

Aerobus

v1.2

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<b>1 Introduction</b>	<b>1</b>
1.1 HOW TO	1
1.1.1 Unit Test	2
1.1.2 Benchmarks	2
1.2 Structures	2
1.2.1 Predefined discrete euclidean domains	2
1.2.2 Polynomials	3
1.2.3 Known polynomials	3
1.2.4 Conway polynomials	3
1.2.5 Taylor series	4
1.3 Operations	5
1.3.1 Field of fractions	5
1.3.2 Quotient	6
1.4 Misc	6
1.4.1 Continued Fractions	6
1.5 CUDA	6
<b>2 Namespace Index</b>	<b>7</b>
2.1 Namespace List	7
<b>3 Concept Index</b>	<b>9</b>
3.1 Concepts	9
<b>4 Class Index</b>	<b>11</b>
4.1 Class List	11
<b>5 File Index</b>	<b>13</b>
5.1 File List	13
<b>6 Namespace Documentation</b>	<b>15</b>
6.1 aerobus Namespace Reference	15
6.1.1 Detailed Description	19
6.1.2 Typedef Documentation	20
6.1.2.1 abs_t	20
6.1.2.2 add_t	20
6.1.2.3 addfractions_t	20
6.1.2.4 alternate_t	20
6.1.2.5 asin	21
6.1.2.6 asinh	21
6.1.2.7 atan	21
6.1.2.8 atanh	21
6.1.2.9 bell_t	23
6.1.2.10 bernoulli_t	23
6.1.2.11 combination_t	23

6.1.2.12 cos	23
6.1.2.13 cosh	25
6.1.2.14 div_t	25
6.1.2.15 E_fraction	25
6.1.2.16 embed_int_poly_in_fractions_t	25
6.1.2.17 exp	26
6.1.2.18 expm1	26
6.1.2.19 factorial_t	26
6.1.2.20 fpq32	26
6.1.2.21 fpq64	27
6.1.2.22 FractionField	27
6.1.2.23 gcd_t	27
6.1.2.24 geometric_sum	27
6.1.2.25 lnp1	28
6.1.2.26 make_frac_polynomial_t	28
6.1.2.27 make_int_polynomial_t	28
6.1.2.28 make_q32_t	28
6.1.2.29 make_q64_t	29
6.1.2.30 makefraction_t	29
6.1.2.31 mul_t	29
6.1.2.32 mulfractions_t	30
6.1.2.33 pi64	30
6.1.2.34 PI_fraction	30
6.1.2.35 pow_t	30
6.1.2.36 pq64	30
6.1.2.37 q32	31
6.1.2.38 q64	31
6.1.2.39 sin	31
6.1.2.40 sinh	31
6.1.2.41 SQRT2_fraction	31
6.1.2.42 SQRT3_fraction	32
6.1.2.43 stirling_1_signed_t	32
6.1.2.44 stirling_1_unsigned_t	32
6.1.2.45 stirling_2_t	32
6.1.2.46 sub_t	33
6.1.2.47 tan	33
6.1.2.48 tanh	33
6.1.2.49 taylor	33
6.1.2.50 vadd_t	34
6.1.2.51 vmul_t	34
6.1.3 Function Documentation	34
6.1.3.1 aligned_malloc()	34

6.1.3.2 field()	34
6.1.4 Variable Documentation	35
6.1.4.1 alternate_v	35
6.1.4.2 bernoulli_v	35
6.1.4.3 combination_v	35
6.1.4.4 factorial_v	36
6.2 aerobus::internal Namespace Reference	36
6.2.1 Detailed Description	39
6.2.2 Typedef Documentation	39
6.2.2.1 make_index_sequence_reverse	39
6.2.2.2 type_at_t	40
6.2.3 Function Documentation	40
6.2.3.1 index_sequence_reverse()	40
6.2.4 Variable Documentation	40
6.2.4.1 is_instantiation_of_v	40
6.3 aerobus::known_polynomials Namespace Reference	40
6.3.1 Detailed Description	40
6.3.2 Enumeration Type Documentation	40
6.3.2.1 hermite_kind	40
<b>7 Concept Documentation</b>	<b>41</b>
7.1 aerobus::IsEuclideanDomain Concept Reference	41
7.1.1 Concept definition	41
7.1.2 Detailed Description	41
7.2 aerobus::IsField Concept Reference	41
7.2.1 Concept definition	41
7.2.2 Detailed Description	42
7.3 aerobus::IsRing Concept Reference	42
7.3.1 Concept definition	42
7.3.2 Detailed Description	42
<b>8 Class Documentation</b>	<b>43</b>
8.1 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E > Struct Template Reference	43
8.2 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0  index > 0)> > Struct Template Reference	43
8.2.1 Member Typedef Documentation	43
8.2.1.1 type	43
8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference	44
8.3.1 Member Typedef Documentation	44
8.3.1.1 type	44
8.4 aerobus::ContinuedFraction< values > Struct Template Reference	44
8.4.1 Detailed Description	44

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference	45
8.5.1 Detailed Description	45
8.5.2 Member Typedef Documentation	45
8.5.2.1 type	45
8.5.3 Member Data Documentation	46
8.5.3.1 val	46
8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference	46
8.6.1 Detailed Description	46
8.6.2 Member Typedef Documentation	47
8.6.2.1 type	47
8.6.3 Member Data Documentation	47
8.6.3.1 val	47
8.7 aerobus::ConwayPolynomial Struct Reference	47
8.8 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference	47
8.8.1 Member Function Documentation	48
8.8.1.1 func()	48
8.9 aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference	48
8.9.1 Member Function Documentation	48
8.9.1.1 func()	48
8.10 aerobus::Embed< Small, Large, E > Struct Template Reference	49
8.10.1 Detailed Description	49
8.11 aerobus::Embed< i32, i64 > Struct Reference	49
8.11.1 Detailed Description	49
8.11.2 Member Typedef Documentation	49
8.11.2.1 type	49
8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference	50
8.12.1 Detailed Description	50
8.12.2 Member Typedef Documentation	50
8.12.2.1 type	50
8.13 aerobus::Embed< q32, q64 > Struct Reference	51
8.13.1 Detailed Description	51
8.13.2 Member Typedef Documentation	51
8.13.2.1 type	51
8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference	52
8.14.1 Detailed Description	52
8.14.2 Member Typedef Documentation	52
8.14.2.1 type	52
8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference	53
8.15.1 Detailed Description	53
8.15.2 Member Typedef Documentation	53
8.15.2.1 type	53

8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference . . . . .	53
8.16.1 Detailed Description . . . . .	54
8.16.2 Member Typedef Documentation . . . . .	54
8.16.2.1 type . . . . .	54
8.17 aerobus::polynomial< Ring >::horner_reduction_t< P > Struct Template Reference . . . . .	54
8.17.1 Detailed Description . . . . .	55
8.18 aerobus::i32 Struct Reference . . . . .	55
8.18.1 Detailed Description . . . . .	56
8.18.2 Member Typedef Documentation . . . . .	56
8.18.2.1 add_t . . . . .	56
8.18.2.2 div_t . . . . .	57
8.18.2.3 eq_t . . . . .	57
8.18.2.4 gcd_t . . . . .	57
8.18.2.5 gt_t . . . . .	57
8.18.2.6 inject_constant_t . . . . .	59
8.18.2.7 inject_ring_t . . . . .	59
8.18.2.8 inner_type . . . . .	59
8.18.2.9 lt_t . . . . .	59
8.18.2.10 mod_t . . . . .	59
8.18.2.11 mul_t . . . . .	60
8.18.2.12 one . . . . .	60
8.18.2.13 pos_t . . . . .	60
8.18.2.14 sub_t . . . . .	60
8.18.2.15 zero . . . . .	61
8.18.3 Member Data Documentation . . . . .	61
8.18.3.1 eq_v . . . . .	61
8.18.3.2 is_euclidean_domain . . . . .	61
8.18.3.3 is_field . . . . .	61
8.18.3.4 pos_v . . . . .	61
8.19 aerobus::i64 Struct Reference . . . . .	62
8.19.1 Detailed Description . . . . .	63
8.19.2 Member Typedef Documentation . . . . .	63
8.19.2.1 add_t . . . . .	63
8.19.2.2 div_t . . . . .	63
8.19.2.3 eq_t . . . . .	64
8.19.2.4 gcd_t . . . . .	64
8.19.2.5 gt_t . . . . .	64
8.19.2.6 inject_constant_t . . . . .	64
8.19.2.7 inject_ring_t . . . . .	65
8.19.2.8 inner_type . . . . .	65
8.19.2.9 lt_t . . . . .	65
8.19.2.10 mod_t . . . . .	65

8.19.2.11 mul_t	65
8.19.2.12 one	66
8.19.2.13 pos_t	66
8.19.2.14 sub_t	66
8.19.2.15 zero	66
8.19.3 Member Data Documentation	67
8.19.3.1 eq_v	67
8.19.3.2 gt_v	67
8.19.3.3 is_euclidean_domain	67
8.19.3.4 is_field	67
8.19.3.5 lt_v	67
8.19.3.6 pos_v	68
8.20 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop > Struct Template Reference	68
8.20.1 Member Typedef Documentation	68
8.20.1.1 type	68
8.21 aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop > Struct Template Reference	69
8.21.1 Member Typedef Documentation	69
8.21.1.1 type	69
8.22 aerobus::is_prime< n > Struct Template Reference	69
8.22.1 Detailed Description	69
8.22.2 Member Data Documentation	70
8.22.2.1 value	70
8.23 aerobus::polynomial< Ring > Struct Template Reference	70
8.23.1 Detailed Description	72
8.23.2 Member Typedef Documentation	72
8.23.2.1 add_t	72
8.23.2.2 derive_t	72
8.23.2.3 div_t	72
8.23.2.4 eq_t	73
8.23.2.5 gcd_t	73
8.23.2.6 gt_t	73
8.23.2.7 inject_constant_t	74
8.23.2.8 inject_ring_t	74
8.23.2.9 lt_t	74
8.23.2.10 mod_t	74
8.23.2.11 monomial_t	75
8.23.2.12 mul_t	75
8.23.2.13 one	75
8.23.2.14 pos_t	75
8.23.2.15 simplify_t	76
8.23.2.16 sub_t	76



8.23.2.17 X	76
8.23.2.18 zero	76
8.23.3 Member Data Documentation	77
8.23.3.1 is_euclidean_domain	77
8.23.3.2 is_field	77
8.23.3.3 pos_v	77
8.24 aerobus::type_list< Ts >::pop_front Struct Reference	77
8.24.1 Detailed Description	77
8.24.2 Member Typedef Documentation	78
8.24.2.1 tail	78
8.24.2.2 type	78
8.25 aerobus::Quotient< Ring, X > Struct Template Reference	78
8.25.1 Detailed Description	79
8.25.2 Member Typedef Documentation	79
8.25.2.1 add_t	79
8.25.2.2 div_t	80
8.25.2.3 eq_t	80
8.25.2.4 inject_constant_t	80
8.25.2.5 inject_ring_t	81
8.25.2.6 mod_t	81
8.25.2.7 mul_t	81
8.25.2.8 one	81
8.25.2.9 pos_t	82
8.25.2.10 zero	82
8.25.3 Member Data Documentation	82
8.25.3.1 eq_v	82
8.25.3.2 is_euclidean_domain	82
8.25.3.3 pos_v	82
8.26 aerobus::type_list< Ts >::split< index > Struct Template Reference	83
8.26.1 Detailed Description	83
8.26.2 Member Typedef Documentation	83
8.26.2.1 head	83
8.26.2.2 tail	83
8.27 aerobus::type_list< Ts > Struct Template Reference	84
8.27.1 Detailed Description	84
8.27.2 Member Typedef Documentation	85
8.27.2.1 at	85
8.27.2.2 concat	85
8.27.2.3 insert	85
8.27.2.4 push_back	86
8.27.2.5 push_front	86
8.27.2.6 remove	86

8.27.3 Member Data Documentation	86
8.27.3.1 length	86
8.28 aerobus::type_list<> Struct Reference	87
8.28.1 Detailed Description	87
8.28.2 Member Typedef Documentation	87
8.28.2.1 concat	87
8.28.2.2 insert	87
8.28.2.3 push_back	87
8.28.2.4 push_front	87
8.28.3 Member Data Documentation	88
8.28.3.1 length	88
8.29 aerobus::i32::val< x > Struct Template Reference	88
8.29.1 Detailed Description	88
8.29.2 Member Typedef Documentation	89
8.29.2.1 enclosing_type	89
8.29.2.2 is_zero_t	89
8.29.3 Member Function Documentation	89
8.29.3.1 get()	89
8.29.3.2 to_string()	89
8.29.4 Member Data Documentation	89
8.29.4.1 v	89
8.30 aerobus::i64::val< x > Struct Template Reference	90
8.30.1 Detailed Description	90
8.30.2 Member Typedef Documentation	91
8.30.2.1 enclosing_type	91
8.30.2.2 inner_type	91
8.30.2.3 is_zero_t	91
8.30.3 Member Function Documentation	91
8.30.3.1 get()	91
8.30.3.2 to_string()	91
8.30.4 Member Data Documentation	92
8.30.4.1 v	92
8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference	92
8.31.1 Detailed Description	93
8.31.2 Member Typedef Documentation	93
8.31.2.1 aN	93
8.31.2.2 coeff_at_t	93
8.31.2.3 enclosing_type	94
8.31.2.4 is_zero_t	94
8.31.2.5 ring_type	94
8.31.2.6 strip	94
8.31.2.7 value_at_t	94

8.31.3 Member Function Documentation	94
8.31.3.1 compensated_eval()	94
8.31.3.2 eval()	95
8.31.3.3 to_string()	95
8.31.4 Member Data Documentation	96
8.31.4.1 degree	96
8.31.4.2 is_zero_v	96
8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference	96
8.32.1 Detailed Description	96
8.32.2 Member Typedef Documentation	97
8.32.2.1 raw_t	97
8.32.2.2 type	97
8.33 aerobus::zpz< p >::val< x > Struct Template Reference	97
8.33.1 Detailed Description	97
8.33.2 Member Typedef Documentation	98
8.33.2.1 enclosing_type	98
8.33.2.2 is_zero_t	98
8.33.3 Member Function Documentation	98
8.33.3.1 get()	98
8.33.3.2 to_string()	98
8.33.4 Member Data Documentation	99
8.33.4.1 is_zero_v	99
8.33.4.2 v	99
8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference	99
8.34.1 Detailed Description	100
8.34.2 Member Typedef Documentation	100
8.34.2.1 aN	100
8.34.2.2 coeff_at_t	100
8.34.2.3 enclosing_type	100
8.34.2.4 is_zero_t	101
8.34.2.5 ring_type	101
8.34.2.6 strip	101
8.34.2.7 value_at_t	101
8.34.3 Member Function Documentation	101
8.34.3.1 eval()	101
8.34.3.2 to_string()	101
8.34.4 Member Data Documentation	102
8.34.4.1 degree	102
8.34.4.2 is_zero_v	102
8.35 aerobus::zpz< p > Struct Template Reference	102
8.35.1 Detailed Description	103
8.35.2 Member Typedef Documentation	104

8.35.2.1 add_t . . . . .	104
8.35.2.2 div_t . . . . .	104
8.35.2.3 eq_t . . . . .	104
8.35.2.4 gcd_t . . . . .	105
8.35.2.5 gt_t . . . . .	105
8.35.2.6 inject_constant_t . . . . .	105
8.35.2.7 inner_type . . . . .	105
8.35.2.8 lt_t . . . . .	106
8.35.2.9 mod_t . . . . .	106
8.35.2.10 mul_t . . . . .	106
8.35.2.11 one . . . . .	106
8.35.2.12 pos_t . . . . .	107
8.35.2.13 sub_t . . . . .	107
8.35.2.14 zero . . . . .	107
8.35.3 Member Data Documentation . . . . .	107
8.35.3.1 eq_v . . . . .	107
8.35.3.2 gt_v . . . . .	108
8.35.3.3 is_euclidean_domain . . . . .	108
8.35.3.4 is_field . . . . .	108
8.35.3.5 lt_v . . . . .	108
8.35.3.6 pos_v . . . . .	108
<b>9 File Documentation</b>	<b>111</b>
9.1 README.md File Reference . . . . .	111
9.2 src/aerobus.h File Reference . . . . .	111
9.3 aerobus.h . . . . .	111
9.4 src/examples.h File Reference . . . . .	204
9.5 examples.h . . . . .	204
<b>10 Examples</b>	<b>205</b>
10.1 examples/hermite.cpp . . . . .	205
10.2 examples/custom_taylor.cpp . . . . .	205
10.3 examples/fp16.cu . . . . .	206
10.4 examples/continued_fractions.cpp . . . . .	206
10.5 examples/modular_arithmetic.cpp . . . . .	206
10.6 examples/make_polynomial.cpp . . . . .	207
10.7 examples/polynomials_over_finite_field.cpp . . . . .	207
10.8 examples/compensated_horner.cpp . . . . .	208
<b>Index</b>	<b>209</b>

# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.



For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its **field of fractions**.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , `Aerobus` provides automatic implementation for **quotient ring**  $R/X$  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

`Aerobus` gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, `aerobus` gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

## 1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, `Aerobus` provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

<a href="#">aerobus</a>	Main namespace for all publicly exposed types or functions . . . . .	15
<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	36
<a href="#">aerobus::known_polynomials</a>	Families of well known polynomials such as Hermite or Bernstein . . . . .	40



## Chapter 3

# Concept Index

### 3.1 Concepts

Here is a list of all concepts with brief descriptions:

<a href="#">aerobus::IsEuclideanDomain</a>	
Concept to express R is an euclidean domain . . . . .	41
<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	41
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	42



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, E &gt; . . . . .</a>	<a href="#">43</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt; 43</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt; . . . . .</a>	<a href="#">44</a>
<a href="#">aerobus::ContinuedFraction&lt; values &gt;</a>	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$ . . . . .	<a href="#">44</a>
<a href="#">aerobus::ContinuedFraction&lt; a0 &gt;</a>	
Specialization for only one coefficient, technically just 'a0' . . . . .	<a href="#">45</a>
<a href="#">aerobus::ContinuedFraction&lt; a0, rest... &gt;</a>	
Specialization for multiple coefficients (strictly more than one) . . . . .	<a href="#">46</a>
<a href="#">aerobus::ConwayPolynomial</a> . . . . .	<a href="#">47</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::compensated_horner&lt; arithmeticType, P &gt;::EFTHorner&lt; index, ghost &gt;</a>	<a href="#">47</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::compensated_horner&lt; arithmeticType, P &gt;::EFTHorner&lt;-1, ghost &gt; . .</a>	<a href="#">48</a>
<a href="#">aerobus::Embed&lt; Small, Large, E &gt;</a>	
Embedding - struct forward declaration . . . . .	<a href="#">49</a>
<a href="#">aerobus::Embed&lt; i32, i64 &gt;</a>	
Embeds i32 into i64 . . . . .	<a href="#">49</a>
<a href="#">aerobus::Embed&lt; polynomial&lt; Small &gt;, polynomial&lt; Large &gt; &gt;</a>	
Embeds polynomial<Small> into polynomial<Large> . . . . .	<a href="#">50</a>
<a href="#">aerobus::Embed&lt; q32, q64 &gt;</a>	
Embeds q32 into q64 . . . . .	<a href="#">51</a>
<a href="#">aerobus::Embed&lt; Quotient&lt; Ring, X &gt;, Ring &gt;</a>	
Embeds Quotient<Ring, X> into Ring . . . . .	<a href="#">52</a>
<a href="#">aerobus::Embed&lt; Ring, FractionField&lt; Ring &gt; &gt;</a>	
Embeds values from Ring to its field of fractions . . . . .	<a href="#">53</a>
<a href="#">aerobus::Embed&lt; zpz&lt; x &gt;, i32 &gt;</a>	
Embeds zpz values into i32 . . . . .	<a href="#">53</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;</a>	
Used to evaluate polynomials over a value in Ring . . . . .	<a href="#">54</a>
<a href="#">aerobus::i32</a>	
32 bits signed integers, seen as a algebraic ring with related operations . . . . .	<a href="#">55</a>
<a href="#">aerobus::i64</a>	
64 bits signed integers, seen as a algebraic ring with related operations . . . . .	<a href="#">62</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;::inner&lt; index, stop &gt;</a>	<a href="#">68</a>
<a href="#">aerobus::polynomial&lt; Ring &gt;::horner_reduction_t&lt; P &gt;::inner&lt; stop, stop &gt;</a>	<a href="#">69</a>

<a href="#">aerobus::is_prime&lt; n &gt;</a>	69
Checks if n is prime	
<a href="#">aerobus::polynomial&lt; Ring &gt;</a>	70
<a href="#">aerobus::type_list&lt; Ts &gt;::pop_front</a>	77
Removes types from head of the list	
<a href="#">aerobus::Quotient&lt; Ring, X &gt;</a>	78
Quotient ring by the principal ideal generated by 'X' With <a href="#">i32</a> as Ring and <a href="#">i32::val&lt;2&gt;</a> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	
<a href="#">aerobus::type_list&lt; Ts &gt;::split&lt; index &gt;</a>	83
Splits list at index	
<a href="#">aerobus::type_list&lt; Ts &gt;</a>	84
Empty pure template struct to handle type list	
<a href="#">aerobus::type_list&lt;&gt;</a>	87
Specialization for empty type list	
<a href="#">aerobus::i32::val&lt; x &gt;</a>	88
Values in <a href="#">i32</a> , again represented as types	
<a href="#">aerobus::i64::val&lt; x &gt;</a>	90
Values in <a href="#">i64</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN, coeffs &gt;</a>	92
Values (seen as types) in polynomial ring	
<a href="#">aerobus::Quotient&lt; Ring, X &gt;::val&lt; V &gt;</a>	96
Projection values in the quotient ring	
<a href="#">aerobus::zpz&lt; p &gt;::val&lt; x &gt;</a>	97
Values in <a href="#">zpz</a>	
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;</a>	99
Specialization for constants	
<a href="#">aerobus::zpz&lt; p &gt;</a>	102
Congruence classes of integers modulo p (32 bits)	



# Chapter 5

## File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

src/ <a href="#">aerobus.h</a> . . . . .	111
src/ <a href="#">examples.h</a> . . . . .	204



## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial<Small> into polynomial<Large>*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient<Ring, X> into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
  - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
  - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)
  - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
  - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)
  - Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)
  - specialization for empty type list*
- struct [zpz](#)
  - congruence classes of integers modulo p (32 bits)*

## Concepts

- concept [IsRing](#)
  - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
  - Concept to express R is an euclidean domain.*
- concept [IsField](#)
  - Concept to express R is a field.*

## Typedefs

- template<typename T , typename A , typename B >
 using [gcd\\_t](#) = typename internal::gcd< T >::template type< A, B >
  - computes the greatest common divisor of A and B*
- template<typename... vals>
 using [vadd\\_t](#) = typename internal::vadd< vals... >::type
  - adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an [add\\_t](#) binary operator*
- template<typename... vals>
 using [vmul\\_t](#) = typename internal::vmul< vals... >::type
  - multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an [mul\\_t](#) binary operator*
- template<typename val >
 using [abs\\_t](#) = std::conditional\_t< val::enclosing\_type::template pos\_v< val >, val, typename val::enclosing\_type::template [sub\\_t](#)< typename val::enclosing\_type::zero, val > >
  - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >
 using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type
  - Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .*
- template<typename X , typename Y >
 using [add\\_t](#) = typename X::enclosing\_type::template [add\\_t](#)< X, Y >
  - generic addition*
- template<typename X , typename Y >
 using [sub\\_t](#) = typename X::enclosing\_type::template [sub\\_t](#)< X, Y >

- generic subtraction*
- `template<typename X , typename Y >`  
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
- generic multiplication*
- `template<typename X , typename Y >`  
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
- generic division*
- `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
- `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
- `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
- `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
- `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
- `template<typename Ring , typename v1 , typename v2 >`  
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
- `template<typename v >`  
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
- `template<int64_t p, int64_t q>`  
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<typename Ring , typename v1 , typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- `template<typename Ring , typename v1 , typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- `template<typename Ring , auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
- `template<typename Ring , auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField< Ring>*
- `template<typename T , size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*

- `template<typename T , size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T , size_t n>`  
`using bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T , size_t n>`  
`using bell_t = typename internal::bell_helper< T, n >::type`  
*Bell numbers.*
- `template<typename T , int k>`  
`using alternate_t = typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T , int n, int k>`  
`using stirling_1_signed_t = typename internal::stirling_1_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T , int n, int k>`  
`using stirling_1_unsigned_t = abs_t< typename internal::stirling_1_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T , int n, int k>`  
`using stirling_2_t = typename internal::stirling_2_helper< T, n, k >::type`  
*Stirling number of second kind – as types.*
- `template<typename T , typename p , size_t n>`  
`using pow_t = typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`  
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<`  
`deg+1 > >::type`
- `template<typename Integers , size_t deg>`  
`using exp = taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers , size_t deg>`  
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg`  
`>, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers , size_t deg>`  
`using lnp1 = taylor< Integers, internal::lnp1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`  
`using atan = taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers , size_t deg>`  
`using sin = taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers , size_t deg>`  
`using sinh = taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers , size_t deg>`  
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers , size_t deg>`  
`using cos = taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*
- `template<typename Integers , size_t deg>`  
`using geometric_sum = taylor< Integers, internal::geom_coeff, deg >`

- $\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$   
 • template<typename Integers , size\_t deg>  
 using **asin** = **taylor**< Integers, internal::asin\_coeff, deg >  
 arcsin( $x$ ) *arc sinus*
- template<typename Integers , size\_t deg>  
 using **asinh** = **taylor**< Integers, internal::asinh\_coeff, deg >  
 arcsinh( $x$ ) *arc hyperbolic sinus*
- template<typename Integers , size\_t deg>  
 using **atanh** = **taylor**< Integers, internal::atanh\_coeff, deg >  
 arctanh( $x$ ) *arc hyperbolic tangent*
- template<typename Integers , size\_t deg>  
 using **tan** = **taylor**< Integers, internal::tan\_coeff, deg >  
 tan( $x$ ) *tangent*
- template<typename Integers , size\_t deg>  
 using **tanh** = **taylor**< Integers, internal::tanh\_coeff, deg >  
 tanh( $x$ ) *hyperbolic tangent*
- using **PI\_fraction** = **ContinuedFraction**< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >
- using **E\_fraction** = **ContinuedFraction**< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >  
*approximation of  $e$*
- using **SQRT2\_fraction** = **ContinuedFraction**< 1, 2 >  
*approximation of  $\sqrt{2}$*
- using **SQRT3\_fraction** = **ContinuedFraction**< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >  
*approximation of*

## Functions

- template<typename T >  
 T \* **aligned\_malloc** (size\_t count, size\_t alignment)
- brief Conway polynomials tparam p characteristic of the **field** (prime number) @tparam n degree of extension  
 template< int p

## Variables

- template<typename T , size\_t i>  
 constexpr T::inner\_type **factorial\_v** = internal::factorial<T, i>::value  
*computes factorial( $i$ ) as value in  $T$*
- template<typename T , size\_t k, size\_t n>  
 constexpr T::inner\_type **combination\_v** = internal::combination<T, k, n>::value  
*computes binomial coefficients ( $k$  among  $n$ ) as value*
- template<typename FloatType , typename T , size\_t n>  
 constexpr FloatType **bernoulli\_v** = internal::bernoulli<T, n>::template value<FloatType>  
 *$n$ th bernoulli number as value in  $FloatType$*
- template<typename T , size\_t k>  
 constexpr T::inner\_type **alternate\_v** = internal::alternate<T, k>::value  
 *$(-1)^k$  as value from  $T$*

### 6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

## 6.1.2 Typedef Documentation

### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

#### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T



## Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example
----------	---

## 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh( $x$ )` arc hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

## Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

## 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

## 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

## 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  `cosinus`

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

## Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

## 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of  $e$

## 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

## Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

## 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rational coefficients (32 bits numerator and denominator)

#### 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

#### 6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

##### Template Parameters

<i>Ring</i>	
-------------	--

#### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

##### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

#### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

##### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator



## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

**6.1.2.29 make\_q64\_t**

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

**6.1.2.30 makefraction\_t**

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

## Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

**6.1.2.31 mul\_t**

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

### 6.1.2.32 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

### 6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

### 6.1.2.34 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of  $\pi$  as a continued fraction

### 6.1.2.35 pow\_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

#### Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

### 6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.37 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.38 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

**6.1.2.39 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.40 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.41 SQRT2\_fraction**

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of  $\sqrt{2}$

#### 6.1.2.42 Sqrt3\_fraction

```
using aerobus::Sqrt3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

approximation of

#### 6.1.2.43 stirling\_1\_signed\_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_signed_t = typedef typename internal::stirling_1_helper<T, n, k>::type
```

Stirling number of first king (signed) – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

#### 6.1.2.44 stirling\_1\_unsigned\_t

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T, n, k>::type>
```

Stirling number of first king (unsigned) – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

#### 6.1.2.45 stirling\_2\_t

```
template<typename T , int n, int k>
using aerobus::stirling_2_t = typedef typename internal::stirling_2_helper<T, n, k>::type
```

Stirling number of second king – as types.

##### Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

**6.1.2.46 sub\_t**

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

**Template Parameters**

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

**6.1.2.47 tan**

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.48 tanh**

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.49 taylor**

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

**Template Parameters**

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff<sub>↔</sub> _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

### 6.1.2.50 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.2.51 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

### 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

#### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

#### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

### 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<Float↵
Type> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

##### Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, I >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >



- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **fma\_helper**
- struct **fma\_helper**< double >
- struct **fma\_helper**< float >
- struct **fma\_helper**< int16\_t >
- struct **fma\_helper**< int32\_t >

- struct **fma\_helper**< int64\_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**
  - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **staticcast**
- struct **stirling\_1\_helper**
- struct **stirling\_1\_helper**< T, 0, 0 >
- struct **stirling\_1\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, k, std::enable\_if\_t<(k > 0) && (n > 0)> >
- struct **stirling\_2\_helper**
- struct **stirling\_2\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >

- struct **stirling\_2\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling\_2\_helper**< T, n, n, std::enable\_if\_t<(n >=0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **touchard\_coeff**
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N>{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

## 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

## 6.2.2 Typedef Documentation

### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Enumeration Type Documentation

#### 6.3.2.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```

#### Enumerator

probabilist	
physicist	

# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename Ring::zero

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using [type](#) = [aN](#)

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

#### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

##### Template Parameters

<i>...values</i>	are <code>int64_t</code>
------------------	-----------------------------

#### Examples

[examples/continued\\_fractions.cpp](#).



The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
represented value as double

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

#### Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

### 8.5.2 Member Typedef Documentation

#### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template [add\\_t](#)< typename q64::template inject\_constant\_t< a0 >, typename q64::template [div\\_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

#### Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINED** void [func](#) (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.8.1 Member Function Documentation

### 8.8.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE** **DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.9.1 Member Function Documentation

### 8.9.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P >::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.10 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.10.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.11 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

### Public Types

- template<typename val >  
using [type](#) = [i64::val](#)< static\_cast< int64\_t >(val::v)>  
*the [i64](#) representation of val*

### 8.11.1 Detailed Description

embeds [i32](#) into [i64](#)

### 8.11.2 Member Typedef Documentation

#### 8.11.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

## Template Parameters

<i>val</i>	a value in <a href="#">i32</a>
------------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.12 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

## Public Types

- `template<typename v >`  
using `type` = `typename at_low< v, typename internal::make\_index\_sequence\_reverse< v::degree+1 > >::type`  
*the polynomial<Large> representation of v*

### 8.12.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

## Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

### 8.12.2 Member Typedef Documentation

#### 8.12.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make\_index\_sequence\_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

## Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

## Public Types

- `template<typename v >`  
using `type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>`  
*q64 representation of v*

### 8.13.1 Detailed Description

embeds q32 into q64

### 8.13.2 Member Typedef Documentation

#### 8.13.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_←
cast<int64_t>(v::y::v)>
```

q64 representation of v

## Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = typename val::raw_t`  
*Ring representation of val.*

### 8.14.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`



## 8.15 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = typename `FractionField< Ring >::template val< v, typename Ring::one >`  
*FractionField<Ring> representation of v.*

### 8.15.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

<i>Ring</i>	an integers ring, such as <a href="#">i32</a>
-------------	---

### 8.15.2 Member Typedef Documentation

#### 8.15.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

`FractionField<Ring>` representation of v.

#### Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.16 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the i32 representation of val*

### 8.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into i32

#### Template Parameters

<i>x</i>	an integer
----------	------------

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the i32 representation of val

#### Template Parameters

<i>val</i>	a value in zpz<x>
------------	-------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.17 aerobus::polynomial< Ring >::horner\_reduction\_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

## Classes

- struct [inner](#)
- struct [inner](#)< [stop](#), [stop](#) >

### 8.17.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

#### Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.18 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

## Public Types

- using [inner\\_type](#) = int32\_t
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*inject a native constant*
- template<typename v >  
using [inject\\_ring\\_t](#) = v
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator yields v1 + v2*

- `template<typename v1 , typename v2 >`  
`using sub\_t = typename sub< v1, v2 >::type`  
*subtraction operator yields  $v1 - v2$*
- `template<typename v1 , typename v2 >`  
`using mul\_t = typename mul< v1, v2 >::type`  
*multiplication operator yields  $v1 * v2$*
- `template<typename v1 , typename v2 >`  
`using div\_t = typename div< v1, v2 >::type`  
*division operator yields  $v1 / v2$*
- `template<typename v1 , typename v2 >`  
`using mod\_t = typename remainder< v1, v2 >::type`  
*modulus operator yields  $v1 \% v2$*
- `template<typename v1 , typename v2 >`  
`using gt\_t = typename gt< v1, v2 >::type`  
*strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$*
- `template<typename v1 , typename v2 >`  
`using lt\_t = typename lt< v1, v2 >::type`  
*strict less operator ( $v1 < v2$ ) yields  $v1 < v2$*
- `template<typename v1 , typename v2 >`  
`using eq\_t = typename eq< v1, v2 >::type`  
*equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`*
- `template<typename v1 , typename v2 >`  
`using gcd\_t = gcd\_t< i32, v1, v2 >`  
*greatest common divisor yields  $GCD(v1, v2)$*
- `template<typename v >`  
`using pos\_t = typename pos< v >::type`  
*positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`*

### Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator (boolean value)*
- `template<typename v >`  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity (boolean value) yields  $v > 0$  as boolean value*

## 8.18.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::add\_t = typename add<v1, v2>::type

addition operator yields  $v1 + v2$ 
```

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

8.18.2.2 `div_t`

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

division operator yields  $v1 / v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

8.18.2.3 `eq_t`

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

8.18.2.4 `gcd_t`

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields  $GCD(v1, v2)$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

8.18.2.5 `gt_t`

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

## 8.18.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

inject a native constant

## Template Parameters

<i>x</i>	
----------	--

## 8.18.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

## 8.18.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

## 8.18.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

strict less operator ( $v1 < v2$ ) yields  $v1 < v2$

## Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

## 8.18.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields  $v1 * v2$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

**8.18.2.12 one**

```
using aerobus::i32::one = val<1>
```

constant one

**8.18.2.13 pos\_t**

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`

**Template Parameters**

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

**8.18.2.14 sub\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields  $v1 - v2$

**Template Parameters**

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>



### 8.18.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

## 8.18.3 Member Data Documentation

### 8.18.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.18.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.18.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

### 8.18.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields  $v > 0$  as boolean value

#### Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in [i64](#)*

### Public Types

- using [inner\\_type](#) = int64\_t  
*type of represented values*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int64\_t >(x)>  
*injects constant as an [i64](#) value*
- template<typename v >  
using [inject\\_ring\\_t](#) = v  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> [i64::val<1>](#)*
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = typename mul< v1, v2 >::type  
*multiplication operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = typename div< v1, v2 >::type  
*division operator integer division*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = typename remainder< v1, v2 >::type  
*modulus operator*
- template<typename v1 , typename v2 >  
using [gt\\_t](#) = typename gt< v1, v2 >::type  
*strictly greater operator yields v1 > v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [lt\\_t](#) = typename lt< v1, v2 >::type  
*strict less operator yields v1 < v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename eq< v1, v2 >::type  
*equality operator yields v1 == v2 as std::true\_type or std::false\_type*
- template<typename v1 , typename v2 >  
using [gcd\\_t](#) = [gcd\\_t](#)< [i64](#), v1, v2 >  
*greatest common divisor yields GCD(v1, v2) as instantiation of [i64::val](#)*
- template<typename v >  
using [pos\\_t](#) = typename pos< v >::type  
*is v positive yields v > 0 as std::true\_type or std::false\_type*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- template<typename v1 , typename v2 >  
static constexpr bool [gt\\_v](#) = [gt\\_t](#)<v1, v2>::value  
*strictly greater operator yields  $v1 > v2$  as boolean value*
- template<typename v1 , typename v2 >  
static constexpr bool [lt\\_v](#) = [lt\\_t](#)<v1, v2>::value  
*strictly smaller operator yields  $v1 < v2$  as boolean value*
- template<typename v1 , typename v2 >  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator yields  $v1 == v2$  as boolean value*
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity yields  $v > 0$  as boolean value*

### 8.19.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::add\_t = typename add<v1, v2>::type
```

addition operator

#### Template Parameters

<a href="#">v1</a>	: an element of <a href="#">aerobus::i64::val</a>
<a href="#">v2</a>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.2 [div\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i64::div\_t = typename div<v1, v2>::type
```

division operator integer division

#### Template Parameters

<a href="#">v1</a>	: an element of <a href="#">aerobus::i64::val</a>
<a href="#">v2</a>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields GCD( $v1, v2$ ) as instantiation of [i64::val](#)

#### Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

#### Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

### 8.19.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

#### Template Parameters

<i>x</i>	
----------	--

### 8.19.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`  
 -> `i64::val<1>`

#### Template Parameters

<code>v</code>	a value in <code>i64</code>
----------------	-----------------------------

### 8.19.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

### 8.19.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields `v1 < v2` as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
<code>v2</code>	: an element of <code>aerobus::i64::val</code>

### 8.19.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

#### Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
<code>v2</code>	: an element of <code>aerobus::i64::val</code>

### 8.19.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

#### 8.19.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
-----------------	---

#### 8.19.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

### 8.19.3 Member Data Documentation

#### 8.19.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields  $v1 == v2$  as boolean value

##### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

##### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

#### 8.19.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.19.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

#### 8.19.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields  $v1 < v2$  as boolean value

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

8.19.3.6 `pos_v`

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos\_t<v>::value [static], [constexpr]
```

positivity yields  $v > 0$  as boolean value

## Template Parameters

<code>v</code>	: an element of <a href="#">aerobus::i64::val</a>
----------------	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 `aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >` Struct Template Reference

```
#include <aerobus.h>
```

## Public Types

- `template<typename accum , typename x >`  
using `type` = `typename horner\_reduction\_t< P >::template inner< index+1, stop > ::template type< typename Ring::template add\_t< typename Ring::template mul\_t< x, accum >, typename P::template coeff\_↵  
at\_t< P::degree - index > >, x >`

### 8.20.1 Member Typedef Documentation

#### 8.20.1.1 `type`

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner\_reduction\_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add\_t< typename Ring::template mul\_t<x, accum>, typename P::template coeff\_↵
at\_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## 8.21 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
`using type = accum`

### 8.21.1 Member Typedef Documentation

#### 8.21.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.22 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- `static constexpr bool value = internal::_is_prime<n, 5>::value`  
*true iff n is prime*

### 8.22.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

## Template Parameters

<i>n</i>	
----------	--

## 8.22.2 Member Data Documentation

### 8.22.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime<n>::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.23 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [horner\\_reduction\\_t](#)  
*Used to evaluate polynomials over a value in Ring.*
- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val<coeffN>](#)  
*specialization for constants*

### Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*constant zero*
- using [one](#) = [val](#)< typename Ring::one >  
*constant one*
- using [X](#) = [val](#)< typename Ring::one, typename Ring::zero >  
*generator*
- template<typename P >  
using [simplify\\_t](#) = typename simplify< P >::type  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*adds two polynomials*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type

- subtraction of two polynomials*

  - template<typename v1 , typename v2 >  
using `mul_t` = typename mul< v1, v2 >::type
- multiplication of two polynomials*

  - template<typename v1 , typename v2 >  
using `eq_t` = typename eq\_helper< v1, v2 >::type
- equality operator*

  - template<typename v1 , typename v2 >  
using `lt_t` = typename lt\_helper< v1, v2 >::type
- strict less operator*

  - template<typename v1 , typename v2 >  
using `gt_t` = typename gt\_helper< v1, v2 >::type
- strict greater operator*

  - template<typename v1 , typename v2 >  
using `div_t` = typename div< v1, v2 >::q\_type
- division operator*

  - template<typename v1 , typename v2 >  
using `mod_t` = typename div\_helper< v1, v2, `zero`, v1 >::mod\_type
- modulo operator*

  - template<typename coeff , size\_t deg>  
using `monomial_t` = typename monomial< coeff, deg >::type
- monomial : coeff X<sup>deg</sup>*

  - template<typename v >  
using `derive_t` = typename derive\_helper< v >::type
- derivation operator*

  - template<typename v >  
using `pos_t` = typename Ring::template `pos_t`< typename v::aN >
- checks for positivity (an > 0)*

  - template<typename v1 , typename v2 >  
using `gcd_t` = std::conditional\_t< Ring::is\_euclidean\_domain, typename make\_unit< `gcd_t`< `polynomial`< Ring >, v1, v2 > >::type, void >
- greatest common divisor of two polynomials*

  - template<auto x>  
using `inject_constant_t` = val< typename Ring::template `inject_constant_t`< x > >
- makes the constant (native type) polynomial a\_0*

  - template<typename v >  
using `inject_ring_t` = val< v >
- makes the constant (ring type) polynomial a\_0*

### Static Public Attributes

- static constexpr bool `is_field` = false
  - static constexpr bool `is_euclidean_domain` = Ring::is\_euclidean\_domain
  - template<typename v >  
static constexpr bool `pos_v` = `pos_t`<v>::value
- positivity operator*

### 8.23.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

#### Examples

[examples/compensated\\_horner.cpp](#), [examples/make\\_polynomial.cpp](#), and [examples/modular\\_arithmetic.cpp](#).

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

##### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.23.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

##### Template Parameters

<i>v</i>	
----------	--

#### 8.23.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.23.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.23.2.5 gcd\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.23.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.23.2.7 inject\_constant\_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

makes the constant (native type) polynomial a\_0

#### Template Parameters

x	
---	--

### 8.23.2.8 inject\_ring\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial a\_0

#### Template Parameters

v	
---	--

### 8.23.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

v1	
v2	

### 8.23.2.10 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.23.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X<sup>deg</sup>

## Template Parameters

<i>coeff</i>	
<i>deg</i>	

## 8.23.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.23.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

## 8.23.2.14 pos\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

<i>v</i>	
----------	--

**8.23.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

<i>P</i>	
----------	--

**8.23.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.23.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.23.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero



### 8.23.3 Member Data Documentation

#### 8.23.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.23.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.23.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<i>v</i>	a value in <a href="#">polynomial::val</a>
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.24 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.24.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.24.2 Member Typedef Documentation

### 8.24.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.24.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.25 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*zero value*
- using [one](#) = [val](#)< typename Ring::one >  
*one*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = [val](#)< typename Ring::template [add\\_t](#)< typename v1::type, typename v2::type > >  
*addition operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = [val](#)< typename Ring::template [mul\\_t](#)< typename v1::type, typename v2::type > >  
*subtraction operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = [val](#)< typename Ring::template [div\\_t](#)< typename v1::type, typename v2::type > >  
*division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = [val](#)< typename Ring::template [mod\\_t](#)< typename v1::type, typename v2::type > >

- modulus operator*
- template<typename v1 , typename v2 >  
using `eq_t` = typename Ring::template `eq_t`< typename v1::type, typename v2::type >  
*equality operator (as type)*
- template<typename v1 >  
using `pos_t` = std::true\_type  
*positivity operator always true*
- template<auto x>  
using `inject_constant_t` = val< typename Ring::template `inject_constant_t`< x > >  
*inject a 'constant' in quotient ring\**
- template<typename v >  
using `inject_ring_t` = val< v >  
*projects a value of Ring onto the quotient*

## Static Public Attributes

- template<typename v1 , typename v2 >  
static constexpr bool `eq_v` = Ring::template `eq_t`<typename v1::type, typename v2::type>::value  
*addition operator (as boolean value)*
- template<typename v >  
static constexpr bool `pos_v` = `pos_t`<v>::value  
*positivity operator always true*
- static constexpr bool `is_euclidean_domain` = true  
*quotien rings are euclidean domain*

### 8.25.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as ' <code>i32</code> ', must satisfy the <code>IsRing</code> concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1↔
::type, typename v2::type> >
```

addition operator

**Template Parameters**

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

**8.25.2.2 div\_t**

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

**Template Parameters**

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

**8.25.2.3 eq\_t**

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

**Template Parameters**

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

**8.25.2.4 inject\_constant\_t**

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring\*

**Template Parameters**

<i>x</i>	a 'constant' from Ring point of view
----------	--------------------------------------

### 8.25.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

#### Template Parameters

<i>v</i>	a value in Ring
----------	-----------------

### 8.25.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.25.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.25.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.25.3 Member Data Documentation

### 8.25.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.25.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.25.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.26.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.26.2 Member Typedef Documentation

#### 8.26.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.26.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.27 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = [internal::type\\_at\\_t](#)< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename [concat\\_h](#)< U >::type  
*concatenates two list into one*
- template<typename T, size\_t index>  
using [insert](#) = typename [internal::insert\\_h](#)< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename [internal::remove\\_h](#)< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*

### Static Public Attributes

- static constexpr size\_t [length](#) = sizeof...(Ts)  
*length of list*

#### 8.27.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.



## Template Parameters

<i>...Ts</i>	types to store and manipulate at compile time
--------------	---

## 8.27.2 Member Typedef Documentation

## 8.27.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

## Template Parameters

<i>index</i>	
--------------	--

## 8.27.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

## Template Parameters

<i>U</i>	
----------	--

## 8.27.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

## Template Parameters

<i>index</i>	
<i>T</i>	

### 8.27.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.5 push\_front

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

<i>T</i>	
----------	--

### 8.27.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

<i>index</i>	
--------------	--

## 8.27.3 Member Data Documentation

### 8.27.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.28 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t [length](#) = 0

### 8.28.1 Detailed Description

specialization for empty type list

### 8.28.2 Member Typedef Documentation

#### 8.28.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

#### 8.28.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

#### 8.28.2.3 push\_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

#### 8.28.2.4 push\_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

### 8.28.3 Member Data Documentation

#### 8.28.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.29 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using [enclosing\\_type](#) = [i32](#)  
*Enclosing ring type.*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [DEVICE](#) valueType [get](#) ()  
*cast x into valueType*
- static std::string [to\\_string](#) ()  
*string representation of value*

### Static Public Attributes

- static constexpr int32\_t [v](#) = x  
*actual value stored in val type*

#### 8.29.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 8.29.2 Member Typedef Documentation

### 8.29.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

### 8.29.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.29.3 Member Function Documentation

### 8.29.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

## Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

### 8.29.3.2 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

## 8.29.4 Member Data Documentation

### 8.29.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.30 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using [inner\\_type](#) = int32\_t  
*type of represented values*
- using [enclosing\\_type](#) = [i64](#)  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr [INLINED\\_DEVICE](#) valueType [get](#) ()  
*cast value in valueType*
- static std::string [to\\_string](#) ()  
*string representation*

### Static Public Attributes

- static constexpr int64\_t [v](#) = x  
*actual value*

#### 8.30.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

<a href="#">x</a>	an actual integer
-------------------	-------------------

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.30.2 Member Typedef Documentation

### 8.30.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.30.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.30.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.30.3 Member Function Documentation

### 8.30.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

#### Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.30.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

### 8.30.4 Member Data Documentation

#### 8.30.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.31 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

### Public Types

- using `ring_type` = Ring  
*ring coefficients live in*
- using `enclosing_type` = polynomial< Ring >  
*enclosing ring type*
- using `aN` = coeffN  
*heavy weight coefficient (non zero)*
- using `strip` = val< coeffs... >  
*remove largest coefficient*
- using `is_zero_t` = std::bool\_constant<(degree==0) &&(aN::is\_zero\_t::value)>  
*true\_type if polynomial is constant zero*
- template<size\_t index>  
using `coeff_at_t` = typename coeff\_at< index >::type  
*type of coefficient at index*
- template<typename x >  
using `value_at_t` = horner\_reduction\_t< val >::template inner< 0, degree+1 >::template type< typename Ring::zero, x >

### Static Public Member Functions

- static std::string `to_string` ()  
*get a string representation of polynomial*
- template<typename arithmeticType >  
static constexpr `DEVICE INLINED` arithmeticType `eval` (const arithmeticType &x)  
*evaluates polynomial seen as a function operating on arithmeticType*
- template<typename arithmeticType >  
static `DEVICE INLINED` arithmeticType `compensated_eval` (const arithmeticType &x)  
*Evaluate polynomial on x using compensated horner scheme This is twice as accurate as simple eval (horner) but cannot be constexpr Please not this makes no sense on integer types as arithmetic on integers is exact in IEEE.*



## Static Public Attributes

- static constexpr size\_t [degree](#) = sizeof...(coeffs)  
*degree of the polynomial*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value  
*true if polynomial is constant zero*

### 8.31.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<i>coeffN</i>	high degree coefficient
<i>...coeffs</i>	lower degree coefficients

#### Examples

[examples/compensated\\_horner.cpp](#).

### 8.31.2 Member Typedef Documentation

#### 8.31.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

#### 8.31.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.31.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.31.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.31.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

### 8.31.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

### 8.31.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

## 8.31.3 Member Function Documentation

### 8.31.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme This is twice as accurate as simple eval (horner) but cannot be constexpr Please not this makes no sense on integer types as arithmetic on integers is exact in IEEE.

## Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

## Parameters

x	
---	--

## 8.31.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

## Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

## Parameters

x	value
---	-------

## Returns

$P(x)$

## 8.31.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

## Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

### 8.31.4 Member Data Documentation

#### 8.31.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

#### 8.31.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.32 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- using [raw\\_t](#) = V
- using [type](#) = [abs\\_t](#)< typename Ring::template [mod\\_t](#)< V, X > >

#### 8.32.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

<a href="#">V</a>	a value from 'Ring'
-------------------	---------------------

## 8.32.2 Member Typedef Documentation

### 8.32.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.32.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.33 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- using enclosing\_type = zpz< p >  
*enclosing ring type*
- using is\_zero\_t = std::bool\_constant< v==0 >  
*true\_type if zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr INLINED\_DEVICE valueType get ()  
*get value as valueType*
- static std::string to\_string ()  
*string representation*

### Static Public Attributes

- static constexpr int32\_t v = x % p  
*actual value*
- static constexpr bool is\_zero\_v = v == 0  
*true if zero*

### 8.33.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

## Template Parameters

<code>x</code>	an integer
----------------	------------

## 8.33.2 Member Typedef Documentation

### 8.33.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.33.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.33.3 Member Function Documentation

### 8.33.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

## Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

### 8.33.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

**Returns**

a string representation

**8.33.4 Member Data Documentation****8.33.4.1 is\_zero\_v**

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

**8.33.4.2 v**

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/aerobus.h

**8.34 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference**

specialization for constants

```
#include <aerobus.h>
```

**Classes**

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

**Public Types**

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = [polynomial< Ring >](#)  
*enclosing ring type*
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is\\_zero\\_t](#) = std::bool\_constant< [aN::is\\_zero\\_t::value](#) >
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename [coeff\\_at< index >::type](#)
- template<typename x >  
using [value\\_at\\_t](#) = [coeffN](#)

### Static Public Member Functions

- static std::string [to\\_string](#) ()
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)

### Static Public Attributes

- static constexpr size\_t [degree](#) = 0  
*degree*
- static constexpr bool [is\\_zero\\_v](#) = is\_zero\_t::value

## 8.34.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<i>coeffN</i>	
---------------	--

## 8.34.2 Member Typedef Documentation

### 8.34.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.34.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.34.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type



#### 8.34.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

#### 8.34.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

#### 8.34.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

#### 8.34.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

### 8.34.3 Member Function Documentation

#### 8.34.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >↵
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

#### 8.34.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

### 8.34.4 Member Data Documentation

#### 8.34.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

#### 8.34.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.35 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in zpz*

### Public Types

- using [inner\\_type](#) = int32\_t  
*underlying type for values*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*injects a constant integer into zpz*
- using [zero](#) = [val](#)< 0 >  
*zero value*
- using [one](#) = [val](#)< 1 >  
*one value*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type

- subtraction operator*
- template<typename v1 , typename v2 >  
using **mul\_t** = typename mul< v1, v2 >::type
- multiplication operator*
- template<typename v1 , typename v2 >  
using **div\_t** = typename div< v1, v2 >::type
- division operator*
- template<typename v1 , typename v2 >  
using **mod\_t** = typename remainder< v1, v2 >::type
- modulo operator*
- template<typename v1 , typename v2 >  
using **gt\_t** = typename gt< v1, v2 >::type
- strictly greater operator (type)*
- template<typename v1 , typename v2 >  
using **lt\_t** = typename lt< v1, v2 >::type
- strictly smaller operator (type)*
- template<typename v1 , typename v2 >  
using **eq\_t** = typename eq< v1, v2 >::type
- equality operator (type)*
- template<typename v1 , typename v2 >  
using **gcd\_t** = **gcd\_t**< i32, v1, v2 >
- greatest common divisor*
- template<typename v1 >  
using **pos\_t** = typename pos< v1 >::type
- positivity operator (type)*

### Static Public Attributes

- static constexpr bool **is\_field** = **is\_prime**<p>::value  
*true iff p is prime*
- static constexpr bool **is\_euclidean\_domain** = true  
*always true*
- template<typename v1 , typename v2 >  
static constexpr bool **gt\_v** = **gt\_t**<v1, v2>::value  
*strictly greater operator (booleanvalue)*
- template<typename v1 , typename v2 >  
static constexpr bool **lt\_v** = **lt\_t**<v1, v2>::value  
*strictly smaller operator (booleanvalue)*
- template<typename v1 , typename v2 >  
static constexpr bool **eq\_v** = **eq\_t**<v1, v2>::value  
*equality operator (booleanvalue)*
- template<typename v >  
static constexpr bool **pos\_v** = **pos\_t**<v>::value  
*positivity operator (boolean value)*

### 8.35.1 Detailed Description

**template<int32\_t p>**  
**struct aerobus::zpz< p >**

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

## Template Parameters

<i>p</i>	a integer
----------	-----------

## Examples

[examples/modular\\_arithmetic.cpp](#), and [examples/polynomials\\_over\\_finite\\_field.cpp](#).

## 8.35.2 Member Typedef Documentation

### 8.35.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

#### 8.35.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.35.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.35.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

##### Template Parameters

x	an integer
---	------------

#### 8.35.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

### 8.35.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

### 8.35.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

### 8.35.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.35.3 Member Data Documentation

### 8.35.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.35.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.35.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.35.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.35.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)



## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
Include dependency graph for aerobus.h:
```

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051     template<typename T>
00052     T* aligned_malloc(size_t count, size_t alignment) {
00053         #ifdef _MSC_VER
00054             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055         #else
00056             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057         #endif
00058     }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064     template <typename R>
00065     concept IsRing = requires {
00066         typename R::one;
00067         typename R::zero;
00068         typename R::template add_t<typename R::one, typename R::one>;
00069         typename R::template sub_t<typename R::one, typename R::one>;
00070         typename R::template mul_t<typename R::one, typename R::one>;
00071     };
00072
00074     template <typename R>
00075     concept IsEuclideanDomain = IsRing<R> && requires {
00076         typename R::template div_t<typename R::one, typename R::one>;
00077         typename R::template mod_t<typename R::one, typename R::one>;
00078         typename R::template gcd_t<typename R::one, typename R::one>;
00079         typename R::template eq_t<typename R::one, typename R::one>;
00080         typename R::template pos_t<typename R::one>;
00081
00082         R::template pos_v<typename R::one> == true;
00083         // typename R::template gt_t<typename R::one, typename R::zero>;
00084         R::is_euclidean_domain == true;
00085     };
00086
00088     template<typename R>
00089     concept IsField = IsEuclideanDomain<R> && requires {
00090         R::is_field == true;
00091     };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097     namespace internal {
00098         static constexpr DEVICE uint16_t my_internal_float2half(
00099             const float f, uint32_t &sign, uint32_t &remainder) {
00100             uint32_t x;
00101             uint32_t u;
00102             uint32_t result;
00103             x = std::bit_cast<int32_t>(f);
00104             u = (x & 0x7fffffffU);
00105             sign = ((x > 0) & 0x800000U);
00106             // NaN/+Inf/-Inf
00107             if (u >= 0x7f800000U) {
00108                 remainder = 0U;
00109                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110             } else if (u > 0x477ffffU) { // Overflows
00111                 remainder = 0x80000000U;
00112                 result = (sign | 0x7bfffU);
00113             } else if (u >= 0x38800000U) { // Normal numbers
00114                 remainder = u << 19U;
00115                 u -= 0x38000000U;
00116                 result = (sign | (u >> 13U));
00117             } else if (u < 0x33000001U) { // +0/-0
00118                 remainder = u;
00119                 result = sign;
00120             } else { // Denormal numbers
00121                 const uint32_t exponent = u >> 23U;
00122                 const uint32_t shift = 0x7eU - exponent;
00123                 uint32_t mantissa = (u & 0x7ffffU);
00124                 mantissa |= 0x800000U;
00125                 remainder = mantissa << (32U - shift);
00126                 result = (sign | (mantissa >> shift));
00127                 result &= 0x0000ffffU;

```

```

00128         }
00129         return static_cast<uint16_t>(result);
00130     }
00131
00132     static constexpr DEVICE __half my_float2half_rn(const float a) {
00133         __half val;
00134         __half_raw r;
00135         uint32_t sign = 0U;
00136         uint32_t remainder = 0U;
00137         r.x = my_internal_float2half(a, sign, remainder);
00138         if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139             r.x++;
00140         }
00141
00142         val = std::bit_cast<__half>(r);
00143         return val;
00144     }
00145
00146     template<int16_t i>
00147     static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148
00149
00150     template<typename Out, int16_t x, typename E = void>
00151     struct int16_convert_helper;
00152
00153     template<typename Out, int16_t x>
00154     struct int16_convert_helper<Out, x,
00155         std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00156         static constexpr Out value() {
00157             return static_cast<Out>(x);
00158         }
00159     };
00160
00161     template<int16_t x>
00162     struct int16_convert_helper<__half, x> {
00163         static constexpr __half value() {
00164             return convert_int16_to_half<x>;
00165         }
00166     };
00167
00168     template<int16_t x>
00169     struct int16_convert_helper<__half2, x> {
00170         static constexpr __half2 value() {
00171             return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00172         }
00173     };
00174 } // namespace internal
00175 } // namespace aerobus
00176 #endif
00177
00178 // cast
00179 namespace aerobus {
00180     namespace internal {
00181         template<typename Out, typename In>
00182         struct staticcast {
00183             template<auto x>
00184             static constexpr INLINED DEVICE Out func() {
00185                 return static_cast<Out>(x);
00186             }
00187         };
00188
00189         #ifdef WITH_CUDA_FP16
00190         template<>
00191         struct staticcast<__half, int16_t> {
00192             template<int16_t x>
00193             static constexpr INLINED DEVICE __half func() {
00194                 return int16_convert_helper<__half, x>::value();
00195             }
00196         };
00197
00198         template<>
00199         struct staticcast<__half2, int16_t> {
00200             template<int16_t x>
00201             static constexpr INLINED DEVICE __half2 func() {
00202                 return int16_convert_helper<__half2, x>::value();
00203             }
00204         };
00205         #endif
00206     } // namespace internal
00207 } // namespace aerobus
00208
00209 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00210 namespace aerobus {
00211     namespace internal {
00212         template<typename T>
00213         struct fma_helper;
00214     }

```

```

00215     template<>
00216     struct fma_helper<double> {
00217         static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00218             return x * y + z;
00219         }
00220     };
00221
00222     template<>
00223     struct fma_helper<float> {
00224         static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00225             return x * y + z;
00226         }
00227     };
00228
00229     template<>
00230     struct fma_helper<int32_t> {
00231         static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00232             return x * y + z;
00233         }
00234     };
00235
00236     template<>
00237     struct fma_helper<int16_t> {
00238         static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00239             return x * y + z;
00240         }
00241     };
00242
00243     template<>
00244     struct fma_helper<int64_t> {
00245         static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00246             return x * y + z;
00247         }
00248     };
00249
00250     #ifdef WITH_CUDA_FP16
00251     template<>
00252     struct fma_helper<__half> {
00253         static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00254             #ifdef __CUDA_ARCH__
00255                 return __hfma(x, y, z);
00256             #else
00257                 return x * y + z;
00258             #endif
00259         }
00260     };
00261     template<>
00262     struct fma_helper<__half2> {
00263         static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00264             #ifdef __CUDA_ARCH__
00265                 return __hfma2(x, y, z);
00266             #else
00267                 return x * y + z;
00268             #endif
00269         }
00270     };
00271     #endif
00272 } // namespace internal
00273 } // namespace aerobus
00274
00275 // compensated horner utilities
00276 namespace aerobus {
00277     namespace internal {
00278         template <typename T>
00279         struct FloatLayout;
00280
00281         template <>
00282         struct FloatLayout<double> {
00283             static constexpr uint8_t exponent = 11;
00284             static constexpr uint8_t mantissa = 53;
00285             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00286         };
00287
00288         template <>
00289         struct FloatLayout<float> {
00290             static constexpr uint8_t exponent = 8;
00291             static constexpr uint8_t mantissa = 24;
00292             static constexpr uint8_t r = 12; // ceil(mantissa/2)
00293         };
00294
00295         #ifdef WITH_CUDA_FP16

```

```

00296     template <>
00297     struct FloatLayout<__half> {
00298         static constexpr uint8_t exponent = 5;
00299         static constexpr uint8_t mantissa = 11; // 10 explicitly stored
00300         static constexpr uint8_t r = 6; // ceil(mantissa/2)
00301     };
00302 #endif
00303
00304     template<typename T>
00305     static constexpr INLINED_DEVICE void split(T a, T *x, T *y) {
00306         T z = a * ((1 < FloatLayout<T>::r) + 1);
00307         *x = z - (z - a);
00308         *y = a - *x;
00309     }
00310
00311     template<typename T>
00312     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00313         *x = a + b;
00314         T z = *x - a;
00315         *y = (a - (*x - z)) + (b - z);
00316     }
00317
00318     template<typename T>
00319     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00320         *x = a * b;
00321         T ah, al, bh, bl;
00322         split(a, &ah, &al);
00323         split(b, &bh, &bl);
00324         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00325     }
00326
00327
00328     template<typename T, size_t N>
00329     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00330         T r = p1[0] + p2[0];
00331         for (int64_t i = N - 1; i >= 0; --i) {
00332             r = r * x + p1[N - i] + p2[N - i];
00333         }
00334
00335         return r;
00336     }
00337 } // namespace internal
00338 } // namespace aerobus
00339
00340 // utilities
00341 namespace aerobus {
00342     namespace internal {
00343         template<template<typename...> typename TT, typename T>
00344         struct is_instantiation_of : std::false_type { };
00345
00346         template<template<typename...> typename TT, typename... Ts>
00347         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00348
00349         template<template<typename...> typename TT, typename T>
00350         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00351
00352         template <int64_t i, typename T, typename... Ts>
00353         struct type_at {
00354             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00355             using type = typename type_at<i - 1, Ts...>::type;
00356         };
00357
00358         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00359             using type = T;
00360         };
00361
00362         template <size_t i, typename... Ts>
00363         using type_at_t = typename type_at<i, Ts...>::type;
00364
00365
00366         template<size_t n, size_t i, typename E = void>
00367         struct _is_prime {};
00368
00369         template<size_t i>
00370         struct _is_prime<0, i> {
00371             static constexpr bool value = false;
00372         };
00373
00374         template<size_t i>
00375         struct _is_prime<1, i> {
00376             static constexpr bool value = false;
00377         };
00378
00379         template<size_t i>
00380         struct _is_prime<2, i> {
00381             static constexpr bool value = true;
00382         };

```

```

00383
00384     template<size_t i>
00385     struct _is_prime<3, i> {
00386         static constexpr bool value = true;
00387     };
00388
00389     template<size_t i>
00390     struct _is_prime<5, i> {
00391         static constexpr bool value = true;
00392     };
00393
00394     template<size_t i>
00395     struct _is_prime<7, i> {
00396         static constexpr bool value = true;
00397     };
00398
00399     template<size_t n, size_t i>
00400     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00401         static constexpr bool value = false;
00402     };
00403
00404     template<size_t n, size_t i>
00405     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00406         static constexpr bool value = false;
00407     };
00408
00409     template<size_t n, size_t i>
00410     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00411         static constexpr bool value = true;
00412     };
00413
00414     template<size_t n, size_t i>
00415     struct _is_prime<n, i, std::enable_if_t<(
00416         n % i == 0 &&
00417         n >= 9 &&
00418         n % 3 != 0 &&
00419         n % 2 != 0 &&
00420         i * i > n)> {
00421         static constexpr bool value = true;
00422     };
00423
00424     template<size_t n, size_t i>
00425     struct _is_prime<n, i, std::enable_if_t<(
00426         n % (i+2) == 0 &&
00427         n >= 9 &&
00428         n % 3 != 0 &&
00429         n % 2 != 0 &&
00430         i * i <= n)> {
00431         static constexpr bool value = true;
00432     };
00433
00434     template<size_t n, size_t i>
00435     struct _is_prime<n, i, std::enable_if_t<(
00436         n % (i+2) != 0 &&
00437         n % i != 0 &&
00438         n >= 9 &&
00439         n % 3 != 0 &&
00440         n % 2 != 0 &&
00441         (i * i <= n))> {
00442         static constexpr bool value = _is_prime<n, i+6>::value;
00443     };
00444 } // namespace internal
00445
00446 template<size_t n>
00447 struct is_prime {
00448     static constexpr bool value = internal::_is_prime<n, 5>::value;
00449 };
00450
00451 template<size_t n>
00452 static constexpr bool is_prime_v = is_prime<n>::value;
00453
00454 // gcd
00455 namespace internal {
00456     template <std::size_t... Is>
00457     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00458     -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00459
00460     template <std::size_t N>
00461     using make_index_sequence_reverse
00462     = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00463
00464     template<typename Ring, typename E = void>
00465     struct gcd;
00466
00467     template<typename Ring>
00468     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00469         template<typename A, typename B, typename E = void>

```



```

00481     struct gcd_helper {};
00482
00483     // B = 0, A > 0
00484     template<typename A, typename B>
00485     struct gcd_helper<A, B, std::enable_if_t<
00486         ((B::is_zero_t::value) &&
00487         (Ring::template gt_t<A, typename Ring::zero>::value))> {
00488         using type = A;
00489     };
00490
00491     // B = 0, A < 0
00492     template<typename A, typename B>
00493     struct gcd_helper<A, B, std::enable_if_t<
00494         ((B::is_zero_t::value) &&
00495         !(Ring::template gt_t<A, typename Ring::zero>::value))> {
00496         using type = typename Ring::template sub_t<typename Ring::zero, A>;
00497     };
00498
00499     // B != 0
00500     template<typename A, typename B>
00501     struct gcd_helper<A, B, std::enable_if_t<
00502         (!B::is_zero_t::value)
00503         >> {
00504     private: // NOLINT
00505         // A / B
00506         using k = typename Ring::template div_t<A, B>;
00507         // A - (A/B)*B = A % B
00508         using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00509
00510     public:
00511         using type = typename gcd_helper<B, m>::type;
00512     };
00513
00514     template<typename A, typename B>
00515     using type = typename gcd_helper<A, B>::type;
00516 };
00517 } // namespace internal
00518
00519 // vadd and vmul
00520 namespace internal {
00521     template<typename... vals>
00522     struct vmul {};
00523
00524     template<typename v1, typename... vals>
00525     struct vmul<v1, vals...> {
00526         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00527     };
00528
00529     template<typename v1>
00530     struct vmul<v1> {
00531         using type = v1;
00532     };
00533
00534     template<typename... vals>
00535     struct vadd {};
00536
00537     template<typename v1, typename... vals>
00538     struct vadd<v1, vals...> {
00539         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00540     };
00541
00542     template<typename v1>
00543     struct vadd<v1> {
00544         using type = v1;
00545     };
00546 } // namespace internal
00547
00550     template<typename T, typename A, typename B>
00551     using gcd_t = typename internal::gcd<T>::template type<A, B>;
00552
00556     template<typename... vals>
00557     using vadd_t = typename internal::vadd<vals...>::type;
00558
00562     template<typename... vals>
00563     using vmul_t = typename internal::vmul<vals...>::type;
00564
00568     template<typename val>
00569     requires IsEuclideanDomain<typename val::enclosing_type>
00570     using abs_t = std::conditional_t<
00571         val::enclosing_type::template pos_v<val>,
00572         val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00573 } // namespace aerobus
00574
00575 // embedding

```

```

00576 namespace aerobus {
00581     template<typename Small, typename Large, typename E = void>
00582     struct Embed;
00583 } // namespace aerobus
00584
00585 namespace aerobus {
00590     template<typename Ring, typename X>
00591     requires IsRing<Ring>
00592     struct Quotient {
00595         template <typename V>
00596         struct val {
00597             public:
00598                 using raw_t = V;
00599                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00600         };
00601
00603         using zero = val<typename Ring::zero>;
00604
00606         using one = val<typename Ring::one>;
00607
00611         template<typename v1, typename v2>
00612         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00613
00617         template<typename v1, typename v2>
00618         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00619
00623         template<typename v1, typename v2>
00624         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00625
00629         template<typename v1, typename v2>
00630         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00631
00635         template<typename v1, typename v2>
00636         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00637
00641         template<typename v1, typename v2>
00642         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00643
00647         template<typename v1>
00648         using pos_t = std::true_type;
00649
00653         template<typename v>
00654         static constexpr bool pos_v = pos_t<v>::value;
00655
00657         static constexpr bool is_euclidean_domain = true;
00658
00662         template<auto x>
00663         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00664
00668         template<typename v>
00669         using inject_ring_t = val<v>;
00670     };
00671
00675     template<typename Ring, typename X>
00676     struct Embed<Quotient<Ring, X>, Ring> {
00679         template<typename val>
00680         using type = typename val::raw_t;
00681     };
00682 } // namespace aerobus
00683
00684 // type_list
00685 namespace aerobus {
00687     template <typename... Ts>
00688     struct type_list;
00689
00690     namespace internal {
00691         template <typename T, typename... Us>
00692         struct pop_front_h {
00693             using tail = type_list<Us...>;
00694             using head = T;
00695         };
00696
00697         template <size_t index, typename L1, typename L2>
00698         struct split_h {
00699             private:
00700                 static_assert(index <= L2::length, "index out of bounds");
00701                 using a = typename L2::pop_front::type;
00702                 using b = typename L2::pop_front::tail;
00703                 using c = typename L1::template push_back<a>;
00704
00705             public:
00706                 using head = typename split_h<index - 1, c, b>::head;
00707                 using tail = typename split_h<index - 1, c, b>::tail;
00708         };
00709
00710         template <typename L1, typename L2>
00711         struct split_h<0, L1, L2> {

```

```

00712         using head = L1;
00713         using tail = L2;
00714     };
00715
00716     template <size_t index, typename L, typename T>
00717     struct insert_h {
00718         static_assert(index <= L::length, "index out of bounds");
00719         using s = typename L::template split<index>;
00720         using left = typename s::head;
00721         using right = typename s::tail;
00722         using ll = typename left::template push_back<T>;
00723         using type = typename ll::template concat<right>;
00724     };
00725
00726     template <size_t index, typename L>
00727     struct remove_h {
00728         using s = typename L::template split<index>;
00729         using left = typename s::head;
00730         using right = typename s::tail;
00731         using rr = typename right::pop_front::tail;
00732         using type = typename left::template concat<rr>;
00733     };
00734 } // namespace internal
00735
00736 template <typename... Ts>
00737 struct type_list {
00738 private:
00739     template <typename T>
00740     struct concat_h;
00741
00742     template <typename... Us>
00743     struct concat_h<type_list<Us...> {
00744         using type = type_list<Ts..., Us...>;
00745     };
00746
00747 public:
00748     static constexpr size_t length = sizeof...(Ts);
00749
00750     template <typename T>
00751     using push_front = type_list<T, Ts...>;
00752
00753     template <size_t index>
00754     using at = internal::type_at_t<index, Ts...>;
00755
00756     struct pop_front {
00757         using type = typename internal::pop_front_h<Ts...>::head;
00758         using tail = typename internal::pop_front_h<Ts...>::tail;
00759     };
00760
00761     template <typename T>
00762     using push_back = type_list<Ts..., T>;
00763
00764     template <typename U>
00765     using concat = typename concat_h<U>::type;
00766
00767     template <size_t index>
00768     struct split {
00769     private:
00770         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00771
00772     public:
00773         using head = typename inner::head;
00774         using tail = typename inner::tail;
00775     };
00776
00777     template <typename T, size_t index>
00778     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00779
00780     template <size_t index>
00781     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00782 };
00783
00784 template <>
00785 struct type_list<> {
00786     static constexpr size_t length = 0;
00787
00788     template <typename T>
00789     using push_front = type_list<T>;
00790
00791     template <typename T>
00792     using push_back = type_list<T>;
00793
00794     template <typename U>
00795     using concat = U;
00796
00797     // TODO(jewave): assert index == 0
00798     template <typename T, size_t index>

```

```

00821         using insert = type_list<T>;
00822     };
00823 } // namespace aerobus
00824
00825 // i16
00826 #ifndef WITH_CUDA_FP16
00827 // i16
00828 namespace aerobus {
00829     struct i16 {
00830         using inner_type = int16_t;
00831         template<int16_t x>
00832         struct val {
00833             using enclosing_type = i16;
00834             static constexpr int16_t v = x;
00835
00836             template<typename valueType>
00837             static constexpr INLINED_DEVICE valueType get() {
00838                 return internal::template int16_convert_helper<valueType, x>::value();
00839             }
00840
00841             using is_zero_t = std::bool_constant<x == 0>;
00842
00843             static std::string to_string() {
00844                 return std::to_string(x);
00845             }
00846         };
00847     };
00848
00849     using zero = val<0>;
00850     using one = val<1>;
00851     static constexpr bool is_field = false;
00852     static constexpr bool is_euclidean_domain = true;
00853     template<auto x>
00854     using inject_constant_t = val<static_cast<int16_t>(x)>;
00855
00856     template<typename v>
00857     using inject_ring_t = v;
00858
00859 private:
00860     template<typename v1, typename v2>
00861     struct add {
00862         using type = val<v1::v + v2::v>;
00863     };
00864
00865     template<typename v1, typename v2>
00866     struct sub {
00867         using type = val<v1::v - v2::v>;
00868     };
00869
00870     template<typename v1, typename v2>
00871     struct mul {
00872         using type = val<v1::v * v2::v>;
00873     };
00874
00875     template<typename v1, typename v2>
00876     struct div {
00877         using type = val<v1::v / v2::v>;
00878     };
00879
00880     template<typename v1, typename v2>
00881     struct remainder {
00882         using type = val<v1::v % v2::v>;
00883     };
00884
00885     template<typename v1, typename v2>
00886     struct gt {
00887         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00888     };
00889
00890     template<typename v1, typename v2>
00891     struct lt {
00892         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00893     };
00894
00895     template<typename v1, typename v2>
00896     struct eq {
00897         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00898     };
00899
00900     template<typename v1>
00901     struct pos {
00902         using type = std::bool_constant<(v1::v > 0)>;
00903     };
00904
00905 public:
00906     template<typename v1, typename v2>
00907     using add_t = typename add<v1, v2>::type;
00908
00909     template<typename v1, typename v2>
00910     using sub_t = typename sub<v1, v2>::type;
00911
00912     template<typename v1, typename v2>
00913     using mul_t = typename mul<v1, v2>::type;
00914
00915     template<typename v1, typename v2>
00916     using div_t = typename div<v1, v2>::type;
00917
00918     template<typename v1, typename v2>
00919     using remainder_t = typename remainder<v1, v2>::type;
00920
00921     template<typename v1, typename v2>
00922     using gt_t = typename gt<v1, v2>::type;
00923
00924     template<typename v1, typename v2>
00925     using lt_t = typename lt<v1, v2>::type;
00926
00927     template<typename v1>
00928     using pos_t = typename pos<v1>::type;

```

```

00931     template<typename v1, typename v2>
00932     using sub_t = typename sub<v1, v2>::type;
00933
00938     template<typename v1, typename v2>
00939     using mul_t = typename mul<v1, v2>::type;
00940
00945     template<typename v1, typename v2>
00946     using div_t = typename div<v1, v2>::type;
00947
00952     template<typename v1, typename v2>
00953     using mod_t = typename remainder<v1, v2>::type;
00954
00959     template<typename v1, typename v2>
00960     using gt_t = typename gt<v1, v2>::type;
00961
00966     template<typename v1, typename v2>
00967     using lt_t = typename lt<v1, v2>::type;
00968
00973     template<typename v1, typename v2>
00974     using eq_t = typename eq<v1, v2>::type;
00975
00979     template<typename v1, typename v2>
00980     static constexpr bool eq_v = eq_t<v1, v2>::value;
00981
00986     template<typename v1, typename v2>
00987     using gcd_t = gcd_t<i16, v1, v2>;
00988
00992     template<typename v>
00993     using pos_t = typename pos<v>::type;
00994
00998     template<typename v>
00999     static constexpr bool pos_v = pos_t<v>::value;
01000 };
01001 } // namespace aerobus
01002 #endif
01003
01004 // i32
01005 namespace aerobus {
01006     struct i32 {
01007         using inner_type = int32_t;
01008         template<int32_t x>
01009         struct val {
01010             using enclosing_type = i32;
01011             static constexpr int32_t v = x;
01012
01013             template<typename valueType>
01014             static constexpr DEVICE valueType get() {
01015                 return static_cast<valueType>(x);
01016             }
01017
01020             using is_zero_t = std::bool_constant<x == 0>;
01021
01024             static std::string to_string() {
01025                 return std::to_string(x);
01026             }
01027         };
01028     };
01029
01030     using zero = val<0>;
01031     using one = val<1>;
01032
01033     static constexpr bool is_field = false;
01034     static constexpr bool is_euclidean_domain = true;
01035
01036     template<auto x>
01037     using inject_constant_t = val<static_cast<int32_t>(x)>;
01038
01039     template<typename v>
01040     using inject_ring_t = v;
01041
01042 private:
01043     template<typename v1, typename v2>
01044     struct add {
01045         using type = val<v1::v + v2::v>;
01046     };
01047
01048     template<typename v1, typename v2>
01049     struct sub {
01050         using type = val<v1::v - v2::v>;
01051     };
01052
01053     template<typename v1, typename v2>
01054     struct mul {
01055         using type = val<v1::v * v2::v>;
01056     };
01057
01058     template<typename v1, typename v2>
01059     struct div {
01060         using type = val<v1::v / v2::v>;
01061     };
01062 }

```

```

01070
01071     template<typename v1, typename v2>
01072     struct remainder {
01073         using type = val<v1::v % v2::v>;
01074     };
01075
01076     template<typename v1, typename v2>
01077     struct gt {
01078         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01079     };
01080
01081     template<typename v1, typename v2>
01082     struct lt {
01083         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01084     };
01085
01086     template<typename v1, typename v2>
01087     struct eq {
01088         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01089     };
01090
01091     template<typename v1>
01092     struct pos {
01093         using type = std::bool_constant<(v1::v > 0)>;
01094     };
01095
01096     public:
01101     template<typename v1, typename v2>
01102     using add_t = typename add<v1, v2>::type;
01103
01108     template<typename v1, typename v2>
01109     using sub_t = typename sub<v1, v2>::type;
01110
01115     template<typename v1, typename v2>
01116     using mul_t = typename mul<v1, v2>::type;
01117
01122     template<typename v1, typename v2>
01123     using div_t = typename div<v1, v2>::type;
01124
01129     template<typename v1, typename v2>
01130     using mod_t = typename remainder<v1, v2>::type;
01131
01136     template<typename v1, typename v2>
01137     using gt_t = typename gt<v1, v2>::type;
01138
01143     template<typename v1, typename v2>
01144     using lt_t = typename lt<v1, v2>::type;
01145
01150     template<typename v1, typename v2>
01151     using eq_t = typename eq<v1, v2>::type;
01152
01156     template<typename v1, typename v2>
01157     static constexpr bool eq_v = eq_t<v1, v2>::value;
01158
01163     template<typename v1, typename v2>
01164     using gcd_t = gcd_t<i32, v1, v2>;
01165
01169     template<typename v>
01170     using pos_t = typename pos<v>::type;
01171
01175     template<typename v>
01176     static constexpr bool pos_v = pos_t<v>::value;
01177 };
01178 } // namespace aerobus
01179
01180 // i64
01181 namespace aerobus {
01182     struct i64 {
01185         using inner_type = int64_t;
01188         template<int64_t x>
01189         struct val {
01191             using inner_type = int32_t;
01193             using enclosing_type = i64;
01195             static constexpr int64_t v = x;
01196
01199             template<typename valueType>
01200             static constexpr INLINED_DEVICE valueType get() {
01201                 return static_cast<valueType>(x);
01202             }
01203
01205             using is_zero_t = std::bool_constant<x == 0>;
01206
01208             static std::string to_string() {
01209                 return std::to_string(x);
01210             }
01211         };
01212     };

```

```

01215     template<auto x>
01216     using inject_constant_t = val<static_cast<int64_t>(x)>;
01217
01222     template<typename v>
01223     using inject_ring_t = v;
01224
01226     using zero = val<0>;
01228     using one = val<1>;
01230     static constexpr bool is_field = false;
01232     static constexpr bool is_euclidean_domain = true;
01233
01234 private:
01235     template<typename v1, typename v2>
01236     struct add {
01237         using type = val<v1::v + v2::v>;
01238     };
01239
01240     template<typename v1, typename v2>
01241     struct sub {
01242         using type = val<v1::v - v2::v>;
01243     };
01244
01245     template<typename v1, typename v2>
01246     struct mul {
01247         using type = val<v1::v * v2::v>;
01248     };
01249
01250     template<typename v1, typename v2>
01251     struct div {
01252         using type = val<v1::v / v2::v>;
01253     };
01254
01255     template<typename v1, typename v2>
01256     struct remainder {
01257         using type = val<v1::v % v2::v>;
01258     };
01259
01260     template<typename v1, typename v2>
01261     struct gt {
01262         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01263     };
01264
01265     template<typename v1, typename v2>
01266     struct lt {
01267         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01268     };
01269
01270     template<typename v1, typename v2>
01271     struct eq {
01272         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01273     };
01274
01275     template<typename v>
01276     struct pos {
01277         using type = std::bool_constant<(v::v > 0)>;
01278     };
01279
01280 public:
01284     template<typename v1, typename v2>
01285     using add_t = typename add<v1, v2>::type;
01286
01290     template<typename v1, typename v2>
01291     using sub_t = typename sub<v1, v2>::type;
01292
01296     template<typename v1, typename v2>
01297     using mul_t = typename mul<v1, v2>::type;
01298
01303     template<typename v1, typename v2>
01304     using div_t = typename div<v1, v2>::type;
01305
01309     template<typename v1, typename v2>
01310     using mod_t = typename remainder<v1, v2>::type;
01311
01316     template<typename v1, typename v2>
01317     using gt_t = typename gt<v1, v2>::type;
01318
01323     template<typename v1, typename v2>
01324     static constexpr bool gt_v = gt_t<v1, v2>::value;
01325
01330     template<typename v1, typename v2>
01331     using lt_t = typename lt<v1, v2>::type;
01332
01337     template<typename v1, typename v2>
01338     static constexpr bool lt_v = lt_t<v1, v2>::value;
01339
01344     template<typename v1, typename v2>
01345     using eq_t = typename eq<v1, v2>::type;

```

```

01346
01351     template<typename v1, typename v2>
01352     static constexpr bool eq_v = eq_t<v1, v2>::value;
01353
01358     template<typename v1, typename v2>
01359     using gcd_t = gcd_t<i64, v1, v2>;
01360
01364     template<typename v>
01365     using pos_t = typename pos<v>::type;
01366
01370     template<typename v>
01371     static constexpr bool pos_v = pos_t<v>::value;
01372 };
01373
01375 template<>
01376 struct Embed<i32, i64> {
01377     template<typename val>
01380     using type = i64::val<static_cast<int64_t>(val::v)>;
01381 };
01382 } // namespace aerobus
01383
01384 // z/pz
01385 namespace aerobus {
01391     template<int32_t p>
01392     struct zpz {
01394         using inner_type = int32_t;
01395
01398         template<int32_t x>
01399         struct val {
01401             using enclosing_type = zpz<p>;
01403             static constexpr int32_t v = x % p;
01404
01407             template<typename valueType>
01408             static constexpr INLINED_DEVICE valueType get() {
01409                 return static_cast<valueType>(x % p);
01410             }
01411
01413             using is_zero_t = std::bool_constant<v == 0>;
01414
01416             static constexpr bool is_zero_v = v == 0;
01417
01420             static std::string to_string() {
01421                 return std::to_string(x % p);
01422             }
01423         };
01424
01427         template<auto x>
01428         using inject_constant_t = val<static_cast<int32_t>(x)>;
01429
01431         using zero = val<0>;
01432
01434         using one = val<1>;
01435
01437         static constexpr bool is_field = is_prime<p>::value;
01438
01440         static constexpr bool is_euclidean_domain = true;
01441
01442     private:
01443         template<typename v1, typename v2>
01444         struct add {
01445             using type = val<(v1::v + v2::v) % p>;
01446         };
01447
01448         template<typename v1, typename v2>
01449         struct sub {
01450             using type = val<(v1::v - v2::v) % p>;
01451         };
01452
01453         template<typename v1, typename v2>
01454         struct mul {
01455             using type = val<(v1::v * v2::v) % p>;
01456         };
01457
01458         template<typename v1, typename v2>
01459         struct div {
01460             using type = val<(v1::v % p) / (v2::v % p)>;
01461         };
01462
01463         template<typename v1, typename v2>
01464         struct remainder {
01465             using type = val<(v1::v % v2::v) % p>;
01466         };
01467
01468         template<typename v1, typename v2>
01469         struct gt {
01470             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01471         };

```



```

01472
01473     template<typename v1, typename v2>
01474     struct lt {
01475         using type = std::conditional_t<(v1::v% p < v2::v% p), std::true_type, std::false_type>;
01476     };
01477
01478     template<typename v1, typename v2>
01479     struct eq {
01480         using type = std::conditional_t<(v1::v% p == v2::v % p), std::true_type, std::false_type>;
01481     };
01482
01483     template<typename v1>
01484     struct pos {
01485         using type = std::bool_constant<(v1::v > 0)>;
01486     };
01487
01488     public:
01492     template<typename v1, typename v2>
01493     using add_t = typename add<v1, v2>::type;
01494
01498     template<typename v1, typename v2>
01499     using sub_t = typename sub<v1, v2>::type;
01500
01504     template<typename v1, typename v2>
01505     using mul_t = typename mul<v1, v2>::type;
01506
01510     template<typename v1, typename v2>
01511     using div_t = typename div<v1, v2>::type;
01512
01516     template<typename v1, typename v2>
01517     using mod_t = typename remainder<v1, v2>::type;
01518
01522     template<typename v1, typename v2>
01523     using gt_t = typename gt<v1, v2>::type;
01524
01528     template<typename v1, typename v2>
01529     static constexpr bool gt_v = gt_t<v1, v2>::value;
01530
01534     template<typename v1, typename v2>
01535     using lt_t = typename lt<v1, v2>::type;
01536
01540     template<typename v1, typename v2>
01541     static constexpr bool lt_v = lt_t<v1, v2>::value;
01542
01546     template<typename v1, typename v2>
01547     using eq_t = typename eq<v1, v2>::type;
01548
01552     template<typename v1, typename v2>
01553     static constexpr bool eq_v = eq_t<v1, v2>::value;
01554
01558     template<typename v1, typename v2>
01559     using gcd_t = gcd_t<i32, v1, v2>;
01560
01563     template<typename v1>
01564     using pos_t = typename pos<v1>::type;
01565
01568     template<typename v>
01569     static constexpr bool pos_v = pos_t<v>::value;
01570 };
01571
01574     template<int32_t x>
01575     struct Embed<zpz<x>, i32> {
01576         template <typename val>
01577         using type = i32::val<val::v>;
01578     };
01581 } // namespace aerobus
01582
01583 // polynomial
01584 namespace aerobus {
01585     // coeffN x^N + ...
01586     template<typename Ring>
01587     requires IsEuclideanDomain<Ring>
01588     struct polynomial {
01593         static constexpr bool is_field = false;
01594         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01595
01598         template<typename P>
01599         struct horner_reduction_t {
01600             template<size_t index, size_t stop>
01601             struct inner {
01602                 template<typename accum, typename x>
01603                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01604                     ::template type<
01605                         typename Ring::template add_t<
01606                             typename Ring::template mul_t<x, accum>,
01607                             typename P::template coeff_at_t<P::degree - index>
01608                             >, x>;

```

```

01609         };
01610
01611         template<size_t stop>
01612         struct inner<stop, stop> {
01613             template<typename accum, typename x>
01614             using type = accum;
01615         };
01616     };
01617
01621     template<typename coeffN, typename... coeffs>
01622     struct val {
01623         using ring_type = Ring;
01624         using enclosing_type = polynomial<Ring>;
01625         static constexpr size_t degree = sizeof...(coeffs);
01626         using aN = coeffN;
01627         using strip = val<coeffs...>;
01628         using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01629         static constexpr bool is_zero_v = is_zero_t::value;
01630
01631     private:
01632         template<size_t index, typename E = void>
01633         struct coeff_at {};
01634
01635         template<size_t index>
01636         struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01637             using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01638         };
01639
01640         template<size_t index>
01641         struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01642             using type = typename Ring::zero;
01643         };
01644
01645     public:
01646         template<size_t index>
01647         using coeff_at_t = typename coeff_at<index>::type;
01648
01649         static std::string to_string() {
01650             return string_helper<coeffN, coeffs...>::func();
01651         }
01652
01653         template<typename arithmeticType>
01654         static constexpr DEVICE INLINEED arithmeticType eval(const arithmeticType& x) {
01655             #ifdef WITH_CUDA_FP16
01656             arithmeticType start;
01657             if constexpr (std::is_same_v<arithmeticType, __half2>) {
01658                 start = __half2(0, 0);
01659             } else {
01660                 start = static_cast<arithmeticType>(0);
01661             }
01662             #else
01663             arithmeticType start = static_cast<arithmeticType>(0);
01664             #endif
01665             return horner_evaluation<arithmeticType, val>
01666                 ::template inner<0, degree + 1>
01667                 ::func(start, x);
01668         }
01669
01670         template<typename arithmeticType>
01671         static DEVICE INLINEED arithmeticType compensated_eval(const arithmeticType& x) {
01672             return compensated_horner<arithmeticType, val>::func(x);
01673         }
01674
01675         template<typename x>
01676         using value_at_t = horner_reduction_t<val>
01677             ::template inner<0, degree + 1>
01678             ::template type<typename Ring::zero, x>;
01679     };
01680
01681     template<typename coeffN>
01682     struct val<coeffN> {
01683         using ring_type = Ring;
01684         using enclosing_type = polynomial<Ring>;
01685         static constexpr size_t degree = 0;
01686         using aN = coeffN;
01687         using strip = val<coeffN>;
01688         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01689
01690         static constexpr bool is_zero_v = is_zero_t::value;
01691
01692         template<size_t index, typename E = void>
01693         struct coeff_at {};
01694
01695         template<size_t index>
01696         struct coeff_at<index, std::enable_if_t<(index == 0)> {
01697             using type = aN;
01698         };
01699     };

```

```

01724
01725     template<size_t index>
01726     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01727         using type = typename Ring::zero;
01728     };
01729
01730     template<size_t index>
01731     using coeff_at_t = typename coeff_at<index>::type;
01732
01733     static std::string to_string() {
01734         return string_helper<coeffN>::func();
01735     }
01736
01737     template<typename arithmeticType>
01738     static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01739         return coeffN::template get<arithmeticType>();
01740     }
01741
01742     template<typename x>
01743     using value_at_t = coeffN;
01744 };
01745
01746 using zero = val<typename Ring::zero>;
01747 using one = val<typename Ring::one>;
01748 using X = val<typename Ring::one, typename Ring::zero>;
01749
01750 private:
01751     template<typename P, typename E = void>
01752     struct simplify;
01753
01754     template<typename P1, typename P2, typename I>
01755     struct add_low;
01756
01757     template<typename P1, typename P2>
01758     struct add {
01759         using type = typename simplify<typename add_low<
01760             P1,
01761             P2,
01762             internal::make_index_sequence_reverse<
01763                 std::max(P1::degree, P2::degree) + 1
01764             >::type>::type;
01765     };
01766
01767     template<typename P1, typename P2, typename I>
01768     struct sub_low;
01769
01770     template<typename P1, typename P2, typename I>
01771     struct mul_low;
01772
01773     template<typename v1, typename v2>
01774     struct mul {
01775         using type = typename mul_low<
01776             v1,
01777             v2,
01778             internal::make_index_sequence_reverse<
01779                 v1::degree + v2::degree + 1
01780             >::type;
01781     };
01782
01783     template<typename coeff, size_t deg>
01784     struct monomial;
01785
01786     template<typename v, typename E = void>
01787     struct derive_helper {};
01788
01789     template<typename v>
01790     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01791         using type = zero;
01792     };
01793
01794     template<typename v>
01795     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01796         using type = typename add<
01797             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01798             typename monomial<
01799                 typename Ring::template mul_t<
01800                     typename v::aN,
01801                     typename Ring::template inject_constant_t<(v::degree)>
01802                 >,
01803                 v::degree - 1
01804             >::type
01805         >::type;
01806     };
01807
01808     template<typename v1, typename v2, typename E = void>
01809     struct eq_helper {};
01810
01811
01812
01813

```

```

01814     template<typename v1, typename v2>
01815     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01816         using type = std::false_type;
01817     };
01818
01819
01820     template<typename v1, typename v2>
01821     struct eq_helper<v1, v2, std::enable_if_t<
01822         v1::degree == v2::degree &&
01823         (v1::degree != 0 || v2::degree != 0) &&
01824         std::is_same<
01825             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01826             std::false_type
01827         >::value
01828     >
01829     > {
01830         using type = std::false_type;
01831     };
01832
01833     template<typename v1, typename v2>
01834     struct eq_helper<v1, v2, std::enable_if_t<
01835         v1::degree == v2::degree &&
01836         (v1::degree != 0 || v2::degree != 0) &&
01837         std::is_same<
01838             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01839             std::true_type
01840         >::value
01841     >> {
01842         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01843     };
01844
01845     template<typename v1, typename v2>
01846     struct eq_helper<v1, v2, std::enable_if_t<
01847         v1::degree == v2::degree &&
01848         (v1::degree == 0)
01849     >> {
01850         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01851     };
01852
01853     template<typename v1, typename v2, typename E = void>
01854     struct lt_helper {};
01855
01856     template<typename v1, typename v2>
01857     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01858         using type = std::true_type;
01859     };
01860
01861     template<typename v1, typename v2>
01862     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01863         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01864     };
01865
01866     template<typename v1, typename v2>
01867     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01868         using type = std::false_type;
01869     };
01870
01871     template<typename v1, typename v2, typename E = void>
01872     struct gt_helper {};
01873
01874     template<typename v1, typename v2>
01875     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01876         using type = std::true_type;
01877     };
01878
01879     template<typename v1, typename v2>
01880     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01881         using type = std::false_type;
01882     };
01883
01884     template<typename v1, typename v2>
01885     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01886         using type = std::false_type;
01887     };
01888
01889     // when high power is zero : strip
01890     template<typename P>
01891     struct simplify<P, std::enable_if_t<
01892         std::is_same<
01893             typename Ring::zero,
01894             typename P::aN
01895         >::value && (P::degree > 0)
01896     >> {
01897         using type = typename simplify<typename P::strip>::type;
01898     };
01899
01900     // otherwise : do nothing

```

```

01901     template<typename P>
01902     struct simplify<P, std::enable_if_t<
01903         !std::is_same<
01904             typename Ring::zero,
01905             typename P::aN
01906         >::value && (P::degree > 0)
01907     > {
01908         using type = P;
01909     };
01910
01911     // do not simplify constants
01912     template<typename P>
01913     struct simplify<P, std::enable_if_t<P::degree == 0> {
01914         using type = P;
01915     };
01916
01917     // addition at
01918     template<typename P1, typename P2, size_t index>
01919     struct add_at {
01920         using type =
01921             typename Ring::template add_t<
01922                 typename P1::template coeff_at_t<index>,
01923                 typename P2::template coeff_at_t<index>;
01924     };
01925
01926     template<typename P1, typename P2, size_t index>
01927     using add_at_t = typename add_at<P1, P2, index>::type;
01928
01929     template<typename P1, typename P2, std::size_t... I>
01930     struct add_low<P1, P2, std::index_sequence<I...> {
01931         using type = val<add_at_t<P1, P2, I>...>;
01932     };
01933
01934     // subtraction at
01935     template<typename P1, typename P2, size_t index>
01936     struct sub_at {
01937         using type =
01938             typename Ring::template sub_t<
01939                 typename P1::template coeff_at_t<index>,
01940                 typename P2::template coeff_at_t<index>;
01941     };
01942
01943     template<typename P1, typename P2, size_t index>
01944     using sub_at_t = typename sub_at<P1, P2, index>::type;
01945
01946     template<typename P1, typename P2, std::size_t... I>
01947     struct sub_low<P1, P2, std::index_sequence<I...> {
01948         using type = val<sub_at_t<P1, P2, I>...>;
01949     };
01950
01951     template<typename P1, typename P2>
01952     struct sub {
01953         using type = typename simplify<typename sub_low<
01954             P1,
01955             P2,
01956             internal::make_index_sequence_reverse<
01957                 std::max(P1::degree, P2::degree) + 1
01958             >::type>::type;
01959     };
01960
01961     // multiplication at
01962     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01963     struct mul_at_loop_helper {
01964         using type = typename Ring::template add_t<
01965             typename Ring::template mul_t<
01966                 typename v1::template coeff_at_t<index>,
01967                 typename v2::template coeff_at_t<k - index>
01968             >,
01969             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01970         >;
01971     };
01972
01973     template<typename v1, typename v2, size_t k, size_t stop>
01974     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01975         using type = typename Ring::template mul_t<
01976             typename v1::template coeff_at_t<stop>,
01977             typename v2::template coeff_at_t<0>;
01978     };
01979
01980     template <typename v1, typename v2, size_t k, typename E = void>
01981     struct mul_at {};
01982
01983     template<typename v1, typename v2, size_t k>
01984     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01985         using type = typename Ring::zero;
01986     };
01987

```

```

01988     template<typename v1, typename v2, size_t k>
01989     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01990         using type = typename mul_at_loop_helper<v1, v2, k, 0, k::type>;
01991     };
01992
01993     template<typename P1, typename P2, size_t index>
01994     using mul_at_t = typename mul_at<P1, P2, index>::type;
01995
01996     template<typename P1, typename P2, std::size_t... I>
01997     struct mul_low<P1, P2, std::index_sequence<I...> {
01998         using type = val<mul_at_t<P1, P2, I>...>;
01999     };
02000
02001     // division helper
02002     template< typename A, typename B, typename Q, typename R, typename E = void>
02003     struct div_helper {};
02004
02005     template<typename A, typename B, typename Q, typename R>
02006     struct div_helper<A, B, Q, R, std::enable_if_t<
02007         (R::degree < B::degree) ||
02008         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02009         using q_type = Q;
02010         using mod_type = R;
02011         using gcd_type = B;
02012     };
02013
02014     template<typename A, typename B, typename Q, typename R>
02015     struct div_helper<A, B, Q, R, std::enable_if_t<
02016         (R::degree >= B::degree) &&
02017         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02018     private: // NOLINT
02019         using rN = typename R::aN;
02020         using bN = typename B::aN;
02021         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02022             B::degree>::type;
02023         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02024         using qq = typename add<Q, pT>::type;
02025     public:
02026         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02027         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02028         using gcd_type = rr;
02029     };
02030
02031     template<typename A, typename B>
02032     struct div {
02033         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02034         using q_type = typename div_helper<A, B, zero, A>::q_type;
02035         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02036     };
02037
02038     template<typename P>
02039     struct make_unit {
02040         using type = typename div<P, val<typename P::aN>::q_type>;
02041     };
02042
02043     template<typename coeff, size_t deg>
02044     struct monomial {
02045         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02046     };
02047
02048     template<typename coeff>
02049     struct monomial<coeff, 0> {
02050         using type = val<coeff>;
02051     };
02052
02053     template<typename arithmeticType, typename P>
02054     struct horner_evaluation {
02055         template<size_t index, size_t stop>
02056         struct inner {
02057             static constexpr DEVICE INLINED arithmeticType func(
02058                 const arithmeticType& accum, const arithmeticType& x) {
02059                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02060 stop>::func(
02061                     internal::fma_helper<arithmeticType>::eval(
02062                         x,
02063                         accum,
02064                         P::template coeff_at_t<P::degree - index>::template
02065 get<arithmeticType>(), x);
02066                     }
02067             };
02068         template<size_t stop>
02069         struct inner<stop, stop> {
02070             static constexpr DEVICE INLINED arithmeticType func(
02071                 const arithmeticType& accum, const arithmeticType& x) {
02072                 return accum;
02073             }
02074         };
02075     };

```

```

02072     }
02073     };
02074 };
02075
02076 template<typename arithmeticType, typename P>
02077 struct compensated_horner {
02078     template<int64_t index, int ghost>
02079     struct EFTHorner {
02080         static INLINED void func(
02081             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02082             *r) {
02083             arithmeticType p;
02084             internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02085             constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
02086             get<arithmeticType>();
02087             internal::two_sum<arithmeticType>(
02088                 p, coeff,
02089                 r, sigma + P::degree - index - 1);
02090             EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02091         }
02092     };
02093     template<int ghost>
02094     struct EFTHorner<-1, ghost> {
02095         static INLINED DEVICE void func(
02096             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02097             *r) {
02098         }
02099     };
02100     static INLINED DEVICE arithmeticType func(arithmeticType x) {
02101         arithmeticType pi[P::degree], sigma[P::degree];
02102         arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02103         EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02104         arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02105         return r + c;
02106     }
02107 };
02108
02109 template<typename coeff, typename... coeffs>
02110 struct string_helper {
02111     static std::string func() {
02112         std::string tail = string_helper<coeffs...>::func();
02113         std::string result = "";
02114         if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02115             return tail;
02116         } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02117             if (sizeof...(coeffs) == 1) {
02118                 result += "x";
02119             } else {
02120                 result += "x^" + std::to_string(sizeof...(coeffs));
02121             }
02122         } else {
02123             if (sizeof...(coeffs) == 1) {
02124                 result += coeff::to_string() + " x";
02125             } else {
02126                 result += coeff::to_string()
02127                     + " x^" + std::to_string(sizeof...(coeffs));
02128             }
02129         }
02130         if (!tail.empty()) {
02131             if (tail.at(0) != '-') {
02132                 result += " + " + tail;
02133             } else {
02134                 result += " - " + tail.substr(1);
02135             }
02136         }
02137         return result;
02138     }
02139 };
02140 };
02141
02142 template<typename coeff>
02143 struct string_helper<coeff> {
02144     static std::string func() {
02145         if (!std::is_same<coeff, typename Ring::zero>::value) {
02146             return coeff::to_string();
02147         } else {
02148             return "";
02149         }
02150     }
02151 };
02152
02153 public:
02154     template<typename P>
02155     using simplify_t = typename simplify<P>::type;

```

```

02158
02162     template<typename v1, typename v2>
02163     using add_t = typename add<v1, v2>::type;
02164
02168     template<typename v1, typename v2>
02169     using sub_t = typename sub<v1, v2>::type;
02170
02174     template<typename v1, typename v2>
02175     using mul_t = typename mul<v1, v2>::type;
02176
02180     template<typename v1, typename v2>
02181     using eq_t = typename eq_helper<v1, v2>::type;
02182
02186     template<typename v1, typename v2>
02187     using lt_t = typename lt_helper<v1, v2>::type;
02188
02192     template<typename v1, typename v2>
02193     using gt_t = typename gt_helper<v1, v2>::type;
02194
02198     template<typename v1, typename v2>
02199     using div_t = typename div<v1, v2>::q_type;
02200
02204     template<typename v1, typename v2>
02205     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02206
02210     template<typename coeff, size_t deg>
02211     using monomial_t = typename monomial<coeff, deg>::type;
02212
02215     template<typename v>
02216     using derive_t = typename derive_helper<v>::type;
02217
02220     template<typename v>
02221     using pos_t = typename Ring::template pos_t<typename v::aN>;
02222
02225     template<typename v>
02226     static constexpr bool pos_v = pos_t<v>::value;
02227
02231     template<typename v1, typename v2>
02232     using gcd_t = std::conditional_t<
02233         Ring::is_euclidean_domain,
02234         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02235         void>;
02236
02239     template<auto x>
02240     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02241
02244     template<typename v>
02245     using inject_ring_t = val<v>;
02246 };
02247 } // namespace aerobus
02248
02249 // fraction field
02250 namespace aerobus {
02251     namespace internal {
02252         template<typename Ring, typename E = void>
02253         requires IsEuclideanDomain<Ring>
02254         struct _FractionField {};
02255
02256         template<typename Ring>
02257         requires IsEuclideanDomain<Ring>
02258         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
02259             static constexpr bool is_field = true;
02260             static constexpr bool is_euclidean_domain = true;
02261
02262         private:
02263             template<typename val1, typename val2, typename E = void>
02264             struct to_string_helper {};
02265
02266             template<typename val1, typename val2>
02267             struct to_string_helper<val1, val2,
02268                 std::enable_if_t<
02269                     Ring::template eq_t<
02270                         val2, typename Ring::one
02271                         >::value
02272                     >
02273             > {
02274                 static std::string func() {
02275                     return val1::to_string();
02276                 }
02277             };
02278
02279             template<typename val1, typename val2>
02280             struct to_string_helper<val1, val2,
02281                 std::enable_if_t<
02282                     !Ring::template eq_t<
02283                         val2,
02284                         typename Ring::one

```



```

02286         >::value
02287     >
02288     > {
02289         static std::string func() {
02290             return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
02291         }
02292     };
02293
02294 public:
02295     template<typename val1, typename val2>
02296     struct val {
02301         using x = val1;
02303         using y = val2;
02305         using is_zero_t = typename val1::is_zero_t;
02307         static constexpr bool is_zero_v = val1::is_zero_t::value;
02308
02310         using ring_type = Ring;
02311         using enclosing_type = _FractionField<Ring>;
02312
02315         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02316
02320         template<typename valueType>
02321         static constexpr INLINED_DEVICE valueType get() {
02322             return internal::staticcast<valueType, typename ring_type::inner_type>::template
func<x::v>() /
02323             internal::staticcast<valueType, typename ring_type::inner_type>::template
func<y::v>();
02324         }
02325
02328         static std::string to_string() {
02329             return to_string_helper<val1, val2>::func();
02330         }
02331
02336         template<typename arithmeticType>
02337         static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02338             return x::eval(v) / y::eval(v);
02339         }
02340     };
02341
02343     using zero = val<typename Ring::zero, typename Ring::one>;
02345     using one = val<typename Ring::one, typename Ring::one>;
02346
02349     template<typename v>
02350     using inject_t = val<v, typename Ring::one>;
02351
02354     template<auto x>
02355     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02356
02359     template<typename v>
02360     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02361
02363     using ring_type = Ring;
02364
02365 private:
02366     template<typename v, typename E = void>
02367     struct simplify {};
02368
02369     // x = 0
02370     template<typename v>
02371     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02372         using type = typename _FractionField<Ring>::zero;
02373     };
02374
02375     // x != 0
02376     template<typename v>
02377     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02378     private:
02379         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02380         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02381         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02382
02383         using posx = std::conditional_t<
02384             !Ring::template pos_v<newy>,
02385             typename Ring::template sub_t<typename Ring::zero, newx>,
02386             newx>;
02387         using posy = std::conditional_t<
02388             !Ring::template pos_v<newy>,
02389             typename Ring::template sub_t<typename Ring::zero, newy>,
02390             newy>;
02391     public:
02392         using type = typename _FractionField<Ring>::template val<posx, posy>;
02393     };
02394
02395 public:
02396     template<typename v>
02397     using simplify_t = typename simplify<v>::type;

```

```

02400
02401     private:
02402         template<typename v1, typename v2>
02403         struct add {
02404             private:
02405                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02406                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02407                 using dividend = typename Ring::template add_t<a, b>;
02408                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02409                 using g = typename Ring::template gcd_t<dividend, diviser>;
02410
02411             public:
02412                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
02413         };
02414
02415         template<typename v>
02416         struct pos {
02417             using type = std::conditional_t<
02418                 (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02419                 (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02420                 std::true_type,
02421                 std::false_type>;
02422         };
02423
02424         template<typename v1, typename v2>
02425         struct sub {
02426             private:
02427                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02428                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02429                 using dividend = typename Ring::template sub_t<a, b>;
02430                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02431                 using g = typename Ring::template gcd_t<dividend, diviser>;
02432
02433             public:
02434                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>>;
02435         };
02436
02437         template<typename v1, typename v2>
02438         struct mul {
02439             private:
02440                 using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02441                 using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02442
02443             public:
02444                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02445         };
02446
02447         template<typename v1, typename v2, typename E = void>
02448         struct div {};
02449
02450         template<typename v1, typename v2>
02451         struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> > {
02452             private:
02453                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02454                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02455
02456             public:
02457                 using type = typename _FractionField<Ring>::template simplify_t<val<a, b>>;
02458         };
02459
02460         template<typename v1, typename v2>
02461         struct div<v1, v2, std::enable_if_t<
02462             std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> > {
02463             using type = one;
02464         };
02465
02466         template<typename v1, typename v2>
02467         struct eq {
02468             using type = std::conditional_t<
02469                 std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02470                 std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02471                 std::true_type,
02472                 std::false_type>;
02473         };
02474
02475         template<typename v1, typename v2, typename E = void>
02476         struct gt;
02477
02478         template<typename v1, typename v2>
02479         struct gt<v1, v2, std::enable_if_t<
02480             (eq<v1, v2>::type::value)
02481             >> {
02482             using type = std::false_type;
02483         };

```

```

02484
02485     template<typename v1, typename v2>
02486     struct gt<v1, v2, std::enable_if_t<
02487         (!eq<v1, v2>::type::value) &&
02488         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02489         >> {
02490         using type = typename gt<
02491             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02492             >::type;
02493     };
02494
02495     template<typename v1, typename v2>
02496     struct gt<v1, v2, std::enable_if_t<
02497         (!eq<v1, v2>::type::value) &&
02498         (pos<v1>::type::value) && (!pos<v2>::type::value)
02499         >> {
02500         using type = std::true_type;
02501     };
02502
02503     template<typename v1, typename v2>
02504     struct gt<v1, v2, std::enable_if_t<
02505         (!eq<v1, v2>::type::value) &&
02506         (!pos<v1>::type::value) && (pos<v2>::type::value)
02507         >> {
02508         using type = std::false_type;
02509     };
02510
02511     template<typename v1, typename v2>
02512     struct gt<v1, v2, std::enable_if_t<
02513         (!eq<v1, v2>::type::value) &&
02514         (pos<v1>::type::value) && (pos<v2>::type::value)
02515         >> {
02516         using type = typename Ring::template gt_t<
02517             typename Ring::template mul_t<v1::x, v2::y>,
02518             typename Ring::template mul_t<v2::y, v2::x>
02519         >;
02520     };
02521
02522 public:
02523     template<typename v1, typename v2>
02524     using add_t = typename add<v1, v2>::type;
02525
02526     template<typename v1, typename v2>
02527     using mod_t = zero;
02528
02529     template<typename v1, typename v2>
02530     using gcd_t = v1;
02531
02532     template<typename v1, typename v2>
02533     using sub_t = typename sub<v1, v2>::type;
02534
02535     template<typename v1, typename v2>
02536     using mul_t = typename mul<v1, v2>::type;
02537
02538     template<typename v1, typename v2>
02539     using div_t = typename div<v1, v2>::type;
02540
02541     template<typename v1, typename v2>
02542     using eq_t = typename eq<v1, v2>::type;
02543
02544     template<typename v1, typename v2>
02545     static constexpr bool eq_v = eq<v1, v2>::type::value;
02546
02547     template<typename v1, typename v2>
02548     using gt_t = typename gt<v1, v2>::type;
02549
02550     template<typename v1, typename v2>
02551     static constexpr bool gt_v = gt<v1, v2>::type::value;
02552
02553     template<typename v1>
02554     using pos_t = typename pos<v1>::type;
02555
02556     template<typename v>
02557     static constexpr bool pos_v = pos_t<v>::value;
02558 };
02559
02560 template<typename Ring, typename E = void>
02561 requires IsEuclideanDomain<Ring>
02562 struct FractionFieldImpl {};
02563
02564 // fraction field of a field is the field itself
02565 template<typename Field>
02566 requires IsEuclideanDomain<Field>
02567 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02568     using type = Field;
02569     template<typename v>
02570     using inject_t = v;
02571 };

```

```

02607     };
02608
02609     // fraction field of a ring is the actual fraction field
02610     template<typename Ring>
02611     requires IsEuclideanDomain<Ring>
02612     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02613         using type = _FractionField<Ring>;
02614     };
02615 } // namespace internal
02616
02617 template<typename Ring>
02618 requires IsEuclideanDomain<Ring>
02619 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02620
02621 template<typename Ring>
02622 struct Embed<Ring, FractionField<Ring> {
02623     template<typename v>
02624     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02625 };
02626 } // namespace aerobus
02627
02628 // short names for common types
02629 namespace aerobus {
02630     template<typename X, typename Y>
02631     requires IsRing<typename X::enclosing_type> &&
02632     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02633     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02634
02635     template<typename X, typename Y>
02636     requires IsRing<typename X::enclosing_type> &&
02637     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02638     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02639
02640     template<typename X, typename Y>
02641     requires IsRing<typename X::enclosing_type> &&
02642     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02643     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02644
02645     template<typename X, typename Y>
02646     requires IsEuclideanDomain<typename X::enclosing_type> &&
02647     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02648     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02649
02650     using q32 = FractionField<i32>;
02651
02652     using fpq32 = FractionField<polynomial<q32>>;
02653
02654     using q64 = FractionField<i64>;
02655
02656     using pi64 = polynomial<i64>;
02657
02658     using pq64 = polynomial<q64>;
02659
02660     using fpq64 = FractionField<polynomial<q64>>;
02661
02662     template<typename Ring, typename v1, typename v2>
02663     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02664
02665     template<typename v>
02666     using embed_int_poly_in_fractions_t =
02667         typename Embed<
02668             polynomial<typename v::ring_type>,
02669             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02670
02671     template<int64_t p, int64_t q>
02672     using make_q64_t = typename q64::template simplify_t<
02673         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02674
02675     template<int32_t p, int32_t q>
02676     using make_q32_t = typename q32::template simplify_t<
02677         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02678
02679     template<typename Ring, typename v1, typename v2>
02680     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02681     template<typename Ring, typename v1, typename v2>
02682     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02683
02684     template<>
02685     struct Embed<q32, q64> {
02686         template<typename v>
02687         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02688     };
02689
02690     template<typename Small, typename Large>
02691     struct Embed<polynomial<Small>, polynomial<Large> {
02692     private:

```

```

02751     template<typename v, typename i>
02752     struct at_low;
02753
02754     template<typename v, size_t i>
02755     struct at_index {
02756         using type = typename Embed<Small, Large>::template
type<typename v::template coeff_at_t<i>>;
02757     };
02758
02759     template<typename v, size_t... Is>
02760     struct at_low<v, std::index_sequence<Is...> {
02761         using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02762     };
02763
02764     public:
02765     template<typename v>
02766     using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
1>::type;
02769     };
02770
02774     template<typename Ring, auto... xs>
02775     using make_int_polynomial_t = typename polynomial<Ring>::template val<
typename Ring::template inject_constant_t<xs>...>;
02776
02777     template<typename Ring, auto... xs>
02778     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
typename FractionField<Ring>::template inject_constant_t<xs>...>;
02784 } // namespace aerobus
02785
02786 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02787 namespace aerobus {
02788     namespace internal {
02789         template<typename T, size_t x, typename E = void>
02790         struct factorial {};
02791
02792         template<typename T, size_t x>
02793         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02794             private:
02795                 template<typename, size_t, typename>
02796                 friend struct factorial;
02797             public:
02798                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02799                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02800             };
02801
02802         template<typename T>
02803         struct factorial<T, 0> {
02804             public:
02805                 using type = typename T::one;
02806                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02807             };
02808     } // namespace internal
02809
02813     template<typename T, size_t i>
02814     using factorial_t = typename internal::factorial<T, i>::type;
02815
02819     template<typename T, size_t i>
02820     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02821
02822     namespace internal {
02823         template<typename T, size_t k, size_t n, typename E = void>
02824         struct combination_helper {};
02825
02826         template<typename T, size_t k, size_t n>
02827         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02828             using type = typename FractionField<T>::template mul_t<
typename combination_helper<T, k - 1, n - 1>::type,
makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02831         };
02832
02833         template<typename T, size_t k, size_t n>
02834         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02835             using type = typename combination_helper<T, n - k, n>::type;
02836         };
02837
02838         template<typename T, size_t n>
02839         struct combination_helper<T, 0, n> {
02840             using type = typename FractionField<T>::one;
02841         };
02842
02843         template<typename T, size_t k, size_t n>
02844         struct combination {
02845             using type = typename internal::combination_helper<T, k, n>::type::x;
02846             static constexpr typename T::inner_type value =

```

```

02847         internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>());
02848     };
02849     } // namespace internal
02850
02853     template<typename T, size_t k, size_t n>
02854     using combination_t = typename internal::combination<T, k, n>::type;
02855
02860     template<typename T, size_t k, size_t n>
02861     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02862
02863     namespace internal {
02864         template<typename T, size_t m>
02865         struct bernoulli;
02866
02867         template<typename T, typename accum, size_t k, size_t m>
02868         struct bernoulli_helper {
02869             using type = typename bernoulli_helper<
02870                 T,
02871                 addfractions_t<T,
02872                     accum,
02873                     mulfractions_t<T,
02874                         makefraction_t<T,
02875                             combination_t<T, k, m + 1>,
02876                             typename T::one>,
02877                             typename bernoulli<T, k>::type
02878                         >,
02879                     >,
02880                     k + 1,
02881                     m>::type;
02882         };
02883
02884         template<typename T, typename accum, size_t m>
02885         struct bernoulli_helper<T, accum, m, m> {
02886             using type = accum;
02887         };
02888
02889
02890
02891         template<typename T, size_t m>
02892         struct bernoulli {
02893             using type = typename FractionField<T>::template mul_t<
02894                 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02895                 makefraction_t<T,
02896                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02897                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02898                 >
02899             >;
02900
02901         template<typename floatType>
02902         static constexpr floatType value = type::template get<floatType>();
02903     };
02904
02905     template<typename T>
02906     struct bernoulli<T, 0> {
02907         using type = typename FractionField<T>::one;
02908
02909         template<typename floatType>
02910         static constexpr floatType value = type::template get<floatType>();
02911     };
02912     } // namespace internal
02913
02917     template<typename T, size_t n>
02918     using bernoulli_t = typename internal::bernoulli<T, n>::type;
02919
02924     template<typename FloatType, typename T, size_t n>
02925     inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02926
02927     // bell numbers
02928     namespace internal {
02929         template<typename T, size_t n, typename E = void>
02930         struct bell_helper;
02931
02932         template<typename T, size_t n>
02933         struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02934             template<typename accum, size_t i, size_t stop>
02935             struct sum_helper {
02936             private:
02937                 using left = typename T::template mul_t<
02938                     combination_t<T, i, n-1>,
02939                     typename bell_helper<T, i>::type>;
02940                 using new_accum = typename T::template add_t<accum, left>;
02941             public:
02942                 using type = typename sum_helper<new_accum, i+1, stop>::type;
02943             };
02944
02945         template<typename accum, size_t stop>

```

```

02946         struct sum_helper<accum, stop, stop> {
02947             using type = accum;
02948         };
02949
02950         using type = typename sum_helper<typename T::zero, 0, n>::type;
02951     };
02952
02953     template<typename T>
02954     struct bell_helper<T, 0> {
02955         using type = typename T::one;
02956     };
02957
02958     template<typename T>
02959     struct bell_helper<T, 1> {
02960         using type = typename T::one;
02961     };
02962 } // namespace internal
02963
02964 template<typename T, size_t n>
02965 using bell_t = typename internal::bell_helper<T, n>::type;
02966
02967 template<typename T, size_t n>
02968 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02969
02970 namespace internal {
02971     template<typename T, int k, typename E = void>
02972     struct alternate {};
02973
02974     template<typename T, int k>
02975     struct alternate<T, k, std::enable_if_t<k % 2 == 0> > {
02976         using type = typename T::one;
02977         static constexpr typename T::inner_type value = type::template get<typename
02978 T::inner_type>();
02979     };
02980
02981     template<typename T, int k>
02982     struct alternate<T, k, std::enable_if_t<k % 2 != 0> > {
02983         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02984         static constexpr typename T::inner_type value = type::template get<typename
02985 T::inner_type>();
02986     };
02987 } // namespace internal
02988
02989 template<typename T, int k>
02990 using alternate_t = typename internal::alternate<T, k>::type;
02991
02992 template<typename T, size_t k>
02993 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02994
02995 namespace internal {
02996     template<typename T, int n, int k, typename E = void>
02997     struct stirling_l_helper {};
02998
02999     template<typename T>
03000     struct stirling_l_helper<T, 0, 0> {
03001         using type = typename T::one;
03002     };
03003
03004     template<typename T, int n>
03005     struct stirling_l_helper<T, n, 0, std::enable_if_t<(n > 0)> > {
03006         using type = typename T::zero;
03007     };
03008
03009     template<typename T, int n>
03010     struct stirling_l_helper<T, 0, n, std::enable_if_t<(n > 0)> > {
03011         using type = typename T::zero;
03012     };
03013
03014     template<typename T, int n, int k>
03015     struct stirling_l_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> > {
03016         using type = typename T::template sub_t<
03017             typename stirling_l_helper<T, n-1, k-1>::type,
03018             typename T::template mul_t<
03019                 typename T::template inject_constant_t<n-1>,
03020                 typename stirling_l_helper<T, n-1, k>::type
03021             >;
03022     };
03023 } // namespace internal
03024
03025 template<typename T, int n, int k>
03026 using stirling_l_signed_t = typename internal::stirling_l_helper<T, n, k>::type;
03027
03028 template<typename T, int n, int k>
03029 using stirling_l_unsigned_t = abs_t<typename internal::stirling_l_helper<T, n, k>::type>;
03030
03031 template<typename T, int n, int k>
03032 static constexpr typename T::inner_type stirling_l_unsigned_v = stirling_l_unsigned_t<T, n, k>::v;

```

```

03053
03058     template<typename T, int n, int k>
03059     static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03060
03061     namespace internal {
03062         template<typename T, int n, int k, typename E = void>
03063         struct stirling_2_helper {};
03064
03065         template<typename T, int n>
03066         struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)>> {
03067             using type = typename T::one;
03068         };
03069
03070         template<typename T, int n>
03071         struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)>> {
03072             using type = typename T::zero;
03073         };
03074
03075         template<typename T, int n>
03076         struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)>> {
03077             using type = typename T::zero;
03078         };
03079
03080         template<typename T, int n, int k>
03081         struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)>> {
03082             using type = typename T::template add_t<
03083                 typename stirling_2_helper<T, n-1, k-1>::type,
03084                 typename T::template mul_t<
03085                     typename T::template inject_constant_t<k>,
03086                     typename stirling_2_helper<T, n-1, k>::type
03087                 >>;
03088         };
03089     } // namespace internal
03090
03095     template<typename T, int n, int k>
03096     using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03097
03102     template<typename T, int n, int k>
03103     static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03104
03105     namespace internal {
03106         template<typename T>
03107         struct pow_scalar {
03108             template<size_t p>
03109             static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03110                 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03111                 x * func<p/2>(x) * func<p/2>(x);
03112         };
03113     };
03114
03115     template<typename T, typename p, size_t n, typename E = void>
03116     requires IsEuclideanDomain<T>
03117     struct pow;
03118
03119     template<typename T, typename p, size_t n>
03120     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)>> {
03121         using type = typename T::template mul_t<
03122             typename pow<T, p, n/2>::type,
03123             typename pow<T, p, n/2>::type
03124         >;
03125     };
03126
03127     template<typename T, typename p, size_t n>
03128     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)>> {
03129         using type = typename T::template mul_t<
03130             p,
03131             typename T::template mul_t<
03132                 typename pow<T, p, n/2>::type,
03133                 typename pow<T, p, n/2>::type
03134             >
03135         >;
03136     };
03137
03138     template<typename T, typename p, size_t n>
03139     struct pow<T, p, n, std::enable_if_t<n == 0>> { using type = typename T::one; };
03140 } // namespace internal
03141
03146     template<typename T, typename p, size_t n>
03147     using pow_t = typename internal::pow<T, p, n>::type;
03148
03153     template<typename T, typename p, size_t n>
03154     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03155
03156     template<typename T, size_t p>
03157     static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03158         internal::pow_scalar<T>::template func<p>(x); }

```



```

03159     namespace internal {
03160         template<typename, template<typename, size_t> typename, class>
03161             struct make_taylor_impl;
03162
03163         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03164             struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03165                 using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
Is>::type...>;
03166             };
03167     }
03168
03173     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03174     using taylor = typename internal::make_taylor_impl<
03175         T,
03176         coeff_at,
03177         internal::make_index_sequence_reverse<deg + 1>::type>;
03178
03179     namespace internal {
03180         template<typename T, size_t i>
03181         struct exp_coeff {
03182             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03183         };
03184
03185         template<typename T, size_t i, typename E = void>
03186         struct sin_coeff_helper {};
03187
03188         template<typename T, size_t i>
03189         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03190             using type = typename FractionField<T>::zero;
03191         };
03192
03193         template<typename T, size_t i>
03194         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03195             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03196         };
03197
03198         template<typename T, size_t i>
03199         struct sin_coeff {
03200             using type = typename sin_coeff_helper<T, i>::type;
03201         };
03202
03203         template<typename T, size_t i, typename E = void>
03204         struct sh_coeff_helper {};
03205
03206         template<typename T, size_t i>
03207         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03208             using type = typename FractionField<T>::zero;
03209         };
03210
03211         template<typename T, size_t i>
03212         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03213             using type = makefraction_t<T, typename T::one, factorial_t<T, i>;>;
03214         };
03215
03216         template<typename T, size_t i>
03217         struct sh_coeff {
03218             using type = typename sh_coeff_helper<T, i>::type;
03219         };
03220
03221         template<typename T, size_t i, typename E = void>
03222         struct cos_coeff_helper {};
03223
03224         template<typename T, size_t i>
03225         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03226             using type = typename FractionField<T>::zero;
03227         };
03228
03229         template<typename T, size_t i>
03230         struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03231             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;>;
03232         };
03233
03234         template<typename T, size_t i>
03235         struct cos_coeff {
03236             using type = typename cos_coeff_helper<T, i>::type;
03237         };
03238
03239         template<typename T, size_t i, typename E = void>
03240         struct cosh_coeff_helper {};
03241
03242         template<typename T, size_t i>
03243         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03244             using type = typename FractionField<T>::zero;
03245         };
03246
03247         template<typename T, size_t i>
03248         struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {

```

```

03249         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
03250     };
03251
03252     template<typename T, size_t i>
03253     struct cosh_coeff {
03254         using type = typename cosh_coeff_helper<T, i>::type;
03255     };
03256
03257     template<typename T, size_t i>
03258     struct geom_coeff { using type = typename FractionField<T>::one; };
03259
03260
03261     template<typename T, size_t i, typename E = void>
03262     struct atan_coeff_helper;
03263
03264     template<typename T, size_t i>
03265     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03266         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
03267     };
03268
03269     template<typename T, size_t i>
03270     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03271         using type = typename FractionField<T>::zero;
03272     };
03273
03274     template<typename T, size_t i>
03275     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03276
03277     template<typename T, size_t i, typename E = void>
03278     struct asin_coeff_helper;
03279
03280     template<typename T, size_t i>
03281     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03282         using type = makefraction_t<T,
03283             factorial_t<T, i - 1>,
03284             typename T::template mul_t<
03285                 typename T::template val<i>,
03286                 T::template mul_t<
03287                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03288                     pow<T, factorial_t<T, i / 2>, 2
03289                 >
03290             >
03291         >;
03292     };
03293
03294     template<typename T, size_t i>
03295     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03296         using type = typename FractionField<T>::zero;
03297     };
03298
03299     template<typename T, size_t i>
03300     struct asin_coeff {
03301         using type = typename asin_coeff_helper<T, i>::type;
03302     };
03303
03304     template<typename T, size_t i>
03305     struct lnpl_coeff {
03306         using type = makefraction_t<T,
03307             alternate_t<T, i + 1>,
03308             typename T::template val<i>;
03309     };
03310
03311     template<typename T>
03312     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03313
03314     template<typename T, size_t i, typename E = void>
03315     struct asinh_coeff_helper;
03316
03317     template<typename T, size_t i>
03318     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03319         using type = makefraction_t<T,
03320             typename T::template mul_t<
03321                 alternate_t<T, i / 2>,
03322                 factorial_t<T, i - 1>
03323             >,
03324             typename T::template mul_t<
03325                 typename T::template mul_t<
03326                     typename T::template val<i>,
03327                     pow_t<T, factorial_t<T, i / 2>, 2>
03328                 >,
03329                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03330             >
03331         >;
03332     };
03333
03334     template<typename T, size_t i>
03335     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {

```

```

03336         using type = typename FractionField<T>::zero;
03337     };
03338
03339     template<typename T, size_t i>
03340     struct asinh_coeff {
03341         using type = typename asinh_coeff_helper<T, i>::type;
03342     };
03343
03344     template<typename T, size_t i, typename E = void>
03345     struct atanh_coeff_helper;
03346
03347     template<typename T, size_t i>
03348     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03349         // 1/i
03350         using type = typename FractionField<T>::template val<
03351             typename T::one,
03352             typename T::template inject_constant_t<i>;
03353     };
03354
03355     template<typename T, size_t i>
03356     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03357         using type = typename FractionField<T>::zero;
03358     };
03359
03360     template<typename T, size_t i>
03361     struct atanh_coeff {
03362         using type = typename atanh_coeff_helper<T, i>::type;
03363     };
03364
03365     template<typename T, size_t i, typename E = void>
03366     struct tan_coeff_helper;
03367
03368     template<typename T, size_t i>
03369     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03370         using type = typename FractionField<T>::zero;
03371     };
03372
03373     template<typename T, size_t i>
03374     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03375     private:
03376         // 4^((i+1)/2)
03377         using _4p = typename FractionField<T>::template inject_t<
03378             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03379         // 4^((i+1)/2) - 1
03380         using _4pml = typename FractionField<T>::template
03381             sub_t<_4p, typename FractionField<T>::one>;
03382         // (-1)^((i-1)/2)
03383         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03384         using dividend = typename FractionField<T>::template mul_t<
03385             altp,
03386             FractionField<T>::template mul_t<
03387                 _4p,
03388                 FractionField<T>::template mul_t<
03389                     _4pml,
03390                     bernoulli_t<T, (i + 1)>
03391                 >
03392             >;
03393     public:
03394         using type = typename FractionField<T>::template div_t<dividend,
03395             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03396     };
03397
03398     template<typename T, size_t i>
03399     struct tan_coeff {
03400         using type = typename tan_coeff_helper<T, i>::type;
03401     };
03402
03403     template<typename T, size_t i, typename E = void>
03404     struct tanh_coeff_helper;
03405
03406     template<typename T, size_t i>
03407     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03408         using type = typename FractionField<T>::zero;
03409     };
03410
03411     template<typename T, size_t i>
03412     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03413     private:
03414         using _4p = typename FractionField<T>::template inject_t<
03415             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03416         using _4pml = typename FractionField<T>::template
03417             sub_t<_4p, typename FractionField<T>::one>;
03418         using dividend =
03419             typename FractionField<T>::template mul_t<
03420                 _4p,
03421                 typename FractionField<T>::template mul_t<

```

```

03421         _4pml,
03422         bernoulli_t<T, (i + 1)>>::type;
03423     public:
03424         using type = typename FractionField<T>::template div_t<dividend,
03425         FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03426     };
03427
03428     template<typename T, size_t i>
03429     struct tanh_coeff {
03430         using type = typename tanh_coeff_helper<T, i>::type;
03431     };
03432 } // namespace internal
03433
03437 template<typename Integers, size_t deg>
03438 using exp = taylor<Integers, internal::exp_coeff, deg>;
03439
03443 template<typename Integers, size_t deg>
03444 using expm1 = typename polynomial<FractionField<Integers>>::template sub_t<
03445     exp<Integers, deg>,
03446     typename polynomial<FractionField<Integers>>::one>;
03447
03451 template<typename Integers, size_t deg>
03452 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03453
03457 template<typename Integers, size_t deg>
03458 using atan = taylor<Integers, internal::atan_coeff, deg>;
03459
03463 template<typename Integers, size_t deg>
03464 using sin = taylor<Integers, internal::sin_coeff, deg>;
03465
03469 template<typename Integers, size_t deg>
03470 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03471
03476 template<typename Integers, size_t deg>
03477 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03478
03483 template<typename Integers, size_t deg>
03484 using cos = taylor<Integers, internal::cos_coeff, deg>;
03485
03490 template<typename Integers, size_t deg>
03491 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03492
03497 template<typename Integers, size_t deg>
03498 using asin = taylor<Integers, internal::asin_coeff, deg>;
03499
03504 template<typename Integers, size_t deg>
03505 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03506
03511 template<typename Integers, size_t deg>
03512 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03513
03518 template<typename Integers, size_t deg>
03519 using tan = taylor<Integers, internal::tan_coeff, deg>;
03520
03525 template<typename Integers, size_t deg>
03526 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03527 } // namespace aerobus
03528
03529 // continued fractions
03530 namespace aerobus {
03533     template<int64_t... values>
03534     struct ContinuedFraction {};
03535
03538     template<int64_t a0>
03539     struct ContinuedFraction<a0> {
03541         using type = typename q64::template inject_constant_t<a0>;
03543         static constexpr double val = static_cast<double>(a0);
03544     };
03545
03549     template<int64_t a0, int64_t... rest>
03550     struct ContinuedFraction<a0, rest...> {
03552         using type = q64::template add_t<
03553             typename q64::template inject_constant_t<a0>,
03554             typename q64::template div_t<
03555                 typename q64::one,
03556                 typename ContinuedFraction<rest...>::type
03557             >>;
03558
03560         static constexpr double val = type::template get<double>();
03561     };
03562
03566     using PI_fraction =
03567     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03568     using E_fraction =
03569     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03570     using SQRT2_fraction =
03571     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;

```

```

03572     using Sqrt3_fraction =
03573     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03574     // NOLINT
03575 } // namespace aerobus
03576 // known polynomials
03577 namespace aerobus {
03578     // CChebyshev
03579     namespace internal {
03580         template<int kind, size_t deg, typename I>
03581         struct chebyshev_helper {
03582             using type = typename polynomial<I>::template sub_t<
03583                 typename polynomial<I>::template mul_t<
03584                     typename polynomial<I>::template inject_constant_t<2>,
03585                     typename polynomial<I>::X>,
03586                     typename chebyshev_helper<kind, deg - 1, I>::type
03587                 >,
03588             typename chebyshev_helper<kind, deg - 2, I>::type
03589         >;
03590     };
03591
03592     template<typename I>
03593     struct chebyshev_helper<1, 0, I> {
03594         using type = typename polynomial<I>::one;
03595     };
03596
03597     template<typename I>
03598     struct chebyshev_helper<1, 1, I> {
03599         using type = typename polynomial<I>::X;
03600     };
03601
03602     template<typename I>
03603     struct chebyshev_helper<2, 0, I> {
03604         using type = typename polynomial<I>::one;
03605     };
03606
03607     template<typename I>
03608     struct chebyshev_helper<2, 1, I> {
03609         using type = typename polynomial<I>::template mul_t<
03610             typename polynomial<I>::template inject_constant_t<2>,
03611             typename polynomial<I>::X>;
03612     };
03613 } // namespace internal
03614
03615 // Laguerre
03616 namespace internal {
03617     template<size_t deg, typename I>
03618     struct laguerre_helper {
03619         using Q = FractionField<I>;
03620         using PQ = polynomial<Q>;
03621
03622     private:
03623         // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)lkm2)
03624         using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03625         using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03626         // -x + 2k-1
03627         using p = typename PQ::template val<
03628             typename Q::template inject_constant_t<-1>,
03629             typename Q::template inject_constant_t<2 * deg - 1>;
03630         // 1/n
03631         using factor = typename PQ::template inject_ring_t<
03632             typename Q::template val<typename I::one, typename I::template
03633             inject_constant_t<deg>>;
03634
03635     public:
03636         using type = typename PQ::template mul_t <
03637             factor,
03638             typename PQ::template sub_t<
03639                 typename PQ::template mul_t<
03640                     p,
03641                     lnm1
03642                 >,
03643             typename PQ::template mul_t<
03644                 typename PQ::template inject_constant_t<deg-1>,
03645                 lnm2
03646             >
03647         >;
03648     };
03649
03650     template<typename I>
03651     struct laguerre_helper<0, I> {
03652         using type = typename polynomial<FractionField<I>::one>;
03653     };
03654
03655     template<typename I>

```

```

03656     struct laguerre_helper<l, I> {
03657     private:
03658         using PQ = polynomial<FractionField<I>;
03659     public:
03660         using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03661     };
03662 } // namespace internal
03663
03664 // Bernstein
03665 namespace internal {
03666     template<size_t i, size_t m, typename I, typename E = void>
03667     struct bernstein_helper {};
03668
03669     template<typename I>
03670     struct bernstein_helper<0, 0, I> {
03671         using type = typename polynomial<I>::one;
03672     };
03673
03674     template<size_t i, size_t m, typename I>
03675     struct bernstein_helper<i, m, I, std::enable_if_t<
03676         (m > 0) && (i == 0)> {
03677     private:
03678         using P = polynomial<I>;
03679     public:
03680         using type = typename P::template mul_t<
03681             typename P::template sub_t<typename P::one, typename P::X>,
03682             typename bernstein_helper<i, m-1, I>::type>;
03683     };
03684
03685     template<size_t i, size_t m, typename I>
03686     struct bernstein_helper<i, m, I, std::enable_if_t<
03687         (m > 0) && (i == m)> {
03688     private:
03689         using P = polynomial<I>;
03690     public:
03691         using type = typename P::template mul_t<
03692             typename P::X,
03693             typename bernstein_helper<i-1, m-1, I>::type>;
03694     };
03695
03696     template<size_t i, size_t m, typename I>
03697     struct bernstein_helper<i, m, I, std::enable_if_t<
03698         (m > 0) && (i > 0) && (i < m)> {
03699     private:
03700         using P = polynomial<I>;
03701     public:
03702         using type = typename P::template add_t<
03703             typename P::template mul_t<
03704                 typename P::template sub_t<typename P::one, typename P::X>,
03705                 typename bernstein_helper<i, m-1, I>::type>,
03706                 typename P::template mul_t<
03707                     typename P::X,
03708                     typename bernstein_helper<i-1, m-1, I>::type>;
03709             >;
03710     };
03711 } // namespace internal
03712
03713 // AllOne polynomials
03714 namespace internal {
03715     template<size_t deg, typename I>
03716     struct AllOneHelper {
03717         using type = aerobus::add_t<
03718             typename polynomial<I>::one,
03719             typename aerobus::mul_t<
03720                 typename polynomial<I>::X,
03721                 typename AllOneHelper<deg-1, I>::type
03722             >;
03723     };
03724
03725     template<typename I>
03726     struct AllOneHelper<0, I> {
03727         using type = typename polynomial<I>::one;
03728     };
03729 } // namespace internal
03730
03731 // Bessel polynomials
03732 namespace internal {
03733     template<size_t deg, typename I>
03734     struct BesselHelper {
03735     private:
03736         using P = polynomial<I>;
03737         using factor = typename P::template monomial_t<
03738             typename I::template inject_constant_t<(2*deg - 1)>,
03739             1>;
03740     public:
03741         using type = typename P::template add_t<
03742             typename P::template mul_t<

```

```

03743         typename BesselHelper<deg-1, I>::type
03744     >,
03745     typename BesselHelper<deg-2, I>::type
03746 >;
03747 };
03748
03749 template<typename I>
03750 struct BesselHelper<0, I> {
03751     using type = typename polynomial<I>::one;
03752 };
03753
03754 template<typename I>
03755 struct BesselHelper<1, I> {
03756     private:
03757         using P = polynomial<I>;
03758     public:
03759         using type = typename P::template add_t<
03760             typename P::one,
03761             typename P::X
03762         >;
03763 };
03764 } // namespace internal
03765
03766 namespace known_polynomials {
03767     enum hermite_kind {
03768         probabilist,
03769         physicist
03770     };
03771 }
03772
03773 // hermite
03774 namespace internal {
03775     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03776     struct hermite_helper {};
03777
03778     template<size_t deg, typename I>
03779     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03780     private:
03781         using hnm1 = typename hermite_helper<deg - 1,
03782             known_polynomials::hermite_kind::probabilist, I>::type;
03783         using hnm2 = typename hermite_helper<deg - 2,
03784             known_polynomials::hermite_kind::probabilist, I>::type;
03785     public:
03786         using type = typename polynomial<I>::template sub_t<
03787             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03788             typename polynomial<I>::template mul_t<
03789                 typename polynomial<I>::template inject_constant_t<deg - 1>,
03790                 hnm2
03791             >
03792         >;
03793     };
03794
03795     template<size_t deg, typename I>
03796     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03797     private:
03798         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03799             I>::type;
03800         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03801             I>::type;
03802     public:
03803         using type = typename polynomial<I>::template sub_t<
03804             // 2X Hn-1
03805             typename polynomial<I>::template mul_t<
03806                 typename pi64::val<typename I::template inject_constant_t<2>,
03807                 typename I::zero>, hnm1>,
03808                 typename polynomial<I>::template mul_t<
03809                     typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03810                     hnm2
03811                 >
03812             >
03813         >;
03814     };
03815
03816     template<typename I>
03817     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03818         using type = typename polynomial<I>::one;
03819     };
03820
03821     template<typename I>
03822     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03823         using type = typename polynomial<I>::X;
03824     };
03825
03826     template<typename I>
03827     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {

```

```

03829         using type = typename pi64::one;
03830     };
03831
03832     template<typename I>
03833     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03834         // 2X
03835         using type = typename polynomial<I>::template val<
03836             typename I::template inject_constant_t<2>,
03837             typename I::zero>;
03838     };
03839 } // namespace internal
03840
03841 // legendre
03842 namespace internal {
03843     template<size_t n, typename I>
03844     struct legendre_helper {
03845     private:
03846         using Q = FractionField<I>;
03847         using PQ = polynomial<Q>;
03848         // 1/n constant
03849         // (2n-1)/n X
03850         using fact_left = typename PQ::template monomial_t<
03851             makefraction_t<I,
03852                 typename I::template inject_constant_t<2*n-1>,
03853                 typename I::template inject_constant_t<n>
03854             >,
03855             1>;
03856         // (n-1) / n
03857         using fact_right = typename PQ::template val<
03858             makefraction_t<I,
03859                 typename I::template inject_constant_t<n-1>,
03860                 typename I::template inject_constant_t<n>>;
03861
03862     public:
03863         using type = PQ::template sub_t<
03864             typename PQ::template mul_t<
03865                 fact_left,
03866                 typename legendre_helper<n-1, I>::type
03867             >,
03868             typename PQ::template mul_t<
03869                 fact_right,
03870                 typename legendre_helper<n-2, I>::type
03871             >
03872         >;
03873     };
03874
03875     template<typename I>
03876     struct legendre_helper<0, I> {
03877         using type = typename polynomial<FractionField<I>::one>;
03878     };
03879
03880     template<typename I>
03881     struct legendre_helper<1, I> {
03882         using type = typename polynomial<FractionField<I>::X>;
03883     };
03884 } // namespace internal
03885
03886 // bernoulli polynomials
03887 namespace internal {
03888     template<size_t n>
03889     struct bernoulli_coeff {
03890         template<typename T, size_t i>
03891         struct inner {
03892         private:
03893             using F = FractionField<T>;
03894         public:
03895             using type = typename F::template mul_t<
03896                 typename F::template inject_ring_t<combination_t<T, i, n>,
03897                 bernoulli_t<T, n-i>
03898             >;
03899         };
03900     };
03901 } // namespace internal
03902
03903 namespace internal {
03904     template<size_t n>
03905     struct touchard_coeff {
03906         template<typename T, size_t i>
03907         struct inner {
03908             using type = stirling_2_t<T, n, i>;
03909         };
03910     };
03911 } // namespace internal
03912
03913 namespace internal {
03914     template<typename I = aerobus::i64>
03915     struct AbelHelper {

```



```

03916     private:
03917         using P = aerobus::polynomial<I>;
03918
03919     public:
03920         // to keep recursion working, we need to operate on a*n and not just a
03921         template<size_t deg, I::inner_type an>
03922         struct Inner {
03923             // Abel(n, a) = (x-an) * Abel(n-1, a)
03924             using type = typename aerobus::mul_t<
03925                 typename Inner<deg-1, an>::type,
03926                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
03927             >;
03928         };
03929
03930         // Abel(0, a) = 1
03931         template<I::inner_type an>
03932         struct Inner<0, an> {
03933             using type = P::one;
03934         };
03935
03936         // Abel(1, a) = X
03937         template<I::inner_type an>
03938         struct Inner<1, an> {
03939             using type = P::X;
03940         };
03941     };
03942 } // namespace internal
03943
03944 namespace known_polynomials {
03945
03946     template<size_t n, auto a, typename I = aerobus::i64>
03947     using Abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
03948
03949     template<size_t deg, typename I = aerobus::i64>
03950     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03951
03952     template<size_t deg, typename I = aerobus::i64>
03953     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03954
03955     template<size_t deg, typename I = aerobus::i64>
03956     using laguerre = typename internal::laguerre_helper<deg, I>::type;
03957
03958     template<size_t deg, typename I = aerobus::i64>
03959     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03960         I>::type;
03961
03962     template<size_t deg, typename I = aerobus::i64>
03963     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03964
03965     template<size_t i, size_t m, typename I = aerobus::i64>
03966     using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03967
03968     template<size_t deg, typename I = aerobus::i64>
03969     using legendre = typename internal::legendre_helper<deg, I>::type;
03970
03971     template<size_t deg, typename I = aerobus::i64>
03972     using bernoulli = typename internal::bernoulli_helper<deg>::template inner, deg>;
03973
03974     template<size_t deg, typename I = aerobus::i64>
03975     using allone = typename internal::AllOneHelper<deg, I>::type;
03976
03977     template<size_t deg, typename I = aerobus::i64>
03978     using bessel = typename internal::BesselHelper<deg, I>::type;
03979
03980     template<size_t deg, typename I = aerobus::i64>
03981     using touchard = typename internal::touchard_helper<deg>::template inner, deg>;
03982 } // namespace known_polynomials
03983 } // namespace aerobus
03984
03985 #ifdef AEROBUS_CONWAY_IMPORTS
03986
03987 // Conway polynomials
03988 namespace aerobus {
03989     template<int p, int n>
03990     struct ConwayPolynomial {};
03991
03992 #ifndef DO_NOT_DOCUMENT
03993     #define ZPV ZPZ::template val
03994     #define POLYV aerobus::polynomial<ZPV>::template val
03995     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
03996         POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
03997     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type =
03998         POLYV<ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03999     template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type =
04000         POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
04001     template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type =

```

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## 9.4 src/examples.h File Reference

## 9.5 examples.h

[Go to the documentation of this file.](#)

```
00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_
```



# Chapter 10

## Examples

### 10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

### 10.2 examples/custom\_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```



```
#include "../src/aerobus.h"

using FIELD = aerobus::zpz<2>;
using POLYNOMIALS = aerobus::polynomial<FIELD>;
using FRACTIONS = aerobus::FractionField<POLYNOMIALS>;

// x^3 + 2x^2 + 1, with coefficients in Z/2Z, actually x^3 + 1
using P = aerobus::make_int_polynomial_t<FIELD, 1, 2, 0, 1>;
// x^3 + 5x^2 + 7x + 11 with coefficients in Z/17Z, meaning actually x^3 + x^2 + 1
using Q = aerobus::make_int_polynomial_t<FIELD, 1, 5, 8, 1>;

// P/Q in the field of fractions of polynomials
using F = aerobus::makefraction_t<POLYNOMIALS, P, Q>;

int main() {
    const double v = F::eval<double>(1.0);
    std::cout << "expected = " << 2.0/3.0 << std::endl;
    std::cout << "value = " << v << std::endl;
    return 0;
}
```

## 10.6 examples/make\_polynomial.cpp

How to build your own sequence of known polynomials, here [Abel polynomials](#)

```
#include <iostream>
#include "../src/aerobus.h"

// let's build Abel polynomials from scratch using Aerobus
// note : it's now integrated in the main library, but still serves as an example

template<typename I = aerobus::i64>
struct AbelHelper {
private:
    using P = aerobus::polynomial<I>;

public:
    // to keep recursion working, we need to operate on a*n and not just a
    template<size_t deg, I::inner_type an>
    struct Inner {
        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}
```

## 10.7 examples/polynomials\_over\_finite\_field.cpp

How to build a known polynomial (here aerobus::known\_polynomials::allone) with coefficients in a finite field (here aerobus::zpz<2>) and get its value when evaluated at a value in this field (here 1).

```
#include <iostream>
```

```
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}
```

## 10.8 examples/compensated\_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```
// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {
    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE INLINED T exact(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    // (0.75 - x)^5 * (1 - x)^11
    using P = mul_t<
        pow_t<pq64, pq64::val<
            typename q64::template inject_constant_t<-1>,
            q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
        >;

    ::printf("polynomial = %s\n", P::to_string().c_str());

    using FLOAT = double;
    run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat");
    run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat");

    run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat");
    run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat");

    run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat");
    run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat");
}
```

# Index

abs\_t  
  aerobus, [20](#)  
add\_t  
  aerobus, [20](#)  
  aerobus::i32, [56](#)  
  aerobus::i64, [63](#)  
  aerobus::polynomial< Ring >, [72](#)  
  aerobus::Quotient< Ring, X >, [79](#)  
  aerobus::zpz< p >, [104](#)  
addfractions\_t  
  aerobus, [20](#)  
aerobus, [15](#)  
  abs\_t, [20](#)  
  add\_t, [20](#)  
  addfractions\_t, [20](#)  
  aligned\_malloc, [34](#)  
  alternate\_t, [20](#)  
  alternate\_v, [35](#)  
  asin, [21](#)  
  asinh, [21](#)  
  atan, [21](#)  
  atanh, [21](#)  
  bell\_t, [23](#)  
  bernoulli\_t, [23](#)  
  bernoulli\_v, [35](#)  
  combination\_t, [23](#)  
  combination\_v, [35](#)  
  cos, [23](#)  
  cosh, [25](#)  
  div\_t, [25](#)  
  E\_fraction, [25](#)  
  embed\_int\_poly\_in\_fractions\_t, [25](#)  
  exp, [26](#)  
  expm1, [26](#)  
  factorial\_t, [26](#)  
  factorial\_v, [35](#)  
  field, [34](#)  
  fpq32, [26](#)  
  fpq64, [27](#)  
  FractionField, [27](#)  
  gcd\_t, [27](#)  
  geometric\_sum, [27](#)  
  lnp1, [27](#)  
  make\_frac\_polynomial\_t, [28](#)  
  make\_int\_polynomial\_t, [28](#)  
  make\_q32\_t, [28](#)  
  make\_q64\_t, [29](#)  
  makefraction\_t, [29](#)  
  mul\_t, [29](#)  
  mulfractions\_t, [29](#)  
  pi64, [30](#)  
  PI\_fraction, [30](#)  
  pow\_t, [30](#)  
  pq64, [30](#)  
  q32, [30](#)  
  q64, [31](#)  
  sin, [31](#)  
  sinh, [31](#)  
  SQRT2\_fraction, [31](#)  
  SQRT3\_fraction, [31](#)  
  stirling\_1\_signed\_t, [32](#)  
  stirling\_1\_unsigned\_t, [32](#)  
  stirling\_2\_t, [32](#)  
  sub\_t, [33](#)  
  tan, [33](#)  
  tanh, [33](#)  
  taylor, [33](#)  
  vadd\_t, [34](#)  
  vmul\_t, [34](#)  
aerobus::ContinuedFraction< a0 >, [45](#)  
  type, [45](#)  
  val, [46](#)  
aerobus::ContinuedFraction< a0, rest... >, [46](#)  
  type, [47](#)  
  val, [47](#)  
aerobus::ContinuedFraction< values >, [44](#)  
aerobus::ConwayPolynomial, [47](#)  
aerobus::Embed< i32, i64 >, [49](#)  
  type, [49](#)  
aerobus::Embed< polynomial< Small >, polynomial< Large > >, [50](#)  
  type, [50](#)  
aerobus::Embed< q32, q64 >, [51](#)  
  type, [51](#)  
aerobus::Embed< Quotient< Ring, X >, Ring >, [52](#)  
  type, [52](#)  
aerobus::Embed< Ring, FractionField< Ring > >, [53](#)  
  type, [53](#)  
aerobus::Embed< Small, Large, E >, [49](#)  
aerobus::Embed< zpz< x >, i32 >, [53](#)  
  type, [54](#)  
aerobus::i32, [55](#)  
  add\_t, [56](#)  
  div\_t, [57](#)  
  eq\_t, [57](#)  
  eq\_v, [61](#)  
  gcd\_t, [57](#)  
  gt\_t, [57](#)

- inject\_constant\_t, 59
- inject\_ring\_t, 59
- inner\_type, 59
- is\_euclidean\_domain, 61
- is\_field, 61
- lt\_t, 59
- mod\_t, 59
- mul\_t, 60
- one, 60
- pos\_t, 60
- pos\_v, 61
- sub\_t, 60
- zero, 61
- aerobus::i32::val< x >, 88
  - enclosing\_type, 89
  - get, 89
  - is\_zero\_t, 89
  - to\_string, 89
  - v, 89
- aerobus::i64, 62
  - add\_t, 63
  - div\_t, 63
  - eq\_t, 64
  - eq\_v, 67
  - gcd\_t, 64
  - gt\_t, 64
  - gt\_v, 67
  - inject\_constant\_t, 64
  - inject\_ring\_t, 64
  - inner\_type, 65
  - is\_euclidean\_domain, 67
  - is\_field, 67
  - lt\_t, 65
  - lt\_v, 67
  - mod\_t, 65
  - mul\_t, 65
  - one, 66
  - pos\_t, 66
  - pos\_v, 68
  - sub\_t, 66
  - zero, 66
- aerobus::i64::val< x >, 90
  - enclosing\_type, 91
  - get, 91
  - inner\_type, 91
  - is\_zero\_t, 91
  - to\_string, 91
  - v, 92
- aerobus::internal, 36
  - index\_sequence\_reverse, 40
  - is\_instantiation\_of\_v, 40
  - make\_index\_sequence\_reverse, 39
  - type\_at\_t, 39
- aerobus::is\_prime< n >, 69
  - value, 70
- aerobus::IsEuclideanDomain, 41
- aerobus::IsField, 41
- aerobus::IsRing, 42
- aerobus::known\_polynomials, 40
  - hermite\_kind, 40
  - physicist, 40
  - probabilist, 40
- aerobus::polynomial< Ring >, 70
  - add\_t, 72
  - derive\_t, 72
  - div\_t, 72
  - eq\_t, 73
  - gcd\_t, 73
  - gt\_t, 73
  - inject\_constant\_t, 73
  - inject\_ring\_t, 74
  - is\_euclidean\_domain, 77
  - is\_field, 77
  - lt\_t, 74
  - mod\_t, 74
  - monomial\_t, 75
  - mul\_t, 75
  - one, 75
  - pos\_t, 75
  - pos\_v, 77
  - simplify\_t, 76
  - sub\_t, 76
  - X, 76
  - zero, 76
- aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost >, 47
  - func, 48
- aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost >, 48
  - func, 48
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >, 54
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop >, 68
  - type, 68
- aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop >, 69
  - type, 69
- aerobus::polynomial< Ring >::val< coeffN >, 99
  - aN, 100
  - coeff\_at\_t, 100
  - degree, 102
  - enclosing\_type, 100
  - eval, 101
  - is\_zero\_t, 100
  - is\_zero\_v, 102
  - ring\_type, 101
  - strip, 101
  - to\_string, 101
  - value\_at\_t, 101
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, E >, 43
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index < 0 || index >

- 0)> >, [43](#)
- type, [43](#)
- aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> >, [44](#)
- type, [44](#)
- aerobus::polynomial< Ring >::val< coeffN, coeffs >, [92](#)
- aN, [93](#)
- coeff\_at\_t, [93](#)
- compensated\_eval, [94](#)
- degree, [96](#)
- enclosing\_type, [93](#)
- eval, [95](#)
- is\_zero\_t, [94](#)
- is\_zero\_v, [96](#)
- ring\_type, [94](#)
- strip, [94](#)
- to\_string, [95](#)
- value\_at\_t, [94](#)
- aerobus::Quotient< Ring, X >, [78](#)
- add\_t, [79](#)
- div\_t, [80](#)
- eq\_t, [80](#)
- eq\_v, [82](#)
- inject\_constant\_t, [80](#)
- inject\_ring\_t, [80](#)
- is\_euclidean\_domain, [82](#)
- mod\_t, [81](#)
- mul\_t, [81](#)
- one, [81](#)
- pos\_t, [81](#)
- pos\_v, [82](#)
- zero, [82](#)
- aerobus::Quotient< Ring, X >::val< V >, [96](#)
- raw\_t, [97](#)
- type, [97](#)
- aerobus::type\_list< Ts >, [84](#)
- at, [85](#)
- concat, [85](#)
- insert, [85](#)
- length, [86](#)
- push\_back, [85](#)
- push\_front, [86](#)
- remove, [86](#)
- aerobus::type\_list< Ts >::pop\_front, [77](#)
- tail, [78](#)
- type, [78](#)
- aerobus::type\_list< Ts >::split< index >, [83](#)
- head, [83](#)
- tail, [83](#)
- aerobus::type\_list<>, [87](#)
- concat, [87](#)
- insert, [87](#)
- length, [88](#)
- push\_back, [87](#)
- push\_front, [87](#)
- aerobus::zpz< p >, [102](#)
- add\_t, [104](#)
- div\_t, [104](#)
- eq\_t, [104](#)
- eq\_v, [107](#)
- gcd\_t, [105](#)
- gt\_t, [105](#)
- gt\_v, [107](#)
- inject\_constant\_t, [105](#)
- inner\_type, [105](#)
- is\_euclidean\_domain, [108](#)
- is\_field, [108](#)
- lt\_t, [105](#)
- lt\_v, [108](#)
- mod\_t, [106](#)
- mul\_t, [106](#)
- one, [106](#)
- pos\_t, [106](#)
- pos\_v, [108](#)
- sub\_t, [107](#)
- zero, [107](#)
- aerobus::zpz< p >::val< x >, [97](#)
- enclosing\_type, [98](#)
- get, [98](#)
- is\_zero\_t, [98](#)
- is\_zero\_v, [99](#)
- to\_string, [98](#)
- v, [99](#)
- aligned\_malloc
  - aerobus, [34](#)
- alternate\_t
  - aerobus, [20](#)
- alternate\_v
  - aerobus, [35](#)
- aN
  - aerobus::polynomial< Ring >::val< coeffN >, [100](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [93](#)
- asin
  - aerobus, [21](#)
- asinh
  - aerobus, [21](#)
- at
  - aerobus::type\_list< Ts >, [85](#)
- atan
  - aerobus, [21](#)
- atanh
  - aerobus, [21](#)
- bell\_t
  - aerobus, [23](#)
- bernoulli\_t
  - aerobus, [23](#)
- bernoulli\_v
  - aerobus, [35](#)
- coeff\_at\_t
  - aerobus::polynomial< Ring >::val< coeffN >, [100](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [93](#)
- combination\_t

- aerobus, [23](#)
- combination\_v
  - aerobus, [35](#)
- compensated\_eval
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [94](#)
- concat
  - aerobus::type\_list< Ts >, [85](#)
  - aerobus::type\_list<>, [87](#)
- cos
  - aerobus, [23](#)
- cosh
  - aerobus, [25](#)
- degree
  - aerobus::polynomial< Ring >::val< coeffN >, [102](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [96](#)
- derive\_t
  - aerobus::polynomial< Ring >, [72](#)
- div\_t
  - aerobus, [25](#)
  - aerobus::i32, [57](#)
  - aerobus::i64, [63](#)
  - aerobus::polynomial< Ring >, [72](#)
  - aerobus::Quotient< Ring, X >, [80](#)
  - aerobus::zpz< p >, [104](#)
- E\_fraction
  - aerobus, [25](#)
- embed\_int\_poly\_in\_fractions\_t
  - aerobus, [25](#)
- enclosing\_type
  - aerobus::i32::val< x >, [89](#)
  - aerobus::i64::val< x >, [91](#)
  - aerobus::polynomial< Ring >::val< coeffN >, [100](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [93](#)
  - aerobus::zpz< p >::val< x >, [98](#)
- eq\_t
  - aerobus::i32, [57](#)
  - aerobus::i64, [64](#)
  - aerobus::polynomial< Ring >, [73](#)
  - aerobus::Quotient< Ring, X >, [80](#)
  - aerobus::zpz< p >, [104](#)
- eq\_v
  - aerobus::i32, [61](#)
  - aerobus::i64, [67](#)
  - aerobus::Quotient< Ring, X >, [82](#)
  - aerobus::zpz< p >, [107](#)
- eval
  - aerobus::polynomial< Ring >::val< coeffN >, [101](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [95](#)
- exp
  - aerobus, [26](#)
- expm1
  - aerobus, [26](#)
- factorial\_t
  - aerobus, [26](#)
- factorial\_v
  - aerobus, [35](#)
- field
  - aerobus, [34](#)
- fpq32
  - aerobus, [26](#)
- fpq64
  - aerobus, [27](#)
- FractionField
  - aerobus, [27](#)
- func
  - aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost >, [48](#)
  - aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost >, [48](#)
- gcd\_t
  - aerobus, [27](#)
  - aerobus::i32, [57](#)
  - aerobus::i64, [64](#)
  - aerobus::polynomial< Ring >, [73](#)
  - aerobus::zpz< p >, [105](#)
- geometric\_sum
  - aerobus, [27](#)
- get
  - aerobus::i32::val< x >, [89](#)
  - aerobus::i64::val< x >, [91](#)
  - aerobus::zpz< p >::val< x >, [98](#)
- gt\_t
  - aerobus::i32, [57](#)
  - aerobus::i64, [64](#)
  - aerobus::polynomial< Ring >, [73](#)
  - aerobus::zpz< p >, [105](#)
- gt\_v
  - aerobus::i64, [67](#)
  - aerobus::zpz< p >, [107](#)
- head
  - aerobus::type\_list< Ts >::split< index >, [83](#)
- hermite\_kind
  - aerobus::known\_polynomials, [40](#)
- index\_sequence\_reverse
  - aerobus::internal, [40](#)
- inject\_constant\_t
  - aerobus::i32, [59](#)
  - aerobus::i64, [64](#)
  - aerobus::polynomial< Ring >, [73](#)
  - aerobus::Quotient< Ring, X >, [80](#)
  - aerobus::zpz< p >, [105](#)
- inject\_ring\_t
  - aerobus::i32, [59](#)
  - aerobus::i64, [64](#)
  - aerobus::polynomial< Ring >, [74](#)
  - aerobus::Quotient< Ring, X >, [80](#)



- inner\_type
  - aerobus::i32, [59](#)
  - aerobus::i64, [65](#)
  - aerobus::i64::val< x >, [91](#)
  - aerobus::zpz< p >, [105](#)
- insert
  - aerobus::type\_list< Ts >, [85](#)
  - aerobus::type\_list<>, [87](#)
- Introduction, [1](#)
- is\_euclidean\_domain
  - aerobus::i32, [61](#)
  - aerobus::i64, [67](#)
  - aerobus::polynomial< Ring >, [77](#)
  - aerobus::Quotient< Ring, X >, [82](#)
  - aerobus::zpz< p >, [108](#)
- is\_field
  - aerobus::i32, [61](#)
  - aerobus::i64, [67](#)
  - aerobus::polynomial< Ring >, [77](#)
  - aerobus::zpz< p >, [108](#)
- is\_instantiation\_of\_v
  - aerobus::internal, [40](#)
- is\_zero\_t
  - aerobus::i32::val< x >, [89](#)
  - aerobus::i64::val< x >, [91](#)
  - aerobus::polynomial< Ring >::val< coeffN >, [100](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [94](#)
  - aerobus::zpz< p >::val< x >, [98](#)
- is\_zero\_v
  - aerobus::polynomial< Ring >::val< coeffN >, [102](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [96](#)
  - aerobus::zpz< p >::val< x >, [99](#)
- length
  - aerobus::type\_list< Ts >, [86](#)
  - aerobus::type\_list<>, [88](#)
- Inp1
  - aerobus, [27](#)
- lt\_t
  - aerobus::i32, [59](#)
  - aerobus::i64, [65](#)
  - aerobus::polynomial< Ring >, [74](#)
  - aerobus::zpz< p >, [105](#)
- lt\_v
  - aerobus::i64, [67](#)
  - aerobus::zpz< p >, [108](#)
- make\_frac\_polynomial\_t
  - aerobus, [28](#)
- make\_index\_sequence\_reverse
  - aerobus::internal, [39](#)
- make\_int\_polynomial\_t
  - aerobus, [28](#)
- make\_q32\_t
  - aerobus, [28](#)
- make\_q64\_t
  - aerobus, [29](#)
- makefraction\_t
  - aerobus, [29](#)
- mod\_t
  - aerobus::i32, [59](#)
  - aerobus::i64, [65](#)
  - aerobus::polynomial< Ring >, [74](#)
  - aerobus::Quotient< Ring, X >, [81](#)
  - aerobus::zpz< p >, [106](#)
- monomial\_t
  - aerobus::polynomial< Ring >, [75](#)
- mul\_t
  - aerobus, [29](#)
  - aerobus::i32, [60](#)
  - aerobus::i64, [65](#)
  - aerobus::polynomial< Ring >, [75](#)
  - aerobus::Quotient< Ring, X >, [81](#)
  - aerobus::zpz< p >, [106](#)
- mulfractions\_t
  - aerobus, [29](#)
- one
  - aerobus::i32, [60](#)
  - aerobus::i64, [66](#)
  - aerobus::polynomial< Ring >, [75](#)
  - aerobus::Quotient< Ring, X >, [81](#)
  - aerobus::zpz< p >, [106](#)
- physicist
  - aerobus::known\_polynomials, [40](#)
- pi64
  - aerobus, [30](#)
- PI\_fraction
  - aerobus, [30](#)
- pos\_t
  - aerobus::i32, [60](#)
  - aerobus::i64, [66](#)
  - aerobus::polynomial< Ring >, [75](#)
  - aerobus::Quotient< Ring, X >, [81](#)
  - aerobus::zpz< p >, [106](#)
- pos\_v
  - aerobus::i32, [61](#)
  - aerobus::i64, [68](#)
  - aerobus::polynomial< Ring >, [77](#)
  - aerobus::Quotient< Ring, X >, [82](#)
  - aerobus::zpz< p >, [108](#)
- pow\_t
  - aerobus, [30](#)
- pq64
  - aerobus, [30](#)
- probabilist
  - aerobus::known\_polynomials, [40](#)
- push\_back
  - aerobus::type\_list< Ts >, [85](#)
  - aerobus::type\_list<>, [87](#)
- push\_front
  - aerobus::type\_list< Ts >, [86](#)
  - aerobus::type\_list<>, [87](#)
- q32

- aerobus, [30](#)
- q64
  - aerobus, [31](#)
- raw\_t
  - aerobus::Quotient< Ring, X >::val< V >, [97](#)
- README.md, [111](#)
- remove
  - aerobus::type\_list< Ts >, [86](#)
- ring\_type
  - aerobus::polynomial< Ring >::val< coeffN >, [101](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [94](#)
- simplify\_t
  - aerobus::polynomial< Ring >, [76](#)
- sin
  - aerobus, [31](#)
- sinh
  - aerobus, [31](#)
- SQRT2\_fraction
  - aerobus, [31](#)
- SQRT3\_fraction
  - aerobus, [31](#)
- src/aerobus.h, [111](#)
- src/examples.h, [204](#)
- stirling\_1\_signed\_t
  - aerobus, [32](#)
- stirling\_1\_unsigned\_t
  - aerobus, [32](#)
- stirling\_2\_t
  - aerobus, [32](#)
- strip
  - aerobus::polynomial< Ring >::val< coeffN >, [101](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [94](#)
- sub\_t
  - aerobus, [33](#)
  - aerobus::i32, [60](#)
  - aerobus::i64, [66](#)
  - aerobus::polynomial< Ring >, [76](#)
  - aerobus::zpz< p >, [107](#)
- tail
  - aerobus::type\_list< Ts >::pop\_front, [78](#)
  - aerobus::type\_list< Ts >::split< index >, [83](#)
- tan
  - aerobus, [33](#)
- tanh
  - aerobus, [33](#)
- taylor
  - aerobus, [33](#)
- to\_string
  - aerobus::i32::val< x >, [89](#)
  - aerobus::i64::val< x >, [91](#)
  - aerobus::polynomial< Ring >::val< coeffN >, [101](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [95](#)
  - aerobus::zpz< p >::val< x >, [98](#)
- type
  - aerobus::ContinuedFraction< a0 >, [45](#)
  - aerobus::ContinuedFraction< a0, rest... >, [47](#)
  - aerobus::Embed< i32, i64 >, [49](#)
  - aerobus::Embed< polynomial< Small >, polynomial< Large > >, [50](#)
  - aerobus::Embed< q32, q64 >, [51](#)
  - aerobus::Embed< Quotient< Ring, X >, Ring >, [52](#)
  - aerobus::Embed< Ring, FractionField< Ring > >, [53](#)
  - aerobus::Embed< zpz< x >, i32 >, [54](#)
  - aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop >, [68](#)
  - aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop >, [69](#)
  - aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index< 0 || index > 0)> >, [43](#)
  - aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> >, [44](#)
  - aerobus::Quotient< Ring, X >::val< V >, [97](#)
  - aerobus::type\_list< Ts >::pop\_front, [78](#)
- type\_at\_t
  - aerobus::internal, [39](#)
- v
  - aerobus::i32::val< x >, [89](#)
  - aerobus::i64::val< x >, [92](#)
  - aerobus::zpz< p >::val< x >, [99](#)
- vadd\_t
  - aerobus, [34](#)
- val
  - aerobus::ContinuedFraction< a0 >, [46](#)
  - aerobus::ContinuedFraction< a0, rest... >, [47](#)
- value
  - aerobus::is\_prime< n >, [70](#)
- value\_at\_t
  - aerobus::polynomial< Ring >::val< coeffN >, [101](#)
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, [94](#)
- vmul\_t
  - aerobus, [34](#)
- X
  - aerobus::polynomial< Ring >, [76](#)
- zero
  - aerobus::i32, [61](#)
  - aerobus::i64, [66](#)
  - aerobus::polynomial< Ring >, [76](#)
  - aerobus::Quotient< Ring, X >, [82](#)
  - aerobus::zpz< p >, [107](#)