

Aerobus

v1.2

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<b>1 Concept Index</b>	<b>1</b>
1.1 Concepts	1
<b>2 Class Index</b>	<b>3</b>
2.1 Class List	3
<b>3 File Index</b>	<b>5</b>
3.1 File List	5
<b>4 Concept Documentation</b>	<b>7</b>
4.1 aerobus::IsEuclideanDomain Concept Reference	7
4.1.1 Concept definition	7
4.1.2 Detailed Description	7
4.2 aerobus::IsField Concept Reference	7
4.2.1 Concept definition	7
4.2.2 Detailed Description	8
4.3 aerobus::IsRing Concept Reference	8
4.3.1 Concept definition	8
4.3.2 Detailed Description	8
<b>5 Class Documentation</b>	<b>9</b>
5.1 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E > Struct Template Reference	9
5.2 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index < 0  index > 0)> > Struct Template Reference	9
5.3 aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> > Struct Template Reference	9
5.4 aerobus::ContinuedFraction< values > Struct Template Reference	10
5.4.1 Detailed Description	10
5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference	10
5.5.1 Detailed Description	10
5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference	11
5.6.1 Detailed Description	11
5.7 aerobus::i32 Struct Reference	11
5.7.1 Detailed Description	12
5.7.2 Member Typedef Documentation	13
5.7.2.1 mod_t	13
5.8 aerobus::i64 Struct Reference	13
5.8.1 Detailed Description	14
5.8.2 Member Typedef Documentation	14
5.8.2.1 inject_ring_t	14
5.8.3 Member Data Documentation	15
5.8.3.1 gt_v	15
5.9 aerobus::is_prime< n > Struct Template Reference	15
5.9.1 Detailed Description	15

5.10 aerobus::polynomial< Ring > Struct Template Reference	16
5.10.1 Detailed Description	17
5.10.2 Member Typedef Documentation	17
5.10.2.1 add_t	17
5.10.2.2 derive_t	17
5.10.2.3 div_t	18
5.10.2.4 eq_t	18
5.10.2.5 gcd_t	18
5.10.2.6 gt_t	19
5.10.2.7 lt_t	19
5.10.2.8 mod_t	19
5.10.2.9 monomial_t	19
5.10.2.10 mul_t	20
5.10.2.11 pos_t	20
5.10.2.12 simplify_t	20
5.10.2.13 sub_t	20
5.10.3 Member Data Documentation	21
5.10.3.1 pos_v	21
5.11 aerobus::type_list< Ts >::pop_front Struct Reference	21
5.11.1 Detailed Description	21
5.12 aerobus::Quotient< Ring, X > Struct Template Reference	22
5.12.1 Detailed Description	23
5.12.2 Member Typedef Documentation	23
5.12.2.1 add_t	23
5.12.2.2 div_t	23
5.12.2.3 eq_t	23
5.12.2.4 mod_t	24
5.12.2.5 mul_t	24
5.12.2.6 pos_t	24
5.12.3 Member Data Documentation	25
5.12.3.1 eq_v	25
5.12.3.2 pos_v	25
5.13 aerobus::type_list< Ts >::split< index > Struct Template Reference	25
5.13.1 Detailed Description	26
5.14 aerobus::type_list< Ts > Struct Template Reference	26
5.14.1 Detailed Description	27
5.14.2 Member Typedef Documentation	27
5.14.2.1 at	27
5.14.2.2 concat	27
5.14.2.3 insert	27
5.14.2.4 push_back	28
5.14.2.5 push_front	28

5.14.2.6 remove . . . . .	28
5.15 aerobus::type_list<> Struct Reference . . . . .	29
5.15.1 Detailed Description . . . . .	29
5.16 aerobus::i32::val< x > Struct Template Reference . . . . .	29
5.16.1 Detailed Description . . . . .	30
5.16.2 Member Function Documentation . . . . .	30
5.16.2.1 eval() . . . . .	30
5.16.2.2 get() . . . . .	30
5.17 aerobus::i64::val< x > Struct Template Reference . . . . .	31
5.17.1 Detailed Description . . . . .	31
5.17.2 Member Function Documentation . . . . .	32
5.17.2.1 eval() . . . . .	32
5.17.2.2 get() . . . . .	32
5.18 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference . . . . .	32
5.18.1 Detailed Description . . . . .	33
5.18.2 Member Typedef Documentation . . . . .	33
5.18.2.1 coeff_at_t . . . . .	33
5.18.3 Member Function Documentation . . . . .	34
5.18.3.1 eval() . . . . .	34
5.18.3.2 to_string() . . . . .	34
5.19 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference . . . . .	35
5.19.1 Detailed Description . . . . .	35
5.20 aerobus::zpz< p >::val< x > Struct Template Reference . . . . .	35
5.21 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference . . . . .	36
5.21.1 Detailed Description . . . . .	36
5.22 aerobus::zpz< p > Struct Template Reference . . . . .	37
5.22.1 Detailed Description . . . . .	38
5.22.2 Member Typedef Documentation . . . . .	38
5.22.2.1 add_t . . . . .	38
5.22.2.2 div_t . . . . .	38
5.22.2.3 eq_t . . . . .	39
5.22.2.4 gcd_t . . . . .	39
5.22.2.5 gt_t . . . . .	39
5.22.2.6 lt_t . . . . .	40
5.22.2.7 mod_t . . . . .	40
5.22.2.8 mul_t . . . . .	40
5.22.2.9 pos_t . . . . .	40
5.22.2.10 sub_t . . . . .	41
5.22.3 Member Data Documentation . . . . .	41
5.22.3.1 eq_v . . . . .	41
5.22.3.2 gt_v . . . . .	41
5.22.3.3 lt_v . . . . .	42

5.22.3.4 pos_v	42
<b>6 File Documentation</b>	<b>43</b>
6.1 src/aerobus.h File Reference	43
6.2 aerobus.h	43
<b>7 Examples</b>	<b>127</b>
7.1 QuotientRing	127
7.2 type_list	127
7.3 i32::template	127
7.4 i32::add_t	128
7.5 i32::sub_t	128
7.6 i32::mul_t	128
7.7 i32::div_t	128
7.8 i32::gt_t	129
7.9 i32::eq_t	129
7.10 i32::eq_v	129
7.11 i32::gcd_t	129
7.12 i32::pos_t	130
7.13 i32::pos_v	130
7.14 i64::template	130
7.15 i64::add_t	130
7.16 i64::sub_t	131
7.17 i64::mul_t	131
7.18 i64::div_t	131
7.19 i64::mod_t	131
7.20 i64::gt_t	132
7.21 i64::lt_t	132
7.22 i64::lt_v	132
7.23 i64::eq_t	132
7.24 i64::eq_v	133
7.25 i64::gcd_t	133
7.26 i64::pos_t	133
7.27 i64::pos_v	133
7.28 polynomial	134
7.29 q32::add_t	134
7.30 FractionField	134
7.31 PI_fraction::val	134
7.32 E_fraction::val	134
<b>Index</b>	<b>135</b>

# Chapter 1

## Concept Index

### 1.1 Concepts

Here is a list of all documented concepts with brief descriptions:

<a href="#">aerobus::IsEuclideanDomain</a>	
Concept to express R is an euclidean domain . . . . .	7
<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	7
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring (ordered) . . . . .	8





## Chapter 2

# Class Index

### 2.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, E &gt;</a>	9
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt;</a>	9
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt;</a>	9
<a href="#">aerobus::ContinuedFraction&lt; values &gt;</a>	
Continued fraction $a_0 + 1/(a_1 + 1/(...))$	10
<a href="#">aerobus::ContinuedFraction&lt; a0 &gt;</a>	
Specialization for only one coefficient, technically just 'a0'	10
<a href="#">aerobus::ContinuedFraction&lt; a0, rest... &gt;</a>	
Specialization for multiple coefficients (strictly more than one)	11
<a href="#">aerobus::i32</a>	
32 bits signed integers, seen as a algebraic ring with related operations	11
<a href="#">aerobus::i64</a>	
64 bits signed integers, seen as a algebraic ring with related operations	13
<a href="#">aerobus::is_prime&lt; n &gt;</a>	
Checks if n is prime	15
<a href="#">aerobus::polynomial&lt; Ring &gt;</a>	16
<a href="#">aerobus::type_list&lt; Ts &gt;::pop_front</a>	
Removes types from head of the list	21
<a href="#">aerobus::Quotient&lt; Ring, X &gt;</a>	
Quotient ring by the principal ideal generated by 'X' With <a href="#">i32</a> as Ring and <a href="#">i32::val&lt;2&gt;</a> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	22
<a href="#">aerobus::type_list&lt; Ts &gt;::split&lt; index &gt;</a>	
Splits list at index	25
<a href="#">aerobus::type_list&lt; Ts &gt;</a>	
Empty pure template struct to handle type list	26
<a href="#">aerobus::type_list&lt;&gt;</a>	
Specialization for empty type list	29
<a href="#">aerobus::i32::val&lt; x &gt;</a>	
Values in <a href="#">i32</a> , again represented as types	29
<a href="#">aerobus::i64::val&lt; x &gt;</a>	
Values in <a href="#">i64</a>	31
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN, coeffs &gt;</a>	
Values (seen as types) in polynomial ring	32
<a href="#">aerobus::Quotient&lt; Ring, X &gt;::val&lt; V &gt;</a>	
Projection values in the quotient ring	35

<a href="#">aerobus::zpz&lt; p &gt;::val&lt; x &gt;</a> . . . . .	35
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;</a>	
Specialization for constants . . . . .	36
<a href="#">aerobus::zpz&lt; p &gt;</a> . . . . .	37

## Chapter 3

# File Index

### 3.1 File List

Here is a list of all documented files with brief descriptions:

src/ <a href="#">aerobus.h</a> . . . . .	43
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# Chapter 4

## Concept Documentation

### 4.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 4.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 4.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 4.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 4.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 4.2.2 Detailed Description

Concept to express R is a field.

## 4.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring (ordered)

```
#include <aerobus.h>
```

### 4.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 4.3.2 Detailed Description

Concept to express R is a Ring (ordered)

## Chapter 5

# Class Documentation

### 5.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 5.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

#### Public Types

- `using type = typename Ring::zero`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 5.3 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)> >` Struct Template Reference

#### Public Types

- `using type = aN`

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.4 aerobus::ContinuedFraction< values > Struct Template Reference

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

```
#include <aerobus.h>
```

### 5.4.1 Detailed Description

```
template<int64_t... values>
struct aerobus::ContinuedFraction< values >
```

represents a continued fraction  $a_0 + 1/(a_1 + 1/(...))$

Template Parameters

<code>...values</code>	are <a href="#">aerobus::i64</a>
------------------------	----------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

### Public Types

- using **type** = typename q64::template inject\_constant\_t< a0 >

### Static Public Attributes

- static constexpr double **val** = type::template get<double>()

### 5.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.



## Template Parameters

<i>a0</i>	an integer ( <a href="#">aerobus::i64</a> )
-----------	---

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

## Public Types

- using **type** = q64::template add\_t< typename q64::template inject\_constant\_t< a0 >, typename q64::template div\_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >

## Static Public Attributes

- static constexpr double **val** = type::template get<double>()

### 5.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

## Template Parameters

<i>a0</i>	an integer ( <a href="#">aerobus::i64</a> )
<i>...rest</i>	integers ( <a href="#">aerobus::i64</a> )

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.7 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*values in [i32](#), again represented as types*

## Public Types

- using **inner\_type** = int32\_t
- using **zero** = [val](#)< 0 >  
*constant zero*
- using **one** = [val](#)< 1 >  
*constant one*
- template<auto x>  
using **inject\_constant\_t** = [val](#)< static\_cast< int32\_t >(x)>
- template<typename v >  
using **inject\_ring\_t** = v
- template<typename v1, typename v2 >  
using **add\_t** = typename add< v1, v2 >::type
- template<typename v1, typename v2 >  
using **sub\_t** = typename sub< v1, v2 >::type
- template<typename v1, typename v2 >  
using **mul\_t** = typename mul< v1, v2 >::type
- template<typename v1, typename v2 >  
using **div\_t** = typename div< v1, v2 >::type
- template<typename v1, typename v2 >  
using **mod\_t** = typename remainder< v1, v2 >::type  
*modulus operator yields v1 % v2 for example : [i32::mod\\_t](#)<[i32::val](#)<7>, [i32::val](#)<2>>*
- template<typename v1, typename v2 >  
using **gt\_t** = typename gt< v1, v2 >::type
- template<typename v1, typename v2 >  
using **lt\_t** = typename lt< v1, v2 >::type
- template<typename v1, typename v2 >  
using **eq\_t** = typename eq< v1, v2 >::type
- template<typename v1, typename v2 >  
using **gcd\_t** = gcd\_t< [i32](#), v1, v2 >
- template<typename v >  
using **pos\_t** = typename pos< v >::type

## Static Public Attributes

- static constexpr bool **is\_field** = false  
*integers are not a field*
- static constexpr bool **is\_euclidean\_domain** = true  
*integers are an euclidean domain*
- template<typename v1, typename v2 >  
static constexpr bool **eq\_v** = eq\_t<v1, v2>::value
- template<typename v >  
static constexpr bool **pos\_v** = pos\_t<v>::value

### 5.7.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 5.7.2 Member Typedef Documentation

### 5.7.2.1 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type
```

modulus operator yields  $v1 \% v2$  for example : `i32::mod_t<i32::val<7>, i32::val<2>>`

#### Template Parameters

<code>v1</code>	a value in <code>i32</code>
<code>v2</code>	a value in <code>i32</code>

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 5.8 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct `val`  
values in `i64`

### Public Types

- using `inner_type` = `int64_t`  
type for actual values
- template<auto x>  
using `inject_constant_t` = `val< static_cast< int64_t >(x)>`
- template<typename v >  
using `inject_ring_t` = v  
injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>> -> i64::val<1>`
- using `zero` = `val< 0 >`  
constant zero
- using `one` = `val< 1 >`  
constant one
- template<typename v1 , typename v2 >  
using `add_t` = `typename add< v1, v2 >::type`
- template<typename v1 , typename v2 >  
using `sub_t` = `typename sub< v1, v2 >::type`

- `template<typename v1 , typename v2 >`  
using **mul\_t** = typename mul< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **div\_t** = typename div< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **mod\_t** = typename remainder< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **gt\_t** = typename gt< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **lt\_t** = typename lt< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **eq\_t** = typename eq< v1, v2 >::type
- `template<typename v1 , typename v2 >`  
using **gcd\_t** = gcd\_t< i64, v1, v2 >
- `template<typename v >`  
using **pos\_t** = typename pos< v >::type

### Static Public Attributes

- static constexpr bool **is\_field** = false  
*integers are not a field*
- static constexpr bool **is\_euclidean\_domain** = true  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
static constexpr bool **gt\_v** = gt\_t<v1, v2>::value  
*strictly greater operator yields v1 > v2 as boolean value*
- `template<typename v1 , typename v2 >`  
static constexpr bool **lt\_v** = lt\_t<v1, v2>::value
- `template<typename v1 , typename v2 >`  
static constexpr bool **eq\_v** = eq\_t<v1, v2>::value
- `template<typename v >`  
static constexpr bool **pos\_v** = pos\_t<v>::value

## 5.8.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

## 5.8.2 Member Typedef Documentation

### 5.8.2.1 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64↔::val<1>> -> i64::val<1>`

#### Template Parameters

<i>v</i>	a value in i64
----------	----------------

## 5.8.3 Member Data Documentation

### 5.8.3.1 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a>

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.9 aerobus::is\_prime< n > Struct Template Reference

checks if  $n$  is prime

```
#include <aerobus.h>
```

#### Static Public Attributes

- static constexpr bool **value** = internal::\_is\_prime< $n$ , 5>::value  
*true iff  $n$  is prime*

### 5.9.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if  $n$  is prime

#### Template Parameters

<i>n</i>	
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.10 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1, typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1, typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*
- [template<typename v1, typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication of two polynomials*
- [template<typename v1, typename v2 >](#)  
[using eq\\_t = typename eq\\_helper< v1, v2 >::type](#)  
*equality operator*
- [template<typename v1, typename v2 >](#)  
[using lt\\_t = typename lt\\_helper< v1, v2 >::type](#)  
*strict less operator*
- [template<typename v1, typename v2 >](#)  
[using gt\\_t = typename gt\\_helper< v1, v2 >::type](#)  
*strict greater operator*
- [template<typename v1, typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::q\\_type](#)  
*division operator*
- [template<typename v1, typename v2 >](#)  
[using mod\\_t = typename div\\_helper< v1, v2, zero, v1 >::mod\\_type](#)  
*modulo operator*
- [template<typename coeff, size\\_t deg>](#)  
[using monomial\\_t = typename monomial< coeff, deg >::type](#)  
*monomial : coeff X<sup>deg</sup>*
- [template<typename v >](#)  
[using derive\\_t = typename derive\\_helper< v >::type](#)

- derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity (an > 0)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial<`  
`Ring >, v1, v2 > >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

### Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

## 5.10.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain

## 5.10.2 Member Typedef Documentation

### 5.10.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

<code>v1</code>	
<code>v2</code>	

### 5.10.2.2 derive\_t

```
template<typename Ring >
```

```
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 5.10.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 5.10.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 5.10.2.5 gcd\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	



### 5.10.2.6 gt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

#### Template Parameters

v1	
v2	

### 5.10.2.7 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

v1	
v2	

### 5.10.2.8 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

v1	
v2	

### 5.10.2.9 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial : coeff X^deg

**Template Parameters**

<i>coeff</i>	
<i>deg</i>	

**5.10.2.10 mul\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**5.10.2.11 pos\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

**Template Parameters**

<i>v</i>	
----------	--

**5.10.2.12 simplify\_t**

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

**Template Parameters**

<i>P</i>	
----------	--

**5.10.2.13 sub\_t**

```
template<typename Ring >
```

```
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 5.10.3 Member Data Documentation

#### 5.10.3.1 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

#### Template Parameters

<i>v</i>	a value in <a href="#">polynomial::val</a>
----------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.11 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

#### Public Types

- using **type** = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using **tail** = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

#### 5.11.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.12 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*projection values in the quotient ring*

### Public Types

- using **zero** = [val](#)< [typename](#) Ring::zero >  
*zero value*
- using **one** = [val](#)< [typename](#) Ring::one >  
*one*
- template<[typename](#) v1 , [typename](#) v2 >  
using **add\_t** = [val](#)< [typename](#) Ring::template [add\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*addition operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using **mul\_t** = [val](#)< [typename](#) Ring::template [mul\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*subtraction operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using **div\_t** = [val](#)< [typename](#) Ring::template [div\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*division operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using **mod\_t** = [val](#)< [typename](#) Ring::template [mod\\_t](#)< [typename](#) v1::type, [typename](#) v2::type > >  
*modulus operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using **eq\_t** = [typename](#) Ring::template [eq\\_t](#)< [typename](#) v1::type, [typename](#) v2::type >  
*equality operator (as type)*
- template<[typename](#) v1 >  
using **pos\_t** = `std::true_type`  
*positivity operator always true*
- template<`auto` x>  
using **inject\_constant\_t** = [val](#)< [typename](#) Ring::template [inject\\_constant\\_t](#)< x > >
- template<[typename](#) v >  
using **inject\_ring\_t** = [val](#)< v >

### Static Public Attributes

- template<[typename](#) v1 , [typename](#) v2 >  
**static constexpr bool eq\_v** = Ring::template [eq\\_t](#)<[typename](#) v1::type, [typename](#) v2::type>::value  
*addition operator (as boolean value)*
- template<[typename](#) v >  
**static constexpr bool pos\_v** = [pos\\_t](#)<v>::value  
*positivity operator always true*
- **static constexpr bool is\_euclidean\_domain** = `true`  
*quotien rings are euclidean domain*

### 5.12.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and `i32::val<2>` as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as ' <a href="#">i32</a> ', must satisfy the <code>IsRing</code> concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

### 5.12.2 Member Typedef Documentation

#### 5.12.2.1 `add_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 5.12.2.2 `div_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 5.12.2.3 `eq_t`

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
```

```
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 5.12.2.4 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 5.12.2.5 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

#### 5.12.2.6 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

## Template Parameters

<code>v1</code>	a value in quotient ring
-----------------	--------------------------

## 5.12.3 Member Data Documentation

## 5.12.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

## Template Parameters

<code>v1</code>	a value in quotient ring
<code>v2</code>	a value in quotient ring

## 5.12.3.2 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<code>v1</code>	a value in quotient ring
-----------------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.13 aerobus::type\_list&lt; Ts &gt;::split&lt; index &gt; Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using **head** = typename inner::head
- using **tail** = typename inner::tail

### 5.13.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

Template Parameters

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.14 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = internal::type\_at\_t< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename concat\_h< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using [insert](#) = typename internal::insert\_h< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename internal::remove\_h< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*



### Static Public Attributes

- static constexpr size\_t **length** = sizeof...(Ts)  
*length of list*

## 5.14.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

## 5.14.2 Member Typedef Documentation

### 5.14.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

#### Template Parameters

<i>index</i>	
--------------	--

### 5.14.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

#### Template Parameters

<i>U</i>	
----------	--

### 5.14.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>,
T>::type
```

inserts type at index

**Template Parameters**

<i>index</i>	
<i>T</i>	

**5.14.2.4 push\_back**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

<i>T</i>	
----------	--

**5.14.2.5 push\_front**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

**Template Parameters**

<i>T</i>	
----------	--

**5.14.2.6 remove**

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>::type
```

removes type at index

**Template Parameters**

<i>index</i>	
--------------	--

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.15 aerobus::type\_list<> Struct Reference

specialization for empty type list

```
#include <aerobus.h>
```

### Public Types

- template<typename T >  
using **push\_front** = [type\\_list](#)< T >
- template<typename T >  
using **push\_back** = [type\\_list](#)< T >
- template<typename U >  
using **concat** = U
- template<typename T , size\_t index>  
using **insert** = [type\\_list](#)< T >

### Static Public Attributes

- static constexpr size\_t **length** = 0

#### 5.15.1 Detailed Description

specialization for empty type list

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.16 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- using **ring\_type** = [i32](#)  
*Enclosing ring type.*
- using **is\_zero\_t** = std::bool\_constant< x==0 >  
*is value zero*

## Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`  
*cast x into valueType*
- `static std::string to\_string ()`  
*string representation of value*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`  
*cast x into valueRing*

## Static Public Attributes

- `static constexpr int32_t v = x`  
*actual value stored in val type*

### 5.16.1 Detailed Description

```
template<int32_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types

#### Template Parameters

<code>x</code>	an actual integer
----------------	-------------------

### 5.16.2 Member Function Documentation

#### 5.16.2.1 [eval\(\)](#)

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

#### Template Parameters

<code>valueRing</code>	double for example
------------------------	--------------------

#### 5.16.2.2 [get\(\)](#)

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

#### Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.17 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- using **ring\_type** = [i64](#)  
*enclosing ring type*
- using **is\_zero\_t** = std::bool\_constant< x==0 >  
*is value zero*

### Static Public Member Functions

- template<typename valueType >  
static constexpr valueType **get** ()  
*cast value in valueType*
- static std::string **to\_string** ()  
*string representation*
- template<typename valueRing >  
static constexpr valueRing **eval** (const valueRing &v)  
*cast value in valueRing*

### Static Public Attributes

- static constexpr int64\_t **v** = x  
*actual value*

### 5.17.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 5.17.2 Member Function Documentation

### 5.17.2.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

## Template Parameters

<i>valueRing</i>	(double for example)
------------------	----------------------

### 5.17.2.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

## Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 5.18 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

## Public Types

- `using ring_type = polynomial< Ring >`  
*enclosing ring type*
- `using aN = coeffN`  
*heavy weight coefficient (non zero)*
- `using strip = val< coeffs... >`  
*remove largest coefficient*
- `using is_zero_t = std::bool_constant<(degree==0) &&(aN::is_zero_t::value)>`  
*true\_type if polynomial is constant zero*
- `template<size_t index>`  
`using coeff_at_t = typename coeff_at< index >::type`  
*type of coefficient at index*

## Static Public Member Functions

- `static std::string to_string ()`  
*get a string representation of polynomial*
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &x)`  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree = sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v = is_zero_t::value`  
*true if polynomial is constant zero*

### 5.18.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

### 5.18.2 Member Typedef Documentation

#### 5.18.2.1 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
```

```
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_at_
at<index>::type
```

type of coefficient at index

#### Template Parameters

<i>index</i>	
--------------	--

## 5.18.3 Member Function Documentation

### 5.18.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

#### Parameters

<i>x</i>	value
----------	-------

#### Returns

$P(x)$

### 5.18.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## 5.19 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

### 5.19.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

#### Template Parameters

V	a value from 'Ring'
---	---------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.20 aerobus::zpz< p >::val< x > Struct Template Reference

### Public Types

- `using ring_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< x% p==0 >`

### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 5.21 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

### Classes

- struct [coeff\\_at](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index< 0||index > 0\)> >](#)
- struct [coeff\\_at< index, std::enable\\_if\\_t<\(index==0\)> >](#)

### Public Types

- [using ring\\_type](#) = [polynomial< Ring >](#)  
*enclosing ring type*
- [using aN](#) = [coeffN](#)
- [using strip](#) = [val< coeffN >](#)
- [using is\\_zero\\_t](#) = [std::bool\\_constant< aN::is\\_zero\\_t::value >](#)
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t](#) = [typename](#) [coeff\\_at< index >::type](#)

### Static Public Member Functions

- [static](#) [std::string to\\_string](#) ()
- [template<typename valueRing >](#)  
[static constexpr valueRing eval](#) ([const valueRing &x](#))

### Static Public Attributes

- [static constexpr size\\_t degree](#) = 0  
*degree*
- [static constexpr bool is\\_zero\\_v](#) = [is\\_zero\\_t::value](#)

#### 5.21.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

#### Template Parameters

<a href="#">coeffN</a>	
------------------------	--

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 5.22 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)

### Public Types

- using [inner\\_type](#) = [int32\\_t](#)
- template<[auto](#) x>  
using [inject\\_constant\\_t](#) = [val](#)< [static\\_cast](#)< [int32\\_t](#) >(x)>
- using [zero](#) = [val](#)< 0 >
- using [one](#) = [val](#)< 1 >
- template<[typename](#) v1 , [typename](#) v2 >  
using [add\\_t](#) = [typename](#) [add](#)< v1, v2 >::type  
*addition operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [sub\\_t](#) = [typename](#) [sub](#)< v1, v2 >::type  
*subtraction operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [mul\\_t](#) = [typename](#) [mul](#)< v1, v2 >::type  
*multiplication operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [div\\_t](#) = [typename](#) [div](#)< v1, v2 >::type  
*division operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [mod\\_t](#) = [typename](#) [remainder](#)< v1, v2 >::type  
*modulo operator*
- template<[typename](#) v1 , [typename](#) v2 >  
using [gt\\_t](#) = [typename](#) [gt](#)< v1, v2 >::type  
*strictly greater operator (type)*
- template<[typename](#) v1 , [typename](#) v2 >  
using [lt\\_t](#) = [typename](#) [lt](#)< v1, v2 >::type  
*strictly smaller operator (type)*
- template<[typename](#) v1 , [typename](#) v2 >  
using [eq\\_t](#) = [typename](#) [eq](#)< v1, v2 >::type  
*equality operator (type)*
- template<[typename](#) v1 , [typename](#) v2 >  
using [gcd\\_t](#) = [gcd\\_t](#)< [i32](#), v1, v2 >  
*greatest common divisor*
- template<[typename](#) v1 >  
using [pos\\_t](#) = [typename](#) [pos](#)< v1 >::type  
*positivity operator (type)*

## Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

### 5.22.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

### 5.22.2 Member Typedef Documentation

#### 5.22.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

#### Template Parameters

v1	a value in <code>zpz::val</code>
v2	a value in <code>zpz::val</code>

#### 5.22.2.2 div\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 5.22.2.3 eq\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 5.22.2.4 gcd\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 5.22.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 5.22.2.6 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 5.22.2.7 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 5.22.2.8 mul\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 5.22.2.9 pos\_t

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

## 5.22.2.10 sub\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 5.22.3 Member Data Documentation

## 5.22.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

## 5.22.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 5.22.3.3 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 5.22.3.4 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)



## Chapter 6

# File Documentation

### 6.1 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 6.2 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016
00017 #ifdef _MSC_VER
00018 #define ALIGNED(x) __declspec(align(x))
00019 #define INLINED __forceinline
00020 #else
00021 #define ALIGNED(x) __attribute__((aligned(x)))
00022 #define INLINED __attribute__((always_inline)) inline
00023 #endif
00024
00027
00028 // aligned allocation
00029 namespace aerobus {
00036     template<typename T>
00037     T* aligned_malloc(size_t count, size_t alignment) {
00038         #ifdef _MSC_VER
```

```

00039         return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00040     #else
00041     return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00042     #endif
00043     }
00044 } // namespace aerobus
00045
00046 // concepts
00047 namespace aerobus {
00048     template <typename R>
00049     concept IsRing = requires {
00050         typename R::one;
00051         typename R::zero;
00052         typename R::template add_t<typename R::one, typename R::one>;
00053         typename R::template sub_t<typename R::one, typename R::one>;
00054         typename R::template mul_t<typename R::one, typename R::one>;
00055     };
00056
00057     template <typename R>
00058     concept IsEuclideanDomain = IsRing<R> && requires {
00059         typename R::template div_t<typename R::one, typename R::one>;
00060         typename R::template mod_t<typename R::one, typename R::one>;
00061         typename R::template gcd_t<typename R::one, typename R::one>;
00062         typename R::template eq_t<typename R::one, typename R::one>;
00063         typename R::template pos_t<typename R::one>;
00064
00065         R::template pos_v<typename R::one> == true;
00066         // typename R::template gt_t<typename R::one, typename R::zero>;
00067         R::is_euclidean_domain == true;
00068     };
00069
00070     template<typename R>
00071     concept IsField = IsEuclideanDomain<R> && requires {
00072         R::is_field == true;
00073     };
00074 } // namespace aerobus
00075
00076 // utilities
00077 namespace aerobus {
00078     namespace internal {
00079         template<template<typename...> typename TT, typename T>
00080         struct is_instantiation_of : std::false_type { };
00081
00082         template<template<typename...> typename TT, typename... Ts>
00083         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00084
00085         template<template<typename...> typename TT, typename T>
00086         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00087
00088         template<int64_t i, typename T, typename... Ts>
00089         struct type_at {
00090             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00091             using type = typename type_at<i - 1, Ts...>::type;
00092         };
00093
00094         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00095             using type = T;
00096         };
00097
00098         template <size_t i, typename... Ts>
00099         using type_at_t = typename type_at<i, Ts...>::type;
00100
00101         template<size_t n, size_t i, typename E = void>
00102         struct _is_prime {};
00103
00104         template<size_t i>
00105         struct _is_prime<0, i> {
00106             static constexpr bool value = false;
00107         };
00108
00109         template<size_t i>
00110         struct _is_prime<1, i> {
00111             static constexpr bool value = false;
00112         };
00113
00114         template<size_t i>
00115         struct _is_prime<2, i> {
00116             static constexpr bool value = true;
00117         };
00118
00119         template<size_t i>
00120         struct _is_prime<3, i> {
00121             static constexpr bool value = true;
00122         };
00123
00124         template<size_t i>

```

```

00129     struct _is_prime<5, i> {
00130         static constexpr bool value = true;
00131     };
00132
00133     template<size_t i>
00134     struct _is_prime<7, i> {
00135         static constexpr bool value = true;
00136     };
00137
00138     template<size_t n, size_t i>
00139     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00140         static constexpr bool value = false;
00141     };
00142
00143     template<size_t n, size_t i>
00144     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00145         static constexpr bool value = false;
00146     };
00147
00148     template<size_t n, size_t i>
00149     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00150         static constexpr bool value = true;
00151     };
00152
00153     template<size_t n, size_t i>
00154     struct _is_prime<n, i, std::enable_if_t<(
00155         n % i == 0 &&
00156         n >= 9 &&
00157         n % 3 != 0 &&
00158         n % 2 != 0 &&
00159         i * i > n)>> {
00160         static constexpr bool value = true;
00161     };
00162
00163     template<size_t n, size_t i>
00164     struct _is_prime<n, i, std::enable_if_t<(
00165         n % (i+2) == 0 &&
00166         n >= 9 &&
00167         n % 3 != 0 &&
00168         n % 2 != 0 &&
00169         i * i <= n)>> {
00170         static constexpr bool value = true;
00171     };
00172
00173     template<size_t n, size_t i>
00174     struct _is_prime<n, i, std::enable_if_t<(
00175         n % (i+2) != 0 &&
00176         n % i != 0 &&
00177         n >= 9 &&
00178         n % 3 != 0 &&
00179         n % 2 != 0 &&
00180         (i * i <= n))>> {
00181         static constexpr bool value = _is_prime<n, i+6>::value;
00182     };
00183
00184 } // namespace internal
00185
00186 template<size_t n>
00187 struct is_prime {
00188     static constexpr bool value = internal::_is_prime<n, 5>::value;
00189 };
00190
00191 template<size_t n>
00192 static constexpr bool is_prime_v = is_prime<n>::value;
00193
00194 namespace internal {
00195     template <std::size_t... Is>
00196     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&)
00197         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{});
00198
00199     template <std::size_t N>
00200     using make_index_sequence_reverse
00201         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00202
00203     template<typename Ring, typename E = void>
00204     struct gcd;
00205
00206     template<typename Ring>
00207     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00208         template<typename A, typename B, typename E = void>
00209         struct gcd_helper {};
00210
00211         // B = 0, A > 0
00212         template<typename A, typename B>
00213         struct gcd_helper<A, B, std::enable_if_t<
00214             (B::is_zero_t::value) &&
00215             (Ring::template gt_t<A, typename Ring::zero>::value)>> {

```

```

00227         using type = A;
00228     };
00229
00230     // B = 0, A < 0
00231     template<typename A, typename B>
00232     struct gcd_helper<A, B, std::enable_if_t<
00233         ((B::is_zero_t::value) &&
00234         !(Ring::template gt_t<A, typename Ring::zero>::value))>> {
00235         using type = typename Ring::template sub_t<typename Ring::zero, A>;
00236     };
00237
00238     // B != 0
00239     template<typename A, typename B>
00240     struct gcd_helper<A, B, std::enable_if_t<
00241         (!B::is_zero_t::value)
00242         >> {
00243     private: // NOLINT
00244         // A / B
00245         using k = typename Ring::template div_t<A, B>;
00246         // A - (A/B)*B = A % B
00247         using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00248
00249     public:
00250         using type = typename gcd_helper<B, m>::type;
00251     };
00252
00253     template<typename A, typename B>
00254     using type = typename gcd_helper<A, B>::type;
00255 }
00256 // namespace internal
00257
00260 template<typename T, typename A, typename B>
00261 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00262
00266 template<typename val>
00267 requires IsEuclideanDomain<typename val::ring_type>
00268 using abs_t = std::conditional_t<
00269     val::ring_type::template pos_v<val>,
00270     val, typename val::ring_type::template sub_t<typename val::ring_type::zero, val>;
00271 } // namespace aerobus
00272
00273 namespace aerobus {
00274     template<typename Ring, typename X>
00275     requires IsRing<Ring>
00276     struct Quotient {
00277     template <typename V>
00278     struct val {
00279     public:
00280         using type = abs_t<typename Ring::template mod_t<V, X>>;
00281     };
00282
00283     using zero = val<typename Ring::zero>;
00284
00285     using one = val<typename Ring::one>;
00286
00287     template<typename v1, typename v2>
00288     using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00289
00290     template<typename v1, typename v2>
00291     using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00292
00293     template<typename v1, typename v2>
00294     using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00295
00296     template<typename v1, typename v2>
00297     using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00298
00299     template<typename v1, typename v2>
00300     using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00301
00302     template<typename v1, typename v2>
00303     static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00304
00305     template<typename v1>
00306     using pos_t = std::true_type;
00307
00308     template<typename v>
00309     static constexpr bool pos_v = pos_t<v>::value;
00310
00311     static constexpr bool is_euclidean_domain = true;
00312
00313     template<auto x>
00314     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00315
00316     template<typename v>
00317     using inject_ring_t = val<v>;
00318 }

```

```

00358 } // namespace aerobus
00359
00360 // type_list
00361 namespace aerobus {
00362     template <typename... Ts>
00363     struct type_list;
00364
00365     namespace internal {
00366         template <typename T, typename... Us>
00367         struct pop_front_h {
00368             using tail = type_list<Us...>;
00369             using head = T;
00370         };
00371
00372         template <size_t index, typename L1, typename L2>
00373         struct split_h {
00374             private:
00375                 static_assert(index <= L2::length, "index ouf of bounds");
00376                 using a = typename L2::pop_front::type;
00377                 using b = typename L2::pop_front::tail;
00378                 using c = typename L1::template push_back<a>;
00379
00380             public:
00381                 using head = typename split_h<index - 1, c, b>::head;
00382                 using tail = typename split_h<index - 1, c, b>::tail;
00383             };
00384
00385             template <typename L1, typename L2>
00386             struct split_h<0, L1, L2> {
00387                 using head = L1;
00388                 using tail = L2;
00389             };
00390
00391             template <size_t index, typename L, typename T>
00392             struct insert_h {
00393                 static_assert(index <= L::length, "index ouf of bounds");
00394                 using s = typename L::template split<index>;
00395                 using left = typename s::head;
00396                 using right = typename s::tail;
00397                 using ll = typename left::template push_back<T>;
00398                 using type = typename ll::template concat<right>;
00399             };
00400
00401             template <size_t index, typename L>
00402             struct remove_h {
00403                 using s = typename L::template split<index>;
00404                 using left = typename s::head;
00405                 using right = typename s::tail;
00406                 using rr = typename right::pop_front::tail;
00407                 using type = typename left::template concat<rr>;
00408             };
00409         } // namespace internal
00410
00411     template <typename... Ts>
00412     struct type_list {
00413     private:
00414         template <typename T>
00415         struct concat_h;
00416
00417         template <typename... Us>
00418         struct concat_h<type_list<Us...> {
00419             using type = type_list<Ts..., Us...>;
00420         };
00421
00422     public:
00423         static constexpr size_t length = sizeof...(Ts);
00424
00425         template <typename T>
00426         using push_front = type_list<T, Ts...>;
00427
00428         template <size_t index>
00429         using at = internal::type_at_t<index, Ts...>;
00430
00431         struct pop_front {
00432             using type = typename internal::pop_front_h<Ts...>::head;
00433             using tail = typename internal::pop_front_h<Ts...>::tail;
00434         };
00435
00436         template <typename T>
00437         using push_back = type_list<Ts..., T>;
00438
00439         template <typename U>
00440         using concat = typename concat_h<U>::type;
00441
00442         template <size_t index>
00443         struct split {
00444             private:

```

```

00463         using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00464
00465     public:
00466         using head = typename inner::head;
00467         using tail = typename inner::tail;
00468     };
00469
00473     template <typename T, size_t index>
00474     using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00475
00478     template <size_t index>
00479     using remove = typename internal::remove_h<index, type_list<Ts...>>::type;
00480 };
00481
00483 template <>
00484 struct type_list<> {
00485     static constexpr size_t length = 0;
00486
00487     template <typename T>
00488     using push_front = type_list<T>;
00489
00490     template <typename T>
00491     using push_back = type_list<T>;
00492
00493     template <typename U>
00494     using concat = U;
00495
00496     // TODO(jewave): assert index == 0
00497     template <typename T, size_t index>
00498     using insert = type_list<T>;
00499 };
00500 } // namespace aerobus
00501
00502 // i32
00503 namespace aerobus {
00504     struct i32 {
00505         using inner_type = int32_t;
00506         template<int32_t x>
00507         struct val {
00508             using ring_type = i32;
00509             static constexpr int32_t v = x;
00510
00511             template<typename valueType>
00512             static constexpr valueType get() { return static_cast<valueType>(x); }
00513
00514             using is_zero_t = std::bool_constant<x == 0>;
00515
00516             static std::string to_string() {
00517                 return std::to_string(x);
00518             }
00519
00520             template<typename valueRing>
00521             static constexpr valueRing eval(const valueRing& v) {
00522                 return static_cast<valueRing>(x);
00523             }
00524         };
00525
00526         using zero = val<0>;
00527         using one = val<1>;
00528         static constexpr bool is_field = false;
00529         static constexpr bool is_euclidean_domain = true;
00530         template<auto x>
00531         using inject_constant_t = val<static_cast<int32_t>(x)>;
00532
00533         template<typename v>
00534         using inject_ring_t = v;
00535     private:
00536         template<typename v1, typename v2>
00537         struct add {
00538             using type = val<v1::v + v2::v>;
00539         };
00540
00541         template<typename v1, typename v2>
00542         struct sub {
00543             using type = val<v1::v - v2::v>;
00544         };
00545
00546         template<typename v1, typename v2>
00547         struct mul {
00548             using type = val<v1::v * v2::v>;
00549         };
00550
00551         template<typename v1, typename v2>
00552         struct div {
00553             using type = val<v1::v / v2::v>;
00554         };
00555     };

```

```

00574
00575     template<typename v1, typename v2>
00576     struct remainder {
00577         using type = val<v1::v % v2::v>;
00578     };
00579
00580     template<typename v1, typename v2>
00581     struct gt {
00582         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00583     };
00584
00585     template<typename v1, typename v2>
00586     struct lt {
00587         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00588     };
00589
00590     template<typename v1, typename v2>
00591     struct eq {
00592         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00593     };
00594
00595     template<typename v1>
00596     struct pos {
00597         using type = std::bool_constant<(v1::v > 0)>;
00598     };
00599
00600     public:
00601     template<typename v1, typename v2>
00602     using add_t = typename add<v1, v2>::type;
00603
00604     template<typename v1, typename v2>
00605     using sub_t = typename sub<v1, v2>::type;
00606
00607     template<typename v1, typename v2>
00608     using mul_t = typename mul<v1, v2>::type;
00609
00610     template<typename v1, typename v2>
00611     using div_t = typename div<v1, v2>::type;
00612
00613     template<typename v1, typename v2>
00614     using mod_t = typename remainder<v1, v2>::type;
00615
00616     template<typename v1, typename v2>
00617     using gt_t = typename gt<v1, v2>::type;
00618
00619     template<typename v1, typename v2>
00620     using lt_t = typename lt<v1, v2>::type;
00621
00622     template<typename v1, typename v2>
00623     using eq_t = typename eq<v1, v2>::type;
00624
00625     template<typename v1, typename v2>
00626     static constexpr bool eq_v = eq_t<v1, v2>::value;
00627
00628     template<typename v1, typename v2>
00629     using gcd_t = gcd_t<i32, v1, v2>;
00630
00631     template<typename v>
00632     using pos_t = typename pos<v>::type;
00633
00634     template<typename v>
00635     static constexpr bool pos_v = pos_t<v>::value;
00636 };
00637 } // namespace aerobus
00638
00639 // i64
00640 namespace aerobus {
00641     struct i64 {
00642         using inner_type = int64_t;
00643         template<int64_t x>
00644         struct val {
00645             using ring_type = i64;
00646             static constexpr int64_t v = x;
00647
00648             template<typename valueType>
00649             static constexpr valueType get() { return static_cast<valueType>(x); }
00650
00651             using is_zero_t = std::bool_constant<x == 0>;
00652
00653             static std::string to_string() {
00654                 return std::to_string(x);
00655             }
00656
00657             template<typename valueRing>
00658             static constexpr valueRing eval(const valueRing& v) {
00659                 return static_cast<valueRing>(x);
00660             }
00661         };
00662     };
00663 }

```

```

00730     };
00731
00732     template<auto x>
00733     using inject_constant_t = val<static_cast<int64_t>(x)>;
00734
00735     template<typename v>
00736     using inject_ring_t = v;
00737
00738     using zero = val<0>;
00739     using one = val<1>;
00740     static constexpr bool is_field = false;
00741     static constexpr bool is_euclidean_domain = true;
00742
00743 private:
00744     template<typename v1, typename v2>
00745     struct add {
00746         using type = val<v1::v + v2::v>;
00747     };
00748
00749     template<typename v1, typename v2>
00750     struct sub {
00751         using type = val<v1::v - v2::v>;
00752     };
00753
00754     template<typename v1, typename v2>
00755     struct mul {
00756         using type = val<v1::v * v2::v>;
00757     };
00758
00759     template<typename v1, typename v2>
00760     struct div {
00761         using type = val<v1::v / v2::v>;
00762     };
00763
00764     template<typename v1, typename v2>
00765     struct remainder {
00766         using type = val<v1::v % v2::v>;
00767     };
00768
00769     template<typename v1, typename v2>
00770     struct gt {
00771         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00772     };
00773
00774     template<typename v1, typename v2>
00775     struct lt {
00776         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00777     };
00778
00779     template<typename v1, typename v2>
00780     struct eq {
00781         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00782     };
00783
00784     template<typename v>
00785     struct pos {
00786         using type = std::bool_constant<(v::v > 0)>;
00787     };
00788
00789 public:
00790     template<typename v1, typename v2>
00791     using add_t = typename add<v1, v2>::type;
00792
00793     template<typename v1, typename v2>
00794     using sub_t = typename sub<v1, v2>::type;
00795
00796     template<typename v1, typename v2>
00797     using mul_t = typename mul<v1, v2>::type;
00798
00799     template<typename v1, typename v2>
00800     using div_t = typename div<v1, v2>::type;
00801
00802     template<typename v1, typename v2>
00803     using mod_t = typename remainder<v1, v2>::type;
00804
00805     template<typename v1, typename v2>
00806     using gt_t = typename gt<v1, v2>::type;
00807
00808     template<typename v1, typename v2>
00809     static constexpr bool gt_v = gt_t<v1, v2>::value;
00810
00811     template<typename v1, typename v2>
00812     using lt_t = typename lt<v1, v2>::type;
00813
00814     template<typename v1, typename v2>
00815     static constexpr bool lt_v = lt_t<v1, v2>::value;
00816
00817

```



```

00873     template<typename v1, typename v2>
00874     using eq_t = typename eq<v1, v2>::type;
00875
00881     template<typename v1, typename v2>
00882     static constexpr bool eq_v = eq_t<v1, v2>::value;
00883
00889     template<typename v1, typename v2>
00890     using gcd_t = gcd_t<i64, v1, v2>;
00891
00896     template<typename v>
00897     using pos_t = typename pos<v>::type;
00898
00903     template<typename v>
00904     static constexpr bool pos_v = pos_t<v>::value;
00905 };
00906 } // namespace aerobus
00907
00908 // z/pz
00909 namespace aerobus {
00914     template<int32_t p>
00915     struct zp {
00916         using inner_type = int32_t;
00917         template<int32_t x>
00918         struct val {
00920             using ring_type = zp<p>;
00922             static constexpr int32_t v = x % p;
00923
00924             template<typename valueType>
00925             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00926
00927             using is_zero_t = std::bool_constant<x % p == 0>;
00928             static std::string to_string() {
00929                 return std::to_string(x % p);
00930             }
00931
00932             template<typename valueRing>
00933             static constexpr valueRing eval(const valueRing& v) {
00934                 return static_cast<valueRing>(x % p);
00935             }
00936         };
00937
00938         template<auto x>
00939         using inject_constant_t = val<static_cast<int32_t>(x)>;
00940
00941         using zero = val<0>;
00942         using one = val<1>;
00943         static constexpr bool is_field = is_prime<p>::value;
00944         static constexpr bool is_euclidean_domain = true;
00945
00946     private:
00947         template<typename v1, typename v2>
00948         struct add {
00949             using type = val<(v1::v + v2::v) % p>;
00950         };
00951
00952         template<typename v1, typename v2>
00953         struct sub {
00954             using type = val<(v1::v - v2::v) % p>;
00955         };
00956
00957         template<typename v1, typename v2>
00958         struct mul {
00959             using type = val<(v1::v * v2::v) % p>;
00960         };
00961
00962         template<typename v1, typename v2>
00963         struct div {
00964             using type = val<(v1::v % p) / (v2::v % p)>;
00965         };
00966
00967         template<typename v1, typename v2>
00968         struct remainder {
00969             using type = val<(v1::v % v2::v) % p>;
00970         };
00971
00972         template<typename v1, typename v2>
00973         struct gt {
00974             using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
00975         };
00976
00977         template<typename v1, typename v2>
00978         struct lt {
00979             using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
00980         };
00981
00982         template<typename v1, typename v2>
00983         struct eq {

```

```

00984         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
00985     };
00986
00987     template<typename v1>
00988     struct pos {
00989         using type = std::bool_constant<(v1::v > 0)>;
00990     };
00991
00992     public:
00993     template<typename v1, typename v2>
00994     using add_t = typename add<v1, v2>::type;
00995
00996     template<typename v1, typename v2>
00997     using sub_t = typename sub<v1, v2>::type;
00998
00999     template<typename v1, typename v2>
01000     using mul_t = typename mul<v1, v2>::type;
01001
01002     template<typename v1, typename v2>
01003     using div_t = typename div<v1, v2>::type;
01004
01005     template<typename v1, typename v2>
01006     using mod_t = typename remainder<v1, v2>::type;
01007
01008     template<typename v1, typename v2>
01009     using gt_t = typename gt<v1, v2>::type;
01010
01011     template<typename v1, typename v2>
01012     static constexpr bool gt_v = gt_t<v1, v2>::value;
01013
01014     template<typename v1, typename v2>
01015     using lt_t = typename lt<v1, v2>::type;
01016
01017     template<typename v1, typename v2>
01018     static constexpr bool lt_v = lt_t<v1, v2>::value;
01019
01020     template<typename v1, typename v2>
01021     using eq_t = typename eq<v1, v2>::type;
01022
01023     template<typename v1, typename v2>
01024     static constexpr bool eq_v = eq_t<v1, v2>::value;
01025
01026     template<typename v1, typename v2>
01027     using gcd_t = gcd_t<i32, v1, v2>;
01028
01029     template<typename v1>
01030     using pos_t = typename pos<v1>::type;
01031
01032     template<typename v>
01033     static constexpr bool pos_v = pos_t<v>::value;
01034 };
01035 } // namespace aerobus
01036
01037 // polynomial
01038 namespace aerobus {
01039     // coeffN x^N + ...
01040     template<typename Ring>
01041     requires IsEuclideanDomain<Ring>
01042     struct polynomial {
01043     public:
01044         static constexpr bool is_field = false;
01045         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01046
01047         template<typename coeffN, typename... coeffs>
01048         struct val {
01049         public:
01050             using ring_type = polynomial<Ring>;
01051             static constexpr size_t degree = sizeof...(coeffs);
01052             using aN = coeffN;
01053             using strip = val<coeffs...>;
01054             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01055             static constexpr bool is_zero_v = is_zero_t::value;
01056
01057         private:
01058             template<size_t index, typename E = void>
01059             struct coeff_at {};
01060
01061             template<size_t index>
01062             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01063             public:
01064                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01065             };
01066
01067             template<size_t index>
01068             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01069             public:
01070                 using type = typename Ring::zero;
01071             };
01072
01073         public:
01074             template<size_t index>

```

```

01126         using coeff_at_t = typename coeff_at<index>::type;
01127
01130     static std::string to_string() {
01131         return string_helper<coeffN, coeffs...>::func();
01132     }
01133
01138     template<typename valueRing>
01139     static constexpr valueRing eval(const valueRing& x) {
01140         return horner_evaluation<valueRing, val>
01141             ::template inner<0, degree + 1>
01142             ::func(static_cast<valueRing>(0), x);
01143     }
01144 };
01145
01148 template<typename coeffN>
01149 struct val<coeffN> {
01150     using ring_type = polynomial<Ring>;
01151     static constexpr size_t degree = 0;
01152     using aN = coeffN;
01153     using strip = val<coeffN>;
01154     using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01155
01156     static constexpr bool is_zero_v = is_zero_t::value;
01157
01158     template<size_t index, typename E = void>
01159     struct coeff_at {};
01160
01161     template<size_t index>
01162     struct coeff_at<index, std::enable_if_t<(index == 0)>> {
01163         using type = aN;
01164     };
01165
01166     template<size_t index>
01167     struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)>> {
01168         using type = typename Ring::zero;
01169     };
01170
01171     template<size_t index>
01172     using coeff_at_t = typename coeff_at<index>::type;
01173
01174     static std::string to_string() {
01175         return string_helper<coeffN>::func();
01176     }
01177
01178     template<typename valueRing>
01179     static constexpr valueRing eval(const valueRing& x) {
01180         return static_cast<valueRing>(aN::template get<valueRing>());
01181     }
01182 };
01183
01184 using zero = val<typename Ring::zero>;
01185 using one = val<typename Ring::one>;
01186 using X = val<typename Ring::one, typename Ring::zero>;
01187
01188 private:
01189     template<typename P, typename E = void>
01190     struct simplify;
01191
01192     template<typename P1, typename P2, typename I>
01193     struct add_low;
01194
01195     template<typename P1, typename P2>
01196     struct add {
01197         using type = typename simplify<typename add_low<
01198             P1,
01199             P2,
01200             internal::make_index_sequence_reverse<
01201                 std::max(P1::degree, P2::degree) + 1
01202             >::type>::type;
01203     };
01204
01205     template<typename P1, typename P2, typename I>
01206     struct sub_low;
01207
01208     template<typename P1, typename P2, typename I>
01209     struct mul_low;
01210
01211     template<typename v1, typename v2>
01212     struct mul {
01213         using type = typename mul_low<
01214             v1,
01215             v2,
01216             internal::make_index_sequence_reverse<
01217                 v1::degree + v2::degree + 1
01218             >::type;
01219     };
01220
01221 };
01222
01223
01224
01225

```

```

01226     template<typename coeff, size_t deg>
01227     struct monomial;
01228
01229     template<typename v, typename E = void>
01230     struct derive_helper {};
01231
01232     template<typename v>
01233     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01234         using type = zero;
01235     };
01236
01237     template<typename v>
01238     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01239         using type = typename add<
01240             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01241             typename monomial<
01242                 typename Ring::template mul_t<
01243                     typename v::aN,
01244                     typename Ring::template inject_constant_t<(v::degree)>
01245                 >,
01246                 v::degree - 1
01247             >::type
01248         >::type;
01249     };
01250
01251     template<typename v1, typename v2, typename E = void>
01252     struct eq_helper {};
01253
01254     template<typename v1, typename v2>
01255     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01256         using type = std::false_type;
01257     };
01258
01259     template<typename v1, typename v2>
01260     struct eq_helper<v1, v2, std::enable_if_t<
01261         v1::degree == v2::degree &&
01262         (v1::degree != 0 || v2::degree != 0) &&
01263         std::is_same<
01264             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01265             std::false_type
01266         >::value
01267     >
01268     > {
01269         using type = std::false_type;
01270     };
01271
01272     template<typename v1, typename v2>
01273     struct eq_helper<v1, v2, std::enable_if_t<
01274         v1::degree == v2::degree &&
01275         (v1::degree != 0 || v2::degree != 0) &&
01276         std::is_same<
01277             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01278             std::true_type
01279         >::value
01280     > {
01281         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01282     };
01283
01284     template<typename v1, typename v2>
01285     struct eq_helper<v1, v2, std::enable_if_t<
01286         v1::degree == v2::degree &&
01287         (v1::degree == 0)
01288     > {
01289         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01290     };
01291
01292     template<typename v1, typename v2, typename E = void>
01293     struct lt_helper {};
01294
01295     template<typename v1, typename v2>
01296     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)> {
01297         using type = std::true_type;
01298     };
01299
01300     template<typename v1, typename v2>
01301     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)> {
01302         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01303     };
01304
01305     template<typename v1, typename v2>
01306     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)> {
01307         using type = std::false_type;
01308     };
01309
01310     template<typename v1, typename v2, typename E = void>
01311     struct gt_helper {};
01312

```

```

01313
01314     template<typename v1, typename v2>
01315     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01316         using type = std::true_type;
01317     };
01318
01319     template<typename v1, typename v2>
01320     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01321         using type = std::false_type;
01322     };
01323
01324     template<typename v1, typename v2>
01325     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01326         using type = std::false_type;
01327     };
01328
01329     // when high power is zero : strip
01330     template<typename P>
01331     struct simplify<P, std::enable_if_t<
01332         std::is_same<
01333             typename Ring::zero,
01334             typename P::aN
01335             >::value && (P::degree > 0)
01336     >> {
01337         using type = typename simplify<typename P::strip>::type;
01338     };
01339
01340     // otherwise : do nothing
01341     template<typename P>
01342     struct simplify<P, std::enable_if_t<
01343         !std::is_same<
01344             typename Ring::zero,
01345             typename P::aN
01346             >::value && (P::degree > 0)
01347     >> {
01348         using type = P;
01349     };
01350
01351     // do not simplify constants
01352     template<typename P>
01353     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01354         using type = P;
01355     };
01356
01357     // addition at
01358     template<typename P1, typename P2, size_t index>
01359     struct add_at {
01360         using type =
01361             typename Ring::template add_t<
01362                 typename P1::template coeff_at_t<index>,
01363                 typename P2::template coeff_at_t<index>>;
01364     };
01365
01366     template<typename P1, typename P2, size_t index>
01367     using add_at_t = typename add_at<P1, P2, index>::type;
01368
01369     template<typename P1, typename P2, std::size_t... I>
01370     struct add_low<P1, P2, std::index_sequence<I...>> {
01371         using type = val<add_at_t<P1, P2, I>...>;
01372     };
01373
01374     // subtraction at
01375     template<typename P1, typename P2, size_t index>
01376     struct sub_at {
01377         using type =
01378             typename Ring::template sub_t<
01379                 typename P1::template coeff_at_t<index>,
01380                 typename P2::template coeff_at_t<index>>;
01381     };
01382
01383     template<typename P1, typename P2, size_t index>
01384     using sub_at_t = typename sub_at<P1, P2, index>::type;
01385
01386     template<typename P1, typename P2, std::size_t... I>
01387     struct sub_low<P1, P2, std::index_sequence<I...>> {
01388         using type = val<sub_at_t<P1, P2, I>...>;
01389     };
01390
01391     template<typename P1, typename P2>
01392     struct sub {
01393         using type = typename simplify<typename sub_low<
01394             P1,
01395             P2,
01396             internal::make_index_sequence_reverse<
01397                 std::max(P1::degree, P2::degree) + 1
01398             >::type>::type;
01399     };

```

```

01400
01401 // multiplication at
01402 template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01403 struct mul_at_loop_helper {
01404     using type = typename Ring::template add_t<
01405         typename Ring::template mul_t<
01406             typename v1::template coeff_at_t<index>,
01407             typename v2::template coeff_at_t<k - index>
01408         >,
01409         typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01410     >;
01411 };
01412
01413 template<typename v1, typename v2, size_t k, size_t stop>
01414 struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01415     using type = typename Ring::template mul_t<
01416         typename v1::template coeff_at_t<stop>,
01417         typename v2::template coeff_at_t<0>>;
01418 };
01419
01420 template <typename v1, typename v2, size_t k, typename E = void>
01421 struct mul_at {};
01422
01423 template<typename v1, typename v2, size_t k>
01424 struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)>> {
01425     using type = typename Ring::zero;
01426 };
01427
01428 template<typename v1, typename v2, size_t k>
01429 struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)>> {
01430     using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01431 };
01432
01433 template<typename P1, typename P2, size_t index>
01434 using mul_at_t = typename mul_at<P1, P2, index>::type;
01435
01436 template<typename P1, typename P2, std::size_t... I>
01437 struct mul_low<P1, P2, std::index_sequence<I...> {
01438     using type = val<mul_at_t<P1, P2, I>...>;
01439 };
01440
01441 // division helper
01442 template< typename A, typename B, typename Q, typename R, typename E = void>
01443 struct div_helper {};
01444
01445 template<typename A, typename B, typename Q, typename R>
01446 struct div_helper<A, B, Q, R, std::enable_if_t<
01447     (R::degree < B::degree) ||
01448     (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01449     using q_type = Q;
01450     using mod_type = R;
01451     using gcd_type = B;
01452 };
01453
01454 template<typename A, typename B, typename Q, typename R>
01455 struct div_helper<A, B, Q, R, std::enable_if_t<
01456     (R::degree >= B::degree) &&
01457     !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01458     private: // NOLINT
01459         using rN = typename R::aN;
01460         using bN = typename B::aN;
01461         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01462             B::degree>::type;
01463         using rr = typename sub<R, typename mul<pT, B>::type>::type;
01464         using qq = typename add<Q, pT>::type;
01465     public:
01466         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01467         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01468         using gcd_type = rr;
01469 };
01470
01471 template<typename A, typename B>
01472 struct div {
01473     static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01474     using q_type = typename div_helper<A, B, zero, A>::q_type;
01475     using m_type = typename div_helper<A, B, zero, A>::mod_type;
01476 };
01477
01478 template<typename P>
01479 struct make_unit {
01480     using type = typename div<P, val<typename P::aN>>::q_type;
01481 };
01482
01483 template<typename coeff, size_t deg>
01484 struct monomial {
01485     using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;

```

```

01486     };
01487
01488     template<typename coeff>
01489     struct monomial<coeff, 0> {
01490         using type = val<coeff>;
01491     };
01492
01493     template<typename valueRing, typename P>
01494     struct horner_evaluation {
01495         template<size_t index, size_t stop>
01496         struct inner {
01497             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01498                 constexpr valueRing coeff =
01499                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
get<valueRing>());
01500                 return horner_evaluation<valueRing, P::template inner<index + 1, stop>::func(x *
accum + coeff, x);
01501             }
01502         };
01503
01504         template<size_t stop>
01505         struct inner<stop, stop> {
01506             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01507                 return accum;
01508             }
01509         };
01510     };
01511
01512     template<typename coeff, typename... coeffs>
01513     struct string_helper {
01514         static std::string func() {
01515             std::string tail = string_helper<coeffs...>::func();
01516             std::string result = "";
01517             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01518                 return tail;
01519             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01520                 if (sizeof...(coeffs) == 1) {
01521                     result += "x";
01522                 } else {
01523                     result += "x^" + std::to_string(sizeof...(coeffs));
01524                 }
01525             } else {
01526                 if (sizeof...(coeffs) == 1) {
01527                     result += coeff::to_string() + " x";
01528                 } else {
01529                     result += coeff::to_string()
01530                         + " x^" + std::to_string(sizeof...(coeffs));
01531                 }
01532             }
01533
01534             if (!tail.empty()) {
01535                 result += " + " + tail;
01536             }
01537
01538             return result;
01539         }
01540     };
01541
01542     template<typename coeff>
01543     struct string_helper<coeff> {
01544         static std::string func() {
01545             if (!std::is_same<coeff, typename Ring::zero>::value) {
01546                 return coeff::to_string();
01547             } else {
01548                 return "";
01549             }
01550         }
01551     };
01552
01553     public:
01554     template<typename P>
01555     using simplify_t = typename simplify<P>::type;
01556
01557     template<typename v1, typename v2>
01558     using add_t = typename add<v1, v2>::type;
01559
01560     template<typename v1, typename v2>
01561     using sub_t = typename sub<v1, v2>::type;
01562
01563     template<typename v1, typename v2>
01564     using mul_t = typename mul<v1, v2>::type;
01565
01566     template<typename v1, typename v2>
01567     using eq_t = typename eq_helper<v1, v2>::type;
01568
01569     template<typename v1, typename v2>
01570     using lt_t = typename lt_helper<v1, v2>::type;

```

```

01588
01592     template<typename v1, typename v2>
01593     using gt_t = typename gt_helper<v1, v2>::type;
01594
01598     template<typename v1, typename v2>
01599     using div_t = typename div<v1, v2>::q_type;
01600
01604     template<typename v1, typename v2>
01605     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01606
01610     template<typename coeff, size_t deg>
01611     using monomial_t = typename monomial<coeff, deg>::type;
01612
01615     template<typename v>
01616     using derive_t = typename derive_helper<v>::type;
01617
01620     template<typename v>
01621     using pos_t = typename Ring::template pos_t<typename v::aN>;
01622
01625     template<typename v>
01626     static constexpr bool pos_v = pos_t<v>::value;
01627
01631     template<typename v1, typename v2>
01632     using gcd_t = std::conditional_t<
01633         Ring::is_euclidean_domain,
01634         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01635         void>;
01636
01640     template<auto x>
01641     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01642
01646     template<typename v>
01647     using inject_ring_t = val<v>;
01648 };
01649 } // namespace aerobus
01650
01651 // fraction field
01652 namespace aerobus {
01653     namespace internal {
01654         template<typename Ring, typename E = void>
01655         requires IsEuclideanDomain<Ring>
01656         struct _FractionField {};
01657
01658         template<typename Ring>
01659         requires IsEuclideanDomain<Ring>
01660         struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
01661             static constexpr bool is_field = true;
01662             static constexpr bool is_euclidean_domain = true;
01663
01664         private:
01665             template<typename val1, typename val2, typename E = void>
01666             struct to_string_helper {};
01667
01668             template<typename val1, typename val2>
01669             struct to_string_helper<val1, val2,
01670                 std::enable_if_t<
01671                     Ring::template eq_t<
01672                         val2, typename Ring::one
01673                         >::value
01674                     >
01675                 > {
01676                 static std::string func() {
01677                     return val1::to_string();
01678                 }
01679             };
01680
01681             template<typename val1, typename val2>
01682             struct to_string_helper<val1, val2,
01683                 std::enable_if_t<
01684                     !Ring::template eq_t<
01685                         val2,
01686                         typename Ring::one
01687                         >::value
01688                     >
01689                 > {
01690                 static std::string func() {
01691                     return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01692                 }
01693             };
01694         };
01695
01696     public:
01697         template<typename val1, typename val2>
01698         struct val {
01699             using x = val1;
01700             using y = val2;
01701             using is_zero_t = typename val1::is_zero_t;
01702             static constexpr bool is_zero_v = val1::is_zero_t::value;
01703         };

```



```

01710
01712         using ring_type = Ring;
01713         using field_type = _FractionField<Ring>;
01714
01717         static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01718
01722         template<typename valueType>
01723         static constexpr valueType get() { return static_cast<valueType>(x::v) /
static_cast<valueType>(y::v); }
01724
01727         static std::string to_string() {
01728             return to_string_helper<val1, val2>::func();
01729         }
01730
01735         template<typename valueRing>
01736         static constexpr valueRing eval(const valueRing& v) {
01737             return x::eval(v) / y::eval(v);
01738         }
01739     };
01740
01742     using zero = val<typename Ring::zero, typename Ring::one>;
01744     using one = val<typename Ring::one, typename Ring::one>;
01745
01748     template<typename v>
01749     using inject_t = val<v, typename Ring::one>;
01750
01753     template<auto x>
01754     using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
Ring::one>;
01755
01758     template<typename v>
01759     using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01760
01762     using ring_type = Ring;
01763
01764     private:
01765         template<typename v, typename E = void>
01766         struct simplify {};
01767
01768         // x = 0
01769         template<typename v>
01770         struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {
01771             using type = typename _FractionField<Ring>::zero;
01772         };
01773
01774         // x != 0
01775         template<typename v>
01776         struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01777             private:
01778                 using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01779                 using newx = typename Ring::template div_t<typename v::x, _gcd>;
01780                 using newy = typename Ring::template div_t<typename v::y, _gcd>;
01781
01782                 using posx = std::conditional_t<
01783                     !Ring::template pos_v<newx>,
01784                     typename Ring::template sub_t<typename Ring::zero, newx>,
01785                     newx>;
01786                 using posy = std::conditional_t<
01787                     !Ring::template pos_v<newy>,
01788                     typename Ring::template sub_t<typename Ring::zero, newy>,
01789                     newy>;
01790             public:
01791                 using type = typename _FractionField<Ring>::template val<posx, posy>;
01792         };
01793
01794     public:
01797         template<typename v>
01798         using simplify_t = typename simplify<v>::type;
01799
01800     private:
01801         template<typename v1, typename v2>
01802         struct add {
01803             private:
01804                 using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01805                 using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01806                 using dividend = typename Ring::template add_t<a, b>;
01807                 using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01808                 using g = typename Ring::template gcd_t<dividend, diviser>;
01809             public:
01810                 using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01811
01812         };
01813
01814         template<typename v>
01815         struct pos {
01816             using type = std::conditional_t<

```

```

01817         (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01818         (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01819         std::true_type,
01820         std::false_type>;
01821     };
01822
01823     template<typename v1, typename v2>
01824     struct sub {
01825     private:
01826         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01827         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01828         using dividend = typename Ring::template sub_t<a, b>;
01829         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01830         using g = typename Ring::template gcd_t<dividend, diviser>;
01831
01832     public:
01833         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
diviser>;
01834     };
01835
01836     template<typename v1, typename v2>
01837     struct mul {
01838     private:
01839         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01840         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01841
01842     public:
01843         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01844     };
01845
01846     template<typename v1, typename v2, typename E = void>
01847     struct div {};
01848
01849     template<typename v1, typename v2>
01850     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
_FractionField<Ring>::zero>::value> {
01851     private:
01852         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01853         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01854
01855     public:
01856         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01857     };
01858
01859     template<typename v1, typename v2>
01860     struct div<v1, v2, std::enable_if_t<
std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01861         using type = one;
01862     };
01863
01864     template<typename v1, typename v2>
01865     struct eq {
01866     private:
01867         using type = std::conditional_t<
std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
std::true_type,
std::false_type>;
01871     };
01872
01873     template<typename TL, typename E = void>
01874     struct vadd {};
01875
01876     template<typename TL>
01877     struct vadd<TL, std::enable_if_t<(TL::length > 1)> {
01878         using head = typename TL::pop_front::type;
01879         using tail = typename TL::pop_front::tail;
01880         using type = typename add<head, typename vadd<tail>::type>::type;
01881     };
01882
01883     template<typename TL>
01884     struct vadd<TL, std::enable_if_t<(TL::length == 1)> {
01885         using type = typename TL::template at<0>;
01886     };
01887
01888     template<typename... vals>
01889     struct vmul {};
01890
01891     template<typename v1, typename... vals>
01892     struct vmul<v1, vals...> {
01893         using type = typename mul<v1, typename vmul<vals...>::type>::type;
01894     };
01895
01896     template<typename v1>
01897     struct vmul<v1> {
01898         using type = v1;
01899     };
01900
01901

```

```

01902
01903     template<typename v1, typename v2, typename E = void>
01904     struct gt;
01905
01906     template<typename v1, typename v2>
01907     struct gt<v1, v2, std::enable_if_t<
01908         (eq<v1, v2>::type::value)
01909         >> {
01910         using type = std::false_type;
01911     };
01912
01913     template<typename v1, typename v2>
01914     struct gt<v1, v2, std::enable_if_t<
01915         (!eq<v1, v2>::type::value) &&
01916         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01917         >> {
01918         using type = typename gt<
01919             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01920             >::type;
01921     };
01922
01923     template<typename v1, typename v2>
01924     struct gt<v1, v2, std::enable_if_t<
01925         (!eq<v1, v2>::type::value) &&
01926         (pos<v1>::type::value) && (!pos<v2>::type::value)
01927         >> {
01928         using type = std::true_type;
01929     };
01930
01931     template<typename v1, typename v2>
01932     struct gt<v1, v2, std::enable_if_t<
01933         (!eq<v1, v2>::type::value) &&
01934         (!pos<v1>::type::value) && (pos<v2>::type::value)
01935         >> {
01936         using type = std::false_type;
01937     };
01938
01939     template<typename v1, typename v2>
01940     struct gt<v1, v2, std::enable_if_t<
01941         (!eq<v1, v2>::type::value) &&
01942         (pos<v1>::type::value) && (pos<v2>::type::value)
01943         >> {
01944         using type = typename Ring::template gt_t<
01945             typename Ring::template mul_t<v1::x, v2::y>,
01946             typename Ring::template mul_t<v2::y, v2::x>
01947         >;
01948     };
01949
01950     public:
01951     template<typename v1, typename v2>
01952     using add_t = typename add<v1, v2>::type;
01953
01954     template<typename v1, typename v2>
01955     using mod_t = zero;
01956
01957     template<typename v1, typename v2>
01958     using gcd_t = v1;
01959
01960     template<typename... vs>
01961     using vadd_t = typename vadd<vs...>::type;
01962
01963     template<typename... vs>
01964     using vmul_t = typename vmul<vs...>::type;
01965
01966     template<typename v1, typename v2>
01967     using sub_t = typename sub<v1, v2>::type;
01968
01969     template<typename v1, typename v2>
01970     using mul_t = typename mul<v1, v2>::type;
01971
01972     template<typename v1, typename v2>
01973     using div_t = typename div<v1, v2>::type;
01974
01975     template<typename v1, typename v2>
01976     using eq_t = typename eq<v1, v2>::type;
01977
01978     template<typename v1, typename v2>
01979     static constexpr bool eq_v = eq<v1, v2>::type::value;
01980
01981     template<typename v1, typename v2>
01982     using gt_t = typename gt<v1, v2>::type;
01983
01984     template<typename v1, typename v2>
01985     static constexpr bool gt_v = gt<v1, v2>::type::value;
01986
01987     template<typename v1>
01988     using pos_t = typename pos<v1>::type;

```

```

02028
02031     template<typename v>
02032         static constexpr bool pos_v = pos_t<v>::value;
02033 };
02034
02035     template<typename Ring, typename E = void>
02036     requires IsEuclideanDomain<Ring>
02037     struct FractionFieldImpl {};
02038
02039     // fraction field of a field is the field itself
02040     template<typename Field>
02041     requires IsEuclideanDomain<Field>
02042     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02043         using type = Field;
02044         template<typename v>
02045             using inject_t = v;
02046     };
02047
02048     // fraction field of a ring is the actual fraction field
02049     template<typename Ring>
02050     requires IsEuclideanDomain<Ring>
02051     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02052         using type = _FractionField<Ring>;
02053     };
02054 } // namespace internal
02055
02056     template<typename Ring>
02057     requires IsEuclideanDomain<Ring>
02058     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02059 } // namespace aerobus
02060
02061 // short names for common types
02062 namespace aerobus {
02063     using q32 = FractionField<i32>;
02064     using fpq32 = FractionField<polynomial<q32>;
02065     using q64 = FractionField<i64>;
02066     using pi64 = polynomial<i64>;
02067     using pq64 = polynomial<q64>;
02068     using fpq64 = FractionField<polynomial<q64>;
02069     template<typename Ring, typename v1, typename v2>
02070     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02071
02072     template<typename Ring, typename v1, typename v2>
02073     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02074     template<typename Ring, typename v1, typename v2>
02075     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02076 } // namespace aerobus
02077
02078 // taylor series and common integers (factorial, bernouilli...) appearing in taylor coefficients
02079 namespace aerobus {
02080     namespace internal {
02081         template<typename T, size_t x, typename E = void>
02082         struct factorial {};
02083
02084         template<typename T, size_t x>
02085         struct factorial<T, x, std::enable_if_t<(x > 0)> {
02086             private:
02087                 template<typename, size_t, typename>
02088                 friend struct factorial;
02089             public:
02090                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
02091 x - 1>::type>;
02092                 static constexpr typename T::inner_type value = type::template get<typename
02093 T::inner_type>();
02094         };
02095
02096         template<typename T>
02097         struct factorial<T, 0> {
02098             public:
02099                 using type = typename T::one;
02100                 static constexpr typename T::inner_type value = type::template get<typename
02101 T::inner_type>();
02102         };
02103     } // namespace internal
02104
02105     template<typename T, size_t i>
02106     using factorial_t = typename internal::factorial<T, i>::type;
02107
02108     template<typename T, size_t i>
02109     inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02110
02111     namespace internal {
02112         template<typename T, size_t k, size_t n, typename E = void>
02113         struct combination_helper {};
02114
02115         template<typename T, size_t k, size_t n>
02116         struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {

```

```

02144         using type = typename FractionField<T>::template mul_t<
02145             typename combination_helper<T, k - 1, n - 1>::type,
02146             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02147     };
02148
02149     template<typename T, size_t k, size_t n>
02150     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)>> {
02151         using type = typename combination_helper<T, n - k, n>::type;
02152     };
02153
02154     template<typename T, size_t n>
02155     struct combination_helper<T, 0, n> {
02156         using type = typename FractionField<T>::one;
02157     };
02158
02159     template<typename T, size_t k, size_t n>
02160     struct combination {
02161         using type = typename internal::combination_helper<T, k, n>::type::x;
02162         static constexpr typename T::inner_type value =
02163             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02164     };
02165     } // namespace internal
02166
02167     template<typename T, size_t k, size_t n>
02168     using combination_t = typename internal::combination<T, k, n>::type;
02169
02170     template<typename T, size_t k, size_t n>
02171     inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02172
02173     namespace internal {
02174         template<typename T, size_t m>
02175         struct bernouilli;
02176
02177         template<typename T, typename accum, size_t k, size_t m>
02178         struct bernouilli_helper {
02179             using type = typename bernouilli_helper<
02180                 T,
02181                 addfractions_t<T,
02182                     accum,
02183                     mulfractions_t<T,
02184                         makefraction_t<T,
02185                             combination_t<T, k, m + 1>,
02186                             typename T::one>,
02187                             typename bernouilli<T, k>::type
02188                         >,
02189                     k + 1,
02190                     m>::type;
02191         };
02192
02193         template<typename T, typename accum, size_t m>
02194         struct bernouilli_helper<T, accum, m, m> {
02195             using type = accum;
02196         };
02197
02198         template<typename T, size_t m>
02199         struct bernouilli {
02200             using type = typename FractionField<T>::template mul_t<
02201                 typename internal::bernouilli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02202                 makefraction_t<T,
02203                     typename T::template val<static_cast<typename T::inner_type>(-1)>,
02204                     typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02205                 >
02206             >;
02207
02208             template<typename floatType>
02209             static constexpr floatType value = type::template get<floatType>();
02210         };
02211
02212         template<typename T>
02213         struct bernouilli<T, 0> {
02214             using type = typename FractionField<T>::one;
02215
02216             template<typename floatType>
02217             static constexpr floatType value = type::template get<floatType>();
02218         };
02219     } // namespace internal
02220
02221     template<typename T, size_t n>
02222     using bernouilli_t = typename internal::bernouilli<T, n>::type;
02223
02224     template<typename FloatType, typename T, size_t n>
02225     inline constexpr FloatType bernouilli_v = internal::bernouilli<T, n>::template value<FloatType>;
02226

```

```

02243     namespace internal {
02244         template<typename T, int k, typename E = void>
02245         struct alternate {};
02246
02247         template<typename T, int k>
02248         struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02249             using type = typename T::one;
02250             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02251         };
02252
02253         template<typename T, int k>
02254         struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02255             using type = typename T::template sub_t<typename T::zero, typename T::one>;
02256             static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02257         };
02258     } // namespace internal
02259
02260     template<typename T, int k>
02261     using alternate_t = typename internal::alternate<T, k>::type;
02262
02263     namespace internal {
02264         template<typename T, int n, int k, typename E = void>
02265         struct stirling_helper {};
02266
02267         template<typename T>
02268         struct stirling_helper<T, 0, 0> {
02269             using type = typename T::one;
02270         };
02271
02272         template<typename T, int n>
02273         struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02274             using type = typename T::zero;
02275         };
02276
02277         template<typename T, int n>
02278         struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02279             using type = typename T::zero;
02280         };
02281
02282         template<typename T, int n, int k>
02283         struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02284             using type = typename T::template sub_t<
02285                 typename stirling_helper<T, n-1, k-1>::type,
02286                 typename T::template mul_t<
02287                     typename T::template inject_constant_t<n-1>,
02288                     typename stirling_helper<T, n-1, k>::type
02289                 >;
02290         };
02291     } // namespace internal
02292
02293     template<typename T, int n, int k>
02294     using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02295
02296     template<typename T, int n, int k>
02297     using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02298
02299     template<typename T, int n, int k>
02300     static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02301
02302     template<typename T, int n, int k>
02303     static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02304
02305     template<typename T, size_t k>
02306     inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02307
02308     namespace internal {
02309         template<typename T, auto p, auto n, typename E = void>
02310         struct pow {};
02311
02312         template<typename T, auto p, auto n>
02313         struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02314             using type = typename T::template mul_t<
02315                 typename pow<T, p, n/2>::type,
02316                 typename pow<T, p, n/2>::type
02317             >;
02318         };
02319
02320         template<typename T, auto p, auto n>
02321         struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02322             using type = typename T::template mul_t<
02323                 typename T::template inject_constant_t<p>,
02324                 typename T::template mul_t<
02325                     typename pow<T, p, n/2>::type,
02326                     typename pow<T, p, n/2>::type
02327                 >;
02328         };
02329     }

```

```

02348         >
02349         >;
02350     };
02351
02352     template<typename T, auto p>
02353     struct pow<T, p, 0> { using type = typename T::one; };
02354 } // namespace internal
02355
02360 template<typename T, auto p, auto n>
02361 using pow_t = typename internal::pow<T, p, n>::type;
02362
02367 template<typename T, auto p, auto n>
02368 static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02369
02370 namespace internal {
02371     template<typename, template<typename, size_t> typename, class>
02372     struct make_taylor_impl;
02373
02374     template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02375     struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02376         using type = typename polynomial<FractionField<T>>::template val<typename coeff_at<T,
Is>::type...>;
02377     };
02378 }
02379
02384 template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02385 using taylor = typename internal::make_taylor_impl<
02386     T,
02387     coeff_at,
02388     internal::make_index_sequence_reverse<deg + 1>::type;
02389
02390 namespace internal {
02391     template<typename T, size_t i>
02392     struct exp_coeff {
02393         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02394     };
02395
02396     template<typename T, size_t i, typename E = void>
02397     struct sin_coeff_helper {};
02398
02399     template<typename T, size_t i>
02400     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02401         using type = typename FractionField<T>::zero;
02402     };
02403
02404     template<typename T, size_t i>
02405     struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02406         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02407     };
02408
02409     template<typename T, size_t i>
02410     struct sin_coeff {
02411         using type = typename sin_coeff_helper<T, i>::type;
02412     };
02413
02414     template<typename T, size_t i, typename E = void>
02415     struct sh_coeff_helper {};
02416
02417     template<typename T, size_t i>
02418     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02419         using type = typename FractionField<T>::zero;
02420     };
02421
02422     template<typename T, size_t i>
02423     struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02424         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02425     };
02426
02427     template<typename T, size_t i>
02428     struct sh_coeff {
02429         using type = typename sh_coeff_helper<T, i>::type;
02430     };
02431
02432     template<typename T, size_t i, typename E = void>
02433     struct cos_coeff_helper {};
02434
02435     template<typename T, size_t i>
02436     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02437         using type = typename FractionField<T>::zero;
02438     };
02439
02440     template<typename T, size_t i>
02441     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02442         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02443     };
02444
02445     template<typename T, size_t i>

```

```

02446     struct cos_coeff {
02447         using type = typename cos_coeff_helper<T, i>::type;
02448     };
02449
02450     template<typename T, size_t i, typename E = void>
02451     struct cosh_coeff_helper {};
02452
02453     template<typename T, size_t i>
02454     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02455         using type = typename FractionField<T>::zero;
02456     };
02457
02458     template<typename T, size_t i>
02459     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02460         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02461     };
02462
02463     template<typename T, size_t i>
02464     struct cosh_coeff {
02465         using type = typename cosh_coeff_helper<T, i>::type;
02466     };
02467
02468     template<typename T, size_t i>
02469     struct geom_coeff { using type = typename FractionField<T>::one; };
02470
02471
02472     template<typename T, size_t i, typename E = void>
02473     struct atan_coeff_helper;
02474
02475     template<typename T, size_t i>
02476     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02477         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02478     };
02479
02480     template<typename T, size_t i>
02481     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02482         using type = typename FractionField<T>::zero;
02483     };
02484
02485     template<typename T, size_t i>
02486     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02487
02488     template<typename T, size_t i, typename E = void>
02489     struct asin_coeff_helper;
02490
02491     template<typename T, size_t i>
02492     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02493         using type = makefraction_t<T,
02494             factorial_t<T, i - 1>,
02495             typename T::template mul_t<
02496                 typename T::template val<i>,
02497                 T::template mul_t<
02498                     pow_t<T, 4, i / 2>,
02499                     pow<T, factorial<T, i / 2>::value, 2
02500                 >
02501             >
02502         >>;
02503     };
02504
02505     template<typename T, size_t i>
02506     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02507         using type = typename FractionField<T>::zero;
02508     };
02509
02510     template<typename T, size_t i>
02511     struct asin_coeff {
02512         using type = typename asin_coeff_helper<T, i>::type;
02513     };
02514
02515     template<typename T, size_t i>
02516     struct lnpl_coeff {
02517         using type = makefraction_t<T,
02518             alternate_t<T, i + 1>,
02519             typename T::template val<i>;
02520     };
02521
02522     template<typename T>
02523     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02524
02525     template<typename T, size_t i, typename E = void>
02526     struct asinh_coeff_helper;
02527
02528     template<typename T, size_t i>
02529     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02530         using type = makefraction_t<T,
02531             typename T::template mul_t<
02532                 alternate_t<T, i / 2>,

```



```

02533         factorial_t<T, i - 1>
02534     >,
02535     typename T::template mul_t<
02536         T::template mul_t<
02537             typename T::template val<i>,
02538             pow_t<T, (factorial<T, i / 2>::value), 2>
02539         >,
02540         pow_t<T, 4, i / 2>
02541     >
02542 >;
02543 };
02544
02545 template<typename T, size_t i>
02546 struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02547     using type = typename FractionField<T>::zero;
02548 };
02549
02550 template<typename T, size_t i>
02551 struct asinh_coeff {
02552     using type = typename asinh_coeff_helper<T, i>::type;
02553 };
02554
02555 template<typename T, size_t i, typename E = void>
02556 struct atanh_coeff_helper;
02557
02558 template<typename T, size_t i>
02559 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02560     // 1/i
02561     using type = typename FractionField<T>::template val<
02562         typename T::one,
02563         typename T::template val<static_cast<typename T::inner_type>(i)>;
02564     };
02565
02566 template<typename T, size_t i>
02567 struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02568     using type = typename FractionField<T>::zero;
02569 };
02570
02571 template<typename T, size_t i>
02572 struct atanh_coeff {
02573     using type = typename asinh_coeff_helper<T, i>::type;
02574 };
02575
02576 template<typename T, size_t i, typename E = void>
02577 struct tan_coeff_helper;
02578
02579 template<typename T, size_t i>
02580 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02581     using type = typename FractionField<T>::zero;
02582 };
02583
02584 template<typename T, size_t i>
02585 struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02586 private:
02587     // 4^((i+1)/2)
02588     using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02589     // 4^((i+1)/2) - 1
02590     using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02591     // (-1)^((i-1)/2)
02592     using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02593     using dividend = typename FractionField<T>::template mul_t<
02594         altp,
02595         FractionField<T>::template mul_t<
02596             _4p,
02597             FractionField<T>::template mul_t<
02598                 _4pml,
02599                 bernouilli_t<T, (i + 1)>
02600         >
02601     >
02602 >;
02603 public:
02604     using type = typename FractionField<T>::template div_t<dividend,
02605         typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02606 };
02607
02608 template<typename T, size_t i>
02609 struct tan_coeff {
02610     using type = typename tan_coeff_helper<T, i>::type;
02611 };
02612
02613 template<typename T, size_t i, typename E = void>
02614 struct tanh_coeff_helper;
02615
02616 template<typename T, size_t i>
02617 struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02618     using type = typename FractionField<T>::zero;

```

```

02619     };
02620
02621     template<typename T, size_t i>
02622     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02623     private:
02624         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02625         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02626         using dividend =
02627             typename FractionField<T>::template mul_t<
02628                 _4p,
02629                 typename FractionField<T>::template mul_t<
02630                     _4pml,
02631                     bernouilli_t<T, (i + 1)>
02632                 >
02633             >::type;
02634     public:
02635         using type = typename FractionField<T>::template div_t<dividend,
02636             FractionField<T>::template inject_t<factorial_t<T, i + 1>>;
02637     };
02638
02639     template<typename T, size_t i>
02640     struct tanh_coeff {
02641         using type = typename tanh_coeff_helper<T, i>::type;
02642     };
02643 } // namespace internal
02644
02645 template<typename T, size_t deg>
02646 using exp = taylor<T, internal::exp_coeff, deg>;
02647
02648 template<typename T, size_t deg>
02649 using expml = typename polynomial<FractionField<T>>::template sub_t<
02650     exp<T, deg>,
02651     typename polynomial<FractionField<T>>::one>;
02652
02653 template<typename T, size_t deg>
02654 using lnpl = taylor<T, internal::lnpl_coeff, deg>;
02655
02656 template<typename T, size_t deg>
02657 using atan = taylor<T, internal::atan_coeff, deg>;
02658
02659 template<typename T, size_t deg>
02660 using sin = taylor<T, internal::sin_coeff, deg>;
02661
02662 template<typename T, size_t deg>
02663 using sinh = taylor<T, internal::sh_coeff, deg>;
02664
02665 template<typename T, size_t deg>
02666 using cosh = taylor<T, internal::cosh_coeff, deg>;
02667
02668 template<typename T, size_t deg>
02669 using cos = taylor<T, internal::cos_coeff, deg>;
02670
02671 template<typename T, size_t deg>
02672 using geometric_sum = taylor<T, internal::geom_coeff, deg>;
02673
02674 template<typename T, size_t deg>
02675 using asin = taylor<T, internal::asin_coeff, deg>;
02676
02677 template<typename T, size_t deg>
02678 using asinh = taylor<T, internal::asinh_coeff, deg>;
02679
02680 template<typename T, size_t deg>
02681 using atanh = taylor<T, internal::atanh_coeff, deg>;
02682
02683 template<typename T, size_t deg>
02684 using tan = taylor<T, internal::tan_coeff, deg>;
02685
02686 template<typename T, size_t deg>
02687 using tanh = taylor<T, internal::tanh_coeff, deg>;
02688 } // namespace aerobus
02689
02690 // continued fractions
02691 namespace aerobus {
02692     template<int64_t... values>
02693     struct ContinuedFraction {};
02694
02695     template<int64_t a0>
02696     struct ContinuedFraction<a0> {
02697         using type = typename q64::template inject_constant_t<a0>;
02698         static constexpr double val = type::template get<double>();
02699     };
02700
02701     template<int64_t a0, int64_t... rest>
02702     struct ContinuedFraction<a0, rest...> {
02703         using type = q64::template add_t<
02704             typename q64::template inject_constant_t<a0>,

```

```

02754         typename q64::template div_t<
02755             typename q64::one,
02756             typename ContinuedFraction<rest...>::type
02757         >>;
02758         static constexpr double val = type::template get<double>();
02759     };
02760
02761     using PI_fraction =
02762     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02763     using E_fraction =
02764     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02765     using SQRT2_fraction =
02766     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02767     using SQRT3_fraction =
02768     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02769     // NOLINT
02770 } // namespace aerobus
02771
02772 // known polynomials
02773 namespace aerobus {
02774     // CChebyshev
02775     namespace internal {
02776         template<int kind, size_t deg>
02777         struct chebyshev_helper {
02778             using type = typename pi64::template sub_t<
02779                 typename pi64::template mul_t<
02780                     typename pi64::template mul_t<
02781                         pi64::inject_constant_t<2>,
02782                         typename pi64::X>,
02783                     typename chebyshev_helper<kind, deg - 1>::type
02784                 >,
02785                 typename chebyshev_helper<kind, deg - 2>::type
02786             >;
02787         };
02788
02789         template<>
02790         struct chebyshev_helper<1, 0> {
02791             using type = typename pi64::one;
02792         };
02793
02794         template<>
02795         struct chebyshev_helper<1, 1> {
02796             using type = typename pi64::X;
02797         };
02798
02799         template<>
02800         struct chebyshev_helper<2, 0> {
02801             using type = typename pi64::one;
02802         };
02803
02804         template<>
02805         struct chebyshev_helper<2, 1> {
02806             using type = typename pi64::template mul_t<
02807                 typename pi64::inject_constant_t<2>,
02808                 typename pi64::X>;
02809         };
02810     } // namespace internal
02811
02812     // Laguerre
02813     namespace internal {
02814         template<size_t deg>
02815         struct laguerre_helper {
02816             private:
02817                 // Lk = (1 / k) * ((2 * k - 1 - x) * lkm1 - (k - 2)lkm2)
02818                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02819                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02820                 // -x + 2k-1
02821                 using p = typename pq64::template val<
02822                     typename q64::template inject_constant_t<-1>,
02823                     typename q64::template inject_constant_t<2 * deg - 1>,
02824                     // 1/n
02825                     using factor = typename pq64::template inject_ring_t<
02826                         q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>;
02827                 >
02828             public:
02829                 using type = typename pq64::template mul_t <
02830                     factor,
02831                     typename pq64::template sub_t<
02832                         typename pq64::template mul_t<
02833                             p,
02834                             lnm1
02835                         >,
02836                     typename pq64::template mul_t<
02837                         typename pq64::template inject_constant_t<deg-1>,
02838                         lnm2
02839                     >
02840                 >
02841         };
02842     }
02843 }

```

```

02844         >;
02845
02846     };
02847
02848     template<>
02849     struct laguerre_helper<0> {
02850         using type = typename pq64::one;
02851     };
02852
02853     template<>
02854     struct laguerre_helper<1> {
02855         using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02856     };
02857 } // namespace internal
02858
02859 // Bernstein
02860 namespace internal {
02861     template<size_t i, size_t m, typename E = void>
02862     struct bernstein_helper {};
02863
02864     template<>
02865     struct bernstein_helper<0, 0> {
02866         using type = typename pi64::one;
02867     };
02868
02869     template<size_t i, size_t m>
02870     struct bernstein_helper<i, m, std::enable_if_t<
02871         (m > 0) && (i == 0)>> {
02872         using type = typename pi64::mul_t<
02873             typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02874             typename bernstein_helper<i, m-1>::type>;
02875     };
02876
02877     template<size_t i, size_t m>
02878     struct bernstein_helper<i, m, std::enable_if_t<
02879         (m > 0) && (i == m)>> {
02880         using type = typename pi64::template mul_t<
02881             typename pi64::X,
02882             typename bernstein_helper<i-1, m-1>::type>;
02883     };
02884
02885     template<size_t i, size_t m>
02886     struct bernstein_helper<i, m, std::enable_if_t<
02887         (m > 0) && (i > 0) && (i < m)>> {
02888         using type = typename pi64::add_t<
02889             typename pi64::mul_t<
02890                 typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02891                 typename bernstein_helper<i, m-1>::type>,
02892             typename pi64::mul_t<
02893                 typename pi64::X,
02894                 typename bernstein_helper<i-1, m-1>::type>;
02895     };
02896 } // namespace internal
02897
02898 namespace known_polynomials {
02899     enum hermite_kind {
02900         probabilist,
02901         physicist
02902     };
02903 }
02904
02905 namespace internal {
02906     template<size_t deg, known_polynomials::hermite_kind kind>
02907     struct hermite_helper {};
02908
02909     template<size_t deg>
02910     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02911     private:
02912         using hnm1 = typename hermite_helper<deg - 1,
02913             known_polynomials::hermite_kind::probabilist>::type;
02914         using hnm2 = typename hermite_helper<deg - 2,
02915             known_polynomials::hermite_kind::probabilist>::type;
02916     public:
02917         using type = typename pi64::template sub_t<
02918             typename pi64::template mul_t<typename pi64::X, hnm1>,
02919             typename pi64::template mul_t<
02920                 typename pi64::template inject_constant_t<deg - 1>,
02921                 hnm2
02922             >
02923         >;
02924     };
02925
02926     template<size_t deg>
02927     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02928     private:
02929         using hnm1 = typename hermite_helper<deg - 1,

```

```

known_polynomials::hermite_kind::physicist>::type;
02930     using hnm2 = typename hermite_helper<deg - 2,
known_polynomials::hermite_kind::physicist>::type;
02931
02932     public:
02933     using type = typename pi64::template sub_t<
02934         // 2X Hn-1
02935         typename pi64::template mul_t<
02936             typename pi64::val<typename i64::template inject_constant_t<2>,
02937             typename i64::zero>, hnm1>,
02938
02939             typename pi64::template mul_t<
02940                 typename pi64::template inject_constant_t<2*(deg - 1)>,
02941                 hnm2
02942             >
02943     >;
02944 };
02945
02946 template<>
02947 struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02948     using type = typename pi64::one;
02949 };
02950
02951 template<>
02952 struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02953     using type = typename pi64::X;
02954 };
02955
02956 template<>
02957 struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
02958     using type = typename pi64::one;
02959 };
02960
02961 template<>
02962 struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
02963     // 2X
02964     using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
02965     typename i64::zero>;
02966 };
02967 } // namespace internal
02968
02969 namespace known_polynomials {
02970     template <size_t deg>
02971     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
02972
02973     template <size_t deg>
02974     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
02975
02976     template <size_t deg>
02977     using laguerre = typename internal::laguerre_helper<deg>::type;
02978
02979     template <size_t deg>
02980     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
02981
02982     template <size_t deg>
02983     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
02984
02985     template<size_t i, size_t m>
02986     using bernstein = typename internal::bernstein_helper<i, m>::type;
02987 } // namespace known_polynomials
03000 } // namespace aerobus
03001
03002
03003 #ifdef AEROBUS_CONWAY_IMPORTS
03004 template<int p, int n>
03005 struct ConwayPolynomial;
03006
03007 #define ZPZV ZPZ::template val
03008 #define POLYV aerobus::polynomial<ZPZ>::template val
03009 template<> struct ConwayPolynomial<2, 1> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>; }; // NOLINT
03010 template<> struct ConwayPolynomial<2, 2> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<1>, ZPZV<1>; }; // NOLINT
03011 template<> struct ConwayPolynomial<2, 3> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03012 template<> struct ConwayPolynomial<2, 4> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03013 template<> struct ConwayPolynomial<2, 5> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03014 template<> struct ConwayPolynomial<2, 6> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03015 template<> struct ConwayPolynomial<2, 7> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>; }; // NOLINT
03016 template<> struct ConwayPolynomial<2, 8> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<1>, ZPZV<0>, ZPZV<1>; }; // NOLINT
03017 template<> struct ConwayPolynomial<2, 9> { using ZPZ = aerobus::zpz<2>; using type = POLYV<ZPZV<1>,
ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<1>; }; // NOLINT

```



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```
04952 #endif // __INC_AEROBUS__ // NOLINT
```



## Chapter 7

# Examples

### 7.1 QuotientRing

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject\_constant\_t<1>

inject a 'constant' in quotient ring <i32, i32::val<2>>::inject\_constant\_t<1>

Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

### 7.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

...Ts	types to store and manipulate at compile time
-------	---

### 7.3 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

## 7.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

## 7.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

## 7.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

## 7.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

$v1$	a value in i32
$v2$	a value in i32

## 7.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  `<i32::val<7>, i32::val<2>>`

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  `<i32::val<7>, i32::val<2>>`

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 7.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` `<i32::val<2>, i32::val<2>>`

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` `<i32::val<2>, i32::val<2>>`

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 7.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

<i>v1</i>	
<i>v2</i>	<code>&lt;i32::val&lt;1&gt;, i32::val&lt;1&gt;&gt;</code>

## 7.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  `<i32::val<6>, i32::val<15>>`

greatest common divisor yields  $GCD(v1, v2)$  `<i32::val<6>, i32::val<15>>`

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 7.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

$v$	a value in i32
-----	----------------

## 7.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

$v$	a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code>
-----	---

## 7.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

$x$	<code>inject_constant_t&lt;2&gt;</code>
-----	---

## 7.15 i64::add\_t

addition operator

addition operator

Template Parameters

$v1$	: an element of <code>aerobus::i64::val</code>
$v2$	: an element of <code>aerobus::i64::val</code> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>



## 7.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

v1	: an element of <a href="#">aerobus::i64::val</a>
v2	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 7.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

v1	: an element of <a href="#">aerobus::i64::val</a>
v2	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 7.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

v1	: an element of <a href="#">aerobus::i64::val</a>
v2	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 7.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

v1	: an element of <a href="#">aerobus::i64::val</a>
v2	: an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>>

## 7.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code>

## 7.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 7.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 7.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code>

## 7.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code>

## 7.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of `i64::val`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;6&gt;, i64::val&lt;15&gt;&gt;</code>

## 7.26 i64::pos\_t

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

is v positive yields  $v > 0$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code>
-----------------	--

## 7.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

### Template Parameters

<code>v</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;&gt;</code>
----------------	--

## 7.28 polynomial

makes the constant (native type) polynomial a\_0

makes the constant (native type) polynomial a\_0

Template Parameters

x	<i32>::template inject_constant_t<2>
---	--------------------------------------

## 7.29 q32::add\_t

addition operator

addition operator

Template Parameters

v1	a value
v2	a value <q32::val<i32::val<1>, i32::val<2>>, q32::val<i32::val<1>, i32::val<3>>>

## 7.30 FractionField

Fraction field of an euclidean domain, such as Q for Z.

Fraction field of an euclidean domain, such as Q for Z

Template Parameters

Ring	<i64> is q64 (rationals with 64 bits numerator and denominator)
------	---

## 7.31 PI\_fraction::val

representation of PI as a continued fraction -> 3.14...

## 7.32 E\_fraction::val

approximation of e -> 2.718...

approximation of e -> 2.718...

# Index

add\_t  
  aerobus::polynomial< Ring >, 17  
  aerobus::Quotient< Ring, X >, 23  
  aerobus::zpz< p >, 38  
aerobus::ContinuedFraction< a0 >, 10  
aerobus::ContinuedFraction< a0, rest... >, 11  
aerobus::ContinuedFraction< values >, 10  
aerobus::i32, 11  
  mod\_t, 13  
aerobus::i32::val< x >, 29  
  eval, 30  
  get, 30  
aerobus::i64, 13  
  gt\_v, 15  
  inject\_ring\_t, 14  
aerobus::i64::val< x >, 31  
  eval, 32  
  get, 32  
aerobus::is\_prime< n >, 15  
aerobus::IsEuclideanDomain, 7  
aerobus::IsField, 7  
aerobus::IsRing, 8  
aerobus::polynomial< Ring >, 16  
  add\_t, 17  
  derive\_t, 17  
  div\_t, 18  
  eq\_t, 18  
  gcd\_t, 18  
  gt\_t, 19  
  lt\_t, 19  
  mod\_t, 19  
  monomial\_t, 19  
  mul\_t, 20  
  pos\_t, 20  
  pos\_v, 21  
  simplify\_t, 20  
  sub\_t, 20  
aerobus::polynomial< Ring >::val< coeffN >, 36  
aerobus::polynomial< Ring >::val< coeffN >::coeff\_at<  
  index, E >, 9  
aerobus::polynomial< Ring >::val< coeffN >::coeff\_at<  
  index, std::enable\_if\_t<(index< 0 || index >  
  0)>, 9  
aerobus::polynomial< Ring >::val< coeffN >::coeff\_at<  
  index, std::enable\_if\_t<(index==0)>, 9  
aerobus::polynomial< Ring >::val< coeffN, coeffs >,  
  32  
  coeff\_at\_t, 33  
  eval, 34  
  to\_string, 34  
aerobus::Quotient< Ring, X >, 22  
  add\_t, 23  
  div\_t, 23  
  eq\_t, 23  
  eq\_v, 25  
  mod\_t, 24  
  mul\_t, 24  
  pos\_t, 24  
  pos\_v, 25  
aerobus::Quotient< Ring, X >::val< V >, 35  
aerobus::type\_list< Ts >, 26  
  at, 27  
  concat, 27  
  insert, 27  
  push\_back, 28  
  push\_front, 28  
  remove, 28  
aerobus::type\_list< Ts >::pop\_front, 21  
aerobus::type\_list< Ts >::split< index >, 25  
aerobus::type\_list<>, 29  
aerobus::zpz< p >, 37  
  add\_t, 38  
  div\_t, 38  
  eq\_t, 39  
  eq\_v, 41  
  gcd\_t, 39  
  gt\_t, 39  
  gt\_v, 41  
  lt\_t, 39  
  lt\_v, 41  
  mod\_t, 40  
  mul\_t, 40  
  pos\_t, 40  
  pos\_v, 42  
  sub\_t, 41  
aerobus::zpz< p >::val< x >, 35  
at  
  aerobus::type\_list< Ts >, 27  
coeff\_at\_t  
  aerobus::polynomial< Ring >::val< coeffN, coeffs  
  >, 33  
concat  
  aerobus::type\_list< Ts >, 27  
derive\_t  
  aerobus::polynomial< Ring >, 17  
div\_t  
  aerobus::polynomial< Ring >, 18

- aerobus::Quotient< Ring, X >, 23
- aerobus::zpz< p >, 38
- eq\_t
  - aerobus::polynomial< Ring >, 18
  - aerobus::Quotient< Ring, X >, 23
  - aerobus::zpz< p >, 39
- eq\_v
  - aerobus::Quotient< Ring, X >, 25
  - aerobus::zpz< p >, 41
- eval
  - aerobus::i32::val< x >, 30
  - aerobus::i64::val< x >, 32
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 34
- gcd\_t
  - aerobus::polynomial< Ring >, 18
  - aerobus::zpz< p >, 39
- get
  - aerobus::i32::val< x >, 30
  - aerobus::i64::val< x >, 32
- gt\_t
  - aerobus::polynomial< Ring >, 19
  - aerobus::zpz< p >, 39
- gt\_v
  - aerobus::i64, 15
  - aerobus::zpz< p >, 41
- inject\_ring\_t
  - aerobus::i64, 14
- insert
  - aerobus::type\_list< Ts >, 27
- lt\_t
  - aerobus::polynomial< Ring >, 19
  - aerobus::zpz< p >, 39
- lt\_v
  - aerobus::zpz< p >, 41
- mod\_t
  - aerobus::i32, 13
  - aerobus::polynomial< Ring >, 19
  - aerobus::Quotient< Ring, X >, 24
  - aerobus::zpz< p >, 40
- monomial\_t
  - aerobus::polynomial< Ring >, 19
- mul\_t
  - aerobus::polynomial< Ring >, 20
  - aerobus::Quotient< Ring, X >, 24
  - aerobus::zpz< p >, 40
- pos\_t
  - aerobus::polynomial< Ring >, 20
  - aerobus::Quotient< Ring, X >, 24
  - aerobus::zpz< p >, 40
- pos\_v
  - aerobus::polynomial< Ring >, 21
  - aerobus::Quotient< Ring, X >, 25
  - aerobus::zpz< p >, 42
- push\_back
  - aerobus::type\_list< Ts >, 28
- push\_front
  - aerobus::type\_list< Ts >, 28
- remove
  - aerobus::type\_list< Ts >, 28
- simplify\_t
  - aerobus::polynomial< Ring >, 20
- src/aerobus.h, 43
- sub\_t
  - aerobus::polynomial< Ring >, 20
  - aerobus::zpz< p >, 41
- to\_string
  - aerobus::polynomial< Ring >::val< coeffN, coeffs >, 34