

Aerobus

v1.2

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Chapter 1

Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

Everything is expressed as types

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials.

It is designed to be 'quite easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

1.1 HOW TO

- Clone or download the repository somewhere, or just download [aerobus.h](#)
- In your code, add : `#include "aerobus.h"`
- Compile with `-std=c++20` (at least) `-I<install_location>`

`Aerobus` provides a definition for low-degree (up to 997) Conway polynomials. To use them, define `AEROBUS↔_CONWAY_IMPORTS` before including [aerobus.h](#).

1.1.1 Unit Test

Install [Cmake](#) Install a recent compiler (supporting c++20), such as MSVC, G++ or Clang++

Move to the top directory then :

```
cmake -S . -B build
cmake --build build
cd build && ctest
```

Terminal should write :

```
100% tests passed, 0 tests failed out of 48
```

Alternate way :

```
make tests
```

From top directory.

1.1.2 Benchmarks

Benchmarks are written for Intel CPUs having AVX512f and AVX512vl flags, they work only on Linux operating system using g++.

In addition of Cmake and compiler, install [OpenMP](#). Then move to top directory :

```
rm -rf build
mkdir build
cd build
cmake ..
make aerobus_benchmarks
./aerobus_benchmarks
```

results on my laptop :

```
./benchmarks_avx512.exe
[std math] 5.358e-01 Gsin/s
[std fast math] 3.389e+00 Gsin/s
[aerobus deg 1] 1.871e+01 Gsin/s
average error (vs std) : 4.36e-02
max error (vs std) : 1.50e-01
[aerobus deg 3] 1.943e+01 Gsin/s
average error (vs std) : 1.85e-04
max error (vs std) : 8.17e-04
[aerobus deg 5] 1.335e+01 Gsin/s
average error (vs std) : 6.07e-07
max error (vs std) : 3.63e-06
[aerobus deg 7] 8.634e+00 Gsin/s
average error (vs std) : 1.27e-09
max error (vs std) : 9.75e-09
[aerobus deg 9] 6.171e+00 Gsin/s
average error (vs std) : 1.89e-12
max error (vs std) : 1.78e-11
[aerobus deg 11] 4.731e+00 Gsin/s
average error (vs std) : 2.12e-15
max error (vs std) : 2.40e-14
[aerobus deg 13] 3.862e+00 Gsin/s
average error (vs std) : 3.16e-17
max error (vs std) : 3.33e-16
[aerobus deg 15] 3.359e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
[aerobus deg 17] 2.947e+00 Gsin/s
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
average error (vs std) : 3.13e-17
max error (vs std) : 3.33e-16
```


1.2 Structures

1.2.1 Predefined discrete euclidean domains

Aerobus predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::mpz<p>` : integers modulo p (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where `i` is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

1.2.2 Polynomials

Aerobus defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

1.2.3 Known polynomials

Aerobus predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

They have their coefficients either in `aerobus::i64` or `aerobus::q64`. Complete list is (but is meant to be extended):

- chebyshev_T
- chebyshev_U
- laguerre
- hermite_prob
- hermite_phys
- bernstein
- legendre
- bernoulli

1.2.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), aerobus provides definition for all Conway polynomials $CP(p, n)$ for p up to 997 and low values for n (usually less than 10).

They can be used to construct finite fields of order p^n (\mathbb{F}_{p^n}):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

1.2.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `Fraction<Field<Integers>>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Exposed functions are:

- `exp`
- `expm1` $e^x - 1$
- `lnp1` $\ln(x + 1)$
- `geom` $\frac{1}{1-x}$
- `sin`

- cos
- tan
- sh
- cosh
- tanh
- asin
- acos
- acosh
- asinh
- atanh

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in `Integers (typename)` and `index (size_t)`;
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`.

For example, to define the serie $1 + x + x^2 + x^3 + \dots$, users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, size_t degree>
using my_serie = taylor<Integers, my_coeff_at, degree>;

static constexpr double x = my_serie<i64, 3>::eval(3.0);
```

On x86-64 and CUDA platforms at least, using proper compiler directives, these functions yield very performant assembly, similar or better than standard library implementation in fast math. For example, this code:

```
double compute_expml(const size_t N, double* in, double* out) {
    using V = aerobus::expml<aerobus::i64, 13>;
    for (size_t i = 0; i < N; ++i) {
        out[i] = V::eval(in[i]);
    }
}
```

Yields this assembly (clang 17, `-mavx2 -O3`) where we can see a pile of Fused-Multiply-Add vector instructions, generated because we unrolled completely the Horner evaluation loop:

```
compute_expml(unsigned long, double const*, double*):
    lea     rax, [rdi-1]
    cmp     rax, 2
    jbe     .L5
    mov     rcx, rdi
    xor     eax, eax
    vxorpd  xmm1, xmm1, xmm1
    vbroadcastsd ymm14, QWORD PTR .LC1[rip]
    vbroadcastsd ymm13, QWORD PTR .LC3[rip]
    shr     rcx, 2
    vbroadcastsd ymm12, QWORD PTR .LC5[rip]
    vbroadcastsd ymm11, QWORD PTR .LC7[rip]
    sal     rcx, 5
    vbroadcastsd ymm10, QWORD PTR .LC9[rip]
    vbroadcastsd ymm9, QWORD PTR .LC11[rip]
    vbroadcastsd ymm8, QWORD PTR .LC13[rip]
    vbroadcastsd ymm7, QWORD PTR .LC15[rip]
    vbroadcastsd ymm6, QWORD PTR .LC17[rip]
    vbroadcastsd ymm5, QWORD PTR .LC19[rip]
    vbroadcastsd ymm4, QWORD PTR .LC21[rip]
```

```

vbroadcastsd    ymm3, QWORD PTR .LC23[rip]
vbroadcastsd    ymm2, QWORD PTR .LC25[rip]
.L3:
vmovupd ymm15, YMMWORD PTR [rsi+rax]
vmovapd ymm0, ymm15
vmadd132pd      ymm0, ymm14, ymm1
vmadd132pd      ymm0, ymm13, ymm15
vmadd132pd      ymm0, ymm12, ymm15
vmadd132pd      ymm0, ymm11, ymm15
vmadd132pd      ymm0, ymm10, ymm15
vmadd132pd      ymm0, ymm9, ymm15
vmadd132pd      ymm0, ymm8, ymm15
vmadd132pd      ymm0, ymm7, ymm15
vmadd132pd      ymm0, ymm6, ymm15
vmadd132pd      ymm0, ymm5, ymm15
vmadd132pd      ymm0, ymm4, ymm15
vmadd132pd      ymm0, ymm3, ymm15
vmadd132pd      ymm0, ymm2, ymm15
vmadd132pd      ymm0, ymm1, ymm15
vmovupd YMMWORD PTR [rdx+rax], ymm0
add    rax, 32
cmp    rcx, rax
jne    .L3
mov    rax, rdi
and    rax, -4
vzeroupper

```

1.3 Operations

1.3.1 Field of fractions

Given a set (type) satisfies the `IsEuclideanDomain` concept, `Aerobus` allows to define its `field of fractions`.

This new type is again a euclidean domain, especially a field, and therefore we can define polynomials over it.

For example, integers modulo p is not a field when p is not prime. We then can define its field of fraction and polynomials over it this way:

```

using namespace aerobus;
using ZmZ = zp<8>;
using Fzmz = FractionField<ZmZ>;
using Pfzmz = polynomial<Fzmz>;

```

The same operation would stand for any set that users would have implemented in place of `ZmZ`.

For example, we can easily define `rational functions` by taking the ring of fractions of polynomials:

```

using namespace aerobus;
using RF64 = FractionField<polynomial<q64>>;

```

Which also have an evaluation function, as polynomial do.

1.3.2 Quotient

Given a ring R , `Aerobus` provides automatic implementation for `quotient ring R/X` where X is a principal ideal generated by some element, as we know this kind of ideal is two-sided as long as R is commutative (and we assume it is).

For example, if we want R to be \mathbb{Z} represented as `aerobus::i64`, we can express arithmetic modulo 17 using:

```

using namespace aerobus;
using ZpZ = Quotient<i64, i64::val<17>>;

```

As we could have using `zp<17>`.

This is mainly used to define finite fields of order p^n using Conway polynomials but may have other applications.

1.4 Misc

1.4.1 Continued Fractions

Aerobus gives an implementation for [continued fractions](#). It can be used this way:

```
using namespace aerobus;  
using T = ContinuedFraction<1,2,3,4>;  
constexpr double x = T::val;
```

As practical examples, aerobus gives continued fractions of π , e , $\sqrt{2}$ and $\sqrt{3}$:

```
constexpr double A_SQRT3 = aerobus::SQRT3_fraction::val; // 1.7320508075688772935
```

1.5 CUDA

When compiled with `nvcc` and the flag `WITH_CUDA_FP16`, Aerobus provides some kind of support of 16 bits integers and floats (aka `__half`).

Unfortunately, NVIDIA did not put enough `constexpr` in its `cuda_fp16.h` header, so we had to implement our own `constexpr static_cast` from `int16_t` to `__half` to make integers polynomials work with `__half`. See [this bug](#).

More, it's (at this time), not possible to make it work for `__half2` because of [another bug](#).

Please push to make these bug fixed by NVIDIA.

Chapter 2

Namespace Index

2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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aerobus::internal	Internal implementations, subject to breaking changes without notice	36
aerobus::known_polynomials	Families of well known polynomials such as Hermite or Bernstein	40

Chapter 3

Concept Index

3.1 Concepts

Here is a list of all concepts with brief descriptions:

aerobus::IsEuclideanDomain	
Concept to express R is an euclidean domain	45
aerobus::IsField	
Concept to express R is a field	45
aerobus::IsRing	
Concept to express R is a Ring	46

Chapter 4

Class Index

4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

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aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >	48
aerobus::ContinuedFraction< values >	
Continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$	48
aerobus::ContinuedFraction< a0 >	
Specialization for only one coefficient, technically just 'a0'	49
aerobus::ContinuedFraction< a0, rest... >	
Specialization for multiple coefficients (strictly more than one)	50
aerobus::ConwayPolynomial	51
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Embeds i32 into i64	52
aerobus::Embed< polynomial< Small >, polynomial< Large > >	
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aerobus::Embed< q32, q64 >	
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aerobus::Embed< Quotient< Ring, X >, Ring >	
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aerobus::Embed< Ring, FractionField< Ring > >	
Embeds values from Ring to its field of fractions	55
aerobus::Embed< zpz< x >, i32 >	
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aerobus::i32	
32 bits signed integers, seen as a algebraic ring with related operations	57
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Removes types from head of the list	72

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Quotient ring by the principal ideal generated by 'X' With i32 as Ring and i32::val<2> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	73
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aerobus::type_list< Ts >	
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aerobus::i64::val< x >	
Values in i64	84
aerobus::polynomial< Ring >::val< coeffN, coeffs >	
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aerobus::Quotient< Ring, X >::val< V >	
Projection values in the quotient ring	90
aerobus::zpz< p >::val< x >	
Values in zpz	91
aerobus::polynomial< Ring >::val< coeffN >	
Specialization for constants	93
aerobus::zpz< p >	
Congruence classes of integers modulo p (32 bits)	96

Chapter 5

File Index

5.1 File List

Here is a list of all files with brief descriptions:

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Chapter 6

Namespace Documentation

6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

Namespaces

- namespace [internal](#)
internal implementations, subject to breaking changes without notice
- namespace [known_polynomials](#)
families of well known polynomials such as Hermite or Bernstein

Classes

- struct [ContinuedFraction](#)
represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$
- struct [ContinuedFraction< a0 >](#)
Specialization for only one coefficient, technically just 'a0'.
- struct [ContinuedFraction< a0, rest... >](#)
specialization for multiple coefficients (strictly more than one)
- struct [ConwayPolynomial](#)
- struct [Embed](#)
embedding - struct forward declaration
- struct [Embed< i32, i64 >](#)
embeds i32 into i64
- struct [Embed< polynomial< Small >, polynomial< Large > >](#)
embeds polynomial<Small> into polynomial<Large>
- struct [Embed< q32, q64 >](#)
embeds q32 into q64
- struct [Embed< Quotient< Ring, X >, Ring >](#)
embeds Quotient<Ring, X> into Ring
- struct [Embed< Ring, FractionField< Ring > >](#)
embeds values from Ring to its field of fractions
- struct [Embed< zpz< x >, i32 >](#)

- embeds zpz values into [i32](#)*
- struct [i32](#)
 - 32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)
 - 64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is_prime](#)
 - checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)
 - [Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is $\mathbb{Z}/2\mathbb{Z}$.*
- struct [type_list](#)
 - Empty pure template struct to handle type list.*
- struct [type_list<>](#)
 - specialization for empty type list*
- struct [zpz](#)
 - congruence classes of integers modulo p (32 bits)*

Concepts

- concept [IsRing](#)
 - Concept to express R is a Ring.*
- concept [IsEuclideanDomain](#)
 - Concept to express R is an euclidean domain.*
- concept [IsField](#)
 - Concept to express R is a field.*

Typedefs

- template<[typename T](#) , [typename A](#) , [typename B](#) >
 - [using gcd_t = typename internal::gcd< T >::template type< A, B >](#)
 - computes the greatest common divisor of A and B*
- template<[typename... vals](#)>
 - [using vadd_t = typename internal::vadd< vals... >::type](#)
 - adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator*
- template<[typename... vals](#)>
 - [using vmul_t = typename internal::vmul< vals... >::type](#)
 - multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator*
- template<[typename val](#) >
 - [using abs_t = std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template sub_t< typename val::enclosing_type::zero, val > >](#)
 - computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- template<[typename Ring](#) >
 - [using FractionField = typename internal::FractionFieldImpl< Ring >::type](#)
- template<[typename X](#) , [typename Y](#) >
 - [using add_t = typename X::enclosing_type::template add_t< X, Y >](#)
 - generic addition*
- template<[typename X](#) , [typename Y](#) >
 - [using sub_t = typename X::enclosing_type::template sub_t< X, Y >](#)
 - generic subtraction*

- `template<typename X, typename Y >`
`using mul_t = typename X::enclosing_type::template mul_t< X, Y >`
generic multiplication
- `template<typename X, typename Y >`
`using div_t = typename X::enclosing_type::template div_t< X, Y >`
generic division
- `using q32 = FractionField< i32 >`
32 bits rationals rationals with 32 bits numerator and denominator
- `using fpq32 = FractionField< polynomial< q32 > >`
rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)
- `using q64 = FractionField< i64 >`
64 bits rationals rationals with 64 bits numerator and denominator
- `using pi64 = polynomial< i64 >`
polynomial with 64 bits integers coefficients
- `using pq64 = polynomial< q64 >`
polynomial with 64 bits rationals coefficients
- `using fpq64 = FractionField< polynomial< q64 > >`
polynomial with 64 bits rational coefficients
- `template<typename Ring, typename v1, typename v2 >`
`using makefraction_t = typename FractionField< Ring >::template val< v1, v2 >`
helper type : the rational V1/V2 in the field of fractions of Ring
- `template<typename v >`
`using embed_int_poly_in_fractions_t = typename Embed< polynomial< typename v::ring_type >, polynomial< FractionField< typename v::ring_type > > >::template type< v >`
embed a polynomial with integers coefficients into rational coefficients polynomials
- `template<int64_t p, int64_t q>`
`using make_q64_t = typename q64::template simplify_t< typename q64::val< i64::inject_constant_t< p >, i64::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<int32_t p, int32_t q>`
`using make_q32_t = typename q32::template simplify_t< typename q32::val< i32::inject_constant_t< p >, i32::inject_constant_t< q > > >`
helper type : make a fraction from numerator and denominator
- `template<typename Ring, typename v1, typename v2 >`
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`
helper type : adds two fractions
- `template<typename Ring, typename v1, typename v2 >`
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`
helper type : multiplies two fractions
- `template<typename Ring, auto... xs>`
`using make_int_polynomial_t = typename polynomial< Ring >::template val< typename Ring::template inject_constant_t< xs >... >`
make a polynomial with coefficients in Ring
- `template<typename Ring, auto... xs>`
`using make_frac_polynomial_t = typename polynomial< FractionField< Ring > >::template val< typename FractionField< Ring >::template inject_constant_t< xs >... >`
make a polynomial with coefficients in FractionField<Ring>
- `template<typename T, size_t i>`
`using factorial_t = typename internal::factorial< T, i >::type`
computes factorial(i), as type
- `template<typename T, size_t k, size_t n>`
`using combination_t = typename internal::combination< T, k, n >::type`

- computes binomial coefficient (k among n) as type*

```
template<typename T, size_t n>
using bernoulli_t = typename internal::bernoulli< T, n >::type
```

nth bernoulli number as type in T
- ```
template<typename T, size_t n>
using bell_t = typename internal::bell_helper< T, n >::type
```

*Bell numbers.*
- ```
template<typename T, int k>
using alternate_t = typename internal::alternate< T, k >::type
```

$(-1)^k$ as type in T
- ```
template<typename T, int n, int k>
using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type
```

*Stirling number of first kind (signed) – as types.*
- ```
template<typename T, int n, int k>
using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >
```

Stirling number of first kind (unsigned) – as types.
- ```
template<typename T, typename p, size_t n>
using pow_t = typename internal::pow< T, p, n >::type
```

*$p^n$  (as 'val' type in T)*
- ```
template<typename T, template< typename, size_t index > typename coeff_at, size_t deg>
using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<
deg+1 > >::type
```
- ```
template<typename Integers, size_t deg>
using exp = taylor< Integers, internal::exp_coeff, deg >
```

*$e^x$*
- ```
template<typename Integers, size_t deg>
using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg
>, typename polynomial< FractionField< Integers > >::one >
```

$e^x - 1$
- ```
template<typename Integers, size_t deg>
using ln1 = taylor< Integers, internal::ln1_coeff, deg >
```

*$\ln(1 + x)$*
- ```
template<typename Integers, size_t deg>
using atan = taylor< Integers, internal::atan_coeff, deg >
```

$\arctan(x)$
- ```
template<typename Integers, size_t deg>
using sin = taylor< Integers, internal::sin_coeff, deg >
```

*$\sin(x)$*
- ```
template<typename Integers, size_t deg>
using sinh = taylor< Integers, internal::sh_coeff, deg >
```

$\sinh(x)$
- ```
template<typename Integers, size_t deg>
using cosh = taylor< Integers, internal::cosh_coeff, deg >
```

*$\cosh(x)$  hyperbolic cosine*
- ```
template<typename Integers, size_t deg>
using cos = taylor< Integers, internal::cos_coeff, deg >
```

$\cos(x)$ cosinus
- ```
template<typename Integers, size_t deg>
using geometric_sum = taylor< Integers, internal::geom_coeff, deg >
```

*$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$*
- ```
template<typename Integers, size_t deg>
using asin = taylor< Integers, internal::asin_coeff, deg >
```

$\arcsin(x)$ arc sinus

- `template<typename Integers , size_t deg>`
`using asinh = taylor< Integers, internal::asinh_coeff, deg >`
`arcsinh(x)` *arc hyperbolic sinus*
- `template<typename Integers , size_t deg>`
`using atanh = taylor< Integers, internal::atanh_coeff, deg >`
`arctanh(x)` *arc hyperbolic tangent*
- `template<typename Integers , size_t deg>`
`using tan = taylor< Integers, internal::tan_coeff, deg >`
`tan(x)` *tangent*
- `template<typename Integers , size_t deg>`
`using tanh = taylor< Integers, internal::tanh_coeff, deg >`
`tanh(x)` *hyperbolic tangent*
- `using PI_fraction = ContinuedFraction< 3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1 >`
- `using E_fraction = ContinuedFraction< 2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1 >`
- `using SQRT2_fraction = ContinuedFraction< 1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2 >`
approximation of $\sqrt{2}$
- `using SQRT3_fraction = ContinuedFraction< 1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2 >`
approximation of

Functions

- `template<typename T >`
`T * aligned_malloc (size_t count, size_t alignment)`
- brief Conway polynomials tparam p characteristic of the field (prime number) @tparam n degree of extension
`template< int p`

Variables

- `template<typename T , size_t i>`
`constexpr T::inner_type factorial_v = internal::factorial<T, i>::value`
computes factorial(i) as value in T
- `template<typename T , size_t k, size_t n>`
`constexpr T::inner_type combination_v = internal::combination<T, k, n>::value`
computes binomial coefficients (k among n) as value
- `template<typename FloatType , typename T , size_t n>`
`constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>`
 n th bernoulli number as value in $FloatType$
- `template<typename T , size_t k>`
`constexpr T::inner_type alternate_v = internal::alternate<T, k>::value`
 $(-1)^k$ as value from T

6.1.1 Detailed Description

main namespace for all publicly exposed types or functions

6.1.2 Typedef Documentation

6.1.2.1 abs_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

Template Parameters

<i>val</i>	a value in a RIng, such as <code>i64::val<-2></code>
------------	------------------------------------------------------------

6.1.2.2 add_t

```
template<typename X , typename Y >
using aerobus::add_t = typedef typename X::enclosing_type::template add_t<X, Y>
```

generic addition

Template Parameters

<i>X</i>	a value in a ring providing add_t operator
<i>Y</i>	a value in same ring

6.1.2.3 addfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FranctionField<Ring>

6.1.2.4 alternate_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$ as type in T

Template Parameters

<i>T</i>	Ring type, <code>aerobus::i64</code> for example
----------	--------------------------------------------------

6.1.2.5 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$ arc sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.6 asinh

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$ arc hyperbolic sinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.7 atan

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\operatorname{arctan}(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.8 atanh

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$ arc hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.9 bell_t

```
template<typename T , size_t n>
using aerobus::bell_t = typedef typename internal::bell_helper<T, n>::type
```

Bell numbers.

Template Parameters

<i>T</i>	ring type, such as aerobus::i64
<i>n</i>	index

6.1.2.10 bernoulli_t

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

Template Parameters

<i>T</i>	Ring type (i64)
<i>n</i>	

6.1.2.11 combination_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

Template Parameters

<i>T</i>	Ring type (i32 for example)
----------	----------------------------------------------

6.1.2.12 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$ cosinus

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.13 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$ hyperbolic cosine

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.14 `div_t`

```
template<typename X , typename Y >
using aerobus::div_t = typedef typename X::enclosing_type::template div_t<X, Y>
```

generic division

Template Parameters

<i>X</i>	a value in a euclidean domain
<i>Y</i>	a value in same Euclidean domain

6.1.2.15 `E_fraction`

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

6.1.2.16 `embed_int_poly_in_fractions_t`

```
template<typename v >
using aerobus::embed_int_poly_in_fractions_t = typedef typename Embed< polynomial<typename v↔
::ring_type>, polynomial<FractionField<typename v::ring_type> >>::template type<v>
```

embed a polynomial with integers coefficients into rational coefficients polynomials

Lives in `polynomial<FractionField<Ring>>`

Template Parameters

<i>Ring</i>	Integers
<i>a</i>	value in <code>polynomial<Ring></code>

6.1.2.17 `exp`

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

e^x

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.18 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_t<
exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.19 factorial_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

Template Parameters

<i>T</i>	Ring type (e.g. i32)
<i>i</i>	

6.1.2.20 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)

6.1.2.21 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

6.1.2.22 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

6.1.2.23 gcd_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

6.1.2.24 geometric_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$ zero development of $\frac{1}{1-x}$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.25 ln1

```
template<typename Integers , size_t deg>
using aerobus::ln1 = typedef taylor<Integers, internal::ln1_coeff, deg>
```

$\ln(1+x)$

Template Parameters

<i>T</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.26 make_frac_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_frac_polynomial_t = typedef typename polynomial<FractionField<Ring> >←
::template val< typename FractionField<Ring>::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in FractionField<Ring>

Template Parameters

<i>Ring</i>	integers
<i>...xs</i>	values

6.1.2.27 make_int_polynomial_t

```
template<typename Ring , auto... xs>
using aerobus::make_int_polynomial_t = typedef typename polynomial<Ring>::template val< typename
Ring::template inject_constant_t<xs>...>
```

make a polynomial with coefficients in Ring

Template Parameters

<i>Ring</i>	integers
...xs	coefficients

6.1.2.28 make_q32_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.29 make_q64_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

Template Parameters

<i>p</i>	numerator
<i>q</i>	denominator

6.1.2.30 makefraction_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

Template Parameters

<i>Ring</i>	the base ring
<i>v1</i>	value 1 in Ring
<i>v2</i>	value 2 in Ring

6.1.2.31 mul_t

```
template<typename X , typename Y >
using aerobus::mul_t = typedef typename X::enclosing_type::template mul_t<X, Y>
```

generic multiplication

Template Parameters

<i>X</i>	a value in a ring providing mul_t operator
<i>Y</i>	a value in same ring

6.1.2.32 mulfractions_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

6.1.2.33 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

6.1.2.34 PI_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

6.1.2.35 pow_t

```
template<typename T , typename p , size_t n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

p^n (as 'val' type in T)

Template Parameters

T	(some ring type, such as aerobus::i64)
p	must be an instantiation of $T::val$
n	power

6.1.2.36 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

6.1.2.37 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator

6.1.2.38 q64

```
using aerobus::q64 = typedef FractionField<i64>
```

64 bits rationals rationals with 64 bits numerator and denominator

6.1.2.39 sin

```
template<typename Integers , size_t deg>
using aerobus::sin = typedef taylor<Integers, internal::sin_coeff, deg>
```

$\sin(x)$

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.40 sinh

```
template<typename Integers , size_t deg>
using aerobus::sinh = typedef taylor<Integers, internal::sh_coeff, deg>
```

$\sinh(x)$

6.1.2.45 sub_t

```
template<typename X , typename Y >
using aerobus::sub_t = typedef typename X::enclosing_type::template sub_t<X, Y>
```

generic subtraction

Template Parameters

<i>X</i>	a value in a ring providing sub_t operator
<i>Y</i>	a value in same ring

6.1.2.46 tan

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$ tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.47 tanh

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$ hyperbolic tangent

Template Parameters

<i>Integers</i>	Ring type (for example i64)
<i>deg</i>	taylor approximation degree

6.1.2.48 taylor

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
deg + 1> >::type
```

Template Parameters

<i>T</i>	Used Ring type (aerobus::i64 for example)
<i>coeff_at</i>	- implementation giving the 'value' (seen as type in FractionField<T>)
<i>deg</i>	

6.1.2.49 vadd_t

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an add_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.2.50 vmul_t

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ($v_1 + v_2 + \dots + v_n$) vals must have same "enclosing_type" and "enclosing_type" must have an mul_t binary operator

Template Parameters

<i>...vals</i>	
----------------	--

6.1.3 Function Documentation

6.1.3.1 aligned_malloc()

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

6.1.3.2 field()

```
brief Conway polynomials tparam p characteristic of the aerobus::field (
```



```
prime number )
```

6.1.4 Variable Documentation

6.1.4.1 alternate_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$ as value from T

Template Parameters

<i>T</i>	Ring type, aerobus::i64 for example, then result will be an <code>int64_t</code>
----------	--------------------------------------------------------------------------------------------------

6.1.4.2 bernoulli_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>
[inline], [constexpr]
```

nth bernoulli number as value in FloatType

Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	(aerobus::i64 for example)
<i>n</i>	

6.1.4.3 combination_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

Template Parameters

<i>T</i>	(aerobus::i32 for example)
<i>k</i>	
<i>n</i>	

6.1.4.4 factorial_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

Template Parameters

<i>T</i>	(aerobus::i64 for example)
<i>i</i>	

6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

Classes

- struct **_FractionField**
- struct **_FractionField**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **_is_prime**
- struct **_is_prime**< 0, i >
- struct **_is_prime**< 1, i >
- struct **_is_prime**< 2, i >
- struct **_is_prime**< 3, i >
- struct **_is_prime**< 5, i >
- struct **_is_prime**< 7, i >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n !=2 &&n % 2==0)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i > n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i *i<=n))> >
- struct **_is_prime**< n, i, std::enable_if_t<(n %(i+2)==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i *i<=n)> >
- struct **_is_prime**< n, i, std::enable_if_t<(n >=9 &&i *i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable_if_t< k % 2 !=0 > >
- struct **alternate**< T, k, std::enable_if_t< k % 2==0 > >
- struct **asin_coeff**
- struct **asin_coeff_helper**
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asin_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **asinh_coeff**
- struct **asinh_coeff_helper**
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **asinh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atan_coeff**
- struct **atan_coeff_helper**
- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >

- struct **atan_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **atanh_coeff**
- struct **atanh_coeff_helper**
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **atanh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **bell_helper**
- struct **bell_helper**< T, 0 >
- struct **bell_helper**< T, 1 >
- struct **bell_helper**< T, n, std::enable_if_t<(n > 1)> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli_coeff**
- struct **bernoulli_helper**
- struct **bernoulli_helper**< T, accum, m, m >
- struct **bernstein_helper**
- struct **bernstein_helper**< 0, 0, l >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i > 0) &&(i < m)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==0)> >
- struct **bernstein_helper**< i, m, l, std::enable_if_t<(m > 0) &&(i==m)> >
- struct **chebyshev_helper**
- struct **chebyshev_helper**< 1, 0, l >
- struct **chebyshev_helper**< 1, 1, l >
- struct **chebyshev_helper**< 2, 0, l >
- struct **chebyshev_helper**< 2, 1, l >
- struct **combination**
- struct **combination_helper**
- struct **combination_helper**< T, 0, n >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k >(n/2) &&k > 0)> >
- struct **combination_helper**< T, k, n, std::enable_if_t<(n >=0 &&k <=(n/2) &&k > 0)> >
- struct **cos_coeff**
- struct **cos_coeff_helper**
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cos_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **cosh_coeff**
- struct **cosh_coeff_helper**
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==0 > >
- struct **cosh_coeff_helper**< T, i, std::enable_if_t<(i &1)==1 > >
- struct **exp_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable_if_t<(x > 0)> >
- struct **fma_helper**
- struct **fma_helper**< double >
- struct **fma_helper**< float >
- struct **fma_helper**< int32_t >
- struct **fma_helper**< int64_t >
- struct **FractionFieldImpl**
- struct **FractionFieldImpl**< Field, std::enable_if_t< Field::is_field > >
- struct **FractionFieldImpl**< Ring, std::enable_if_t<!Ring::is_field > >
- struct **gcd**
 - greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*
- struct **gcd**< Ring, std::enable_if_t< Ring::is_euclidean_domain > >
- struct **geom_coeff**

- struct **hermite_helper**
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 0, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< 1, known_polynomials::hermite_kind::probabilist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::physicist, I >
- struct **hermite_helper**< deg, known_polynomials::hermite_kind::probabilist, I >
- struct **insert_h**
- struct **is_instantiation_of**
- struct **is_instantiation_of**< TT, TT< Ts... > >
- struct **laguerre_helper**
- struct **laguerre_helper**< 0, I >
- struct **laguerre_helper**< 1, I >
- struct **legendre_helper**
- struct **legendre_helper**< 0, I >
- struct **legendre_helper**< 1, I >
- struct **lnp1_coeff**
- struct **lnp1_coeff**< T, 0 >
- struct **make_taylor_impl**
- struct **make_taylor_impl**< T, coeff_at, std::integer_sequence< size_t, Is... > >
- struct **pop_front_h**
- struct **pow**
- struct **pow**< T, p, n, std::enable_if_t< n==0 > >
- struct **pow**< T, p, n, std::enable_if_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable_if_t<(n > 0 && n % 2==0)> >
- struct **pow_scalar**
- struct **remove_h**
- struct **sh_coeff**
- struct **sh_coeff_helper**
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i & 1)==0 > >
- struct **sh_coeff_helper**< T, i, std::enable_if_t<(i & 1)==1 > >
- struct **sin_coeff**
- struct **sin_coeff_helper**
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i & 1)==0 > >
- struct **sin_coeff_helper**< T, i, std::enable_if_t<(i & 1)==1 > >
- struct **split_h**
- struct **split_h**< 0, L1, L2 >
- struct **stirling_helper**
- struct **stirling_helper**< T, 0, 0 >
- struct **stirling_helper**< T, 0, n, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, 0, std::enable_if_t<(n > 0)> >
- struct **stirling_helper**< T, n, k, std::enable_if_t<(k > 0) && (n > 0)> >
- struct **tan_coeff**
- struct **tan_coeff_helper**
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2) != 0 > >
- struct **tan_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **tanh_coeff**
- struct **tanh_coeff_helper**
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2) != 0 > >
- struct **tanh_coeff_helper**< T, i, std::enable_if_t<(i % 2)==0 > >
- struct **type_at**
- struct **type_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

Typedefs

- `template<size_t i, typename... Ts>`
`using type_at_t = typename type_at< i, Ts... >::type`
- `template<std::size_t N>`
`using make_index_sequence_reverse = decltype(index_sequence_reverse(std::make_index_sequence< N>{}))`

Functions

- `template<std::size_t... Is>`
`constexpr auto index_sequence_reverse (std::index_sequence< Is... > const &) -> decltype(std::index_sequence< sizeof...(Is) - 1U - Is... >{})`

Variables

- `template<template< typename... > typename TT, typename T >`
`constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value`

6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

6.2.2 Typedef Documentation

6.2.2.1 make_index_sequence_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

6.2.2.2 type_at_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

6.2.3 Function Documentation

6.2.3.1 index_sequence_reverse()

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

6.2.4 Variable Documentation

6.2.4.1 is_instantiation_of_v

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

6.3 aerobus::known_polynomials Namespace Reference

families of well known polynomials such as Hermite or Bernstein

Typedefs

- template<size_t deg, typename I = aerobus::i64>
using chebyshev_T = typename internal::chebyshev_helper< 1, deg, I >::type
Chebyshev polynomials of first kind.
- template<size_t deg, typename I = aerobus::i64>
using chebyshev_U = typename internal::chebyshev_helper< 2, deg, I >::type
Chebyshev polynomials of second kind.
- template<size_t deg, typename I = aerobus::i64>
using laguerre = typename internal::laguerre_helper< deg, I >::type
Laguerre polynomials.
- template<size_t deg, typename I = aerobus::i64>
using hermite_prob = typename internal::hermite_helper< deg, hermite_kind::probabilist, I >::type
Hermite polynomials - probabilist form.
- template<size_t deg, typename I = aerobus::i64>
using hermite_phys = typename internal::hermite_helper< deg, hermite_kind::physicist, I >::type
Hermite polynomials - physicist form.
- template<size_t i, size_t m, typename I = aerobus::i64>
using bernstein = typename internal::bernstein_helper< i, m, I >::type
Bernstein polynomials.
- template<size_t deg, typename I = aerobus::i64>
using legendre = typename internal::legendre_helper< deg, I >::type
Legendre polynomials.
- template<size_t deg, typename I = aerobus::i64>
using bernoulli = taylor< I, internal::bernoulli_coeff< deg >::template inner, deg >
Bernoulli polynomials.

Enumerations

- enum hermite_kind { probabilist , physicist }

6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

6.3.2 Typedef Documentation

6.3.2.1 bernoulli

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::bernoulli = typedef taylor<I, internal::bernoulli_coeff<deg><←
::template inner, deg>
```

Bernoulli polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to aerobus::i64)

6.3.2.2 bernstein

```
template<size_t i, size_t m, typename I = aerobus::i64>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m, I>::type
```

Bernstein polynomials.

Lives in polynomial

See also

[See in Wikipedia](#)

Template Parameters

<i>i</i>	<i>index of polynomial (between 0 and m)</i>
<i>m</i>	<i>degree of polynomial</i>
<i>I</i>	<i>Integers ring (defaults to aerobus::i64)</i>

6.3.2.3 chebyshev_T

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg, I>::type
```

Chebyshev polynomials of first kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>integer</i>	rings (defaults to aerobus::i64)

6.3.2.4 chebyshev_U

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg, I>::type
```

Chebyshev polynomials of second kind.

Lives in polynomial

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	<i>degree of polynomial</i>
<i>integer</i>	<i>rings (defaults to aerobus::i64)</i>

6.3.2.5 hermite_phys

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist, I>::type
```

Hermite polynomials - physicist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.6 hermite_prob

```
template<size_t deg, typename I = aerobus::i64>
```



```
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist, I>::type
```

Hermite polynomials - probabilist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

6.3.2.7 laguerre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg,
I>::type
```

Laguerre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers ring (defaults to aerobus::i64)

6.3.2.8 legendre

```
template<size_t deg, typename I = aerobus::i64>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg,
I>::type
```

Legendre polynomials.

Lives in polynomial<FractionField<I>>

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
<i>I</i>	Integers Ring (defaults to aerobus::i64)

6.3.3 Enumeration Type Documentation

6.3.3.1 hermite_kind

```
enum aerobus::known_polynomials::hermite_kind
```

Enumerator

probabilist	
physicist	

Chapter 7

Concept Documentation

7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

7.1.2 Detailed Description

Concept to express R is an euclidean domain.

7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

7.2.2 Detailed Description

Concept to express R is a field.

7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

7.3.2 Detailed Description

Concept to express R is a Ring.

Chapter 8

Class Documentation

8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- `using type = typename Ring::zero`

8.2.1 Member Typedef Documentation

8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

Public Types

- [using type = aN](#)

8.3.1 Member Typedef Documentation

8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

```
#include <aerobus.h>
```

8.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

Template Parameters

<i>...values</i>	are int64_t
------------------	----------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

Public Types

- `using type = typename q64::template inject_constant_t< a0 >`
represented value as `aerobus::q64`

Static Public Attributes

- `static constexpr double val = static_cast<double>(a0)`
represented value as `double`

8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

8.5.2 Member Typedef Documentation

8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

8.5.3 Member Data Documentation

8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as `double`

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

Public Types

- `using type = q64::template add_t< typename q64::template inject_constant_t< a0 >, typename q64::template div_t< typename q64::one, typename ContinuedFraction< rest... >::type > >`
represented value as `aerobus::q64`

Static Public Attributes

- `static constexpr double val = type::template get<double>`
represented value as `double`

8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

Template Parameters

<code>a0</code>	integer (int64_t)
<code>...rest</code>	integers (int64_t)

8.6.2 Member Typedef Documentation

8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename ContinuedFraction<rest...>::type > >
```

represented value as `aerobus::q64`

8.6.3 Member Data Documentation

8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.8 aerobus::Embed< Small, Large, E > Struct Template Reference

embedding - struct forward declaration

8.8.1 Detailed Description

```
template<typename Small, typename Large, typename E = void>
struct aerobus::Embed< Small, Large, E >
```

embedding - struct forward declaration

Template Parameters

<i>Small</i>	a ring which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded
<i>E</i>	some default type (unused – implementation related)

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.9 aerobus::Embed< i32, i64 > Struct Reference

embeds [i32](#) into [i64](#)

```
#include <aerobus.h>
```

Public Types

- `template<typename val >`
`using type = i64::val< static_cast< int64_t >(val::v)>`
the [i64](#) representation of val

8.9.1 Detailed Description

embeds [i32](#) into [i64](#)

8.9.2 Member Typedef Documentation

8.9.2.1 type

```
template<typename val >
using aerobus::Embed< i32, i64 >::type = i64::val<static_cast<int64_t>(val::v)>
```

the [i64](#) representation of val

Template Parameters

<i>val</i>	a value in i32
------------	--------------------------------

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

8.10 aerobus::Embed< polynomial< Small >, polynomial< Large > > Struct Template Reference

embeds `polynomial<Small>` into `polynomial<Large>`

```
#include <aerobus.h>
```

Public Types

- `template<typename v >`
`using type = typename at_low< v, typename internal::make_index_sequence_reverse< v::degree+1 > >::type`
the `polynomial<Large>` representation of v

8.10.1 Detailed Description

```
template<typename Small, typename Large>
struct aerobus::Embed< polynomial< Small >, polynomial< Large > >
```

embeds polynomial<Small> into polynomial<Large>

Template Parameters

<i>Small</i>	a rings which can be embedded in Large
<i>Large</i>	a ring in which Small can be embedded

8.10.2 Member Typedef Documentation

8.10.2.1 type

```
template<typename Small , typename Large >
template<typename v >
using aerobus::Embed< polynomial< Small >, polynomial< Large > >::type = typename at_low<v,
typename internal::make_index_sequence_reverse<v::degree + 1> >::type
```

the polynomial<Large> reprensentation of v

Template Parameters

<i>v</i>	a value in polynomial<Small>
----------	------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

8.11 aerobus::Embed< q32, q64 > Struct Reference

embeds q32 into q64

```
#include <aerobus.h>
```

Public Types

- ```
template<typename v >
using type = make_q64_t< static_cast< int64_t >(v::x::v), static_cast< int64_t >(v::y::v)>
```

*q64 representation of v*

### 8.11.1 Detailed Description

embeds q32 into q64

## 8.11.2 Member Typedef Documentation

### 8.11.2.1 type

```
template<typename v >
using aerobus::Embed< q32, q64 >::type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>
```

q64 representation of v

#### Template Parameters

|   |                |
|---|----------------|
| v | a value in q32 |
|---|----------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.12 aerobus::Embed< Quotient< Ring, X >, Ring > Struct Template Reference

embeds Quotient<Ring, X> into Ring

```
#include <aerobus.h>
```

### Public Types

- template<typename val >  
using type = typename val::raw\_t  
*Ring representation of val.*

### 8.12.1 Detailed Description

```
template<typename Ring, typename X>
struct aerobus::Embed< Quotient< Ring, X >, Ring >
```

embeds Quotient<Ring, X> into Ring

#### Template Parameters

|             |                  |
|-------------|------------------|
| <i>Ring</i> | a Euclidean ring |
| <i>X</i>    | a value in Ring  |

## 8.12.2 Member Typedef Documentation

### 8.12.2.1 type

```
template<typename Ring , typename X >
template<typename val >
using aerobus::Embed< Quotient< Ring, X >, Ring >::type = typename val::raw_t
```

Ring representation of val.

#### Template Parameters

|            |                              |
|------------|------------------------------|
| <i>val</i> | a value in Quotient<Ring, X> |
|------------|------------------------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.13 aerobus::Embed< Ring, FractionField< Ring > > Struct Template Reference

embeds values from Ring to its field of fractions

```
#include <aerobus.h>
```

### Public Types

- template<typename v >  
using type = typename FractionField< Ring >::template val< v, typename Ring::one >  
*FractionField<Ring> representation of v.*

### 8.13.1 Detailed Description

```
template<typename Ring>
struct aerobus::Embed< Ring, FractionField< Ring > >
```

embeds values from Ring to its field of fractions

#### Template Parameters

|             |                                               |
|-------------|-----------------------------------------------|
| <i>Ring</i> | an integers ring, such as <a href="#">i32</a> |
|-------------|-----------------------------------------------|

## 8.13.2 Member Typedef Documentation

### 8.13.2.1 type

```
template<typename Ring >
template<typename v >
using aerobus::Embed< Ring, FractionField< Ring > >::type = typename FractionField<Ring>↔
::template val<v, typename Ring::one>
```

FractionField<Ring> representation of v.

#### Template Parameters

|   |              |
|---|--------------|
| v | a Ring value |
|---|--------------|

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.14 aerobus::Embed< zpz< x >, i32 > Struct Template Reference

embeds zpz values into [i32](#)

```
#include <aerobus.h>
```

### Public Types

- `template<typename val >`  
`using type = i32::val< val::v >`  
*the [i32](#) representation of val*

### 8.14.1 Detailed Description

```
template<int32_t x>
struct aerobus::Embed< zpz< x >, i32 >
```

embeds zpz values into [i32](#)

#### Template Parameters

|   |            |
|---|------------|
| x | an integer |
|---|------------|

## 8.14.2 Member Typedef Documentation

### 8.14.2.1 type

```
template<int32_t x>
template<typename val >
using aerobus::Embed< zpz< x >, i32 >::type = i32::val<val::v>
```

the `i32` representation of `val`

#### Template Parameters

|                  |                                      |
|------------------|--------------------------------------|
| <code>val</code> | a value in <code>zpz&lt;x&gt;</code> |
|------------------|--------------------------------------|

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.15 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in `i32`, again represented as types*

### Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`

- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator yields v1 % v2 for example : i32::mod\_t<i32::val<7>, i32::val<2>>*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

### Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

## 8.15.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::add_t = typename add<v1, v2>::type
```

### 8.15.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

### 8.15.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```



#### 8.15.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

#### 8.15.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

#### 8.15.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

#### 8.15.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

#### 8.15.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

#### 8.15.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

#### 8.15.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

##### Template Parameters

|                 |                             |
|-----------------|-----------------------------|
| <code>v1</code> | a value in <code>i32</code> |
| <code>v2</code> | a value in <code>i32</code> |

#### 8.15.2.11 mul\_t

```
template<typename v1 , typename v2 >
```

```
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

#### 8.15.2.12 one

```
using aerobus::i32::one = val<1>
```

constant one

#### 8.15.2.13 pos\_t

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

#### 8.15.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

#### 8.15.2.15 zero

```
using aerobus::i32::zero = val<0>
```

constant zero

### 8.15.3 Member Data Documentation

#### 8.15.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

#### 8.15.3.2 is\_euclidean\_domain

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

#### 8.15.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

### 8.15.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.16 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in i64*

### Public Types

- [using inner\\_type = int64\\_t](#)  
*type of represented values*
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int64\\_t >\(x\)>](#)
- [template<typename v >](#)  
[using inject\\_ring\\_t = v](#)  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)*  
*-> [i64::val<1>](#)*
- [using zero = val< 0 >](#)  
*constant zero*
- [using one = val< 1 >](#)  
*constant one*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mod\\_t = typename remainder< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using gcd\\_t = gcd\\_t< i64, v1, v2 >](#)
- [template<typename v >](#)  
[using pos\\_t = typename pos< v >::type](#)

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields  $v1 > v2$  as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 8.16.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 `add_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

#### 8.16.2.2 `div_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

#### 8.16.2.3 `eq_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

#### 8.16.2.4 `gcd_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

#### 8.16.2.5 `gt_t`

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

### 8.16.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

### 8.16.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>>`  
`-> i64::val<1>`

#### Template Parameters

|                |                             |
|----------------|-----------------------------|
| <code>v</code> | a value in <code>i64</code> |
|----------------|-----------------------------|

### 8.16.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type of represented values

### 8.16.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

### 8.16.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

### 8.16.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

### 8.16.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

### 8.16.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

### 8.16.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

### 8.16.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.16.3 Member Data Documentation

### 8.16.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

### 8.16.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

|                 |                                                   |
|-----------------|---------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a> |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> |

### 8.16.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.16.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

## 8.16.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

## 8.16.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.17 aerobus::is\_prime&lt; n &gt; Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

## Static Public Attributes

- `static constexpr bool value` = `internal::_is_prime<n, 5>::value`  
*true iff n is prime*

## 8.17.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

## Template Parameters

|          |  |
|----------|--|
| <i>n</i> |  |
|----------|--|

## 8.17.2 Member Data Documentation

## 8.17.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.18 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using eq\\_t = typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1 , typename v2 >](#)  
[using lt\\_t = typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1 , typename v2 >](#)  
[using gt\\_t = typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::q\\_type](#)  
*division operator*



- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial : coeff X^deg*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity (an > 0)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`  
`Ring >, v1, v2 > >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

### Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

## 8.18.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.2 `derive_t`

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

|                |  |
|----------------|--|
| <code>v</code> |  |
|----------------|--|

### 8.18.2.3 `div_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.4 `eq_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

|                 |  |
|-----------------|--|
| <code>v1</code> |  |
| <code>v2</code> |  |

### 8.18.2.5 `gcd_t`

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

## 8.18.2.7 inject\_constant\_t

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.18.2.8 inject\_ring\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

## 8.18.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

## Template Parameters

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.10 mod\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.11 monomial\_t**

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

**Template Parameters**

|              |  |
|--------------|--|
| <i>coeff</i> |  |
| <i>deg</i>   |  |

**8.18.2.12 mul\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.13 one**

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

**8.18.2.14 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

|          |  |
|----------|--|
| <i>v</i> |  |
|----------|--|

**8.18.2.15 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

|          |  |
|----------|--|
| <i>P</i> |  |
|----------|--|

**8.18.2.16 sub\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

**Template Parameters**

|           |  |
|-----------|--|
| <i>v1</i> |  |
| <i>v2</i> |  |

**8.18.2.17 X**

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

**8.18.2.18 zero**

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

### 8.18.3 Member Data Documentation

#### 8.18.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

#### 8.18.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

#### 8.18.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

|          |                                            |
|----------|--------------------------------------------|
| <i>v</i> | a value in <a href="#">polynomial::val</a> |
|----------|--------------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

### Public Types

- [using type](#) = [typename](#) internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- [using tail](#) = [typename](#) internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 8.19.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

## 8.19.2 Member Typedef Documentation

### 8.19.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

### 8.19.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- [using zero](#) = [val](#)< [typename](#) Ring::zero >  
*zero value*
- [using one](#) = [val](#)< [typename](#) Ring::one >  
*one*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using add\\_t](#) = [val](#)< [typename](#) Ring::template [add\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*addition operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using mul\\_t](#) = [val](#)< [typename](#) Ring::template [mul\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*subtraction operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using div\\_t](#) = [val](#)< [typename](#) Ring::template [div\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*division operator*
- [template](#)<[typename](#) v1 , [typename](#) v2 >  
[using mod\\_t](#) = [val](#)< [typename](#) Ring::template [mod\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >

- modulus operator*  
 • `template<typename v1 , typename v2 >`  
   `using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
   *equality operator (as type)*
- `template<typename v1 >`  
   `using pos_t = std::true_type`  
   *positivity operator always true*
- `template<auto x>`  
   `using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
   `using inject_ring_t = val< v >`

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
   `static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
   *addition operator (as boolean value)*
- `template<typename v >`  
   `static constexpr bool pos_v = pos_t<v>::value`  
   *positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
   *quotien rings are euclidean domain*

## 8.20.1 Detailed Description

`template<typename Ring, typename X>`  
**requires** `IsRing<Ring>`  
**struct** `aerobus::Quotient< Ring, X >`

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

### Template Parameters

|             |                                                                                         |
|-------------|-----------------------------------------------------------------------------------------|
| <i>Ring</i> | A ring type, such as ' <code>i32</code> ', must satisfy the <code>IsRing</code> concept |
| <i>X</i>    | a value in Ring, such as <code>i32::val&lt;2&gt;</code>                                 |

## 8.20.2 Member Typedef Documentation

### 8.20.2.1 add\_t

`template<typename Ring , typename X >`  
`template<typename v1 , typename v2 >`  
`using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,`  
`typename v2::type> >`

addition operator



## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

## Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

## 8.20.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

## 8.20.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

### 8.20.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
| v2 | a value in quotient ring |

### 8.20.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.20.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

|    |                          |
|----|--------------------------|
| v1 | a value in quotient ring |
|----|--------------------------|

### 8.20.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.20.3 Member Data Documentation

### 8.20.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
| <i>v2</i> | a value in quotient ring |

### 8.20.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.20.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

#### Template Parameters

|           |                          |
|-----------|--------------------------|
| <i>v1</i> | a value in quotient ring |
|-----------|--------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

### Public Types

- [using head](#) = [typename](#) inner::head
- [using tail](#) = [typename](#) inner::tail

### 8.21.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.21.2 Member Typedef Documentation

#### 8.21.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.21.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.22 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

## Classes

- struct `pop_front`  
*removes types from head of the list*
- struct `split`  
*splits list at index*

## Public Types

- template<typename T >  
using `push_front` = `type_list`< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using `at` = `internal::type_at_t`< index, Ts... >  
*returns type at index*
- template<typename T >  
using `push_back` = `type_list`< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using `concat` = `typename concat_h`< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using `insert` = `typename internal::insert_h`< index, `type_list`< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using `remove` = `typename internal::remove_h`< index, `type_list`< Ts... > >::type  
*removes type at index*

## Static Public Attributes

- `static constexpr size_t length` = `sizeof...(Ts)`  
*length of list*

### 8.22.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

### 8.22.2 Member Typedef Documentation

#### 8.22.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

**8.22.2.2 concat**

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

**Template Parameters**

|          |  |
|----------|--|
| <i>U</i> |  |
|----------|--|

**8.22.2.3 insert**

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

|              |  |
|--------------|--|
| <i>index</i> |  |
| <i>T</i>     |  |

**8.22.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

**8.22.2.5 push\_front**

```
template<typename... Ts>
```

```
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

#### Template Parameters

|          |  |
|----------|--|
| <i>T</i> |  |
|----------|--|

#### 8.22.2.6 remove

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

#### Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.22.3 Member Data Documentation

#### 8.22.3.1 length

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.23 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

#### Public Types

- `template<typename T >`  
`using push_front = type_list< T >`
- `template<typename T >`  
`using push_back = type_list< T >`
- `template<typename U >`  
`using concat = U`
- `template<typename T , size_t index>`  
`using insert = type_list< T >`

## Static Public Attributes

- `static constexpr size_t length = 0`

### 8.23.1 Detailed Description

specialization for empty type list

### 8.23.2 Member Typedef Documentation

#### 8.23.2.1 concat

```
template<typename U >
using aerobus::type_list<>::concat = U
```

#### 8.23.2.2 insert

```
template<typename T , size_t index>
using aerobus::type_list<>::insert = type_list<T>
```

#### 8.23.2.3 push\_back

```
template<typename T >
using aerobus::type_list<>::push_back = type_list<T>
```

#### 8.23.2.4 push\_front

```
template<typename T >
using aerobus::type_list<>::push_front = type_list<T>
```

### 8.23.3 Member Data Documentation

#### 8.23.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.24 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```



## Public Types

- `using enclosing_type = i32`  
*Enclosing ring type.*
- `using is_zero_t = std::bool_constant< x==0 >`  
*is value zero*

## Static Public Member Functions

- `static std::string to_string ()`  
*string representation of value*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*
- `template<typename valueType >`  
`static constexpr valueType get = static_cast<valueType>(x)`  
*cast x into valueType*

### 8.24.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in `i32`, again represented as types

#### Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

### 8.24.2 Member Typedef Documentation

#### 8.24.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

#### 8.24.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.24.3 Member Function Documentation

#### 8.24.3.1 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string () [inline], [static]
```

string representation of value

### 8.24.4 Member Data Documentation

#### 8.24.4.1 get

```
template<int32_t x>
template<typename valueType >
constexpr valueType aerobus::i32::val< x >::get = static_cast<valueType>(x) [static], [constexpr]
```

cast x into valueType

##### Template Parameters

|                  |                    |
|------------------|--------------------|
| <i>valueType</i> | double for example |
|------------------|--------------------|

#### 8.24.4.2 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.25 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- [using inner\\_type = int32\\_t](#)  
*type of represented values*
- [using enclosing\\_type = i64](#)  
*enclosing ring type*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

## Static Public Member Functions

- `static std::string to_string ()`  
*string representation*

## Static Public Attributes

- `static constexpr int64_t v = x`  
*actual value*
- `template<typename valueType >`  
`static constexpr valueType get = static_cast<valueType>(x)`  
*cast value in valueType*

## 8.25.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in `i64`

Template Parameters

|                |                   |
|----------------|-------------------|
| <code>x</code> | an actual integer |
|----------------|-------------------|

## 8.25.2 Member Typedef Documentation

### 8.25.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.25.2.2 inner\_type

```
template<int64_t x>
using aerobus::i64::val< x >::inner_type = int32_t
```

type of represented values

### 8.25.2.3 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

### 8.25.3 Member Function Documentation

#### 8.25.3.1 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string () [inline], [static]
```

string representation

### 8.25.4 Member Data Documentation

#### 8.25.4.1 get

```
template<int64_t x>
template<typename valueType >
constexpr valueType aerobus::i64::val< x >::get = static_cast<valueType>(x) [static], [constexpr]
```

cast value in valueType

##### Template Parameters

|                  |                      |
|------------------|----------------------|
| <i>valueType</i> | (double for example) |
|------------------|----------------------|

#### 8.25.4.2 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.26 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- `using ring_type = Ring`  
*ring coefficients live in*
- `using enclosing_type = polynomial< Ring >`  
*enclosing ring type*
- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true\_type if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr DEVICE INLINED valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 8.26.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

|                        |                           |
|------------------------|---------------------------|
| <code>coeffN</code>    | high degree coefficient   |
| <code>...coeffs</code> | lower degree coefficients |

## 8.26.2 Member Typedef Documentation

### 8.26.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

### 8.26.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

Template Parameters

|              |  |
|--------------|--|
| <i>index</i> |  |
|--------------|--|

### 8.26.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.26.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.26.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::ring_type = Ring
```

ring coefficients live in

### 8.26.2.6 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

## 8.26.3 Member Function Documentation

### 8.26.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr DEVICE INLINED valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs
>::eval (
 const valueRing & x) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

|                  |                         |
|------------------|-------------------------|
| <i>valueRing</i> | usually float or double |
|------------------|-------------------------|

#### Parameters

|          |       |
|----------|-------|
| <i>x</i> | value |
|----------|-------|

#### Returns

$P(x)$

### 8.26.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string () [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.26.4 Member Data Documentation

### 8.26.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

### 8.26.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.27 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- [using raw\\_t = V](#)
- [using type = abs\\_t< typename Ring::template mod\\_t< V, X > >](#)

### 8.27.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

Template Parameters

|                   |                     |
|-------------------|---------------------|
| <a href="#">V</a> | a value from 'Ring' |
|-------------------|---------------------|



## 8.27.2 Member Typedef Documentation

### 8.27.2.1 raw\_t

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::raw_t = V
```

### 8.27.2.2 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.28 aerobus::zpz< p >::val< x > Struct Template Reference

values in zpz

```
#include <aerobus.h>
```

### Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< v==0 >`  
*true\_type if zero*

### Static Public Member Functions

- `static std::string to_string ()`  
*string representation*

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*
- `template<typename valueType >`  
`static constexpr valueType get = static_cast<valueType>(x % p)`  
*get value as valueType*
- `static constexpr bool is_zero_v = v == 0`  
*true if zero*

## 8.28.1 Detailed Description

```
template<int32_t p>
template<int32_t x>
struct aerobus::zpz< p >::val< x >
```

values in zpz

## Template Parameters

|   |            |
|---|------------|
| x | an integer |
|---|------------|

## 8.28.2 Member Typedef Documentation

### 8.28.2.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type

### 8.28.2.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<v == 0>
```

true\_type if zero

## 8.28.3 Member Function Documentation

### 8.28.3.1 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string () [inline], [static]
```

string representation

#### Returns

a string representation

## 8.28.4 Member Data Documentation

### 8.28.4.1 get

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
constexpr valueType aerobus::zpz< p >::val< x >::get = static_cast<valueType>(x % p) [static],
[constexpr]
```

get value as valueType

## Template Parameters

|                  |                                   |
|------------------|-----------------------------------|
| <i>valueType</i> | an arithmetic type, such as float |
|------------------|-----------------------------------|

## 8.28.4.2 is\_zero\_v

```
template<int32_t p>
template<int32_t x>
constexpr bool aerobus::zpz< p >::val< x >::is_zero_v = v == 0 [static], [constexpr]
```

true if zero

## 8.28.4.3 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.29 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

## Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

## Public Types

- [using ring\\_type = Ring](#)  
*ring coefficients live in*
- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)
- [using strip = val< coeffN >](#)
- [using is\\_zero\\_t = std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)

## Static Public Member Functions

- `static std::string to_string ()`

## Static Public Attributes

- `static constexpr size_t degree = 0`  
*degree*
- `static constexpr bool is_zero_v = is_zero_t::value`

### 8.29.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

|               |  |
|---------------|--|
| <i>coeffN</i> |  |
|---------------|--|

### 8.29.2 Member Typedef Documentation

#### 8.29.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

#### 8.29.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

#### 8.29.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.29.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_←
zero_t::value>
```

### 8.29.2.5 ring\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::ring_type = Ring
```

ring coefficients live in

### 8.29.2.6 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

## 8.29.3 Member Function Documentation

### 8.29.3.1 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string () [inline], [static]
```

## 8.29.4 Member Data Documentation

### 8.29.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

### 8.29.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.30 aerobus::zpz< p > Struct Template Reference

congruence classes of integers modulo p (32 bits)

```
#include <aerobus.h>
```

### Classes

- struct `val`  
*values in zpz*

### Public Types

- `using inner_type = int32_t`  
*underlying type for values*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`  
*injects a constant integer into zpz*
- `using zero = val< 0 >`  
*zero value*
- `using one = val< 1 >`  
*one value*
- `template<typename v1, typename v2 >`  
`using add_t = typename add< v1, v2 >::type`  
*addition operator*
- `template<typename v1, typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`  
*subtraction operator*
- `template<typename v1, typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication operator*
- `template<typename v1, typename v2 >`  
`using div_t = typename div< v1, v2 >::type`  
*division operator*
- `template<typename v1, typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1, typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1, typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1, typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1, typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos_t = typename pos< v1 >::type`  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`  
*true iff p is prime*
- `static constexpr bool is_euclidean_domain = true`  
*always true*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

### 8.30.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers modulo p (32 bits)

if p is prime, zpz

is a field

#### Template Parameters

|          |           |
|----------|-----------|
| <i>p</i> | a integer |
|----------|-----------|

### 8.30.2 Member Typedef Documentation

#### 8.30.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

### 8.30.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)



## Template Parameters

|                 |                                  |
|-----------------|----------------------------------|
| <code>v1</code> | a value in <code>zpz::val</code> |
| <code>v2</code> | a value in <code>zpz::val</code> |

## 8.30.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

injects a constant integer into zpz

## Template Parameters

|                |            |
|----------------|------------|
| <code>x</code> | an integer |
|----------------|------------|

## 8.30.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

underlying type for values

## 8.30.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

## Template Parameters

|                 |                                  |
|-----------------|----------------------------------|
| <code>v1</code> | a value in <code>zpz::val</code> |
| <code>v2</code> | a value in <code>zpz::val</code> |

## 8.30.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

#### 8.30.2.10 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

#### 8.30.2.11 one

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

one value

#### 8.30.2.12 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

#### Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
|-----------|-------------------------------------|

#### 8.30.2.13 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

zero value

## 8.30.3 Member Data Documentation

## 8.30.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

## Template Parameters

|           |                                     |
|-----------|-------------------------------------|
| <i>v1</i> | a value in <a href="#">zpz::val</a> |
| <i>v2</i> | a value in <a href="#">zpz::val</a> |

## 8.30.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

always true

### 8.30.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

true iff p is prime

### 8.30.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
| v2 | a value in <a href="#">zpz::val</a> |

### 8.30.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

|    |                                     |
|----|-------------------------------------|
| v1 | a value in <a href="#">zpz::val</a> |
|----|-------------------------------------|

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 README.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015 #ifdef WITH_CUDA_FP16
00016 #include <bit>
00017 #include <cuda_fp16.h>
00018 #endif
00019
00023 #ifdef _MSC_VER
00024 #define ALIGNED(x) __declspec(align(x))
00025 #define INLINED __forceinline
00026 #else
00027 #define ALIGNED(x) __attribute__((aligned(x)))
00028 #define INLINED __attribute__((always_inline)) inline
```

```

00029 #endif
00030
00031 #ifdef __CUDACC__
00032 #define DEVICE __host__ __device__
00033 #else
00034 #define DEVICE
00035 #endif
00036
00038
00040
00042
00043 // aligned allocation
00044 namespace aerobus {
00051 template<typename T>
00052 T* aligned_malloc(size_t count, size_t alignment) {
00053 #ifdef _MSC_VER
00054 return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00055 #else
00056 return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00057 #endif
00058 }
00059 } // namespace aerobus
00060
00061 // concepts
00062 namespace aerobus {
00064 template <typename R>
00065 concept IsRing = requires {
00066 typename R::one;
00067 typename R::zero;
00068 typename R::template add_t<typename R::one, typename R::one>;
00069 typename R::template sub_t<typename R::one, typename R::one>;
00070 typename R::template mul_t<typename R::one, typename R::one>;
00071 };
00072
00074 template <typename R>
00075 concept IsEuclideanDomain = IsRing<R> && requires {
00076 typename R::template div_t<typename R::one, typename R::one>;
00077 typename R::template mod_t<typename R::one, typename R::one>;
00078 typename R::template gcd_t<typename R::one, typename R::one>;
00079 typename R::template eq_t<typename R::one, typename R::one>;
00080 typename R::template pos_t<typename R::one>;
00081
00082 R::template pos_v<typename R::one> == true;
00083 // typename R::template gt_t<typename R::one, typename R::zero>;
00084 R::is_euclidean_domain == true;
00085 };
00086
00088 template<typename R>
00089 concept IsField = IsEuclideanDomain<R> && requires {
00090 R::is_field == true;
00091 };
00092 } // namespace aerobus
00093
00094 #ifdef WITH_CUDA_FP16
00095 // all this shit is required because of NVIDIA bug https://developer.nvidia.com/bugs/4863696
00096 namespace aerobus {
00097 namespace internal {
00098 static constexpr uint16_t my_internal_float2half(
00099 const float f, uint32_t &sign, uint32_t &remainder) {
00100 uint32_t x;
00101 uint32_t u;
00102 uint32_t result;
00103 x = std::bit_cast<int32_t>(f);
00104 u = (x & 0x7fffffffU);
00105 sign = ((x > 0) & 0x800000U);
00106 // NaN/+Inf/-Inf
00107 if (u >= 0x7f800000U) {
00108 remainder = 0U;
00109 result = ((u == 0x7f800000U) ? (sign | 0x7c00U) : 0x7fffU);
00110 } else if (u > 0x477fefffU) { // Overflows
00111 remainder = 0x80000000U;
00112 result = (sign | 0x7bfffU);
00113 } else if (u >= 0x38800000U) { // Normal numbers
00114 remainder = u << 19U;
00115 u -= 0x38000000U;
00116 result = (sign | (u >> 13U));
00117 } else if (u < 0x33000001U) { // +0/-0
00118 remainder = u;
00119 result = sign;
00120 } else { // Denormal numbers
00121 const uint32_t exponent = u >> 23U;
00122 const uint32_t shift = 0x7eU - exponent;
00123 uint32_t mantissa = (u & 0x7ffffU);
00124 mantissa |= 0x800000U;
00125 remainder = mantissa << (32U - shift);
00126 result = (sign | (mantissa >> shift));
00127 result &= 0x0000ffffU;

```

```

00128 }
00129 return static_cast<uint16_t>(result);
00130 }
00131
00132 static constexpr __half my_float2half_rn(const float a) {
00133 __half val;
00134 __half_raw r;
00135 uint32_t sign = 0U;
00136 uint32_t remainder = 0U;
00137 r.x = my_internal_float2half(a, sign, remainder);
00138 if ((remainder > 0x80000000U) || ((remainder == 0x80000000U) && ((r.x & 0x1U) != 0U))) {
00139 r.x++;
00140 }
00141
00142 val = std::bit_cast<__half>(r);
00143 return val;
00144 }
00145
00146 template<int16_t i>
00147 static constexpr __half convert_int16_to_half = my_float2half_rn(static_cast<float>(i));
00148 } // namespace internal
00149 } // namespace aerobus
00150 #endif
00151
00152 // fma_helper, required because nvidia fails to reconstruct fma
00153 namespace aerobus {
00154 namespace internal {
00155 template<typename T>
00156 struct fma_helper;
00157
00158 template<>
00159 struct fma_helper<double> {
00160 static constexpr INLINED_DEVICE double eval(const double x, const double y, const double
00161 z) {
00162 return x * y + z;
00163 }
00164 };
00165
00166 template<>
00167 struct fma_helper<float> {
00168 static constexpr INLINED_DEVICE float eval(const float x, const float y, const float z) {
00169 return x * y + z;
00170 }
00171 };
00172
00173 template<>
00174 struct fma_helper<int32_t> {
00175 static constexpr INLINED_DEVICE int32_t eval(const int32_t x, const int32_t y, const
00176 int32_t z) {
00177 return x * y + z;
00178 }
00179 };
00180
00181 template<>
00182 struct fma_helper<int64_t> {
00183 static constexpr INLINED_DEVICE int64_t eval(const int64_t x, const int64_t y, const
00184 int64_t z) {
00185 return x * y + z;
00186 }
00187 };
00188
00189 #ifdef WITH_CUDA_FP16
00190 template<>
00191 struct fma_helper<__half> {
00192 static constexpr INLINED_DEVICE __half eval(const __half x, const __half y, const __half
00193 z) {
00194 #ifdef __CUDA_ARCH__
00195 return __hmma(x, y, z);
00196 #else
00197 return x * y + z;
00198 #endif
00199 }
00200 };
00201
00202 template<>
00203 struct fma_helper<__half2> {
00204 static constexpr INLINED_DEVICE __half2 eval(const __half2 x, const __half2 y, const
00205 __half2 z) {
00206 #ifdef __CUDA_ARCH__
00207 return __hmma2(x, y, z);
00208 #else
00209 return x * y + z;
00210 #endif
00211 }
00212 };
00213 } // namespace internal
00214 } // namespace aerobus

```

```

00210
00211 // utilities
00212 namespace aerobus {
00213 namespace internal {
00214 template<template<typename...> typename TT, typename T>
00215 struct is_instantiation_of : std::false_type { };
00216
00217 template<template<typename...> typename TT, typename... Ts>
00218 struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00219
00220 template<template<typename...> typename TT, typename T>
00221 inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00222
00223 template<int64_t i, typename T, typename... Ts>
00224 struct type_at {
00225 static_assert(i < sizeof...(Ts) + 1, "index out of range");
00226 using type = typename type_at<i - 1, Ts...>::type;
00227 };
00228
00229 template<typename T, typename... Ts> struct type_at<0, T, Ts...> {
00230 using type = T;
00231 };
00232
00233 template<size_t i, typename... Ts>
00234 using type_at_t = typename type_at<i, Ts...>::type;
00235
00236
00237 template<size_t n, size_t i, typename E = void>
00238 struct _is_prime {};
00239
00240 template<size_t i>
00241 struct _is_prime<0, i> {
00242 static constexpr bool value = false;
00243 };
00244
00245 template<size_t i>
00246 struct _is_prime<1, i> {
00247 static constexpr bool value = false;
00248 };
00249
00250 template<size_t i>
00251 struct _is_prime<2, i> {
00252 static constexpr bool value = true;
00253 };
00254
00255 template<size_t i>
00256 struct _is_prime<3, i> {
00257 static constexpr bool value = true;
00258 };
00259
00260 template<size_t i>
00261 struct _is_prime<5, i> {
00262 static constexpr bool value = true;
00263 };
00264
00265 template<size_t i>
00266 struct _is_prime<7, i> {
00267 static constexpr bool value = true;
00268 };
00269
00270 template<size_t n, size_t i>
00271 struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)> {
00272 static constexpr bool value = false;
00273 };
00274
00275 template<size_t n, size_t i>
00276 struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)> {
00277 static constexpr bool value = false;
00278 };
00279
00280 template<size_t n, size_t i>
00281 struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)> {
00282 static constexpr bool value = true;
00283 };
00284
00285 template<size_t n, size_t i>
00286 struct _is_prime<n, i, std::enable_if_t<(
00287 n % i == 0 &&
00288 n >= 9 &&
00289 n % 3 != 0 &&
00290 n % 2 != 0 &&
00291 i * i > n)> {
00292 static constexpr bool value = true;
00293 };
00294
00295 template<size_t n, size_t i>
00296 struct _is_prime<n, i, std::enable_if_t<

```



```

00297 n % (i+2) == 0 &&
00298 n >= 9 &&
00299 n % 3 != 0 &&
00300 n % 2 != 0 &&
00301 i * i <= n)» {
00302 static constexpr bool value = true;
00303 };
00304
00305 template<size_t n, size_t i>
00306 struct _is_prime<n, i, std::enable_if_t<(
00307 n % (i+2) != 0 &&
00308 n % i != 0 &&
00309 n >= 9 &&
00310 n % 3 != 0 &&
00311 n % 2 != 0 &&
00312 (i * i <= n))» {
00313 static constexpr bool value = _is_prime<n, i+6>::value;
00314 };
00315
00316 } // namespace internal
00317
00320 template<size_t n>
00321 struct is_prime {
00322 static constexpr bool value = internal::_is_prime<n, 5>::value;
00323 };
00324
00325
00329 template<size_t n>
00330 static constexpr bool is_prime_v = is_prime<n>::value;
00331
00332 // gcd
00333 namespace internal {
00334 template <std::size_t... Is>
00335 constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00336 -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00337
00338 template <std::size_t N>
00339 using make_index_sequence_reverse
00340 = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00341
00342 template<typename Ring, typename E = void>
00343 struct gcd;
00344
00345 template<typename Ring>
00346 struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
00347 template<typename A, typename B, typename E = void>
00348 struct gcd_helper {};
00349
00350 // B = 0, A > 0
00351 template<typename A, typename B>
00352 struct gcd_helper<A, B, std::enable_if_t<
00353 ((B::is_zero_t::value) &&
00354 (Ring::template gt_t<A, typename Ring::zero>::value))> {
00355 using type = A;
00356 };
00357
00358 // B = 0, A < 0
00359 template<typename A, typename B>
00360 struct gcd_helper<A, B, std::enable_if_t<
00361 ((B::is_zero_t::value) &&
00362 !(Ring::template gt_t<A, typename Ring::zero>::value))> {
00363 using type = typename Ring::template sub_t<typename Ring::zero, A>;
00364 };
00365
00366 // B != 0
00367 template<typename A, typename B>
00368 struct gcd_helper<A, B, std::enable_if_t<
00369 (!B::is_zero_t::value)
00370 >> {
00371 private: // NOLINT
00372 // A / B
00373 using k = typename Ring::template div_t<A, B>;
00374 // A - (A/B)*B = A % B
00375 using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00376
00377 public:
00378 using type = typename gcd_helper<B, m>::type;
00379 };
00380
00381 template<typename A, typename B>
00382 using type = typename gcd_helper<A, B>::type;
00383 };
00384
00385 } // namespace internal
00386
00387 // vadd and vmul
00388 namespace internal {
00389 template<typename... vals>
00390 struct vmul {};

```

```

00395
00396 template<typename v1, typename... vals>
00397 struct vmul<v1, vals...> {
00398 using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00399 };
00400
00401 template<typename v1>
00402 struct vmul<v1> {
00403 using type = v1;
00404 };
00405
00406 template<typename... vals>
00407 struct vadd {};
00408
00409 template<typename v1, typename... vals>
00410 struct vadd<v1, vals...> {
00411 using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00412 };
00413
00414 template<typename v1>
00415 struct vadd<v1> {
00416 using type = v1;
00417 };
00418 } // namespace internal
00419
00422 template<typename T, typename A, typename B>
00423 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00424
00428 template<typename... vals>
00429 using vadd_t = typename internal::vadd<vals...>::type;
00430
00434 template<typename... vals>
00435 using vmul_t = typename internal::vmul<vals...>::type;
00436
00440 template<typename val>
00441 requires IsEuclideanDomain<typename val::enclosing_type>
00442 using abs_t = std::conditional_t<
00443 val::enclosing_type::template pos_v<val>,
00444 val, typename val::enclosing_type::template
sub_t<typename val::enclosing_type::zero, val>>;
00445 } // namespace aerobus
00446
00447 // embedding
00448 namespace aerobus {
00453 template<typename Small, typename Large, typename E = void>
00454 struct Embed;
00455 } // namespace aerobus
00456
00457 namespace aerobus {
00462 template<typename Ring, typename X>
00463 requires IsRing<Ring>
00464 struct Quotient {
00467 template <typename V>
00468 struct val {
00469 public:
00470 using raw_t = V;
00471 using type = abs_t<typename Ring::template mod_t<V, X>>;
00472 };
00473
00475 using zero = val<typename Ring::zero>;
00476
00478 using one = val<typename Ring::one>;
00479
00483 template<typename v1, typename v2>
00484 using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00485
00489 template<typename v1, typename v2>
00490 using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00491
00495 template<typename v1, typename v2>
00496 using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00497
00501 template<typename v1, typename v2>
00502 using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00503
00507 template<typename v1, typename v2>
00508 using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00509
00513 template<typename v1, typename v2>
00514 static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00515
00519 template<typename v1>
00520 using pos_t = std::true_type;
00521
00525 template<typename v>

```

```

00526 static constexpr bool pos_v = pos_t<v>::value;
00527
00529 static constexpr bool is_euclidean_domain = true;
00530
00536 template<auto x>
00537 using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00538
00544 template<typename v>
00545 using inject_ring_t = val<v>;
00546 };
00547
00551 template<typename Ring, typename X>
00552 struct Embed<Quotient<Ring, X>, Ring> {
00555 template<typename val>
00556 using type = typename val::raw_t;
00557 };
00558 } // namespace aerobus
00559
00560 // type_list
00561 namespace aerobus {
00563 template <typename... Ts>
00564 struct type_list;
00565
00566 namespace internal {
00567 template <typename T, typename... Us>
00568 struct pop_front_h {
00569 using tail = type_list<Us...>;
00570 using head = T;
00571 };
00572
00573 template <size_t index, typename L1, typename L2>
00574 struct split_h {
00575 private:
00576 static_assert(index <= L2::length, "index ouf of bounds");
00577 using a = typename L2::pop_front::type;
00578 using b = typename L2::pop_front::tail;
00579 using c = typename L1::template push_back<a>;
00580
00581 public:
00582 using head = typename split_h<index - 1, c, b>::head;
00583 using tail = typename split_h<index - 1, c, b>::tail;
00584 };
00585
00586 template <typename L1, typename L2>
00587 struct split_h<0, L1, L2> {
00588 using head = L1;
00589 using tail = L2;
00590 };
00591
00592 template <size_t index, typename L, typename T>
00593 struct insert_h {
00594 static_assert(index <= L::length, "index ouf of bounds");
00595 using s = typename L::template split<index>;
00596 using left = typename s::head;
00597 using right = typename s::tail;
00598 using ll = typename left::template push_back<T>;
00599 using type = typename ll::template concat<right>;
00600 };
00601
00602 template <size_t index, typename L>
00603 struct remove_h {
00604 using s = typename L::template split<index>;
00605 using left = typename s::head;
00606 using right = typename s::tail;
00607 using rr = typename right::pop_front::tail;
00608 using type = typename left::template concat<rr>;
00609 };
00610 } // namespace internal
00611
00615 template <typename... Ts>
00616 struct type_list {
00617 private:
00618 template <typename T>
00619 struct concat_h;
00620
00621 template <typename... Us>
00622 struct concat_h<type_list<Us...> {
00623 using type = type_list<Ts..., Us...>;
00624 };
00625
00626 public:
00628 static constexpr size_t length = sizeof...(Ts);
00629
00632 template <typename T>
00633 using push_front = type_list<T, Ts...>;
00634
00637 template <size_t index>

```

```

00638 using at = internal::type_at_t<index, Ts...>;
00639
00641 struct pop_front {
00642 using type = typename internal::pop_front_h<Ts...>::head;
00643 using tail = typename internal::pop_front_h<Ts...>::tail;
00644 };
00645
00647 template <typename T>
00650 using push_back = type_list<Ts..., T>;
00651
00652 template <typename U>
00655 using concat = typename concat_h<U>::type;
00656
00657 template <size_t index>
00660 struct split {
00661 private:
00662 using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00663
00664 public:
00665 using head = typename inner::head;
00666 using tail = typename inner::tail;
00667 };
00668
00669 template <typename T, size_t index>
00673 using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00674
00675 template <size_t index>
00678 using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00679
00680 };
00681
00683 template <>
00684 struct type_list<> {
00685 static constexpr size_t length = 0;
00686
00687 template <typename T>
00688 using push_front = type_list<T>;
00689
00690 template <typename T>
00691 using push_back = type_list<T>;
00692
00693 template <typename U>
00694 using concat = U;
00695
00696 // TODO(jewave): assert index == 0
00697 template <typename T, size_t index>
00698 using insert = type_list<T>;
00699 };
00700 } // namespace aerobus
00701
00702 // i16
00703 #ifdef WITH_CUDA_FP16
00704 // i16
00705 namespace aerobus {
00706 namespace internal {
00707 template<int16_t x, typename Out, typename E = void>
00708 struct int16_convert_helper;
00709
00710 template<int16_t x, typename Out>
00711 struct int16_convert_helper<x, Out, std::enable_if_t<
00712 !std::is_same_v<Out, __half> &&
00713 !std::is_same_v<Out, __half2>
00714 > {
00715 static constexpr Out value = static_cast<Out>(x);
00716 };
00717
00718 template<int16_t x>
00719 struct int16_convert_helper<x, __half> {
00720 static constexpr __half value = internal::convert_int16_to_half<x>;
00721 };
00722
00723 // this won't compile because of NVIDIA https://developer.nvidia.com/bugs/4872028
00724 template<int16_t x>
00725 struct int16_convert_helper<x, __half2> {
00726 static constexpr __half2 value = {
00727 internal::convert_int16_to_half<x>,
00728 internal::convert_int16_to_half<x>
00729 };
00730 };
00731 } // namespace internal
00732
00733 struct i16 {
00734 using inner_type = int16_t;
00735 template<int16_t x>
00736 struct val {
00737 using enclosing_type = i16;
00738 static constexpr int16_t v = x;
00739
00740 template<typename valueType>

```

```

00747 static constexpr valueType get = internal::template int16_convert_helper<x,
valueType>::value;
00748
00750 using is_zero_t = std::bool_constant<x == 0>;
00751
00753 static std::string to_string() {
00754 return std::to_string(x);
00755 }
00756 };
00757
00759 using zero = val<0>;
00761 using one = val<1>;
00763 static constexpr bool is_field = false;
00765 static constexpr bool is_euclidean_domain = true;
00769 template<auto x>
00770 using inject_constant_t = val<static_cast<int16_t>(x)>;
00771
00772 template<typename v>
00773 using inject_ring_t = v;
00774
00775 private:
00776 template<typename v1, typename v2>
00777 struct add {
00778 using type = val<v1::v + v2::v>;
00779 };
00780
00781 template<typename v1, typename v2>
00782 struct sub {
00783 using type = val<v1::v - v2::v>;
00784 };
00785
00786 template<typename v1, typename v2>
00787 struct mul {
00788 using type = val<v1::v * v2::v>;
00789 };
00790
00791 template<typename v1, typename v2>
00792 struct div {
00793 using type = val<v1::v / v2::v>;
00794 };
00795
00796 template<typename v1, typename v2>
00797 struct remainder {
00798 using type = val<v1::v % v2::v>;
00799 };
00800
00801 template<typename v1, typename v2>
00802 struct gt {
00803 using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00804 };
00805
00806 template<typename v1, typename v2>
00807 struct lt {
00808 using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00809 };
00810
00811 template<typename v1, typename v2>
00812 struct eq {
00813 using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00814 };
00815
00816 template<typename v1>
00817 struct pos {
00818 using type = std::bool_constant<(v1::v > 0)>;
00819 };
00820
00821 public:
00827 template<typename v1, typename v2>
00828 using add_t = typename add<v1, v2>::type;
00829
00835 template<typename v1, typename v2>
00836 using sub_t = typename sub<v1, v2>::type;
00837
00843 template<typename v1, typename v2>
00844 using mul_t = typename mul<v1, v2>::type;
00845
00851 template<typename v1, typename v2>
00852 using div_t = typename div<v1, v2>::type;
00853
00859 template<typename v1, typename v2>
00860 using mod_t = typename remainder<v1, v2>::type;
00861
00867 template<typename v1, typename v2>
00868 using gt_t = typename gt<v1, v2>::type;
00869
00875 template<typename v1, typename v2>
00876 using lt_t = typename lt<v1, v2>::type;

```

```

00877
00883 template<typename v1, typename v2>
00884 using eq_t = typename eq<v1, v2>::type;
00885
00890 template<typename v1, typename v2>
00891 static constexpr bool eq_v = eq_t<v1, v2>::value;
00892
00898 template<typename v1, typename v2>
00899 using gcd_t = gcd_t<i16, v1, v2>;
00900
00905 template<typename v>
00906 using pos_t = typename pos<v>::type;
00907
00912 template<typename v>
00913 static constexpr bool pos_v = pos_t<v>::value;
00914 };
00915 } // namespace aerobus
00916 #endif
00917
00918 // i32
00919 namespace aerobus {
00921 struct i32 {
00922 using inner_type = int32_t;
00923 template<int32_t x>
00924 struct val {
00925 using enclosing_type = i32;
00926 static constexpr int32_t v = x;
00927
00934 template<typename valueType>
00935 static constexpr valueType get = static_cast<valueType>(x);
00936
00938 using is_zero_t = std::bool_constant<x == 0>;
00939
00941 static std::string to_string() {
00942 return std::to_string(x);
00943 }
00944 };
00945
00947 using zero = val<0>;
00949 using one = val<1>;
00951 static constexpr bool is_field = false;
00953 static constexpr bool is_euclidean_domain = true;
00957 template<auto x>
00958 using inject_constant_t = val<static_cast<int32_t>(x)>;
00959
00960 template<typename v>
00961 using inject_ring_t = v;
00962
00963 private:
00964 template<typename v1, typename v2>
00965 struct add {
00966 using type = val<v1::v + v2::v>;
00967 };
00968
00969 template<typename v1, typename v2>
00970 struct sub {
00971 using type = val<v1::v - v2::v>;
00972 };
00973
00974 template<typename v1, typename v2>
00975 struct mul {
00976 using type = val<v1::v * v2::v>;
00977 };
00978
00979 template<typename v1, typename v2>
00980 struct div {
00981 using type = val<v1::v / v2::v>;
00982 };
00983
00984 template<typename v1, typename v2>
00985 struct remainder {
00986 using type = val<v1::v % v2::v>;
00987 };
00988
00989 template<typename v1, typename v2>
00990 struct gt {
00991 using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00992 };
00993
00994 template<typename v1, typename v2>
00995 struct lt {
00996 using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00997 };
00998
00999 template<typename v1, typename v2>
01000 struct eq {
01001 using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;

```

```

01002 };
01003
01004 template<typename v1>
01005 struct pos {
01006 using type = std::bool_constant<(v1::v > 0)>;
01007 };
01008
01009 public:
01010 template<typename v1, typename v2>
01011 using add_t = typename add<v1, v2>::type;
01012
01013 template<typename v1, typename v2>
01014 using sub_t = typename sub<v1, v2>::type;
01015
01016 template<typename v1, typename v2>
01017 using mul_t = typename mul<v1, v2>::type;
01018
01019 template<typename v1, typename v2>
01020 using div_t = typename div<v1, v2>::type;
01021
01022 template<typename v1, typename v2>
01023 using mod_t = typename remainder<v1, v2>::type;
01024
01025 template<typename v1, typename v2>
01026 using gt_t = typename gt<v1, v2>::type;
01027
01028 template<typename v1, typename v2>
01029 using lt_t = typename lt<v1, v2>::type;
01030
01031 template<typename v1, typename v2>
01032 using eq_t = typename eq<v1, v2>::type;
01033
01034 template<typename v1, typename v2>
01035 static constexpr bool eq_v = eq_t<v1, v2>::value;
01036
01037 template<typename v1, typename v2>
01038 using gcd_t = gcd_t<i32, v1, v2>;
01039
01040 template<typename v>
01041 using pos_t = typename pos<v>::type;
01042
01043 template<typename v>
01044 static constexpr bool pos_v = pos_t<v>::value;
01045 };
01046 } // namespace aerobus
01047
01048 // i64
01049 namespace aerobus {
01050 struct i64 {
01051 using inner_type = int64_t;
01052 template<int64_t x>
01053 struct val {
01054 using inner_type = int32_t;
01055 using enclosing_type = i64;
01056 static constexpr int64_t v = x;
01057
01058 template<typename valueType>
01059 static constexpr valueType get = static_cast<valueType>(x);
01060
01061 using is_zero_t = std::bool_constant<x == 0>;
01062
01063 static std::string to_string() {
01064 return std::to_string(x);
01065 }
01066 };
01067
01068 template<auto x>
01069 using inject_constant_t = val<static_cast<int64_t>(x)>;
01070
01071 template<typename v>
01072 using inject_ring_t = v;
01073
01074 using zero = val<0>;
01075 using one = val<1>;
01076 static constexpr bool is_field = false;
01077 static constexpr bool is_euclidean_domain = true;
01078
01079 private:
01080 template<typename v1, typename v2>
01081 struct add {
01082 using type = val<v1::v + v2::v>;
01083 };
01084
01085 template<typename v1, typename v2>
01086 struct sub {
01087 using type = val<v1::v - v2::v>;
01088 };
01089 };

```

```

01168
01169 template<typename v1, typename v2>
01170 struct mul {
01171 using type = val<v1::v* v2::v>;
01172 };
01173
01174 template<typename v1, typename v2>
01175 struct div {
01176 using type = val<v1::v / v2::v>;
01177 };
01178
01179 template<typename v1, typename v2>
01180 struct remainder {
01181 using type = val<v1::v% v2::v>;
01182 };
01183
01184 template<typename v1, typename v2>
01185 struct gt {
01186 using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
01187 };
01188
01189 template<typename v1, typename v2>
01190 struct lt {
01191 using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
01192 };
01193
01194 template<typename v1, typename v2>
01195 struct eq {
01196 using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
01197 };
01198
01199 template<typename v>
01200 struct pos {
01201 using type = std::bool_constant<(v::v > 0)>;
01202 };
01203
01204 public:
01209 template<typename v1, typename v2>
01210 using add_t = typename add<v1, v2>::type;
01211
01216 template<typename v1, typename v2>
01217 using sub_t = typename sub<v1, v2>::type;
01218
01223 template<typename v1, typename v2>
01224 using mul_t = typename mul<v1, v2>::type;
01225
01231 template<typename v1, typename v2>
01232 using div_t = typename div<v1, v2>::type;
01233
01238 template<typename v1, typename v2>
01239 using mod_t = typename remainder<v1, v2>::type;
01240
01246 template<typename v1, typename v2>
01247 using gt_t = typename gt<v1, v2>::type;
01248
01253 template<typename v1, typename v2>
01254 static constexpr bool gt_v = gt_t<v1, v2>::value;
01255
01261 template<typename v1, typename v2>
01262 using lt_t = typename lt<v1, v2>::type;
01263
01269 template<typename v1, typename v2>
01270 static constexpr bool lt_v = lt_t<v1, v2>::value;
01271
01277 template<typename v1, typename v2>
01278 using eq_t = typename eq<v1, v2>::type;
01279
01285 template<typename v1, typename v2>
01286 static constexpr bool eq_v = eq_t<v1, v2>::value;
01287
01293 template<typename v1, typename v2>
01294 using gcd_t = gcd_t<i64, v1, v2>;
01295
01300 template<typename v>
01301 using pos_t = typename pos<v>::type;
01302
01307 template<typename v>
01308 static constexpr bool pos_v = pos_t<v>::value;
01309 };
01310
01312 template<>
01313 struct Embed<i32, i64> {
01316 template<typename val>
01317 using type = i64::val<static_cast<int64_t>(val::v)>;
01318 };
01319 } // namespace aerobus
01320

```



```

01321 // z/pz
01322 namespace aerobus {
01323 template<int32_t p>
01324 struct zpz {
01325 using inner_type = int32_t;
01326
01327 template<int32_t x>
01328 struct val {
01329 using enclosing_type = zpz<p>;
01330 static constexpr int32_t v = x % p;
01331
01332 template<typename valueType>
01333 static constexpr valueType get = static_cast<valueType>(x % p);
01334
01335 using is_zero_t = std::bool_constant<v == 0>;
01336
01337 static constexpr bool is_zero_v = v == 0;
01338
01339 static std::string to_string() {
01340 return std::to_string(x % p);
01341 }
01342 };
01343
01344 template<auto x>
01345 using inject_constant_t = val<static_cast<int32_t>(x)>;
01346
01347 using zero = val<0>;
01348
01349 using one = val<1>;
01350
01351 static constexpr bool is_field = is_prime<p>::value;
01352
01353 static constexpr bool is_euclidean_domain = true;
01354
01355 private:
01356 template<typename v1, typename v2>
01357 struct add {
01358 using type = val<(v1::v + v2::v) % p>;
01359 };
01360
01361 template<typename v1, typename v2>
01362 struct sub {
01363 using type = val<(v1::v - v2::v) % p>;
01364 };
01365
01366 template<typename v1, typename v2>
01367 struct mul {
01368 using type = val<(v1::v * v2::v) % p>;
01369 };
01370
01371 template<typename v1, typename v2>
01372 struct div {
01373 using type = val<(v1::v % p) / (v2::v % p)>;
01374 };
01375
01376 template<typename v1, typename v2>
01377 struct remainder {
01378 using type = val<(v1::v % v2::v) % p>;
01379 };
01380
01381 template<typename v1, typename v2>
01382 struct gt {
01383 using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01384 };
01385
01386 template<typename v1, typename v2>
01387 struct lt {
01388 using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01389 };
01390
01391 template<typename v1, typename v2>
01392 struct eq {
01393 using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01394 };
01395
01396 template<typename v1>
01397 struct pos {
01398 using type = std::bool_constant<(v1::v > 0)>;
01399 };
01400
01401 public:
01402 template<typename v1, typename v2>
01403 using add_t = typename add<v1, v2>::type;
01404
01405 template<typename v1, typename v2>
01406 using sub_t = typename sub<v1, v2>::type;
01407
01408 template<typename v1, typename v2>
01409 using mul_t = typename mul<v1, v2>::type;
01410
01411 template<typename v1, typename v2>
01412 using div_t = typename div<v1, v2>::type;
01413
01414 template<typename v1, typename v2>
01415 using remainder_t = typename remainder<v1, v2>::type;
01416
01417 template<typename v1, typename v2>
01418 using gt_t = typename gt<v1, v2>::type;
01419
01420 template<typename v1, typename v2>
01421 using lt_t = typename lt<v1, v2>::type;
01422
01423 template<typename v1, typename v2>
01424 using eq_t = typename eq<v1, v2>::type;
01425
01426 template<typename v1>
01427 using pos_t = typename pos<v1>::type;
01428
01429 template<typename v1, typename v2>
01430 using is_zero_t = is_zero_t;
01431
01432 static constexpr bool is_zero_v = is_zero_v;
01433
01434 static std::string to_string() {
01435 return std::to_string(x % p);
01436 }
01437 };
01438 }

```

```

01439 template<typename v1, typename v2>
01440 using mul_t = typename mul<v1, v2>::type;
01441
01445 template<typename v1, typename v2>
01446 using div_t = typename div<v1, v2>::type;
01447
01451 template<typename v1, typename v2>
01452 using mod_t = typename remainder<v1, v2>::type;
01453
01457 template<typename v1, typename v2>
01458 using gt_t = typename gt<v1, v2>::type;
01459
01463 template<typename v1, typename v2>
01464 static constexpr bool gt_v = gt_t<v1, v2>::value;
01465
01469 template<typename v1, typename v2>
01470 using lt_t = typename lt<v1, v2>::type;
01471
01475 template<typename v1, typename v2>
01476 static constexpr bool lt_v = lt_t<v1, v2>::value;
01477
01481 template<typename v1, typename v2>
01482 using eq_t = typename eq<v1, v2>::type;
01483
01487 template<typename v1, typename v2>
01488 static constexpr bool eq_v = eq_t<v1, v2>::value;
01489
01493 template<typename v1, typename v2>
01494 using gcd_t = gcd_t<i32, v1, v2>;
01495
01498 template<typename v1>
01499 using pos_t = typename pos<v1>::type;
01500
01503 template<typename v>
01504 static constexpr bool pos_v = pos_t<v>::value;
01505 };
01506
01509 template<int32_t x>
01510 struct Embed<zp<x>, i32> {
01513 template <typename val>
01514 using type = i32::val<val::v>;
01515 };
01516 } // namespace aerobus
01517
01518 // polynomial
01519 namespace aerobus {
01520 // coeffN x^N + ...
01525 template<typename Ring>
01526 requires IsEuclideanDomain<Ring>
01527 struct polynomial {
01528 static constexpr bool is_field = false;
01529 static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01530
01534 template<typename coeffN, typename... coeffs>
01535 struct val {
01537 using ring_type = Ring;
01539 using enclosing_type = polynomial<Ring>;
01541 static constexpr size_t degree = sizeof...(coeffs);
01543 using aN = coeffN;
01545 using strip = val<coeffs...>;
01547 using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01549 static constexpr bool is_zero_v = is_zero_t::value;
01550
01551 private:
01552 template<size_t index, typename E = void>
01553 struct coeff_at {};
01554
01555 template<size_t index>
01556 struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01557 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01558 };
01559
01560 template<size_t index>
01561 struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01562 using type = typename Ring::zero;
01563 };
01564
01565 public:
01568 template<size_t index>
01569 using coeff_at_t = typename coeff_at<index>::type;
01570
01573 static std::string to_string() {
01574 return string_helper<coeffN, coeffs...>::func();
01575 }
01576
01581 template<typename valueRing>
01582 static constexpr DEVICE INLINED valueRing eval(const valueRing& x) {

```

```

01583 #ifdef WITH_CUDA_FP16
01584 valueRing start;
01585 if constexpr (std::is_same_v<valueRing, __half2>) {
01586 start = { 0, 0 };
01587 } else {
01588 start = static_cast<valueRing>(0);
01589 }
01590 #else
01591 valueRing start = static_cast<valueRing>(0);
01592 #endif
01593 return horner_evaluation<valueRing, val>
01594 ::template inner<0, degree + 1>
01595 ::func(start, x);
01596 }
01597 };
01598
01601 template<typename coeffN>
01602 struct val<coeffN> {
01603 using ring_type = Ring;
01604 using enclosing_type = polynomial<Ring>;
01605 static constexpr size_t degree = 0;
01606 using aN = coeffN;
01607 using strip = val<coeffN>;
01608 using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01609
01610 static constexpr bool is_zero_v = is_zero_t::value;
01611
01612 template<size_t index, typename E = void>
01613 struct coeff_at {};
01614
01615 template<size_t index>
01616 struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01617 using type = aN;
01618 };
01619
01620 template<size_t index>
01621 struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01622 using type = typename Ring::zero;
01623 };
01624
01625 template<size_t index>
01626 using coeff_at_t = typename coeff_at<index>::type;
01627
01628 static std::string to_string() {
01629 return string_helper<coeffN>::func();
01630 }
01631 };
01632
01633 using zero = val<typename Ring::zero>;
01634 using one = val<typename Ring::one>;
01635 using X = val<typename Ring::one, typename Ring::zero>;
01636
01637 private:
01638 template<typename P, typename E = void>
01639 struct simplify;
01640
01641 template <typename P1, typename P2, typename I>
01642 struct add_low;
01643
01644 template<typename P1, typename P2>
01645 struct add {
01646 using type = typename simplify<typename add_low<
01647 P1,
01648 P2,
01649 internal::make_index_sequence_reverse<
01650 std::max(P1::degree, P2::degree) + 1
01651 >::type>::type;
01652 };
01653
01654 template <typename P1, typename P2, typename I>
01655 struct sub_low;
01656
01657 template <typename P1, typename P2, typename I>
01658 struct mul_low;
01659
01660 template<typename v1, typename v2>
01661 struct mul {
01662 using type = typename mul_low<
01663 v1,
01664 v2,
01665 internal::make_index_sequence_reverse<
01666 v1::degree + v2::degree + 1
01667 >::type;
01668 };
01669
01670 template<typename coeff, size_t deg>
01671 struct monomial;

```

```

01678
01679 template<typename v, typename E = void>
01680 struct derive_helper {};
01681
01682 template<typename v>
01683 struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01684 using type = zero;
01685 };
01686
01687 template<typename v>
01688 struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01689 using type = typename add<
01690 typename derive_helper<typename simplify<typename v::strip>::type>::type,
01691 typename monomial<
01692 typename Ring::template mul_t<
01693 typename v::aN,
01694 typename Ring::template inject_constant_t<(v::degree)>
01695 >,
01696 v::degree - 1
01697 >::type
01698 >::type;
01699 };
01700
01701 template<typename v1, typename v2, typename E = void>
01702 struct eq_helper {};
01703
01704 template<typename v1, typename v2>
01705 struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01706 using type = std::false_type;
01707 };
01708
01709 template<typename v1, typename v2>
01710 struct eq_helper<v1, v2, std::enable_if_t<
01711 v1::degree == v2::degree &&
01712 (v1::degree != 0 || v2::degree != 0) &&
01713 std::is_same<
01714 typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01715 std::false_type
01716 >::value
01717 > {
01718 > {
01719 using type = std::false_type;
01720 };
01721
01722 template<typename v1, typename v2>
01723 struct eq_helper<v1, v2, std::enable_if_t<
01724 v1::degree == v2::degree &&
01725 (v1::degree != 0 || v2::degree != 0) &&
01726 std::is_same<
01727 typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01728 std::true_type
01729 >::value
01730 > {
01731 > {
01732 using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01733 };
01734
01735 template<typename v1, typename v2>
01736 struct eq_helper<v1, v2, std::enable_if_t<
01737 v1::degree == v2::degree &&
01738 (v1::degree == 0)
01739 > {
01740 using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01741 };
01742
01743 template<typename v1, typename v2, typename E = void>
01744 struct lt_helper {};
01745
01746 template<typename v1, typename v2>
01747 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01748 using type = std::true_type;
01749 };
01750
01751 template<typename v1, typename v2>
01752 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01753 using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01754 };
01755
01756 template<typename v1, typename v2>
01757 struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01758 using type = std::false_type;
01759 };
01760
01761 template<typename v1, typename v2, typename E = void>
01762 struct gt_helper {};
01763
01764 template<typename v1, typename v2>

```

```

01765 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01766 using type = std::true_type;
01767 };
01768
01769 template<typename v1, typename v2>
01770 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01771 using type = std::false_type;
01772 };
01773
01774 template<typename v1, typename v2>
01775 struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01776 using type = std::false_type;
01777 };
01778
01779 // when high power is zero : strip
01780 template<typename P>
01781 struct simplify<P, std::enable_if_t<
01782 std::is_same<
01783 typename Ring::zero,
01784 typename P::aN
01785 >::value && (P::degree > 0)
01786 >> {
01787 using type = typename simplify<typename P::strip>::type;
01788 };
01789
01790 // otherwise : do nothing
01791 template<typename P>
01792 struct simplify<P, std::enable_if_t<
01793 !std::is_same<
01794 typename Ring::zero,
01795 typename P::aN
01796 >::value && (P::degree > 0)
01797 >> {
01798 using type = P;
01799 };
01800
01801 // do not simplify constants
01802 template<typename P>
01803 struct simplify<P, std::enable_if_t<P::degree == 0>> {
01804 using type = P;
01805 };
01806
01807 // addition at
01808 template<typename P1, typename P2, size_t index>
01809 struct add_at {
01810 using type =
01811 typename Ring::template add_t<
01812 typename P1::template coeff_at_t<index>,
01813 typename P2::template coeff_at_t<index>>;
01814 };
01815
01816 template<typename P1, typename P2, size_t index>
01817 using add_at_t = typename add_at<P1, P2, index>::type;
01818
01819 template<typename P1, typename P2, std::size_t... I>
01820 struct add_low<P1, P2, std::index_sequence<I...>> {
01821 using type = val<add_at_t<P1, P2, I>...>;
01822 };
01823
01824 // subtraction at
01825 template<typename P1, typename P2, size_t index>
01826 struct sub_at {
01827 using type =
01828 typename Ring::template sub_t<
01829 typename P1::template coeff_at_t<index>,
01830 typename P2::template coeff_at_t<index>>;
01831 };
01832
01833 template<typename P1, typename P2, size_t index>
01834 using sub_at_t = typename sub_at<P1, P2, index>::type;
01835
01836 template<typename P1, typename P2, std::size_t... I>
01837 struct sub_low<P1, P2, std::index_sequence<I...>> {
01838 using type = val<sub_at_t<P1, P2, I>...>;
01839 };
01840
01841 template<typename P1, typename P2>
01842 struct sub {
01843 using type = typename simplify<typename sub_low<
01844 P1,
01845 P2,
01846 internal::make_index_sequence_reverse<
01847 std::max(P1::degree, P2::degree) + 1
01848 >::type>::type;
01849 };
01850
01851 // multiplication at

```

```

01852 template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01853 struct mul_at_loop_helper {
01854 using type = typename Ring::template add_t<
01855 typename Ring::template mul_t<
01856 typename v1::template coeff_at_t<index>,
01857 typename v2::template coeff_at_t<k - index>
01858 >,
01859 typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01860 >;
01861 };
01862
01863 template<typename v1, typename v2, size_t k, size_t stop>
01864 struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01865 using type = typename Ring::template mul_t<
01866 typename v1::template coeff_at_t<stop>,
01867 typename v2::template coeff_at_t<0>>;
01868 };
01869
01870 template <typename v1, typename v2, size_t k, typename E = void>
01871 struct mul_at {};
01872
01873 template<typename v1, typename v2, size_t k>
01874 struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01875 using type = typename Ring::zero;
01876 };
01877
01878 template<typename v1, typename v2, size_t k>
01879 struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01880 using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01881 };
01882
01883 template<typename P1, typename P2, size_t index>
01884 using mul_at_t = typename mul_at<P1, P2, index>::type;
01885
01886 template<typename P1, typename P2, std::size_t... I>
01887 struct mul_low<P1, P2, std::index_sequence<I...> {
01888 using type = val<mul_at_t<P1, P2, I>...>;
01889 };
01890
01891 // division helper
01892 template< typename A, typename B, typename Q, typename R, typename E = void>
01893 struct div_helper {};
01894
01895 template<typename A, typename B, typename Q, typename R>
01896 struct div_helper<A, B, Q, R, std::enable_if_t<
01897 (R::degree < B::degree) ||
01898 (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01899 using q_type = Q;
01900 using mod_type = R;
01901 using gcd_type = B;
01902 };
01903
01904 template<typename A, typename B, typename Q, typename R>
01905 struct div_helper<A, B, Q, R, std::enable_if_t<
01906 (R::degree >= B::degree) &&
01907 !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01908 private: // NOLINT
01909 using rN = typename R::aN;
01910 using bN = typename B::aN;
01911 using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
B::degree>::type;
01912 using rr = typename sub<R, typename mul<pT, B>::type>::type;
01913 using qq = typename add<Q, pT>::type;
01914
01915 public:
01916 using q_type = typename div_helper<A, B, qq, rr>::q_type;
01917 using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01918 using gcd_type = rr;
01919 };
01920
01921 template<typename A, typename B>
01922 struct div {
01923 static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01924 using q_type = typename div_helper<A, B, zero, A>::q_type;
01925 using m_type = typename div_helper<A, B, zero, A>::mod_type;
01926 };
01927
01928 template<typename P>
01929 struct make_unit {
01930 using type = typename div<P, val<typename P::aN>::q_type>;
01931 };
01932
01933 template<typename coeff, size_t deg>
01934 struct monomial {
01935 using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01936 };
01937

```

```

01938 template<typename coeff>
01939 struct monomial<coeff, 0> {
01940 using type = val<coeff>;
01941 };
01942
01943 template<typename valueRing, typename P>
01944 struct horner_evaluation {
01945 template<size_t index, size_t stop>
01946 struct inner {
01947 static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01948
01949 constexpr valueRing coeff =
static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
get<valueRing>());
01950 return horner_evaluation<valueRing, P::template inner<index + 1, stop>::func(
internal::fma_helper<valueRing>::eval(x, accum, coeff), x);
01951 }
01952 };
01953 };
01954
01955 template<size_t stop>
01956 struct inner<stop, stop> {
01957 static constexpr DEVICE INLINED valueRing func(const valueRing& accum, const
valueRing& x) {
01958 return accum;
01959 }
01960 };
01961 };
01962
01963 template<typename coeff, typename... coeffs>
01964 struct string_helper {
01965 static std::string func() {
01966 std::string tail = string_helper<coeffs...>::func();
01967 std::string result = "";
01968 if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01969 return tail;
01970 } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01971 if (sizeof...(coeffs) == 1) {
01972 result += "x";
01973 } else {
01974 result += "x^" + std::to_string(sizeof...(coeffs));
01975 }
01976 } else {
01977 if (sizeof...(coeffs) == 1) {
01978 result += coeff::to_string() + " x";
01979 } else {
01980 result += coeff::to_string()
+ " x^" + std::to_string(sizeof...(coeffs));
01981 }
01982 }
01983 }
01984
01985 if (!tail.empty()) {
01986 result += " + " + tail;
01987 }
01988
01989 return result;
01990 }
01991 };
01992
01993 template<typename coeff>
01994 struct string_helper<coeff> {
01995 static std::string func() {
01996 if (!std::is_same<coeff, typename Ring::zero>::value) {
01997 return coeff::to_string();
01998 } else {
01999 return "";
02000 }
02001 }
02002 };
02003
02004 public:
02005 template<typename P>
02006 using simplify_t = typename simplify<P>::type;
02007
02008 template<typename v1, typename v2>
02009 using add_t = typename add<v1, v2>::type;
02010
02011 template<typename v1, typename v2>
02012 using sub_t = typename sub<v1, v2>::type;
02013
02014 template<typename v1, typename v2>
02015 using mul_t = typename mul<v1, v2>::type;
02016
02017 template<typename v1, typename v2>
02018 using eq_t = typename eq_helper<v1, v2>::type;
02019
02020 template<typename v1, typename v2>
02021 using lt_t = typename lt_helper<v1, v2>::type;

```

```

02039
02043 template<typename v1, typename v2>
02044 using gt_t = typename gt_helper<v1, v2>::type;
02045
02049 template<typename v1, typename v2>
02050 using div_t = typename div<v1, v2>::q_type;
02051
02055 template<typename v1, typename v2>
02056 using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
02057
02061 template<typename coeff, size_t deg>
02062 using monomial_t = typename monomial<coeff, deg>::type;
02063
02066 template<typename v>
02067 using derive_t = typename derive_helper<v>::type;
02068
02071 template<typename v>
02072 using pos_t = typename Ring::template pos_t<typename v::aN>;
02073
02076 template<typename v>
02077 static constexpr bool pos_v = pos_t<v>::value;
02078
02082 template<typename v1, typename v2>
02083 using gcd_t = std::conditional_t<
02084 Ring::is_euclidean_domain,
02085 typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
02086 void>;
02087
02091 template<auto x>
02092 using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
02093
02097 template<typename v>
02098 using inject_ring_t = val<v>;
02099 };
02100 } // namespace aerobus
02101
02102 // fraction field
02103 namespace aerobus {
02104 namespace internal {
02105 template<typename Ring, typename E = void>
02106 requires IsEuclideanDomain<Ring>
02107 struct _FractionField {};
02108
02109 template<typename Ring>
02110 requires IsEuclideanDomain<Ring>
02111 struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
02112 static constexpr bool is_field = true;
02113 static constexpr bool is_euclidean_domain = true;
02114
02115 private:
02116 template<typename val1, typename val2, typename E = void>
02117 struct to_string_helper {};
02118
02119 template<typename val1, typename val2>
02120 struct to_string_helper<val1, val2,
02121 std::enable_if_t<
02122 Ring::template eq_t<
02123 val2, typename Ring::one
02124 >::value
02125 >
02126 > {
02127 static std::string func() {
02128 return val1::to_string();
02129 }
02130 };
02131
02132 template<typename val1, typename val2>
02133 struct to_string_helper<val1, val2,
02134 std::enable_if_t<
02135 !Ring::template eq_t<
02136 val2,
02137 typename Ring::one
02138 >::value
02139 >
02140 > {
02141 static std::string func() {
02142 return "(" + val1::to_string() + ")" / "(" + val2::to_string() + ")";
02143 }
02144 };
02145 };
02146
02147 public:
02151 template<typename val1, typename val2>
02152 struct val {
02153 using x = val1;
02154 using y = val2;
02155 using is_zero_t = typename val1::is_zero_t;
02156 static constexpr bool is_zero_v = val1::is_zero_t::value;
02160

```



```

02161
02163 using ring_type = Ring;
02164 using enclosing_type = _FractionField<Ring>;
02165
02166 static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
02169
02173 template<typename valueType>
02174 static constexpr valueType get = static_cast<valueType>(x::v) /
static_cast<valueType>(y::v);
02175
02178 static std::string to_string() {
02179 return to_string_helper<val1, val2>::func();
02180 }
02181
02186 template<typename valueRing>
02187 static constexpr DEVICE INLINED valueRing eval(const valueRing& v) {
02188 return x::eval(v) / y::eval(v);
02189 }
02190 };
02191
02193 using zero = val<typename Ring::zero, typename Ring::one>;
02195 using one = val<typename Ring::one, typename Ring::one>;
02196
02199 template<typename v>
02200 using inject_t = val<v, typename Ring::one>;
02201
02204 template<auto x>
02205 using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
02206
02209 template<typename v>
02210 using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
02211
02213 using ring_type = Ring;
02214
02215 private:
02216 template<typename v, typename E = void>
02217 struct simplify {};
02218
02219 // x = 0
02220 template<typename v>
02221 struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
02222 using type = typename _FractionField<Ring>::zero;
02223 };
02224
02225 // x != 0
02226 template<typename v>
02227 struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
02228 private:
02229 using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
02230 using newx = typename Ring::template div_t<typename v::x, _gcd>;
02231 using newy = typename Ring::template div_t<typename v::y, _gcd>;
02232
02233 using posx = std::conditional_t<
02234 !Ring::template pos_v<newx>,
02235 typename Ring::template sub_t<typename Ring::zero, newx>,
02236 newx>;
02237 using posy = std::conditional_t<
02238 !Ring::template pos_v<newy>,
02239 typename Ring::template sub_t<typename Ring::zero, newy>,
02240 newy>;
02241 public:
02242 using type = typename _FractionField<Ring>::template val<posx, posy>;
02243 };
02244
02245 public:
02248 template<typename v>
02249 using simplify_t = typename simplify<v>::type;
02250
02251 private:
02252 template<typename v1, typename v2>
02253 struct add {
02254 private:
02255 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
02256 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
02257 using dividend = typename Ring::template add_t<a, b>;
02258 using divider = typename Ring::template mul_t<typename v1::y, typename v2::y>;
02259 using g = typename Ring::template gcd_t<dividend, divider>;
02260
02261 public:
02262 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
02263 };
02264
02265 template<typename v>
02266 struct pos {
02267 using type = std::conditional_t<

```



```

02353 template<typename v1, typename v2>
02354 struct gt<v1, v2, std::enable_if_t<
02355 (!eq<v1, v2>::type::value) &&
02356 (!pos<v1>::type::value) && (pos<v2>::type::value)
02357 >> {
02358 using type = std::false_type;
02359 };
02360
02361 template<typename v1, typename v2>
02362 struct gt<v1, v2, std::enable_if_t<
02363 (!eq<v1, v2>::type::value) &&
02364 (pos<v1>::type::value) && (pos<v2>::type::value)
02365 >> {
02366 using type = typename Ring::template gt_t<
02367 typename Ring::template mul_t<v1::x, v2::y>,
02368 typename Ring::template mul_t<v2::y, v2::x>
02369 >;
02370 };
02371
02372 public:
02373 template<typename v1, typename v2>
02374 using add_t = typename add<v1, v2>::type;
02375
02376 template<typename v1, typename v2>
02377 using mod_t = zero;
02378
02379 template<typename v1, typename v2>
02380 using gcd_t = v1;
02381
02382 template<typename v1, typename v2>
02383 using sub_t = typename sub<v1, v2>::type;
02384
02385 template<typename v1, typename v2>
02386 using mul_t = typename mul<v1, v2>::type;
02387
02388 template<typename v1, typename v2>
02389 using div_t = typename div<v1, v2>::type;
02390
02391 template<typename v1, typename v2>
02392 using eq_t = typename eq<v1, v2>::type;
02393
02394 template<typename v1, typename v2>
02395 static constexpr bool eq_v = eq<v1, v2>::type::value;
02396
02397 template<typename v1, typename v2>
02398 using gt_t = typename gt<v1, v2>::type;
02399
02400 template<typename v1, typename v2>
02401 static constexpr bool gt_v = gt<v1, v2>::type::value;
02402
02403 template<typename v1>
02404 using pos_t = typename pos<v1>::type;
02405
02406 template<typename v>
02407 static constexpr bool pos_v = pos_t<v>::value;
02408 };
02409
02410 template<typename Ring, typename E = void>
02411 requires IsEuclideanDomain<Ring>
02412 struct FractionFieldImpl {};
02413
02414 // fraction field of a field is the field itself
02415 template<typename Field>
02416 requires IsEuclideanDomain<Field>
02417 struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field>> {
02418 using type = Field;
02419 template<typename v>
02420 using inject_t = v;
02421 };
02422
02423 // fraction field of a ring is the actual fraction field
02424 template<typename Ring>
02425 requires IsEuclideanDomain<Ring>
02426 struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field>> {
02427 using type = _FractionField<Ring>;
02428 };
02429 } // namespace internal
02430
02431 template<typename Ring>
02432 requires IsEuclideanDomain<Ring>
02433 using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02434
02435 template<typename Ring>
02436 struct Embed<Ring, FractionField<Ring>> {
02437 template<typename v>
02438 using type = typename FractionField<Ring>::template val<v, typename Ring::one>;
02439 };

```

```

02484 } // namespace aerobus
02485
02486
02487 // short names for common types
02488 namespace aerobus {
02492 template<typename X, typename Y>
02493 requires IsRing<typename X::enclosing_type> &&
02494 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02495 using add_t = typename X::enclosing_type::template add_t<X, Y>;
02496
02500 template<typename X, typename Y>
02501 requires IsRing<typename X::enclosing_type> &&
02502 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02503 using sub_t = typename X::enclosing_type::template sub_t<X, Y>;
02504
02508 template<typename X, typename Y>
02509 requires IsRing<typename X::enclosing_type> &&
02510 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02511 using mul_t = typename X::enclosing_type::template mul_t<X, Y>;
02512
02516 template<typename X, typename Y>
02517 requires IsEuclideanDomain<typename X::enclosing_type> &&
02518 (std::is_same_v<typename X::enclosing_type, typename Y::enclosing_type>)
02519 using div_t = typename X::enclosing_type::template div_t<X, Y>;
02520
02523 using q32 = FractionField<i32>;
02524
02527 using fpq32 = FractionField<polynomial<q32>>;
02528
02531 using q64 = FractionField<i64>;
02532
02534 using pi64 = polynomial<i64>;
02535
02537 using pq64 = polynomial<q64>;
02538
02540 using fpq64 = FractionField<polynomial<q64>>;
02541
02546 template<typename Ring, typename v1, typename v2>
02547 using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02548
02555 template<typename v>
02556 using embed_int_poly_in_fractions_t =
02557 typename Embed<
02558 polynomial<typename v::ring_type>,
02559 polynomial<FractionField<typename v::ring_type>>::template type<v>;
02560
02564 template<int64_t p, int64_t q>
02565 using make_q64_t = typename q64::template simplify_t<
02566 typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02567
02571 template<int32_t p, int32_t q>
02572 using make_q32_t = typename q32::template simplify_t<
02573 typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02574
02579 template<typename Ring, typename v1, typename v2>
02580 using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02585 template<typename Ring, typename v1, typename v2>
02586 using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02587
02589 template<>
02590 struct Embed<q32, q64> {
02593 template<typename v>
02594 using type = make_q64_t<static_cast<int64_t>(v::x::v), static_cast<int64_t>(v::y::v)>;
02595 };
02596
02600 template<typename Small, typename Large>
02601 struct Embed<polynomial<Small>, polynomial<Large>> {
02602 private:
02603 template<typename v, typename i>
02604 struct at_low;
02605
02606 template<typename v, size_t i>
02607 struct at_index {
02608 using type = typename Embed<Small, Large>::template
02609 type<typename v::template coeff_at_t<i>>;
02610 };
02611
02612 template<typename v, size_t... Is>
02613 struct at_low<v, std::index_sequence<Is...>> {
02614 using type = typename polynomial<Large>::template val<typename at_index<v, Is>::type...>;
02615 };
02616 public:
02619 template<typename v>
02620 using type = typename
02621 at_low<v, typename internal::make_index_sequence_reverse<v::degree + 1>::type>;
02621 };

```

```

02622
02626 template<typename Ring, auto... xs>
02627 using make_int_polynomial_t = typename polynomial<Ring>::template val<
02628 typename Ring::template inject_constant_t<xs>...>;
02629
02633 template<typename Ring, auto... xs>
02634 using make_frac_polynomial_t = typename polynomial<FractionField<Ring>>::template val<
02635 typename FractionField<Ring>::template inject_constant_t<xs>...>;
02636 } // namespace aerobus
02637
02638 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02639 namespace aerobus {
02640 namespace internal {
02641 template<typename T, size_t x, typename E = void>
02642 struct factorial {};
02643
02644 template<typename T, size_t x>
02645 struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02646 private:
02647 template<typename, size_t, typename>
02648 friend struct factorial;
02649 public:
02650 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02651 static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>;
02652 };
02653
02654 template<typename T>
02655 struct factorial<T, 0> {
02656 public:
02657 using type = typename T::one;
02658 static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>;
02659 };
02660 } // namespace internal
02661
02665 template<typename T, size_t i>
02666 using factorial_t = typename internal::factorial<T, i>::type;
02667
02671 template<typename T, size_t i>
02672 inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02673
02674 namespace internal {
02675 template<typename T, size_t k, size_t n, typename E = void>
02676 struct combination_helper {};
02677
02678 template<typename T, size_t k, size_t n>
02679 struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)>> {
02680 using type = typename FractionField<T>::template mul_t<
02681 typename combination_helper<T, k - 1, n - 1>::type,
02682 makefraction_t<T, typename T::template val<n>, typename T::template val<k>>;
02683 };
02684
02685 template<typename T, size_t k, size_t n>
02686 struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02687 using type = typename combination_helper<T, n - k, n>::type;
02688 };
02689
02690 template<typename T, size_t n>
02691 struct combination_helper<T, 0, n> {
02692 using type = typename FractionField<T>::one;
02693 };
02694
02695 template<typename T, size_t k, size_t n>
02696 struct combination {
02697 using type = typename internal::combination_helper<T, k, n>::type::x;
02698 static constexpr typename T::inner_type value =
02699 internal::combination_helper<T, k, n>::type::template
get<typename T::inner_type>;
02700 };
02701 } // namespace internal
02702
02705 template<typename T, size_t k, size_t n>
02706 using combination_t = typename internal::combination<T, k, n>::type;
02707
02712 template<typename T, size_t k, size_t n>
02713 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02714
02715 namespace internal {
02716 template<typename T, size_t m>
02717 struct bernoulli;
02718
02719 template<typename T, typename accum, size_t k, size_t m>
02720 struct bernoulli_helper {
02721 using type = typename bernoulli_helper<
02722 T,

```

```

02723 addfractions_t<T,
02724 accum,
02725 mulfractions_t<T,
02726 makefraction_t<T,
02727 combination_t<T, k, m + 1>,
02728 typename T::one>,
02729 typename bernoulli<T, k>::type
02730 >
02731 >,
02732 k + 1,
02733 m>::type;
02734 };
02735
02736 template<typename T, typename accum, size_t m>
02737 struct bernoulli_helper<T, accum, m, m> {
02738 using type = accum;
02739 };
02740
02741
02742
02743 template<typename T, size_t m>
02744 struct bernoulli {
02745 using type = typename FractionField<T>::template mul_t<
02746 typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02747 makefraction_t<T,
02748 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02749 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02750 >
02751 >;
02752
02753 template<typename floatType>
02754 static constexpr floatType value = type::template get<floatType>;
02755 };
02756
02757 template<typename T>
02758 struct bernoulli<T, 0> {
02759 using type = typename FractionField<T>::one;
02760
02761 template<typename floatType>
02762 static constexpr floatType value = type::template get<floatType>;
02763 };
02764 } // namespace internal
02765
02766 template<typename T, size_t n>
02767 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02768
02769 template<typename FloatType, typename T, size_t n>
02770 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02771
02772 // bell numbers
02773 namespace internal {
02774 template<typename T, size_t n, typename E = void>
02775 struct bell_helper;
02776
02777 template<typename T, size_t n>
02778 struct bell_helper<T, n, std::enable_if_t<(n > 1)>> {
02779 template<typename accum, size_t i, size_t stop>
02780 struct sum_helper {
02781 private:
02782 using left = typename T::template mul_t<
02783 combination_t<T, i, n-1>,
02784 typename bell_helper<T, i>::type>;
02785 using new_accum = typename T::template add_t<accum, left>;
02786 public:
02787 using type = typename sum_helper<new_accum, i+1, stop>::type;
02788 };
02789
02790 template<typename accum, size_t stop>
02791 struct sum_helper<accum, stop, stop> {
02792 using type = accum;
02793 };
02794
02795 using type = typename sum_helper<typename T::zero, 0, n>::type;
02796 };
02797
02798 template<typename T>
02799 struct bell_helper<T, 0> {
02800 using type = typename T::one;
02801 };
02802
02803 template<typename T>
02804 struct bell_helper<T, 1> {
02805 using type = typename T::one;
02806 };
02807 } // namespace internal
02808
02809 template<typename T, size_t n>

```

```

02820 using bell_t = typename internal::bell_helper<T, n>::type;
02821
02825 template<typename T, size_t n>
02826 static constexpr typename T::inner_type bell_v = bell_t<T, n>::v;
02827
02828 namespace internal {
02829 template<typename T, int k, typename E = void>
02830 struct alternate {};
02831
02832 template<typename T, int k>
02833 struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02834 using type = typename T::one;
02835 static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>;
02836 };
02837
02838 template<typename T, int k>
02839 struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02840 using type = typename T::template sub_t<typename T::zero, typename T::one>;
02841 static constexpr typename T::inner_type value = type::template
get<typename T::inner_type>;
02842 };
02843 } // namespace internal
02844
02847 template<typename T, int k>
02848 using alternate_t = typename internal::alternate<T, k>::type;
02849
02850 namespace internal {
02851 template<typename T, int n, int k, typename E = void>
02852 struct stirling_helper {};
02853
02854 template<typename T>
02855 struct stirling_helper<T, 0, 0> {
02856 using type = typename T::one;
02857 };
02858
02859 template<typename T, int n>
02860 struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02861 using type = typename T::zero;
02862 };
02863
02864 template<typename T, int n>
02865 struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02866 using type = typename T::zero;
02867 };
02868
02869 template<typename T, int n, int k>
02870 struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02871 using type = typename T::template sub_t<
02872 typename stirling_helper<T, n-1, k-1>::type,
02873 typename T::template mul_t<
02874 typename T::template inject_constant_t<n-1>,
02875 typename stirling_helper<T, n-1, k>::type
02876 >;
02877 };
02878 } // namespace internal
02879
02884 template<typename T, int n, int k>
02885 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02886
02891 template<typename T, int n, int k>
02892 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02893
02898 template<typename T, int n, int k>
02899 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02900
02906 template<typename T, int n, int k>
02907 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02908
02911 template<typename T, size_t k>
02912 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02913
02914 namespace internal {
02915 template<typename T>
02916 struct pow_scalar {
02917 template<size_t p>
02918 static constexpr DEVICE INLINED T func(const T& x) { return p == 0 ? static_cast<T>(1) :
02919 p % 2 == 0 ? func<p/2>(x) * func<p/2>(x) :
02920 x * func<p/2>(x) * func<p/2>(x);
02921 }
02922 };
02923
02924 template<typename T, typename p, size_t n, typename E = void>
02925 requires IsEuclideanDomain<T>
02926 struct pow;
02927

```

```

02928 template<typename T, typename p, size_t n>
02929 struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02930 using type = typename T::template mul_t<
02931 typename pow<T, p, n/2>::type,
02932 typename pow<T, p, n/2>::type
02933 >;
02934 };
02935
02936 template<typename T, typename p, size_t n>
02937 struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02938 using type = typename T::template mul_t<
02939 p,
02940 typename T::template mul_t<
02941 typename pow<T, p, n/2>::type,
02942 typename pow<T, p, n/2>::type
02943 >
02944 >;
02945 };
02946
02947 template<typename T, typename p, size_t n>
02948 struct pow<T, p, n, std::enable_if_t<n == 0> { using type = typename T::one; };
02949 } // namespace internal
02950
02951 template<typename T, typename p, size_t n>
02952 using pow_t = typename internal::pow<T, p, n>::type;
02953
02954 template<typename T, typename p, size_t n>
02955 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02956
02957 template<typename T, size_t p>
02958 static constexpr DEVICE INLINED T pow_scalar(const T& x) { return
02959 internal::pow_scalar<T>::template func<p>(x); }
02960
02961 namespace internal {
02962 template<typename, template<typename, size_t> typename, class>
02963 struct make_taylor_impl;
02964
02965 template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02966 struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02967 using type = typename polynomial<FractionField<T>>::template
02968 val<typename coeff_at<T, Is>::type...>;
02969 };
02970 }
02971
02972 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02973 using taylor = typename internal::make_taylor_impl<
02974 T,
02975 coeff_at,
02976 internal::make_index_sequence_reverse<deg + 1>::type;
02977
02978 namespace internal {
02979 template<typename T, size_t i>
02980 struct exp_coeff {
02981 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
02982 };
02983
02984 template<typename T, size_t i, typename E = void>
02985 struct sin_coeff_helper {};
02986
02987 template<typename T, size_t i>
02988 struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02989 using type = typename FractionField<T>::zero;
02990 };
02991
02992 template<typename T, size_t i>
02993 struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02994 using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>;
02995 };
02996
02997 template<typename T, size_t i>
02998 struct sin_coeff {
02999 using type = typename sin_coeff_helper<T, i>::type;
03000 };
03001
03002 template<typename T, size_t i, typename E = void>
03003 struct sh_coeff_helper {};
03004
03005 template<typename T, size_t i>
03006 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03007 using type = typename FractionField<T>::zero;
03008 };
03009
03010 template<typename T, size_t i>
03011 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03012 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03013 };
03014 }
03015
03016 template<typename T, size_t i>
03017 struct sh_coeff {
03018 using type = typename sh_coeff_helper<T, i>::type;
03019 };
03020
03021 template<typename T, size_t i>
03022 struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03023 using type = makefraction_t<T, typename T::one, factorial_t<T, i>;
03024 };

```



```

03025 template<typename T, size_t i>
03026 struct sh_coeff {
03027 using type = typename sh_coeff_helper<T, i>::type;
03028 };
03029
03030 template<typename T, size_t i, typename E = void>
03031 struct cos_coeff_helper {};
03032
03033 template<typename T, size_t i>
03034 struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03035 using type = typename FractionField<T>::zero;
03036 };
03037
03038 template<typename T, size_t i>
03039 struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03040 using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
03041 };
03042
03043 template<typename T, size_t i>
03044 struct cosh_coeff {
03045 using type = typename cosh_coeff_helper<T, i>::type;
03046 };
03047
03048 template<typename T, size_t i, typename E = void>
03049 struct cosh_coeff_helper {};
03050
03051 template<typename T, size_t i>
03052 struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03053 using type = typename FractionField<T>::zero;
03054 };
03055
03056 template<typename T, size_t i>
03057 struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03058 using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
03059 };
03060
03061 template<typename T, size_t i>
03062 struct cosh_coeff {
03063 using type = typename cosh_coeff_helper<T, i>::type;
03064 };
03065
03066 template<typename T, size_t i>
03067 struct geom_coeff { using type = typename FractionField<T>::one; };
03068
03069
03070 template<typename T, size_t i, typename E = void>
03071 struct atan_coeff_helper;
03072
03073 template<typename T, size_t i>
03074 struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03075 using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>>;
03076 };
03077
03078 template<typename T, size_t i>
03079 struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03080 using type = typename FractionField<T>::zero;
03081 };
03082
03083 template<typename T, size_t i>
03084 struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
03085
03086 template<typename T, size_t i, typename E = void>
03087 struct asin_coeff_helper;
03088
03089 template<typename T, size_t i>
03090 struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03091 using type = makefraction_t<T,
03092 factorial_t<T, i - 1>,
03093 typename T::template mul_t<
03094 typename T::template val<i>,
03095 T::template mul_t<
03096 pow_t<T, typename T::template inject_constant_t<4>, i / 2>,
03097 pow<T, factorial_t<T, i / 2>, 2
03098 >
03099 >
03100 >>;
03101 };
03102
03103 template<typename T, size_t i>
03104 struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03105 using type = typename FractionField<T>::zero;
03106 };
03107
03108 template<typename T, size_t i>
03109 struct asin_coeff {
03110 using type = typename asin_coeff_helper<T, i>::type;
03111 };

```

```

03112
03113 template<typename T, size_t i>
03114 struct lnpl_coeff {
03115 using type = makefraction_t<T,
03116 alternate_t<T, i + 1>,
03117 typename T::template val<i>>;
03118 };
03119
03120 template<typename T>
03121 struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
03122
03123 template<typename T, size_t i, typename E = void>
03124 struct asinh_coeff_helper;
03125
03126 template<typename T, size_t i>
03127 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03128 using type = makefraction_t<T,
03129 typename T::template mul_t<
03130 alternate_t<T, i / 2>,
03131 factorial_t<T, i - 1>
03132 >,
03133 typename T::template mul_t<
03134 typename T::template mul_t<
03135 typename T::template val<i>,
03136 pow_t<T, factorial_t<T, i / 2>, 2>
03137 >,
03138 pow_t<T, typename T::template inject_constant_t<4>, i / 2>
03139 >
03140 >;
03141 };
03142
03143 template<typename T, size_t i>
03144 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03145 using type = typename FractionField<T>::zero;
03146 };
03147
03148 template<typename T, size_t i>
03149 struct asinh_coeff {
03150 using type = typename asinh_coeff_helper<T, i>::type;
03151 };
03152
03153 template<typename T, size_t i, typename E = void>
03154 struct atanh_coeff_helper;
03155
03156 template<typename T, size_t i>
03157 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
03158 // 1/i
03159 using type = typename FractionField<T>::template val<
03160 typename T::one,
03161 typename T::template inject_constant_t<i>>;
03162 };
03163
03164 template<typename T, size_t i>
03165 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
03166 using type = typename FractionField<T>::zero;
03167 };
03168
03169 template<typename T, size_t i>
03170 struct atanh_coeff {
03171 using type = typename atanh_coeff_helper<T, i>::type;
03172 };
03173
03174 template<typename T, size_t i, typename E = void>
03175 struct tan_coeff_helper;
03176
03177 template<typename T, size_t i>
03178 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03179 using type = typename FractionField<T>::zero;
03180 };
03181
03182 template<typename T, size_t i>
03183 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03184 private:
03185 // 4^((i+1)/2)
03186 using _4p = typename FractionField<T>::template inject_t<
03187 pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>;
03188 // 4^((i+1)/2) - 1
03189 using _4pml = typename FractionField<T>::template
03190 sub_t<_4p, typename FractionField<T>::one>;
03191 // (-1)^((i-1)/2)
03192 using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
03193 using dividend = typename FractionField<T>::template mul_t<
03194 altp,
03195 FractionField<T>::template mul_t<
03196 _4p,
03197 FractionField<T>::template mul_t<

```

```

03198 bernoulli_t<T, (i + 1)>
03199 >
03200 >
03201 >;
03202 public:
03203 using type = typename FractionField<T>::template div_t<dividend,
03204 typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03205 };
03206
03207 template<typename T, size_t i>
03208 struct tan_coeff {
03209 using type = typename tan_coeff_helper<T, i>::type;
03210 };
03211
03212 template<typename T, size_t i, typename E = void>
03213 struct tanh_coeff_helper;
03214
03215 template<typename T, size_t i>
03216 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
03217 using type = typename FractionField<T>::zero;
03218 };
03219
03220 template<typename T, size_t i>
03221 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
03222 private:
03223 using _4p = typename FractionField<T>::template inject_t<
03224 pow_t<T, typename T::template inject_constant_t<4>, (i + 1) / 2>>,
03225 using _4pml = typename FractionField<T>::template
03226 sub_t<_4p, typename FractionField<T>::one>;
03227 using dividend =
03228 typename FractionField<T>::template mul_t<
03229 _4p,
03230 typename FractionField<T>::template mul_t<
03231 _4pml,
03232 bernoulli_t<T, (i + 1)>>>::type;
03233 public:
03234 using type = typename FractionField<T>::template div_t<dividend,
03235 FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
03236 };
03237
03238 template<typename T, size_t i>
03239 struct tanh_coeff {
03240 using type = typename tanh_coeff_helper<T, i>::type;
03241 };
03242 } // namespace internal
03243
03244 template<typename Integers, size_t deg>
03245 using exp = taylor<Integers, internal::exp_coeff, deg>;
03246
03247 template<typename Integers, size_t deg>
03248 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
03249 exp<Integers, deg>,
03250 typename polynomial<FractionField<Integers>>::one>;
03251
03252 template<typename Integers, size_t deg>
03253 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
03254
03255 template<typename Integers, size_t deg>
03256 using atan = taylor<Integers, internal::atan_coeff, deg>;
03257
03258 template<typename Integers, size_t deg>
03259 using sin = taylor<Integers, internal::sin_coeff, deg>;
03260
03261 template<typename Integers, size_t deg>
03262 using sinh = taylor<Integers, internal::sh_coeff, deg>;
03263
03264 template<typename Integers, size_t deg>
03265 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
03266
03267 template<typename Integers, size_t deg>
03268 using cos = taylor<Integers, internal::cos_coeff, deg>;
03269
03270 template<typename Integers, size_t deg>
03271 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
03272
03273 template<typename Integers, size_t deg>
03274 using asin = taylor<Integers, internal::asin_coeff, deg>;
03275
03276 template<typename Integers, size_t deg>
03277 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
03278
03279 template<typename Integers, size_t deg>
03280 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
03281
03282 template<typename Integers, size_t deg>
03283 using tan = taylor<Integers, internal::tan_coeff, deg>;
03284
03285

```

```

03334 template<typename Integers, size_t deg>
03335 using tanh = Taylor<Integers, internal::tanh_coeff, deg>;
03336 } // namespace aerobus
03337
03338 // continued fractions
03339 namespace aerobus {
03340 template<int64_t... values>
03341 struct ContinuedFraction {};
03342
03343 template<int64_t a0>
03344 struct ContinuedFraction<a0> {
03345 using type = typename q64::template inject_constant_t<a0>;
03346 static constexpr double val = static_cast<double>(a0);
03347 };
03348
03349 template<int64_t a0, int64_t... rest>
03350 struct ContinuedFraction<a0, rest...> {
03351 using type = q64::template add_t<
03352 typename q64::template inject_constant_t<a0>,
03353 typename q64::template div_t<
03354 typename q64::one,
03355 typename ContinuedFraction<rest...>::type
03356 >>;
03357
03358 static constexpr double val = type::template get<double>;
03359 };
03360
03361 using PI_fraction =
03362 ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
03363 using E_fraction =
03364 ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
03365 using SQRT2_fraction =
03366 ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
03367 using SQRT3_fraction =
03368 ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
03369 // NOLINT
03370 } // namespace aerobus
03371
03372 // known polynomials
03373 namespace aerobus {
03374 // CChebyshev
03375 namespace internal {
03376 template<int kind, size_t deg, typename I>
03377 struct chebyshev_helper {
03378 using type = typename polynomial<I>::template sub_t<
03379 typename polynomial<I>::template mul_t<
03380 typename polynomial<I>::template mul_t<
03381 typename polynomial<I>::template inject_constant_t<2>,
03382 typename polynomial<I>::X>,
03383 typename chebyshev_helper<kind, deg - 1, I>::type
03384 >,
03385 typename chebyshev_helper<kind, deg - 2, I>::type
03386 >>;
03387 };
03388
03389 template<typename I>
03390 struct chebyshev_helper<1, 0, I> {
03391 using type = typename polynomial<I>::one;
03392 };
03393
03394 template<typename I>
03395 struct chebyshev_helper<1, 1, I> {
03396 using type = typename polynomial<I>::X;
03397 };
03398
03399 template<typename I>
03400 struct chebyshev_helper<2, 0, I> {
03401 using type = typename polynomial<I>::one;
03402 };
03403
03404 template<typename I>
03405 struct chebyshev_helper<2, 1, I> {
03406 using type = typename polynomial<I>::template mul_t<
03407 typename polynomial<I>::template inject_constant_t<2>,
03408 typename polynomial<I>::X>;
03409 };
03410 } // namespace internal
03411
03412 // Laguerre
03413 namespace internal {
03414 template<size_t deg, typename I>
03415 struct laguerre_helper {
03416 using Q = FractionField<I>;
03417 using PQ = polynomial<Q>;
03418
03419 private:
03420 // Lk = (1 / k) * ((2 * k - 1 - x) * Lk-1 - (k - 2) Lk-2)

```

```

03435 using lnm2 = typename laguerre_helper<deg - 2, I>::type;
03436 using lnm1 = typename laguerre_helper<deg - 1, I>::type;
03437 // -x + 2k-1
03438 using p = typename PQ::template val<
03439 typename Q::template inject_constant_t<-1>,
03440 typename Q::template inject_constant_t<2 * deg - 1>;
03441 // 1/n
03442 using factor = typename PQ::template inject_ring_t<
03443 typename Q::template
03444 val<typename I::one, typename I::template inject_constant_t<deg>>;
03445
03446 public:
03447 using type = typename PQ::template mul_t <
03448 factor,
03449 typename PQ::template sub_t<
03450 typename PQ::template mul_t<
03451 p,
03452 lnm1
03453 >,
03454 typename PQ::template mul_t<
03455 typename PQ::template inject_constant_t<deg-1>,
03456 lnm2
03457 >
03458 >;
03459 };
03460
03461 template<typename I>
03462 struct laguerre_helper<0, I> {
03463 using type = typename polynomial<FractionField<I>>::one;
03464 };
03465
03466 template<typename I>
03467 struct laguerre_helper<1, I> {
03468 private:
03469 using PQ = polynomial<FractionField<I>>;
03470 public:
03471 using type = typename PQ::template sub_t<typename PQ::one, typename PQ::X>;
03472 };
03473 } // namespace internal
03474
03475 // Bernstein
03476 namespace internal {
03477 template<size_t i, size_t m, typename I, typename E = void>
03478 struct bernstein_helper {};
03479
03480 template<typename I>
03481 struct bernstein_helper<0, 0, I> {
03482 using type = typename polynomial<I>::one;
03483 };
03484
03485 template<size_t i, size_t m, typename I>
03486 struct bernstein_helper<i, m, I, std::enable_if_t<
03487 (m > 0) && (i == 0)> {
03488 private:
03489 using P = polynomial<I>;
03490 public:
03491 using type = typename P::template mul_t<
03492 typename P::template sub_t<typename P::one, typename P::X>,
03493 typename bernstein_helper<i, m-1, I>::type>;
03494 };
03495
03496 template<size_t i, size_t m, typename I>
03497 struct bernstein_helper<i, m, I, std::enable_if_t<
03498 (m > 0) && (i == m)> {
03499 private:
03500 using P = polynomial<I>;
03501 public:
03502 using type = typename P::template mul_t<
03503 typename P::X,
03504 typename bernstein_helper<i-1, m-1, I>::type>;
03505 };
03506
03507 template<size_t i, size_t m, typename I>
03508 struct bernstein_helper<i, m, I, std::enable_if_t<
03509 (m > 0) && (i > 0) && (i < m)> {
03510 private:
03511 using P = polynomial<I>;
03512 public:
03513 using type = typename P::template add_t<
03514 typename P::template mul_t<
03515 typename P::template sub_t<typename P::one, typename P::X>,
03516 typename bernstein_helper<i, m-1, I>::type>,
03517 typename P::template mul_t<
03518 typename P::X,
03519 typename bernstein_helper<i-1, m-1, I>::type>;
03520 };

```

```

03521 } // namespace internal
03522
03523 namespace known_polynomials {
03524 enum hermite_kind {
03525 probabilist,
03526 physicist
03527 };
03528 }
03529
03530 // hermite
03531 namespace internal {
03532 template<size_t deg, known_polynomials::hermite_kind kind, typename I>
03533 struct hermite_helper {};
03534
03535 template<size_t deg, typename I>
03536 struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist, I> {
03537 private:
03538 using hnm1 = typename hermite_helper<deg - 1,
03539 known_polynomials::hermite_kind::probabilist, I>::type;
03540 using hnm2 = typename hermite_helper<deg - 2,
03541 known_polynomials::hermite_kind::probabilist, I>::type;
03542
03543 public:
03544 using type = typename polynomial<I>::template sub_t<
03545 typename polynomial<I>::template mul_t<typename polynomial<I>::X, hnm1>,
03546 typename polynomial<I>::template mul_t<
03547 typename polynomial<I>::template inject_constant_t<deg - 1>,
03548 hnm2
03549 >
03550 >;
03551 };
03552
03553 template<size_t deg, typename I>
03554 struct hermite_helper<deg, known_polynomials::hermite_kind::physicist, I> {
03555 private:
03556 using hnm1 = typename hermite_helper<deg - 1, known_polynomials::hermite_kind::physicist,
03557 I>::type;
03558 using hnm2 = typename hermite_helper<deg - 2, known_polynomials::hermite_kind::physicist,
03559 I>::type;
03560
03561 public:
03562 using type = typename polynomial<I>::template sub_t<
03563 // 2X Hn-1
03564 typename polynomial<I>::template mul_t<
03565 typename pi64::val<typename I::template inject_constant_t<2>,
03566 typename I::zero>, hnm1>,
03567 typename polynomial<I>::template mul_t<
03568 typename polynomial<I>::template inject_constant_t<2*(deg - 1)>,
03569 hnm2
03570 >
03571 >;
03572 };
03573
03574 template<typename I>
03575 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist, I> {
03576 using type = typename polynomial<I>::one;
03577 };
03578
03579 template<typename I>
03580 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist, I> {
03581 using type = typename polynomial<I>::X;
03582 };
03583
03584 template<typename I>
03585 struct hermite_helper<0, known_polynomials::hermite_kind::physicist, I> {
03586 using type = typename pi64::one;
03587 };
03588
03589 template<typename I>
03590 struct hermite_helper<1, known_polynomials::hermite_kind::physicist, I> {
03591 // 2X
03592 using type = typename polynomial<I>::template val<
03593 typename I::template inject_constant_t<2>,
03594 typename I::zero>;
03595 };
03596 } // namespace internal
03597
03598 // legendre
03599 namespace internal {
03600 template<size_t n, typename I>
03601 struct legendre_helper {
03602 private:
03603 using Q = FractionField<I>;
03604 using PQ = polynomial<Q>;
03605 // 1/n constant
03606 // (2n-1)/n X

```

```

03607 using fact_left = typename PQ::template monomial_t<
03608 makefraction_t<I,
03609 typename I::template inject_constant_t<2*n-1>,
03610 typename I::template inject_constant_t<n>
03611 >,
03612 1>;
03613 // (n-1) / n
03614 using fact_right = typename PQ::template val<
03615 makefraction_t<I,
03616 typename I::template inject_constant_t<n-1>,
03617 typename I::template inject_constant_t<n>»>;
03618
03619 public:
03620 using type = PQ::template sub_t<
03621 typename PQ::template mul_t<
03622 fact_left,
03623 typename legendre_helper<n-1, I>::type
03624 >,
03625 typename PQ::template mul_t<
03626 fact_right,
03627 typename legendre_helper<n-2, I>::type
03628 >
03629 >;
03630
03631 };
03632
03633 template<typename I>
03634 struct legendre_helper<0, I> {
03635 using type = typename polynomial<FractionField<I>>::one;
03636 };
03637
03638 template<typename I>
03639 struct legendre_helper<1, I> {
03640 using type = typename polynomial<FractionField<I>>::X;
03641 };
03642
03643 // bernoulli polynomials
03644 namespace internal {
03645 template<size_t n>
03646 struct bernoulli_coeff {
03647 template<typename T, size_t i>
03648 struct inner {
03649 private:
03650 using F = FractionField<T>;
03651 public:
03652 using type = typename F::template mul_t<
03653 typename F::template inject_ring_t<combination_t<T, i, n>>,
03654 bernoulli_t<T, n-i>
03655 >;
03656 };
03657 };
03658 } // namespace internal
03659
03660 namespace known_polynomials {
03661 template <size_t deg, typename I = aerobus::i64>
03662 using chebyshev_T = typename internal::chebyshev_helper<1, deg, I>::type;
03663
03664 template <size_t deg, typename I = aerobus::i64>
03665 using chebyshev_U = typename internal::chebyshev_helper<2, deg, I>::type;
03666
03667 template <size_t deg, typename I = aerobus::i64>
03668 using laguerre = typename internal::laguerre_helper<deg, I>::type;
03669
03670 template <size_t deg, typename I = aerobus::i64>
03671 using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist,
03672 I>::type;
03673
03674 template <size_t deg, typename I = aerobus::i64>
03675 using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist, I>::type;
03676
03677 template<size_t i, size_t m, typename I = aerobus::i64>
03678 using bernstein = typename internal::bernstein_helper<i, m, I>::type;
03679
03680 template<size_t deg, typename I = aerobus::i64>
03681 using legendre = typename internal::legendre_helper<deg, I>::type;
03682
03683 template<size_t deg, typename I = aerobus::i64>
03684 using bernoulli = taylor<I, internal::bernoulli_coeff<deg>::template inner, deg>;
03685 } // namespace known_polynomials
03686 } // namespace aerobus
03687
03688 #ifndef AEROBUS_CONWAY_IMPORTS
03689 // conway polynomials
03690 namespace aerobus {
03691 template<int p, int n>

```





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```
05704 template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>>; }; // NOLINT
05705 template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>>; }; // NOLINT
05706 template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>>; }; //
NOLINT
05707 template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>,
ZPZV<990>>; }; // NOLINT
05708 #endif // DO_NOT_DOCUMENT
05709 } // namespace aerobus
05710 #endif // AEROBUS_CONWAY_IMPORTS
05711
05712 #endif // __INC_AEROBUS__ // NOLINT
```

## Chapter 10

# Examples

### 10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject\_constant\_t<1>>

Template Parameters

|   |                                      |
|---|--------------------------------------|
| x | a 'constant' from Ring point of view |
|---|--------------------------------------|

### 10.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

|       |                                               |
|-------|-----------------------------------------------|
| ...Ts | types to store and manipulate at compile time |
|-------|-----------------------------------------------|

### 10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

|   |                                              |
|---|----------------------------------------------|
| x | inject_constant_2<2> -> i32::template val<2> |
|---|----------------------------------------------|

## 10.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |

## 10.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

|      |                |
|------|----------------|
| $v1$ | a value in i32 |
| $v2$ | a value in i32 |



## 10.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

|           |                            |
|-----------|----------------------------|
| <i>v1</i> |                            |
| <i>v2</i> | <i32::val<1>, i32::val<1>> |

## 10.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

### Template Parameters

|           |                |
|-----------|----------------|
| <i>v1</i> | a value in i32 |
| <i>v2</i> | a value in i32 |

## 10.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

|     |                |
|-----|----------------|
| $v$ | a value in i32 |
|-----|----------------|

## 10.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

|     |                                                       |
|-----|-------------------------------------------------------|
| $v$ | a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code> |
|-----|-------------------------------------------------------|

## 10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

|     |                                         |
|-----|-----------------------------------------|
| $x$ | <code>inject_constant_t&lt;2&gt;</code> |
|-----|-----------------------------------------|

## 10.15 i64::add\_t

addition operator

addition operator

Template Parameters

|      |                                                                                                          |
|------|----------------------------------------------------------------------------------------------------------|
| $v1$ | : an element of <code>aerobus::i64::val</code>                                                           |
| $v2$ | : an element of <code>aerobus::i64::val</code> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

|           |                                                                              |
|-----------|------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>> |

## 10.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

|           |                                                                               |
|-----------|-------------------------------------------------------------------------------|
| <i>v1</i> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <i>v2</i> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 10.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code> |

## 10.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

|                 |                                                                                                             |
|-----------------|-------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                                                           |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code> |

## 10.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

|                 |                                                                              |
|-----------------|------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                            |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<2>, i64::val<2>> |

## 10.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of [i64::val](#)

### Template Parameters

|                 |                                                                               |
|-----------------|-------------------------------------------------------------------------------|
| <code>v1</code> | : an element of <a href="#">aerobus::i64::val</a>                             |
| <code>v2</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>> |

## 10.26 i64::pos\_t

is v positive yields  $v > 0$  as [std::true\\_type](#) or [std::false\\_type](#)

is v positive yields  $v > 0$  as [std::true\\_type](#) or [std::false\\_type](#)

### Template Parameters

|                |                                                                 |
|----------------|-----------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>> |
|----------------|-----------------------------------------------------------------|

## 10.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

|                |                                                                 |
|----------------|-----------------------------------------------------------------|
| <code>v</code> | : an element of <a href="#">aerobus::i64::val</a> <i64::val<1>> |
|----------------|-----------------------------------------------------------------|

## 10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

|                |                                                               |
|----------------|---------------------------------------------------------------|
| <code>x</code> | <code>&lt;i32&gt;::template inject_constant_t&lt;2&gt;</code> |
|----------------|---------------------------------------------------------------|

## 10.29 q32::add\_t

addition operator

addition operator

Template Parameters

|                 |                                                                                                                                         |
|-----------------|-----------------------------------------------------------------------------------------------------------------------------------------|
| <code>v1</code> | a value                                                                                                                                 |
| <code>v2</code> | a value <code>&lt;q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;2&gt;&gt;, q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;3&gt;&gt;&gt;</code> |

## 10.30 FractionField

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$

Template Parameters

|             |                                                                                    |
|-------------|------------------------------------------------------------------------------------|
| <i>Ring</i> | <code>&lt;i64&gt;</code> is q64 (rationals with 64 bits numerator and denominator) |
|-------------|------------------------------------------------------------------------------------|

## 10.31 PI\_fraction::val

representation of  $\pi$  as a continued fraction -> 3.14...

## 10.32 E\_fraction::val

approximation of  $e$  -> 2.718...

approximation of  $e$  -> 2.718...

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