

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download the [aerobus.h](#)
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including [aerobus.h](#).

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

results on my laptop :

```
./benchmarks_avx512.exe
[std math] 5.358e-01 Gsin/s
[std fast math] 3.389e+00 Gsin/s
[aerobus deg 1] 1.871e+01 Gsin/s
average error (vs std) : 4.36e-02
max error (vs std) : 1.50e-01
[aerobus deg 3] 1.943e+01 Gsin/s
average error (vs std) : 1.85e-04
max error (vs std) : 8.17e-04
[aerobus deg 5] 1.335e+01 Gsin/s
average error (vs std) : 6.07e-07
max error (vs std) : 3.63e-06
[aerobus deg 7] 8.634e+00 Gsin/s
average error (vs std) : 1.27e-09
max error (vs std) : 9.75e-09
[aerobus deg 9] 6.171e+00 Gsin/s
average error (vs std) : 1.89e-12
max error (vs std) : 1.78e-11
[aerobus deg 11] 4.731e+00 Gsin/s
average error (vs std) : 2.12e-15
max error (vs std) : 2.40e-14
[aerobus deg 13] 3.862e+00 Gsin/s
average error (vs std) : 3.16e-17
max error (vs std) : 3.33e-16
[aerobus deg 15] 3.359e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
[aerobus deg 17] 2.947e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
```

1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `Fraction<Field<Integers>>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`

- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in `Integers (typename)` and `index (size_t)`;
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
```

```

vbroadcastsd    ymm3, QWORD PTR .LC23[rip]
vbroadcastsd    ymm2, QWORD PTR .LC25[rip]
.L3:
vmovupd ymm15, YMMWORD PTR [rsi+rax]
vmovapd ymm0, ymm15
vmadd132pd      ymm0, ymm14, ymm1
vmadd132pd      ymm0, ymm13, ymm15
vmadd132pd      ymm0, ymm12, ymm15
vmadd132pd      ymm0, ymm11, ymm15
vmadd132pd      ymm0, ymm10, ymm15
vmadd132pd      ymm0, ymm9, ymm15
vmadd132pd      ymm0, ymm8, ymm15
vmadd132pd      ymm0, ymm7, ymm15
vmadd132pd      ymm0, ymm6, ymm15
vmadd132pd      ymm0, ymm5, ymm15
vmadd132pd      ymm0, ymm4, ymm15
vmadd132pd      ymm0, ymm3, ymm15
vmadd132pd      ymm0, ymm2, ymm15
vmadd132pd      ymm0, ymm1, ymm15
vmovupd YMMWORD PTR [rdx+rax], ymm0
add    rax, 32
cmp    rcx, rax
jne    .L3
mov    rax, rdi
and    rax, -4
vzeroupper

```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, `Aerobus` allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```

using namespace aerobus;
using ZmZ = zp<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;

```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define `rational functions` by taking the ring of fractions of polynomials:

```

using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;

```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for `quotient ring R/X` where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```

using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;

```

As we could have using `zp<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

Aerobus gives an implementation for `continued fractions`. It can be used this way:

```
using namespace aerobus;  
using T = ContinuedFraction<1,2,3,4>;  
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```


Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

aerobus	Main namespace for all publicly exposed types or functions	17
aerobus::internal	Internal implementations, subject to breaking changes without notice	33
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	36

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	41
aerobus::IsField	
Concept to express R is a field	41
aerobus::IsRing	
Concept to express R is a Ring	42

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0 index > 0)> >	43
aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	44
aerobus::ContinuedFraction< values >	44
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	44
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	45
aerobus::ConwayPolynomial	46
aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	46
aerobus::i64	
64 bits signed integers, seen as a algebraic ring with related operations	50
aerobus::is_prime< n >	
Checks if n is prime	54
aerobus::polynomial< Ring >	55
aerobus::type_list< Ts >::pop_front	
Removes types from head of the list	61
aerobus::Quotient< Ring, X >	
Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	62
aerobus::type_list< Ts >::split< index >	
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aerobus::type_list< Ts >	
Empty pure template struct to handle type list	68
aerobus::type_list<>	
Specialization for empty type list	70
aerobus::i32::val< x >	
Values in i32 , again represented as types	72
aerobus::i64::val< x >	
Values in i64	74
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
Values (seen as types) in polynomial ring	76
aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	79

aerobus::zpz< p >::val< x >	80
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Specialization for constants	82
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Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

src/ aerobus.h	91
--	----

Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [i32](#)
32 bits signed integers, seen as a algebraic ring with related operations
- struct [i64](#)
64 bits signed integers, seen as a algebraic ring with related operations
- struct [is_prime](#)
checks if n is prime
- struct [polynomial](#)
- struct [Quotient](#)
[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.
- struct [type_list](#)
Empty pure template struct to handle type list.
- struct [type_list<>](#)
specialization for empty type list
- struct [zpz](#)

Concepts

- concept [IsRing](#)
Concept to express R is a Ring.
- concept [IsEuclideanDomain](#)
Concept to express R is an euclidean domain.
- concept [IsField](#)
Concept to express R is a field.

Typedefs

- `template<typename T , typename A , typename B >`
`using gcd_t = typename internal::gcd< T >::template type< A, B >`
computes the greatest common divisor of A and B
- `template<typename... vals>`
`using vadd_t = typename internal::vadd< vals... >::type`
adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an `add_t` binary operator
- `template<typename... vals>`
`using vmul_t = typename internal::vmul< vals... >::type`
multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an `mul_t` binary operator
- `template<typename val >`
`using abs_t = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template sub_t< typename val::enclosing_type::zero, val > >`
computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept
- `template<typename Ring >`
`using FractionField = typename internal::FractionFieldImpl< Ring >::type`
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring , typename v1 , typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V_1/V_2 in the field of fractions of Ring
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p >, i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p >, i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator

- `template<typename Ring , typename v1 , typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring , typename v1 , typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename T , size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type
- `template<typename T , size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`
computes binomial coefficient (k among n) as type
- `template<typename T , size_t n>`
`using bernoulli_t = typename internal::bernoulli< T, n >::type`
nth bernoulli number as type in T
- `template<typename T , size_t n>`
`using bell_t = typename internal::bell_helper< T, n >::type`
Bell numbers.
- `template<typename T , int k>`
`using alternate_t = typename internal::alternate< T, k >::type`
 $(-1)^k$ as type in T
- `template<typename T , int n, int k>`
`using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type`
Stirling number of first kind (signed) – as types.
- `template<typename T , int n, int k>`
`using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >`
Stirling number of first kind (unsigned) – as types.
- `template<typename T , typename p , size_t n>`
`using pow_t = typename internal::pow< T, p, n >::type`
 p^n (as 'val' type in T)
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers , size_t deg>`
`using exp = taylor< Integers, internal::exp_coeff, deg >`
 e^x
- `template<typename Integers , size_t deg>`
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`
 $e^x - 1$
- `template<typename Integers , size_t deg>`
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`
`using atan = taylor< Integers, internal::atan_coeff, deg >`
 $\arctan(x)$
- `template<typename Integers , size_t deg>`
`using sin = taylor< Integers, internal::sin_coeff, deg >`
 $\sin(x)$
- `template<typename Integers , size_t deg>`
`using sinh = taylor< Integers, internal::sh_coeff, deg >`
 $\sinh(x)$
- `template<typename Integers , size_t deg>`
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`

- $\cosh(x)$ *hyperbolic cosine*
 • template<typename Integers , size_t deg>
 using **cos** = **taylor**< Integers, internal::cos_coeff, deg >
 $\cos(x)$ *cosinus*
- template<typename Integers , size_t deg>
 using **geometric_sum** = **taylor**< Integers, internal::geom_coeff, deg >
 $\frac{1}{1-x}$ *zero development of $\frac{1}{1-x}$*
- template<typename Integers , size_t deg>
 using **asin** = **taylor**< Integers, internal::asin_coeff, deg >
 $\arcsin(x)$ *arc sinus*
- template<typename Integers , size_t deg>
 using **asinh** = **taylor**< Integers, internal::asinh_coeff, deg >
 $\operatorname{arcsinh}(x)$ *arc hyperbolic sinus*
- template<typename Integers , size_t deg>
 using **atanh** = **taylor**< Integers, internal::atanh_coeff, deg >
 $\operatorname{arctanh}(x)$ *arc hyperbolic tangent*
- template<typename Integers , size_t deg>
 using **tan** = **taylor**< Integers, internal::tan_coeff, deg >
 $\tan(x)$ *tangent*
- template<typename Integers , size_t deg>
 using **tanh** = **taylor**< Integers, internal::tanh_coeff, deg >
 $\tanh(x)$ *hyperbolic tangent*
- using **PI_fraction** = **ContinuedFraction**< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >
- using **E_fraction** = **ContinuedFraction**< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >
- using **SQRT2_fraction** = **ContinuedFraction**< 1, 2 >
approximation of $\sqrt{2}$
- using **SQRT3_fraction** = **ContinuedFraction**< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >
approximation of

Functions

- template<typename T >
 T * **aligned_malloc** (size_t count, size_t alignment)
- brief Conway polynomials tparam p characteristic of the **field** (prime number) @tparam n degree of extension
 template< int p

Variables

- template<typename T , size_t i>
 constexpr T::inner_type **factorial_v** = internal::factorial<T, i>::value
computes factorial(i) as value in T
- template<typename T , size_t k, size_t n>
 constexpr T::inner_type **combination_v** = internal::combination<T, k, n>::value
computes binomial coefficients (k among n) as value
- template<typename FloatType , typename T , size_t n>
 constexpr FloatType **bernoulli_v** = internal::bernoulli<T, n>::template value<FloatType>
nth bernoulli number as value in FloatType
- template<typename T , size_t k>
 constexpr T::inner_type **alternate_v** = internal::alternate<T, k>::value
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val<-2></code>
------------	--

6.1.2.2 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.3 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, <code>aerobus::i64</code> for example
----------	--

6.1.2.4 asin

```
template<typename Integers , size_t deg>
```

```
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.5 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$ arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.9 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

*n*th bernoulli number as type in *T*

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.10 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	--

6.1.2.11 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ cosinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.12 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 E_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

6.1.2.14 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

e^x

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.15 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_↔
t< exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$e^x - 1$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.16 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.17 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

6.1.2.18 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.19 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

6.1.2.20 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.21 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.22 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.23 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.24 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.25 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.26 `mulfractions_t`

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.27 `pi64`

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.28 `PI_fraction`

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

6.1.2.29 `pow_t`

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

6.1.2.30 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.31 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.32 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.33 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.34 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.35 SQRT2_fraction

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2,
```


Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.41 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make\_index\_sequence
+ 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff↔ _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.42 vadd_t

```
template<typename... vals>
using aerobus::vadd\_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.43 vmul_t

```
template<typename... vals>
```

```
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
    prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>
[inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i <=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i <=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0 >

- struct **bernstein_helper**< i, m, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0 >
- struct **chebyshev_helper**< 1, 1 >
- struct **chebyshev_helper**< 2, 0 >
- struct **chebyshev_helper**< 2, 1 >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0> >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1> >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0> >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1> >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**

greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain

- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**
- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0 >
- struct **laguerre_helper**< 1 >
- struct **legendre_helper**
- struct **legendre_helper**< 0 >
- struct **legendre_helper**< 1 >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, ls... > >
- struct **pop_front_h**

- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **stirling_helper**
- struct **stirling_helper**< T, 0, 0 >
- struct **stirling_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, k, std::enable_if_t<(k > 0) &&(n > 0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) !=0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- template<size_t i, typename... Ts>
using **type_at_t** = typename type_at< i, Ts... >::type
- template<std::size_t N>
using **make_index_sequence_reverse** = decltype(index_sequence_reverse(std::make_index_sequence< N >{}))

Functions

- template<std::size_t... Is>
constexpr auto **index_sequence_reverse** (std::index_sequence< Is... > const &) -> decltype(std::index_↵
sequence< sizeof...(Is) - 1U - Is... >{})

Variables

- `template<template< typename... > typename TT, typename T >`
`constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value`

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 `make_index_sequence_reverse`

```
template<std::size_t N>
using aerobus::internal::make\_index\_sequence\_reverse = typedef decltype(index\_sequence\_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 `type_at_t`

```
template<size_t i, typename... Ts>
using aerobus::internal::type\_at\_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 `index_sequence_reverse()`

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 `is_instantiation_of_v`

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 `aerobus::known_polynomials` Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Typedefs

- `template<size_t deg>`
using `chebyshev_T` = `typename internal::chebyshev_helper< 1, deg >::type`
Chebyshev polynomials of first kind.
- `template<size_t deg>`
using `chebyshev_U` = `typename internal::chebyshev_helper< 2, deg >::type`
Chebyshev polynomials of second kind.
- `template<size_t deg>`
using `laguerre` = `typename internal::laguerre_helper< deg >::type`
Laguerre polynomials.
- `template<size_t deg>`
using `hermite_prob` = `typename internal::hermite_helper< deg, hermite_kind::probabilist >::type`
Hermite polynomials - probabilist form.
- `template<size_t deg>`
using `hermite_phys` = `typename internal::hermite_helper< deg, hermite_kind::physicist >::type`
Hermite polynomials - physicist form.
- `template<size_t i, size_t m>`
using `bernstein` = `typename internal::bernstein_helper< i, m >::type`
Bernstein polynomials.
- `template<size_t deg>`
using `legendre` = `typename internal::legendre_helper< deg >::type`
Legendre polynomials.
- `template<size_t deg>`
using `bernoulli` = `taylor< i64, internal::bernoulli_coeff< deg >::template inner, deg >`
Bernoulli polynomials.

Enumerations

- enum `hermite_kind` { `probabilist` , `physicist` }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Typedef Documentation

6.3.2.1 bernoulli

```
template<size_t deg>
using aerobus::known_polynomials::bernoulli = typedef Taylor<i64, internal::bernoulli_coeff<deg>↔
::template inner, deg>
```

Bernoulli polynomials.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.2 **bernstein**

```
template<size_t i, size_t m>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m>::type
```

Bernstein polynomials.

See also

[See in Wikipedia](#)

Template Parameters

<i>i</i>	index of polynomial (between 0 and m)
<i>m</i>	degree of polynomial

6.3.2.3 **chebyshev_T**

```
template<size_t deg>
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg>::type
```

Chebyshev polynomials of first kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.4 **chebyshev_U**

```
template<size_t deg>
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg>::type
```

Chebyshev polynomials of second kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.5 hermite_phys

```
template<size_t deg>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist>::type
```

Hermite polynomials - physicist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.6 hermite_prob

```
template<size_t deg>
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist>::type
```

Hermite polynomials - probabilist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.7 laguerre

```
template<size_t deg>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg>↵
::type
```

Laguerre polynomials.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.8 legendre

```
template<size_t deg>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg>↵
::type
```

Legendre polynomials.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.3 Enumeration Type Documentation

6.3.3.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `using type = typename Ring::zero`

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- using [type](#) = [aN](#)

8.3.1 Member Typedef Documentation

8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)>
>::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.4 aerobus::ContinuedFraction< values > Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename [q64::template inject_constant_t< a0 >](#)
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = [static_cast<double>\(a0\)](#)
represented value as double

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using [type](#) = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >
represented value as [aerobus::q64](#)

Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()
represented value as double

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in `i32`, again represented as types

Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator yields $v1 \% v2$ for example : `i32::mod_t<i32::val<7>, i32::val<2>>`
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
- `template<typename v >`
`using pos_t = typename pos< v >::type`

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

8.8.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

8.8.2 Member Typedef Documentation

8.8.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i32::add_t = typename add<v1, v2>::type
```

8.8.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

8.8.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

8.8.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

8.8.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

8.8.2.6 inject_constant_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

8.8.2.7 inject_ring_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

8.8.2.8 inner_type

```
using aerobus::i32::inner_type = int32_t
```

8.8.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

8.8.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type

modulus operator yields v1 % v2 for example : i32::mod_t<i32::val<7>, i32::val<2>>
```

Template Parameters

<code>v1</code>	a value in i32
<code>v2</code>	a value in i32

8.8.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

8.8.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

8.8.2.13 pos_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

8.8.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

8.8.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

8.8.3 Member Data Documentation**8.8.3.1 eq_v**

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

8.8.3.2 is_euclidean_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.8.3.3 is_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

8.8.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i64](#)

Public Types

- [using inner_type = int64_t](#)
type of represented values
- [template<auto x>](#)
[using inject_constant_t = val< static_cast< int64_t >\(x\)>](#)
- [template<typename v >](#)
[using inject_ring_t = v](#)
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject_ring_t<i64::val<1>>](#)
-> [i64::val<1>](#)*
- [using zero = val< 0 >](#)
constant zero
- [using one = val< 1 >](#)
constant one
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)
[using div_t = typename div< v1, v2 >::type](#)

- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i64, v1, v2 >`
- `template<typename v >`
`using pos_t = typename pos< v >::type`

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator yields v1 > v2 as boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

8.9.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

8.9.2 Member Typedef Documentation

8.9.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

8.9.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

8.9.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

8.9.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

8.9.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

8.9.2.6 inject_constant_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

8.9.2.7 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`
 -> `i64::val<1>`

Template Parameters

<code>v</code>	a value in <code>i64</code>
----------------	-----------------------------

8.9.2.8 inner_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

8.9.2.9 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

8.9.2.10 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```


8.9.2.11 mul_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

8.9.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

8.9.2.13 pos_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

8.9.2.14 sub_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

8.9.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

8.9.3 Member Data Documentation

8.9.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

8.9.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields $v1 > v2$ as boolean value

Template Parameters

$v1$: an element of aerobus::i64::val
$v2$: an element of aerobus::i64::val

8.9.3.3 is_euclidean_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

8.9.3.4 is_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

8.9.3.5 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

8.9.3.6 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.10 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool [value](#) = internal::_is_prime<n, 5>::value
true iff n is prime

8.10.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

8.10.2 Member Data Documentation

8.10.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.11 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero = val< typename Ring::zero >](#)
constant zero
- [using one = val< typename Ring::one >](#)
constant one
- [using X = val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t = typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction of two polynomials

- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication of two polynomials
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq_helper< v1, v2 >::type`
equality operator
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt_helper< v1, v2 >::type`
strict less operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt_helper< v1, v2 >::type`
strict greater operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::q_type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`
modulo operator
- `template<typename coeff , size_t deg>`
`using monomial_t = typename monomial< coeff, deg >::type`
monomial : $\text{coeff } X^{\text{deg}}$
- `template<typename v >`
`using derive_t = typename derive_helper< v >::type`
derivation operator
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity ($an > 0$)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial< Ring >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator

8.11.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

8.11.2 Member Typedef Documentation

8.11.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

8.11.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.7 inject_constant_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

8.11.2.8 inject_ring_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

8.11.2.9 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.10 mod_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.11 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : $\text{coeff } X^{\text{deg}}$

Template Parameters

<i>coeff</i>	
<i>deg</i>	

8.11.2.12 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

8.11.2.14 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ($a_n > 0$)

Template Parameters

<i>v</i>	
----------	--

8.11.2.15 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

8.11.2.16 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

8.11.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

8.11.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

8.11.3 Member Data Documentation

8.11.3.1 is_euclidean_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

8.11.3.2 is_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

8.11.3.3 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<code>v</code>	a value in <code>polynomial::val</code>
----------------	---

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.12 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using [type](#) = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using [tail](#) = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

8.12.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

8.12.2 Member Typedef Documentation

8.12.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

8.12.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.13 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- `using zero = val< typename Ring::zero >`
zero value
- `using one = val< typename Ring::one >`
one
- `template<typename v1 , typename v2 >`
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`
addition operator
- `template<typename v1 , typename v2 >`
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`
subtraction operator
- `template<typename v1 , typename v2 >`
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`
modulus operator
- `template<typename v1 , typename v2 >`
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
equality operator (as type)
- `template<typename v1 >`
`using pos_t = std::true_type`
positivity operator always true
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
addition operator (as boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator always true
- `static constexpr bool is_euclidean_domain = true`
quotient rings are euclidean domain

8.13.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as 'i32', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

8.13.2 Member Typedef Documentation

8.13.2.1 add_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

8.13.2.2 div_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

8.13.2.3 eq_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

8.13.2.4 inject_constant_t

```
template<typename Ring , typename X >
```

```
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

8.13.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

8.13.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.13.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.13.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

8.13.2.9 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

8.13.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

8.13.3 Member Data Documentation

8.13.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

8.13.3.2 is_euclidean_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

8.13.3.3 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.14 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

8.14.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

8.14.2 Member Typedef Documentation

8.14.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

8.14.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.15 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = [internal::type_at_t](#)< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename [concat_h](#)< U >::type
concatenates two list into one
- template<typename T, size_t index>
using [insert](#) = typename [internal::insert_h](#)< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename [internal::remove_h](#)< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

8.15.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

8.15.2 Member Typedef Documentation

8.15.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

8.15.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

8.15.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

8.15.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

8.15.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

8.15.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

8.15.3 Member Data Documentation**8.15.3.1 length**

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.16 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using `push_front` = `type_list`< T >
- template<typename T >
using `push_back` = `type_list`< T >
- template<typename U >
using `concat` = U
- template<typename T , size_t index>
using `insert` = `type_list`< T >

Static Public Attributes

- static constexpr size_t `length` = 0

8.16.1 Detailed Description

specialization for empty type list

8.16.2 Member Typedef Documentation

8.16.2.1 concat

```
template<typename U >  
using aerobus::type_list<>::concat = U
```

8.16.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type_list<>::insert = type_list<T>
```

8.16.2.3 push_back

```
template<typename T >  
using aerobus::type_list<>::push_back = type_list<T>
```

8.16.2.4 push_front

```
template<typename T >  
using aerobus::type_list<>::push_front = type_list<T>
```

8.16.3 Member Data Documentation

8.16.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.17 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- [using enclosing_type = i32](#)
Enclosing ring type.
- [using is_zero_t = std::bool_constant< x==0 >](#)
is value zero

Static Public Member Functions

- [template<typename valueType >](#)
[static constexpr valueType get \(\)](#)
cast x into valueType
- [static std::string to_string \(\)](#)
string representation of value
- [template<typename valueRing >](#)
[static constexpr valueRing eval \(const valueRing &v\)](#)
cast x into valueRing

Static Public Attributes

- [static constexpr int32_t v = x](#)
actual value stored in val type

8.17.1 Detailed Description

```
template<int32\_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<i>x</i>	an actual integer
----------	-------------------

8.17.2 Member Typedef Documentation

8.17.2.1 enclosing_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

8.17.2.2 is_zero_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.17.3 Member Function Documentation

8.17.3.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<i>valueRing</i>	double for example
------------------	--------------------

8.17.3.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

8.17.3.3 to_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

8.17.4 Member Data Documentation

8.17.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.18 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- [using inner_type = int32_t](#)
type of represented values
- [using enclosing_type = i64](#)
enclosing ring type
- [using is_zero_t = std::bool_constant< x==0 >](#)
is value zero

Static Public Member Functions

- [template<typename valueType > static constexpr valueType get \(\)](#)
cast value in valueType
- [static std::string to_string \(\)](#)
string representation
- [template<typename valueRing > static constexpr valueRing eval \(const valueRing &v\)](#)
cast value in valueRing

Static Public Attributes

- [static constexpr int64_t v = x](#)
actual value

8.18.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

<i>x</i>	an actual integer
----------	-------------------

8.18.2 Member Typedef Documentation

8.18.2.1 enclosing_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

8.18.2.2 inner_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

8.18.2.3 is_zero_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

8.18.3 Member Function Documentation

8.18.3.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<i>valueRing</i>	(double for example)
------------------	----------------------

8.18.3.2 get()

```
template<int64_t x>
```

```
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.18.3.3 to_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

8.18.4 Member Data Documentation

8.18.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.19 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using enclosing_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

8.19.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

8.19.2 Member Typedef Documentation

8.19.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

8.19.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

8.19.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.19.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true_type if polynomial is constant zero

8.19.2.5 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

8.19.3 Member Function Documentation

8.19.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

x	value
---	-------

Returns

$P(x)$

8.19.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

8.19.4 Member Data Documentation

8.19.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

8.19.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t←
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.20 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

8.20.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

<code>V</code>	a value from 'Ring'
----------------	---------------------

8.20.2 Member Typedef Documentation

8.20.2.1 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.21 aerobus::zpz< p >::val< x > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `using enclosing_type = zpz< p >`
enclosing ring type
- `using is_zero_t = std::bool_constant< x% p==0 >`

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value

8.21.1 Member Typedef Documentation

8.21.1.1 enclosing_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

8.21.1.2 is_zero_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<x% p == 0>
```

8.21.2 Member Function Documentation

8.21.2.1 eval()

```
template<int32_t p>
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::zpz< p >::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

8.21.2.2 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::zpz< p >::val< x >::get ( ) [inline], [static], [constexpr]
```

8.21.2.3 to_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

8.21.3 Member Data Documentation

8.21.3.1 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

8.22 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- [using enclosing_type = polynomial< Ring >](#)
enclosing ring type
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is_zero_t = std::bool_constant< aN::is_zero_t::value >](#)
- [template<size_t index>](#)
[using coeff_at_t = typename coeff_at< index >::type](#)

Static Public Member Functions

- [static std::string to_string \(\)](#)
- [template<typename valueRing >](#)
[static constexpr valueRing eval \(const valueRing &x\)](#)

Static Public Attributes

- [static constexpr size_t degree = 0](#)
degree
- [static constexpr bool is_zero_v = is_zero_t::value](#)

8.22.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

8.22.2 Member Typedef Documentation

8.22.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

8.22.2.2 coeff_at_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

8.22.2.3 enclosing_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

8.22.2.4 is_zero_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

8.22.2.5 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

8.22.3 Member Function Documentation

8.22.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

8.22.3.2 to_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

8.22.4 Member Data Documentation

8.22.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

8.22.4.2 is_zero_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

8.23 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- `using inner_type = int32_t`
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `using zero = val< 0 >`
- `using one = val< 1 >`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulo operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (type)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strictly smaller operator (type)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v1 >`
`using pos_t = typename pos< v1 >::type`
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

8.23.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

8.23.2 Member Typedef Documentation

8.23.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.23.2.2 div_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.23.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.23.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.23.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

8.23.2.6 inject_constant_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

8.23.2.7 inner_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

8.23.2.8 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.23.2.9 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.23.2.10 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.23.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

8.23.2.12 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<code>v1</code>	a value in zpz::val
-----------------	-------------------------------------

8.23.2.13 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<code>v1</code>	a value in zpz::val
<code>v2</code>	a value in zpz::val

8.23.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

8.23.3 Member Data Documentation

8.23.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

<code>v1</code>	a value in zpz::val
<code>v2</code>	a value in zpz::val

8.23.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.23.3.3 is_euclidean_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

8.23.3.4 is_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

8.23.3.5 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

8.23.3.6 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 9

File Documentation

9.1 README.md File Reference

9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00018 #ifdef _MSC_VER
00019 #define ALIGNED(x) __declspec(align(x))
00020 #define INLINED __forceinline
00021 #else
00022 #define ALIGNED(x) __attribute__((aligned(x)))
00023 #define INLINED __attribute__((always_inline)) inline
00024 #endif
00025
00027
00029
```

```

00031
00032 // aligned allocation
00033 namespace aerobus {
00040     template<typename T>
00041     T* aligned_malloc(size_t count, size_t alignment) {
00042         #ifdef _MSC_VER
00043             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00044         #else
00045             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00046         #endif
00047     }
00048 } // namespace aerobus
00049
00050 // concepts
00051 namespace aerobus {
00053     template <typename R>
00054     concept IsRing = requires {
00055         typename R::one;
00056         typename R::zero;
00057         typename R::template add_t<typename R::one, typename R::one>;
00058         typename R::template sub_t<typename R::one, typename R::one>;
00059         typename R::template mul_t<typename R::one, typename R::one>;
00060     };
00061
00063     template <typename R>
00064     concept IsEuclideanDomain = IsRing<R> && requires {
00065         typename R::template div_t<typename R::one, typename R::one>;
00066         typename R::template mod_t<typename R::one, typename R::one>;
00067         typename R::template gcd_t<typename R::one, typename R::one>;
00068         typename R::template eq_t<typename R::one, typename R::one>;
00069         typename R::template pos_t<typename R::one>;
00070
00071         R::template pos_v<typename R::one> == true;
00072         // typename R::template gt_t<typename R::one, typename R::zero>;
00073         R::is_euclidean_domain == true;
00074     };
00075
00077     template<typename R>
00078     concept IsField = IsEuclideanDomain<R> && requires {
00079         R::is_field == true;
00080     };
00081 } // namespace aerobus
00082
00083 // utilities
00084 namespace aerobus {
00085     namespace internal {
00086         template<template<typename...> typename TT, typename T>
00087         struct is_instantiation_of : std::false_type { };
00088
00089         template<template<typename...> typename TT, typename... Ts>
00090         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00091
00092         template<template<typename...> typename TT, typename T>
00093         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00094
00095         template <int64_t i, typename T, typename... Ts>
00096         struct type_at {
00097             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00098             using type = typename type_at<i - 1, Ts...>::type;
00099         };
00100
00101         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00102             using type = T;
00103         };
00104
00105         template <size_t i, typename... Ts>
00106         using type_at_t = typename type_at<i, Ts...>::type;
00107
00108
00109         template<size_t n, size_t i, typename E = void>
00110         struct _is_prime {};
00111
00112         template<size_t i>
00113         struct _is_prime<0, i> {
00114             static constexpr bool value = false;
00115         };
00116
00117         template<size_t i>
00118         struct _is_prime<1, i> {
00119             static constexpr bool value = false;
00120         };
00121
00122         template<size_t i>
00123         struct _is_prime<2, i> {
00124             static constexpr bool value = true;
00125         };
00126

```



```

00127     template<size_t i>
00128     struct _is_prime<3, i> {
00129         static constexpr bool value = true;
00130     };
00131
00132     template<size_t i>
00133     struct _is_prime<5, i> {
00134         static constexpr bool value = true;
00135     };
00136
00137     template<size_t i>
00138     struct _is_prime<7, i> {
00139         static constexpr bool value = true;
00140     };
00141
00142     template<size_t n, size_t i>
00143     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00144         static constexpr bool value = false;
00145     };
00146
00147     template<size_t n, size_t i>
00148     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00149         static constexpr bool value = false;
00150     };
00151
00152     template<size_t n, size_t i>
00153     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00154         static constexpr bool value = true;
00155     };
00156
00157     template<size_t n, size_t i>
00158     struct _is_prime<n, i, std::enable_if_t<(
00159         n % i == 0 &&
00160         n >= 9 &&
00161         n % 3 != 0 &&
00162         n % 2 != 0 &&
00163         i * i > n)>> {
00164         static constexpr bool value = true;
00165     };
00166
00167     template<size_t n, size_t i>
00168     struct _is_prime<n, i, std::enable_if_t<(
00169         n % (i+2) == 0 &&
00170         n >= 9 &&
00171         n % 3 != 0 &&
00172         n % 2 != 0 &&
00173         i * i <= n)>> {
00174         static constexpr bool value = true;
00175     };
00176
00177     template<size_t n, size_t i>
00178     struct _is_prime<n, i, std::enable_if_t<(
00179         n % (i+2) != 0 &&
00180         n % i != 0 &&
00181         n >= 9 &&
00182         n % 3 != 0 &&
00183         n % 2 != 0 &&
00184         (i * i <= n))>> {
00185         static constexpr bool value = _is_prime<n, i+6>::value;
00186     };
00187
00188 } // namespace internal
00189
00190 template<size_t n>
00191 struct is_prime {
00192     static constexpr bool value = internal::_is_prime<n, 5>::value;
00193 };
00194
00195 template<size_t n>
00196 static constexpr bool is_prime_v = is_prime<n>::value;
00197
00201 // gcd
00202 namespace internal {
00203     template <std::size_t... Is>
00204     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&
00205         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{}));
00206
00207     template <std::size_t N>
00208     using make_index_sequence_reverse
00209         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00210
00211     template<typename Ring, typename E = void>
00212     struct gcd;
00213
00214     template<typename Ring>
00215     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00216         template<typename A, typename B, typename E = void>

```

```

00225         struct gcd_helper {};
00226
00227         // B = 0, A > 0
00228         template<typename A, typename B>
00229         struct gcd_helper<A, B, std::enable_if_t<
00230             ((B::is_zero_t::value) &&
00231              (Ring::template gt_t<A, typename Ring::zero>::value))>> {
00232             using type = A;
00233         };
00234
00235         // B = 0, A < 0
00236         template<typename A, typename B>
00237         struct gcd_helper<A, B, std::enable_if_t<
00238             ((B::is_zero_t::value) &&
00239              !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00240             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00241         };
00242
00243         // B != 0
00244         template<typename A, typename B>
00245         struct gcd_helper<A, B, std::enable_if_t<
00246             (!B::is_zero_t::value)
00247             >> {
00248             private: // NOLINT
00249                 // A / B
00250                 using k = typename Ring::template div_t<A, B>;
00251                 // A - (A/B)*B = A % B
00252                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00253
00254             public:
00255                 using type = typename gcd_helper<B, m>::type;
00256         };
00257
00258         template<typename A, typename B>
00259         using type = typename gcd_helper<A, B>::type;
00260     };
00261 } // namespace internal
00262
00263 // vadd and vmul
00264 namespace internal {
00265     template<typename... vals>
00266     struct vmul {};
00267
00268     template<typename v1, typename... vals>
00269     struct vmul<v1, vals...> {
00270         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00271     };
00272
00273     template<typename v1>
00274     struct vmul<v1> {
00275         using type = v1;
00276     };
00277
00278     template<typename... vals>
00279     struct vadd {};
00280
00281     template<typename v1, typename... vals>
00282     struct vadd<v1, vals...> {
00283         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00284     };
00285
00286     template<typename v1>
00287     struct vadd<v1> {
00288         using type = v1;
00289     };
00290 } // namespace internal
00291
00292 template<typename T, typename A, typename B>
00293 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00294
00295 template<typename... vals>
00296 using vadd_t = typename internal::vadd<vals...>::type;
00297
00298 template<typename... vals>
00299 using vmul_t = typename internal::vmul<vals...>::type;
00300
00301 template<typename val>
00302 requires IsEuclideanDomain<typename val::enclosing_type>
00303 using abs_t = std::conditional_t<
00304     val::enclosing_type::template pos_v<val>,
00305     val, typename val::enclosing_type::template sub_t<typename
val::enclosing_type::zero, val>;
00306 } // namespace aerobus
00307
00308 namespace aerobus {

```

```

00324     template<typename Ring, typename X>
00325     requires IsRing<Ring>
00326     struct Quotient {
00329         template <typename V>
00330         struct val {
00331             public:
00332                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00333         };
00334
00336         using zero = val<typename Ring::zero>;
00337
00339         using one = val<typename Ring::one>;
00340
00344         template<typename v1, typename v2>
00345         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00346
00350         template<typename v1, typename v2>
00351         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00352
00356         template<typename v1, typename v2>
00357         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00358
00362         template<typename v1, typename v2>
00363         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00364
00368         template<typename v1, typename v2>
00369         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00370
00374         template<typename v1, typename v2>
00375         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00376
00380         template<typename v1>
00381         using pos_t = std::true_type;
00382
00386         template<typename v>
00387         static constexpr bool pos_v = pos_t<v>::value;
00388
00390         static constexpr bool is_euclidean_domain = true;
00391
00397         template<auto x>
00398         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00399
00405         template<typename v>
00406         using inject_ring_t = val<v>;
00407     };
00408 } // namespace aerobus
00409
00410 // type_list
00411 namespace aerobus {
00412     template <typename... Ts>
00413     struct type_list;
00414
00415     namespace internal {
00416         template <typename T, typename... Us>
00417         struct pop_front_h {
00418             using tail = type_list<Us...>;
00419             using head = T;
00420         };
00421     };
00422
00423     template <size_t index, typename L1, typename L2>
00424     struct split_h {
00425     private:
00426         static_assert(index <= L2::length, "index ouf of bounds");
00427         using a = typename L2::pop_front::type;
00428         using b = typename L2::pop_front::tail;
00429         using c = typename L1::template push_back<a>;
00430
00431     public:
00432         using head = typename split_h<index - 1, c, b>::head;
00433         using tail = typename split_h<index - 1, c, b>::tail;
00434     };
00435
00436     template <typename L1, typename L2>
00437     struct split_h<0, L1, L2> {
00438         using head = L1;
00439         using tail = L2;
00440     };
00441
00442     template <size_t index, typename L, typename T>
00443     struct insert_h {
00444         static_assert(index <= L::length, "index ouf of bounds");
00445         using s = typename L::template split<index>;
00446         using left = typename s::head;
00447         using right = typename s::tail;
00448         using ll = typename left::template push_back<T>;
00449         using type = typename ll::template concat<right>;
00450     };

```

```

00451
00452     template <size_t index, typename L>
00453     struct remove_h {
00454         using s = typename L::template split<index>;
00455         using left = typename s::head;
00456         using right = typename s::tail;
00457         using rr = typename right::pop_front::tail;
00458         using type = typename left::template concat<rr>;
00459     };
00460 } // namespace internal
00461
00462 template <typename... Ts>
00463 struct type_list {
00464     private:
00465         template <typename T>
00466         struct concat_h;
00467
00468         template <typename... Us>
00469         struct concat_h<type_list<Us...> {
00470             using type = type_list<Ts..., Us...>;
00471         };
00472
00473     public:
00474         static constexpr size_t length = sizeof...(Ts);
00475
00476         template <typename T>
00477         using push_front = type_list<T, Ts...>;
00478
00479         template <size_t index>
00480         using at = internal::type_at_t<index, Ts...>;
00481
00482         struct pop_front {
00483             using type = typename internal::pop_front_h<Ts...>::head;
00484             using tail = typename internal::pop_front_h<Ts...>::tail;
00485         };
00486
00487         template <typename T>
00488         using push_back = type_list<Ts..., T>;
00489
00490         template <typename U>
00491         using concat = typename concat_h<U>::type;
00492
00493         template <size_t index>
00494         struct split {
00495             private:
00496                 using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00497
00498             public:
00499                 using head = typename inner::head;
00500                 using tail = typename inner::tail;
00501         };
00502
00503         template <typename T, size_t index>
00504         using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00505
00506         template <size_t index>
00507         using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00508     };
00509
00510 template <>
00511 struct type_list<> {
00512     static constexpr size_t length = 0;
00513
00514     template <typename T>
00515     using push_front = type_list<T>;
00516
00517     template <typename T>
00518     using push_back = type_list<T>;
00519
00520     template <typename U>
00521     using concat = U;
00522
00523     // TODO(jewave): assert index == 0
00524     template <typename T, size_t index>
00525     using insert = type_list<T>;
00526 };
00527 } // namespace aerobus
00528
00529 // i32
00530 namespace aerobus {
00531     struct i32 {
00532         using inner_type = int32_t;
00533         template<int32_t x>
00534         struct val {
00535             using enclosing_type = i32;
00536             static constexpr int32_t v = x;
00537         };
00538     };
00539 }

```

```

00568         template<typename valueType>
00569         static constexpr valueType get() { return static_cast<valueType>(x); }
00570
00572         using is_zero_t = std::bool_constant<x == 0>;
00573
00575         static std::string to_string() {
00576             return std::to_string(x);
00577         }
00578
00581         template<typename valueRing>
00582         static constexpr valueRing eval(const valueRing& v) {
00583             return static_cast<valueRing>(x);
00584         }
00585     };
00586
00588     using zero = val<0>;
00590     using one = val<1>;
00592     static constexpr bool is_field = false;
00594     static constexpr bool is_euclidean_domain = true;
00598     template<auto x>
00599     using inject_constant_t = val<static_cast<int32_t>(x)>;
00600
00601     template<typename v>
00602     using inject_ring_t = v;
00603
00604 private:
00605     template<typename v1, typename v2>
00606     struct add {
00607         using type = val<v1::v + v2::v>;
00608     };
00609
00610     template<typename v1, typename v2>
00611     struct sub {
00612         using type = val<v1::v - v2::v>;
00613     };
00614
00615     template<typename v1, typename v2>
00616     struct mul {
00617         using type = val<v1::v * v2::v>;
00618     };
00619
00620     template<typename v1, typename v2>
00621     struct div {
00622         using type = val<v1::v / v2::v>;
00623     };
00624
00625     template<typename v1, typename v2>
00626     struct remainder {
00627         using type = val<v1::v % v2::v>;
00628     };
00629
00630     template<typename v1, typename v2>
00631     struct gt {
00632         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00633     };
00634
00635     template<typename v1, typename v2>
00636     struct lt {
00637         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00638     };
00639
00640     template<typename v1, typename v2>
00641     struct eq {
00642         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00643     };
00644
00645     template<typename v1>
00646     struct pos {
00647         using type = std::bool_constant<(v1::v > 0)>;
00648     };
00649
00650 public:
00656     template<typename v1, typename v2>
00657     using add_t = typename add<v1, v2>::type;
00658
00664     template<typename v1, typename v2>
00665     using sub_t = typename sub<v1, v2>::type;
00666
00672     template<typename v1, typename v2>
00673     using mul_t = typename mul<v1, v2>::type;
00674
00680     template<typename v1, typename v2>
00681     using div_t = typename div<v1, v2>::type;
00682
00688     template<typename v1, typename v2>
00689     using mod_t = typename remainder<v1, v2>::type;
00690

```

```

00696     template<typename v1, typename v2>
00697     using gt_t = typename gt<v1, v2>::type;
00698
00704     template<typename v1, typename v2>
00705     using lt_t = typename lt<v1, v2>::type;
00706
00712     template<typename v1, typename v2>
00713     using eq_t = typename eq<v1, v2>::type;
00714
00719     template<typename v1, typename v2>
00720     static constexpr bool eq_v = eq_t<v1, v2>::value;
00721
00727     template<typename v1, typename v2>
00728     using gcd_t = gcd_t<i32, v1, v2>;
00729
00734     template<typename v>
00735     using pos_t = typename pos<v>::type;
00736
00741     template<typename v>
00742     static constexpr bool pos_v = pos_t<v>::value;
00743 };
00744 } // namespace aerobus
00745
00746 // i64
00747 namespace aerobus {
00748     struct i64 {
00749         using inner_type = int64_t;
00750         template<int64_t x>
00751         struct val {
00752             using inner_type = int32_t;
00753             using enclosing_type = i64;
00754             static constexpr int64_t v = x;
00755
00756             template<typename valueType>
00757             static constexpr valueType get() {
00758                 return static_cast<valueType>(x);
00759             }
00760
00761             using is_zero_t = std::bool_constant<x == 0>;
00762
00763             static std::string to_string() {
00764                 return std::to_string(x);
00765             }
00766
00767             template<typename valueRing>
00768             static constexpr valueRing eval(const valueRing& v) {
00769                 return static_cast<valueRing>(x);
00770             }
00771         };
00772     };
00773
00774     template<auto x>
00775     using inject_constant_t = val<static_cast<int64_t>(x)>;
00776
00777     template<typename v>
00778     using inject_ring_t = v;
00779
00780     using zero = val<0>;
00781     using one = val<1>;
00782     static constexpr bool is_field = false;
00783     static constexpr bool is_euclidean_domain = true;
00784
00785 private:
00786     template<typename v1, typename v2>
00787     struct add {
00788         using type = val<v1::v + v2::v>;
00789     };
00790
00791     template<typename v1, typename v2>
00792     struct sub {
00793         using type = val<v1::v - v2::v>;
00794     };
00795
00796     template<typename v1, typename v2>
00797     struct mul {
00798         using type = val<v1::v * v2::v>;
00799     };
00800
00801     template<typename v1, typename v2>
00802     struct div {
00803         using type = val<v1::v / v2::v>;
00804     };
00805
00806     template<typename v1, typename v2>
00807     struct remainder {
00808         using type = val<v1::v % v2::v>;
00809     };
00810 }

```

```

00834     template<typename v1, typename v2>
00835     struct gt {
00836         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00837     };
00838
00839     template<typename v1, typename v2>
00840     struct lt {
00841         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00842     };
00843
00844     template<typename v1, typename v2>
00845     struct eq {
00846         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00847     };
00848
00849     template<typename v>
00850     struct pos {
00851         using type = std::bool_constant<(v::v > 0)>;
00852     };
00853
00854     public:
00855     template<typename v1, typename v2>
00856     using add_t = typename add<v1, v2>::type;
00857
00858     template<typename v1, typename v2>
00859     using sub_t = typename sub<v1, v2>::type;
00860
00861     template<typename v1, typename v2>
00862     using mul_t = typename mul<v1, v2>::type;
00863
00864     template<typename v1, typename v2>
00865     using div_t = typename div<v1, v2>::type;
00866
00867     template<typename v1, typename v2>
00868     using mod_t = typename remainder<v1, v2>::type;
00869
00870     template<typename v1, typename v2>
00871     using gt_t = typename gt<v1, v2>::type;
00872
00873     template<typename v1, typename v2>
00874     static constexpr bool gt_v = gt_t<v1, v2>::value;
00875
00876     template<typename v1, typename v2>
00877     using lt_t = typename lt<v1, v2>::type;
00878
00879     template<typename v1, typename v2>
00880     static constexpr bool lt_v = lt_t<v1, v2>::value;
00881
00882     template<typename v1, typename v2>
00883     using eq_t = typename eq<v1, v2>::type;
00884
00885     template<typename v1, typename v2>
00886     static constexpr bool eq_v = eq_t<v1, v2>::value;
00887
00888     template<typename v1, typename v2>
00889     using gcd_t = gcd_t<i64, v1, v2>;
00890
00891     template<typename v>
00892     using pos_t = typename pos<v>::type;
00893
00894     template<typename v>
00895     static constexpr bool pos_v = pos_t<v>::value;
00896 };
00897 } // namespace aerobus
00898
00899 // z/pz
00900 namespace aerobus {
00901     template<int32_t p>
00902     struct zp {
00903         using inner_type = int32_t;
00904         template<int32_t x>
00905         struct val {
00906             using enclosing_type = zp<p>;
00907             static constexpr int32_t v = x % p;
00908
00909             template<typename valueType>
00910             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00911
00912             using is_zero_t = std::bool_constant<x % p == 0>;
00913             static std::string to_string() {
00914                 return std::to_string(x % p);
00915             }
00916         }
00917
00918         template<typename valueRing>
00919         static constexpr valueRing eval(const valueRing& v) {
00920             return static_cast<valueRing>(x % p);
00921         }
00922     }
00923 }

```

```

00990     };
00991
00992     template<auto x>
00993     using inject_constant_t = val<static_cast<int32_t>(x)>;
00994
00995     using zero = val<0>;
00996     using one = val<1>;
00997     static constexpr bool is_prime<p>::value;
00998     static constexpr bool is_euclidean_domain = true;
00999
01000 private:
01001     template<typename v1, typename v2>
01002     struct add {
01003         using type = val<(v1::v + v2::v) % p>;
01004     };
01005
01006     template<typename v1, typename v2>
01007     struct sub {
01008         using type = val<(v1::v - v2::v) % p>;
01009     };
01010
01011     template<typename v1, typename v2>
01012     struct mul {
01013         using type = val<(v1::v * v2::v) % p>;
01014     };
01015
01016     template<typename v1, typename v2>
01017     struct div {
01018         using type = val<(v1::v % p) / (v2::v % p)>;
01019     };
01020
01021     template<typename v1, typename v2>
01022     struct remainder {
01023         using type = val<(v1::v % v2::v) % p>;
01024     };
01025
01026     template<typename v1, typename v2>
01027     struct gt {
01028         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01029     };
01030
01031     template<typename v1, typename v2>
01032     struct lt {
01033         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01034     };
01035
01036     template<typename v1, typename v2>
01037     struct eq {
01038         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01039     };
01040
01041     template<typename v1>
01042     struct pos {
01043         using type = std::bool_constant<(v1::v > 0)>;
01044     };
01045
01046 public:
01050     template<typename v1, typename v2>
01051     using add_t = typename add<v1, v2>::type;
01052
01056     template<typename v1, typename v2>
01057     using sub_t = typename sub<v1, v2>::type;
01058
01062     template<typename v1, typename v2>
01063     using mul_t = typename mul<v1, v2>::type;
01064
01068     template<typename v1, typename v2>
01069     using div_t = typename div<v1, v2>::type;
01070
01074     template<typename v1, typename v2>
01075     using mod_t = typename remainder<v1, v2>::type;
01076
01080     template<typename v1, typename v2>
01081     using gt_t = typename gt<v1, v2>::type;
01082
01086     template<typename v1, typename v2>
01087     static constexpr bool gt_v = gt_t<v1, v2>::value;
01088
01092     template<typename v1, typename v2>
01093     using lt_t = typename lt<v1, v2>::type;
01094
01098     template<typename v1, typename v2>
01099     static constexpr bool lt_v = lt_t<v1, v2>::value;
01100
01104     template<typename v1, typename v2>
01105     using eq_t = typename eq<v1, v2>::type;
01106

```



```

01110     template<typename v1, typename v2>
01111     static constexpr bool eq_v = eq_t<v1, v2>::value;
01112
01116     template<typename v1, typename v2>
01117     using gcd_t = gcd_t<i32, v1, v2>;
01118
01121     template<typename v1>
01122     using pos_t = typename pos<v1>::type;
01123
01126     template<typename v>
01127     static constexpr bool pos_v = pos_t<v>::value;
01128 };
01129 } // namespace aerobus
01130
01131 // polynomial
01132 namespace aerobus {
01133     // coeffN x^N + ...
01138     template<typename Ring>
01139     requires IsEuclideanDomain<Ring>
01140     struct polynomial {
01141         static constexpr bool is_field = false;
01142         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01143
01147         template<typename coeffN, typename... coeffs>
01148         struct val {
01150             using enclosing_type = polynomial<Ring>;
01152             static constexpr size_t degree = sizeof...(coeffs);
01154             using aN = coeffN;
01156             using strip = val<coeffs...>;
01158             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01160             static constexpr bool is_zero_v = is_zero_t::value;
01161
01162         private:
01163             template<size_t index, typename E = void>
01164             struct coeff_at {};
01165
01166             template<size_t index>
01167             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01168                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01169             };
01170
01171             template<size_t index>
01172             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01173                 using type = typename Ring::zero;
01174             };
01175
01176         public:
01179             template<size_t index>
01180             using coeff_at_t = typename coeff_at<index>::type;
01181
01184             static std::string to_string() {
01185                 return string_helper<coeffN, coeffs...>::func();
01186             }
01187
01192             template<typename valueRing>
01193             static constexpr valueRing eval(const valueRing& x) {
01194                 return horner_evaluation<valueRing, val>
01195                     ::template inner<0, degree + 1>
01196                     ::func(static_cast<valueRing>(0), x);
01197             }
01198 };
01199
01202     template<typename coeffN>
01203     struct val<coeffN> {
01205         using enclosing_type = polynomial<Ring>;
01207         static constexpr size_t degree = 0;
01208         using aN = coeffN;
01209         using strip = val<coeffN>;
01210         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01211
01212         static constexpr bool is_zero_v = is_zero_t::value;
01213
01214         template<size_t index, typename E = void>
01215         struct coeff_at {};
01216
01217         template<size_t index>
01218         struct coeff_at<index, std::enable_if_t<(index == 0)> {
01219             using type = aN;
01220         };
01221
01222         template<size_t index>
01223         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01224             using type = typename Ring::zero;
01225         };
01226
01227         template<size_t index>
01228         using coeff_at_t = typename coeff_at<index>::type;

```

```

01229         static std::string to_string() {
01230             return string_helper<coeffN>::func();
01231         }
01232     }
01233
01234     template<typename valueRing>
01235     static constexpr valueRing eval(const valueRing& x) {
01236         return static_cast<valueRing>(aN::template get<valueRing>());
01237     }
01238 };
01239
01240 using zero = val<typename Ring::zero>;
01241 using one = val<typename Ring::one>;
01242 using X = val<typename Ring::one, typename Ring::zero>;
01243
01244 private:
01245     template<typename P, typename E = void>
01246     struct simplify;
01247
01248     template<typename P1, typename P2, typename I>
01249     struct add_low;
01250
01251     template<typename P1, typename P2>
01252     struct add {
01253         using type = typename simplify<typename add_low<
01254             P1,
01255             P2,
01256             internal::make_index_sequence_reverse<
01257                 std::max(P1::degree, P2::degree) + 1
01258             >::type>::type;
01259     };
01260
01261     template<typename P1, typename P2, typename I>
01262     struct sub_low;
01263
01264     template<typename P1, typename P2, typename I>
01265     struct mul_low;
01266
01267     template<typename v1, typename v2>
01268     struct mul {
01269         using type = typename mul_low<
01270             v1,
01271             v2,
01272             internal::make_index_sequence_reverse<
01273                 v1::degree + v2::degree + 1
01274             >::type;
01275     };
01276
01277     template<typename coeff, size_t deg>
01278     struct monomial;
01279
01280     template<typename v, typename E = void>
01281     struct derive_helper {};
01282
01283     template<typename v>
01284     struct derive_helper<v, std::enable_if_t<v::degree == 0>> {
01285         using type = zero;
01286     };
01287
01288     template<typename v>
01289     struct derive_helper<v, std::enable_if_t<v::degree != 0>> {
01290         using type = typename add<
01291             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01292             typename monomial<
01293                 typename Ring::template mul_t<
01294                     typename v::aN,
01295                     typename Ring::template inject_constant_t<(v::degree)>
01296                 >,
01297                 v::degree - 1
01298             >::type
01299         >::type;
01300     };
01301
01302     template<typename v1, typename v2, typename E = void>
01303     struct eq_helper {};
01304
01305     template<typename v1, typename v2>
01306     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree>> {
01307         using type = std::false_type;
01308     };
01309
01310     template<typename v1, typename v2>
01311     struct eq_helper<v1, v2, std::enable_if_t<
01312         v1::degree == v2::degree &&
01313         (v1::degree != 0 || v2::degree != 0) &&
01314         std::is_same<

```

```

01319         typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01320         std::false_type
01321     >::value
01322 >
01323 > {
01324     using type = std::false_type;
01325 };
01326
01327 template<typename v1, typename v2>
01328 struct eq_helper<v1, v2, std::enable_if_t<
01329     v1::degree == v2::degree &&
01330     (v1::degree != 0 || v2::degree != 0) &&
01331     std::is_same<
01332     typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01333     std::true_type
01334     >::value
01335     > {
01336     using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01337 };
01338
01339 template<typename v1, typename v2>
01340 struct eq_helper<v1, v2, std::enable_if_t<
01341     v1::degree == v2::degree &&
01342     (v1::degree == 0)
01343     > {
01344     using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01345 };
01346
01347 template<typename v1, typename v2, typename E = void>
01348 struct lt_helper {};
01349
01350 template<typename v1, typename v2>
01351 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01352     using type = std::true_type;
01353 };
01354
01355 template<typename v1, typename v2>
01356 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01357     using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01358 };
01359
01360 template<typename v1, typename v2>
01361 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01362     using type = std::false_type;
01363 };
01364
01365 template<typename v1, typename v2, typename E = void>
01366 struct gt_helper {};
01367
01368 template<typename v1, typename v2>
01369 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01370     using type = std::true_type;
01371 };
01372
01373 template<typename v1, typename v2>
01374 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01375     using type = std::false_type;
01376 };
01377
01378 template<typename v1, typename v2>
01379 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01380     using type = std::false_type;
01381 };
01382
01383 // when high power is zero : strip
01384 template<typename P>
01385 struct simplify<P, std::enable_if_t<
01386     std::is_same<
01387     typename Ring::zero,
01388     typename P::aN
01389     >::value && (P::degree > 0)
01390     > {
01391     using type = typename simplify<typename P::strip>::type;
01392 };
01393
01394 // otherwise : do nothing
01395 template<typename P>
01396 struct simplify<P, std::enable_if_t<
01397     !std::is_same<
01398     typename Ring::zero,
01399     typename P::aN
01400     >::value && (P::degree > 0)
01401     > {
01402     using type = P;
01403 };
01404
01405 // do not simplify constants

```

```

01406     template<typename P>
01407     struct simplify<P, std::enable_if_t<P::degree == 0> {
01408         using type = P;
01409     };
01410
01411     // addition at
01412     template<typename P1, typename P2, size_t index>
01413     struct add_at {
01414         using type =
01415             typename Ring::template add_t<
01416                 typename P1::template coeff_at_t<index>,
01417                 typename P2::template coeff_at_t<index>>;
01418     };
01419
01420     template<typename P1, typename P2, size_t index>
01421     using add_at_t = typename add_at<P1, P2, index>::type;
01422
01423     template<typename P1, typename P2, std::size_t... I>
01424     struct add_low<P1, P2, std::index_sequence<I...> {
01425         using type = val<add_at_t<P1, P2, I>...>;
01426     };
01427
01428     // subtraction at
01429     template<typename P1, typename P2, size_t index>
01430     struct sub_at {
01431         using type =
01432             typename Ring::template sub_t<
01433                 typename P1::template coeff_at_t<index>,
01434                 typename P2::template coeff_at_t<index>>;
01435     };
01436
01437     template<typename P1, typename P2, size_t index>
01438     using sub_at_t = typename sub_at<P1, P2, index>::type;
01439
01440     template<typename P1, typename P2, std::size_t... I>
01441     struct sub_low<P1, P2, std::index_sequence<I...> {
01442         using type = val<sub_at_t<P1, P2, I>...>;
01443     };
01444
01445     template<typename P1, typename P2>
01446     struct sub {
01447         using type = typename simplify<typename sub_low<
01448             P1,
01449             P2,
01450             internal::make_index_sequence_reverse<
01451                 std::max(P1::degree, P2::degree) + 1
01452             >::type>::type;
01453     };
01454
01455     // multiplication at
01456     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01457     struct mul_at_loop_helper {
01458         using type = typename Ring::template add_t<
01459             typename Ring::template mul_t<
01460                 typename v1::template coeff_at_t<index>,
01461                 typename v2::template coeff_at_t<k - index>
01462             >,
01463             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01464         >;
01465     };
01466
01467     template<typename v1, typename v2, size_t k, size_t stop>
01468     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01469         using type = typename Ring::template mul_t<
01470             typename v1::template coeff_at_t<stop>,
01471             typename v2::template coeff_at_t<0>>;
01472     };
01473
01474     template <typename v1, typename v2, size_t k, typename E = void>
01475     struct mul_at {};
01476
01477     template<typename v1, typename v2, size_t k>
01478     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01479         using type = typename Ring::zero;
01480     };
01481
01482     template<typename v1, typename v2, size_t k>
01483     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01484         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01485     };
01486
01487     template<typename P1, typename P2, size_t index>
01488     using mul_at_t = typename mul_at<P1, P2, index>::type;
01489
01490     template<typename P1, typename P2, std::size_t... I>
01491     struct mul_low<P1, P2, std::index_sequence<I...> {
01492         using type = val<mul_at_t<P1, P2, I>...>;

```

```

01493     };
01494
01495     // division helper
01496     template< typename A, typename B, typename Q, typename R, typename E = void>
01497     struct div_helper {};
01498
01499     template<typename A, typename B, typename Q, typename R>
01500     struct div_helper<A, B, Q, R, std::enable_if_t<
01501         (R::degree < B::degree) ||
01502         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01503         using q_type = Q;
01504         using mod_type = R;
01505         using gcd_type = B;
01506     };
01507
01508     template<typename A, typename B, typename Q, typename R>
01509     struct div_helper<A, B, Q, R, std::enable_if_t<
01510         (R::degree >= B::degree) &&
01511         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01512     private: // NOLINT
01513         using rN = typename R::aN;
01514         using bN = typename B::aN;
01515         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01516             B::degree>::type;
01517         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01518         using qq = typename add<Q, pT>::type;
01519     public:
01520         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01521         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01522         using gcd_type = rr;
01523     };
01524
01525     template<typename A, typename B>
01526     struct div {
01527         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01528         using q_type = typename div_helper<A, B, zero, A>::q_type;
01529         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01530     };
01531
01532     template<typename P>
01533     struct make_unit {
01534         using type = typename div<P, val<typename P::aN>::q_type>;
01535     };
01536
01537     template<typename coeff, size_t deg>
01538     struct monomial {
01539         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01540     };
01541
01542     template<typename coeff>
01543     struct monomial<coeff, 0> {
01544         using type = val<coeff>;
01545     };
01546
01547     template<typename valueRing, typename P>
01548     struct horner_evaluation {
01549         template<size_t index, size_t stop>
01550         struct inner {
01551             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01552                 constexpr valueRing coeff =
01553                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01554             get<valueRing>());
01555                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01556             accum + coeff, x);
01557             };
01558         };
01559         template<size_t stop>
01560         struct inner<stop, stop> {
01561             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01562                 return accum;
01563             };
01564         };
01565     };
01566
01567     template<typename coeff, typename... coeffs>
01568     struct string_helper {
01569         static std::string func() {
01570             std::string tail = string_helper<coeffs...>::func();
01571             std::string result = "";
01572             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01573                 return tail;
01574             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01575                 if (sizeof...(coeffs) == 1) {
01576                     result += "x";
01577                 } else {

```

```

01577         result += "x^" + std::to_string(sizeof...(coeffs));
01578     }
01579     } else {
01580         if (sizeof...(coeffs) == 1) {
01581             result += coeff::to_string() + " x";
01582         } else {
01583             result += coeff::to_string()
01584                 + " x^" + std::to_string(sizeof...(coeffs));
01585         }
01586     }
01587
01588     if (!tail.empty()) {
01589         result += " + " + tail;
01590     }
01591
01592     return result;
01593 }
01594 };
01595
01596 template<typename coeff>
01597 struct string_helper<coeff> {
01598     static std::string func() {
01599         if (!std::is_same<coeff, typename Ring::zero>::value) {
01600             return coeff::to_string();
01601         } else {
01602             return "";
01603         }
01604     }
01605 };
01606
01607 public:
01610     template<typename P>
01611     using simplify_t = typename simplify<P>::type;
01612
01616     template<typename v1, typename v2>
01617     using add_t = typename add<v1, v2>::type;
01618
01622     template<typename v1, typename v2>
01623     using sub_t = typename sub<v1, v2>::type;
01624
01628     template<typename v1, typename v2>
01629     using mul_t = typename mul<v1, v2>::type;
01630
01634     template<typename v1, typename v2>
01635     using eq_t = typename eq_helper<v1, v2>::type;
01636
01640     template<typename v1, typename v2>
01641     using lt_t = typename lt_helper<v1, v2>::type;
01642
01646     template<typename v1, typename v2>
01647     using gt_t = typename gt_helper<v1, v2>::type;
01648
01652     template<typename v1, typename v2>
01653     using div_t = typename div<v1, v2>::q_type;
01654
01658     template<typename v1, typename v2>
01659     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01660
01664     template<typename coeff, size_t deg>
01665     using monomial_t = typename monomial<coeff, deg>::type;
01666
01669     template<typename v>
01670     using derive_t = typename derive_helper<v>::type;
01671
01674     template<typename v>
01675     using pos_t = typename Ring::template pos_t<typename v::aN>;
01676
01679     template<typename v>
01680     static constexpr bool pos_v = pos_t<v>::value;
01681
01685     template<typename v1, typename v2>
01686     using gcd_t = std::conditional_t<
01687         Ring::is_euclidean_domain,
01688         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01689         void>;
01690
01694     template<auto x>
01695     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01696
01700     template<typename v>
01701     using inject_ring_t = val<v>;
01702 };
01703 } // namespace aerobus
01704
01705 // fraction field
01706 namespace aerobus {
01707     namespace internal {

```

```

01708     template<typename Ring, typename E = void>
01709     requires IsEuclideanDomain<Ring>
01710     struct _FractionField {};
01711
01712     template<typename Ring>
01713     requires IsEuclideanDomain<Ring>
01714     struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
01715         static constexpr bool is_field = true;
01716         static constexpr bool is_euclidean_domain = true;
01717
01718     private:
01719         template<typename val1, typename val2, typename E = void>
01720         struct to_string_helper {};
01721
01722         template<typename val1, typename val2>
01723         struct to_string_helper <val1, val2,
01724             std::enable_if_t<
01725                 Ring::template eq_t<
01726                     val2, typename Ring::one
01727                 >::value
01728             >
01729         > {
01730             static std::string func() {
01731                 return val1::to_string();
01732             }
01733         };
01734
01735         template<typename val1, typename val2>
01736         struct to_string_helper<val1, val2,
01737             std::enable_if_t<
01738                 !Ring::template eq_t<
01739                     val2,
01740                     typename Ring::one
01741                 >::value
01742             >
01743         > {
01744             static std::string func() {
01745                 return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01746             }
01747         };
01748     };
01749
01750     public:
01751         template<typename val1, typename val2>
01752         struct val {
01753             using x = val1;
01754             using y = val2;
01755             using is_zero_t = typename val1::is_zero_t;
01756             static constexpr bool is_zero_v = val1::is_zero_t::value;
01757
01758             using ring_type = Ring;
01759             using enclosing_type = _FractionField<Ring>;
01760
01761             static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01762
01763             template<typename valueType>
01764             static constexpr valueType get() { return static_cast<valueType>(x::v) /
01765                 static_cast<valueType>(y::v); }
01766
01767             static std::string to_string() {
01768                 return to_string_helper<val1, val2>::func();
01769             }
01770
01771             template<typename valueRing>
01772             static constexpr valueRing eval(const valueRing& v) {
01773                 return x::eval(v) / y::eval(v);
01774             }
01775         };
01776
01777         using zero = val<typename Ring::zero, typename Ring::one>;
01778         using one = val<typename Ring::one, typename Ring::one>;
01779
01780         template<typename v>
01781         using inject_t = val<v, typename Ring::one>;
01782
01783         template<auto x>
01784         using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01785         Ring::one>;
01786
01787         template<typename v>
01788         using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01789
01790         using ring_type = Ring;
01791
01792     private:
01793         template<typename v, typename E = void>
01794         struct simplify {};
01795
01796

```

```

01822         // x = 0
01823         template<typename v>
01824         struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01825             using type = typename _FractionField<Ring>::zero;
01826         };
01827
01828         // x != 0
01829         template<typename v>
01830         struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01831             private:
01832                 using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01833                 using newx = typename Ring::template div_t<typename v::x, _gcd>;
01834                 using newy = typename Ring::template div_t<typename v::y, _gcd>;
01835
01836                 using posx = std::conditional_t<
01837                     !Ring::template pos_v<newy>,
01838                     typename Ring::template sub_t<typename Ring::zero, newx>,
01839                     newx>;
01840                 using posy = std::conditional_t<
01841                     !Ring::template pos_v<newy>,
01842                     typename Ring::template sub_t<typename Ring::zero, newy>,
01843                     newy>;
01844             public:
01845                 using type = typename _FractionField<Ring>::template val<posx, posy>;
01846         };
01847
01848     public:
01849         template<typename v>
01850         using simplify_t = typename simplify<v>::type;
01851
01852     private:
01853         template<typename v1, typename v2>
01854         struct add {
01855             private:
01856                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01857                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01858                 using dividend = typename Ring::template add_t<a, b>;
01859                 using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01860                 using g = typename Ring::template gcd_t<dividend, divider>;
01861
01862             public:
01863                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01864                     divider>;
01865         };
01866
01867         template<typename v>
01868         struct pos {
01869             using type = std::conditional_t<
01870                 (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01871                 (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01872                 std::true_type,
01873                 std::false_type>;
01874         };
01875
01876         template<typename v1, typename v2>
01877         struct sub {
01878             private:
01879                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01880                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01881                 using dividend = typename Ring::template sub_t<a, b>;
01882                 using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01883                 using g = typename Ring::template gcd_t<dividend, divider>;
01884
01885             public:
01886                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01887                     divider>;
01888         };
01889
01890         template<typename v1, typename v2>
01891         struct mul {
01892             private:
01893                 using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01894                 using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01895
01896             public:
01897                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01898         };
01899
01900         template<typename v1, typename v2, typename E = void>
01901         struct div {};
01902
01903         template<typename v1, typename v2>
01904         struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
01905             _FractionField<Ring>::zero>::value> {
01906             private:
01907                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01908                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;

```



```

01908
01909         public:
01910             using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01911     };
01912
01913     template<typename v1, typename v2>
01914     struct div<v1, v2, std::enable_if_t<
01915         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01916         using type = one;
01917     };
01918
01919     template<typename v1, typename v2>
01920     struct eq {
01921         using type = std::conditional_t<
01922             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01923             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01924             std::true_type,
01925             std::false_type>;
01926     };
01927
01928     template<typename v1, typename v2, typename E = void>
01929     struct gt;
01930
01931     template<typename v1, typename v2>
01932     struct gt<v1, v2, std::enable_if_t<
01933         (eq<v1, v2>::type::value)
01934         > {
01935         using type = std::false_type;
01936     };
01937
01938     template<typename v1, typename v2>
01939     struct gt<v1, v2, std::enable_if_t<
01940         (!eq<v1, v2>::type::value) &&
01941         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01942         > {
01943         using type = typename gt<
01944             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01945             >::type;
01946     };
01947
01948     template<typename v1, typename v2>
01949     struct gt<v1, v2, std::enable_if_t<
01950         (!eq<v1, v2>::type::value) &&
01951         (pos<v1>::type::value) && (!pos<v2>::type::value)
01952         > {
01953         using type = std::true_type;
01954     };
01955
01956     template<typename v1, typename v2>
01957     struct gt<v1, v2, std::enable_if_t<
01958         (!eq<v1, v2>::type::value) &&
01959         (!pos<v1>::type::value) && (pos<v2>::type::value)
01960         > {
01961         using type = std::false_type;
01962     };
01963
01964     template<typename v1, typename v2>
01965     struct gt<v1, v2, std::enable_if_t<
01966         (!eq<v1, v2>::type::value) &&
01967         (pos<v1>::type::value) && (pos<v2>::type::value)
01968         > {
01969         using type = typename Ring::template gt_t<
01970             typename Ring::template mul_t<v1::x, v2::y>,
01971             typename Ring::template mul_t<v2::y, v2::x>
01972             >;
01973     };
01974
01975     public:
01976     template<typename v1, typename v2>
01977     using add_t = typename add<v1, v2>::type;
01978
01979     template<typename v1, typename v2>
01980     using mod_t = zero;
01981
01982     template<typename v1, typename v2>
01983     using gcd_t = v1;
01984
01985     template<typename v1, typename v2>
01986     using sub_t = typename sub<v1, v2>::type;
01987
01988     template<typename v1, typename v2>
01989     using mul_t = typename mul<v1, v2>::type;
01990
01991     template<typename v1, typename v2>
01992     using div_t = typename div<v1, v2>::type;
01993
01994     template<typename v1, typename v2>

```

```

02019         using eq_t = typename eq<v1, v2>::type;
02020
02024         template<typename v1, typename v2>
02025         static constexpr bool eq_v = eq<v1, v2>::type::value;
02026
02030         template<typename v1, typename v2>
02031         using gt_t = typename gt<v1, v2>::type;
02032
02036         template<typename v1, typename v2>
02037         static constexpr bool gt_v = gt<v1, v2>::type::value;
02038
02041         template<typename v1>
02042         using pos_t = typename pos<v1>::type;
02043
02046         template<typename v>
02047         static constexpr bool pos_v = pos_t<v>::value;
02048     };
02049
02050     template<typename Ring, typename E = void>
02051     requires IsEuclideanDomain<Ring>
02052     struct FractionFieldImpl {};
02053
02054     // fraction field of a field is the field itself
02055     template<typename Field>
02056     requires IsEuclideanDomain<Field>
02057     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02058         using type = Field;
02059         template<typename v>
02060         using inject_t = v;
02061     };
02062
02063     // fraction field of a ring is the actual fraction field
02064     template<typename Ring>
02065     requires IsEuclideanDomain<Ring>
02066     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02067         using type = _FractionField<Ring>;
02068     };
02069 } // namespace internal
02070
02074     template<typename Ring>
02075     requires IsEuclideanDomain<Ring>
02076     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02077 } // namespace aerobus
02078
02079 // short names for common types
02080 namespace aerobus {
02083     using q32 = FractionField<i32>;
02086     using fpq32 = FractionField<polynomial<q32>>;
02089     using q64 = FractionField<i64>;
02091     using pi64 = polynomial<i64>;
02093     using pq64 = polynomial<q64>;
02095     using fpq64 = FractionField<polynomial<q64>>;
02100     template<typename Ring, typename v1, typename v2>
02101     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02102
02106     template<int64_t p, int64_t q>
02107     using make_q64_t = typename q64::template simplify_t<
02108         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02109
02113     template<int32_t p, int32_t q>
02114     using make_q32_t = typename q32::template simplify_t<
02115         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02116
02121     template<typename Ring, typename v1, typename v2>
02122     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02127     template<typename Ring, typename v1, typename v2>
02128     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02129 } // namespace aerobus
02130
02131 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02132 namespace aerobus {
02133     namespace internal {
02134         template<typename T, size_t x, typename E = void>
02135         struct factorial {};
02136
02137         template<typename T, size_t x>
02138         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02139             private:
02140                 template<typename, size_t, typename>
02141                 friend struct factorial;
02142             public:
02143                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02144 x - 1>::type>;
02145                 static constexpr typename T::inner_type value = type::template get<typename
02146 T::inner_type>();
02147             };
02148         };
02149     }
02150 }

```

```

02147     template<typename T>
02148     struct factorial<T, 0> {
02149     public:
02150         using type = typename T::one;
02151         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02152     };
02153     } // namespace internal
02154
02155     template<typename T, size_t i>
02156     using factorial_t = typename internal::factorial<T, i>::type;
02157
02158     template<typename T, size_t i>
02159     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02160
02161     namespace internal {
02162         template<typename T, size_t k, size_t n, typename E = void>
02163         struct combination_helper {};
02164
02165         template<typename T, size_t k, size_t n>
02166         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02167             using type = typename FractionField<T>::template mul_t<
02168                 typename combination_helper<T, k - 1, n - 1>::type,
02169                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02170         };
02171
02172         template<typename T, size_t k, size_t n>
02173         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02174             using type = typename combination_helper<T, n - k, n>::type;
02175         };
02176
02177         template<typename T, size_t n>
02178         struct combination_helper<T, 0, n> {
02179             using type = typename FractionField<T>::one;
02180         };
02181
02182         template<typename T, size_t k, size_t n>
02183         struct combination {
02184             using type = typename internal::combination_helper<T, k, n>::type::x;
02185             static constexpr typename T::inner_type value =
02186                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02187         };
02188     } // namespace internal
02189
02190     template<typename T, size_t k, size_t n>
02191     using combination_t = typename internal::combination<T, k, n>::type;
02192
02193     template<typename T, size_t k, size_t n>
02194     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02195
02196     namespace internal {
02197         template<typename T, size_t m>
02198         struct bernoulli;
02199
02200         template<typename T, typename accum, size_t k, size_t m>
02201         struct bernoulli_helper {
02202             using type = typename bernoulli_helper<
02203                 T,
02204                 addfractions_t<T,
02205                     accum,
02206                     mulfractions_t<T,
02207                         makefraction_t<T,
02208                             combination_t<T, k, m + 1>,
02209                             typename T::one>,
02210                             typename bernoulli<T, k>::type
02211                         >,
02212                     >,
02213                     k + 1,
02214                     m>::type;
02215         };
02216
02217         template<typename T, typename accum, size_t m>
02218         struct bernoulli_helper<T, accum, m, m> {
02219             using type = accum;
02220         };
02221
02222         template<typename T, size_t m>
02223         struct bernoulli {
02224             using type = typename FractionField<T>::template mul_t<
02225                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02226                 makefraction_t<T,
02227                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02228                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02229                 >
02230             >
02231         };
02232     }

```

```

02244         >;
02245
02246         template<typename floatType>
02247         static constexpr floatType value = type::template get<floatType>();
02248     };
02249
02250     template<typename T>
02251     struct bernoulli<T, 0> {
02252         using type = typename FractionField<T>::one;
02253
02254         template<typename floatType>
02255         static constexpr floatType value = type::template get<floatType>();
02256     };
02257 } // namespace internal
02258
02262 template<typename T, size_t n>
02263 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02264
02269 template<typename FloatType, typename T, size_t n>
02270 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02271
02272 // bell numbers
02273 namespace internal {
02274     template<typename T, size_t n, typename E = void>
02275     struct bell_helper;
02276
02277     template<typename T, size_t n>
02278     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02279         template<typename accum, size_t i, size_t stop>
02280         struct sum_helper {
02281             private:
02282                 using left = typename T::template mul_t<
02283                     combination_t<T, i, n-1>,
02284                     typename bell_helper<T, i>::type>;
02285                 using new_accum = typename T::template add_t<accum, left>;
02286             public:
02287                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02288         };
02289
02290         template<typename accum, size_t stop>
02291         struct sum_helper<accum, stop, stop> {
02292             using type = accum;
02293         };
02294
02295         using type = typename sum_helper<typename T::zero, 0, n>::type;
02296     };
02297
02298     template<typename T>
02299     struct bell_helper<T, 0> {
02300         using type = typename T::one;
02301     };
02302
02303     template<typename T>
02304     struct bell_helper<T, 1> {
02305         using type = typename T::one;
02306     };
02307 } // namespace internal
02308
02312 template<typename T, size_t n>
02313 using bell_t = typename internal::bell_helper<T, n>::type;
02314
02318 template<typename T, size_t n>
02319 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02320
02321 namespace internal {
02322     template<typename T, int k, typename E = void>
02323     struct alternate {};
02324
02325     template<typename T, int k>
02326     struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
02327         using type = typename T::one;
02328         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02329     };
02330
02331     template<typename T, int k>
02332     struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
02333         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02334         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02335     };
02336 } // namespace internal
02337
02340 template<typename T, int k>
02341 using alternate_t = typename internal::alternate<T, k>::type;
02342
02343 namespace internal {

```

```

02344     template<typename T, int n, int k, typename E = void>
02345     struct stirling_helper {};
02346
02347     template<typename T>
02348     struct stirling_helper<T, 0, 0> {
02349         using type = typename T::one;
02350     };
02351
02352     template<typename T, int n>
02353     struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
02354         using type = typename T::zero;
02355     };
02356
02357     template<typename T, int n>
02358     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
02359         using type = typename T::zero;
02360     };
02361
02362     template<typename T, int n, int k>
02363     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)>> {
02364         using type = typename T::template sub_t<
02365             typename stirling_helper<T, n-1, k-1>::type,
02366             typename T::template mul_t<
02367                 typename T::template inject_constant_t<n-1>,
02368                 typename stirling_helper<T, n-1, k>::type
02369             >;
02370     };
02371 } // namespace internal
02372
02373 template<typename T, int n, int k>
02374 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02375
02376 template<typename T, int n, int k>
02377 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02378
02379 template<typename T, int n, int k>
02380 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02381
02382 template<typename T, int n, int k>
02383 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02384
02385 template<typename T, size_t k>
02386 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02387
02388 namespace internal {
02389     template<typename T>
02390     struct pow_scalar {
02391         template<size_t p>
02392         static constexpr T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02393             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02394             x * func<p/2>(x) * func<p/2>(x);
02395         }
02396     };
02397
02398     template<typename T, typename p, size_t n, typename E = void>
02399     requires IsEuclideanDomain<T>
02400     struct pow;
02401
02402     template<typename T, typename p, size_t n>
02403     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
02404         using type = typename T::template mul_t<
02405             typename pow<T, p, n/2>::type,
02406             typename pow<T, p, n/2>::type
02407         >;
02408     };
02409
02410     template<typename T, typename p, size_t n>
02411     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
02412         using type = typename T::template mul_t<
02413             p,
02414             typename T::template mul_t<
02415                 typename pow<T, p, n/2>::type,
02416                 typename pow<T, p, n/2>::type
02417             >
02418         >;
02419     };
02420
02421     template<typename T, typename p, size_t n>
02422     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
02423 } // namespace internal
02424
02425 template<typename T, typename p, size_t n>
02426 using pow_t = typename internal::pow<T, p, n>::type;
02427
02428 template<typename T, typename p, size_t n>
02429 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;

```

```

02457
02458     template<typename T, size_t p>
02459     static constexpr T pow_scalar(const T& x) { return internal::pow_scalar<T>::template func<p>(x); }
02460
02461     namespace internal {
02462         template<typename, template<typename, size_t> typename, class>
02463         struct make_taylor_impl;
02464
02465         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02466         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02467             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
02468         };
02469     }
02470
02471     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02472     using taylor = typename internal::make_taylor_impl<
02473         T,
02474         coeff_at,
02475         internal::make_index_sequence_reverse<deg + 1>::type;
02476
02477     namespace internal {
02478         template<typename T, size_t i>
02479         struct exp_coeff {
02480             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02481         };
02482
02483         template<typename T, size_t i, typename E = void>
02484         struct sin_coeff_helper {};
02485
02486         template<typename T, size_t i>
02487         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02488             using type = typename FractionField<T>::zero;
02489         };
02490
02491         template<typename T, size_t i>
02492         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02493             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02494         };
02495
02496         template<typename T, size_t i>
02497         struct sin_coeff {
02498             using type = typename sin_coeff_helper<T, i>::type;
02499         };
02500
02501         template<typename T, size_t i, typename E = void>
02502         struct sh_coeff_helper {};
02503
02504         template<typename T, size_t i>
02505         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02506             using type = typename FractionField<T>::zero;
02507         };
02508
02509         template<typename T, size_t i>
02510         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02511             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02512         };
02513
02514         template<typename T, size_t i>
02515         struct sh_coeff {
02516             using type = typename sh_coeff_helper<T, i>::type;
02517         };
02518
02519         template<typename T, size_t i, typename E = void>
02520         struct cos_coeff_helper {};
02521
02522         template<typename T, size_t i>
02523         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02524             using type = typename FractionField<T>::zero;
02525         };
02526
02527         template<typename T, size_t i>
02528         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02529             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02530         };
02531
02532         template<typename T, size_t i>
02533         struct cos_coeff {
02534             using type = typename cos_coeff_helper<T, i>::type;
02535         };
02536
02537         template<typename T, size_t i, typename E = void>
02538         struct cosh_coeff_helper {};
02539
02540         template<typename T, size_t i>
02541         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02542             using type = typename FractionField<T>::zero;
02543         };
02544     }

```

```

02547     };
02548
02549     template<typename T, size_t i>
02550     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02551         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02552     };
02553
02554     template<typename T, size_t i>
02555     struct cosh_coeff {
02556         using type = typename cosh_coeff_helper<T, i>::type;
02557     };
02558
02559     template<typename T, size_t i>
02560     struct geom_coeff { using type = typename FractionField<T>::one; };
02561
02562
02563     template<typename T, size_t i, typename E = void>
02564     struct atan_coeff_helper;
02565
02566     template<typename T, size_t i>
02567     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02568         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02569     };
02570
02571     template<typename T, size_t i>
02572     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02573         using type = typename FractionField<T>::zero;
02574     };
02575
02576     template<typename T, size_t i>
02577     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02578
02579     template<typename T, size_t i, typename E = void>
02580     struct asin_coeff_helper;
02581
02582     template<typename T, size_t i>
02583     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02584         using type = makefraction_t<T,
02585             factorial_t<T, i - 1>,
02586             typename T::template mul_t<
02587                 typename T::template val<i>,
02588                 T::template mul_t<
02589                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
02590                     pow<T, factorial_t<T, i / 2>, 2
02591                 >
02592             >
02593         >>;
02594     };
02595
02596     template<typename T, size_t i>
02597     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02598         using type = typename FractionField<T>::zero;
02599     };
02600
02601     template<typename T, size_t i>
02602     struct asin_coeff {
02603         using type = typename asin_coeff_helper<T, i>::type;
02604     };
02605
02606     template<typename T, size_t i>
02607     struct lnpl_coeff {
02608         using type = makefraction_t<T,
02609             alternate_t<T, i + 1>,
02610             typename T::template val<i>;
02611     };
02612
02613     template<typename T>
02614     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02615
02616     template<typename T, size_t i, typename E = void>
02617     struct asinh_coeff_helper;
02618
02619     template<typename T, size_t i>
02620     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02621         using type = makefraction_t<T,
02622             typename T::template mul_t<
02623                 alternate_t<T, i / 2>,
02624                 factorial_t<T, i - 1>
02625             >,
02626             typename T::template mul_t<
02627                 typename T::template mul_t<
02628                     typename T::template val<i>,
02629                     pow_t<T, factorial_t<T, i / 2>, 2>
02630                 >,
02631                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
02632             >
02633         >>;

```

```

02634     };
02635
02636     template<typename T, size_t i>
02637     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02638         using type = typename FractionField<T>::zero;
02639     };
02640
02641     template<typename T, size_t i>
02642     struct asinh_coeff {
02643         using type = typename asinh_coeff_helper<T, i>::type;
02644     };
02645
02646     template<typename T, size_t i, typename E = void>
02647     struct atanh_coeff_helper;
02648
02649     template<typename T, size_t i>
02650     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02651         // 1/i
02652         using type = typename FractionField<T>::template val<
02653             typename T::one,
02654             typename T::template inject_constant_t<i>;
02655     };
02656
02657     template<typename T, size_t i>
02658     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02659         using type = typename FractionField<T>::zero;
02660     };
02661
02662     template<typename T, size_t i>
02663     struct atanh_coeff {
02664         using type = typename atanh_coeff_helper<T, i>::type;
02665     };
02666
02667     template<typename T, size_t i, typename E = void>
02668     struct tan_coeff_helper;
02669
02670     template<typename T, size_t i>
02671     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02672         using type = typename FractionField<T>::zero;
02673     };
02674
02675     template<typename T, size_t i>
02676     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02677     private:
02678         // 4^((i+1)/2)
02679         using _4p = typename FractionField<T>::template inject_t<
02680             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
02681         // 4^((i+1)/2) - 1
02682         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02683         // (-1)^((i-1)/2)
02684         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02685         using dividend = typename FractionField<T>::template mul_t<
02686             altp,
02687             FractionField<T>::template mul_t<
02688                 _4p,
02689                 FractionField<T>::template mul_t<
02690                     _4pml,
02691                     bernoulli_t<T, (i + 1)>
02692                 >
02693             >
02694         >;
02695     public:
02696         using type = typename FractionField<T>::template div_t<dividend,
02697             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02698     };
02699
02700     template<typename T, size_t i>
02701     struct tan_coeff {
02702         using type = typename tan_coeff_helper<T, i>::type;
02703     };
02704
02705     template<typename T, size_t i, typename E = void>
02706     struct tanh_coeff_helper;
02707
02708     template<typename T, size_t i>
02709     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02710         using type = typename FractionField<T>::zero;
02711     };
02712
02713     template<typename T, size_t i>
02714     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02715     private:
02716         using _4p = typename FractionField<T>::template inject_t<
02717             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
02718         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;

```



```

02719         using dividend =
02720             typename FractionField<T>::template mul_t<
02721                 _4p,
02722                 typename FractionField<T>::template mul_t<
02723                     _4pml,
02724                     bernoulli_t<T, (i + 1)>>::type;
02725     public:
02726         using type = typename FractionField<T>::template div_t<dividend,
02727             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
02728     };
02729
02730     template<typename T, size_t i>
02731     struct tanh_coeff {
02732         using type = typename tanh_coeff_helper<T, i>::type;
02733     };
02734 } // namespace internal
02735
02736 template<typename Integers, size_t deg>
02737 using exp = taylor<Integers, internal::exp_coeff, deg>;
02738
02739 template<typename Integers, size_t deg>
02740 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
02741     exp<Integers, deg>,
02742     typename polynomial<FractionField<Integers>>::one>;
02743
02744 template<typename Integers, size_t deg>
02745 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
02746
02747 template<typename Integers, size_t deg>
02748 using atan = taylor<Integers, internal::atan_coeff, deg>;
02749
02750 template<typename Integers, size_t deg>
02751 using sin = taylor<Integers, internal::sin_coeff, deg>;
02752
02753 template<typename Integers, size_t deg>
02754 using sinh = taylor<Integers, internal::sh_coeff, deg>;
02755
02756 template<typename Integers, size_t deg>
02757 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
02758
02759 template<typename Integers, size_t deg>
02760 using cos = taylor<Integers, internal::cos_coeff, deg>;
02761
02762 template<typename Integers, size_t deg>
02763 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
02764
02765 template<typename Integers, size_t deg>
02766 using asin = taylor<Integers, internal::asin_coeff, deg>;
02767
02768 template<typename Integers, size_t deg>
02769 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
02770
02771 template<typename Integers, size_t deg>
02772 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
02773
02774 template<typename Integers, size_t deg>
02775 using tan = taylor<Integers, internal::tan_coeff, deg>;
02776
02777 template<typename Integers, size_t deg>
02778 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
02779 } // namespace aerobus
02780
02781 // continued fractions
02782 namespace aerobus {
02783     template<int64_t... values>
02784     struct ContinuedFraction {};
02785
02786     template<int64_t a0>
02787     struct ContinuedFraction<a0> {
02788         using type = typename q64::template inject_constant_t<a0>;
02789         static constexpr double val = static_cast<double>(a0);
02790     };
02791
02792     template<int64_t a0, int64_t... rest>
02793     struct ContinuedFraction<a0, rest...> {
02794         using type = q64::template add_t<
02795             typename q64::template inject_constant_t<a0>,
02796             typename q64::template div_t<
02797                 typename q64::one,
02798                 typename ContinuedFraction<rest...>::type
02799             >>;
02800         static constexpr double val = type::template get<double>();
02801     };
02802
02803     using PI_fraction =
02804         ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02805     using E_fraction =

```

```

ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02879     using Sqrt2_fraction =
ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02881     using Sqrt3_fraction =
ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
// NOLINT
02882 } // namespace aerobus
02883
02884 // known polynomials
02885 namespace aerobus {
02886     // CChebyshev
02887     namespace internal {
02888         template<int kind, size_t deg>
02889         struct chebyshev_helper {
02890             using type = typename pi64::template sub_t<
02891                 typename pi64::template mul_t<
02892                     typename pi64::template mul_t<
02893                         pi64::inject_constant_t<2>,
02894                         typename pi64::X>,
02895                     typename chebyshev_helper<kind, deg - 1>::type
02896                 >,
02897                 typename chebyshev_helper<kind, deg - 2>::type
02898             >;
02899         };
02900
02901         template<>
02902         struct chebyshev_helper<1, 0> {
02903             using type = typename pi64::one;
02904         };
02905
02906         template<>
02907         struct chebyshev_helper<1, 1> {
02908             using type = typename pi64::X;
02909         };
02910
02911         template<>
02912         struct chebyshev_helper<2, 0> {
02913             using type = typename pi64::one;
02914         };
02915
02916         template<>
02917         struct chebyshev_helper<2, 1> {
02918             using type = typename pi64::template mul_t<
02919                 typename pi64::inject_constant_t<2>,
02920                 typename pi64::X>;
02921         };
02922     } // namespace internal
02923
02924     // Laguerre
02925     namespace internal {
02926         template<size_t deg>
02927         struct laguerre_helper {
02928             private:
02929                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
02930                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02931                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02932                 // -x + 2k-1
02933                 using p = typename pq64::template val<
02934                     typename q64::template inject_constant_t<-1>,
02935                     typename q64::template inject_constant_t<2 * deg - 1>,
02936                 > // 1/n
02937                 using factor = typename pq64::template inject_ring_t<
02938                     q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02939
02940             public:
02941                 using type = typename pq64::template mul_t <
02942                     factor,
02943                     typename pq64::template sub_t<
02944                         typename pq64::template mul_t<
02945                             p,
02946                             lnm1
02947                         >,
02948                     typename pq64::template mul_t<
02949                         typename pq64::template inject_constant_t<deg-1>,
02950                         lnm2
02951                     >
02952                 >
02953             >;
02954         };
02955
02956         template<>
02957         struct laguerre_helper<0> {
02958             using type = typename pq64::one;
02959         };
02960
02961         template<>
02962         struct laguerre_helper<1> {

```

```

02963         using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02964     };
02965 } // namespace internal
02966
02967 // Bernstein
02968 namespace internal {
02969     template<size_t i, size_t m, typename E = void>
02970     struct bernstein_helper {};
02971
02972     template<>
02973     struct bernstein_helper<0, 0> {
02974         using type = typename pi64::one;
02975     };
02976
02977     template<size_t i, size_t m>
02978     struct bernstein_helper<i, m, std::enable_if_t<
02979         (m > 0) && (i == 0)>> {
02980         using type = typename pi64::mul_t<
02981             typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02982             typename bernstein_helper<i, m-1>::type>;
02983     };
02984
02985     template<size_t i, size_t m>
02986     struct bernstein_helper<i, m, std::enable_if_t<
02987         (m > 0) && (i == m)>> {
02988         using type = typename pi64::template mul_t<
02989             typename pi64::X,
02990             typename bernstein_helper<i-1, m-1>::type>;
02991     };
02992
02993     template<size_t i, size_t m>
02994     struct bernstein_helper<i, m, std::enable_if_t<
02995         (m > 0) && (i > 0) && (i < m)>> {
02996         using type = typename pi64::add_t<
02997             typename pi64::mul_t<
02998                 typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02999                 typename bernstein_helper<i, m-1>::type>,
03000             typename pi64::mul_t<
03001                 typename pi64::X,
03002                 typename bernstein_helper<i-1, m-1>::type>>;
03003     };
03004 } // namespace internal
03005
03006 namespace known_polynomials {
03007     enum hermite_kind {
03008         probabilist,
03009         physicist
03010     };
03011 }
03012
03013 // hermite
03014 namespace internal {
03015     template<size_t deg, known_polynomials::hermite_kind kind>
03016     struct hermite_helper {};
03017
03018     template<size_t deg>
03019     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
03020     private:
03021         using hnm1 = typename hermite_helper<deg - 1,
03022             known_polynomials::hermite_kind::probabilist>::type;
03023         using hnm2 = typename hermite_helper<deg - 2,
03024             known_polynomials::hermite_kind::probabilist>::type;
03025
03026     public:
03027         using type = typename pi64::template sub_t<
03028             typename pi64::template mul_t<typename pi64::X, hnm1>,
03029             typename pi64::template mul_t<
03030                 typename pi64::template inject_constant_t<deg - 1>,
03031                 hnm2
03032             >
03033         >;
03034     };
03035
03036     template<size_t deg>
03037     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
03038     private:
03039         using hnm1 = typename hermite_helper<deg - 1,
03040             known_polynomials::hermite_kind::physicist>::type;
03041         using hnm2 = typename hermite_helper<deg - 2,
03042             known_polynomials::hermite_kind::physicist>::type;
03043
03044     public:
03045         using type = typename pi64::template sub_t<
03046             // 2X Hn-1
03047             typename pi64::template mul_t<
03048                 typename pi64::val<typename i64::template inject_constant_t<2>,
03049                 typename i64::zero>, hnm1>,

```

```

03049
03050         typename pi64::template mul_t<
03051             typename pi64::template inject_constant_t<2*(deg - 1)>,
03052             hnm2
03053         >
03054     >;
03055 };
03056
03057 template<>
03058 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
03059     using type = typename pi64::one;
03060 };
03061
03062 template<>
03063 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
03064     using type = typename pi64::X;
03065 };
03066
03067 template<>
03068 struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
03069     using type = typename pi64::one;
03070 };
03071
03072 template<>
03073 struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
03074     // 2X
03075     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
typename i64::zero>;
03076 };
03077 } // namespace internal
03078
03079 // legendre
03080 namespace internal {
03081     template<size_t n>
03082     struct legendre_helper {
03083     private:
03084         // 1/n constant
03085         // (2n-1)/n X
03086         using fact_left = typename pq64::monomial_t<make_q64_t<2*n-1, n>, 1>;
03087         // (n-1) / n
03088         using fact_right = typename pq64::val<make_q64_t<n-1, n>;
03089     public:
03090         using type = pq64::template sub_t<
03091             typename pq64::template mul_t<
03092                 fact_left,
03093                 typename legendre_helper<n-1>::type
03094             >,
03095             typename pq64::template mul_t<
03096                 fact_right,
03097                 typename legendre_helper<n-2>::type
03098             >
03099         >;
03100     };
03101
03102     template<>
03103     struct legendre_helper<0> {
03104         using type = typename pq64::one;
03105     };
03106
03107     template<>
03108     struct legendre_helper<1> {
03109         using type = typename pq64::X;
03110     };
03111 } // namespace internal
03112
03113 // bernoulli polynomials
03114 namespace internal {
03115     template<size_t n>
03116     struct bernoulli_coeff {
03117         template<typename T, size_t i>
03118         struct inner {
03119         private:
03120             using F = FractionField<T>;
03121         public:
03122             using type = typename F::template mul_t<
03123                 typename F::template inject_ring_t<combination_t<T, i, n>,
03124                 bernoulli_t<T, n-i>
03125             >;
03126         };
03127     };
03128 } // namespace internal
03129
03130 namespace known_polynomials {
03131     template <size_t deg>
03132     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
03133
03134     template <size_t deg>

```

```

03148     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
03149
03156     template <size_t deg>
03157     using laguerre = typename internal::laguerre_helper<deg>::type;
03158
03165     template <size_t deg>
03166     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
03167
03174     template <size_t deg>
03175     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
03176
03184     template<size_t i, size_t m>
03185     using bernstein = typename internal::bernstein_helper<i, m>::type;
03186
03193     template<size_t deg>
03194     using legendre = typename internal::legendre_helper<deg>::type;
03195
03202     template<size_t deg>
03203     using bernoulli = taylor<i64, internal::bernoulli_coeff<deg>::template inner, deg>;
03204 } // namespace known_polynomials
03205 } // namespace aerobus
03206
03207
03208 #ifdef AEROBUS_CONWAY_IMPORTS
03209
03210 // conway polynomials
03211 namespace aerobus {
03212     template<int p, int n>
03213     struct ConwayPolynomial {};
03214
03215 #ifndef DO_NOT_DOCUMENT
03216     #define ZPZV ZPZ::template val
03217     #define POLYV aerobus::polynomial<ZPZ>::template val
03218     template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>; }; // NOLINT
03219     template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03220     template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03221     template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03222     template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03223     template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03224     template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03225     template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03226     template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; //
NOLINT
03227     template<> struct ConwayPolynomial<2, 10> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<1>,
ZPZV<1>; }; // NOLINT
03228     template<> struct ConwayPolynomial<2, 11> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>,
ZPZV<1>; }; // NOLINT
03229     template<> struct ConwayPolynomial<2, 12> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>,
ZPZV<1>, ZPZV<1>; }; // NOLINT
03230     template<> struct ConwayPolynomial<2, 13> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>,
ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03231     template<> struct ConwayPolynomial<2, 14> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>,
ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03232     template<> struct ConwayPolynomial<2, 15> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>,
ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03233     template<> struct ConwayPolynomial<2, 16> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>,
ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03234     template<> struct ConwayPolynomial<2, 17> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>,
ZPZV<0>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03235     template<> struct ConwayPolynomial<2, 18> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>,
ZPZV<0>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03236     template<> struct ConwayPolynomial<2, 19> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; //
NOLINT
03237     template<> struct ConwayPolynomial<2, 20> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>,
ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>,
ZPZV<1>; }; // NOLINT
03238
03239 // NOLINT
03240

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05149     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
05150     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
05151     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
05152     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
05153     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
05154     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
05155     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
05156     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
05157     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
05158     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
05159     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
05160     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
05161     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
05162 #endif // DO_NOT_DOCUMENT
05163 } // namespace aerobus
05164 #endif // AEROBUS_CONWAY_IMPORTS
05165
05166 #endif // __INC_AEROBUS__ // NOLINT

```


Chapter 10

Examples

10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject_constant_t<1>>

Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

10.2 type_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

...Ts	types to store and manipulate at compile time
-------	---

10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

10.4 i32::add_t

addition operator yields $v1 + v2$ $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields $v1 + v2$ $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.5 i32::sub_t

subtraction operator yields $v1 - v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields $v1 - v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.6 i32::mul_t

multiplication operator yields $v1 * v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields $v1 * v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.7 i32::div_t

division operator yields $v1 / v2$ $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields $v1 / v2$ $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.8 i32::gt_t

strictly greater operator ($v1 > v2$) yields $v1 > v2$ `<i32::val<7>, i32::val<2>>`

strictly greater operator ($v1 > v2$) yields $v1 > v2$ `<i32::val<7>, i32::val<2>>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.9 i32::eq_t

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool> <i32::val<2>, i32::val<2>>`

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool> <i32::val<2>, i32::val<2>>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.10 i32::eq_v

equality operator (boolean value)

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	<code><i32::val<1>, i32::val<1>></code>

10.11 i32::gcd_t

greatest common divisor yields $GCD(v1, v2)$ `<i32::val<6>, i32::val<15>>`

greatest common divisor yields $GCD(v1, v2)$ `<i32::val<6>, i32::val<15>>`

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

10.12 i32::pos_t

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

v	a value in i32
-----	----------------

10.13 i32::pos_v

positivity (boolean value) yields $v > 0$ as boolean value

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

v	a value in i32 <code><i32::val<1>></code>
-----	---

10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	<code>inject_constant_t<2></code>
-----	---

10.15 i64::add_t

addition operator

addition operator

Template Parameters

$v1$: an element of <code>aerobus::i64::val</code>
$v2$: an element of <code>aerobus::i64::val</code> <code><i64::val<1>, i64::val<2>></code>

10.16 i64::sub_t

subtraction operator

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

10.17 i64::mul_t

multiplication operator

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

10.18 i64::div_t

division operator integer division

division operator integer division

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

10.19 i64::mod_t

modulus operator

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<6>, i64::val<15>>

10.20 i64::gt_t

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<2>, i64::val<1>></code>

10.21 i64::lt_t

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<1>, i64::val<2>></code>

10.22 i64::lt_v

strictly smaller operator yields $v1 < v2$ as boolean value

strictly smaller operator yields $v1 < v2$ as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<1>, i64::val<2>></code>

10.23 i64::eq_t

equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`

equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<2>, i64::val<2>></code>

10.24 i64::eq_v

equality operator yields $v1 == v2$ as boolean value

equality operator yields $v1 == v2$ as boolean value

Template Parameters

$v1$: an element of aerobus::i64::val
$v2$: an element of aerobus::i64::val <i64::val<2>, i64::val<2>>

10.25 i64::gcd_t

greatest common divisor yields $GCD(v1, v2)$ as instantiation of i64::val

greatest common divisor yields $GCD(v1, v2)$ as instantiation of i64::val

Template Parameters

$v1$: an element of aerobus::i64::val
$v2$: an element of aerobus::i64::val <i64::val<6>, i64::val<15>>

10.26 i64::pos_t

is v positive yields $v > 0$ as std::true_type or std::false_type

is v positive yields $v > 0$ as std::true_type or std::false_type

Template Parameters

v	: an element of aerobus::i64::val <i64::val<1>>
-----	---

10.27 i64::pos_v

positivity yields $v > 0$ as boolean value

positivity yields $v > 0$ as boolean value

Template Parameters

v	: an element of aerobus::i64::val <i64::val<1>>
-----	---

10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

<code>x</code>	<code><i32>::template inject_constant_t<2></code>
----------------	---

10.29 q32::add_t

addition operator

addition operator

Template Parameters

<code>v1</code>	a value
<code>v2</code>	a value <code><q32::val<i32::val<1>, i32::val<2>>, q32::val<i32::val<1>, i32::val<3>>></code>

10.30 FractionField

Fraction field of an euclidean domain, such as \mathbb{Q} for \mathbb{Z} .

Fraction field of an euclidean domain, such as \mathbb{Q} for \mathbb{Z}

Template Parameters

<code>Ring</code>	<code><i64></code> is q64 (rationals with 64 bits numerator and denominator)
-------------------	--

10.31 aerobus::ContinuedFraction

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$ [https://en.wikipedia.org/wiki/Continued_fraction](See in Wikipedia)

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

$\langle 1, 1, 1 \rangle$ represents $1 + \frac{1}{1}$

10.32 PI_fraction::val

representation of π as a continued fraction -> 3.14...

10.33 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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