I...> {
02049         using type = val<mul_at_t<P1, P2, I>...>;
02050     };
02051
02052     // division helper
02053     template< typename A, typename B, typename Q, typename R, typename E = void>
02054     struct div_helper {};
02055
02056     template<typename A, typename B, typename Q, typename R>
02057     struct div_helper<A, B, Q, R, std::enable_if_t<
02058         (R::degree < B::degree) ||
02059         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02060         using q_type = Q;
02061         using mod_type = R;
02062         using gcd_type = B;
02063     };
02064
02065     template<typename A, typename B, typename Q, typename R>
02066     struct div_helper<A, B, Q, R, std::enable_if_t<
02067         (R::degree >= B::degree) &&
02068         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
02069     private: // NOLINT
02070         using rN = typename R::aN;
02071         using bN = typename B::aN;
02072         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02073             B::degree>::type;
02074         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02075         using qq = typename add<Q, pT>::type;
02076     public:
02077         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02078         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02079         using gcd_type = rr;
02080     };

```



```

02081
02082     template<typename A, typename B>
02083     struct div {
02084         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02085         using q_type = typename div_helper<A, B, zero, A>::q_type;
02086         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02087     };
02088
02089     template<typename P>
02090     struct make_unit {
02091         using type = typename div<P, val<typename P::aN>>::q_type;
02092     };
02093
02094     template<typename coeff, size_t deg>
02095     struct monomial {
02096         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02097     };
02098
02099     template<typename coeff>
02100     struct monomial<coeff, 0> {
02101         using type = val<coeff>;
02102     };
02103
02104     template<typename arithmeticType, typename P>
02105     struct horner_evaluation {
02106         template<size_t index, size_t stop>
02107         struct inner {
02108             static constexpr DEVICE INLINED arithmeticType func(
02109                 const arithmeticType& accum, const arithmeticType& x) {
02110                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02111 stop>::func(
02112                     internal::fma_helper<arithmeticType>::eval(
02113                         x,
02114                         accum,
02115                         P::template coeff_at_t<P::degree - index>::template
02116 get<arithmeticType>()), x);
02117             }
02118         };
02119         template<size_t stop>
02120         struct inner<stop, stop> {
02121             static constexpr DEVICE INLINED arithmeticType func(
02122                 const arithmeticType& accum, const arithmeticType& x) {
02123                 return accum;
02124             }
02125         };
02126     };
02127
02128     template<typename arithmeticType, typename P>
02129     struct compensated_horner {
02130         template<int64_t index, int ghost>
02131         struct EFTHorner {
02132             static INLINED void func(
02133                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02134 *r) {
02135                 arithmeticType p;
02136                 internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02137                 constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
02138 get<arithmeticType>();
02139                 internal::two_sum<arithmeticType>(
02140                     p, coeff,
02141                     r, sigma + P::degree - index - 1);
02142                 EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02143             }
02144         };
02145         template<int ghost>
02146         struct EFTHorner<-1, ghost> {
02147             static INLINED DEVICE void func(
02148                 arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02149 *r) {
02150             }
02151         };
02152     };
02153
02154     static INLINED DEVICE arithmeticType func(arithmeticType x) {
02155         arithmeticType pi[P::degree], sigma[P::degree];
02156         arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02157         EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02158         arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02159         return r + c;
02160     }
02161
02162     template<typename coeff, typename... coeffs>
02163     struct string_helper {
02164         static std::string func() {
02165             std::string tail = string_helper<coeffs...>::func();

```

```

02163         std::string result = "";
02164         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02165             return tail;
02166         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02167             if (sizeof...(coeffs) == 1) {
02168                 result += "x";
02169             } else {
02170                 result += "x^" + std::to_string(sizeof...(coeffs));
02171             }
02172         } else {
02173             if (sizeof...(coeffs) == 1) {
02174                 result += coeff::to_string() + " x";
02175             } else {
02176                 result += coeff::to_string()
02177                     + " x^" + std::to_string(sizeof...(coeffs));
02178             }
02179         }
02180
02181         if (!tail.empty()) {
02182             if (tail.at(0) != '-') {
02183                 result += " + " + tail;
02184             } else {
02185                 result += " - " + tail.substr(1);
02186             }
02187         }
02188
02189         return result;
02190     }
02191 };
02192
02193 template<typename coeff>
02194 struct string_helper<coeff> {
02195     static std::string func() {
02196         if (!std::is_same<coeff, typename Ring::zero>::value) {
02197             return coeff::to_string();
02198         } else {
02199             return "";
02200         }
02201     }
02202 };
02203
02204 public:
02205     template<typename P>
02206     using simplify_t = typename simplify<P>::type;
02207
02208     template<typename v1, typename v2>
02209     using add_t = typename add<v1, v2>::type;
02210
02211     template<typename v1, typename v2>
02212     using sub_t = typename sub<v1, v2>::type;
02213
02214     template<typename v1, typename v2>
02215     using mul_t = typename mul<v1, v2>::type;
02216
02217     template<typename v1, typename v2>
02218     using eq_t = typename eq_helper<v1, v2>::type;
02219
02220     template<typename v1, typename v2>
02221     using lt_t = typename lt_helper<v1, v2>::type;
02222
02223     template<typename v1, typename v2>
02224     using gt_t = typename gt_helper<v1, v2>::type;
02225
02226     template<typename v1, typename v2>
02227     using div_t = typename div<v1, v2>::q_type;
02228
02229     template<typename v1, typename v2>
02230     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02231
02232     template<typename coeff, size_t deg>
02233     using monomial_t = typename monomial<coeff, deg>::type;
02234
02235     template<typename v>
02236     using derive_t = typename derive_helper<v>::type;
02237
02238     template<typename v>
02239     using pos_t = typename Ring::template pos_t<typename v::aN>;
02240
02241     template<typename v>
02242     static constexpr bool pos_v = pos_t<v>::value;
02243
02244     template<typename v1, typename v2>
02245     using gcd_t = std::conditional_t<
02246         Ring::is_euclidean_domain,
02247         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02248         void>;
02249 
```

```

02290     template<auto x>
02291     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02292
02293     template<typename v>
02294     using inject_ring_t = val<v>;
02295 };
02296 } // namespace aerobus
02297
02298 // fraction field
02299 namespace aerobus {
02300     namespace internal {
02301         template<typename Ring, typename E = void>
02302         requires IsEuclideanDomain<Ring>
02303         struct _FractionField {};
02304
02305         template<typename Ring>
02306         requires IsEuclideanDomain<Ring>
02307         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
02308             static constexpr bool is_field = true;
02309             static constexpr bool is_euclidean_domain = true;
02310
02311         private:
02312             template<typename vall, typename val2, typename E = void>
02313             struct to_string_helper {};
02314
02315             template<typename vall, typename val2>
02316             struct to_string_helper <vall, val2,
02317                 std::enable_if_t<
02318                     Ring::template eq_t<
02319                         val2, typename Ring::one
02320                         >::value
02321                     > {
02322                 static std::string func() {
02323                     return vall::to_string();
02324                 }
02325             };
02326
02327             template<typename vall, typename val2>
02328             struct to_string_helper<vall, val2,
02329                 std::enable_if_t<
02330                     !Ring::template eq_t<
02331                         val2,
02332                         typename Ring::one
02333                         >::value
02334                     > {
02335                 static std::string func() {
02336                     return "(" + vall::to_string() + ") / (" + val2::to_string() + ")";
02337                 }
02338             };
02339
02340         public:
02341             template<typename vall, typename val2>
02342             struct val {
02343                 using x = vall;
02344                 using y = val2;
02345                 using is_zero_t = typename vall::is_zero_t;
02346                 static constexpr bool is_zero_v = vall::is_zero_t::value;
02347
02348                 using ring_type = Ring;
02349                 using enclosing_type = _FractionField<Ring>;
02350
02351                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02352
02353                 template<typename valueType, int ghost = 0>
02354                 struct get_helper {
02355                     static constexpr INLINED_DEVICE valueType get() {
02356                         return internal::staticcast<valueType, typename
02357                             ring_type::inner_type>::template func<x::v>() /
02358                             internal::staticcast<valueType, typename ring_type::inner_type>::template
02359                             func<y::v>();
02360                     }
02361                 };
02362
02363                 #ifdef WITH_CUDA_FP16
02364                 template<int ghost>
02365                 struct get_helper<__half, ghost> {
02366                     static constexpr INLINED_DEVICE __half get() {
02367                         return internal::my_float2half_rn(
02368                             internal::staticcast<float, typename ring_type::inner_type>::template
02369                             func<x::v>() /
02370                             internal::staticcast<float, typename ring_type::inner_type>::template
02371                             func<y::v>());
02372                     }
02373                 };
02374             };
02375
02376             #ifdef WITH_CUDA_FP16
02377             template<int ghost>
02378             struct get_helper<__half, ghost> {
02379                 static constexpr INLINED_DEVICE __half get() {
02380                     return internal::my_float2half_rn(
02381                         internal::staticcast<float, typename ring_type::inner_type>::template
02382                         func<x::v>() /
02383                         internal::staticcast<float, typename ring_type::inner_type>::template
02384                         func<y::v>());
02385             };
02386         };
02387     };
02388 }
02389

```

```

02386         template<int ghost>
02387         struct get_helper<__half2, ghost> {
02388             static constexpr INLINED_DEVICE __half2 get() {
02389                 constexpr __half tmp = internal::my_float2half_rn(
02390                     internal::staticcast<float, typename ring_type::inner_type>::template
02391 func<x::v>() /
02391                     internal::staticcast<float, typename ring_type::inner_type>::template
02392 func<y::v>());
02392                 return __half2(tmp, tmp);
02393             }
02394         };
02395     #endif
02396
02400     template<typename valueType>
02401     static constexpr INLINED_DEVICE valueType get() {
02402         return get_helper<valueType, 0>::get();
02403     }
02404
02407     static std::string to_string() {
02408         return to_string_helper<vall, val2>::func();
02409     }
02410
02415     template<typename arithmeticType>
02416     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02417         return x::eval(v) / y::eval(v);
02418     }
02419 };
02420
02422 using zero = val<typename Ring::zero, typename Ring::one>;
02424 using one = val<typename Ring::one, typename Ring::one>;
02425
02428 template<typename v>
02429 using inject_t = val<v, typename Ring::one>;
02430
02433 template<auto x>
02434 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02435
02438 template<typename v>
02439 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02440
02442 using ring_type = Ring;
02443
02444 private:
02445     template<typename v, typename E = void>
02446     struct simplify {};
02447
02448     // x = 0
02449     template<typename v>
02450     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02451         using type = typename _FractionField<Ring>::zero;
02452     };
02453
02454     // x != 0
02455     template<typename v>
02456     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02457     private:
02458         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02459         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02460         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02461
02462         using posx = std::conditional_t<
02463             !Ring::template pos_v<newx>,
02464             typename Ring::template sub_t<typename Ring::zero, newx>,
02465             newx>;
02466         using posy = std::conditional_t<
02467             !Ring::template pos_v<newy>,
02468             typename Ring::template sub_t<typename Ring::zero, newy>,
02469             newy>;
02470     public:
02471         using type = typename _FractionField<Ring>::template val<posx, posy>;
02472     };
02473
02474 public:
02477     template<typename v>
02478     using simplify_t = typename simplify<v>::type;
02479
02480 private:
02481     template<typename v1, typename v2>
02482     struct add {
02483     private:
02484         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02485         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02486         using dividend = typename Ring::template add_t<a, b>;
02487         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02488         using g = typename Ring::template gcd_t<dividend, diviser>;
02489

```

```

02490         public:
02491             using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02492     };
02493
02494     template<typename v>
02495     struct pos {
02496         using type = std::conditional_t<
02497             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02498             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02499             std::true_type,
02500             std::false_type>;
02501     };
02502
02503     template<typename v1, typename v2>
02504     struct sub {
02505     private:
02506         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02507         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02508         using dividend = typename Ring::template sub_t<a, b>;
02509         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02510         using g = typename Ring::template gcd_t<dividend, diviser>;
02511
02512     public:
02513         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02514     };
02515
02516     template<typename v1, typename v2>
02517     struct mul {
02518     private:
02519         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02520         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02521
02522     public:
02523         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02524     };
02525
02526     template<typename v1, typename v2, typename E = void>
02527     struct div {};
02528
02529     template<typename v1, typename v2>
02530     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
02531     private:
02532         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02533         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02534
02535     public:
02536         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02537     };
02538
02539     template<typename v1, typename v2>
02540     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02541         using type = one;
02542     };
02543
02544     template<typename v1, typename v2>
02545     struct eq {
02546     private:
02547         using type = std::conditional_t<
02548             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02549             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02550             std::true_type,
02551             std::false_type>;
02552     };
02553
02554     template<typename v1, typename v2, typename E = void>
02555     struct gt;
02556
02557     template<typename v1, typename v2>
02558     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
02559         >> {
02560         using type = std::false_type;
02561     };
02562
02563     template<typename v1, typename v2>
02564     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
02565         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02566         >> {
02567         using type = typename gt<
02568             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02569             >::type;
02570     };
02571
02572     };
02573

```

```

02574     template<typename v1, typename v2>
02575     struct gt<v1, v2, std::enable_if_t<
02576         (!eq<v1, v2>::type::value) &&
02577         (pos<v1>::type::value) && (!pos<v2>::type::value)
02578         >> {
02579         using type = std::true_type;
02580     };
02581
02582     template<typename v1, typename v2>
02583     struct gt<v1, v2, std::enable_if_t<
02584         (!eq<v1, v2>::type::value) &&
02585         (!pos<v1>::type::value) && (pos<v2>::type::value)
02586         >> {
02587         using type = std::false_type;
02588     };
02589
02590     template<typename v1, typename v2>
02591     struct gt<v1, v2, std::enable_if_t<
02592         (!eq<v1, v2>::type::value) &&
02593         (pos<v1>::type::value) && (pos<v2>::type::value)
02594         >> {
02595         using type = typename Ring::template gt_t<
02596             typename Ring::template mul_t<v1::x, v2::y>,
02597             typename Ring::template mul_t<v2::y, v2::x>
02598         >;
02599     };
02600
02601 public:
02602     template<typename v1, typename v2>
02603     using add_t = typename add<v1, v2>::type;
02604
02605     template<typename v1, typename v2>
02606     using mod_t = zero;
02607
02608     template<typename v1, typename v2>
02609     using gcd_t = v1;
02610
02611     template<typename v1, typename v2>
02612     using sub_t = typename sub<v1, v2>::type;
02613
02614     template<typename v1, typename v2>
02615     using mul_t = typename mul<v1, v2>::type;
02616
02617     template<typename v1, typename v2>
02618     using div_t = typename div<v1, v2>::type;
02619
02620     template<typename v1, typename v2>
02621     using eq_t = typename eq<v1, v2>::type;
02622
02623     template<typename v1, typename v2>
02624     static constexpr bool eq_v = eq<v1, v2>::type::value;
02625
02626     template<typename v1, typename v2>
02627     using gt_t = typename gt<v1, v2>::type;
02628
02629     template<typename v1, typename v2>
02630     static constexpr bool gt_v = gt<v1, v2>::type::value;
02631
02632     template<typename v1>
02633     using pos_t = typename pos<v1>::type;
02634
02635     template<typename v>
02636     static constexpr bool pos_v = pos_t<v>::value;
02637
02638     template<typename Ring, typename E = void>
02639     requires IsEuclideanDomain<Ring>
02640     struct FractionFieldImpl {};
02641
02642     // fraction field of a field is the field itself
02643     template<typename Field>
02644     requires IsEuclideanDomain<Field>
02645     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field>> {
02646         using type = Field;
02647         template<typename v>
02648         using inject_t = v;
02649     };
02650
02651     // fraction field of a ring is the actual fraction field
02652     template<typename Ring>
02653     requires IsEuclideanDomain<Ring>
02654     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field>> {
02655         using type = _FractionField<Ring>;
02656     };
02657 } // namespace internal
02658
02659 template<typename Ring>

```

```

02699     requires IsEuclideanDomain<Ring>
02700     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02701
02702     template<typename Ring>
02703     struct Embed<Ring, FractionField<Ring> > {
02704         template<typename v>
02705         using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02706     };
02707 } // namespace aerobus
02708
02709 // short names for common types
02710 namespace aerobus {
02711     template<typename X, typename Y>
02712     requires IsRing<typename X::enclosing_type> &&
02713     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02714     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02715
02716     template<typename X, typename Y>
02717     requires IsRing<typename X::enclosing_type> &&
02718     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02719     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02720
02721     template<typename X, typename Y>
02722     requires IsRing<typename X::enclosing_type> &&
02723     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02724     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02725
02726     template<typename X, typename Y>
02727     requires IsEuclideanDomain<typename X::enclosing_type> &&
02728     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02729     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02730
02731     using q32 = FractionField<i32>;
02732
02733     using fpq32 = FractionField<polynomial<q32>>;
02734
02735     using q64 = FractionField<i64>;
02736
02737     using pi64 = polynomial<i64>;
02738
02739     using pq64 = polynomial<q64>;
02740
02741     using fpq64 = FractionField<polynomial<q64>>;
02742
02743     template<typename Ring, typename v1, typename v2>
02744     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02745
02746     template<typename v>
02747     using embed_int_poly_in_fractions_t =
02748         typename Embed<
02749             polynomial<typename v::ring_type>,
02750             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02751
02752     template<int64_t p, int64_t q>
02753     using make_q64_t = typename q64::template simplify_t<
02754         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02755
02756     template<int32_t p, int32_t q>
02757     using make_q32_t = typename q32::template simplify_t<
02758         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02759
02760     template<typename Ring, typename v1, typename v2>
02761     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02762     template<typename Ring, typename v1, typename v2>
02763     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02764
02765     template<>
02766     struct Embed<q32, q64> {
02767         template<typename v>
02768         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02769     };
02770
02771     template<typename Small, typename Large>
02772     struct Embed<polynomial<Small>, polynomial<Large> > {
02773     private:
02774         template<typename v, typename i>
02775         struct at_low;
02776
02777         template<typename v, size_t i>
02778         struct at_index {
02779             using type = typename Embed<Small, Large>::template
02780                 type<typename v::template coeff_at_t<i>>;
02781         };
02782
02783         template<typename v, size_t... Is>
02784         struct at_low<v, std::index_sequence<Is...> > {

```

```

02840         using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02841     };
02842
02843     public:
02844         template<typename v>
02845         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
1>::type;
02846     };
02847
02848     template<typename Ring, auto... xs>
02849     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02850         typename Ring::template inject_constant_t<xs>...>;
02851
02852     template<typename Ring, auto... xs>
02853     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02854         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02855 } // namespace aerobus
02856
02857 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02858 namespace aerobus {
02859     namespace internal {
02860         template<typename T, size_t x, typename E = void>
02861         struct factorial {};
02862
02863         template<typename T, size_t x>
02864         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02865             private:
02866                 template<typename, size_t, typename>
02867                 friend struct factorial;
02868             public:
02869                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02870             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02871         };
02872
02873         template<typename T>
02874         struct factorial<T, 0> {
02875             public:
02876                 using type = typename T::one;
02877             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02878         };
02879     } // namespace internal
02880
02881     template<typename T, size_t i>
02882     using factorial_t = typename internal::factorial<T, i>::type;
02883
02884     template<typename T, size_t i>
02885     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02886
02887     namespace internal {
02888         template<typename T, size_t k, size_t n, typename E = void>
02889         struct combination_helper {};
02890
02891         template<typename T, size_t k, size_t n>
02892         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02893             using type = typename FractionField<T>::template mul_t<
02894                 typename combination_helper<T, k - 1, n - 1>::type,
02895                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02896         };
02897
02898         template<typename T, size_t k, size_t n>
02899         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02900             using type = typename combination_helper<T, n - k, n>::type;
02901         };
02902
02903         template<typename T, size_t n>
02904         struct combination_helper<T, 0, n> {
02905             using type = typename FractionField<T>::one;
02906         };
02907
02908         template<typename T, size_t k, size_t n>
02909         struct combination {
02910             using type = typename internal::combination_helper<T, k, n>::type::x;
02911             static constexpr typename T::inner_type value =
02912                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02913         };
02914     } // namespace internal
02915
02916     template<typename T, size_t k, size_t n>
02917     using combination_t = typename internal::combination<T, k, n>::type;
02918
02919     template<typename T, size_t k, size_t n>
02920     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02921

```



```

02942     namespace internal {
02943         template<typename T, size_t m>
02944         struct bernoulli;
02945
02946         template<typename T, typename accum, size_t k, size_t m>
02947         struct bernoulli_helper {
02948             using type = typename bernoulli_helper<
02949                 T,
02950                 addfractions_t<T,
02951                     accum,
02952                     mulfractions_t<T,
02953                         makefraction_t<T,
02954                             combination_t<T, k, m + 1>,
02955                             typename T::one>,
02956                             typename bernoulli<T, k>::type
02957                         >,
02958                     k + 1,
02959                     m>::type;
02960         };
02961
02962         template<typename T, typename accum, size_t m>
02963         struct bernoulli_helper<T, accum, m, m> {
02964             using type = accum;
02965         };
02966
02967
02968
02969         template<typename T, size_t m>
02970         struct bernoulli {
02971             using type = typename FractionField<T>::template mul_t<
02972                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02973                 makefraction_t<T,
02974                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02975                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02976             >
02977         >;
02978
02979         template<typename floatType>
02980         static constexpr floatType value = type::template get<floatType>();
02981     };
02982
02983     template<typename T>
02984     struct bernoulli<T, 0> {
02985         using type = typename FractionField<T>::one;
02986
02987         template<typename floatType>
02988         static constexpr floatType value = type::template get<floatType>();
02989     };
02990 } // namespace internal
02991
02992 template<typename T, size_t n>
02993 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02994
03003 template<typename FloatType, typename T, size_t n>
03004 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
03005
03006 // bell numbers
03007 namespace internal {
03008     template<typename T, size_t n, typename E = void>
03009     struct bell_helper;
03010
03011     template<typename T, size_t n>
03012     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
03013         template<typename accum, size_t i, size_t stop>
03014         struct sum_helper {
03015             private:
03016                 using left = typename T::template mul_t<
03017                     combination_t<T, i, n-1>,
03018                     typename bell_helper<T, i>::type>;
03019                 using new_accum = typename T::template add_t<accum, left>;
03020             public:
03021                 using type = typename sum_helper<new_accum, i+1, stop>::type;
03022         };
03023
03024         template<typename accum, size_t stop>
03025         struct sum_helper<accum, stop, stop> {
03026             using type = accum;
03027         };
03028
03029         using type = typename sum_helper<typename T::zero, 0, n>::type;
03030     };
03031
03032     template<typename T>
03033     struct bell_helper<T, 0> {
03034         using type = typename T::one;
03035     };

```

```

03036
03037     template<typename T>
03038     struct bell_helper<T, 1> {
03039         using type = typename T::one;
03040     };
03041 } // namespace internal
03042
03043 template<typename T, size_t n>
03044 using bell_t = typename internal::bell_helper<T, n>::type;
03045
03046 template<typename T, size_t n>
03047 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03048
03049 namespace internal {
03050     template<typename T, int k, typename E = void>
03051     struct alternate {};
03052
03053     template<typename T, int k>
03054     struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
03055         using type = typename T::one;
03056         static constexpr typename T::inner_type value = type::template get<typename
03057 T::inner_type>();
03058     };
03059
03060     template<typename T, int k>
03061     struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
03062         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03063         static constexpr typename T::inner_type value = type::template get<typename
03064 T::inner_type>();
03065     };
03066 } // namespace internal
03067
03068 template<typename T, int k>
03069 using alternate_t = typename internal::alternate<T, k>::type;
03070
03071 template<typename T, size_t k>
03072 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03073
03074 namespace internal {
03075     template<typename T, int n, int k, typename E = void>
03076     struct stirling_1_helper {};
03077
03078     template<typename T>
03079     struct stirling_1_helper<T, 0, 0> {
03080         using type = typename T::one;
03081     };
03082
03083     template<typename T, int n>
03084     struct stirling_1_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
03085         using type = typename T::zero;
03086     };
03087
03088     template<typename T, int n>
03089     struct stirling_1_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
03090         using type = typename T::zero;
03091     };
03092
03093     template<typename T, int n, int k>
03094     struct stirling_1_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)>> {
03095         using type = typename T::template sub_t<
03096             typename stirling_1_helper<T, n-1, k-1>::type,
03097             typename T::template mul_t<
03098                 typename T::template inject_constant_t<n-1>,
03099                 typename stirling_1_helper<T, n-1, k>::type
03100             >>;
03101     };
03102 } // namespace internal
03103
03104 template<typename T, int n, int k>
03105 using stirling_1_signed_t = typename internal::stirling_1_helper<T, n, k>::type;
03106
03107 template<typename T, int n, int k>
03108 using stirling_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03109
03110 template<typename T, int n, int k>
03111 static constexpr typename T::inner_type stirling_1_unsigned_v = stirling_1_unsigned_t<T, n, k>::v;
03112
03113 template<typename T, int n, int k>
03114 static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03115
03116 namespace internal {
03117     template<typename T, int n, int k, typename E = void>
03118     struct stirling_2_helper {};
03119
03120     template<typename T, int n>
03121     struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)>> {
03122         using type = typename T::one;
03123     };

```

```

03147     };
03148
03149     template<typename T, int n>
03150     struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03151         using type = typename T::zero;
03152     };
03153
03154     template<typename T, int n>
03155     struct stirling_2_helper<T, n, n, std::enable_if_t<(n > 0)> {
03156         using type = typename T::zero;
03157     };
03158
03159     template<typename T, int n, int k>
03160     struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03161         using type = typename T::template add_t<
03162             typename stirling_2_helper<T, n-1, k-1>::type,
03163             typename T::template mul_t<
03164                 typename T::template inject_constant_t<k>,
03165                 typename stirling_2_helper<T, n-1, k>::type
03166             >;
03167     };
03168 } // namespace internal
03169
03174 template<typename T, int n, int k>
03175 using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03176
03181 template<typename T, int n, int k>
03182 static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03183
03184 namespace internal {
03185     template<typename T>
03186     struct pow_scalar {
03187         template<size_t p>
03188         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03189             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03190             x * func<p/2>(x) * func<p/2>(x);
03191         }
03192     };
03193
03194     template<typename T, typename p, size_t n, typename E = void>
03195     requires IsEuclideanDomain<T>
03196     struct pow;
03197
03198     template<typename T, typename p, size_t n>
03199     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03200         using type = typename T::template mul_t<
03201             typename pow<T, p, n/2>::type,
03202             typename pow<T, p, n/2>::type
03203         >;
03204     };
03205
03206     template<typename T, typename p, size_t n>
03207     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03208         using type = typename T::template mul_t<
03209             p,
03210             typename T::template mul_t<
03211                 typename pow<T, p, n/2>::type,
03212                 typename pow<T, p, n/2>::type
03213             >
03214         >;
03215     };
03216
03217     template<typename T, typename p, size_t n>
03218     struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03219 } // namespace internal
03220
03225 template<typename T, typename p, size_t n>
03226 using pow_t = typename internal::pow<T, p, n>::type;
03227
03232 template<typename T, typename p, size_t n>
03233 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03234
03235     template<typename T, size_t p>
03236     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03237 internal::pow_scalar<T>::template func<p>(x); }
03238
03238     namespace internal {
03239         template<typename, template<typename, size_t> typename, class>
03240         struct make_taylor_impl;
03241
03242         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03243         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03244             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
03245 Is>::type...>;
03246         };
03247     }

```

```

03252     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03253     using taylor = typename internal::make_taylor_impl<
03254         T,
03255         coeff_at,
03256         internal::make_index_sequence_reverse<deg + 1>::type;
03257
03258     namespace internal {
03259         template<typename T, size_t i>
03260         struct exp_coeff {
03261             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03262         };
03263
03264         template<typename T, size_t i, typename E = void>
03265         struct sin_coeff_helper {};
03266
03267         template<typename T, size_t i>
03268         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03269             using type = typename FractionField<T>::zero;
03270         };
03271
03272         template<typename T, size_t i>
03273         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03274             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03275         };
03276
03277         template<typename T, size_t i>
03278         struct sin_coeff {
03279             using type = typename sin_coeff_helper<T, i>::type;
03280         };
03281
03282         template<typename T, size_t i, typename E = void>
03283         struct sh_coeff_helper {};
03284
03285         template<typename T, size_t i>
03286         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03287             using type = typename FractionField<T>::zero;
03288         };
03289
03290         template<typename T, size_t i>
03291         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03292             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03293         };
03294
03295         template<typename T, size_t i>
03296         struct sh_coeff {
03297             using type = typename sh_coeff_helper<T, i>::type;
03298         };
03299
03300         template<typename T, size_t i, typename E = void>
03301         struct cos_coeff_helper {};
03302
03303         template<typename T, size_t i>
03304         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03305             using type = typename FractionField<T>::zero;
03306         };
03307
03308         template<typename T, size_t i>
03309         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03310             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03311         };
03312
03313         template<typename T, size_t i>
03314         struct cos_coeff {
03315             using type = typename cos_coeff_helper<T, i>::type;
03316         };
03317
03318         template<typename T, size_t i, typename E = void>
03319         struct cosh_coeff_helper {};
03320
03321         template<typename T, size_t i>
03322         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03323             using type = typename FractionField<T>::zero;
03324         };
03325
03326         template<typename T, size_t i>
03327         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03328             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03329         };
03330
03331         template<typename T, size_t i>
03332         struct cosh_coeff {
03333             using type = typename cosh_coeff_helper<T, i>::type;
03334         };
03335
03336         template<typename T, size_t i>
03337         struct geom_coeff { using type = typename FractionField<T>::one; };
03338

```

```

03339
03340     template<typename T, size_t i, typename E = void>
03341     struct atan_coeff_helper;
03342
03343     template<typename T, size_t i>
03344     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03345         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03346     };
03347
03348     template<typename T, size_t i>
03349     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03350         using type = typename FractionField<T>::zero;
03351     };
03352
03353     template<typename T, size_t i>
03354     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03355
03356     template<typename T, size_t i, typename E = void>
03357     struct asin_coeff_helper;
03358
03359     template<typename T, size_t i>
03360     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03361         using type = makefraction_t<T,
03362             factorial_t<T, i - 1>,
03363             typename T::template mul_t<
03364                 typename T::template val<i>,
03365                 T::template mul_t<
03366                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03367                     pow<T, factorial_t<T, i / 2>, 2
03368                 >
03369             >
03370         >>;
03371     };
03372
03373     template<typename T, size_t i>
03374     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03375         using type = typename FractionField<T>::zero;
03376     };
03377
03378     template<typename T, size_t i>
03379     struct asin_coeff {
03380         using type = typename asin_coeff_helper<T, i>::type;
03381     };
03382
03383     template<typename T, size_t i>
03384     struct lnpl_coeff {
03385         using type = makefraction_t<T,
03386             alternate_t<T, i + 1>,
03387             typename T::template val<i>;
03388     };
03389
03390     template<typename T>
03391     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03392
03393     template<typename T, size_t i, typename E = void>
03394     struct asinh_coeff_helper;
03395
03396     template<typename T, size_t i>
03397     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03398         using type = makefraction_t<T,
03399             typename T::template mul_t<
03400                 alternate_t<T, i / 2>,
03401                 factorial_t<T, i - 1>
03402             >,
03403             typename T::template mul_t<
03404                 typename T::template mul_t<
03405                     typename T::template val<i>,
03406                     pow_t<T, factorial_t<T, i / 2>, 2>
03407                 >,
03408                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03409             >
03410         >;
03411     };
03412
03413     template<typename T, size_t i>
03414     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03415         using type = typename FractionField<T>::zero;
03416     };
03417
03418     template<typename T, size_t i>
03419     struct asinh_coeff {
03420         using type = typename asinh_coeff_helper<T, i>::type;
03421     };
03422
03423     template<typename T, size_t i, typename E = void>
03424     struct atanh_coeff_helper;
03425

```

```

03426     template<typename T, size_t i>
03427     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03428         // 1/i
03429         using type = typename FractionField<T>::template val<
03430             typename T::one,
03431             typename T::template inject_constant_t<i>;
03432     };
03433
03434     template<typename T, size_t i>
03435     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03436         using type = typename FractionField<T>::zero;
03437     };
03438
03439     template<typename T, size_t i>
03440     struct atanh_coeff {
03441         using type = typename atanh_coeff_helper<T, i>::type;
03442     };
03443
03444     template<typename T, size_t i, typename E = void>
03445     struct tan_coeff_helper;
03446
03447     template<typename T, size_t i>
03448     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03449         using type = typename FractionField<T>::zero;
03450     };
03451
03452     template<typename T, size_t i>
03453     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03454     private:
03455         // 4^((i+1)/2)
03456         using _4p = typename FractionField<T>::template inject_t<
03457             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03458         // 4^((i+1)/2) - 1
03459         using _4pml = typename FractionField<T>::template
03460             sub_t<_4p, typename FractionField<T>::one>;
03461         // (-1)^((i-1)/2)
03462         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03463         using dividend = typename FractionField<T>::template mul_t<
03464             altp,
03465             FractionField<T>::template mul_t<
03466                 _4p,
03467                 FractionField<T>::template mul_t<
03468                     _4pml,
03469                     bernoulli_t<T, (i + 1)>
03470                 >
03471             >;
03472     public:
03473         using type = typename FractionField<T>::template div_t<dividend,
03474             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03475     };
03476
03477     template<typename T, size_t i>
03478     struct tan_coeff {
03479         using type = typename tan_coeff_helper<T, i>::type;
03480     };
03481
03482     template<typename T, size_t i, typename E = void>
03483     struct tanh_coeff_helper;
03484
03485     template<typename T, size_t i>
03486     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03487         using type = typename FractionField<T>::zero;
03488     };
03489
03490     template<typename T, size_t i>
03491     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03492     private:
03493         using _4p = typename FractionField<T>::template inject_t<
03494             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03495         using _4pml = typename FractionField<T>::template
03496             sub_t<_4p, typename FractionField<T>::one>;
03497         using dividend =
03498             typename FractionField<T>::template mul_t<
03499                 _4p,
03500                 typename FractionField<T>::template mul_t<
03501                     _4pml,
03502                     bernoulli_t<T, (i + 1)>>::type;
03503     public:
03504         using type = typename FractionField<T>::template div_t<dividend,
03505             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03506     };
03507
03508     template<typename T, size_t i>
03509     struct tanh_coeff {
03510         using type = typename tanh_coeff_helper<T, i>::type;
03511     };

```

```

03511     } // namespace internal
03512
03516     template<typename Integers, size_t deg>
03517     using exp = taylor<Integers, internal::exp_coeff, deg>;
03518
03522     template<typename Integers, size_t deg>
03523     using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03524         exp<Integers, deg>,
03525         typename polynomial<FractionField<Integers>>::one>;
03526
03530     template<typename Integers, size_t deg>
03531     using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03532
03536     template<typename Integers, size_t deg>
03537     using atan = taylor<Integers, internal::atan_coeff, deg>;
03538
03542     template<typename Integers, size_t deg>
03543     using sin = taylor<Integers, internal::sin_coeff, deg>;
03544
03548     template<typename Integers, size_t deg>
03549     using sinh = taylor<Integers, internal::sh_coeff, deg>;
03550
03555     template<typename Integers, size_t deg>
03556     using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03557
03562     template<typename Integers, size_t deg>
03563     using cos = taylor<Integers, internal::cos_coeff, deg>;
03564
03569     template<typename Integers, size_t deg>
03570     using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03571
03576     template<typename Integers, size_t deg>
03577     using asin = taylor<Integers, internal::asin_coeff, deg>;
03578
03583     template<typename Integers, size_t deg>
03584     using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03585
03590     template<typename Integers, size_t deg>
03591     using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03592
03597     template<typename Integers, size_t deg>
03598     using tan = taylor<Integers, internal::tan_coeff, deg>;
03599
03604     template<typename Integers, size_t deg>
03605     using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03606 } // namespace aerobus
03607
03608 // continued fractions
03609 namespace aerobus {
03612     template<int64_t... values>
03613     struct ContinuedFraction {};
03614
03617     template<int64_t a0>
03618     struct ContinuedFraction<a0> {
03620         using type = typename q64::template inject_constant_t<a0>;
03622         static constexpr double val = static_cast<double>(a0);
03623     };
03624
03628     template<int64_t a0, int64_t... rest>
03629     struct ContinuedFraction<a0, rest...> {
03631         using type = q64::template add_t<
03632             typename q64::template inject_constant_t<a0>,
03633             typename q64::template div_t<
03634                 typename q64::one,
03635                 typename ContinuedFraction<rest...>::type
03636             >;
03637
03639         static constexpr double val = type::template get<double>();
03640     };
03641
03645     using PI_fraction =
03646     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03647     using E_fraction =
03648     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03649     using SQRT2_fraction =
03650     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03651     using SQRT3_fraction =
03652     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03653 // NOLINT
03652 } // namespace aerobus
03653
03654 // known polynomials
03655 namespace aerobus {
03656     // CChebyshev
03657     namespace internal {
03658         template<int kind, size_t deg, typename I>
03659         struct chebyshev_helper {

```

```

03660         using type = typename polynomial<I>::template sub_t<
03661             typename polynomial<I>::template mul_t<
03662                 typename polynomial<I>::template mul_t<
03663                     typename polynomial<I>::template inject_constant_t<2>,
03664                     typename polynomial<I>::X>,
03665                     typename chebyshev_helper<kind, deg - 1, I>::type
03666                 >,
03667                 typename chebyshev_helper<kind, deg - 2, I>::type
03668             >;
03669     };
03670
03671     template<typename I>
03672     struct chebyshev_helper<1, 0, I> {
03673         using type = typename polynomial<I>::one;
03674     };
03675
03676     template<typename I>
03677     struct chebyshev_helper<1, 1, I> {
03678         using type = typename polynomial<I>::X;
03679     };
03680
03681     template<typename I>
03682     struct chebyshev_helper<2, 0, I> {
03683         using type = typename polynomial<I>::one;
03684     };
03685
03686     template<typename I>
03687     struct chebyshev_helper<2, 1, I> {
03688         using type = typename polynomial<I>::template mul_t<
03689             typename polynomial<I>::template inject_constant_t<2>,
03690             typename polynomial<I>::X>;
03691     };
03692 } // namespace internal
03693
03694 // Laguerre
03695 namespace internal {
03696     template<size_t deg, typename I>
03697     struct laguerre_helper {
03698         using Q = FractionField<I>;
03699         using PQ = polynomial<Q>;
03700
03701     private:
03702         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)Lkm2)
03703         using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03704         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03705         // -x + 2k-1
03706         using p = typename PQ::template val<
03707             typename Q::template inject_constant_t<-1>,
03708             typename Q::template inject_constant_t<2 * deg - 1>;
03709         // 1/n
03710         using factor = typename PQ::template inject_ring_t<
03711             typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03712
03713     public:
03714         using type = typename PQ::template mul_t <
03715             factor,
03716             typename PQ::template sub_t<
03717                 typename PQ::template mul_t<
03718                     p,
03719                     lnm1
03720                 >,
03721                 typename PQ::template mul_t<
03722                     typename PQ::template inject_constant_t<deg-1>,
03723                     lnm2
03724                 >
03725             >
03726         >;
03727     };
03728
03729     template<typename I>
03730     struct laguerre_helper<0, I> {
03731         using type = typename polynomial<FractionField<I>::one;
03732     };
03733
03734     template<typename I>
03735     struct laguerre_helper<1, I> {
03736     private:
03737         using PQ = polynomial<FractionField<I>::one;
03738     public:
03739         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03740     };
03741 } // namespace internal
03742
03743 // Bernstein
03744 namespace internal {
03745     template<size_t i, size_t m, typename I, typename E = void>

```



```

03746     struct bernstein_helper {};
03747
03748     template<typename I>
03749     struct bernstein_helper<0, 0, I> {
03750         using type = typename polynomial<I>::one;
03751     };
03752
03753     template<size_t i, size_t m, typename I>
03754     struct bernstein_helper<i, m, I, std::enable_if_t<
03755         (m > 0) && (i == 0)>> {
03756     private:
03757         using P = polynomial<I>;
03758     public:
03759         using type = typename P::template mul_t<
03760             typename P::template sub_t<typename P::one, typename P::X>,
03761             typename bernstein_helper<i, m-1, I>::type>;
03762     };
03763
03764     template<size_t i, size_t m, typename I>
03765     struct bernstein_helper<i, m, I, std::enable_if_t<
03766         (m > 0) && (i == m)>> {
03767     private:
03768         using P = polynomial<I>;
03769     public:
03770         using type = typename P::template mul_t<
03771             typename P::X,
03772             typename bernstein_helper<i-1, m-1, I>::type>;
03773     };
03774
03775     template<size_t i, size_t m, typename I>
03776     struct bernstein_helper<i, m, I, std::enable_if_t<
03777         (m > 0) && (i > 0) && (i < m)>> {
03778     private:
03779         using P = polynomial<I>;
03780     public:
03781         using type = typename P::template add_t<
03782             typename P::template mul_t<
03783                 typename P::template sub_t<typename P::one, typename P::X>,
03784                 typename bernstein_helper<i, m-1, I>::type>,
03785                 typename P::template mul_t<
03786                     typename P::X,
03787                     typename bernstein_helper<i-1, m-1, I>::type>;
03788             >>;
03789     } // namespace internal
03790
03791     // AllOne polynomials
03792     namespace internal {
03793         template<size_t deg, typename I>
03794         struct AllOneHelper {
03795             using type = aerobus::add_t<
03796                 typename polynomial<I>::one,
03797                 aerobus::mul_t<
03798                     typename polynomial<I>::X,
03799                     typename AllOneHelper<deg-1, I>::type
03800                 >>;
03801         };
03802
03803         template<typename I>
03804         struct AllOneHelper<0, I> {
03805             using type = typename polynomial<I>::one;
03806         };
03807     } // namespace internal
03808
03809     // Bessel polynomials
03810     namespace internal {
03811         template<size_t deg, typename I>
03812         struct BesselHelper {
03813     private:
03814             using P = polynomial<I>;
03815             using factor = typename P::template monomial_t<
03816                 typename I::template inject_constant_t<(2*deg - 1)>,
03817                 1>;
03818     public:
03819             using type = typename P::template add_t<
03820                 typename P::template mul_t<
03821                     factor,
03822                     typename BesselHelper<deg-1, I>::type
03823                 >,
03824                 typename BesselHelper<deg-2, I>::type
03825             >;
03826         };
03827
03828         template<typename I>
03829         struct BesselHelper<0, I> {
03830             using type = typename polynomial<I>::one;
03831         };
03832

```

```

03833     template<typename I>
03834     struct BesselHelper<1, I> {
03835     private:
03836         using P = polynomial<I>;
03837     public:
03838         using type = typename P::template add_t<
03839             typename P::one,
03840             typename P::X
03841         >;
03842     };
03843 } // namespace internal
03844
03845 namespace known_polynomials {
03846     enum hermite_kind {
03847         probabilist,
03848         physicist
03849     };
03850 }
03851
03852 // hermite
03853 namespace internal {
03854     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03855     struct hermite_helper {};
03856
03857     template<size_t deg, typename I>
03858     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03859     private:
03860         using hnm1 = typename hermite_helper<deg - 1,
03861             known_polynomials::hermite_kind::probabilist, I>::type;
03862         using hnm2 = typename hermite_helper<deg - 2,
03863             known_polynomials::hermite_kind::probabilist, I>::type;
03864     public:
03865         using type = typename polynomial<I>::template sub_t<
03866             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03867             typename polynomial<I>::template mul_t<
03868                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03869                 hnm2
03870             >
03871         >;
03872     };
03873
03874     template<size_t deg, typename I>
03875     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03876     private:
03877         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03878             I>::type;
03879         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03880             I>::type;
03881     public:
03882         using type = typename polynomial<I>::template sub_t<
03883             // 2X Hn-1
03884             typename polynomial<I>::template mul_t<
03885                 typename pi64::val<typename I::template inject_constant_t<2>,
03886                 typename I::zero>, hnm1>,
03887             typename polynomial<I>::template mul_t<
03888                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03889                 hnm2
03890             >
03891         >;
03892     };
03893
03894     template<typename I>
03895     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03896     using type = typename polynomial<I>::one;
03897     };
03898
03899     template<typename I>
03900     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03901     using type = typename polynomial<I>::X;
03902     };
03903
03904     template<typename I>
03905     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03906     using type = typename pi64::one;
03907     };
03908
03909     template<typename I>
03910     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03911     // 2X
03912     using type = typename polynomial<I>::template val<
03913         typename I::template inject_constant_t<2>,
03914         typename I::zero>;
03915     };
03916 } // namespace internal

```

```

03919
03920 // legendre
03921 namespace internal {
03922     template<size_t n, typename I>
03923     struct legendre_helper {
03924     private:
03925         using Q = FractionField<I>;
03926         using PQ = polynomial<Q>;
03927         // 1/n constant
03928         // (2n-1)/n X
03929         using fact_left = typename PQ::template monomial_t<
03930             makefraction_t<I,
03931                 typename I::template inject_constant_t<2*n-1>,
03932                 typename I::template inject_constant_t<n>
03933             >,
03934             1>;
03935         // (n-1) / n
03936         using fact_right = typename PQ::template val<
03937             makefraction_t<I,
03938                 typename I::template inject_constant_t<n-1>,
03939                 typename I::template inject_constant_t<n>>;
03940
03941     public:
03942         using type = PQ::template sub_t<
03943             typename PQ::template mul_t<
03944                 fact_left,
03945                 typename legendre_helper<n-1, I>::type
03946             >,
03947             typename PQ::template mul_t<
03948                 fact_right,
03949                 typename legendre_helper<n-2, I>::type
03950             >
03951         >;
03952     };
03953
03954     template<typename I>
03955     struct legendre_helper<0, I> {
03956     public:
03957         using type = typename polynomial<FractionField<I>::one>;
03958     };
03959
03960     template<typename I>
03961     struct legendre_helper<1, I> {
03962     public:
03963         using type = typename polynomial<FractionField<I>::X>;
03964     };
03965 } // namespace internal
03966
03967 // bernoulli polynomials
03968 namespace internal {
03969     template<size_t n>
03970     struct bernoulli_coeff {
03971     private:
03972         template<typename T, size_t i>
03973         struct inner {
03974         private:
03975             using F = FractionField<T>;
03976         public:
03977             using type = typename F::template mul_t<
03978                 typename F::template inject_ring_t<combination_t<T, i, n>,
03979                 bernoulli_t<T, n-i>
03980             >;
03981         };
03982     };
03983 } // namespace internal
03984
03985 namespace internal {
03986     template<size_t n>
03987     struct touchard_coeff {
03988     private:
03989         template<typename T, size_t i>
03990         struct inner {
03991         public:
03992             using type = stirling_2_t<T, n, i>;
03993         };
03994     };
03995 } // namespace internal
03996
03997 namespace internal {
03998     template<typename I = aerobus::i64>
03999     struct AbelHelper {
04000     private:
04001         using P = aerobus::polynomial<I>;
04002     public:
04003         // to keep recursion working, we need to operate on a*n and not just a
04004         template<size_t deg, I::inner_type an>
04005         struct Inner {
04006         public:
04007             // abel(n, a) = (x-an) * abel(n-1, a)
04008             using type = typename aerobus::mul_t<
04009                 typename Inner<deg-1, an>::type,
04010                 aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>

```

```

04006         >;
04007     };
04008
04009     // abel(0, a) = 1
04010     template<I::inner_type an>
04011     struct Inner<0, an> {
04012         using type = P::one;
04013     };
04014
04015     // abel(1, a) = X
04016     template<I::inner_type an>
04017     struct Inner<1, an> {
04018         using type = P::X;
04019     };
04020 };
04021 } // namespace internal
04022
04023 namespace known_polynomials {
04024
04025     template<size_t n, auto a, typename I = aerobus::i64>
04026     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
04027
04028     template<size_t deg, typename I = aerobus::i64>
04029     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
04030
04031     template<size_t deg, typename I = aerobus::i64>
04032     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
04033
04034     template<size_t deg, typename I = aerobus::i64>
04035     using laguerre = typename internal::laguerre_helper<deg, I>::type;
04036
04037     template<size_t deg, typename I = aerobus::i64>
04038     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
04039 I>::type;
04040
04041     template<size_t deg, typename I = aerobus::i64>
04042     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04043
04044     template<size_t i, size_t m, typename I = aerobus::i64>
04045     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04046
04047     template<size_t deg, typename I = aerobus::i64>
04048     using legendre = typename internal::legendre_helper<deg, I>::type;
04049
04050     template<size_t deg, typename I = aerobus::i64>
04051     using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
04052
04053     template<size_t deg, typename I = aerobus::i64>
04054     using allone = typename internal::AllOneHelper<deg, I>::type;
04055
04056     template<size_t deg, typename I = aerobus::i64>
04057     using bessel = typename internal::BesselHelper<deg, I>::type;
04058
04059     template<size_t deg, typename I = aerobus::i64>
04060     using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
04061 } // namespace known_polynomials
04062 } // namespace aerobus
04063
04064 #ifdef AEROBUS_CONWAY_IMPORTS
04065 // conway polynomials
04066 namespace aerobus {
04067     template<int p, int n>
04068     struct ConwayPolynomial {};
04069 }
04070 #ifndef DO_NOT_DOCUMENT
04071 #define ZPZV ZPZ::template val
04072 #define POLYV aerobus::polynomial<ZPZ>::template val
04073 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>; }; // NOLINT
04074 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04075 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04076 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04077 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04078 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04079 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04080 template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04081 template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; //

```

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```

06083     template<> struct ConwayPolynomial<977, 8> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<855>, ZPZV<807>, ZPZV<77>, ZPZV<3>; }; //
NOLINT
06084     template<> struct ConwayPolynomial<977, 9> { using ZPZ = aerobus::zpz<977>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<450>, ZPZV<740>, ZPZV<974>;
}; // NOLINT
06085     template<> struct ConwayPolynomial<983, 1> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<978>; }; // NOLINT
06086     template<> struct ConwayPolynomial<983, 2> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<981>, ZPZV<5>; }; // NOLINT
06087     template<> struct ConwayPolynomial<983, 3> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<978>; }; // NOLINT
06088     template<> struct ConwayPolynomial<983, 4> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<5>, ZPZV<567>, ZPZV<5>; }; // NOLINT
06089     template<> struct ConwayPolynomial<983, 5> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<8>, ZPZV<978>; }; // NOLINT
06090     template<> struct ConwayPolynomial<983, 6> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<849>, ZPZV<296>, ZPZV<228>, ZPZV<5>; }; // NOLINT
06091     template<> struct ConwayPolynomial<983, 7> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<978>; }; // NOLINT
06092     template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<738>, ZPZV<276>, ZPZV<530>, ZPZV<5>; }; //
NOLINT
06093     template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<858>, ZPZV<87>, ZPZV<978>;
}; // NOLINT
06094     template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<985>; }; // NOLINT
06095     template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<989>, ZPZV<6>; }; // NOLINT
06096     template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<985>; }; // NOLINT
06097     template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<10>, ZPZV<794>, ZPZV<6>; }; // NOLINT
06098     template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<985>; }; // NOLINT
06099     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
06100     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
06101     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
06102     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
06103     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
06104     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
06105     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
06106     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
06107     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
06108     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
06109     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
06110     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
06111     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
06112 #endif // DO_NOT_DOCUMENT
06113 } // namespace aerobus
06114 #endif // AEROBUS_CONWAY_IMPORTS
06115
06116 #endif // __INC_AEROBUS__ // NOLINT

```

## 9.4 src/examples.h File Reference

## 9.5 examples.h

[Go to the documentation of this file.](#)

```

00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_

```

# Chapter 10

## Examples

### 10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

### 10.2 examples/custom\_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

## 10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
#include <cstdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

/*
change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
to see how good is the generated assembly compared to what nvcc generates for 16 bits
*/
using int_type = aerobus::i16;
using float_type = __half2;

constexpr size_t N = 1 << 24;

template<typename T>
struct ExpmlDegree;

template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};

template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};

template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};

double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

template<typename T>
struct GetRandT;

template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};

template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};

template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};

using EXPM1 = aerobus::expml<int_type, ExpmlDegree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}
```

```

}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        out[i] = f(f(f(f(f(f(in[i]))))))) ;
    }
}

int main() {
    float_type *d_in, *d_out;
    cudaMalloc<float_type>(&d_in, N * sizeof(float_type));
    cudaMalloc<float_type>(&d_out, N * sizeof(float_type));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice);

    run<<128, 512>>>(N, d_in, d_out);

    cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost);

    cudaFree(d_in);
    cudaFree(d_out);
}

```

## 10.4 examples/continued\_fractions.cpp

## How to use `aerobus::ContinuedFraction` to get approximations of known numbers

[illegible]

## 10.5 examples/modular\_arithmetic.cpp

## How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

// x^3 + 2x^2 + 1, with coefficients in Z/2Z, actually x^3 + 1
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
// x^3 + 5x^2 + 7x + 11 with coefficients in Z/17Z, meaning actually x^3 + x^2 + 1
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

## 10.6 examples/make\_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

## 10.7 examples/polynomials\_over\_finite\_field.cpp

How to build a known polynomial (here aerobus::known\_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```

## 10.8 examples/compensated\_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

## See also

[publication](#)

```
// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;
        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}
```





# Index

abs\_t  
  aerobus, 20  
add\_t  
  aerobus, 20  
  aerobus::i32, 59  
  aerobus::i64, 65  
  aerobus::polynomial< Ring >, 74  
  aerobus::Quotient< Ring, X >, 82  
  aerobus::zpz< p >, 107  
addfractions\_t  
  aerobus, 20  
aerobus, 15  
  abs\_t, 20  
  add\_t, 20  
  addfractions\_t, 20  
  aligned\_malloc, 34  
  alternate\_t, 20  
  alternate\_v, 35  
  asin, 21  
  asinh, 21  
  atan, 21  
  atanh, 21  
  bell\_t, 23  
  bernoulli\_t, 23  
  bernoulli\_v, 35  
  combination\_t, 23  
  combination\_v, 35  
  cos, 23  
  cosh, 25  
  div\_t, 25  
  E\_fraction, 25  
  embed\_int\_poly\_in\_fractions\_t, 25  
  exp, 26  
  expm1, 26  
  factorial\_t, 26  
  factorial\_v, 35  
  field, 34  
  fpq32, 26  
  fpq64, 27  
  FractionField, 27  
  gcd\_t, 27  
  geometric\_sum, 27  
  lnp1, 27  
  make\_frac\_polynomial\_t, 28  
  make\_int\_polynomial\_t, 28  
  make\_q32\_t, 28  
  make\_q64\_t, 29  
  makefraction\_t, 29  
  mul\_t, 29  
  mulfractions\_t, 29  
  pi64, 30  
  PI\_fraction, 30  
  pow\_t, 30  
  pq64, 30  
  q32, 30  
  q64, 31  
  sin, 31  
  sinh, 31  
  SQRT2\_fraction, 31  
  SQRT3\_fraction, 31  
  stirling\_1\_signed\_t, 32  
  stirling\_1\_unsigned\_t, 32  
  stirling\_2\_t, 32  
  sub\_t, 33  
  tan, 33  
  tanh, 33  
  taylor, 33  
  vadd\_t, 34  
  vmul\_t, 34  
aerobus::ContinuedFraction< a0 >, 47  
  type, 47  
  val, 48  
aerobus::ContinuedFraction< a0, rest... >, 48  
  type, 49  
  val, 49  
aerobus::ContinuedFraction< values >, 46  
aerobus::ConwayPolynomial, 49  
aerobus::Embed< i32, i64 >, 51  
  type, 51  
aerobus::Embed< polynomial< Small >, polynomial< Large > >, 52  
  type, 52  
aerobus::Embed< q32, q64 >, 53  
  type, 53  
aerobus::Embed< Quotient< Ring, X >, Ring >, 54  
  type, 54  
aerobus::Embed< Ring, FractionField< Ring > >, 55  
  type, 55  
aerobus::Embed< Small, Large, E >, 51  
aerobus::Embed< zpz< x >, i32 >, 55  
  type, 56  
aerobus::i32, 57  
  add\_t, 59  
  div\_t, 59  
  eq\_t, 59  
  eq\_v, 62  
  gcd\_t, 59  
  gt\_t, 60

- inject\_constant\_t, 60
- inject\_ring\_t, 60
- inner\_type, 60
- is\_euclidean\_domain, 62
- is\_field, 62
- lt\_t, 60
- mod\_t, 61
- mul\_t, 61
- one, 61
- pos\_t, 61
- pos\_v, 62
- sub\_t, 62
- zero, 62
- aerobus::i32::val< x >, 91
  - enclosing\_type, 92
  - get, 92
  - is\_zero\_t, 92
  - to\_string, 92
  - v, 92
- aerobus::i64, 64
  - add\_t, 65
  - div\_t, 66
  - eq\_t, 66
  - eq\_v, 69
  - gcd\_t, 66
  - gt\_t, 66
  - gt\_v, 69
  - inject\_constant\_t, 67
  - inject\_ring\_t, 67
  - inner\_type, 67
  - is\_euclidean\_domain, 69
  - is\_field, 69
  - lt\_t, 67
  - lt\_v, 70
  - mod\_t, 68
  - mul\_t, 68
  - one, 68
  - pos\_t, 68
  - pos\_v, 70
  - sub\_t, 68
  - zero, 69
- aerobus::i64::val< x >, 93
  - enclosing\_type, 94
  - get, 94
  - inner\_type, 94
  - is\_zero\_t, 94
  - to\_string, 94
  - v, 95
- aerobus::internal, 36
  - index\_sequence\_reverse, 40
  - is\_instantiation\_of\_v, 40
  - make\_index\_sequence\_reverse, 40
  - type\_at\_t, 40
- aerobus::is\_prime< n >, 71
  - value, 72
- aerobus::IsEuclideanDomain, 43
- aerobus::IsField, 43
- aerobus::IsRing, 44
- aerobus::known\_polynomials, 40
  - hermite\_kind, 40
  - physicist, 41
  - probabilist, 41
- aerobus::polynomial< Ring >, 72
  - add\_t, 74
  - derive\_t, 74
  - div\_t, 74
  - eq\_t, 75
  - gcd\_t, 75
  - gt\_t, 75
  - inject\_constant\_t, 76
  - inject\_ring\_t, 76
  - is\_euclidean\_domain, 80
  - is\_field, 80
  - lt\_t, 76
  - mod\_t, 76
  - monomial\_t, 77
  - mul\_t, 77
  - one, 77
  - pos\_t, 77
  - pos\_v, 80
  - simplify\_t, 79
  - sub\_t, 79
  - X, 79
  - zero, 79
- aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost >, 49
  - func, 50
- aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
  - func, 50
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >, 56
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop >, 70
  - type, 71
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop >, 71
  - type, 71
- aerobus::polynomial< Ring >::val< coeffN >, 102
  - aN, 103
  - coeff\_at\_t, 103
  - compensated\_eval, 104
  - degree, 105
  - enclosing\_type, 103
  - eval, 104
  - is\_zero\_t, 103
  - is\_zero\_v, 105
  - ring\_type, 104
  - strip, 104
  - to\_string, 104
  - value\_at\_t, 104
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, E >, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at<

- index, std::enable\_if\_t<(index < 0 || index > 0)>, 45
- type, 45
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index == 0)>>, 46
- type, 46
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 95
- aN, 96
- coeff\_at\_t, 96
- compensated\_eval, 97
- degree, 99
- enclosing\_type, 96
- eval, 98
- is\_zero\_t, 97
- is\_zero\_v, 99
- ring\_type, 97
- strip, 97
- to\_string, 98
- value\_at\_t, 97
- aerobus::Quotient< Ring, X >, 81
- add\_t, 82
- div\_t, 83
- eq\_t, 83
- eq\_v, 85
- inject\_constant\_t, 83
- inject\_ring\_t, 83
- is\_euclidean\_domain, 85
- mod\_t, 84
- mul\_t, 84
- one, 84
- pos\_t, 84
- pos\_v, 85
- zero, 85
- aerobus::Quotient< Ring, X >::val< V >, 99
- raw\_t, 100
- type, 100
- aerobus::type\_list< Ts >, 87
- at, 88
- concat, 88
- insert, 88
- length, 89
- push\_back, 88
- push\_front, 89
- remove, 89
- aerobus::type\_list< Ts >::pop\_front, 80
- tail, 81
- type, 81
- aerobus::type\_list< Ts >::split< index >, 86
- head, 86
- tail, 86
- aerobus::type\_list<>, 90
- concat, 90
- insert, 90
- length, 91
- push\_back, 90
- push\_front, 90
- aerobus::zpz< p >, 105
- add\_t, 107
- div\_t, 107
- eq\_t, 108
- eq\_v, 111
- gcd\_t, 108
- gt\_t, 108
- gt\_v, 111
- inject\_constant\_t, 109
- inner\_type, 109
- is\_euclidean\_domain, 111
- is\_field, 111
- lt\_t, 109
- lt\_v, 111
- mod\_t, 109
- mul\_t, 109
- one, 110
- pos\_t, 110
- pos\_v, 112
- sub\_t, 110
- zero, 110
- aerobus::zpz< p >::val< x >, 100
- enclosing\_type, 101
- get, 101
- is\_zero\_t, 101
- is\_zero\_v, 102
- to\_string, 101
- v, 102
- aligned\_malloc
- aerobus, 34
- alternate\_t
- aerobus, 20
- alternate\_v
- aerobus, 35
- aN
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
- asin
- aerobus, 21
- asinh
- aerobus, 21
- at
- aerobus::type\_list< Ts >, 88
- atan
- aerobus, 21
- atanh
- aerobus, 21
- bell\_t
- aerobus, 23
- bernoulli\_t
- aerobus, 23
- bernoulli\_v
- aerobus, 35
- coeff\_at\_t
- aerobus::polynomial< Ring >::val< coeffN >, 103
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96

- combination\_t
  - aerobus, 23
- combination\_v
  - aerobus, 35
- compensated\_eval
  - aerobus::polynomial< Ring >::val< coeffN >, 104
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
- concat
  - aerobus::type\_list< Ts >, 88
  - aerobus::type\_list<>, 90
- cos
  - aerobus, 23
- cosh
  - aerobus, 25
- degree
  - aerobus::polynomial< Ring >::val< coeffN >, 105
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
- derive\_t
  - aerobus::polynomial< Ring >, 74
- div\_t
  - aerobus, 25
  - aerobus::i32, 59
  - aerobus::i64, 66
  - aerobus::polynomial< Ring >, 74
  - aerobus::Quotient< Ring, X >, 83
  - aerobus::zpz< p >, 107
- E\_fraction
  - aerobus, 25
- embed\_int\_poly\_in\_fractions\_t
  - aerobus, 25
- enclosing\_type
  - aerobus::i32::val< x >, 92
  - aerobus::i64::val< x >, 94
  - aerobus::polynomial< Ring >::val< coeffN >, 103
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 96
  - aerobus::zpz< p >::val< x >, 101
- eq\_t
  - aerobus::i32, 59
  - aerobus::i64, 66
  - aerobus::polynomial< Ring >, 75
  - aerobus::Quotient< Ring, X >, 83
  - aerobus::zpz< p >, 108
- eq\_v
  - aerobus::i32, 62
  - aerobus::i64, 69
  - aerobus::Quotient< Ring, X >, 85
  - aerobus::zpz< p >, 111
- eval
  - aerobus::polynomial< Ring >::val< coeffN >, 104
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 98
- exp
  - aerobus, 26
- expm1
  - aerobus, 26
- factorial\_t
  - aerobus, 26
- factorial\_v
  - aerobus, 35
- field
  - aerobus, 34
- fpq32
  - aerobus, 26
- fpq64
  - aerobus, 27
- FractionField
  - aerobus, 27
- func
  - aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost >, 50
  - aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 50
- gcd\_t
  - aerobus, 27
  - aerobus::i32, 59
  - aerobus::i64, 66
  - aerobus::polynomial< Ring >, 75
  - aerobus::zpz< p >, 108
- geometric\_sum
  - aerobus, 27
- get
  - aerobus::i32::val< x >, 92
  - aerobus::i64::val< x >, 94
  - aerobus::zpz< p >::val< x >, 101
- gt\_t
  - aerobus::i32, 60
  - aerobus::i64, 66
  - aerobus::polynomial< Ring >, 75
  - aerobus::zpz< p >, 108
- gt\_v
  - aerobus::i64, 69
  - aerobus::zpz< p >, 111
- head
  - aerobus::type\_list< Ts >::split< index >, 86
- hermite\_kind
  - aerobus::known\_polynomials, 40
- index\_sequence\_reverse
  - aerobus::internal, 40
- inject\_constant\_t
  - aerobus::i32, 60
  - aerobus::i64, 67
  - aerobus::polynomial< Ring >, 76
  - aerobus::Quotient< Ring, X >, 83
  - aerobus::zpz< p >, 109
- inject\_ring\_t
  - aerobus::i32, 60
  - aerobus::i64, 67

- aerobus::polynomial< Ring >, 76
- aerobus::Quotient< Ring, X >, 83
- inner\_type
  - aerobus::i32, 60
  - aerobus::i64, 67
  - aerobus::i64::val< x >, 94
  - aerobus::zpz< p >, 109
- insert
  - aerobus::type\_list< Ts >, 88
  - aerobus::type\_list<>, 90
- Introduction, 1
- is\_euclidean\_domain
  - aerobus::i32, 62
  - aerobus::i64, 69
  - aerobus::polynomial< Ring >, 80
  - aerobus::Quotient< Ring, X >, 85
  - aerobus::zpz< p >, 111
- is\_field
  - aerobus::i32, 62
  - aerobus::i64, 69
  - aerobus::polynomial< Ring >, 80
  - aerobus::zpz< p >, 111
- is\_instantiation\_of\_v
  - aerobus::internal, 40
- is\_zero\_t
  - aerobus::i32::val< x >, 92
  - aerobus::i64::val< x >, 94
  - aerobus::polynomial< Ring >::val< coeffN >, 103
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 97
  - aerobus::zpz< p >::val< x >, 101
- is\_zero\_v
  - aerobus::polynomial< Ring >::val< coeffN >, 105
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 99
  - aerobus::zpz< p >::val< x >, 102
- length
  - aerobus::type\_list< Ts >, 89
  - aerobus::type\_list<>, 91
- Inp1
  - aerobus, 27
- lt\_t
  - aerobus::i32, 60
  - aerobus::i64, 67
  - aerobus::polynomial< Ring >, 76
  - aerobus::zpz< p >, 109
- lt\_v
  - aerobus::i64, 70
  - aerobus::zpz< p >, 111
- make\_frac\_polynomial\_t
  - aerobus, 28
- make\_index\_sequence\_reverse
  - aerobus::internal, 40
- make\_int\_polynomial\_t
  - aerobus, 28
- make\_q32\_t
  - aerobus, 28
- make\_q64\_t
  - aerobus, 29
- makefraction\_t
  - aerobus, 29
- mod\_t
  - aerobus::i32, 61
  - aerobus::i64, 68
  - aerobus::polynomial< Ring >, 76
  - aerobus::Quotient< Ring, X >, 84
  - aerobus::zpz< p >, 109
- monomial\_t
  - aerobus::polynomial< Ring >, 77
- mul\_t
  - aerobus, 29
  - aerobus::i32, 61
  - aerobus::i64, 68
  - aerobus::polynomial< Ring >, 77
  - aerobus::Quotient< Ring, X >, 84
  - aerobus::zpz< p >, 109
- mulfractions\_t
  - aerobus, 29
- one
  - aerobus::i32, 61
  - aerobus::i64, 68
  - aerobus::polynomial< Ring >, 77
  - aerobus::Quotient< Ring, X >, 84
  - aerobus::zpz< p >, 110
- physicist
  - aerobus::known\_polynomials, 41
- pi64
  - aerobus, 30
- PI\_fraction
  - aerobus, 30
- pos\_t
  - aerobus::i32, 61
  - aerobus::i64, 68
  - aerobus::polynomial< Ring >, 77
  - aerobus::Quotient< Ring, X >, 84
  - aerobus::zpz< p >, 110
- pos\_v
  - aerobus::i32, 62
  - aerobus::i64, 70
  - aerobus::polynomial< Ring >, 80
  - aerobus::Quotient< Ring, X >, 85
  - aerobus::zpz< p >, 112
- pow\_t
  - aerobus, 30
- pq64
  - aerobus, 30
- probabilist
  - aerobus::known\_polynomials, 41
- push\_back
  - aerobus::type\_list< Ts >, 88
  - aerobus::type\_list<>, 90
- push\_front
  - aerobus::type\_list< Ts >, 89
  - aerobus::type\_list<>, 90

- q32
  - aerobus, [30](#)
- q64
  - aerobus, [31](#)
- raw\_t
  - aerobus::Quotient< Ring, X >::val< V >, [100](#)
- README.md, [113](#)
- remove
  - aerobus::type\_list< Ts >, [89](#)
- ring\_type
  - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- simplify\_t
  - aerobus::polynomial< Ring >, [79](#)
- sin
  - aerobus, [31](#)
- sinh
  - aerobus, [31](#)
- SQRT2\_fraction
  - aerobus, [31](#)
- SQRT3\_fraction
  - aerobus, [31](#)
- src/aerobus.h, [113](#)
- src/examples.h, [206](#)
- stirling\_1\_signed\_t
  - aerobus, [32](#)
- stirling\_1\_unsigned\_t
  - aerobus, [32](#)
- stirling\_2\_t
  - aerobus, [32](#)
- strip
  - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- sub\_t
  - aerobus, [33](#)
  - aerobus::i32, [62](#)
  - aerobus::i64, [68](#)
  - aerobus::polynomial< Ring >, [79](#)
  - aerobus::zpz< p >, [110](#)
- tail
  - aerobus::type\_list< Ts >::pop\_front, [81](#)
  - aerobus::type\_list< Ts >::split< index >, [86](#)
- tan
  - aerobus, [33](#)
- tanh
  - aerobus, [33](#)
- taylor
  - aerobus, [33](#)
- to\_string
  - aerobus::i32::val< x >, [92](#)
  - aerobus::i64::val< x >, [94](#)
  - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [98](#)
- aerobus::zpz< p >::val< x >, [101](#)
- type
  - aerobus::ContinuedFraction< a0 >, [47](#)
  - aerobus::ContinuedFraction< a0, rest... >, [49](#)
  - aerobus::Embed< i32, i64 >, [51](#)
  - aerobus::Embed< polynomial< Small >, polynomial< Large > >, [52](#)
  - aerobus::Embed< q32, q64 >, [53](#)
  - aerobus::Embed< Quotient< Ring, X >, Ring >, [54](#)
  - aerobus::Embed< Ring, FractionField< Ring > >, [55](#)
  - aerobus::Embed< zpz< x >, i32 >, [56](#)
  - aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop >, [71](#)
  - aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop >, [71](#)
  - aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index< 0 || index > 0)> >, [45](#)
  - aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> >, [46](#)
  - aerobus::Quotient< Ring, X >::val< V >, [100](#)
  - aerobus::type\_list< Ts >::pop\_front, [81](#)
- type\_at\_t
  - aerobus::internal, [40](#)
- v
  - aerobus::i32::val< x >, [92](#)
  - aerobus::i64::val< x >, [95](#)
  - aerobus::zpz< p >::val< x >, [102](#)
- vadd\_t
  - aerobus, [34](#)
- val
  - aerobus::ContinuedFraction< a0 >, [48](#)
  - aerobus::ContinuedFraction< a0, rest... >, [49](#)
- value
  - aerobus::is\_prime< n >, [72](#)
- value\_at\_t
  - aerobus::polynomial< Ring >::val< coeffN >, [104](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [97](#)
- vmul\_t
  - aerobus, [34](#)
- X
  - aerobus::polynomial< Ring >, [79](#)
- zero
  - aerobus::i32, [62](#)
  - aerobus::i64, [69](#)
  - aerobus::polynomial< Ring >, [79](#)
  - aerobus::Quotient< Ring, X >, [85](#)
  - aerobus::zpz< p >, [110](#)