

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

Here is a list of all documented concepts with brief descriptions:

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Class Index

2.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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--	----

Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are aerobus::i64
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
-----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using **type** = `q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`

Static Public Attributes

- static constexpr double **val** = `type::template get<double>()`

5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
<i>...rest</i>	integers (aerobus::i64)

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in [i32](#), again represented as types

Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity (type)(v > 0)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Data Documentation

5.7.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.7.2.2 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<i>v</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in i64

Public Types

- `using inner_type = int64_t`
type for actual values
- `template<auto x>`
`using inject_constant_t = val< static_cast< int64_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2) - type
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i64, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
is v positive (type)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator ($v1 > v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator ($v1 < v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Typedef Documentation

5.8.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) - type

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.6 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.7 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.8 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.9 pos_t

```
template<typename v >  
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive (type)

Template Parameters

v1	: an element of aerobus::i64::val
----	---

5.8.2.10 sub_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3 Member Data Documentation

5.8.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator ($v1 > v2$) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.3 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator ($v1 < v2$) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<code>v</code>	: an element of aerobus::i64::val
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.9 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< index, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.10 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< stop, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.11 aerobus::is_prime< n > Struct Template Reference

checks if `n` is prime

```
#include <aerobus.h>
```

Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`
true iff `n` is prime

5.11.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if `n` is prime

Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.12 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero = val< typename Ring::zero >](#)
constant zero
- [using one = val< typename Ring::one >](#)
constant one
- [using X = val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t = typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt_helper< v1, v2 >::type](#)
strict greater operator

- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::q_type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`
modulo operator
- `template<typename coeff , size_t deg>`
`using monomial_t = typename monomial< coeff, deg >::type`
monomial : coeff X^{deg}
- `template<typename v >`
`using derive_t = typename derive_helper< v >::type`
derivation operator
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity (an > 0)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

5.12.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.12.2 Member Typedef Documentation

5.12.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.12.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.5 gcd_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.7 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.8 mod_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.9 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^{deg}

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.12.2.10 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.11 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.12.2.12 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.12.2.13 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.13 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.13.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.14 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

```
#include <aerobus.h>
```

Classes

- struct [val](#)
projection values in the quotient ring

Public Types

- using [zero](#) = [val](#)< [typename](#) [Ring::zero](#) >
zero value
- using [one](#) = [val](#)< [typename](#) [Ring::one](#) >
one
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
using [add_t](#) = [val](#)< [typename](#) [Ring::template add_t](#)< [typename](#) [v1::type](#), [typename](#) [v2::type](#) > >
addition operator
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
using [mul_t](#) = [val](#)< [typename](#) [Ring::template mul_t](#)< [typename](#) [v1::type](#), [typename](#) [v2::type](#) > >
subtraction operator
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
using [div_t](#) = [val](#)< [typename](#) [Ring::template div_t](#)< [typename](#) [v1::type](#), [typename](#) [v2::type](#) > >
division operator
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
using [mod_t](#) = [val](#)< [typename](#) [Ring::template mod_t](#)< [typename](#) [v1::type](#), [typename](#) [v2::type](#) > >
modulus operator
- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
using [eq_t](#) = [typename](#) [Ring::template eq_t](#)< [typename](#) [v1::type](#), [typename](#) [v2::type](#) >
equality operator (as type)
- template<[typename](#) [v1](#) >
using [pos_t](#) = [std::true_type](#)
positivity operator always true
- template<[auto](#) [x](#)>
using [inject_constant_t](#) = [val](#)< [typename](#) [Ring::template inject_constant_t](#)< [x](#) > >
inject a 'constant' in quotient ring for example : [QuotientRing](#)<[i32](#), [i32::val](#)<2>>::[inject_constant_t](#)<1>
- template<[typename](#) [v](#) >
using [inject_ring_t](#) = [val](#)< [v](#) >
projects a value of Ring onto the quotient for example : [QuotientRing](#)<[i32](#), [i32::val](#)<2>>::[inject_ring_t](#)<[i32::val](#)<1>>

Static Public Attributes

- template<[typename](#) [v1](#) , [typename](#) [v2](#) >
static constexpr bool [eq_v](#) = [Ring::template eq_t](#)<[typename](#) [v1::type](#), [typename](#) [v2::type](#)>::value
addition operator (as boolean value)
- template<[typename](#) [v](#) >
static constexpr bool [pos_v](#) = [pos_t](#)<[v](#)>::value
positivity operator always true
- static constexpr bool [is_euclidean_domain](#) = true
quotien rings are euclidean domain

5.14.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.

Template Parameters

<i>Ring</i>	A ring type, such as ' i32 ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as i32::val<2>

5.14.2 Member Typedef Documentation

5.14.2.1 `add_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.14.2.2 `div_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.14.2.3 `eq_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
```

```
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

<code>v1</code>	a value in quotient ring
<code>v2</code>	a value in quotient ring

5.14.2.4 inject_constant_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring for example : `QuotientRing<i32, i32::val<2>>::inject_constant_t<1>`

Template Parameters

<code>x</code>	a 'constant' from Ring point of view
----------------	--------------------------------------

5.14.2.5 inject_ring_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient for example : `QuotientRing<i32, i32::val<2>>::inject_ring_t<i32::val<1>>`

Template Parameters

<code>v</code>	a value in Ring
----------------	-----------------

5.14.2.6 mod_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

Template Parameters

<code>v1</code>	a value in quotient ring
<code>v2</code>	a value in quotient ring

5.14.2.7 mul_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.14.2.8 pos_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

5.14.3 Member Data Documentation

5.14.3.1 eq_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

5.14.3.2 pos_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.15 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- using **head** = typename inner::head
- using **tail** = typename inner::tail

5.15.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.16 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- template<typename T >
using [push_front](#) = [type_list](#)< T, Ts... >
Adds T to front of the list.
- template<size_t index>
using [at](#) = internal::type_at_t< index, Ts... >
returns type at index
- template<typename T >
using [push_back](#) = [type_list](#)< Ts..., T >
pushes T at the tail of the list
- template<typename U >
using [concat](#) = typename concat_h< U >::type
concatenates two list into one
- template<typename T , size_t index>
using [insert](#) = typename internal::insert_h< index, [type_list](#)< Ts... >, T >::type
inserts type at index
- template<size_t index>
using [remove](#) = typename internal::remove_h< index, [type_list](#)< Ts... > >::type
removes type at index

Static Public Attributes

- static constexpr size_t [length](#) = sizeof...(Ts)
length of list

5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types for example `type_list<int, double, float>`

Template Parameters

<code>...Ts</code>	types to store and manipulate at compile time
--------------------	---

5.16.2 Member Typedef Documentation

5.16.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.16.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.16.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>, T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.16.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.16.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.16.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.17 aerobus::type_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

Public Types

- template<typename T >
using **push_front** = type_list< T >
- template<typename T >
using **push_back** = type_list< T >
- template<typename U >
using **concat** = U
- template<typename T , size_t index>
using **insert** = type_list< T >

Static Public Attributes

- static constexpr size_t **length** = 0

5.17.1 Detailed Description

specialization for empty type list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.18 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

Public Types

- using **ring_type** = [i32](#)
Enclosing ring type.
- using **is_zero_t** = std::bool_constant< x==0 >
is value zero

Static Public Member Functions

- template<typename [valueType](#) >
static constexpr [valueType](#) **get** ()
cast x into valueType
- static std::string **to_string** ()
string representation of value
- template<typename [valueRing](#) >
static constexpr [valueRing](#) **eval** (const [valueRing](#) &v)
cast x into valueRing

Static Public Attributes

- static constexpr [int32_t](#) **v** = x
actual value stored in val type

5.18.1 Detailed Description

```
template<int32\_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.18.2 Member Function Documentation

5.18.2.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.18.2.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<code>valueType</code>	double for example
------------------------	--------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.19 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

Public Types

- `using ring_type = i64`
enclosing ring type
- `using is_zero_t = std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast value in valueType
- `static std::string to_string ()`
string representation
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast value in valueRing

Static Public Attributes

- `static constexpr int64_t v = x`
actual value

5.19.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.19.2 Member Function Documentation

5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.20 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

5.20.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

5.20.2 Member Typedef Documentation

5.20.2.1 `coeff_at_t`

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.20.3 Member Function Documentation

5.20.3.1 `eval()`

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.20.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

5.21.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- `using ring_type = zpz< p >`
enclosing ring type
- `using is_zero_t = std::bool_constant< x% p==0 >`

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.23 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct `coeff_at`
- struct `coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >`
- struct `coeff_at< index, std::enable_if_t<(index==0)> >`

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
- `using strip = val< coeffN >`
- `using is_zero_t = std::bool_constant< aN::is_zero_t::value >`
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`

Static Public Member Functions

- `static std::string to_string ()`
- `template<typename valueRing >
static constexpr valueRing eval (const valueRing &x)`

Static Public Attributes

- `static constexpr size_t degree = 0`
degree
- `static constexpr bool is_zero_v = is_zero_t::value`

5.23.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.24 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct `val`

Public Types

- `using inner_type = int32_t`
- `template<auto x>
using inject_constant_t = val< static_cast< int32_t >(x)>`
- `using zero = val< 0 >`
- `using one = val< 1 >`
- `template<typename v1 , typename v2 >
using add_t = typename add< v1 , v2 >::type`
addition operator

- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulo operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (type)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strictly smaller operator (type)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v1 >`
`using pos_t = typename pos< v1 >::type`
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.24.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

Chapter 6

File Documentation

6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035 } // namespace aerobus
00036
00037 // concepts
00038 namespace aerobus {
00039     template <typename R>
00040     concept IsRing = requires {
00041         typename R::one;
00042         typename R::zero;
00043         typename R::template add_t<typename R::one, typename R::one>;
00044         typename R::template sub_t<typename R::one, typename R::one>;
00045         typename R::template mul_t<typename R::one, typename R::one>;
00046     };
00047
00048     template <typename R>
00049     concept IsEuclideanDomain = IsRing<R> && requires {
00050         typename R::template div_t<typename R::one, typename R::one>;
00051         typename R::template mod_t<typename R::one, typename R::one>;
00052         typename R::template gcd_t<typename R::one, typename R::one>;
00053         typename R::template eq_t<typename R::one, typename R::one>;
00054         typename R::template pos_t<typename R::one>;
00055     };
00056
00057     R::template pos_v<typename R::one> == true;
00058     // typename R::template gt_t<typename R::one, typename R::zero>;
00059     R::is_euclidean_domain == true;
```

```

00067     };
00068
00070     template<typename R>
00071     concept IsField = IsEuclideanDomain<R> && requires {
00072         R::is_field == true;
00073     };
00074 } // namespace aerobus
00075
00076 // utilities
00077 namespace aerobus {
00078     namespace internal {
00079         template<template<typename...> typename TT, typename T>
00080             struct is_instantiation_of : std::false_type { };
00081
00082         template<template<typename...> typename TT, typename... Ts>
00083             struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00084
00085         template<template<typename...> typename TT, typename T>
00086             inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00087
00088         template<int64_t i, typename T, typename... Ts>
00089         struct type_at {
00090             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00091             using type = typename type_at<i - 1, Ts...>::type;
00092         };
00093
00094         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00095             using type = T;
00096         };
00097
00098         template<size_t i, typename... Ts>
00099         using type_at_t = typename type_at<i, Ts...>::type;
00100
00101
00102         template<size_t n, size_t i, typename E = void>
00103         struct _is_prime {};
00104
00105         template<size_t i>
00106         struct _is_prime<0, i> {
00107             static constexpr bool value = false;
00108         };
00109
00110         template<size_t i>
00111         struct _is_prime<1, i> {
00112             static constexpr bool value = false;
00113         };
00114
00115         template<size_t i>
00116         struct _is_prime<2, i> {
00117             static constexpr bool value = true;
00118         };
00119
00120         template<size_t i>
00121         struct _is_prime<3, i> {
00122             static constexpr bool value = true;
00123         };
00124
00125         template<size_t i>
00126         struct _is_prime<5, i> {
00127             static constexpr bool value = true;
00128         };
00129
00130         template<size_t i>
00131         struct _is_prime<7, i> {
00132             static constexpr bool value = true;
00133         };
00134
00135         template<size_t n, size_t i>
00136         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00137             static constexpr bool value = false;
00138         };
00139
00140         template<size_t n, size_t i>
00141         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00142             static constexpr bool value = false;
00143         };
00144
00145         template<size_t n, size_t i>
00146         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00147             static constexpr bool value = true;
00148         };
00149
00150         template<size_t n, size_t i>
00151         struct _is_prime<n, i, std::enable_if_t<(
00152             n % i == 0 &&
00153             n >= 9 &&
00154             n % 3 != 0 &&

```

```

00155         n % 2 != 0 &&
00156         i * i > n)» {
00157     static constexpr bool value = true;
00158 };
00159
00160 template<size_t n, size_t i>
00161 struct _is_prime<n, i, std::enable_if_t<(
00162     n % (i+2) == 0 &&
00163     n >= 9 &&
00164     n % 3 != 0 &&
00165     n % 2 != 0 &&
00166     i * i <= n)» {
00167     static constexpr bool value = true;
00168 };
00169
00170 template<size_t n, size_t i>
00171 struct _is_prime<n, i, std::enable_if_t<(
00172     n % (i+2) != 0 &&
00173     n % i != 0 &&
00174     n >= 9 &&
00175     n % 3 != 0 &&
00176     n % 2 != 0 &&
00177     (i * i <= n))» {
00178     static constexpr bool value = _is_prime<n, i+6>::value;
00179 };
00180
00181 } // namespace internal
00182
00183 template<size_t n>
00184 struct is_prime {
00185     static constexpr bool value = internal::_is_prime<n, 5>::value;
00186 };
00187
00188 template<size_t n>
00189 static constexpr bool is_prime_v = is_prime<n>::value;
00190
00191 namespace internal {
00192     template <std::size_t... Is>
00193     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00194     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00195
00196     template <std::size_t N>
00197     using make_index_sequence_reverse
00198     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00199
00200     template<typename Ring, typename E = void>
00201     struct gcd;
00202
00203     template<typename Ring>
00204     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00205         template<typename A, typename B, typename E = void>
00206         struct gcd_helper {};
00207
00208         // B = 0, A > 0
00209         template<typename A, typename B>
00210         struct gcd_helper<A, B, std::enable_if_t<
00211             ((B::is_zero_t::value) &&
00212              (Ring::template gt_t<A, typename Ring::zero>::value))> {
00213             using type = A;
00214         };
00215
00216         // B = 0, A < 0
00217         template<typename A, typename B>
00218         struct gcd_helper<A, B, std::enable_if_t<
00219             ((B::is_zero_t::value) &&
00220              !(Ring::template gt_t<A, typename Ring::zero>::value))> {
00221             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00222         };
00223
00224         // B != 0
00225         template<typename A, typename B>
00226         struct gcd_helper<A, B, std::enable_if_t<
00227             (!B::is_zero_t::value)
00228             >> {
00229             private: // NOLINT
00230                 // A / B
00231                 using k = typename Ring::template div_t<A, B>;
00232                 // A - (A/B)*B = A % B
00233                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00234
00235             public:
00236                 using type = typename gcd_helper<B, m>::type;
00237         };
00238
00239         template<typename A, typename B>
00240         using type = typename gcd_helper<A, B>::type;
00241     };
00242 }

```

```

00253     } // namespace internal
00254
00257     template<typename T, typename A, typename B>
00258     using gcd_t = typename internal::gcd<T>::template type<A, B>;
00259
00263     template<typename val>
00264     requires IsEuclideanDomain<typename val::ring_type>
00265     using abs_t = std::conditional_t<
00266         val::ring_type::template pos_v<val>,
00267         val, typename val::ring_type::template sub_t<typename val::ring_type::zero, val>;
00268 } // namespace aerobus
00269
00270 namespace aerobus {
00271     template<typename Ring, typename X>
00272     requires IsRing<Ring>
00273     struct Quotient {
00274         template <typename V>
00275         struct val {
00276             public:
00277                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00278         };
00279
00287         using zero = val<typename Ring::zero>;
00288
00290         using one = val<typename Ring::one>;
00291
00295         template<typename v1, typename v2>
00296         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00297
00301         template<typename v1, typename v2>
00302         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00303
00307         template<typename v1, typename v2>
00308         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00309
00313         template<typename v1, typename v2>
00314         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00315
00319         template<typename v1, typename v2>
00320         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00321
00325         template<typename v1, typename v2>
00326         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00327
00331         template<typename v1>
00332         using pos_t = std::true_type;
00333
00337         template<typename v>
00338         static constexpr bool pos_v = pos_t<v>::value;
00339
00341         static constexpr bool is_euclidean_domain = true;
00342
00346         template<auto x>
00347         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00348
00352         template<typename v>
00353         using inject_ring_t = val<v>;
00354     };
00355 } // namespace aerobus
00356
00357 // type_list
00358 namespace aerobus {
00359     template <typename... Ts>
00360     struct type_list;
00361
00362     namespace internal {
00363         template <typename T, typename... Us>
00364         struct pop_front_h {
00365             using tail = type_list<Us...>;
00366             using head = T;
00367         };
00368
00369         template <size_t index, typename L1, typename L2>
00370         struct split_h {
00371             private:
00372                 static_assert(index <= L2::length, "index out of bounds");
00373                 using a = typename L2::pop_front::type;
00374                 using b = typename L2::pop_front::tail;
00375                 using c = typename L1::template push_back<a>;
00376
00377             public:
00378                 using head = typename split_h<index - 1, c, b>::head;
00379                 using tail = typename split_h<index - 1, c, b>::tail;
00380         };
00381
00382         template <typename L1, typename L2>
00383         struct split_h<0, L1, L2> {

```

```

00385         using head = L1;
00386         using tail = L2;
00387     };
00388
00389     template <size_t index, typename L, typename T>
00390     struct insert_h {
00391         static_assert(index <= L::length, "index out of bounds");
00392         using s = typename L::template split<index>;
00393         using left = typename s::head;
00394         using right = typename s::tail;
00395         using ll = typename left::template push_back<T>;
00396         using type = typename ll::template concat<right>;
00397     };
00398
00399     template <size_t index, typename L>
00400     struct remove_h {
00401         using s = typename L::template split<index>;
00402         using left = typename s::head;
00403         using right = typename s::tail;
00404         using rr = typename right::pop_front::tail;
00405         using type = typename left::template concat<rr>;
00406     };
00407 } // namespace internal
00408
00412 template <typename... Ts>
00413 struct type_list {
00414 private:
00415     template <typename T>
00416     struct concat_h;
00417
00418     template <typename... Us>
00419     struct concat_h<type_list<Us...> {
00420         using type = type_list<Ts..., Us...>;
00421     };
00422
00423 public:
00425     static constexpr size_t length = sizeof...(Ts);
00426
00429     template <typename T>
00430     using push_front = type_list<T, Ts...>;
00431
00434     template <size_t index>
00435     using at = internal::type_at_t<index, Ts...>;
00436
00438     struct pop_front {
00440         using type = typename internal::pop_front_h<Ts...>::head;
00442         using tail = typename internal::pop_front_h<Ts...>::tail;
00443     };
00444
00447     template <typename T>
00448     using push_back = type_list<Ts..., T>;
00449
00452     template <typename U>
00453     using concat = typename concat_h<U>::type;
00454
00457     template <size_t index>
00458     struct split {
00459     private:
00460         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00461
00462     public:
00463         using head = typename inner::head;
00464         using tail = typename inner::tail;
00465     };
00466
00470     template <typename T, size_t index>
00471     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00472
00475     template <size_t index>
00476     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00477 };
00478
00480 template <>
00481 struct type_list<> {
00482     static constexpr size_t length = 0;
00483
00484     template <typename T>
00485     using push_front = type_list<T>;
00486
00487     template <typename T>
00488     using push_back = type_list<T>;
00489
00490     template <typename U>
00491     using concat = U;
00492
00493     // TODO(jewave): assert index == 0
00494     template <typename T, size_t index>

```

```

00495         using insert = type_list<T>;
00496     };
00497 } // namespace aerobus
00498
00499 // i32
00500 namespace aerobus {
00501     struct i32 {
00502         using inner_type = int32_t;
00503         template<int32_t x>
00504         struct val {
00505             using ring_type = i32;
00506             static constexpr int32_t v = x;
00507
00508             template<typename valueType>
00509             static constexpr valueType get() { return static_cast<valueType>(x); }
00510
00511             using is_zero_t = std::bool_constant<x == 0>;
00512
00513             static std::string to_string() {
00514                 return std::to_string(x);
00515             }
00516
00517             template<typename valueRing>
00518             static constexpr valueRing eval(const valueRing& v) {
00519                 return static_cast<valueRing>(x);
00520             }
00521         };
00522     };
00523
00524     using zero = val<0>;
00525     using one = val<1>;
00526     static constexpr bool is_field = false;
00527     static constexpr bool is_euclidean_domain = true;
00528     template<auto x>
00529     using inject_constant_t = val<static_cast<int32_t>(x)>;
00530
00531     template<typename v>
00532     using inject_ring_t = v;
00533
00534 private:
00535     template<typename v1, typename v2>
00536     struct add {
00537         using type = val<v1::v + v2::v>;
00538     };
00539
00540     template<typename v1, typename v2>
00541     struct sub {
00542         using type = val<v1::v - v2::v>;
00543     };
00544
00545     template<typename v1, typename v2>
00546     struct mul {
00547         using type = val<v1::v * v2::v>;
00548     };
00549
00550     template<typename v1, typename v2>
00551     struct div {
00552         using type = val<v1::v / v2::v>;
00553     };
00554
00555     template<typename v1, typename v2>
00556     struct remainder {
00557         using type = val<v1::v % v2::v>;
00558     };
00559
00560     template<typename v1, typename v2>
00561     struct gt {
00562         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00563     };
00564
00565     template<typename v1, typename v2>
00566     struct lt {
00567         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00568     };
00569
00570     template<typename v1, typename v2>
00571     struct eq {
00572         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00573     };
00574
00575     template<typename v1>
00576     struct pos {
00577         using type = std::bool_constant<(v1::v > 0)>;
00578     };
00579
00580 public:
00581     template<typename v1, typename v2>
00582     using add_t = typename add<v1, v2>::type;
00583
00584     template<typename v1, typename v2>
00585     using sub_t = typename sub<v1, v2>::type;
00586
00587     template<typename v1, typename v2>
00588     using mul_t = typename mul<v1, v2>::type;
00589
00590     template<typename v1, typename v2>
00591     using div_t = typename div<v1, v2>::type;
00592
00593     template<typename v1, typename v2>
00594     using remainder_t = typename remainder<v1, v2>::type;
00595
00596     template<typename v1, typename v2>
00597     using gt_t = typename gt<v1, v2>::type;
00598
00599     template<typename v1, typename v2>
00600     using lt_t = typename lt<v1, v2>::type;
00601
00602     template<typename v1, typename v2>
00603     using eq_t = typename eq<v1, v2>::type;
00604
00605     template<typename v1>
00606     using pos_t = typename pos<v1>::type;

```

```

00601
00602     template<typename v1, typename v2>
00603     using sub_t = typename sub<v1, v2>::type;
00604
00605     template<typename v1, typename v2>
00606     using mul_t = typename mul<v1, v2>::type;
00607
00608     template<typename v1, typename v2>
00609     using div_t = typename div<v1, v2>::type;
00610
00611     template<typename v1, typename v2>
00612     using mod_t = typename remainder<v1, v2>::type;
00613
00614     template<typename v1, typename v2>
00615     using gt_t = typename gt<v1, v2>::type;
00616
00617     template<typename v1, typename v2>
00618     using lt_t = typename lt<v1, v2>::type;
00619
00620     template<typename v1, typename v2>
00621     using eq_t = typename eq<v1, v2>::type;
00622
00623     template<typename v1, typename v2>
00624     static constexpr bool eq_v = eq_t<v1, v2>::value;
00625
00626     template<typename v1, typename v2>
00627     using gcd_t = gcd_t<i32, v1, v2>;
00628
00629     template<typename v>
00630     using pos_t = typename pos<v>::type;
00631
00632     template<typename v>
00633     static constexpr bool pos_v = pos_t<v>::value;
00634 };
00635 } // namespace aerobus
00636
00637 // i64
00638 namespace aerobus {
00639     struct i64 {
00640         using inner_type = int64_t;
00641         template<int64_t x>
00642         struct val {
00643             using ring_type = i64;
00644             static constexpr int64_t v = x;
00645
00646             template<typename valueType>
00647             static constexpr valueType get() { return static_cast<valueType>(x); }
00648
00649             using is_zero_t = std::bool_constant<x == 0>;
00650
00651             static std::string to_string() {
00652                 return std::to_string(x);
00653             }
00654
00655             template<typename valueRing>
00656             static constexpr valueRing eval(const valueRing& v) {
00657                 return static_cast<valueRing>(x);
00658             }
00659         };
00660     };
00661
00662     template<auto x>
00663     using inject_constant_t = val<static_cast<int64_t>(x)>;
00664
00665     template<typename v>
00666     using inject_ring_t = v;
00667
00668     using zero = val<0>;
00669     using one = val<1>;
00670     static constexpr bool is_field = false;
00671     static constexpr bool is_euclidean_domain = true;
00672
00673 private:
00674     template<typename v1, typename v2>
00675     struct add {
00676         using type = val<v1::v + v2::v>;
00677     };
00678
00679     template<typename v1, typename v2>
00680     struct sub {
00681         using type = val<v1::v - v2::v>;
00682     };
00683
00684     template<typename v1, typename v2>
00685     struct mul {
00686         using type = val<v1::v * v2::v>;
00687     };
00688 }
00689
00690
00691
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00693
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```

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00721     template<typename v1, typename v2>
00722     struct div {
00723         using type = val<v1::v / v2::v>;
00724     };
00725
00726     template<typename v1, typename v2>
00727     struct remainder {
00728         using type = val<v1::v% v2::v>;
00729     };
00730
00731     template<typename v1, typename v2>
00732     struct gt {
00733         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00734     };
00735
00736     template<typename v1, typename v2>
00737     struct lt {
00738         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00739     };
00740
00741     template<typename v1, typename v2>
00742     struct eq {
00743         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00744     };
00745
00746     template<typename v>
00747     struct pos {
00748         using type = std::bool_constant<(v::v > 0)>;
00749     };
00750
00751 public:
00752     template<typename v1, typename v2>
00753     using add_t = typename add<v1, v2>::type;
00754
00755     template<typename v1, typename v2>
00756     using sub_t = typename sub<v1, v2>::type;
00757
00758     template<typename v1, typename v2>
00759     using mul_t = typename mul<v1, v2>::type;
00760
00761     template<typename v1, typename v2>
00762     using div_t = typename div<v1, v2>::type;
00763
00764     template<typename v1, typename v2>
00765     using mod_t = typename remainder<v1, v2>::type;
00766
00767     template<typename v1, typename v2>
00768     using gt_t = typename gt<v1, v2>::type;
00769
00770     template<typename v1, typename v2>
00771     static constexpr bool gt_v = gt_t<v1, v2>::value;
00772
00773     template<typename v1, typename v2>
00774     using lt_t = typename lt<v1, v2>::type;
00775
00776     template<typename v1, typename v2>
00777     static constexpr bool lt_v = lt_t<v1, v2>::value;
00778
00779     template<typename v1, typename v2>
00780     using eq_t = typename eq<v1, v2>::type;
00781
00782     template<typename v1, typename v2>
00783     static constexpr bool eq_v = eq_t<v1, v2>::value;
00784
00785     template<typename v1, typename v2>
00786     using gcd_t = gcd_t<i64, v1, v2>;
00787
00788     template<typename v>
00789     using pos_t = typename pos<v>::type;
00790
00791     template<typename v>
00792     static constexpr bool pos_v = pos_t<v>::value;
00793 };
00794 } // namespace aerobus
00795
00796 // z/pz
00797 namespace aerobus {
00798     template<int32_t p>
00799     struct zp {
00800         using inner_type = int32_t;
00801         template<int32_t x>
00802         struct val {
00803             using ring_type = zp<p>;
00804             static constexpr int32_t v = x % p;
00805
00806             template<typename valueType>
00807             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00808         };
00809     };
00810 }

```



```

00854
00855     using is_zero_t = std::bool_constant<x% p == 0>;
00856     static std::string to_string() {
00857         return std::to_string(x % p);
00858     }
00859
00860     template<typename valueRing>
00861     static constexpr valueRing eval(const valueRing& v) {
00862         return static_cast<valueRing>(x % p);
00863     }
00864 };
00865
00866 template<auto x>
00867 using inject_constant_t = val<static_cast<int32_t>(x)>;
00868
00869 using zero = val<0>;
00870 using one = val<1>;
00871 static constexpr bool is_field = is_prime<p>::value;
00872 static constexpr bool is_euclidean_domain = true;
00873
00874 private:
00875     template<typename v1, typename v2>
00876     struct add {
00877         using type = val<(v1::v + v2::v) % p>;
00878     };
00879
00880     template<typename v1, typename v2>
00881     struct sub {
00882         using type = val<(v1::v - v2::v) % p>;
00883     };
00884
00885     template<typename v1, typename v2>
00886     struct mul {
00887         using type = val<(v1::v * v2::v) % p>;
00888     };
00889
00890     template<typename v1, typename v2>
00891     struct div {
00892         using type = val<(v1::v % p) / (v2::v % p)>;
00893     };
00894
00895     template<typename v1, typename v2>
00896     struct remainder {
00897         using type = val<(v1::v % v2::v) % p>;
00898     };
00899
00900     template<typename v1, typename v2>
00901     struct gt {
00902         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00903     };
00904
00905     template<typename v1, typename v2>
00906     struct lt {
00907         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00908     };
00909
00910     template<typename v1, typename v2>
00911     struct eq {
00912         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00913     };
00914
00915     template<typename v1>
00916     struct pos {
00917         using type = std::bool_constant<(v1::v > 0)>;
00918     };
00919
00920 public:
00921     template<typename v1, typename v2>
00922     using add_t = typename add<v1, v2>::type;
00923
00924     template<typename v1, typename v2>
00925     using sub_t = typename sub<v1, v2>::type;
00926
00927     template<typename v1, typename v2>
00928     using mul_t = typename mul<v1, v2>::type;
00929
00930     template<typename v1, typename v2>
00931     using div_t = typename div<v1, v2>::type;
00932
00933     template<typename v1, typename v2>
00934     using mod_t = typename remainder<v1, v2>::type;
00935
00936     template<typename v1, typename v2>
00937     using gt_t = typename gt<v1, v2>::type;
00938
00939     template<typename v1, typename v2>
00940     using lt_t = typename lt<v1, v2>::type;
00941
00942     template<typename v1, typename v2>
00943     using eq_t = typename eq<v1, v2>::type;
00944
00945     template<typename v1, typename v2>
00946     static constexpr bool gt_v = gt_t<v1, v2>::value;
00947

```

```

00948
00950     template<typename v1, typename v2>
00951     using lt_t = typename lt<v1, v2>::type;
00952
00954     template<typename v1, typename v2>
00955     static constexpr bool lt_v = lt_t<v1, v2>::value;
00956
00958     template<typename v1, typename v2>
00959     using eq_t = typename eq<v1, v2>::type;
00960
00962     template<typename v1, typename v2>
00963     static constexpr bool eq_v = eq_t<v1, v2>::value;
00964
00966     template<typename v1, typename v2>
00967     using gcd_t = gcd_t<i32, v1, v2>;
00968
00970     template<typename v1>
00971     using pos_t = typename pos<v1>::type;
00972
00974     template<typename v>
00975     static constexpr bool pos_v = pos_t<v>::value;
00976 };
00977 } // namespace aerobus
00978
00979 // polynomial
00980 namespace aerobus {
00981     // coeffN x^N + ...
00982     template<typename Ring>
00983     requires IsEuclideanDomain<Ring>
00984     struct polynomial {
00985         static constexpr bool is_field = false;
00986         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
00987
00988         template<typename coeffN, typename... coeffs>
00989         struct val {
00990             using ring_type = polynomial<Ring>;
00991             static constexpr size_t degree = sizeof...(coeffs);
00992             using aN = coeffN;
00993             using strip = val<coeffs...>;
00994             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
00995             static constexpr bool is_zero_v = is_zero_t::value;
00996
00997         private:
00998             template<size_t index, typename E = void>
00999             struct coeff_at {};
01000
01001             template<size_t index>
01002             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01003                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01004             };
01005
01006             template<size_t index>
01007             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01008                 using type = typename Ring::zero;
01009             };
01010
01011         public:
01012             template<size_t index>
01013             using coeff_at_t = typename coeff_at<index>::type;
01014
01015             static std::string to_string() {
01016                 return string_helper<coeffN, coeffs...>::func();
01017             }
01018
01019             template<typename valueRing>
01020             static constexpr valueRing eval(const valueRing& x) {
01021                 return horner_evaluation<valueRing, val>
01022                     ::template inner<0, degree + 1>
01023                     ::func(static_cast<valueRing>(0), x);
01024             }
01025         };
01026     };
01027
01028     template<typename coeffN>
01029     struct val<coeffN> {
01030         using ring_type = polynomial<Ring>;
01031         static constexpr size_t degree = 0;
01032         using aN = coeffN;
01033         using strip = val<coeffN>;
01034         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01035
01036         static constexpr bool is_zero_v = is_zero_t::value;
01037
01038         template<size_t index, typename E = void>
01039         struct coeff_at {};
01040
01041         template<size_t index>
01042         struct coeff_at<index, std::enable_if_t<(index == 0)> {

```

```

01067         using type = aN;
01068     };
01069
01070     template<size_t index>
01071     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01072         using type = typename Ring::zero;
01073     };
01074
01075     template<size_t index>
01076     using coeff_at_t = typename coeff_at<index>::type;
01077
01078     static std::string to_string() {
01079         return string_helper<coeffN>::func();
01080     }
01081
01082     template<typename valueRing>
01083     static constexpr valueRing eval(const valueRing& x) {
01084         return static_cast<valueRing>(aN::template get<valueRing>());
01085     }
01086 };
01087
01088 using zero = val<typename Ring::zero>;
01089 using one = val<typename Ring::one>;
01090 using X = val<typename Ring::one, typename Ring::zero>;
01091
01092 private:
01093     template<typename P, typename E = void>
01094     struct simplify;
01095
01096     template<typename P1, typename P2, typename I>
01097     struct add_low;
01098
01099     template<typename P1, typename P2>
01100     struct add {
01101         using type = typename simplify<typename add_low<
01102             P1,
01103             P2,
01104             internal::make_index_sequence_reverse<
01105                 std::max(P1::degree, P2::degree) + 1
01106             >::type>::type;
01107     };
01108
01109     template<typename P1, typename P2, typename I>
01110     struct sub_low;
01111
01112     template<typename P1, typename P2, typename I>
01113     struct mul_low;
01114
01115     template<typename v1, typename v2>
01116     struct mul {
01117         using type = typename mul_low<
01118             v1,
01119             v2,
01120             internal::make_index_sequence_reverse<
01121                 v1::degree + v2::degree + 1
01122             >::type>
01123     };
01124
01125     template<typename coeff, size_t deg>
01126     struct monomial;
01127
01128     template<typename v, typename E = void>
01129     struct derive_helper {};
01130
01131     template<typename v>
01132     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01133         using type = zero;
01134     };
01135
01136     template<typename v>
01137     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01138         using type = typename add<
01139             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01140             typename monomial<
01141                 typename Ring::template mul_t<
01142                     typename v::aN,
01143                     typename Ring::template inject_constant_t<(v::degree)>
01144                 >,
01145                 v::degree - 1
01146             >::type
01147         >::type;
01148     };
01149
01150     template<typename v1, typename v2, typename E = void>
01151     struct eq_helper {};
01152
01153     template<typename v1, typename v2>

```

```

01157     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01158         using type = std::false_type;
01159     };
01160
01161
01162     template<typename v1, typename v2>
01163     struct eq_helper<v1, v2, std::enable_if_t<
01164         v1::degree == v2::degree &&
01165         (v1::degree != 0 || v2::degree != 0) &&
01166         std::is_same<
01167             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01168             std::false_type
01169         >::value
01170     > {
01171     > {
01172         using type = std::false_type;
01173     };
01174
01175     template<typename v1, typename v2>
01176     struct eq_helper<v1, v2, std::enable_if_t<
01177         v1::degree == v2::degree &&
01178         (v1::degree != 0 || v2::degree != 0) &&
01179         std::is_same<
01180             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01181             std::true_type
01182         >::value
01183     >> {
01184         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01185     };
01186
01187     template<typename v1, typename v2>
01188     struct eq_helper<v1, v2, std::enable_if_t<
01189         v1::degree == v2::degree &&
01190         (v1::degree == 0)
01191     >> {
01192         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01193     };
01194
01195     template<typename v1, typename v2, typename E = void>
01196     struct lt_helper {};
01197
01198     template<typename v1, typename v2>
01199     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01200         using type = std::true_type;
01201     };
01202
01203     template<typename v1, typename v2>
01204     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01205         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01206     };
01207
01208     template<typename v1, typename v2>
01209     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01210         using type = std::false_type;
01211     };
01212
01213     template<typename v1, typename v2, typename E = void>
01214     struct gt_helper {};
01215
01216     template<typename v1, typename v2>
01217     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01218         using type = std::true_type;
01219     };
01220
01221     template<typename v1, typename v2>
01222     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01223         using type = std::false_type;
01224     };
01225
01226     template<typename v1, typename v2>
01227     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01228         using type = std::false_type;
01229     };
01230
01231     // when high power is zero : strip
01232     template<typename P>
01233     struct simplify<P, std::enable_if_t<
01234         std::is_same<
01235             typename Ring::zero,
01236             typename P::aN
01237         >::value && (P::degree > 0)
01238     >> {
01239         using type = typename simplify<typename P::strip>::type;
01240     };
01241
01242     // otherwise : do nothing
01243     template<typename P>

```

```

01244     struct simplify<P, std::enable_if_t<
01245         !std::is_same<
01246             typename Ring::zero,
01247             typename P::aN
01248         >::value && (P::degree > 0)
01249     > {
01250         using type = P;
01251     };
01252
01253     // do not simplify constants
01254     template<typename P>
01255     struct simplify<P, std::enable_if_t<P::degree == 0> {
01256         using type = P;
01257     };
01258
01259     // addition at
01260     template<typename P1, typename P2, size_t index>
01261     struct add_at {
01262         using type =
01263             typename Ring::template add_t<
01264                 typename P1::template coeff_at_t<index>,
01265                 typename P2::template coeff_at_t<index>>;
01266     };
01267
01268     template<typename P1, typename P2, size_t index>
01269     using add_at_t = typename add_at<P1, P2, index>::type;
01270
01271     template<typename P1, typename P2, std::size_t... I>
01272     struct add_low<P1, P2, std::index_sequence<I...> {
01273         using type = val<add_at_t<P1, P2, I>...>;
01274     };
01275
01276     // subtraction at
01277     template<typename P1, typename P2, size_t index>
01278     struct sub_at {
01279         using type =
01280             typename Ring::template sub_t<
01281                 typename P1::template coeff_at_t<index>,
01282                 typename P2::template coeff_at_t<index>>;
01283     };
01284
01285     template<typename P1, typename P2, size_t index>
01286     using sub_at_t = typename sub_at<P1, P2, index>::type;
01287
01288     template<typename P1, typename P2, std::size_t... I>
01289     struct sub_low<P1, P2, std::index_sequence<I...> {
01290         using type = val<sub_at_t<P1, P2, I>...>;
01291     };
01292
01293     template<typename P1, typename P2>
01294     struct sub {
01295         using type = typename simplify<typename sub_low<
01296             P1,
01297             P2,
01298             internal::make_index_sequence_reverse<
01299                 std::max(P1::degree, P2::degree) + 1
01300             >::type>::type;
01301     };
01302
01303     // multiplication at
01304     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01305     struct mul_at_loop_helper {
01306         using type = typename Ring::template add_t<
01307             typename Ring::template mul_t<
01308                 typename v1::template coeff_at_t<index>,
01309                 typename v2::template coeff_at_t<k - index>
01310             >,
01311             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01312         >;
01313     };
01314
01315     template<typename v1, typename v2, size_t k, size_t stop>
01316     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01317         using type = typename Ring::template mul_t<
01318             typename v1::template coeff_at_t<stop>,
01319             typename v2::template coeff_at_t<0>>;
01320     };
01321
01322     template<typename v1, typename v2, size_t k, typename E = void>
01323     struct mul_at {};
01324
01325     template<typename v1, typename v2, size_t k>
01326     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01327         using type = typename Ring::zero;
01328     };
01329
01330     template<typename v1, typename v2, size_t k>

```

```

01331     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01332         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01333     };
01334
01335     template<typename P1, typename P2, size_t index>
01336     using mul_at_t = typename mul_at<P1, P2, index>::type;
01337
01338     template<typename P1, typename P2, std::size_t... I>
01339     struct mul_low<P1, P2, std::index_sequence<I...> {
01340         using type = val<mul_at_t<P1, P2, I>...>;
01341     };
01342
01343     // division helper
01344     template< typename A, typename B, typename Q, typename R, typename E = void>
01345     struct div_helper {};
01346
01347     template<typename A, typename B, typename Q, typename R>
01348     struct div_helper<A, B, Q, R, std::enable_if_t<
01349         (R::degree < B::degree) ||
01350         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01351         using q_type = Q;
01352         using mod_type = R;
01353         using gcd_type = B;
01354     };
01355
01356     template<typename A, typename B, typename Q, typename R>
01357     struct div_helper<A, B, Q, R, std::enable_if_t<
01358         (R::degree >= B::degree) &&
01359         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)> {
01360     private: // NOLINT
01361         using rN = typename R::aN;
01362         using bN = typename B::aN;
01363         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01364         B::degree>::type;
01365         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01366         using qq = typename add<Q, pT>::type;
01367     public:
01368         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01369         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01370         using gcd_type = rr;
01371     };
01372
01373     template<typename A, typename B>
01374     struct div {
01375         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01376         using q_type = typename div_helper<A, B, zero, A>::q_type;
01377         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01378     };
01379
01380     template<typename P>
01381     struct make_unit {
01382         using type = typename div<P, val<typename P::aN>::q_type>;
01383     };
01384
01385     template<typename coeff, size_t deg>
01386     struct monomial {
01387         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01388     };
01389
01390     template<typename coeff>
01391     struct monomial<coeff, 0> {
01392         using type = val<coeff>;
01393     };
01394
01395     template<typename valueRing, typename P>
01396     struct horner_evaluation {
01397         template<size_t index, size_t stop>
01398         struct inner {
01399             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01400                 constexpr valueRing coeff =
01401                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01402                     get<valueRing>());
01403                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01404                     accum + coeff, x);
01405             };
01406
01407             template<size_t stop>
01408             struct inner<stop, stop> {
01409                 static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01410                     return accum;
01411                 };
01412             };
01413         };
01414     };
01415     template<typename coeff, typename... coeffs>

```

```

01416     struct string_helper {
01417     static std::string func() {
01418         std::string tail = string_helper<coeffs...>::func();
01419         std::string result = "";
01420         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01421             return tail;
01422         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01423             if (sizeof...(coeffs) == 1) {
01424                 result += "x";
01425             } else {
01426                 result += "x^" + std::to_string(sizeof...(coeffs));
01427             }
01428         } else {
01429             if (sizeof...(coeffs) == 1) {
01430                 result += coeff::to_string() + " x";
01431             } else {
01432                 result += coeff::to_string()
01433                     + " x^" + std::to_string(sizeof...(coeffs));
01434             }
01435         }
01436     }
01437     if (!tail.empty()) {
01438         result += " + " + tail;
01439     }
01440     return result;
01441 }
01442 };
01443
01444 template<typename coeff>
01445 struct string_helper<coeff> {
01446     static std::string func() {
01447         if (!std::is_same<coeff, typename Ring::zero>::value) {
01448             return coeff::to_string();
01449         } else {
01450             return "";
01451         }
01452     }
01453 };
01454 };
01455
01456 public:
01457     template<typename P>
01458     using simplify_t = typename simplify<P>::type;
01459
01460     template<typename v1, typename v2>
01461     using add_t = typename add<v1, v2>::type;
01462
01463     template<typename v1, typename v2>
01464     using sub_t = typename sub<v1, v2>::type;
01465
01466     template<typename v1, typename v2>
01467     using mul_t = typename mul<v1, v2>::type;
01468
01469     template<typename v1, typename v2>
01470     using eq_t = typename eq_helper<v1, v2>::type;
01471
01472     template<typename v1, typename v2>
01473     using lt_t = typename lt_helper<v1, v2>::type;
01474
01475     template<typename v1, typename v2>
01476     using gt_t = typename gt_helper<v1, v2>::type;
01477
01478     template<typename v1, typename v2>
01479     using div_t = typename div<v1, v2>::q_type;
01480
01481     template<typename v1, typename v2>
01482     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01483
01484     template<typename coeff, size_t deg>
01485     using monomial_t = typename monomial<coeff, deg>::type;
01486
01487     template<typename v>
01488     using derive_t = typename derive_helper<v>::type;
01489
01490     template<typename v>
01491     using pos_t = typename Ring::template pos_t<typename v::aN>;
01492
01493     template<typename v>
01494     static constexpr bool pos_v = pos_t<v>::value;
01495
01496     template<typename v1, typename v2>
01497     using gcd_t = std::conditional_t<
01498         Ring::is_euclidean_domain,
01499         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01500         void>;
01501
01502     template<auto x>

```

```

01542         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01543
01544         template<typename v>
01545         using inject_ring_t = val<v>;
01546     };
01547 } // namespace aerobus
01548
01549 // fraction field
01550 namespace aerobus {
01551     namespace internal {
01552         template<typename Ring, typename E = void>
01553         requires IsEuclideanDomain<Ring>
01554         struct _FractionField {};
01555
01556         template<typename Ring>
01557         requires IsEuclideanDomain<Ring>
01558         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01559             static constexpr bool is_field = true;
01560             static constexpr bool is_euclidean_domain = true;
01561
01562         private:
01563             template<typename val1, typename val2, typename E = void>
01564             struct to_string_helper {};
01565
01566             template<typename val1, typename val2>
01567             struct to_string_helper<val1, val2,
01568                 std::enable_if_t<
01569                     Ring::template eq_t<
01570                         val2, typename Ring::one
01571                     >::value
01572                 >
01573             > {
01574                 static std::string func() {
01575                     return val1::to_string();
01576                 }
01577             };
01578
01579             template<typename val1, typename val2>
01580             struct to_string_helper<val1, val2,
01581                 std::enable_if_t<
01582                     !Ring::template eq_t<
01583                         val2,
01584                         typename Ring::one
01585                     >::value
01586                 >
01587             > {
01588                 static std::string func() {
01589                     return "(" + val1::to_string() + " / (" + val2::to_string() + ")";
01590                 }
01591             };
01592
01593         public:
01594             template<typename val1, typename val2>
01595             struct val {
01596                 using x = val1;
01597                 using y = val2;
01598                 using is_zero_t = typename val1::is_zero_t;
01599                 static constexpr bool is_zero_v = val1::is_zero_t::value;
01600
01601                 using ring_type = Ring;
01602                 using field_type = _FractionField<Ring>;
01603
01604                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01605
01606                 template<typename valueType>
01607                 static constexpr valueType get() { return static_cast<valueType>(x::v) /
01608                     static_cast<valueType>(y::v); }
01609
01610                 static std::string to_string() {
01611                     return to_string_helper<val1, val2>::func();
01612                 }
01613
01614                 template<typename valueRing>
01615                 static constexpr valueRing eval(const valueRing& v) {
01616                     return x::eval(v) / y::eval(v);
01617                 }
01618             };
01619
01620             using zero = val<typename Ring::zero, typename Ring::one>;
01621             using one = val<typename Ring::one, typename Ring::one>;
01622
01623             template<typename v>
01624             using inject_t = val<v, typename Ring::one>;
01625
01626             template<auto x>
01627             using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01628                 Ring::one>;

```



```

01656
01659     template<typename v>
01660     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01661
01663     using ring_type = Ring;
01664
01665 private:
01666     template<typename v, typename E = void>
01667     struct simplify {};
01668
01669     // x = 0
01670     template<typename v>
01671     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> > {
01672         using type = typename _FractionField<Ring>::zero;
01673     };
01674
01675     // x != 0
01676     template<typename v>
01677     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> > {
01678     private:
01679         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01680         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01681         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01682
01683         using posx = std::conditional_t<
01684             !Ring::template pos_v<newy>,
01685             typename Ring::template sub_t<typename Ring::zero, newx>,
01686             newx>;
01687         using posy = std::conditional_t<
01688             !Ring::template pos_v<newy>,
01689             typename Ring::template sub_t<typename Ring::zero, newy>,
01690             newy>;
01691     public:
01692         using type = typename _FractionField<Ring>::template val<posx, posy>;
01693     };
01694
01695 public:
01696     template<typename v>
01697     using simplify_t = typename simplify<v>::type;
01698
01699 private:
01700     template<typename v1, typename v2>
01701     struct add {
01702     private:
01703         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01704         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01705         using dividend = typename Ring::template add_t<a, b>;
01706         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01707         using g = typename Ring::template gcd_t<dividend, diviser>;
01708
01709     public:
01710         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01711             diviser>;
01712     };
01713
01714     template<typename v>
01715     struct pos {
01716     private:
01717         using type = std::conditional_t<
01718             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01719             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01720             std::true_type,
01721             std::false_type>;
01722     };
01723
01724     template<typename v1, typename v2>
01725     struct sub {
01726     private:
01727         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01728         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01729         using dividend = typename Ring::template sub_t<a, b>;
01730         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01731         using g = typename Ring::template gcd_t<dividend, diviser>;
01732
01733     public:
01734         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01735             diviser>;
01736     };
01737
01738     template<typename v1, typename v2>
01739     struct mul {
01740     private:
01741         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01742         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01743
01744     public:
01745         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01746     };

```

```

01746
01747     template<typename v1, typename v2, typename E = void>
01748     struct div {};
01749
01750     template<typename v1, typename v2>
01751     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01752     private:
01753         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01754         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01755
01756     public:
01757         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01758     };
01759
01760     template<typename v1, typename v2>
01761     struct div<v1, v2, std::enable_if_t<
01762         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01763         using type = one;
01764     };
01765
01766     template<typename v1, typename v2>
01767     struct eq {
01768         using type = std::conditional_t<
01769             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01770             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01771             std::true_type,
01772             std::false_type>;
01773     };
01774
01775     template<typename TL, typename E = void>
01776     struct vadd {};
01777
01778     template<typename TL>
01779     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01780         using head = typename TL::pop_front::type;
01781         using tail = typename TL::pop_front::tail;
01782         using type = typename add<head, typename vadd<tail>::type>::type;
01783     };
01784
01785     template<typename TL>
01786     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01787         using type = typename TL::template at<0>;
01788     };
01789
01790     template<typename... vals>
01791     struct vmul {};
01792
01793     template<typename v1, typename... vals>
01794     struct vmul<v1, vals...> {
01795         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01796     };
01797
01798     template<typename v1>
01799     struct vmul<v1> {
01800         using type = v1;
01801     };
01802
01803     template<typename v1, typename v2, typename E = void>
01804     struct gt;
01805
01806     template<typename v1, typename v2>
01807     struct gt<v1, v2, std::enable_if_t<
01808         (eq<v1, v2>::type::value)
01809         >> {
01810         using type = std::false_type;
01811     };
01812
01813     template<typename v1, typename v2>
01814     struct gt<v1, v2, std::enable_if_t<
01815         (!eq<v1, v2>::type::value) &&
01816         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01817         >> {
01818         using type = typename gt<
01819             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01820             >::type;
01821     };
01822
01823     template<typename v1, typename v2>
01824     struct gt<v1, v2, std::enable_if_t<
01825         (!eq<v1, v2>::type::value) &&
01826         (pos<v1>::type::value) && (!pos<v2>::type::value)
01827         >> {
01828         using type = std::true_type;
01829     };
01830
01831

```

```

01832     template<typename v1, typename v2>
01833     struct gt<v1, v2, std::enable_if_t<
01834         (!eq<v1, v2>::type::value) &&
01835         (!pos<v1>::type::value) && (pos<v2>::type::value)
01836         >> {
01837         using type = std::false_type;
01838     };
01839
01840     template<typename v1, typename v2>
01841     struct gt<v1, v2, std::enable_if_t<
01842         (!eq<v1, v2>::type::value) &&
01843         (pos<v1>::type::value) && (pos<v2>::type::value)
01844         >> {
01845         using type = typename Ring::template gt_t<
01846             typename Ring::template mul_t<v1::x, v2::y>,
01847             typename Ring::template mul_t<v2::y, v2::x>
01848         >;
01849     };
01850
01851     public:
01852     template<typename v1, typename v2>
01853     using add_t = typename add<v1, v2>::type;
01854     template<typename v1, typename v2>
01855     using mod_t = zero;
01856     template<typename v1, typename v2>
01857     using gcd_t = v1;
01858     template<typename... vs>
01859     using vadd_t = typename vadd<vs...>::type;
01860     template<typename... vs>
01861     using vmul_t = typename vmul<vs...>::type;
01862     template<typename v1, typename v2>
01863     using sub_t = typename sub<v1, v2>::type;
01864     template<typename v1, typename v2>
01865     using mul_t = typename mul<v1, v2>::type;
01866     template<typename v1, typename v2>
01867     using div_t = typename div<v1, v2>::type;
01868     template<typename v1, typename v2>
01869     using eq_t = typename eq<v1, v2>::type;
01870     template<typename v1, typename v2>
01871     static constexpr bool eq_v = eq<v1, v2>::type::value;
01872     template<typename v1, typename v2>
01873     using gt_t = typename gt<v1, v2>::type;
01874     template<typename v1, typename v2>
01875     static constexpr bool gt_v = gt<v1, v2>::type::value;
01876     template<typename v1>
01877     using pos_t = typename pos<v1>::type;
01878     template<typename v>
01879     static constexpr bool pos_v = pos<v>::value;
01880 };
01881
01882     template<typename Ring, typename E = void>
01883     requires IsEuclideanDomain<Ring>
01884     struct FractionFieldImpl {};
01885
01886     // fraction field of a field is the field itself
01887     template<typename Field>
01888     requires IsEuclideanDomain<Field>
01889     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01890         using type = Field;
01891         template<typename v>
01892         using inject_t = v;
01893     };
01894
01895     // fraction field of a ring is the actual fraction field
01896     template<typename Ring>
01897     requires IsEuclideanDomain<Ring>
01898     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01899         using type = _FractionField<Ring>;
01900     };
01901 } // namespace internal
01902
01903     template<typename Ring>
01904     requires IsEuclideanDomain<Ring>
01905     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01906 } // namespace aerobus
01907
01908 // short names for common types
01909 namespace aerobus {
01910     using q32 = FractionField<i32>;
01911     using fpq32 = FractionField<polynomial<q32>>;
01912     using q64 = FractionField<i64>;
01913     using pi64 = polynomial<i64>;
01914     using pq64 = polynomial<q64>;
01915     using fpq64 = FractionField<polynomial<q64>>;
01916     template<typename Ring, typename v1, typename v2>
01917     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01918 }

```

```

01951     template<typename Ring, typename v1, typename v2>
01952     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
01957     template<typename Ring, typename v1, typename v2>
01958     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01959 } // namespace aerobus
01960
01961 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
01962 namespace aerobus {
01963     namespace internal {
01964         template<typename T, size_t x, typename E = void>
01965         struct factorial {};
01966
01967         template<typename T, size_t x>
01968         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
01969             private:
01970                 template<typename, size_t, typename>
01971                 friend struct factorial;
01972             public:
01973                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
01974                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01975             };
01976
01977         template<typename T>
01978         struct factorial<T, 0> {
01979             public:
01980                 using type = typename T::one;
01981                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
01982             };
01983     } // namespace internal
01984
01988     template<typename T, size_t i>
01989     using factorial_t = typename internal::factorial<T, i>::type;
01990
01994     template<typename T, size_t i>
01995     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01996
01997     namespace internal {
01998         template<typename T, size_t k, size_t n, typename E = void>
01999         struct combination_helper {};
02000
02001         template<typename T, size_t k, size_t n>
02002         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02003             using type = typename FractionField<T>::template mul_t<
02004                 typename combination_helper<T, k - 1, n - 1>::type,
02005                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02006             };
02007
02008         template<typename T, size_t k, size_t n>
02009         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02010             using type = typename combination_helper<T, n - k, n>::type;
02011             };
02012
02013         template<typename T, size_t n>
02014         struct combination_helper<T, 0, n> {
02015             using type = typename FractionField<T>::one;
02016             };
02017
02018         template<typename T, size_t k, size_t n>
02019         struct combination {
02020             using type = typename internal::combination_helper<T, k, n>::type::x;
02021             static constexpr typename T::inner_type value =
02022                 internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02023             };
02024     } // namespace internal
02025
02028     template<typename T, size_t k, size_t n>
02029     using combination_t = typename internal::combination<T, k, n>::type;
02030
02035     template<typename T, size_t k, size_t n>
02036     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02037
02038     namespace internal {
02039         template<typename T, size_t m>
02040         struct bernouilli;
02041
02042         template<typename T, typename accum, size_t k, size_t m>
02043         struct bernouilli_helper {
02044             using type = typename bernouilli_helper<
02045                 T,
02046                 addfractions_t<T,
02047                     accum,
02048                     mulfractions_t<T,
02049                     makefraction_t<T,

```

```

02050             combination_t<T, k, m + 1>,
02051             typename T::one>,
02052             typename bernouilli<T, k>::type
02053         >
02054     >,
02055     k + 1,
02056     m>::type;
02057 };
02058
02059 template<typename T, typename accum, size_t m>
02060 struct bernouilli_helper<T, accum, m, m> {
02061     using type = accum;
02062 };
02063
02064
02065
02066 template<typename T, size_t m>
02067 struct bernouilli {
02068     using type = typename FractionField<T>::template mul_t<
02069         typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02070         makefraction_t<T,
02071         typename T::template val<static_cast<typename T::inner_type>(-1)>,
02072         typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02073     >
02074     >;
02075
02076     template<typename floatType>
02077     static constexpr floatType value = type::template get<floatType>();
02078 };
02079
02080 template<typename T>
02081 struct bernouilli<T, 0> {
02082     using type = typename FractionField<T>::one;
02083
02084     template<typename floatType>
02085     static constexpr floatType value = type::template get<floatType>();
02086 };
02087 } // namespace internal
02088
02092 template<typename T, size_t n>
02093 using bernouilli_t = typename internal::bernouilli<T, n>::type;
02094
02099 template<typename FloatType, typename T, size_t n>
02100 inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02101
02102 namespace internal {
02103     template<typename T, int k, typename E = void>
02104     struct alternate {};
02105
02106     template<typename T, int k>
02107     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02108         using type = typename T::one;
02109         static constexpr typename T::inner_type value = type::template get<typename
02110 T::inner_type>();
02111     };
02112
02113     template<typename T, int k>
02114     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02115         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02116         static constexpr typename T::inner_type value = type::template get<typename
02117 T::inner_type>();
02118     };
02119 } // namespace internal
02120
02121 template<typename T, int k>
02122 using alternate_t = typename internal::alternate<T, k>::type;
02123
02124 namespace internal {
02125     template<typename T, int n, int k, typename E = void>
02126     struct stirling_helper {};
02127
02128     template<typename T>
02129     struct stirling_helper<T, 0, 0> {
02130         using type = typename T::one;
02131     };
02132
02133     template<typename T, int n>
02134     struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02135         using type = typename T::zero;
02136     };
02137
02138     template<typename T, int n>
02139     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02140         using type = typename T::zero;
02141     };
02142
02143     template<typename T, int n, int k>

```

```

02144     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02145         using type = typename T::template sub_t<
02146             typename stirling_helper<T, n-1, k-1>::type,
02147             typename T::template mul_t<
02148                 typename T::template inject_constant_t<n-1>,
02149                 typename stirling_helper<T, n-1, k>::type
02150             >>;
02151     };
02152 } // namespace internal
02153
02154 template<typename T, int n, int k>
02155 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02156
02157 template<typename T, int n, int k>
02158 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02159
02160 template<typename T, int n, int k>
02161 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02162
02163 template<typename T, int n, int k>
02164 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02165
02166 template<typename T, size_t k>
02167 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02168
02169 namespace internal {
02170     template<typename T, auto p, auto n, typename E = void>
02171     struct pow {};
02172
02173     template<typename T, auto p, auto n>
02174     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02175         using type = typename T::template mul_t<
02176             typename pow<T, p, n/2>::type,
02177             typename pow<T, p, n/2>::type
02178         >;
02179     };
02180
02181     template<typename T, auto p, auto n>
02182     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02183         using type = typename T::template mul_t<
02184             typename T::template inject_constant_t<p>,
02185             typename T::template mul_t<
02186                 typename pow<T, p, n/2>::type,
02187                 typename pow<T, p, n/2>::type
02188             >
02189         >;
02190     };
02191
02192     template<typename T, auto p>
02193     struct pow<T, p, 0> { using type = typename T::one; };
02194 } // namespace internal
02195
02196 template<typename T, auto p, auto n>
02197 using pow_t = typename internal::pow<T, p, n>::type;
02198
02199 template<typename T, auto p, auto n>
02200 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02201
02202 namespace internal {
02203     template<typename, template<typename, size_t> typename, class>
02204     struct make_taylor_impl;
02205
02206     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02207     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02208         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
02209             Is>::type...>;
02210     };
02211 }
02212
02213 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02214 using taylor = typename internal::make_taylor_impl<
02215     T,
02216     coeff_at,
02217     internal::make_index_sequence_reverse<deg + 1>::type;
02218
02219 namespace internal {
02220     template<typename T, size_t i>
02221     struct exp_coeff {
02222         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02223     };
02224
02225     template<typename T, size_t i, typename E = void>
02226     struct sin_coeff_helper {};
02227
02228     template<typename T, size_t i>
02229     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {

```

```

02260         using type = typename FractionField<T>::zero;
02261     };
02262
02263     template<typename T, size_t i>
02264     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02265         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02266     };
02267
02268     template<typename T, size_t i>
02269     struct sin_coeff {
02270         using type = typename sin_coeff_helper<T, i>::type;
02271     };
02272
02273     template<typename T, size_t i, typename E = void>
02274     struct sh_coeff_helper {};
02275
02276     template<typename T, size_t i>
02277     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02278         using type = typename FractionField<T>::zero;
02279     };
02280
02281     template<typename T, size_t i>
02282     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02283         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02284     };
02285
02286     template<typename T, size_t i>
02287     struct sh_coeff {
02288         using type = typename sh_coeff_helper<T, i>::type;
02289     };
02290
02291     template<typename T, size_t i, typename E = void>
02292     struct cos_coeff_helper {};
02293
02294     template<typename T, size_t i>
02295     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02296         using type = typename FractionField<T>::zero;
02297     };
02298
02299     template<typename T, size_t i>
02300     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02301         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02302     };
02303
02304     template<typename T, size_t i>
02305     struct cos_coeff {
02306         using type = typename cos_coeff_helper<T, i>::type;
02307     };
02308
02309     template<typename T, size_t i, typename E = void>
02310     struct cosh_coeff_helper {};
02311
02312     template<typename T, size_t i>
02313     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02314         using type = typename FractionField<T>::zero;
02315     };
02316
02317     template<typename T, size_t i>
02318     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02319         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02320     };
02321
02322     template<typename T, size_t i>
02323     struct cosh_coeff {
02324         using type = typename cosh_coeff_helper<T, i>::type;
02325     };
02326
02327     template<typename T, size_t i>
02328     struct geom_coeff { using type = typename FractionField<T>::one; };
02329
02330
02331     template<typename T, size_t i, typename E = void>
02332     struct atan_coeff_helper;
02333
02334     template<typename T, size_t i>
02335     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02336         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02337     };
02338
02339     template<typename T, size_t i>
02340     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02341         using type = typename FractionField<T>::zero;
02342     };
02343
02344     template<typename T, size_t i>
02345     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02346

```

```

02347     template<typename T, size_t i, typename E = void>
02348     struct asin_coeff_helper;
02349
02350     template<typename T, size_t i>
02351     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02352         using type = makefraction_t<T,
02353             factorial_t<T, i - 1>,
02354             typename T::template mul_t<
02355                 typename T::template val<i>,
02356                 T::template mul_t<
02357                     pow_t<T, 4, i / 2>,
02358                     pow<T, factorial<T, i / 2>::value, 2
02359                 >
02360             >
02361         >>;
02362     };
02363
02364     template<typename T, size_t i>
02365     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02366         using type = typename FractionField<T>::zero;
02367     };
02368
02369     template<typename T, size_t i>
02370     struct asin_coeff {
02371         using type = typename asin_coeff_helper<T, i>::type;
02372     };
02373
02374     template<typename T, size_t i>
02375     struct lnpl_coeff {
02376         using type = makefraction_t<T,
02377             alternate_t<T, i + 1>,
02378             typename T::template val<i>;
02379     };
02380
02381     template<typename T>
02382     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02383
02384     template<typename T, size_t i, typename E = void>
02385     struct asinh_coeff_helper;
02386
02387     template<typename T, size_t i>
02388     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02389         using type = makefraction_t<T,
02390             typename T::template mul_t<
02391                 alternate_t<T, i / 2>,
02392                 factorial_t<T, i - 1>
02393             >,
02394             typename T::template mul_t<
02395                 T::template mul_t<
02396                     typename T::template val<i>,
02397                     pow_t<T, (factorial<T, i / 2>::value), 2>
02398                 >,
02399                 pow_t<T, 4, i / 2>
02400             >
02401         >>;
02402     };
02403
02404     template<typename T, size_t i>
02405     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02406         using type = typename FractionField<T>::zero;
02407     };
02408
02409     template<typename T, size_t i>
02410     struct asinh_coeff {
02411         using type = typename asinh_coeff_helper<T, i>::type;
02412     };
02413
02414     template<typename T, size_t i, typename E = void>
02415     struct atanh_coeff_helper;
02416
02417     template<typename T, size_t i>
02418     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02419         // 1/i
02420         using type = typename FractionField<T>::template val<
02421             typename T::one,
02422             typename T::template val<static_cast<typename T::inner_type>(i)>;
02423     };
02424
02425     template<typename T, size_t i>
02426     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02427         using type = typename FractionField<T>::zero;
02428     };
02429
02430     template<typename T, size_t i>
02431     struct atanh_coeff {
02432         using type = typename asinh_coeff_helper<T, i>::type;
02433     };

```



```

02434
02435     template<typename T, size_t i, typename E = void>
02436     struct tan_coeff_helper;
02437
02438     template<typename T, size_t i>
02439     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02440         using type = typename FractionField<T>::zero;
02441     };
02442
02443     template<typename T, size_t i>
02444     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02445     private:
02446         //  $4^{((i+1)/2)}$ 
02447         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02448         //  $4^{((i+1)/2)} - 1$ 
02449         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02450         //  $(-1)^{((i-1)/2)}$ 
02451         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02452         using dividend = typename FractionField<T>::template mul_t<
02453             altp,
02454             FractionField<T>::template mul_t<
02455                 _4p,
02456                 FractionField<T>::template mul_t<
02457                     _4pml,
02458                     bernouilli_t<T, (i + 1)>
02459                 >
02460             >
02461         >;
02462     public:
02463         using type = typename FractionField<T>::template div_t<dividend,
02464             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02465     };
02466
02467     template<typename T, size_t i>
02468     struct tan_coeff {
02469         using type = typename tan_coeff_helper<T, i>::type;
02470     };
02471
02472     template<typename T, size_t i, typename E = void>
02473     struct tanh_coeff_helper;
02474
02475     template<typename T, size_t i>
02476     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02477         using type = typename FractionField<T>::zero;
02478     };
02479
02480     template<typename T, size_t i>
02481     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02482     private:
02483         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02484         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02485         using dividend =
02486             typename FractionField<T>::template mul_t<
02487                 _4p,
02488                 typename FractionField<T>::template mul_t<
02489                     _4pml,
02490                     bernouilli_t<T, (i + 1)>
02491                 >
02492             >::type;
02493     public:
02494         using type = typename FractionField<T>::template div_t<dividend,
02495             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02496     };
02497
02498     template<typename T, size_t i>
02499     struct tanh_coeff {
02500         using type = typename tanh_coeff_helper<T, i>::type;
02501     };
02502 } // namespace internal
02503
02504     template<typename T, size_t deg>
02505     using exp = taylor<T, internal::exp_coeff, deg>;
02506
02507     template<typename T, size_t deg>
02508     using expml = typename polynomial<FractionField<T>::template sub_t<
02509         exp<T, deg>,
02510         typename polynomial<FractionField<T>::one>;
02511
02512     template<typename T, size_t deg>
02513     using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02514
02515     template<typename T, size_t deg>
02516     using atan = taylor<T, internal::atan_coeff, deg>;
02517
02518     template<typename T, size_t deg>

```

```

02534     using sin = taylor<T, internal::sin_coeff, deg>;
02535
02539     template<typename T, size_t deg>
02540     using sinh = taylor<T, internal::sh_coeff, deg>;
02541
02545     template<typename T, size_t deg>
02546     using cosh = taylor<T, internal::cosh_coeff, deg>;
02547
02551     template<typename T, size_t deg>
02552     using cos = taylor<T, internal::cos_coeff, deg>;
02553
02557     template<typename T, size_t deg>
02558     using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02559
02563     template<typename T, size_t deg>
02564     using asin = taylor<T, internal::asin_coeff, deg>;
02565
02569     template<typename T, size_t deg>
02570     using asinh = taylor<T, internal::asinh_coeff, deg>;
02571
02575     template<typename T, size_t deg>
02576     using atanh = taylor<T, internal::atanh_coeff, deg>;
02577
02581     template<typename T, size_t deg>
02582     using tan = taylor<T, internal::tan_coeff, deg>;
02583
02587     template<typename T, size_t deg>
02588     using tanh = taylor<T, internal::tanh_coeff, deg>;
02589 } // namespace aerobus
02590
02591 // continued fractions
02592 namespace aerobus {
02595     template<int64_t... values>
02596     struct ContinuedFraction {};
02597
02600     template<int64_t a0>
02601     struct ContinuedFraction<a0> {
02602         using type = typename q64::template inject_constant_t<a0>;
02603         static constexpr double val = type::template get<double>();
02604     };
02605
02609     template<int64_t a0, int64_t... rest>
02610     struct ContinuedFraction<a0, rest...> {
02611         using type = q64::template add_t<
02612             typename q64::template inject_constant_t<a0>,
02613             typename q64::template div_t<
02614                 typename q64::one,
02615                 typename ContinuedFraction<rest...>::type
02616             >;
02617         static constexpr double val = type::template get<double>();
02618     };
02619
02624     using PI_fraction =
02625     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02627     using E_fraction =
02628     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02629     using SQRT2_fraction =
02630     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02631     using SQRT3_fraction =
02632     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02633 // NOLINT
02632 } // namespace aerobus
02633
02634 // known polynomials
02635 namespace aerobus {
02636     // CChebyshev
02637     namespace internal {
02638         template<int kind, int deg>
02639         struct chebyshev_helper {
02640             using type = typename pi64::template sub_t<
02641                 typename pi64::template mul_t<
02642                     typename pi64::template mul_t<
02643                         pi64::inject_constant_t<2>,
02644                         typename pi64::X
02645                     >,
02646                     typename chebyshev_helper<kind, deg - 1>::type
02647                 >,
02648                 typename chebyshev_helper<kind, deg - 2>::type
02649             >;
02650         };
02651
02652         template<>
02653         struct chebyshev_helper<1, 0> {
02654             using type = typename pi64::one;
02655         };
02656
02657         template<>

```

```

02658     struct chebyshev_helper<1, 1> {
02659         using type = typename pi64::X;
02660     };
02661
02662     template<>
02663     struct chebyshev_helper<2, 0> {
02664         using type = typename pi64::one;
02665     };
02666
02667     template<>
02668     struct chebyshev_helper<2, 1> {
02669         using type = typename pi64::template mul_t<
02670             typename pi64::inject_constant_t<2>,
02671             typename pi64::X>;
02672     };
02673 } // namespace internal
02674
02675 // Laguerre
02676 namespace internal {
02677     template<size_t deg>
02678     struct laguerre_helper {
02679     private:
02680         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
02681         using lnm2 = typename laguerre_helper<deg - 2>::type;
02682         using lnm1 = typename laguerre_helper<deg - 1>::type;
02683         // -x + 2k-1
02684         using p = typename pq64::template val<
02685             typename q64::template inject_constant_t<-1>,
02686             typename q64::template inject_constant_t<2 * deg - 1>;
02687         // 1/n
02688         using factor = typename pq64::template inject_ring_t<
02689             q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02690
02691     public:
02692         using type = typename pq64::template mul_t <
02693             factor,
02694             typename pq64::template sub_t<
02695                 typename pq64::template mul_t<
02696                     p,
02697                     lnm1
02698                 >,
02699                 typename pq64::template mul_t<
02700                     typename pq64::template inject_constant_t<deg-1>,
02701                     lnm2
02702                 >
02703             >
02704         >;
02705     };
02706
02707     template<>
02708     struct laguerre_helper<0> {
02709         using type = typename pq64::one;
02710     };
02711
02712     template<>
02713     struct laguerre_helper<1> {
02714         using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02715     };
02716 } // namespace internal
02717
02718 namespace known_polynomials {
02719     enum hermite_kind {
02720         probabilist,
02721         physicist
02722     };
02723 }
02724
02725 namespace internal {
02726     template<size_t deg, known_polynomials::hermite_kind kind>
02727     struct hermite_helper {};
02728
02729     template<size_t deg>
02730     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02731     private:
02732         using hnm1 = typename hermite_helper<deg - 1,
02733             known_polynomials::hermite_kind::probabilist>::type;
02734         using hnm2 = typename hermite_helper<deg - 2,
02735             known_polynomials::hermite_kind::probabilist>::type;
02736
02737     public:
02738         using type = typename pi64::template sub_t<
02739             typename pi64::template mul_t<typename pi64::X, hnm1>,
02740             typename pi64::template mul_t<
02741                 typename pi64::template inject_constant_t<deg - 1>,
02742                 hnm2
02743             >

```

```

02744         >;
02745     };
02746
02747     template<size_t deg>
02748     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02749     private:
02750         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02751         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02752
02753     public:
02754         using type = typename pi64::template sub_t<
// 2X Hn-1
02755             typename pi64::template mul_t<
02756                 typename pi64::val<typename i64::template inject_constant_t<2>,
02757                     typename i64::zero>, hnm1>,
02758                 typename pi64::template mul_t<
02759                     typename pi64::template inject_constant_t<2*(deg - 1)>,
02760                     hnm2
02761                 >
02762             >;
02763     };
02764
02765     template<>
02766     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02767     using type = typename pi64::one;
02768     };
02769
02770     template<>
02771     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02772     using type = typename pi64::X;
02773     };
02774
02775     template<>
02776     struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02777     using type = typename pi64::one;
02778     };
02779
02780     template<>
02781     struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02782     // 2X
02783     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
02784         typename i64::zero>;
02785     };
02786     } // namespace internal
02787
02788     namespace known_polynomials {
02789     template <size_t deg>
02790     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02791
02792     template <size_t deg>
02793     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02794
02795     template <size_t deg>
02796     using laguerre = typename internal::laguerre_helper<deg>::type;
02797
02798     template <size_t deg>
02799     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02800
02801     template <size_t deg>
02802     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
02803     } // namespace known_polynomials
02804 } // namespace aerobus
02805
02806 #ifndef AEROBUS_CONWAY_IMPORTS
02807 template<int p, int n>
02808 struct ConwayPolynomial;
02809
02810 #define ZPZV ZPZ::template val
02811 #define POLYV aerobus::polynomial<ZPZ>::template val
02812 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>; }; // NOLINT
02813 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>, ZPZV<1>; }; // NOLINT
02814 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02815 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02816 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
02817 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
02818 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT

```

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```
        POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;  
    }; // NOLINT  
04765 #endif // AEROBUS_CONWAY_IMPORTS  
04766  
04767 #endif // __INC_AEROBUS__ // NOLINT
```

Chapter 7

Examples

7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

7.3 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.4 `PI_fraction::val`

representation of PI as a continued fraction -> 3.14...

7.5 `E_fraction::val`

approximation of e -> 2.718...

approximation of e -> 2.718...

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