

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

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Class Index

2.1 Class List

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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--	----

Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are aerobus::i64
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
-----------	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using **type** = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
<i>...rest</i>	integers (aerobus::i64)

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values in [i32](#), again represented as types

Public Types

- using **inner_type** = int32_t
- using **zero** = [val](#)< 0 >
constant zero
- using **one** = [val](#)< 1 >
constant one
- template<auto x>
using **inject_constant_t** = [val](#)< static_cast< int32_t >(x)>
- template<typename v >
using **inject_ring_t** = v
- template<typename v1, typename v2 >
using **add_t** = typename add< v1, v2 >::type
- template<typename v1, typename v2 >
using **sub_t** = typename sub< v1, v2 >::type
- template<typename v1, typename v2 >
using **mul_t** = typename mul< v1, v2 >::type
- template<typename v1, typename v2 >
using **div_t** = typename div< v1, v2 >::type
- template<typename v1, typename v2 >
using **mod_t** = typename remainder< v1, v2 >::type
modulus operator yields v1 % v2 for example : [i32::mod_t](#)<[i32::val](#)<7>, [i32::val](#)<2>>
- template<typename v1, typename v2 >
using **gt_t** = typename gt< v1, v2 >::type
- template<typename v1, typename v2 >
using **lt_t** = typename lt< v1, v2 >::type
- template<typename v1, typename v2 >
using **eq_t** = typename eq< v1, v2 >::type
- template<typename v1, typename v2 >
using **gcd_t** = gcd_t< [i32](#), v1, v2 >
- template<typename v >
using **pos_t** = typename pos< v >::type

Static Public Attributes

- static constexpr bool **is_field** = false
integers are not a field
- static constexpr bool **is_euclidean_domain** = true
integers are an euclidean domain
- template<typename v1, typename v2 >
static constexpr bool **eq_v** = eq_t<v1, v2>::value
- template<typename v >
static constexpr bool **pos_v** = pos_t<v>::value

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Typedef Documentation

5.7.2.1 mod_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields $v1 \% v2$ for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

Template Parameters

<code>v1</code>	a value in <code>i32</code>
<code>v2</code>	a value in <code>i32</code>

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in `i64`

Public Types

- using `inner_type` = `int64_t`
type for actual values
- template<auto x>
using `inject_constant_t` = `val< static_cast< int64_t >(x)>`
- template<typename v >
using `inject_ring_t` = v
injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>> -> i64::val<1>`
- using `zero` = `val< 0 >`
constant zero
- using `one` = `val< 1 >`
constant one
- template<typename v1 , typename v2 >
using `add_t` = `typename add< v1, v2 >::type`
- template<typename v1 , typename v2 >
using `sub_t` = `typename sub< v1, v2 >::type`

- `template<typename v1 , typename v2 >`
using **mul_t** = typename mul< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **div_t** = typename div< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **mod_t** = typename remainder< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **gt_t** = typename gt< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **lt_t** = typename lt< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **eq_t** = typename eq< v1, v2 >::type
- `template<typename v1 , typename v2 >`
using **gcd_t** = gcd_t< i64, v1, v2 >
- `template<typename v >`
using **pos_t** = typename pos< v >::type

Static Public Attributes

- static constexpr bool **is_field** = false
integers are not a field
- static constexpr bool **is_euclidean_domain** = true
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
static constexpr bool **gt_v** = gt_t<v1, v2>::value
strictly greater operator yields v1 > v2 as boolean value
- `template<typename v1 , typename v2 >`
static constexpr bool **lt_v** = lt_t<v1, v2>::value
- `template<typename v1 , typename v2 >`
static constexpr bool **eq_v** = eq_t<v1, v2>::value
- `template<typename v >`
static constexpr bool **pos_v** = pos_t<v>::value

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Typedef Documentation

5.8.2.1 inject_ring_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64↔::val<1>> -> i64::val<1>`

Template Parameters

<i>v</i>	a value in i64
----------	----------------

5.8.3 Member Data Documentation

5.8.3.1 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields $v1 > v2$ as boolean value

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.9 aerobus::is_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

Static Public Attributes

- static constexpr bool **value** = internal::_is_prime< n , 5>::value
true iff n is prime

5.9.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.10 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero = val< typename Ring::zero >](#)
constant zero
- [using one = val< typename Ring::one >](#)
constant one
- [using X = val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t = typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1, typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1, typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1, typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1, typename v2 >](#)
[using eq_t = typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1, typename v2 >](#)
[using lt_t = typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1, typename v2 >](#)
[using gt_t = typename gt_helper< v1, v2 >::type](#)
strict greater operator
- [template<typename v1, typename v2 >](#)
[using div_t = typename div< v1, v2 >::q_type](#)
division operator
- [template<typename v1, typename v2 >](#)
[using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type](#)
modulo operator
- [template<typename coeff, size_t deg>](#)
[using monomial_t = typename monomial< coeff, deg >::type](#)
monomial : coeff X^{deg}
- [template<typename v >](#)
[using derive_t = typename derive_helper< v >::type](#)

- derivation operator*
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity (an > 0)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator

5.10.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.10.2 Member Typedef Documentation

5.10.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.2 derive_t

```
template<typename Ring >
```

```
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.10.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.5 gcd_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.7 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.8 mod_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.9 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.10.2.10 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.2.11 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.10.2.12 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.10.2.13 sub_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.10.3 Member Data Documentation

5.10.3.1 pos_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in polynomial::val
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.11 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.11.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.12 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< [typename](#) Ring::zero >
zero value
- using [one](#) = [val](#)< [typename](#) Ring::one >
one
- template<[typename](#) v1 , [typename](#) v2 >
using [add_t](#) = [val](#)< [typename](#) Ring::template [add_t](#)< [typename](#) v1::type, [typename](#) v2::type > >
addition operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mul_t](#) = [val](#)< [typename](#) Ring::template [mul_t](#)< [typename](#) v1::type, [typename](#) v2::type > >
subtraction operator
- template<[typename](#) v1 , [typename](#) v2 >
using [div_t](#) = [val](#)< [typename](#) Ring::template [div_t](#)< [typename](#) v1::type, [typename](#) v2::type > >
division operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mod_t](#) = [val](#)< [typename](#) Ring::template [mod_t](#)< [typename](#) v1::type, [typename](#) v2::type > >
modulus operator
- template<[typename](#) v1 , [typename](#) v2 >
using [eq_t](#) = [typename](#) Ring::template [eq_t](#)< [typename](#) v1::type, [typename](#) v2::type >
equality operator (as type)
- template<[typename](#) v1 >
using [pos_t](#) = `std::true_type`
positivity operator always true
- template<[auto](#) x>
using [inject_constant_t](#) = [val](#)< [typename](#) Ring::template [inject_constant_t](#)< x > >
- template<[typename](#) v >
using [inject_ring_t](#) = [val](#)< v >

Static Public Attributes

- template<[typename](#) v1 , [typename](#) v2 >
[static constexpr bool](#) [eq_v](#) = Ring::template [eq_t](#)<[typename](#) v1::type, [typename](#) v2::type>::value
addition operator (as boolean value)
- template<[typename](#) v >
[static constexpr bool](#) [pos_v](#) = [pos_t](#)<v>::value
positivity operator always true
- [static constexpr bool](#) [is_euclidean_domain](#) = `true`
quotien rings are euclidean domain

5.12.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val<2></code>

5.12.2 Member Typedef Documentation

5.12.2.1 `add_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.12.2.2 `div_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.12.2.3 `eq_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
```

```
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.12.2.4 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.12.2.5 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.12.2.6 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<code>v1</code>	a value in quotient ring
-----------------	--------------------------

5.12.3 Member Data Documentation

5.12.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<code>v1</code>	a value in quotient ring
<code>v2</code>	a value in quotient ring

5.12.3.2 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<code>v1</code>	a value in quotient ring
-----------------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.13 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using **head** = typename inner::head
- using **tail** = typename inner::tail

5.13.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

5.14 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = internal::type_at_t< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename concat_h< U >::type
concatenates two list into one
- template<typename T , size_t index>
using [insert](#) = typename internal::insert_h< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename internal::remove_h< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t **length** = sizeof...(Ts)
length of list

5.14.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

5.14.2 Member Typedef Documentation

5.14.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.14.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.14.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.14.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.14.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.14.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.15 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using **push_front** = [type_list](#)< T >
- template<typename T >
using **push_back** = [type_list](#)< T >
- template<typename U >
using **concat** = U
- template<typename T , size_t index>
using **insert** = [type_list](#)< T >

Static Public Attributes

- static constexpr size_t **length** = 0

5.15.1 Detailed Description

specialization for empty type list

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

5.16 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using **enclosing_type** = [i32](#)
Enclosing ring type.
- using **is_zero_t** = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast x into valueType
- `static std::string to_string ()`
string representation of value
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast x into valueRing

Static Public Attributes

- `static constexpr int32_t v = x`
actual value stored in val type

5.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.16.2 Member Function Documentation

5.16.2.1 [eval\(\)](#)

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.16.2.2 [get\(\)](#)

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

5.17 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- using **enclosing_type** = [i64](#)
enclosing ring type
- using **is_zero_t** = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename valueType >
static constexpr valueType **get** ()
cast value in valueType
- static std::string **to_string** ()
string representation
- template<typename valueRing >
static constexpr valueRing **eval** (const valueRing &v)
cast value in valueRing

Static Public Attributes

- static constexpr int64_t **v** = x
actual value

5.17.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

<i>x</i>	an actual integer
----------	-------------------

5.17.2 Member Function Documentation

5.17.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<i>valueRing</i>	(double for example)
------------------	----------------------

5.17.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.18 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using enclosing_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

5.18.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

5.18.2 Member Typedef Documentation

5.18.2.1 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_at_
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.18.3 Member Function Documentation

5.18.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.18.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.19 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

5.19.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.20 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- `using enclosing_type = zpz< p >`
enclosing ring type
- `using is_zero_t = std::bool_constant< x% p==0 >`

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

5.21 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct [coeff_at](#)
- struct [coeff_at< index, std::enable_if_t<\(index< 0||index > 0\)> >](#)
- struct [coeff_at< index, std::enable_if_t<\(index==0\)> >](#)

Public Types

- [using enclosing_type = polynomial< Ring >](#)
enclosing ring type
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is_zero_t = std::bool_constant< aN::is_zero_t::value >](#)
- [template<size_t index>](#)
[using coeff_at_t = typename coeff_at< index >::type](#)

Static Public Member Functions

- [static std::string to_string \(\)](#)
- [template<typename valueRing >](#)
[static constexpr valueRing eval \(const valueRing &x\)](#)

Static Public Attributes

- [static constexpr size_t degree = 0](#)
degree
- [static constexpr bool is_zero_v = is_zero_t::value](#)

5.21.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

coeffN	
------------------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- using [inner_type](#) = [int32_t](#)
- template<[auto](#) x>
using [inject_constant_t](#) = [val](#)< [static_cast](#)< [int32_t](#) >(x)>
- using [zero](#) = [val](#)< 0 >
- using [one](#) = [val](#)< 1 >
- template<[typename](#) v1 , [typename](#) v2 >
using [add_t](#) = [typename](#) [add](#)< v1, v2 >::type
addition operator
- template<[typename](#) v1 , [typename](#) v2 >
using [sub_t](#) = [typename](#) [sub](#)< v1, v2 >::type
subtraction operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mul_t](#) = [typename](#) [mul](#)< v1, v2 >::type
multiplication operator
- template<[typename](#) v1 , [typename](#) v2 >
using [div_t](#) = [typename](#) [div](#)< v1, v2 >::type
division operator
- template<[typename](#) v1 , [typename](#) v2 >
using [mod_t](#) = [typename](#) [remainder](#)< v1, v2 >::type
modulo operator
- template<[typename](#) v1 , [typename](#) v2 >
using [gt_t](#) = [typename](#) [gt](#)< v1, v2 >::type
strictly greater operator (type)
- template<[typename](#) v1 , [typename](#) v2 >
using [lt_t](#) = [typename](#) [lt](#)< v1, v2 >::type
strictly smaller operator (type)
- template<[typename](#) v1 , [typename](#) v2 >
using [eq_t](#) = [typename](#) [eq](#)< v1, v2 >::type
equality operator (type)
- template<[typename](#) v1 , [typename](#) v2 >
using [gcd_t](#) = [gcd_t](#)< [i32](#), v1, v2 >
greatest common divisor
- template<[typename](#) v1 >
using [pos_t](#) = [typename](#) [pos](#)< v1 >::type
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.22.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

5.22.2 Member Typedef Documentation

5.22.2.1 add_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	a value in <code>zpz::val</code>
<code>v2</code>	a value in <code>zpz::val</code>

5.22.2.2 div_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.2.3 eq_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.2.4 gcd_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.2.5 gt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.2.6 lt_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

5.22.2.7 mod_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

5.22.2.8 mul_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	a value in zpz::val
v2	a value in zpz::val

5.22.2.9 pos_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

5.22.2.10 sub_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.3 Member Data Documentation

5.22.3.1 eq_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.3.2 gt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.3.3 lt_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

Template Parameters

<i>v1</i>	a value in zpz::val
<i>v2</i>	a value in zpz::val

5.22.3.4 pos_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

Template Parameters

<i>v1</i>	a value in zpz::val
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

Chapter 6

File Documentation

6.1 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

6.2 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00027
00028 // aligned allocation
00029 namespace aerobus {
00036     template<typename T>
00037     T* aligned_malloc(size_t count, size_t alignment) {
00038         #ifdef _MSC_VER
```

```

00039         return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00040     #else
00041         return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00042     #endif
00043     }
00044 } // namespace aerobus
00045
00046 // concepts
00047 namespace aerobus {
00048     template <typename R>
00049     concept IsRing = requires {
00050         typename R::one;
00051         typename R::zero;
00052         typename R::template add_t<typename R::one, typename R::one>;
00053         typename R::template sub_t<typename R::one, typename R::one>;
00054         typename R::template mul_t<typename R::one, typename R::one>;
00055     };
00056
00057     template <typename R>
00058     concept IsEuclideanDomain = IsRing<R> && requires {
00059         typename R::template div_t<typename R::one, typename R::one>;
00060         typename R::template mod_t<typename R::one, typename R::one>;
00061         typename R::template gcd_t<typename R::one, typename R::one>;
00062         typename R::template eq_t<typename R::one, typename R::one>;
00063         typename R::template pos_t<typename R::one>;
00064
00065         R::template pos_v<typename R::one> == true;
00066         // typename R::template gt_t<typename R::one, typename R::zero>;
00067         R::is_euclidean_domain == true;
00068     };
00069
00070     template<typename R>
00071     concept IsField = IsEuclideanDomain<R> && requires {
00072         R::is_field == true;
00073     };
00074 } // namespace aerobus
00075
00076 // utilities
00077 namespace aerobus {
00078     namespace internal {
00079         template<template<typename...> typename TT, typename T>
00080         struct is_instantiation_of : std::false_type { };
00081
00082         template<template<typename...> typename TT, typename... Ts>
00083         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00084
00085         template<template<typename...> typename TT, typename T>
00086         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00087
00088         template<int64_t i, typename T, typename... Ts>
00089         struct type_at {
00090             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00091             using type = typename type_at<i - 1, Ts...>::type;
00092         };
00093
00094         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00095             using type = T;
00096         };
00097
00098         template <size_t i, typename... Ts>
00099         using type_at_t = typename type_at<i, Ts...>::type;
00100
00101         template<size_t n, size_t i, typename E = void>
00102         struct _is_prime {};
00103
00104         template<size_t i>
00105         struct _is_prime<0, i> {
00106             static constexpr bool value = false;
00107         };
00108
00109         template<size_t i>
00110         struct _is_prime<1, i> {
00111             static constexpr bool value = false;
00112         };
00113
00114         template<size_t i>
00115         struct _is_prime<2, i> {
00116             static constexpr bool value = true;
00117         };
00118
00119         template<size_t i>
00120         struct _is_prime<3, i> {
00121             static constexpr bool value = true;
00122         };
00123
00124         template<size_t i>

```

```

00129     struct _is_prime<5, i> {
00130         static constexpr bool value = true;
00131     };
00132
00133     template<size_t i>
00134     struct _is_prime<7, i> {
00135         static constexpr bool value = true;
00136     };
00137
00138     template<size_t n, size_t i>
00139     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00140         static constexpr bool value = false;
00141     };
00142
00143     template<size_t n, size_t i>
00144     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00145         static constexpr bool value = false;
00146     };
00147
00148     template<size_t n, size_t i>
00149     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00150         static constexpr bool value = true;
00151     };
00152
00153     template<size_t n, size_t i>
00154     struct _is_prime<n, i, std::enable_if_t<(
00155         n % i == 0 &&
00156         n >= 9 &&
00157         n % 3 != 0 &&
00158         n % 2 != 0 &&
00159         i * i > n)>> {
00160         static constexpr bool value = true;
00161     };
00162
00163     template<size_t n, size_t i>
00164     struct _is_prime<n, i, std::enable_if_t<(
00165         n % (i+2) == 0 &&
00166         n >= 9 &&
00167         n % 3 != 0 &&
00168         n % 2 != 0 &&
00169         i * i <= n)>> {
00170         static constexpr bool value = true;
00171     };
00172
00173     template<size_t n, size_t i>
00174     struct _is_prime<n, i, std::enable_if_t<(
00175         n % (i+2) != 0 &&
00176         n % i != 0 &&
00177         n >= 9 &&
00178         n % 3 != 0 &&
00179         n % 2 != 0 &&
00180         (i * i <= n))>> {
00181         static constexpr bool value = _is_prime<n, i+6>::value;
00182     };
00183
00184 } // namespace internal
00185
00186 template<size_t n>
00187 struct is_prime {
00188     static constexpr bool value = internal::_is_prime<n, 5>::value;
00189 };
00190
00191 template<size_t n>
00192 static constexpr bool is_prime_v = is_prime<n>::value;
00193
00194 // gcd
00195 namespace internal {
00196     template <std::size_t... Is>
00197     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00198         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00199
00200     template <std::size_t N>
00201     using make_index_sequence_reverse
00202         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00203
00204     template<typename Ring, typename E = void>
00205     struct gcd;
00206
00207     template<typename Ring>
00208     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00209         template<typename A, typename B, typename E = void>
00210         struct gcd_helper {};
00211
00212         // B = 0, A > 0
00213         template<typename A, typename B>
00214         struct gcd_helper<A, B, std::enable_if_t<
00215             (B::is_zero_t::value) &&

```

```

00227         (Ring::template gt_t<A, typename Ring::zero>::value))» {
00228             using type = A;
00229         };
00230
00231         // B = 0, A < 0
00232         template<typename A, typename B>
00233         struct gcd_helper<A, B, std::enable_if_t<
00234             ((B::is_zero_t::value) &&
00235              !(Ring::template gt_t<A, typename Ring::zero>::value))» {
00236             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00237         };
00238
00239         // B != 0
00240         template<typename A, typename B>
00241         struct gcd_helper<A, B, std::enable_if_t<
00242             (!B::is_zero_t::value)
00243             » {
00244             private: // NOLINT
00245                 // A / B
00246                 using k = typename Ring::template div_t<A, B>;
00247                 // A - (A/B)*B = A % B
00248                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00249
00250             public:
00251                 using type = typename gcd_helper<B, m>::type;
00252             };
00253
00254         template<typename A, typename B>
00255         using type = typename gcd_helper<A, B>::type;
00256     };
00257 } // namespace internal
00258
00259 // vadd and vmul
00260 namespace internal {
00261     template<typename... vals>
00262     struct vmul {};
00263
00264     template<typename v1, typename... vals>
00265     struct vmul<v1, vals...> {
00266         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00267     };
00268
00269     template<typename v1>
00270     struct vmul<v1> {
00271         using type = v1;
00272     };
00273
00274     template<typename... vals>
00275     struct vadd {};
00276
00277     template<typename v1, typename... vals>
00278     struct vadd<v1, vals...> {
00279         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00280     };
00281
00282     template<typename v1>
00283     struct vadd<v1> {
00284         using type = v1;
00285     };
00286 } // namespace internal
00287
00288 template<typename T, typename A, typename B>
00289 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00290
00291 template<typename... vals>
00292 using vadd_t = typename internal::vadd<vals...>::type;
00293
00294 template<typename... vals>
00295 using vmul_t = typename internal::vmul<vals...>::type;
00296
00297 template<typename val>
00298 requires IsEuclideanDomain<typename val::enclosing_type>
00299 using abs_t = std::conditional_t<
00300     val::enclosing_type::template pos_v<val>,
00301     val, typename val::enclosing_type::template sub_t<typename
val::enclosing_type::zero, val>;
00302 } // namespace aerobus
00303
00304 namespace aerobus {
00305     template<typename Ring, typename X>
00306     requires IsRing<Ring>
00307     struct Quotient {
00308         template <typename V>
00309         struct val {
00310             public:

```

```

00328         using type = abs_t<typename Ring::template mod_t<V, X>>;
00329     };
00330
00332     using zero = val<typename Ring::zero>;
00333
00335     using one = val<typename Ring::one>;
00336
00340     template<typename v1, typename v2>
00341     using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00342
00346     template<typename v1, typename v2>
00347     using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00348
00352     template<typename v1, typename v2>
00353     using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00354
00358     template<typename v1, typename v2>
00359     using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00360
00364     template<typename v1, typename v2>
00365     using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00366
00370     template<typename v1, typename v2>
00371     static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00372
00376     template<typename v1>
00377     using pos_t = std::true_type;
00378
00382     template<typename v>
00383     static constexpr bool pos_v = pos_t<v>::value;
00384
00386     static constexpr bool is_euclidean_domain = true;
00387
00391     template<auto x>
00392     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00393
00397     template<typename v>
00398     using inject_ring_t = val<v>;
00399 };
00400 } // namespace aerobus
00401
00402 // type_list
00403 namespace aerobus {
00404     template <typename... Ts>
00405     struct type_list;
00406
00407     namespace internal {
00408         template <typename T, typename... Us>
00409         struct pop_front_h {
00410             using tail = type_list<Us...>;
00411             using head = T;
00412         };
00413     };
00414
00415     template <size_t index, typename L1, typename L2>
00416     struct split_h {
00417     private:
00418         static_assert(index <= L2::length, "index out of bounds");
00419         using a = typename L2::pop_front::type;
00420         using b = typename L2::pop_front::tail;
00421         using c = typename L1::template push_back<a>;
00422
00423     public:
00424         using head = typename split_h<index - 1, c, b>::head;
00425         using tail = typename split_h<index - 1, c, b>::tail;
00426     };
00427
00428     template <typename L1, typename L2>
00429     struct split_h<0, L1, L2> {
00430         using head = L1;
00431         using tail = L2;
00432     };
00433
00434     template <size_t index, typename L, typename T>
00435     struct insert_h {
00436         static_assert(index <= L::length, "index out of bounds");
00437         using s = typename L::template split<index>;
00438         using left = typename s::head;
00439         using right = typename s::tail;
00440         using ll = typename left::template push_back<T>;
00441         using type = typename ll::template concat<right>;
00442     };
00443
00444     template <size_t index, typename L>
00445     struct remove_h {
00446         using s = typename L::template split<index>;
00447         using left = typename s::head;
00448         using right = typename s::tail;

```

```

00449         using rr = typename right::pop_front::tail;
00450         using type = typename left::template concat<rr>;
00451     };
00452 } // namespace internal
00453
00454 template <typename... Ts>
00455 struct type_list {
00456 private:
00457     template <typename T>
00458     struct concat_h;
00459
00460     template <typename... Us>
00461     struct concat_h<type_list<Us...> {
00462         using type = type_list<Ts..., Us...>;
00463     };
00464
00465 public:
00466     static constexpr size_t length = sizeof...(Ts);
00467
00468     template <typename T>
00469     using push_front = type_list<T, Ts...>;
00470
00471     template <size_t index>
00472     using at = internal::type_at_t<index, Ts...>;
00473
00474     struct pop_front {
00475         using type = typename internal::pop_front_h<Ts...>::head;
00476         using tail = typename internal::pop_front_h<Ts...>::tail;
00477     };
00478
00479     template <typename T>
00480     using push_back = type_list<Ts..., T>;
00481
00482     template <typename U>
00483     using concat = typename concat_h<U>::type;
00484
00485     template <size_t index>
00486     struct split {
00487     private:
00488         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00489
00490     public:
00491         using head = typename inner::head;
00492         using tail = typename inner::tail;
00493     };
00494
00495     template <typename T, size_t index>
00496     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00497
00498     template <size_t index>
00499     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00500 };
00501
00502 template <>
00503 struct type_list<> {
00504     static constexpr size_t length = 0;
00505
00506     template <typename T>
00507     using push_front = type_list<T>;
00508
00509     template <typename T>
00510     using push_back = type_list<T>;
00511
00512     template <typename U>
00513     using concat = U;
00514
00515     // TODO(jewave): assert index == 0
00516     template <typename T, size_t index>
00517     using insert = type_list<T>;
00518 };
00519 } // namespace aerobus
00520
00521 // i32
00522 namespace aerobus {
00523     struct i32 {
00524         using inner_type = int32_t;
00525         template<int32_t x>
00526         struct val {
00527             using enclosing_type = i32;
00528             static constexpr int32_t v = x;
00529
00530             template<typename valueType>
00531             static constexpr valueType get() { return static_cast<valueType>(x); }
00532
00533             using is_zero_t = std::bool_constant<x == 0>;
00534
00535             static std::string to_string() {

```

```

00568         return std::to_string(x);
00569     }
00570
00573     template<typename valueRing>
00574     static constexpr valueRing eval(const valueRing& v) {
00575         return static_cast<valueRing>(x);
00576     }
00577 };
00578
00580 using zero = val<0>;
00582 using one = val<1>;
00584 static constexpr bool is_field = false;
00586 static constexpr bool is_euclidean_domain = true;
00590 template<auto x>
00591 using inject_constant_t = val<static_cast<int32_t>(x)>;
00592
00593 template<typename v>
00594 using inject_ring_t = v;
00595
00596 private:
00597     template<typename v1, typename v2>
00598     struct add {
00599         using type = val<v1::v + v2::v>;
00600     };
00601
00602     template<typename v1, typename v2>
00603     struct sub {
00604         using type = val<v1::v - v2::v>;
00605     };
00606
00607     template<typename v1, typename v2>
00608     struct mul {
00609         using type = val<v1::v * v2::v>;
00610     };
00611
00612     template<typename v1, typename v2>
00613     struct div {
00614         using type = val<v1::v / v2::v>;
00615     };
00616
00617     template<typename v1, typename v2>
00618     struct remainder {
00619         using type = val<v1::v % v2::v>;
00620     };
00621
00622     template<typename v1, typename v2>
00623     struct gt {
00624         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00625     };
00626
00627     template<typename v1, typename v2>
00628     struct lt {
00629         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00630     };
00631
00632     template<typename v1, typename v2>
00633     struct eq {
00634         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00635     };
00636
00637     template<typename v1>
00638     struct pos {
00639         using type = std::bool_constant<(v1::v > 0)>;
00640     };
00641
00642 public:
00643     template<typename v1, typename v2>
00644     using add_t = typename add<v1, v2>::type;
00645
00646     template<typename v1, typename v2>
00647     using sub_t = typename sub<v1, v2>::type;
00648
00649     template<typename v1, typename v2>
00650     using mul_t = typename mul<v1, v2>::type;
00651
00652     template<typename v1, typename v2>
00653     using div_t = typename div<v1, v2>::type;
00654
00655     template<typename v1, typename v2>
00656     using mod_t = typename remainder<v1, v2>::type;
00657
00658     template<typename v1, typename v2>
00659     using gt_t = typename gt<v1, v2>::type;
00660
00661     template<typename v1, typename v2>
00662     using lt_t = typename lt<v1, v2>::type;
00663
00664     template<typename v1, typename v2>
00665     using eq_t = typename eq<v1, v2>::type;
00666
00667     template<typename v1>
00668     using pos_t = typename pos<v1>::type;

```

```

00704     template<typename v1, typename v2>
00705     using eq_t = typename eq<v1, v2>::type;
00706
00711     template<typename v1, typename v2>
00712     static constexpr bool eq_v = eq_t<v1, v2>::value;
00713
00719     template<typename v1, typename v2>
00720     using gcd_t = gcd_t<i32, v1, v2>;
00721
00726     template<typename v>
00727     using pos_t = typename pos<v>::type;
00728
00733     template<typename v>
00734     static constexpr bool pos_v = pos_t<v>::value;
00735 };
00736 } // namespace aerobus
00737
00738 // i64
00739 namespace aerobus {
00741     struct i64 {
00743         using inner_type = int64_t;
00744         template<int64_t x>
00745         struct val {
00747             using enclosing_type = i64;
00748             static constexpr int64_t v = x;
00749
00755             template<typename valueType>
00756             static constexpr valueType get() { return static_cast<valueType>(x); }
00757
00759             using is_zero_t = std::bool_constant<x == 0>;
00760
00762             static std::string to_string() {
00763                 return std::to_string(x);
00764             }
00765
00768             template<typename valueRing>
00769             static constexpr valueRing eval(const valueRing& v) {
00770                 return static_cast<valueRing>(x);
00771             }
00772         };
00773
00777         template<auto x>
00778         using inject_constant_t = val<static_cast<int64_t>(x)>;
00779
00784         template<typename v>
00785         using inject_ring_t = v;
00786
00788         using zero = val<0>;
00789         using one = val<1>;
00792         static constexpr bool is_field = false;
00794         static constexpr bool is_euclidean_domain = true;
00795
00796     private:
00797         template<typename v1, typename v2>
00798         struct add {
00799             using type = val<v1::v + v2::v>;
00800         };
00801
00802         template<typename v1, typename v2>
00803         struct sub {
00804             using type = val<v1::v - v2::v>;
00805         };
00806
00807         template<typename v1, typename v2>
00808         struct mul {
00809             using type = val<v1::v * v2::v>;
00810         };
00811
00812         template<typename v1, typename v2>
00813         struct div {
00814             using type = val<v1::v / v2::v>;
00815         };
00816
00817         template<typename v1, typename v2>
00818         struct remainder {
00819             using type = val<v1::v % v2::v>;
00820         };
00821
00822         template<typename v1, typename v2>
00823         struct gt {
00824             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00825         };
00826
00827         template<typename v1, typename v2>
00828         struct lt {
00829             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00830         };

```



```

00831
00832     template<typename v1, typename v2>
00833     struct eq {
00834         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00835     };
00836
00837     template<typename v>
00838     struct pos {
00839         using type = std::bool_constant<(v::v > 0)>;
00840     };
00841
00842 public:
00843     template<typename v1, typename v2>
00844     using add_t = typename add<v1, v2>::type;
00845
00846     template<typename v1, typename v2>
00847     using sub_t = typename sub<v1, v2>::type;
00848
00849     template<typename v1, typename v2>
00850     using mul_t = typename mul<v1, v2>::type;
00851
00852     template<typename v1, typename v2>
00853     using div_t = typename div<v1, v2>::type;
00854
00855     template<typename v1, typename v2>
00856     using mod_t = typename remainder<v1, v2>::type;
00857
00858     template<typename v1, typename v2>
00859     using gt_t = typename gt<v1, v2>::type;
00860
00861     template<typename v1, typename v2>
00862     static constexpr bool gt_v = gt_t<v1, v2>::value;
00863
00864     template<typename v1, typename v2>
00865     using lt_t = typename lt<v1, v2>::type;
00866
00867     template<typename v1, typename v2>
00868     static constexpr bool lt_v = lt_t<v1, v2>::value;
00869
00870     template<typename v1, typename v2>
00871     using eq_t = typename eq<v1, v2>::type;
00872
00873     template<typename v1, typename v2>
00874     static constexpr bool eq_v = eq_t<v1, v2>::value;
00875
00876     template<typename v1, typename v2>
00877     using gcd_t = gcd_t<i64, v1, v2>;
00878
00879     template<typename v>
00880     using pos_t = typename pos<v>::type;
00881
00882     template<typename v>
00883     static constexpr bool pos_v = pos_t<v>::value;
00884 };
00885 } // namespace aerobus
00886
00887 // z/pz
00888 namespace aerobus {
00889     template<int32_t p>
00890     struct zp {
00891         using inner_type = int32_t;
00892         template<int32_t x>
00893         struct val {
00894             using enclosing_type = zp<p>;
00895             static constexpr int32_t v = x % p;
00896
00897             template<typename valueType>
00898             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00899
00900             using is_zero_t = std::bool_constant<x % p == 0>;
00901             static std::string to_string() {
00902                 return std::to_string(x % p);
00903             }
00904
00905             template<typename valueRing>
00906             static constexpr valueRing eval(const valueRing& v) {
00907                 return static_cast<valueRing>(x % p);
00908             }
00909         };
00910     };
00911
00912     template<auto x>
00913     using inject_constant_t = val<static_cast<int32_t>(x)>;
00914
00915     using zero = val<0>;
00916     using one = val<1>;
00917     static constexpr bool is_prime = is_prime<p>::value;
00918     static constexpr bool is_euclidean_domain = true;
00919 }

```

```

00987
00988     private:
00989         template<typename v1, typename v2>
00990         struct add {
00991             using type = val<(v1::v + v2::v) % p>;
00992         };
00993
00994         template<typename v1, typename v2>
00995         struct sub {
00996             using type = val<(v1::v - v2::v) % p>;
00997         };
00998
00999         template<typename v1, typename v2>
01000         struct mul {
01001             using type = val<(v1::v* v2::v) % p>;
01002         };
01003
01004         template<typename v1, typename v2>
01005         struct div {
01006             using type = val<(v1::v% p) / (v2::v % p)>;
01007         };
01008
01009         template<typename v1, typename v2>
01010         struct remainder {
01011             using type = val<(v1::v% v2::v) % p>;
01012         };
01013
01014         template<typename v1, typename v2>
01015         struct gt {
01016             using type = std::conditional_t<(v1::v% p > v2::v% p), std::true_type, std::false_type>;
01017         };
01018
01019         template<typename v1, typename v2>
01020         struct lt {
01021             using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01022         };
01023
01024         template<typename v1, typename v2>
01025         struct eq {
01026             using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01027         };
01028
01029         template<typename v1>
01030         struct pos {
01031             using type = std::bool_constant<(v1::v > 0)>;
01032         };
01033
01034     public:
01038         template<typename v1, typename v2>
01039         using add_t = typename add<v1, v2>::type;
01040
01044         template<typename v1, typename v2>
01045         using sub_t = typename sub<v1, v2>::type;
01046
01050         template<typename v1, typename v2>
01051         using mul_t = typename mul<v1, v2>::type;
01052
01056         template<typename v1, typename v2>
01057         using div_t = typename div<v1, v2>::type;
01058
01062         template<typename v1, typename v2>
01063         using mod_t = typename remainder<v1, v2>::type;
01064
01068         template<typename v1, typename v2>
01069         using gt_t = typename gt<v1, v2>::type;
01070
01074         template<typename v1, typename v2>
01075         static constexpr bool gt_v = gt_t<v1, v2>::value;
01076
01080         template<typename v1, typename v2>
01081         using lt_t = typename lt<v1, v2>::type;
01082
01086         template<typename v1, typename v2>
01087         static constexpr bool lt_v = lt_t<v1, v2>::value;
01088
01092         template<typename v1, typename v2>
01093         using eq_t = typename eq<v1, v2>::type;
01094
01098         template<typename v1, typename v2>
01099         static constexpr bool eq_v = eq_t<v1, v2>::value;
01100
01104         template<typename v1, typename v2>
01105         using gcd_t = gcd_t<i32, v1, v2>;
01106
01109         template<typename v1>
01110         using pos_t = typename pos<v1>::type;
01111

```

```

01114     template<typename v>
01115     static constexpr bool pos_v = pos_t<v>::value;
01116 };
01117 } // namespace aerobus
01118
01119 // polynomial
01120 namespace aerobus {
01121     // coeffN x^N + ...
01122     template<typename Ring>
01123     requires IsEuclideanDomain<Ring>
01124     struct polynomial {
01125         static constexpr bool is_field = false;
01126         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01127
01128         template<typename coeffN, typename... coeffs>
01129         struct val {
01130             using enclosing_type = polynomial<Ring>;
01131             static constexpr size_t degree = sizeof...(coeffs);
01132             using aN = coeffN;
01133             using strip = val<coeffs...>;
01134             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01135             static constexpr bool is_zero_v = is_zero_t::value;
01136
01137         private:
01138             template<size_t index, typename E = void>
01139             struct coeff_at {};
01140
01141             template<size_t index>
01142             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))>> {
01143                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01144             };
01145
01146             template<size_t index>
01147             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))>> {
01148                 using type = typename Ring::zero;
01149             };
01150
01151         public:
01152             template<size_t index>
01153             using coeff_at_t = typename coeff_at<index>::type;
01154
01155             static std::string to_string() {
01156                 return string_helper<coeffN, coeffs...>::func();
01157             }
01158
01159             template<typename valueRing>
01160             static constexpr valueRing eval(const valueRing& x) {
01161                 return horner_evaluation<valueRing, val>
01162                     ::template inner<0, degree + 1>
01163                     ::func(static_cast<valueRing>(0), x);
01164             }
01165         };
01166     };
01167
01168     template<typename coeffN>
01169     struct val<coeffN> {
01170         using enclosing_type = polynomial<Ring>;
01171         static constexpr size_t degree = 0;
01172         using aN = coeffN;
01173         using strip = val<coeffN>;
01174         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01175         static constexpr bool is_zero_v = is_zero_t::value;
01176
01177         template<size_t index, typename E = void>
01178         struct coeff_at {};
01179
01180         template<size_t index>
01181         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01182             using type = aN;
01183         };
01184
01185         template<size_t index>
01186         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01187             using type = typename Ring::zero;
01188         };
01189
01190         template<size_t index>
01191         using coeff_at_t = typename coeff_at<index>::type;
01192
01193         static std::string to_string() {
01194             return string_helper<coeffN>::func();
01195         }
01196
01197         template<typename valueRing>
01198         static constexpr valueRing eval(const valueRing& x) {
01199             return static_cast<valueRing>(aN::template get<valueRing>());
01200         }
01201     };
01202 }

```

```

01226     };
01227
01229     using zero = val<typename Ring::zero>;
01231     using one = val<typename Ring::one>;
01233     using X = val<typename Ring::one, typename Ring::zero>;
01234
01235 private:
01236     template<typename P, typename E = void>
01237     struct simplify;
01238
01239     template<typename P1, typename P2, typename I>
01240     struct add_low;
01241
01242     template<typename P1, typename P2>
01243     struct add {
01244         using type = typename simplify<typename add_low<
01245             P1,
01246             P2,
01247             internal::make_index_sequence_reverse<
01248                 std::max(P1::degree, P2::degree) + 1
01249             >::type>::type;
01250     };
01251
01252     template<typename P1, typename P2, typename I>
01253     struct sub_low;
01254
01255     template<typename P1, typename P2, typename I>
01256     struct mul_low;
01257
01258     template<typename v1, typename v2>
01259     struct mul {
01260         using type = typename mul_low<
01261             v1,
01262             v2,
01263             internal::make_index_sequence_reverse<
01264                 v1::degree + v2::degree + 1
01265             >::type;
01266     };
01267
01268     template<typename coeff, size_t deg>
01269     struct monomial;
01270
01271     template<typename v, typename E = void>
01272     struct derive_helper {};
01273
01274     template<typename v>
01275     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01276         using type = zero;
01277     };
01278
01279     template<typename v>
01280     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01281         using type = typename add<
01282             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01283             typename monomial<
01284                 typename Ring::template mul_t<
01285                     typename v::aN,
01286                     typename Ring::template inject_constant_t<(v::degree)>
01287                 >,
01288                 v::degree - 1
01289             >::type
01290         >::type;
01291     };
01292
01293     template<typename v1, typename v2, typename E = void>
01294     struct eq_helper {};
01295
01296     template<typename v1, typename v2>
01297     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01298         using type = std::false_type;
01299     };
01300
01301     template<typename v1, typename v2>
01302     struct eq_helper<v1, v2, std::enable_if_t<
01303         v1::degree == v2::degree &&
01304         (v1::degree != 0 || v2::degree != 0) &&
01305         std::is_same<
01306             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01307             std::false_type
01308         >::value
01309     > {
01310     > {
01311         using type = std::false_type;
01312     };
01313
01314     template<typename v1, typename v2>

```

```

01316     struct eq_helper<v1, v2, std::enable_if_t<
01317         v1::degree == v2::degree &&
01318         (v1::degree != 0 || v2::degree != 0) &&
01319         std::is_same<
01320             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01321             std::true_type
01322         >::value
01323     > {
01324         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01325     };
01326
01327     template<typename v1, typename v2>
01328     struct eq_helper<v1, v2, std::enable_if_t<
01329         v1::degree == v2::degree &&
01330         (v1::degree == 0)
01331     > {
01332         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01333     };
01334
01335     template<typename v1, typename v2, typename E = void>
01336     struct lt_helper {};
01337
01338     template<typename v1, typename v2>
01339     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01340         using type = std::true_type;
01341     };
01342
01343     template<typename v1, typename v2>
01344     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01345         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01346     };
01347
01348     template<typename v1, typename v2>
01349     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01350         using type = std::false_type;
01351     };
01352
01353     template<typename v1, typename v2, typename E = void>
01354     struct gt_helper {};
01355
01356     template<typename v1, typename v2>
01357     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01358         using type = std::true_type;
01359     };
01360
01361     template<typename v1, typename v2>
01362     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01363         using type = std::false_type;
01364     };
01365
01366     template<typename v1, typename v2>
01367     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01368         using type = std::false_type;
01369     };
01370
01371     // when high power is zero : strip
01372     template<typename P>
01373     struct simplify<P, std::enable_if_t<
01374         std::is_same<
01375             typename Ring::zero,
01376             typename P::aN
01377         >::value && (P::degree > 0)
01378     > {
01379         using type = typename simplify<typename P::strip>::type;
01380     };
01381
01382     // otherwise : do nothing
01383     template<typename P>
01384     struct simplify<P, std::enable_if_t<
01385         !std::is_same<
01386             typename Ring::zero,
01387             typename P::aN
01388         >::value && (P::degree > 0)
01389     > {
01390         using type = P;
01391     };
01392
01393     // do not simplify constants
01394     template<typename P>
01395     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01396         using type = P;
01397     };
01398
01399     // addition at
01400     template<typename P1, typename P2, size_t index>
01401     struct add_at {
01402         using type =

```

```

01403         typename Ring::template add_t<
01404             typename P1::template coeff_at_t<index>,
01405             typename P2::template coeff_at_t<index>>;
01406     };
01407
01408     template<typename P1, typename P2, size_t index>
01409     using add_at_t = typename add_at<P1, P2, index>::type;
01410
01411     template<typename P1, typename P2, std::size_t... I>
01412     struct add_low<P1, P2, std::index_sequence<I...> {
01413         using type = val<add_at_t<P1, P2, I>...>;
01414     };
01415
01416     // subtraction at
01417     template<typename P1, typename P2, size_t index>
01418     struct sub_at {
01419         using type =
01420             typename Ring::template sub_t<
01421                 typename P1::template coeff_at_t<index>,
01422                 typename P2::template coeff_at_t<index>>;
01423     };
01424
01425     template<typename P1, typename P2, size_t index>
01426     using sub_at_t = typename sub_at<P1, P2, index>::type;
01427
01428     template<typename P1, typename P2, std::size_t... I>
01429     struct sub_low<P1, P2, std::index_sequence<I...> {
01430         using type = val<sub_at_t<P1, P2, I>...>;
01431     };
01432
01433     template<typename P1, typename P2>
01434     struct sub {
01435         using type = typename simplify<typename sub_low<
01436             P1,
01437             P2,
01438             internal::make_index_sequence_reverse<
01439                 std::max(P1::degree, P2::degree) + 1
01440             >::type>::type;
01441     };
01442
01443     // multiplication at
01444     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01445     struct mul_at_loop_helper {
01446         using type = typename Ring::template add_t<
01447             typename Ring::template mul_t<
01448                 typename v1::template coeff_at_t<index>,
01449                 typename v2::template coeff_at_t<k - index>
01450             >,
01451             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01452         >;
01453     };
01454
01455     template<typename v1, typename v2, size_t k, size_t stop>
01456     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01457         using type = typename Ring::template mul_t<
01458             typename v1::template coeff_at_t<stop>,
01459             typename v2::template coeff_at_t<0>>;
01460     };
01461
01462     template<typename v1, typename v2, size_t k, typename E = void>
01463     struct mul_at {};
01464
01465     template<typename v1, typename v2, size_t k>
01466     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01467         using type = typename Ring::zero;
01468     };
01469
01470     template<typename v1, typename v2, size_t k>
01471     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01472         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01473     };
01474
01475     template<typename P1, typename P2, size_t index>
01476     using mul_at_t = typename mul_at<P1, P2, index>::type;
01477
01478     template<typename P1, typename P2, std::size_t... I>
01479     struct mul_low<P1, P2, std::index_sequence<I...> {
01480         using type = val<mul_at_t<P1, P2, I>...>;
01481     };
01482
01483     // division helper
01484     template<typename A, typename B, typename Q, typename R, typename E = void>
01485     struct div_helper {};
01486
01487     template<typename A, typename B, typename Q, typename R>
01488     struct div_helper<A, B, Q, R, std::enable_if_t<
01489         (R::degree < B::degree) ||

```

```

01490         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01491             using q_type = Q;
01492             using mod_type = R;
01493             using gcd_type = B;
01494         };
01495
01496         template<typename A, typename B, typename Q, typename R>
01497         struct div_helper<A, B, Q, R, std::enable_if_t<
01498             (R::degree >= B::degree) &&
01499             !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01500             private: // NOLINT
01501                 using rN = typename R::aN;
01502                 using bN = typename B::aN;
01503                 using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01504                     B::degree>::type;
01505                 using rr = typename sub<R, typename mul<pT, B>::type>::type;
01506                 using qq = typename add<Q, pT>::type;
01507             public:
01508                 using q_type = typename div_helper<A, B, qq, rr>::q_type;
01509                 using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01510                 using gcd_type = rr;
01511         };
01512
01513         template<typename A, typename B>
01514         struct div {
01515             static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01516             using q_type = typename div_helper<A, B, zero, A>::q_type;
01517             using m_type = typename div_helper<A, B, zero, A>::mod_type;
01518         };
01519
01520         template<typename P>
01521         struct make_unit {
01522             using type = typename div<P, val<typename P::aN>>::q_type;
01523         };
01524
01525         template<typename coeff, size_t deg>
01526         struct monomial {
01527             using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01528         };
01529
01530         template<typename coeff>
01531         struct monomial<coeff, 0> {
01532             using type = val<coeff>;
01533         };
01534
01535         template<typename valueRing, typename P>
01536         struct horner_evaluation {
01537             template<size_t index, size_t stop>
01538             struct inner {
01539                 static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01540                     constexpr valueRing coeff =
01541                         static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01542                             get<valueRing>());
01543                     return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01544                         accum + coeff, x);
01545                 }
01546             };
01547             template<size_t stop>
01548             struct inner<stop, stop> {
01549                 static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01550                     return accum;
01551                 }
01552             };
01553         };
01554
01555         template<typename coeff, typename... coeffs>
01556         struct string_helper {
01557             static std::string func() {
01558                 std::string tail = string_helper<coeffs...>::func();
01559                 std::string result = "";
01560                 if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01561                     return tail;
01562                 } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01563                     if (sizeof...(coeffs) == 1) {
01564                         result += "x";
01565                     } else {
01566                         result += "x^" + std::to_string(sizeof...(coeffs));
01567                     }
01568                 } else {
01569                     if (sizeof...(coeffs) == 1) {
01570                         result += coeff::to_string() + " x";
01571                     } else {
01572                         result += coeff::to_string()
01573                             + " x^" + std::to_string(sizeof...(coeffs));
01574                     }
01575                 }
01576             }
01577         };

```

```

01574         }
01575
01576         if (!tail.empty()) {
01577             result += " + " + tail;
01578         }
01579
01580         return result;
01581     }
01582 };
01583
01584 template<typename coeff>
01585 struct string_helper<coeff> {
01586     static std::string func() {
01587         if (!std::is_same<coeff, typename Ring::zero>::value) {
01588             return coeff::to_string();
01589         } else {
01590             return "";
01591         }
01592     }
01593 };
01594
01595 public:
01596     template<typename P>
01597     using simplify_t = typename simplify<P>::type;
01598
01599     template<typename v1, typename v2>
01600     using add_t = typename add<v1, v2>::type;
01601
01602     template<typename v1, typename v2>
01603     using sub_t = typename sub<v1, v2>::type;
01604
01605     template<typename v1, typename v2>
01606     using mul_t = typename mul<v1, v2>::type;
01607
01608     template<typename v1, typename v2>
01609     using eq_t = typename eq_helper<v1, v2>::type;
01610
01611     template<typename v1, typename v2>
01612     using lt_t = typename lt_helper<v1, v2>::type;
01613
01614     template<typename v1, typename v2>
01615     using gt_t = typename gt_helper<v1, v2>::type;
01616
01617     template<typename v1, typename v2>
01618     using div_t = typename div<v1, v2>::q_type;
01619
01620     template<typename v1, typename v2>
01621     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01622
01623     template<typename coeff, size_t deg>
01624     using monomial_t = typename monomial<coeff, deg>::type;
01625
01626     template<typename v>
01627     using derive_t = typename derive_helper<v>::type;
01628
01629     template<typename v>
01630     using pos_t = typename Ring::template pos_t<typename v::aN>;
01631
01632     template<typename v>
01633     static constexpr bool pos_v = pos_t<v>::value;
01634
01635     template<typename v1, typename v2>
01636     using gcd_t = std::conditional_t<
01637         Ring::is_euclidean_domain,
01638         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01639         void>;
01640
01641     template<auto x>
01642     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01643
01644     template<typename v>
01645     using inject_ring_t = val<v>;
01646 };
01647 } // namespace aerobus
01648
01649 // fraction field
01650 namespace aerobus {
01651     namespace internal {
01652         template<typename Ring, typename E = void>
01653         requires IsEuclideanDomain<Ring>
01654         struct _FractionField {};
01655
01656         template<typename Ring>
01657         requires IsEuclideanDomain<Ring>
01658         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01659             static constexpr bool is_field = true;
01660             static constexpr bool is_euclidean_domain = true;
01661         };
01662     }
01663 }

```



```

01706
01707 private:
01708     template<typename val1, typename val2, typename E = void>
01709     struct to_string_helper {};
01710
01711     template<typename val1, typename val2>
01712     struct to_string_helper <val1, val2,
01713         std::enable_if_t<
01714             Ring::template eq_t<
01715                 val2, typename Ring::one
01716             >::value
01717         >
01718     > {
01719         static std::string func() {
01720             return val1::to_string();
01721         }
01722     };
01723
01724     template<typename val1, typename val2>
01725     struct to_string_helper<val1, val2,
01726         std::enable_if_t<
01727             !Ring::template eq_t<
01728                 val2,
01729                 typename Ring::one
01730             >::value
01731         >
01732     > {
01733         static std::string func() {
01734             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01735         }
01736     };
01737
01738 public:
01742     template<typename val1, typename val2>
01743     struct val {
01744         using x = val1;
01745         using y = val2;
01746         using is_zero_t = typename val1::is_zero_t;
01747         static constexpr bool is_zero_v = val1::is_zero_t::value;
01748
01749         using ring_type = Ring;
01750         using enclosing_type = _FractionField<Ring>;
01751
01752         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01753
01754         template<typename valueType>
01755         static constexpr valueType get() { return static_cast<valueType>(x::v) /
01756             static_cast<valueType>(y::v); }
01757
01758         static std::string to_string() {
01759             return to_string_helper<val1, val2>::func();
01760         }
01761
01762         template<typename valueRing>
01763         static constexpr valueRing eval(const valueRing& v) {
01764             return x::eval(v) / y::eval(v);
01765         }
01766     };
01767
01768     using zero = val<typename Ring::zero, typename Ring::one>;
01769     using one = val<typename Ring::one, typename Ring::one>;
01770
01771     template<typename v>
01772     using inject_t = val<v, typename Ring::one>;
01773
01774     template<auto x>
01775     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01776     Ring::one>;
01777
01778     template<typename v>
01779     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01780
01781     using ring_type = Ring;
01782
01783 private:
01784     template<typename v, typename E = void>
01785     struct simplify {};
01786
01787     // x = 0
01788     template<typename v>
01789     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> > {
01790         using type = typename _FractionField<Ring>::zero;
01791     };
01792
01793     // x != 0
01794     template<typename v>
01795     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> > {

```

```

01819     private:
01820         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01821         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01822         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01823
01824         using posx = std::conditional_t<
01825             !Ring::template pos_v<newy>,
01826             typename Ring::template sub_t<typename Ring::zero, newx>,
01827             newx>;
01828         using posy = std::conditional_t<
01829             !Ring::template pos_v<newy>,
01830             typename Ring::template sub_t<typename Ring::zero, newy>,
01831             newy>;
01832     public:
01833         using type = typename _FractionField<Ring>::template val<posx, posy>;
01834     };
01835
01836 public:
01837     template<typename v>
01838     using simplify_t = typename simplify<v>::type;
01839
01840 private:
01841     template<typename v1, typename v2>
01842     struct add {
01843     private:
01844         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01845         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01846         using dividend = typename Ring::template add_t<a, b>;
01847         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01848         using g = typename Ring::template gcd_t<dividend, diviser>;
01849
01850     public:
01851         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01852             dividend>, diviser>;
01853     };
01854
01855     template<typename v>
01856     struct pos {
01857     private:
01858         using type = std::conditional_t<
01859             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01860             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01861             std::true_type,
01862             std::false_type>;
01863     };
01864
01865     template<typename v1, typename v2>
01866     struct sub {
01867     private:
01868         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01869         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01870         using dividend = typename Ring::template sub_t<a, b>;
01871         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01872         using g = typename Ring::template gcd_t<dividend, diviser>;
01873
01874     public:
01875         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01876             dividend>, diviser>;
01877     };
01878
01879     template<typename v1, typename v2>
01880     struct mul {
01881     private:
01882         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01883         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01884
01885     public:
01886         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01887     };
01888
01889     template<typename v1, typename v2, typename E = void>
01890     struct div {};
01891
01892     template<typename v1, typename v2>
01893     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
01894         _FractionField<Ring>::zero>::value> > {
01895     private:
01896         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01897         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01898
01899     public:
01900         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01901     };
01902
01903     template<typename v1, typename v2>
01904     struct div<v1, v2, std::enable_if_t<
01905         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> > {
01906         using type = one;
01907     };

```

```

01905         };
01906
01907     template<typename v1, typename v2>
01908     struct eq {
01909         using type = std::conditional_t<
01910             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01911             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01912             std::true_type,
01913             std::false_type>;
01914     };
01915
01916     template<typename v1, typename v2, typename E = void>
01917     struct gt;
01918
01919     template<typename v1, typename v2>
01920     struct gt<v1, v2, std::enable_if_t<
01921         (eq<v1, v2>::type::value)
01922         >> {
01923         using type = std::false_type;
01924     };
01925
01926     template<typename v1, typename v2>
01927     struct gt<v1, v2, std::enable_if_t<
01928         (!eq<v1, v2>::type::value) &&
01929         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01930         >> {
01931         using type = typename gt<
01932             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01933             >::type;
01934     };
01935
01936     template<typename v1, typename v2>
01937     struct gt<v1, v2, std::enable_if_t<
01938         (!eq<v1, v2>::type::value) &&
01939         (pos<v1>::type::value) && (!pos<v2>::type::value)
01940         >> {
01941         using type = std::true_type;
01942     };
01943
01944     template<typename v1, typename v2>
01945     struct gt<v1, v2, std::enable_if_t<
01946         (!eq<v1, v2>::type::value) &&
01947         (!pos<v1>::type::value) && (pos<v2>::type::value)
01948         >> {
01949         using type = std::false_type;
01950     };
01951
01952     template<typename v1, typename v2>
01953     struct gt<v1, v2, std::enable_if_t<
01954         (!eq<v1, v2>::type::value) &&
01955         (pos<v1>::type::value) && (pos<v2>::type::value)
01956         >> {
01957         using type = typename Ring::template gt_t<
01958             typename Ring::template mul_t<v1::x, v2::y>,
01959             typename Ring::template mul_t<v2::y, v2::x>
01960             >;
01961     };
01962
01963     public:
01964     template<typename v1, typename v2>
01965     using add_t = typename add<v1, v2>::type;
01966
01967     template<typename v1, typename v2>
01968     using mod_t = zero;
01969
01970     template<typename v1, typename v2>
01971     using gcd_t = v1;
01972
01973     template<typename v1, typename v2>
01974     using sub_t = typename sub<v1, v2>::type;
01975
01976     template<typename v1, typename v2>
01977     using mul_t = typename mul<v1, v2>::type;
01978
01979     template<typename v1, typename v2>
01980     using div_t = typename div<v1, v2>::type;
01981
01982     template<typename v1, typename v2>
01983     using eq_t = typename eq<v1, v2>::type;
01984
01985     template<typename v1, typename v2>
01986     static constexpr bool eq_v = eq<v1, v2>::type::value;
01987
01988     template<typename v1, typename v2>
01989     using gt_t = typename gt<v1, v2>::type;
01990
01991     template<typename v1, typename v2>

```

```

02025         static constexpr bool gt_v = gt<v1, v2>::type::value;
02026
02029         template<typename v1>
02030         using pos_t = typename pos<v1>::type;
02031
02034         template<typename v>
02035         static constexpr bool pos_v = pos_t<v>::value;
02036     };
02037
02038     template<typename Ring, typename E = void>
02039     requires IsEuclideanDomain<Ring>
02040     struct FractionFieldImpl {};
02041
02042     // fraction field of a field is the field itself
02043     template<typename Field>
02044     requires IsEuclideanDomain<Field>
02045     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02046         using type = Field;
02047         template<typename v>
02048         using inject_t = v;
02049     };
02050
02051     // fraction field of a ring is the actual fraction field
02052     template<typename Ring>
02053     requires IsEuclideanDomain<Ring>
02054     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02055         using type = _FractionField<Ring>;
02056     };
02057 } // namespace internal
02058
02062     template<typename Ring>
02063     requires IsEuclideanDomain<Ring>
02064     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02065 } // namespace aerobus
02066
02067 // short names for common types
02068 namespace aerobus {
02071     using q32 = FractionField<i32>;
02074     using fpq32 = FractionField<polynomial<q32>;
02077     using q64 = FractionField<i64>;
02079     using pi64 = polynomial<i64>;
02081     using pq64 = polynomial<q64>;
02083     using fpq64 = FractionField<polynomial<q64>;
02088     template<typename Ring, typename v1, typename v2>
02089     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02090
02094     template<int64_t p, int64_t q>
02095     using make_q64_t = typename q64::template simplify_t<
02096         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>;
02097
02101     template<int32_t p, int32_t q>
02102     using make_q32_t = typename q32::template simplify_t<
02103         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>;
02104
02109     template<typename Ring, typename v1, typename v2>
02110     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02115     template<typename Ring, typename v1, typename v2>
02116     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02117 } // namespace aerobus
02118
02119 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02120 namespace aerobus {
02121     namespace internal {
02122         template<typename T, size_t x, typename E = void>
02123         struct factorial {};
02124
02125         template<typename T, size_t x>
02126         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02127         private:
02128             template<typename, size_t, typename>
02129             friend struct factorial;
02130         public:
02131             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02132 x - 1>::type>;
02132             static constexpr typename T::inner_type value = type::template get<typename
02133 T::inner_type>();
02134         };
02135
02136         template<typename T>
02137         struct factorial<T, 0> {
02138         public:
02139             using type = typename T::one;
02139             static constexpr typename T::inner_type value = type::template get<typename
02140 T::inner_type>();
02141         };
02141     } // namespace internal
02142

```

```

02146     template<typename T, size_t i>
02147     using factorial_t = typename internal::factorial<T, i>::type;
02148
02152     template<typename T, size_t i>
02153     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02154
02155     namespace internal {
02156         template<typename T, size_t k, size_t n, typename E = void>
02157         struct combination_helper {};
02158
02159         template<typename T, size_t k, size_t n>
02160         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02161             using type = typename FractionField<T>::template mul_t<
02162                 typename combination_helper<T, k - 1, n - 1>::type,
02163                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02164         };
02165
02166         template<typename T, size_t k, size_t n>
02167         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02168             using type = typename combination_helper<T, n - k, n>::type;
02169         };
02170
02171         template<typename T, size_t n>
02172         struct combination_helper<T, 0, n> {
02173             using type = typename FractionField<T>::one;
02174         };
02175
02176         template<typename T, size_t k, size_t n>
02177         struct combination {
02178             using type = typename internal::combination_helper<T, k, n>::type::x;
02179             static constexpr typename T::inner_type value =
02180                 internal::combination_helper<T, k, n>::type::template get<typename
02181                 T::inner_type>();
02182         };
02183     } // namespace internal
02184
02186     template<typename T, size_t k, size_t n>
02187     using combination_t = typename internal::combination<T, k, n>::type;
02188
02193     template<typename T, size_t k, size_t n>
02194     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02195
02196     namespace internal {
02197         template<typename T, size_t m>
02198         struct bernoulli;
02199
02200         template<typename T, typename accum, size_t k, size_t m>
02201         struct bernoulli_helper {
02202             using type = typename bernoulli_helper<
02203                 T,
02204                 addfractions_t<T,
02205                     accum,
02206                     mulfractions_t<T,
02207                         makefraction_t<T,
02208                             combination_t<T, k, m + 1>,
02209                             typename T::one>,
02210                             typename bernoulli<T, k>::type
02211                         >,
02212                     >,
02213                     k + 1,
02214                     m>::type;
02215         };
02216
02217         template<typename T, typename accum, size_t m>
02218         struct bernoulli_helper<T, accum, m, m> {
02219             using type = accum;
02220         };
02221
02222
02223
02224         template<typename T, size_t m>
02225         struct bernoulli {
02226             using type = typename FractionField<T>::template mul_t<
02227                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02228                 makefraction_t<T,
02229                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02230                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02231                 >
02232             >;
02233
02234             template<typename floatType>
02235             static constexpr floatType value = type::template get<floatType>();
02236         };
02237
02238         template<typename T>
02239         struct bernoulli<T, 0> {
02240             using type = typename FractionField<T>::one;

```

```

02241
02242     template<typename floatType>
02243     static constexpr floatType value = type::template get<floatType>();
02244 };
02245 } // namespace internal
02246
02250 template<typename T, size_t n>
02251 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02252
02257 template<typename FloatType, typename T, size_t n>
02258 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02259
02260 namespace internal {
02261     template<typename T, int k, typename E = void>
02262     struct alternate {};
02263
02264     template<typename T, int k>
02265     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02266         using type = typename T::one;
02267         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02268     };
02269
02270     template<typename T, int k>
02271     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02272         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02273         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02274     };
02275 } // namespace internal
02276
02279 template<typename T, int k>
02280 using alternate_t = typename internal::alternate<T, k>::type;
02281
02282 namespace internal {
02283     template<typename T, int n, int k, typename E = void>
02284     struct stirling_helper {};
02285
02286     template<typename T>
02287     struct stirling_helper<T, 0, 0> {
02288         using type = typename T::one;
02289     };
02290
02291     template<typename T, int n>
02292     struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02293         using type = typename T::zero;
02294     };
02295
02296     template<typename T, int n>
02297     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02298         using type = typename T::zero;
02299     };
02300
02301     template<typename T, int n, int k>
02302     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02303         using type = typename T::template sub_t<
02304             typename stirling_helper<T, n-1, k-1>::type,
02305             typename T::template mul_t<
02306                 typename T::template inject_constant_t<n-1>,
02307                 typename stirling_helper<T, n-1, k>::type
02308             >;
02309     };
02310 } // namespace internal
02311
02316 template<typename T, int n, int k>
02317 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02318
02323 template<typename T, int n, int k>
02324 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02325
02330 template<typename T, int n, int k>
02331 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02332
02333
02338 template<typename T, int n, int k>
02339 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02340
02343 template<typename T, size_t k>
02344 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02345
02346 namespace internal {
02347     template<typename T, auto p, auto n, typename E = void>
02348     struct pow {};
02349
02350     template<typename T, auto p, auto n>
02351     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02352         using type = typename T::template mul_t<

```

```

02353         typename pow<T, p, n/2>::type,
02354         typename pow<T, p, n/2>::type
02355     >;
02356 };
02357
02358 template<typename T, auto p, auto n>
02359 struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
02360     using type = typename T::template mul_t<
02361         typename T::template inject_constant_t<p>,
02362         typename T::template mul_t<
02363             typename pow<T, p, n/2>::type,
02364             typename pow<T, p, n/2>::type
02365         >
02366     >;
02367 };
02368
02369 template<typename T, auto p>
02370 struct pow<T, p, 0> { using type = typename T::one; };
02371 } // namespace internal
02372
02373 template<typename T, auto p, auto n>
02374 using pow_t = typename internal::pow<T, p, n>::type;
02375
02376 template<typename T, auto p, auto n>
02377 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02378
02379 namespace internal {
02380     template<typename, template<typename, size_t> typename, class>
02381     struct make_taylor_impl;
02382
02383     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02384     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...>> {
02385         using type = typename polynomial<FractionField<T>>::template val<typename coeff_at<T,
02386         Is>::type...>;
02387     };
02388 }
02389
02390 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02391 using taylor = typename internal::make_taylor_impl<
02392     T,
02393     coeff_at,
02394     internal::make_index_sequence_reverse<deg + 1>::type>;
02395
02396 namespace internal {
02397     template<typename T, size_t i>
02398     struct exp_coeff {
02399         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02400     };
02401
02402     template<typename T, size_t i, typename E = void>
02403     struct sin_coeff_helper {};
02404
02405     template<typename T, size_t i>
02406     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0>> {
02407         using type = typename FractionField<T>::zero;
02408     };
02409
02410     template<typename T, size_t i>
02411     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1>> {
02412         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02413     };
02414
02415     template<typename T, size_t i>
02416     struct sin_coeff {
02417         using type = typename sin_coeff_helper<T, i>::type;
02418     };
02419
02420     template<typename T, size_t i, typename E = void>
02421     struct sh_coeff_helper {};
02422
02423     template<typename T, size_t i>
02424     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0>> {
02425         using type = typename FractionField<T>::zero;
02426     };
02427
02428     template<typename T, size_t i>
02429     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1>> {
02430         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02431     };
02432
02433     template<typename T, size_t i>
02434     struct sh_coeff {
02435         using type = typename sh_coeff_helper<T, i>::type;
02436     };
02437
02438     template<typename T, size_t i, typename E = void>
02439     struct cos_coeff_helper {};
02440
02441     template<typename T, size_t i, typename E = void>
02442     struct cos_coeff_helper {};
02443
02444     template<typename T, size_t i, typename E = void>
02445     struct cos_coeff_helper {};
02446
02447     template<typename T, size_t i, typename E = void>
02448     struct cos_coeff_helper {};
02449
02450     template<typename T, size_t i, typename E = void>
02451     struct cos_coeff_helper {};

```

```

02451
02452     template<typename T, size_t i>
02453     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02454         using type = typename FractionField<T>::zero;
02455     };
02456
02457     template<typename T, size_t i>
02458     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02459         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02460     };
02461
02462     template<typename T, size_t i>
02463     struct cos_coeff {
02464         using type = typename cos_coeff_helper<T, i>::type;
02465     };
02466
02467     template<typename T, size_t i, typename E = void>
02468     struct cosh_coeff_helper {};
02469
02470     template<typename T, size_t i>
02471     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02472         using type = typename FractionField<T>::zero;
02473     };
02474
02475     template<typename T, size_t i>
02476     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02477         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02478     };
02479
02480     template<typename T, size_t i>
02481     struct cosh_coeff {
02482         using type = typename cosh_coeff_helper<T, i>::type;
02483     };
02484
02485     template<typename T, size_t i>
02486     struct geom_coeff { using type = typename FractionField<T>::one; };
02487
02488
02489     template<typename T, size_t i, typename E = void>
02490     struct atan_coeff_helper;
02491
02492     template<typename T, size_t i>
02493     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02494         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02495     };
02496
02497     template<typename T, size_t i>
02498     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02499         using type = typename FractionField<T>::zero;
02500     };
02501
02502     template<typename T, size_t i>
02503     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02504
02505     template<typename T, size_t i, typename E = void>
02506     struct asin_coeff_helper;
02507
02508     template<typename T, size_t i>
02509     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02510         using type = makefraction_t<T,
02511             factorial_t<T, i - 1>,
02512             typename T::template mul_t<
02513                 typename T::template val<i>,
02514                 T::template mul_t<
02515                     pow_t<T, 4, i / 2>,
02516                     pow<T, factorial<T, i / 2>::value, 2
02517                 >
02518             >
02519         >>;
02520     };
02521
02522     template<typename T, size_t i>
02523     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02524         using type = typename FractionField<T>::zero;
02525     };
02526
02527     template<typename T, size_t i>
02528     struct asin_coeff {
02529         using type = typename asin_coeff_helper<T, i>::type;
02530     };
02531
02532     template<typename T, size_t i>
02533     struct lnpl_coeff {
02534         using type = makefraction_t<T,
02535             alternate_t<T, i + 1>,
02536             typename T::template val<i>;
02537     };

```



```

02538
02539     template<typename T>
02540     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02541
02542     template<typename T, size_t i, typename E = void>
02543     struct asinh_coeff_helper;
02544
02545     template<typename T, size_t i>
02546     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02547         using type = makefraction_t<T,
02548             typename T::template mul_t<
02549                 alternate_t<T, i / 2>,
02550                 factorial_t<T, i - 1>
02551             >,
02552             typename T::template mul_t<
02553                 T::template mul_t<
02554                     typename T::template val<i>,
02555                     pow_t<T, (factorial<T, i / 2>::value), 2>
02556                 >,
02557                 pow_t<T, 4, i / 2>
02558             >
02559         >;
02560     };
02561
02562     template<typename T, size_t i>
02563     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02564         using type = typename FractionField<T>::zero;
02565     };
02566
02567     template<typename T, size_t i>
02568     struct asinh_coeff {
02569         using type = typename asinh_coeff_helper<T, i>::type;
02570     };
02571
02572     template<typename T, size_t i, typename E = void>
02573     struct atanh_coeff_helper;
02574
02575     template<typename T, size_t i>
02576     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02577         // 1/i
02578         using type = typename FractionField<T>::template val<
02579             typename T::one,
02580             typename T::template val<static_cast<typename T::inner_type>(i)>
02581         >;
02582     };
02583
02584     template<typename T, size_t i>
02585     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02586         using type = typename FractionField<T>::zero;
02587     };
02588
02589     template<typename T, size_t i>
02590     struct atanh_coeff {
02591         using type = typename asinh_coeff_helper<T, i>::type;
02592     };
02593
02594     template<typename T, size_t i, typename E = void>
02595     struct tan_coeff_helper;
02596
02597     template<typename T, size_t i>
02598     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02599         using type = typename FractionField<T>::zero;
02600     };
02601
02602     template<typename T, size_t i>
02603     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02604     private:
02605         // 4^((i+1)/2)
02606         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02607         // 4^((i+1)/2) - 1
02608         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
02609             FractionField<T>::one>;
02610         // (-1)^((i-1)/2)
02611         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02612         using dividend = typename FractionField<T>::template mul_t<
02613             altp,
02614             FractionField<T>::template mul_t<
02615                 _4p,
02616                 FractionField<T>::template mul_t<
02617                     _4pml,
02618                     bernoulli_t<T, (i + 1)>
02619                 >
02620             >
02621         >;
02622     public:
02623         using type = typename FractionField<T>::template div_t<dividend,
02624             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02625     };

```

```

02624
02625     template<typename T, size_t i>
02626     struct tan_coeff {
02627         using type = typename tan_coeff_helper<T, i>::type;
02628     };
02629
02630     template<typename T, size_t i, typename E = void>
02631     struct tanh_coeff_helper;
02632
02633     template<typename T, size_t i>
02634     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02635         using type = typename FractionField<T>::zero;
02636     };
02637
02638     template<typename T, size_t i>
02639     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02640     private:
02641         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02642         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02643         using dividend =
02644             typename FractionField<T>::template mul_t<
02645                 _4p,
02646                 typename FractionField<T>::template mul_t<
02647                     _4pml,
02648                     bernoulli_t<T, (i + 1)>::type;
02649     public:
02650         using type = typename FractionField<T>::template div_t<dividend,
02651             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02652     };
02653
02654     template<typename T, size_t i>
02655     struct tanh_coeff {
02656         using type = typename tanh_coeff_helper<T, i>::type;
02657     };
02658 } // namespace internal
02659
02660 template<typename T, size_t deg>
02661 using exp = taylor<T, internal::exp_coeff, deg>;
02662
02663 template<typename T, size_t deg>
02664 using expml = typename polynomial<FractionField<T>>::template sub_t<
02665     exp<T, deg>,
02666     typename polynomial<FractionField<T>>::one>;
02667
02668 template<typename T, size_t deg>
02669 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02670
02671 template<typename T, size_t deg>
02672 using atan = taylor<T, internal::atan_coeff, deg>;
02673
02674 template<typename T, size_t deg>
02675 using sin = taylor<T, internal::sin_coeff, deg>;
02676
02677 template<typename T, size_t deg>
02678 using sinh = taylor<T, internal::sh_coeff, deg>;
02679
02680 template<typename T, size_t deg>
02681 using cosh = taylor<T, internal::cosh_coeff, deg>;
02682
02683 template<typename T, size_t deg>
02684 using cos = taylor<T, internal::cos_coeff, deg>;
02685
02686 template<typename T, size_t deg>
02687 using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02688
02689 template<typename T, size_t deg>
02690 using asin = taylor<T, internal::asin_coeff, deg>;
02691
02692 template<typename T, size_t deg>
02693 using asinh = taylor<T, internal::asinh_coeff, deg>;
02694
02695 template<typename T, size_t deg>
02696 using atanh = taylor<T, internal::atanh_coeff, deg>;
02697
02698 template<typename T, size_t deg>
02699 using tan = taylor<T, internal::tan_coeff, deg>;
02700
02701 template<typename T, size_t deg>
02702 using tanh = taylor<T, internal::tanh_coeff, deg>;
02703 } // namespace aerobus
02704
02705 // continued fractions
02706 namespace aerobus {
02707     template<int64_t... values>
02708     struct ContinuedFraction {};
02709 }

```

```

02756     template<int64_t a0>
02757     struct ContinuedFraction<a0> {
02758         using type = typename q64::template inject_constant_t<a0>;
02759         static constexpr double val = type::template get<double>();
02760     };
02761
02762     template<int64_t a0, int64_t... rest>
02763     struct ContinuedFraction<a0, rest...> {
02764         using type = q64::template add_t<
02765             typename q64::template inject_constant_t<a0>,
02766             typename q64::template div_t<
02767                 typename q64::one,
02768                 typename ContinuedFraction<rest...>::type
02769             >;
02770         static constexpr double val = type::template get<double>();
02771     };
02772
02773     using PI_fraction =
02774     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02775
02776     using E_fraction =
02777     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02778
02779     using SQRT2_fraction =
02780     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02781
02782     using SQRT3_fraction =
02783     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02784     // NOLINT
02785 } // namespace aerobus
02786
02787 // known polynomials
02788 namespace aerobus {
02789     // CChebyshev
02790     namespace internal {
02791         template<int kind, size_t deg>
02792         struct chebyshev_helper {
02793             using type = typename pi64::template sub_t<
02794                 typename pi64::template mul_t<
02795                     typename pi64::template mul_t<
02796                         pi64::inject_constant_t<2>,
02797                         typename pi64::X>,
02798                     typename chebyshev_helper<kind, deg - 1>::type
02799                 >,
02800                 typename chebyshev_helper<kind, deg - 2>::type
02801             >;
02802         };
02803
02804         template<>
02805         struct chebyshev_helper<1, 0> {
02806             using type = typename pi64::one;
02807         };
02808
02809         template<>
02810         struct chebyshev_helper<1, 1> {
02811             using type = typename pi64::X;
02812         };
02813
02814         template<>
02815         struct chebyshev_helper<2, 0> {
02816             using type = typename pi64::one;
02817         };
02818
02819         template<>
02820         struct chebyshev_helper<2, 1> {
02821             using type = typename pi64::template mul_t<
02822                 typename pi64::inject_constant_t<2>,
02823                 typename pi64::X>;
02824         };
02825     } // namespace internal
02826
02827     // Laguerre
02828     namespace internal {
02829         template<size_t deg>
02830         struct laguerre_helper {
02831             private:
02832                 // Lk = (1 / k) * ((2 * k - 1 - x) * Lk-1 - (k - 2) Lk-2)
02833                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02834                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02835                 // -x + 2k-1
02836                 using p = typename pq64::template val<
02837                     typename q64::template inject_constant_t<-1>,
02838                     typename q64::template inject_constant_t<2 * deg - 1>
02839                 >;
02840                 // 1/n
02841                 using factor = typename pq64::template inject_ring_t<
02842                     q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>
02843                 >;
02844             public:
02845                 using type = typename pq64::template mul_t <
02846                     factor,

```

```

02849         typename pq64::template sub_t<
02850             typename pq64::template mul_t<
02851                 p,
02852                 lnm1
02853             >,
02854             typename pq64::template mul_t<
02855                 typename pq64::template inject_constant_t<deg-1>,
02856                 lnm2
02857             >
02858         >
02859     >;
02860
02861 };
02862
02863 template<>
02864 struct laguerre_helper<0> {
02865     using type = typename pq64::one;
02866 };
02867
02868 template<>
02869 struct laguerre_helper<1> {
02870     using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02871 };
02872 } // namespace internal
02873
02874 // Bernstein
02875 namespace internal {
02876     template<size_t i, size_t m, typename E = void>
02877     struct bernstein_helper {};
02878
02879     template<>
02880     struct bernstein_helper<0, 0> {
02881         using type = typename pi64::one;
02882     };
02883
02884     template<size_t i, size_t m>
02885     struct bernstein_helper<i, m, std::enable_if_t<
02886         (m > 0) && (i == 0)>> {
02887         using type = typename pi64::mul_t<
02888             typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02889             typename bernstein_helper<i, m-1>::type>;
02890     };
02891
02892     template<size_t i, size_t m>
02893     struct bernstein_helper<i, m, std::enable_if_t<
02894         (m > 0) && (i == m)>> {
02895         using type = typename pi64::template mul_t<
02896             typename pi64::X,
02897             typename bernstein_helper<i-1, m-1>::type>;
02898     };
02899
02900     template<size_t i, size_t m>
02901     struct bernstein_helper<i, m, std::enable_if_t<
02902         (m > 0) && (i > 0) && (i < m)>> {
02903         using type = typename pi64::add_t<
02904             typename pi64::mul_t<
02905                 typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02906                 typename bernstein_helper<i, m-1>::type>,
02907             typename pi64::mul_t<
02908                 typename pi64::X,
02909                 typename bernstein_helper<i-1, m-1>::type>;
02910     };
02911 } // namespace internal
02912
02913 namespace known_polynomials {
02914     enum hermite_kind {
02915         probabilist,
02916         physicist
02917     };
02918 }
02919
02920 // hermite
02921 namespace internal {
02922     template<size_t deg, known_polynomials::hermite_kind kind>
02923     struct hermite_helper {};
02924
02925     template<size_t deg>
02926     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02927     private:
02928         using hnm1 = typename hermite_helper<deg - 1,
02929             known_polynomials::hermite_kind::probabilist>::type;
02930         using hnm2 = typename hermite_helper<deg - 2,
02931             known_polynomials::hermite_kind::probabilist>::type;
02932     public:
02933         using type = typename pi64::template sub_t<
02934             typename pi64::template mul_t<typename pi64::X, hnm1>,

```

```

02935         typename pi64::template mul_t<
02936             typename pi64::template inject_constant_t<deg - 1>,
02937             hnm2
02938         >
02939     >;
02940 };
02941
02942 template<size_t deg>
02943 struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02944     private:
02945         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02946         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02947
02948     public:
02949         using type = typename pi64::template sub_t<
02950             // 2X Hn-1
02951             typename pi64::template mul_t<
02952                 typename pi64::val<typename i64::template inject_constant_t<2>,
02953                     typename i64::zero>, hnm1>,
02954
02955                 typename pi64::template mul_t<
02956                     typename pi64::template inject_constant_t<2*(deg - 1)>,
02957                     hnm2
02958                 >
02959             >;
02960 };
02961
02962 template<>
02963 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02964     using type = typename pi64::one;
02965 };
02966
02967 template<>
02968 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02969     using type = typename pi64::X;
02970 };
02971
02972 template<>
02973 struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02974     using type = typename pi64::one;
02975 };
02976
02977 template<>
02978 struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02979     // 2X
02980     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
typename i64::zero>;
02981 };
02982 } // namespace internal
02983
02984 // legendre
02985 namespace internal {
02986     template<size_t n>
02987     struct legendre_helper {
02988     private:
02989         // 1/n constant
02990         // (2n-1)/n X
02991         using fact_left = typename pq64::monomial_t<make_q64_t<2*n-1, n>, 1>;
02992         // (n-1) / n
02993         using fact_right = typename pq64::val<make_q64_t<n-1, n>;
02994     public:
02995         using type = pq64::template sub_t<
02996             typename pq64::template mul_t<
02997                 fact_left,
02998                 typename legendre_helper<n-1>::type
02999             >,
03000             typename pq64::template mul_t<
03001                 fact_right,
03002                 typename legendre_helper<n-2>::type
03003             >
03004         >;
03005 };
03006
03007 template<>
03008 struct legendre_helper<0> {
03009     using type = typename pq64::one;
03010 };
03011
03012 template<>
03013 struct legendre_helper<1> {
03014     using type = typename pq64::X;
03015 };
03016 } // namespace internal
03017
03018 // bernoulli polynomials

```

```

03019     namespace internal {
03020         template<size_t n>
03021         struct bernoulli_coeff {
03022             template<typename T, size_t i>
03023             struct inner {
03024                 using type = typename q64::template mul_t<
03025                     q64::inject_ring_t<combination_t<i64, i, n>,
03026                     bernoulli_t<i64, n-i>
03027                 >;
03028             };
03029         };
03030     } // namespace internal
03031
03032     namespace known_polynomials {
03033         template <size_t deg>
03034         using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
03035
03036         template <size_t deg>
03037         using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
03038
03039         template <size_t deg>
03040         using laguerre = typename internal::laguerre_helper<deg>::type;
03041
03042         template <size_t deg>
03043         using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
03044
03045         template <size_t deg>
03046         using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
03047
03048         template<size_t i, size_t m>
03049         using bernstein = typename internal::bernstein_helper<i, m>::type;
03050
03051         template<size_t deg>
03052         using legendre = typename internal::legendre_helper<deg>::type;
03053
03054         template<size_t deg>
03055         using bernoulli = taylor<i64, internal::bernoulli_coeff<deg>::template inner, deg>;
03056     } // namespace known_polynomials
03057 } // namespace aerobus
03058
03059 #ifdef AEROBUS_CONWAY_IMPORTS
03060 template<int p, int n>
03061 struct ConwayPolynomial;
03062
03063 #define ZPV ZPZ::template val
03064 #define POLYV aerobus::polynomial<ZPV>::template val
03065 template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03066     ZPV<1>; }; // NOLINT
03067 template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03068     ZPV<1>, ZPV<1>; }; // NOLINT
03069 template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03070     ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03071 template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03072     ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03073 template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03074     ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03075 template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03076     ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03077 template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03078     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03079 template<> struct ConwayPolynomial<2, 8> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03080     ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03081 template<> struct ConwayPolynomial<2, 9> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03082     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03083 template<> struct ConwayPolynomial<2, 10> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03084     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>; }; //
03085     NOLINT
03086 template<> struct ConwayPolynomial<2, 11> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03087     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; };
03088     // NOLINT
03089 template<> struct ConwayPolynomial<2, 12> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03090     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>,
03091     ZPV<1>; }; // NOLINT
03092 template<> struct ConwayPolynomial<2, 13> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03093     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>,
03094     ZPV<1>, ZPV<1>; }; // NOLINT
03095 template<> struct ConwayPolynomial<2, 14> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03096     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>,
03097     ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03098 template<> struct ConwayPolynomial<2, 15> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03099     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>,
03100     ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03101 template<> struct ConwayPolynomial<2, 16> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,
03102     ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>,
03103     ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03104 template<> struct ConwayPolynomial<2, 17> { using ZPV = aerobus::zpv<2>; using type = POLYV<ZPV<1>,

```

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Chapter 7

Examples

7.1 QuotientRing

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject_constant_t<1>

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject_constant_t<1>

Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

7.2 type_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

...Ts	types to store and manipulate at compile time
-------	---

7.3 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.4 i32::add_t

addition operator yields $v1 + v2$ $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields $v1 + v2$ $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

7.5 i32::sub_t

subtraction operator yields $v1 - v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields $v1 - v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

7.6 i32::mul_t

multiplication operator yields $v1 * v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields $v1 * v2$ $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

7.7 i32::div_t

division operator yields $v1 / v2$ $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields $v1 / v2$ $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

7.8 i32::gt_t

strictly greater operator ($v1 > v2$) yields $v1 > v2$ <i32::val<7>, i32::val<2>>

strictly greater operator ($v1 > v2$) yields $v1 > v2$ <i32::val<7>, i32::val<2>>

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

7.9 i32::eq_t

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields $v1 == v2$ as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

7.10 i32::eq_v

equality operator (boolean value)

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	<i32::val<1>, i32::val<1>>

7.11 i32::gcd_t

greatest common divisor yields $GCD(v1, v2)$ <i32::val<6>, i32::val<15>>

greatest common divisor yields $GCD(v1, v2)$ <i32::val<6>, i32::val<15>>

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

7.12 i32::pos_t

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields $v > 0$ as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

v	a value in i32
-----	----------------

7.13 i32::pos_v

positivity (boolean value) yields $v > 0$ as boolean value

positivity (boolean value) yields $v > 0$ as boolean value

Template Parameters

v	a value in i32 <code><i32::val<1>></code>
-----	---

7.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	<code>inject_constant_t<2></code>
-----	---

7.15 i64::add_t

addition operator

addition operator

Template Parameters

$v1$: an element of <code>aerobus::i64::val</code>
$v2$: an element of <code>aerobus::i64::val</code> <code><i64::val<1>, i64::val<2>></code>

7.16 i64::sub_t

subtraction operator

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

7.17 i64::mul_t

multiplication operator

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

7.18 i64::div_t

division operator integer division

division operator integer division

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<1>, i64::val<2>>

7.19 i64::mod_t

modulus operator

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val <i64::val<6>, i64::val<15>>

7.20 i64::gt_t

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

strictly greater operator yields $v1 > v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<2>, i64::val<1>></code>

7.21 i64::lt_t

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

strict less operator yields $v1 < v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<1>, i64::val<2>></code>

7.22 i64::lt_v

strictly smaller operator yields $v1 < v2$ as boolean value

strictly smaller operator yields $v1 < v2$ as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<1>, i64::val<2>></code>

7.23 i64::eq_t

equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`

equality operator yields $v1 == v2$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<2>, i64::val<2>></code>

7.24 i64::eq_v

equality operator yields $v1 == v2$ as boolean value

equality operator yields $v1 == v2$ as boolean value

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<2>, i64::val<2>></code>

7.25 i64::gcd_t

greatest common divisor yields $GCD(v1, v2)$ as instantiation of `i64::val`

greatest common divisor yields $GCD(v1, v2)$ as instantiation of `i64::val`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val <code><i64::val<6>, i64::val<15>></code>

7.26 i64::pos_t

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

is v positive yields $v > 0$ as `std::true_type` or `std::false_type`

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val <code><i64::val<1>></code>
-----------------	--

7.27 i64::pos_v

positivity yields $v > 0$ as boolean value

positivity yields $v > 0$ as boolean value

Template Parameters

<code>v</code>	: an element of aerobus::i64::val <code><i64::val<1>></code>
----------------	--

7.28 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.29 q32::add_t

addition operator

addition operator

Template Parameters

v1	a value
v2	a value <q32::val<i32::val<1>, i32::val<2>>, q32::val<i32::val<1>, i32::val<3>>>

7.30 FractionField

Fraction field of an euclidean domain, such as Q for Z.

Fraction field of an euclidean domain, such as Q for Z

Template Parameters

Ring	<i64> is q64 (rationals with 64 bits numerator and denominator)
------	---

7.31 PI_fraction::val

representation of PI as a continued fraction -> 3.14...

7.32 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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