

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download the [aerobus.h](#)
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including [aerobus.h](#).

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

results on my laptop :

```
./benchmarks_avx512.exe
[std math] 5.358e-01 Gsin/s
[std fast math] 3.389e+00 Gsin/s
[aerobus deg 1] 1.871e+01 Gsin/s
average error (vs std) : 4.36e-02
max error (vs std) : 1.50e-01
[aerobus deg 3] 1.943e+01 Gsin/s
average error (vs std) : 1.85e-04
max error (vs std) : 8.17e-04
[aerobus deg 5] 1.335e+01 Gsin/s
average error (vs std) : 6.07e-07
max error (vs std) : 3.63e-06
[aerobus deg 7] 8.634e+00 Gsin/s
average error (vs std) : 1.27e-09
max error (vs std) : 9.75e-09
[aerobus deg 9] 6.171e+00 Gsin/s
average error (vs std) : 1.89e-12
max error (vs std) : 1.78e-11
[aerobus deg 11] 4.731e+00 Gsin/s
average error (vs std) : 2.12e-15
max error (vs std) : 2.40e-14
[aerobus deg 13] 3.862e+00 Gsin/s
average error (vs std) : 3.16e-17
max error (vs std) : 3.33e-16
[aerobus deg 15] 3.359e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
[aerobus deg 17] 2.947e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `Fraction<Field<Integers>>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`



- cos
- tan
- sh
- cosh
- tanh
- asin
- acos
- acosh
- asinh
- atanh

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in `Integers (typename)` and `index (size_t)`;
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
```

```

vbroadcastsd    ymm3, QWORD PTR .LC23[rip]
vbroadcastsd    ymm2, QWORD PTR .LC25[rip]
.L3:
vmovupd ymm15, YMMWORD PTR [rsi+rax]
vmovapd ymm0, ymm15
vmaddl32pd      ymm0, ymm14, ymm1
vmaddl32pd      ymm0, ymm13, ymm15
vmaddl32pd      ymm0, ymm12, ymm15
vmaddl32pd      ymm0, ymm11, ymm15
vmaddl32pd      ymm0, ymm10, ymm15
vmaddl32pd      ymm0, ymm9, ymm15
vmaddl32pd      ymm0, ymm8, ymm15
vmaddl32pd      ymm0, ymm7, ymm15
vmaddl32pd      ymm0, ymm6, ymm15
vmaddl32pd      ymm0, ymm5, ymm15
vmaddl32pd      ymm0, ymm4, ymm15
vmaddl32pd      ymm0, ymm3, ymm15
vmaddl32pd      ymm0, ymm2, ymm15
vmaddl32pd      ymm0, ymm1, ymm15
vmovupd YMMWORD PTR [rdx+rax], ymm0
add    rax, 32
cmp    rcx, rax
jne    .L3
mov    rax, rdi
and    rax, -4
vzeroupper

```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, `Aerobus` allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```

using namespace aerobus;
using ZmZ = zp<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;

```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define `rational functions` by taking the ring of fractions of polynomials:

```

using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;

```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , `Aerobus` provides automatic implementation for `quotient ring  $R/X$`  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```

using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;

```

As we could have using `zp<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

Aerobus gives an implementation for `continued fractions`. It can be used this way:

```
using namespace aerobus;  
using T = ContinuedFraction<1,2,3,4>;  
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	35
<a href="#">aerobus::known_polynomials</a>	Families of well known polynomials such as Hermite or Bernstein . . . . .	39



## Chapter 3

# Concept Index

### 3.1 Concepts

Here is a list of all concepts with brief descriptions:

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<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	43
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	44





## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt;</a>	46
<a href="#">aerobus::ContinuedFraction&lt; values &gt;</a>	46
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Quotient ring by the principal ideal generated by 'X' With <a href="#">i32</a> as Ring and <a href="#">i32::val&lt;2&gt;</a> as X, Quotient is Z/2Z	68
<a href="#">aerobus::type_list&lt; Ts &gt;::split&lt; index &gt;</a>	
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## Chapter 5

# File Index

### 5.1 File List

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--	----



## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)
- struct [Embed< i32, i64 >](#)
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
- struct [Embed< q32, q64 >](#)
- struct [Embed< Quotient< Ring, X >, Ring >](#)
- struct [Embed< Ring, FractionField< Ring > >](#)
- struct [Embed< zpz< x >, i32 >](#)
- struct [i32](#)  
*32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)  
*64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)  
*checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)  
*Quotient ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)  
*Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)  
*specialization for empty type list*
- struct [zpz](#)

## Concepts

- concept [IsRing](#)  
*Concept to express  $R$  is a Ring.*
- concept [IsEuclideanDomain](#)  
*Concept to express  $R$  is an euclidean domain.*
- concept [IsField](#)  
*Concept to express  $R$  is a field.*

## Typedefs

- `template<typename T , typename A , typename B >`  
`using gcd\_t = typename internal::gcd< T >::template type< A, B >`  
*computes the greatest common divisor of A and B*
- `template<typename... vals>`  
`using vadd\_t = typename internal::vadd< vals... >::type`  
*adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an `add_t` binary operator*
- `template<typename... vals>`  
`using vmul\_t = typename internal::vmul< vals... >::type`  
*multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an `mul_t` binary operator*
- `template<typename val >`  
`using abs\_t = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template sub_t< typename val::enclosing_type::zero, val > >`  
*computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- `template<typename Ring >`  
`using FractionField = typename internal::FractionFieldImpl< Ring >::type`
- `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
- `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
- `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
- `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
- `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
- `template<typename Ring , typename v1 , typename v2 >`  
`using makefraction\_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational  $V_1/V_2$  in the field of fractions of Ring*
- `template<typename v >`  
`using embed\_int\_poly\_in\_fractions\_t = typename Embed< polynomial< typename v::ring_type >, polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
- `template<int64_t p, int64_t q>`  
`using make\_q64\_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p >, i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*

- `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p >, i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<typename Ring, typename v1, typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- `template<typename Ring, typename v1, typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- `template<typename Ring, auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
- `template<typename Ring, auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField<Ring>*
- `template<typename T, size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*
- `template<typename T, size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T, size_t n>`  
`using bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T, size_t n>`  
`using bell_t = typename internal::bell_helper< T, n >::type`  
*Bell numbers.*
- `template<typename T, int k>`  
`using alternate_t = typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T, int n, int k>`  
`using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T, int n, int k>`  
`using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T, typename p, size_t n>`  
`using pow_t = typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>`  
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers, size_t deg>`  
`using exp = taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers, size_t deg>`  
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers, size_t deg>`  
`using ln1 = taylor< Integers, internal::ln1_coeff, deg >`

- $\ln(1 + x)$ 
  - `template<typename Integers, size_t deg>`  
`using atan = taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
  - `template<typename Integers, size_t deg>`  
`using sin = taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
  - `template<typename Integers, size_t deg>`  
`using sinh = taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
  - `template<typename Integers, size_t deg>`  
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
  - `template<typename Integers, size_t deg>`  
`using cos = taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*
  - `template<typename Integers, size_t deg>`  
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`  
 $\frac{1}{1-x}$  *zero development of  $\frac{1}{1-x}$*
  - `template<typename Integers, size_t deg>`  
`using asin = taylor< Integers, internal::asin_coeff, deg >`  
 $\arcsin(x)$  *arc sinus*
  - `template<typename Integers, size_t deg>`  
`using asinh = taylor< Integers, internal::asinh_coeff, deg >`  
 $\operatorname{arcsinh}(x)$  *arc hyperbolic sinus*
  - `template<typename Integers, size_t deg>`  
`using atanh = taylor< Integers, internal::atanh_coeff, deg >`  
 $\operatorname{arctanh}(x)$  *arc hyperbolic tangent*
  - `template<typename Integers, size_t deg>`  
`using tan = taylor< Integers, internal::tan_coeff, deg >`  
 $\tan(x)$  *tangent*
  - `template<typename Integers, size_t deg>`  
`using tanh = taylor< Integers, internal::tanh_coeff, deg >`  
 $\tanh(x)$  *hyperbolic tangent*
  - `using PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 1 >`
  - `using E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`
  - `using SQRT2_fraction = ContinuedFraction< 1, 2 >`  
*approximation of  $\sqrt{2}$*
  - `using SQRT3_fraction = ContinuedFraction< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >`  
*approximation of*

## Functions

- `template<typename T >`  
`T * aligned\_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the [field](#) (prime number) @tparam n degree of extension  
`template< int p`



## Variables

- `template<typename T, size_t i>`  
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`  
*computes factorial(i) as value in T*
- `template<typename T, size_t k, size_t n>`  
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`  
*computes binomial coefficients (k among n) as value*
- `template<typename FloatType, typename T, size_t n>`  
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`  
*nth bernoulli number as value in FloatType*
- `template<typename T, size_t k>`  
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`  
 *$(-1)^k$  as value from T*

### 6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

### 6.1.2 Typedef Documentation

#### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val&lt;-2&gt;</code>
------------	--

#### 6.1.2.2 addfractions\_t

```
template<typename Ring, typename v1, typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

### 6.1.2.3 `alternate_t`

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example
----------	---

### 6.1.2.4 `asin`

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.5 `asinh`

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.6 `atan`

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.7 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$  arc hyperbolic tangent

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.8 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

#### Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

### 6.1.2.9 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

$n$ th bernoulli number as type in  $T$

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

### 6.1.2.10 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient ( $k$  among  $n$ ) as type

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

### 6.1.2.11 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  cosinus

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.12 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.13 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

### 6.1.2.14 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in polynomial<FractionField<Ring>>

#### Template Parameters

<i>Ring</i>	Integers
<i>a</i>	valu in polynomial<Ring>

### 6.1.2.15 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.16 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_←
t< exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.17 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

#### Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

### 6.1.2.18 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

### 6.1.2.19 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

### 6.1.2.20 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

### 6.1.2.21 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

#### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

### 6.1.2.22 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.23 ln1

```
template<typename Integers , size_t deg>
using aerobus::ln1 = typedef taylor<Integers, internal::ln1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.24 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

## Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

**6.1.2.25 make\_int\_polynomial\_t**

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

## Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

**6.1.2.26 make\_q32\_t**

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

**6.1.2.27 make\_q64\_t**

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator



**6.1.2.28 makefraction\_t**

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

**Template Parameters**

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

**6.1.2.29 mulfractions\_t**

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

**Template Parameters**

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

**6.1.2.30 pi64**

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

**6.1.2.31 PI\_fraction**

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

**6.1.2.32 pow\_t**

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

## Template Parameters

$T$	(some ring type, such as <a href="#">aerobus::i64</a> )
$p$	must be an instantiation of $T::val$
$n$	power

**6.1.2.33 pq64**

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.34 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.35 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

**6.1.2.36 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.37 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.38 SQRT2\_fraction

[illegible]approximation of  $\sqrt{2}$ 

### 6.1.2.39 SQRT3\_fraction

```
using aerobus::SQRT3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1,  
2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

#### 6.1.2.40 stirling\_signed\_t

```
template<typename T , int n, int k>
using aerobus::stirling_signed_t = typedef typename internal::stirling_helper<T, n, k>::type
```

Stirling number of first kind (signed) – as types.

## Template Parameters

$T$	(ring type, such as <code>aerobus::i64</code> )
$n$	(integer)
$k$	(integer)

#### 6.1.2.41 `stirling_unsigned_t`

```
template<typename T , int n, int k>
using aerobus::stirling_unsigned_t = typedef abs_t<typename internal::stirling_helper<T, n,
k>::type>
```

Stirling number of first kind (unsigned) – as types.

## Template Parameters

$T$	(ring type, such as <code>aerobus::i64</code> )
$n$	(integer)
$k$	(integer)

#### 6.1.2.42 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

##### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.43 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

##### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.44 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

##### Template Parameters

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff<sub>↔</sub> _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

#### 6.1.2.45 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

## Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.2.46 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

## Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

## 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

## Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

## Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

## 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
    prime number )
```

## 6.1.4 Variable Documentation

## 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
```

```
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

#### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

#### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

## Template Parameters

<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 && n !=3 && n % 2 !=0 && n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 && n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 && n >=9 && n % 3 !=0 && n % 2 !=0 && i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 && n % i !=0 && n >=9 && n % 3 !=0 && n % 2 !=0 && (i \*i <=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 && n >=9 && n % 3 !=0 && n % 2 !=0 && i \*i <=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 && i \*i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >

- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i == 0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i == m)> >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k > (n/2) && k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k <= (n/2) && k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t<Field::is\_field> >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field> >
- struct **gcd**

*greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*

- struct **gcd**< Ring, std::enable\_if\_t<Ring::is\_euclidean\_domain> >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, l >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, l >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, l >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, l >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, l >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, l >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, l >
- struct **laguerre\_helper**< 1, l >



- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **stirling\_helper**
- struct **stirling\_helper**< T, 0, 0 >
- struct **stirling\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, k, std::enable\_if\_t<(k > 0) && (n > 0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N >{}))

## Functions

- `template<std::size_t... Is>  
constexpr auto index\_sequence\_reverse (std::index_sequence< Is... > const &) -> decltype(std::index_↵  
sequence< sizeof...(Is) - 1U - Is... >{})`

## Variables

- `template<template< typename... > typename TT, typename T >  
constexpr bool is\_instantiation\_of\_v = is_instantiation_of<TT, T>::value`

### 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

### 6.2.2 Typedef Documentation

#### 6.2.2.1 `make_index_sequence_reverse`

```
template<std::size_t N>
using aerobus::internal::make\_index\_sequence\_reverse = typedef decltype(index\_sequence\_reverse(std↵
::make_index_sequence<N>{}))
```

#### 6.2.2.2 `type_at_t`

```
template<size_t i, typename... Ts>
using aerobus::internal::type\_at\_t = typedef typename type_at<i, Ts...>::type
```

### 6.2.3 Function Documentation

#### 6.2.3.1 `index_sequence_reverse()`

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index\_sequence\_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

### 6.2.4 Variable Documentation

#### 6.2.4.1 `is_instantiation_of_v`

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is\_instantiation\_of\_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Typedefs

- `template<size_t deg, typename I = aerobus::i64>`  
`using chebyshev\_T = typename internal::chebyshev_helper< 1, deg, I >::type`  
*Chebyshev polynomials of first kind.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using chebyshev\_U = typename internal::chebyshev_helper< 2, deg, I >::type`  
*Chebyshev polynomials of second kind.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using laguerre = typename internal::laguerre_helper< deg, I >::type`  
*Laguerre polynomials.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using hermite\_prob = typename internal::hermite_helper< deg, hermite\_kind::probabilist, I >::type`  
*Hermite polynomials - probabilist form.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using hermite\_phys = typename internal::hermite_helper< deg, hermite\_kind::physicist, I >::type`  
*Hermite polynomials - physicist form.*
- `template<size_t i, size_t m, typename I = aerobus::i64>`  
`using bernstein = typename internal::bernstein_helper< i, m, I >::type`  
*Bernstein polynomials.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using legendre = typename internal::legendre_helper< deg, I >::type`  
*Legendre polynomials.*
- `template<size_t deg, typename I = aerobus::i64>`  
`using bernoulli = taylor< I, internal::bernoulli_coeff< deg >::template inner, deg >`  
*Bernoulli polynomials.*

### Enumerations

- `enum hermite\_kind { probabilist , physicist }`

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Typedef Documentation

#### 6.3.2.1 bernoulli

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::bernoulli = typedef taylor<I, internal::bernoulli_coeff<deg><
::template inner, deg>
```

Bernoulli polynomials.

Lives in `polynomial<FractionField<I>>`

See also

[See in Wikipedia](#)

## Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to <a href="#">aerobus::i64</a> )

## 6.3.2.2 bernstein

```
template<size_t i, size_t m, typename I = aerobus::i64>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m, I>::type
```

Bernstein polynomials.

Lives in polynomial

## See also

[See in Wikipedia](#)

## Template Parameters

<i>i</i>	<i>index of polynomial (between 0 and m)</i>
<i>m</i>	<i>degree of polynomial</i>
<i>I</i>	<i>Integers ring (defaults to <a href="#">aerobus::i64</a>)</i>

## 6.3.2.3 chebyshev\_T

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg, I>::type
```

Chebyshev polynomials of first kind.

## See also

[See in Wikipedia](#)

## Template Parameters

<i>deg</i>	degree of polynomial
<i>integer</i>	rings (defaults to <a href="#">aerobus::i64</a> )

## 6.3.2.4 chebyshev\_U

```
template<size_t deg, typename I = aerobus::i64>
```

```
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg, I>::type
```

Chebyshev polynomials of second kind.

Lives in polynomial

**See also**

[See in Wikipedia](#)

#### Template Parameters

deg	<i>degree of polynomial</i>
integer	<i>rings (defaults to <a href="#">aerobus::i64</a>)</i>

#### 6.3.2.5 hermite\_phys

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist, I>::type
```

Hermite polynomials - physicist form.

**See also**

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	<i>degree of polynomial</i>
------------	-----------------------------

#### 6.3.2.6 hermite\_prob

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist, I>::type
```

Hermite polynomials - probabilist form.

**See also**

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	<i>degree of polynomial</i>
------------	-----------------------------

### 6.3.2.7 laguerre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg,
I>::type
```

Laguerre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to <a href="#">aerobus::i64</a> )

### 6.3.2.8 legendre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg,
I>::type
```

Legendre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers Ring (defaults to <a href="#">aerobus::i64</a> )

## 6.3.3 Enumeration Type Documentation

### 6.3.3.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.



## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- `using type = typename Ring::zero`

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- [using type = aN](#)

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)>
>::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

#### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
*represented value as [aerobus::q64](#)*

#### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
*represented value as double*

#### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

## Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

## 8.5.2 Member Typedef Documentation

### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template add\_t< typename q64::template inject\_constant\_t< a0 >, typename q64::template div\_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

## Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::Embed< Small, Large, E > Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::Embed< i32, i64 > Struct Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i64::val< static_cast< int64_t >(val::v)>`

### 8.9.1 Member Typedef Documentation

#### 8.9.1.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.10 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
`using type = typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`

### 8.10.1 Member Typedef Documentation

#### 8.10.1.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.11 aerobus::Embed< q32, q64 > Struct Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
`using type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`

### 8.11.1 Member Typedef Documentation

#### 8.11.1.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_↵
cast<int64_t>(v::y::v)>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.12 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = typename val::raw_t`

### 8.12.1 Member Typedef Documentation

#### 8.12.1.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.13 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
`using type = typename FractionField< Ring >::template val< v, typename Ring::one >`

### 8.13.1 Member Typedef Documentation

#### 8.13.1.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>←
::template val<v, typename Ring::one>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.14 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`

### 8.14.1 Member Typedef Documentation

#### 8.14.1.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.15 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

### Public Types

- [using inner\\_type = int32\\_t](#)
- [using zero = val< 0 >](#)  
*constant zero*
- [using one = val< 1 >](#)  
*constant one*
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int32\\_t >\(x\)>](#)
- [template<typename v >](#)  
[using inject\\_ring\\_t = v](#)
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mod\\_t = typename remainder< v1, v2 >::type](#)  
*modulus operator yields v1 % v2 for example : [i32::mod\\_t<i32::val<7>](#), [i32::val<2>](#)>*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gcd\\_t = gcd\\_t< i32, v1, v2 >](#)
- [template<typename v >](#)  
[using pos\\_t = typename pos< v >::type](#)

### Static Public Attributes

- [static constexpr bool is\\_field = false](#)  
*integers are not a field*
- [static constexpr bool is\\_euclidean\\_domain = true](#)  
*integers are an euclidean domain*
- [template<typename v1 , typename v2 >](#)  
[static constexpr bool eq\\_v = eq\\_t<v1, v2>::value](#)
- [template<typename v >](#)  
[static constexpr bool pos\\_v = pos\\_t<v>::value](#)



### 8.15.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

### 8.15.2 Member Typedef Documentation

#### 8.15.2.1 add\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

#### 8.15.2.2 div\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

#### 8.15.2.3 eq\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

#### 8.15.2.4 gcd\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

#### 8.15.2.5 gt\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

#### 8.15.2.6 inject\_constant\_t

```
template<auto x>  
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

#### 8.15.2.7 inject\_ring\_t

```
template<typename v >  
using aerobus::i32::inject_ring_t = v
```

#### 8.15.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

**8.15.2.9 lt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

**8.15.2.10 mod\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

**Template Parameters**

<code>v1</code>	a value in <code>i32</code>
<code>v2</code>	a value in <code>i32</code>

**8.15.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

**8.15.2.12 one**

```
using aerobus::i32::one = val<1>
```

constant one

**8.15.2.13 pos\_t**

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

**8.15.2.14 sub\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

**8.15.2.15 zero**

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.15.3 Member Data Documentation

#### 8.15.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

#### 8.15.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.15.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

#### 8.15.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.16 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
values in [i64](#)

## Public Types

- `using inner_type = int64_t`  
*type of represented values*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int64_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`  
*injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`  
-> `i64::val<1>`*
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i64, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields `v1 > v2` as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 8.16.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

## 8.16.2 Member Typedef Documentation

### 8.16.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

### 8.16.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

### 8.16.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

### 8.16.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

### 8.16.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

### 8.16.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

### 8.16.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`  
 -> `i64::val<1>`

#### Template Parameters

<code>v</code>	a value in <code>i64</code>
----------------	-----------------------------

#### 8.16.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

#### 8.16.2.9 lt\_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

#### 8.16.2.10 mod\_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

#### 8.16.2.11 mul\_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

#### 8.16.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

#### 8.16.2.13 pos\_t

```
template<typename v >  
using aerobus::i64::pos_t = typename pos<v>::type
```

#### 8.16.2.14 sub\_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

#### 8.16.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.16.3 Member Data Documentation

### 8.16.3.1 eq\_v

```
template<typename v1 , typename v2 >  
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.2 gt\_v

```
template<typename v1 , typename v2 >  
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.16.3.3 is\_euclidean\_domain**

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

**8.16.3.4 is\_field**

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

**8.16.3.5 lt\_v**

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

**8.16.3.6 pos\_v**

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

**8.17 aerobus::is\_prime< n > Struct Template Reference**

checks if n is prime

```
#include <aerobus.h>
```

**Static Public Attributes**

- static constexpr bool [value](#) = internal::\_is\_prime<n, 5>::value  
*true iff n is prime*

**8.17.1 Detailed Description**

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime



## Template Parameters

<i>n</i>	
----------	--

## 8.17.2 Member Data Documentation

### 8.17.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.18 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*

- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication of two polynomials*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq_helper< v1, v2 >::type`  
*equality operator*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt_helper< v1, v2 >::type`  
*strict less operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt_helper< v1, v2 >::type`  
*strict greater operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::q_type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial :  $\text{coeff } X^{\text{deg}}$*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity ( $an > 0$ )*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial< Ring >, v1, v2 > >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

## Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

### 8.18.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 8.18.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.18.2.5 gcd\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.18.2.6 gt\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.18.2.7 inject\_constant\_t**

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

**8.18.2.8 inject\_ring\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

### 8.18.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.2.10 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial :  $\text{coeff } X^{\text{deg}}$

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

### 8.18.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

**8.18.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

**8.18.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ( $a_n > 0$ )

## Template Parameters

<i>v</i>	
----------	--

**8.18.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

## Template Parameters

<i>P</i>	
----------	--

**8.18.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

### 8.18.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

## 8.18.3 Member Data Documentation

### 8.18.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

### 8.18.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

### 8.18.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<code>v</code>	a value in <code>polynomial::val</code>
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.19 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

## Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

### 8.19.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

#### 8.19.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*projection values in the quotient ring*



## Public Types

- `using zero = val< typename Ring::zero >`  
*zero value*
- `using one = val< typename Ring::one >`  
*one*
- `template<typename v1 , typename v2 >`  
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`  
*modulus operator*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
*equality operator (as type)*
- `template<typename v1 >`  
`using pos_t = std::true_type`  
*positivity operator always true*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
*addition operator (as boolean value)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
*quotien rings are euclidean domain*

### 8.20.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as 'i32', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

## 8.20.2 Member Typedef Documentation

### 8.20.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

#### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.20.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

#### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.20.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

#### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.20.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
```

```
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

### 8.20.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

### 8.20.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.20.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.20.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.20.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.20.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.20.3 Member Data Documentation

### 8.20.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.20.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.20.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.21.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.21.2 Member Typedef Documentation

#### 8.21.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.21.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = [internal::type\\_at\\_t](#)< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename [concat\\_h](#)< U >::type  
*concatenates two list into one*
- template<typename T, size\_t index>  
using [insert](#) = typename [internal::insert\\_h](#)< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename [internal::remove\\_h](#)< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*

### Static Public Attributes

- static constexpr size\_t [length](#) = sizeof...(Ts)  
*length of list*

### 8.22.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

## 8.22.2 Member Typedef Documentation

### 8.22.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.22.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

#### Template Parameters

<i>U</i>	
----------	--

### 8.22.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

#### Template Parameters

<i>index</i>	
<i>T</i>	

### 8.22.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

<i>T</i>	
----------	--

**8.22.2.5 push\_front**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

**Template Parameters**

<i>T</i>	
----------	--

**8.22.2.6 remove**

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

**Template Parameters**

<i>index</i>	
--------------	--

**8.22.3 Member Data Documentation****8.22.3.1 length**

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

**8.23 aerobus::type\_list<> Struct Reference**

specialization for empty type list

```
#include <aerobus.h>
```



## Public Types

- template<typename T >  
using `push_front` = `type_list`< T >
- template<typename T >  
using `push_back` = `type_list`< T >
- template<typename U >  
using `concat` = U
- template<typename T , size\_t index>  
using `insert` = `type_list`< T >

## Static Public Attributes

- static constexpr size\_t `length` = 0

### 8.23.1 Detailed Description

specialization for empty type list

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 concat

```
template<typename U >  
using aerobus::type_list<>::concat = U
```

#### 8.23.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type_list<>::insert = type_list<T>
```

#### 8.23.2.3 push\_back

```
template<typename T >  
using aerobus::type_list<>::push_back = type_list<T>
```

#### 8.23.2.4 push\_front

```
template<typename T >  
using aerobus::type_list<>::push_front = type_list<T>
```

### 8.23.3 Member Data Documentation

#### 8.23.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.24 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- [using enclosing\\_type = i32](#)  
*Enclosing ring type.*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

### Static Public Member Functions

- [template<typename valueType >](#)  
[static constexpr INLINED DEVICE valueType get \(\)](#)  
*cast x into valueType*
- [static std::string to\\_string \(\)](#)  
*string representation of value*
- [template<typename valueRing >](#)  
[static constexpr DEVICE INLINED valueRing eval \(const valueRing &v\)](#)  
*cast x into valueRing*

### Static Public Attributes

- [static constexpr int32\\_t v = x](#)  
*actual value stored in val type*

#### 8.24.1 Detailed Description

```
template<int32\_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 8.24.2 Member Typedef Documentation

### 8.24.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

### 8.24.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.24.3 Member Function Documentation

### 8.24.3.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

## Template Parameters

<i>valueRing</i>	double for example
------------------	--------------------

### 8.24.3.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr INLINED DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast x into valueType

### Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

#### 8.24.3.3 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

### 8.24.4 Member Data Documentation

#### 8.24.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.25 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- [using inner\\_type = int32\\_t](#)  
*type of represented values*
- [using enclosing\\_type = i64](#)  
*enclosing ring type*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

### Static Public Member Functions

- [template<typename valueType >](#)  
[static constexpr DEVICE INLINED valueType get \( \)](#)  
*cast value in valueType*
- [static std::string to\\_string \( \)](#)  
*string representation*
- [template<typename valueRing >](#)  
[static constexpr DEVICE INLINED valueRing eval \(const valueRing &v\)](#)  
*cast value in valueRing*

## Static Public Attributes

- `static constexpr int64_t v = x`  
*actual value*

### 8.25.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

#### 8.25.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

#### 8.25.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.25.3 Member Function Documentation

#### 8.25.3.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

## Template Parameters

<i>valueRing</i>	(double for example)
------------------	----------------------

8.25.3.2 `get()`

```
template<int64_t x>
template<typename valueType >
static constexpr DEVICE INLINED valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

## Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

8.25.3.3 `to_string()`

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

## 8.25.4 Member Data Documentation

8.25.4.1 `v`

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- `using ring_type = Ring`  
*ring coefficients live in*
- `using enclosing_type = polynomial< Ring >`  
*enclosing ring type*
- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true\_type if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 8.26.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

## 8.26.2 Member Typedef Documentation

### 8.26.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

### 8.26.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

### 8.26.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.26.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.26.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in



### 8.26.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

## 8.26.3 Member Function Documentation

### 8.26.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs
>::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

#### Parameters

<i>x</i>	value
----------	-------

#### Returns

$P(x)$

### 8.26.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.26.4 Member Data Documentation

### 8.26.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

### 8.26.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.27 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- [using raw\\_t = V](#)
- [using type = abs\\_t< typename Ring::template mod\\_t< V, X > >](#)

### 8.27.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

<a href="#">V</a>	a value from 'Ring'
-------------------	---------------------

## 8.27.2 Member Typedef Documentation

### 8.27.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.27.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.28 aerobus::zpz< p >::val< x > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< x% p==0 >`

### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr DEVICE INLINED valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &v)`

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*

## 8.28.1 Member Typedef Documentation

### 8.28.1.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.28.1.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<x% p == 0>
```

## 8.28.2 Member Function Documentation

### 8.28.2.1 eval()

```
template<int32_t p>
template<int32_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::zpz< p >::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

### 8.28.2.2 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr DEVICE INLINED valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

### 8.28.2.3 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

## 8.28.3 Member Data Documentation

### 8.28.3.1 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.29 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

### Public Types

- [using ring\\_type = Ring](#)  
*ring coefficients live in*
- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is\\_zero\\_t = std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)

### Static Public Member Functions

- [static std::string to\\_string \(\)](#)
- [template<typename valueRing >](#)  
[static constexpr DEVICE INLINED valueRing eval \(const valueRing &x\)](#)

### Static Public Attributes

- [static constexpr size\\_t degree = 0](#)  
*degree*
- [static constexpr bool is\\_zero\\_v = is\\_zero\\_t::value](#)

### 8.29.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<a href="#">coeffN</a>	
------------------------	--

## 8.29.2 Member Typedef Documentation

### 8.29.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.29.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.29.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.29.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

### 8.29.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

### 8.29.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

### 8.29.3 Member Function Documentation

#### 8.29.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

#### 8.29.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

### 8.29.4 Member Data Documentation

#### 8.29.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

#### 8.29.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.30 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)

## Public Types

- `using inner_type = int32_t`
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `using zero = val< 0 >`
- `using one = val< 1 >`
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos_t = typename pos< v1 >::type`  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*



### 8.30.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

### 8.30.2 Member Typedef Documentation

#### 8.30.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.30.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.30.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.30.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.30.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.30.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

### 8.30.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

### 8.30.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.30.2.9 mod\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.30.2.10 mul\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.30.2.11 one**

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

**8.30.2.12 pos\_t**

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

## 8.30.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.30.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

## 8.30.3 Member Data Documentation

## 8.30.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.30.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

## Template Parameters

<code>v1</code>	a value in <a href="#">zpz::val</a>
<code>v2</code>	a value in <a href="#">zpz::val</a>

**8.30.3.3 is\_euclidean\_domain**

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

**8.30.3.4 is\_field**

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

**8.30.3.5 lt\_v**

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

## Template Parameters

<code>v1</code>	a value in <a href="#">zpz::val</a>
<code>v2</code>	a value in <a href="#">zpz::val</a>

**8.30.3.6 pos\_v**

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

## Template Parameters

<code>v1</code>	a value in <a href="#">zpz::val</a>
-----------------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00019 #ifdef _MSC_VER
00020 #define ALIGNED(x) __declspec(align(x))
00021 #define INLINED __forceinline
00022 #else
00023 #define ALIGNED(x) __attribute__((aligned(x)))
00024 #define INLINED __attribute__((always_inline)) inline
00025 #endif
00026
00027 #ifdef __CUDACC__
00028 #define DEVICE __host__ __device__
```

```

00029 #else
00030 #define DEVICE
00031 #endif
00032
00033
00034
00035
00036
00037
00038
00039 // aligned allocation
00040 namespace aerobus {
00041     template<typename T>
00042     T* aligned_malloc(size_t count, size_t alignment) {
00043         #ifdef _MSC_VER
00044             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00045         #else
00046             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00047         #endif
00048     }
00049 } // namespace aerobus
00050
00051 // concepts
00052 namespace aerobus {
00053     template <typename R>
00054     concept IsRing = requires {
00055         typename R::one;
00056         typename R::zero;
00057         typename R::template add_t<typename R::one, typename R::one>;
00058         typename R::template sub_t<typename R::one, typename R::one>;
00059         typename R::template mul_t<typename R::one, typename R::one>;
00060     };
00061
00062     template <typename R>
00063     concept IsEuclideanDomain = IsRing<R> && requires {
00064         typename R::template div_t<typename R::one, typename R::one>;
00065         typename R::template mod_t<typename R::one, typename R::one>;
00066         typename R::template gcd_t<typename R::one, typename R::one>;
00067         typename R::template eq_t<typename R::one, typename R::one>;
00068         typename R::template pos_t<typename R::one>;
00069
00070         R::template pos_v<typename R::one> == true;
00071         // typename R::template gt_t<typename R::one, typename R::zero>;
00072         R::is_euclidean_domain == true;
00073     };
00074
00075     template<typename R>
00076     concept IsField = IsEuclideanDomain<R> && requires {
00077         R::is_field == true;
00078     };
00079 } // namespace aerobus
00080
00081 // utilities
00082 namespace aerobus {
00083     namespace internal {
00084         template<template<typename...> typename TT, typename T>
00085         struct is_instantiation_of : std::false_type { };
00086
00087         template<template<typename...> typename TT, typename... Ts>
00088         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00089
00090         template<template<typename...> typename TT, typename T>
00091         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00092
00093         template <int64_t i, typename T, typename... Ts>
00094         struct type_at {
00095             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00096             using type = typename type_at<i - 1, Ts...>::type;
00097         };
00098
00099         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00100             using type = T;
00101         };
00102
00103         template <size_t i, typename... Ts>
00104         using type_at_t = typename type_at<i, Ts...>::type;
00105
00106         template<size_t n, size_t i, typename E = void>
00107         struct _is_prime {};
00108
00109         template<size_t i>
00110         struct _is_prime<0, i> {
00111             static constexpr bool value = false;
00112         };
00113
00114         template<size_t i>
00115         struct _is_prime<1, i> {
00116             static constexpr bool value = false;
00117         };
00118     }
00119 }

```



```

00128
00129     template<size_t i>
00130     struct _is_prime<2, i> {
00131         static constexpr bool value = true;
00132     };
00133
00134     template<size_t i>
00135     struct _is_prime<3, i> {
00136         static constexpr bool value = true;
00137     };
00138
00139     template<size_t i>
00140     struct _is_prime<5, i> {
00141         static constexpr bool value = true;
00142     };
00143
00144     template<size_t i>
00145     struct _is_prime<7, i> {
00146         static constexpr bool value = true;
00147     };
00148
00149     template<size_t n, size_t i>
00150     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00151         static constexpr bool value = false;
00152     };
00153
00154     template<size_t n, size_t i>
00155     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00156         static constexpr bool value = false;
00157     };
00158
00159     template<size_t n, size_t i>
00160     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00161         static constexpr bool value = true;
00162     };
00163
00164     template<size_t n, size_t i>
00165     struct _is_prime<n, i, std::enable_if_t<(
00166         n % i == 0 &&
00167         n >= 9 &&
00168         n % 3 != 0 &&
00169         n % 2 != 0 &&
00170         i * i > n)>> {
00171         static constexpr bool value = true;
00172     };
00173
00174     template<size_t n, size_t i>
00175     struct _is_prime<n, i, std::enable_if_t<(
00176         n % (i+2) == 0 &&
00177         n >= 9 &&
00178         n % 3 != 0 &&
00179         n % 2 != 0 &&
00180         i * i <= n)>> {
00181         static constexpr bool value = true;
00182     };
00183
00184     template<size_t n, size_t i>
00185     struct _is_prime<n, i, std::enable_if_t<(
00186         n % (i+2) != 0 &&
00187         n % i != 0 &&
00188         n >= 9 &&
00189         n % 3 != 0 &&
00190         n % 2 != 0 &&
00191         (i * i <= n))>> {
00192         static constexpr bool value = _is_prime<n, i+6>::value;
00193     };
00194
00195 } // namespace internal
00196
00197 template<size_t n>
00198 struct is_prime {
00199     static constexpr bool value = internal::_is_prime<n, 5>::value;
00200 };
00201
00202 template<size_t n>
00203 static constexpr bool is_prime_v = is_prime<n>::value;
00204
00205 // gcd
00206 namespace internal {
00207     template <std::size_t... Is>
00208     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00209         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00210
00211     template <std::size_t N>
00212     using make_index_sequence_reverse
00213         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00214
00215 }
00216
00217
00218
00219
00220

```

```

00226     template<typename Ring, typename E = void>
00227     struct gcd;
00228
00229     template<typename Ring>
00230     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00231         template<typename A, typename B, typename E = void>
00232         struct gcd_helper {};
00233
00234         // B = 0, A > 0
00235         template<typename A, typename B>
00236         struct gcd_helper<A, B, std::enable_if_t<
00237             (B::is_zero_t::value) &&
00238             (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00239             using type = A;
00240         };
00241
00242         // B = 0, A < 0
00243         template<typename A, typename B>
00244         struct gcd_helper<A, B, std::enable_if_t<
00245             (B::is_zero_t::value) &&
00246             !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00247             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00248         };
00249
00250         // B != 0
00251         template<typename A, typename B>
00252         struct gcd_helper<A, B, std::enable_if_t<
00253             (!B::is_zero_t::value)
00254             >> {
00255             private: // NOLINT
00256                 // A / B
00257                 using k = typename Ring::template div_t<A, B>;
00258                 // A - (A/B)*B = A % B
00259                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00260
00261             public:
00262                 using type = typename gcd_helper<B, m>::type;
00263         };
00264
00265         template<typename A, typename B>
00266         using type = typename gcd_helper<A, B>::type;
00267     };
00268 } // namespace internal
00269
00270 // vadd and vmul
00271 namespace internal {
00272     template<typename... vals>
00273     struct vmul {};
00274
00275     template<typename v1, typename... vals>
00276     struct vmul<v1, vals...> {
00277         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00278     };
00279
00280     template<typename v1>
00281     struct vmul<v1> {
00282         using type = v1;
00283     };
00284
00285     template<typename... vals>
00286     struct vadd {};
00287
00288     template<typename v1, typename... vals>
00289     struct vadd<v1, vals...> {
00290         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00291     };
00292
00293     template<typename v1>
00294     struct vadd<v1> {
00295         using type = v1;
00296     };
00297 } // namespace internal
00298
00299 template<typename T, typename A, typename B>
00300 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00301
00302 template<typename... vals>
00303 using vadd_t = typename internal::vadd<vals...>::type;
00304
00305 template<typename... vals>
00306 using vmul_t = typename internal::vmul<vals...>::type;
00307
00308 template<typename val>
00309 requires IsEuclideanDomain<typename val::enclosing_type>
00310 using abs_t = std::conditional_t<

```

```

00322         val::enclosing_type::template pos_v<val>,
00323         val, typename val::enclosing_type::template sub_t<typename
    val::enclosing_type::zero, val>;
00324 } // namespace aerobus
00325
00326 // embedding
00327 namespace aerobus {
00328     template<typename Small, typename Large, typename E = void>
00329     struct Embed;
00330 } // namespace aerobus
00331
00332 namespace aerobus {
00333     template<typename Ring, typename X>
00334     requires IsRing<Ring>
00335     struct Quotient {
00336         template <typename V>
00337         struct val {
00338             public:
00339                 using raw_t = V;
00340                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00341         };
00342
00343         using zero = val<typename Ring::zero>;
00344
00345         using one = val<typename Ring::one>;
00346
00347         template<typename v1, typename v2>
00348         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00349
00350         template<typename v1, typename v2>
00351         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00352
00353         template<typename v1, typename v2>
00354         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00355
00356         template<typename v1, typename v2>
00357         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00358
00359         template<typename v1, typename v2>
00360         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00361
00362         template<typename v1, typename v2>
00363         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00364
00365         template<typename v1>
00366         using pos_t = std::true_type;
00367
00368         template<typename v>
00369         static constexpr bool pos_v = pos_t<v>::value;
00370
00371         static constexpr bool is_euclidean_domain = true;
00372
00373         template<auto x>
00374         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00375
00376         template<typename v>
00377         using inject_ring_t = val<v>;
00378     };
00379
00380     template<typename Ring, typename X>
00381     struct Embed<Quotient<Ring, X>, Ring> {
00382         template<typename val>
00383         using type = typename val::raw_t;
00384     };
00385 } // namespace aerobus
00386
00387 // type_list
00388 namespace aerobus {
00389     template <typename... Ts>
00390     struct type_list;
00391
00392     namespace internal {
00393         template <typename T, typename... Us>
00394         struct pop_front_h {
00395             using tail = type_list<Us...>;
00396             using head = T;
00397         };
00398
00399         template <size_t index, typename L1, typename L2>
00400         struct split_h {
00401             private:
00402                 static_assert(index <= L2::length, "index out of bounds");
00403                 using a = typename L2::pop_front::type;
00404                 using b = typename L2::pop_front::tail;
00405                 using c = typename L1::template push_back<a>;
00406             public:

```

```

00452         using head = typename split_h<index - 1, c, b>::head;
00453         using tail = typename split_h<index - 1, c, b>::tail;
00454     };
00455
00456     template <typename L1, typename L2>
00457     struct split_h<0, L1, L2> {
00458         using head = L1;
00459         using tail = L2;
00460     };
00461
00462     template <size_t index, typename L, typename T>
00463     struct insert_h {
00464         static_assert(index <= L::length, "index out of bounds");
00465         using s = typename L::template split<index>;
00466         using left = typename s::head;
00467         using right = typename s::tail;
00468         using ll = typename left::template push_back<T>;
00469         using type = typename ll::template concat<right>;
00470     };
00471
00472     template <size_t index, typename L>
00473     struct remove_h {
00474         using s = typename L::template split<index>;
00475         using left = typename s::head;
00476         using right = typename s::tail;
00477         using rr = typename right::pop_front::tail;
00478         using type = typename left::template concat<rr>;
00479     };
00480 } // namespace internal
00481
00482 template <typename... Ts>
00483 struct type_list {
00484 private:
00485     template <typename T>
00486     struct concat_h;
00487
00488     template <typename... Us>
00489     struct concat_h<type_list<Us...> {
00490         using type = type_list<Ts..., Us...>;
00491     };
00492
00493 public:
00494     static constexpr size_t length = sizeof...(Ts);
00495
00496     template <typename T>
00497     using push_front = type_list<T, Ts...>;
00498
00499     template <size_t index>
00500     using at = internal::type_at_t<index, Ts...>;
00501
00502     struct pop_front {
00503         using type = typename internal::pop_front_h<Ts...>::head;
00504         using tail = typename internal::pop_front_h<Ts...>::tail;
00505     };
00506
00507     template <typename T>
00508     using push_back = type_list<Ts..., T>;
00509
00510     template <typename U>
00511     using concat = typename concat_h<U>::type;
00512
00513     template <size_t index>
00514     struct split {
00515 private:
00516         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00517
00518     public:
00519         using head = typename inner::head;
00520         using tail = typename inner::tail;
00521     };
00522
00523     template <typename T, size_t index>
00524     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00525
00526     template <size_t index>
00527     using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00528 };
00529
00530 template <>
00531 struct type_list<> {
00532     static constexpr size_t length = 0;
00533
00534     template <typename T>
00535     using push_front = type_list<T>;
00536
00537     template <typename T>
00538     using push_back = type_list<T>;
00539

```

```

00562
00563     template <typename U>
00564     using concat = U;
00565
00566     // TODO(jewave): assert index == 0
00567     template <typename T, size_t index>
00568     using insert = type_list<T>;
00569 };
00570 } // namespace aerobus
00571
00572 // i32
00573 namespace aerobus {
00574     struct i32 {
00575         using inner_type = int32_t;
00576         template<int32_t x>
00577         struct val {
00578             using enclosing_type = i32;
00579             static constexpr int32_t v = x;
00580
00581             template<typename valueType>
00582             static constexpr INLINED DEVICE valueType get() { return static_cast<valueType>(x); }
00583
00584             using is_zero_t = std::bool_constant<x == 0>;
00585
00586             static std::string to_string() {
00587                 return std::to_string(x);
00588             }
00589
00590             template<typename valueRing>
00591             static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
00592                 return static_cast<valueRing>(x);
00593             }
00594         };
00595     };
00596
00597     using zero = val<0>;
00598     using one = val<1>;
00599     static constexpr bool is_field = false;
00600     static constexpr bool is_euclidean_domain = true;
00601     template<auto x>
00602     using inject_constant_t = val<static_cast<int32_t>(x)>;
00603
00604     template<typename v>
00605     using inject_ring_t = v;
00606
00607 private:
00608     template<typename v1, typename v2>
00609     struct add {
00610         using type = val<v1::v + v2::v>;
00611     };
00612
00613     template<typename v1, typename v2>
00614     struct sub {
00615         using type = val<v1::v - v2::v>;
00616     };
00617
00618     template<typename v1, typename v2>
00619     struct mul {
00620         using type = val<v1::v * v2::v>;
00621     };
00622
00623     template<typename v1, typename v2>
00624     struct div {
00625         using type = val<v1::v / v2::v>;
00626     };
00627
00628     template<typename v1, typename v2>
00629     struct remainder {
00630         using type = val<v1::v % v2::v>;
00631     };
00632
00633     template<typename v1, typename v2>
00634     struct gt {
00635         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00636     };
00637
00638     template<typename v1, typename v2>
00639     struct lt {
00640         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00641     };
00642
00643     template<typename v1, typename v2>
00644     struct eq {
00645         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00646     };
00647
00648     template<typename v1>
00649     struct pos {

```

```

00667         using type = std::bool_constant<(v1::v > 0)>;
00668     };
00669
00670     public:
00671         template<typename v1, typename v2>
00672         using add_t = typename add<v1, v2>::type;
00673
00674         template<typename v1, typename v2>
00675         using sub_t = typename sub<v1, v2>::type;
00676
00677         template<typename v1, typename v2>
00678         using mul_t = typename mul<v1, v2>::type;
00679
00680         template<typename v1, typename v2>
00681         using div_t = typename div<v1, v2>::type;
00682
00683         template<typename v1, typename v2>
00684         using mod_t = typename remainder<v1, v2>::type;
00685
00686         template<typename v1, typename v2>
00687         using gt_t = typename gt<v1, v2>::type;
00688
00689         template<typename v1, typename v2>
00690         using lt_t = typename lt<v1, v2>::type;
00691
00692         template<typename v1, typename v2>
00693         using eq_t = typename eq<v1, v2>::type;
00694
00695         template<typename v1, typename v2>
00696         static constexpr bool eq_v = eq_t<v1, v2>::value;
00697
00698         template<typename v1, typename v2>
00699         using gcd_t = gcd_t<i32, v1, v2>;
00700
00701         template<typename v>
00702         using pos_t = typename pos<v>::type;
00703
00704         template<typename v>
00705         static constexpr bool pos_v = pos_t<v>::value;
00706     };
00707 } // namespace aerobus
00708
00709 // i64
00710 namespace aerobus {
00711     struct i64 {
00712         using inner_type = int64_t;
00713         template<int64_t x>
00714         struct val {
00715             using inner_type = int32_t;
00716             using enclosing_type = i64;
00717             static constexpr int64_t v = x;
00718
00719             template<typename valueType>
00720             static constexpr DEVICE INLINE valueType get() {
00721                 return static_cast<valueType>(x);
00722             }
00723
00724             using is_zero_t = std::bool_constant<x == 0>;
00725
00726             static std::string to_string() {
00727                 return std::to_string(x);
00728             }
00729
00730             template<typename valueRing>
00731             static constexpr DEVICE INLINE valueRing eval(const valueRing& v) {
00732                 return static_cast<valueRing>(x);
00733             }
00734         };
00735
00736         template<auto x>
00737         using inject_constant_t = val<static_cast<int64_t>(x)>;
00738
00739         template<typename v>
00740         using inject_ring_t = v;
00741
00742         using zero = val<0>;
00743         using one = val<1>;
00744         static constexpr bool is_field = false;
00745         static constexpr bool is_euclidean_domain = true;
00746
00747     private:
00748         template<typename v1, typename v2>
00749         struct add {
00750             using type = val<v1::v + v2::v>;
00751         };
00752
00753         template<typename v1, typename v2>

```

```

00835     struct sub {
00836         using type = val<v1::v - v2::v>;
00837     };
00838
00839     template<typename v1, typename v2>
00840     struct mul {
00841         using type = val<v1::v* v2::v>;
00842     };
00843
00844     template<typename v1, typename v2>
00845     struct div {
00846         using type = val<v1::v / v2::v>;
00847     };
00848
00849     template<typename v1, typename v2>
00850     struct remainder {
00851         using type = val<v1::v% v2::v>;
00852     };
00853
00854     template<typename v1, typename v2>
00855     struct gt {
00856         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00857     };
00858
00859     template<typename v1, typename v2>
00860     struct lt {
00861         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00862     };
00863
00864     template<typename v1, typename v2>
00865     struct eq {
00866         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00867     };
00868
00869     template<typename v>
00870     struct pos {
00871         using type = std::bool_constant<(v::v > 0)>;
00872     };
00873
00874 public:
00875     template<typename v1, typename v2>
00876     using add_t = typename add<v1, v2>::type;
00877
00878     template<typename v1, typename v2>
00879     using sub_t = typename sub<v1, v2>::type;
00880
00881     template<typename v1, typename v2>
00882     using mul_t = typename mul<v1, v2>::type;
00883
00884     template<typename v1, typename v2>
00885     using div_t = typename div<v1, v2>::type;
00886
00887     template<typename v1, typename v2>
00888     using mod_t = typename remainder<v1, v2>::type;
00889
00890     template<typename v1, typename v2>
00891     using gt_t = typename gt<v1, v2>::type;
00892
00893     template<typename v1, typename v2>
00894     static constexpr bool gt_v = gt_t<v1, v2>::value;
00895
00896     template<typename v1, typename v2>
00897     using lt_t = typename lt<v1, v2>::type;
00898
00899     template<typename v1, typename v2>
00900     static constexpr bool lt_v = lt_t<v1, v2>::value;
00901
00902     template<typename v1, typename v2>
00903     using eq_t = typename eq<v1, v2>::type;
00904
00905     template<typename v1, typename v2>
00906     static constexpr bool eq_v = eq_t<v1, v2>::value;
00907
00908     template<typename v1, typename v2>
00909     using gcd_t = gcd_t<i64, v1, v2>;
00910
00911     template<typename v>
00912     using pos_t = typename pos<v>::type;
00913
00914     template<typename v>
00915     static constexpr bool pos_v = pos_t<v>::value;
00916 };
00917
00918 template<>
00919 struct Embed<i32, i64> {
00920     template<typename val>
00921     using type = i64::val<static_cast<int64_t>(val::v)>;
00922 };

```

```

00985     };
00986 } // namespace aerobus
00987
00988 // z/pz
00989 namespace aerobus {
00994     template<int32_t p>
00995     struct zpz {
00996         using inner_type = int32_t;
00997         template<int32_t x>
00998         struct val {
01000             using enclosing_type = zpz<p>;
01002             static constexpr int32_t v = x % p;
01003
01004             template<typename valueType>
01005             static constexpr DEVICE INLINED valueType get() { return static_cast<valueType>(x % p); }
01006
01007             using is_zero_t = std::bool_constant<x % p == 0>;
01008             static std::string to_string() {
01009                 return std::to_string(x % p);
01010             }
01011
01012             template<typename valueRing>
01013             static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
01014                 return static_cast<valueRing>(x % p);
01015             }
01016         };
01017
01018         template<auto x>
01019         using inject_constant_t = val<static_cast<int32_t>(x)>;
01020
01021         using zero = val<0>;
01022         using one = val<1>;
01023         static constexpr bool is_field = is_prime<p>::value;
01024         static constexpr bool is_euclidean_domain = true;
01025
01026     private:
01027         template<typename v1, typename v2>
01028         struct add {
01029             using type = val<(v1::v + v2::v) % p>;
01030         };
01031
01032         template<typename v1, typename v2>
01033         struct sub {
01034             using type = val<(v1::v - v2::v) % p>;
01035         };
01036
01037         template<typename v1, typename v2>
01038         struct mul {
01039             using type = val<(v1::v * v2::v) % p>;
01040         };
01041
01042         template<typename v1, typename v2>
01043         struct div {
01044             using type = val<(v1::v % p) / (v2::v % p)>;
01045         };
01046
01047         template<typename v1, typename v2>
01048         struct remainder {
01049             using type = val<(v1::v % v2::v) % p>;
01050         };
01051
01052         template<typename v1, typename v2>
01053         struct gt {
01054             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01055         };
01056
01057         template<typename v1, typename v2>
01058         struct lt {
01059             using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01060         };
01061
01062         template<typename v1, typename v2>
01063         struct eq {
01064             using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01065         };
01066
01067         template<typename v1>
01068         struct pos {
01069             using type = std::bool_constant<(v1::v > 0)>;
01070         };
01071
01072     public:
01076         template<typename v1, typename v2>
01077         using add_t = typename add<v1, v2>::type;
01078
01082         template<typename v1, typename v2>
01083         using sub_t = typename sub<v1, v2>::type;

```



```

01084
01088     template<typename v1, typename v2>
01089     using mul_t = typename mul<v1, v2>::type;
01090
01094     template<typename v1, typename v2>
01095     using div_t = typename div<v1, v2>::type;
01096
01100     template<typename v1, typename v2>
01101     using mod_t = typename remainder<v1, v2>::type;
01102
01106     template<typename v1, typename v2>
01107     using gt_t = typename gt<v1, v2>::type;
01108
01112     template<typename v1, typename v2>
01113     static constexpr bool gt_v = gt_t<v1, v2>::value;
01114
01118     template<typename v1, typename v2>
01119     using lt_t = typename lt<v1, v2>::type;
01120
01124     template<typename v1, typename v2>
01125     static constexpr bool lt_v = lt_t<v1, v2>::value;
01126
01130     template<typename v1, typename v2>
01131     using eq_t = typename eq<v1, v2>::type;
01132
01136     template<typename v1, typename v2>
01137     static constexpr bool eq_v = eq_t<v1, v2>::value;
01138
01142     template<typename v1, typename v2>
01143     using gcd_t = gcd_t<i32, v1, v2>;
01144
01147     template<typename v1>
01148     using pos_t = typename pos<v1>::type;
01149
01152     template<typename v>
01153     static constexpr bool pos_v = pos_t<v>::value;
01154 };
01155
01156 template<int32_t x>
01157 struct Embed<zp<x>, i32> {
01158     template <typename val>
01159     using type = i32::val<val::v>;
01160 };
01161 } // namespace aerobus
01162
01163 // polynomial
01164 namespace aerobus {
01165     // coeffN x^N + ...
01170     template<typename Ring>
01171     requires IsEuclideanDomain<Ring>
01172     struct polynomial {
01173         static constexpr bool is_field = false;
01174         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01175
01179         template<typename coeffN, typename... coeffs>
01180         struct val {
01182             using ring_type = Ring;
01184             using enclosing_type = polynomial<Ring>;
01186             static constexpr size_t degree = sizeof...(coeffs);
01188             using aN = coeffN;
01190             using strip = val<coeffs...>;
01192             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01194             static constexpr bool is_zero_v = is_zero_t::value;
01195
01196         private:
01197             template<size_t index, typename E = void>
01198             struct coeff_at {};
01199
01200             template<size_t index>
01201             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01202                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01203             };
01204
01205             template<size_t index>
01206             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01207                 using type = typename Ring::zero;
01208             };
01209
01210         public:
01213             template<size_t index>
01214             using coeff_at_t = typename coeff_at<index>::type;
01215
01218             static std::string to_string() {
01219                 return string_helper<coeffN, coeffs...>::func();
01220             }
01221
01226             template<typename valueRing>

```

```

01227         static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {
01228             return horner_evaluation<valueRing, val>
01229                 ::template inner<0, degree + 1>
01230                 ::func(static_cast<valueRing>(0), x);
01231         }
01232     };
01233
01234     template<typename coeffN>
01235     struct val<coeffN> {
01236         using ring_type = Ring;
01237         using enclosing_type = polynomial<Ring>;
01238         static constexpr size_t degree = 0;
01239         using aN = coeffN;
01240         using strip = val<coeffN>;
01241         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01242
01243         static constexpr bool is_zero_v = is_zero_t::value;
01244
01245         template<size_t index, typename E = void>
01246         struct coeff_at {};
01247
01248         template<size_t index>
01249         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01250             using type = aN;
01251         };
01252
01253         template<size_t index>
01254         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01255             using type = typename Ring::zero;
01256         };
01257
01258         template<size_t index>
01259         using coeff_at_t = typename coeff_at<index>::type;
01260
01261         static std::string to_string() {
01262             return string_helper<coeffN>::func();
01263         }
01264
01265         template<typename valueRing>
01266         static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {
01267             return static_cast<valueRing>(aN::template get<valueRing>());
01268         }
01269     };
01270
01271     using zero = val<typename Ring::zero>;
01272     using one = val<typename Ring::one>;
01273     using X = val<typename Ring::one, typename Ring::zero>;
01274
01275 private:
01276     template<typename P, typename E = void>
01277     struct simplify;
01278
01279     template<typename P1, typename P2, typename I>
01280     struct add_low;
01281
01282     template<typename P1, typename P2>
01283     struct add {
01284         using type = typename simplify<typename add_low<
01285             P1,
01286             P2,
01287             internal::make_index_sequence_reverse<
01288                 std::max(P1::degree, P2::degree) + 1
01289             >::type>::type;
01290     };
01291
01292     template<typename P1, typename P2, typename I>
01293     struct sub_low;
01294
01295     template<typename P1, typename P2, typename I>
01296     struct mul_low;
01297
01298     template<typename v1, typename v2>
01299     struct mul {
01300         using type = typename mul_low<
01301             v1,
01302             v2,
01303             internal::make_index_sequence_reverse<
01304                 v1::degree + v2::degree + 1
01305             >::type;
01306     };
01307
01308     template<typename coeff, size_t deg>
01309     struct monomial;
01310
01311     template<typename v, typename E = void>
01312     struct derive_helper {};
01313
01314

```

```

01322     template<typename v>
01323     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01324         using type = zero;
01325     };
01326
01327     template<typename v>
01328     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01329         using type = typename add<
01330             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01331             typename monomial<
01332                 typename Ring::template mul_t<
01333                     typename v::aN,
01334                     typename Ring::template inject_constant_t<(v::degree)>
01335                 >,
01336                 v::degree - 1
01337             >::type
01338         >::type;
01339     };
01340
01341     template<typename v1, typename v2, typename E = void>
01342     struct eq_helper {};
01343
01344     template<typename v1, typename v2>
01345     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01346         using type = std::false_type;
01347     };
01348
01349
01350     template<typename v1, typename v2>
01351     struct eq_helper<v1, v2, std::enable_if_t<
01352         v1::degree == v2::degree &&
01353         (v1::degree != 0 || v2::degree != 0) &&
01354         std::is_same<
01355             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01356             std::false_type
01357         >::value
01358     >
01359     > {
01360         using type = std::false_type;
01361     };
01362
01363     template<typename v1, typename v2>
01364     struct eq_helper<v1, v2, std::enable_if_t<
01365         v1::degree == v2::degree &&
01366         (v1::degree != 0 || v2::degree != 0) &&
01367         std::is_same<
01368             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01369             std::true_type
01370         >::value
01371     > {
01372         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01373     };
01374
01375     template<typename v1, typename v2>
01376     struct eq_helper<v1, v2, std::enable_if_t<
01377         v1::degree == v2::degree &&
01378         (v1::degree == 0)
01379     > {
01380         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01381     };
01382
01383     template<typename v1, typename v2, typename E = void>
01384     struct lt_helper {};
01385
01386     template<typename v1, typename v2>
01387     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01388         using type = std::true_type;
01389     };
01390
01391     template<typename v1, typename v2>
01392     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01393         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01394     };
01395
01396     template<typename v1, typename v2>
01397     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01398         using type = std::false_type;
01399     };
01400
01401     template<typename v1, typename v2, typename E = void>
01402     struct gt_helper {};
01403
01404     template<typename v1, typename v2>
01405     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01406         using type = std::true_type;
01407     };
01408

```

```

01409     template<typename v1, typename v2>
01410     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01411         using type = std::false_type;
01412     };
01413
01414     template<typename v1, typename v2>
01415     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01416         using type = std::false_type;
01417     };
01418
01419     // when high power is zero : strip
01420     template<typename P>
01421     struct simplify<P, std::enable_if_t<
01422         std::is_same<
01423             typename Ring::zero,
01424             typename P::aN
01425         >::value && (P::degree > 0)
01426     >> {
01427         using type = typename simplify<typename P::strip>::type;
01428     };
01429
01430     // otherwise : do nothing
01431     template<typename P>
01432     struct simplify<P, std::enable_if_t<
01433         !std::is_same<
01434             typename Ring::zero,
01435             typename P::aN
01436         >::value && (P::degree > 0)
01437     >> {
01438         using type = P;
01439     };
01440
01441     // do not simplify constants
01442     template<typename P>
01443     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01444         using type = P;
01445     };
01446
01447     // addition at
01448     template<typename P1, typename P2, size_t index>
01449     struct add_at {
01450         using type =
01451             typename Ring::template add_t<
01452                 typename P1::template coeff_at_t<index>,
01453                 typename P2::template coeff_at_t<index>>;
01454     };
01455
01456     template<typename P1, typename P2, size_t index>
01457     using add_at_t = typename add_at<P1, P2, index>::type;
01458
01459     template<typename P1, typename P2, std::size_t... I>
01460     struct add_low<P1, P2, std::index_sequence<I...>> {
01461         using type = val<add_at_t<P1, P2, I>...>;
01462     };
01463
01464     // subtraction at
01465     template<typename P1, typename P2, size_t index>
01466     struct sub_at {
01467         using type =
01468             typename Ring::template sub_t<
01469                 typename P1::template coeff_at_t<index>,
01470                 typename P2::template coeff_at_t<index>>;
01471     };
01472
01473     template<typename P1, typename P2, size_t index>
01474     using sub_at_t = typename sub_at<P1, P2, index>::type;
01475
01476     template<typename P1, typename P2, std::size_t... I>
01477     struct sub_low<P1, P2, std::index_sequence<I...>> {
01478         using type = val<sub_at_t<P1, P2, I>...>;
01479     };
01480
01481     template<typename P1, typename P2>
01482     struct sub {
01483         using type = typename simplify<typename sub_low<
01484             P1,
01485             P2,
01486             internal::make_index_sequence_reverse<
01487                 std::max(P1::degree, P2::degree) + 1
01488             >::type>::type;
01489     };
01490
01491     // multiplication at
01492     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01493     struct mul_at_loop_helper {
01494         using type = typename Ring::template add_t<
01495             typename Ring::template mul_t<

```

```

01496         typename v1::template coeff_at_t<index>,
01497         typename v2::template coeff_at_t<k - index>
01498     >,
01499     typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01500 >;
01501 };
01502
01503 template<typename v1, typename v2, size_t k, size_t stop>
01504 struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01505     using type = typename Ring::template mul_t<
01506         typename v1::template coeff_at_t<stop>,
01507         typename v2::template coeff_at_t<0>>;
01508 };
01509
01510 template <typename v1, typename v2, size_t k, typename E = void>
01511 struct mul_at {};
01512
01513 template<typename v1, typename v2, size_t k>
01514 struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01515     using type = typename Ring::zero;
01516 };
01517
01518 template<typename v1, typename v2, size_t k>
01519 struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01520     using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01521 };
01522
01523 template<typename P1, typename P2, size_t index>
01524 using mul_at_t = typename mul_at<P1, P2, index>::type;
01525
01526 template<typename P1, typename P2, std::size_t... I>
01527 struct mul_low<P1, P2, std::index_sequence<I...> {
01528     using type = val<mul_at_t<P1, P2, I>...>;
01529 };
01530
01531 // division helper
01532 template< typename A, typename B, typename Q, typename R, typename E = void>
01533 struct div_helper {};
01534
01535 template<typename A, typename B, typename Q, typename R>
01536 struct div_helper<A, B, Q, R, std::enable_if_t<
01537     (R::degree < B::degree) ||
01538     (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01539     using q_type = Q;
01540     using mod_type = R;
01541     using gcd_type = B;
01542 };
01543
01544 template<typename A, typename B, typename Q, typename R>
01545 struct div_helper<A, B, Q, R, std::enable_if_t<
01546     (R::degree >= B::degree) &&
01547     !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01548     private: // NOLINT
01549         using rN = typename R::aN;
01550         using bN = typename B::aN;
01551         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01552             B::degree>::type;
01553         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01554         using qq = typename add<Q, pT>::type;
01555     public:
01556         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01557         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01558         using gcd_type = rr;
01559 };
01560
01561 template<typename A, typename B>
01562 struct div {
01563     static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01564     using q_type = typename div_helper<A, B, zero, A>::q_type;
01565     using m_type = typename div_helper<A, B, zero, A>::mod_type;
01566 };
01567
01568 template<typename P>
01569 struct make_unit {
01570     using type = typename div<P, val<typename P::aN>::q_type>;
01571 };
01572
01573 template<typename coeff, size_t deg>
01574 struct monomial {
01575     using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01576 };
01577
01578 template<typename coeff>
01579 struct monomial<coeff, 0> {
01580     using type = val<coeff>;
01581 };

```

```

01582
01583     template<typename valueRing, typename P>
01584     struct horner_evaluation {
01585         template<size_t index, size_t stop>
01586         struct inner {
01587             static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01588                 constexpr valueRing coeff =
01589                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
get<valueRing>());
01590                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
accum + coeff, x);
01591             }
01592         };
01593
01594         template<size_t stop>
01595         struct inner<stop, stop> {
01596             static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01597                 return accum;
01598             }
01599         };
01600     };
01601
01602     template<typename coeff, typename... coeffs>
01603     struct string_helper {
01604         static std::string func() {
01605             std::string tail = string_helper<coeffs...>::func();
01606             std::string result = "";
01607             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01608                 return tail;
01609             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01610                 if (sizeof...(coeffs) == 1) {
01611                     result += "x";
01612                 } else {
01613                     result += "x^" + std::to_string(sizeof...(coeffs));
01614                 }
01615             } else {
01616                 if (sizeof...(coeffs) == 1) {
01617                     result += coeff::to_string() + " x";
01618                 } else {
01619                     result += coeff::to_string()
01620                         + " x^" + std::to_string(sizeof...(coeffs));
01621                 }
01622             }
01623
01624             if (!tail.empty()) {
01625                 result += " + " + tail;
01626             }
01627
01628             return result;
01629         }
01630     };
01631
01632     template<typename coeff>
01633     struct string_helper<coeff> {
01634         static std::string func() {
01635             if (!std::is_same<coeff, typename Ring::zero>::value) {
01636                 return coeff::to_string();
01637             } else {
01638                 return "";
01639             }
01640         }
01641     };
01642
01643 public:
01644     template<typename P>
01645     using simplify_t = typename simplify<P>::type;
01646
01647     template<typename v1, typename v2>
01648     using add_t = typename add<v1, v2>::type;
01649
01650     template<typename v1, typename v2>
01651     using sub_t = typename sub<v1, v2>::type;
01652
01653     template<typename v1, typename v2>
01654     using mul_t = typename mul<v1, v2>::type;
01655
01656     template<typename v1, typename v2>
01657     using eq_t = typename eq_helper<v1, v2>::type;
01658
01659     template<typename v1, typename v2>
01660     using lt_t = typename lt_helper<v1, v2>::type;
01661
01662     template<typename v1, typename v2>
01663     using gt_t = typename gt_helper<v1, v2>::type;
01664

```

```

01688     template<typename v1, typename v2>
01689     using div_t = typename div<v1, v2>::q_type;
01690
01694     template<typename v1, typename v2>
01695     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01696
01700     template<typename coeff, size_t deg>
01701     using monomial_t = typename monomial<coeff, deg>::type;
01702
01705     template<typename v>
01706     using derive_t = typename derive_helper<v>::type;
01707
01710     template<typename v>
01711     using pos_t = typename Ring::template pos_t<typename v::aN>;
01712
01715     template<typename v>
01716     static constexpr bool pos_v = pos_t<v>::value;
01717
01721     template<typename v1, typename v2>
01722     using gcd_t = std::conditional_t<
01723         Ring::is_euclidean_domain,
01724         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01725         void>;
01726
01730     template<auto x>
01731     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01732
01736     template<typename v>
01737     using inject_ring_t = val<v>;
01738 };
01739 } // namespace aerobus
01740
01741 // fraction field
01742 namespace aerobus {
01743     namespace internal {
01744         template<typename Ring, typename E = void>
01745         requires IsEuclideanDomain<Ring>
01746         struct _FractionField {};
01747
01748         template<typename Ring>
01749         requires IsEuclideanDomain<Ring>
01750         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
01751             static constexpr bool is_field = true;
01752             static constexpr bool is_euclidean_domain = true;
01753
01754         private:
01755             template<typename vall, typename val2, typename E = void>
01756             struct to_string_helper {};
01757
01758             template<typename vall, typename val2>
01759             struct to_string_helper <vall, val2,
01760                 std::enable_if_t<
01761                     Ring::template eq_t<
01762                         val2, typename Ring::one
01763                         >::value
01764                     >
01765             > {
01766                 static std::string func() {
01767                     return "(" + vall::to_string() + ") / (" + val2::to_string() + ")";
01768                 }
01769             };
01770
01771             template<typename vall, typename val2>
01772             struct to_string_helper<vall, val2,
01773                 std::enable_if_t<
01774                     !Ring::template eq_t<
01775                         val2,
01776                         typename Ring::one
01777                         >::value
01778                     >
01779             > {
01780                 static std::string func() {
01781                     return "(" + vall::to_string() + ") / (" + val2::to_string() + ")";
01782                 }
01783             };
01784         };
01785
01786     public:
01787         template<typename vall, typename val2>
01788         struct val {
01789             using x = vall;
01790             using y = val2;
01791             using is_zero_t = typename vall::is_zero_t;
01792             static constexpr bool is_zero_v = vall::is_zero_t::value;
01793
01794             using ring_type = Ring;
01795             using enclosing_type = _FractionField<Ring>;
01796         };
01797     };
01804

```

```

01807         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01808
01812     template<typename valueType>
01813     static constexpr DEVICE INLINED valueType get() {
01814         return static_cast<valueType>(x::v) / static_cast<valueType>(y::v);
01815     }
01816
01819     static std::string to_string() {
01820         return to_string_helper<vall, val2>::func();
01821     }
01822
01827     template<typename valueRing>
01828     static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
01829         return x::eval(v) / y::eval(v);
01830     }
01831 };
01832
01834 using zero = val<typename Ring::zero, typename Ring::one>;
01836 using one = val<typename Ring::one, typename Ring::one>;
01837
01840 template<typename v>
01841 using inject_t = val<v, typename Ring::one>;
01842
01845 template<auto x>
01846 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
01847
01850 template<typename v>
01851 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01852
01854 using ring_type = Ring;
01855
01856 private:
01857     template<typename v, typename E = void>
01858     struct simplify {};
01859
01860     // x = 0
01861     template<typename v>
01862     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01863         using type = typename _FractionField<Ring>::zero;
01864     };
01865
01866     // x != 0
01867     template<typename v>
01868     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01869     private:
01870         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01871         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01872         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01873
01874         using posx = std::conditional_t<
01875             !Ring::template pos_v<newy>,
01876             typename Ring::template sub_t<typename Ring::zero, newx>,
01877             newx>;
01878         using posy = std::conditional_t<
01879             !Ring::template pos_v<newy>,
01880             typename Ring::template sub_t<typename Ring::zero, newy>,
01881             newy>;
01882     public:
01883         using type = typename _FractionField<Ring>::template val<posx, posy>;
01884     };
01885
01886 public:
01887     template<typename v>
01888     using simplify_t = typename simplify<v>::type;
01889
01892 private:
01893     template<typename v1, typename v2>
01894     struct add {
01895     private:
01896         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01897         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01898         using dividend = typename Ring::template add_t<a, b>;
01899         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01900         using g = typename Ring::template gcd_t<dividend, diviser>;
01901
01902     public:
01903         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01904     };
01905
01906     template<typename v>
01907     struct pos {
01908     using type = std::conditional_t<
01909         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01910         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01911         std::true_type,

```



```

01912         std::false_type>;
01913     };
01914
01915     template<typename v1, typename v2>
01916     struct sub {
01917     private:
01918         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01919         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01920         using dividend = typename Ring::template sub_t<a, b>;
01921         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01922         using g = typename Ring::template gcd_t<dividend, divider>;
01923
01924     public:
01925         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
01926     };
01927
01928     template<typename v1, typename v2>
01929     struct mul {
01930     private:
01931         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01932         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01933
01934     public:
01935         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
01936     };
01937
01938     template<typename v1, typename v2, typename E = void>
01939     struct div {};
01940
01941     template<typename v1, typename v2>
01942     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01943     private:
01944         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01945         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01946
01947     public:
01948         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
01949     };
01950
01951     template<typename v1, typename v2>
01952     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01953         using type = one;
01954     };
01955
01956     template<typename v1, typename v2>
01957     struct eq {
01958     private:
01959         using type = std::conditional_t<
std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
std::true_type,
std::false_type>;
01960     };
01961
01962     template<typename v1, typename v2, typename E = void>
01963     struct gt;
01964
01965     template<typename v1, typename v2>
01966     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
>> {
01967         using type = std::false_type;
01968     };
01969
01970     template<typename v1, typename v2>
01971     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(!pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
01972         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
01973     };
01974
01975     template<typename v1, typename v2>
01976     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
01977         using type = std::true_type;
01978     };
01979
01980     template<typename v1, typename v2>
01981     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
01982     };
01983
01984     template<typename v1, typename v2>
01985     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
01986     };
01987
01988     template<typename v1, typename v2>
01989     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
01990     };
01991
01992     template<typename v1, typename v2>
01993     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
01994     };
01995
01996     template<typename v1, typename v2>
01997     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
01998     };

```

```

01997         (!pos<v1>::type::value) && (pos<v2>::type::value)
01998     >> {
01999         using type = std::false_type;
02000     };
02001
02002     template<typename v1, typename v2>
02003     struct gt<v1, v2, std::enable_if_t<
02004         (!eq<v1, v2>::type::value) &&
02005         (pos<v1>::type::value) && (pos<v2>::type::value)
02006     >> {
02007         using type = typename Ring::template gt_t<
02008             typename Ring::template mul_t<v1::x, v2::y>,
02009             typename Ring::template mul_t<v2::y, v2::x>
02010         >;
02011     };
02012
02013     public:
02014         template<typename v1, typename v2>
02015         using add_t = typename add<v1, v2>::type;
02016
02017         template<typename v1, typename v2>
02018         using mod_t = zero;
02019
02020         template<typename v1, typename v2>
02021         using gcd_t = v1;
02022
02023         template<typename v1, typename v2>
02024         using sub_t = typename sub<v1, v2>::type;
02025
02026         template<typename v1, typename v2>
02027         using mul_t = typename mul<v1, v2>::type;
02028
02029         template<typename v1, typename v2>
02030         using div_t = typename div<v1, v2>::type;
02031
02032         template<typename v1, typename v2>
02033         using eq_t = typename eq<v1, v2>::type;
02034
02035         template<typename v1, typename v2>
02036         static constexpr bool eq_v = eq<v1, v2>::type::value;
02037
02038         template<typename v1, typename v2>
02039         using gt_t = typename gt<v1, v2>::type;
02040
02041         template<typename v1, typename v2>
02042         static constexpr bool gt_v = gt<v1, v2>::type::value;
02043
02044         template<typename v1>
02045         using pos_t = typename pos<v1>::type;
02046
02047         template<typename v>
02048         static constexpr bool pos_v = pos_t<v>::value;
02049     };
02050
02051     template<typename Ring, typename E = void>
02052     requires IsEuclideanDomain<Ring>
02053     struct FractionFieldImpl {};
02054
02055     // fraction field of a field is the field itself
02056     template<typename Field>
02057     requires IsEuclideanDomain<Field>
02058     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02059         using type = Field;
02060         template<typename v>
02061         using inject_t = v;
02062     };
02063
02064     // fraction field of a ring is the actual fraction field
02065     template<typename Ring>
02066     requires IsEuclideanDomain<Ring>
02067     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02068         using type = _FractionField<Ring>;
02069     };
02070 } // namespace internal
02071
02072     template<typename Ring>
02073     requires IsEuclideanDomain<Ring>
02074     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02075
02076     template<typename Ring>
02077     struct Embed<Ring, FractionField<Ring> {
02078         template<typename v>
02079         using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02080     };
02081 } // namespace aerobus
02082
02083
02084

```

```

02124 // short names for common types
02125 namespace aerobus {
02128     using q32 = FractionField<i32>;
02129
02132     using fpq32 = FractionField<polynomial<q32>>;
02133
02136     using q64 = FractionField<i64>;
02137
02139     using pi64 = polynomial<i64>;
02140
02142     using pq64 = polynomial<q64>;
02143
02145     using fpq64 = FractionField<polynomial<q64>>;
02146
02151     template<typename Ring, typename v1, typename v2>
02152     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02153
02160     template<typename v>
02161     using embed_int_poly_in_fractions_t =
02162         typename Embed<
02163             polynomial<typename v::ring_type>,
02164             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02165
02169     template<int64_t p, int64_t q>
02170     using make_q64_t = typename q64::template simplify_t<
02171         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02172
02176     template<int32_t p, int32_t q>
02177     using make_q32_t = typename q32::template simplify_t<
02178         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02179
02184     template<typename Ring, typename v1, typename v2>
02185     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02190     template<typename Ring, typename v1, typename v2>
02191     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02192
02193     template<>
02194     struct Embed<q32, q64> {
02195         template<typename v>
02196         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02197     };
02198
02199     template<typename Small, typename Large>
02200     struct Embed<polynomial<Small>, polynomial<Large>> {
02201     private:
02202         template<typename v, typename i>
02203         struct at_low;
02204
02205         template<typename v, size_t i>
02206         struct at_index {
02207             using type = typename Embed<Small, Large>::template
02208             type<typename v::template coeff_at_t<i>>;
02209         };
02210
02211         template<typename v, size_t... Is>
02212         struct at_low<v, std::index_sequence<Is...>> {
02213             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02214         };
02215     public:
02216         template<typename v>
02217         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02218         1>::type;
02219     };
02220
02223     template<typename Ring, auto... xs>
02224     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02225         typename Ring::template inject_constant_t<xs>...>;
02226
02230     template<typename Ring, auto... xs>
02231     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02232         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02233 } // namespace aerobus
02234
02235 // Taylor series and common integers (factorial, bernoulli...) appearing in Taylor coefficients
02236 namespace aerobus {
02237     namespace internal {
02238         template<typename T, size_t x, typename E = void>
02239         struct factorial {};
02240
02241         template<typename T, size_t x>
02242         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02243     private:
02244         template<typename, size_t, typename>
02245         friend struct factorial;
02246     public:
02247         using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,

```

```

    x - 1>::type>;
02248         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02249     };
02250
02251     template<typename T>
02252     struct factorial<T, 0> {
02253     public:
02254         using type = typename T::one;
02255         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02256     };
02257     } // namespace internal
02258
02262     template<typename T, size_t i>
02263     using factorial_t = typename internal::factorial<T, i>::type;
02264
02268     template<typename T, size_t i>
02269     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02270
02271     namespace internal {
02272         template<typename T, size_t k, size_t n, typename E = void>
02273         struct combination_helper {};
02274
02275         template<typename T, size_t k, size_t n>
02276         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02277             using type = typename FractionField<T>::template mul_t<
02278                 typename combination_helper<T, k - 1, n - 1>::type,
02279                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02280         };
02281
02282         template<typename T, size_t k, size_t n>
02283         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02284             using type = typename combination_helper<T, n - k, n>::type;
02285         };
02286
02287         template<typename T, size_t n>
02288         struct combination_helper<T, 0, n> {
02289             using type = typename FractionField<T>::one;
02290         };
02291
02292         template<typename T, size_t k, size_t n>
02293         struct combination {
02294             using type = typename internal::combination_helper<T, k, n>::type::x;
02295             static constexpr typename T::inner_type value =
02296                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02297         };
02298     } // namespace internal
02299
02302     template<typename T, size_t k, size_t n>
02303     using combination_t = typename internal::combination<T, k, n>::type;
02304
02309     template<typename T, size_t k, size_t n>
02310     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02311
02312     namespace internal {
02313         template<typename T, size_t m>
02314         struct bernoulli;
02315
02316         template<typename T, typename accum, size_t k, size_t m>
02317         struct bernoulli_helper {
02318             using type = typename bernoulli_helper<
02319                 T,
02320                 addfractions_t<T,
02321                     accum,
02322                     mulfractions_t<T,
02323                         makefraction_t<T,
02324                             combination_t<T, k, m + 1>,
02325                             typename T::one>,
02326                             typename bernoulli<T, k>::type
02327                         >,
02328                     >,
02329                     k + 1,
02330                     m>::type;
02331         };
02332
02333         template<typename T, typename accum, size_t m>
02334         struct bernoulli_helper<T, accum, m, m> {
02335             using type = accum;
02336         };
02337
02338
02339
02340         template<typename T, size_t m>
02341         struct bernoulli {
02342             using type = typename FractionField<T>::template mul_t<

```

```

02343         typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02344         makefraction_t<T,
02345         typename T::template val<static_cast<typename T::inner_type>(-1)>,
02346         typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02347         >
02348     >;
02349
02350     template<typename floatType>
02351     static constexpr floatType value = type::template get<floatType>();
02352 };
02353
02354     template<typename T>
02355     struct bernoulli<T, 0> {
02356         using type = typename FractionField<T>::one;
02357
02358         template<typename floatType>
02359         static constexpr floatType value = type::template get<floatType>();
02360     };
02361 } // namespace internal
02362
02363     template<typename T, size_t n>
02364     using bernoulli_t = typename internal::bernoulli<T, n>::type;
02365
02366     template<typename FloatType, typename T, size_t n>
02367     inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02368
02369 // bell numbers
02370 namespace internal {
02371     template<typename T, size_t n, typename E = void>
02372     struct bell_helper;
02373
02374     template<typename T, size_t n>
02375     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02376         template<typename accum, size_t i, size_t stop>
02377         struct sum_helper {
02378             private:
02379                 using left = typename T::template mul_t<
02380                     combination_t<T, i, n-1>,
02381                     typename bell_helper<T, i>::type>;
02382                 using new_accum = typename T::template add_t<accum, left>;
02383             public:
02384                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02385         };
02386
02387         template<typename accum, size_t stop>
02388         struct sum_helper<accum, stop, stop> {
02389             using type = accum;
02390         };
02391
02392         using type = typename sum_helper<typename T::zero, 0, n>::type;
02393     };
02394
02395     template<typename T>
02396     struct bell_helper<T, 0> {
02397         using type = typename T::one;
02398     };
02399
02400     template<typename T>
02401     struct bell_helper<T, 1> {
02402         using type = typename T::one;
02403     };
02404 } // namespace internal
02405
02406     template<typename T, size_t n>
02407     using bell_t = typename internal::bell_helper<T, n>::type;
02408
02409     template<typename T, size_t n>
02410     static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02411
02412 namespace internal {
02413     template<typename T, int k, typename E = void>
02414     struct alternate {};
02415
02416     template<typename T, int k>
02417     struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
02418         using type = typename T::one;
02419         static constexpr typename T::inner_type value = type::template get<typename
02420             T::inner_type>();
02421     };
02422
02423     template<typename T, int k>
02424     struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
02425         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02426         static constexpr typename T::inner_type value = type::template get<typename
02427             T::inner_type>();
02428     };
02429 } // namespace internal

```

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02441
02442 template<typename T, int k>
02443 using alternate_t = typename internal::alternate<T, k>::type;
02444
02445 namespace internal {
02446     template<typename T, int n, int k, typename E = void>
02447     struct stirling_helper {};
02448
02449     template<typename T>
02450     struct stirling_helper<T, 0, 0> {
02451         using type = typename T::one;
02452     };
02453
02454     template<typename T, int n>
02455     struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
02456         using type = typename T::zero;
02457     };
02458
02459     template<typename T, int n>
02460     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
02461         using type = typename T::zero;
02462     };
02463
02464     template<typename T, int n, int k>
02465     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)>> {
02466         using type = typename T::template sub_t<
02467             typename stirling_helper<T, n-1, k-1>::type,
02468             typename T::template mul_t<
02469                 typename T::template inject_constant_t<n-1>,
02470                 typename stirling_helper<T, n-1, k>::type
02471             >>;
02472     };
02473 } // namespace internal
02474
02475 template<typename T, int n, int k>
02476 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02477
02478 template<typename T, int n, int k>
02479 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02480
02481 template<typename T, int n, int k>
02482 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02483
02484 template<typename T, int n, int k>
02485 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02486
02487 template<typename T, size_t k>
02488 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02489
02490 namespace internal {
02491     template<typename T>
02492     struct pow_scalar {
02493         template<size_t p>
02494         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02495             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02496             x * func<p/2>(x) * func<p/2>(x);
02497         }
02498     };
02499
02500     template<typename T, typename p, size_t n, typename E = void>
02501     requires IsEuclideanDomain<T>
02502     struct pow;
02503
02504     template<typename T, typename p, size_t n>
02505     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
02506         using type = typename T::template mul_t<
02507             typename pow<T, p, n/2>::type,
02508             typename pow<T, p, n/2>::type
02509         >;
02510     };
02511
02512     template<typename T, typename p, size_t n>
02513     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
02514         using type = typename T::template mul_t<
02515             p,
02516             typename T::template mul_t<
02517                 typename pow<T, p, n/2>::type,
02518                 typename pow<T, p, n/2>::type
02519             >
02520         >;
02521     };
02522
02523     template<typename T, typename p, size_t n>
02524     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
02525 } // namespace internal
02526
02527

```

```

02552     template<typename T, typename p, size_t n>
02553     using pow_t = typename internal::pow<T, p, n>::type;
02554
02555     template<typename T, typename p, size_t n>
02556     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02561
02562     template<typename T, size_t p>
02563     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
internal::pow_scalar<T>::template func<p>(x); }
02564
02565     namespace internal {
02566         template<typename, template<typename, size_t> typename, class>
02567         struct make_taylor_impl;
02568
02569         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02570         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02571             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
02572         };
02573     }
02574
02575     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02576     using taylor = typename internal::make_taylor_impl<
02577         T,
02578         coeff_at,
02579         internal::make_index_sequence_reverse<deg + 1>::type;
02584
02585     namespace internal {
02586         template<typename T, size_t i>
02587         struct exp_coeff {
02588             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02589         };
02590
02591         template<typename T, size_t i, typename E = void>
02592         struct sin_coeff_helper {};
02593
02594         template<typename T, size_t i>
02595         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02596             using type = typename FractionField<T>::zero;
02597         };
02598
02599         template<typename T, size_t i>
02600         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02601             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02602         };
02603
02604         template<typename T, size_t i>
02605         struct sin_coeff {
02606             using type = typename sin_coeff_helper<T, i>::type;
02607         };
02608
02609         template<typename T, size_t i, typename E = void>
02610         struct sh_coeff_helper {};
02611
02612         template<typename T, size_t i>
02613         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02614             using type = typename FractionField<T>::zero;
02615         };
02616
02617         template<typename T, size_t i>
02618         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02619             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02620         };
02621
02622         template<typename T, size_t i>
02623         struct sh_coeff {
02624             using type = typename sh_coeff_helper<T, i>::type;
02625         };
02626
02627         template<typename T, size_t i, typename E = void>
02628         struct cos_coeff_helper {};
02629
02630         template<typename T, size_t i>
02631         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02632             using type = typename FractionField<T>::zero;
02633         };
02634
02635         template<typename T, size_t i>
02636         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02637             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02638         };
02639
02640         template<typename T, size_t i>
02641         struct cos_coeff {
02642             using type = typename cos_coeff_helper<T, i>::type;
02643         };
02644

```

```

02645     template<typename T, size_t i, typename E = void>
02646     struct cosh_coeff_helper {};
02647
02648     template<typename T, size_t i>
02649     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02650         using type = typename FractionField<T>::zero;
02651     };
02652
02653     template<typename T, size_t i>
02654     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02655         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02656     };
02657
02658     template<typename T, size_t i>
02659     struct cosh_coeff {
02660         using type = typename cosh_coeff_helper<T, i>::type;
02661     };
02662
02663     template<typename T, size_t i>
02664     struct geom_coeff { using type = typename FractionField<T>::one; };
02665
02666
02667     template<typename T, size_t i, typename E = void>
02668     struct atan_coeff_helper;
02669
02670     template<typename T, size_t i>
02671     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02672         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>>;
02673     };
02674
02675     template<typename T, size_t i>
02676     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02677         using type = typename FractionField<T>::zero;
02678     };
02679
02680     template<typename T, size_t i>
02681     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02682
02683     template<typename T, size_t i, typename E = void>
02684     struct asin_coeff_helper;
02685
02686     template<typename T, size_t i>
02687     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02688         using type = makefraction_t<T,
02689             factorial_t<T, i - 1>,
02690             typename T::template mul_t<
02691                 typename T::template val<i>,
02692                 T::template mul_t<
02693                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
02694                     pow<T, factorial_t<T, i / 2>, 2
02695                 >
02696             >
02697             >>;
02698     };
02699
02700     template<typename T, size_t i>
02701     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02702         using type = typename FractionField<T>::zero;
02703     };
02704
02705     template<typename T, size_t i>
02706     struct asin_coeff {
02707         using type = typename asin_coeff_helper<T, i>::type;
02708     };
02709
02710     template<typename T, size_t i>
02711     struct lnpl_coeff {
02712         using type = makefraction_t<T,
02713             alternate_t<T, i + 1>,
02714             typename T::template val<i>>;
02715     };
02716
02717     template<typename T>
02718     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02719
02720     template<typename T, size_t i, typename E = void>
02721     struct asinh_coeff_helper;
02722
02723     template<typename T, size_t i>
02724     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02725         using type = makefraction_t<T,
02726             typename T::template mul_t<
02727                 alternate_t<T, i / 2>,
02728                 factorial_t<T, i - 1>
02729             >,
02730             typename T::template mul_t<
02731                 typename T::template mul_t<

```



```

02732         typename T::template val<i>,
02733         pow_t<T, factorial_t<T, i / 2>, 2>
02734     >,
02735     pow_t<T, typename T::template inject_constant_t<4>, i / 2>
02736 >
02737 >;
02738 };
02739
02740 template<typename T, size_t i>
02741 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02742     using type = typename FractionField<T>::zero;
02743 };
02744
02745 template<typename T, size_t i>
02746 struct asinh_coeff {
02747     using type = typename asinh_coeff_helper<T, i>::type;
02748 };
02749
02750 template<typename T, size_t i, typename E = void>
02751 struct atanh_coeff_helper;
02752
02753 template<typename T, size_t i>
02754 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02755     // 1/i
02756     using type = typename FractionField<T>::template val<
02757         typename T::one,
02758         typename T::template inject_constant_t<i>;
02759 };
02760
02761 template<typename T, size_t i>
02762 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02763     using type = typename FractionField<T>::zero;
02764 };
02765
02766 template<typename T, size_t i>
02767 struct atanh_coeff {
02768     using type = typename atanh_coeff_helper<T, i>::type;
02769 };
02770
02771 template<typename T, size_t i, typename E = void>
02772 struct tan_coeff_helper;
02773
02774 template<typename T, size_t i>
02775 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02776     using type = typename FractionField<T>::zero;
02777 };
02778
02779 template<typename T, size_t i>
02780 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02781 private:
02782     // 4^((i+1)/2)
02783     using _4p = typename FractionField<T>::template inject_t<
02784         pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
02785     // 4^((i+1)/2) - 1
02786     using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02787     // (-1)^((i-1)/2)
02788     using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02789     using dividend = typename FractionField<T>::template mul_t<
02790         altp,
02791         FractionField<T>::template mul_t<
02792             _4p,
02793             FractionField<T>::template mul_t<
02794                 _4pml,
02795                 bernoulli_t<T, (i + 1)>
02796             >
02797         >
02798     >;
02799 public:
02800     using type = typename FractionField<T>::template div_t<dividend,
02801         typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02802 };
02803
02804 template<typename T, size_t i>
02805 struct tan_coeff {
02806     using type = typename tan_coeff_helper<T, i>::type;
02807 };
02808
02809 template<typename T, size_t i, typename E = void>
02810 struct tanh_coeff_helper;
02811
02812 template<typename T, size_t i>
02813 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02814     using type = typename FractionField<T>::zero;
02815 };
02816
02817 template<typename T, size_t i>

```

```

02818     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02819     private:
02820         using _4p = typename FractionField<T>::template inject_t<
02821             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
02822         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02823         using dividend =
02824             typename FractionField<T>::template mul_t<
02825                 _4p,
02826                 typename FractionField<T>::template mul_t<
02827                     _4pml,
02828                     bernoulli_t<T, (i + 1)>>::type;
02829     public:
02830         using type = typename FractionField<T>::template div_t<dividend,
02831             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02832     };
02833
02834     template<typename T, size_t i>
02835     struct tanh_coeff {
02836         using type = typename tanh_coeff_helper<T, i>::type;
02837     };
02838 } // namespace internal
02839
02840 template<typename Integers, size_t deg>
02841 using exp = taylor<Integers, internal::exp_coeff, deg>;
02842
02843 template<typename Integers, size_t deg>
02844 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
02845     exp<Integers, deg>,
02846     typename polynomial<FractionField<Integers>>::one>;
02847
02848 template<typename Integers, size_t deg>
02849 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
02850
02851 template<typename Integers, size_t deg>
02852 using atan = taylor<Integers, internal::atan_coeff, deg>;
02853
02854 template<typename Integers, size_t deg>
02855 using sin = taylor<Integers, internal::sin_coeff, deg>;
02856
02857 template<typename Integers, size_t deg>
02858 using sinh = taylor<Integers, internal::sh_coeff, deg>;
02859
02860 template<typename Integers, size_t deg>
02861 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
02862
02863 template<typename Integers, size_t deg>
02864 using cos = taylor<Integers, internal::cos_coeff, deg>;
02865
02866 template<typename Integers, size_t deg>
02867 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
02868
02869 template<typename Integers, size_t deg>
02870 using asin = taylor<Integers, internal::asin_coeff, deg>;
02871
02872 template<typename Integers, size_t deg>
02873 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
02874
02875 template<typename Integers, size_t deg>
02876 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
02877
02878 template<typename Integers, size_t deg>
02879 using tan = taylor<Integers, internal::tan_coeff, deg>;
02880
02881 template<typename Integers, size_t deg>
02882 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
02883 } // namespace aerobus
02884
02885 // continued fractions
02886 namespace aerobus {
02887     template<int64_t... values>
02888     struct ContinuedFraction {};
02889
02890     template<int64_t a0>
02891     struct ContinuedFraction<a0> {
02892         using type = typename q64::template inject_constant_t<a0>;
02893         static constexpr double val = static_cast<double>(a0);
02894     };
02895
02896     template<int64_t a0, int64_t... rest>
02897     struct ContinuedFraction<a0, rest...> {
02898         using type = q64::template add_t<
02899             typename q64::template inject_constant_t<a0>,
02900             typename q64::template div_t<
02901                 typename q64::one,
02902                 typename ContinuedFraction<rest...>::type
02903             >;
02904     };

```

```

02971     static constexpr double val = type::template get<double>();
02972 };
02973
02974     using PI_fraction =
02975     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02981     using E_fraction =
02982     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02983     using SQRT2_fraction =
02984     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02985     using SQRT3_fraction =
02986     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02987 // NOLINT
02988 } // namespace aerobus
02989
02990 // known polynomials
02991 namespace aerobus {
02992     // CChebyshev
02993     namespace internal {
02994         template<int kind, size_t deg, typename I>
02995         struct chebyshev_helper {
02996             using type = typename polynomial<I>::template sub_t<
02997                 typename polynomial<I>::template mul_t<
02998                     typename polynomial<I>::template mul_t<
02999                         typename polynomial<I>::template inject_constant_t<2>,
03000                         typename polynomial<I>::X>,
03001                         typename chebyshev_helper<kind, deg - 1, I>::type
03002                     >,
03003                     typename chebyshev_helper<kind, deg - 2, I>::type
03004                 >;
03005         };
03006
03007         template<typename I>
03008         struct chebyshev_helper<1, 0, I> {
03009             using type = typename polynomial<I>::one;
03010         };
03011
03012         template<typename I>
03013         struct chebyshev_helper<1, 1, I> {
03014             using type = typename polynomial<I>::X;
03015         };
03016
03017         template<typename I>
03018         struct chebyshev_helper<2, 0, I> {
03019             using type = typename polynomial<I>::one;
03020         };
03021
03022         template<typename I>
03023         struct chebyshev_helper<2, 1, I> {
03024             using type = typename polynomial<I>::template mul_t<
03025                 typename polynomial<I>::template inject_constant_t<2>,
03026                 typename polynomial<I>::X>;
03027         };
03028     } // namespace internal
03029
03030     // Laguerre
03031     namespace internal {
03032         template<size_t deg, typename I>
03033         struct laguerre_helper {
03034             using Q = FractionField<I>;
03035             using PQ = polynomial<Q>;
03036
03037             private:
03038                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03039                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03040                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03041                 // -x + 2k-1
03042                 using p = typename PQ::template val<
03043                     typename Q::template inject_constant_t<-1>,
03044                     typename Q::template inject_constant_t<2 * deg - 1>;
03045                 // 1/n
03046                 using factor = typename PQ::template inject_ring_t<
03047                     typename Q::template val<typename I::one, typename I::template
03048                         inject_constant_t<deg>>;
03049
03050             public:
03051                 using type = typename PQ::template mul_t <
03052                     factor,
03053                     typename PQ::template sub_t<
03054                         typename PQ::template mul_t<
03055                             p,
03056                             lnm1
03057                         >,
03058                         typename PQ::template mul_t<
03059                             typename PQ::template inject_constant_t<deg-1>,
03060                             lnm2
03061                         >
03062                     >
03063         };
03064     } // namespace internal
03065 }

```

```

03060         >;
03061     };
03062
03063     template<typename I>
03064     struct laguerre_helper<0, I> {
03065         using type = typename polynomial<FractionField<I>::one>;
03066     };
03067
03068     template<typename I>
03069     struct laguerre_helper<1, I> {
03070     private:
03071         using PQ = polynomial<FractionField<I>;
03072     public:
03073         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03074     };
03075 } // namespace internal
03076
03077 // Bernstein
03078 namespace internal {
03079     template<size_t i, size_t m, typename I, typename E = void>
03080     struct bernstein_helper {};
03081
03082     template<typename I>
03083     struct bernstein_helper<0, 0, I> {
03084         using type = typename polynomial<I>::one;
03085     };
03086
03087     template<size_t i, size_t m, typename I>
03088     struct bernstein_helper<i, m, I, std::enable_if_t<
03089         (m > 0) && (i == 0)> {
03090     private:
03091         using P = polynomial<I>;
03092     public:
03093         using type = typename P::template mul_t<
03094             typename P::template sub_t<typename P::one, typename P::X>,
03095             typename bernstein_helper<i, m-1, I>::type>;
03096     };
03097
03098     template<size_t i, size_t m, typename I>
03099     struct bernstein_helper<i, m, I, std::enable_if_t<
03100         (m > 0) && (i == m)> {
03101     private:
03102         using P = polynomial<I>;
03103     public:
03104         using type = typename P::template mul_t<
03105             typename P::X,
03106             typename bernstein_helper<i-1, m-1, I>::type>;
03107     };
03108
03109     template<size_t i, size_t m, typename I>
03110     struct bernstein_helper<i, m, I, std::enable_if_t<
03111         (m > 0) && (i > 0) && (i < m)> {
03112     private:
03113         using P = polynomial<I>;
03114     public:
03115         using type = typename P::template add_t<
03116             typename P::template mul_t<
03117                 typename P::template sub_t<typename P::one, typename P::X>,
03118                 typename bernstein_helper<i, m-1, I>::type>,
03119                 typename P::template mul_t<
03120                     typename P::X,
03121                     typename bernstein_helper<i-1, m-1, I>::type>;
03122     };
03123 } // namespace internal
03124
03125 namespace known_polynomials {
03126     enum hermite_kind {
03127         probabilist,
03128         physicist
03129     };
03130 }
03131
03132 // hermite
03133 namespace internal {
03134     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03135     struct hermite_helper {};
03136
03137     template<size_t deg, typename I>
03138     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03139     private:
03140         using hnm1 = typename hermite_helper<deg - 1,
03141             known_polynomials::hermite_kind::probabilist, I>::type;
03142         using hnm2 = typename hermite_helper<deg - 2,
03143             known_polynomials::hermite_kind::probabilist, I>::type;
03144     public:
03145         using type = typename polynomial<I>::template sub_t<

```

```

03148         typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03149         typename polynomial<I>::template mul_t<
03150             typename polynomial<I>::template inject_constant_t<deg - 1>,
03151             hnm2
03152         >
03153     >;
03154 };
03155
03156 template<size_t deg, typename I>
03157 struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03158     private:
03159         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
I>::type;
03160         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
I>::type;
03161     public:
03162         using type = typename polynomial<I>::template sub_t<
03163             // 2X Hn-1
03164             typename polynomial<I>::template mul_t<
03165                 typename pi64::val<typename I::template inject_constant_t<2>,
03166                 typename I::zero>, hnm1>,
03167                 typename polynomial<I>::template mul_t<
03168                     typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03169                     hnm2
03170                 >
03171             >
03172         >;
03173 };
03174
03175 template<typename I>
03176 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03177     using type = typename polynomial<I>::one;
03178 };
03179
03180 template<typename I>
03181 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03182     using type = typename polynomial<I>::X;
03183 };
03184
03185 template<typename I>
03186 struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03187     using type = typename pi64::one;
03188 };
03189
03190 template<typename I>
03191 struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03192     // 2X
03193     using type = typename polynomial<I>::template val<
03194         typename I::template inject_constant_t<2>,
03195         typename I::zero>;
03196 };
03197 } // namespace internal
03198
03199 // legendre
03200 namespace internal {
03201     template<size_t n, typename I>
03202     struct legendre_helper {
03203     private:
03204         using Q = FractionField<I>;
03205         using PQ = polynomial<Q>;
03206         // 1/n constant
03207         // (2n-1)/n X
03208         using fact_left = typename PQ::template monomial_t<
03209             makefraction_t<I,
03210                 typename I::template inject_constant_t<2*n-1>,
03211                 typename I::template inject_constant_t<n>
03212             >,
03213             1>;
03214         // (n-1) / n
03215         using fact_right = typename PQ::template val<
03216             makefraction_t<I,
03217                 typename I::template inject_constant_t<n-1>,
03218                 typename I::template inject_constant_t<n>>;
03219     public:
03220         using type = PQ::template sub_t<
03221             typename PQ::template mul_t<
03222                 fact_left,
03223                 typename legendre_helper<n-1, I>::type
03224             >,
03225             typename PQ::template mul_t<
03226                 fact_right,
03227                 typename legendre_helper<n-2, I>::type
03228             >
03229         >;
03230     };
03231 };
03232

```

```

03233
03234     template<typename I>
03235     struct legendre_helper<0, I> {
03236         using type = typename polynomial<FractionField<I>::one>;
03237     };
03238
03239     template<typename I>
03240     struct legendre_helper<1, I> {
03241         using type = typename polynomial<FractionField<I>::X>;
03242     };
03243 } // namespace internal
03244
03245 // bernoulli polynomials
03246 namespace internal {
03247     template<size_t n>
03248     struct bernoulli_coeff {
03249         template<typename T, size_t i>
03250         struct inner {
03251             private:
03252                 using F = FractionField<T>;
03253             public:
03254                 using type = typename F::template mul_t<
03255                     typename F::template inject_ring_t<combination_t<T, i, n>,
03256                         bernoulli_t<T, n-i>
03257                 >;
03258             };
03259         };
03260     } // namespace internal
03261
03262     namespace known_polynomials {
03263         template <size_t deg, typename I = aerobus::i64>
03264         using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03265
03266         template <size_t deg, typename I = aerobus::i64>
03267         using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03268
03269         template <size_t deg, typename I = aerobus::i64>
03270         using laguerre = typename internal::laguerre_helper<deg, I>::type;
03271
03272         template <size_t deg, typename I = aerobus::i64>
03273         using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03274             I>::type;
03275
03276         template <size_t deg, typename I = aerobus::i64>
03277         using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03278
03279         template<size_t i, size_t m, typename I = aerobus::i64>
03280         using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03281
03282         template<size_t deg, typename I = aerobus::i64>
03283         using legendre = typename internal::legendre_helper<deg, I>::type;
03284
03285         template<size_t deg, typename I = aerobus::i64>
03286         using bernoulli =aylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03287     } // namespace known_polynomials
03288 } // namespace aerobus
03289
03290 #ifdef AEROBUS_CONWAY_IMPORTS
03291 // conway polynomials
03292 namespace aerobus {
03293     template<int p, int n>
03294     struct ConwayPolynomial {};
03295
03296 #ifndef DO_NOT_DOCUMENT
03297     #define ZPV ZPZ::template val
03298     #define POLYV aerobus::polynomial<ZPV>::template val
03299     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
03300 POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
03301     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type =
03302 POLYV<ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03303     template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type =
03304 POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03305     template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type =
03306 POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03307     template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpv<2>; using type =
03308 POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03309     template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpv<2>; using type =
03310 POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03311     template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpv<2>; using type =
03312 POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03313     template<> struct ConwayPolynomial<2, 8> { using ZPV = aerobus::zpv<2>; using type =
03314 POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03315     template<> struct ConwayPolynomial<2, 9> { using ZPV = aerobus::zpv<2>; using type =
03316 POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>; }; //
03317 NOLINT

```

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## Chapter 10

# Examples

### 10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject\_constant\_t<1>>

Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

### 10.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

...Ts	types to store and manipulate at compile time
-------	---

### 10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

## 10.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

<i>v1</i>	
<i>v2</i>	<i32::val<1>, i32::val<1>>

## 10.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

$v$	a value in i32
-----	----------------

## 10.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

$v$	a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code>
-----	---

## 10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

$x$	<code>inject_constant_t&lt;2&gt;</code>
-----	---

## 10.15 i64::add\_t

addition operator

addition operator

Template Parameters

$v1$	: an element of <code>aerobus::i64::val</code>
$v2$	: an element of <code>aerobus::i64::val &lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>



## 10.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>>

## 10.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code>

## 10.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<2>, i64::val<2>>

## 10.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of i64::val

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of i64::val

### Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>>

## 10.26 i64::pos\_t

is v positive yields  $v > 0$  as std::true\_type or std::false\_type

is v positive yields  $v > 0$  as std::true\_type or std::false\_type

### Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>>
-----	---

## 10.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>>
-----	---

## 10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

<code>x</code>	<code>&lt;i32&gt;::template inject_constant_t&lt;2&gt;</code>
----------------	---

## 10.29 q32::add\_t

addition operator

addition operator

Template Parameters

<code>v1</code>	a value
<code>v2</code>	a value <code>&lt;q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;2&gt;&gt;, q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;3&gt;&gt;&gt;</code>

## 10.30 FractionField

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$

Template Parameters

<code>Ring</code>	<code>&lt;i64&gt;</code> is q64 (rationals with 64 bits numerator and denominator)
-------------------	--

## 10.31 aerobus::ContinuedFraction

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$  [ [https://en.wikipedia.org/wiki/Continued\\_fraction](https://en.wikipedia.org/wiki/Continued_fraction)](See in Wikipedia)

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

$\langle 1, 1, 1 \rangle$  represents  $1 + \frac{1}{1}$

## 10.32 PI\_fraction::val

representation of  $\pi$  as a continued fraction -> 3.14...

## 10.33 E\_fraction::val

approximation of  $e$  -> 2.718...

approximation of  $e$  -> 2.718...



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