

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 HOW TO

- Clone or download the repository somewhere, or just download `aerobus.h`
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including `aerobus.h`.

### 1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

### 1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). And Google's [Benchmark library](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make benchmarks
./benchmarks
```

## 1.2 Structures

### 1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`



### 1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev\_T
- chebyshev\_U
- laguerre
- hermite\_prob
- hermite\_phys
- bernstein
- legendre
- bernoulli

### 1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1`  $e^x - 1$
- `lnp1`  $\ln(x + 1)$
- `geom`  $\frac{1}{1-x}$
- `sin`
- `cos`
- `tan`
- `sh`
- `cosh`
- `tanh`
- `asin`
- `acos`
- `acosh`
- `asinh`
- `atanh`

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`);
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_series = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_series<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
    vbroadcastsd ymm3, QWORD PTR .LC23[rip]
    vbroadcastsd ymm2, QWORD PTR .LC25[rip]
.L3:
    vmovupd ymm15, YMMWORD PTR [rsi+rax]
    vmovapd ymm0, ymm15
    vfmadd132pd ymm0, ymm14, ymm1
    vfmadd132pd ymm0, ymm13, ymm15
    vfmadd132pd ymm0, ymm12, ymm15
    vfmadd132pd ymm0, ymm11, ymm15
    vfmadd132pd ymm0, ymm10, ymm15
    vfmadd132pd ymm0, ymm9, ymm15
    vfmadd132pd ymm0, ymm8, ymm15
    vfmadd132pd ymm0, ymm7, ymm15
    vfmadd132pd ymm0, ymm6, ymm15
    vfmadd132pd ymm0, ymm5, ymm15
    vfmadd132pd ymm0, ymm4, ymm15
    vfmadd132pd ymm0, ymm3, ymm15
    vfmadd132pd ymm0, ymm2, ymm15
    vfmadd132pd ymm0, ymm1, ymm15
    vmovupd YMMWORD PTR [rdx+rax], ymm0
    add     rax, 32
    cmp     rcx, rax
    jne     .L3
    mov     rax, rdi
    and     rax, -4
    vzeroupper
```

## 1.3 Operations

### 1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, Aerobus allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo  $p$  is not a field when  $p$  is not prime. We then can define its field of fraction and polynomials over it this way:

```
using namespace aerobus;
using ZmZ = zpz<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;
```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define **rational functions** by taking the ring of fractions of polynomials:

```
using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;
```

Which also have an evaluation function, as polynomial do.

### 1.3.2 Quotient

Given a ring  $R$ , Aerobus provides automatic implementation for **quotient ring**  $R/X$  where  $X$  is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as  $R$  is commutative (and we assume it is).

For example, if we want  $R$  to be  $\mathbb{Z}$  represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```
using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;
```

As we could have using `zpz<17>`.

This is mainly used to define finite fields of order  $p^n$  using Conway polynomials but may have other applications.

## 1.4 Misc

### 1.4.1 Continued Fractions

Aerobus gives an implementation for **continued fractions**. It can be used this way:

```
using namespace aerobus;
using T = ContinuedFraction<1,2,3,4>;
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of  $\pi$ ,  $e$ ,  $\sqrt{2}$  and  $\sqrt{3}$ :

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

## 1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, Aerobus provides some support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough constexpr in its `cuda_fp16.h` header, so we had to implement our own constexpr `static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not easily possible to make it work for `__half2` because of [another bug](#).

A workaround is to modify `cuda_fp16.h` and add a constexpr modifier to line 5039. This works but only tested on Linux with CUDA 16.1.

Once done, `nvcc` generates splendid assembly, same as for double or float:

```
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R7, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R7, R5, R7, 0.008331298828125, 0.008331298828125 ;
HFMA2 R7, R5, R7, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R7, R5, R7, 0.1666259765625, 0.1666259765625 ;
HFMA2 R7, R5, R7, 0.5, 0.5 ;
HFMA2.MMA R7, R5, R7, 1, 1 ;
HFMA2 R7, R5, R7, RZ.H0_H0 ;
```

Please push to make these bug fixed by NVIDIA.



## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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## Chapter 3

# Concept Index

### 3.1 Concepts

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Concept to express R is a field . . . . .	45
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# Chapter 4

## Class Index

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## Chapter 5

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## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*
- namespace [libm](#)

#### Classes

- struct [ContinuedFraction](#)  
*represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$*
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [Embed](#)  
*embedding - struct forward declaration*
- struct [Embed< i32, i64 >](#)  
*embeds i32 into i64*
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)  
*embeds polynomial<Small> into polynomial<Large>*
- struct [Embed< q32, q64 >](#)  
*embeds q32 into q64*
- struct [Embed< Quotient< Ring, X >, Ring >](#)  
*embeds Quotient<Ring, X> into Ring*
- struct [Embed< Ring, FractionField< Ring > >](#)  
*embeds values from Ring to its field of fractions*

- struct [Embed< zpz< x >, i32 >](#)  
*embeds zpz values into i32*
- struct [i32](#)  
*32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)  
*64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)  
*checks if n is prime*
- struct [meta\\_libm](#)
- struct [polynomial](#)
- struct [Quotient](#)  
*Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is Z/2Z.*
- struct [type\\_list](#)  
*Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)  
*specialization for empty type list*
- struct [zpz](#)  
*congruence classes of integers modulo p (32 bits)*

## Concepts

- concept [IsRing](#)  
*Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)  
*Concept to express R is an euclidean domain.*
- concept [IsField](#)  
*Concept to express R is a field.*

## Typedefs

- template<typename T , typename A , typename B >  
using [gcd\\_t](#) = typename internal::gcd< T >::template type< A, B >  
*computes the greatest common divisor of A and B*
- template<typename... vals>  
using [vadd\\_t](#) = typename internal::vadd< vals... >::type  
*adds multiple values (v1 + v2 + ... + vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator*
- template<typename... vals>  
using [vmul\\_t](#) = typename internal::vmul< vals... >::type  
*multiplies multiple values (v1 \* v2 + ... + vn) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator*
- template<typename val >  
using [abs\\_t](#) = std::conditional\_t< val::enclosing\_type::template pos\_v< val >, val, typename val::enclosing\_type::template [sub\\_t](#)< typename val::enclosing\_type::zero, val > >  
*computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<typename Ring >  
using [FractionField](#) = typename internal::FractionFieldImpl< Ring >::type  
*Fraction field of an euclidean domain, such as Q for Z.*
- template<typename X , typename Y >  
using [add\\_t](#) = typename X::enclosing\_type::template [add\\_t](#)< X, Y >  
*generic addition*



- `template<typename X , typename Y >`  
`using sub_t = typename X::enclosing_type::template sub_t< X, Y >`  
*generic subtraction*
- `template<typename X , typename Y >`  
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`  
*generic multiplication*
- `template<typename X , typename Y >`  
`using div_t = typename X::enclosing_type::template div_t< X, Y >`  
*generic division*
- `using q32 = FractionField< i32 >`  
*32 bits rationals rationals with 32 bits numerator and denominator*
- `using fpq32 = FractionField< polynomial< q32 > >`  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- `using q64 = FractionField< i64 >`  
*64 bits rationals rationals with 64 bits numerator and denominator*
- `using pi64 = polynomial< i64 >`  
*polynomial with 64 bits integers coefficients*
- `using pq64 = polynomial< q64 >`  
*polynomial with 64 bits rationals coefficients*
- `using fpq64 = FractionField< polynomial< q64 > >`  
*polynomial with 64 bits rational coefficients*
- `template<typename Ring , typename v1 , typename v2 >`  
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational V1/V2 in the field of fractions of Ring*
- `template<typename v >`  
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type > , polynomial< FractionField< typename v::ring_type > > >::template type< v >`  
*embed a polynomial with integers coefficients into rational coefficients polynomials*
- `template<int64_t p, int64_t q>`  
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p > , i64::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<int32_t p, int32_t q>`  
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p > , i32::inject_constant_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<typename Ring , typename v1 , typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- `template<typename Ring , typename v1 , typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- `template<typename Ring , auto... xs>`  
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in Ring*
- `template<typename Ring , auto... xs>`  
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`  
*make a polynomial with coefficients in FractionField<Ring>*
- `template<typename T , size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`

- computes factorial(i), as type*

  - `template<typename T, size_t k, size_t n>`  
using `combination_t` = `typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T, size_t n>`  
using `bernoulli_t` = `typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T, size_t n>`  
using `bell_t` = `typename internal::bell_helper< T, n >::type`  
*Bell numbers.*
- `template<typename T, int k>`  
using `alternate_t` = `typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T, int n, int k>`  
using `stirling_1_signed_t` = `typename internal::stirling_1_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T, int n, int k>`  
using `stirling_1_unsigned_t` = `abs_t< typename internal::stirling_1_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T, int n, int k>`  
using `stirling_2_t` = `typename internal::stirling_2_helper< T, n, k >::type`  
*Stirling number of second kind – as types.*
- `template<typename T, typename p, size_t n>`  
using `pow_t` = `typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>`  
using `taylor` = `typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse< deg+1 > >::type`
- `template<typename Integers, size_t deg>`  
using `exp` = `taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers, size_t deg>`  
using `expm1` = `typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg >, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers, size_t deg>`  
using `lnp1` = `taylor< Integers, internal::lnp1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers, size_t deg>`  
using `atan` = `taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers, size_t deg>`  
using `sin` = `taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers, size_t deg>`  
using `sinh` = `taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers, size_t deg>`  
using `cosh` = `taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers, size_t deg>`  
using `cos` = `taylor< Integers, internal::cos_coeff, deg >`  
 $\cos(x)$  *cosinus*

- `template<typename Integers , size_t deg>`  
using `geometric_sum = Taylor< Integers, internal::geom_coeff, deg >`  

$$\frac{1}{1-x}$$
*zero development of  $\frac{1}{1-x}$*
- `template<typename Integers , size_t deg>`  
using `asin = Taylor< Integers, internal::asin_coeff, deg >`  
`arcsin(x)` *arc sinus*
- `template<typename Integers , size_t deg>`  
using `asinh = Taylor< Integers, internal::asinh_coeff, deg >`  
`arsinh(x)` *arc hyperbolic sinus*
- `template<typename Integers , size_t deg>`  
using `atanh = Taylor< Integers, internal::atanh_coeff, deg >`  
`arctanh(x)` *arc hyperbolic tangent*
- `template<typename Integers , size_t deg>`  
using `tan = Taylor< Integers, internal::tan_coeff, deg >`  
`tan(x)` *tangent*
- `template<typename Integers , size_t deg>`  
using `tanh = Taylor< Integers, internal::tanh_coeff, deg >`  
`tanh(x)` *hyperbolic tangent*
- using `PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >`
- using `E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`  
*approximation of e*
- using `SQRT2_fraction = ContinuedFraction< 1, 2 >`  
*approximation of  $\sqrt{2}$*
- using `SQRT3_fraction = ContinuedFraction< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >`  
*approximation of*

## Functions

- `template<typename T >`  
`T * aligned\_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the [field](#) (prime number) @tparam n degree of extension  
`template< int p`

## Variables

- `template<typename T , size_t i>`  
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`  
*computes factorial(i) as value in T*
- `template<typename T , size_t k, size_t n>`  
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`  
*computes binomial coefficients (k among n) as value*
- `template<typename FloatType , typename T , size_t n>`  
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`  
*nth bernoulli number as value in FloatType*
- `template<typename T , size_t k>`  
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`  
 *$(-1)^k$  as value from T*

### 6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

## 6.1.2 Typedef Documentation

### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 add\_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

#### Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

### 6.1.2.3 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

### 6.1.2.4 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T

## Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example
----------	---

## 6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\arctan(x)$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

`atanh( $x$ )` arc hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.9 bell\_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

## Template Parameters

<i>T</i>	ring type, such as <a href="#">aerobus::i64</a>
<i>n</i>	index

## 6.1.2.10 bernoulli\_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

## 6.1.2.11 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

## Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

## 6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  `cosinus`



## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.14 div\_t

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div\_t<X, Y>
```

generic division

## Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

## 6.1.2.15 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

approximation of  $e$

## 6.1.2.16 embed\_int\_poly\_in\_fractions\_t

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

## Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in polynomial<Ring>

## 6.1.2.17 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

## Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

## 6.1.2.19 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

## Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

## 6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

#### 6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

#### 6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

Fraction field of an euclidean domain, such as Q for Z.

##### Template Parameters

<i>Ring</i>	
-------------	--

#### 6.1.2.23 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

##### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

#### 6.1.2.24 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

##### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.25 Inp1

```
template<typename Integers , size_t deg>
using aerobus::lnp1 = typedef taylor<Integers, internal::lnp1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.26 make\_frac\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

### 6.1.2.27 make\_int\_polynomial\_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

#### Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	coefficients

### 6.1.2.28 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.29 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

## Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

## 6.1.2.30 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

## Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

## 6.1.2.31 mul\_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

## Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

### 6.1.2.32 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

### 6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

### 6.1.2.34 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

representation of  $\pi$  as a continued fraction

### 6.1.2.35 pow\_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

#### Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	must be an instantiation of T::val
<i>n</i>	power

### 6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

**6.1.2.37 q32**

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

**6.1.2.38 q64**

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

**6.1.2.39 sin**

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.40 sinh**

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.41 SQRT2\_fraction**

```
using aerobus::SQRT2_fraction = typedef ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>
```

approximation of  $\sqrt{2}$

```
using aerobus::SQRT3_fraction = typedef ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1,  
2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>
```

#### 6.1.2.43 `stirling_1_signed_t`

Stirling number of first kind (signed) – as types.

$T$	(ring type, such as <code>aerobus::i64</code> )
$n$	(integer)
$k$	(integer)

```
template<typename T , int n, int k>
using aerobus::stirling_1_unsigned_t = typedef abs_t<typename internal::stirling_1_helper<T,
n, k>::type>
```

$T$	(ring type, such as <code>aerobus::i64</code> )
$n$	(integer)
$k$	(integer)

```
template<typename T , int n, int k>
using aerobus::stirling_2_t = typedef typename internal::stirling_2_helper<T, n, k>::type
```

$T$	(ring type, such as <code>aerobus::i64</code> )
$n$	(integer)
$k$	(integer)



**6.1.2.46 sub\_t**

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

**Template Parameters**

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

**6.1.2.47 tan**

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.48 tanh**

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.49 taylor**

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

**Template Parameters**

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff<sub>↔</sub> _at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

### 6.1.2.50 vadd\_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.2.51 vmul\_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator

#### Template Parameters

<i>...vals</i>	
----------------	--

## 6.1.3 Function Documentation

### 6.1.3.1 aligned\_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

#### Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

#### Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

### 6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```

```
prime number )
```

## 6.1.4 Variable Documentation

### 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

#### Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an <code>int64_t</code>
----------	--

### 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

#### Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

#### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

##### Template Parameters

<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i<=n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >=9 &&i \*i > n)> >
- struct **AbelHelper**
- struct **AllOneHelper**
- struct **AllOneHelper**< 0, l >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable\_if\_t< k % 2==0 > >
- struct **arithmetic\_helpers**
- struct **arithmetic\_helpers**< double >
- struct **arithmetic\_helpers**< float >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **asinh\_coeff**

- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **bell\_helper**
- struct **bell\_helper**< T, 0 >
- struct **bell\_helper**< T, 1 >
- struct **bell\_helper**< T, n, std::enable\_if\_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0, l >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==0)> >
- struct **bernstein\_helper**< i, m, l, std::enable\_if\_t<(m > 0) &&(i==m)> >
- struct **BesselHelper**
- struct **BesselHelper**< 0, l >
- struct **BesselHelper**< 1, l >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0, l >
- struct **chebyshev\_helper**< 1, 1, l >
- struct **chebyshev\_helper**< 2, 0, l >
- struct **chebyshev\_helper**< 2, 1, l >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **double\_double**
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FloatLayout**
- struct **FloatLayout**< double >
- struct **FloatLayout**< float >
- struct **FloatLayout**< long double >

- struct **fma\_helper**
- struct **fma\_helper**< double >
- struct **fma\_helper**< float >
- struct **fma\_helper**< int16\_t >
- struct **fma\_helper**< int32\_t >
- struct **fma\_helper**< int64\_t >
- struct **fma\_helper**< long double >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**
  - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist, I >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist, I >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0, I >
- struct **laguerre\_helper**< 1, I >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0, I >
- struct **legendre\_helper**< 1, I >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, Is... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable\_if\_t< n==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **pow\_scalar**
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **Split**
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **staticcast**

- struct **stirling\_1\_helper**
- struct **stirling\_1\_helper**< T, 0, 0 >
- struct **stirling\_1\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_1\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0)> >
- struct **stirling\_2\_helper**
- struct **stirling\_2\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_2\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0) &&(k < n)> >
- struct **stirling\_2\_helper**< T, n, n, std::enable\_if\_t<(n >=0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **touchard\_coeff**
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N >{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

### 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

## 6.2.2 Typedef Documentation

### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 index\_sequence\_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 is\_instantiation\_of\_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 aerobus::known\_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Enumeration Type Documentation

#### 6.3.2.1 hermite\_kind

```
enum aerobus::known_polynomials::hermite_kind
```



## Enumerator

probabilist	
physicist	

## 6.4 aerobus::libm Namespace Reference

## Namespaces

- namespace [internal](#)

## Functions

- brief computes [x](#) (NOT TESTED YET) @tparam T @param x @return template< typename T > static T exp2(const T &x)
- brief computes sine following IEEE recommendations exact values for and correct at epsilon for other values tparam T [arithmetic\\_type](#) (float only so far) @param [x](#) input value @return [sin\(x\)](#) template< typename T > static T [sin](#)(const T &x)

## Variables

- brief computes sine [function](#)
- brief computes sine following IEEE recommendations exact values for and [infinities](#)

### 6.4.1 Function Documentation

#### 6.4.1.1 arithmetic\_type()

```
brief computes cosine following IEEE recommendations exact values for and correct at epsilon
for other values tparam T aerobus::libm::arithmetic_type (
    float only so far ) const &
```

#### 6.4.1.2 x()

```
brief computes aerobus::libm::x (
    NOT TESTED YET ) const &
```

### 6.4.2 Variable Documentation

#### 6.4.2.1 function

```
brief computes cosine aerobus::libm::function
```

#### 6.4.2.2 infinities

brief computes cosine following IEEE recommendations exact values for and aerobus::libm←  
::infinities

## 6.5 aerobus::libm::internal Namespace Reference

### Classes

- struct [cos\\_poly](#)
- struct [cos\\_poly< double >](#)
- struct [cos\\_poly< float >](#)
- struct [exp2\\_poly](#)
- struct [exp2\\_poly< double >](#)
- struct [exp2\\_poly< float >](#)
- struct [sin\\_poly](#)
- struct [sin\\_poly< double >](#)
- struct [sin\\_poly< float >](#)

# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

## 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename Ring::zero

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = `aN`

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 `type`

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

### 8.4 `aerobus::ContinuedFraction< values >` Struct Template Reference

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

#### 8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

#### Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

#### Examples

[examples/continued\\_fractions.cpp](#).

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename q64::template inject\_constant\_t< a0 >  
*represented value as [aerobus::q64](#)*

### Static Public Attributes

- static constexpr double [val](#) = static\_cast<double>(a0)  
*represented value as double*

### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

#### Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

### 8.5.2 Member Typedef Documentation

#### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
```

represented value as [aerobus::q64](#)

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template [add\\_t](#)< typename q64::template inject\_constant\_t< a0 >, typename q64::template [div\\_t](#)< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
*represented value as [aerobus::q64](#)*

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
*represented value as double*

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

#### Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)



## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64↔
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::libm::internal::cos\_poly< T > Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::libm::internal::cos\_poly< double > Struct Reference

```
#include <aerobus.h>
```

## Public Types

- using `type` = typename `aerobus::polynomial< aerobus::q64 >::simplify_t< typename aerobus::polynomial< aerobus::q64 >::template val< aerobus::make_q64_t< 14, 407833194230757 >, aerobus::make_q64_t<-283, 24728864074278 >, aerobus::make_q64_t< 136, 65144855767 >, aerobus::make_q64_t<-4089, 14838163607 >, aerobus::make_q64_t< 452396, 18240606721 >, aerobus::make_q64_t<-6405119470037555, 4611686018427387904 >, aerobus::make_q64_t< 6004799503160661, 144115188075855872 > > >`

## 8.9.1 Member Typedef Documentation

### 8.9.1.1 type

```
using aerobus::libm::internal::cos_poly< double >::type = typename aerobus::polynomial<aerobus::q64>↵
::simplify_t< typename aerobus::polynomial<aerobus::q64>:: template val< aerobus::make_q64_t<14,
407833194230757>, aerobus::make_q64_t<-283, 24728864074278>, aerobus::make_q64_t<136, 65144855767>,
aerobus::make_q64_t<-4089, 14838163607>, aerobus::make_q64_t<452396, 18240606721>, aerobus::make_q64_t<-6405
4611686018427387904>, aerobus::make_q64_t<6004799503160661, 144115188075855872> >>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.10 aerobus::libm::internal::cos\_poly< float > Struct Reference

```
#include <aerobus.h>
```

## Public Types

- using `type` = typename `aerobus::polynomial< aerobus::q32 >::simplify_t< typename aerobus::polynomial< aerobus::q32 >::template val< aerobus::make_q32_t<-1, 3112816 >, aerobus::make_q32_t< 1, 40237 >, aerobus::make_q32_t<-119, 85679 >, aerobus::make_q32_t< 11184811, 268435456 > > >`

## 8.10.1 Member Typedef Documentation

### 8.10.1.1 type

```
using aerobus::libm::internal::cos_poly< float >::type = typename aerobus::polynomial<aerobus::q32>↵
::simplify_t< typename aerobus::polynomial<aerobus::q32>:: template val< aerobus::make_q32_t<-1,
3112816>, aerobus::make_q32_t<1, 40237>, aerobus::make_q32_t<-119, 85679>, aerobus::make_q32_t<11184811,
268435456> >>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.11 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner< index, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

### 8.11.1 Member Function Documentation

#### 8.11.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int64_t index, int ghost>
static INLINE DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P
>::EFTHorner< index, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.12 aerobus::polynomial< Ring >::compensated\_horner< arithmeticType, P >::EFTHorner<-1, ghost > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static **INLINE DEVICE** void **func** (arithmeticType x, arithmeticType \*pi, arithmeticType \*sigma, arithmeticType \*r)

## 8.12.1 Member Function Documentation

### 8.12.1.1 func()

```
template<typename Ring >
template<typename arithmeticType , typename P >
template<int ghost>
static INLINED DEVICE void aerobus::polynomial< Ring >::compensated_horner< arithmeticType, P
>::EFTHorner<-1, ghost >::func (
    arithmeticType x,
    arithmeticType * pi,
    arithmeticType * sigma,
    arithmeticType * r ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

### 8.13.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

#### Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

## Public Types

- template<typename val >  
using **type** = i64::val< static\_cast< int64\_t >(val::v)>  
*the i64 representation of val*

### 8.14.1 Detailed Description

embeds i32 into i64

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the i64 representation of val

#### Template Parameters

<i>val</i>	a value in i32
------------	----------------

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.15 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds polynomial<Small> into polynomial<Large>

```
#include <aerobus.h>
```

## Public Types

- template<typename v >  
using **type** = typename at\_low< v, typename internal::make\_index\_sequence\_reverse< v::degree+1 > >::type  
*the polynomial<Large> representation of v*

### 8.15.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

### Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
```

the polynomial<Large> representation of v

### Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.16 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

### Public Types

- template<typename v >  
using type = make\_q64\_t< static\_cast< int64\_t >(v::x::v), static\_cast< int64\_t >(v::y::v)>  
*q64 representation of v*

### 8.16.1 Detailed Description

embeds q32 into q64

## 8.16.2 Member Typedef Documentation

### 8.16.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_↵
cast<int64_t>(v::y::v)>
```

q64 representation of v

## Template Parameters

<i>v</i>	a value in q32
----------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.17 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = typename val::raw_t`  
*Ring representation of val.*

### 8.17.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

## Template Parameters

<i>Ring</i>	a Euclidean ring
<i>X</i>	a value in Ring

### 8.17.2 Member Typedef Documentation

#### 8.17.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

### Template Parameters

<i>val</i>	a value in Quotient<Ring, X>
------------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.18 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- `template<typename v >`  
using `type` = typename `FractionField< Ring >::template val< v, typename Ring::one >`  
*FractionField<Ring> representation of v.*

### 8.18.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

### Template Parameters

<i>Ring</i>	an integers ring, such as <a href="#">i32</a>
-------------	---

### 8.18.2 Member Typedef Documentation

#### 8.18.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

FractionField<Ring> representation of v.



## Template Parameters

<i>v</i>	a Ring value
----------	--------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

## Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the [i32](#) representation of val*

### 8.19.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into [i32](#)

## Template Parameters

<i>x</i>	an integer
----------	------------

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the [i32](#) representation of val

## Template Parameters

<i>val</i>	a value in zpz<x>
------------	-------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::libm::internal::exp2\_poly< T > Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::libm::internal::exp2\_poly< double > Struct Reference

```
#include <aerobus.h>
```

### Public Types

- using `type` = `aerobus::polynomial< aerobus::q64 >::template val< aerobus::make_q64_t< 388, 10641171255427 >, aerobus::make_q64_t< 2296, 5579977897299 >, aerobus::make_q64_t< 4963, 697799188641 >, aerobus::make_q64_t< 15551, 152884735543 >, aerobus::make_q64_t< 21273, 16096444733 >, aerobus::make_q64_t< 683500, 44811710339 >, aerobus::make_q64_t< 44397656049575, 288230376151711744 >, aerobus::make_q64_t< 192156823709857, 144115188075855872 >, aerobus::make_q64_t< 693059242663871, 72057594037927936 >, aerobus::make_q64_t< 1999746264802375, 36028797018963968 >, aerobus::make_q64_t< 4327536028902111, 18014398509481984 >, aerobus::make_q64_t< 6243314768165359, 9007199254740992 >, aerobus::q64::one >`

### 8.21.1 Member Typedef Documentation

#### 8.21.1.1 type

```
using aerobus::libm::internal::exp2_poly< double >::type = aerobus::polynomial<aerobus::q64>↵
::template val< aerobus::make_q64_t<388, 10641171255427>, aerobus::make_q64_t<2296, 5579977897299>,
aerobus::make_q64_t<4963, 697799188641>, aerobus::make_q64_t<15551, 152884735543>, aerobus::make_q64_t<21273
16096444733>, aerobus::make_q64_t<683500, 44811710339>, aerobus::make_q64_t<44397656049575,
288230376151711744>, aerobus::make_q64_t<192156823709857, 144115188075855872>, aerobus::make_q64_t<693059242
72057594037927936>, aerobus::make_q64_t<1999746264802375, 36028797018963968>, aerobus::make_q64_t<4327536028
18014398509481984>, aerobus::make_q64_t<6243314768165359, 9007199254740992>, aerobus::q64↵
::one>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::libm::internal::exp2\_poly< float > Struct Reference

```
#include <aerobus.h>
```

## Public Types

- using `type` = `aerobus::polynomial< aerobus::q32 >::template val< aerobus::make_q32_t< 8, 375115 >, aerobus::make_q32_t< 30, 208117 >, aerobus::make_q32_t< 109, 81261 >, aerobus::make_q32_t< 5161841, 536870912 >, aerobus::make_q32_t< 3724869, 67108864 >, aerobus::make_q32_t< 16121323, 67108864 >, aerobus::make_q32_t< 1453635, 2097152 >, aerobus::q32::one >`

## 8.22.1 Member Typedef Documentation

### 8.22.1.1 `type`

```
using aerobus::libm::internal::exp2_poly< float >::type = aerobus::polynomial<aerobus::q32>↵
::template val< aerobus::make_q32_t<8, 375115>, aerobus::make_q32_t<30, 208117>, aerobus::make_q32_t<109,↵
81261>, aerobus::make_q32_t<5161841, 536870912>, aerobus::make_q32_t<3724869, 67108864>,↵
aerobus::make_q32_t<16121323, 67108864>, aerobus::make_q32_t<1453635, 2097152>, aerobus↵
::q32::one>
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.23 aerobus::polynomial< Ring >::horner\_reduction\_t< P > Struct Template Reference

Used to evaluate polynomials over a value in Ring.

```
#include <aerobus.h>
```

## Classes

- struct `inner`
- struct `inner< stop, stop >`

### 8.23.1 Detailed Description

```
template<typename Ring>
template<typename P>
struct aerobus::polynomial< Ring >::horner_reduction_t< P >
```

Used to evaluate polynomials over a value in Ring.

#### Template Parameters

<i>P</i>	a value in polynomial<Ring>
----------	-----------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.24 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

### Public Types

- using [inner\\_type](#) = `int32_t`
- using [zero](#) = [val](#)< 0 >  
*constant zero*
- using [one](#) = [val](#)< 1 >  
*constant one*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*inject a native constant*
- template<typename v >  
using [inject\\_ring\\_t](#) = v
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator yields v1 + v2*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction operator yields v1 - v2*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = typename mul< v1, v2 >::type  
*multiplication operator yields v1 \* v2*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = typename div< v1, v2 >::type  
*division operator yields v1 / v2*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = typename remainder< v1, v2 >::type  
*modulus operator yields v1 % v2*
- template<typename v1 , typename v2 >  
using [gt\\_t](#) = typename gt< v1, v2 >::type  
*strictly greater operator (v1 > v2) yields v1 > v2*
- template<typename v1 , typename v2 >  
using [lt\\_t](#) = typename lt< v1, v2 >::type  
*strict less operator (v1 < v2) yields v1 < v2*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename eq< v1, v2 >::type  
*equality operator (type) yields v1 == v2 as std::integral\_constant<bool>*
- template<typename v1 , typename v2 >  
using [gcd\\_t](#) = [gcd\\_t](#)< [i32](#), v1, v2 >  
*greatest common divisor yields GCD(v1, v2)*
- template<typename v >  
using [pos\\_t](#) = typename pos< v >::type  
*positivity operator yields v > 0 as std::true\_type or std::false\_type*

## Static Public Attributes

- static constexpr bool [is\\_field](#) = false  
*integers are not a field*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*integers are an euclidean domain*
- template<typename v1 , typename v2 >  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator (boolean value)*
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity (boolean value) yields  $v > 0$  as boolean value*

### 8.24.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

#### Examples

[examples/compensated\\_horner.cpp](#).

### 8.24.2 Member Typedef Documentation

#### 8.24.2.1 [add\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::add\_t = typename add<v1, v2>::type
```

addition operator yields  $v1 + v2$

##### Template Parameters

<a href="#">v1</a>	a value in <a href="#">i32</a>
<a href="#">v2</a>	a value in <a href="#">i32</a>

#### 8.24.2.2 [div\\_t](#)

```
template<typename v1 , typename v2 >
using aerobus::i32::div\_t = typename div<v1, v2>::type
```

division operator yields  $v1 / v2$

##### Template Parameters

<a href="#">v1</a>	a value in <a href="#">i32</a>
<a href="#">v2</a>	a value in <a href="#">i32</a>

### 8.24.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>`

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor yields `GCD(v1, v2)`

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

inject a native constant

#### Template Parameters

<i>x</i>	
----------	--

### 8.24.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

### 8.24.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

### 8.24.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

strict less operator ( $v1 < v2$ ) yields  $v1 < v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

multiplication operator yields  $v1 * v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

### 8.24.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type`

#### Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

### 8.24.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

subtraction operator yields  $v1 - v2$

#### Template Parameters

<i>v1</i>	a value in <a href="#">i32</a>
<i>v2</i>	a value in <a href="#">i32</a>

### 8.24.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

## 8.24.3 Member Data Documentation

### 8.24.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (boolean value)

#### Template Parameters

<i>v1</i>	
<i>v2</i>	



### 8.24.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.24.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

### 8.24.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity (boolean value) yields  $v > 0$  as boolean value

#### Template Parameters

<i>v</i>	a value in <a href="#">i32</a>
----------	--------------------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.25 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in [i64](#)*

### Public Types

- using [inner\\_type](#) = int64\_t  
*type of represented values*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int64\_t >(x)>  
*injects constant as an [i64](#) value*

- `template<typename v >`  
`using inject_ring_t = v`  
*injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`*  
*-> `i64::val<1>`*
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator integer division*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator yields `v1 > v2` as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strict less operator yields `v1 < v2` as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator yields `v1 == v2` as `std::true_type` or `std::false_type`*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i64, v1, v2 >`  
*greatest common divisor yields `GCD(v1, v2)` as instantiation of `i64::val`*
- `template<typename v >`  
`using pos_t = typename pos< v >::type`  
*is v positive yields `v > 0` as `std::true_type` or `std::false_type`*

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields `v1 > v2` as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator yields `v1 < v2` as boolean value*

- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator yields v1 == v2 as boolean value*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity yields v > 0 as boolean value*

### 8.25.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.25.2 Member Typedef Documentation

#### 8.25.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

addition operator

Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
<code>v2</code>	: an element of <code>aerobus::i64::val</code>

#### 8.25.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

division operator integer division

Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
<code>v2</code>	: an element of <code>aerobus::i64::val</code>

#### 8.25.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

equality operator yields v1 == v2 as `std::true_type` or `std::false_type`

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.2.4 gcd\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

greatest common divisor yields GCD(v1, v2) as instantiation of [i64::val](#)

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.2.5 gt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

strictly greater operator yields `v1 > v2` as `std::true_type` or `std::false_type`

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.2.6 inject\_constant\_t**

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

injects constant as an [i64](#) value

## Template Parameters

<code>x</code>	
----------------	--

**8.25.2.7 inject\_ring\_t**

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)  
-> i64::val<1>

## Template Parameters

<code>v</code>	a value in <a href="#">i64</a>
----------------	--------------------------------

**8.25.2.8 inner\_type**

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

**8.25.2.9 lt\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

strict less operator yields `v1 < v2` as `std::true_type` or `std::false_type`

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.2.10 mod\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

modulus operator

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

multiplication operator

## Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

### 8.25.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

### 8.25.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

#### Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
-----------------	--

### 8.25.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

<code>v1</code>	: an element of <code>aerobus::i64::val</code>
<code>v2</code>	: an element of <code>aerobus::i64::val</code>

### 8.25.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.25.3 Member Data Documentation

### 8.25.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator yields  $v1 == v2$  as boolean value

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.3.2 gt\_v**

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt\_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.3.3 is\_euclidean\_domain**

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

**8.25.3.4 is\_field**

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

**8.25.3.5 lt\_v**

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt\_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator yields  $v1 < v2$  as boolean value

**Template Parameters**

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a>

**8.25.3.6 pos\_v**

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos\_t<v>::value [static], [constexpr]
```



positivity yields  $v > 0$  as boolean value

#### Template Parameters

<code>v</code>	: an element of <a href="#">aerobus::i64::val</a>
----------------	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< index, stop > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `template<typename accum , typename x >`  
using `type` = `typename horner_reduction_t< P >::template inner< index+1, stop > ::template type< typename Ring::template add_t< typename Ring::template mul_t< x, accum >, typename P::template coeff_↵`  
`at_t< P::degree - index > >, x >`

### 8.26.1 Member Typedef Documentation

#### 8.26.1.1 type

```
template<typename Ring >
template<typename P >
template<size_t index, size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< index, stop >::type =
typename horner_reduction_t<P>::template inner<index + 1, stop> ::template type< typename
Ring::template add_t< typename Ring::template mul_t<x, accum>, typename P::template coeff_↵
at_t<P::degree - index> >, x>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.27 aerobus::polynomial< Ring >::horner\_reduction\_t< P >::inner< stop, stop > Struct Template Reference

```
#include <aerobus.h>
```

## Public Types

- `template<typename accum , typename x >`  
`using type = accum`

## 8.27.1 Member Typedef Documentation

### 8.27.1.1 `type`

```
template<typename Ring >
template<typename P >
template<size_t stop>
template<typename accum , typename x >
using aerobus::polynomial< Ring >::horner_reduction_t< P >::inner< stop, stop >::type =
accum
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.28 `aerobus::is_prime< n >` Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool [value](#) = `internal::_is_prime<n, 5>::value`  
*true iff n is prime*

### 8.28.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

#### Template Parameters

<i>n</i>	
----------	--

## 8.28.2 Member Data Documentation

### 8.28.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.29 aerobus::meta\_libm< T > Struct Template Reference

```
#include <aerobus.h>
```

### Static Public Member Functions

- static `INLINED DEVICE` T floor (const T &f)
- static `INLINED DEVICE` T fmod (const T &x, const T &d)

## 8.29.1 Member Function Documentation

### 8.29.1.1 floor()

```
template<typename T >
static INLINED DEVICE T aerobus::meta_libm< T >::floor (
    const T & f ) [inline], [static]
```

### 8.29.1.2 fmod()

```
template<typename T >
static INLINED DEVICE T aerobus::meta_libm< T >::fmod (
    const T & x,
    const T & d ) [inline], [static]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.30 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

## Classes

- struct [horner\\_reduction\\_t](#)  
*Used to evaluate polynomials over a value in Ring.*
- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val<coeffN >](#)  
*specialization for constants*

## Public Types

- using [zero](#) = [val](#)< typename Ring::zero >  
*constant zero*
- using [one](#) = [val](#)< typename Ring::one >  
*constant one*
- using [X](#) = [val](#)< typename Ring::one, typename Ring::zero >  
*generator*
- template<typename P >  
using [simplify\\_t](#) = typename simplify< P >::type  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*adds two polynomials*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type  
*subtraction of two polynomials*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = typename mul< v1, v2 >::type  
*multiplication of two polynomials*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename eq\_helper< v1, v2 >::type  
*equality operator*
- template<typename v1 , typename v2 >  
using [lt\\_t](#) = typename lt\_helper< v1, v2 >::type  
*strict less operator*
- template<typename v1 , typename v2 >  
using [gt\\_t](#) = typename gt\_helper< v1, v2 >::type  
*strict greater operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = typename div< v1, v2 >::q\_type  
*division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = typename div\_helper< v1, v2, [zero](#), v1 >::mod\_type  
*modulo operator*
- template<typename coeff , size\_t deg>  
using [monomial\\_t](#) = typename monomial< coeff, deg >::type  
*monomial : coeff X<sup>deg</sup>*
- template<typename v >  
using [derive\\_t](#) = typename derive\_helper< v >::type  
*derivation operator*
- template<typename v >  
using [pos\\_t](#) = typename Ring::template [pos\\_t](#)< typename v::aN >

- checks for positivity ( $an > 0$ )*
  - template<typename v1 , typename v2 >  
 using gcd\_t = std::conditional\_t< Ring::is\_euclidean\_domain, typename make\_unit< gcd\_t< polynomial< Ring >, v1, v2 > >::type, void >
  - greatest common divisor of two polynomials*
- template<auto x>  
 using inject\_constant\_t = val< typename Ring::template inject\_constant\_t< x > >
  - makes the constant (native type) polynomial a\_0*
- template<typename v >  
 using inject\_ring\_t = val< v >
  - makes the constant (ring type) polynomial a\_0*

### Static Public Attributes

- static constexpr bool is\_field = false
- static constexpr bool is\_euclidean\_domain = Ring::is\_euclidean\_domain
- template<typename v >  
 static constexpr bool pos\_v = pos\_t<v>::value  
*positivity operator*

## 8.30.1 Detailed Description

template<typename Ring>  
 requires IsEuclideanDomain<Ring>  
 struct aerobus::polynomial< Ring >

polynomial with coefficients in Ring Ring must be an integral domain

### Examples

[examples/compensated\\_horner.cpp](#), [examples/make\\_polynomial.cpp](#), and [examples/modular\\_arithmetic.cpp](#).

## 8.30.2 Member Typedef Documentation

### 8.30.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

### Template Parameters

v1	
v2	

### 8.30.2.2 `derive_t`

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 8.30.2.3 `div_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.30.2.4 `eq_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.30.2.5 `gcd_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.30.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.30.2.7 inject\_constant\_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x> >
```

makes the constant (native type) polynomial a\_0

## Template Parameters

<i>x</i>	
----------	--

## 8.30.2.8 inject\_ring\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

makes the constant (ring type) polynomial a\_0

## Template Parameters

<i>v</i>	
----------	--

## 8.30.2.9 lt\_t

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.30.2.10 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

#### 8.30.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

#### 8.30.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	



**8.30.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

**8.30.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

<i>v</i>	
----------	--

**8.30.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

<i>P</i>	
----------	--

**8.30.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.30.2.17 X**

```
template<typename Ring >
```

```
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

### 8.30.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

## 8.30.3 Member Data Documentation

### 8.30.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

### 8.30.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

### 8.30.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

#### Template Parameters

<i>v</i>	a value in <a href="#">polynomial::val</a>
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.31 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

## Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

### 8.31.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

### 8.31.2 Member Typedef Documentation

#### 8.31.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

#### 8.31.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.32 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*projection values in the quotient ring*

## Public Types

- using `zero` = `val`< typename Ring::zero >  
*zero value*
- using `one` = `val`< typename Ring::one >  
*one*
- template<typename v1 , typename v2 >  
using `add_t` = `val`< typename Ring::template `add_t`< typename v1::type, typename v2::type > >  
*addition operator*
- template<typename v1 , typename v2 >  
using `mul_t` = `val`< typename Ring::template `mul_t`< typename v1::type, typename v2::type > >  
*subtraction operator*
- template<typename v1 , typename v2 >  
using `div_t` = `val`< typename Ring::template `div_t`< typename v1::type, typename v2::type > >  
*division operator*
- template<typename v1 , typename v2 >  
using `mod_t` = `val`< typename Ring::template `mod_t`< typename v1::type, typename v2::type > >  
*modulus operator*
- template<typename v1 , typename v2 >  
using `eq_t` = typename Ring::template `eq_t`< typename v1::type, typename v2::type >  
*equality operator (as type)*
- template<typename v1 >  
using `pos_t` = std::true\_type  
*positivity operator always true*
- template<auto x>  
using `inject_constant_t` = `val`< typename Ring::template `inject_constant_t`< x > >  
*inject a 'constant' in quotient ring\**
- template<typename v >  
using `inject_ring_t` = `val`< v >  
*projects a value of Ring onto the quotient*

## Static Public Attributes

- template<typename v1 , typename v2 >  
static constexpr bool `eq_v` = Ring::template `eq_t`<typename v1::type, typename v2::type>::value  
*addition operator (as boolean value)*
- template<typename v >  
static constexpr bool `pos_v` = `pos_t`<v>::value  
*positivity operator always true*
- static constexpr bool `is_euclidean_domain` = true  
*quotien rings are euclidean domain*

### 8.32.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

## Template Parameters

<i>Ring</i>	A ring type, such as 'i32', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as i32::val<2>

## 8.32.2 Member Typedef Documentation

## 8.32.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1↔
::type, typename v2::type> >
```

addition operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.32.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1↔
::type, typename v2::type> >
```

division operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

## 8.32.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 8.32.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

inject a 'constant' in quotient ring\*

##### Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

#### 8.32.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

projects a value of Ring onto the quotient

##### Template Parameters

v	a value in Ring
---	-----------------

#### 8.32.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1↔
::type, typename v2::type> >
```

modulus operator

##### Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

#### 8.32.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1↔
::type, typename v2::type> >
```

substraction operator

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

**8.32.2.8 one**

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

**8.32.2.9 pos\_t**

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

**8.32.2.10 zero**

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

**8.32.3 Member Data Documentation****8.32.3.1 eq\_v**

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

## Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.32.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.32.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.33 aerobus::libm::internal::sin\_poly< P > Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.34 aerobus::libm::internal::sin\_poly< double > Struct Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = typename `aerobus::polynomial< aerobus::q64 >::simplify_t` < typename `aerobus::polynomial< aerobus::q64 >::template val` < `aerobus::make_q64_t` < -43, 1042171195712159 >, `aerobus::make_q64_t` < -89, 136637767615782 >, `aerobus::make_q64_t` < 123, 766493207966 >, `aerobus::make_q64_t` < -18133, 723813242548 >, `aerobus::make_q64_t` < 122341, 44395102410 >, `aerobus::make_q64_t` < -11252871, 56714469841 >, `aerobus::make_q64_t` < 2401919801264179, 288230376151711744 >, `aerobus::make_q64_t` < -6004799503160661, 36028797018963968 >, `aerobus::q64::one` > >



### 8.34.1 Member Typedef Documentation

#### 8.34.1.1 type

```
using aerobus::libm::internal::sin_poly< double >::type = typename aerobus::polynomial<aerobus::q64>↵
::simplify_t< typename aerobus::polynomial<aerobus::q64>:: template val< aerobus::make_q64_t<-43,
1042171195712159>, aerobus::make_q64_t<-89, 136637767615782>, aerobus::make_q64_t<123, 766493207966>,
aerobus::make_q64_t<-18133, 723813242548>, aerobus::make_q64_t<122341, 44395102410>, aerobus::make_q64_t<-11
56714469841>, aerobus::make_q64_t<2401919801264179, 288230376151711744>, aerobus::make_q64_t<-60047995031606
36028797018963968>, aerobus::q64::one> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.35 aerobus::libm::internal::sin\_poly< float > Struct Reference

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = typename [aerobus::polynomial< aerobus::q32 >::simplify\\_t< typename aerobus::polynomial< aerobus::q32 >::template val< aerobus::make\\_q32\\_t< 1, 357073 >, aerobus::make\\_q32\\_t<-67, 337533 >, aerobus::make\\_q32\\_t< 4473945, 536870912 >, aerobus::make\\_q32\\_t<-11184811, 67108864 >, aerobus::q32::one > >](#)

### 8.35.1 Member Typedef Documentation

#### 8.35.1.1 type

```
using aerobus::libm::internal::sin_poly< float >::type = typename aerobus::polynomial<aerobus::q32>↵
::simplify_t< typename aerobus::polynomial<aerobus::q32>:: template val< aerobus::make_q32_t<1,
357073>, aerobus::make_q32_t<-67, 337533>, aerobus::make_q32_t<4473945, 536870912>, aerobus::make_q32_t<-11
67108864>, aerobus::q32::one> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.36 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.36.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

### 8.36.2 Member Typedef Documentation

#### 8.36.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.36.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.37 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

## Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

## Public Types

- template<typename T >  
using `push_front` = `type_list`< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using `at` = `internal::type_at_t`< index, Ts... >  
*returns type at index*
- template<typename T >  
using `push_back` = `type_list`< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using `concat` = typename `concat_h`< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using `insert` = typename `internal::insert_h`< index, `type_list`< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using `remove` = typename `internal::remove_h`< index, `type_list`< Ts... > >::type  
*removes type at index*

## Static Public Attributes

- static constexpr size\_t `length` = sizeof...(Ts)  
*length of list*

### 8.37.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

A list of types.

#### Template Parameters

<code>...Ts</code>	types to store and manipulate at compile time
--------------------	---

### 8.37.2 Member Typedef Documentation

#### 8.37.2.1 `at`

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

**Template Parameters**

<i>index</i>	
--------------	--

**8.37.2.2 concat**

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

**Template Parameters**

<i>U</i>	
----------	--

**8.37.2.3 insert**

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

<i>index</i>	
<i>T</i>	

**8.37.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

<i>T</i>	
----------	--

**8.37.2.5 push\_front**

```
template<typename... Ts>
```

```
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

<i>T</i>	
----------	--

### 8.37.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

<i>index</i>	
--------------	--

## 8.37.3 Member Data Documentation

### 8.37.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.38 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

#### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

## Static Public Attributes

- static constexpr size\_t `length` = 0

### 8.38.1 Detailed Description

specialization for empty type list

### 8.38.2 Member Typedef Documentation

#### 8.38.2.1 `concat`

```
template<typename U >
using aerobus::type_list<>::concat = U
```

#### 8.38.2.2 `insert`

```
template<typename T , size_t index>
using aerobus::type_list<>::insert = type_list<T>
```

#### 8.38.2.3 `push_back`

```
template<typename T >
using aerobus::type_list<>::push_back = type_list<T>
```

#### 8.38.2.4 `push_front`

```
template<typename T >
using aerobus::type_list<>::push_front = type_list<T>
```

### 8.38.3 Member Data Documentation

#### 8.38.3.1 `length`

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.39 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

## Public Types

- using `enclosing_type` = `i32`  
*Enclosing ring type.*
- using `is_zero_t` = `std::bool_constant< x==0 >`  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr DEVICE valueType get ()`  
*cast x into valueType*
- `static std::string to_string ()`  
*string representation of value*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*

### 8.39.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

#### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

### 8.39.2 Member Typedef Documentation

#### 8.39.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

#### 8.39.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.39.3 Member Function Documentation

#### 8.39.3.1 get()

```
template<int32_t x>
template<typename valueType >
static constexpr DEVICE valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

##### Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

#### 8.39.3.2 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

### 8.39.4 Member Data Documentation

#### 8.39.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.40 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using [inner\\_type](#) = int32\_t  
*type of represented values*
- using [enclosing\\_type](#) = [i64](#)  
*enclosing ring type*
- using [is\\_zero\\_t](#) = std::bool\_constant< x==0 >  
*is value zero*



### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr INLINED_DEVICE valueType get ()`  
*cast value in valueType*
- `static std::string to_string ()`  
*string representation*

### Static Public Attributes

- `static constexpr int64_t v = x`  
*actual value*

## 8.40.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

#### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

#### Examples

[examples/compensated\\_horner.cpp](#).

## 8.40.2 Member Typedef Documentation

### 8.40.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.40.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.40.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.40.3 Member Function Documentation

### 8.40.3.1 get()

```
template<int64_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::i64::val< x >::get ( ) [inline], [static],
[constexpr]
```

cast value in valueType

Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.40.3.2 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

## 8.40.4 Member Data Documentation

### 8.40.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.41 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- using `ring_type` = Ring  
*ring coefficients live in*
- using `enclosing_type` = polynomial< Ring >  
*enclosing ring type*
- using `aN` = coeffN  
*heavy weight coefficient (non zero)*
- using `strip` = val< coeffs... >  
*remove largest coefficient*
- using `is_zero_t` = std::bool\_constant<(degree==0) &&(aN::is\_zero\_t::value)>  
*true\_type if polynomial is constant zero*
- template<size\_t index>  
using `coeff_at_t` = typename coeff\_at< index >::type  
*type of coefficient at index*
- template<typename x >  
using `value_at_t` = horner\_reduction\_t< val > ::template inner< 0, degree+1 > ::template type< typename Ring::zero, x >

## Static Public Member Functions

- static std::string `to_string` ()  
*get a string representation of polynomial*
- template<typename arithmeticType >  
static constexpr `DEVICE INLINED` arithmeticType `eval` (const arithmeticType &x)  
*evaluates polynomial seen as a function operating on arithmeticType*
- template<typename arithmeticType >  
static `DEVICE INLINED` arithmeticType `compensated_eval` (const arithmeticType &x)  
*Evaluate polynomial on x using compensated horner scheme.*

## Static Public Attributes

- static constexpr size\_t `degree` = sizeof...(coeffs)  
*degree of the polynomial*
- static constexpr bool `is_zero_v` = is\_zero\_t::value  
*true if polynomial is constant zero*

### 8.41.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

## Examples

[examples/compensated\\_horner.cpp](#).

## 8.41.2 Member Typedef Documentation

### 8.41.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

### 8.41.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.41.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.41.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.41.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

### 8.41.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

### 8.41.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::value_at_t = horner_reduction_t<val>
::template inner<0, degree + 1> ::template type<typename Ring::zero, x>
```

## 8.41.3 Member Function Documentation

### 8.41.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename arithmeticType >
static DEVICE INLINE arithmeticType aerobus::polynomial< Ring >::val< coeffN, coeffs >↔
::compensated_eval (
    const arithmeticType & x ) [inline], [static]
```

Evaluate polynomial on x using compensated horner scheme.

This is twice as accurate as simple eval (horner) but cannot be constexpr

Please note this makes no sense on integer types as arithmetic on integers is exact in IEEE

WARNING : this does not work with gcc with -O3 optimization level because gcc does illegal stuff with floating point arithmetic

#### Template Parameters

<i>arithmeticType</i>	float for example
-----------------------	-------------------

#### Parameters

x	
---	--

### 8.41.3.2 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN,
coeffs >::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on arithmeticType

#### Template Parameters

<i>arithmeticType</i>	usually float or double
-----------------------	-------------------------

#### Parameters

<i>x</i>	value
----------	-------

#### Returns

P(x)

### 8.41.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.41.4 Member Data Documentation

### 8.41.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

### 8.41.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t←
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.42 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- using [raw\\_t](#) = V
- using [type](#) = [abs\\_t](#)< typename Ring::template [mod\\_t](#)< V, X > >

### 8.42.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

V	a value from 'Ring'
---	---------------------

### 8.42.2 Member Typedef Documentation

#### 8.42.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

#### 8.42.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs\_t<typename Ring::template mod\_t<V, X> >
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.43 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

## Public Types

- using `enclosing_type` = `zpz< p >`  
*enclosing ring type*
- using `is_zero_t` = `std::bool_constant< v==0 >`  
*true\_type if zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr INLINED_DEVICE valueType get ()`  
*get value as valueType*
- `static std::string to_string ()`  
*string representation*

## Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*
- `static constexpr bool is_zero_v = v == 0`  
*true if zero*

### 8.43.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

Template Parameters

<code>x</code>	an integer
----------------	------------

### 8.43.2 Member Typedef Documentation

#### 8.43.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type



### 8.43.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.43.3 Member Function Documentation

### 8.43.3.1 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr INLINED_DEVICE valueType aerobus::zpz< p >::val< x >::get ( ) [inline],
[static], [constexpr]
```

get value as valueType

#### Template Parameters

<i>valueType</i>	an arithmetic type, such as float
------------------	-----------------------------------

### 8.43.3.2 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

string representation

#### Returns

a string representation

## 8.43.4 Member Data Documentation

### 8.43.4.1 is\_zero\_v

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

#### 8.43.4.2 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.44 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

### Public Types

- using [ring\\_type](#) = Ring  
*ring coefficients live in*
- using [enclosing\\_type](#) = [polynomial< Ring >](#)  
*enclosing ring type*
- using [aN](#) = [coeffN](#)
- using [strip](#) = [val< coeffN >](#)
- using [is\\_zero\\_t](#) = std::bool\_constant< [aN::is\\_zero\\_t::value](#) >
- template<size\_t index>  
using [coeff\\_at\\_t](#) = typename [coeff\\_at< index >::type](#)
- template<typename x >  
using [value\\_at\\_t](#) = [coeffN](#)

### Static Public Member Functions

- static std::string [to\\_string](#) ()
- template<typename arithmeticType >  
static constexpr [DEVICE INLINED](#) arithmeticType [eval](#) (const arithmeticType &x)
- template<typename arithmeticType >  
static [DEVICE INLINED](#) arithmeticType [compensated\\_eval](#) (const arithmeticType &x)

### Static Public Attributes

- static constexpr size\_t `degree` = 0
- static constexpr bool `is_zero_v` = `is_zero_t::value`

## 8.44.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

### Template Parameters

<code>coeffN</code>	
---------------------	--

## 8.44.2 Member Typedef Documentation

### 8.44.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.44.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.44.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.44.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

#### 8.44.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

#### 8.44.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

#### 8.44.2.7 value\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<typename x >
using aerobus::polynomial< Ring >::val< coeffN >::value_at_t = coeffN
```

### 8.44.3 Member Function Documentation

#### 8.44.3.1 compensated\_eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >::compensated←
_eval (
    const arithmeticType & x ) [inline], [static]
```

#### 8.44.3.2 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename arithmeticType >
static constexpr DEVICE INLINED arithmeticType aerobus::polynomial< Ring >::val< coeffN >←
::eval (
    const arithmeticType & x ) [inline], [static], [constexpr]
```

#### 8.44.3.3 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

### 8.44.4 Member Data Documentation

#### 8.44.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

#### 8.44.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.45 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in zpz*

### Public Types

- using [inner\\_type](#) = int32\_t  
*underlying type for values*
- template<auto x>  
using [inject\\_constant\\_t](#) = [val](#)< static\_cast< int32\_t >(x)>  
*injects a constant integer into zpz*
- using [zero](#) = [val](#)< 0 >  
*zero value*
- using [one](#) = [val](#)< 1 >  
*one value*
- template<typename v1 , typename v2 >  
using [add\\_t](#) = typename add< v1, v2 >::type  
*addition operator*
- template<typename v1 , typename v2 >  
using [sub\\_t](#) = typename sub< v1, v2 >::type

- subtraction operator*
- template<typename v1 , typename v2 >  
using [mul\\_t](#) = typename mul< v1, v2 >::type
- multiplication operator*
- template<typename v1 , typename v2 >  
using [div\\_t](#) = typename div< v1, v2 >::type
- division operator*
- template<typename v1 , typename v2 >  
using [mod\\_t](#) = typename remainder< v1, v2 >::type
- modulo operator*
- template<typename v1 , typename v2 >  
using [gt\\_t](#) = typename gt< v1, v2 >::type
- strictly greater operator (type)*
- template<typename v1 , typename v2 >  
using [lt\\_t](#) = typename lt< v1, v2 >::type
- strictly smaller operator (type)*
- template<typename v1 , typename v2 >  
using [eq\\_t](#) = typename eq< v1, v2 >::type
- equality operator (type)*
- template<typename v1 , typename v2 >  
using [gcd\\_t](#) = [gcd\\_t](#)< [i32](#), v1, v2 >
- greatest common divisor*
- template<typename v1 >  
using [pos\\_t](#) = typename pos< v1 >::type
- positivity operator (type)*

### Static Public Attributes

- static constexpr bool [is\\_field](#) = [is\\_prime](#)<p>::value  
*true iff p is prime*
- static constexpr bool [is\\_euclidean\\_domain](#) = true  
*always true*
- template<typename v1 , typename v2 >  
static constexpr bool [gt\\_v](#) = [gt\\_t](#)<v1, v2>::value  
*strictly greater operator (booleanvalue)*
- template<typename v1 , typename v2 >  
static constexpr bool [lt\\_v](#) = [lt\\_t](#)<v1, v2>::value  
*strictly smaller operator (booleanvalue)*
- template<typename v1 , typename v2 >  
static constexpr bool [eq\\_v](#) = [eq\\_t](#)<v1, v2>::value  
*equality operator (booleanvalue)*
- template<typename v >  
static constexpr bool [pos\\_v](#) = [pos\\_t](#)<v>::value  
*positivity operator (boolean value)*

### 8.45.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

## Template Parameters

<i>p</i>	a integer
----------	-----------

## Examples

[examples/modular\\_arithmetic.cpp](#), and [examples/polynomials\\_over\\_finite\\_field.cpp](#).

## 8.45.2 Member Typedef Documentation

## 8.45.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.45.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 8.45.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

#### 8.45.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.45.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

##### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

#### 8.45.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

##### Template Parameters

x	an integer
---	------------

#### 8.45.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values



### 8.45.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.45.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.45.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.45.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

### 8.45.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

### 8.45.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.45.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.45.3 Member Data Documentation

### 8.45.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq\_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.45.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.45.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.45.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.45.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.45.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
#include <cmath>
#include <limits>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #include <cmath>
00016 #include <limits>
00017 #ifdef WITH_CUDA_FP16
00018 #include <bit>
00019 #include <cuda_fp16.h>
00020 #endif
00021
00025 #ifdef _MSC_VER
```

```

00026 #define ALIGNED(x) __declspec(align(x))
00027 #define INLINED __forceinline
00028 #else
00029 #define ALIGNED(x) __attribute__((aligned(x)))
00030 #define INLINED __attribute__((always_inline)) inline
00031 #endif
00032
00033 #ifdef __CUDACC__
00034 #define DEVICE __host__ __device__
00035 #else
00036 #define DEVICE
00037 #endif
00038
00040
00042
00044
00045 // aligned allocation
00046 namespace aerobus {
00053     template<typename T>
00054     T* aligned_malloc(size_t count, size_t alignment) {
00055         #ifdef _MSC_VER
00056             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00057         #else
00058             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00059         #endif
00060     }
00061 } // namespace aerobus
00062
00063 // concepts
00064 namespace aerobus {
00066     template <typename R>
00067     concept IsRing = requires {
00068         typename R::one;
00069         typename R::zero;
00070         typename R::template add_t<typename R::one, typename R::one>;
00071         typename R::template sub_t<typename R::one, typename R::one>;
00072         typename R::template mul_t<typename R::one, typename R::one>;
00073     };
00074
00076     template <typename R>
00077     concept IsEuclideanDomain = IsRing<R> && requires {
00078         typename R::template div_t<typename R::one, typename R::one>;
00079         typename R::template mod_t<typename R::one, typename R::one>;
00080         typename R::template gcd_t<typename R::one, typename R::one>;
00081         typename R::template eq_t<typename R::one, typename R::one>;
00082         typename R::template pos_t<typename R::one>;
00083
00084         R::template pos_v<typename R::one> == true;
00085         // typename R::template gt_t<typename R::one, typename R::zero>;
00086         R::is_euclidean_domain == true;
00087     };
00088
00090     template<typename R>
00091     concept IsField = IsEuclideanDomain<R> && requires {
00092         R::is_field == true;
00093     };
00094 } // namespace aerobus
00095
00096 #ifdef WITH_CUDA_FP16
00097 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00098 namespace aerobus {
00099     namespace internal {
00100         static constexpr DEVICE uint16_t my_internal_float2half(
00101             const float f, uint32_t &sign, uint32_t &remainder) {
00102             uint32_t x;
00103             uint32_t u;
00104             uint32_t result;
00105             x = std::bit_cast<int32_t>(f);
00106             u = (x & 0x7fffffffU);
00107             sign = ((x > 16U) & 0x8000U);
00108             // NaN/+Inf/-Inf
00109             if (u >= 0x7f800000U) {
00110                 remainder = 0U;
00111                 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00112             } else if (u > 0x477ffffU) { // Overflows
00113                 remainder = 0x80000000U;
00114                 result = (sign | 0x7bffU);
00115             } else if (u >= 0x38800000U) { // Normal numbers
00116                 remainder = u << 19U;
00117                 u -= 0x38000000U;
00118                 result = (sign | (u >> 13U));
00119             } else if (u < 0x33000001U) { // +0/-0
00120                 remainder = u;
00121                 result = sign;
00122             } else { // Denormal numbers
00123                 const uint32_t exponent = u >> 23U;
00124                 const uint32_t shift = 0x7eU - exponent;

```

```

00125         uint32_t mantissa = (u & 0x7ffffU);
00126         mantissa |= 0x800000U;
00127         remainder = mantissa « (32U - shift);
00128         result = (sign | (mantissa » shift));
00129         result &= 0x0000FFFFU;
00130     }
00131     return static_cast<uint16_t>(result);
00132 }
00133
00134 static constexpr DEVICE __half my_float2half_rn(const float a) {
00135     __half val;
00136     __half_raw r;
00137     uint32_t sign = 0U;
00138     uint32_t remainder = 0U;
00139     r.x = my_internal_float2half(a, sign, remainder);
00140     if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00141         r.x++;
00142     }
00143
00144     val = std::bit_cast<__half>(r);
00145     return val;
00146 }
00147
00148 template <int16_t i>
00149 static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00150
00151
00152 template <typename Out, int16_t x, typename E = void>
00153 struct int16_convert_helper;
00154
00155 template <typename Out, int16_t x>
00156 struct int16_convert_helper<Out, x,
00157     std::enable_if_t<!std::is_same_v<Out, __half> && !std::is_same_v<Out, __half2>> {
00158     static constexpr Out value() {
00159         return static_cast<Out>(x);
00160     }
00161 };
00162
00163 template <int16_t x>
00164 struct int16_convert_helper<__half, x> {
00165     static constexpr __half value() {
00166         return convert_int16_to_half<x>;
00167     }
00168 };
00169
00170 template <int16_t x>
00171 struct int16_convert_helper<__half2, x> {
00172     static constexpr __half2 value() {
00173         return __half2(convert_int16_to_half<x>, convert_int16_to_half<x>);
00174     }
00175 };
00176
00177 } // namespace internal
00178 } // namespace aerobus
00179 #endif
00180
00181 // cast
00182 namespace aerobus {
00183     namespace internal {
00184         template<typename Out, typename In>
00185         struct staticcast {
00186             template<auto x>
00187             static constexpr INLINED_DEVICE Out func() {
00188                 return static_cast<Out>(x);
00189             }
00190         };
00191
00192         #ifdef WITH_CUDA_FP16
00193         template<>
00194         struct staticcast<__half, int16_t> {
00195             template<int16_t x>
00196             static constexpr INLINED_DEVICE __half func() {
00197                 return int16_convert_helper<__half, x>::value();
00198             }
00199         };
00200
00201         template<>
00202         struct staticcast<__half2, int16_t> {
00203             template<int16_t x>
00204             static constexpr INLINED_DEVICE __half2 func() {
00205                 return int16_convert_helper<__half2, x>::value();
00206             }
00207         };
00208         #endif
00209     } // namespace internal
00210 } // namespace aerobus
00211

```

```

00212 // fma_helper, required because nvidia fails to reconstruct fma for fp16 types
00213 namespace aerobus {
00214     namespace internal {
00215         template<typename T>
00216         struct fma_helper;
00217
00218         template<>
00219         struct fma_helper<double> {
00220             static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
z) {
00221                 return x * y + z;
00222             }
00223         };
00224
00225         template<>
00226         struct fma_helper<long double> {
00227             static constexpr INLINED_DEVICE long double eval(
00228                 const long double x, const long double y, const long double z) {
00229                 return x * y + z;
00230             }
00231         };
00232
00233         template<>
00234         struct fma_helper<float> {
00235             static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00236                 return x * y + z;
00237             }
00238         };
00239
00240         template<>
00241         struct fma_helper<int32_t> {
00242             static constexpr INLINED_DEVICE int16_t eval(const int16_t x, const int16_t y, const
int16_t z) {
00243                 return x * y + z;
00244             }
00245         };
00246
00247         template<>
00248         struct fma_helper<int16_t> {
00249             static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
int32_t z) {
00250                 return x * y + z;
00251             }
00252         };
00253
00254         template<>
00255         struct fma_helper<int64_t> {
00256             static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
int64_t z) {
00257                 return x * y + z;
00258             }
00259         };
00260
00261         #ifdef WITH_CUDA_FP16
00262         template<>
00263         struct fma_helper<__half> {
00264             static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
z) {
00265                 #ifdef __CUDA_ARCH__
00266                 return __hfma(x, y, z);
00267                 #else
00268                 return x * y + z;
00269                 #endif
00270             }
00271         };
00272         template<>
00273         struct fma_helper<__half2> {
00274             static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
__half2 z) {
00275                 #ifdef __CUDA_ARCH__
00276                 return __hfma2(x, y, z);
00277                 #else
00278                 return x * y + z;
00279                 #endif
00280             }
00281         };
00282         #endif
00283     } // namespace internal
00284 } // namespace aerobus
00285
00286 namespace aerobus {
00287     namespace internal {
00288         struct double_double {};
00289
00290         template<typename T>
00291         struct arithmetic_helpers;
00292

```



```

00293     template<>
00294     struct arithmetic_helpers<double> {
00295         using integers = int64_t;
00296         using upper_type = double_double;
00297         static constexpr double one = 1.0;
00298         static constexpr double zero = 0.0;
00299         static constexpr double pi = 0x1.921fb54442d18p1;
00300         static constexpr double pi_2 = 0x1.921fb54442d18p0;
00301         static constexpr double pi_4 = 0x1.921fb54442d18p-1;
00302         static constexpr double two_pi = 0x1.921fb54442d18p2;
00303         static constexpr double inv_two_pi = 0x1.45f306dc9c883p-3;
00304         static constexpr double half = 0x1p-1;
00305     };
00306
00307     template<>
00308     struct arithmetic_helpers<float> {
00309         using integers = int32_t;
00310         using upper_type = double;
00311         static constexpr double one = 1.0F;
00312         static constexpr double zero = 0.0F;
00313         static constexpr float pi = 0x1.921fb6p1f;
00314         static constexpr double pi_2 = 0x1.921fb6p0f;
00315         static constexpr double pi_4 = 0x1.921fb6p-1f;
00316         static constexpr double two_pi = 0x1.921fb6p2f;
00317         static constexpr double inv_two_pi = 0x1.45f306p-3f;
00318         static constexpr double half = 0x1p-1f;
00319     };
00320
00321     #ifdef WITH_CUDA_FP16
00322     template<>
00323     struct arithmetic_helpers<__half> {
00324         using integers = int16_t;
00325         using upper_type = float;
00326         const __half one = CUDART_ONE_FP16;
00327         const __half zero = CUDART_ZERO_FP16;
00328         const __half pi = __half_raw(0x4248);
00329         const __half pi_2 = __half_raw(0b011111001001000);
00330         const __half pi_4 = __half_raw(0b011101001001000);
00331         const __half two_pi = __half_raw(0b100011001001000);
00332         const __half inv_two_pi = __half_raw(0b011000100011000);
00333         const __half half = __half_raw(0b011100000000000);
00334     };
00335
00336     template<>
00337     struct arithmetic_helpers<__half2> {
00338         using integers = int16_t;
00339         using upper_type = float; // TODO(JeWaVe) : check for float2
00340         const __half2 one = __half2(CUDART_ONE_FP16, CUDART_ONE_FP16);
00341         const __half2 zero = __half2(CUDART_ZERO_FP16, CUDART_ZERO_FP16);
00342         const __half2 pi = __half2(__half_raw(0x4248), __half_raw(0x4248));
00343         const __half2 pi_2 = __half2(__half_raw(0b011111001001000),
00344             __half_raw(0b011111001001000));
00345         const __half2 pi_4 = __half2(__half_raw(0b011101001001000),
00346             __half_raw(0b011101001001000));
00347         const __half2 two_pi = __half2(__half_raw(0b100011001001000),
00348             __half_raw(0b100011001001000));
00349         const __half2 half = __half2(__half_raw(0b011100000000000),
00350             __half_raw(0b011100000000000));
00351     };
00352     #endif
00353     } // namespace internal
00354
00355     template<typename T>
00356     struct meta_libm {
00357         static INLINED_DEVICE T floor(const T& f) {
00358             return std::floor(f);
00359         }
00360         static INLINED_DEVICE T fmod(const T& x, const T& d) {
00361             return std::fmod(x, d);
00362         }
00363     };
00364
00365     // TODO(JeWaVe) : investigate as hfloor is pure device -- should be replaced by something
00366     different and constexpr
00367     #ifdef WITH_CUDA_FP16
00368     template<>
00369     struct meta_libm<__half> {
00370         static INLINED __device__ __half floor(const __half& f) {
00371             return hfloor(f);
00372         }
00373
00374         static INLINED __device__ __half fmod(const __half& x, const __half& d) {
00375             __half i = meta_libm<__half>::floor(x / d);
00376             return x - d * i;
00377         }
00378     };
00379     #endif
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01873     };
01874
01875     };
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01879     };
01880
01881     };
01882
01883     };
01884
01885     };
01886
01887     };
```

```

00375     template<>
00376     struct meta_libm<__half2> {
00377         static INLINED __device__ __half2 floor(const __half2& f) {
00378             return h2floor(f);
00379         }
00380
00381         static INLINED __device__ __half2 fmod(const __half2& x, const __half2& d) {
00382             __half2 i = meta_libm<__half2>::floor(x / d);
00383             return x - d * i;
00384         }
00385     };
00386 #endif
00387 } // namespace aerobus
00388
00389 // compensated horner utilities
00390 namespace aerobus {
00391     namespace internal {
00392         template <typename T>
00393         struct FloatLayout;
00394
00395         #ifdef _MSC_VER
00396         template <>
00397         struct FloatLayout<long double> {
00398             static constexpr uint8_t exponent = 11;
00399             static constexpr uint8_t mantissa = 52;
00400             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00401             static constexpr long double shift = (1LL « r) + 1;
00402         };
00403         #else
00404         template <>
00405         struct FloatLayout<long double> {
00406             static constexpr uint8_t exponent = 15;
00407             static constexpr uint8_t mantissa = 64;
00408             static constexpr uint8_t r = 32; // ceil(mantissa/2)
00409             static constexpr long double shift = (1LL « r) + 1;
00410         };
00411         #endif
00412
00413         template <>
00414         struct FloatLayout<double> {
00415             static constexpr uint8_t exponent = 11;
00416             static constexpr uint8_t mantissa = 52;
00417             static constexpr uint8_t r = 27; // ceil(mantissa/2)
00418             static constexpr double shift = (1LL « r) + 1;
00419         };
00420
00421         template <>
00422         struct FloatLayout<float> {
00423             static constexpr uint8_t exponent = 8;
00424             static constexpr uint8_t mantissa = 23;
00425             static constexpr uint8_t r = 11; // ceil(mantissa/2)
00426             static constexpr float shift = (1 « r) + 1;
00427         };
00428
00429         #ifdef WITH_CUDA_FP16
00430         template<>
00431         struct FloatLayout<__half> {
00432             static constexpr uint8_t exponent = 5;
00433             static constexpr uint8_t mantissa = 10;
00434         };
00435
00436         template<>
00437         struct FloatLayout<__half2> {
00438             static constexpr uint8_t exponent = 5;
00439             static constexpr uint8_t mantissa = 10;
00440         };
00441         #endif
00442
00443         template<typename T>
00444         struct Split {
00445             static constexpr INLINED DEVICE void func(T a, T *x, T *y) {
00446                 T z = a * FloatLayout<T>::shift;
00447                 *x = z - (z - a);
00448                 *y = a - *x;
00449             }
00450         };
00451
00452         #ifdef WITH_CUDA_FP16
00453         template<>
00454         struct Split<__half> {
00455             static constexpr INLINED DEVICE void func(__half a, __half *x, __half *y) {
00456                 __half z = a * __half_raw(0x5280); // TODO(JeWaVe): check this value
00457                 *x = z - (z - a);
00458                 *y = a - *x;
00459             }
00460         };
00461

```

```

00462     template<>
00463     struct Split<__half2> {
00464         static constexpr INLINED_DEVICE void func(__half2 a, __half2 *x, __half2 *y) {
00465             __half2 z = a * __half2(__half_raw(0x5280), __half_raw(0x5280)); // TODO(JeWaVe):
check this value
00466             *x = z - (z - a);
00467             *y = a - *x;
00468         }
00469     };
00470 #endif
00471
00472     template<typename T>
00473     static constexpr INLINED_DEVICE void two_sum(T a, T b, T *x, T *y) {
00474         *x = a + b;
00475         T z = *x - a;
00476         *y = (a - (*x - z)) + (b - z);
00477     }
00478
00479     template<typename T>
00480     static constexpr INLINED_DEVICE void two_prod(T a, T b, T *x, T *y) {
00481         *x = a * b;
00482         #ifdef __clang__
00483         *y = fma_helper<T>::eval(a, b, -*x);
00484         #else
00485         T ah, al, bh, bl;
00486         Split<T>::func(a, &ah, &al);
00487         Split<T>::func(b, &bh, &bl);
00488         *y = al * bl - ((*x - ah * bh) - al * bh) - ah * bl;
00489         #endif
00490     }
00491
00492     template<typename T, size_t N>
00493     static INLINED_DEVICE T horner(T *p1, T *p2, T x) {
00494         T r = p1[0] + p2[0];
00495         for (int64_t i = N - 1; i >= 0; --i) {
00496             r = r * x + p1[N - i] + p2[N - i];
00497         }
00498
00499         return r;
00500     }
00501 } // namespace internal
00502 } // namespace aerobus
00503
00504 // utilities
00505 namespace aerobus {
00506     namespace internal {
00507         template<template<typename...> typename TT, typename T>
00508         struct is_instantiation_of : std::false_type { };
00509
00510         template<template<typename...> typename TT, typename... Ts>
00511         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00512
00513         template<template<typename...> typename TT, typename T>
00514         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00515
00516         template<int64_t i, typename T, typename... Ts>
00517         struct type_at {
00518             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00519             using type = typename type_at<i - 1, Ts...>::type;
00520         };
00521
00522         template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00523             using type = T;
00524         };
00525
00526         template<size_t i, typename... Ts>
00527         using type_at_t = typename type_at<i, Ts...>::type;
00528
00529         template<size_t n, size_t i, typename E = void>
00530         struct _is_prime {};
00531
00532         template<size_t i>
00533         struct _is_prime<0, i> {
00534             static constexpr bool value = false;
00535         };
00536
00537         template<size_t i>
00538         struct _is_prime<1, i> {
00539             static constexpr bool value = false;
00540         };
00541
00542         template<size_t i>
00543         struct _is_prime<2, i> {
00544             static constexpr bool value = true;
00545         };
00546     }
00547

```

```

00548     template<size_t i>
00549     struct _is_prime<3, i> {
00550         static constexpr bool value = true;
00551     };
00552
00553     template<size_t i>
00554     struct _is_prime<5, i> {
00555         static constexpr bool value = true;
00556     };
00557
00558     template<size_t i>
00559     struct _is_prime<7, i> {
00560         static constexpr bool value = true;
00561     };
00562
00563     template<size_t n, size_t i>
00564     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00565         static constexpr bool value = false;
00566     };
00567
00568     template<size_t n, size_t i>
00569     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00570         static constexpr bool value = false;
00571     };
00572
00573     template<size_t n, size_t i>
00574     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00575         static constexpr bool value = true;
00576     };
00577
00578     template<size_t n, size_t i>
00579     struct _is_prime<n, i, std::enable_if_t<(
00580         n % i == 0 &&
00581         n >= 9 &&
00582         n % 3 != 0 &&
00583         n % 2 != 0 &&
00584         i * i > n)>> {
00585         static constexpr bool value = true;
00586     };
00587
00588     template<size_t n, size_t i>
00589     struct _is_prime<n, i, std::enable_if_t<(
00590         n % (i+2) == 0 &&
00591         n >= 9 &&
00592         n % 3 != 0 &&
00593         n % 2 != 0 &&
00594         i * i <= n)>> {
00595         static constexpr bool value = true;
00596     };
00597
00598     template<size_t n, size_t i>
00599     struct _is_prime<n, i, std::enable_if_t<(
00600         n % (i+2) != 0 &&
00601         n % i != 0 &&
00602         n >= 9 &&
00603         n % 3 != 0 &&
00604         n % 2 != 0 &&
00605         (i * i <= n))>> {
00606         static constexpr bool value = _is_prime<n, i+6>::value;
00607     };
00608 } // namespace internal
00609
00610 template<size_t n>
00611 struct is_prime {
00612     static constexpr bool value = internal::_is_prime<n, 5>::value;
00613 };
00614
00615 template<size_t n>
00616 static constexpr bool is_prime_v = is_prime<n>::value;
00617
00618 // gcd
00619 namespace internal {
00620     template <std::size_t... Is>
00621     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00622         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00623
00624     template <std::size_t N>
00625     using make_index_sequence_reverse
00626         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00627
00628     template<typename Ring, typename E = void>
00629     struct gcd;
00630
00631     template<typename Ring>
00632     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00633         template<typename A, typename B, typename E = void>
00634         struct gcd_helper {};
00635     };

```

```

00646
00647 // B = 0, A > 0
00648 template<typename A, typename B>
00649 struct gcd_helper<A, B, std::enable_if_t<
00650     ((B::is_zero_t::value) &&
00651     (Ring::template gt_t<A, typename Ring::zero>::value))>> {
00652     using type = A;
00653 };
00654
00655 // B = 0, A < 0
00656 template<typename A, typename B>
00657 struct gcd_helper<A, B, std::enable_if_t<
00658     ((B::is_zero_t::value) &&
00659     !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00660     using type = typename Ring::template sub_t<typename Ring::zero, A>;
00661 };
00662
00663 // B != 0
00664 template<typename A, typename B>
00665 struct gcd_helper<A, B, std::enable_if_t<
00666     (!B::is_zero_t::value)
00667     >> {
00668 private: // NOLINT
00669     // A / B
00670     using k = typename Ring::template div_t<A, B>;
00671     // A - (A/B)*B = A % B
00672     using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00673
00674 public:
00675     using type = typename gcd_helper<B, m>::type;
00676 };
00677
00678 template<typename A, typename B>
00679 using type = typename gcd_helper<A, B>::type;
00680 };
00681 } // namespace internal
00682
00683 // vadd and vmul
00684 namespace internal {
00685     template<typename... vals>
00686     struct vmul {};
00687
00688     template<typename v1, typename... vals>
00689     struct vmul<v1, vals...> {
00690         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00691     };
00692
00693     template<typename v1>
00694     struct vmul<v1> {
00695         using type = v1;
00696     };
00697
00698     template<typename... vals>
00699     struct vadd {};
00700
00701     template<typename v1, typename... vals>
00702     struct vadd<v1, vals...> {
00703         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00704     };
00705
00706     template<typename v1>
00707     struct vadd<v1> {
00708         using type = v1;
00709     };
00710 } // namespace internal
00711
00712 template<typename T, typename A, typename B>
00713 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00714
00715 template<typename... vals>
00716 using vadd_t = typename internal::vadd<vals...>::type;
00717
00718 template<typename... vals>
00719 using vmul_t = typename internal::vmul<vals...>::type;
00720
00721 template<typename val>
00722 requires IsEuclideanDomain<typename val::enclosing_type>
00723 using abs_t = std::conditional_t<
00724     val::enclosing_type::template pos_v<val>,
00725     val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>;
00726 } // namespace aerobus
00727
00728 // embedding
00729 namespace aerobus {

```

```

00745     template<typename Small, typename Large, typename E = void>
00746     struct Embed;
00747 } // namespace aerobus
00748
00749 namespace aerobus {
00750     template<typename Ring, typename X>
00751     requires IsRing<Ring>
00752     struct Quotient {
00753         template <typename V>
00754         struct val {
00755             public:
00756                 using raw_t = V;
00757                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00758             };
00759
00760             using zero = val<typename Ring::zero>;
00761
00762             using one = val<typename Ring::one>;
00763
00764             template<typename v1, typename v2>
00765             using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00766
00767             template<typename v1, typename v2>
00768             using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00769
00770             template<typename v1, typename v2>
00771             using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00772
00773             template<typename v1, typename v2>
00774             using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00775
00776             template<typename v1, typename v2>
00777             using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00778
00779             template<typename v1, typename v2>
00780             static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00781
00782             template<typename v1>
00783             using pos_t = std::true_type;
00784
00785             template<typename v>
00786             static constexpr bool pos_v = pos_t<v>::value;
00787
00788             static constexpr bool is_euclidean_domain = true;
00789
00790             template<auto x>
00791             using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00792
00793             template<typename v>
00794             using inject_ring_t = val<v>;
00795         };
00796
00797         template<typename Ring, typename X>
00798         struct Embed<Quotient<Ring, X>, Ring> {
00799             template<typename val>
00800             using type = typename val::raw_t;
00801         };
00802     } // namespace aerobus
00803
00804     // type_list
00805     namespace aerobus {
00806         template <typename... Ts>
00807         struct type_list;
00808
00809         namespace internal {
00810             template <typename T, typename... Us>
00811             struct pop_front_h {
00812                 using tail = type_list<Us...>;
00813                 using head = T;
00814             };
00815
00816             template <size_t index, typename L1, typename L2>
00817             struct split_h {
00818             private:
00819                 static_assert(index <= L2::length, "index out of bounds");
00820                 using a = typename L2::pop_front::type;
00821                 using b = typename L2::pop_front::tail;
00822                 using c = typename L1::template push_back<a>;
00823
00824             public:
00825                 using head = typename split_h<index - 1, c, b>::head;
00826                 using tail = typename split_h<index - 1, c, b>::tail;
00827             };
00828
00829             template <typename L1, typename L2>
00830             struct split_h<0, L1, L2> {
00831                 using head = L1;
00832             };
00833         }
00834     }

```

```

00877         using tail = L2;
00878     };
00879
00880     template <size_t index, typename L, typename T>
00881     struct insert_h {
00882         static_assert(index <= L::length, "index ouf of bounds");
00883         using s = typename L::template split<index>;
00884         using left = typename s::head;
00885         using right = typename s::tail;
00886         using ll = typename left::template push_back<T>;
00887         using type = typename ll::template concat<right>;
00888     };
00889
00890     template <size_t index, typename L>
00891     struct remove_h {
00892         using s = typename L::template split<index>;
00893         using left = typename s::head;
00894         using right = typename s::tail;
00895         using rr = typename right::pop_front::tail;
00896         using type = typename left::template concat<rr>;
00897     };
00898 } // namespace internal
00899
00900 template <typename... Ts>
00901 struct type_list {
00902 private:
00903     template <typename T>
00904     struct concat_h;
00905
00906     template <typename... Us>
00907     struct concat_h<type_list<Us...> {
00908         using type = type_list<Ts..., Us...>;
00909     };
00910
00911 public:
00912     static constexpr size_t length = sizeof...(Ts);
00913
00914     template <typename T>
00915     using push_front = type_list<T, Ts...>;
00916
00917     template <size_t index>
00918     using at = internal::type_at_t<index, Ts...>;
00919
00920     struct pop_front {
00921         using type = typename internal::pop_front_h<Ts...>::head;
00922         using tail = typename internal::pop_front_h<Ts...>::tail;
00923     };
00924
00925     template <typename T>
00926     using push_back = type_list<Ts..., T>;
00927
00928     template <typename U>
00929     using concat = typename concat_h<U>::type;
00930
00931     template <size_t index>
00932     struct split {
00933     private:
00934         using inner = internal::split_h<index, type_list<>, type_list<Ts...>>;
00935
00936     public:
00937         using head = typename inner::head;
00938         using tail = typename inner::tail;
00939     };
00940
00941     template <typename T, size_t index>
00942     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00943
00944     template <size_t index>
00945     using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00946 };
00947
00948 template <>
00949 struct type_list<> {
00950     static constexpr size_t length = 0;
00951
00952     template <typename T>
00953     using push_front = type_list<T>;
00954
00955     template <typename T>
00956     using push_back = type_list<T>;
00957
00958     template <typename U>
00959     using concat = U;
00960
00961     // TODO(jewave): assert index == 0
00962     template <typename T, size_t index>
00963     using insert = type_list<T>;
00964

```

```

00986     };
00987 } // namespace aerobus
00988
00989 // i16
00990 #ifdef WITH_CUDA_FP16
00991 // i16
00992 namespace aerobus {
00993     struct i16 {
00994         using inner_type = int16_t;
00995         template<int16_t x>
00996         struct val {
01001             using enclosing_type = i16;
01003             static constexpr int16_t v = x;
01004
01007             template<typename valueType>
01008             static constexpr INLINED_DEVICE valueType get() {
01009                 return internal::template int16_convert_helper<valueType, x>::value();
01010             }
01011
01013             using is_zero_t = std::bool_constant<x == 0>;
01014
01016             static std::string to_string() {
01017                 return std::to_string(x);
01018             }
01019         };
01020
01022         using zero = val<0>;
01024         using one = val<1>;
01026         static constexpr bool is_field = false;
01028         static constexpr bool is_euclidean_domain = true;
01031         template<auto x>
01032         using inject_constant_t = val<static_cast<int16_t>(x)>;
01033
01034         template<typename v>
01035         using inject_ring_t = v;
01036
01037     private:
01038         template<typename v1, typename v2>
01039         struct add {
01040             using type = val<v1::v + v2::v>;
01041         };
01042
01043         template<typename v1, typename v2>
01044         struct sub {
01045             using type = val<v1::v - v2::v>;
01046         };
01047
01048         template<typename v1, typename v2>
01049         struct mul {
01050             using type = val<v1::v * v2::v>;
01051         };
01052
01053         template<typename v1, typename v2>
01054         struct div {
01055             using type = val<v1::v / v2::v>;
01056         };
01057
01058         template<typename v1, typename v2>
01059         struct remainder {
01060             using type = val<v1::v % v2::v>;
01061         };
01062
01063         template<typename v1, typename v2>
01064         struct gt {
01065             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01066         };
01067
01068         template<typename v1, typename v2>
01069         struct lt {
01070             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01071         };
01072
01073         template<typename v1, typename v2>
01074         struct eq {
01075             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01076         };
01077
01078         template<typename v1>
01079         struct pos {
01080             using type = std::bool_constant<(v1::v > 0)>;
01081         };
01082
01083     public:
01084         template<typename v1, typename v2>
01085         using add_t = typename add<v1, v2>::type;
01086
01087         template<typename v1, typename v2>

```



```

01096     using sub_t = typename sub<v1, v2>::type;
01097
01102     template<typename v1, typename v2>
01103     using mul_t = typename mul<v1, v2>::type;
01104
01109     template<typename v1, typename v2>
01110     using div_t = typename div<v1, v2>::type;
01111
01116     template<typename v1, typename v2>
01117     using mod_t = typename remainder<v1, v2>::type;
01118
01123     template<typename v1, typename v2>
01124     using gt_t = typename gt<v1, v2>::type;
01125
01130     template<typename v1, typename v2>
01131     using lt_t = typename lt<v1, v2>::type;
01132
01137     template<typename v1, typename v2>
01138     using eq_t = typename eq<v1, v2>::type;
01139
01143     template<typename v1, typename v2>
01144     static constexpr bool eq_v = eq_t<v1, v2>::value;
01145
01150     template<typename v1, typename v2>
01151     using gcd_t = gcd_t<i16, v1, v2>;
01152
01156     template<typename v>
01157     using pos_t = typename pos<v>::type;
01158
01162     template<typename v>
01163     static constexpr bool pos_v = pos_t<v>::value;
01164 };
01165 } // namespace aerobus
01166 #endif
01167
01168 // i32
01169 namespace aerobus {
01171     struct i32 {
01172         using inner_type = int32_t;
01173         template<int32_t x>
01174         struct val {
01175             using enclosing_type = i32;
01176             static constexpr int32_t v = x;
01177
01184             template<typename valueType>
01185             static constexpr DEVICE valueType get() {
01186                 return static_cast<valueType>(x);
01187             }
01188
01190             using is_zero_t = std::bool_constant<x == 0>;
01191
01193             static std::string to_string() {
01194                 return std::to_string(x);
01195             }
01196         };
01197
01199         using zero = val<0>;
01201         using one = val<1>;
01203         static constexpr bool is_field = false;
01205         static constexpr bool is_euclidean_domain = true;
01208         template<auto x>
01209         using inject_constant_t = val<static_cast<int32_t>(x)>;
01210
01211         template<typename v>
01212         using inject_ring_t = v;
01213
01214     private:
01215         template<typename v1, typename v2>
01216         struct add {
01217             using type = val<v1::v + v2::v>;
01218         };
01219
01220         template<typename v1, typename v2>
01221         struct sub {
01222             using type = val<v1::v - v2::v>;
01223         };
01224
01225         template<typename v1, typename v2>
01226         struct mul {
01227             using type = val<v1::v * v2::v>;
01228         };
01229
01230         template<typename v1, typename v2>
01231         struct div {
01232             using type = val<v1::v / v2::v>;
01233         };
01234

```

```

01235     template<typename v1, typename v2>
01236     struct remainder {
01237         using type = val<v1::v % v2::v>;
01238     };
01239
01240     template<typename v1, typename v2>
01241     struct gt {
01242         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01243     };
01244
01245     template<typename v1, typename v2>
01246     struct lt {
01247         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01248     };
01249
01250     template<typename v1, typename v2>
01251     struct eq {
01252         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01253     };
01254
01255     template<typename v1>
01256     struct pos {
01257         using type = std::bool_constant<(v1::v > 0)>;
01258     };
01259
01260     public:
01261         template<typename v1, typename v2>
01262         using add_t = typename add<v1, v2>::type;
01263
01264         template<typename v1, typename v2>
01265         using sub_t = typename sub<v1, v2>::type;
01266
01267         template<typename v1, typename v2>
01268         using mul_t = typename mul<v1, v2>::type;
01269
01270         template<typename v1, typename v2>
01271         using div_t = typename div<v1, v2>::type;
01272
01273         template<typename v1, typename v2>
01274         using mod_t = typename remainder<v1, v2>::type;
01275
01276         template<typename v1, typename v2>
01277         using gt_t = typename gt<v1, v2>::type;
01278
01279         template<typename v1, typename v2>
01280         using lt_t = typename lt<v1, v2>::type;
01281
01282         template<typename v1, typename v2>
01283         using eq_t = typename eq<v1, v2>::type;
01284
01285         template<typename v1, typename v2>
01286         static constexpr bool eq_v = eq_t<v1, v2>::value;
01287
01288         template<typename v1, typename v2>
01289         using gcd_t = gcd_t<i32, v1, v2>;
01290
01291         template<typename v>
01292         using pos_t = typename pos<v>::type;
01293
01294         template<typename v>
01295         static constexpr bool pos_v = pos_t<v>::value;
01296     };
01297 } // namespace aerobus
01298
01299 // i64
01300 namespace aerobus {
01301     struct i64 {
01302         using inner_type = int64_t;
01303         template<int64_t x>
01304         struct val {
01305             using inner_type = int32_t;
01306             using enclosing_type = i64;
01307             static constexpr int64_t v = x;
01308
01309             template<typename valueType>
01310             static constexpr INLINED_DEVICE valueType get() {
01311                 return static_cast<valueType>(x);
01312             }
01313
01314             using is_zero_t = std::bool_constant<x == 0>;
01315
01316             static std::string to_string() {
01317                 return std::to_string(x);
01318             }
01319         };
01320     };
01321
01322     template<auto x>

```

```

01380         using inject_constant_t = val<static_cast<int64_t>(x)>;
01381
01386         template<typename v>
01387         using inject_ring_t = v;
01388
01390         using zero = val<0>;
01392         using one = val<1>;
01394         static constexpr bool is_field = false;
01396         static constexpr bool is_euclidean_domain = true;
01397
01398     private:
01399         template<typename v1, typename v2>
01400         struct add {
01401             using type = val<v1::v + v2::v>;
01402         };
01403
01404         template<typename v1, typename v2>
01405         struct sub {
01406             using type = val<v1::v - v2::v>;
01407         };
01408
01409         template<typename v1, typename v2>
01410         struct mul {
01411             using type = val<v1::v * v2::v>;
01412         };
01413
01414         template<typename v1, typename v2>
01415         struct div {
01416             using type = val<v1::v / v2::v>;
01417         };
01418
01419         template<typename v1, typename v2>
01420         struct remainder {
01421             using type = val<v1::v % v2::v>;
01422         };
01423
01424         template<typename v1, typename v2>
01425         struct gt {
01426             using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01427         };
01428
01429         template<typename v1, typename v2>
01430         struct lt {
01431             using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01432         };
01433
01434         template<typename v1, typename v2>
01435         struct eq {
01436             using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01437         };
01438
01439         template<typename v>
01440         struct pos {
01441             using type = std::bool_constant<(v::v > 0)>;
01442         };
01443
01444     public:
01448         template<typename v1, typename v2>
01449         using add_t = typename add<v1, v2>::type;
01450
01454         template<typename v1, typename v2>
01455         using sub_t = typename sub<v1, v2>::type;
01456
01460         template<typename v1, typename v2>
01461         using mul_t = typename mul<v1, v2>::type;
01462
01467         template<typename v1, typename v2>
01468         using div_t = typename div<v1, v2>::type;
01469
01473         template<typename v1, typename v2>
01474         using mod_t = typename remainder<v1, v2>::type;
01475
01480         template<typename v1, typename v2>
01481         using gt_t = typename gt<v1, v2>::type;
01482
01487         template<typename v1, typename v2>
01488         static constexpr bool gt_v = gt_t<v1, v2>::value;
01489
01494         template<typename v1, typename v2>
01495         using lt_t = typename lt<v1, v2>::type;
01496
01501         template<typename v1, typename v2>
01502         static constexpr bool lt_v = lt_t<v1, v2>::value;
01503
01508         template<typename v1, typename v2>
01509         using eq_t = typename eq<v1, v2>::type;
01510

```

```

01515     template<typename v1, typename v2>
01516     static constexpr bool eq_v = eq_t<v1, v2>::value;
01517
01522     template<typename v1, typename v2>
01523     using gcd_t = gcd_t<i64, v1, v2>;
01524
01528     template<typename v>
01529     using pos_t = typename pos<v>::type;
01530
01534     template<typename v>
01535     static constexpr bool pos_v = pos_t<v>::value;
01536 };
01537
01539 template<>
01540 struct Embed<i32, i64> {
01543     template<typename val>
01544     using type = i64::val<static_cast<int64_t>(val::v)>;
01545 };
01546 } // namespace aerobus
01547
01548 // z/pz
01549 namespace aerobus {
01555     template<int32_t p>
01556     struct zpz {
01558         using inner_type = int32_t;
01559
01562         template<int32_t x>
01563         struct val {
01565             using enclosing_type = zpz<p>;
01567             static constexpr int32_t v = x % p;
01568
01571             template<typename valueType>
01572             static constexpr INLINED_DEVICE valueType get() {
01573                 return static_cast<valueType>(x % p);
01574             }
01575
01577             using is_zero_t = std::bool_constant<v == 0>;
01578
01580             static constexpr bool is_zero_v = v == 0;
01581
01584             static std::string to_string() {
01585                 return std::to_string(x % p);
01586             }
01587         };
01588
01591         template<auto x>
01592         using inject_constant_t = val<static_cast<int32_t>(x)>;
01593
01595         using zero = val<0>;
01596
01598         using one = val<1>;
01599
01601         static constexpr bool is_field = is_prime<p>::value;
01602
01604         static constexpr bool is_euclidean_domain = true;
01605
01606     private:
01607         template<typename v1, typename v2>
01608         struct add {
01609             using type = val<(v1::v + v2::v) % p>;
01610         };
01611
01612         template<typename v1, typename v2>
01613         struct sub {
01614             using type = val<(v1::v - v2::v) % p>;
01615         };
01616
01617         template<typename v1, typename v2>
01618         struct mul {
01619             using type = val<(v1::v * v2::v) % p>;
01620         };
01621
01622         template<typename v1, typename v2>
01623         struct div {
01624             using type = val<(v1::v % p) / (v2::v % p)>;
01625         };
01626
01627         template<typename v1, typename v2>
01628         struct remainder {
01629             using type = val<(v1::v % v2::v) % p>;
01630         };
01631
01632         template<typename v1, typename v2>
01633         struct gt {
01634             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01635         };
01636

```

```

01637     template<typename v1, typename v2>
01638     struct lt {
01639         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01640     };
01641
01642     template<typename v1, typename v2>
01643     struct eq {
01644         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01645     };
01646
01647     template<typename v1>
01648     struct pos {
01649         using type = std::bool_constant<(v1::v > 0)>;
01650     };
01651
01652     public:
01653     template<typename v1, typename v2>
01654     using add_t = typename add<v1, v2>::type;
01655
01656     template<typename v1, typename v2>
01657     using sub_t = typename sub<v1, v2>::type;
01658
01659     template<typename v1, typename v2>
01660     using mul_t = typename mul<v1, v2>::type;
01661
01662     template<typename v1, typename v2>
01663     using div_t = typename div<v1, v2>::type;
01664
01665     template<typename v1, typename v2>
01666     using mod_t = typename remainder<v1, v2>::type;
01667
01668     template<typename v1, typename v2>
01669     using gt_t = typename gt<v1, v2>::type;
01670
01671     template<typename v1, typename v2>
01672     static constexpr bool gt_v = gt_t<v1, v2>::value;
01673
01674     template<typename v1, typename v2>
01675     using lt_t = typename lt<v1, v2>::type;
01676
01677     template<typename v1, typename v2>
01678     static constexpr bool lt_v = lt_t<v1, v2>::value;
01679
01680     template<typename v1, typename v2>
01681     using eq_t = typename eq<v1, v2>::type;
01682
01683     template<typename v1, typename v2>
01684     static constexpr bool eq_v = eq_t<v1, v2>::value;
01685
01686     template<typename v1, typename v2>
01687     using gcd_t = gcd_t<i32, v1, v2>;
01688
01689     template<typename v1>
01690     using pos_t = typename pos<v1>::type;
01691
01692     template<typename v>
01693     static constexpr bool pos_v = pos_t<v>::value;
01694 };
01695
01696 template<int32_t x>
01697 struct Embed<zp<x>, i32> {
01698     template<typename val>
01699     using type = i32::val<val::v>;
01700 };
01701 } // namespace aerobus
01702
01703 // polynomial
01704 namespace aerobus {
01705     // coeffN x^N + ...
01706     template<typename Ring>
01707     requires IsEuclideanDomain<Ring>
01708     struct polynomial {
01709         static constexpr bool is_field = false;
01710         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01711
01712         template<typename P>
01713         struct horner_reduction_t {
01714             template<size_t index, size_t stop>
01715             struct inner {
01716                 template<typename accum, typename x>
01717                 using type = typename horner_reduction_t<P>::template inner<index + 1, stop>
01718                     ::template type<
01719                         typename Ring::template add_t<
01720                             typename Ring::template mul_t<x, accum>,
01721                             typename P::template coeff_at_t<P::degree - index>
01722                             >, x>;
01723             };
01724         };
01725     };

```

```

01774         template<size_t stop>
01775         struct inner<stop, stop> {
01776             template<typename accum, typename x>
01777                 using type = accum;
01778         };
01779     };
01780
01781     template<typename coeffN, typename... coeffs>
01782     struct val {
01783         using ring_type = Ring;
01784         using enclosing_type = polynomial<Ring>;
01785         static constexpr size_t degree = sizeof...(coeffs);
01786         using aN = coeffN;
01787         using strip = val<coeffs...>;
01788         using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01789         static constexpr bool is_zero_v = is_zero_t::value;
01790
01791     private:
01792         template<size_t index, typename E = void>
01793         struct coeff_at {};
01794
01795         template<size_t index>
01796         struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01797             using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01798         };
01799
01800         template<size_t index>
01801         struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01802             using type = typename Ring::zero;
01803         };
01804
01805     public:
01806         template<size_t index>
01807         using coeff_at_t = typename coeff_at<index>::type;
01808
01809         static std::string to_string() {
01810             return string_helper<coeffN, coeffs...>::func();
01811         }
01812
01813         template<typename arithmeticType>
01814         static constexpr DEVICE INLINE arithmeticType eval(const arithmeticType& x) {
01815             #ifdef WITH_CUDA_FP16
01816             arithmeticType start;
01817             if constexpr (std::is_same_v<arithmeticType, __half2>) {
01818                 start = __half2(0, 0);
01819             } else {
01820                 start = static_cast<arithmeticType>(0);
01821             }
01822             #else
01823             arithmeticType start = static_cast<arithmeticType>(0);
01824             #endif
01825             return horner_evaluation<arithmeticType, val>
01826                 ::template inner<0, degree + 1>
01827                 ::func(start, x);
01828         }
01829
01830         template<typename arithmeticType>
01831         static constexpr DEVICE INLINE arithmeticType compensated_eval(const arithmeticType& x) {
01832             return compensated_horner<arithmeticType, val>::func(x);
01833         }
01834
01835         template<typename x>
01836         using value_at_t = horner_reduction_t<val>
01837             ::template inner<0, degree + 1>
01838             ::template type<typename Ring::zero, x>;
01839     };
01840
01841     template<typename coeffN>
01842     struct val<coeffN> {
01843         using ring_type = Ring;
01844         using enclosing_type = polynomial<Ring>;
01845         static constexpr size_t degree = 0;
01846         using aN = coeffN;
01847         using strip = val<coeffN>;
01848         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01849
01850         static constexpr bool is_zero_v = is_zero_t::value;
01851
01852         template<size_t index, typename E = void>
01853         struct coeff_at {};
01854
01855         template<size_t index>
01856         struct coeff_at<index, std::enable_if_t<(index == 0)> {
01857             using type = aN;
01858         };
01859     };

```

```

01896     template<size_t index>
01897     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)» {
01898         using type = typename Ring::zero;
01899     };
01900
01901     template<size_t index>
01902     using coeff_at_t = typename coeff_at<index>::type;
01903
01904     static std::string to_string() {
01905         return string_helper<coeffN>::func();
01906     }
01907
01908     template<typename arithmeticType>
01909     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& x) {
01910         return coeffN::template get<arithmeticType>();
01911     }
01912
01913     template<typename arithmeticType>
01914     static DEVICE INLINED arithmeticType compensated_eval(const arithmeticType& x) {
01915         return coeffN::template get<arithmeticType>();
01916     }
01917
01918     template<typename x>
01919     using value_at_t = coeffN;
01920 };
01921
01922 using zero = val<typename Ring::zero>;
01923 using one = val<typename Ring::one>;
01924 using X = val<typename Ring::one, typename Ring::zero>;
01925
01926 private:
01927     template<typename P, typename E = void>
01928     struct simplify;
01929
01930     template<typename P1, typename P2, typename I>
01931     struct add_low;
01932
01933     template<typename P1, typename P2>
01934     struct add {
01935         using type = typename simplify<typename add_low<
01936             P1,
01937             P2,
01938             internal::make_index_sequence_reverse<
01939                 std::max(P1::degree, P2::degree) + 1
01940             >::type>::type;
01941     };
01942
01943     template<typename P1, typename P2, typename I>
01944     struct sub_low;
01945
01946     template<typename P1, typename P2, typename I>
01947     struct mul_low;
01948
01949     template<typename v1, typename v2>
01950     struct mul {
01951         using type = typename mul_low<
01952             v1,
01953             v2,
01954             internal::make_index_sequence_reverse<
01955                 v1::degree + v2::degree + 1
01956             >::type;
01957     };
01958
01959     template<typename coeff, size_t deg>
01960     struct monomial;
01961
01962     template<typename v, typename E = void>
01963     struct derive_helper {};
01964
01965     template<typename v>
01966     struct derive_helper<v, std::enable_if_t<v::degree == 0>» {
01967         using type = zero;
01968     };
01969
01970     template<typename v>
01971     struct derive_helper<v, std::enable_if_t<v::degree != 0>» {
01972         using type = typename add<
01973             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01974             typename monomial<
01975                 typename Ring::template mul_t<
01976                     typename v::aN,
01977                     typename Ring::template inject_constant_t<(v::degree)>
01978                 >,
01979                 v::degree - 1
01980             >::type
01981         >::type;
01982     };
01983
01984 };
01985

```

```

01986
01987     template<typename v1, typename v2, typename E = void>
01988     struct eq_helper {};
01989
01990     template<typename v1, typename v2>
01991     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01992         using type = std::false_type;
01993     };
01994
01995
01996     template<typename v1, typename v2>
01997     struct eq_helper<v1, v2, std::enable_if_t<
01998         v1::degree == v2::degree &&
01999         (v1::degree != 0 || v2::degree != 0) &&
02000         std::is_same<
02001             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
02002             std::false_type
02003         >::value
02004     >
02005     > {
02006         using type = std::false_type;
02007     };
02008
02009     template<typename v1, typename v2>
02010     struct eq_helper<v1, v2, std::enable_if_t<
02011         v1::degree == v2::degree &&
02012         (v1::degree != 0 || v2::degree != 0) &&
02013         std::is_same<
02014             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
02015             std::true_type
02016         >::value
02017     > {
02018         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
02019     };
02020
02021     template<typename v1, typename v2>
02022     struct eq_helper<v1, v2, std::enable_if_t<
02023         v1::degree == v2::degree &&
02024         (v1::degree == 0)
02025     > {
02026         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
02027     };
02028
02029     template<typename v1, typename v2, typename E = void>
02030     struct lt_helper {};
02031
02032     template<typename v1, typename v2>
02033     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
02034         using type = std::true_type;
02035     };
02036
02037     template<typename v1, typename v2>
02038     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
02039         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
02040     };
02041
02042     template<typename v1, typename v2>
02043     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
02044         using type = std::false_type;
02045     };
02046
02047     template<typename v1, typename v2, typename E = void>
02048     struct gt_helper {};
02049
02050     template<typename v1, typename v2>
02051     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
02052         using type = std::true_type;
02053     };
02054
02055     template<typename v1, typename v2>
02056     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
02057         using type = std::false_type;
02058     };
02059
02060     template<typename v1, typename v2>
02061     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
02062         using type = std::false_type;
02063     };
02064
02065     // when high power is zero : strip
02066     template<typename P>
02067     struct simplify<P, std::enable_if_t<
02068         std::is_same<
02069             typename Ring::zero,
02070             typename P::aN
02071         >::value && (P::degree > 0)
02072     > {

```



```

02073         using type = typename simplify<typename P::strip>::type;
02074     };
02075
02076     // otherwise : do nothing
02077     template<typename P>
02078     struct simplify<P, std::enable_if_t<
02079         !std::is_same<
02080             typename Ring::zero,
02081             typename P::aN
02082         >::value && (P::degree > 0)
02083     > {
02084         using type = P;
02085     };
02086
02087     // do not simplify constants
02088     template<typename P>
02089     struct simplify<P, std::enable_if_t<P::degree == 0> {
02090         using type = P;
02091     };
02092
02093     // addition at
02094     template<typename P1, typename P2, size_t index>
02095     struct add_at {
02096         using type =
02097             typename Ring::template add_t<
02098                 typename P1::template coeff_at_t<index>,
02099                 typename P2::template coeff_at_t<index>;
02100     };
02101
02102     template<typename P1, typename P2, size_t index>
02103     using add_at_t = typename add_at<P1, P2, index>::type;
02104
02105     template<typename P1, typename P2, std::size_t... I>
02106     struct add_low<P1, P2, std::index_sequence<I...> {
02107         using type = val<add_at_t<P1, P2, I>...>;
02108     };
02109
02110     // subtraction at
02111     template<typename P1, typename P2, size_t index>
02112     struct sub_at {
02113         using type =
02114             typename Ring::template sub_t<
02115                 typename P1::template coeff_at_t<index>,
02116                 typename P2::template coeff_at_t<index>;
02117     };
02118
02119     template<typename P1, typename P2, size_t index>
02120     using sub_at_t = typename sub_at<P1, P2, index>::type;
02121
02122     template<typename P1, typename P2, std::size_t... I>
02123     struct sub_low<P1, P2, std::index_sequence<I...> {
02124         using type = val<sub_at_t<P1, P2, I>...>;
02125     };
02126
02127     template<typename P1, typename P2>
02128     struct sub {
02129         using type = typename simplify<typename sub_low<
02130             P1,
02131             P2,
02132             internal::make_index_sequence_reverse<
02133                 std::max(P1::degree, P2::degree) + 1
02134             >::type>::type;
02135     };
02136
02137     // multiplication at
02138     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
02139     struct mul_at_loop_helper {
02140         using type = typename Ring::template add_t<
02141             typename Ring::template mul_t<
02142                 typename v1::template coeff_at_t<index>,
02143                 typename v2::template coeff_at_t<k - index>
02144             >,
02145             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
02146         >;
02147     };
02148
02149     template<typename v1, typename v2, size_t k, size_t stop>
02150     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
02151         using type = typename Ring::template mul_t<
02152             typename v1::template coeff_at_t<stop>,
02153             typename v2::template coeff_at_t<0>;
02154     };
02155
02156     template<typename v1, typename v2, size_t k, typename E = void>
02157     struct mul_at {};
02158
02159     template<typename v1, typename v2, size_t k>

```

```

02160     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
02161         using type = typename Ring::zero;
02162     };
02163
02164     template<typename v1, typename v2, size_t k>
02165     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
02166         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
02167     };
02168
02169     template<typename P1, typename P2, size_t index>
02170     using mul_at_t = typename mul_at<P1, P2, index>::type;
02171
02172     template<typename P1, typename P2, std::size_t... I>
02173     struct mul_low<P1, P2, std::index_sequence<I...>> {
02174         using type = val<mul_at_t<P1, P2, I>...>;
02175     };
02176
02177     // division helper
02178     template< typename A, typename B, typename Q, typename R, typename E = void>
02179     struct div_helper {};
02180
02181     template<typename A, typename B, typename Q, typename R>
02182     struct div_helper<A, B, Q, R, std::enable_if_t<
02183         (R::degree < B::degree) ||
02184         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02185         using q_type = Q;
02186         using mod_type = R;
02187         using gcd_type = B;
02188     };
02189
02190     template<typename A, typename B, typename Q, typename R>
02191     struct div_helper<A, B, Q, R, std::enable_if_t<
02192         (R::degree >= B::degree) &&
02193         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
02194     private: // NOLINT
02195         using rN = typename R::aN;
02196         using bN = typename B::aN;
02197         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
02198         B::degree>::type;
02199         using rr = typename sub<R, typename mul<pT, B>::type>::type;
02200         using qq = typename add<Q, pT>::type;
02201     public:
02202         using q_type = typename div_helper<A, B, qq, rr>::q_type;
02203         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
02204         using gcd_type = rr;
02205     };
02206
02207     template<typename A, typename B>
02208     struct div {
02209         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
02210         using q_type = typename div_helper<A, B, zero, A>::q_type;
02211         using m_type = typename div_helper<A, B, zero, A>::mod_type;
02212     };
02213
02214     template<typename P>
02215     struct make_unit {
02216         using type = typename div<P, val<typename P::aN>::q_type>;
02217     };
02218
02219     template<typename coeff, size_t deg>
02220     struct monomial {
02221         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
02222     };
02223
02224     template<typename coeff>
02225     struct monomial<coeff, 0> {
02226         using type = val<coeff>;
02227     };
02228
02229     template<typename arithmeticType, typename P>
02230     struct horner_evaluation {
02231         template<size_t index, size_t stop>
02232         struct inner {
02233             static constexpr DEVICE INLINED arithmeticType func(
02234                 const arithmeticType& accum, const arithmeticType& x) {
02235                 return horner_evaluation<arithmeticType, P>::template inner<index + 1,
02236                 stop>::func(
02237                     internal::fma_helper<arithmeticType>::eval(
02238                         x,
02239                         accum,
02240                         P::template coeff_at_t<P::degree - index>::template
02241                         get<arithmeticType>()), x);
02242             };
02243         };
02244     };

```

```

02244     struct inner<stop, stop> {
02245         static constexpr DEVICE INLINED arithmeticType func(
02246             const arithmeticType& accum, const arithmeticType& x) {
02247             return accum;
02248         }
02249     };
02250 };
02251
02252 template<typename arithmeticType, typename P>
02253 struct compensated_horner {
02254     template<int64_t index, int ghost>
02255     struct EFTHorner {
02256         static INLINED DEVICE void func(
02257             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02258 *r) {
02259             arithmeticType p;
02259             internal::two_prod(*r, x, &p, pi + P::degree - index - 1);
02260             constexpr arithmeticType coeff = P::template coeff_at_t<index>::template
get<arithmeticType>();
02261             internal::two_sum<arithmeticType>(
02262                 p, coeff,
02263                 r, sigma + P::degree - index - 1);
02264             EFTHorner<index - 1, ghost>::func(x, pi, sigma, r);
02265         }
02266     };
02267
02268     template<int ghost>
02269     struct EFTHorner<-1, ghost> {
02270         static INLINED DEVICE void func(
02271             arithmeticType x, arithmeticType *pi, arithmeticType *sigma, arithmeticType
02272 *r) {
02273             }
02274         };
02275
02276         static INLINED DEVICE arithmeticType func(arithmeticType x) {
02277             arithmeticType pi[P::degree], sigma[P::degree];
02278             arithmeticType r = P::template coeff_at_t<P::degree>::template get<arithmeticType>();
02279             EFTHorner<P::degree - 1, 0>::func(x, pi, sigma, &r);
02280             arithmeticType c = internal::horner<arithmeticType, P::degree - 1>(pi, sigma, x);
02281             return r + c;
02282         }
02283     };
02284
02285     template<typename coeff, typename... coeffs>
02286     struct string_helper {
02287         static std::string func() {
02288             std::string tail = string_helper<coeffs...>::func();
02289             std::string result = "";
02290             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
02291                 return tail;
02292             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
02293                 if (sizeof...(coeffs) == 1) {
02294                     result += "x";
02295                 } else {
02296                     result += "x^" + std::to_string(sizeof...(coeffs));
02297                 }
02298             } else {
02299                 if (sizeof...(coeffs) == 1) {
02300                     result += coeff::to_string() + " x";
02301                 } else {
02302                     result += coeff::to_string()
02303                         + " x^" + std::to_string(sizeof...(coeffs));
02304                 }
02305             }
02306
02307             if (!tail.empty()) {
02308                 if (tail.at(0) != '-') {
02309                     result += " + " + tail;
02310                 } else {
02311                     result += " - " + tail.substr(1);
02312                 }
02313             }
02314
02315             return result;
02316         }
02317     };
02318
02319     template<typename coeff>
02320     struct string_helper<coeff> {
02321         static std::string func() {
02322             if (!std::is_same<coeff, typename Ring::zero>::value) {
02323                 return coeff::to_string();
02324             } else {
02325                 return "";
02326             }
02327         }
02328     };

```

```

02328
02329     public:
02332         template<typename P>
02333         using simplify_t = typename simplify<P>::type;
02334
02338         template<typename v1, typename v2>
02339         using add_t = typename add<v1, v2>::type;
02340
02344         template<typename v1, typename v2>
02345         using sub_t = typename sub<v1, v2>::type;
02346
02350         template<typename v1, typename v2>
02351         using mul_t = typename mul<v1, v2>::type;
02352
02356         template<typename v1, typename v2>
02357         using eq_t = typename eq_helper<v1, v2>::type;
02358
02362         template<typename v1, typename v2>
02363         using lt_t = typename lt_helper<v1, v2>::type;
02364
02368         template<typename v1, typename v2>
02369         using gt_t = typename gt_helper<v1, v2>::type;
02370
02374         template<typename v1, typename v2>
02375         using div_t = typename div<v1, v2>::q_type;
02376
02380         template<typename v1, typename v2>
02381         using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02382
02386         template<typename coeff, size_t deg>
02387         using monomial_t = typename monomial<coeff, deg>::type;
02388
02391         template<typename v>
02392         using derive_t = typename derive_helper<v>::type;
02393
02396         template<typename v>
02397         using pos_t = typename Ring::template pos_t<typename v::aN>;
02398
02401         template<typename v>
02402         static constexpr bool pos_v = pos_t<v>::value;
02403
02407         template<typename v1, typename v2>
02408         using gcd_t = std::conditional_t<
02409             Ring::is_euclidean_domain,
02410             typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02411                 void>;
02412
02415         template<auto x>
02416         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02417
02420         template<typename v>
02421         using inject_ring_t = val<v>;
02422     };
02423 } // namespace aerobus
02424
02425 // fraction field
02426 namespace aerobus {
02427     namespace internal {
02428         template<typename Ring, typename E = void>
02429         requires IsEuclideanDomain<Ring>
02430         struct _FractionField {};
02431
02432         template<typename Ring>
02433         requires IsEuclideanDomain<Ring>
02434         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
02435             static constexpr bool is_field = true;
02436             static constexpr bool is_euclidean_domain = true;
02437
02438         private:
02439             template<typename val1, typename val2, typename E = void>
02440             struct to_string_helper {};
02441
02442             template<typename val1, typename val2>
02443             struct to_string_helper <val1, val2,
02444                 std::enable_if_t<
02445                     Ring::template eq_t<
02446                         val2, typename Ring::one
02447                         >::value
02448                     >
02449             > {
02450                 static std::string func() {
02451                     return val1::to_string();
02452                 }
02453             };
02454
02455             template<typename val1, typename val2>
02456             struct to_string_helper<val1, val2,
02457 
```

```

02458         std::enable_if_t<
02459         !Ring::template eq_t<
02460         val2,
02461         typename Ring::one
02462         >::value
02463         >
02464     > {
02465         static std::string func() {
02466             return "(" + val1::to_string() + ") / (" + val2::to_string() + ")";
02467         }
02468     };
02469
02470 public:
02471     template<typename val1, typename val2>
02472     struct val {
02473         using x = val1;
02474         using y = val2;
02475         using is_zero_t = typename val1::is_zero_t;
02476         static constexpr bool is_zero_v = val1::is_zero_t::value;
02477
02478         using ring_type = Ring;
02479         using enclosing_type = _FractionField<Ring>;
02480
02481         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02482
02483         template<typename valueType, int ghost = 0>
02484         struct get_helper {
02485             static constexpr INLINED_DEVICE valueType get() {
02486                 return internal::staticcast<valueType, typename
ring_type::inner_type>::template func<x::v>() /
02487                 internal::staticcast<valueType, typename ring_type::inner_type>::template
func<y::v>());
02488             }
02489         };
02490
02491         #ifdef WITH_CUDA_FP16
02492         template<int ghost>
02493         struct get_helper<__half, ghost> {
02494             static constexpr INLINED_DEVICE __half get() {
02495                 return internal::my_float2half_rn(
02496                 internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /
02497                 internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02498             }
02499         };
02500
02501         template<int ghost>
02502         struct get_helper<__half2, ghost> {
02503             static constexpr INLINED_DEVICE __half2 get() {
02504                 constexpr __half tmp = internal::my_float2half_rn(
02505                 internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /
02506                 internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02507             }
02508         };
02509
02510         template<int ghost>
02511         struct get_helper<__half2, ghost> {
02512             static constexpr INLINED_DEVICE __half2 get() {
02513                 constexpr __half tmp = internal::my_float2half_rn(
02514                 internal::staticcast<float, typename ring_type::inner_type>::template
func<x::v>() /
02515                 internal::staticcast<float, typename ring_type::inner_type>::template
func<y::v>());
02516                 return __half2(tmp, tmp);
02517             }
02518         };
02519     };
02520 #endif
02521
02522     template<typename valueType>
02523     static constexpr INLINED_DEVICE valueType get() {
02524         return get_helper<valueType, 0>::get();
02525     }
02526
02527     static std::string to_string() {
02528         return to_string_helper<val1, val2>::func();
02529     }
02530
02531     template<typename arithmeticType>
02532     static constexpr DEVICE INLINED arithmeticType eval(const arithmeticType& v) {
02533         return x::eval(v) / y::eval(v);
02534     }
02535 };
02536
02537 using zero = val<typename Ring::zero, typename Ring::one>;
02538 using one = val<typename Ring::one, typename Ring::one>;
02539
02540 template<typename v>
02541 using inject_t = val<v, typename Ring::one>;
02542
02543 template<auto x>
02544 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02545
02546 template<typename v>
02547 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;

```

```

02565         using ring_type = Ring;
02566
02567     private:
02568     template<typename v, typename E = void>
02569     struct simplify {};
02570
02571     // x = 0
02572     template<typename v>
02573     struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02574         using type = typename _FractionField<Ring>::zero;
02575     };
02576
02577     // x != 0
02578     template<typename v>
02579     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02580     private:
02581         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02582         using newx = typename Ring::template div_t<typename v::x, _gcd>;
02583         using newy = typename Ring::template div_t<typename v::y, _gcd>;
02584
02585         using posx = std::conditional_t<
02586             !Ring::template pos_v<newx>,
02587             typename Ring::template sub_t<typename Ring::zero, newx>,
02588             newx>;
02589         using posy = std::conditional_t<
02590             !Ring::template pos_v<newy>,
02591             typename Ring::template sub_t<typename Ring::zero, newy>,
02592             newy>;
02593     public:
02594         using type = typename _FractionField<Ring>::template val<posx, posy>;
02595     };
02596
02597     public:
02598     template<typename v>
02599     using simplify_t = typename simplify<v>::type;
02600
02601     private:
02602     template<typename v1, typename v2>
02603     struct add {
02604     private:
02605         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02606         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02607         using dividend = typename Ring::template add_t<a, b>;
02608         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02609         using g = typename Ring::template gcd_t<dividend, diviser>;
02610
02611     public:
02612         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02613             diviser>;
02614     };
02615
02616     template<typename v>
02617     struct pos {
02618     using type = std::conditional_t<
02619         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
02620         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
02621         std::true_type,
02622         std::false_type>;
02623     };
02624
02625     template<typename v1, typename v2>
02626     struct sub {
02627     private:
02628         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02629         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02630         using dividend = typename Ring::template sub_t<a, b>;
02631         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02632         using g = typename Ring::template gcd_t<dividend, diviser>;
02633
02634     public:
02635         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
02636             diviser>;
02637     };
02638
02639     template<typename v1, typename v2>
02640     struct mul {
02641     private:
02642         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
02643         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02644
02645     public:
02646         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02647     };
02648
02649     template<typename v1, typename v2, typename E = void>
02650     struct div {};

```

```

02653
02654     template<typename v1, typename v2>
02655     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
02656     private:
02657         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02658         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02659
02660     public:
02661         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
02662     };
02663
02664     template<typename v1, typename v2>
02665     struct div<v1, v2, std::enable_if_t<
02666         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
02667         using type = one;
02668     };
02669
02670     template<typename v1, typename v2>
02671     struct eq {
02672         using type = std::conditional_t<
02673             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
02674             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
02675             std::true_type,
02676             std::false_type>;
02677     };
02678
02679     template<typename v1, typename v2, typename E = void>
02680     struct gt;
02681
02682     template<typename v1, typename v2>
02683     struct gt<v1, v2, std::enable_if_t<
02684         (eq<v1, v2>::type::value)
02685         > {
02686         using type = std::false_type;
02687     };
02688
02689     template<typename v1, typename v2>
02690     struct gt<v1, v2, std::enable_if_t<
02691         (!eq<v1, v2>::type::value) &&
02692         (!pos<v1>::type::value) && (!pos<v2>::type::value)
02693         > {
02694         using type = typename gt<
02695             typename sub<zero, v1>::type, typename sub<zero, v2>::type
02696             >::type;
02697     };
02698
02699     template<typename v1, typename v2>
02700     struct gt<v1, v2, std::enable_if_t<
02701         (!eq<v1, v2>::type::value) &&
02702         (pos<v1>::type::value) && (!pos<v2>::type::value)
02703         > {
02704         using type = std::true_type;
02705     };
02706
02707     template<typename v1, typename v2>
02708     struct gt<v1, v2, std::enable_if_t<
02709         (!eq<v1, v2>::type::value) &&
02710         (!pos<v1>::type::value) && (pos<v2>::type::value)
02711         > {
02712         using type = std::false_type;
02713     };
02714
02715     template<typename v1, typename v2>
02716     struct gt<v1, v2, std::enable_if_t<
02717         (!eq<v1, v2>::type::value) &&
02718         (pos<v1>::type::value) && (pos<v2>::type::value)
02719         > {
02720         using type = typename Ring::template gt_t<
02721             typename Ring::template mul_t<v1::x, v2::y>,
02722             typename Ring::template mul_t<v2::y, v2::x>
02723         >;
02724     };
02725
02726     public:
02727     template<typename v1, typename v2>
02728     using add_t = typename add<v1, v2>::type;
02729
02730     template<typename v1, typename v2>
02731     using mod_t = zero;
02732
02733     template<typename v1, typename v2>
02734     using gcd_t = v1;
02735
02736     template<typename v1, typename v2>
02737     using sub_t = typename sub<v1, v2>::type;
02738
02739
02740
02741
02742
02743
02744
02745
02746
02747
02748
02749
02750
02751
02752

```

```

02756     template<typename v1, typename v2>
02757     using mul_t = typename mul<v1, v2>::type;
02758
02762     template<typename v1, typename v2>
02763     using div_t = typename div<v1, v2>::type;
02764
02768     template<typename v1, typename v2>
02769     using eq_t = typename eq<v1, v2>::type;
02770
02774     template<typename v1, typename v2>
02775     static constexpr bool eq_v = eq<v1, v2>::type::value;
02776
02780     template<typename v1, typename v2>
02781     using gt_t = typename gt<v1, v2>::type;
02782
02786     template<typename v1, typename v2>
02787     static constexpr bool gt_v = gt<v1, v2>::type::value;
02788
02791     template<typename v>
02792     using pos_t = typename pos<v>::type;
02793
02796     template<typename v>
02797     static constexpr bool pos_v = pos<v>::value;
02798 };
02799
02800 template<typename Ring, typename E = void>
02801 requires IsEuclideanDomain<Ring>
02802 struct FractionFieldImpl {};
02803
02804 // fraction field of a field is the field itself
02805 template<typename Field>
02806 requires IsEuclideanDomain<Field>
02807 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02808     using type = Field;
02809     template<typename v>
02810     using inject_t = v;
02811 };
02812
02813 // fraction field of a ring is the actual fraction field
02814 template<typename Ring>
02815 requires IsEuclideanDomain<Ring>
02816 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02817     using type = _FractionField<Ring>;
02818 };
02819 } // namespace internal
02820
02823 template<typename Ring>
02824 requires IsEuclideanDomain<Ring>
02825 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02826
02829 template<typename Ring>
02830 struct Embed<Ring, FractionField<Ring> {
02831     template<typename v>
02832     using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02833 };
02834 } // namespace aerobus
02835
02838 // short names for common types
02839 namespace aerobus {
02840     template<typename X, typename Y>
02841     requires IsRing<typename X::enclosing_type> &&
02842     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02843     using add_t = typename X::enclosing_type::template add_t<X, Y>;
02844
02847     template<typename X, typename Y>
02848     requires IsRing<typename X::enclosing_type> &&
02849     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02850     using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02851
02854     template<typename X, typename Y>
02855     requires IsRing<typename X::enclosing_type> &&
02856     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02857     using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02858
02861     template<typename X, typename Y>
02862     requires IsEuclideanDomain<typename X::enclosing_type> &&
02863     (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02864     using div_t = typename X::enclosing_type::template div_t<X, Y>;
02865
02868     using q32 = FractionField<i32>;
02869
02872     using fpq32 = FractionField<polynomial<q32>>;
02873
02876     using q64 = FractionField<i64>;
02877
02880     using pi64 = polynomial<i64>;

```



```

02887
02888     using pq64 = polynomial<q64>;
02889
02890
02891     using fpq64 = FractionField<polynomial<q64>>;
02892
02893     template<typename Ring, typename v1, typename v2>
02894     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02895
02896     template<typename v>
02897     using embed_int_poly_in_fractions_t =
02898         typename Embed<
02899             polynomial<typename v::ring_type>,
02900             polynomial<FractionField<typename v::ring_type>>::template type<v>;
02901
02902     template<int64_t p, int64_t q>
02903     using make_q64_t = typename q64::template simplify_t<
02904         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02905
02906     template<int32_t p, int32_t q>
02907     using make_q32_t = typename q32::template simplify_t<
02908         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02909
02910     #ifdef WITH_CUDA_FP16
02911     using q16 = FractionField<i16>;
02912
02913     template<int16_t p, int16_t q>
02914     using make_q16_t = typename q16::template simplify_t<
02915         typename q16::val<i16::inject_constant_t<p>, i16::inject_constant_t<q>>;
02916
02917     #endif
02918     template<typename Ring, typename v1, typename v2>
02919     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02920     template<typename Ring, typename v1, typename v2>
02921     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02922
02923     template<>
02924     struct Embed<q32, q64> {
02925         template<typename v>
02926         using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02927     };
02928
02929     template<typename Small, typename Large>
02930     struct Embed<polynomial<Small>, polynomial<Large>> {
02931     private:
02932         template<typename v, typename i>
02933         struct at_low;
02934
02935         template<typename v, size_t i>
02936         struct at_index {
02937             using type = typename Embed<Small, Large>::template
02938             type<typename v::template coeff_at_t<i>>;
02939         };
02940
02941         template<typename v, size_t... Is>
02942         struct at_low<v, std::index_sequence<Is...>> {
02943             using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02944         };
02945
02946     public:
02947         template<typename v>
02948         using type = typename at_low<v, typename internal::make_index_sequence_reverse<v::degree +
02949         1>::type>;
02950     };
02951
02952     template<typename Ring, auto... xs>
02953     using make_int_polynomial_t = typename polynomial<Ring>::template val<
02954         typename Ring::template inject_constant_t<xs>...>;
02955
02956     template<typename Ring, auto... xs>
02957     using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02958         typename FractionField<Ring>::template inject_constant_t<xs>...>;
02959 } // namespace aerobus
02960
02961 // Taylor series and common integers (factorial, bernoulli...) appearing in Taylor coefficients
02962 namespace aerobus {
02963     namespace internal {
02964         template<typename T, size_t x, typename E = void>
02965         struct factorial {};
02966
02967         template<typename T, size_t x>
02968         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02969     private:
02970             template<typename, size_t, typename>
02971             friend struct factorial;
02972     public:
02973             using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02974             x - 1>::type>;

```

```

03015         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>());
03016     };
03017
03018     template<typename T>
03019     struct factorial<T, 0> {
03020     public:
03021         using type = typename T::one;
03022         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>());
03023     };
03024     } // namespace internal
03025
03026     template<typename T, size_t i>
03027     using factorial_t = typename internal::factorial<T, i>::type;
03028
03029     template<typename T, size_t i>
03030     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
03031
03032     namespace internal {
03033     template<typename T, size_t k, size_t n, typename E = void>
03034     struct combination_helper {};
03035
03036     template<typename T, size_t k, size_t n>
03037     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
03038         using type = typename FractionField<T>::template mul_t<
03039             typename combination_helper<T, k - 1, n - 1>::type,
03040             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
03041     };
03042
03043     template<typename T, size_t k, size_t n>
03044     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
03045         using type = typename combination_helper<T, n - k, n>::type;
03046     };
03047
03048     template<typename T, size_t n>
03049     struct combination_helper<T, 0, n> {
03050         using type = typename FractionField<T>::one;
03051     };
03052
03053     template<typename T, size_t k, size_t n>
03054     struct combination {
03055         using type = typename internal::combination_helper<T, k, n>::type::x;
03056         static constexpr typename T::inner_type value =
03057             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>());
03058     };
03059     } // namespace internal
03060
03061     template<typename T, size_t k, size_t n>
03062     using combination_t = typename internal::combination<T, k, n>::type;
03063
03064     template<typename T, size_t k, size_t n>
03065     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
03066
03067     namespace internal {
03068     template<typename T, size_t m>
03069     struct bernoulli;
03070
03071     template<typename T, typename accum, size_t k, size_t m>
03072     struct bernoulli_helper {
03073         using type = typename bernoulli_helper<
03074             T,
03075             addfractions_t<T,
03076                 accum,
03077                 mulfractions_t<T,
03078                     makefraction_t<T,
03079                         combination_t<T, k, m + 1>,
03080                         typename T::one>,
03081                         typename bernoulli<T, k>::type
03082                     >,
03083                     >,
03084                     k + 1,
03085                     m>::type;
03086     };
03087
03088     template<typename T, typename accum, size_t m>
03089     struct bernoulli_helper<T, accum, m, m> {
03090         using type = accum;
03091     };
03092
03093     template<typename T, size_t m>
03094     struct bernoulli {
03095         using type = typename FractionField<T>::template mul_t<
03096             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,

```

```

03111         makefraction_t<T,
03112             typename T::template val<static_cast<typename T::inner_type>(-1)>,
03113             typename T::template val<static_cast<typename T::inner_type>(m + 1)>
03114         >
03115     >;
03116
03117     template<typename floatType>
03118     static constexpr floatType value = type::template get<floatType>();
03119 };
03120
03121     template<typename T>
03122     struct bernoulli<T, 0> {
03123         using type = typename FractionField<T>::one;
03124
03125         template<typename floatType>
03126         static constexpr floatType value = type::template get<floatType>();
03127     };
03128 } // namespace internal
03129
03130 template<typename T, size_t n>
03131 using bernoulli_t = typename internal::bernoulli<T, n>::type;
03132
03133 template<typename FloatType, typename T, size_t n>
03134 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
03135
03136 // bell numbers
03137 namespace internal {
03138     template<typename T, size_t n, typename E = void>
03139     struct bell_helper;
03140
03141     template<typename T, size_t n>
03142     struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
03143         template<typename accum, size_t i, size_t stop>
03144         struct sum_helper {
03145             private:
03146                 using left = typename T::template mul_t<
03147                     combination_t<T, i, n-1>,
03148                     typename bell_helper<T, i>::type>;
03149                 using new_accum = typename T::template add_t<accum, left>;
03150             public:
03151                 using type = typename sum_helper<new_accum, i+1, stop>::type;
03152         };
03153
03154         template<typename accum, size_t stop>
03155         struct sum_helper<accum, stop, stop> {
03156             using type = accum;
03157         };
03158
03159         using type = typename sum_helper<typename T::zero, 0, n>::type;
03160     };
03161
03162     template<typename T>
03163     struct bell_helper<T, 0> {
03164         using type = typename T::one;
03165     };
03166
03167     template<typename T>
03168     struct bell_helper<T, 1> {
03169         using type = typename T::one;
03170     };
03171 } // namespace internal
03172
03173 template<typename T, size_t n>
03174 using bell_t = typename internal::bell_helper<T, n>::type;
03175
03176 template<typename T, size_t n>
03177 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
03178
03179 namespace internal {
03180     template<typename T, int k, typename E = void>
03181     struct alternate {};
03182
03183     template<typename T, int k>
03184     struct alternate<T, k, std::enable_if_t<k % 2 == 0>> {
03185         using type = typename T::one;
03186         static constexpr typename T::inner_type value = type::template get<typename
03187 T::inner_type>();
03188     };
03189
03190     template<typename T, int k>
03191     struct alternate<T, k, std::enable_if_t<k % 2 != 0>> {
03192         using type = typename T::template sub_t<typename T::zero, typename T::one>;
03193         static constexpr typename T::inner_type value = type::template get<typename
03194 T::inner_type>();
03195     };
03196 } // namespace internal
03197
03198
03199

```

```

03211     template<typename T, int k>
03212     using alternate_t = typename internal::alternate<T, k>::type;
03213
03214     template<typename T, size_t k>
03215     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
03216
03217     namespace internal {
03218         template<typename T, int n, int k, typename E = void>
03219         struct stirling_1_helper {};
03220
03221         template<typename T>
03222         struct stirling_1_helper<T, 0, 0> {
03223             using type = typename T::one;
03224         };
03225
03226         template<typename T, int n>
03227         struct stirling_1_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03228             using type = typename T::zero;
03229         };
03230
03231         template<typename T, int n>
03232         struct stirling_1_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03233             using type = typename T::zero;
03234         };
03235
03236         template<typename T, int n, int k>
03237         struct stirling_1_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
03238             using type = typename T::template sub_t<
03239                 typename stirling_1_helper<T, n-1, k-1>::type,
03240                 typename T::template mul_t<
03241                     typename T::template inject_constant_t<n-1>,
03242                     typename stirling_1_helper<T, n-1, k>::type
03243                 >;
03244         };
03245     } // namespace internal
03246
03247     template<typename T, int n, int k>
03248     using stirling_1_signed_t = typename internal::stirling_1_helper<T, n, k>::type;
03249
03250     template<typename T, int n, int k>
03251     using stirling_1_unsigned_t = abs_t<typename internal::stirling_1_helper<T, n, k>::type>;
03252
03253     template<typename T, int n, int k>
03254     static constexpr typename T::inner_type stirling_1_unsigned_v = stirling_1_unsigned_t<T, n, k>::v;
03255
03256     template<typename T, int n, int k>
03257     static constexpr typename T::inner_type stirling_1_signed_v = stirling_1_signed_t<T, n, k>::v;
03258
03259     namespace internal {
03260         template<typename T, int n, int k, typename E = void>
03261         struct stirling_2_helper {};
03262
03263         template<typename T, int n>
03264         struct stirling_2_helper<T, n, n, std::enable_if_t<(n >= 0)> {
03265             using type = typename T::one;
03266         };
03267
03268         template<typename T, int n>
03269         struct stirling_2_helper<T, n, 0, std::enable_if_t<(n > 0)> {
03270             using type = typename T::zero;
03271         };
03272
03273         template<typename T, int n>
03274         struct stirling_2_helper<T, 0, n, std::enable_if_t<(n > 0)> {
03275             using type = typename T::zero;
03276         };
03277
03278         template<typename T, int n, int k>
03279         struct stirling_2_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0) && (k < n)> {
03280             using type = typename T::template add_t<
03281                 typename stirling_2_helper<T, n-1, k-1>::type,
03282                 typename T::template mul_t<
03283                     typename T::template inject_constant_t<k>,
03284                     typename stirling_2_helper<T, n-1, k>::type
03285                 >;
03286         };
03287     } // namespace internal
03288
03289     template<typename T, int n, int k>
03290     using stirling_2_t = typename internal::stirling_2_helper<T, n, k>::type;
03291
03292     template<typename T, int n, int k>
03293     static constexpr typename T::inner_type stirling_2_v = stirling_2_t<T, n, k>::v;
03294
03295     namespace internal {
03296         template<typename T>
03297         struct pow_scalar {
03298

```

```

03324         template<size_t p>
03325         static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
03326             p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
03327             x * func<p/2>(x) * func<p/2>(x);
03328         }
03329     };
03330
03331     template<typename T, typename p, size_t n, typename E = void>
03332     requires IsEuclideanDomain<T>
03333     struct pow;
03334
03335     template<typename T, typename p, size_t n>
03336     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
03337         using type = typename T::template mul_t<
03338             typename pow<T, p, n/2>::type,
03339             typename pow<T, p, n/2>::type
03340         >;
03341     };
03342
03343     template<typename T, typename p, size_t n>
03344     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
03345         using type = typename T::template mul_t<
03346             p,
03347             typename T::template mul_t<
03348                 typename pow<T, p, n/2>::type,
03349                 typename pow<T, p, n/2>::type
03350             >
03351         >;
03352     };
03353
03354     template<typename T, typename p, size_t n>
03355     struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
03356 } // namespace internal
03357
03358 template<typename T, typename p, size_t n>
03359 using pow_t = typename internal::pow<T, p, n>::type;
03360
03361 template<typename T, typename p, size_t n>
03362 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
03363
03364 template<typename T, size_t p>
03365 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
03366     internal::pow_scalar<T>::template func<p>(x); }
03367
03368 namespace internal {
03369     template<typename, template<typename, size_t> typename, class>
03370     struct make_taylor_impl;
03371
03372     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
03373     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
03374         using type = typename polynomial<FractionField<T>::template val<typename coeff_at<T,
03375             Is>::type...>;
03376     };
03377 }
03378
03379 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
03380 using taylor = typename internal::make_taylor_impl<
03381     T,
03382     coeff_at,
03383     internal::make_index_sequence_reverse<deg + 1>::type;
03384
03385 namespace internal {
03386     template<typename T, size_t i>
03387     struct exp_coeff {
03388         using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03389     };
03390
03391     template<typename T, size_t i, typename E = void>
03392     struct sin_coeff_helper {};
03393
03394     template<typename T, size_t i>
03395     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03396         using type = typename FractionField<T>::zero;
03397     };
03398
03399     template<typename T, size_t i>
03400     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03401         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
03402     };
03403
03404     template<typename T, size_t i>
03405     struct sin_coeff {
03406         using type = typename sin_coeff_helper<T, i>::type;
03407     };
03408
03409     template<typename T, size_t i, typename E = void>
03410     struct sh_coeff_helper {};
03411
03412     template<typename T, size_t i, typename E = void>
03413     struct sh_coeff_helper {};

```

```

03421
03422     template<typename T, size_t i>
03423     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03424         using type = typename FractionField<T>::zero;
03425     };
03426
03427     template<typename T, size_t i>
03428     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03429         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
03430     };
03431
03432     template<typename T, size_t i>
03433     struct sh_coeff {
03434         using type = typename sh_coeff_helper<T, i>::type;
03435     };
03436
03437     template<typename T, size_t i, typename E = void>
03438     struct cos_coeff_helper {};
03439
03440     template<typename T, size_t i>
03441     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03442         using type = typename FractionField<T>::zero;
03443     };
03444
03445     template<typename T, size_t i>
03446     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03447         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
03448     };
03449
03450     template<typename T, size_t i>
03451     struct cos_coeff {
03452         using type = typename cos_coeff_helper<T, i>::type;
03453     };
03454
03455     template<typename T, size_t i, typename E = void>
03456     struct cosh_coeff_helper {};
03457
03458     template<typename T, size_t i>
03459     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03460         using type = typename FractionField<T>::zero;
03461     };
03462
03463     template<typename T, size_t i>
03464     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03465         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
03466     };
03467
03468     template<typename T, size_t i>
03469     struct cosh_coeff {
03470         using type = typename cosh_coeff_helper<T, i>::type;
03471     };
03472
03473     template<typename T, size_t i>
03474     struct geom_coeff { using type = typename FractionField<T>::one; };
03475
03476
03477     template<typename T, size_t i, typename E = void>
03478     struct atan_coeff_helper;
03479
03480     template<typename T, size_t i>
03481     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03482         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>>;
03483     };
03484
03485     template<typename T, size_t i>
03486     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03487         using type = typename FractionField<T>::zero;
03488     };
03489
03490     template<typename T, size_t i>
03491     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03492
03493     template<typename T, size_t i, typename E = void>
03494     struct asin_coeff_helper;
03495
03496     template<typename T, size_t i>
03497     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03498         using type = makefraction_t<T,
03499             factorial_t<T, i - 1>,
03500             typename T::template mul_t<
03501                 typename T::template val<i>,
03502                 T::template mul_t<
03503                     pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03504                     pow<T, factorial_t<T, i / 2>, 2
03505                 >
03506             >
03507         >>;

```

```

03508     };
03509
03510     template<typename T, size_t i>
03511     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03512         using type = typename FractionField<T>::zero;
03513     };
03514
03515     template<typename T, size_t i>
03516     struct asin_coeff {
03517         using type = typename asin_coeff_helper<T, i>::type;
03518     };
03519
03520     template<typename T, size_t i>
03521     struct lnpl_coeff {
03522         using type = makefraction_t<T,
03523             alternate_t<T, i + 1>,
03524             typename T::template val<i>;
03525     };
03526
03527     template<typename T>
03528     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03529
03530     template<typename T, size_t i, typename E = void>
03531     struct asinh_coeff_helper;
03532
03533     template<typename T, size_t i>
03534     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03535         using type = makefraction_t<T,
03536             typename T::template mul_t<
03537                 alternate_t<T, i / 2>,
03538                 factorial_t<T, i - 1>
03539             >,
03540             typename T::template mul_t<
03541                 typename T::template mul_t<
03542                     typename T::template val<i>,
03543                     pow_t<T, factorial_t<T, i / 2>, 2>
03544                 >,
03545                 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03546             >
03547         >;
03548     };
03549
03550     template<typename T, size_t i>
03551     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03552         using type = typename FractionField<T>::zero;
03553     };
03554
03555     template<typename T, size_t i>
03556     struct asinh_coeff {
03557         using type = typename asinh_coeff_helper<T, i>::type;
03558     };
03559
03560     template<typename T, size_t i, typename E = void>
03561     struct atanh_coeff_helper;
03562
03563     template<typename T, size_t i>
03564     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03565         // 1/i
03566         using type = typename FractionField<T>::template val<
03567             typename T::one,
03568             typename T::template inject_constant_t<i>;
03569     };
03570
03571     template<typename T, size_t i>
03572     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03573         using type = typename FractionField<T>::zero;
03574     };
03575
03576     template<typename T, size_t i>
03577     struct atanh_coeff {
03578         using type = typename atanh_coeff_helper<T, i>::type;
03579     };
03580
03581     template<typename T, size_t i, typename E = void>
03582     struct tan_coeff_helper;
03583
03584     template<typename T, size_t i>
03585     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03586         using type = typename FractionField<T>::zero;
03587     };
03588
03589     template<typename T, size_t i>
03590     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03591     private:
03592         // 4^((i+1)/2)
03593         using _4p = typename FractionField<T>::template inject_t<
03594             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;

```

```

03595         // 4^((i+1)/2) - 1
03596         using _4pml = typename FractionField<T>::template
sub_t<_4p, typename FractionField<T>::one>;
03597         // (-1)^((i-1)/2)
03598         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03599         using dividend = typename FractionField<T>::template mul_t<
03600             altp,
03601             FractionField<T>::template mul_t<
03602                 _4p,
03603                 FractionField<T>::template mul_t<
03604                     _4pml,
03605                     bernoulli_t<T, (i + 1)>
03606                 >
03607             >
03608         >;
03609     public:
03610         using type = typename FractionField<T>::template div_t<dividend,
03611             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03612     };
03613
03614     template<typename T, size_t i>
03615     struct tan_coeff {
03616         using type = typename tan_coeff_helper<T, i>::type;
03617     };
03618
03619     template<typename T, size_t i, typename E = void>
03620     struct tanh_coeff_helper;
03621
03622     template<typename T, size_t i>
03623     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03624         using type = typename FractionField<T>::zero;
03625     };
03626
03627     template<typename T, size_t i>
03628     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03629     private:
03630         using _4p = typename FractionField<T>::template inject_t<
03631             pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03632         using _4pml = typename FractionField<T>::template
sub_t<_4p, typename FractionField<T>::one>;
03633         using dividend =
03634             typename FractionField<T>::template mul_t<
03635                 _4p,
03636                 typename FractionField<T>::template mul_t<
03637                     _4pml,
03638                     bernoulli_t<T, (i + 1)>>::type;
03639     public:
03640         using type = typename FractionField<T>::template div_t<dividend,
03641             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
03642     };
03643
03644     template<typename T, size_t i>
03645     struct tanh_coeff {
03646         using type = typename tanh_coeff_helper<T, i>::type;
03647     };
03648 } // namespace internal
03649
03650 template<typename Integers, size_t deg>
03651 using exp = taylor<Integers, internal::exp_coeff, deg>;
03652
03653 template<typename Integers, size_t deg>
03654 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03655     exp<Integers, deg>,
03656     typename polynomial<FractionField<Integers>>::one>;
03657
03658 template<typename Integers, size_t deg>
03659 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03660
03661 template<typename Integers, size_t deg>
03662 using atan = taylor<Integers, internal::atan_coeff, deg>;
03663
03664 template<typename Integers, size_t deg>
03665 using sin = taylor<Integers, internal::sin_coeff, deg>;
03666
03667 template<typename Integers, size_t deg>
03668 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03669
03670 template<typename Integers, size_t deg>
03671 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03672
03673 template<typename Integers, size_t deg>
03674 using cos = taylor<Integers, internal::cos_coeff, deg>;
03675
03676 template<typename Integers, size_t deg>
03677 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03678
03679 template<typename Integers, size_t deg>

```



```

03714     using asin = taylor<Integers, internal::asin_coeff, deg>;
03715
03720     template<typename Integers, size_t deg>
03721     using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03722
03727     template<typename Integers, size_t deg>
03728     using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03729
03734     template<typename Integers, size_t deg>
03735     using tan = taylor<Integers, internal::tan_coeff, deg>;
03736
03741     template<typename Integers, size_t deg>
03742     using tanh = taylor<Integers, internal::tanh_coeff, deg>;
03743 } // namespace aerobus
03744
03745 // continued fractions
03746 namespace aerobus {
03749     template<int64_t... values>
03750     struct ContinuedFraction {};
03751
03754     template<int64_t a0>
03755     struct ContinuedFraction<a0> {
03757         using type = typename q64::template inject_constant_t<a0>;
03759         static constexpr double val = static_cast<double>(a0);
03760     };
03761
03765     template<int64_t a0, int64_t... rest>
03766     struct ContinuedFraction<a0, rest...> {
03768         using type = q64::template add_t<
03769             typename q64::template inject_constant_t<a0>,
03770             typename q64::template div_t<
03771                 typename q64::one,
03772                 typename ContinuedFraction<rest...>::type
03773             >;
03774
03776         static constexpr double val = type::template get<double>();
03777     };
03778
03782     using PI_fraction =
03783     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03784     using E_fraction =
03785     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03786     using SQRT2_fraction =
03787     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03788     using SQRT3_fraction =
03789     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03789 } // namespace aerobus
03790
03791 // known polynomials
03792 namespace aerobus {
03793     // CChebyshev
03794     namespace internal {
03795         template<int kind, size_t deg, typename I>
03796         struct chebyshev_helper {
03797             using type = typename polynomial<I>::template sub_t<
03798                 typename polynomial<I>::template mul_t<
03799                     typename polynomial<I>::template mul_t<
03800                         typename polynomial<I>::template inject_constant_t<2>,
03801                         typename polynomial<I>::X>,
03802                         typename chebyshev_helper<kind, deg - 1, I>::type
03803                     >,
03804                     typename chebyshev_helper<kind, deg - 2, I>::type
03805                 >;
03806         };
03807
03808         template<typename I>
03809         struct chebyshev_helper<1, 0, I> {
03810             using type = typename polynomial<I>::one;
03811         };
03812
03813         template<typename I>
03814         struct chebyshev_helper<1, 1, I> {
03815             using type = typename polynomial<I>::X;
03816         };
03817
03818         template<typename I>
03819         struct chebyshev_helper<2, 0, I> {
03820             using type = typename polynomial<I>::one;
03821         };
03822
03823         template<typename I>
03824         struct chebyshev_helper<2, 1, I> {
03825             using type = typename polynomial<I>::template mul_t<
03826                 typename polynomial<I>::template inject_constant_t<2>,
03827                 typename polynomial<I>::X>;
03828         };

```

```

03829     } // namespace internal
03830
03831     // Laguerre
03832     namespace internal {
03833         template<size_t deg, typename I>
03834         struct laguerre_helper {
03835             using Q = FractionField<I>;
03836             using PQ = polynomial<Q>;
03837
03838             private:
03839                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)lkm2)
03840                 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03841                 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03842                 // -x + 2k-1
03843                 using p = typename PQ::template val<
03844                     typename Q::template inject_constant_t<-1>,
03845                     typename Q::template inject_constant_t<2 * deg - 1>;
03846                 // 1/n
03847                 using factor = typename PQ::template inject_ring_t<
03848                     typename Q::template val<typename I::one, typename I::template
inject_constant_t<deg>>>;
03849
03850             public:
03851                 using type = typename PQ::template mul_t <
03852                     factor,
03853                     typename PQ::template sub_t<
03854                         typename PQ::template mul_t<
03855                             p,
03856                             lnm1
03857                         >,
03858                     typename PQ::template mul_t<
03859                         typename PQ::template inject_constant_t<deg-1>,
03860                         lnm2
03861                     >
03862                 >
03863                 >;
03864         };
03865
03866         template<typename I>
03867         struct laguerre_helper<0, I> {
03868             using type = typename polynomial<FractionField<I>::one>;
03869         };
03870
03871         template<typename I>
03872         struct laguerre_helper<1, I> {
03873             private:
03874                 using PQ = polynomial<FractionField<I>;
03875             public:
03876                 using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03877         };
03878     } // namespace internal
03879
03880     // Bernstein
03881     namespace internal {
03882         template<size_t i, size_t m, typename I, typename E = void>
03883         struct bernstein_helper {};
03884
03885         template<typename I>
03886         struct bernstein_helper<0, 0, I> {
03887             using type = typename polynomial<I>::one;
03888         };
03889
03890         template<size_t i, size_t m, typename I>
03891         struct bernstein_helper<i, m, I, std::enable_if_t<
03892             (m > 0) && (i == 0)>> {
03893             private:
03894                 using P = polynomial<I>;
03895             public:
03896                 using type = typename P::template mul_t<
03897                     typename P::template sub_t<typename P::one, typename P::X>,
03898                     typename bernstein_helper<i, m-1, I>::type>;
03899         };
03900
03901         template<size_t i, size_t m, typename I>
03902         struct bernstein_helper<i, m, I, std::enable_if_t<
03903             (m > 0) && (i == m)>> {
03904             private:
03905                 using P = polynomial<I>;
03906             public:
03907                 using type = typename P::template mul_t<
03908                     typename P::X,
03909                     typename bernstein_helper<i-1, m-1, I>::type>;
03910         };
03911
03912         template<size_t i, size_t m, typename I>
03913         struct bernstein_helper<i, m, I, std::enable_if_t<
03914             (m > 0) && (i > 0) && (i < m)>> {

```

```

03915     private:
03916         using P = polynomial<I>;
03917     public:
03918         using type = typename P::template add_t<
03919             typename P::template mul_t<
03920                 typename P::template sub_t<typename P::one, typename P::X>,
03921                 typename bernstein_helper<i, m-1, I>::type>,
03922                 typename P::template mul_t<
03923                     typename P::X,
03924                     typename bernstein_helper<i-1, m-1, I>::type>
03925             >>;
03926 } // namespace internal
03927
03928 // AllOne polynomials
03929 namespace internal {
03930     template<size_t deg, typename I>
03931     struct AllOneHelper {
03932         using type = aerobus::add_t<
03933             typename polynomial<I>::one,
03934             typename aerobus::mul_t<
03935                 typename polynomial<I>::X,
03936                 typename AllOneHelper<deg-1, I>::type
03937             >>;
03938     };
03939
03940     template<typename I>
03941     struct AllOneHelper<0, I> {
03942         using type = typename polynomial<I>::one;
03943     };
03944 } // namespace internal
03945
03946 // Bessel polynomials
03947 namespace internal {
03948     template<size_t deg, typename I>
03949     struct BesselHelper {
03950     private:
03951         using P = polynomial<I>;
03952         using factor = typename P::template monomial_t<
03953             typename I::template inject_constant_t<(2*deg - 1)>,
03954             1>;
03955     public:
03956         using type = typename P::template add_t<
03957             typename P::template mul_t<
03958                 factor,
03959                 typename BesselHelper<deg-1, I>::type
03960             >,
03961             typename BesselHelper<deg-2, I>::type
03962         >;
03963     };
03964
03965     template<typename I>
03966     struct BesselHelper<0, I> {
03967         using type = typename polynomial<I>::one;
03968     };
03969
03970     template<typename I>
03971     struct BesselHelper<1, I> {
03972     private:
03973         using P = polynomial<I>;
03974     public:
03975         using type = typename P::template add_t<
03976             typename P::one,
03977             typename P::X
03978         >;
03979     };
03980 } // namespace internal
03981
03982 namespace known_polynomials {
03983     enum hermite_kind {
03984         probabilist,
03985         physicist
03986     };
03987 }
03988
03989 // hermite
03990 namespace internal {
03991     template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03992     struct hermite_helper {};
03993
03994     template<size_t deg, typename I>
03995     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03996     private:
03997         using hnm1 = typename hermite_helper<deg - 1,
03998             known_polynomials::hermite_kind::probabilist, I>::type;
03999         using hnm2 = typename hermite_helper<deg - 2,
04000             known_polynomials::hermite_kind::probabilist, I>::type;
04001     };
04002

```

```

04003     public:
04004         using type = typename polynomial<I>::template sub_t<
04005             typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
04006             typename polynomial<I>::template mul_t<
04007                 typename polynomial<I>::template inject_constant_t<deg - 1>,
04008                 hnm2
04009             >
04010         >;
04011     };
04012
04013     template<size_t deg, typename I>
04014     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
04015     private:
04016         using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
I>::type;
04017         using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
I>::type;
04018
04019     public:
04020         using type = typename polynomial<I>::template sub_t<
04021             // 2X Hn-1
04022             typename polynomial<I>::template mul_t<
04023                 typename pi64::val<typename I::template inject_constant_t<2>,
04024                 typename I::zero>, hnm1>,
04025
04026             typename polynomial<I>::template mul_t<
04027                 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
04028                 hnm2
04029             >
04030         >;
04031     };
04032
04033     template<typename I>
04034     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
04035     private:
04036         using type = typename polynomial<I>::one;
04037     };
04038
04039     template<typename I>
04040     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
04041     private:
04042         using type = typename polynomial<I>::X;
04043     };
04044
04045     template<typename I>
04046     struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
04047     private:
04048         using type = typename pi64::one;
04049     };
04050
04051     template<typename I>
04052     struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
04053     private:
04054         // 2X
04055         using type = typename polynomial<I>::template val<
04056             typename I::template inject_constant_t<2>,
04057             typename I::zero>;
04058     };
04059 } // namespace internal
04060
04061 // legendre
04062 namespace internal {
04063     template<size_t n, typename I>
04064     struct legendre_helper {
04065     private:
04066         using Q = FractionField<I>;
04067         using PQ = polynomial<Q>;
04068         // 1/n constant
04069         // (2n-1)/n X
04070         using fact_left = typename PQ::template monomial_t<
04071             makefraction_t<I,
04072                 typename I::template inject_constant_t<2*n-1>,
04073                 typename I::template inject_constant_t<n>
04074             >,
04075             1>;
04076         // (n-1) / n
04077         using fact_right = typename PQ::template val<
04078             makefraction_t<I,
04079                 typename I::template inject_constant_t<n-1>,
04080                 typename I::template inject_constant_t<n>>;
04081     };
04082
04083     public:
04084         using type = PQ::template sub_t<
04085             typename PQ::template mul_t<
04086                 fact_left,
04087                 typename legendre_helper<n-1, I>::type
04088             >,
04089             typename PQ::template mul_t<
04090                 fact_right,
04091                 typename legendre_helper<n-2, I>::type
04092             >
04093         >;

```

```

04088         >;
04089     };
04090
04091     template<typename I>
04092     struct legendre_helper<0, I> {
04093         using type = typename polynomial<FractionField<I>::one>;
04094     };
04095
04096     template<typename I>
04097     struct legendre_helper<1, I> {
04098         using type = typename polynomial<FractionField<I>::X>;
04099     };
04100 } // namespace internal
04101
04102 // bernoulli polynomials
04103 namespace internal {
04104     template<size_t n>
04105     struct bernoulli_coeff {
04106         template<typename T, size_t i>
04107         struct inner {
04108             private:
04109                 using F = FractionField<T>;
04110             public:
04111                 using type = typename F::template mul_t<
04112                     typename F::template inject_ring_t<combination_t<T, i, n>,
04113                     bernoulli_t<T, n-i>
04114                 >;
04115         };
04116     };
04117 } // namespace internal
04118
04119 namespace internal {
04120     template<size_t n>
04121     struct touchard_coeff {
04122         template<typename T, size_t i>
04123         struct inner {
04124             using type = stirling_2_t<T, n, i>;
04125         };
04126     };
04127 } // namespace internal
04128
04129 namespace internal {
04130     template<typename I = aerobus::i64>
04131     struct AbelHelper {
04132     private:
04133         using P = aerobus::polynomial<I>;
04134
04135     public:
04136         // to keep recursion working, we need to operate on a*n and not just a
04137         template<size_t deg, I::inner_type an>
04138         struct Inner {
04139             // abel(n, a) = (x-an) * abel(n-1, a)
04140             using type = typename aerobus::mul_t<
04141                 typename Inner<deg-1, an>::type,
04142                 typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
04143             >;
04144         };
04145
04146         // abel(0, a) = 1
04147         template<I::inner_type an>
04148         struct Inner<0, an> {
04149             using type = P::one;
04150         };
04151
04152         // abel(1, a) = X
04153         template<I::inner_type an>
04154         struct Inner<1, an> {
04155             using type = P::X;
04156         };
04157     };
04158 } // namespace internal
04159
04160 namespace known_polynomials {
04161
04162     template<size_t n, auto a, typename I = aerobus::i64>
04163     using abel = typename internal::AbelHelper<I>::template Inner<n, a*n>::type;
04164
04165     template<size_t deg, typename I = aerobus::i64>
04166     using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
04167
04168     template<size_t deg, typename I = aerobus::i64>
04169     using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
04170
04171     template<size_t deg, typename I = aerobus::i64>
04172     using laguerre = typename internal::laguerre_helper<deg, I>::type;
04173
04174     template<size_t deg, typename I = aerobus::i64>

```

```

04215         using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
I>::type;
04216
04223         template <size_t deg, typename I = aerobus::i64>
04224         using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
04225
04236         template<size_t i, size_t m, typename I = aerobus::i64>
04237         using bernstein = typename internal::bernstein_helper<i, m, I>::type;
04238
04248         template<size_t deg, typename I = aerobus::i64>
04249         using legendre = typename internal::legendre_helper<deg, I>::type;
04250
04260         template<size_t deg, typename I = aerobus::i64>
04261         using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
04262
04269         template<size_t deg, typename I = aerobus::i64>
04270         using allone = typename internal::AllOneHelper<deg, I>::type;
04271
04279         template<size_t deg, typename I = aerobus::i64>
04280         using bessel = typename internal::BesselHelper<deg, I>::type;
04281
04289         template<size_t deg, typename I = aerobus::i64>
04290         using touchard = taylor<I, internal::touchard_coeff<deg>::template inner, deg>;
04291     } // namespace known_polynomials
04292 } // namespace aerobus
04293
04294 namespace aerobus {
04295     namespace libm {
04296         namespace internal {
04297             template<typename T>
04298             struct exp2_poly;
04299
04300             template<>
04301             struct exp2_poly<double> {
04302                 using type = aerobus::polynomial<aerobus::q64>::template val<
04303                     aerobus::make_q64_t<388, 10641171255427>,
04304                     aerobus::make_q64_t<2296, 5579977897299>,
04305                     aerobus::make_q64_t<4963, 697799188641>,
04306                     aerobus::make_q64_t<15551, 152884735543>,
04307                     aerobus::make_q64_t<21273, 16096444733>,
04308                     aerobus::make_q64_t<683500, 44811710339>,
04309                     aerobus::make_q64_t<44397656049575, 288230376151711744>,
04310                     aerobus::make_q64_t<192156823709857, 144115188075855872>,
04311                     aerobus::make_q64_t<693059242663871, 72057594037927936>,
04312                     aerobus::make_q64_t<1999746264802375, 36028797018963968>,
04313                     aerobus::make_q64_t<4327536028902111, 18014398509481984>,
04314                     aerobus::make_q64_t<6243314768165359, 9007199254740992>,
04315                     aerobus::q64::one>;
04316             };
04317
04318             template<>
04319             struct exp2_poly<float> {
04320                 using type = aerobus::polynomial<aerobus::q32>::template val<
04321                     aerobus::make_q32_t<8, 375115>,
04322                     aerobus::make_q32_t<30, 208117>,
04323                     aerobus::make_q32_t<109, 81261>,
04324                     aerobus::make_q32_t<5161841, 536870912>,
04325                     aerobus::make_q32_t<3724869, 67108864>,
04326                     aerobus::make_q32_t<16121323, 67108864>,
04327                     aerobus::make_q32_t<1453635, 2097152>,
04328                     aerobus::q32::one>;
04329             };
04330
04331             #ifdef WITH_CUDA_FP16
04332             template<>
04333             struct exp2_poly<__half> {
04334                 using type = aerobus::polynomial<aerobus::q16>::template val<
04335                     aerobus::make_q16_t<3, 212>,
04336                     aerobus::make_q16_t<13, 256>,
04337                     aerobus::make_q16_t<31, 128>,
04338                     aerobus::make_q16_t<1419, 2048>,
04339                     aerobus::q16::one>;
04340             };
04341
04342             template<>
04343             struct exp2_poly<__half2> {
04344                 using type = aerobus::polynomial<aerobus::q16>::template val<
04345                     aerobus::make_q16_t<3, 212>,
04346                     aerobus::make_q16_t<13, 256>,
04347                     aerobus::make_q16_t<31, 128>,
04348                     aerobus::make_q16_t<1419, 2048>,
04349                     aerobus::q16::one>;
04350             };
04351             #endif
04352
04353             template<typename P>
04354             struct sin_poly;

```

```

04355
04356     template<>
04357     struct sin_poly<double> {
04358         // approximates sin(x)/x over [-pi/4, pi/4] with precision 9.318608669702093e-20
04359         using type = typename aerobus::polynomial<aerobus::q64>::simplify_t<
04360             typename aerobus::polynomial<aerobus::q64>:: template val<
04361                 aerobus::make_q64_t<-43, 1042171195712159>,
04362                 aerobus::make_q64_t<-89, 136637767615782>,
04363                 aerobus::make_q64_t<123, 766493207966>,
04364                 aerobus::make_q64_t<-18133, 723813242548>,
04365                 aerobus::make_q64_t<122341, 44395102410>,
04366                 aerobus::make_q64_t<-11252871, 56714469841>,
04367                 aerobus::make_q64_t<2401919801264179, 288230376151711744>,
04368                 aerobus::make_q64_t<-6004799503160661, 36028797018963968>,
04369                 aerobus::q64::one>;
04370     };
04371
04372     template<>
04373     struct sin_poly<float> {
04374         // approximates sin(x)/x over [-pi/4, pi/4] with precision 1.941356e-10
04375         // must be evaluated in x*x as we removed half the coefficients to have a dense
04376     polynomial
04377         using type = typename aerobus::polynomial<aerobus::q32>::simplify_t<
04378             typename aerobus::polynomial<aerobus::q32>:: template val<
04379                 aerobus::make_q32_t<1, 357073>,
04380                 aerobus::make_q32_t<-67, 337533>,
04381                 aerobus::make_q32_t<4473945, 536870912>,
04382                 aerobus::make_q32_t<-11184811, 67108864>,
04383                 aerobus::q32::one>;
04384     };
04385
04386     #ifdef WITH_CUDA_FP16
04387     template<>
04388     struct sin_poly<__half> {
04389         // approximates sin(x)/x over [-pi/4, pi/4] with precision 6.389e-6
04390         // must be evaluated in x*x as we removed half the coefficients to have a dense
04391     polynomial
04392         using type = typename aerobus::polynomial<aerobus::q16>::simplify_t<
04393             typename aerobus::polynomial<aerobus::q16>:: template val<
04394                 aerobus::make_q16_t<1, 123>,
04395                 aerobus::make_q16_t<-1365, 8192>,
04396                 aerobus::q16::one>;
04397     };
04398
04399     template<>
04400     struct sin_poly<__half2> {
04401         // approximates sin(x)/x over [-pi/4, pi/4] with precision 6.389e-6
04402         // must be evaluated in x*x as we removed half the coefficients to have a dense
04403     polynomial
04404         using type = typename aerobus::polynomial<aerobus::q16>::simplify_t<
04405             typename aerobus::polynomial<aerobus::q16>:: template val<
04406                 aerobus::make_q16_t<1, 123>,
04407                 aerobus::make_q16_t<-1365, 8192>,
04408                 aerobus::q16::one>;
04409     };
04410
04411     #endif
04412
04413     template<typename T>
04414     struct cos_poly;
04415
04416     template <>
04417     struct cos_poly<double> {
04418         // approximates (cos(x) - 1 + x^2)/x^4
04419         // must be evaluated in x*x as we removed half the coefficients to have a dense
04420     polynomial
04421         using type = typename aerobus::polynomial<aerobus::q64>::simplify_t<
04422             typename aerobus::polynomial<aerobus::q64>:: template val<
04423                 aerobus::make_q64_t<14, 407833194230757>,
04424                 aerobus::make_q64_t<-283, 24728864074278>,
04425                 aerobus::make_q64_t<136, 65144855767>,
04426                 aerobus::make_q64_t<-4089, 14838163607>,
04427                 aerobus::make_q64_t<452396, 18240606721>,
04428                 aerobus::make_q64_t<-6405119470037555, 4611686018427387904>,
04429                 aerobus::make_q64_t<6004799503160661, 144115188075855872>;
04430     };
04431
04432     template <>
04433     struct cos_poly<float> {
04434         // approximates (cos(x) - 1 + x^2)/x^4
04435         // must be evaluated in x*x as we removed half the coefficients to have a dense
04436     polynomial
04437         using type = typename aerobus::polynomial<aerobus::q32>::simplify_t<
04438             typename aerobus::polynomial<aerobus::q32>:: template val<
04439                 aerobus::make_q32_t<-1, 3112816>,
04440                 aerobus::make_q32_t<1, 40237>,
04441                 aerobus::make_q32_t<-119, 85679>,
04442                 aerobus::make_q32_t<11184811, 268435456>;
04443     };

```

```

04437         };
04438
04439 #ifdef WITH_CUDA_FP16
04440     template<>
04441     struct cos_poly<__half> {
04442         // approximates (cos(x) - 1 + x^2)/x^4
04443         // must be evaluated in x*x as we removed half the coefficients to have a dense
04444         polynomial
04445             using type = typename aerobus::polynomial<aerobus::q16>::simplify_t<
04446                 typename aerobus::polynomial<aerobus::q16>::template val<
04447                     aerobus::make_q16_t<-1, 16321>,
04448                     aerobus::make_q16_t<-1, 756>,
04449                     aerobus::make_q16_t<1, 24>>>;
04450     };
04451
04452     template<>
04453     struct cos_poly<__half2> {
04454         // approximates (cos(x) - 1 + x^2)/x^4
04455         // must be evaluated in x*x as we removed half the coefficients to have a dense
04456         polynomial
04457             using type = typename aerobus::polynomial<aerobus::q16>::simplify_t<
04458                 typename aerobus::polynomial<aerobus::q16>::template val<
04459                     aerobus::make_q16_t<-1, 16321>,
04460                     aerobus::make_q16_t<-1, 756>,
04461                     aerobus::make_q16_t<1, 24>>>;
04462     };
04463 #endif
04464 } // namespace internal
04465
04466 template<typename T>
04467 static T exp2(const T&x) {
04468     using poly = internal::exp2_poly<T>::type;
04469     if (x >= (aerobus::internal::arithmetic_helpers<T>::zero) &&
04470         x < (aerobus::internal::arithmetic_helpers<T>::one)) {
04471         return poly::eval(x);
04472     } else {
04473         // TODO(JeWaVe): handle denormals
04474         auto i = static_cast<typename aerobus::internal::arithmetic_helpers<T>::integers>(
04475             aerobus::meta_libm<T>::floor(x));
04476         T eps = x - i;
04477         T mantissa = poly::eval(eps);
04478         uint64_t* p = reinterpret_cast<uint64_t*>(&mantissa);
04479         uint64_t exponent = (*p >> 52) & 0x7FF;
04480         exponent += i;
04481         *p &= ~(0x7FFULL << 52);
04482         *p |= (exponent & 0x7FF) << 52;
04483         return mantissa;
04484     }
04485 }
04486
04487 template<typename T>
04488 static T cos(const T& x);
04489
04490 template<typename T>
04491 static T sin(const T& x) {
04492     using upper_type = aerobus::internal::arithmetic_helpers<T>::upper_type;
04493     using constants = aerobus::internal::arithmetic_helpers<upper_type>;
04494     using poly = internal::sin_poly<T>::type;
04495     constexpr upper_type zero = constants::zero;
04496     const upper_type pi_4 = constants::pi_4;
04497     const upper_type pi = constants::pi;
04498     const upper_type two_pi = constants::two_pi;
04499     const upper_type pi_2 = constants::pi_2;
04500     upper_type X = static_cast<upper_type>(x);
04501     // TODO(JeWaVe): infinities
04502     if (X == 0 || X == -0) {
04503         return X;
04504     } else if (X < zero) {
04505         return -sin(static_cast<T>(-X));
04506     } else if (X <= std::numeric_limits<T>::epsilon() || std::isnan(X)) {
04507         return X;
04508     } else if (X > pi_4 && X <= pi_2) {
04509         return aerobus::libm::cos(static_cast<T>(pi_2 - X));
04510     } else if (X >= pi_2 && X <= pi) {
04511         return sin(static_cast<T>(pi - X));
04512     } else if (X > pi && X <= two_pi) {
04513         return -sin(static_cast<T>(X - pi));
04514     } else if (X > two_pi) {
04515         T i = static_cast<T>(aerobus::meta_libm<upper_type>::fmod(X, two_pi));
04516         return sin(i);
04517     } else {
04518         return X * poly::eval(X*X);
04519     }
04520 }
04521
04522 template<typename T>

```



```

04540     static T cos(const T& x) {
04541         using poly = internal::cos_poly<T>::type;
04542         using upper_type = aerobus::internal::arithmetic_helpers<T>::upper_type;
04543         using constants = aerobus::internal::arithmetic_helpers<upper_type>;
04544         constexpr upper_type zero = constants::zero;
04545         constexpr upper_type half = constants::half;
04546         constexpr upper_type one = constants::one;
04547         constexpr upper_type pi_4 = constants::pi_4;
04548         constexpr upper_type pi = constants::pi;
04549         constexpr upper_type two_pi = constants::two_pi;
04550         constexpr upper_type pi_2 = constants::pi_2;
04551         upper_type X = static_cast<upper_type>(x);
04552         // domain reduction
04553         if (x == 0 || x == -0) {
04554             return aerobus::internal::arithmetic_helpers<T>::one;
04555         } else if (x < zero) {
04556             return cos(static_cast<T>(-X));
04557         } else if (std::isnan(x)) {
04558             return x;
04559         } else if (x <= std::numeric_limits<T>::epsilon()) {
04560             return constants::one;
04561         } else if (X > pi_4 && X <= pi_2) {
04562             return aerobus::libm::sin(static_cast<T>(pi_2 - X));
04563         } else if (X >= pi_2 && X < pi) {
04564             return -cos(static_cast<T>(X - pi));
04565         } else if (X >= pi && X < two_pi) {
04566             return -cos(static_cast<T>(pi - X));
04567         } else if (X >= two_pi) {
04568             T i = static_cast<T>(aerobus::meta_libm<upper_type>::fmod(X, two_pi));
04569             return cos(i);
04570         } else {
04571             T x2 = x*x;
04572             T x4 = x2*x2;
04573             return x4 * poly::eval(x2) + one - x2*half;
04574         }
04575     } // namespace libm
04576 } // namespace aerobus
04577
04578 #ifdef AEROBUS_CONWAY_IMPORTS
04579 // conway polynomials
04580 namespace aerobus {
04581     template<int p, int n>
04582     struct ConwayPolynomial {};
04583
04584 #ifndef DO_NOT_DOCUMENT
04585     #define ZPZV ZPZ::template val
04586     #define POLYV aerobus::polynomial<ZPZ>::template val
04587     template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>; }; // NOLINT
04588     template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04589     template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04590     template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04591     template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04592     template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04593     template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04594     template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04595     template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04596     template<> struct ConwayPolynomial<2, 10> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04597     template<> struct ConwayPolynomial<2, 11> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04598     template<> struct ConwayPolynomial<2, 12> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04599     template<> struct ConwayPolynomial<2, 13> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
04600     template<> struct ConwayPolynomial<2, 14> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
04601     template<> struct ConwayPolynomial<2, 15> { using ZPZ = aerobus::zpz<2>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>; }; // NOLINT
04602     template<> struct ConwayPolynomial<2, 16> { using ZPZ = aerobus::zpz<2>; using type =

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```

06513     template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<738>, ZPZV<276>, ZPZV<530>, ZPZV<5>; }; //
NOLINT
06514     template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<858>, ZPZV<87>, ZPZV<978>;
}; // NOLINT
06515     template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<985>; }; // NOLINT
06516     template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<989>, ZPZV<6>; }; // NOLINT
06517     template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<985>; }; // NOLINT
06518     template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<10>, ZPZV<794>, ZPZV<6>; }; // NOLINT
06519     template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<985>; }; // NOLINT
06520     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
06521     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
06522     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
06523     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
06524     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
06525     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
06526     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
06527     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
06528     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
06529     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
06530     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
06531     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
06532     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
06533 #endif // DO_NOT_DOCUMENT
06534 } // namespace aerobus
06535 #endif // AEROBUS_CONWAY_IMPORTS
06536
06537 #endif // __INC_AEROBUS__ // NOLINT

```

## 9.4 src/examples.h File Reference

## 9.5 examples.h

[Go to the documentation of this file.](#)

```

00001 #ifndef SRC_EXAMPLES_H_
00002 #define SRC_EXAMPLES_H_
00050 #endif // SRC_EXAMPLES_H_

```





# Chapter 10

## Examples

### 10.1 examples/hermite.cpp

How to use `aerobus::known_polynomials::hermite_phys` polynomials

```
#include <cmath>
#include <iostream>
#include "../src/aerobus.h"

namespace standardlib {
    double H3(double x) {
        return 8 * std::pow(x, 3) - 12 * x;
    }

    double H4(double x) {
        return 16 * std::pow(x, 4) - 48 * x * x + 12;
    }
}

namespace aerobuslib {
    double H3(double x) {
        return 8 * aerobus::pow_scalar<double, 3>(x) - 12 * x;
    }

    double H4(double x) {
        return 16 * aerobus::pow_scalar<double, 4>(x) - 48 * x * x + 12;
    }
}

int main() {
    std::cout << std::hermite(3, 10) << '=' << standardlib::H3(10) << '\n'
              << std::hermite(4, 10) << '=' << standardlib::H4(10) << '\n';
    std::cout << aerobus::known_polynomials::hermite_phys<3>::eval(10) << '=' << aerobuslib::H3(10) << '\n'
              << aerobus::known_polynomials::hermite_phys<4>::eval(10) << '=' << aerobuslib::H4(10) << '\n';
}
```

### 10.2 examples/custom\_taylor.cpp

How to implement your own Taylor serie using `aerobus::taylor`

```
#include <cmath>
#include <iostream>
#include <iomanip>
#include "../src/aerobus.h"

template<typename T, size_t i>
struct my_coeff {
    using type = aerobus::makefraction_t<T, aerobus::bell_t<T, i>, aerobus::factorial_t<T, i>>;
};

template<size_t deg>
```

```
using F = aerobus::taylor<aerobus::i64, my_coeff, deg>;

int main() {
    constexpr double x = F<15>::eval(0.1);
    double xx = std::exp(std::exp(0.1) - 1);
    std::cout << std::setprecision(18) << x << " == " << xx << std::endl;
}
```

## 10.3 examples/fp16.cu

How to leverage CUDA `__half` and `__half2` 16 bits floating points number using `aerobus::i16` Warning : due to an NVIDIA bug (lack of `constexpr` operators), performance is not good

// TO compile with `nvcc -O3 -std=c++20 -arch=sm_90 fp16.cu`  
 // Beforehand, you need to modify `cuda_fp16.h` by adding `__CUDA_FP16_CONSTEXPR__` to line 5039 (version 12.6)  
 #include <stdio>

```
#define WITH_CUDA_FP16
#include "../src/aerobus.h"
```

```
/*
You may want to change int_type to aerobus::i32 (or i64) and float_type to float (resp. double)
*/
using int_type = aerobus::i16;
using float_type = __half2;
```

```
constexpr size_t N = 1 << 26;
```

```
template<typename T>
struct ExpmlDegree;
```

```
template<>
struct ExpmlDegree<double> {
    static constexpr size_t val = 18;
};
```

```
template<>
struct ExpmlDegree<float> {
    static constexpr size_t val = 11;
};
```

```
template<>
struct ExpmlDegree<__half2> {
    static constexpr size_t val = 6;
};
```

```
template<>
struct ExpmlDegree<__half> {
    static constexpr size_t val = 6;
};
```

```
double rand(double min, double max) {
    double range = (max - min);
    double div = RAND_MAX / range;
    return min + (rand() / div); // NOLINT
}
```

```
template<typename T>
struct GetRandT;
```

```
template<>
struct GetRandT<double> {
    static double func(double min, double max) {
        return rand(min, max);
    }
};
```

```
template<>
struct GetRandT<float> {
    static float func(double min, double max) {
        return (float) rand(min, max);
    }
};
```

```
template<>
struct GetRandT<__half2> {
    static __half2 func(double min, double max) {
        return __half2(__float2half((float)rand(min, max)), __float2half((float)rand(min, max)));
    }
};
```

```
template<>
```

```

struct GetRandT<__half>
{
    static __half func(double min, double max) {
        return __float2half((float)rand(min, max));
    }
};

using EXPM1 = aerobus::expm1<int_type, Expm1Degree<float_type>::val>;

__device__ INLINED float_type f(float_type x) {
    return EXPM1::eval(x);
}

__global__ void run(size_t N, float_type* in, float_type* out) {
    for(size_t i = threadIdx.x + blockDim.x * blockIdx.x; i < N; i += blockDim.x * gridDim.x) {
        // fp16 FMA pipeline is quite wide so we need to feed it with a LOT of computations
        out[i] = f(f(f(f(f(f(f(f(f(f(f(f(f(f(f(in[i])))))))))))))));
    }
}

#define cudaErrorCheck(ans) { gpuAssert((ans), __FILE__, __LINE__); }
inline void gpuAssert(cudaError_t code, const char *file, int line, bool abort=true)
{
    if (code != cudaSuccess)
    {
        fprintf(stderr, "GPUassert: %s %s %d\n", cudaGetErrorString(code), file, line);
        if (abort) exit(code);
    }
}

int main() {
    // configure CUDA devices
    int deviceCount;
    int device = -1;
    int maxProcCount = 0;
    cudaErrorCheck(cudaGetDeviceCount(&deviceCount));
    for(int i = 0; i < deviceCount; ++i) {
        cudaDeviceProp prop;
        cudaErrorCheck(cudaGetDeviceProperties(&prop, i));
        int procCount = prop.multiProcessorCount;
        if(procCount > maxProcCount) {
            maxProcCount = procCount;
            device = i;
        }
    }

    if(device == -1) {
        ::printf("CANNOT FIND CUDA CAPABLE DEVICE -- aborting\n");
        ::abort();
    }

    cudaErrorCheck(cudaSetDevice(device));
    int blockSize; // The launch configurator returned block size
    int minGridSize; // The minimum grid size needed to achieve the
                    // maximum occupancy for a full device launch

    cudaErrorCheck(cudaOccupancyMaxPotentialBlockSize( &minGridSize, &blockSize, &run, 0, 0));

    ::printf("configure launch bounds to %d-%d\n", minGridSize, blockSize);

    // allocate and populate memory
    float_type *d_in, *d_out;
    cudaErrorCheck(cudaMalloc<float_type>(&d_in, N * sizeof(float_type)));
    cudaErrorCheck(cudaMalloc<float_type>(&d_out, N * sizeof(float_type)));

    float_type *in = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));
    float_type *out = reinterpret_cast<float_type*>(malloc(N * sizeof(float_type)));

    for(size_t i = 0; i < N; ++i) {
        in[i] = GetRandT<float_type>::func(-0.01, 0.01);
    }

    cudaErrorCheck(cudaMemcpy(d_in, in, N * sizeof(float_type), cudaMemcpyHostToDevice));

    // execute kernel and get memory back from device
    run<<minGridSize, blockSize>>(N, d_in, d_out);
    cudaErrorCheck(cudaPeekAtLastError());
    cudaErrorCheck(cudaMemcpy(out, d_out, N * sizeof(float_type), cudaMemcpyDeviceToHost));

    cudaErrorCheck(cudaFree(d_in));
    cudaErrorCheck(cudaFree(d_out));
}

// Example of generated SASS :

/*
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;

```

```
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R7, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R7, R5, R7, 0.008331298828125, 0.008331298828125 ;
HFMA2 R7, R5, R7, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R7, R5, R7, 0.1666259765625, 0.1666259765625 ;
HFMA2 R7, R5, R7, 0.5, 0.5 ;
HFMA2.MMA R7, R5, R7, 1, 1 ;
HFMA2 R7, R5, R7, RZ.H0_H0 ;
HFMA2.MMA R5, R7, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R7, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R7, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R7, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R7, R5, 0.5, 0.5 ;
HFMA2 R5, R7, R5, 1, 1 ;
HFMA2.MMA R5, R7, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R5, R6, R5, RZ ;
HFMA2 R6, R5, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R6, R5, R6, 0.008331298828125, 0.008331298828125 ;
HFMA2 R6, R5, R6, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R6, R5, R6, 0.1666259765625, 0.1666259765625 ;
HFMA2 R6, R5, R6, 0.5, 0.5 ;
HFMA2.MMA R6, R5, R6, 1, 1 ;
HFMA2 R6, R5, R6, RZ.H0_H0 ;
HFMA2.MMA R5, R6, RZ, 0.0013885498046875, 0.0013885498046875 ;
HFMA2 R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2.MMA R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2 R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2.MMA R5, R6, R5, 0.5, 0.5 ;
HFMA2 R5, R6, R5, 1, 1 ;
HFMA2.MMA R6, R6, R5, RZ ;
HFMA2 R5, R6, RZ.H0_H0, 0.0013885498046875, 0.0013885498046875 ;
HFMA2.MMA R5, R6, R5, 0.008331298828125, 0.008331298828125 ;
HFMA2 R5, R6, R5, 0.041656494140625, 0.041656494140625 ;
HFMA2.MMA R5, R6, R5, 0.1666259765625, 0.1666259765625 ;
HFMA2 R5, R6, R5, 0.5, 0.5 ;
HFMA2.MMA R7, R6, R5, 1, 1 ;
IADD3.X R5, R8, UR11, RZ, P0, !PT ;
IADD3 R3, P0, R2, R3, RZ ;
IADD3.X R0, RZ, R0, RZ, P0, !PT ;
ISETP.GE.U32.AND P0, PT, R3, UR8, PT ;
HFMA2 R7, R6, R7, RZ.H0_H0 ;
```



```

        // abel(n, a) = (x-an) * abel(n-1, a)
        using type = typename aerobus::mul_t<
            typename Inner<deg-1, an>::type,
            typename aerobus::sub_t<typename P::X, typename P::template inject_constant_t<an>>
        >;
    };

    // abel(0, a) = 1
    template<I::inner_type an>
    struct Inner<0, an> {
        using type = P::one;
    };

    // abel(1, a) = X
    template<I::inner_type an>
    struct Inner<1, an> {
        using type = P::X;
    };
};

template<size_t n, auto a, typename I = aerobus::i64>
using AbelPolynomials = typename AbelHelper<I>::template Inner<n, a*n>::type;

using A2_3 = AbelPolynomials<3, 2>;

int main() {
    std::cout << "expected = x^3 - 12 x^2 + 36 x" << std::endl;
    std::cout << "aerobus = " << A2_3::to_string() << std::endl;
    return 0;
}

```

## 10.7 examples/polynomials\_over\_finite\_field.cpp

How to build a known polynomial (here `aerobus::known_polynomials::allone`) with coefficients in a finite field (here `aerobus::zpz<2>`) and get its value when evaluated at a value in this field (here 1).

```

#include <iostream>
#include "../src/aerobus.h"

using GF2 = aerobus::zpz<2>;
using P = aerobus::known_polynomials::allone<8, GF2>;

int main() {
    // at this point, value_at_1 is an instantiation of zpz<2>::val
    using value_at_1 = P::template value_at_t<GF2::template inject_constant_t<1>;
    // here we get its value in an arithmetic type, here int32_t
    constexpr int32_t x = value_at_1::template get<int32_t>();
    // ensure that 1+1+1+1+1+1+1 in Z/2Z is equal to one
    std::cout << "expected = " << 1 << std::endl;
    std::cout << "computed = " << x << std::endl;
    return 0;
}

```

## 10.8 examples/compensated\_horner.cpp

How to use compensated horner evaluation scheme to get better accuracy when evaluating polynomials close to its roots

See also

[publication](#)

```

// run with ./generate_comp_horner.sh in this directory
// that will compile and run this sample and plot all the generated data
#include "../src/aerobus.h"

using namespace aerobus; // NOLINT

constexpr size_t NB_POINTS = 400;

template<typename P, typename T, bool compensated>
DEVICE INLINED T eval(const T& x) {

```

```

    if constexpr (compensated) {
        return P::template compensated_eval<T>(x);
    } else {
        return P::template eval<T>(x);
    }
}

template<typename T>
DEVICE T exact_large(const T& x) {
    return pow_scalar<T, 5>(0.75 - x) * pow_scalar<T, 11>(1 - x);
}

template<typename T>
DEVICE T exact_small(const T& x) {
    return pow_scalar<T, 3>(x - 1);
}

template<typename P, typename T, bool compensated>
void run(T left, T right, const char *file_name, T (*exact)(const T&)) {
    FILE *f = ::fopen(file_name, "w+");
    T step = (right - left) / NB_POINTS;
    T x = left;
    for (size_t i = 0; i <= NB_POINTS; ++i) {
        ::fprintf(f, "%e %e %e\n", x, eval<P, T, compensated>(x), exact(x));
        x += step;
    }
    ::fclose(f);
}

int main() {
    {
        // (0.75 - x)^5 * (1 - x)^11
        using P = mul_t<
            pow_t<pq64, pq64::val<
                typename q64::template inject_constant_t<-1>,
                q64::val<i64::val<3>, i64::val<4>>, 5>,
            pow_t<pq64, pq64::val<typename q64::template inject_constant_t<-1>, typename q64::one>, 11>
            >;
        using FLOAT = double;
        run<P, FLOAT, false>(0.68, 1.15, "plots/large_sample_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.68, 1.15, "plots/large_sample_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.74995, 0.75005, "plots/first_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.74995, 0.75005, "plots/first_root_comp_horner.dat", &exact_large);

        run<P, FLOAT, false>(0.9935, 1.0065, "plots/second_root_horner.dat", &exact_large);
        run<P, FLOAT, true>(0.9935, 1.0065, "plots/second_root_comp_horner.dat", &exact_large);
    }
    {
        // (x - 1) ^ 3
        using P = make_int_polynomial_t<i32, 1, -3, 3, -1>;

        run<P, double, false>(1-0.00005, 1+0.00005, "plots/double.dat", &exact_small);
        run<P, float, true>(1-0.00005, 1+0.00005, "plots/float_comp.dat", &exact_small);
    }
}

```





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