

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download the [aerobus.h](#)
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including [aerobus.h](#).

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

results on my laptop :

```
./benchmarks_avx512.exe
[std math] 5.358e-01 Gsin/s
[std fast math] 3.389e+00 Gsin/s
[aerobus deg 1] 1.871e+01 Gsin/s
average error (vs std) : 4.36e-02
max error (vs std) : 1.50e-01
[aerobus deg 3] 1.943e+01 Gsin/s
average error (vs std) : 1.85e-04
max error (vs std) : 8.17e-04
[aerobus deg 5] 1.335e+01 Gsin/s
average error (vs std) : 6.07e-07
max error (vs std) : 3.63e-06
[aerobus deg 7] 8.634e+00 Gsin/s
average error (vs std) : 1.27e-09
max error (vs std) : 9.75e-09
[aerobus deg 9] 6.171e+00 Gsin/s
average error (vs std) : 1.89e-12
max error (vs std) : 1.78e-11
[aerobus deg 11] 4.731e+00 Gsin/s
average error (vs std) : 2.12e-15
max error (vs std) : 2.40e-14
[aerobus deg 13] 3.862e+00 Gsin/s
average error (vs std) : 3.16e-17
max error (vs std) : 3.33e-16
[aerobus deg 15] 3.359e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
[aerobus deg 17] 2.947e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
```



## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::mpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `Fraction<Field<Integers>>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`

- cos
- tan
- sh
- cosh
- tanh
- asin
- acos
- acosh
- asinh
- atanh

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in `Integers` (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_serie = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_serie<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
```

```

vbroadcastsd    ymm3, QWORD PTR .LC23[rip]
vbroadcastsd    ymm2, QWORD PTR .LC25[rip]
.L3:
vmovupd ymm15, YMMWORD PTR [rsi+rax]
vmovapd ymm0, ymm15
vmadd132pd      ymm0, ymm14, ymm1
vmadd132pd      ymm0, ymm13, ymm15
vmadd132pd      ymm0, ymm12, ymm15
vmadd132pd      ymm0, ymm11, ymm15
vmadd132pd      ymm0, ymm10, ymm15
vmadd132pd      ymm0, ymm9, ymm15
vmadd132pd      ymm0, ymm8, ymm15
vmadd132pd      ymm0, ymm7, ymm15
vmadd132pd      ymm0, ymm6, ymm15
vmadd132pd      ymm0, ymm5, ymm15
vmadd132pd      ymm0, ymm4, ymm15
vmadd132pd      ymm0, ymm3, ymm15
vmadd132pd      ymm0, ymm2, ymm15
vmadd132pd      ymm0, ymm1, ymm15
vmovupd YMMWORD PTR [rdx+rax], ymm0
add    rax, 32
cmp    rcx, rax
jne    .L3
mov    rax, rdi
and    rax, -4
vzeroupper

```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, `Aerobus` allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```

using namespace aerobus;
using ZmZ = zp<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;

```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define `rational functions` by taking the ring of fractions of polynomials:

```

using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;

```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , `Aerobus` provides automatic implementation for `quotient ring  $R/X$`  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```

using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;

```

As we could have using `zp<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

Aerobus gives an implementation for `continued fractions`. It can be used this way:

```
using namespace aerobus;  
using T = ContinuedFraction<1,2,3,4>;  
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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## Chapter 3

# Concept Index

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# Chapter 4

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### 4.1 Class List

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## Chapter 5

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### 5.1 File List

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## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial<Small> into polynomial<Large>*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient<Ring, X> into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
  - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
  - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)
  - checks if  $n$  is prime*
- struct [polynomial](#)
- struct [Quotient](#)
  - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)
  - Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)
  - specialization for empty type list*
- struct [zpz](#)
  - congruence classes of integers modulo  $p$  (32 bits)*

## Concepts

- concept [IsRing](#)
  - Concept to express  $R$  is a Ring.*
- concept [IsEuclideanDomain](#)
  - Concept to express  $R$  is an euclidean domain.*
- concept [IsField](#)
  - Concept to express  $R$  is a field.*

## Typedefs

- template<[typename T](#) , [typename A](#) , [typename B](#) >
  - [using gcd\\_t = typename internal::gcd< T >::template type< A, B >](#)
  - computes the greatest common divisor of A and B*
- template<[typename... vals](#)>
  - [using vadd\\_t = typename internal::vadd< vals... >::type](#)
  - adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator*
- template<[typename... vals](#)>
  - [using vmul\\_t = typename internal::vmul< vals... >::type](#)
  - multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator*
- template<[typename val](#) >
  - [using abs\\_t = std::conditional\\_t< val::enclosing\\_type::template pos\\_v< val >, val, typename val::enclosing\\_type::template sub\\_t< typename val::enclosing\\_type::zero, val > >](#)
  - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<[typename Ring](#) >
  - [using FractionField = typename internal::FractionFieldImpl< Ring >::type](#)
- template<[typename X](#) , [typename Y](#) >
  - [using add\\_t = typename X::enclosing\\_type::template add\\_t< X, Y >](#)
  - generic addition*
- template<[typename X](#) , [typename Y](#) >
  - [using sub\\_t = typename X::enclosing\\_type::template sub\\_t< X, Y >](#)
  - generic subtraction*



- `template<typename X, typename Y >`  
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`  
*generic multiplication*
- `template<typename X, typename Y >`  
`using div_t = typename X::enclosing_type::template div_t< X, Y >`  
*generic division*
- `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
- `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
- `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
- `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
- `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
- `template<typename Ring, typename v1, typename v2 >`  
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
- `template<typename v >`  
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type >, polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
- `template<int64_t p, int64_t q>`  
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p >, i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p >, i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<typename Ring, typename v1, typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- `template<typename Ring, typename v1, typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- `template<typename Ring, auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
- `template<typename Ring, auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField<Ring>*
- `template<typename T, size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*
- `template<typename T, size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`

- computes binomial coefficient (k among n) as type*

```
template<typename T, size_t n>
using bernoulli_t = typename internal::bernoulli< T, n >::type
```

*nth bernoulli number as type in T*
- ```
template<typename T, size_t n>
using bell_t = typename internal::bell_helper< T, n >::type
```

*Bell numbers.*
- ```
template<typename T, int k>
using alternate_t = typename internal::alternate< T, k >::type
```

*$(-1)^k$  as type in T*
- ```
template<typename T, int n, int k>
using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type
```

*Stirling number of first kind (signed) – as types.*
- ```
template<typename T, int n, int k>
using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >
```

*Stirling number of first kind (unsigned) – as types.*
- ```
template<typename T, typename p, size_t n>
using pow_t = typename internal::pow< T, p, n >::type
```

*$p^n$  (as 'val' type in T)*
- ```
template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>
using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<
deg+1 > >::type
```
- ```
template<typename Integers, size_t deg>
using exp = taylor< Integers, internal::exp_coeff, deg >
```

*$e^x$*
- ```
template<typename Integers, size_t deg>
using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg
>, typename polynomial< FractionField< Integers > >::one >
```

*$e^x - 1$*
- ```
template<typename Integers, size_t deg>
using ln1 = taylor< Integers, internal::ln1_coeff, deg >
```

*$\ln(1 + x)$*
- ```
template<typename Integers, size_t deg>
using atan = taylor< Integers, internal::atan_coeff, deg >
```

*$\arctan(x)$*
- ```
template<typename Integers, size_t deg>
using sin = taylor< Integers, internal::sin_coeff, deg >
```

*$\sin(x)$*
- ```
template<typename Integers, size_t deg>
using sinh = taylor< Integers, internal::sh_coeff, deg >
```

*$\sinh(x)$*
- ```
template<typename Integers, size_t deg>
using cosh = taylor< Integers, internal::cosh_coeff, deg >
```

*$\cosh(x)$  hyperbolic cosine*
- ```
template<typename Integers, size_t deg>
using cos = taylor< Integers, internal::cos_coeff, deg >
```

*$\cos(x)$  cosinus*
- ```
template<typename Integers, size_t deg>
using geometric_sum = taylor< Integers, internal::geom_coeff, deg >
```

*$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$*
- ```
template<typename Integers, size_t deg>
using asin = taylor< Integers, internal::asin_coeff, deg >
```

*$\arcsin(x)$  arc sinus*

- `template<typename Integers , size_t deg>`  
`using asinh = taylor< Integers, internal::asinh_coeff, deg >`  
`arcsinh( $x$ )` *arc hyperbolic sinus*
- `template<typename Integers , size_t deg>`  
`using atanh = taylor< Integers, internal::atanh_coeff, deg >`  
`arctanh( $x$ )` *arc hyperbolic tangent*
- `template<typename Integers , size_t deg>`  
`using tan = taylor< Integers, internal::tan_coeff, deg >`  
`tan( $x$ )` *tangent*
- `template<typename Integers , size_t deg>`  
`using tanh = taylor< Integers, internal::tanh_coeff, deg >`  
`tanh( $x$ )` *hyperbolic tangent*
- `using PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >`
- `using E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`
- `using SQRT2_fraction = ContinuedFraction< 1, 2 >`  
*approximation of  $\sqrt{2}$*
- `using SQRT3_fraction = ContinuedFraction< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >`  
*approximation of*

## Functions

- `template<typename T >`  
`T * aligned_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the field (prime number) @tparam n degree of extension  
`template< int p`

## Variables

- `template<typename T , size_t i>`  
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`  
*computes factorial( $i$ ) as value in  $T$*
- `template<typename T , size_t k, size_t n>`  
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`  
*computes binomial coefficients ( $k$  among  $n$ ) as value*
- `template<typename FloatType , typename T , size_t n>`  
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`  
 *$n$ th bernoulli number as value in  $FloatType$*
- `template<typename T , size_t k>`  
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`  
 *$(-1)^k$  as value from  $T$*

### 6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

### 6.1.2 Typedef Documentation

#### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

### Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val&lt;-2&gt;</code>
------------	--

#### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

#### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

#### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T

### Template Parameters

<i>T</i>	Ring type, <code>aerobus::i64</code> for example
----------	--

#### 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\operatorname{arctan}(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$  arc hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

#### Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

#### 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

#### 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

#### 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  cosinus

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div_t<X, Y>
```

generic division

#### Template Parameters

<i>X</i>	a value in a ring providing div_t operator
<i>Y</i>	a value in same ring

#### 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

#### 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in polynomial<FractionField<Ring>>

#### Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

#### 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$e^x$



## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

## 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

## 6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

#### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.25 ln1

```
template<typename Integers , size_t deg>
using aerobus::ln1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

#### Template Parameters

<i>Ring</i>	integers
...xs	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

#### Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

### 6.1.2.29 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

#### Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

### 6.1.2.30 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

## Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

**6.1.2.31 mul\_t**

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

**6.1.2.32 mulfractions\_t**

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

## Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

**6.1.2.33 pi64**

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

**6.1.2.34 PI\_fraction**

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

**6.1.2.35 pow\_t**

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

## Template Parameters

$T$	(some ring type, such as <a href="#">aerobus::i64</a> )
$p$	must be an instantiation of $T::val$
$n$	power

**6.1.2.36 pq64**

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.37 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.38 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

**6.1.2.39 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.40 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.41 Sqrt2\_fraction**

```
using aerobus::Sqrt2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of  $\sqrt{2}$

**6.1.2.42 Sqrt3\_fraction**

```
using aerobus::Sqrt3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

**6.1.2.43 stirling\_signed\_t**

```
template<typename T , int n, int k>
using aerobus::stirling_signed_t = typedef typename internal::stirling_helper<T, n, k>::type
```

Stirling number of first kind (signed) – as types.

## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

**6.1.2.44 stirling\_unsigned\_t**

```
template<typename T , int n, int k>
using aerobus::stirling_unsigned_t = typedef abs_t<typename internal::stirling_helper<T, n, k>::type>
```

Stirling number of first kind (unsigned) – as types.

## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

## 6.1.2.45 sub\_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

## Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

## 6.1.2.46 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.47 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.48 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
deg + 1> >::type
```

## Template Parameters

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff_at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

#### 6.1.2.49 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

##### Template Parameters

<i>...vals</i>	
----------------	--

#### 6.1.2.50 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

##### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.3 Function Documentation

#### 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

##### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

##### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

#### 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```



```
prime number )
```

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>
[inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

##### Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >

- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==m)> >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**

*greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*

- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, l >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, l >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, l >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, l >

- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **stirling\_helper**
- struct **stirling\_helper**< T, 0, 0 >
- struct **stirling\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- `template<size_t i, typename... Ts>`  
`using type_at_t = typename type_at<i, Ts...>::type`
- `template<std::size_t N>`  
`using make_index_sequence_reverse = decltype(index_sequence_reverse(std::make_index_sequence< N>{}))`

## Functions

- `template<std::size_t... Is>`  
`constexpr auto index_sequence_reverse (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})`

## Variables

- `template<template< typename... > typename TT, typename T >`  
`constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value`

### 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

### 6.2.2 Typedef Documentation

#### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

#### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

### 6.2.3 Function Documentation

#### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Typedefs

- template<size\_t deg, typename I = aerobus::i64>  
using chebyshev\_T = typename internal::chebyshev\_helper< 1, deg, I >::type  
*Chebyshev polynomials of first kind.*
- template<size\_t deg, typename I = aerobus::i64>  
using chebyshev\_U = typename internal::chebyshev\_helper< 2, deg, I >::type  
*Chebyshev polynomials of second kind.*
- template<size\_t deg, typename I = aerobus::i64>  
using laguerre = typename internal::laguerre\_helper< deg, I >::type  
*Laguerre polynomials.*
- template<size\_t deg, typename I = aerobus::i64>  
using hermite\_prob = typename internal::hermite\_helper< deg, hermite\_kind::probabilist, I >::type  
*Hermite polynomials - probabilist form.*
- template<size\_t deg, typename I = aerobus::i64>  
using hermite\_phys = typename internal::hermite\_helper< deg, hermite\_kind::physicist, I >::type  
*Hermite polynomials - physicist form.*
- template<size\_t i, size\_t m, typename I = aerobus::i64>  
using bernstein = typename internal::bernstein\_helper< i, m, I >::type  
*Bernstein polynomials.*
- template<size\_t deg, typename I = aerobus::i64>  
using legendre = typename internal::legendre\_helper< deg, I >::type  
*Legendre polynomials.*
- template<size\_t deg, typename I = aerobus::i64>  
using bernoulli = taylor< I, internal::bernoulli\_coeff< deg >::template inner, deg >  
*Bernoulli polynomials.*

### Enumerations

- enum hermite\_kind { probabilist , physicist }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

## 6.3.2 Typedef Documentation

### 6.3.2.1 bernoulli

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::bernoulli = typedef taylor<I, internal::bernoulli_coeff<deg><←
::template inner, deg>
```

Bernoulli polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to <a href="#">aerobus::i64</a> )

### 6.3.2.2 bernstein

```
template<size_t i, size_t m, typename I = aerobus::i64>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m, I>::type
```

Bernstein polynomials.

Lives in polynomial

See also

[See in Wikipedia](#)

#### Template Parameters

<i>i</i>	<i>index of polynomial (between 0 and m)</i>
<i>m</i>	<i>degree of polynomial</i>
<i>I</i>	<i>Integers ring (defaults to <a href="#">aerobus::i64</a>)</i>

### 6.3.2.3 chebyshev\_T

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg, I>::type
```

Chebyshev polynomials of first kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>integer</i>	rings (defaults to <a href="#">aerobus::i64</a> )

#### 6.3.2.4 chebyshev\_U

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg, I>::type
```

Chebyshev polynomials of second kind.

Lives in polynomial

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	<i>degree of polynomial</i>
<i>integer</i>	<i>rings (defaults to <a href="#">aerobus::i64</a>)</i>

#### 6.3.2.5 hermite\_phys

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist, I>::type
```

Hermite polynomials - physicist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

#### 6.3.2.6 hermite\_prob

```
template<size_t deg, typename I = aerobus::i64>
```



```
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist, I>::type
```

Hermite polynomials - probabilist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

### 6.3.2.7 laguerre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg,
I>::type
```

Laguerre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to <a href="#">aerobus::i64</a> )

### 6.3.2.8 legendre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg,
I>::type
```

Legendre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

## Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers Ring (defaults to <a href="#">aerobus::i64</a> )

## 6.3.3 Enumeration Type Documentation

### 6.3.3.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```

## Enumerator

probabilist	
physicist	

# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- `using type = typename Ring::zero`

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- [using type = aN](#)

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

#### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

##### Template Parameters

<i>...values</i>	are int64_t
------------------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- `using type = typename q64::template inject_constant_t< a0 >`  
represented value as `aerobus::q64`

### Static Public Attributes

- `static constexpr double val = static_cast<double>(a0)`  
represented value as `double`

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

#### Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

### 8.5.2 Member Typedef Documentation

#### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

### 8.5.3 Member Data Documentation

#### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as `double`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- `using type = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`  
represented value as `aerobus::q64`

### Static Public Attributes

- `static constexpr double val = type::template get<double>()`  
represented value as `double`

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

#### Template Parameters

<code>a0</code>	integer (int64_t)
<code>...rest</code>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename ContinuedFraction<rest...>::type > >
```

represented value as `aerobus::q64`



## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.8.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i64::val< static_cast< int64_t >(val::v)>`  
*the [i64](#) representation of val*

### 8.9.1 Detailed Description

embeds [i32](#) into [i64](#)

### 8.9.2 Member Typedef Documentation

#### 8.9.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

#### Template Parameters

<i>val</i>	a value in <a href="#">i32</a>
------------	--------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.10 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds `polynomial<Small>` into `polynomial<Large>`

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
`using type = typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`  
*the `polynomial<Large>` representation of v*

### 8.10.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

### 8.10.2 Member Typedef Documentation

#### 8.10.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
```

the polynomial<Large> reprensentation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.11 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

### Public Types

- ```
template<typename v >
using type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>
```

  
*q64 representation of v*

### 8.11.1 Detailed Description

embeds q32 into q64

## 8.11.2 Member Typedef Documentation

### 8.11.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>
```

q64 representation of v

#### Template Parameters

|   |                |
|---|----------------|
| v | a value in q32 |
|---|----------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.12 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = typename val::raw_t`  
*Ring representation of val.*

### 8.12.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

|             |                  |
|-------------|------------------|
| <i>Ring</i> | a Euclidean ring |
| <i>X</i>    | a value in Ring  |

## 8.12.2 Member Typedef Documentation

### 8.12.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

|            |                              |
|------------|------------------------------|
| <i>val</i> | a value in Quotient<Ring, X> |
|------------|------------------------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.13 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- template<typename v >  
using type = typename FractionField< Ring >::template val< v, typename Ring::one >  
*FractionField<Ring> representation of v.*

### 8.13.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

|             |                                               |
|-------------|-----------------------------------------------|
| <i>Ring</i> | an integers ring, such as <a href="#">i32</a> |
|-------------|-----------------------------------------------|

## 8.13.2 Member Typedef Documentation

### 8.13.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

FractionField<Ring> representation of v.

#### Template Parameters

|   |              |
|---|--------------|
| v | a Ring value |
|---|--------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.14 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the [i32](#) representation of val*

### 8.14.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into [i32](#)

#### Template Parameters

|   |            |
|---|------------|
| x | an integer |
|---|------------|

## 8.14.2 Member Typedef Documentation

### 8.14.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the `i32` representation of `val`

#### Template Parameters

|                  |                                      |
|------------------|--------------------------------------|
| <code>val</code> | a value in <code>zpz&lt;x&gt;</code> |
|------------------|--------------------------------------|

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.15 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in `i32`, again represented as types*

### Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`

- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator yields v1 % v2 for example : `i32::mod_t<i32::val<7>, i32::val<2>>`*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

### Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

## 8.15.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::add_t = typename add<v1, v2>::type
```

### 8.15.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

### 8.15.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```



#### 8.15.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

#### 8.15.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

#### 8.15.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

#### 8.15.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

#### 8.15.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

#### 8.15.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

#### 8.15.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

##### Template Parameters

|                 |                             |
|-----------------|-----------------------------|
| <code>v1</code> | a value in <code>i32</code> |
| <code>v2</code> | a value in <code>i32</code> |

#### 8.15.2.11 mul\_t

```
template<typename v1 , typename v2 >
```

```
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

#### 8.15.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

#### 8.15.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

#### 8.15.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

#### 8.15.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.15.3 Member Data Documentation

#### 8.15.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

#### 8.15.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.15.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

### 8.15.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.16 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in i64*

### Public Types

- [using inner\\_type = int64\\_t](#)  
*type of represented values*
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int64\\_t >\(x\)>](#)
- [template<typename v >](#)  
[using inject\\_ring\\_t = v](#)  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)*  
*-> [i64::val<1>](#)*
- [using zero = val< 0 >](#)  
*constant zero*
- [using one = val< 1 >](#)  
*constant one*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mod\\_t = typename remainder< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gcd\\_t = gcd\\_t< i64, v1, v2 >](#)
- [template<typename v >](#)  
[using pos\\_t = typename pos< v >::type](#)

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields v1 > v2 as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 8.16.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

#### 8.16.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

#### 8.16.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

#### 8.16.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

#### 8.16.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

### 8.16.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

### 8.16.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>>`  
 -> `i64::val<1>`

#### Template Parameters

|                |                             |
|----------------|-----------------------------|
| <code>v</code> | a value in <code>i64</code> |
|----------------|-----------------------------|

### 8.16.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

### 8.16.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

### 8.16.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

### 8.16.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

### 8.16.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

### 8.16.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

### 8.16.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

### 8.16.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.16.3 Member Data Documentation

### 8.16.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

|                 |                                                   |
|-----------------|---------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a> |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> |

### 8.16.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.16.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

### 8.16.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.17 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- `static constexpr bool value` = `internal::_is_prime<n, 5>::value`  
*true iff n is prime*

### 8.17.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

|          |  |
|----------|--|
| <i>n</i> |  |
|----------|--|

### 8.17.2 Member Data Documentation

#### 8.17.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.18 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::q\\_type](#)  
*division operator*



- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial : coeff X^deg*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity (an > 0)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`  
`Ring >, v1, v2 > >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

### Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

## 8.18.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.2 `derive_t`

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

|                |  |
|----------------|--|
| <code>v</code> |  |
|----------------|--|

### 8.18.2.3 `div_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.4 `eq_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.5 `gcd_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.7 inject\_constant\_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.18.2.8 inject\_ring\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

## 8.18.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.10 mod\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.11 monomial\_t**

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

**Template Parameters**

|              |  |
|--------------|--|
| <i>coeff</i> |  |
| <i>deg</i>   |  |

**8.18.2.12 mul\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

**8.18.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

|          |  |
|----------|--|
| <i>v</i> |  |
|----------|--|

**8.18.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

|          |  |
|----------|--|
| <i>P</i> |  |
|----------|--|

**8.18.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.18.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

### 8.18.3 Member Data Documentation

#### 8.18.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.18.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.18.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

|          |                                            |
|----------|--------------------------------------------|
| <i>v</i> | a value in <a href="#">polynomial::val</a> |
|----------|--------------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- [using type](#) = [typename](#) internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- [using tail](#) = [typename](#) internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.19.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.19.2 Member Typedef Documentation

### 8.19.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.19.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- [using zero](#) = [val](#)< [typename](#) Ring::zero >  
*zero value*
- [using one](#) = [val](#)< [typename](#) Ring::one >  
*one*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using add\\_t](#) = [val](#)< [typename](#) Ring::template [add\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*addition operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using mul\\_t](#) = [val](#)< [typename](#) Ring::template [mul\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*subtraction operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using div\\_t](#) = [val](#)< [typename](#) Ring::template [div\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*division operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using mod\\_t](#) = [val](#)< [typename](#) Ring::template [mod\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >

- modulus operator*  
 • `template<typename v1 , typename v2 >`  
   `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
   *equality operator (as type)*
- `template<typename v1 >`  
   `using pos_t = std::true_type`  
   *positivity operator always true*
- `template<auto x>`  
   `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
   `using inject_ring_t = val< v >`

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
   `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
   *addition operator (as boolean value)*
- `template<typename v >`  
   `static constexpr bool pos_v = pos_t<v>::value`  
   *positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
   *quotien rings are euclidean domain*

## 8.20.1 Detailed Description

`template<typename Ring, typename X>`  
**requires** `IsRing<Ring>`  
**struct** `aerobus::Quotient< Ring, X >`

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

### Template Parameters

|             |                                                                                         |
|-------------|-----------------------------------------------------------------------------------------|
| <i>Ring</i> | A ring type, such as ' <code>i32</code> ', must satisfy the <code>IsRing</code> concept |
| <i>X</i>    | a value in Ring, such as <code>i32::val&lt;2&gt;</code>                                 |

## 8.20.2 Member Typedef Documentation

### 8.20.2.1 add\_t

`template<typename Ring , typename X >`  
`template<typename v1 , typename v2 >`  
`using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,`  
`typename v2::type> >`

addition operator



## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.20.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

### 8.20.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.20.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
|----|--------------------------|

### 8.20.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.20.3 Member Data Documentation

### 8.20.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

### 8.20.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.20.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
|-----------|--------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

### Public Types

- [using head](#) = [typename](#) inner::head
- [using tail](#) = [typename](#) inner::tail

### 8.21.1 Detailed Description

```
template<typename... Ts>
template<size\_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.21.2 Member Typedef Documentation

#### 8.21.2.1 head

```
template<typename... Ts>
template<size\_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.21.2.2 tail

```
template<typename... Ts>
template<size\_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

## Classes

- struct `pop_front`  
*removes types from head of the list*
- struct `split`  
*splits list at index*

## Public Types

- template<typename T >  
using `push_front` = `type_list`< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using `at` = `internal::type_at_t`< index, Ts... >  
*returns type at index*
- template<typename T >  
using `push_back` = `type_list`< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using `concat` = `typename concat_h`< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using `insert` = `typename internal::insert_h`< index, `type_list`< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using `remove` = `typename internal::remove_h`< index, `type_list`< Ts... > >::type  
*removes type at index*

## Static Public Attributes

- `static constexpr size_t length` = `sizeof...(Ts)`  
*length of list*

### 8.22.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

### 8.22.2 Member Typedef Documentation

#### 8.22.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

**8.22.2.2 concat**

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

**Template Parameters**

|          |  |
|----------|--|
| <i>U</i> |  |
|----------|--|

**8.22.2.3 insert**

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
| <i>T</i>     |  |

**8.22.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

**8.22.2.5 push\_front**

```
template<typename... Ts>
```

```
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

#### 8.22.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.22.3 Member Data Documentation

#### 8.22.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.23 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

#### Public Types

- `template<typename T >`  
`using push_front = type_list< T >`
- `template<typename T >`  
`using push_back = type_list< T >`
- `template<typename U >`  
`using concat = U`
- `template<typename T , size_t index>`  
`using insert = type_list< T >`

## Static Public Attributes

- `static constexpr size_t length = 0`

### 8.23.1 Detailed Description

specialization for empty type list

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 concat

```
template<typename U >
using aerobus::type_list<>::concat = U
```

#### 8.23.2.2 insert

```
template<typename T , size_t index>
using aerobus::type_list<>::insert = type_list<T>
```

#### 8.23.2.3 push\_back

```
template<typename T >
using aerobus::type_list<>::push_back = type_list<T>
```

#### 8.23.2.4 push\_front

```
template<typename T >
using aerobus::type_list<>::push_front = type_list<T>
```

### 8.23.3 Member Data Documentation

#### 8.23.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.24 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```



## Public Types

- `using enclosing_type = i32`  
*Enclosing ring type.*
- `using is_zero_t = std::bool_constant< x==0 >`  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr INLINED DEVICE valueType get ()`  
*cast x into valueType*
- `static std::string to_string ()`  
*string representation of value*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &v)`  
*cast x into valueRing*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*

### 8.24.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

#### Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

### 8.24.2 Member Typedef Documentation

#### 8.24.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

#### 8.24.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.24.3 Member Function Documentation

#### 8.24.3.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

|                  |                    |
|------------------|--------------------|
| <i>valueRing</i> | double for example |
|------------------|--------------------|

#### 8.24.3.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr INLINED DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast x into valueType

Template Parameters

|                  |                    |
|------------------|--------------------|
| <i>valueType</i> | double for example |
|------------------|--------------------|

#### 8.24.3.3 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

### 8.24.4 Member Data Documentation

#### 8.24.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- [using inner\\_type = int32\\_t](#)  
*type of represented values*
- [using enclosing\\_type = i64](#)  
*enclosing ring type*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

### Static Public Member Functions

- [template<typename valueType >](#)  
[static constexpr DEVICE INLINED valueType get \(\)](#)  
*cast value in valueType*
- [static std::string to\\_string \(\)](#)  
*string representation*
- [template<typename valueRing >](#)  
[static constexpr DEVICE INLINED valueRing eval \(const valueRing &v\)](#)  
*cast value in valueRing*

### Static Public Attributes

- [static constexpr int64\\_t v = x](#)  
*actual value*

### 8.25.1 Detailed Description

```
template<int64\_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

|                   |                   |
|-------------------|-------------------|
| <a href="#">x</a> | an actual integer |
|-------------------|-------------------|

## 8.25.2 Member Typedef Documentation

### 8.25.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.25.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.25.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.25.3 Member Function Documentation

### 8.25.3.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueRing</i> | (double for example) |
|------------------|----------------------|

### 8.25.3.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr DEVICE INLINED valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

## Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueType</i> | (double for example) |
|------------------|----------------------|

## 8.25.3.3 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

## 8.25.4 Member Data Documentation

## 8.25.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN, coeffs &gt; Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- [using ring\\_type = Ring](#)  
*ring coefficients live in*
- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)  
*heavy weight coefficient (non zero)*
- [using strip = val< coeffs... >](#)  
*remove largest coefficient*
- [using is\\_zero\\_t = std::bool\\_constant<\(degree==0\) &&\(aN::is\\_zero\\_t::value\)>](#)  
*true\_type if polynomial is constant zero*
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 8.26.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

|                        |                           |
|------------------------|---------------------------|
| <code>coeffN</code>    | high degree coefficient   |
| <code>...coeffs</code> | lower degree coefficients |

### 8.26.2 Member Typedef Documentation

#### 8.26.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

#### 8.26.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

## Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

**8.26.2.3 enclosing\_type**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

**8.26.2.4 is\_zero\_t**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

**8.26.2.5 ring\_type**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

**8.26.2.6 strip**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

**8.26.3 Member Function Documentation****8.26.3.1 eval()**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs
>::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

## Template Parameters

|                  |                         |
|------------------|-------------------------|
| <i>valueRing</i> | usually float or double |
|------------------|-------------------------|

## Parameters

|          |       |
|----------|-------|
| <i>x</i> | value |
|----------|-------|

## Returns

$P(x)$

## 8.26.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

## Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.26.4 Member Data Documentation

## 8.26.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

## 8.26.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t←
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## 8.27 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- `using raw_t = V`
- `using type = abs_t< typename Ring::template mod_t< V, X > >`

### 8.27.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

|   |                     |
|---|---------------------|
| V | a value from 'Ring' |
|---|---------------------|

### 8.27.2 Member Typedef Documentation

#### 8.27.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

#### 8.27.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.28 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

## Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< v==0 >`  
*true\_type if zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr DEVICE INLINED valueType get ()`  
*get value as valueType*
- `static std::string to_string ()`  
*string representation*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &v)`

## Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*
- `static constexpr bool is_zero_v = v == 0`  
*true if zero*

### 8.28.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

|          |            |
|----------|------------|
| <i>x</i> | an integer |
|----------|------------|

### 8.28.2 Member Typedef Documentation

#### 8.28.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.28.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.28.3 Member Function Documentation

### 8.28.3.1 eval()

```
template<int32_t p>
template<int32_t x>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::zpz< p >::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

### 8.28.3.2 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr DEVICE INLINED valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

#### Template Parameters

|                  |                                   |
|------------------|-----------------------------------|
| <i>valueType</i> | an arithmetic type, such as float |
|------------------|-----------------------------------|

### 8.28.3.3 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

#### Returns

a string representation

## 8.28.4 Member Data Documentation

### 8.28.4.1 is\_zero\_v

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

### 8.28.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.29 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

### Public Types

- [using ring\\_type = Ring](#)  
*ring coefficients live in*
- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is\\_zero\\_t = std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)

### Static Public Member Functions

- [static std::string to\\_string \(\)](#)
- [template<typename valueRing >](#)  
[static constexpr DEVICE INLINED valueRing eval \(const valueRing &x\)](#)

### Static Public Attributes

- [static constexpr size\\_t degree = 0](#)  
*degree*
- [static constexpr bool is\\_zero\\_v = is\\_zero\\_t::value](#)

### 8.29.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

## Template Parameters

|               |  |
|---------------|--|
| <i>coeffN</i> |  |
|---------------|--|

## 8.29.2 Member Typedef Documentation

### 8.29.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.29.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.29.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.29.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

### 8.29.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

### 8.29.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

### 8.29.3 Member Function Documentation

#### 8.29.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

#### 8.29.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

### 8.29.4 Member Data Documentation

#### 8.29.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

#### 8.29.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.30 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

#### Classes

- struct [val](#)  
*values in zpz*

## Public Types

- `using inner_type = int32_t`  
*underlying type for values*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`  
*injects a constant integer into zpz*
- `using zero = val< 0 >`  
*zero value*
- `using one = val< 1 >`  
*one value*
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos_t = typename pos< v1 >::type`  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`  
*true iff p is prime*
- `static constexpr bool is_euclidean_domain = true`  
*always true*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*

- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

### 8.30.1 Detailed Description

`template<int32_t p>`  
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

|          |           |
|----------|-----------|
| <i>p</i> | a integer |
|----------|-----------|

### 8.30.2 Member Typedef Documentation

#### 8.30.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

#### 8.30.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator



## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

**8.30.2.3 eq\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

**8.30.2.4 gcd\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

**8.30.2.5 gt\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

#### Template Parameters

|          |            |
|----------|------------|
| <i>x</i> | an integer |
|----------|------------|

### 8.30.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

### 8.30.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

### 8.30.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
|----|-------------------------------------|

### 8.30.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.30.3 Member Data Documentation

### 8.30.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.30.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.30.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (boolean value)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00019 #ifdef _MSC_VER
00020 #define ALIGNED(x) __declspec(align(x))
00021 #define INLINED __forceinline
00022 #else
00023 #define ALIGNED(x) __attribute__((aligned(x)))
00024 #define INLINED __attribute__((always_inline)) inline
00025 #endif
00026
00027 #ifdef __CUDACC__
00028 #define DEVICE __host__ __device__
```

```

00029 #else
00030 #define DEVICE
00031 #endif
00032
00033
00034
00035
00036
00037
00038
00039 // aligned allocation
00040 namespace aerobus {
00041     template<typename T>
00042     T* aligned_malloc(size_t count, size_t alignment) {
00043         #ifdef _MSC_VER
00044             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00045         #else
00046             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00047         #endif
00048     }
00049 } // namespace aerobus
00050
00051 // concepts
00052 namespace aerobus {
00053     template <typename R>
00054     concept IsRing = requires {
00055         typename R::one;
00056         typename R::zero;
00057         typename R::template add_t<typename R::one, typename R::one>;
00058         typename R::template sub_t<typename R::one, typename R::one>;
00059         typename R::template mul_t<typename R::one, typename R::one>;
00060     };
00061
00062     template <typename R>
00063     concept IsEuclideanDomain = IsRing<R> && requires {
00064         typename R::template div_t<typename R::one, typename R::one>;
00065         typename R::template mod_t<typename R::one, typename R::one>;
00066         typename R::template gcd_t<typename R::one, typename R::one>;
00067         typename R::template eq_t<typename R::one, typename R::one>;
00068         typename R::template pos_t<typename R::one>;
00069
00070         R::template pos_v<typename R::one> == true;
00071         // typename R::template gt_t<typename R::one, typename R::zero>;
00072         R::is_euclidean_domain == true;
00073     };
00074
00075     template<typename R>
00076     concept IsField = IsEuclideanDomain<R> && requires {
00077         R::is_field == true;
00078     };
00079 } // namespace aerobus
00080
00081 // utilities
00082 namespace aerobus {
00083     namespace internal {
00084         template<template<typename...> typename TT, typename T>
00085         struct is_instantiation_of : std::false_type { };
00086
00087         template<template<typename...> typename TT, typename... Ts>
00088         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00089
00090         template<template<typename...> typename TT, typename T>
00091         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00092
00093         template <int64_t i, typename T, typename... Ts>
00094         struct type_at {
00095             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00096             using type = typename type_at<i - 1, Ts...>::type;
00097         };
00098
00099         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00100             using type = T;
00101         };
00102
00103         template <size_t i, typename... Ts>
00104         using type_at_t = typename type_at<i, Ts...>::type;
00105
00106         template<size_t n, size_t i, typename E = void>
00107         struct _is_prime {};
00108
00109         template<size_t i>
00110         struct _is_prime<0, i> {
00111             static constexpr bool value = false;
00112         };
00113
00114         template<size_t i>
00115         struct _is_prime<1, i> {
00116             static constexpr bool value = false;
00117         };
00118     }
00119 }

```



```

00128
00129     template<size_t i>
00130     struct _is_prime<2, i> {
00131         static constexpr bool value = true;
00132     };
00133
00134     template<size_t i>
00135     struct _is_prime<3, i> {
00136         static constexpr bool value = true;
00137     };
00138
00139     template<size_t i>
00140     struct _is_prime<5, i> {
00141         static constexpr bool value = true;
00142     };
00143
00144     template<size_t i>
00145     struct _is_prime<7, i> {
00146         static constexpr bool value = true;
00147     };
00148
00149     template<size_t n, size_t i>
00150     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00151         static constexpr bool value = false;
00152     };
00153
00154     template<size_t n, size_t i>
00155     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00156         static constexpr bool value = false;
00157     };
00158
00159     template<size_t n, size_t i>
00160     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00161         static constexpr bool value = true;
00162     };
00163
00164     template<size_t n, size_t i>
00165     struct _is_prime<n, i, std::enable_if_t<(
00166         n % i == 0 &&
00167         n >= 9 &&
00168         n % 3 != 0 &&
00169         n % 2 != 0 &&
00170         i * i > n)>> {
00171         static constexpr bool value = true;
00172     };
00173
00174     template<size_t n, size_t i>
00175     struct _is_prime<n, i, std::enable_if_t<(
00176         n % (i+2) == 0 &&
00177         n >= 9 &&
00178         n % 3 != 0 &&
00179         n % 2 != 0 &&
00180         i * i <= n)>> {
00181         static constexpr bool value = true;
00182     };
00183
00184     template<size_t n, size_t i>
00185     struct _is_prime<n, i, std::enable_if_t<(
00186         n % (i+2) != 0 &&
00187         n % i != 0 &&
00188         n >= 9 &&
00189         n % 3 != 0 &&
00190         n % 2 != 0 &&
00191         (i * i <= n))>> {
00192         static constexpr bool value = _is_prime<n, i+6>::value;
00193     };
00194
00195 } // namespace internal
00196
00197 template<size_t n>
00198 struct is_prime {
00199     static constexpr bool value = internal::_is_prime<n, 5>::value;
00200 };
00201
00202 template<size_t n>
00203 static constexpr bool is_prime_v = is_prime<n>::value;
00204
00205 // gcd
00206 namespace internal {
00207     template <std::size_t... Is>
00208     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00209         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00210
00211     template <std::size_t N>
00212     using make_index_sequence_reverse
00213         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00214
00215 }
00216
00217
00218
00219
00220

```

```

00226     template<typename Ring, typename E = void>
00227     struct gcd;
00228
00229     template<typename Ring>
00230     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00231         template<typename A, typename B, typename E = void>
00232         struct gcd_helper {};
00233
00234         // B = 0, A > 0
00235         template<typename A, typename B>
00236         struct gcd_helper<A, B, std::enable_if_t<
00237             (B::is_zero_t::value) &&
00238             (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00239             using type = A;
00240         };
00241
00242         // B = 0, A < 0
00243         template<typename A, typename B>
00244         struct gcd_helper<A, B, std::enable_if_t<
00245             (B::is_zero_t::value) &&
00246             !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00247             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00248         };
00249
00250         // B != 0
00251         template<typename A, typename B>
00252         struct gcd_helper<A, B, std::enable_if_t<
00253             (!B::is_zero_t::value)
00254             >> {
00255             private: // NOLINT
00256                 // A / B
00257                 using k = typename Ring::template div_t<A, B>;
00258                 // A - (A/B)*B = A % B
00259                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00260
00261             public:
00262                 using type = typename gcd_helper<B, m>::type;
00263         };
00264
00265         template<typename A, typename B>
00266         using type = typename gcd_helper<A, B>::type;
00267     };
00268 } // namespace internal
00269
00270 // vadd and vmul
00271 namespace internal {
00272     template<typename... vals>
00273     struct vmul {};
00274
00275     template<typename v1, typename... vals>
00276     struct vmul<v1, vals...> {
00277         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00278     };
00279
00280     template<typename v1>
00281     struct vmul<v1> {
00282         using type = v1;
00283     };
00284
00285     template<typename... vals>
00286     struct vadd {};
00287
00288     template<typename v1, typename... vals>
00289     struct vadd<v1, vals...> {
00290         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00291     };
00292
00293     template<typename v1>
00294     struct vadd<v1> {
00295         using type = v1;
00296     };
00297 } // namespace internal
00298
00299 template<typename T, typename A, typename B>
00300 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00301
00302 template<typename... vals>
00303 using vadd_t = typename internal::vadd<vals...>::type;
00304
00305 template<typename... vals>
00306 using vmul_t = typename internal::vmul<vals...>::type;
00307
00308 template<typename val>
00309 requires IsEuclideanDomain<typename val::enclosing_type>
00310 using abs_t = std::conditional_t<

```

```

00322         val::enclosing_type::template pos_v<val>,
00323         val, typename val::enclosing_type::template
    sub_t<typename val::enclosing_type::zero, val>;
00324 } // namespace aerobus
00325
00326 // embedding
00327 namespace aerobus {
00332     template<typename Small, typename Large, typename E = void>
00333     struct Embed;
00334 } // namespace aerobus
00335
00336 namespace aerobus {
00341     template<typename Ring, typename X>
00342     requires IsRing<Ring>
00343     struct Quotient {
00346         template <typename V>
00347         struct val {
00348             public:
00349                 using raw_t = V;
00350                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00351         };
00352
00354         using zero = val<typename Ring::zero>;
00355
00357         using one = val<typename Ring::one>;
00358
00362         template<typename v1, typename v2>
00363         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00364
00368         template<typename v1, typename v2>
00369         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00370
00374         template<typename v1, typename v2>
00375         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00376
00380         template<typename v1, typename v2>
00381         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00382
00386         template<typename v1, typename v2>
00387         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00388
00392         template<typename v1, typename v2>
00393         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00394
00398         template<typename v1>
00399         using pos_t = std::true_type;
00400
00404         template<typename v>
00405         static constexpr bool pos_v = pos_t<v>::value;
00406
00408         static constexpr bool is_euclidean_domain = true;
00409
00415         template<auto x>
00416         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00417
00423         template<typename v>
00424         using inject_ring_t = val<v>;
00425     };
00426
00430     template<typename Ring, typename X>
00431     struct Embed<Quotient<Ring, X>, Ring> {
00434         template<typename val>
00435         using type = typename val::raw_t;
00436     };
00437 } // namespace aerobus
00438
00439 // type_list
00440 namespace aerobus {
00442     template <typename... Ts>
00443     struct type_list;
00444
00445     namespace internal {
00446         template <typename T, typename... Us>
00447         struct pop_front_h {
00448             using tail = type_list<Us...>;
00449             using head = T;
00450         };
00451
00452         template <size_t index, typename L1, typename L2>
00453         struct split_h {
00454             private:
00455                 static_assert(index <= L2::length, "index out of bounds");
00456                 using a = typename L2::pop_front::type;
00457                 using b = typename L2::pop_front::tail;
00458                 using c = typename L1::template push_back<a>;
00459
00460             public:

```

```

00461         using head = typename split_h<index - 1, c, b>::head;
00462         using tail = typename split_h<index - 1, c, b>::tail;
00463     };
00464
00465     template <typename L1, typename L2>
00466     struct split_h<0, L1, L2> {
00467         using head = L1;
00468         using tail = L2;
00469     };
00470
00471     template <size_t index, typename L, typename T>
00472     struct insert_h {
00473         static_assert(index <= L::length, "index out of bounds");
00474         using s = typename L::template split<index>;
00475         using left = typename s::head;
00476         using right = typename s::tail;
00477         using ll = typename left::template push_back<T>;
00478         using type = typename ll::template concat<right>;
00479     };
00480
00481     template <size_t index, typename L>
00482     struct remove_h {
00483         using s = typename L::template split<index>;
00484         using left = typename s::head;
00485         using right = typename s::tail;
00486         using rr = typename right::pop_front::tail;
00487         using type = typename left::template concat<rr>;
00488     };
00489 } // namespace internal
00490
00491 template <typename... Ts>
00492 struct type_list {
00493 private:
00494     template <typename T>
00495     struct concat_h;
00496
00497     template <typename... Us>
00498     struct concat_h<type_list<Us...> {
00499         using type = type_list<Ts..., Us...>;
00500     };
00501
00502 public:
00503     static constexpr size_t length = sizeof...(Ts);
00504
00505     template <typename T>
00506     using push_front = type_list<T, Ts...>;
00507
00508     template <size_t index>
00509     using at = internal::type_at_t<index, Ts...>;
00510
00511     struct pop_front {
00512         using type = typename internal::pop_front_h<Ts...>::head;
00513         using tail = typename internal::pop_front_h<Ts...>::tail;
00514     };
00515
00516     template <typename T>
00517     using push_back = type_list<Ts..., T>;
00518
00519     template <typename U>
00520     using concat = typename concat_h<U>::type;
00521
00522     template <size_t index>
00523     struct split {
00524 private:
00525         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00526
00527     public:
00528         using head = typename inner::head;
00529         using tail = typename inner::tail;
00530     };
00531
00532     template <typename T, size_t index>
00533     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00534
00535     template <size_t index>
00536     using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00537 };
00538
00539 template <>
00540 struct type_list<> {
00541     static constexpr size_t length = 0;
00542
00543     template <typename T>
00544     using push_front = type_list<T>;
00545
00546     template <typename T>
00547     using push_back = type_list<T>;
00548
00549
00550
00551
00552
00553
00554
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00556
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00566
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00569
00570

```

```

00571
00572     template <typename U>
00573     using concat = U;
00574
00575     // TODO(jewave): assert index == 0
00576     template <typename T, size_t index>
00577     using insert = type_list<T>;
00578 };
00579 } // namespace aerobus
00580
00581 // i32
00582 namespace aerobus {
00583     struct i32 {
00584         using inner_type = int32_t;
00585         template<int32_t x>
00586         struct val {
00587             using enclosing_type = i32;
00588             static constexpr int32_t v = x;
00589
00590             template<typename valueType>
00591             static constexpr INLINED DEVICE valueType get() { return static_cast<valueType>(x); }
00592
00593             using is_zero_t = std::bool_constant<x == 0>;
00594
00595             static std::string to_string() {
00596                 return std::to_string(x);
00597             }
00598
00599             template<typename valueRing>
00600             static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
00601                 return static_cast<valueRing>(x);
00602             }
00603         };
00604     };
00605
00606     using zero = val<0>;
00607     using one = val<1>;
00608     static constexpr bool is_field = false;
00609     static constexpr bool is_euclidean_domain = true;
00610     template<auto x>
00611     using inject_constant_t = val<static_cast<int32_t>(x)>;
00612
00613     template<typename v>
00614     using inject_ring_t = v;
00615
00616 private:
00617     template<typename v1, typename v2>
00618     struct add {
00619         using type = val<v1::v + v2::v>;
00620     };
00621
00622     template<typename v1, typename v2>
00623     struct sub {
00624         using type = val<v1::v - v2::v>;
00625     };
00626
00627     template<typename v1, typename v2>
00628     struct mul {
00629         using type = val<v1::v * v2::v>;
00630     };
00631
00632     template<typename v1, typename v2>
00633     struct div {
00634         using type = val<v1::v / v2::v>;
00635     };
00636
00637     template<typename v1, typename v2>
00638     struct remainder {
00639         using type = val<v1::v % v2::v>;
00640     };
00641
00642     template<typename v1, typename v2>
00643     struct gt {
00644         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00645     };
00646
00647     template<typename v1, typename v2>
00648     struct lt {
00649         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00650     };
00651
00652     template<typename v1, typename v2>
00653     struct eq {
00654         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00655     };
00656
00657     template<typename v1>
00658     struct pos {

```

```

00676         using type = std::bool_constant<(v1::v > 0)>;
00677     };
00678
00679     public:
00685         template<typename v1, typename v2>
00686         using add_t = typename add<v1, v2>::type;
00687
00693         template<typename v1, typename v2>
00694         using sub_t = typename sub<v1, v2>::type;
00695
00701         template<typename v1, typename v2>
00702         using mul_t = typename mul<v1, v2>::type;
00703
00709         template<typename v1, typename v2>
00710         using div_t = typename div<v1, v2>::type;
00711
00717         template<typename v1, typename v2>
00718         using mod_t = typename remainder<v1, v2>::type;
00719
00725         template<typename v1, typename v2>
00726         using gt_t = typename gt<v1, v2>::type;
00727
00733         template<typename v1, typename v2>
00734         using lt_t = typename lt<v1, v2>::type;
00735
00741         template<typename v1, typename v2>
00742         using eq_t = typename eq<v1, v2>::type;
00743
00748         template<typename v1, typename v2>
00749         static constexpr bool eq_v = eq_t<v1, v2>::value;
00750
00756         template<typename v1, typename v2>
00757         using gcd_t = gcd_t<i32, v1, v2>;
00758
00763         template<typename v>
00764         using pos_t = typename pos<v>::type;
00765
00770         template<typename v>
00771         static constexpr bool pos_v = pos_t<v>::value;
00772     };
00773 } // namespace aerobus
00774
00775 // i64
00776 namespace aerobus {
00777     struct i64 {
00780         using inner_type = int64_t;
00781         template<int64_t x>
00782         struct val {
00786             using inner_type = int32_t;
00787             using enclosing_type = i64;
00788             static constexpr int64_t v = x;
00791
00794             template<typename valueType>
00795             static constexpr DEVICE INLINED valueType get() {
00796                 return static_cast<valueType>(x);
00797             }
00798
00800             using is_zero_t = std::bool_constant<x == 0>;
00801
00803             static std::string to_string() {
00804                 return std::to_string(x);
00805             }
00806
00809             template<typename valueRing>
00810             static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
00811                 return static_cast<valueRing>(x);
00812             }
00813         };
00814
00818         template<auto x>
00819         using inject_constant_t = val<static_cast<int64_t>(x)>;
00820
00825         template<typename v>
00826         using inject_ring_t = v;
00827
00829         using zero = val<0>;
00831         using one = val<1>;
00833         static constexpr bool is_field = false;
00835         static constexpr bool is_euclidean_domain = true;
00836
00837     private:
00838         template<typename v1, typename v2>
00839         struct add {
00840             using type = val<v1::v + v2::v>;
00841         };
00842
00843         template<typename v1, typename v2>

```

```

00844     struct sub {
00845         using type = val<v1::v - v2::v>;
00846     };
00847
00848     template<typename v1, typename v2>
00849     struct mul {
00850         using type = val<v1::v* v2::v>;
00851     };
00852
00853     template<typename v1, typename v2>
00854     struct div {
00855         using type = val<v1::v / v2::v>;
00856     };
00857
00858     template<typename v1, typename v2>
00859     struct remainder {
00860         using type = val<v1::v% v2::v>;
00861     };
00862
00863     template<typename v1, typename v2>
00864     struct gt {
00865         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00866     };
00867
00868     template<typename v1, typename v2>
00869     struct lt {
00870         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00871     };
00872
00873     template<typename v1, typename v2>
00874     struct eq {
00875         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00876     };
00877
00878     template<typename v>
00879     struct pos {
00880         using type = std::bool_constant<(v::v > 0)>;
00881     };
00882
00883 public:
00884     template<typename v1, typename v2>
00885     using add_t = typename add<v1, v2>::type;
00886
00887     template<typename v1, typename v2>
00888     using sub_t = typename sub<v1, v2>::type;
00889
00890     template<typename v1, typename v2>
00891     using mul_t = typename mul<v1, v2>::type;
00892
00893     template<typename v1, typename v2>
00894     using div_t = typename div<v1, v2>::type;
00895
00896     template<typename v1, typename v2>
00897     using mod_t = typename remainder<v1, v2>::type;
00898
00899     template<typename v1, typename v2>
00900     using gt_t = typename gt<v1, v2>::type;
00901
00902     template<typename v1, typename v2>
00903     static constexpr bool gt_v = gt_t<v1, v2>::value;
00904
00905     template<typename v1, typename v2>
00906     using lt_t = typename lt<v1, v2>::type;
00907
00908     template<typename v1, typename v2>
00909     static constexpr bool lt_v = lt_t<v1, v2>::value;
00910
00911     template<typename v1, typename v2>
00912     using eq_t = typename eq<v1, v2>::type;
00913
00914     template<typename v1, typename v2>
00915     static constexpr bool eq_v = eq_t<v1, v2>::value;
00916
00917     template<typename v1, typename v2>
00918     using gcd_t = gcd_t<i64, v1, v2>;
00919
00920     template<typename v>
00921     using pos_t = typename pos<v>::type;
00922
00923     template<typename v>
00924     static constexpr bool pos_v = pos_t<v>::value;
00925 };
00926
00927 template<>
00928 struct Embed<i32, i64> {
00929     template<typename val>
00930     using type = i64::val<static_cast<int64_t>(val::v)>;

```

```

00997     };
00998 } // namespace aerobus
00999
01000 // z/pz
01001 namespace aerobus {
01002     template<int32_t p>
01003     struct zpz {
01004         using inner_type = int32_t;
01005
01006         template<int32_t x>
01007         struct val {
01008             using enclosing_type = zpz<p>;
01009             static constexpr int32_t v = x % p;
01010
01011             template<typename valueType>
01012             static constexpr DEVICE INLINED valueType get() { return static_cast<valueType>(x % p); }
01013
01014             using is_zero_t = std::bool_constant<v == 0>;
01015
01016             static constexpr bool is_zero_v = v == 0;
01017
01018             static std::string to_string() {
01019                 return std::to_string(x % p);
01020             }
01021
01022             template<typename valueRing>
01023             static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
01024                 return static_cast<valueRing>(x % p);
01025             }
01026         };
01027     };
01028
01029     template<auto x>
01030     using inject_constant_t = val<static_cast<int32_t>(x)>;
01031
01032     using zero = val<0>;
01033
01034     using one = val<1>;
01035
01036     static constexpr bool is_field = is_prime<p>::value;
01037
01038     static constexpr bool is_euclidean_domain = true;
01039
01040 private:
01041     template<typename v1, typename v2>
01042     struct add {
01043         using type = val<(v1::v + v2::v) % p>;
01044     };
01045
01046     template<typename v1, typename v2>
01047     struct sub {
01048         using type = val<(v1::v - v2::v) % p>;
01049     };
01050
01051     template<typename v1, typename v2>
01052     struct mul {
01053         using type = val<(v1::v * v2::v) % p>;
01054     };
01055
01056     template<typename v1, typename v2>
01057     struct div {
01058         using type = val<(v1::v % p) / (v2::v % p)>;
01059     };
01060
01061     template<typename v1, typename v2>
01062     struct remainder {
01063         using type = val<(v1::v % v2::v) % p>;
01064     };
01065
01066     template<typename v1, typename v2>
01067     struct gt {
01068         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01069     };
01070
01071     template<typename v1, typename v2>
01072     struct lt {
01073         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01074     };
01075
01076     template<typename v1, typename v2>
01077     struct eq {
01078         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01079     };
01080
01081     template<typename v1>
01082     struct pos {
01083         using type = std::bool_constant<(v1::v > 0)>;
01084     };
01085 }
01086

```



```

01106
01107     public:
01111         template<typename v1, typename v2>
01112             using add_t = typename add<v1, v2>::type;
01113
01117         template<typename v1, typename v2>
01118             using sub_t = typename sub<v1, v2>::type;
01119
01123         template<typename v1, typename v2>
01124             using mul_t = typename mul<v1, v2>::type;
01125
01129         template<typename v1, typename v2>
01130             using div_t = typename div<v1, v2>::type;
01131
01135         template<typename v1, typename v2>
01136             using mod_t = typename remainder<v1, v2>::type;
01137
01141         template<typename v1, typename v2>
01142             using gt_t = typename gt<v1, v2>::type;
01143
01147         template<typename v1, typename v2>
01148             static constexpr bool gt_v = gt_t<v1, v2>::value;
01149
01153         template<typename v1, typename v2>
01154             using lt_t = typename lt<v1, v2>::type;
01155
01159         template<typename v1, typename v2>
01160             static constexpr bool lt_v = lt_t<v1, v2>::value;
01161
01165         template<typename v1, typename v2>
01166             using eq_t = typename eq<v1, v2>::type;
01167
01171         template<typename v1, typename v2>
01172             static constexpr bool eq_v = eq_t<v1, v2>::value;
01173
01177         template<typename v1, typename v2>
01178             using gcd_t = gcd_t<i32, v1, v2>;
01179
01182         template<typename v1>
01183             using pos_t = typename pos<v1>::type;
01184
01187         template<typename v>
01188             static constexpr bool pos_v = pos_t<v>::value;
01189     };
01190
01193     template<int32_t x>
01194     struct Embed<zpz<x>, i32> {
01197         template <typename val>
01198             using type = i32::val<val::v>;
01199     };
01200 } // namespace aerobus
01201
01202 // polynomial
01203 namespace aerobus {
01204     // coeffN x^N + ...
01209     template<typename Ring>
01210     requires IsEuclideanDomain<Ring>
01211     struct polynomial {
01212         static constexpr bool is_field = false;
01213         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01214
01218         template<typename coeffN, typename... coeffs>
01219         struct val {
01221             using ring_type = Ring;
01222             using enclosing_type = polynomial<Ring>;
01225             static constexpr size_t degree = sizeof...(coeffs);
01227             using aN = coeffN;
01229             using strip = val<coeffs...>;
01231             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01233             static constexpr bool is_zero_v = is_zero_t::value;
01234
01235         private:
01236             template<size_t index, typename E = void>
01237                 struct coeff_at {};
01238
01239             template<size_t index>
01240             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01241                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01242             };
01243
01244             template<size_t index>
01245             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01246                 using type = typename Ring::zero;
01247             };
01248
01249         public:
01252             template<size_t index>

```

```

01253         using coeff_at_t = typename coeff_at<index>::type;
01254
01257     static std::string to_string() {
01258         return string_helper<coeffN, coeffs...>::func();
01259     }
01260
01265     template<typename valueRing>
01266     static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {
01267         return horner_evaluation<valueRing, val>
01268             ::template inner<0, degree + 1>
01269             ::func(static_cast<valueRing>(0), x);
01270     }
01271 };
01272
01275 template<typename coeffN>
01276 struct val<coeffN> {
01277     using ring_type = Ring;
01278     using enclosing_type = polynomial<Ring>;
01282     static constexpr size_t degree = 0;
01283     using aN = coeffN;
01284     using strip = val<coeffN>;
01285     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01286
01287     static constexpr bool is_zero_v = is_zero_t::value;
01288
01289     template<size_t index, typename E = void>
01290     struct coeff_at {};
01291
01292     template<size_t index>
01293     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01294         using type = aN;
01295     };
01296
01297     template<size_t index>
01298     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01299         using type = typename Ring::zero;
01300     };
01301
01302     template<size_t index>
01303     using coeff_at_t = typename coeff_at<index>::type;
01304
01305     static std::string to_string() {
01306         return string_helper<coeffN>::func();
01307     }
01308
01309     template<typename valueRing>
01310     static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {
01311         return static_cast<valueRing>(aN::template get<valueRing>());
01312     }
01313 };
01314
01316 using zero = val<typename Ring::zero>;
01318 using one = val<typename Ring::one>;
01320 using X = val<typename Ring::one, typename Ring::zero>;
01321
01322 private:
01323     template<typename P, typename E = void>
01324     struct simplify;
01325
01326     template <typename P1, typename P2, typename I>
01327     struct add_low;
01328
01329     template<typename P1, typename P2>
01330     struct add {
01331         using type = typename simplify<typename add_low<
01332             P1,
01333             P2,
01334             internal::make_index_sequence_reverse<
01335                 std::max(P1::degree, P2::degree) + 1
01336             >::type>::type;
01337     };
01338
01339     template <typename P1, typename P2, typename I>
01340     struct sub_low;
01341
01342     template <typename P1, typename P2, typename I>
01343     struct mul_low;
01344
01345     template<typename v1, typename v2>
01346     struct mul {
01347         using type = typename mul_low<
01348             v1,
01349             v2,
01350             internal::make_index_sequence_reverse<
01351                 v1::degree + v2::degree + 1
01352             >::type>;
01353     };

```

```

01354
01355     template<typename coeff, size_t deg>
01356     struct monomial;
01357
01358     template<typename v, typename E = void>
01359     struct derive_helper {};
01360
01361     template<typename v>
01362     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01363         using type = zero;
01364     };
01365
01366     template<typename v>
01367     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01368         using type = typename add<
01369             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01370             typename monomial<
01371                 typename Ring::template mul_t<
01372                     typename v::aN,
01373                     typename Ring::template inject_constant_t<(v::degree)>
01374                 >,
01375                 v::degree - 1
01376             >::type
01377         >::type;
01378     };
01379
01380     template<typename v1, typename v2, typename E = void>
01381     struct eq_helper {};
01382
01383     template<typename v1, typename v2>
01384     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01385         using type = std::false_type;
01386     };
01387
01388     template<typename v1, typename v2>
01389     struct eq_helper<v1, v2, std::enable_if_t<
01390         v1::degree == v2::degree &&
01391         (v1::degree != 0 || v2::degree != 0) &&
01392         std::is_same<
01393             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01394             std::false_type
01395         >::value
01396     > {
01397     > {
01398         using type = std::false_type;
01399     };
01400
01401     template<typename v1, typename v2>
01402     struct eq_helper<v1, v2, std::enable_if_t<
01403         v1::degree == v2::degree &&
01404         (v1::degree != 0 || v2::degree != 0) &&
01405         std::is_same<
01406             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01407             std::true_type
01408         >::value
01409     > {
01410     > {
01411         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01412     };
01413
01414     template<typename v1, typename v2>
01415     struct eq_helper<v1, v2, std::enable_if_t<
01416         v1::degree == v2::degree &&
01417         (v1::degree == 0)
01418     > {
01419         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01420     };
01421
01422     template<typename v1, typename v2, typename E = void>
01423     struct lt_helper {};
01424
01425     template<typename v1, typename v2>
01426     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01427         using type = std::true_type;
01428     };
01429
01430     template<typename v1, typename v2>
01431     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01432         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01433     };
01434
01435     template<typename v1, typename v2>
01436     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01437         using type = std::false_type;
01438     };
01439
01440     template<typename v1, typename v2, typename E = void>

```

```

01441     struct gt_helper {};
01442
01443     template<typename v1, typename v2>
01444     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01445         using type = std::true_type;
01446     };
01447
01448     template<typename v1, typename v2>
01449     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01450         using type = std::false_type;
01451     };
01452
01453     template<typename v1, typename v2>
01454     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01455         using type = std::false_type;
01456     };
01457
01458     // when high power is zero : strip
01459     template<typename P>
01460     struct simplify<P, std::enable_if_t<
01461         std::is_same<
01462             typename Ring::zero,
01463             typename P::aN
01464         >::value && (P::degree > 0)
01465     >> {
01466         using type = typename simplify<typename P::strip>::type;
01467     };
01468
01469     // otherwise : do nothing
01470     template<typename P>
01471     struct simplify<P, std::enable_if_t<
01472         !std::is_same<
01473             typename Ring::zero,
01474             typename P::aN
01475         >::value && (P::degree > 0)
01476     >> {
01477         using type = P;
01478     };
01479
01480     // do not simplify constants
01481     template<typename P>
01482     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01483         using type = P;
01484     };
01485
01486     // addition at
01487     template<typename P1, typename P2, size_t index>
01488     struct add_at {
01489         using type =
01490             typename Ring::template add_t<
01491                 typename P1::template coeff_at_t<index>,
01492                 typename P2::template coeff_at_t<index>>;
01493     };
01494
01495     template<typename P1, typename P2, size_t index>
01496     using add_at_t = typename add_at<P1, P2, index>::type;
01497
01498     template<typename P1, typename P2, std::size_t... I>
01499     struct add_low<P1, P2, std::index_sequence<I...>> {
01500         using type = val<add_at_t<P1, P2, I>...>;
01501     };
01502
01503     // subtraction at
01504     template<typename P1, typename P2, size_t index>
01505     struct sub_at {
01506         using type =
01507             typename Ring::template sub_t<
01508                 typename P1::template coeff_at_t<index>,
01509                 typename P2::template coeff_at_t<index>>;
01510     };
01511
01512     template<typename P1, typename P2, size_t index>
01513     using sub_at_t = typename sub_at<P1, P2, index>::type;
01514
01515     template<typename P1, typename P2, std::size_t... I>
01516     struct sub_low<P1, P2, std::index_sequence<I...>> {
01517         using type = val<sub_at_t<P1, P2, I>...>;
01518     };
01519
01520     template<typename P1, typename P2>
01521     struct sub {
01522         using type = typename simplify<typename sub_low<
01523             P1,
01524             P2,
01525             internal::make_index_sequence<
01526                 std::max(P1::degree, P2::degree) + 1
01527             >::type>::type;

```

```

01528     };
01529
01530     // multiplication at
01531     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01532     struct mul_at_loop_helper {
01533         using type = typename Ring::template add_t<
01534             typename Ring::template mul_t<
01535                 typename v1::template coeff_at_t<index>,
01536                 typename v2::template coeff_at_t<k - index>
01537             >,
01538             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01539         >;
01540     };
01541
01542     template<typename v1, typename v2, size_t k, size_t stop>
01543     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01544         using type = typename Ring::template mul_t<
01545             typename v1::template coeff_at_t<stop>,
01546             typename v2::template coeff_at_t<0>>;
01547     };
01548
01549     template<typename v1, typename v2, size_t k, typename E = void>
01550     struct mul_at {};
01551
01552     template<typename v1, typename v2, size_t k>
01553     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01554         using type = typename Ring::zero;
01555     };
01556
01557     template<typename v1, typename v2, size_t k>
01558     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01559         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01560     };
01561
01562     template<typename P1, typename P2, size_t index>
01563     using mul_at_t = typename mul_at<P1, P2, index>::type;
01564
01565     template<typename P1, typename P2, std::size_t... I>
01566     struct mul_low<P1, P2, std::index_sequence<I...> {
01567         using type = val<mul_at_t<P1, P2, I>...>;
01568     };
01569
01570     // division helper
01571     template<typename A, typename B, typename Q, typename R, typename E = void>
01572     struct div_helper {};
01573
01574     template<typename A, typename B, typename Q, typename R>
01575     struct div_helper<A, B, Q, R, std::enable_if_t<
01576         (R::degree < B::degree) ||
01577         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01578         using q_type = Q;
01579         using mod_type = R;
01580         using gcd_type = B;
01581     };
01582
01583     template<typename A, typename B, typename Q, typename R>
01584     struct div_helper<A, B, Q, R, std::enable_if_t<
01585         (R::degree >= B::degree) &&
01586         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01587     private: // NOLINT
01588         using rN = typename R::aN;
01589         using bN = typename B::aN;
01590         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01591             B::degree>::type;
01592         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01593         using qq = typename add<Q, pT>::type;
01594
01595     public:
01596         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01597         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01598         using gcd_type = rr;
01599     };
01600
01601     template<typename A, typename B>
01602     struct div {
01603         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01604         using q_type = typename div_helper<A, B, zero, A>::q_type;
01605         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01606     };
01607
01608     template<typename P>
01609     struct make_unit {
01610         using type = typename div<P, val<typename P::aN>>::q_type;
01611     };
01612
01613     template<typename coeff, size_t deg>
01614     struct monomial {

```

```

01614         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01615     };
01616
01617     template<typename coeff>
01618     struct monomial<coeff, 0> {
01619         using type = val<coeff>;
01620     };
01621
01622     template<typename valueRing, typename P>
01623     struct horner_evaluation {
01624         template<size_t index, size_t stop>
01625         struct inner {
01626             static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01627                 constexpr valueRing coeff =
01628                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
get<valueRing>());
01629                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
accum + coeff, x);
01630             }
01631         };
01632
01633         template<size_t stop>
01634         struct inner<stop, stop> {
01635             static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01636                 return accum;
01637             }
01638         };
01639     };
01640
01641     template<typename coeff, typename... coeffs>
01642     struct string_helper {
01643         static std::string func() {
01644             std::string tail = string_helper<coeffs...>::func();
01645             std::string result = "";
01646             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01647                 return tail;
01648             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01649                 if (sizeof...(coeffs) == 1) {
01650                     result += "x";
01651                 } else {
01652                     result += "x^" + std::to_string(sizeof...(coeffs));
01653                 }
01654             } else {
01655                 if (sizeof...(coeffs) == 1) {
01656                     result += coeff::to_string() + " x";
01657                 } else {
01658                     result += coeff::to_string()
+ " x^" + std::to_string(sizeof...(coeffs));
01659                 }
01660             }
01661             if (!tail.empty()) {
01662                 result += " + " + tail;
01663             }
01664             return result;
01665         }
01666     };
01667
01668     template<typename coeff>
01669     struct string_helper<coeff> {
01670         static std::string func() {
01671             if (!std::is_same<coeff, typename Ring::zero>::value) {
01672                 return coeff::to_string();
01673             } else {
01674                 return "";
01675             }
01676         }
01677     };
01678
01679     public:
01680     template<typename P>
01681     using simplify_t = typename simplify<P>::type;
01682
01683     template<typename v1, typename v2>
01684     using add_t = typename add<v1, v2>::type;
01685
01686     template<typename v1, typename v2>
01687     using sub_t = typename sub<v1, v2>::type;
01688
01689     template<typename v1, typename v2>
01690     using mul_t = typename mul<v1, v2>::type;
01691
01692     template<typename v1, typename v2>
01693     using eq_t = typename eq_helper<v1, v2>::type;

```

```

01711
01715     template<typename v1, typename v2>
01716     using lt_t = typename lt_helper<v1, v2>::type;
01717
01721     template<typename v1, typename v2>
01722     using gt_t = typename gt_helper<v1, v2>::type;
01723
01727     template<typename v1, typename v2>
01728     using div_t = typename div<v1, v2>::q_type;
01729
01733     template<typename v1, typename v2>
01734     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01735
01739     template<typename coeff, size_t deg>
01740     using monomial_t = typename monomial<coeff, deg>::type;
01741
01744     template<typename v>
01745     using derive_t = typename derive_helper<v>::type;
01746
01749     template<typename v>
01750     using pos_t = typename Ring::template pos_t<typename v::aN>;
01751
01754     template<typename v>
01755     static constexpr bool pos_v = pos_t<v>::value;
01756
01760     template<typename v1, typename v2>
01761     using gcd_t = std::conditional_t<
01762         Ring::is_euclidean_domain,
01763         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01764         void>;
01765
01769     template<auto x>
01770     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01771
01775     template<typename v>
01776     using inject_ring_t = val<v>;
01777 };
01778 } // namespace aerobus
01779
01780 // fraction field
01781 namespace aerobus {
01782     namespace internal {
01783         template<typename Ring, typename E = void>
01784         requires IsEuclideanDomain<Ring>
01785         struct _FractionField {};
01786
01787         template<typename Ring>
01788         requires IsEuclideanDomain<Ring>
01789         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01790             static constexpr bool is_field = true;
01791             static constexpr bool is_euclidean_domain = true;
01792
01793         private:
01794             template<typename val1, typename val2, typename E = void>
01795             struct to_string_helper {};
01796
01797             template<typename val1, typename val2>
01798             struct to_string_helper<val1, val2,
01799                 std::enable_if_t<
01800                     Ring::template eq_t<
01801                         val2, typename Ring::one
01802                     >::value
01803                 >
01804             > {
01805                 static std::string func() {
01806                     return val1::to_string();
01807                 }
01808             };
01809
01810             template<typename val1, typename val2>
01811             struct to_string_helper<val1, val2,
01812                 std::enable_if_t<
01813                     !Ring::template eq_t<
01814                         val2,
01815                         typename Ring::one
01816                     >::value
01817                 >
01818             > {
01819                 static std::string func() {
01820                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01821                 }
01822             };
01823         };
01824
01825     public:
01829         template<typename val1, typename val2>
01830         struct val {
01831             using x = val1;

```

```

01834         using y = val2;
01836         using is_zero_t = typename val1::is_zero_t;
01838         static constexpr bool is_zero_v = val1::is_zero_t::value;
01839
01841         using ring_type = Ring;
01842         using enclosing_type = _FractionField<Ring>;
01843
01846         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01847
01851         template<typename valueType>
01852         static constexpr DEVICE INLINED valueType get() {
01853             return static_cast<valueType>(x::v) / static_cast<valueType>(y::v);
01854         }
01855
01858         static std::string to_string() {
01859             return to_string_helper<val1, val2>::func();
01860         }
01861
01866         template<typename valueRing>
01867         static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
01868             return x::eval(v) / y::eval(v);
01869         }
01870     };
01871
01873     using zero = val<typename Ring::zero, typename Ring::one>;
01875     using one = val<typename Ring::one, typename Ring::one>;
01876
01879     template<typename v>
01880     using inject_t = val<v, typename Ring::one>;
01881
01884     template<auto x>
01885     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
01886
01889     template<typename v>
01890     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01891
01893     using ring_type = Ring;
01894
01895 private:
01896     template<typename v, typename E = void>
01897     struct simplify {};
01898
01899     // x = 0
01900     template<typename v>
01901     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01902         using type = typename _FractionField<Ring>::zero;
01903     };
01904
01905     // x != 0
01906     template<typename v>
01907     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01908     private:
01909         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01910         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01911         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01912
01913         using posx = std::conditional_t<
01914             !Ring::template pos_v<newy>,
01915             typename Ring::template sub_t<typename Ring::zero, newx>,
01916             newx>;
01917         using posy = std::conditional_t<
01918             !Ring::template pos_v<newy>,
01919             typename Ring::template sub_t<typename Ring::zero, newy>,
01920             newy>;
01921     public:
01922         using type = typename _FractionField<Ring>::template val<posx, posy>;
01923     };
01924
01925 public:
01926     template<typename v>
01927     using simplify_t = typename simplify<v>::type;
01928
01931 private:
01932     template<typename v1, typename v2>
01933     struct add {
01934     private:
01935         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01936         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01937         using dividend = typename Ring::template add_t<a, b>;
01938         using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01939         using g = typename Ring::template gcd_t<dividend, divider>;
01940
01941     public:
01942         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
01943     };

```



```

01944
01945     template<typename v>
01946     struct pos {
01947         using type = std::conditional_t<
01948             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01949             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01950             std::true_type,
01951             std::false_type>;
01952     };
01953
01954     template<typename v1, typename v2>
01955     struct sub {
01956     private:
01957         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01958         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01959         using dividend = typename Ring::template sub_t<a, b>;
01960         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01961         using g = typename Ring::template gcd_t<dividend, diviser>;
01962
01963     public:
01964         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01965     };
01966
01967     template<typename v1, typename v2>
01968     struct mul {
01969     private:
01970         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01971         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01972
01973     public:
01974         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01975     };
01976
01977     template<typename v1, typename v2, typename E = void>
01978     struct div {};
01979
01980     template<typename v1, typename v2>
01981     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01982     private:
01983         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01984         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01985
01986     public:
01987         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01988     };
01989
01990     template<typename v1, typename v2>
01991     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01992         using type = one;
01993     };
01994
01995
01996     template<typename v1, typename v2>
01997     struct eq {
01998     private:
01999         using type = std::conditional_t<
std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
std::true_type,
std::false_type>;
02000
02001
02002     };
02003
02004
02005     template<typename v1, typename v2, typename E = void>
02006     struct gt;
02007
02008     template<typename v1, typename v2>
02009     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
>> {
02010         using type = std::false_type;
02011     };
02012
02013
02014
02015     template<typename v1, typename v2>
02016     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(!pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
02017         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02018     };
02019
02020
02021     template<typename v1, typename v2>
02022     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
02023         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02024     };
02025
02026     template<typename v1, typename v2>
02027     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
02028         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
02029     };

```

```

02029         » {
02030             using type = std::true_type;
02031         };
02032
02033     template<typename v1, typename v2>
02034     struct gt<v1, v2, std::enable_if_t<
02035         (!eq<v1, v2>::type::value) &&
02036         (!pos<v1>::type::value) && (pos<v2>::type::value)
02037     > {
02038         using type = std::false_type;
02039     };
02040
02041     template<typename v1, typename v2>
02042     struct gt<v1, v2, std::enable_if_t<
02043         (!eq<v1, v2>::type::value) &&
02044         (pos<v1>::type::value) && (pos<v2>::type::value)
02045     > {
02046         using type = typename Ring::template gt_t<
02047             typename Ring::template mul_t<v1::x, v2::y>,
02048             typename Ring::template mul_t<v2::y, v2::x>
02049         >;
02050     };
02051
02052     public:
02053     template<typename v1, typename v2>
02054     using add_t = typename add<v1, v2>::type;
02055
02056     template<typename v1, typename v2>
02057     using mod_t = zero;
02058
02059     template<typename v1, typename v2>
02060     using gcd_t = v1;
02061
02062     template<typename v1, typename v2>
02063     using sub_t = typename sub<v1, v2>::type;
02064
02065     template<typename v1, typename v2>
02066     using mul_t = typename mul<v1, v2>::type;
02067
02068     template<typename v1, typename v2>
02069     using div_t = typename div<v1, v2>::type;
02070
02071     template<typename v1, typename v2>
02072     using eq_t = typename eq<v1, v2>::type;
02073
02074     template<typename v1, typename v2>
02075     static constexpr bool eq_v = eq<v1, v2>::type::value;
02076
02077     template<typename v1, typename v2>
02078     using gt_t = typename gt<v1, v2>::type;
02079
02080     template<typename v1, typename v2>
02081     static constexpr bool gt_v = gt<v1, v2>::type::value;
02082
02083     template<typename v1>
02084     using pos_t = typename pos<v1>::type;
02085
02086     template<typename v>
02087     static constexpr bool pos_v = pos<t<v>::value;
02088 };
02089
02090 template<typename Ring, typename E = void>
02091 requires IsEuclideanDomain<Ring>
02092 struct FractionFieldImpl {};
02093
02094 // fraction field of a field is the field itself
02095 template<typename Field>
02096 requires IsEuclideanDomain<Field>
02097 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02098     using type = Field;
02099     template<typename v>
02100     using inject_t = v;
02101 };
02102
02103 // fraction field of a ring is the actual fraction field
02104 template<typename Ring>
02105 requires IsEuclideanDomain<Ring>
02106 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02107     using type = _FractionField<Ring>;
02108 };
02109 } // namespace internal
02110
02111 template<typename Ring>
02112 requires IsEuclideanDomain<Ring>
02113 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02114
02115 template<typename Ring>

```

```

02158     struct Embed<Ring, FractionField<Ring>> {
02161         template<typename v>
02162             using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02163     };
02164 } // namespace aerobus
02165
02166 // short names for common types
02167 namespace aerobus {
02172     template<typename X, typename Y>
02173     requires(std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02174     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02175
02176     template<typename X, typename Y>
02177     requires(std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02178     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02179
02180     template<typename X, typename Y>
02181     requires(std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02182     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02183
02184     template<typename X, typename Y>
02185     requires(std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02186     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02187
02188     using q32 = FractionField<i32>;
02189
02190     using fpq32 = FractionField<polynomial<q32>>;
02191
02192     using q64 = FractionField<i64>;
02193
02194     using pi64 = polynomial<i64>;
02195
02196     using pq64 = polynomial<q64>;
02197
02198     using fpq64 = FractionField<polynomial<q64>>;
02199
02200     template<typename Ring, typename v1, typename v2>
02201     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02202
02203     template<typename v>
02204     using embed_int_poly_in_fractions_t =
02205         typename Embed<
02206             polynomial<typename v::ring_type>,
02207             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02208         >
02209
02210     template<int64_t p, int64_t q>
02211     using make_q64_t = typename q64::template simplify_t<
02212         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02213     >
02214
02215     template<int32_t p, int32_t q>
02216     using make_q32_t = typename q32::template simplify_t<
02217         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02218     >
02219
02220     template<typename Ring, typename v1, typename v2>
02221     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02222
02223     template<typename Ring, typename v1, typename v2>
02224     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02225
02226     template<>
02227     struct Embed<q32, q64> {
02228         template<typename v>
02229             using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02230     };
02231
02232     template<typename Small, typename Large>
02233     struct Embed<polynomial<Small>, polynomial<Large>> {
02234     private:
02235         template<typename v, typename i>
02236         struct at_low;
02237
02238         template<typename v, size_t i>
02239         struct at_index {
02240             using type = typename Embed<Small, Large>::template
02241                 type<typename v::template coeff_at_t<i>>;
02242         };
02243
02244         template<typename v, size_t... Is>
02245         struct at_low<v, std::index_sequence<Is...>> {
02246             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02247         };
02248
02249     public:
02250         template<typename v>
02251         using type = typename
02252             at_low<v, typename internal::make_index_sequence_reverse<v::degree + 1>::type>;
02253     };

```

```

02298
02302     template<typename Ring, auto... xs>
02303     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02304         typename Ring::template inject_constant_t<xs>...>;
02305
02309     template<typename Ring, auto... xs>
02310     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02311         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02312 } // namespace aerobus
02313
02314 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02315 namespace aerobus {
02316     namespace internal {
02317         template<typename T, size_t x, typename E = void>
02318         struct factorial {};
02319
02320         template<typename T, size_t x>
02321         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02322         private:
02323             template<typename, size_t, typename>
02324             friend struct factorial;
02325         public:
02326             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02327             static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>();
02328         };
02329
02330         template<typename T>
02331         struct factorial<T, 0> {
02332         public:
02333             using type = typename T::one;
02334             static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>();
02335         };
02336     } // namespace internal
02337
02341     template<typename T, size_t i>
02342     using factorial_t = typename internal::factorial<T, i>::type;
02343
02347     template<typename T, size_t i>
02348     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02349
02350     namespace internal {
02351         template<typename T, size_t k, size_t n, typename E = void>
02352         struct combination_helper {};
02353
02354         template<typename T, size_t k, size_t n>
02355         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02356         using type = typename FractionField<T>::template mul_t<
02357             typename combination_helper<T, k - 1, n - 1>::type,
02358             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02359         };
02360
02361         template<typename T, size_t k, size_t n>
02362         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02363         using type = typename combination_helper<T, n - k, n>::type;
02364         };
02365
02366         template<typename T, size_t n>
02367         struct combination_helper<T, 0, n> {
02368         using type = typename FractionField<T>::one;
02369         };
02370
02371         template<typename T, size_t k, size_t n>
02372         struct combination {
02373         using type = typename internal::combination_helper<T, k, n>::type::x;
02374         static constexpr typename T::inner_type value =
02375             internal::combination_helper<T, k, n>::type::template
get<typename T::inner_type>();
02376         };
02377     } // namespace internal
02378
02381     template<typename T, size_t k, size_t n>
02382     using combination_t = typename internal::combination<T, k, n>::type;
02383
02388     template<typename T, size_t k, size_t n>
02389     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02390
02391     namespace internal {
02392         template<typename T, size_t m>
02393         struct bernoulli;
02394
02395         template<typename T, typename accum, size_t k, size_t m>
02396         struct bernoulli_helper {
02397         using type = typename bernoulli_helper<
02398             T,

```

```

02399         addfractions_t<T,
02400             accum,
02401             mulfractions_t<T,
02402                 makefraction_t<T,
02403                     combination_t<T, k, m + 1>,
02404                     typename T::one>,
02405                     typename bernoulli<T, k>::type
02406             >
02407         >,
02408         k + 1,
02409         m>::type;
02410     };
02411
02412     template<typename T, typename accum, size_t m>
02413     struct bernoulli_helper<T, accum, m, m> {
02414         using type = accum;
02415     };
02416
02417
02418
02419     template<typename T, size_t m>
02420     struct bernoulli {
02421         using type = typename FractionField<T>::template mul_t<
02422             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02423             makefraction_t<T,
02424                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02425                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02426             >
02427         >;
02428
02429         template<typename floatType>
02430         static constexpr floatType value = type::template get<floatType>();
02431     };
02432
02433     template<typename T>
02434     struct bernoulli<T, 0> {
02435         using type = typename FractionField<T>::one;
02436
02437         template<typename floatType>
02438         static constexpr floatType value = type::template get<floatType>();
02439     };
02440 } // namespace internal
02441
02442     template<typename T, size_t n>
02443     using bernoulli_t = typename internal::bernoulli<T, n>::type;
02444
02445     template<typename FloatType, typename T, size_t n>
02446     inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02447
02448 // bell numbers
02449 namespace internal {
02450     template<typename T, size_t n, typename E = void>
02451     struct bell_helper;
02452
02453     template<typename T, size_t n>
02454     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02455         template<typename accum, size_t i, size_t stop>
02456         struct sum_helper {
02457             private:
02458                 using left = typename T::template mul_t<
02459                     combination_t<T, i, n-1>,
02460                     typename bell_helper<T, i>::type>;
02461                 using new_accum = typename T::template add_t<accum, left>;
02462             public:
02463                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02464         };
02465
02466         template<typename accum, size_t stop>
02467         struct sum_helper<accum, stop, stop> {
02468             using type = accum;
02469         };
02470
02471         using type = typename sum_helper<typename T::zero, 0, n>::type;
02472     };
02473
02474     template<typename T>
02475     struct bell_helper<T, 0> {
02476         using type = typename T::one;
02477     };
02478
02479     template<typename T>
02480     struct bell_helper<T, 1> {
02481         using type = typename T::one;
02482     };
02483 } // namespace internal
02484
02485     template<typename T, size_t n>

```

```

02496     using bell_t = typename internal::bell_helper<T, n>::type;
02497
02501     template<typename T, size_t n>
02502     static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02503
02504     namespace internal {
02505         template<typename T, int k, typename E = void>
02506         struct alternate {};
02507
02508         template<typename T, int k>
02509         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02510             using type = typename T::one;
02511             static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>();
02512         };
02513
02514         template<typename T, int k>
02515         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02516             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02517             static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>();
02518         };
02519     } // namespace internal
02520
02523     template<typename T, int k>
02524     using alternate_t = typename internal::alternate<T, k>::type;
02525
02526     namespace internal {
02527         template<typename T, int n, int k, typename E = void>
02528         struct stirling_helper {};
02529
02530         template<typename T>
02531         struct stirling_helper<T, 0, 0> {
02532             using type = typename T::one;
02533         };
02534
02535         template<typename T, int n>
02536         struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02537             using type = typename T::zero;
02538         };
02539
02540         template<typename T, int n>
02541         struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02542             using type = typename T::zero;
02543         };
02544
02545         template<typename T, int n, int k>
02546         struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02547             using type = typename T::template sub_t<
02548                 typename stirling_helper<T, n-1, k-1>::type,
02549                 typename T::template mul_t<
02550                     typename T::template inject_constant_t<n-1>,
02551                     typename stirling_helper<T, n-1, k>::type
02552                 >;
02553         };
02554     } // namespace internal
02555
02560     template<typename T, int n, int k>
02561     using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02562
02567     template<typename T, int n, int k>
02568     using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02569
02574     template<typename T, int n, int k>
02575     static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02576
02577
02582     template<typename T, int n, int k>
02583     static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02584
02587     template<typename T, size_t k>
02588     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02589
02590     namespace internal {
02591         template<typename T>
02592         struct pow_scalar {
02593             template<size_t p>
02594             static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02595                 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02596                 x * func<p/2>(x) * func<p/2>(x);
02597             }
02598         };
02599
02600         template<typename T, typename p, size_t n, typename E = void>
02601         requires IsEuclideanDomain<T>
02602         struct pow;
02603

```

```

02604     template<typename T, typename p, size_t n>
02605     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02606         using type = typename T::template mul_t<
02607             typename pow<T, p, n/2>::type,
02608             typename pow<T, p, n/2>::type
02609         >;
02610     };
02611
02612     template<typename T, typename p, size_t n>
02613     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02614         using type = typename T::template mul_t<
02615             p,
02616             typename T::template mul_t<
02617                 typename pow<T, p, n/2>::type,
02618                 typename pow<T, p, n/2>::type
02619             >
02620         >;
02621     };
02622
02623     template<typename T, typename p, size_t n>
02624     struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
02625 } // namespace internal
02626
02627 template<typename T, typename p, size_t n>
02628 using pow_t = typename internal::pow<T, p, n>::type;
02629
02630 template<typename T, typename p, size_t n>
02631 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02632
02633 template<typename T, size_t p>
02634 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
02635     internal::pow_scalar<T>::template func<p>(x); }
02636
02637 namespace internal {
02638     template<typename, template<typename, size_t> typename, class>
02639     struct make_taylor_impl;
02640
02641     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02642     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02643         using type = typename polynomial<FractionField<T>>::template
02644             val<typename coeff_at<T, Is>::type...>;
02645     };
02646 }
02647
02648 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02649 using taylor = typename internal::make_taylor_impl<
02650     T,
02651     coeff_at,
02652     internal::make_index_sequence_reverse<deg + 1>::type;
02653
02654 namespace internal {
02655     template<typename T, size_t i>
02656     struct exp_coeff {
02657         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02658     };
02659
02660     template<typename T, size_t i, typename E = void>
02661     struct sin_coeff_helper {};
02662
02663     template<typename T, size_t i>
02664     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02665         using type = typename FractionField<T>::zero;
02666     };
02667
02668     template<typename T, size_t i>
02669     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02670         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02671     };
02672
02673     template<typename T, size_t i>
02674     struct sin_coeff {
02675         using type = typename sin_coeff_helper<T, i>::type;
02676     };
02677
02678     template<typename T, size_t i, typename E = void>
02679     struct sh_coeff_helper {};
02680
02681     template<typename T, size_t i>
02682     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02683         using type = typename FractionField<T>::zero;
02684     };
02685
02686     template<typename T, size_t i>
02687     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02688         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02689     };
02690 }
02691
02692 template<typename T, size_t i>
02693 struct sh_coeff {
02694     using type = typename sh_coeff_helper<T, i>::type;
02695 };
02696
02697 template<typename T, size_t i>
02698 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02699     using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02700 };

```

```

02701     template<typename T, size_t i>
02702     struct sh_coeff {
02703         using type = typename sh_coeff_helper<T, i>::type;
02704     };
02705
02706     template<typename T, size_t i, typename E = void>
02707     struct cos_coeff_helper {};
02708
02709     template<typename T, size_t i>
02710     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02711         using type = typename FractionField<T>::zero;
02712     };
02713
02714     template<typename T, size_t i>
02715     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02716         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02717     };
02718
02719     template<typename T, size_t i>
02720     struct cos_coeff {
02721         using type = typename cos_coeff_helper<T, i>::type;
02722     };
02723
02724     template<typename T, size_t i, typename E = void>
02725     struct cosh_coeff_helper {};
02726
02727     template<typename T, size_t i>
02728     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02729         using type = typename FractionField<T>::zero;
02730     };
02731
02732     template<typename T, size_t i>
02733     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02734         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02735     };
02736
02737     template<typename T, size_t i>
02738     struct cosh_coeff {
02739         using type = typename cosh_coeff_helper<T, i>::type;
02740     };
02741
02742     template<typename T, size_t i>
02743     struct geom_coeff { using type = typename FractionField<T>::one; };
02744
02745     template<typename T, size_t i, typename E = void>
02746     struct atan_coeff_helper;
02747
02748     template<typename T, size_t i>
02749     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02750         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>>;
02751     };
02752
02753     template<typename T, size_t i>
02754     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02755         using type = typename FractionField<T>::zero;
02756     };
02757
02758     template<typename T, size_t i>
02759     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02760
02761     template<typename T, size_t i, typename E = void>
02762     struct asin_coeff_helper;
02763
02764     template<typename T, size_t i>
02765     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02766         using type = makefraction_t<T,
02767             factorial_t<T, i - 1>,
02768             typename T::template mul_t<
02769                 typename T::template val<i>,
02770                 T::template mul_t<
02771                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
02772                     pow<T, factorial_t<T, i / 2>, 2
02773                 >
02774             >
02775         >
02776         >>;
02777     };
02778
02779     template<typename T, size_t i>
02780     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02781         using type = typename FractionField<T>::zero;
02782     };
02783
02784     template<typename T, size_t i>
02785     struct asin_coeff {
02786         using type = typename asin_coeff_helper<T, i>::type;
02787     };

```



```

02788
02789     template<typename T, size_t i>
02790     struct lnpl_coeff {
02791         using type = makefraction_t<T,
02792             alternate_t<T, i + 1>,
02793             typename T::template val<i>>;
02794     };
02795
02796     template<typename T>
02797     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02798
02799     template<typename T, size_t i, typename E = void>
02800     struct asinh_coeff_helper;
02801
02802     template<typename T, size_t i>
02803     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02804         using type = makefraction_t<T,
02805             typename T::template mul_t<
02806                 alternate_t<T, i / 2>,
02807                 factorial_t<T, i - 1>
02808             >,
02809             typename T::template mul_t<
02810                 typename T::template mul_t<
02811                     typename T::template val<i>,
02812                     pow_t<T, factorial_t<T, i / 2>, 2>
02813                 >,
02814                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
02815             >
02816         >;
02817     };
02818
02819     template<typename T, size_t i>
02820     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02821         using type = typename FractionField<T>::zero;
02822     };
02823
02824     template<typename T, size_t i>
02825     struct asinh_coeff {
02826         using type = typename asinh_coeff_helper<T, i>::type;
02827     };
02828
02829     template<typename T, size_t i, typename E = void>
02830     struct atanh_coeff_helper;
02831
02832     template<typename T, size_t i>
02833     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02834         // 1/i
02835         using type = typename FractionField<T>::template val<
02836             typename T::one,
02837             typename T::template inject_constant_t<i>>;
02838     };
02839
02840     template<typename T, size_t i>
02841     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02842         using type = typename FractionField<T>::zero;
02843     };
02844
02845     template<typename T, size_t i>
02846     struct atanh_coeff {
02847         using type = typename atanh_coeff_helper<T, i>::type;
02848     };
02849
02850     template<typename T, size_t i, typename E = void>
02851     struct tan_coeff_helper;
02852
02853     template<typename T, size_t i>
02854     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02855         using type = typename FractionField<T>::zero;
02856     };
02857
02858     template<typename T, size_t i>
02859     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02860     private:
02861         // 4^((i+1)/2)
02862         using _4p = typename FractionField<T>::template inject_t<
02863             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
02864         // 4^((i+1)/2) - 1
02865         using _4pml = typename FractionField<T>::template
02866 sub_t<_4p, typename FractionField<T>::one>;
02867         // (-1)^((i-1)/2)
02868         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02869         using dividend = typename FractionField<T>::template mul_t<
02870             altp,
02871             FractionField<T>::template mul_t<
02872                 _4p,
02873                 FractionField<T>::template mul_t<
02874                     _4pml,

```



```

03010     template<typename Integers, size_t deg>
03011     using tanh = Taylor<Integers, internal::tanh_coeff, deg>;
03012 } // namespace aerobus
03013
03014 // continued fractions
03015 namespace aerobus {
03016     template<int64_t... values>
03017     struct ContinuedFraction {};
03018
03019     template<int64_t a0>
03020     struct ContinuedFraction<a0> {
03021         using type = typename q64::template inject_constant_t<a0>;
03022         static constexpr double val = static_cast<double>(a0);
03023     };
03024
03025     template<int64_t a0, int64_t... rest>
03026     struct ContinuedFraction<a0, rest...> {
03027         using type = q64::template add_t<
03028             typename q64::template inject_constant_t<a0>,
03029             typename q64::template div_t<
03030                 typename q64::one,
03031                 typename ContinuedFraction<rest...>::type
03032             >>;
03033         static constexpr double val = type::template get<double>();
03034     };
03035
03036     using PI_fraction =
03037     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03038     using E_fraction =
03039     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03040     using SQRT2_fraction =
03041     ContinuedFraction<1, 2>;
03042     using SQRT3_fraction =
03043     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03044     // NOLINT
03045 } // namespace aerobus
03046
03047 // known polynomials
03048 namespace aerobus {
03049     // CChebyshev
03050     namespace internal {
03051         template<int kind, size_t deg, typename I>
03052         struct chebyshev_helper {
03053             using type = typename polynomial<I>::template sub_t<
03054                 typename polynomial<I>::template mul_t<
03055                     typename polynomial<I>::template mul_t<
03056                         typename polynomial<I>::template inject_constant_t<2>,
03057                         typename polynomial<I>::X>,
03058                     typename chebyshev_helper<kind, deg - 1, I>::type
03059                 >,
03060                 typename chebyshev_helper<kind, deg - 2, I>::type
03061             >;
03062         };
03063
03064         template<typename I>
03065         struct chebyshev_helper<1, 0, I> {
03066             using type = typename polynomial<I>::one;
03067         };
03068
03069         template<typename I>
03070         struct chebyshev_helper<1, 1, I> {
03071             using type = typename polynomial<I>::X;
03072         };
03073
03074         template<typename I>
03075         struct chebyshev_helper<2, 0, I> {
03076             using type = typename polynomial<I>::one;
03077         };
03078
03079         template<typename I>
03080         struct chebyshev_helper<2, 1, I> {
03081             using type = typename polynomial<I>::template mul_t<
03082                 typename polynomial<I>::template inject_constant_t<2>,
03083                 typename polynomial<I>::X>;
03084         };
03085     } // namespace internal
03086
03087     // Laguerre
03088     namespace internal {
03089         template<size_t deg, typename I>
03090         struct laguerre_helper {
03091             using Q = FractionField<I>;
03092             using PQ = polynomial<Q>;
03093
03094         private:
03095             // Lk = (1 / k) * ((2 * k - 1 - x) * Lk-1 - (k - 2) Lk-2)

```

```

03111         using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03112         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03113         // -x + 2k-1
03114         using p = typename PQ::template val<
03115             typename Q::template inject_constant_t<-1>,
03116             typename Q::template inject_constant_t<2 * deg - 1>;
03117         // 1/n
03118         using factor = typename PQ::template inject_ring_t<
03119             typename Q::template
03120             val<typename I::one, typename I::template inject_constant_t<deg>>;
03121
03122         public:
03123             using type = typename PQ::template mul_t <
03124                 factor,
03125                 typename PQ::template sub_t<
03126                     typename PQ::template mul_t<
03127                         p,
03128                         lnm1
03129                     >,
03130                     typename PQ::template mul_t<
03131                         typename PQ::template inject_constant_t<deg-1>,
03132                         lnm2
03133                     >
03134                 >
03135             >;
03136
03137         template<typename I>
03138         struct laguerre_helper<0, I> {
03139             using type = typename polynomial<FractionField<I>>::one;
03140         };
03141
03142         template<typename I>
03143         struct laguerre_helper<1, I> {
03144             private:
03145                 using PQ = polynomial<FractionField<I>>;
03146             public:
03147                 using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03148         };
03149     } // namespace internal
03150
03151     // Bernstein
03152     namespace internal {
03153         template<size_t i, size_t m, typename I, typename E = void>
03154         struct bernstein_helper {};
03155
03156         template<typename I>
03157         struct bernstein_helper<0, 0, I> {
03158             using type = typename polynomial<I>::one;
03159         };
03160
03161         template<size_t i, size_t m, typename I>
03162         struct bernstein_helper<i, m, I, std::enable_if_t<
03163             (m > 0) && (i == 0)> {
03164             private:
03165                 using P = polynomial<I>;
03166             public:
03167                 using type = typename P::template mul_t<
03168                     typename P::template sub_t<typename P::one, typename P::X>,
03169                     typename bernstein_helper<i, m-1, I>::type>;
03170         };
03171
03172         template<size_t i, size_t m, typename I>
03173         struct bernstein_helper<i, m, I, std::enable_if_t<
03174             (m > 0) && (i == m)> {
03175             private:
03176                 using P = polynomial<I>;
03177             public:
03178                 using type = typename P::template mul_t<
03179                     typename P::X,
03180                     typename bernstein_helper<i-1, m-1, I>::type>;
03181         };
03182
03183         template<size_t i, size_t m, typename I>
03184         struct bernstein_helper<i, m, I, std::enable_if_t<
03185             (m > 0) && (i > 0) && (i < m)> {
03186             private:
03187                 using P = polynomial<I>;
03188             public:
03189                 using type = typename P::template add_t<
03190                     typename P::template mul_t<
03191                         typename P::template sub_t<typename P::one, typename P::X>,
03192                         typename bernstein_helper<i, m-1, I>::type>,
03193                         typename P::template mul_t<
03194                             typename P::X,
03195                             typename bernstein_helper<i-1, m-1, I>::type>;
03196         };

```

```

03197     } // namespace internal
03198
03199     namespace known_polynomials {
03201         enum hermite_kind {
03203             probabilist,
03205             physicist
03206         };
03207     }
03208
03209     // hermite
03210     namespace internal {
03211         template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03212         struct hermite_helper {};
03213
03214         template<size_t deg, typename I>
03215         struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03216             private:
03217                 using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::probabilist, I>::type;
03218                 using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::probabilist, I>::type;
03219
03220             public:
03221                 using type = typename polynomial<I>::template sub_t<
03222                     typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03223                     typename polynomial<I>::template mul_t<
03224                         typename polynomial<I>::template inject_constant_t<deg - 1>,
03225                         hnm2
03226                     >
03227                 >;
03228         };
03229
03230         template<size_t deg, typename I>
03231         struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03232             private:
03233                 using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
I>::type;
03234                 using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
I>::type;
03235
03236             public:
03237                 using type = typename polynomial<I>::template sub_t<
03238                     // 2X Hn-1
03239                     typename polynomial<I>::template mul_t<
03240                         typename pi64::val<typename I::template inject_constant_t<2>,
03241                         typename I::zero>, hnm1>,
03242
03243                     typename polynomial<I>::template mul_t<
03244                         typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03245                         hnm2
03246                     >
03247                 >;
03248         };
03249
03250         template<typename I>
03251         struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03252             using type = typename polynomial<I>::one;
03253         };
03254
03255         template<typename I>
03256         struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03257             using type = typename polynomial<I>::X;
03258         };
03259
03260         template<typename I>
03261         struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03262             using type = typename pi64::one;
03263         };
03264
03265         template<typename I>
03266         struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03267             // 2X
03268             using type = typename polynomial<I>::template val<
03269                 typename I::template inject_constant_t<2>,
03270                 typename I::zero>;
03271         };
03272     } // namespace internal
03273
03274     // legendre
03275     namespace internal {
03276         template<size_t n, typename I>
03277         struct legendre_helper {
03278             private:
03279                 using Q = FractionField<I>;
03280                 using PQ = polynomial<Q>;
03281                 // 1/n constant
03282                 // (2n-1)/n X

```

```

03283         using fact_left = typename PQ::template monomial_t<
03284             makefraction_t<I,
03285                 typename I::template inject_constant_t<2*n-1>,
03286                 typename I::template inject_constant_t<n>
03287             >,
03288             1>;
03289         // (n-1) / n
03290         using fact_right = typename PQ::template val<
03291             makefraction_t<I,
03292                 typename I::template inject_constant_t<n-1>,
03293                 typename I::template inject_constant_t<n>»>;
03294
03295     public:
03296         using type = PQ::template sub_t<
03297             typename PQ::template mul_t<
03298                 fact_left,
03299                 typename legendre_helper<n-1, I>::type
03300             >,
03301             typename PQ::template mul_t<
03302                 fact_right,
03303                 typename legendre_helper<n-2, I>::type
03304             >
03305         >;
03306 };
03307
03308 template<typename I>
03309 struct legendre_helper<0, I> {
03310     using type = typename polynomial<FractionField<I>>::one;
03311 };
03312
03313 template<typename I>
03314 struct legendre_helper<1, I> {
03315     using type = typename polynomial<FractionField<I>>::X;
03316 };
03317 } // namespace internal
03318
03319 // bernoulli polynomials
03320 namespace internal {
03321     template<size_t n>
03322     struct bernoulli_coeff {
03323         template<typename T, size_t i>
03324         struct inner {
03325             private:
03326                 using F = FractionField<T>;
03327             public:
03328                 using type = typename F::template mul_t<
03329                     typename F::template inject_ring_t<combination_t<T, i, n>>,
03330                     bernoulli_t<T, n-i>
03331                 >;
03332         };
03333     };
03334 } // namespace internal
03335
03336 namespace known_polynomials {
03337     template <size_t deg, typename I = aerobus::i64>
03338     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03339
03340     template <size_t deg, typename I = aerobus::i64>
03341     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03342
03343     template <size_t deg, typename I = aerobus::i64>
03344     using laguerre = typename internal::laguerre_helper<deg, I>::type;
03345
03346     template <size_t deg, typename I = aerobus::i64>
03347     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03348         I>::type;
03349
03350     template <size_t deg, typename I = aerobus::i64>
03351     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03352
03353     template<size_t i, size_t m, typename I = aerobus::i64>
03354     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03355
03356     template<size_t deg, typename I = aerobus::i64>
03357     using legendre = typename internal::legendre_helper<deg, I>::type;
03358
03359     template<size_t deg, typename I = aerobus::i64>
03360     using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03361 } // namespace known_polynomials
03362 } // namespace aerobus
03363
03364 #ifndef AEROBUS_CONWAY_IMPORTS
03365 // conway polynomials
03366 namespace aerobus {
03367     template<int p, int n>

```

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```
05380     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>>; }; // NOLINT
05381     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>>; }; // NOLINT
05382     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>>; }; //
NOLINT
05383     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>,
ZPZV<990>>; }; // NOLINT
05384 #endif // DO_NOT_DOCUMENT
05385 } // namespace aerobus
05386 #endif // AEROBUS_CONWAY_IMPORTS
05387
05388 #endif // __INC_AEROBUS__ // NOLINT
```



## Chapter 10

# Examples

### 10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject\_constant\_t<1>>

Template Parameters

|   |                                      |
|---|--------------------------------------|
| x | a 'constant' from Ring point of view |
|---|--------------------------------------|

### 10.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

|       |                                               |
|-------|-----------------------------------------------|
| ...Ts | types to store and manipulate at compile time |
|-------|-----------------------------------------------|

### 10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

|   |                                              |
|---|----------------------------------------------|
| x | inject_constant_2<2> -> i32::template val<2> |
|---|----------------------------------------------|

## 10.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |



## 10.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

|           |                            |
|-----------|----------------------------|
| <i>v1</i> |                            |
| <i>v2</i> | <i32::val<1>, i32::val<1>> |

## 10.11 i32::gcd\_t

greatest common divisor yields  $\text{GCD}(v1, v2)$  <i32::val<6>, i32::val<15>>

greatest common divisor yields  $\text{GCD}(v1, v2)$  <i32::val<6>, i32::val<15>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

|     |                |
|-----|----------------|
| $v$ | a value in i32 |
|-----|----------------|

## 10.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

|     |                                                       |
|-----|-------------------------------------------------------|
| $v$ | a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code> |
|-----|-------------------------------------------------------|

## 10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

|     |                                         |
|-----|-----------------------------------------|
| $x$ | <code>inject_constant_t&lt;2&gt;</code> |
|-----|-----------------------------------------|

## 10.15 i64::add\_t

addition operator

addition operator

Template Parameters

|      |                                                                                                          |
|------|----------------------------------------------------------------------------------------------------------|
| $v1$ | : an element of <code>aerobus::i64::val</code>                                                           |
| $v2$ | : an element of <code>aerobus::i64::val</code> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

|           |                                                                               |
|-----------|-------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 10.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code> |

## 10.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

### Template Parameters

|                 |                                                                                                              |
|-----------------|--------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                            |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;6&gt;, i64::val&lt;15&gt;&gt;</code> |

## 10.26 i64::pos\_t

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

### Template Parameters

|                |                                                                                          |
|----------------|------------------------------------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code> |
|----------------|------------------------------------------------------------------------------------------|

## 10.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

|                |                                                                                          |
|----------------|------------------------------------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code> |
|----------------|------------------------------------------------------------------------------------------|

## 10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

|                |                                                               |
|----------------|---------------------------------------------------------------|
| <code>x</code> | <code>&lt;i32&gt;::template inject_constant_t&lt;2&gt;</code> |
|----------------|---------------------------------------------------------------|

## 10.29 q32::add\_t

addition operator

addition operator

Template Parameters

|                 |                                                                                                                                         |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | a value                                                                                                                                 |
| <code>v2</code> | a value <code>&lt;q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;2&gt;&gt;, q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;3&gt;&gt;&gt;</code> |

## 10.30 FractionField

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$

Template Parameters

|             |                                                                                    |
|-------------|------------------------------------------------------------------------------------|
| <i>Ring</i> | <code>&lt;i64&gt;</code> is q64 (rationals with 64 bits numerator and denominator) |
|-------------|------------------------------------------------------------------------------------|

## 10.31 PI\_fraction::val

representation of  $\pi$  as a continued fraction -> 3.14...

## 10.32 E\_fraction::val

approximation of  $e$  -> 2.718...

approximation of  $e$  -> 2.718...

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