

Aerobus

v1.2

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Chapter 1

Concept Index

1.1 Concepts

Here is a list of all documented concepts with brief descriptions:

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Class Index

2.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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Chapter 3

File Index

3.1 File List

Here is a list of all documented files with brief descriptions:

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Chapter 4

Concept Documentation

4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

4.1.2 Detailed Description

Concept to express R is an euclidean domain.

4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

4.2.2 Detailed Description

Concept to express R is a field.

4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

Chapter 5

Class Documentation

5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are aerobus::i64
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- using **type** = typename q64::template inject_constant_t< a0 >

Static Public Attributes

- static constexpr double **val** = type::template get<double>()

5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
-----------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- using **type** = `q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`

Static Public Attributes

- static constexpr double **val** = `type::template get<double>()`

5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<i>a0</i>	an integer (aerobus::i64)
<i>...rest</i>	integers (aerobus::i64)

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in [i32](#), again represented as types

Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
positivity (type)(v > 0)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

5.7.2 Member Data Documentation

5.7.2.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.7.2.2 pos_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<i>v</i>	
----------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

Classes

- struct `val`
values in i64

Public Types

- `using inner_type = int64_t`
type for actual values
- `template<auto x>`
`using inject_constant_t = val< static_cast< int64_t >(x)>`
- `template<typename v >`
`using inject_ring_t = v`
- `using zero = val< 0 >`
constant zero
- `using one = val< 1 >`
constant one
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulus operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (v1 > v2) - type
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strict less operator (v1 < v2)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i64, v1, v2 >`
greatest common divisor
- `template<typename v >`
`using pos_t = typename pos< v >::type`
is v positive (type)

Static Public Attributes

- `static constexpr bool is_field = false`
integers are not a field
- `static constexpr bool is_euclidean_domain = true`
integers are an euclidean domain
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator ($v1 > v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator ($v1 < v2$) - boolean value
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (boolean value)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity (boolean value)

5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

5.8.2 Member Typedef Documentation

5.8.2.1 add_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.2 div_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator

Template Parameters

<code>v1</code>	: an element of aerobus::i64::val
<code>v2</code>	: an element of aerobus::i64::val

5.8.2.3 eq_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.4 gcd_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.5 gt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ($v1 > v2$) - type

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.6 lt_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator ($v1 < v2$)

Template Parameters

<i>v1</i>	: an element of aerobus::i64::val
<i>v2</i>	: an element of aerobus::i64::val

5.8.2.7 mod_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.8 mul_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.2.9 pos_t

```
template<typename v >  
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive (type)

Template Parameters

v1	: an element of aerobus::i64::val
----	---

5.8.2.10 sub_t

```
template<typename v1 , typename v2 >  
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3 Member Data Documentation

5.8.3.1 eq_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.2 gt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (v1 > v2) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.3 lt_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (v1 < v2) - boolean value

Template Parameters

v1	: an element of aerobus::i64::val
v2	: an element of aerobus::i64::val

5.8.3.4 pos_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value)

Template Parameters

<code>v</code>	: an element of aerobus::i64::val
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.9 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< index, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.10 aerobus::polynomial< Ring >::horner_evaluation< valueRing, P >::inner< stop, stop > Struct Template Reference

Static Public Member Functions

- `static constexpr valueRing func (const valueRing &accum, const valueRing &x)`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.11 aerobus::is_prime< n > Struct Template Reference

checks if `n` is prime

```
#include <aerobus.h>
```

Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`
true iff `n` is prime

5.11.1 Detailed Description

```
template<int32_t n>
struct aerobus::is_prime< n >
```

checks if `n` is prime

Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.12 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)
values (seen as types) in polynomial ring
- struct [val< coeffN >](#)
specialization for constants

Public Types

- [using zero = val< typename Ring::zero >](#)
constant zero
- [using one = val< typename Ring::one >](#)
constant one
- [using X = val< typename Ring::one, typename Ring::zero >](#)
generator
- [template<typename P >](#)
[using simplify_t = typename simplify< P >::type](#)
simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)
- [template<typename v1 , typename v2 >](#)
[using add_t = typename add< v1, v2 >::type](#)
adds two polynomials
- [template<typename v1 , typename v2 >](#)
[using sub_t = typename sub< v1, v2 >::type](#)
subtraction of two polynomials
- [template<typename v1 , typename v2 >](#)
[using mul_t = typename mul< v1, v2 >::type](#)
multiplication of two polynomials
- [template<typename v1 , typename v2 >](#)
[using eq_t = typename eq_helper< v1, v2 >::type](#)
equality operator
- [template<typename v1 , typename v2 >](#)
[using lt_t = typename lt_helper< v1, v2 >::type](#)
strict less operator
- [template<typename v1 , typename v2 >](#)
[using gt_t = typename gt_helper< v1, v2 >::type](#)
strict greater operator

- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::q_type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`
modulo operator
- `template<typename coeff , size_t deg>`
`using monomial_t = typename monomial< coeff, deg >::type`
monomial : coeff X^{deg}
- `template<typename v >`
`using derive_t = typename derive_helper< v >::type`
derivation operator
- `template<typename v >`
`using pos_t = typename Ring::template pos_t< typename v::aN >`
checks for positivity (an > 0)
- `template<typename v1 , typename v2 >`
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`
`Ring >, v1, v2 > >::type, void >`
greatest common divisor of two polynomials
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`

5.12.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

5.12.2 Member Typedef Documentation

5.12.2.1 add_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.2 derive_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

Template Parameters

<i>v</i>	
----------	--

5.12.2.3 div_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.4 eq_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.5 gcd_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.6 gt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.7 lt_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.8 mod_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.9 monomial_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^{deg}

Template Parameters

<i>coeff</i>	
<i>deg</i>	

5.12.2.10 mul_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

5.12.2.11 pos_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

Template Parameters

<i>v</i>	
----------	--

5.12.2.12 simplify_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

Template Parameters

<i>P</i>	
----------	--

5.12.2.13 sub_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

Template Parameters

<i>v1</i>	
<i>v2</i>	

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.13 aerobus::type_list< Ts >::pop_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

Public Types

- using **type** = typename internal::pop_front_h< Ts... >::head
type that was previously head of the list
- using **tail** = typename internal::pop_front_h< Ts... >::tail
remaining types in parent list when front is removed

5.13.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.14 aerobus::Quotient< Ring, X > Struct Template Reference

Classes

- struct [val](#)

Public Types

- `using zero = val< typename Ring::zero >`
- `using one = val< typename Ring::one >`
- `template<typename v1, typename v2 >`
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`
- `template<typename v1, typename v2 >`
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`
- `template<typename v1 >`
`using pos_t = std::true_type`
- `template<auto x>`
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`
`using inject_ring_t = val< v >`

Static Public Attributes

- `template<typename v1, typename v2 >`
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
- `static constexpr bool is_euclidean_domain = true`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.15 aerobus::type_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

Public Types

- `using head = typename inner::head`
- `using tail = typename inner::tail`

5.15.1 Detailed Description

```
template<typename... Ts>  
template<size_t index>  
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

index	
-----------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.16 aerobus::type_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

Classes

- struct [pop_front](#)
removes types from head of the list
- struct [split](#)
splits list at index

Public Types

- `template<typename T >`
`using push_front = type_list< T, Ts... >`
Adds T to front of the list.
- `template<size_t index>`
`using at = internal::type_at_t< index, Ts... >`
returns type at index
- `template<typename T >`
`using push_back = type_list< Ts..., T >`
pushes T at the tail of the list
- `template<typename U >`
`using concat = typename concat_h< U >::type`
concatenates two list into one
- `template<typename T , size_t index>`
`using insert = typename internal::insert_h< index, type_list< Ts... >, T >::type`
inserts type at index
- `template<size_t index>`
`using remove = typename internal::remove_h< index, type_list< Ts... > >::type`
removes type at index

Static Public Attributes

- `static constexpr size_t length = sizeof...(Ts)`
length of list

5.16.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

Template Parameters

<i>...Ts</i>	
--------------	--

5.16.2 Member Typedef Documentation

5.16.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

Template Parameters

<i>index</i>	
--------------	--

5.16.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

Template Parameters

<i>U</i>	
----------	--

5.16.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

Template Parameters

<i>index</i>	
<i>T</i>	

5.16.2.4 push_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

Template Parameters

<i>T</i>	
----------	--

5.16.2.5 push_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

Template Parameters

<i>T</i>	
----------	--

5.16.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.17 aerobus::type_list<> Struct Reference

Public Types

- template<typename T >
using **push_front** = type_list< T >

- `template<typename T >`
using **push_back** = `type_list< T >`
- `template<typename U >`
using **concat** = `U`
- `template<typename T , size_t index>`
using **insert** = `type_list< T >`

Static Public Attributes

- `static constexpr size_t length = 0`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.18 aerobus::i32::val< x > Struct Template Reference

values in `i32`, again represented as types

```
#include <aerobus.h>
```

Public Types

- using **ring_type** = `i32`
Enclosing ring type.
- using **is_zero_t** = `std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast x into valueType
- `static std::string to_string ()`
string representation of value
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast x into valueRing

Static Public Attributes

- `static constexpr int32_t v = x`
actual value stored in val type

5.18.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.18.2 Member Function Documentation

5.18.2.1 `eval()`

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

5.18.2.2 `get()`

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

Template Parameters

<code>valueType</code>	double for example
------------------------	--------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.19 `aerobus::i64::val< x >` Struct Template Reference

values in `i64`

```
#include <aerobus.h>
```

Public Types

- `using ring_type = i64`
enclosing ring type
- `using is_zero_t = std::bool_constant< x==0 >`
is value zero

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
cast value in valueType
- `static std::string to_string ()`
string representation
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`
cast value in valueRing

Static Public Attributes

- `static constexpr int64_t v = x`
actual value

5.19.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

5.19.2 Member Function Documentation

5.19.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

Template Parameters

<code>valueRing</code>	(double for example)
------------------------	----------------------

5.19.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.20 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
heavy weight coefficient (non zero)
- `using strip = val< coeffs... >`
remove largest coefficient
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`
true_type if polynomial is constant zero
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`
type of coefficient at index

Static Public Member Functions

- `static std::string to_string ()`
get a string representation of polynomial
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`
evaluates polynomial seen as a function operating on ValueRing

Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`
degree of the polynomial
- `static constexpr bool is_zero_v = is_zero_t::value`
true if polynomial is constant zero

5.20.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

5.20.2 Member Typedef Documentation

5.20.2.1 coeff_at_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

<i>index</i>	
--------------	--

5.20.3 Member Function Documentation

5.20.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

Parameters

<i>x</i>	value
----------	-------

Returns

$P(x)$

5.20.3.2 to_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

Returns

something like $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.21 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

Public Types

- `using type = std::conditional_t< Ring::template pos_v< tmp >, tmp, typename Ring::template sub_t< typename Ring::zero, tmp > >`

The documentation for this struct was generated from the following file:

- src/aerobus.h

5.22 aerobus::zpz< p >::val< x > Struct Template Reference

Public Types

- `using ring_type = zpz< p >`
enclosing ring type
- `using is_zero_t = std::bool_constant< x% p==0 >`

Static Public Member Functions

- `template<typename valueType >`
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &v)`

Static Public Attributes

- `static constexpr int32_t v = x % p`
actual value

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.23 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

Classes

- struct `coeff_at`
- struct `coeff_at< index, std::enable_if_t<(index< 0||index > 0)>> >`
- struct `coeff_at< index, std::enable_if_t<(index==0)>> >`

Public Types

- `using ring_type = polynomial< Ring >`
enclosing ring type
- `using aN = coeffN`
- `using strip = val< coeffN >`
- `using is_zero_t = std::bool_constant< aN::is_zero_t::value >`
- `template<size_t index>`
`using coeff_at_t = typename coeff_at< index >::type`

Static Public Member Functions

- `static std::string to_string ()`
- `template<typename valueRing >`
`static constexpr valueRing eval (const valueRing &x)`

Static Public Attributes

- `static constexpr size_t degree = 0`
degree
- `static constexpr bool is_zero_v = is_zero_t::value`

5.23.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<code>coeffN</code>	
---------------------	--

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

5.24 `aerobus::zpz< p >` Struct Template Reference

```
#include <aerobus.h>
```

Classes

- struct [val](#)

Public Types

- `using inner_type = int32_t`
- `template<auto x>`
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `using zero = val< 0 >`
- `using one = val< 1 >`
- `template<typename v1 , typename v2 >`
`using add_t = typename add< v1, v2 >::type`
addition operator
- `template<typename v1 , typename v2 >`
`using sub_t = typename sub< v1, v2 >::type`
subtraction operator
- `template<typename v1 , typename v2 >`
`using mul_t = typename mul< v1, v2 >::type`
multiplication operator
- `template<typename v1 , typename v2 >`
`using div_t = typename div< v1, v2 >::type`
division operator
- `template<typename v1 , typename v2 >`
`using mod_t = typename remainder< v1, v2 >::type`
modulo operator
- `template<typename v1 , typename v2 >`
`using gt_t = typename gt< v1, v2 >::type`
strictly greater operator (type)
- `template<typename v1 , typename v2 >`
`using lt_t = typename lt< v1, v2 >::type`
strictly smaller operator (type)
- `template<typename v1 , typename v2 >`
`using eq_t = typename eq< v1, v2 >::type`
equality operator (type)
- `template<typename v1 , typename v2 >`
`using gcd_t = gcd_t< i32, v1, v2 >`
greatest common divisor
- `template<typename v1 >`
`using pos_t = typename pos< v1 >::type`
positivity operator (type)

Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`
`static constexpr bool gt_v = gt_t<v1, v2>::value`
strictly greater operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool lt_v = lt_t<v1, v2>::value`
strictly smaller operator (booleanvalue)
- `template<typename v1 , typename v2 >`
`static constexpr bool eq_v = eq_t<v1, v2>::value`
equality operator (booleanvalue)
- `template<typename v >`
`static constexpr bool pos_v = pos_t<v>::value`
positivity operator (boolean value)

5.24.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

Chapter 6

File Documentation

6.1 aerobus.h

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00025 // aligned allocation
00026 namespace aerobus {
00027     template<typename T>
00028     T* aligned_malloc(size_t count, size_t alignment) {
00029         #ifdef _MSC_VER
00030             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00031         #else
00032             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00033         #endif
00034     }
00035
00036     template<typename T, size_t N>
00037     constexpr bool contains(const std::array<T, N>& arr, const T& v) {
00038         for (const auto& vv : arr) {
00039             if (v == vv) {
00040                 return true;
00041             }
00042         }
00043         return false;
00044     }
00045 } // namespace aerobus
00046
00047 // concepts
00048 namespace aerobus {
00049     template <typename R>
00050     concept IsRing = requires {
00051         typename R::one;
00052         typename R::zero;
00053         typename R::template add_t<typename R::one, typename R::one>;
00054         typename R::template sub_t<typename R::one, typename R::one>;
00055         typename R::template mul_t<typename R::one, typename R::one>;
00056     };
00057 }
```

```

00074
00075
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00089     template<typename R>
00090     concept IsField = IsEuclideanDomain<R> && requires {
00091         R::is_field == true;
00092     };
00093 } // namespace aerobus
00094
00095 // utilities
00096 namespace aerobus {
00097     namespace internal {
00098         template<typename...> typename TT, typename T>
00099         struct is_instantiation_of : std::false_type { };
00100
00101         template<template<typename...> typename TT, typename... Ts>
00102         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00103
00104         template<template<typename...> typename TT, typename T>
00105         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00106
00107         template <int64_t i, typename T, typename... Ts>
00108         struct type_at {
00109             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00110             using type = typename type_at<i - 1, Ts...>::type;
00111         };
00112
00113         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00114             using type = T;
00115         };
00116
00117         template <size_t i, typename... Ts>
00118         using type_at_t = typename type_at<i, Ts...>::type;
00119
00120         template<int32_t n, int32_t i, typename E = void>
00121         struct _is_prime {};
00122
00123         template<int32_t i>
00124         struct _is_prime<1, i> {
00125             static constexpr bool value = false;
00126         };
00127
00128         template<int32_t i>
00129         struct _is_prime<2, i> {
00130             static constexpr bool value = true;
00131         };
00132
00133         template<int32_t i>
00134         struct _is_prime<3, i> {
00135             static constexpr bool value = true;
00136         };
00137
00138         template<int32_t i>
00139         struct _is_prime<5, i> {
00140             static constexpr bool value = true;
00141         };
00142
00143         template<int32_t i>
00144         struct _is_prime<7, i> {
00145             static constexpr bool value = true;
00146         };
00147
00148         template<int32_t n, int32_t i>
00149         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00150             static constexpr bool value = false;
00151         };
00152
00153         template<int32_t n, int32_t i>
00154         struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00155             static constexpr bool value = false;
00156         };
00157
00158         template<int32_t n, int32_t i>
00159         struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00160             static constexpr bool value = true;
00161         };
00162

```



```

00163     };
00164
00165     template<int32_t n, int32_t i>
00166     struct _is_prime<n, i, std::enable_if_t<(
00167         n % i == 0 &&
00168         n >= 9 &&
00169         n % 3 != 0 &&
00170         n % 2 != 0 &&
00171         i * i > n)> {
00172         static constexpr bool value = true;
00173     };
00174
00175     template<int32_t n, int32_t i>
00176     struct _is_prime<n, i, std::enable_if_t<(
00177         n % (i+2) == 0 &&
00178         n >= 9 &&
00179         n % 3 != 0 &&
00180         n % 2 != 0 &&
00181         i * i <= n)> {
00182         static constexpr bool value = true;
00183     };
00184
00185     template<int32_t n, int32_t i>
00186     struct _is_prime<n, i, std::enable_if_t<(
00187         n % (i+2) != 0 &&
00188         n % i != 0 &&
00189         n >= 9 &&
00190         n % 3 != 0 &&
00191         n % 2 != 0 &&
00192         (i * i <= n))> {
00193         static constexpr bool value = _is_prime<n, i+6>::value;
00194     };
00195
00196     } // namespace internal
00197
00200     template<int32_t n>
00201     struct is_prime {
00202         static constexpr bool value = internal::_is_prime<n, 5>::value;
00203     };
00204
00205     template<int32_t n>
00206     static constexpr bool is_prime_v = is_prime<n>::value;
00207
00208     namespace internal {
00209         template <std::size_t... Is>
00210         constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00211             -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00212
00213         template <std::size_t N>
00214         using make_index_sequence_reverse
00215             = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00216
00217         template<typename Ring, typename E = void>
00218         struct gcd;
00219
00220         template<typename Ring>
00221         struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00222             template<typename A, typename B, typename E = void>
00223             struct gcd_helper {};
00224
00225             // B = 0, A > 0
00226             template<typename A, typename B>
00227             struct gcd_helper<A, B, std::enable_if_t<
00228                 (B::is_zero_t::value) &&
00229                 (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00230                 using type = A;
00231             };
00232
00233             // B = 0, A < 0
00234             template<typename A, typename B>
00235             struct gcd_helper<A, B, std::enable_if_t<
00236                 (B::is_zero_t::value) &&
00237                 !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00238                 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00239             };
00240
00241             // B != 0
00242             template<typename A, typename B>
00243             struct gcd_helper<A, B, std::enable_if_t<
00244                 (!B::is_zero_t::value)
00245                 >> {
00246             private: // NOLINT
00247                 // A / B
00248                 using k = typename Ring::template div_t<A, B>;
00249                 // A - (A/B)*B = A % B
00250                 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00251             };
00252         };
00253     }

```

```

00258         public:
00259             using type = typename gcd_helper<B, m>::type;
00260         };
00261
00262         template<typename A, typename B>
00263         using type = typename gcd_helper<A, B>::type;
00264     };
00265 } // namespace internal
00266
00267 template<typename T, typename A, typename B>
00270 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00271
00272 template<typename val>
00273 requires IsEuclideanDomain<typename val::ring_type>
00274 using abs_t = std::conditional_t<
00275     val::ring_type::template pos_v<val>,
00276     val, typename val::ring_type::template sub_t<typename val::ring_type::zero, val>;
00277 } // namespace aerobus
00278
00279 // quotient ring by the principal ideal generated by X
00280 namespace aerobus {
00281     template<typename Ring, typename X>
00282     requires IsRing<Ring>
00283     struct Quotient {
00284         template <typename V>
00285         struct val {
00286             private: // NOLINT
00287                 using tmp = typename Ring::template mod_t<V, X>;
00288
00289             public:
00290                 using type = std::conditional_t<
00291                     Ring::template pos_v<tmp>,
00292                     tmp,
00293                     typename Ring::template sub_t<typename Ring::zero, tmp>
00294                 >;
00295             };
00296
00297             using zero = val<typename Ring::zero>;
00298             using one = val<typename Ring::one>;
00299
00300             template<typename v1, typename v2>
00301             using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00302             template<typename v1, typename v2>
00303             using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00304             template<typename v1, typename v2>
00305             using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00306             template<typename v1, typename v2>
00307             using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00308             template<typename v1, typename v2>
00309             using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00310             template<typename v1, typename v2>
00311             static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00312             template<typename v1>
00313             using pos_t = std::true_type;
00314
00315             template<typename v>
00316             static constexpr bool pos_v = pos_t<v>::value;
00317
00318             static constexpr bool is_euclidean_domain = true;
00319
00320             template<auto x>
00321             using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00322
00323             template<typename v>
00324             using inject_ring_t = val<v>;
00325         };
00326     } // namespace aerobus
00327
00328 // type_list
00329 namespace aerobus {
00330     template <typename... Ts>
00331     struct type_list;
00332
00333     namespace internal {
00334         template <typename T, typename... Us>
00335         struct pop_front_h {
00336             using tail = type_list<Us...>;
00337             using head = T;
00338         };
00339     };
00340
00341     template <size_t index, typename L1, typename L2>
00342     struct split_h {
00343     private:
00344         static_assert(index <= L2::length, "index out of bounds");
00345         using a = typename L2::pop_front::type;
00346         using b = typename L2::pop_front::tail;
00347         using c = typename L1::template push_back<a>;

```

```

00348
00349     public:
00350         using head = typename split_h<index - 1, c, b>::head;
00351         using tail = typename split_h<index - 1, c, b>::tail;
00352     };
00353
00354     template <typename L1, typename L2>
00355     struct split_h<0, L1, L2> {
00356         using head = L1;
00357         using tail = L2;
00358     };
00359
00360     template <size_t index, typename L, typename T>
00361     struct insert_h {
00362         static_assert(index <= L::length, "index ouf of bounds");
00363         using s = typename L::template split<index>;
00364         using left = typename s::head;
00365         using right = typename s::tail;
00366         using ll = typename left::template push_back<T>;
00367         using type = typename ll::template concat<right>;
00368     };
00369
00370     template <size_t index, typename L>
00371     struct remove_h {
00372         using s = typename L::template split<index>;
00373         using left = typename s::head;
00374         using right = typename s::tail;
00375         using rr = typename right::pop_front::tail;
00376         using type = typename left::template concat<rr>;
00377     };
00378 } // namespace internal
00379
00380
00383     template <typename... Ts>
00384     struct type_list {
00385     private:
00386         template <typename T>
00387         struct concat_h;
00388
00389         template <typename... Us>
00390         struct concat_h<type_list<Us...> {
00391             using type = type_list<Ts..., Us...>;
00392         };
00393
00394     public:
00395         static constexpr size_t length = sizeof...(Ts);
00396
00397         template <typename T>
00400         using push_front = type_list<T, Ts...>;
00401
00402         template <size_t index>
00403         using at = internal::type_at_t<index, Ts...>;
00404
00405         struct pop_front {
00406             using type = typename internal::pop_front_h<Ts...>::head;
00407             using tail = typename internal::pop_front_h<Ts...>::tail;
00408         };
00409
00410         template <typename T>
00411         using push_back = type_list<Ts..., T>;
00412
00413         template <typename U>
00414         using concat = typename concat_h<U>::type;
00415
00416         template <size_t index>
00417         struct split {
00418     private:
00419             using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00420
00421     public:
00422             using head = typename inner::head;
00423             using tail = typename inner::tail;
00424         };
00425
00426         template <typename T, size_t index>
00427         using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00428
00429         template <size_t index>
00430         using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00431     };
00432
00433     template <>
00434     struct type_list<> {
00435         static constexpr size_t length = 0;
00436
00437         template <typename T>
00438         using push_front = type_list<T>;

```

```

00456
00457     template <typename T>
00458     using push_back = type_list<T>;
00459
00460     template <typename U>
00461     using concat = U;
00462
00463     // TODO(jewave): assert index == 0
00464     template <typename T, size_t index>
00465     using insert = type_list<T>;
00466 };
00467 } // namespace aerobus
00468
00469 // i32
00470 namespace aerobus {
00471     struct i32 {
00472         using inner_type = int32_t;
00473         template<int32_t x>
00474         struct val {
00475             using ring_type = i32;
00476             static constexpr int32_t v = x;
00477
00478             template<typename valueType>
00479             static constexpr valueType get() { return static_cast<valueType>(x); }
00480
00481             using is_zero_t = std::bool_constant<x == 0>;
00482
00483             static std::string to_string() {
00484                 return std::to_string(x);
00485             }
00486
00487             template<typename valueRing>
00488             static constexpr valueRing eval(const valueRing& v) {
00489                 return static_cast<valueRing>(x);
00490             }
00491         };
00492     };
00493
00494     using zero = val<0>;
00495     using one = val<1>;
00496     static constexpr bool is_field = false;
00497     static constexpr bool is_euclidean_domain = true;
00498     template<auto x>
00499     using inject_constant_t = val<static_cast<int32_t>(x)>;
00500
00501     template<typename v>
00502     using inject_ring_t = v;
00503
00504 private:
00505     template<typename v1, typename v2>
00506     struct add {
00507         using type = val<v1::v + v2::v>;
00508     };
00509
00510     template<typename v1, typename v2>
00511     struct sub {
00512         using type = val<v1::v - v2::v>;
00513     };
00514
00515     template<typename v1, typename v2>
00516     struct mul {
00517         using type = val<v1::v * v2::v>;
00518     };
00519
00520     template<typename v1, typename v2>
00521     struct div {
00522         using type = val<v1::v / v2::v>;
00523     };
00524
00525     template<typename v1, typename v2>
00526     struct remainder {
00527         using type = val<v1::v % v2::v>;
00528     };
00529
00530     template<typename v1, typename v2>
00531     struct gt {
00532         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00533     };
00534
00535     template<typename v1, typename v2>
00536     struct lt {
00537         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00538     };
00539
00540     template<typename v1, typename v2>
00541     struct eq {
00542         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00543     };
00544 }

```

```

00561
00562     template<typename v1>
00563     struct pos {
00564         using type = std::bool_constant<(v1::v > 0)>;
00565     };
00566
00567     public:
00568         template<typename v1, typename v2>
00569         using add_t = typename add<v1, v2>::type;
00570
00571         template<typename v1, typename v2>
00572         using sub_t = typename sub<v1, v2>::type;
00573
00574         template<typename v1, typename v2>
00575         using mul_t = typename mul<v1, v2>::type;
00576
00577         template<typename v1, typename v2>
00578         using div_t = typename div<v1, v2>::type;
00579
00580         template<typename v1, typename v2>
00581         using mod_t = typename remainder<v1, v2>::type;
00582
00583         template<typename v1, typename v2>
00584         using gt_t = typename gt<v1, v2>::type;
00585
00586         template<typename v1, typename v2>
00587         using lt_t = typename lt<v1, v2>::type;
00588
00589         template<typename v1, typename v2>
00590         using eq_t = typename eq<v1, v2>::type;
00591
00592         template<typename v1, typename v2>
00593         static constexpr bool eq_v = eq_t<v1, v2>::value;
00594
00595         template<typename v1, typename v2>
00596         using gcd_t = gcd_t<i32, v1, v2>;
00597
00598         template<typename v>
00599         using pos_t = typename pos<v>::type;
00600
00601         template<typename v>
00602         static constexpr bool pos_v = pos_t<v>::value;
00603     };
00604 } // namespace aerobus
00605
00606 // i64
00607 namespace aerobus {
00608     struct i64 {
00609         using inner_type = int64_t;
00610         template<int64_t x>
00611         struct val {
00612             using ring_type = i64;
00613             static constexpr int64_t v = x;
00614
00615             template<typename valueType>
00616             static constexpr valueType get() { return static_cast<valueType>(x); }
00617
00618             using is_zero_t = std::bool_constant<x == 0>;
00619
00620             static std::string to_string() {
00621                 return std::to_string(x);
00622             }
00623
00624             template<typename valueRing>
00625             static constexpr valueRing eval(const valueRing& v) {
00626                 return static_cast<valueRing>(x);
00627             }
00628         };
00629
00630         template<auto x>
00631         using inject_constant_t = val<static_cast<int64_t>(x)>;
00632
00633         template<typename v>
00634         using inject_ring_t = v;
00635
00636         using zero = val<0>;
00637         using one = val<1>;
00638         static constexpr bool is_field = false;
00639         static constexpr bool is_euclidean_domain = true;
00640
00641     private:
00642         template<typename v1, typename v2>
00643         struct add {
00644             using type = val<v1::v + v2::v>;
00645         };
00646
00647         template<typename v1, typename v2>

```

```

00682     struct sub {
00683         using type = val<v1::v - v2::v>;
00684     };
00685
00686     template<typename v1, typename v2>
00687     struct mul {
00688         using type = val<v1::v* v2::v>;
00689     };
00690
00691     template<typename v1, typename v2>
00692     struct div {
00693         using type = val<v1::v / v2::v>;
00694     };
00695
00696     template<typename v1, typename v2>
00697     struct remainder {
00698         using type = val<v1::v% v2::v>;
00699     };
00700
00701     template<typename v1, typename v2>
00702     struct gt {
00703         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00704     };
00705
00706     template<typename v1, typename v2>
00707     struct lt {
00708         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00709     };
00710
00711     template<typename v1, typename v2>
00712     struct eq {
00713         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00714     };
00715
00716     template<typename v>
00717     struct pos {
00718         using type = std::bool_constant<(v::v > 0)>;
00719     };
00720
00721 public:
00722     template<typename v1, typename v2>
00723     using add_t = typename add<v1, v2>::type;
00724
00725     template<typename v1, typename v2>
00726     using sub_t = typename sub<v1, v2>::type;
00727
00728     template<typename v1, typename v2>
00729     using mul_t = typename mul<v1, v2>::type;
00730
00731     template<typename v1, typename v2>
00732     using div_t = typename div<v1, v2>::type;
00733
00734     template<typename v1, typename v2>
00735     using mod_t = typename remainder<v1, v2>::type;
00736
00737     template<typename v1, typename v2>
00738     using gt_t = typename gt<v1, v2>::type;
00739
00740     template<typename v1, typename v2>
00741     static constexpr bool gt_v = gt_t<v1, v2>::value;
00742
00743     template<typename v1, typename v2>
00744     using lt_t = typename lt<v1, v2>::type;
00745
00746     template<typename v1, typename v2>
00747     static constexpr bool lt_v = lt_t<v1, v2>::value;
00748
00749     template<typename v1, typename v2>
00750     using eq_t = typename eq<v1, v2>::type;
00751
00752     template<typename v1, typename v2>
00753     static constexpr bool eq_v = eq_t<v1, v2>::value;
00754
00755     template<typename v1, typename v2>
00756     using gcd_t = gcd_t<i64, v1, v2>;
00757
00758     template<typename v>
00759     using pos_t = typename pos<v>::type;
00760
00761     template<typename v>
00762     static constexpr bool pos_v = pos_t<v>::value;
00763 };
00764 } // namespace aerobus
00765
00766 // z/pz
00767 namespace aerobus {
00768     template<int32_t p>

```

```

00813     struct zpz {
00814         using inner_type = int32_t;
00815         template<int32_t x>
00816         struct val {
00817             using ring_type = zpz<p>;
00818             static constexpr int32_t v = x % p;
00819
00820             template<typename valueType>
00821             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00822
00823             using is_zero_t = std::bool_constant<x % p == 0>;
00824             static std::string to_string() {
00825                 return std::to_string(x % p);
00826             }
00827
00828             template<typename valueRing>
00829             static constexpr valueRing eval(const valueRing& v) {
00830                 return static_cast<valueRing>(x % p);
00831             }
00832         };
00833
00834     template<auto x>
00835     using inject_constant_t = val<static_cast<int32_t>(x)>;
00836
00837     using zero = val<0>;
00838     using one = val<1>;
00839     static constexpr bool is_field = is_prime<p>::value;
00840     static constexpr bool is_euclidean_domain = true;
00841
00842 private:
00843     template<typename v1, typename v2>
00844     struct add {
00845         using type = val<(v1::v + v2::v) % p>;
00846     };
00847
00848     template<typename v1, typename v2>
00849     struct sub {
00850         using type = val<(v1::v - v2::v) % p>;
00851     };
00852
00853     template<typename v1, typename v2>
00854     struct mul {
00855         using type = val<(v1::v * v2::v) % p>;
00856     };
00857
00858     template<typename v1, typename v2>
00859     struct div {
00860         using type = val<(v1::v % p) / (v2::v % p)>;
00861     };
00862
00863     template<typename v1, typename v2>
00864     struct remainder {
00865         using type = val<(v1::v % v2::v) % p>;
00866     };
00867
00868     template<typename v1, typename v2>
00869     struct gt {
00870         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00871     };
00872
00873     template<typename v1, typename v2>
00874     struct lt {
00875         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00876     };
00877
00878     template<typename v1, typename v2>
00879     struct eq {
00880         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00881     };
00882
00883     template<typename v1>
00884     struct pos {
00885         using type = std::bool_constant<(v1::v > 0)>;
00886     };
00887
00888 public:
00889     template<typename v1, typename v2>
00890     using add_t = typename add<v1, v2>::type;
00891
00892     template<typename v1, typename v2>
00893     using sub_t = typename sub<v1, v2>::type;
00894
00895     template<typename v1, typename v2>
00896     using mul_t = typename mul<v1, v2>::type;
00897
00898     template<typename v1, typename v2>
00899     using div_t = typename div<v1, v2>::type;
00900
00901     template<typename v1, typename v2>
00902     using remainder_t = typename remainder<v1, v2>::type;
00903
00904     template<typename v1, typename v2>
00905     using gt_t = typename gt<v1, v2>::type;
00906
00907     template<typename v1, typename v2>
00908     using lt_t = typename lt<v1, v2>::type;
00909
00910     template<typename v1, typename v2>
00911     using eq_t = typename eq<v1, v2>::type;
00912
00913     template<typename v1>
00914     using pos_t = typename pos<v1>::type;

```

```

00906
00908     template<typename v1, typename v2>
00909     using mod_t = typename remainder<v1, v2>::type;
00910
00912     template<typename v1, typename v2>
00913     using gt_t = typename gt<v1, v2>::type;
00914
00916     template<typename v1, typename v2>
00917     static constexpr bool gt_v = gt_t<v1, v2>::value;
00918
00920     template<typename v1, typename v2>
00921     using lt_t = typename lt<v1, v2>::type;
00922
00924     template<typename v1, typename v2>
00925     static constexpr bool lt_v = lt_t<v1, v2>::value;
00926
00928     template<typename v1, typename v2>
00929     using eq_t = typename eq<v1, v2>::type;
00930
00932     template<typename v1, typename v2>
00933     static constexpr bool eq_v = eq_t<v1, v2>::value;
00934
00936     template<typename v1, typename v2>
00937     using gcd_t = gcd_t<i32, v1, v2>;
00938
00940     template<typename v1>
00941     using pos_t = typename pos<v1>::type;
00942
00944     template<typename v>
00945     static constexpr bool pos_v = pos_t<v>::value;
00946 };
00947 } // namespace aerobus
00948
00949 // polynomial
00950 namespace aerobus {
00951     // coeffN x^N + ...
00952     template<typename Ring>
00953     requires IsEuclideanDomain<Ring>
00954     struct polynomial {
00955         static constexpr bool is_field = false;
00956         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
00957
00958         template<typename coeffN, typename... coeffs>
00959         struct val {
00960             using ring_type = polynomial<Ring>;
00961             static constexpr size_t degree = sizeof...(coeffs);
00962             using aN = coeffN;
00963             using strip = val<coeffs...>;
00964             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
00965             static constexpr bool is_zero_v = is_zero_t::value;
00966
00967         private:
00968             template<size_t index, typename E = void>
00969             struct coeff_at {};
00970
00971             template<size_t index>
00972             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
00973                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
00974             };
00975
00976             template<size_t index>
00977             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
00978                 using type = typename Ring::zero;
00979             };
00980
00981         public:
00982             template<size_t index>
00983             using coeff_at_t = typename coeff_at<index>::type;
00984
00985             static std::string to_string() {
00986                 return string_helper<coeffN, coeffs...>::func();
00987             }
00988
00989             template<typename valueRing>
00990             static constexpr valueRing eval(const valueRing& x) {
00991                 return horner_evaluation<valueRing, val>
00992                     ::template inner<0, degree + 1>
00993                     ::func(static_cast<valueRing>(0), x);
00994             }
00995         };
00996     };
00997
00998     template<typename coeffN>
00999     struct val<coeffN> {
01000         using ring_type = polynomial<Ring>;
01001         static constexpr size_t degree = 0;
01002         using aN = coeffN;
01003         using strip = val<coeffN>;
01004     };

```



```

01028         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01029
01030         static constexpr bool is_zero_v = is_zero_t::value;
01031
01032         template<size_t index, typename E = void>
01033         struct coeff_at {};
01034
01035         template<size_t index>
01036         struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01037             using type = aN;
01038         };
01039
01040         template<size_t index>
01041         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01042             using type = typename Ring::zero;
01043         };
01044
01045         template<size_t index>
01046         using coeff_at_t = typename coeff_at<index>::type;
01047
01048         static std::string to_string() {
01049             return string_helper<coeffN>::func();
01050         }
01051
01052         template<typename valueRing>
01053         static constexpr valueRing eval(const valueRing& x) {
01054             return static_cast<valueRing>(aN::template get<valueRing>());
01055         }
01056     };
01057
01059     using zero = val<typename Ring::zero>;
01061     using one = val<typename Ring::one>;
01063     using X = val<typename Ring::one, typename Ring::zero>;
01064
01065 private:
01066     template<typename P, typename E = void>
01067     struct simplify;
01068
01069     template<typename P1, typename P2, typename I>
01070     struct add_low;
01071
01072     template<typename P1, typename P2>
01073     struct add {
01074         using type = typename simplify<typename add_low<
01075             P1,
01076             P2,
01077             internal::make_index_sequence_reverse<
01078                 std::max(P1::degree, P2::degree) + 1
01079             >::type>::type;
01080     };
01081
01082     template<typename P1, typename P2, typename I>
01083     struct sub_low;
01084
01085     template<typename P1, typename P2, typename I>
01086     struct mul_low;
01087
01088     template<typename v1, typename v2>
01089     struct mul {
01090         using type = typename mul_low<
01091             v1,
01092             v2,
01093             internal::make_index_sequence_reverse<
01094                 v1::degree + v2::degree + 1
01095             >::type;
01096     };
01097
01098     template<typename coeff, size_t deg>
01099     struct monomial;
01100
01101     template<typename v, typename E = void>
01102     struct derive_helper {};
01103
01104     template<typename v>
01105     struct derive_helper<v, std::enable_if_t<v::degree == 0>> {
01106         using type = zero;
01107     };
01108
01109     template<typename v>
01110     struct derive_helper<v, std::enable_if_t<v::degree != 0>> {
01111         using type = typename add<
01112             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01113             typename monomial<
01114                 typename Ring::template mul_t<
01115                     typename v::aN,
01116                     typename Ring::template inject_constant_t<(v::degree)>
01117                 >,

```

```

01118         v::degree - 1
01119         >::type
01120     >::type;
01121 };
01122
01123 template<typename v1, typename v2, typename E = void>
01124 struct eq_helper {};
01125
01126 template<typename v1, typename v2>
01127 struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01128     using type = std::false_type;
01129 };
01130
01131
01132 template<typename v1, typename v2>
01133 struct eq_helper<v1, v2, std::enable_if_t<
01134     v1::degree == v2::degree &&
01135     (v1::degree != 0 || v2::degree != 0) &&
01136     std::is_same<
01137         typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01138         std::false_type
01139     >::value
01140 > {
01141     > {
01142         using type = std::false_type;
01143     };
01144
01145     template<typename v1, typename v2>
01146     struct eq_helper<v1, v2, std::enable_if_t<
01147         v1::degree == v2::degree &&
01148         (v1::degree != 0 || v2::degree != 0) &&
01149         std::is_same<
01150             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01151             std::true_type
01152         >::value
01153     > {
01154         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01155     };
01156
01157     template<typename v1, typename v2>
01158     struct eq_helper<v1, v2, std::enable_if_t<
01159         v1::degree == v2::degree &&
01160         (v1::degree == 0)
01161     > {
01162         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01163     };
01164
01165     template<typename v1, typename v2, typename E = void>
01166     struct lt_helper {};
01167
01168     template<typename v1, typename v2>
01169     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01170         using type = std::true_type;
01171     };
01172
01173     template<typename v1, typename v2>
01174     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01175         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01176     };
01177
01178     template<typename v1, typename v2>
01179     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01180         using type = std::false_type;
01181     };
01182
01183     template<typename v1, typename v2, typename E = void>
01184     struct gt_helper {};
01185
01186     template<typename v1, typename v2>
01187     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01188         using type = std::true_type;
01189     };
01190
01191     template<typename v1, typename v2>
01192     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01193         using type = std::false_type;
01194     };
01195
01196     template<typename v1, typename v2>
01197     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01198         using type = std::false_type;
01199     };
01200
01201 // when high power is zero : strip
01202 template<typename P>
01203 struct simplify<P, std::enable_if_t<
01204     std::is_same<

```

```

01205         typename Ring::zero,
01206         typename P::aN
01207         >::value && (P::degree > 0)
01208     » {
01209         using type = typename simplify<typename P::strip>::type;
01210     };
01211
01212     // otherwise : do nothing
01213     template<typename P>
01214     struct simplify<P, std::enable_if_t<
01215         !std::is_same<
01216             typename Ring::zero,
01217             typename P::aN
01218             >::value && (P::degree > 0)
01219     » {
01220         using type = P;
01221     };
01222
01223     // do not simplify constants
01224     template<typename P>
01225     struct simplify<P, std::enable_if_t<P::degree == 0» {
01226         using type = P;
01227     };
01228
01229     // addition at
01230     template<typename P1, typename P2, size_t index>
01231     struct add_at {
01232         using type =
01233             typename Ring::template add_t<
01234                 typename P1::template coeff_at_t<index>,
01235                 typename P2::template coeff_at_t<index>>;
01236     };
01237
01238     template<typename P1, typename P2, size_t index>
01239     using add_at_t = typename add_at<P1, P2, index>::type;
01240
01241     template<typename P1, typename P2, std::size_t... I>
01242     struct add_low<P1, P2, std::index_sequence<I...» {
01243         using type = val<add_at_t<P1, P2, I>...>;
01244     };
01245
01246     // subtraction at
01247     template<typename P1, typename P2, size_t index>
01248     struct sub_at {
01249         using type =
01250             typename Ring::template sub_t<
01251                 typename P1::template coeff_at_t<index>,
01252                 typename P2::template coeff_at_t<index>>;
01253     };
01254
01255     template<typename P1, typename P2, size_t index>
01256     using sub_at_t = typename sub_at<P1, P2, index>::type;
01257
01258     template<typename P1, typename P2, std::size_t... I>
01259     struct sub_low<P1, P2, std::index_sequence<I...» {
01260         using type = val<sub_at_t<P1, P2, I>...>;
01261     };
01262
01263     template<typename P1, typename P2>
01264     struct sub {
01265         using type = typename simplify<typename sub_low<
01266             P1,
01267             P2,
01268             internal::make_index_sequence_reverse<
01269                 std::max(P1::degree, P2::degree) + 1
01270             >::type>::type;
01271     };
01272
01273     // multiplication at
01274     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01275     struct mul_at_loop_helper {
01276         using type = typename Ring::template add_t<
01277             typename Ring::template mul_t<
01278                 typename v1::template coeff_at_t<index>,
01279                 typename v2::template coeff_at_t<k - index>
01280             >,
01281             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01282         >;
01283     };
01284
01285     template<typename v1, typename v2, size_t k, size_t stop>
01286     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01287         using type = typename Ring::template mul_t<
01288             typename v1::template coeff_at_t<stop>,
01289             typename v2::template coeff_at_t<0>>;
01290     };
01291

```

```

01292     template <typename v1, typename v2, size_t k, typename E = void>
01293     struct mul_at {};
01294
01295     template<typename v1, typename v2, size_t k>
01296     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01297         using type = typename Ring::zero;
01298     };
01299
01300     template<typename v1, typename v2, size_t k>
01301     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01302         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01303     };
01304
01305     template<typename P1, typename P2, size_t index>
01306     using mul_at_t = typename mul_at<P1, P2, index>::type;
01307
01308     template<typename P1, typename P2, std::size_t... I>
01309     struct mul_low<P1, P2, std::index_sequence<I...> {
01310         using type = val<mul_at_t<P1, P2, I>...>;
01311     };
01312
01313     // division helper
01314     template< typename A, typename B, typename Q, typename R, typename E = void>
01315     struct div_helper {};
01316
01317     template<typename A, typename B, typename Q, typename R>
01318     struct div_helper<A, B, Q, R, std::enable_if_t<
01319         (R::degree < B::degree) ||
01320         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01321         using q_type = Q;
01322         using mod_type = R;
01323         using gcd_type = B;
01324     };
01325
01326     template<typename A, typename B, typename Q, typename R>
01327     struct div_helper<A, B, Q, R, std::enable_if_t<
01328         (R::degree >= B::degree) &&
01329         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01330     private: // NOLINT
01331         using rN = typename R::aN;
01332         using bN = typename B::aN;
01333         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01334         B::degree>::type;
01335         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01336         using qq = typename add<Q, pT>::type;
01337     public:
01338         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01339         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01340         using gcd_type = rr;
01341     };
01342
01343     template<typename A, typename B>
01344     struct div {
01345         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01346         using q_type = typename div_helper<A, B, zero, A>::q_type;
01347         using m_type = typename div_helper<A, B, zero, A>::mod_type;
01348     };
01349
01350     template<typename P>
01351     struct make_unit {
01352         using type = typename div<P, val<typename P::aN>::q_type>;
01353     };
01354
01355     template<typename coeff, size_t deg>
01356     struct monomial {
01357         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01358     };
01359
01360     template<typename coeff>
01361     struct monomial<coeff, 0> {
01362         using type = val<coeff>;
01363     };
01364
01365     template<typename valueRing, typename P>
01366     struct horner_evaluation {
01367         template<size_t index, size_t stop>
01368         struct inner {
01369             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01370                 constexpr valueRing coeff =
01371                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01372                     get<valueRing>());
01373                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01374                     accum + coeff, x);
01375             }
01376         };

```

```

01377         template<size_t stop>
01378         struct inner<stop, stop> {
01379             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01380                 return accum;
01381             }
01382         };
01383     };
01384
01385     template<typename coeff, typename... coeffs>
01386     struct string_helper {
01387         static std::string func() {
01388             std::string tail = string_helper<coeffs...>::func();
01389             std::string result = "";
01390             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01391                 return tail;
01392             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01393                 if (sizeof...(coeffs) == 1) {
01394                     result += "x";
01395                 } else {
01396                     result += "x^" + std::to_string(sizeof...(coeffs));
01397                 }
01398             } else {
01399                 if (sizeof...(coeffs) == 1) {
01400                     result += coeff::to_string() + " x";
01401                 } else {
01402                     result += coeff::to_string()
01403                         + " x^" + std::to_string(sizeof...(coeffs));
01404                 }
01405             }
01406
01407             if (!tail.empty()) {
01408                 result += " + " + tail;
01409             }
01410
01411             return result;
01412         }
01413     };
01414
01415     template<typename coeff>
01416     struct string_helper<coeff> {
01417         static std::string func() {
01418             if (!std::is_same<coeff, typename Ring::zero>::value) {
01419                 return coeff::to_string();
01420             } else {
01421                 return "";
01422             }
01423         }
01424     };
01425
01426     public:
01427         template<typename P>
01428         using simplify_t = typename simplify<P>::type;
01429
01430         template<typename v1, typename v2>
01431         using add_t = typename add<v1, v2>::type;
01432
01433         template<typename v1, typename v2>
01434         using sub_t = typename sub<v1, v2>::type;
01435
01436         template<typename v1, typename v2>
01437         using mul_t = typename mul<v1, v2>::type;
01438
01439         template<typename v1, typename v2>
01440         using eq_t = typename eq_helper<v1, v2>::type;
01441
01442         template<typename v1, typename v2>
01443         using lt_t = typename lt_helper<v1, v2>::type;
01444
01445         template<typename v1, typename v2>
01446         using gt_t = typename gt_helper<v1, v2>::type;
01447
01448         template<typename v1, typename v2>
01449         using div_t = typename div<v1, v2>::q_type;
01450
01451         template<typename v1, typename v2>
01452         using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01453
01454         template<typename coeff, size_t deg>
01455         using monomial_t = typename monomial<coeff, deg>::type;
01456
01457         template<typename v>
01458         using derive_t = typename derive_helper<v>::type;
01459
01460         template<typename v>
01461         using pos_t = typename Ring::template pos_t<typename v::aN>;
01462
01463         template<typename v>

```

```

01497     static constexpr bool pos_v = pos_t<v>::value;
01498
01502     template<typename v1, typename v2>
01503     using gcd_t = std::conditional_t<
01504         Ring::is_euclidean_domain,
01505         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01506         void>;
01507
01511     template<auto x>
01512     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01513
01517     template<typename v>
01518     using inject_ring_t = val<v>;
01519 };
01520 } // namespace aerobus
01521
01522 // fraction field
01523 namespace aerobus {
01524     namespace internal {
01525         template<typename Ring, typename E = void>
01526         requires IsEuclideanDomain<Ring>
01527         struct _FractionField {};
01528
01529         template<typename Ring>
01530         requires IsEuclideanDomain<Ring>
01531         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01532             static constexpr bool is_field = true;
01533             static constexpr bool is_euclidean_domain = true;
01534
01535         private:
01536             template<typename val1, typename val2, typename E = void>
01537             struct to_string_helper {};
01538
01539             template<typename val1, typename val2>
01540             struct to_string_helper<val1, val2,
01541                 std::enable_if_t<
01542                     Ring::template eq_t<
01543                         val2, typename Ring::one
01544                         >::value
01545                     >
01546             > {
01547                 static std::string func() {
01548                     return val1::to_string();
01549                 }
01550             };
01551
01552             template<typename val1, typename val2>
01553             struct to_string_helper<val1, val2,
01554                 std::enable_if_t<
01555                     !Ring::template eq_t<
01556                         val2,
01557                         typename Ring::one
01558                         >::value
01559                     >
01560             > {
01561                 static std::string func() {
01562                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01563                 }
01564             };
01565
01566         public:
01567             template<typename val1, typename val2>
01568             struct val {
01569                 using x = val1;
01570                 using y = val2;
01571                 using is_zero_t = typename val1::is_zero_t;
01572                 static constexpr bool is_zero_v = val1::is_zero_t::value;
01573
01574                 using ring_type = Ring;
01575                 using field_type = _FractionField<Ring>;
01576
01577                 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01578
01579                 template<typename valueType>
01580                 static constexpr valueType get() { return static_cast<valueType>(x::v) /
01581                     static_cast<valueType>(y::v); }
01582
01583                 static std::string to_string() {
01584                     return to_string_helper<val1, val2>::func();
01585                 }
01586
01587                 template<typename valueRing>
01588                 static constexpr valueRing eval(const valueRing& v) {
01589                     return x::eval(v) / y::eval(v);
01590                 }
01591             };
01592
01593     }
01594 }

```

```

01613         using zero = val<typename Ring::zero, typename Ring::one>;
01615         using one = val<typename Ring::one, typename Ring::one>;
01616
01619         template<typename v>
01620         using inject_t = val<v, typename Ring::one>;
01621
01624         template<auto x>
01625         using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
01626
01629         template<typename v>
01630         using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01631
01633         using ring_type = Ring;
01634
01635     private:
01636         template<typename v, typename E = void>
01637         struct simplify {};
01638
01639         // x = 0
01640         template<typename v>
01641         struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01642             using type = typename _FractionField<Ring>::zero;
01643         };
01644
01645         // x != 0
01646         template<typename v>
01647         struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01648             private:
01649                 using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01650                 using newx = typename Ring::template div_t<typename v::x, _gcd>;
01651                 using newy = typename Ring::template div_t<typename v::y, _gcd>;
01652
01653                 using posx = std::conditional_t<
01654                     !Ring::template pos_v<newy>,
01655                     typename Ring::template sub_t<typename Ring::zero, newx>,
01656                     newx>;
01657                 using posy = std::conditional_t<
01658                     !Ring::template pos_v<newy>,
01659                     typename Ring::template sub_t<typename Ring::zero, newy>,
01660                     newy>;
01661             public:
01662                 using type = typename _FractionField<Ring>::template val<posx, posy>;
01663         };
01664
01665     public:
01666         template<typename v>
01667         using simplify_t = typename simplify<v>::type;
01668
01669     private:
01670         template<typename v1, typename v2>
01671         struct add {
01672             private:
01673                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01674                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01675                 using dividend = typename Ring::template add_t<a, b>;
01676                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01677                 using g = typename Ring::template gcd_t<dividend, diviser>;
01678             public:
01679                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01680
01681         };
01682
01683     template<typename v>
01684     struct pos {
01685         using type = std::conditional_t<
01686             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01687             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01688             std::true_type,
01689             std::false_type>;
01690     };
01691
01692     template<typename v1, typename v2>
01693     struct sub {
01694         private:
01695             using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01696             using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01697             using dividend = typename Ring::template sub_t<a, b>;
01698             using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01699             using g = typename Ring::template gcd_t<dividend, diviser>;
01700         public:
01701             using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01702
01703     };
01704
01705     };
01706

```

```

01707     template<typename v1, typename v2>
01708     struct mul {
01709     private:
01710         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01711         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01712
01713     public:
01714         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01715     };
01716
01717     template<typename v1, typename v2, typename E = void>
01718     struct div {};
01719
01720     template<typename v1, typename v2>
01721     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01722     private:
01723         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01724         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01725
01726     public:
01727         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01728     };
01729
01730     template<typename v1, typename v2>
01731     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01732         using type = one;
01733     };
01734
01735     template<typename v1, typename v2>
01736     struct eq {
01737         using type = std::conditional_t<
std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
std::true_type,
std::false_type>;
01743     };
01744
01745     template<typename TL, typename E = void>
01746     struct vadd {};
01747
01748     template<typename TL>
01749     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01750         using head = typename TL::pop_front::type;
01751         using tail = typename TL::pop_front::tail;
01752         using type = typename add<head, typename vadd<tail>::type>::type;
01753     };
01754
01755     template<typename TL>
01756     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01757         using type = typename TL::template at<0>;
01758     };
01759
01760     template<typename... vals>
01761     struct vmul {};
01762
01763     template<typename v1, typename... vals>
01764     struct vmul<v1, vals...> {
01765         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01766     };
01767
01768     template<typename v1>
01769     struct vmul<v1> {
01770         using type = v1;
01771     };
01772
01773     template<typename v1, typename v2, typename E = void>
01774     struct gt;
01775
01776     template<typename v1, typename v2>
01777     struct gt<v1, v2, std::enable_if_t<
(eq<v1, v2>::type::value)
>> {
01778         using type = std::false_type;
01779     };
01780
01781     template<typename v1, typename v2>
01782     struct gt<v1, v2, std::enable_if_t<
(!eq<v1, v2>::type::value) &&
(!pos<v1>::type::value) && (!pos<v2>::type::value)
>> {
01783         using type = typename gt<
typename sub<zero, v1>::type, typename sub<zero, v2>::type
>::type;
01791     };
01792

```



```

01793
01794     template<typename v1, typename v2>
01795     struct gt<v1, v2, std::enable_if_t<
01796         (!eq<v1, v2>::type::value) &&
01797         (pos<v1>::type::value) && (!pos<v2>::type::value)
01798         >> {
01799         using type = std::true_type;
01800     };
01801
01802     template<typename v1, typename v2>
01803     struct gt<v1, v2, std::enable_if_t<
01804         (!eq<v1, v2>::type::value) &&
01805         (!pos<v1>::type::value) && (pos<v2>::type::value)
01806         >> {
01807         using type = std::false_type;
01808     };
01809
01810     template<typename v1, typename v2>
01811     struct gt<v1, v2, std::enable_if_t<
01812         (!eq<v1, v2>::type::value) &&
01813         (pos<v1>::type::value) && (pos<v2>::type::value)
01814         >> {
01815         using type = typename Ring::template gt_t<
01816             typename Ring::template mul_t<v1::x, v2::y>,
01817             typename Ring::template mul_t<v2::y, v2::x>
01818         >;
01819     };
01820
01821     public:
01822     template<typename v1, typename v2>
01823     using add_t = typename add<v1, v2>::type;
01824     template<typename v1, typename v2>
01825     using mod_t = zero;
01826     template<typename v1, typename v2>
01827     using gcd_t = v1;
01828     template<typename... vs>
01829     using vadd_t = typename vadd<vs...>::type;
01830     template<typename... vs>
01831     using vmul_t = typename vmul<vs...>::type;
01832     template<typename v1, typename v2>
01833     using sub_t = typename sub<v1, v2>::type;
01834     template<typename v1, typename v2>
01835     using mul_t = typename mul<v1, v2>::type;
01836     template<typename v1, typename v2>
01837     using div_t = typename div<v1, v2>::type;
01838     template<typename v1, typename v2>
01839     using eq_t = typename eq<v1, v2>::type;
01840     template<typename v1, typename v2>
01841     static constexpr bool eq_v = eq<v1, v2>::type::value;
01842     template<typename v1, typename v2>
01843     using gt_t = typename gt<v1, v2>::type;
01844     template<typename v1, typename v2>
01845     static constexpr bool gt_v = gt<v1, v2>::type::value;
01846     template<typename v1>
01847     using pos_t = typename pos<v1>::type;
01848     template<typename v>
01849     static constexpr bool pos_v = pos_t<v>::value;
01850 };
01851
01852     template<typename Ring, typename E = void>
01853     requires IsEuclideanDomain<Ring>
01854     struct FractionFieldImpl {};
01855
01856     // fraction field of a field is the field itself
01857     template<typename Field>
01858     requires IsEuclideanDomain<Field>
01859     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
01860         using type = Field;
01861         template<typename v>
01862         using inject_t = v;
01863     };
01864
01865     // fraction field of a ring is the actual fraction field
01866     template<typename Ring>
01867     requires IsEuclideanDomain<Ring>
01868     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
01869         using type = _FractionField<Ring>;
01870     };
01871 } // namespace internal
01872
01873     template<typename Ring>
01874     requires IsEuclideanDomain<Ring>
01875     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
01876 } // namespace aerobus
01877
01878 // short names for common types
01879 namespace aerobus {

```

```

01899     using q32 = FractionField<i32>;
01901     using fpq32 = FractionField<polynomial<q32>;
01903     using q64 = FractionField<i64>;
01905     using pi64 = polynomial<i64>;
01907     using pq64 = polynomial<q64>;
01909     using fpq64 = FractionField<polynomial<q64>;
01914     template<typename Ring, typename v1, typename v2>
01915     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
01916
01921     template<typename Ring, typename v1, typename v2>
01922     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
01927     template<typename Ring, typename v1, typename v2>
01928     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
01929 } // namespace aerobus
01930
01931 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
01932 namespace aerobus {
01933     namespace internal {
01934         template<typename T, size_t x, typename E = void>
01935         struct factorial {};
01936
01937         template<typename T, size_t x>
01938         struct factorial<T, x, std::enable_if_t<(x > 0)> {
01939             private:
01940                 template<typename, size_t, typename>
01941                 friend struct factorial;
01942             public:
01943                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
01944 x - 1>::type>;
01945                 static constexpr typename T::inner_type value = type::template get<typename
01946 T::inner_type>();
01947             };
01948         template<typename T>
01949         struct factorial<T, 0> {
01950             public:
01951                 using type = typename T::one;
01952                 static constexpr typename T::inner_type value = type::template get<typename
01953 T::inner_type>();
01954             };
01955     } // namespace internal
01956
01958     template<typename T, size_t i>
01959     using factorial_t = typename internal::factorial<T, i>::type;
01960
01964     template<typename T, size_t i>
01965     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
01966
01967     namespace internal {
01968         template<typename T, size_t k, size_t n, typename E = void>
01969         struct combination_helper {};
01970
01971         template<typename T, size_t k, size_t n>
01972         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
01973             using type = typename FractionField<T>::template mul_t<
01974                 typename combination_helper<T, k - 1, n - 1>::type,
01975                 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
01976             };
01977
01978         template<typename T, size_t k, size_t n>
01979         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
01980             using type = typename combination_helper<T, n - k, n>::type;
01981             };
01982
01983         template<typename T, size_t n>
01984         struct combination_helper<T, 0, n> {
01985             using type = typename FractionField<T>::one;
01986             };
01987
01988         template<typename T, size_t k, size_t n>
01989         struct combination {
01990             using type = typename internal::combination_helper<T, k, n>::type::x;
01991             static constexpr typename T::inner_type value =
01992                 internal::combination_helper<T, k, n>::type::template get<typename
01993 T::inner_type>();
01994             };
01995     } // namespace internal
01996
01998     template<typename T, size_t k, size_t n>
01999     using combination_t = typename internal::combination<T, k, n>::type;
02000
02005     template<typename T, size_t k, size_t n>
02006     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02007
02008     namespace internal {
02009         template<typename T, size_t m>
02010         struct bernouilli;

```

```

02011
02012     template<typename T, typename accum, size_t k, size_t m>
02013     struct bernouilli_helper {
02014         using type = typename bernouilli_helper<
02015             T,
02016             addfractions_t<T,
02017                 accum,
02018                 mulfractions_t<T,
02019                     makefraction_t<T,
02020                         combination_t<T, k, m + 1>,
02021                         typename T::one>,
02022                         typename bernouilli<T, k>::type
02023                     >,
02024                     >,
02025                     k + 1,
02026                     m>::type;
02027     };
02028
02029     template<typename T, typename accum, size_t m>
02030     struct bernouilli_helper<T, accum, m, m> {
02031         using type = accum;
02032     };
02033
02034
02035
02036     template<typename T, size_t m>
02037     struct bernouilli {
02038         using type = typename FractionField<T>::template mul_t<
02039             typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02040             makefraction_t<T,
02041                 typename T::template val<static_cast<typename T::inner_type>>(-1)>,
02042                 typename T::template val<static_cast<typename T::inner_type>>(m + 1)>
02043             >
02044         >;
02045
02046         template<typename floatType>
02047         static constexpr floatType value = type::template get<floatType>();
02048     };
02049
02050     template<typename T>
02051     struct bernouilli<T, 0> {
02052         using type = typename FractionField<T>::one;
02053
02054         template<typename floatType>
02055         static constexpr floatType value = type::template get<floatType>();
02056     };
02057 } // namespace internal
02058
02062     template<typename T, size_t n>
02063     using bernouilli_t = typename internal::bernouilli<T, n>::type;
02064
02069     template<typename FloatType, typename T, size_t n>
02070     inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02071
02072     namespace internal {
02073         template<typename T, int k, typename E = void>
02074         struct alternate {};
02075
02076         template<typename T, int k>
02077         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02078             using type = typename T::one;
02079             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02080         };
02081
02082         template<typename T, int k>
02083         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02084             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02085             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02086         };
02087     } // namespace internal
02088
02091     template<typename T, int k>
02092     using alternate_t = typename internal::alternate<T, k>::type;
02093
02094     namespace internal {
02095         template<typename T, int n, int k, typename E = void>
02096         struct stirling_helper {};
02097
02098         template<typename T>
02099         struct stirling_helper<T, 0, 0> {
02100             using type = typename T::one;
02101         };
02102
02103         template<typename T, int n>
02104         struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {

```

```

02105         using type = typename T::zero;
02106     };
02107
02108     template<typename T, int n>
02109     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
02110         using type = typename T::zero;
02111     };
02112
02113     template<typename T, int n, int k>
02114     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)>> {
02115         using type = typename T::template sub_t<
02116             typename stirling_helper<T, n-1, k-1>::type,
02117             typename T::template mul_t<
02118                 typename T::template inject_constant_t<n-1>,
02119                 typename stirling_helper<T, n-1, k>::type
02120             >>;
02121     };
02122 } // namespace internal
02123
02124 template<typename T, int n, int k>
02125 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02126
02127 template<typename T, int n, int k>
02128 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02129
02130 template<typename T, int n, int k>
02131 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02132
02133 template<typename T, int n, int k>
02134 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02135
02136 template<typename T, size_t k>
02137 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02138
02139 namespace internal {
02140     template<typename T, auto p, auto n, typename E = void>
02141     struct pow {};
02142
02143     template<typename T, auto p, auto n>
02144     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
02145         using type = typename T::template mul_t<
02146             typename pow<T, p, n/2>::type,
02147             typename pow<T, p, n/2>::type
02148         >;
02149     };
02150
02151     template<typename T, auto p, auto n>
02152     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
02153         using type = typename T::template mul_t<
02154             typename T::template inject_constant_t<p>,
02155             typename T::template mul_t<
02156                 typename pow<T, p, n/2>::type,
02157                 typename pow<T, p, n/2>::type
02158             >
02159         >;
02160     };
02161
02162     template<typename T, auto p>
02163     struct pow<T, p, 0> { using type = typename T::one; };
02164 } // namespace internal
02165
02166 template<typename T, auto p, auto n>
02167 using pow_t = typename internal::pow<T, p, n>::type;
02168
02169 template<typename T, auto p, auto n>
02170 static constexpr T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02171
02172 namespace internal {
02173     template<typename, template<typename, size_t> typename, class>
02174     struct make_taylor_impl;
02175
02176     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02177     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...>> {
02178         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
02179             Is>::type...>;
02180     };
02181 }
02182
02183 template<typename T, template<typename, size_t> index> typename coeff_at, size_t deg>
02184 using taylor = typename internal::make_taylor_impl<
02185     T,
02186     coeff_at,
02187     internal::make_index_sequence_reverse<deg + 1>::type;
02188
02189 namespace internal {
02190     template<typename T, size_t i>

```

```

02221     struct exp_coeff {
02222         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02223     };
02224
02225     template<typename T, size_t i, typename E = void>
02226     struct sin_coeff_helper {};
02227
02228     template<typename T, size_t i>
02229     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02230         using type = typename FractionField<T>::zero;
02231     };
02232
02233     template<typename T, size_t i>
02234     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02235         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02236     };
02237
02238     template<typename T, size_t i>
02239     struct sin_coeff {
02240         using type = typename sin_coeff_helper<T, i>::type;
02241     };
02242
02243     template<typename T, size_t i, typename E = void>
02244     struct sh_coeff_helper {};
02245
02246     template<typename T, size_t i>
02247     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02248         using type = typename FractionField<T>::zero;
02249     };
02250
02251     template<typename T, size_t i>
02252     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02253         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02254     };
02255
02256     template<typename T, size_t i>
02257     struct sh_coeff {
02258         using type = typename sh_coeff_helper<T, i>::type;
02259     };
02260
02261     template<typename T, size_t i, typename E = void>
02262     struct cos_coeff_helper {};
02263
02264     template<typename T, size_t i>
02265     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02266         using type = typename FractionField<T>::zero;
02267     };
02268
02269     template<typename T, size_t i>
02270     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02271         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02272     };
02273
02274     template<typename T, size_t i>
02275     struct cos_coeff {
02276         using type = typename cos_coeff_helper<T, i>::type;
02277     };
02278
02279     template<typename T, size_t i, typename E = void>
02280     struct cosh_coeff_helper {};
02281
02282     template<typename T, size_t i>
02283     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02284         using type = typename FractionField<T>::zero;
02285     };
02286
02287     template<typename T, size_t i>
02288     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02289         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02290     };
02291
02292     template<typename T, size_t i>
02293     struct cosh_coeff {
02294         using type = typename cosh_coeff_helper<T, i>::type;
02295     };
02296
02297     template<typename T, size_t i>
02298     struct geom_coeff { using type = typename FractionField<T>::one; };
02299
02300
02301     template<typename T, size_t i, typename E = void>
02302     struct atan_coeff_helper;
02303
02304     template<typename T, size_t i>
02305     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02306         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02307     };

```

```

02308
02309     template<typename T, size_t i>
02310     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02311         using type = typename FractionField<T>::zero;
02312     };
02313
02314     template<typename T, size_t i>
02315     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02316
02317     template<typename T, size_t i, typename E = void>
02318     struct asin_coeff_helper;
02319
02320     template<typename T, size_t i>
02321     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02322         using type = makefraction_t<T,
02323             factorial_t<T, i - 1>,
02324             typename T::template mul_t<
02325                 typename T::template val<i>,
02326                 T::template mul_t<
02327                     pow_t<T, 4, i / 2>,
02328                     pow<T, factorial<T, i / 2>::value, 2
02329                 >
02330             >
02331         >>;
02332     };
02333
02334     template<typename T, size_t i>
02335     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02336         using type = typename FractionField<T>::zero;
02337     };
02338
02339     template<typename T, size_t i>
02340     struct asin_coeff {
02341         using type = typename asin_coeff_helper<T, i>::type;
02342     };
02343
02344     template<typename T, size_t i>
02345     struct lnpl_coeff {
02346         using type = makefraction_t<T,
02347             alternate_t<T, i + 1>,
02348             typename T::template val<i>;
02349     };
02350
02351     template<typename T>
02352     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02353
02354     template<typename T, size_t i, typename E = void>
02355     struct asinh_coeff_helper;
02356
02357     template<typename T, size_t i>
02358     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02359         using type = makefraction_t<T,
02360             typename T::template mul_t<
02361                 alternate_t<T, i / 2>,
02362                 factorial_t<T, i - 1>
02363             >,
02364             typename T::template mul_t<
02365                 T::template mul_t<
02366                     typename T::template val<i>,
02367                     pow_t<T, (factorial<T, i / 2>::value), 2>
02368                 >,
02369                 pow_t<T, 4, i / 2>
02370             >
02371         >>;
02372     };
02373
02374     template<typename T, size_t i>
02375     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02376         using type = typename FractionField<T>::zero;
02377     };
02378
02379     template<typename T, size_t i>
02380     struct asinh_coeff {
02381         using type = typename asinh_coeff_helper<T, i>::type;
02382     };
02383
02384     template<typename T, size_t i, typename E = void>
02385     struct atanh_coeff_helper;
02386
02387     template<typename T, size_t i>
02388     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02389         // 1/i
02390         using type = typename FractionField<T>::template val<
02391             typename T::one,
02392             typename T::template val<static_cast<typename T::inner_type>(i)>;
02393     };
02394

```

```

02395     template<typename T, size_t i>
02396     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02397         using type = typename FractionField<T>::zero;
02398     };
02399
02400     template<typename T, size_t i>
02401     struct atanh_coeff {
02402         using type = typename asinh_coeff_helper<T, i>::type;
02403     };
02404
02405     template<typename T, size_t i, typename E = void>
02406     struct tan_coeff_helper;
02407
02408     template<typename T, size_t i>
02409     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02410         using type = typename FractionField<T>::zero;
02411     };
02412
02413     template<typename T, size_t i>
02414     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02415     private:
02416         //  $4^{((i+1)/2)}$ 
02417         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02418         //  $4^{((i+1)/2)} - 1$ 
02419         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02420         //  $(-1)^{(i-1)/2}$ 
02421         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02422         using dividend = typename FractionField<T>::template mul_t<
02423             altp,
02424             FractionField<T>::template mul_t<
02425                 _4p,
02426                 FractionField<T>::template mul_t<
02427                     _4pml,
02428                     bernouilli_t<T, (i + 1)>
02429                 >
02430             >
02431         >;
02432     public:
02433         using type = typename FractionField<T>::template div_t<dividend,
02434             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02435     };
02436
02437     template<typename T, size_t i>
02438     struct tan_coeff {
02439         using type = typename tan_coeff_helper<T, i>::type;
02440     };
02441
02442     template<typename T, size_t i, typename E = void>
02443     struct tanh_coeff_helper;
02444
02445     template<typename T, size_t i>
02446     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02447         using type = typename FractionField<T>::zero;
02448     };
02449
02450     template<typename T, size_t i>
02451     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02452     private:
02453         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02454         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02455         using dividend =
02456             typename FractionField<T>::template mul_t<
02457                 _4p,
02458                 typename FractionField<T>::template mul_t<
02459                     _4pml,
02460                     bernouilli_t<T, (i + 1)>
02461                 >
02462             >::type;
02463     public:
02464         using type = typename FractionField<T>::template div_t<dividend,
02465             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02466     };
02467
02468     template<typename T, size_t i>
02469     struct tanh_coeff {
02470         using type = typename tanh_coeff_helper<T, i>::type;
02471     };
02472 } // namespace internal
02473
02474     template<typename T, size_t deg>
02475     using exp = taylor<T, internal::exp_coeff, deg>;
02476
02477     template<typename T, size_t deg>
02478     using expml = typename polynomial<FractionField<T>::template sub_t<
02485         exp<T, deg>,

```

```

02486         typename polynomial<FractionField<T>::one>;
02487
02491     template<typename T, size_t deg>
02492     using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02493
02497     template<typename T, size_t deg>
02498     using atan = taylor<T, internal::atan_coeff, deg>;
02499
02503     template<typename T, size_t deg>
02504     using sin = taylor<T, internal::sin_coeff, deg>;
02505
02509     template<typename T, size_t deg>
02510     using sinh = taylor<T, internal::sh_coeff, deg>;
02511
02515     template<typename T, size_t deg>
02516     using cosh = taylor<T, internal::cosh_coeff, deg>;
02517
02521     template<typename T, size_t deg>
02522     using cos = taylor<T, internal::cos_coeff, deg>;
02523
02527     template<typename T, size_t deg>
02528     using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02529
02533     template<typename T, size_t deg>
02534     using asin = taylor<T, internal::asin_coeff, deg>;
02535
02539     template<typename T, size_t deg>
02540     using asinh = taylor<T, internal::asinh_coeff, deg>;
02541
02545     template<typename T, size_t deg>
02546     using atanh = taylor<T, internal::atanh_coeff, deg>;
02547
02551     template<typename T, size_t deg>
02552     using tan = taylor<T, internal::tan_coeff, deg>;
02553
02557     template<typename T, size_t deg>
02558     using tanh = taylor<T, internal::tanh_coeff, deg>;
02559 } // namespace aerobus
02560
02561 // continued fractions
02562 namespace aerobus {
02563     template<int64_t... values>
02564     struct ContinuedFraction {};
02565
02567     template<int64_t a0>
02568     struct ContinuedFraction<a0> {
02569         using type = typename q64::template inject_constant_t<a0>;
02570         static constexpr double val = type::template get<double>();
02571     };
02572
02574     template<int64_t a0, int64_t... rest>
02575     struct ContinuedFraction<a0, rest...> {
02576         using type = q64::template add_t<
02577             typename q64::template inject_constant_t<a0>,
02578             typename q64::template div_t<
02579                 typename q64::one,
02580                 typename ContinuedFraction<rest...>::type
02581             >;
02582         static constexpr double val = type::template get<double>();
02583     };
02584
02586     using PI_fraction =
02587     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02588
02589     using E_fraction =
02590     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02591
02592     using Sqrt2_fraction =
02593     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02594
02595     using Sqrt3_fraction =
02596     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02597 // NOLINT
02598 } // namespace aerobus
02599
02600 // known polynomials
02601 namespace aerobus {
02602     // CChebyshev
02603     namespace internal {
02604         template<int kind, int deg>
02605         struct chebyshev_helper {
02606             using type = typename pi64::template sub_t<
02607                 typename pi64::template mul_t<
02608                     typename pi64::template mul_t<
02609                         pi64::inject_constant_t<2>,
02610                         typename pi64::X
02611                     >,
02612                     typename chebyshev_helper<kind, deg - 1>::type
02613                 >,
02614                 typename chebyshev_helper<kind, deg - 2>::type
02615             >;
02616         };
02617     }
02618 }

```



```

02619         >;
02620     };
02621
02622     template<>
02623     struct chebyshev_helper<1, 0> {
02624         using type = typename pi64::one;
02625     };
02626
02627     template<>
02628     struct chebyshev_helper<1, 1> {
02629         using type = typename pi64::X;
02630     };
02631
02632     template<>
02633     struct chebyshev_helper<2, 0> {
02634         using type = typename pi64::one;
02635     };
02636
02637     template<>
02638     struct chebyshev_helper<2, 1> {
02639         using type = typename pi64::template mul_t<
02640             typename pi64::inject_constant_t<2>,
02641             typename pi64::X>;
02642     };
02643 } // namespace internal
02644
02645 // Laguerre
02646 namespace internal {
02647     template<size_t deg>
02648     struct laguerre_helper {
02649     private:
02650         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
02651         using lnm2 = typename laguerre_helper<deg - 2>::type;
02652         using lnm1 = typename laguerre_helper<deg - 1>::type;
02653         // -x + 2k-1
02654         using p = typename pq64::template val<
02655             typename q64::template inject_constant_t<-1>,
02656             typename q64::template inject_constant_t<2 * deg - 1>>;
02657         // 1/n
02658         using factor = typename pq64::template inject_ring_t<
02659             q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02660
02661     public:
02662         using type = typename pq64::template mul_t <
02663             factor,
02664             typename pq64::template sub_t<
02665                 typename pq64::template mul_t<
02666                     p,
02667                     lnm1
02668                 >,
02669             typename pq64::template mul_t<
02670                 typename pq64::template inject_constant_t<deg-1>,
02671                 lnm2
02672             >
02673         >
02674     >;
02675     };
02676
02677     template<>
02678     struct laguerre_helper<0> {
02679         using type = typename pq64::one;
02680     };
02681
02682     template<>
02683     struct laguerre_helper<1> {
02684         using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02685     };
02686 } // namespace internal
02687
02688 namespace known_polynomials {
02689     enum hermite_kind {
02690         probabilist,
02691         physicist
02692     };
02693 }
02694
02695 namespace internal {
02696     template<size_t deg, known_polynomials::hermite_kind kind>
02697     struct hermite_helper {};
02698
02699     template<size_t deg>
02700     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02701     private:
02702         using hnm1 = typename hermite_helper<deg - 1,
02703             known_polynomials::hermite_kind::probabilist>::type;
02704         using hnm2 = typename hermite_helper<deg - 2,

```

```

known_polynomials::hermite_kind::probabilist>::type;
02706
02707     public:
02708         using type = typename pi64::template sub_t<
02709             typename pi64::template mul_t<typename pi64::X, hnm1>,
02710             typename pi64::template mul_t<
02711                 typename pi64::template inject_constant_t<deg - 1>,
02712                 hnm2
02713             >
02714         >;
02715     };
02716
02717     template<size_t deg>
02718     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02719     private:
02720         using hnm1 = typename hermite_helper<deg - 1,
known_polynomials::hermite_kind::physicist>::type;
02721         using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02722
02723     public:
02724         using type = typename pi64::template sub_t<
02725             // 2X Hn-1
02726             typename pi64::template mul_t<
02727                 typename pi64::val<typename i64::template inject_constant_t<2>,
02728                 typename i64::zero>, hnm1>,
02729
02730             typename pi64::template mul_t<
02731                 typename pi64::template inject_constant_t<2*(deg - 1)>,
02732                 hnm2
02733             >
02734         >;
02735     };
02736
02737     template<>
02738     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02739         using type = typename pi64::one;
02740     };
02741
02742     template<>
02743     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02744         using type = typename pi64::X;
02745     };
02746
02747     template<>
02748     struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02749         using type = typename pi64::one;
02750     };
02751
02752     template<>
02753     struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02754         // 2X
02755         using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
typename i64::zero>;
02756     };
02757     } // namespace internal
02758
02759     namespace known_polynomials {
02760     template <size_t deg>
02761         using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02762
02763     template <size_t deg>
02764         using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02765
02766     template <size_t deg>
02767         using laguerre = typename internal::laguerre_helper<deg>::type;
02768
02769     template <size_t deg>
02770         using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02771
02772     template <size_t deg>
02773         using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
02774     } // namespace known_polynomials
02775 } // namespace aerobus
02776
02777 #ifdef AEROBUS_CONWAY_IMPORTS
02778 template<int p, int n>
02779 struct ConwayPolynomial;
02780
02781 #define ZPZV ZPZ::template val
02782 #define POLYV aerobus::polynomial<ZPZ>::template val
02783 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>; }; // NOLINT
02784 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>, ZPZV<1>; }; // NOLINT
02785 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,

```

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```
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<10>, ZPV<990>; }; // NOLINT
04731 template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<981>, ZPV<58>, ZPV<260>, ZPV<7>; }; // NOLINT
04732 template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<990>; }; // NOLINT
04733 template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<934>, ZPV<473>, ZPV<241>, ZPV<7>; }; //
NOLINT
04734 template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<39>, ZPV<732>, ZPV<616>, ZPV<990>;
}; // NOLINT
04735 #endif // AEROBUS_CONWAY_IMPORTS
04736
04737 #endif // __INC_AEROBUS__ // NOLINT
```


Chapter 7

Examples

7.1 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

7.2 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

x	inject_constant_t<2>
---	----------------------

7.3 polynomial

makes the constant (native type) polynomial a_0

makes the constant (native type) polynomial a_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

7.4 PI_fraction::val

representation of PI as a continued fraction -> 3.14...

7.5 E_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

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