

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.



For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , `Aerobus` provides automatic implementation for **quotient ring**  $R/X$  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

## 1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	36
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## Chapter 3

# Concept Index

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Concept to express R is a field . . . . .	41
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	42



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt; 43</a>	
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<a href="#">aerobus::Embed&lt; q32, q64 &gt;</a>	
Embeds q32 into q64 . . . . .	<a href="#">51</a>
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## Chapter 5

# File Index

### 5.1 File List

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## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial<Small> into polynomial<Large>*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient<Ring, X> into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
  - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
  - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)
  - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
  - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)
  - Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)
  - specialization for empty type list*
- struct [zpz](#)
  - congruence classes of integers modulo p (32 bits)*

## Concepts

- concept [IsRing](#)
  - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
  - Concept to express R is an euclidean domain.*
- concept [IsField](#)
  - Concept to express R is a field.*

## Typedefs

- template<typename T , typename A , typename B >
 using [gcd\\_t](#) = typename internal::gcd< T >::template type< A, B >
  - computes the greatest common divisor of A and B*
- template<typename... vals>
 using [vadd\\_t](#) = typename internal::vadd< vals... >::type
  - adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator*
- template<typename... vals>
 using [vmul\\_t](#) = typename internal::vmul< vals... >::type
  - multiplies multiple values (v1 \* v2 + ... \* vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator*
- template<typename val >
 using [abs\\_t](#) = std::conditional\_t< val::enclosing\_type::template pos\_v< val >, val, typename val::enclosing\_type::template [sub\\_t](#)< typename val::enclosing\_type::zero, val > >
  - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
  - Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >
 using [add\\_t](#) = typename X::enclosing\_type::template [add\\_t](#)< X, Y >
  - generic addition*
- template<typename X , typename Y >
 using [sub\\_t](#) = typename X::enclosing\_type::template [sub\\_t](#)< X, Y >

- generic subtraction*
  - `template<typename X , typename Y >`  
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
  - generic multiplication*
  - `template<typename X , typename Y >`  
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
  - generic division*
  - `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
  - `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
  - `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
  - `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
  - `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
  - `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
  - `template<typename v >`  
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > ,`  
`polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
  - `template<int64_t p, int64_t q>`  
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > ,`  
`i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
  - `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > ,`  
`i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
  - `template<typename Ring , typename v1 , typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
  - `template<typename Ring , auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template`  
`inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
  - `template<typename Ring , auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename`  
`FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField< Ring>*
  - `template<typename T , size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*

- `template<typename T, size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T, size_t n>`  
`using bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T, size_t n>`  
`using bell_t = typename internal::bell_helper< T, n >::type`  
*Bell numbers.*
- `template<typename T, int k>`  
`using alternate_t = typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T, int n, int k>`  
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T, int n, int k>`  
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T, int n, int k>`  
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`  
*Stirling number of second kind – as types.*
- `template<typename T, typename p, size_t n>`  
`using pow_t = typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>`  
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<`  
`deg+1 > >::type`
- `template<typename Integers, size_t deg>`  
`using exp = taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers, size_t deg>`  
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg`  
`>, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers, size_t deg>`  
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers, size_t deg>`  
`using atan = taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers, size_t deg>`  
`using sin = taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers, size_t deg>`  
`using sinh = taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers, size_t deg>`  
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers, size_t deg>`  
`using cos = taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*
- `template<typename Integers, size_t deg>`  
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`



## 6.1.2 Typedef Documentation

### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

#### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T



## Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example
----------	---

## 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh( $x$ )` arc hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

## Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

## 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

## 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

## 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  `cosinus`

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

## Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

## 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of  $e$

## 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

## Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

## 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

#### 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

#### 6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

##### Template Parameters

<i>Ring</i>	
-------------	--

#### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

##### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

#### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

##### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator



## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.29 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.30 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

## Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

## 6.1.2.31 mul\_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

### 6.1.2.32 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

### 6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

### 6.1.2.34 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of  $\pi$  as a continued fraction

### 6.1.2.35 pow\_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

#### Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

### 6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.37 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.38 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

**6.1.2.39 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.40 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.41 SQRT2\_fraction**

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of  $\sqrt{2}$

#### 6.1.2.42 Sqrt3\_fraction

```
using aerobus::Sqrt3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

#### 6.1.2.43 stirling\_1\_signed\_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_signed_t = typedef typename internal::stirling_1_helper<T, n, k>::type
```

Stirling number of first king (signed) – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

#### 6.1.2.44 stirling\_1\_unsigned\_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T, n, k>::type>
```

Stirling number of first king (unsigned) – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

#### 6.1.2.45 stirling\_2\_t

```
template<typename T , int n, int k>
using aerobus::stirling_2_t = typedef typename internal::stirling_2_helper<T, n, k>::type
```

Stirling number of second king – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

**6.1.2.46 sub\_t**

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

**Template Parameters**

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

**6.1.2.47 tan**

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.48 tanh**

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.49 taylor**

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

**Template Parameters**

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff<sub>↔</sub> _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

### 6.1.2.50 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.2.51 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

### 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

#### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

#### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

### 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

##### Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >



- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **fma\_helper**
- struct **fma\_helper**< double >
- struct **fma\_helper**< float >
- struct **fma\_helper**< int16\_t >
- struct **fma\_helper**< int32\_t >

- struct **fma\_helper**< int64\_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**
  - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling\_1\_helper**
- struct **stirling\_1\_helper**< T, 0, 0 >
- struct **stirling\_1\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, k, std::enable\_if\_t<(k > 0) && (n > 0)> >
- struct **stirling\_2\_helper**
- struct **stirling\_2\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >

- struct **stirling\_2\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling\_2\_helper**< T, n, n, std::enable\_if\_t<(n >=0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **touchard\_coeff**
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N>{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

## 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

## 6.2.2 Typedef Documentation

### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Enumeration Type Documentation

#### 6.3.2.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```

#### Enumerator

probabilist	
physicist	

# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename Ring::zero

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using [type](#) = [aN](#)

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

#### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

##### Template Parameters

<i>...values</i>	are <code>int64_t</code>
------------------	-----------------------------

#### Examples

[examples/continued\\_fractions.cpp](#).



The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
represented value as double

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

#### Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

### 8.5.2 Member Typedef Documentation

#### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template [add\\_t](#)< typename q64::template inject\_constant\_t< a0 >, typename q64::template [div\\_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

#### Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.8.1 Member Function Documentation

### 8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 **aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference**

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.9.1 Member Function Documentation

### 8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

### Public Types

- template<typename val >  
using [type](#) = [i64::val](#)< static\_cast< int64\_t >(val::v)>  
*the [i64](#) representation of val*

### 8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

### 8.11.2 Member Typedef Documentation

#### 8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

### Template Parameters

<i>val</i>	a value in <a href="#">i32</a>
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = `typename at_low< v, typename internal::make\_index\_sequence\_reverse< v::degree+1 > >::type`  
*the polynomial<Large> representation of v*

### 8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

### Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

### 8.12.2 Member Typedef Documentation

#### 8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

## Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

## Public Types

- `template<typename v >`  
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`  
*q64 representation of v*

### 8.13.1 Detailed Description

embeds q32 into q64

### 8.13.2 Member Typedef Documentation

#### 8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

## Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = typename val::raw_t`  
*Ring representation of val.*

### 8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`



## 8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = typename `FractionField< Ring >::template val< v, typename Ring::one >`  
*FractionField<Ring> representation of v.*

### 8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

<i>Ring</i>	an integers ring, such as <a href="#">i32</a>
-------------	---

### 8.15.2 Member Typedef Documentation

#### 8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

#### Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the i32 representation of val*

### 8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

#### Template Parameters

<code>x</code>	an integer
----------------	------------

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

#### Template Parameters

<code>val</code>	a value in zpz<x>
------------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.17 aerobus::polynomial< Ring >::horner\_reduction\_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

## Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

### 8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

#### Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

## Public Types

- using [inner\\_type](#) = int32\_t
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*inject a native constant*
- template<typename v >  
using [inject\\_ring\\_t](#) = v
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator yields v1 + v2*

- `template<typename v1 , typename v2 >`  
`using sub\_t = typename sub< v1, v2 >::type`  
*subtraction operator yields  $v1 - v2$*
- `template<typename v1 , typename v2 >`  
`using mul\_t = typename mul< v1, v2 >::type`  
*multiplication operator yields  $v1 * v2$*
- `template<typename v1 , typename v2 >`  
`using div\_t = typename div< v1, v2 >::type`  
*division operator yields  $v1 / v2$*
- `template<typename v1 , typename v2 >`  
`using mod\_t = typename remainder< v1, v2 >::type`  
*modulus operator yields  $v1 \% v2$*
- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator ( $v1 < v2$ ) yields  $v1 < v2$*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i32, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator (boolean value)*
- `template<typename v >`  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity (boolean value) yields  $v > 0$  as boolean value*

## 8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

### Examples

[examples/compensated\\_horner.cpp](#).

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::add_t = typename add<v1, v2>::type
```

addition operator yields  $v1 + v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.2 div\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields  $v1 / v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.3 eq\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.18.2.4 gcd\_t

```
template<typename v1 , typename v2 >  
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields  $GCD(v1, v2)$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.5 gt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.6 inject\_constant\_t**

```
template<auto x>
using aerobus::i32::inject\_constant\_t = val<static_cast<int32_t>(x)>
```

inject a native constant

**Template Parameters**

<i>x</i>	
----------	--

**8.18.2.7 inject\_ring\_t**

```
template<typename v >
using aerobus::i32::inject\_ring\_t = v
```

**8.18.2.8 inner\_type**

```
using aerobus::i32::inner\_type = int32_t
```

**8.18.2.9 lt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::lt\_t = typename lt<v1, v2>::type
```

strict less operator ( $v1 < v2$ ) yields  $v1 < v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

## 8.18.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

## 8.18.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields  $v1 * v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

## 8.18.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

## 8.18.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`

## Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

#### 8.18.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields  $v1 - v2$

##### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

#### 8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.18.3 Member Data Documentation

#### 8.18.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

##### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.18.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.18.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field



#### 8.18.3.4 pos\_v

```
template<typename v >  
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields  $v > 0$  as boolean value

### Template Parameters

<code>v</code>	a value in <a href="#">i32</a>
----------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
values in [i64](#)

### Public Types

- using [inner\\_type](#) = `int64_t`  
*type of represented values*
- template<auto x>  
using [inject\\_constant\\_t](#) = `val< static_cast< int64_t >(x)>`  
*injects constant as an [i64](#) value*
- template<typename v >  
using [inject\\_ring\\_t](#) = v  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> [i64::val<1>](#)*
- using [zero](#) = `val< 0 >`  
*constant zero*
- using [one](#) = `val< 1 >`  
*constant one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = `typename add< v1, v2 >::type`  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = `typename sub< v1, v2 >::type`  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = `typename mul< v1, v2 >::type`  
*multiplication operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = `typename div< v1, v2 >::type`  
*division operator integer division*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = `typename remainder< v1, v2 >::type`

*modulus operator*

- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i64, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt\_v = gt\_t<v1, v2>::value`  
*strictly greater operator yields  $v1 > v2$  as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt\_v = lt\_t<v1, v2>::value`  
*strictly smaller operator yields  $v1 < v2$  as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator yields  $v1 == v2$  as boolean value*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity yields  $v > 0$  as boolean value*

### 8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.2 div\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::div\_t = typename div<v1, v2>::type
```

division operator integer division

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.3 eq\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::eq\_t = typename eq<v1, v2>::type
```

equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.4 gcd\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd\_t = gcd\_t<i64, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)` as instantiation of [i64::val](#)

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.5 gt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gt\_t = typename gt<v1, v2>::type
```

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

#### Template Parameters

<code>x</code>	
----------------	--

#### 8.19.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
 $\rightarrow$  [i64::val<1>](#)

#### Template Parameters

<code>v</code>	a value in <a href="#">i64</a>
----------------	--------------------------------

#### 8.19.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

#### 8.19.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.10 mod\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

**8.19.2.12 one**

```
using aerobus::i64::one = val<1>
```

constant one

**8.19.2.13 pos\_t**

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

**Template Parameters**

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
-----------	---

**8.19.2.14 sub\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

substraction operator

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.19.3 Member Data Documentation

## 8.19.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields `v1 == v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields `v1 > v2` as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

## 8.19.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.19.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

### 8.19.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields  $v_1 < v_2$  as boolean value

#### Template Parameters

$v_1$	: an element of <a href="#">aerobus::i64::val</a>
$v_2$	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity yields  $v > 0$  as boolean value

#### Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a>
-----	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop >::template type< type-name Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↔ at_t< P::degree - index > >, x >`



## 8.20.1 Member Typedef Documentation

### 8.20.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_←
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
`using type = accum`

## 8.21.1 Member Typedef Documentation

### 8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool [value](#) = internal::\_is\_prime<n, 5>::value  
*true iff n is prime*

## 8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

<i>n</i>	
----------	--

## 8.22.2 Member Data Documentation

### 8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [horner\\_reduction\\_t](#)  
*Used to evaluate polynomials over a value in Ring.*
- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

## Public Types

- using `zero` = `val`< typename Ring::zero >  
*constant zero*
- using `one` = `val`< typename Ring::one >  
*constant one*
- using `X` = `val`< typename Ring::one, typename Ring::zero >  
*generator*
- template<typename P >  
using `simplify_t` = typename simplify< P >::type  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- template<typename v1, typename v2 >  
using `add_t` = typename add< v1, v2 >::type  
*adds two polynomials*
- template<typename v1, typename v2 >  
using `sub_t` = typename sub< v1, v2 >::type  
*subtraction of two polynomials*
- template<typename v1, typename v2 >  
using `mul_t` = typename mul< v1, v2 >::type  
*multiplication of two polynomials*
- template<typename v1, typename v2 >  
using `eq_t` = typename eq\_helper< v1, v2 >::type  
*equality operator*
- template<typename v1, typename v2 >  
using `lt_t` = typename lt\_helper< v1, v2 >::type  
*strict less operator*
- template<typename v1, typename v2 >  
using `gt_t` = typename gt\_helper< v1, v2 >::type  
*strict greater operator*
- template<typename v1, typename v2 >  
using `div_t` = typename div< v1, v2 >::q\_type  
*division operator*
- template<typename v1, typename v2 >  
using `mod_t` = typename div\_helper< v1, v2, `zero`, v1 >::mod\_type  
*modulo operator*
- template<typename coeff, size\_t deg>  
using `monomial_t` = typename monomial< coeff, deg >::type  
*monomial : coeff X^deg*
- template<typename v >  
using `derive_t` = typename derive\_helper< v >::type  
*derivation operator*
- template<typename v >  
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >  
*checks for positivity (an > 0)*
- template<typename v1, typename v2 >  
using `gcd_t` = std::conditional\_t< Ring::is\_euclidean\_domain, typename make\_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 >::type, void >  
*greatest common divisor of two polynomials*
- template<auto x>  
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >  
*makes the constant (native type) polynomial a\_0*
- template<typename v >  
using `inject_ring_t` = `val`< v >  
*makes the constant (ring type) polynomial a\_0*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false
- static constexpr bool [is\\_euclidean\\_domain](#) = Ring::is\_euclidean\_domain
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator*

### 8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

#### Examples

[examples/compensated\\_horner.cpp](#), [examples/make\\_polynomial.cpp](#), and [examples/modular\\_arithmetic.cpp](#).

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 8.23.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.5 gcd\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.7 inject\_constant\_t**

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial `a_0`

**Template Parameters**

<i>x</i>	
----------	--

**8.23.2.8 inject\_ring\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial `a_0`

**Template Parameters**

<i>v</i>	
----------	--

**8.23.2.9 lt\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.10 mod\_t**

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

#### 8.23.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

#### 8.23.2.14 pos\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity ( $an > 0$ )



## Template Parameters

<i>v</i>	
----------	--

**8.23.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

## Template Parameters

<i>P</i>	
----------	--

**8.23.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

**8.23.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.23.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

### 8.23.3 Member Data Documentation

#### 8.23.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.23.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.23.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in <a href="#">polynomial::val</a>
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.24 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.24.2 Member Typedef Documentation

### 8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*zero value*
- using [one](#) = [val](#)< typename Ring::one >  
*one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = [val](#)< typename Ring::template [add\\_t](#)< typename v1::type, typename v2::type > >  
*addition operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = [val](#)< typename Ring::template [mul\\_t](#)< typename v1::type, typename v2::type > >  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = [val](#)< typename Ring::template [div\\_t](#)< typename v1::type, typename v2::type > >  
*division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = [val](#)< typename Ring::template [mod\\_t](#)< typename v1::type, typename v2::type > >

- modulus operator*  
 • `template<typename v1 , typename v2 >`  
   `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
   *equality operator (as type)*
- `template<typename v1 >`  
   `using pos_t = std::true_type`  
   *positivity operator always true*
- `template<auto x>`  
   `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`  
   *inject a 'constant' in quotient ring\**
- `template<typename v >`  
   `using inject_ring_t = val< v >`  
   *projects a value of Ring onto the quotient*

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
   `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
   *addition operator (as boolean value)*
- `template<typename v >`  
   `static constexpr bool pos_v = pos_t<v>::value`  
   *positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
   *quotien rings are euclidean domain*

### 8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as ' <a href="#">i32</a> ', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1<
::type, typename v2::type> >
```

addition operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.25.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring\*

## Template Parameters

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

### 8.25.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

#### Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

### 8.25.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.25.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.25.3 Member Data Documentation

### 8.25.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.25.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.26.2 Member Typedef Documentation

#### 8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```



The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.27 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = [internal::type\\_at\\_t](#)< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename [concat\\_h](#)< U >::type  
*concatenates two list into one*
- template<typename T, size\_t index>  
using [insert](#) = typename [internal::insert\\_h](#)< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename [internal::remove\\_h](#)< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*

### Static Public Attributes

- static constexpr size\_t [length](#) = sizeof...(Ts)  
*length of list*

### 8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

## Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

## 8.27.2 Member Typedef Documentation

### 8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

## Template Parameters

<i>U</i>	
----------	--

### 8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

## Template Parameters

<i>index</i>	
<i>T</i>	

### 8.27.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.5 push\_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

<i>index</i>	
--------------	--

## 8.27.3 Member Data Documentation

### 8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.28 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t [length](#) = 0

### 8.28.1 Detailed Description

specialization for empty type list

## 8.28.2 Member Typedef Documentation

### 8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

### 8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

### 8.28.2.3 push\_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

### 8.28.2.4 push\_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

### 8.28.3 Member Data Documentation

#### 8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [i32](#)  
*Enclosing ring type.*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [DEVICE](#) valueType [get](#) ()  
*cast x into valueType*
- static std::string [to\\_string](#) ()  
*string representation of value*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x  
*actual value stored in val type*

### 8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 8.29.2 Member Typedef Documentation

### 8.29.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

### 8.29.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.29.3 Member Function Documentation

### 8.29.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

## Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

### 8.29.3.2 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

## 8.29.4 Member Data Documentation

### 8.29.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using [inner\\_type](#) = int32\_t  
*type of represented values*
- using [enclosing\\_type](#) = [i64](#)  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED\\_DEVICE](#) valueType [get](#) ()  
*cast value in valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int64\_t [v](#) = x  
*actual value*

### 8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

<a href="#">x</a>	an actual integer
-------------------	-------------------

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.30.2 Member Typedef Documentation

### 8.30.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.30.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.30.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.30.3 Member Function Documentation

### 8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

#### Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.30.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation



## 8.30.4 Member Data Documentation

### 8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

### Public Types

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = polynomial< Ring >  
*enclosing ring type*
- using [aN](#) = coeffN  
*heavy weight coefficient (non zero)*
- using [strip](#) = val< coeffs... >  
*remove largest coefficient*
- using [is\\_zero\\_t](#) = std::bool\_constant<(degree==0) &&(aN::is\_zero\_t::value)>  
*true\_type if polynomial is constant zero*
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename coeff\_at< index >::type  
*type of coefficient at index*
- template<typename x >  
using [value\\_at\\_t](#) = horner\_reduction\_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >

### Static Public Member Functions

- static std::string [to\\_string](#) ()  
*get a string representation of polynomial*
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)  
*evaluates polynomial seen as a function operating on arithmeticType*
- template<typename arithmeticType >  
static [DEVICE INLINED](#) arithmeticType [compensated\\_eval](#) (const arithmeticType &x)  
*Evaluate polynomial on x using compensated horner scheme.*

## Static Public Attributes

- static constexpr size\_t [degree](#) = sizeof...(coeffs)  
*degree of the polynomial*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value  
*true if polynomial is constant zero*

### 8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

#### Examples

[examples/compensated\\_horner.cpp](#).

### 8.31.2 Member Typedef Documentation

#### 8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

#### 8.31.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.31.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>

enclosing ring type
```

### 8.31.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>

true_type if polynomial is constant zero
```

### 8.31.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring

ring coefficients live in
```

### 8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>

remove largest coefficient
```

### 8.31.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

## 8.31.3 Member Function Documentation

### 8.31.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↵
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

**Template Parameters**

<i>arithmeticType</i>	float for example
-----------------------	-------------------

**Parameters**

x	
---	--

**8.31.3.2 eval()**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

**Template Parameters**

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

**Parameters**

x	value
---	-------

**Returns**

P(x)

**8.31.3.3 to\_string()**

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

**Returns**

something like  $a_n X^n + \dots + a_1 X + a_0$

### 8.31.4 Member Data Documentation

#### 8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

#### 8.31.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- using [raw\\_t](#) = V
- using [type](#) = [abs\\_t](#)< typename Ring::template [mod\\_t](#)< V, X > >

### 8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

V	a value from 'Ring'
---	---------------------

## 8.32.2 Member Typedef Documentation

### 8.32.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [zpz](#)< p >  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< [v](#)==0 >  
*true\_type if zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED DEVICE](#) valueType [get](#) ()  
*get value as valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x % p  
*actual value*
- static constexpr bool [is\\_zero\\_v](#) = [v](#) == 0  
*true if zero*

### 8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

## Template Parameters

x	an integer
---	------------

## 8.33.2 Member Typedef Documentation

### 8.33.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.33.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.33.3 Member Function Documentation

### 8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

## Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

### 8.33.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

**Returns**

a string representation

**8.33.4 Member Data Documentation****8.33.4.1 is\_zero\_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

**8.33.4.2 v**

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

**8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference**

specialization for constants

```
#include <aerobus.h>
```

**Classes**

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

**Public Types**

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = [polynomial< Ring >](#)  
*enclosing ring type*
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is\\_zero\\_t](#) = std::bool\_constant< [aN::is\\_zero\\_t::value](#) >
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename [coeff\\_at< index >::type](#)
- template<typename x >  
using [value\\_at\\_t](#) = [coeffN](#)



### Static Public Member Functions

- static std::string [to\\_string](#) ()
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >  
static [DEVICE INLINED](#) arithmeticType [compensated\\_eval](#) (const arithmeticType &x)

### Static Public Attributes

- static constexpr size\_t [degree](#) = 0  
*degree*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value

## 8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

## 8.34.2 Member Typedef Documentation

### 8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.34.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↔
::type
```

### 8.34.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

#### 8.34.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

#### 8.34.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

#### 8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

#### 8.34.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

### 8.34.3 Member Function Documentation

#### 8.34.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

#### 8.34.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

### 8.34.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

## 8.34.4 Member Data Documentation

### 8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

### 8.34.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in zpz*

## Public Types

- using `inner_type` = `int32_t`  
*underlying type for values*
- template<auto x>  
using `inject_constant_t` = `val`< `static_cast`< `int32_t` >(x)>  
*injects a constant integer into mpz*
- using `zero` = `val`< 0 >  
*zero value*
- using `one` = `val`< 1 >  
*one value*
- template<typename v1 , typename v2 >  
using `add_t` = `typename add`< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using `sub_t` = `typename sub`< v1, v2 >::type  
*subtraction operator*
- template<typename v1 , typename v2 >  
using `mul_t` = `typename mul`< v1, v2 >::type  
*multiplication operator*
- template<typename v1 , typename v2 >  
using `div_t` = `typename div`< v1, v2 >::type  
*division operator*
- template<typename v1 , typename v2 >  
using `mod_t` = `typename remainder`< v1, v2 >::type  
*modulo operator*
- template<typename v1 , typename v2 >  
using `gt_t` = `typename gt`< v1, v2 >::type  
*strictly greater operator (type)*
- template<typename v1 , typename v2 >  
using `lt_t` = `typename lt`< v1, v2 >::type  
*strictly smaller operator (type)*
- template<typename v1 , typename v2 >  
using `eq_t` = `typename eq`< v1, v2 >::type  
*equality operator (type)*
- template<typename v1 , typename v2 >  
using `gcd_t` = `gcd_t`< `i32`, v1, v2 >  
*greatest common divisor*
- template<typename v1 >  
using `pos_t` = `typename pos`< v1 >::type  
*positivity operator (type)*

## Static Public Attributes

- static constexpr bool `is_field` = `is_prime`<p>::value  
*true iff p is prime*
- static constexpr bool `is_euclidean_domain` = true  
*always true*
- template<typename v1 , typename v2 >  
static constexpr bool `gt_v` = `gt_t`<v1, v2>::value  
*strictly greater operator (booleanvalue)*

- `template<typename v1 , typename v2 >`  
`static constexpr bool lt\_v = lt\_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq\_v = eq\_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos\_v = pos\_t<v>::value`  
*positivity operator (boolean value)*

### 8.35.1 Detailed Description

`template<int32_t p>`  
`struct aerobus::zpz< p >`

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

Template Parameters

<i>p</i>	a integer
----------	-----------

Examples

[examples/modular\\_arithmetic.cpp](#), and [examples/polynomials\\_over\\_finite\\_field.cpp](#).

### 8.35.2 Member Typedef Documentation

#### 8.35.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add\_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

#### 8.35.2.2 div\_t

```
template<int32_t p>
```

```
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

#### Template Parameters

x	an integer
---	------------

### 8.35.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

### 8.35.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

### 8.35.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
----	-------------------------------------

### 8.35.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>



### 8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.35.3 Member Data Documentation

### 8.35.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.35.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.35.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 0) & 0x800000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED_DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED_DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED_DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<float> {
00224         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00225             return x * y + z;
00226         }
00227     };
00228
00229     template<>
00230     struct fma_helper<int32_t> {
00231         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00232             return x * y + z;
00233         }
00234     };
00235
00236     template<>
00237     struct fma_helper<int16_t> {
00238         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00239             return x * y + z;
00240         }
00241     };
00242
00243     template<>
00244     struct fma_helper<int64_t> {
00245         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00246             return x * y + z;
00247         }
00248     };
00249
00250     #ifdef WITH_CUDA_FP16
00251     template<>
00252     struct fma_helper<__half> {
00253         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00254             #ifdef __CUDA_ARCH__
00255                 return __hfma(x, y, z);
00256             #else
00257                 return x * y + z;
00258             #endif
00259         }
00260     };
00261     template<>
00262     struct fma_helper<__half2> {
00263         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00264             #ifdef __CUDA_ARCH__
00265                 return __hfma2(x, y, z);
00266             #else
00267                 return x * y + z;
00268             #endif
00269         }
00270     };
00271     #endif
00272 } // namespace internal
00273 } // namespace aerobus
00274
00275 // compensated horner utilities
00276 namespace aerobus {
00277     namespace internal {
00278         template <typename T>
00279         struct FloatLayout;
00280
00281         template <>
00282         struct FloatLayout<double> {
00283             static constexpr uint8_t exponent = 11;
00284             static constexpr uint8_t mantissa = 53;
00285             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00286         };
00287
00288         template <>
00289         struct FloatLayout<float> {
00290             static constexpr uint8_t exponent = 8;
00291             static constexpr uint8_t mantissa = 24;
00292             static constexpr uint8_t r = 11; // ceil(mantissa/2)
00293         };
00294
00295         #ifdef WITH_CUDA_FP16

```

```

00296     template <>
00297     struct FloatLayout<__half> {
00298         static constexpr uint8_t exponent = 5;
00299         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00300         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00301     };
00302 #endif
00303
00304     template<typename T>
00305     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00306         T z = a * ((1 < FloatLayout<T>::r) + 1);
00307         *x = z - (z - a);
00308         *y = a - *x;
00309     }
00310
00311     template<typename T>
00312     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00313         *x = a + b;
00314         T z = *x - a;
00315         *y = (a - (*x - z)) + (b - z);
00316     }
00317
00318     template<typename T>
00319     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00320         *x = a * b;
00321         #ifdef __clang__
00322         *y = fma_helper<T>::eval(a, b, -*x);
00323         #else
00324         T ah, al, bh, bl;
00325         split(a, &ah, &al);
00326         split(b, &bh, &bl);
00327         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00328         #endif
00329     }
00330
00331     template<typename T, size_t N>
00332     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00333         T r = p1[0] + p2[0];
00334         for (int64_t i = N - 1; i >= 0; --i) {
00335             r = r * x + p1[N - i] + p2[N - i];
00336         }
00337     }
00338     return r;
00339 }
00340 } // namespace internal
00341 } // namespace aerobus
00342
00343 // utilities
00344 namespace aerobus {
00345     namespace internal {
00346         template<template<typename...> typename TT, typename T>
00347         struct is_instantiation_of : std::false_type { };
00348
00349         template<template<typename...> typename TT, typename... Ts>
00350         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00351
00352         template<template<typename...> typename TT, typename T>
00353         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00354
00355         template<int64_t i, typename T, typename... Ts>
00356         struct type_at {
00357             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00358             using type = typename type_at<i - 1, Ts...>::type;
00359         };
00360
00361         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00362             using type = T;
00363         };
00364
00365         template<size_t i, typename... Ts>
00366         using type_at_t = typename type_at<i, Ts...>::type;
00367
00368         template<size_t n, size_t i, typename E = void>
00369         struct _is_prime { };
00370
00371         template<size_t i>
00372         struct _is_prime<0, i> {
00373             static constexpr bool value = false;
00374         };
00375
00376         template<size_t i>
00377         struct _is_prime<1, i> {
00378             static constexpr bool value = false;
00379         };
00380     }
00381
00382     template<size_t i>

```

```

00383     struct _is_prime<2, i> {
00384         static constexpr bool value = true;
00385     };
00386
00387     template<size_t i>
00388     struct _is_prime<3, i> {
00389         static constexpr bool value = true;
00390     };
00391
00392     template<size_t i>
00393     struct _is_prime<5, i> {
00394         static constexpr bool value = true;
00395     };
00396
00397     template<size_t i>
00398     struct _is_prime<7, i> {
00399         static constexpr bool value = true;
00400     };
00401
00402     template<size_t n, size_t i>
00403     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00404         static constexpr bool value = false;
00405     };
00406
00407     template<size_t n, size_t i>
00408     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00409         static constexpr bool value = false;
00410     };
00411
00412     template<size_t n, size_t i>
00413     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00414         static constexpr bool value = true;
00415     };
00416
00417     template<size_t n, size_t i>
00418     struct _is_prime<n, i, std::enable_if_t<(
00419         n % i == 0 &&
00420         n >= 9 &&
00421         n % 3 != 0 &&
00422         n % 2 != 0 &&
00423         i * i > n)>> {
00424         static constexpr bool value = true;
00425     };
00426
00427     template<size_t n, size_t i>
00428     struct _is_prime<n, i, std::enable_if_t<(
00429         n % (i+2) == 0 &&
00430         n >= 9 &&
00431         n % 3 != 0 &&
00432         n % 2 != 0 &&
00433         i * i <= n)>> {
00434         static constexpr bool value = true;
00435     };
00436
00437     template<size_t n, size_t i>
00438     struct _is_prime<n, i, std::enable_if_t<(
00439         n % (i+2) != 0 &&
00440         n % i != 0 &&
00441         n >= 9 &&
00442         n % 3 != 0 &&
00443         n % 2 != 0 &&
00444         (i * i <= n))>> {
00445         static constexpr bool value = _is_prime<n, i+6>::value;
00446     };
00447 } // namespace internal
00448
00449 template<size_t n>
00450 struct is_prime {
00451     static constexpr bool value = internal::_is_prime<n, 5>::value;
00452 };
00453
00454 template<size_t n>
00455 static constexpr bool is_prime_v = is_prime<n>::value;
00456
00457 // gcd
00458 namespace internal {
00459     template <std::size_t... Is>
00460     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&
00461         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{}));
00462
00463     template <std::size_t N>
00464     using make_index_sequence_reverse
00465         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00466
00467     template<typename Ring, typename E = void>
00468     struct gcd;
00469
00470
00471
00472
00473
00474
00475
00476
00477
00478
00479
00480

```



```

00481     template<typename Ring>
00482     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00483         template<typename A, typename B, typename E = void>
00484         struct gcd_helper {};
00485
00486         // B = 0, A > 0
00487         template<typename A, typename B>
00488         struct gcd_helper<A, B, std::enable_if_t<
00489             (B::is_zero_t::value) &&
00490             (Ring::template gt_t<A, typename Ring::zero>::value)> {
00491             using type = A;
00492         };
00493
00494         // B = 0, A < 0
00495         template<typename A, typename B>
00496         struct gcd_helper<A, B, std::enable_if_t<
00497             (B::is_zero_t::value) &&
00498             !(Ring::template gt_t<A, typename Ring::zero>::value)> {
00499             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00500         };
00501
00502         // B != 0
00503         template<typename A, typename B>
00504         struct gcd_helper<A, B, std::enable_if_t<
00505             (!B::is_zero_t::value)
00506             > {
00507         private: // NOLINT
00508             // A / B
00509             using k = typename Ring::template div_t<A, B>;
00510             // A - (A/B)*B = A % B
00511             using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00512
00513         public:
00514             using type = typename gcd_helper<B, m>::type;
00515         };
00516
00517         template<typename A, typename B>
00518         using type = typename gcd_helper<A, B>::type;
00519     };
00520 } // namespace internal
00521
00522 // vadd and vmul
00523 namespace internal {
00524     template<typename... vals>
00525     struct vmul {};
00526
00527     template<typename v1, typename... vals>
00528     struct vmul<v1, vals...> {
00529         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00530     };
00531
00532     template<typename v1>
00533     struct vmul<v1> {
00534         using type = v1;
00535     };
00536
00537     template<typename... vals>
00538     struct vadd {};
00539
00540     template<typename v1, typename... vals>
00541     struct vadd<v1, vals...> {
00542         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00543     };
00544
00545     template<typename v1>
00546     struct vadd<v1> {
00547         using type = v1;
00548     };
00549 } // namespace internal
00550
00553     template<typename T, typename A, typename B>
00554     using gcd_t = typename internal::gcd<T>::template type<A, B>;
00555
00559     template<typename... vals>
00560     using vadd_t = typename internal::vadd<vals...>::type;
00561
00565     template<typename... vals>
00566     using vmul_t = typename internal::vmul<vals...>::type;
00567
00571     template<typename val>
00572     requires IsEuclideanDomain<typename val::enclosing_type>
00573     using abs_t = std::conditional_t<
00574         val::enclosing_type::template pos_v<val>,
00575         val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;

```

```

00576 } // namespace aerobus
00577
00578 // embedding
00579 namespace aerobus {
00584     template<typename Small, typename Large, typename E = void>
00585     struct Embed;
00586 } // namespace aerobus
00587
00588 namespace aerobus {
00593     template<typename Ring, typename X>
00594     requires IsRing<Ring>
00595     struct Quotient {
00598         template <typename V>
00599         struct val {
00600             public:
00601                 using raw_t = V;
00602                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00603             };
00604
00606             using zero = val<typename Ring::zero>;
00607
00609             using one = val<typename Ring::one>;
00610
00614             template<typename v1, typename v2>
00615             using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00616
00620             template<typename v1, typename v2>
00621             using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00622
00626             template<typename v1, typename v2>
00627             using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00628
00632             template<typename v1, typename v2>
00633             using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00634
00638             template<typename v1, typename v2>
00639             using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00640
00644             template<typename v1, typename v2>
00645             static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00646
00650             template<typename v1>
00651             using pos_t = std::true_type;
00652
00656             template<typename v>
00657             static constexpr bool pos_v = pos_t<v>::value;
00658
00660             static constexpr bool is_euclidean_domain = true;
00661
00665             template<auto x>
00666             using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00667
00671             template<typename v>
00672             using inject_ring_t = val<v>;
00673         };
00674
00678         template<typename Ring, typename X>
00679         struct Embed<Quotient<Ring, X>, Ring> {
00682             template<typename val>
00683             using type = typename val::raw_t;
00684         };
00685     } // namespace aerobus
00686
00687 // type_list
00688 namespace aerobus {
00690     template <typename... Ts>
00691     struct type_list;
00692
00693     namespace internal {
00694         template <typename T, typename... Us>
00695         struct pop_front_h {
00696             using tail = type_list<Us...>;
00697             using head = T;
00698         };
00699
00700         template <size_t index, typename L1, typename L2>
00701         struct split_h {
00702             private:
00703                 static_assert(index <= L2::length, "index out of bounds");
00704                 using a = typename L2::pop_front::type;
00705                 using b = typename L2::pop_front::tail;
00706                 using c = typename L1::template push_back<a>;
00707
00708             public:
00709                 using head = typename split_h<index - 1, c, b>::head;
00710                 using tail = typename split_h<index - 1, c, b>::tail;
00711         };

```

```

00712
00713     template <typename L1, typename L2>
00714     struct split_h<0, L1, L2> {
00715         using head = L1;
00716         using tail = L2;
00717     };
00718
00719     template <size_t index, typename L, typename T>
00720     struct insert_h {
00721         static_assert(index <= L::length, "index out of bounds");
00722         using s = typename L::template split<index>;
00723         using left = typename s::head;
00724         using right = typename s::tail;
00725         using ll = typename left::template push_back<T>;
00726         using type = typename ll::template concat<right>;
00727     };
00728
00729     template <size_t index, typename L>
00730     struct remove_h {
00731         using s = typename L::template split<index>;
00732         using left = typename s::head;
00733         using right = typename s::tail;
00734         using rr = typename right::pop_front::tail;
00735         using type = typename left::template concat<rr>;
00736     };
00737 } // namespace internal
00738
00741 template <typename... Ts>
00742 struct type_list {
00743 private:
00744     template <typename T>
00745     struct concat_h;
00746
00747     template <typename... Us>
00748     struct concat_h<type_list<Us...> {
00749         using type = type_list<Ts..., Us...>;
00750     };
00751
00752 public:
00754     static constexpr size_t length = sizeof...(Ts);
00755
00758     template <typename T>
00759     using push_front = type_list<T, Ts...>;
00760
00763     template <size_t index>
00764     using at = internal::type_at_t<index, Ts...>;
00765
00767     struct pop_front {
00769         using type = typename internal::pop_front_h<Ts...>::head;
00771         using tail = typename internal::pop_front_h<Ts...>::tail;
00772     };
00773
00776     template <typename T>
00777     using push_back = type_list<Ts..., T>;
00778
00781     template <typename U>
00782     using concat = typename concat_h<U>::type;
00783
00786     template <size_t index>
00787     struct split {
00788     private:
00789         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00790
00791     public:
00792         using head = typename inner::head;
00793         using tail = typename inner::tail;
00794     };
00795
00799     template <typename T, size_t index>
00800     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00801
00804     template <size_t index>
00805     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00806 };
00807
00809 template <>
00810 struct type_list<> {
00811     static constexpr size_t length = 0;
00812
00813     template <typename T>
00814     using push_front = type_list<T>;
00815
00816     template <typename T>
00817     using push_back = type_list<T>;
00818
00819     template <typename U>
00820     using concat = U;

```

```

00821
00822     // TODO(jewave): assert index == 0
00823     template <typename T, size_t index>
00824     using insert = type_list<T>;
00825 };
00826 } // namespace aerobus
00827
00828 // i16
00829 #ifdef WITH_CUDA_FP16
00830 // i16
00831 namespace aerobus {
00832     struct i16 {
00833         using inner_type = int16_t;
00834         template<int16_t x>
00835         struct val {
00836             using enclosing_type = i16;
00837             static constexpr int16_t v = x;
00838
00839             template<typename valueType>
00840             static constexpr INLINED_DEVICE valueType get() {
00841                 return internal::template int16_convert_helper<valueType, x>::value();
00842             }
00843
00844             using is_zero_t = std::bool_constant<x == 0>;
00845
00846             static std::string to_string() {
00847                 return std::to_string(x);
00848             }
00849         };
00850     };
00851
00852     using zero = val<0>;
00853     using one = val<1>;
00854     static constexpr bool is_field = false;
00855     static constexpr bool is_euclidean_domain = true;
00856     template<auto x>
00857     using inject_constant_t = val<static_cast<int16_t>(x)>;
00858
00859     template<typename v>
00860     using inject_ring_t = v;
00861
00862 private:
00863     template<typename v1, typename v2>
00864     struct add {
00865         using type = val<v1::v + v2::v>;
00866     };
00867
00868     template<typename v1, typename v2>
00869     struct sub {
00870         using type = val<v1::v - v2::v>;
00871     };
00872
00873     template<typename v1, typename v2>
00874     struct mul {
00875         using type = val<v1::v * v2::v>;
00876     };
00877
00878     template<typename v1, typename v2>
00879     struct div {
00880         using type = val<v1::v / v2::v>;
00881     };
00882
00883     template<typename v1, typename v2>
00884     struct remainder {
00885         using type = val<v1::v % v2::v>;
00886     };
00887
00888     template<typename v1, typename v2>
00889     struct gt {
00890         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00891     };
00892
00893     template<typename v1, typename v2>
00894     struct lt {
00895         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00896     };
00897
00898     template<typename v1, typename v2>
00899     struct eq {
00900         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00901     };
00902
00903     template<typename v1>
00904     struct pos {
00905         using type = std::bool_constant<(v1::v > 0)>;
00906     };
00907
00908 public:

```

```

00927     template<typename v1, typename v2>
00928     using add_t = typename add<v1, v2>::type;
00929
00934     template<typename v1, typename v2>
00935     using sub_t = typename sub<v1, v2>::type;
00936
00941     template<typename v1, typename v2>
00942     using mul_t = typename mul<v1, v2>::type;
00943
00948     template<typename v1, typename v2>
00949     using div_t = typename div<v1, v2>::type;
00950
00955     template<typename v1, typename v2>
00956     using mod_t = typename remainder<v1, v2>::type;
00957
00962     template<typename v1, typename v2>
00963     using gt_t = typename gt<v1, v2>::type;
00964
00969     template<typename v1, typename v2>
00970     using lt_t = typename lt<v1, v2>::type;
00971
00976     template<typename v1, typename v2>
00977     using eq_t = typename eq<v1, v2>::type;
00978
00982     template<typename v1, typename v2>
00983     static constexpr bool eq_v = eq_t<v1, v2>::value;
00984
00989     template<typename v1, typename v2>
00990     using gcd_t = gcd_t<i16, v1, v2>;
00991
00995     template<typename v>
00996     using pos_t = typename pos<v>::type;
00997
01001     template<typename v>
01002     static constexpr bool pos_v = pos_t<v>::value;
01003 };
01004 } // namespace aerobus
01005 #endif
01006
01007 // i32
01008 namespace aerobus {
01009     struct i32 {
01010         using inner_type = int32_t;
01011         template<int32_t x>
01012         struct val {
01013             using enclosing_type = i32;
01014             static constexpr int32_t v = x;
01015
01023             template<typename valueType>
01024             static constexpr DEVICE valueType get() {
01025                 return static_cast<valueType>(x);
01026             }
01027
01029             using is_zero_t = std::bool_constant<x == 0>;
01030
01032             static std::string to_string() {
01033                 return std::to_string(x);
01034             }
01035         };
01036
01038         using zero = val<0>;
01040         using one = val<1>;
01042         static constexpr bool is_field = false;
01044         static constexpr bool is_euclidean_domain = true;
01047         template<auto x>
01048         using inject_constant_t = val<static_cast<int32_t>(x)>;
01049
01050         template<typename v>
01051         using inject_ring_t = v;
01052
01053     private:
01054         template<typename v1, typename v2>
01055         struct add {
01056             using type = val<v1::v + v2::v>;
01057         };
01058
01059         template<typename v1, typename v2>
01060         struct sub {
01061             using type = val<v1::v - v2::v>;
01062         };
01063
01064         template<typename v1, typename v2>
01065         struct mul {
01066             using type = val<v1::v * v2::v>;
01067         };
01068
01069         template<typename v1, typename v2>

```

```

01070     struct div {
01071         using type = val<v1::v / v2::v>;
01072     };
01073
01074     template<typename v1, typename v2>
01075     struct remainder {
01076         using type = val<v1::v % v2::v>;
01077     };
01078
01079     template<typename v1, typename v2>
01080     struct gt {
01081         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01082     };
01083
01084     template<typename v1, typename v2>
01085     struct lt {
01086         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01087     };
01088
01089     template<typename v1, typename v2>
01090     struct eq {
01091         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01092     };
01093
01094     template<typename v1>
01095     struct pos {
01096         using type = std::bool_constant<(v1::v > 0)>;
01097     };
01098
01099     public:
01100     template<typename v1, typename v2>
01101     using add_t = typename add<v1, v2>::type;
01102
01103     template<typename v1, typename v2>
01104     using sub_t = typename sub<v1, v2>::type;
01105
01106     template<typename v1, typename v2>
01107     using mul_t = typename mul<v1, v2>::type;
01108
01109     template<typename v1, typename v2>
01110     using div_t = typename div<v1, v2>::type;
01111
01112     template<typename v1, typename v2>
01113     using mod_t = typename remainder<v1, v2>::type;
01114
01115     template<typename v1, typename v2>
01116     using gt_t = typename gt<v1, v2>::type;
01117
01118     template<typename v1, typename v2>
01119     using lt_t = typename lt<v1, v2>::type;
01120
01121     template<typename v1, typename v2>
01122     using eq_t = typename eq<v1, v2>::type;
01123
01124     template<typename v1, typename v2>
01125     static constexpr bool eq_v = eq_t<v1, v2>::value;
01126
01127     template<typename v1, typename v2>
01128     using gcd_t = gcd_t<i32, v1, v2>;
01129
01130     template<typename v>
01131     using pos_t = typename pos<v>::type;
01132
01133     template<typename v>
01134     static constexpr bool pos_v = pos_t<v>::value;
01135 };
01136 } // namespace aerobus
01137
01138 // i64
01139 namespace aerobus {
01140     struct i64 {
01141         using inner_type = int64_t;
01142         template<int64_t x>
01143         struct val {
01144             using inner_type = int32_t;
01145             using enclosing_type = i64;
01146             static constexpr int64_t v = x;
01147
01148             template<typename valueType>
01149             static constexpr INLINED_DEVICE valueType get() {
01150                 return static_cast<valueType>(x);
01151             }
01152
01153             using is_zero_t = std::bool_constant<x == 0>;
01154
01155             static std::string to_string() {
01156                 return std::to_string(x);
01157             }
01158         };
01159     };
01160 }

```

```

01213     }
01214 };
01215
01218     template<auto x>
01219     using inject_constant_t = val<static_cast<int64_t>(x)>;
01220
01225     template<typename v>
01226     using inject_ring_t = v;
01227
01229     using zero = val<0>;
01231     using one = val<1>;
01233     static constexpr bool is_field = false;
01235     static constexpr bool is_euclidean_domain = true;
01236
01237 private:
01238     template<typename v1, typename v2>
01239     struct add {
01240         using type = val<v1::v + v2::v>;
01241     };
01242
01243     template<typename v1, typename v2>
01244     struct sub {
01245         using type = val<v1::v - v2::v>;
01246     };
01247
01248     template<typename v1, typename v2>
01249     struct mul {
01250         using type = val<v1::v * v2::v>;
01251     };
01252
01253     template<typename v1, typename v2>
01254     struct div {
01255         using type = val<v1::v / v2::v>;
01256     };
01257
01258     template<typename v1, typename v2>
01259     struct remainder {
01260         using type = val<v1::v % v2::v>;
01261     };
01262
01263     template<typename v1, typename v2>
01264     struct gt {
01265         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01266     };
01267
01268     template<typename v1, typename v2>
01269     struct lt {
01270         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01271     };
01272
01273     template<typename v1, typename v2>
01274     struct eq {
01275         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01276     };
01277
01278     template<typename v>
01279     struct pos {
01280         using type = std::bool_constant<(v::v > 0)>;
01281     };
01282
01283 public:
01287     template<typename v1, typename v2>
01288     using add_t = typename add<v1, v2>::type;
01289
01293     template<typename v1, typename v2>
01294     using sub_t = typename sub<v1, v2>::type;
01295
01299     template<typename v1, typename v2>
01300     using mul_t = typename mul<v1, v2>::type;
01301
01306     template<typename v1, typename v2>
01307     using div_t = typename div<v1, v2>::type;
01308
01312     template<typename v1, typename v2>
01313     using mod_t = typename remainder<v1, v2>::type;
01314
01319     template<typename v1, typename v2>
01320     using gt_t = typename gt<v1, v2>::type;
01321
01326     template<typename v1, typename v2>
01327     static constexpr bool gt_v = gt_t<v1, v2>::value;
01328
01333     template<typename v1, typename v2>
01334     using lt_t = typename lt<v1, v2>::type;
01335
01340     template<typename v1, typename v2>
01341     static constexpr bool lt_v = lt_t<v1, v2>::value;

```

```

01342
01347     template<typename v1, typename v2>
01348     using eq_t = typename eq<v1, v2>::type;
01349
01354     template<typename v1, typename v2>
01355     static constexpr bool eq_v = eq_t<v1, v2>::value;
01356
01361     template<typename v1, typename v2>
01362     using gcd_t = gcd_t<i64, v1, v2>;
01363
01367     template<typename v>
01368     using pos_t = typename pos<v>::type;
01369
01373     template<typename v>
01374     static constexpr bool pos_v = pos_t<v>::value;
01375 };
01376
01378 template<>
01379 struct Embed<i32, i64> {
01382     template<typename val>
01383     using type = i64::val<static_cast<int64_t>(val::v)>;
01384 };
01385 } // namespace aerobus
01386
01387 // z/pz
01388 namespace aerobus {
01394     template<int32_t p>
01395     struct zpz {
01397         using inner_type = int32_t;
01398
01401         template<int32_t x>
01402         struct val {
01404             using enclosing_type = zpz<p>;
01406             static constexpr int32_t v = x % p;
01407
01410             template<typename valueType>
01411             static constexpr INLINED_DEVICE valueType get() {
01412                 return static_cast<valueType>(x % p);
01413             }
01414
01416             using is_zero_t = std::bool_constant<v == 0>;
01417
01419             static constexpr bool is_zero_v = v == 0;
01420
01423             static std::string to_string() {
01424                 return std::to_string(x % p);
01425             }
01426         };
01427
01430         template<auto x>
01431         using inject_constant_t = val<static_cast<int32_t>(x)>;
01432
01434         using zero = val<0>;
01435
01437         using one = val<1>;
01438
01440         static constexpr bool is_field = is_prime<p>::value;
01441
01443         static constexpr bool is_euclidean_domain = true;
01444
01445     private:
01446         template<typename v1, typename v2>
01447         struct add {
01448             using type = val<(v1::v + v2::v) % p>;
01449         };
01450
01451         template<typename v1, typename v2>
01452         struct sub {
01453             using type = val<(v1::v - v2::v) % p>;
01454         };
01455
01456         template<typename v1, typename v2>
01457         struct mul {
01458             using type = val<(v1::v * v2::v) % p>;
01459         };
01460
01461         template<typename v1, typename v2>
01462         struct div {
01463             using type = val<(v1::v % p) / (v2::v % p)>;
01464         };
01465
01466         template<typename v1, typename v2>
01467         struct remainder {
01468             using type = val<(v1::v % v2::v) % p>;
01469         };
01470
01471         template<typename v1, typename v2>

```



```

01472     struct gt {
01473         using type = std::conditional_t<(v1::v% p > v2::v% p), std::true_type, std::false_type>;
01474     };
01475
01476     template<typename v1, typename v2>
01477     struct lt {
01478         using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01479     };
01480
01481     template<typename v1, typename v2>
01482     struct eq {
01483         using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01484     };
01485
01486     template<typename v1>
01487     struct pos {
01488         using type = std::bool_constant<(v1::v > 0)>;
01489     };
01490
01491 public:
01492     template<typename v1, typename v2>
01493     using add_t = typename add<v1, v2>::type;
01494
01495     template<typename v1, typename v2>
01496     using sub_t = typename sub<v1, v2>::type;
01497
01498     template<typename v1, typename v2>
01499     using mul_t = typename mul<v1, v2>::type;
01500
01501     template<typename v1, typename v2>
01502     using div_t = typename div<v1, v2>::type;
01503
01504     template<typename v1, typename v2>
01505     using mod_t = typename remainder<v1, v2>::type;
01506
01507     template<typename v1, typename v2>
01508     using gt_t = typename gt<v1, v2>::type;
01509
01510     template<typename v1, typename v2>
01511     static constexpr bool gt_v = gt_t<v1, v2>::value;
01512
01513     template<typename v1, typename v2>
01514     using lt_t = typename lt<v1, v2>::type;
01515
01516     template<typename v1, typename v2>
01517     static constexpr bool lt_v = lt_t<v1, v2>::value;
01518
01519     template<typename v1, typename v2>
01520     using eq_t = typename eq<v1, v2>::type;
01521
01522     template<typename v1, typename v2>
01523     static constexpr bool eq_v = eq_t<v1, v2>::value;
01524
01525     template<typename v1, typename v2>
01526     using gcd_t = gcd_t<i32, v1, v2>;
01527
01528     template<typename v1>
01529     using pos_t = typename pos<v1>::type;
01530
01531     template<typename v>
01532     static constexpr bool pos_v = pos_t<v>::value;
01533 };
01534
01535 template<int32_t x>
01536 struct Embed<zp<x>, i32> {
01537     template <typename val>
01538     using type = i32::val<val::v>;
01539 };
01540 } // namespace aerobus
01541
01542 // polynomial
01543 namespace aerobus {
01544     // coeffN x^N + ...
01545     template<typename Ring>
01546     requires IsEuclideanDomain<Ring>
01547     struct polynomial {
01548         static constexpr bool is_field = false;
01549         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01550
01551         template<typename P>
01552         struct horner_reduction_t {
01553             template<size_t index, size_t stop>
01554             struct inner {
01555                 template<typename accum, typename x>
01556                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01557                     ::template type<
01558                         typename Ring::template add_t<

```

```

01609         typename Ring::template mul_t<x, accum>,
01610         typename P::template coeff_at_t<P::degree - index>
01611     >, x>;
01612 };
01613
01614 template<size_t stop>
01615 struct inner<stop, stop> {
01616     template<typename accum, typename x>
01617     using type = accum;
01618 };
01619 };
01620
01624 template<typename coeffN, typename... coeffs>
01625 struct val {
01626     using ring_type = Ring;
01627     using enclosing_type = polynomial<Ring>;
01628     static constexpr size_t degree = sizeof...(coeffs);
01629     using aN = coeffN;
01630     using strip = val<coeffs...>;
01631     using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01632     static constexpr bool is_zero_v = is_zero_t::value;
01633
01634 private:
01635     template<size_t index, typename E = void>
01636     struct coeff_at {};
01637
01638     template<size_t index>
01639     struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01640         using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01641     };
01642
01643     template<size_t index>
01644     struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01645         using type = typename Ring::zero;
01646     };
01647
01648 public:
01649     template<size_t index>
01650     using coeff_at_t = typename coeff_at<index>::type;
01651
01652     static std::string to_string() {
01653         return string_helper<coeffN, coeffs...>::func();
01654     }
01655
01656     template<typename arithmeticType>
01657     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01658         #ifdef WITH_CUDA_FP16
01659         arithmeticType start;
01660         if constexpr (std::is_same_v<arithmeticType, __half2>) {
01661             start = __half2(0, 0);
01662         } else {
01663             start = static_cast<arithmeticType>(0);
01664         }
01665         #else
01666         arithmeticType start = static_cast<arithmeticType>(0);
01667         #endif
01668         return horner_evaluation<arithmeticType, val>
01669             ::template inner<0, degree + 1>
01670             ::func(start, x);
01671     }
01672
01673     template<typename arithmeticType>
01674     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01675         return compensated_horner<arithmeticType, val>::func(x);
01676     }
01677
01678     template<typename x>
01679     using value_at_t = horner_reduction_t<val>
01680         ::template inner<0, degree + 1>
01681         ::template type<typename Ring::zero, x>;
01682 };
01683
01684 template<typename coeffN>
01685 struct val<coeffN> {
01686     using ring_type = Ring;
01687     using enclosing_type = polynomial<Ring>;
01688     static constexpr size_t degree = 0;
01689     using aN = coeffN;
01690     using strip = val<coeffN>;
01691     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01692     static constexpr bool is_zero_v = is_zero_t::value;
01693
01694     template<size_t index, typename E = void>
01695     struct coeff_at {};
01696
01697     template<size_t index>

```

```

01731     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01732         using type = aN;
01733     };
01734
01735     template<size_t index>
01736     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01737         using type = typename Ring::zero;
01738     };
01739
01740     template<size_t index>
01741     using coeff_at_t = typename coeff_at<index>::type;
01742
01743     static std::string to_string() {
01744         return string_helper<coeffN>::func();
01745     }
01746
01747     template<typename arithmeticType>
01748     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01749         return coeffN::template get<arithmeticType>();
01750     }
01751
01752     template<typename arithmeticType>
01753     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01754         return coeffN::template get<arithmeticType>();
01755     }
01756
01757     template<typename x>
01758     using value_at_t = coeffN;
01759 };
01760
01762 using zero = val<typename Ring::zero>;
01764 using one = val<typename Ring::one>;
01766 using X = val<typename Ring::one, typename Ring::zero>;
01767
01768 private:
01769     template<typename P, typename E = void>
01770     struct simplify;
01771
01772     template <typename P1, typename P2, typename I>
01773     struct add_low;
01774
01775     template<typename P1, typename P2>
01776     struct add {
01777         using type = typename simplify<typename add_low<
01778             P1,
01779             P2,
01780             internal::make_index_sequence_reverse<
01781                 std::max(P1::degree, P2::degree) + 1
01782             >::type>::type;
01783     };
01784
01785     template <typename P1, typename P2, typename I>
01786     struct sub_low;
01787
01788     template <typename P1, typename P2, typename I>
01789     struct mul_low;
01790
01791     template<typename v1, typename v2>
01792     struct mul {
01793         using type = typename mul_low<
01794             v1,
01795             v2,
01796             internal::make_index_sequence_reverse<
01797                 v1::degree + v2::degree + 1
01798             >::type;
01799     };
01800
01801     template<typename coeff, size_t deg>
01802     struct monomial;
01803
01804     template<typename v, typename E = void>
01805     struct derive_helper {};
01806
01807     template<typename v>
01808     struct derive_helper<v, std::enable_if_t<v::degree == 0>> {
01809         using type = zero;
01810     };
01811
01812     template<typename v>
01813     struct derive_helper<v, std::enable_if_t<v::degree != 0>> {
01814         using type = typename add<
01815             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01816             typename monomial<
01817                 typename Ring::template mul_t<
01818                     typename v::aN,
01819                     typename Ring::template inject_constant_t<(v::degree)>
01820                 >,

```

```

01821         v::degree - 1
01822         >::type
01823     >::type;
01824 };
01825
01826 template<typename v1, typename v2, typename E = void>
01827 struct eq_helper {};
01828
01829 template<typename v1, typename v2>
01830 struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01831     using type = std::false_type;
01832 };
01833
01834
01835 template<typename v1, typename v2>
01836 struct eq_helper<v1, v2, std::enable_if_t<
01837     v1::degree == v2::degree &&
01838     (v1::degree != 0 || v2::degree != 0) &&
01839     std::is_same<
01840         typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01841         std::false_type
01842     >::value
01843 > {
01844     > {
01845         using type = std::false_type;
01846     };
01847
01848     template<typename v1, typename v2>
01849     struct eq_helper<v1, v2, std::enable_if_t<
01850         v1::degree == v2::degree &&
01851         (v1::degree != 0 || v2::degree != 0) &&
01852         std::is_same<
01853             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01854             std::true_type
01855         >::value
01856     > {
01857         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01858     };
01859
01860     template<typename v1, typename v2>
01861     struct eq_helper<v1, v2, std::enable_if_t<
01862         v1::degree == v2::degree &&
01863         (v1::degree == 0)
01864     > {
01865         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01866     };
01867
01868     template<typename v1, typename v2, typename E = void>
01869     struct lt_helper {};
01870
01871     template<typename v1, typename v2>
01872     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01873         using type = std::true_type;
01874     };
01875
01876     template<typename v1, typename v2>
01877     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01878         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01879     };
01880
01881     template<typename v1, typename v2>
01882     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01883         using type = std::false_type;
01884     };
01885
01886     template<typename v1, typename v2, typename E = void>
01887     struct gt_helper {};
01888
01889     template<typename v1, typename v2>
01890     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01891         using type = std::true_type;
01892     };
01893
01894     template<typename v1, typename v2>
01895     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01896         using type = std::false_type;
01897     };
01898
01899     template<typename v1, typename v2>
01900     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01901         using type = std::false_type;
01902     };
01903
01904     // when high power is zero : strip
01905     template<typename P>
01906     struct simplify<P, std::enable_if_t<
01907         std::is_same<

```

```

01908         typename Ring::zero,
01909         typename P::aN
01910         >::value && (P::degree > 0)
01911     » {
01912         using type = typename simplify<typename P::strip>::type;
01913     };
01914
01915     // otherwise : do nothing
01916     template<typename P>
01917     struct simplify<P, std::enable_if_t<
01918         !std::is_same<
01919             typename Ring::zero,
01920             typename P::aN
01921             >::value && (P::degree > 0)
01922     » {
01923         using type = P;
01924     };
01925
01926     // do not simplify constants
01927     template<typename P>
01928     struct simplify<P, std::enable_if_t<P::degree == 0» {
01929         using type = P;
01930     };
01931
01932     // addition at
01933     template<typename P1, typename P2, size_t index>
01934     struct add_at {
01935         using type =
01936             typename Ring::template add_t<
01937                 typename P1::template coeff_at_t<index>,
01938                 typename P2::template coeff_at_t<index>;
01939     };
01940
01941     template<typename P1, typename P2, size_t index>
01942     using add_at_t = typename add_at<P1, P2, index>::type;
01943
01944     template<typename P1, typename P2, std::size_t... I>
01945     struct add_low<P1, P2, std::index_sequence<I...» {
01946         using type = val<add_at_t<P1, P2, I>...>;
01947     };
01948
01949     // subtraction at
01950     template<typename P1, typename P2, size_t index>
01951     struct sub_at {
01952         using type =
01953             typename Ring::template sub_t<
01954                 typename P1::template coeff_at_t<index>,
01955                 typename P2::template coeff_at_t<index>;
01956     };
01957
01958     template<typename P1, typename P2, size_t index>
01959     using sub_at_t = typename sub_at<P1, P2, index>::type;
01960
01961     template<typename P1, typename P2, std::size_t... I>
01962     struct sub_low<P1, P2, std::index_sequence<I...» {
01963         using type = val<sub_at_t<P1, P2, I>...>;
01964     };
01965
01966     template<typename P1, typename P2>
01967     struct sub {
01968         using type = typename simplify<typename sub_low<
01969             P1,
01970             P2,
01971             internal::make_index_sequence_reverse<
01972                 std::max(P1::degree, P2::degree) + 1
01973             >::type>::type;
01974     };
01975
01976     // multiplication at
01977     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01978     struct mul_at_loop_helper {
01979         using type = typename Ring::template add_t<
01980             typename Ring::template mul_t<
01981                 typename v1::template coeff_at_t<index>,
01982                 typename v2::template coeff_at_t<k - index>
01983             >,
01984             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01985         >;
01986     };
01987
01988     template<typename v1, typename v2, size_t k, size_t stop>
01989     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01990         using type = typename Ring::template mul_t<
01991             typename v1::template coeff_at_t<stop>,
01992             typename v2::template coeff_at_t<0>;
01993     };
01994

```

```

01995     template <typename v1, typename v2, size_t k, typename E = void>
01996     struct mul_at {};
01997
01998     template<typename v1, typename v2, size_t k>
01999     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
02000         using type = typename Ring::zero;
02001     };
02002
02003     template<typename v1, typename v2, size_t k>
02004     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
02005         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02006     };
02007
02008     template<typename P1, typename P2, size_t index>
02009     using mul_at_t = typename mul_at<P1, P2, index>::type;
02010
02011     template<typename P1, typename P2, std::size_t... I>
02012     struct mul_low<P1, P2, std::index_sequence<I...> {
02013         using type = val<mul_at_t<P1, P2, I>...>;
02014     };
02015
02016     // division helper
02017     template< typename A, typename B, typename Q, typename R, typename E = void>
02018     struct div_helper {};
02019
02020     template<typename A, typename B, typename Q, typename R>
02021     struct div_helper<A, B, Q, R, std::enable_if_t<
02022         (R::degree < B::degree) ||
02023         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02024         using q_type = Q;
02025         using mod_type = R;
02026         using gcd_type = B;
02027     };
02028
02029     template<typename A, typename B, typename Q, typename R>
02030     struct div_helper<A, B, Q, R, std::enable_if_t<
02031         (R::degree >= B::degree) &&
02032         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02033     private: // NOLINT
02034         using rN = typename R::aN;
02035         using bN = typename B::aN;
02036         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02037 B::degree>::type;
02038         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02039         using qq = typename add<Q, pT>::type;
02040
02041     public:
02042         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02043         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02044         using gcd_type = rr;
02045     };
02046
02047     template<typename A, typename B>
02048     struct div {
02049         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02050         using q_type = typename div_helper<A, B, zero, A>::q_type;
02051         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02052     };
02053
02054     template<typename P>
02055     struct make_unit {
02056         using type = typename div<P, val<typename P::aN>::q_type>;
02057     };
02058
02059     template<typename coeff, size_t deg>
02060     struct monomial {
02061         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02062     };
02063
02064     template<typename coeff>
02065     struct monomial<coeff, 0> {
02066         using type = val<coeff>;
02067     };
02068
02069     template<typename arithmeticType, typename P>
02070     struct horner_evaluation {
02071         template<size_t index, size_t stop>
02072         struct inner {
02073             static constexpr DEVICE INLINED arithmeticType func(
02074                 const arithmeticType& accum, const arithmeticType& x) {
02075                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02076 stop>::func(
02077                     internal::fma_helper<arithmeticType>::eval(
02078                         x,
02079                         accum,
02080                         P::template coeff_at_t<P::degree - index>::template
02081 get<arithmeticType>()), x);

```

```

02079         }
02080     };
02081
02082     template<size_t stop>
02083     struct inner<stop, stop> {
02084         static constexpr DEVICE INLINED arithmeticType func(
02085             const arithmeticType& accum, const arithmeticType& x) {
02086             return accum;
02087         }
02088     };
02089 };
02090
02091 template<typename arithmeticType, typename P>
02092 struct compensated_horner {
02093     template<int64_t index, int ghost>
02094     struct EFTHorner {
02095         static INLINED void func(
02096             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02097             *r) {
02098             arithmeticType p;
02099             internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02100             constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
02101             get<arithmeticType>();
02102             internal::two_sum<arithmeticType>(
02103                 p, coeff,
02104                 r, sigma + P::degree - index - 1);
02105             EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02106         }
02107     };
02108     template<int ghost>
02109     struct EFTHorner<-1, ghost> {
02110         static INLINED DEVICE void func(
02111             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02112             *r) {
02113         }
02114     };
02115     static INLINED DEVICE arithmeticType func(arithmeticType x) {
02116         arithmeticType pi[P::degree], sigma[P::degree];
02117         arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02118         EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02119         arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02120         return r + c;
02121     }
02122 };
02123
02124 template<typename coeff, typename... coeffs>
02125 struct string_helper {
02126     static std::string func() {
02127         std::string tail = string_helper<coeffs...>::func();
02128         std::string result = "";
02129         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02130             return tail;
02131         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02132             if (sizeof...(coeffs) == 1) {
02133                 result += "x";
02134             } else {
02135                 result += "x^" + std::to_string(sizeof...(coeffs));
02136             }
02137         } else {
02138             if (sizeof...(coeffs) == 1) {
02139                 result += coeff::to_string() + " x";
02140             } else {
02141                 result += coeff::to_string()
02142                     + " x^" + std::to_string(sizeof...(coeffs));
02143             }
02144         }
02145         if (!tail.empty()) {
02146             if (tail.at(0) != '-') {
02147                 result += " + " + tail;
02148             } else {
02149                 result += " - " + tail.substr(1);
02150             }
02151         }
02152     }
02153     return result;
02154 }
02155 };
02156
02157 template<typename coeff>
02158 struct string_helper<coeff> {
02159     static std::string func() {
02160         if (!std::is_same<coeff, typename Ring::zero>::value) {
02161             return coeff::to_string();
02162         } else {

```

```

02163         return "";
02164     }
02165 }
02166 };
02167
02168 public:
02169     template<typename P>
02170     using simplify_t = typename simplify<P>::type;
02171
02172     template<typename v1, typename v2>
02173     using add_t = typename add<v1, v2>::type;
02174
02175     template<typename v1, typename v2>
02176     using sub_t = typename sub<v1, v2>::type;
02177
02178     template<typename v1, typename v2>
02179     using mul_t = typename mul<v1, v2>::type;
02180
02181     template<typename v1, typename v2>
02182     using eq_t = typename eq_helper<v1, v2>::type;
02183
02184     template<typename v1, typename v2>
02185     using lt_t = typename lt_helper<v1, v2>::type;
02186
02187     template<typename v1, typename v2>
02188     using gt_t = typename gt_helper<v1, v2>::type;
02189
02190     template<typename v1, typename v2>
02191     using div_t = typename div<v1, v2>::q_type;
02192
02193     template<typename v1, typename v2>
02194     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02195
02196     template<typename coeff, size_t deg>
02197     using monomial_t = typename monomial<coeff, deg>::type;
02198
02199     template<typename v>
02200     using derive_t = typename derive_helper<v>::type;
02201
02202     template<typename v>
02203     using pos_t = typename Ring::template pos_t<typename v::aN>;
02204
02205     template<typename v>
02206     static constexpr bool pos_v = pos_t<v>::value;
02207
02208     template<typename v1, typename v2>
02209     using gcd_t = std::conditional_t<
02210         Ring::is_euclidean_domain,
02211         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02212         void>;
02213
02214     template<auto x>
02215     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02216
02217     template<typename v>
02218     using inject_ring_t = val<v>;
02219 };
02220 } // namespace aerobus
02221
02222 // fraction field
02223 namespace aerobus {
02224     namespace internal {
02225         template<typename Ring, typename E = void>
02226         requires IsEuclideanDomain<Ring>
02227         struct _FractionField {};
02228
02229         template<typename Ring>
02230         requires IsEuclideanDomain<Ring>
02231         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
02232             static constexpr bool is_field = true;
02233             static constexpr bool is_euclidean_domain = true;
02234         };
02235
02236     private:
02237         template<typename val1, typename val2, typename E = void>
02238         struct to_string_helper {};
02239
02240         template<typename val1, typename val2>
02241         struct to_string_helper <val1, val2,
02242             std::enable_if_t<
02243                 Ring::template eq_t<
02244                     val2, typename Ring::one
02245                     >::value
02246                 >
02247             > {
02248             static std::string func() {
02249                 return val1::to_string();
02250             }
02251         };

```



```

02293     };
02294
02295     template<typename val1, typename val2>
02296     struct to_string_helper<val1, val2,
02297         std::enable_if_t<
02298             !Ring::template eq_t<
02299                 val2,
02300                 typename Ring::one
02301             >::value
02302         >
02303     > {
02304         static std::string func() {
02305             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02306         }
02307     };
02308
02309 public:
02310     template<typename val1, typename val2>
02311     struct val {
02312         using x = val1;
02313         using y = val2;
02314         using is_zero_t = typename val1::is_zero_t;
02315         static constexpr bool is_zero_v = val1::is_zero_t::value;
02316
02317         using ring_type = Ring;
02318         using enclosing_type = _FractionField<Ring>;
02319
02320         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02321
02322         template<typename valueType>
02323         static constexpr INLINED_DEVICE valueType get() {
02324             return internal::staticcast<valueType, typename ring_type::inner_type>::template
02325                 func<x::v>() /
02326                 internal::staticcast<valueType, typename ring_type::inner_type>::template
02327                 func<y::v>();
02328         }
02329
02330         static std::string to_string() {
02331             return to_string_helper<val1, val2>::func();
02332         }
02333
02334         template<typename arithmeticType>
02335         static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02336             return x::eval(v) / y::eval(v);
02337         }
02338     };
02339
02340     using zero = val<typename Ring::zero, typename Ring::one>;
02341     using one = val<typename Ring::one, typename Ring::one>;
02342
02343     template<typename v>
02344     using inject_t = val<v, typename Ring::one>;
02345
02346     template<auto x>
02347     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
02348     Ring::one>;
02349
02350     template<typename v>
02351     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02352
02353     using ring_type = Ring;
02354
02355 private:
02356     template<typename v, typename E = void>
02357     struct simplify {};
02358
02359     // x = 0
02360     template<typename v>
02361     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> > {
02362         using type = typename _FractionField<Ring>::zero;
02363     };
02364
02365     // x != 0
02366     template<typename v>
02367     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> > {
02368     private:
02369         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02370         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02371         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02372
02373         using posx = std::conditional_t<
02374             !Ring::template pos_v<newx>,
02375             typename Ring::template sub_t<typename Ring::zero, newx>,
02376             newx>;
02377         using posy = std::conditional_t<
02378             !Ring::template pos_v<newy>,
02379             typename Ring::template sub_t<typename Ring::zero, newy>,

```

```

02405         newy>;
02406     public:
02407         using type = typename _FractionField<Ring>::template val<posx, posy>;
02408     };
02409
02410     public:
02411         template<typename v>
02412         using simplify_t = typename simplify<v>::type;
02413
02414     private:
02415         template<typename v1, typename v2>
02416         struct add {
02417             private:
02418                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02419                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02420                 using dividend = typename Ring::template add_t<a, b>;
02421                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02422                 using g = typename Ring::template gcd_t<dividend, diviser>;
02423
02424             public:
02425                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02426         divider>>;
02427
02428         };
02429
02430         template<typename v>
02431         struct pos {
02432             using type = std::conditional_t<
02433                 (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02434                 (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02435                 std::true_type,
02436                 std::false_type>;
02437         };
02438
02439         template<typename v1, typename v2>
02440         struct sub {
02441             private:
02442                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02443                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02444                 using dividend = typename Ring::template sub_t<a, b>;
02445                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02446                 using g = typename Ring::template gcd_t<dividend, diviser>;
02447
02448             public:
02449                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02450         divider>>;
02451
02452         };
02453
02454         template<typename v1, typename v2>
02455         struct mul {
02456             private:
02457                 using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02458                 using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02459
02460             public:
02461                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02462         };
02463
02464         template<typename v1, typename v2, typename E = void>
02465         struct div {};
02466
02467         template<typename v1, typename v2>
02468         struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
02469         _FractionField<Ring>::zero>::value>> {
02470             private:
02471                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02472                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02473
02474             public:
02475                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02476         };
02477
02478         template<typename v1, typename v2>
02479         struct div<v1, v2, std::enable_if_t<
02480             std::is_same<zero, v1>::value && std::is_same<v2, zero>::value>> {
02481             using type = one;
02482         };
02483
02484         template<typename v1, typename v2>
02485         struct eq {
02486             using type = std::conditional_t<
02487                 std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02488                 std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02489                 std::true_type,
02490                 std::false_type>;
02491         };
02492
02493         template<typename v1, typename v2, typename E = void>

```

```

02491     struct gt;
02492
02493     template<typename v1, typename v2>
02494     struct gt<v1, v2, std::enable_if_t<
02495         (eq<v1, v2>::type::value)
02496         > {
02497         using type = std::false_type;
02498     };
02499
02500     template<typename v1, typename v2>
02501     struct gt<v1, v2, std::enable_if_t<
02502         (!eq<v1, v2>::type::value) &&
02503         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02504         > {
02505         using type = typename gt<
02506             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02507             >::type;
02508     };
02509
02510     template<typename v1, typename v2>
02511     struct gt<v1, v2, std::enable_if_t<
02512         (!eq<v1, v2>::type::value) &&
02513         (pos<v1>::type::value) && (!pos<v2>::type::value)
02514         > {
02515         using type = std::true_type;
02516     };
02517
02518     template<typename v1, typename v2>
02519     struct gt<v1, v2, std::enable_if_t<
02520         (!eq<v1, v2>::type::value) &&
02521         (!pos<v1>::type::value) && (pos<v2>::type::value)
02522         > {
02523         using type = std::false_type;
02524     };
02525
02526     template<typename v1, typename v2>
02527     struct gt<v1, v2, std::enable_if_t<
02528         (!eq<v1, v2>::type::value) &&
02529         (pos<v1>::type::value) && (pos<v2>::type::value)
02530         > {
02531         using type = typename Ring::template gt_t<
02532             typename Ring::template mul_t<v1::x, v2::y>,
02533             typename Ring::template mul_t<v2::y, v2::x>
02534             >;
02535     };
02536
02537 public:
02538     template<typename v1, typename v2>
02539     using add_t = typename add<v1, v2>::type;
02540
02541     template<typename v1, typename v2>
02542     using mod_t = zero;
02543
02544     template<typename v1, typename v2>
02545     using gcd_t = v1;
02546
02547     template<typename v1, typename v2>
02548     using sub_t = typename sub<v1, v2>::type;
02549
02550     template<typename v1, typename v2>
02551     using mul_t = typename mul<v1, v2>::type;
02552
02553     template<typename v1, typename v2>
02554     using div_t = typename div<v1, v2>::type;
02555
02556     template<typename v1, typename v2>
02557     using eq_t = typename eq<v1, v2>::type;
02558
02559     template<typename v1, typename v2>
02560     static constexpr bool eq_v = eq<v1, v2>::type::value;
02561
02562     template<typename v1, typename v2>
02563     using gt_t = typename gt<v1, v2>::type;
02564
02565     template<typename v1, typename v2>
02566     static constexpr bool gt_v = gt<v1, v2>::type::value;
02567
02568     template<typename v1>
02569     using pos_t = typename pos<v1>::type;
02570
02571     template<typename v>
02572     static constexpr bool pos_v = pos_t<v>::value;
02573
02574 };
02575
02576 template<typename Ring, typename E = void>
02577 requires IsEuclideanDomain<Ring>
02578 struct FractionFieldImpl {};

```

```

02614
02615 // fraction field of a field is the field itself
02616 template<typename Field>
02617 requires IsEuclideanDomain<Field>
02618 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02619     using type = Field;
02620     template<typename v>
02621     using inject_t = v;
02622 };
02623
02624 // fraction field of a ring is the actual fraction field
02625 template<typename Ring>
02626 requires IsEuclideanDomain<Ring>
02627 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02628     using type = _FractionField<Ring>;
02629 };
02630 } // namespace internal
02631
02632 template<typename Ring>
02633 requires IsEuclideanDomain<Ring>
02634 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02635
02636 template<typename Ring>
02637 struct Embed<Ring, FractionField<Ring> {
02638     template<typename v>
02639     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02640 };
02641 } // namespace aerobus
02642
02643 // short names for common types
02644 namespace aerobus {
02645     template<typename X, typename Y>
02646     requires IsRing<typename X::enclosing_type> &&
02647     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02648     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02649
02650     template<typename X, typename Y>
02651     requires IsRing<typename X::enclosing_type> &&
02652     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02653     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02654
02655     template<typename X, typename Y>
02656     requires IsRing<typename X::enclosing_type> &&
02657     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02658     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02659
02660     template<typename X, typename Y>
02661     requires IsEuclideanDomain<typename X::enclosing_type> &&
02662     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02663     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02664
02665     using q32 = FractionField<i32>;
02666
02667     using fpq32 = FractionField<polynomial<q32>>;
02668
02669     using q64 = FractionField<i64>;
02670
02671     using pi64 = polynomial<i64>;
02672
02673     using pq64 = polynomial<q64>;
02674
02675     using fpq64 = FractionField<polynomial<q64>>;
02676
02677     template<typename Ring, typename v1, typename v2>
02678     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02679
02680     template<typename v>
02681     using embed_int_poly_in_fractions_t =
02682         typename Embed<
02683             polynomial<typename v::ring_type>,
02684             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02685
02686     template<int64_t p, int64_t q>
02687     using make_q64_t = typename q64::template simplify_t<
02688         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02689
02690     template<int32_t p, int32_t q>
02691     using make_q32_t = typename q32::template simplify_t<
02692         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02693
02694     template<typename Ring, typename v1, typename v2>
02695     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02696     template<typename Ring, typename v1, typename v2>
02697     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02698
02699     template<>

```

```

02753     struct Embed<q32, q64> {
02754         template<typename v>
02755         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02756     };
02757
02758     template<typename Small, typename Large>
02759     struct Embed<polynomial<Small>, polynomial<Large>> {
02760     private:
02761         template<typename v, typename i>
02762         struct at_low;
02763
02764         template<typename v, size_t i>
02765         struct at_index {
02766             using type = typename Embed<Small, Large>::template
02767             type<typename v::template coeff_at_t<i>>;
02768         };
02769
02770         template<typename v, size_t... Is>
02771         struct at_low<v, std::index_sequence<Is...>> {
02772             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02773         };
02774
02775     public:
02776         template<typename v>
02777         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02778         1>::type>;
02779     };
02780
02781     template<typename Ring, auto... xs>
02782     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02783     typename Ring::template inject_constant_t<xs>...>;
02784
02785     template<typename Ring, auto... xs>
02786     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02787     typename FractionField<Ring>::template inject_constant_t<xs>...>;
02788 } // namespace aerobus
02789
02790 // Taylor series and common integers (factorial, bernoulli...) appearing in Taylor coefficients
02791 namespace aerobus {
02792     namespace internal {
02793         template<typename T, size_t x, typename E = void>
02794         struct factorial {};
02795
02796         template<typename T, size_t x>
02797         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02798         private:
02799             template<typename, size_t, typename>
02800             friend struct factorial;
02801         public:
02802             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02803             x - 1>::type>;
02804             static constexpr typename T::inner_type value = type::template get<typename
02805             T::inner_type>();
02806         };
02807
02808         template<typename T>
02809         struct factorial<T, 0> {
02810         public:
02811             using type = typename T::one;
02812             static constexpr typename T::inner_type value = type::template get<typename
02813             T::inner_type>();
02814         };
02815     } // namespace internal
02816
02817     template<typename T, size_t i>
02818     using factorial_t = typename internal::factorial<T, i>::type;
02819
02820     template<typename T, size_t i>
02821     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02822
02823     namespace internal {
02824         template<typename T, size_t k, size_t n, typename E = void>
02825         struct combination_helper {};
02826
02827         template<typename T, size_t k, size_t n>
02828         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02829             using type = typename FractionField<T>::template mul_t<
02830             typename combination_helper<T, k - 1, n - 1>::type,
02831             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02832         };
02833
02834         template<typename T, size_t k, size_t n>
02835         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02836             using type = typename combination_helper<T, n - k, n>::type;
02837         };
02838     }
02839
02840     template<typename T, size_t n>

```

```

02854     struct combination_helper<T, 0, n> {
02855         using type = typename FractionField<T>::one;
02856     };
02857
02858     template<typename T, size_t k, size_t n>
02859     struct combination {
02860         using type = typename internal::combination_helper<T, k, n>::type::x;
02861         static constexpr typename T::inner_type value =
02862             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02863     };
02864     } // namespace internal
02865
02866     template<typename T, size_t k, size_t n>
02867     using combination_t = typename internal::combination<T, k, n>::type;
02868
02869     template<typename T, size_t k, size_t n>
02870     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02871
02872     namespace internal {
02873         template<typename T, size_t m>
02874         struct bernoulli;
02875
02876         template<typename T, typename accum, size_t k, size_t m>
02877         struct bernoulli_helper {
02878             using type = typename bernoulli_helper<
02879                 T,
02880                 addfractions_t<T,
02881                     accum,
02882                     mulfractions_t<T,
02883                         makefraction_t<T,
02884                             combination_t<T, k, m + 1>,
02885                             typename T::one>,
02886                             typename bernoulli<T, k>::type
02887                         >,
02888                     >,
02889                     k + 1,
02890                     m>::type;
02891         };
02892
02893         template<typename T, typename accum, size_t m>
02894         struct bernoulli_helper<T, accum, m, m> {
02895             using type = accum;
02896         };
02897
02898         template<typename T, size_t m>
02899         struct bernoulli {
02900             using type = typename FractionField<T>::template mul_t<
02901                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02902                 makefraction_t<T,
02903                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02904                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02905                 >
02906             >;
02907
02908             template<typename floatType>
02909             static constexpr floatType value = type::template get<floatType>();
02910         };
02911
02912         template<typename T>
02913         struct bernoulli<T, 0> {
02914             using type = typename FractionField<T>::one;
02915
02916             template<typename floatType>
02917             static constexpr floatType value = type::template get<floatType>();
02918         };
02919     } // namespace internal
02920
02921     template<typename T, size_t n>
02922     using bernoulli_t = typename internal::bernoulli<T, n>::type;
02923
02924     template<typename FloatType, typename T, size_t n>
02925     inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02926
02927     // bell numbers
02928     namespace internal {
02929         template<typename T, size_t n, typename E = void>
02930         struct bell_helper;
02931
02932         template<typename T, size_t n>
02933         struct bell_helper<T, n, std::enable_if_t<(n > 1)> {
02934             template<typename accum, size_t i, size_t stop>
02935             struct sum_helper {
02936             private:
02937                 using left = typename T::template mul_t<

```

```

02953         combination_t<T, i, n-1>,
02954         typename bell_helper<T, i>::type>;
02955     using new_accum = typename T::template add_t<accum, left>;
02956 public:
02957     using type = typename sum_helper<new_accum, i+1, stop>::type;
02958 };
02959
02960     template<typename accum, size_t stop>
02961     struct sum_helper<accum, stop, stop> {
02962         using type = accum;
02963     };
02964
02965     using type = typename sum_helper<typename T::zero, 0, n>::type;
02966 };
02967
02968     template<typename T>
02969     struct bell_helper<T, 0> {
02970         using type = typename T::one;
02971     };
02972
02973     template<typename T>
02974     struct bell_helper<T, 1> {
02975         using type = typename T::one;
02976     };
02977 } // namespace internal
02978
02982 template<typename T, size_t n>
02983 using bell_t = typename internal::bell_helper<T, n>::type;
02984
02988 template<typename T, size_t n>
02989 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02990
02991 namespace internal {
02992     template<typename T, int k, typename E = void>
02993     struct alternate {};
02994
02995     template<typename T, int k>
02996     struct alternate<T, k, std::enable_if_t<k % 2 == 0> > {
02997         using type = typename T::one;
02998         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02999     };
03000
03001     template<typename T, int k>
03002     struct alternate<T, k, std::enable_if_t<k % 2 != 0> > {
03003         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03004         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
03005     };
03006 } // namespace internal
03007
03010 template<typename T, int k>
03011 using alternate_t = typename internal::alternate<T, k>::type;
03012
03015 template<typename T, size_t k>
03016 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03017
03018 namespace internal {
03019     template<typename T, int n, int k, typename E = void>
03020     struct stirling_l_helper {};
03021
03022     template<typename T>
03023     struct stirling_l_helper<T, 0, 0> {
03024         using type = typename T::one;
03025     };
03026
03027     template<typename T, int n>
03028     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> > {
03029         using type = typename T::zero;
03030     };
03031
03032     template<typename T, int n>
03033     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> > {
03034         using type = typename T::zero;
03035     };
03036
03037     template<typename T, int n, int k>
03038     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> > {
03039         using type = typename T::template sub_t<
03040             typename T::template stirling_l_helper<T, n-1, k-1>::type,
03041             typename T::template mul_t<
03042                 typename T::template inject_constant_t<n-1>,
03043                 typename stirling_l_helper<T, n-1, k>::type
03044             >;
03045     };
03046 } // namespace internal
03047

```

```

03052     template<typename T, int n, int k>
03053     using stirling_1_signed_t = typename internal::stirling_1_helper<T, n, k>::type;
03054
03055     template<typename T, int n, int k>
03060     using stirling_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03061
03066     template<typename T, int n, int k>
03067     static constexpr typename T::inner_type stirling_1_unsigned_v = stirling_1_unsigned_t<T, n, k>::v;
03068
03073     template<typename T, int n, int k>
03074     static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03075
03076     namespace internal {
03077         template<typename T, int n, int k, typename E = void>
03078         struct stirling_2_helper {};
03079
03080         template<typename T, int n>
03081         struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
03082             using type = typename T::one;
03083         };
03084
03085         template<typename T, int n>
03086         struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03087             using type = typename T::zero;
03088         };
03089
03090         template<typename T, int n>
03091         struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03092             using type = typename T::zero;
03093         };
03094
03095         template<typename T, int n, int k>
03096         struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03097             using type = typename T::template add_t<
03098                 typename stirling_2_helper<T, n-1, k-1>::type,
03099                 typename T::template mul_t<
03100                     typename T::template inject_constant_t<k>,
03101                     typename stirling_2_helper<T, n-1, k>::type
03102             >;
03103         };
03104     } // namespace internal
03105
03110     template<typename T, int n, int k>
03111     using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03112
03117     template<typename T, int n, int k>
03118     static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03119
03120     namespace internal {
03121         template<typename T>
03122         struct pow_scalar {
03123             template<size_t p>
03124             static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03125                 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03126                 x * func<p/2>(x) * func<p/2>(x);
03127         }
03128     };
03129
03130     template<typename T, typename p, size_t n, typename E = void>
03131     requires IsEuclideanDomain<T>
03132     struct pow;
03133
03134     template<typename T, typename p, size_t n>
03135     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03136         using type = typename T::template mul_t<
03137             typename pow<T, p, n/2>::type,
03138             typename pow<T, p, n/2>::type
03139         >;
03140     };
03141
03142     template<typename T, typename p, size_t n>
03143     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03144         using type = typename T::template mul_t<
03145             p,
03146             typename T::template mul_t<
03147                 typename pow<T, p, n/2>::type,
03148                 typename pow<T, p, n/2>::type
03149             >
03150         >;
03151     };
03152
03153     template<typename T, typename p, size_t n>
03154     struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03155 } // namespace internal
03156
03161     template<typename T, typename p, size_t n>
03162     using pow_t = typename internal::pow<T, p, n>::type;

```



```

03163
03168     template<typename T, typename p, size_t n>
03169     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03170
03171     template<typename T, size_t p>
03172     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
internal::pow_scalar<T>::template func<p>(x); }
03173
03174     namespace internal {
03175         template<typename, template<typename, size_t> typename, class>
03176         struct make_taylor_impl;
03177
03178         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03179         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03180             using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
03181         };
03182     }
03183
03188     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03189     using taylor = typename internal::make_taylor_impl<
03190         T,
03191         coeff_at,
03192         internal::make_index_sequence_reverse<deg + 1>::type;
03193
03194     namespace internal {
03195         template<typename T, size_t i>
03196         struct exp_coeff {
03197             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03198         };
03199
03200         template<typename T, size_t i, typename E = void>
03201         struct sin_coeff_helper {};
03202
03203         template<typename T, size_t i>
03204         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03205             using type = typename FractionField<T>::zero;
03206         };
03207
03208         template<typename T, size_t i>
03209         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03210             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03211         };
03212
03213         template<typename T, size_t i>
03214         struct sin_coeff {
03215             using type = typename sin_coeff_helper<T, i>::type;
03216         };
03217
03218         template<typename T, size_t i, typename E = void>
03219         struct sh_coeff_helper {};
03220
03221         template<typename T, size_t i>
03222         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03223             using type = typename FractionField<T>::zero;
03224         };
03225
03226         template<typename T, size_t i>
03227         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03228             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03229         };
03230
03231         template<typename T, size_t i>
03232         struct sh_coeff {
03233             using type = typename sh_coeff_helper<T, i>::type;
03234         };
03235
03236         template<typename T, size_t i, typename E = void>
03237         struct cos_coeff_helper {};
03238
03239         template<typename T, size_t i>
03240         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03241             using type = typename FractionField<T>::zero;
03242         };
03243
03244         template<typename T, size_t i>
03245         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03246             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03247         };
03248
03249         template<typename T, size_t i>
03250         struct cos_coeff {
03251             using type = typename cos_coeff_helper<T, i>::type;
03252         };
03253
03254         template<typename T, size_t i, typename E = void>
03255         struct cosh_coeff_helper {};

```

```

03256
03257     template<typename T, size_t i>
03258     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03259         using type = typename FractionField<T>::zero;
03260     };
03261
03262     template<typename T, size_t i>
03263     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03264         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03265     };
03266
03267     template<typename T, size_t i>
03268     struct cosh_coeff {
03269         using type = typename cosh_coeff_helper<T, i>::type;
03270     };
03271
03272     template<typename T, size_t i>
03273     struct geom_coeff { using type = typename FractionField<T>::one; };
03274
03275
03276     template<typename T, size_t i, typename E = void>
03277     struct atan_coeff_helper;
03278
03279     template<typename T, size_t i>
03280     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03281         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03282     };
03283
03284     template<typename T, size_t i>
03285     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03286         using type = typename FractionField<T>::zero;
03287     };
03288
03289     template<typename T, size_t i>
03290     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03291
03292     template<typename T, size_t i, typename E = void>
03293     struct asin_coeff_helper;
03294
03295     template<typename T, size_t i>
03296     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03297         using type = makefraction_t<T,
03298             factorial_t<T, i - 1>,
03299             typename T::template mul_t<
03300                 typename T::template val<i>,
03301                 T::template mul_t<
03302                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03303                     pow_t<T, factorial_t<T, i / 2>, 2>
03304                 >
03305             >
03306         >>;
03307     };
03308
03309     template<typename T, size_t i>
03310     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03311         using type = typename FractionField<T>::zero;
03312     };
03313
03314     template<typename T, size_t i>
03315     struct asin_coeff {
03316         using type = typename asin_coeff_helper<T, i>::type;
03317     };
03318
03319     template<typename T, size_t i>
03320     struct lnpl_coeff {
03321         using type = makefraction_t<T,
03322             alternate_t<T, i + 1>,
03323             typename T::template val<i>;
03324     };
03325
03326     template<typename T>
03327     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03328
03329     template<typename T, size_t i, typename E = void>
03330     struct asinh_coeff_helper;
03331
03332     template<typename T, size_t i>
03333     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03334         using type = makefraction_t<T,
03335             typename T::template mul_t<
03336                 alternate_t<T, i / 2>,
03337                 factorial_t<T, i - 1>
03338             >,
03339             typename T::template mul_t<
03340                 typename T::template mul_t<
03341                     typename T::template val<i>,
03342                     pow_t<T, factorial_t<T, i / 2>, 2>

```

```

03343         >,
03344         pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03345     >
03346     >;
03347 };
03348
03349 template<typename T, size_t i>
03350 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03351     using type = typename FractionField<T>::zero;
03352 };
03353
03354 template<typename T, size_t i>
03355 struct asinh_coeff {
03356     using type = typename asinh_coeff_helper<T, i>::type;
03357 };
03358
03359 template<typename T, size_t i, typename E = void>
03360 struct atanh_coeff_helper;
03361
03362 template<typename T, size_t i>
03363 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03364     // 1/i
03365     using type = typename FractionField<T>::template val<
03366         typename T::one,
03367         typename T::template inject_constant_t<i>;
03368 };
03369
03370 template<typename T, size_t i>
03371 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03372     using type = typename FractionField<T>::zero;
03373 };
03374
03375 template<typename T, size_t i>
03376 struct atanh_coeff {
03377     using type = typename atanh_coeff_helper<T, i>::type;
03378 };
03379
03380 template<typename T, size_t i, typename E = void>
03381 struct tan_coeff_helper;
03382
03383 template<typename T, size_t i>
03384 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03385     using type = typename FractionField<T>::zero;
03386 };
03387
03388 template<typename T, size_t i>
03389 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03390 private:
03391     // 4^((i+1)/2)
03392     using _4p = typename FractionField<T>::template inject_t<
03393         pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03394     // 4^((i+1)/2) - 1
03395     using _4pml = typename FractionField<T>::template
03396         sub_t<_4p, typename FractionField<T>::one>;
03397     // (-1)^((i-1)/2)
03398     using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03399     using dividend = typename FractionField<T>::template mul_t<
03400         altp,
03401         _4p,
03402         FractionField<T>::template mul_t<
03403             _4pml,
03404             bernoulli_t<T, (i + 1)>
03405         >
03406     >
03407     >;
03408 public:
03409     using type = typename FractionField<T>::template div_t<dividend,
03410         typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03411 };
03412
03413 template<typename T, size_t i>
03414 struct tan_coeff {
03415     using type = typename tan_coeff_helper<T, i>::type;
03416 };
03417
03418 template<typename T, size_t i, typename E = void>
03419 struct tanh_coeff_helper;
03420
03421 template<typename T, size_t i>
03422 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03423     using type = typename FractionField<T>::zero;
03424 };
03425
03426 template<typename T, size_t i>
03427 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03428 private:

```

```

03429         using _4p = typename FractionField<T>::template inject_t<
03430             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03431         using _4pml = typename FractionField<T>::template
sub_t<_4p, typename FractionField<T>::one>;
03432         using dividend =
03433             typename FractionField<T>::template mul_t<
03434                 _4p,
03435                 typename FractionField<T>::template mul_t<
03436                     _4pml,
03437                     bernoulli_t<T, (i + 1)>>::type;
03438     public:
03439         using type = typename FractionField<T>::template div_t<dividend,
03440             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03441     };
03442
03443     template<typename T, size_t i>
03444     struct tanh_coeff {
03445         using type = typename tanh_coeff_helper<T, i>::type;
03446     };
03447 } // namespace internal
03448
03452 template<typename Integers, size_t deg>
03453 using exp = taylor<Integers, internal::exp_coeff, deg>;
03454
03458 template<typename Integers, size_t deg>
03459 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03460     exp<Integers, deg>,
03461     typename polynomial<FractionField<Integers>>::one>;
03462
03466 template<typename Integers, size_t deg>
03467 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03468
03472 template<typename Integers, size_t deg>
03473 using atan = taylor<Integers, internal::atan_coeff, deg>;
03474
03478 template<typename Integers, size_t deg>
03479 using sin = taylor<Integers, internal::sin_coeff, deg>;
03480
03484 template<typename Integers, size_t deg>
03485 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03486
03491 template<typename Integers, size_t deg>
03492 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03493
03498 template<typename Integers, size_t deg>
03499 using cos = taylor<Integers, internal::cos_coeff, deg>;
03500
03505 template<typename Integers, size_t deg>
03506 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03507
03512 template<typename Integers, size_t deg>
03513 using asin = taylor<Integers, internal::asin_coeff, deg>;
03514
03519 template<typename Integers, size_t deg>
03520 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03521
03526 template<typename Integers, size_t deg>
03527 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03528
03533 template<typename Integers, size_t deg>
03534 using tan = taylor<Integers, internal::tan_coeff, deg>;
03535
03540 template<typename Integers, size_t deg>
03541 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03542 } // namespace aerobus
03543
03544 // continued fractions
03545 namespace aerobus {
03548     template<int64_t... values>
03549     struct ContinuedFraction {};
03550
03553     template<int64_t a0>
03554     struct ContinuedFraction<a0> {
03556         using type = typename q64::template inject_constant_t<a0>;
03558         static constexpr double val = static_cast<double>(a0);
03559     };
03560
03564     template<int64_t a0, int64_t... rest>
03565     struct ContinuedFraction<a0, rest...> {
03567         using type = q64::template add_t<
03568             typename q64::template inject_constant_t<a0>,
03569             typename q64::template div_t<
03570                 typename q64::one,
03571                 typename ContinuedFraction<rest...>::type
03572             >;
03573
03575         static constexpr double val = type::template get<double>();

```

```

03576     };
03577
03581     using PI_fraction =
ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03583     using E_fraction =
ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03585     using SQRT2_fraction =
ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03587     using SQRT3_fraction =
ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
// NOLINT
03588 } // namespace aerobus
03589
03590 // known polynomials
03591 namespace aerobus {
03592     // CChebyshev
03593     namespace internal {
03594         template<int kind, size_t deg, typename I>
03595         struct chebyshev_helper {
03596             using type = typename polynomial<I>::template sub_t<
03597                 typename polynomial<I>::template mul_t<
03598                     typename polynomial<I>::template mul_t<
03599                         typename polynomial<I>::template inject_constant_t<2>,
03600                         typename polynomial<I>::X>,
03601                         typename chebyshev_helper<kind, deg - 1, I>::type
03602                     >,
03603                     typename chebyshev_helper<kind, deg - 2, I>::type
03604                 >;
03605         };
03606
03607         template<typename I>
03608         struct chebyshev_helper<1, 0, I> {
03609             using type = typename polynomial<I>::one;
03610         };
03611
03612         template<typename I>
03613         struct chebyshev_helper<1, 1, I> {
03614             using type = typename polynomial<I>::X;
03615         };
03616
03617         template<typename I>
03618         struct chebyshev_helper<2, 0, I> {
03619             using type = typename polynomial<I>::one;
03620         };
03621
03622         template<typename I>
03623         struct chebyshev_helper<2, 1, I> {
03624             using type = typename polynomial<I>::template mul_t<
03625                 typename polynomial<I>::template inject_constant_t<2>,
03626                 typename polynomial<I>::X>;
03627         };
03628     } // namespace internal
03629
03630     // Laguerre
03631     namespace internal {
03632         template<size_t deg, typename I>
03633         struct laguerre_helper {
03634             using Q = FractionField<I>;
03635             using PQ = polynomial<Q>;
03636
03637             private:
03638                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2) Lkm2)
03639                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03640                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03641                 // -x + 2k-1
03642                 using p = typename PQ::template val<
03643                     typename Q::template inject_constant_t<-1>,
03644                     typename Q::template inject_constant_t<2 * deg - 1>;
03645                 // 1/n
03646                 using factor = typename PQ::template inject_ring_t<
03647                     typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03648
03649             public:
03650                 using type = typename PQ::template mul_t <
03651                     factor,
03652                     typename PQ::template sub_t<
03653                         typename PQ::template mul_t<
03654                             p,
03655                             lnm1
03656                         >,
03657                         typename PQ::template mul_t<
03658                             typename PQ::template inject_constant_t<deg-1>,
03659                             lnm2
03660                         >
03661                     >
03662                 >;

```

```

03663     };
03664
03665     template<typename I>
03666     struct laguerre_helper<0, I> {
03667         using type = typename polynomial<FractionField<I>::one>;
03668     };
03669
03670     template<typename I>
03671     struct laguerre_helper<1, I> {
03672     private:
03673         using PQ = polynomial<FractionField<I>;
03674     public:
03675         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03676     };
03677 } // namespace internal
03678
03679 // Bernstein
03680 namespace internal {
03681     template<size_t i, size_t m, typename I, typename E = void>
03682     struct bernstein_helper {};
03683
03684     template<typename I>
03685     struct bernstein_helper<0, 0, I> {
03686         using type = typename polynomial<I>::one;
03687     };
03688
03689     template<size_t i, size_t m, typename I>
03690     struct bernstein_helper<i, m, I, std::enable_if_t<
03691         (m > 0) && (i == 0)>> {
03692     private:
03693         using P = polynomial<I>;
03694     public:
03695         using type = typename P::template mul_t<
03696             typename P::template sub_t<typename P::one, typename P::X>,
03697             typename bernstein_helper<i, m-1, I>::type>;
03698     };
03699
03700     template<size_t i, size_t m, typename I>
03701     struct bernstein_helper<i, m, I, std::enable_if_t<
03702         (m > 0) && (i == m)>> {
03703     private:
03704         using P = polynomial<I>;
03705     public:
03706         using type = typename P::template mul_t<
03707             typename P::X,
03708             typename bernstein_helper<i-1, m-1, I>::type>;
03709     };
03710
03711     template<size_t i, size_t m, typename I>
03712     struct bernstein_helper<i, m, I, std::enable_if_t<
03713         (m > 0) && (i > 0) && (i < m)>> {
03714     private:
03715         using P = polynomial<I>;
03716     public:
03717         using type = typename P::template add_t<
03718             typename P::template mul_t<
03719                 typename P::template sub_t<typename P::one, typename P::X>,
03720                 typename bernstein_helper<i, m-1, I>::type>,
03721                 typename P::template mul_t<
03722                     typename P::X,
03723                     typename bernstein_helper<i-1, m-1, I>::type>;
03724     };
03725 } // namespace internal
03726
03727 // AllOne polynomials
03728 namespace internal {
03729     template<size_t deg, typename I>
03730     struct AllOneHelper {
03731         using type = aerobus::add_t<
03732             typename polynomial<I>::one,
03733             typename aerobus::mul_t<
03734                 typename polynomial<I>::X,
03735                 typename AllOneHelper<deg-1, I>::type
03736             >;
03737     };
03738
03739     template<typename I>
03740     struct AllOneHelper<0, I> {
03741         using type = typename polynomial<I>::one;
03742     };
03743 } // namespace internal
03744
03745 // Bessel polynomials
03746 namespace internal {
03747     template<size_t deg, typename I>
03748     struct BesselHelper {
03749     private:

```

```

03750         using P = polynomial<I>;
03751         using factor = typename P::template monomial_t<
03752             typename I::template inject_constant_t<(2*deg - 1)>,
03753             1>;
03754     public:
03755         using type = typename P::template add_t<
03756             typename P::template mul_t<
03757                 factor,
03758                 typename BesselHelper<deg-1, I>::type
03759             >,
03760             typename BesselHelper<deg-2, I>::type
03761         >;
03762     };
03763
03764     template<typename I>
03765     struct BesselHelper<0, I> {
03766         using type = typename polynomial<I>::one;
03767     };
03768
03769     template<typename I>
03770     struct BesselHelper<1, I> {
03771     private:
03772         using P = polynomial<I>;
03773     public:
03774         using type = typename P::template add_t<
03775             typename P::one,
03776             typename P::X
03777         >;
03778     };
03779 } // namespace internal
03780
03781 namespace known_polynomials {
03782     enum hermite_kind {
03783         probabilist,
03784         physicist
03785     };
03786 }
03787
03788 // hermite
03789 namespace internal {
03790     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03791     struct hermite_helper {};
03792
03793     template<size_t deg, typename I>
03794     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03795     private:
03796         using hnm1 = typename hermite_helper<deg - 1,
03797             known_polynomials::hermite_kind::probabilist, I>::type;
03798         using hnm2 = typename hermite_helper<deg - 2,
03799             known_polynomials::hermite_kind::probabilist, I>::type;
03800     public:
03801         using type = typename polynomial<I>::template sub_t<
03802             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03803             typename polynomial<I>::template mul_t<
03804                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03805                 hnm2
03806             >
03807         >;
03808     };
03809
03810     template<size_t deg, typename I>
03811     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03812     private:
03813         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03814             I>::type;
03815         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03816             I>::type;
03817     public:
03818         using type = typename polynomial<I>::template sub_t<
03819             // 2X Hn-1
03820             typename polynomial<I>::template mul_t<
03821                 typename pi64::val<typename I::template inject_constant_t<2>,
03822                 typename I::zero>, hnm1>,
03823                 typename polynomial<I>::template mul_t<
03824                     typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03825                     hnm2
03826                 >
03827             >
03828         >;
03829     };
03830
03831     template<typename I>
03832     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03833         using type = typename polynomial<I>::one;
03834     };
03835 }

```

```

03836
03837     template<typename I>
03838     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03839         using type = typename polynomial<I>::X;
03840     };
03841
03842     template<typename I>
03843     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03844         using type = typename pi64::one;
03845     };
03846
03847     template<typename I>
03848     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03849         // 2X
03850         using type = typename polynomial<I>::template val<
03851             typename I::template inject_constant_t<2>,
03852             typename I::zero>;
03853     };
03854 } // namespace internal
03855
03856 // legendre
03857 namespace internal {
03858     template<size_t n, typename I>
03859     struct legendre_helper {
03860     private:
03861         using Q = FractionField<I>;
03862         using PQ = polynomial<Q>;
03863         // 1/n constant
03864         // (2n-1)/n X
03865         using fact_left = typename PQ::template monomial_t<
03866             makefraction_t<I,
03867                 typename I::template inject_constant_t<2*n-1>,
03868                 typename I::template inject_constant_t<n>
03869             >,
03870             1>;
03871         // (n-1) / n
03872         using fact_right = typename PQ::template val<
03873             makefraction_t<I,
03874                 typename I::template inject_constant_t<n-1>,
03875                 typename I::template inject_constant_t<n>>;
03876
03877     public:
03878         using type = PQ::template sub_t<
03879             typename PQ::template mul_t<
03880                 fact_left,
03881                 typename legendre_helper<n-1, I>::type
03882             >,
03883             typename PQ::template mul_t<
03884                 fact_right,
03885                 typename legendre_helper<n-2, I>::type
03886             >
03887         >;
03888     };
03889
03890     template<typename I>
03891     struct legendre_helper<0, I> {
03892         using type = typename polynomial<FractionField<I>::one>;
03893     };
03894
03895     template<typename I>
03896     struct legendre_helper<1, I> {
03897         using type = typename polynomial<FractionField<I>::X>;
03898     };
03899 } // namespace internal
03900
03901 // bernoulli polynomials
03902 namespace internal {
03903     template<size_t n>
03904     struct bernoulli_coeff {
03905         template<typename T, size_t i>
03906         struct inner {
03907         private:
03908             using F = FractionField<T>;
03909         public:
03910             using type = typename F::template mul_t<
03911                 typename F::template inject_ring_t<combination_t<T, i, n>,
03912                 bernoulli_t<T, n-i>
03913             >;
03914         };
03915     };
03916 } // namespace internal
03917
03918 namespace internal {
03919     template<size_t n>
03920     struct touchard_coeff {
03921         template<typename T, size_t i>
03922         struct inner {

```



```

03923         using type = stirling_2_t<T, n, i>;
03924     };
03925 };
03926 } // namespace internal
03927
03928 namespace internal {
03929     template<typename I = aerobus::i64>
03930     struct AbelHelper {
03931     private:
03932         using P = aerobus::polynomial<I>;
03933
03934     public:
03935         // to keep recursion working, we need to operate on a*n and not just a
03936         template<size_t deg, I::inner_type an>
03937         struct Inner {
03938             // abel(n, a) = (x-an) * abel(n-1, a)
03939             using type = typename aerobus::mul_t<
03940                 typename Inner<deg-1, an>::type,
03941                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
03942             >;
03943         };
03944
03945         // abel(0, a) = 1
03946         template<I::inner_type an>
03947         struct Inner<0, an> {
03948             using type = P::one;
03949         };
03950
03951         // abel(1, a) = X
03952         template<I::inner_type an>
03953         struct Inner<1, an> {
03954             using type = P::X;
03955         };
03956     };
03957 } // namespace internal
03958
03959 namespace known_polynomials {
03960
03961     template<size_t n, auto a, typename I = aerobus::i64>
03962     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
03963
03964     template <size_t deg, typename I = aerobus::i64>
03965     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03966
03967     template <size_t deg, typename I = aerobus::i64>
03968     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03969
03970     template <size_t deg, typename I = aerobus::i64>
03971     using laguerre = typename internal::laguerre_helper<deg, I>::type;
03972
03973     template <size_t deg, typename I = aerobus::i64>
03974     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03975 I>::type;
03976
03977     template <size_t deg, typename I = aerobus::i64>
03978     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03979
03980     template<size_t i, size_t m, typename I = aerobus::i64>
03981     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03982
03983     template<size_t deg, typename I = aerobus::i64>
03984     using legendre = typename internal::legendre_helper<deg, I>::type;
03985
03986     template<size_t deg, typename I = aerobus::i64>
03987     using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03988
03989     template<size_t deg, typename I = aerobus::i64>
03990     using allone = typename internal::AllOneHelper<deg, I>::type;
03991
03992     template<size_t deg, typename I = aerobus::i64>
03993     using bessel = typename internal::BesselHelper<deg, I>::type;
03994
03995     template<size_t deg, typename I = aerobus::i64>
03996     using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
03997 } // namespace known_polynomials
03998 } // namespace aerobus
03999
04000 #ifndef AEROBUS_CONWAY_IMPORTS
04001
04002 // conway polynomials
04003 namespace aerobus {
04004     template<int p, int n>
04005     struct ConwayPolynomial {};
04006
04007 #ifndef DO_NOT_DOCUMENT
04008     #define ZPZV ZPZ::template val

```



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```
06050 #endif // AEROBUS_CONWAY_IMPORTS
06051
06052 #endif // __INC_AEROBUS__ // NOLINT
```

## 9.4 src/examples.h File Reference

## 9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_
```



# Chapter 10

## Examples

### 10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

### 10.2 examples/custom\_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

## 10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

```
// TO compile with nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu
#include <cstdio>

#define WITH_CUDA_FP16
#include "../src/aerobus.h"

/*
change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
to see how good is the generated assembly compared to what nvcc generates for 16 bits
*/
using int_type = aerobus::i16;
using float_type = __half2;

constexpr size_t N = 1 << 24;

template<typename T>
struct ExpmlDegree;

template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};

template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};

template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};

double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}

template<typename T>
struct GetRandT;

template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};

template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};

template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};

using EXPM1 = aerobus::expml<int_type, ExpmlDegree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}
```

```

}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        out[i] = f(f(f(f(f(f(in[i]))))))) ;
    }
}

int main() {
    float_type *d_in, *d_out;
    cudaMalloc<float_type>(&d_in, N * sizeof(float_type));
    cudaMalloc<float_type>(&d_out, N * sizeof(float_type));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice);

    run<<128, 512>>>(N, d_in, d_out);

    cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost);

    cudaFree(d_in);
    cudaFree(d_out);
}

```

## 10.4 examples/continued\_fractions.cpp

## How to use `aerobus::ContinuedFraction` to get approximations of known numbers

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

static constexpr double PHI = aerobus::ContinuedFraction<
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1,
    1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1, 1>::val;

static const double phi = (std::sqrt(5.0) + 1.0)/2.0;

int main() {
    std::cout << std::setprecision(15) << "Aerobus PHI : " << PHI << std::endl;
    std::cout << std::setprecision(15) << "Computed PHI : " << phi << std::endl;
    return 0;
}
```

## 10.5 examples/modular\_arithmetic.cpp

## How to use `aerobus::zpz` to perform computations on rational fractions with coefficients in modular rings

```
#include <iostream>
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

//  $x^3 + 2x^2 + 1$ , with coefficients in  $\mathbb{Z}/2\mathbb{Z}$ , actually  $x^3 + 1$ 
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
//  $x^3 + 5x^2 + 7x + 11$  with coefficients in  $\mathbb{Z}/17\mathbb{Z}$ , meaning actually  $x^3 + x^2 + 1$ 
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value      = " << v << std::endl;
    return 0;
}
```

## 10.6 examples/make\_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

## 10.7 examples/polynomials\_over\_finite\_field.cpp

How to build a known polynomial (here aerobus::known\_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```

## 10.8 examples/compensated\_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

## See also

[publication](#)

```
// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;
        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}
```



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