

Aerobus

v1.2

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# Chapter 1

## Introduction

`Aerobus` is a C++-20 pure header library for general algebra on polynomials, discrete rings and associated structures.

Everything in `Aerobus` is expressed as types.

We say that again as it is the most fundamental characteristic of `Aerobus` :

### ***Everything is expressed as types***

The library serves two main purposes :

- Express algebra structures and associated operations in type arithmetic, compile-time;
- Provide portable and fast evaluation functions for polynomials. It is designed to be 'easily' extensible.

Given these functions are "generated" at compile time and do not rely on inline assembly, they are actually platform independent, yielding exact same results if processors have same capabilities (such as Fused-Multiply-Add instructions).

## 1.1 Structures

### 1.1.1 Predefined discrete euclidean domains

`Aerobus` predefines several simple euclidean domains, such as :

- `aerobus::i32` : integers (32 bits)
- `aerobus::i64` : integers (64 bits)
- `aerobus::zp<p>` : integers modulo  $p$  (prime number) on 32 bits

All these types represent the Ring, meaning the algebraic structure. They have a nested type `val<i>` where  $i$  is a scalar native value (`int32_t` or `int64_t`) to represent actual values in the ring. They have the following "operations", required by the `IsEuclideanDomain` concept :

- `add_t` : a type (specialization of `val`), representing addition between two values
- `sub_t` : a type (specialization of `val`), representing subtraction between two values
- `mul_t` : a type (specialization of `val`), representing multiplication between two values
- `div_t` : a type (specialization of `val`), representing division between two values
- `mod_t` : a type (specialization of `val`), representing modulus between two values

and the following "elements" :

- `one` : the neutral element for multiplication, `val<1>`
- `zero` : the neutral element for addition, `val<0>`

### 1.1.2 Polynomials

`Aerobus` defines polynomials as a variadic template structure, with coefficient in an arbitrary discrete euclidean domain. As `i32` or `i64`, they are given same operations and elements, which make them a euclidean domain by themselves. Similarly, `aerobus::polynomial` represents the algebraic structure, actual values are in `aerobus::polynomial::val`.

In addition, values have an evaluation function :

```
template<typename valueRing> static constexpr valueRing eval(const valueRing& x) {...}
```

Which can be used at compile time (constexpr evaluation) or runtime.

### 1.1.3 Known polynomials

`Aerobus` predefines some well known families of polynomials, such as Hermite or Bernstein :

```
using B23 = aerobus::known_polynomials::bernstein<2, 3>; // 3X^2(1-X)
constexpr float x = B32::eval(2.0F); // -12
```

### 1.1.4 Conway polynomials

When the tag `AEROBUS_CONWAY_IMPORTS` is defined at compile time (`-DAEROBUS_CONWAY_IMPORTS`), `aerobus` provides definition for all Conway polynomials  $CP(p, n)$  for  $p$  up to 997 and low values for  $n$  (usually less than 10).

They can be used to construct finite fields of order  $p^n$  ( $\mathbb{F}_{p^n}$ ):

```
using F2 = zpz<2>;
using PF2 = polynomial<F2>;
using F4 = Quotient<PF2, ConwayPolynomial<2, 2>::type>;
```

### 1.1.5 Taylor series

Aerobus provides definition for Taylor expansion of known functions. They are all templates in two parameters, degree of expansion (`size_t`) and Integers (`typename`). Coefficients then live in `FractionField<Integers>`.

They can be used and evaluated:

```
using namespace aerobus;
using aero_atanh = atanh<i64, 6>;
constexpr float val = aero_atanh::eval(0.1F); // approximation of arctanh(0.1) using taylor expansion of
degree 6
```

Having the capacity of specifying the degree is very important, as users may use other formats than `float64` or `float32` which require higher or lower degree to achieve correct or acceptable precision.

It's possible to define Taylor expansion by implementing a `coeff_at` structure which must meet the following requirement :

- Being template in Integers (`typename`) and index (`size_t`)
- Exposing a type alias `type`, some specialization of `FractionField<Integers>::val`

For example, to define the serie  $1 + x + x^2 + x^3 + \dots$ , users may write:

```
template<typename Integers, size_t i>
struct my_coeff_at {
    using type = typename FractionField<Integers>::one;
};

template<typename Integers, typename degree>
using my_serie = taylor<Integers, my_coeff_at, degree>;
```

## 1.2 Operations

### 1.2.1 Fraction Field

### 1.2.2 Quotient





## Chapter 2

# Namespace Index

### 2.1 Namespace List

Here is a list of all namespaces with brief descriptions:

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<a href="#">aerobus::internal</a>	Internal implementations, subject to breaking changes without notice . . . . .	28
<a href="#">aerobus::known_polynomials</a>	Families of well known polynomials such as Hermite or Bernstein . . . . .	32



## Chapter 3

# Concept Index

### 3.1 Concepts

Here is a list of all concepts with brief descriptions:

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<a href="#">aerobus::IsField</a>	
Concept to express R is a field . . . . .	37
<a href="#">aerobus::IsRing</a>	
Concept to express R is a Ring . . . . .	38



## Chapter 4

# Class Index

### 4.1 Class List

Here are the classes, structs, unions and interfaces with brief descriptions:

<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, E &gt;</a>	39
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index&lt; 0  index &gt; 0)&gt; &gt;</a>	39
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;::coeff_at&lt; index, std::enable_if_t&lt;(index==0)&gt; &gt;</a>	40
<a href="#">aerobus::ContinuedFraction&lt; values &gt;</a>	40
<a href="#">aerobus::ContinuedFraction&lt; a0 &gt;</a>	
Specialization for only one coefficient, technically just 'a0'	40
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Specialization for multiple coefficients (strictly more than one)	41
<a href="#">aerobus::ConwayPolynomial</a>	42
<a href="#">aerobus::i32</a>	
32 bits signed integers, seen as a algebraic ring with related operations	42
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<a href="#">aerobus::is_prime&lt; n &gt;</a>	
Checks if n is prime	50
<a href="#">aerobus::polynomial&lt; Ring &gt;</a>	51
<a href="#">aerobus::type_list&lt; Ts &gt;::pop_front</a>	
Removes types from head of the list	57
<a href="#">aerobus::Quotient&lt; Ring, X &gt;</a>	
Quotient ring by the principal ideal generated by 'X' With <a href="#">i32</a> as Ring and <a href="#">i32::val&lt;2&gt;</a> as X, Quotient is $\mathbb{Z}/2\mathbb{Z}$	58
<a href="#">aerobus::type_list&lt; Ts &gt;::split&lt; index &gt;</a>	
Splits list at index	63
<a href="#">aerobus::type_list&lt; Ts &gt;</a>	
Empty pure template struct to handle type list	64
<a href="#">aerobus::type_list&lt;&gt;</a>	
Specialization for empty type list	66
<a href="#">aerobus::i32::val&lt; x &gt;</a>	
Values in <a href="#">i32</a> , again represented as types	68
<a href="#">aerobus::i64::val&lt; x &gt;</a>	
Values in <a href="#">i64</a>	70
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN, coeffs &gt;</a>	
Values (seen as types) in polynomial ring	72
<a href="#">aerobus::Quotient&lt; Ring, X &gt;::val&lt; V &gt;</a>	
Projection values in the quotient ring	75

<a href="#">aerobus::zpz&lt; p &gt;::val&lt; x &gt;</a> . . . . .	76
<a href="#">aerobus::polynomial&lt; Ring &gt;::val&lt; coeffN &gt;</a> Specialization for constants . . . . .	77
<a href="#">aerobus::zpz&lt; p &gt;</a> . . . . .	80

## Chapter 5

# File Index

### 5.1 File List

Here is a list of all files with brief descriptions:

src/ <a href="#">aerobus.h</a> . . . . .	87
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## Chapter 6

# Namespace Documentation

### 6.1 aerobus Namespace Reference

main namespace for all publicly exposed types or functions

#### Namespaces

- namespace [internal](#)  
*internal implementations, subject to breaking changes without notice*
- namespace [known\\_polynomials](#)  
*families of well known polynomials such as Hermite or Bernstein*

#### Classes

- struct [ContinuedFraction](#)
- struct [ContinuedFraction< a0 >](#)  
*Specialization for only one coefficient, technically just 'a0'.*
- struct [ContinuedFraction< a0, rest... >](#)  
*specialization for multiple coefficients (strictly more than one)*
- struct [ConwayPolynomial](#)
- struct [i32](#)  
*32 bits signed integers, seen as a algebraic ring with related operations*
- struct [i64](#)  
*64 bits signed integers, seen as a algebraic ring with related operations*
- struct [is\\_prime](#)  
*checks if n is prime*
- struct [polynomial](#)
- struct [Quotient](#)  
*[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .*
- struct [type\\_list](#)  
*Empty pure template struct to handle type list.*
- struct [type\\_list<>](#)  
*specialization for empty type list*
- struct [zpz](#)

## Concepts

- concept [IsRing](#)  
*Concept to express  $R$  is a Ring.*
- concept [IsEuclideanDomain](#)  
*Concept to express  $R$  is an euclidean domain.*
- concept [IsField](#)  
*Concept to express  $R$  is a field.*

## Typedefs

- `template<typename T , typename A , typename B >`  
using [gcd\\_t](#) = `typename internal::gcd< T >::template type< A, B >`  
*computes the greatest common divisor of A and B*
- `template<typename... vals>`  
using [vadd\\_t](#) = `typename internal::vadd< vals... >::type`  
*adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an add\_t binary operator*
- `template<typename... vals>`  
using [vmul\\_t](#) = `typename internal::vmul< vals... >::type`  
*multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an mul\_t binary operator*
- `template<typename val >`  
using [abs\\_t](#) = `std::conditional_t< val::enclosing_type::template pos_v< val >, val, typename val::enclosing_type::template sub_t< typename val::enclosing_type::zero, val > >`  
*computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept*
- `template<typename Ring >`  
using [FractionField](#) = `typename internal::FractionFieldImpl< Ring >::type`
- using [q32](#) = [FractionField](#)< [i32](#) >  
*32 bits rationals rationals with 32 bits numerator and denominator*
- using [fpq32](#) = [FractionField](#)< [polynomial](#)< [q32](#) > >  
*rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)*
- using [q64](#) = [FractionField](#)< [i64](#) >  
*64 bits rationals rationals with 64 bits numerator and denominator*
- using [pi64](#) = [polynomial](#)< [i64](#) >  
*polynomial with 64 bits integers coefficients*
- using [pq64](#) = [polynomial](#)< [q64](#) >  
*polynomial with 64 bits rationals coefficients*
- using [fpq64](#) = [FractionField](#)< [polynomial](#)< [q64](#) > >  
*polynomial with 64 bits rational coefficients*
- `template<typename Ring , typename v1 , typename v2 >`  
using [makefraction\\_t](#) = `typename FractionField< Ring >::template val< v1, v2 >`  
*helper type : the rational  $V_1/V_2$  in the field of fractions of Ring*
- `template<int64_t p, int64_t q>`  
using [make\\_q64\\_t](#) = `typename q64::template simplify_t< typename q64::val< i64::inject\_constant\_t< p >, i64::inject\_constant\_t< q > > >`  
*helper type : make a fraction from numerator and denominator*
- `template<int32_t p, int32_t q>`  
using [make\\_q32\\_t](#) = `typename q32::template simplify_t< typename q32::val< i32::inject\_constant\_t< p >, i32::inject\_constant\_t< q > > >`  
*helper type : make a fraction from numerator and denominator*

- `template<typename Ring , typename v1 , typename v2 >`  
`using addfractions_t = typename FractionField< Ring >::template add_t< v1, v2 >`  
*helper type : adds two fractions*
- `template<typename Ring , typename v1 , typename v2 >`  
`using mulfractions_t = typename FractionField< Ring >::template mul_t< v1, v2 >`  
*helper type : multiplies two fractions*
- `template<typename T , size_t i>`  
`using factorial_t = typename internal::factorial< T, i >::type`  
*computes factorial(i), as type*
- `template<typename T , size_t k, size_t n>`  
`using combination_t = typename internal::combination< T, k, n >::type`  
*computes binomial coefficient (k among n) as type*
- `template<typename T , size_t n>`  
`using bernoulli_t = typename internal::bernoulli< T, n >::type`  
*nth bernoulli number as type in T*
- `template<typename T , int k>`  
`using alternate_t = typename internal::alternate< T, k >::type`  
 *$(-1)^k$  as type in T*
- `template<typename T , int n, int k>`  
`using stirling_signed_t = typename internal::stirling_helper< T, n, k >::type`  
*Stirling number of first kind (signed) – as types.*
- `template<typename T , int n, int k>`  
`using stirling_unsigned_t = abs_t< typename internal::stirling_helper< T, n, k >::type >`  
*Stirling number of first kind (unsigned) – as types.*
- `template<typename T , auto p, auto n>`  
`using pow_t = typename internal::pow< T, p, n >::type`  
 *$p^n$  (as 'val' type in T)*
- `template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>`  
`using taylor = typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence_reverse<`  
`deg+1 > >::type`
- `template<typename Integers , size_t deg>`  
`using exp = taylor< Integers, internal::exp_coeff, deg >`  
 $e^x$
- `template<typename Integers , size_t deg>`  
`using expm1 = typename polynomial< FractionField< Integers > >::template sub_t< exp< Integers, deg`  
`>, typename polynomial< FractionField< Integers > >::one >`  
 $e^x - 1$
- `template<typename Integers , size_t deg>`  
`using ln1 = taylor< Integers, internal::ln1_coeff, deg >`  
 $\ln(1 + x)$
- `template<typename Integers , size_t deg>`  
`using atan = taylor< Integers, internal::atan_coeff, deg >`  
 $\arctan(x)$
- `template<typename Integers , size_t deg>`  
`using sin = taylor< Integers, internal::sin_coeff, deg >`  
 $\sin(x)$
- `template<typename Integers , size_t deg>`  
`using sinh = taylor< Integers, internal::sh_coeff, deg >`  
 $\sinh(x)$
- `template<typename Integers , size_t deg>`  
`using cosh = taylor< Integers, internal::cosh_coeff, deg >`  
 $\cosh(x)$  *hyperbolic cosine*
- `template<typename Integers , size_t deg>`  
`using cos = taylor< Integers, internal::cos_coeff, deg >`



## 6.1.2 Typedef Documentation

### 6.1.2.1 abs\_t

```
template<typename val >
using aerobus::abs_t = typedef std::conditional_t< val::enclosing_type::template pos_v<val>,
val, typename val::enclosing_type::template sub_t<typename val::enclosing_type::zero, val> >
```

computes absolute value of 'val' val must be a 'value' in a Ring satisfying 'IsEuclideanDomain' concept

#### Template Parameters

<i>val</i>	a value in a Ring, such as <code>i64::val&lt;-2&gt;</code>
------------	--

### 6.1.2.2 addfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::addfractions_t = typedef typename FractionField<Ring>::template add_t<v1, v2>
```

helper type : adds two fractions

#### Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FractionField<Ring>

### 6.1.2.3 alternate\_t

```
template<typename T , int k>
using aerobus::alternate_t = typedef typename internal::alternate<T, k>::type
```

$(-1)^k$  as type in T

#### Template Parameters

<i>T</i>	Ring type, <code>aerobus::i64</code> for example
----------	--

### 6.1.2.4 asin

```
template<typename Integers , size_t deg>
using aerobus::asin = typedef taylor<Integers, internal::asin_coeff, deg>
```

$\arcsin(x)$  arc sinus

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.5 asinh**

```
template<typename Integers , size_t deg>
using aerobus::asinh = typedef taylor<Integers, internal::asinh_coeff, deg>
```

$\operatorname{arcsinh}(x)$  arc hyperbolic sinus

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.6 atan**

```
template<typename Integers , size_t deg>
using aerobus::atan = typedef taylor<Integers, internal::atan_coeff, deg>
```

$\operatorname{arctan}(x)$

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.7 atanh**

```
template<typename Integers , size_t deg>
using aerobus::atanh = typedef taylor<Integers, internal::atanh_coeff, deg>
```

$\operatorname{arctanh}(x)$  arc hyperbolic tangent

**Template Parameters**

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

**6.1.2.8 bernoulli\_t**

```
template<typename T , size_t n>
using aerobus::bernoulli_t = typedef typename internal::bernoulli<T, n>::type
```

nth bernoulli number as type in T

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i64</a> )
<i>n</i>	

#### 6.1.2.9 combination\_t

```
template<typename T , size_t k, size_t n>
using aerobus::combination_t = typedef typename internal::combination<T, k, n>::type
```

computes binomial coefficient (k among n) as type

#### Template Parameters

<i>T</i>	Ring type ( <a href="#">i32</a> for example)
----------	--

#### 6.1.2.10 cos

```
template<typename Integers , size_t deg>
using aerobus::cos = typedef taylor<Integers, internal::cos_coeff, deg>
```

$\cos(x)$  cosinus

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.11 cosh

```
template<typename Integers , size_t deg>
using aerobus::cosh = typedef taylor<Integers, internal::cosh_coeff, deg>
```

$\cosh(x)$  hyperbolic cosine

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.12 E\_fraction

```
using aerobus::E_fraction = typedef ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1,
```

```
1, 10, 1, 1, 12, 1, 1, 14, 1, 1>
```

### 6.1.2.13 exp

```
template<typename Integers , size_t deg>
using aerobus::exp = typedef taylor<Integers, internal::exp_coeff, deg>
```

$$e^x$$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.14 expm1

```
template<typename Integers , size_t deg>
using aerobus::expm1 = typedef typename polynomial<FractionField<Integers> >::template sub_←
t< exp<Integers, deg>, typename polynomial<FractionField<Integers> >::one>
```

$$e^x - 1$$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.15 factorial\_t

```
template<typename T , size_t i>
using aerobus::factorial_t = typedef typename internal::factorial<T, i>::type
```

computes factorial(i), as type

#### Template Parameters

<i>T</i>	Ring type (e.g. <a href="#">i32</a> )
<i>i</i>	

### 6.1.2.16 fpq32

```
using aerobus::fpq32 = typedef FractionField<polynomial<q32> >
```

rational fractions with 32 bits rational coefficients rational fractions with rationals coefficients (32 bits numerator and denominator)



### 6.1.2.17 fpq64

```
using aerobus::fpq64 = typedef FractionField<polynomial<q64> >
```

polynomial with 64 bits rational coefficients

### 6.1.2.18 FractionField

```
template<typename Ring >
using aerobus::FractionField = typedef typename internal::FractionFieldImpl<Ring>::type
```

### 6.1.2.19 gcd\_t

```
template<typename T , typename A , typename B >
using aerobus::gcd_t = typedef typename internal::gcd<T>::template type<A, B>
```

computes the greatest common divisor or A and B

#### Template Parameters

<i>T</i>	Ring type (must be euclidean domain)
----------	--------------------------------------

### 6.1.2.20 geometric\_sum

```
template<typename Integers , size_t deg>
using aerobus::geometric_sum = typedef taylor<Integers, internal::geom_coeff, deg>
```

$\frac{1}{1-x}$  zero development of  $\frac{1}{1-x}$

#### Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

### 6.1.2.21 ln1

```
template<typename Integers , size_t deg>
using aerobus::ln1 = typedef taylor<Integers, internal::ln1_coeff, deg>
```

$\ln(1+x)$

#### Template Parameters

<i>T</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.22 make\_q32\_t

```
template<int32_t p, int32_t q>
using aerobus::make_q32_t = typedef typename q32::template simplify_t< typename q32::val<i32::inject_constant<
i32::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

##### Template Parameters

$p$	numerator
$q$	denominator

#### 6.1.2.23 make\_q64\_t

```
template<int64_t p, int64_t q>
using aerobus::make_q64_t = typedef typename q64::template simplify_t< typename q64::val<i64::inject_constant<
i64::inject_constant_t<q> >>
```

helper type : make a fraction from numerator and denominator

##### Template Parameters

$p$	numerator
$q$	denominator

#### 6.1.2.24 makefraction\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::makefraction_t = typedef typename FractionField<Ring>::template val<v1, v2>
```

helper type : the rational V1/V2 in the field of fractions of Ring

##### Template Parameters

$Ring$	the base ring
$v1$	value 1 in Ring
$v2$	value 2 in Ring

#### 6.1.2.25 mulfractions\_t

```
template<typename Ring , typename v1 , typename v2 >
using aerobus::mulfractions_t = typedef typename FractionField<Ring>::template mul_t<v1, v2>
```

helper type : multiplies two fractions

## Template Parameters

<i>Ring</i>	
<i>v1</i>	belongs to FractionField<Ring>
<i>v2</i>	belongs to FransionField<Ring>

## 6.1.2.26 pi64

```
using aerobus::pi64 = typedef polynomial<i64>
```

polynomial with 64 bits integers coefficients

## 6.1.2.27 PI\_fraction

```
using aerobus::PI_fraction = typedef ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1,
14, 2, 1, 1, 2, 2, 2, 2, 1>
```

## 6.1.2.28 pow\_t

```
template<typename T , auto p, auto n>
using aerobus::pow_t = typedef typename internal::pow<T, p, n>::type
```

$p^n$  (as 'val' type in T)

## Template Parameters

<i>T</i>	(some ring type, such as aerobus::i64)
<i>p</i>	(from T::inner_type, such as int64_t)
<i>n</i>	(from T::inner_type, such as int64_t)

## 6.1.2.29 pq64

```
using aerobus::pq64 = typedef polynomial<q64>
```

polynomial with 64 bits rationals coefficients

## 6.1.2.30 q32

```
using aerobus::q32 = typedef FractionField<i32>
```

32 bits rationals rationals with 32 bits numerator and denominator



## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.37 `stirling_unsigned_t`

```
template<typename T , int n, int k>
using aerobus::stirling\_unsigned\_t = typedef abs\_t<typename internal::stirling_helper<T, n,
k>::type>
```

Stirling number of first kind (unsigned) – as types.

## Template Parameters

<i>T</i>	(ring type, such as <a href="#">aerobus::i64</a> )
<i>n</i>	(integer)
<i>k</i>	(integer)

6.1.2.38 `tan`

```
template<typename Integers , size_t deg>
using aerobus::tan = typedef taylor<Integers, internal::tan_coeff, deg>
```

$\tan(x)$  tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

6.1.2.39 `tanh`

```
template<typename Integers , size_t deg>
using aerobus::tanh = typedef taylor<Integers, internal::tanh_coeff, deg>
```

$\tanh(x)$  hyperbolic tangent

## Template Parameters

<i>Integers</i>	Ring type (for example <a href="#">i64</a> )
<i>deg</i>	taylor approximation degree

#### 6.1.2.40 `taylor`

```
template<typename T , template< typename, size_t index > typename coeff_at, size_t deg>
using aerobus::taylor = typedef typename internal::make_taylor_impl< T, coeff_at, internal::make_index_sequence<
+ 1> >::type
```

##### Template Parameters

<i>T</i>	Used Ring type ( <a href="#">aerobus::i64</a> for example)
<i>coeff_at</i>	- implementation giving the 'value' (seen as type in <code>FractionField&lt;T&gt;</code> )
<i>deg</i>	

#### 6.1.2.41 `vadd_t`

```
template<typename... vals>
using aerobus::vadd_t = typedef typename internal::vadd<vals...>::type
```

adds multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an `add_t` binary operator

##### Template Parameters

<i>...vals</i>	
----------------	--

#### 6.1.2.42 `vmul_t`

```
template<typename... vals>
using aerobus::vmul_t = typedef typename internal::vmul<vals...>::type
```

multiplies multiple values ( $v_1 + v_2 + \dots + v_n$ ) vals must have same "enclosing\_type" and "enclosing\_type" must have an `mul_t` binary operator

##### Template Parameters

<i>...vals</i>	
----------------	--

### 6.1.3 Function Documentation

#### 6.1.3.1 `aligned_malloc()`

```
template<typename T >
T * aerobus::aligned_malloc (
    size_t count,
    size_t alignment )
```

'portable' aligned allocation of count elements of type T

## Template Parameters

<i>T</i>	the type of elements to store
----------	-------------------------------

## Parameters

<i>count</i>	the number of elements
<i>alignment</i>	boundary

## 6.1.3.2 field()

brief Conway polynomials tparam p characteristic of the aerobus::field (  
prime number )

## 6.1.4 Variable Documentation

## 6.1.4.1 alternate\_v

```
template<typename T , size_t k>
constexpr T::inner_type aerobus::alternate_v = internal::alternate<T, k>::value [inline],
[constexpr]
```

$(-1)^k$  as value from T

## Template Parameters

<i>T</i>	Ring type, <a href="#">aerobus::i64</a> for example, then result will be an int64_t
----------	---

## 6.1.4.2 bernoulli\_v

```
template<typename FloatType , typename T , size_t n>
constexpr FloatType aerobus::bernoulli_v = internal::bernoulli<T, n>::template value<Float↔
Type> [inline], [constexpr]
```

nth bernoulli number as value in FloatType

## Template Parameters

<i>FloatType</i>	(double or float for example)
<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>n</i>	

### 6.1.4.3 combination\_v

```
template<typename T , size_t k, size_t n>
constexpr T::inner_type aerobus::combination_v = internal::combination<T, k, n>::value [inline],
[constexpr]
```

computes binomial coefficients (k among n) as value

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i32</a> for example)
<i>k</i>	
<i>n</i>	

### 6.1.4.4 factorial\_v

```
template<typename T , size_t i>
constexpr T::inner_type aerobus::factorial_v = internal::factorial<T, i>::value [inline],
[constexpr]
```

computes factorial(i) as value in T

#### Template Parameters

<i>T</i>	( <a href="#">aerobus::i64</a> for example)
<i>i</i>	

## 6.2 aerobus::internal Namespace Reference

internal implementations, subject to breaking changes without notice

### Classes

- struct **\_FractionField**
- struct **\_FractionField**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **\_is\_prime**
- struct **\_is\_prime**< 0, i >
- struct **\_is\_prime**< 1, i >
- struct **\_is\_prime**< 2, i >
- struct **\_is\_prime**< 3, i >
- struct **\_is\_prime**< 5, i >
- struct **\_is\_prime**< 7, i >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n !=3 &&n % 2 !=0 &&n % 3==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n !=2 &&n % 2==0)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % i==0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&i \*i > n)> >
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n %(i+2) !=0 &&n % i !=0 &&n >=9 &&n % 3 !=0 &&n % 2 !=0 &&(i \*i<=n))> >



- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n % (i+2) == 0 && n >= 9 && n % 3 != 0 && n % 2 != 0 && i \* i <= n)>  
>
- struct **\_is\_prime**< n, i, std::enable\_if\_t<(n >= 9 && i \* i > n)> >
- struct **alternate**
- struct **alternate**< T, k, std::enable\_if\_t<k % 2 != 0> >
- struct **alternate**< T, k, std::enable\_if\_t<k % 2 == 0> >
- struct **asin\_coeff**
- struct **asin\_coeff\_helper**
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **asin\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **asinh\_coeff**
- struct **asinh\_coeff\_helper**
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **asinh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **atan\_coeff**
- struct **atan\_coeff\_helper**
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **atan\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **atanh\_coeff**
- struct **atanh\_coeff\_helper**
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **atanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **bernoulli**
- struct **bernoulli**< T, 0 >
- struct **bernoulli\_coeff**
- struct **bernoulli\_helper**
- struct **bernoulli\_helper**< T, accum, m, m >
- struct **bernstein\_helper**
- struct **bernstein\_helper**< 0, 0 >
- struct **bernstein\_helper**< i, m, std::enable\_if\_t<(m > 0) && (i > 0) && (i < m)> >
- struct **bernstein\_helper**< i, m, std::enable\_if\_t<(m > 0) && (i == 0)> >
- struct **bernstein\_helper**< i, m, std::enable\_if\_t<(m > 0) && (i == m)> >
- struct **chebyshev\_helper**
- struct **chebyshev\_helper**< 1, 0 >
- struct **chebyshev\_helper**< 1, 1 >
- struct **chebyshev\_helper**< 2, 0 >
- struct **chebyshev\_helper**< 2, 1 >
- struct **combination**
- struct **combination\_helper**
- struct **combination\_helper**< T, 0, n >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k > (n/2) && k > 0)> >
- struct **combination\_helper**< T, k, n, std::enable\_if\_t<(n >= 0 && k <= (n/2) && k > 0)> >
- struct **cos\_coeff**
- struct **cos\_coeff\_helper**
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cos\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **cosh\_coeff**
- struct **cosh\_coeff\_helper**
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 0> >
- struct **cosh\_coeff\_helper**< T, i, std::enable\_if\_t<(i & 1) == 1> >
- struct **exp\_coeff**
- struct **factorial**
- struct **factorial**< T, 0 >
- struct **factorial**< T, x, std::enable\_if\_t<(x > 0)> >
- struct **FractionFieldImpl**

- struct **FractionFieldImpl**< Field, std::enable\_if\_t< Field::is\_field > >
- struct **FractionFieldImpl**< Ring, std::enable\_if\_t<!Ring::is\_field > >
- struct **gcd**

*greatest common divisor computes the greatest common divisor exposes it in gcd<A, B>::type as long as Ring type is an integral domain*

- struct **gcd**< Ring, std::enable\_if\_t< Ring::is\_euclidean\_domain > >
- struct **geom\_coeff**
- struct **hermite\_helper**
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::physicist >
- struct **hermite\_helper**< 0, known\_polynomials::hermite\_kind::probabilist >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::physicist >
- struct **hermite\_helper**< 1, known\_polynomials::hermite\_kind::probabilist >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::physicist >
- struct **hermite\_helper**< deg, known\_polynomials::hermite\_kind::probabilist >
- struct **insert\_h**
- struct **is\_instantiation\_of**
- struct **is\_instantiation\_of**< TT, TT< Ts... > >
- struct **laguerre\_helper**
- struct **laguerre\_helper**< 0 >
- struct **laguerre\_helper**< 1 >
- struct **legendre\_helper**
- struct **legendre\_helper**< 0 >
- struct **legendre\_helper**< 1 >
- struct **lnp1\_coeff**
- struct **lnp1\_coeff**< T, 0 >
- struct **make\_taylor\_impl**
- struct **make\_taylor\_impl**< T, coeff\_at, std::integer\_sequence< size\_t, ls... > >
- struct **pop\_front\_h**
- struct **pow**
- struct **pow**< T, n, p, std::enable\_if\_t< p==0 > >
- struct **pow**< T, p, n, std::enable\_if\_t<(n % 2==1)> >
- struct **pow**< T, p, n, std::enable\_if\_t<(n > 0 && n % 2==0)> >
- struct **remove\_h**
- struct **sh\_coeff**
- struct **sh\_coeff\_helper**
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sh\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **sin\_coeff**
- struct **sin\_coeff\_helper**
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==0 > >
- struct **sin\_coeff\_helper**< T, i, std::enable\_if\_t<(i &1)==1 > >
- struct **split\_h**
- struct **split\_h**< 0, L1, L2 >
- struct **stirling\_helper**
- struct **stirling\_helper**< T, 0, 0 >
- struct **stirling\_helper**< T, 0, n, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, 0, std::enable\_if\_t<(n > 0)> >
- struct **stirling\_helper**< T, n, k, std::enable\_if\_t<(k > 0) &&(n > 0)> >
- struct **tan\_coeff**
- struct **tan\_coeff\_helper**
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tan\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **tanh\_coeff**
- struct **tanh\_coeff\_helper**

- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2) !=0 > >
- struct **tanh\_coeff\_helper**< T, i, std::enable\_if\_t<(i % 2)==0 > >
- struct **type\_at**
- struct **type\_at**< 0, T, Ts... >
- struct **vadd**
- struct **vadd**< v1 >
- struct **vadd**< v1, vals... >
- struct **vmul**
- struct **vmul**< v1 >
- struct **vmul**< v1, vals... >

## Typedefs

- template<size\_t i, typename... Ts>  
using **type\_at\_t** = typename type\_at< i, Ts... >::type
- template<std::size\_t N>  
using **make\_index\_sequence\_reverse** = decltype(index\_sequence\_reverse(std::make\_index\_sequence< N >{}))

## Functions

- template<std::size\_t... Is>  
constexpr auto **index\_sequence\_reverse** (std::index\_sequence< Is... > const &) -> decltype(std::index\_sequence< sizeof...(Is) - 1U - Is... >{})

## Variables

- template<template< typename... > typename TT, typename T >  
constexpr bool **is\_instantiation\_of\_v** = is\_instantiation\_of<TT, T>::value

### 6.2.1 Detailed Description

internal implementations, subject to breaking changes without notice

### 6.2.2 Typedef Documentation

#### 6.2.2.1 make\_index\_sequence\_reverse

```
template<std::size_t N>
using aerobus::internal::make_index_sequence_reverse = typedef decltype(index_sequence_reverse(std::make_index_sequence<N>{}))
```

#### 6.2.2.2 type\_at\_t

```
template<size_t i, typename... Ts>
using aerobus::internal::type_at_t = typedef typename type_at<i, Ts...>::type
```

## 6.2.3 Function Documentation

### 6.2.3.1 `index_sequence_reverse()`

```
template<std::size_t... Is>
constexpr auto aerobus::internal::index_sequence_reverse (
    std::index_sequence< Is... > const & ) -> decltype(std::index_sequence< sizeof...(Is)
- 1U - Is... >{}) [constexpr]
```

## 6.2.4 Variable Documentation

### 6.2.4.1 `is_instantiation_of_v`

```
template<template< typename... > typename TT, typename T >
constexpr bool aerobus::internal::is_instantiation_of_v = is_instantiation_of<TT, T>::value
[inline], [constexpr]
```

## 6.3 `aerobus::known_polynomials` Namespace Reference

families of well known polynomials such as Hermite or Bernstein

### Typedefs

- `template<size_t deg>`  
using [chebyshev\\_T](#) = `typename internal::chebyshev_helper< 1, deg >::type`  
*Chebyshev polynomials of first kind.*
- `template<size_t deg>`  
using [chebyshev\\_U](#) = `typename internal::chebyshev_helper< 2, deg >::type`  
*Chebyshev polynomials of second kind.*
- `template<size_t deg>`  
using [laguerre](#) = `typename internal::laguerre_helper< deg >::type`  
*Laguerre polynomials.*
- `template<size_t deg>`  
using [hermite\\_prob](#) = `typename internal::hermite_helper< deg, hermite_kind::probabilist >::type`  
*Hermite polynomials - probabilist form.*
- `template<size_t deg>`  
using [hermite\\_phys](#) = `typename internal::hermite_helper< deg, hermite_kind::physicist >::type`  
*Hermite polynomials - physicist form.*
- `template<size_t i, size_t m>`  
using [bernstein](#) = `typename internal::bernstein_helper< i, m >::type`  
*Bernstein polynomials.*
- `template<size_t deg>`  
using [legendre](#) = `typename internal::legendre_helper< deg >::type`  
*Legendre polynomials.*
- `template<size_t deg>`  
using [bernoulli](#) = `taylor< i64, internal::bernoulli_coeff< deg >::template inner, deg >`  
*Bernoulli polynomials.*

## Enumerations

- enum [hermite\\_kind](#) { [probabilist](#) , [physicist](#) }

### 6.3.1 Detailed Description

families of well known polynomials such as Hermite or Bernstein

### 6.3.2 Typedef Documentation

#### 6.3.2.1 bernoulli

```
template<size_t deg>
using aerobus::known_polynomials::bernoulli = typedef taylor<i64, internal::bernoulli_coeff<deg>>↵
::template inner, deg>
```

Bernoulli polynomials.

#### See also

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

#### 6.3.2.2 bernstein

```
template<size_t i, size_t m>
using aerobus::known_polynomials::bernstein = typedef typename internal::bernstein_helper<i,
m>::type
```

Bernstein polynomials.

#### See also

[See in Wikipedia](#)

#### Template Parameters

<i>i</i>	index of polynomial (between 0 and m)
<i>m</i>	degree of polynomial

#### 6.3.2.3 chebyshev\_T

```
template<size_t deg>
```

```
using aerobus::known_polynomials::chebyshev_T = typedef typename internal::chebyshev_helper<1,
deg>::type
```

Chebyshev polynomials of first kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

#### 6.3.2.4 chebyshev\_U

```
template<size_t deg>
using aerobus::known_polynomials::chebyshev_U = typedef typename internal::chebyshev_helper<2,
deg>::type
```

Chebyshev polynomials of second kind.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

#### 6.3.2.5 hermite\_phys

```
template<size_t deg>
using aerobus::known_polynomials::hermite_phys = typedef typename internal::hermite_helper<deg,
hermite_kind::physicist>::type
```

Hermite polynomials - physicist form.

See also

[See in Wikipedia](#)

Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

### 6.3.2.6 hermite\_prob

```
template<size_t deg>
using aerobus::known_polynomials::hermite_prob = typedef typename internal::hermite_helper<deg,
hermite_kind::probabilist>::type
```

Hermite polynomials - probabilist form.

See also

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

### 6.3.2.7 laguerre

```
template<size_t deg>
using aerobus::known_polynomials::laguerre = typedef typename internal::laguerre_helper<deg>↵
::type
```

Laguerre polynomials.

See also

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

### 6.3.2.8 legendre

```
template<size_t deg>
using aerobus::known_polynomials::legendre = typedef typename internal::legendre_helper<deg>↵
::type
```

Legendre polynomials.

See also

[See in Wikipedia](#)

#### Template Parameters

<i>deg</i>	degree of polynomial
------------	----------------------

### 6.3.3 Enumeration Type Documentation

#### 6.3.3.1 hermite\_kind

enum `aerobus::known_polynomials::hermite_kind`

Enumerator

probabilist	
physicist	



# Chapter 7

## Concept Documentation

### 7.1 aerobus::IsEuclideanDomain Concept Reference

Concept to express R is an euclidean domain.

```
#include <aerobus.h>
```

#### 7.1.1 Concept definition

```
template<typename R>
concept aerobus::IsEuclideanDomain = IsRing<R> && requires {
    typename R::template div_t<typename R::one, typename R::one>;
    typename R::template mod_t<typename R::one, typename R::one>;
    typename R::template gcd_t<typename R::one, typename R::one>;
    typename R::template eq_t<typename R::one, typename R::one>;
    typename R::template pos_t<typename R::one>;

    R::template pos_v<typename R::one> == true;

    R::is_euclidean_domain == true;
}
```

#### 7.1.2 Detailed Description

Concept to express R is an euclidean domain.

### 7.2 aerobus::IsField Concept Reference

Concept to express R is a field.

```
#include <aerobus.h>
```

#### 7.2.1 Concept definition

```
template<typename R>
concept aerobus::IsField = IsEuclideanDomain<R> && requires {
    R::is_field == true;
}
```

### 7.2.2 Detailed Description

Concept to express R is a field.

## 7.3 aerobus::IsRing Concept Reference

Concept to express R is a Ring.

```
#include <aerobus.h>
```

### 7.3.1 Concept definition

```
template<typename R>
concept aerobus::IsRing = requires {
    typename R::one;
    typename R::zero;
    typename R::template add_t<typename R::one, typename R::one>;
    typename R::template sub_t<typename R::one, typename R::one>;
    typename R::template mul_t<typename R::one, typename R::one>;
}
```

### 7.3.2 Detailed Description

Concept to express R is a Ring.

## Chapter 8

# Class Documentation

### 8.1 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, E >` Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.2 `aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >` Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- `using type = typename Ring::zero`

#### 8.2.1 Member Typedef Documentation

##### 8.2.1.1 `type`

```
template<typename Ring >  
template<typename coeffN >  
template<size_t index>  
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index<  
0||index > 0)> >::type = typename Ring::zero
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.3 aerobus::polynomial< Ring >::val< coeffN >::coeff\_at< index, std::enable\_if\_t<(index==0)> > Struct Template Reference

```
#include <aerobus.h>
```

#### Public Types

- using `type` = `aN`

#### 8.3.1 Member Typedef Documentation

##### 8.3.1.1 type

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at< index, std::enable_if_t<(index==0)>
>::type = aN
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.4 aerobus::ContinuedFraction< values > Struct Template Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

### 8.5 aerobus::ContinuedFraction< a0 > Struct Template Reference

Specialization for only one coefficient, technically just 'a0'.

```
#include <aerobus.h>
```

#### Public Types

- using `type` = `typename q64::template inject_constant_t< a0 >`  
*represented value as [aerobus::q64](#)*

#### Static Public Attributes

- static constexpr double `val` = `static_cast<double>(a0)`  
*represented value as double*

#### 8.5.1 Detailed Description

```
template<int64_t a0>
struct aerobus::ContinuedFraction< a0 >
```

Specialization for only one coefficient, technically just 'a0'.

## Template Parameters

<i>a0</i>	an integer int64_t
-----------	-----------------------

## 8.5.2 Member Typedef Documentation

### 8.5.2.1 type

```
template<int64_t a0>
using aerobus::ContinuedFraction< a0 >::type = typename q64::template inject_constant_t<a0>
represented value as aerobus::q64
```

## 8.5.3 Member Data Documentation

### 8.5.3.1 val

```
template<int64_t a0>
constexpr double aerobus::ContinuedFraction< a0 >::val = static_cast<double>(a0) [static],
[constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.6 aerobus::ContinuedFraction< a0, rest... > Struct Template Reference

specialization for multiple coefficients (strictly more than one)

```
#include <aerobus.h>
```

### Public Types

- using [type](#) = q64::template add\_t< typename q64::template inject\_constant\_t< a0 >, typename q64::template div\_t< typename q64::one, typename [ContinuedFraction](#)< rest... >::type > >  
represented value as [aerobus::q64](#)

### Static Public Attributes

- static constexpr double [val](#) = type::template get<double>()  
represented value as double

### 8.6.1 Detailed Description

```
template<int64_t a0, int64_t... rest>
struct aerobus::ContinuedFraction< a0, rest... >
```

specialization for multiple coefficients (strictly more than one)

## Template Parameters

<i>a0</i>	integer (int64_t)
<i>...rest</i>	integers (int64_t)

## 8.6.2 Member Typedef Documentation

### 8.6.2.1 type

```
template<int64_t a0, int64_t... rest>
using aerobus::ContinuedFraction< a0, rest... >::type = q64::template add_t< typename q64::
::template inject_constant_t<a0>, typename q64::template div_t< typename q64::one, typename
ContinuedFraction<rest...>::type > >
```

represented value as [aerobus::q64](#)

## 8.6.3 Member Data Documentation

### 8.6.3.1 val

```
template<int64_t a0, int64_t... rest>
constexpr double aerobus::ContinuedFraction< a0, rest... >::val = type::template get<double>()
[static], [constexpr]
```

represented value as double

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.7 aerobus::ConwayPolynomial Struct Reference

```
#include <aerobus.h>
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.8 aerobus::i32 Struct Reference

32 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

## Classes

- struct `val`  
*values in [i32](#), again represented as types*

## Public Types

- `using inner_type = int32_t`
- `using zero = val< 0 >`  
*constant zero*
- `using one = val< 1 >`  
*constant one*
- `template<auto x>`  
`using inject_constant_t = val< static_cast< int32_t >(x)>`
- `template<typename v >`  
`using inject_ring_t = v`
- `template<typename v1 , typename v2 >`  
`using add_t = typename add< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using sub_t = typename sub< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulus operator yields v1 % v2 for example : `i32::mod_t<i32::val<7>, i32::val<2>>`*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

## Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

### 8.8.1 Detailed Description

32 bits signed integers, seen as a algebraic ring with related operations

## 8.8.2 Member Typedef Documentation

### 8.8.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::add_t = typename add<v1, v2>::type
```

### 8.8.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::div_t = typename div<v1, v2>::type
```

### 8.8.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::eq_t = typename eq<v1, v2>::type
```

### 8.8.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gcd_t = gcd_t<i32, v1, v2>
```

### 8.8.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::gt_t = typename gt<v1, v2>::type
```

### 8.8.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i32::inject_constant_t = val<static_cast<int32_t>(x)>
```

### 8.8.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i32::inject_ring_t = v
```

### 8.8.2.8 inner\_type

```
using aerobus::i32::inner_type = int32_t
```

### 8.8.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::lt_t = typename lt<v1, v2>::type
```

### 8.8.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i32::mod_t = typename remainder<v1, v2>::type

modulus operator yields v1 % v2 for example : i32::mod_t<i32::val<7>, i32::val<2>>
```



## Template Parameters

<code>v1</code>	a value in <a href="#">i32</a>
<code>v2</code>	a value in <a href="#">i32</a>

**8.8.2.11 mul\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::mul_t = typename mul<v1, v2>::type
```

**8.8.2.12 one**

```
using aerobus::i32::one = val<1>
```

constant one

**8.8.2.13 pos\_t**

```
template<typename v >
using aerobus::i32::pos_t = typename pos<v>::type
```

**8.8.2.14 sub\_t**

```
template<typename v1 , typename v2 >
using aerobus::i32::sub_t = typename sub<v1, v2>::type
```

**8.8.2.15 zero**

```
using aerobus::i32::zero = val<0>
```

constant zero

**8.8.3 Member Data Documentation****8.8.3.1 eq\_v**

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i32::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

**8.8.3.2 is\_euclidean\_domain**

```
constexpr bool aerobus::i32::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.8.3.3 is\_field

```
constexpr bool aerobus::i32::is_field = false [static], [constexpr]
```

integers are not a field

### 8.8.3.4 pos\_v

```
template<typename v >
constexpr bool aerobus::i32::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.9 aerobus::i64 Struct Reference

64 bits signed integers, seen as a algebraic ring with related operations

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values in [i64](#)*

### Public Types

- [using inner\\_type = int64\\_t](#)  
*type for actual values*
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int64\\_t >\(x\)>](#)
- [template<typename v >](#)  
[using inject\\_ring\\_t = v](#)  
*injects a value used for internal consistency and quotient rings implementations for example [i64::inject\\_ring\\_t<i64::val<1>>](#)*  
*-> [i64::val<1>](#)*
- [using zero = val< 0 >](#)  
*constant zero*
- [using one = val< 1 >](#)  
*constant one*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)

- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i64, v1, v2 >`
- `template<typename v >`  
`using pos_t = typename pos< v >::type`

### Static Public Attributes

- `static constexpr bool is_field = false`  
*integers are not a field*
- `static constexpr bool is_euclidean_domain = true`  
*integers are an euclidean domain*
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator yields v1 > v2 as boolean value*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`

## 8.9.1 Detailed Description

64 bits signed integers, seen as a algebraic ring with related operations

## 8.9.2 Member Typedef Documentation

### 8.9.2.1 add\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::add_t = typename add<v1, v2>::type
```

### 8.9.2.2 div\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::div_t = typename div<v1, v2>::type
```

### 8.9.2.3 eq\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::eq_t = typename eq<v1, v2>::type
```

#### 8.9.2.4 gcd\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gcd_t = gcd_t<i64, v1, v2>
```

#### 8.9.2.5 gt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::gt_t = typename gt<v1, v2>::type
```

#### 8.9.2.6 inject\_constant\_t

```
template<auto x>
using aerobus::i64::inject_constant_t = val<static_cast<int64_t>(x)>
```

#### 8.9.2.7 inject\_ring\_t

```
template<typename v >
using aerobus::i64::inject_ring_t = v
```

injects a value used for internal consistency and quotient rings implementations for example `i64::inject_ring_t<i64::val<1>>`  
 -> `i64::val<1>`

##### Template Parameters

<code>v</code>	a value in <code>i64</code>
----------------	-----------------------------

#### 8.9.2.8 inner\_type

```
using aerobus::i64::inner_type = int64_t
```

type for actual values

#### 8.9.2.9 lt\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::lt_t = typename lt<v1, v2>::type
```

#### 8.9.2.10 mod\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mod_t = typename remainder<v1, v2>::type
```

### 8.9.2.11 mul\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::mul_t = typename mul<v1, v2>::type
```

### 8.9.2.12 one

```
using aerobus::i64::one = val<1>
```

constant one

### 8.9.2.13 pos\_t

```
template<typename v >
using aerobus::i64::pos_t = typename pos<v>::type
```

### 8.9.2.14 sub\_t

```
template<typename v1 , typename v2 >
using aerobus::i64::sub_t = typename sub<v1, v2>::type
```

### 8.9.2.15 zero

```
using aerobus::i64::zero = val<0>
```

constant zero

## 8.9.3 Member Data Documentation

### 8.9.3.1 eq\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

### 8.9.3.2 gt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator yields  $v1 > v2$  as boolean value

#### Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a>

### 8.9.3.3 is\_euclidean\_domain

```
constexpr bool aerobus::i64::is_euclidean_domain = true [static], [constexpr]
```

integers are an euclidean domain

### 8.9.3.4 is\_field

```
constexpr bool aerobus::i64::is_field = false [static], [constexpr]
```

integers are not a field

### 8.9.3.5 lt\_v

```
template<typename v1 , typename v2 >
constexpr bool aerobus::i64::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

### 8.9.3.6 pos\_v

```
template<typename v >
constexpr bool aerobus::i64::pos_v = pos_t<v>::value [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.10 aerobus::is\_prime< n > Struct Template Reference

checks if n is prime

```
#include <aerobus.h>
```

### Static Public Attributes

- static constexpr bool [value](#) = internal::\_is\_prime<n, 5>::value  
*true iff n is prime*

### 8.10.1 Detailed Description

```
template<size_t n>
struct aerobus::is_prime< n >
```

checks if n is prime

## Template Parameters

<i>n</i>	
----------	--

## 8.10.2 Member Data Documentation

### 8.10.2.1 value

```
template<size_t n>
constexpr bool aerobus::is_prime< n >::value = internal::_is_prime<n, 5>::value [static],
[constexpr]
```

true iff n is prime

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.11 aerobus::polynomial< Ring > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)  
*values (seen as types) in polynomial ring*
- struct [val< coeffN >](#)  
*specialization for constants*

### Public Types

- [using zero = val< typename Ring::zero >](#)  
*constant zero*
- [using one = val< typename Ring::one >](#)  
*constant one*
- [using X = val< typename Ring::one, typename Ring::zero >](#)  
*generator*
- [template<typename P >](#)  
[using simplify\\_t = typename simplify< P >::type](#)  
*simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)*
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*adds two polynomials*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction of two polynomials*

- `template<typename v1 , typename v2 >`  
`using mul_t = typename mul< v1, v2 >::type`  
*multiplication of two polynomials*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq_helper< v1, v2 >::type`  
*equality operator*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt_helper< v1, v2 >::type`  
*strict less operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt_helper< v1, v2 >::type`  
*strict greater operator*
- `template<typename v1 , typename v2 >`  
`using div_t = typename div< v1, v2 >::q_type`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = typename div_helper< v1, v2, zero, v1 >::mod_type`  
*modulo operator*
- `template<typename coeff , size_t deg>`  
`using monomial_t = typename monomial< coeff, deg >::type`  
*monomial :  $\text{coeff } X^{\text{deg}}$*
- `template<typename v >`  
`using derive_t = typename derive_helper< v >::type`  
*derivation operator*
- `template<typename v >`  
`using pos_t = typename Ring::template pos_t< typename v::aN >`  
*checks for positivity ( $an > 0$ )*
- `template<typename v1 , typename v2 >`  
`using gcd_t = std::conditional_t< Ring::is_euclidean_domain, typename make_unit< gcd_t< polynomial< Ring >, v1, v2 >::type, void >`  
*greatest common divisor of two polynomials*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

## Static Public Attributes

- `static constexpr bool is_field = false`
- `static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain`
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator*

### 8.11.1 Detailed Description

```
template<typename Ring>
requires IsEuclideanDomain<Ring>
struct aerobus::polynomial< Ring >
```

polynomial with coefficients in Ring Ring must be an integral domain



## 8.11.2 Member Typedef Documentation

### 8.11.2.1 add\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::add_t = typename add<v1, v2>::type
```

adds two polynomials

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.11.2.2 derive\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::derive_t = typename derive_helper<v>::type
```

derivation operator

#### Template Parameters

<i>v</i>	
----------	--

### 8.11.2.3 div\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::div_t = typename div<v1, v2>::q_type
```

division operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.11.2.4 eq\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::eq_t = typename eq_helper<v1, v2>::type
```

equality operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.11.2.5 gcd\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gcd_t = std::conditional_t< Ring::is_euclidean_domain,
typename make_unit<gcd_t<polynomial<Ring>, v1, v2> >::type, void>
```

greatest common divisor of two polynomials

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.11.2.6 gt\_t**

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::gt_t = typename gt_helper<v1, v2>::type
```

strict greater operator

**Template Parameters**

<i>v1</i>	
<i>v2</i>	

**8.11.2.7 inject\_constant\_t**

```
template<typename Ring >
template<auto x>
using aerobus::polynomial< Ring >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

**8.11.2.8 inject\_ring\_t**

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::inject_ring_t = val<v>
```

### 8.11.2.9 lt\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::lt_t = typename lt_helper<v1, v2>::type
```

strict less operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.11.2.10 mod\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mod_t = typename div_helper<v1, v2, zero, v1>::mod_type
```

modulo operator

#### Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.11.2.11 monomial\_t

```
template<typename Ring >
template<typename coeff , size_t deg>
using aerobus::polynomial< Ring >::monomial_t = typename monomial<coeff, deg>::type
```

monomial :  $\text{coeff } X^{\text{deg}}$

#### Template Parameters

<i>coeff</i>	
<i>deg</i>	

### 8.11.2.12 mul\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::mul_t = typename mul<v1, v2>::type
```

multiplication of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

## 8.11.2.13 one

```
template<typename Ring >
using aerobus::polynomial< Ring >::one = val<typename Ring::one>
```

constant one

## 8.11.2.14 pos\_t

```
template<typename Ring >
template<typename v >
using aerobus::polynomial< Ring >::pos_t = typename Ring::template pos_t<typename v::aN>
```

checks for positivity (an > 0)

## Template Parameters

<i>v</i>	
----------	--

## 8.11.2.15 simplify\_t

```
template<typename Ring >
template<typename P >
using aerobus::polynomial< Ring >::simplify_t = typename simplify<P>::type
```

simplifies a polynomial (recursively deletes highest degree if zero, do nothing otherwise)

## Template Parameters

<i>P</i>	
----------	--

## 8.11.2.16 sub\_t

```
template<typename Ring >
template<typename v1 , typename v2 >
using aerobus::polynomial< Ring >::sub_t = typename sub<v1, v2>::type
```

subtraction of two polynomials

## Template Parameters

<i>v1</i>	
<i>v2</i>	

### 8.11.2.17 X

```
template<typename Ring >
using aerobus::polynomial< Ring >::X = val<typename Ring::one, typename Ring::zero>
```

generator

### 8.11.2.18 zero

```
template<typename Ring >
using aerobus::polynomial< Ring >::zero = val<typename Ring::zero>
```

constant zero

## 8.11.3 Member Data Documentation

### 8.11.3.1 is\_euclidean\_domain

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_euclidean_domain = Ring::is_euclidean_domain
[static], [constexpr]
```

### 8.11.3.2 is\_field

```
template<typename Ring >
constexpr bool aerobus::polynomial< Ring >::is_field = false [static], [constexpr]
```

### 8.11.3.3 pos\_v

```
template<typename Ring >
template<typename v >
constexpr bool aerobus::polynomial< Ring >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator

Template Parameters

<code>v</code>	a value in <code>polynomial::val</code>
----------------	---

The documentation for this struct was generated from the following file:

- `src/aerobus.h`

## 8.12 aerobus::type\_list< Ts >::pop\_front Struct Reference

removes types from head of the list

```
#include <aerobus.h>
```

## Public Types

- using [type](#) = typename internal::pop\_front\_h< Ts... >::head  
*type that was previously head of the list*
- using [tail](#) = typename internal::pop\_front\_h< Ts... >::tail  
*remaining types in parent list when front is removed*

### 8.12.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >::pop_front
```

removes types from head of the list

### 8.12.2 Member Typedef Documentation

#### 8.12.2.1 tail

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::tail = typename internal::pop_front_h<Ts...>::tail
```

remaining types in parent list when front is removed

#### 8.12.2.2 type

```
template<typename... Ts>
using aerobus::type_list< Ts >::pop_front::type = typename internal::pop_front_h<Ts...>::head
```

type that was previously head of the list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.13 aerobus::Quotient< Ring, X > Struct Template Reference

[Quotient](#) ring by the principal ideal generated by 'X' With [i32](#) as Ring and [i32::val<2>](#) as X, [Quotient](#) is  $\mathbb{Z}/2\mathbb{Z}$ .

```
#include <aerobus.h>
```

## Classes

- struct [val](#)  
*projection values in the quotient ring*

## Public Types

- `using zero = val< typename Ring::zero >`  
*zero value*
- `using one = val< typename Ring::one >`  
*one*
- `template<typename v1 , typename v2 >`  
`using add_t = val< typename Ring::template add_t< typename v1::type, typename v2::type > >`  
*addition operator*
- `template<typename v1 , typename v2 >`  
`using mul_t = val< typename Ring::template mul_t< typename v1::type, typename v2::type > >`  
*subtraction operator*
- `template<typename v1 , typename v2 >`  
`using div_t = val< typename Ring::template div_t< typename v1::type, typename v2::type > >`  
*division operator*
- `template<typename v1 , typename v2 >`  
`using mod_t = val< typename Ring::template mod_t< typename v1::type, typename v2::type > >`  
*modulus operator*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename Ring::template eq_t< typename v1::type, typename v2::type >`  
*equality operator (as type)*
- `template<typename v1 >`  
`using pos_t = std::true_type`  
*positivity operator always true*
- `template<auto x>`  
`using inject_constant_t = val< typename Ring::template inject_constant_t< x > >`
- `template<typename v >`  
`using inject_ring_t = val< v >`

## Static Public Attributes

- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value`  
*addition operator (as boolean value)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator always true*
- `static constexpr bool is_euclidean_domain = true`  
*quotien rings are euclidean domain*

### 8.13.1 Detailed Description

```
template<typename Ring, typename X>
requires IsRing<Ring>
struct aerobus::Quotient< Ring, X >
```

`Quotient` ring by the principal ideal generated by 'X' With `i32` as Ring and `i32::val<2>` as X, `Quotient` is  $\mathbb{Z}/2\mathbb{Z}$ .

#### Template Parameters

<i>Ring</i>	A ring type, such as 'i32', must satisfy the IsRing concept
<i>X</i>	a value in Ring, such as <code>i32::val&lt;2&gt;</code>

## 8.13.2 Member Typedef Documentation

### 8.13.2.1 add\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::add_t = val<typename Ring::template add_t<typename v1::type,
typename v2::type> >
```

addition operator

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.13.2.2 div\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::div_t = val<typename Ring::template div_t<typename v1::type,
typename v2::type> >
```

division operator

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.13.2.3 eq\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::eq_t = typename Ring::template eq_t<typename v1::type,
typename v2::type>
```

equality operator (as type)

Template Parameters

v1	a value in quotient ring
v2	a value in quotient ring

### 8.13.2.4 inject\_constant\_t

```
template<typename Ring , typename X >
```



```
template<auto x>
using aerobus::Quotient< Ring, X >::inject_constant_t = val<typename Ring::template inject_constant_t<x>
>
```

### 8.13.2.5 inject\_ring\_t

```
template<typename Ring , typename X >
template<typename v >
using aerobus::Quotient< Ring, X >::inject_ring_t = val<v>
```

### 8.13.2.6 mod\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mod_t = val<typename Ring::template mod_t<typename v1::type,
typename v2::type> >
```

modulus operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.13.2.7 mul\_t

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
using aerobus::Quotient< Ring, X >::mul_t = val<typename Ring::template mul_t<typename v1::type,
typename v2::type> >
```

subtraction operator

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.13.2.8 one

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::one = val<typename Ring::one>
```

one

### 8.13.2.9 pos\_t

```
template<typename Ring , typename X >
template<typename v1 >
using aerobus::Quotient< Ring, X >::pos_t = std::true_type
```

positivity operator always true

#### Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

### 8.13.2.10 zero

```
template<typename Ring , typename X >
using aerobus::Quotient< Ring, X >::zero = val<typename Ring::zero>
```

zero value

## 8.13.3 Member Data Documentation

### 8.13.3.1 eq\_v

```
template<typename Ring , typename X >
template<typename v1 , typename v2 >
constexpr bool aerobus::Quotient< Ring, X >::eq_v = Ring::template eq_t<typename v1::type,
typename v2::type>::value [static], [constexpr]
```

addition operator (as boolean value)

#### Template Parameters

<i>v1</i>	a value in quotient ring
<i>v2</i>	a value in quotient ring

### 8.13.3.2 is\_euclidean\_domain

```
template<typename Ring , typename X >
constexpr bool aerobus::Quotient< Ring, X >::is_euclidean_domain = true [static], [constexpr]
```

quotien rings are euclidean domain

### 8.13.3.3 pos\_v

```
template<typename Ring , typename X >
template<typename v >
constexpr bool aerobus::Quotient< Ring, X >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator always true

## Template Parameters

<i>v1</i>	a value in quotient ring
-----------	--------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.14 aerobus::type\_list< Ts >::split< index > Struct Template Reference

splits list at index

```
#include <aerobus.h>
```

## Public Types

- using [head](#) = typename inner::head
- using [tail](#) = typename inner::tail

### 8.14.1 Detailed Description

```
template<typename... Ts>
template<size_t index>
struct aerobus::type_list< Ts >::split< index >
```

splits list at index

## Template Parameters

<i>index</i>	
--------------	--

### 8.14.2 Member Typedef Documentation

#### 8.14.2.1 head

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::head = typename inner::head
```

#### 8.14.2.2 tail

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::split< index >::tail = typename inner::tail
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.15 aerobus::type\_list< Ts > Struct Template Reference

Empty pure template struct to handle type list.

```
#include <aerobus.h>
```

### Classes

- struct [pop\\_front](#)  
*removes types from head of the list*
- struct [split](#)  
*splits list at index*

### Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T, Ts... >  
*Adds T to front of the list.*
- template<size\_t index>  
using [at](#) = [internal::type\\_at\\_t](#)< index, Ts... >  
*returns type at index*
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< Ts..., T >  
*pushes T at the tail of the list*
- template<typename U >  
using [concat](#) = typename [concat\\_h](#)< U >::type  
*concatenates two list into one*
- template<typename T , size\_t index>  
using [insert](#) = typename [internal::insert\\_h](#)< index, [type\\_list](#)< Ts... >, T >::type  
*inserts type at index*
- template<size\_t index>  
using [remove](#) = typename [internal::remove\\_h](#)< index, [type\\_list](#)< Ts... > >::type  
*removes type at index*

### Static Public Attributes

- static constexpr size\_t [length](#) = sizeof...(Ts)  
*length of list*

#### 8.15.1 Detailed Description

```
template<typename... Ts>
struct aerobus::type_list< Ts >
```

Empty pure template struct to handle type list.

## 8.15.2 Member Typedef Documentation

### 8.15.2.1 at

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::at = internal::type_at_t<index, Ts...>
```

returns type at index

#### Template Parameters

<i>index</i>	
--------------	--

### 8.15.2.2 concat

```
template<typename... Ts>
template<typename U >
using aerobus::type_list< Ts >::concat = typename concat_h<U>::type
```

concatenates two list into one

#### Template Parameters

<i>U</i>	
----------	--

### 8.15.2.3 insert

```
template<typename... Ts>
template<typename T , size_t index>
using aerobus::type_list< Ts >::insert = typename internal::insert_h<index, type_list<Ts...>, T>::type
```

inserts type at index

#### Template Parameters

<i>index</i>	
<i>T</i>	

### 8.15.2.4 push\_back

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_back = type_list<Ts..., T>
```

pushes T at the tail of the list

**Template Parameters**

<i>T</i>	
----------	--

**8.15.2.5 push\_front**

```
template<typename... Ts>
template<typename T >
using aerobus::type_list< Ts >::push_front = type_list<T, Ts...>
```

Adds T to front of the list.

**Template Parameters**

<i>T</i>	
----------	--

**8.15.2.6 remove**

```
template<typename... Ts>
template<size_t index>
using aerobus::type_list< Ts >::remove = typename internal::remove_h<index, type_list<Ts...>
>::type
```

removes type at index

**Template Parameters**

<i>index</i>	
--------------	--

**8.15.3 Member Data Documentation****8.15.3.1 length**

```
template<typename... Ts>
constexpr size_t aerobus::type_list< Ts >::length = sizeof...(Ts) [static], [constexpr]
```

length of list

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

**8.16 aerobus::type\_list<> Struct Reference**

specialization for empty type list

```
#include <aerobus.h>
```

## Public Types

- template<typename T >  
using [push\\_front](#) = [type\\_list](#)< T >
- template<typename T >  
using [push\\_back](#) = [type\\_list](#)< T >
- template<typename U >  
using [concat](#) = U
- template<typename T , size\_t index>  
using [insert](#) = [type\\_list](#)< T >

## Static Public Attributes

- static constexpr size\_t [length](#) = 0

### 8.16.1 Detailed Description

specialization for empty type list

### 8.16.2 Member Typedef Documentation

#### 8.16.2.1 concat

```
template<typename U >  
using aerobus::type\_list<>::concat = U
```

#### 8.16.2.2 insert

```
template<typename T , size_t index>  
using aerobus::type\_list<>::insert = type\_list<T>
```

#### 8.16.2.3 push\_back

```
template<typename T >  
using aerobus::type\_list<>::push_back = type\_list<T>
```

#### 8.16.2.4 push\_front

```
template<typename T >  
using aerobus::type\_list<>::push_front = type\_list<T>
```

### 8.16.3 Member Data Documentation

#### 8.16.3.1 length

```
constexpr size_t aerobus::type_list<>::length = 0 [static], [constexpr]
```

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.17 aerobus::i32::val< x > Struct Template Reference

values in [i32](#), again represented as types

```
#include <aerobus.h>
```

### Public Types

- [using enclosing\\_type = i32](#)  
*Enclosing ring type.*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

### Static Public Member Functions

- [template<typename valueType >](#)  
[static constexpr valueType get \(\)](#)  
*cast x into valueType*
- [static std::string to\\_string \(\)](#)  
*string representation of value*
- [template<typename valueRing >](#)  
[static constexpr valueRing eval \(const valueRing &v\)](#)  
*cast x into valueRing*

### Static Public Attributes

- [static constexpr int32\\_t v = x](#)  
*actual value stored in val type*

#### 8.17.1 Detailed Description

```
template<int32\_t x>
struct aerobus::i32::val< x >
```

values in [i32](#), again represented as types



## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 8.17.2 Member Typedef Documentation

### 8.17.2.1 enclosing\_type

```
template<int32_t x>
using aerobus::i32::val< x >::enclosing_type = i32
```

Enclosing ring type.

### 8.17.2.2 is\_zero\_t

```
template<int32_t x>
using aerobus::i32::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.17.3 Member Function Documentation

### 8.17.3.1 eval()

```
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i32::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast x into valueRing

## Template Parameters

<i>valueRing</i>	double for example
------------------	--------------------

### 8.17.3.2 get()

```
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::i32::val< x >::get ( ) [inline], [static], [constexpr]
```

cast x into valueType

## Template Parameters

<i>valueType</i>	double for example
------------------	--------------------

### 8.17.3.3 to\_string()

```
template<int32_t x>
static std::string aerobus::i32::val< x >::to_string ( ) [inline], [static]
```

string representation of value

## 8.17.4 Member Data Documentation

### 8.17.4.1 v

```
template<int32_t x>
constexpr int32_t aerobus::i32::val< x >::v = x [static], [constexpr]
```

actual value stored in val type

The documentation for this struct was generated from the following file:

- src/aerobus.h

## 8.18 aerobus::i64::val< x > Struct Template Reference

values in [i64](#)

```
#include <aerobus.h>
```

### Public Types

- [using enclosing\\_type = i64](#)  
*enclosing ring type*
- [using is\\_zero\\_t = std::bool\\_constant< x==0 >](#)  
*is value zero*

### Static Public Member Functions

- [template<typename valueType >](#)  
[static constexpr valueType get \( \)](#)  
*cast value in valueType*
- [static std::string to\\_string \( \)](#)  
*string representation*
- [template<typename valueRing >](#)  
[static constexpr valueRing eval \(const valueRing &v\)](#)  
*cast value in valueRing*

### Static Public Attributes

- [static constexpr int64\\_t v = x](#)  
*actual value*

### 8.18.1 Detailed Description

```
template<int64_t x>
struct aerobus::i64::val< x >
```

values in [i64](#)

## Template Parameters

<i>x</i>	an actual integer
----------	-------------------

## 8.18.2 Member Typedef Documentation

### 8.18.2.1 enclosing\_type

```
template<int64_t x>
using aerobus::i64::val< x >::enclosing_type = i64
```

enclosing ring type

### 8.18.2.2 is\_zero\_t

```
template<int64_t x>
using aerobus::i64::val< x >::is_zero_t = std::bool_constant<x == 0>
```

is value zero

## 8.18.3 Member Function Documentation

### 8.18.3.1 eval()

```
template<int64_t x>
template<typename valueRing >
static constexpr valueRing aerobus::i64::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

cast value in valueRing

## Template Parameters

<i>valueRing</i>	(double for example)
------------------	----------------------

### 8.18.3.2 get()

```
template<int64_t x>
template<typename valueType >
static constexpr valueType aerobus::i64::val< x >::get ( ) [inline], [static], [constexpr]
```

cast value in valueType

## Template Parameters

<i>valueType</i>	(double for example)
------------------	----------------------

### 8.18.3.3 to\_string()

```
template<int64_t x>
static std::string aerobus::i64::val< x >::to_string ( ) [inline], [static]
```

string representation

## 8.18.4 Member Data Documentation

### 8.18.4.1 v

```
template<int64_t x>
constexpr int64_t aerobus::i64::val< x >::v = x [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.19 aerobus::polynomial< Ring >::val< coeffN, coeffs > Struct Template Reference

values (seen as types) in polynomial ring

```
#include <aerobus.h>
```

### Public Types

- [using enclosing\\_type = polynomial< Ring >](#)  
*enclosing ring type*
- [using aN = coeffN](#)  
*heavy weight coefficient (non zero)*
- [using strip = val< coeffs... >](#)  
*remove largest coefficient*
- [using is\\_zero\\_t = std::bool\\_constant<\(degree==0\) &&\(aN::is\\_zero\\_t::value\)>](#)  
*true\_type if polynomial is constant zero*
- [template<size\\_t index>](#)  
[using coeff\\_at\\_t = typename coeff\\_at< index >::type](#)  
*type of coefficient at index*

### Static Public Member Functions

- [static std::string to\\_string \( \)](#)  
*get a string representation of polynomial*
- [template<typename valueRing >](#)  
[static constexpr valueRing eval \(const valueRing &x\)](#)  
*evaluates polynomial seen as a function operating on ValueRing*

## Static Public Attributes

- `static constexpr size_t degree` = `sizeof...(coeffs)`  
*degree of the polynomial*
- `static constexpr bool is_zero_v` = `is_zero_t::value`  
*true if polynomial is constant zero*

### 8.19.1 Detailed Description

```
template<typename Ring>
template<typename coeffN, typename... coeffs>
struct aerobus::polynomial< Ring >::val< coeffN, coeffs >
```

values (seen as types) in polynomial ring

#### Template Parameters

<code>coeffN</code>	high degree coefficient
<code>...coeffs</code>	lower degree coefficients

### 8.19.2 Member Typedef Documentation

#### 8.19.2.1 aN

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::aN = coeffN
```

heavy weight coefficient (non zero)

#### 8.19.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::coeff_at_t = typename coeff_↵
at<index>::type
```

type of coefficient at index

#### Template Parameters

<code>index</code>	
--------------------	--

### 8.19.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.19.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_t = std::bool_constant<(degree
== 0) && (aN::is_zero_t::value)>
```

true\_type if polynomial is constant zero

### 8.19.2.5 strip

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
using aerobus::polynomial< Ring >::val< coeffN, coeffs >::strip = val<coeffs...>
```

remove largest coefficient

## 8.19.3 Member Function Documentation

### 8.19.3.1 eval()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN, coeffs >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

evaluates polynomial seen as a function operating on ValueRing

#### Template Parameters

<i>valueRing</i>	usually float or double
------------------	-------------------------

#### Parameters

<i>x</i>	value
----------	-------

#### Returns

$P(x)$

### 8.19.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
static std::string aerobus::polynomial< Ring >::val< coeffN, coeffs >::to_string ( ) [inline],
[static]
```

get a string representation of polynomial

#### Returns

something like  $a_n X^n + \dots + a_1 X + a_0$

## 8.19.4 Member Data Documentation

### 8.19.4.1 degree

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr size_t aerobus::polynomial< Ring >::val< coeffN, coeffs >::degree = sizeof...(coeffs)
[static], [constexpr]
```

degree of the polynomial

### 8.19.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN , typename... coeffs>
constexpr bool aerobus::polynomial< Ring >::val< coeffN, coeffs >::is_zero_v = is_zero_t<
::value [static], [constexpr]
```

true if polynomial is constant zero

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.20 aerobus::Quotient< Ring, X >::val< V > Struct Template Reference

projection values in the quotient ring

```
#include <aerobus.h>
```

### Public Types

- `using type = abs_t< typename Ring::template mod_t< V, X > >`

### 8.20.1 Detailed Description

```
template<typename Ring, typename X>
template<typename V>
struct aerobus::Quotient< Ring, X >::val< V >
```

projection values in the quotient ring

## Template Parameters

V	a value from 'Ring'
---	---------------------

## 8.20.2 Member Typedef Documentation

### 8.20.2.1 type

```
template<typename Ring , typename X >
template<typename V >
using aerobus::Quotient< Ring, X >::val< V >::type = abs_t<typename Ring::template mod_t<V,
X> >
```

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## 8.21 aerobus::zpz< p >::val< x > Struct Template Reference

```
#include <aerobus.h>
```

### Public Types

- `using enclosing_type = zpz< p >`  
*enclosing ring type*
- `using is_zero_t = std::bool_constant< x% p==0 >`

### Static Public Member Functions

- `template<typename valueType >`  
`static constexpr valueType get ()`
- `static std::string to_string ()`
- `template<typename valueRing >`  
`static constexpr valueRing eval (const valueRing &v)`

### Static Public Attributes

- `static constexpr int32_t v = x % p`  
*actual value*

## 8.21.1 Member Typedef Documentation

### 8.21.1.1 enclosing\_type

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::enclosing_type = zpz<p>
```

enclosing ring type



### 8.21.1.2 is\_zero\_t

```
template<int32_t p>
template<int32_t x>
using aerobus::zpz< p >::val< x >::is_zero_t = std::bool_constant<x% p == 0>
```

## 8.21.2 Member Function Documentation

### 8.21.2.1 eval()

```
template<int32_t p>
template<int32_t x>
template<typename valueRing >
static constexpr valueRing aerobus::zpz< p >::val< x >::eval (
    const valueRing & v ) [inline], [static], [constexpr]
```

### 8.21.2.2 get()

```
template<int32_t p>
template<int32_t x>
template<typename valueType >
static constexpr valueType aerobus::zpz< p >::val< x >::get ( ) [inline], [static], [constexpr]
```

### 8.21.2.3 to\_string()

```
template<int32_t p>
template<int32_t x>
static std::string aerobus::zpz< p >::val< x >::to_string ( ) [inline], [static]
```

## 8.21.3 Member Data Documentation

### 8.21.3.1 v

```
template<int32_t p>
template<int32_t x>
constexpr int32_t aerobus::zpz< p >::val< x >::v = x % p [static], [constexpr]
```

actual value

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.22 aerobus::polynomial< Ring >::val< coeffN > Struct Template Reference

specialization for constants

```
#include <aerobus.h>
```

## Classes

- struct `coeff_at`
- struct `coeff_at< index, std::enable_if_t<(index< 0||index > 0)> >`
- struct `coeff_at< index, std::enable_if_t<(index==0)> >`

## Public Types

- using `enclosing_type` = `polynomial< Ring >`  
*enclosing ring type*
- using `aN` = `coeffN`
- using `strip` = `val< coeffN >`
- using `is_zero_t` = `std::bool_constant< aN::is_zero_t::value >`
- template<`size_t` `index`>  
using `coeff_at_t` = `typename coeff_at< index >::type`

## Static Public Member Functions

- static `std::string to_string ()`
- template<`typename valueRing` >  
static `constexpr valueRing eval (const valueRing &x)`

## Static Public Attributes

- static `constexpr size_t degree` = 0  
*degree*
- static `constexpr bool is_zero_v` = `is_zero_t::value`

### 8.22.1 Detailed Description

```
template<typename Ring>
template<typename coeffN>
struct aerobus::polynomial< Ring >::val< coeffN >
```

specialization for constants

Template Parameters

<code>coeffN</code>	
---------------------	--

### 8.22.2 Member Typedef Documentation

#### 8.22.2.1 aN

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::aN = coeffN
```

### 8.22.2.2 coeff\_at\_t

```
template<typename Ring >
template<typename coeffN >
template<size_t index>
using aerobus::polynomial< Ring >::val< coeffN >::coeff_at_t = typename coeff_at<index>↵
::type
```

### 8.22.2.3 enclosing\_type

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::enclosing_type = polynomial<Ring>
```

enclosing ring type

### 8.22.2.4 is\_zero\_t

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::is_zero_t = std::bool_constant<aN::is_↵
zero_t::value>
```

### 8.22.2.5 strip

```
template<typename Ring >
template<typename coeffN >
using aerobus::polynomial< Ring >::val< coeffN >::strip = val<coeffN>
```

## 8.22.3 Member Function Documentation

### 8.22.3.1 eval()

```
template<typename Ring >
template<typename coeffN >
template<typename valueRing >
static constexpr valueRing aerobus::polynomial< Ring >::val< coeffN >::eval (
    const valueRing & x ) [inline], [static], [constexpr]
```

### 8.22.3.2 to\_string()

```
template<typename Ring >
template<typename coeffN >
static std::string aerobus::polynomial< Ring >::val< coeffN >::to_string ( ) [inline], [static]
```

## 8.22.4 Member Data Documentation

### 8.22.4.1 degree

```
template<typename Ring >
template<typename coeffN >
constexpr size_t aerobus::polynomial< Ring >::val< coeffN >::degree = 0 [static], [constexpr]
```

degree

### 8.22.4.2 is\_zero\_v

```
template<typename Ring >
template<typename coeffN >
constexpr bool aerobus::polynomial< Ring >::val< coeffN >::is_zero_v = is_zero_t::value [static],
[constexpr]
```

The documentation for this struct was generated from the following file:

- src/[aerobus.h](#)

## 8.23 aerobus::zpz< p > Struct Template Reference

```
#include <aerobus.h>
```

### Classes

- struct [val](#)

### Public Types

- [using inner\\_type = int32\\_t](#)
- [template<auto x>](#)  
[using inject\\_constant\\_t = val< static\\_cast< int32\\_t >\(x\)>](#)
- [using zero = val< 0 >](#)
- [using one = val< 1 >](#)
- [template<typename v1 , typename v2 >](#)  
[using add\\_t = typename add< v1, v2 >::type](#)  
*addition operator*
- [template<typename v1 , typename v2 >](#)  
[using sub\\_t = typename sub< v1, v2 >::type](#)  
*subtraction operator*
- [template<typename v1 , typename v2 >](#)  
[using mul\\_t = typename mul< v1, v2 >::type](#)  
*multiplication operator*
- [template<typename v1 , typename v2 >](#)  
[using div\\_t = typename div< v1, v2 >::type](#)  
*division operator*

- `template<typename v1 , typename v2 >`  
`using mod_t = typename remainder< v1, v2 >::type`  
*modulo operator*
- `template<typename v1 , typename v2 >`  
`using gt_t = typename gt< v1, v2 >::type`  
*strictly greater operator (type)*
- `template<typename v1 , typename v2 >`  
`using lt_t = typename lt< v1, v2 >::type`  
*strictly smaller operator (type)*
- `template<typename v1 , typename v2 >`  
`using eq_t = typename eq< v1, v2 >::type`  
*equality operator (type)*
- `template<typename v1 , typename v2 >`  
`using gcd_t = gcd_t< i32, v1, v2 >`  
*greatest common divisor*
- `template<typename v1 >`  
`using pos_t = typename pos< v1 >::type`  
*positivity operator (type)*

### Static Public Attributes

- `static constexpr bool is_field = is_prime<p>::value`
- `static constexpr bool is_euclidean_domain = true`
- `template<typename v1 , typename v2 >`  
`static constexpr bool gt_v = gt_t<v1, v2>::value`  
*strictly greater operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool lt_v = lt_t<v1, v2>::value`  
*strictly smaller operator (booleanvalue)*
- `template<typename v1 , typename v2 >`  
`static constexpr bool eq_v = eq_t<v1, v2>::value`  
*equality operator (booleanvalue)*
- `template<typename v >`  
`static constexpr bool pos_v = pos_t<v>::value`  
*positivity operator (boolean value)*

## 8.23.1 Detailed Description

```
template<int32_t p>
struct aerobus::zpz< p >
```

congruence classes of integers for a modulus if p is prime, zpz is a field, otherwise an integral domain with all related operations

## 8.23.2 Member Typedef Documentation

### 8.23.2.1 add\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::add_t = typename add<v1, v2>::type
```

addition operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.23.2.2 div\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::div_t = typename div<v1, v2>::type
```

division operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.23.2.3 eq\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::eq_t = typename eq<v1, v2>::type
```

equality operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.23.2.4 gcd\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gcd_t = gcd_t<i32, v1, v2>
```

greatest common divisor

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.23.2.5 gt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::gt_t = typename gt<v1, v2>::type
```

strictly greater operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.23.2.6 inject\_constant\_t

```
template<int32_t p>
template<auto x>
using aerobus::zpz< p >::inject_constant_t = val<static_cast<int32_t>(x)>
```

### 8.23.2.7 inner\_type

```
template<int32_t p>
using aerobus::zpz< p >::inner_type = int32_t
```

### 8.23.2.8 lt\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::lt_t = typename lt<v1, v2>::type
```

strictly smaller operator (type)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.23.2.9 mod\_t

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mod_t = typename remainder<v1, v2>::type
```

modulo operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.23.2.10 mul\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::mul_t = typename mul<v1, v2>::type
```

multiplication operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

**8.23.2.11 one**

```
template<int32_t p>
using aerobus::zpz< p >::one = val<1>
```

**8.23.2.12 pos\_t**

```
template<int32_t p>
template<typename v1 >
using aerobus::zpz< p >::pos_t = typename pos<v1>::type
```

positivity operator (type)

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

**8.23.2.13 sub\_t**

```
template<int32_t p>
template<typename v1 , typename v2 >
using aerobus::zpz< p >::sub_t = typename sub<v1, v2>::type
```

subtraction operator

## Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>



### 8.23.2.14 zero

```
template<int32_t p>
using aerobus::zpz< p >::zero = val<0>
```

## 8.23.3 Member Data Documentation

### 8.23.3.1 eq\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::eq_v = eq_t<v1, v2>::value [static], [constexpr]
```

equality operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.23.3.2 gt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::gt_v = gt_t<v1, v2>::value [static], [constexpr]
```

strictly greater operator (booleanvalue)

#### Template Parameters

v1	a value in <a href="#">zpz::val</a>
v2	a value in <a href="#">zpz::val</a>

### 8.23.3.3 is\_euclidean\_domain

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_euclidean_domain = true [static], [constexpr]
```

### 8.23.3.4 is\_field

```
template<int32_t p>
constexpr bool aerobus::zpz< p >::is_field = is_prime<p>::value [static], [constexpr]
```

### 8.23.3.5 lt\_v

```
template<int32_t p>
template<typename v1 , typename v2 >
constexpr bool aerobus::zpz< p >::lt_v = lt_t<v1, v2>::value [static], [constexpr]
```

strictly smaller operator (booleanvalue)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
<i>v2</i>	a value in <a href="#">zpz::val</a>

### 8.23.3.6 pos\_v

```
template<int32_t p>
template<typename v >
constexpr bool aerobus::zpz< p >::pos_v = pos_t<v>::value [static], [constexpr]
```

positivity operator (boolean value)

#### Template Parameters

<i>v1</i>	a value in <a href="#">zpz::val</a>
-----------	-------------------------------------

The documentation for this struct was generated from the following file:

- [src/aerobus.h](#)

## Chapter 9

# File Documentation

### 9.1 main\_page.md File Reference

### 9.2 src/aerobus.h File Reference

```
#include <cstdint>
#include <cstddef>
#include <cstring>
#include <type_traits>
#include <utility>
#include <algorithm>
#include <functional>
#include <string>
#include <concepts>
#include <array>
```

Include dependency graph for aerobus.h:

### 9.3 aerobus.h

[Go to the documentation of this file.](#)

```
00001 // -*- lsst-c++ -*-
00002 #ifndef __INC_AEROBUS__ // NOLINT
00003 #define __INC_AEROBUS__
00004
00005 #include <cstdint>
00006 #include <cstddef>
00007 #include <cstring>
00008 #include <type_traits>
00009 #include <utility>
00010 #include <algorithm>
00011 #include <functional>
00012 #include <string>
00013 #include <concepts> // NOLINT
00014 #include <array>
00015
00016 #ifdef _MSC_VER
00017 #define ALIGNED(x) __declspec(align(x))
00018 #define INLINED __forceinline
00019 #else
00020 #define ALIGNED(x) __attribute__((aligned(x)))
00021 #define INLINED __attribute__((always_inline)) inline
00022 #endif
00023
00024
00025
00026
00027
00028
00029
```

```

00031
00032 // aligned allocation
00033 namespace aerobus {
00040     template<typename T>
00041     T* aligned_malloc(size_t count, size_t alignment) {
00042         #ifdef _MSC_VER
00043             return static_cast<T*>(_aligned_malloc(count * sizeof(T), alignment));
00044         #else
00045             return static_cast<T*>(aligned_alloc(alignment, count * sizeof(T)));
00046         #endif
00047     }
00048 } // namespace aerobus
00049
00050 // concepts
00051 namespace aerobus {
00053     template <typename R>
00054     concept IsRing = requires {
00055         typename R::one;
00056         typename R::zero;
00057         typename R::template add_t<typename R::one, typename R::one>;
00058         typename R::template sub_t<typename R::one, typename R::one>;
00059         typename R::template mul_t<typename R::one, typename R::one>;
00060     };
00061
00063     template <typename R>
00064     concept IsEuclideanDomain = IsRing<R> && requires {
00065         typename R::template div_t<typename R::one, typename R::one>;
00066         typename R::template mod_t<typename R::one, typename R::one>;
00067         typename R::template gcd_t<typename R::one, typename R::one>;
00068         typename R::template eq_t<typename R::one, typename R::one>;
00069         typename R::template pos_t<typename R::one>;
00070
00071         R::template pos_v<typename R::one> == true;
00072         // typename R::template gt_t<typename R::one, typename R::zero>;
00073         R::is_euclidean_domain == true;
00074     };
00075
00077     template<typename R>
00078     concept IsField = IsEuclideanDomain<R> && requires {
00079         R::is_field == true;
00080     };
00081 } // namespace aerobus
00082
00083 // utilities
00084 namespace aerobus {
00085     namespace internal {
00086         template<template<typename...> typename TT, typename T>
00087         struct is_instantiation_of : std::false_type { };
00088
00089         template<template<typename...> typename TT, typename... Ts>
00090         struct is_instantiation_of<TT, TT<Ts...> : std::true_type { };
00091
00092         template<template<typename...> typename TT, typename T>
00093         inline constexpr bool is_instantiation_of_v = is_instantiation_of<TT, T>::value;
00094
00095         template <int64_t i, typename T, typename... Ts>
00096         struct type_at {
00097             static_assert(i < sizeof...(Ts) + 1, "index out of range");
00098             using type = typename type_at<i - 1, Ts...>::type;
00099         };
00100
00101         template <typename T, typename... Ts> struct type_at<0, T, Ts...> {
00102             using type = T;
00103         };
00104
00105         template <size_t i, typename... Ts>
00106         using type_at_t = typename type_at<i, Ts...>::type;
00107
00108         template<size_t n, size_t i, typename E = void>
00109         struct _is_prime {};
00110
00111         template<size_t i>
00112         struct _is_prime<0, i> {
00113             static constexpr bool value = false;
00114         };
00115
00116         template<size_t i>
00117         struct _is_prime<1, i> {
00118             static constexpr bool value = false;
00119         };
00120
00121         template<size_t i>
00122         struct _is_prime<2, i> {
00123             static constexpr bool value = true;
00124         };
00125     }
00126

```

```

00127     template<size_t i>
00128     struct _is_prime<3, i> {
00129         static constexpr bool value = true;
00130     };
00131
00132     template<size_t i>
00133     struct _is_prime<5, i> {
00134         static constexpr bool value = true;
00135     };
00136
00137     template<size_t i>
00138     struct _is_prime<7, i> {
00139         static constexpr bool value = true;
00140     };
00141
00142     template<size_t n, size_t i>
00143     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n % 2 == 0)>> {
00144         static constexpr bool value = false;
00145     };
00146
00147     template<size_t n, size_t i>
00148     struct _is_prime<n, i, std::enable_if_t<(n != 2 && n != 3 && n % 2 != 0 && n % 3 == 0)>> {
00149         static constexpr bool value = false;
00150     };
00151
00152     template<size_t n, size_t i>
00153     struct _is_prime<n, i, std::enable_if_t<(n >= 9 && i * i > n)>> {
00154         static constexpr bool value = true;
00155     };
00156
00157     template<size_t n, size_t i>
00158     struct _is_prime<n, i, std::enable_if_t<(
00159         n % i == 0 &&
00160         n >= 9 &&
00161         n % 3 != 0 &&
00162         n % 2 != 0 &&
00163         i * i > n)>> {
00164         static constexpr bool value = true;
00165     };
00166
00167     template<size_t n, size_t i>
00168     struct _is_prime<n, i, std::enable_if_t<(
00169         n % (i+2) == 0 &&
00170         n >= 9 &&
00171         n % 3 != 0 &&
00172         n % 2 != 0 &&
00173         i * i <= n)>> {
00174         static constexpr bool value = true;
00175     };
00176
00177     template<size_t n, size_t i>
00178     struct _is_prime<n, i, std::enable_if_t<(
00179         n % (i+2) != 0 &&
00180         n % i != 0 &&
00181         n >= 9 &&
00182         n % 3 != 0 &&
00183         n % 2 != 0 &&
00184         (i * i <= n))>> {
00185         static constexpr bool value = _is_prime<n, i+6>::value;
00186     };
00187
00188 } // namespace internal
00189
00190 template<size_t n>
00191 struct is_prime {
00192     static constexpr bool value = internal::_is_prime<n, 5>::value;
00193 };
00194
00195 template<size_t n>
00196 static constexpr bool is_prime_v = is_prime<n>::value;
00197
00201 // gcd
00202 namespace internal {
00203     template <std::size_t... Is>
00204     constexpr auto index_sequence_reverse(std::index_sequence<Is...> const&
00205         -> decltype(std::index_sequence<sizeof...(Is) - 1U - Is...>{}));
00206
00207     template <std::size_t N>
00208     using make_index_sequence_reverse
00209         = decltype(index_sequence_reverse(std::make_index_sequence<N>{}));
00210
00211     template<typename Ring, typename E = void>
00212     struct gcd;
00213
00214     template<typename Ring>
00215     struct gcd<Ring, std::enable_if_t<Ring::is_euclidean_domain>> {
00216         template<typename A, typename B, typename E = void>

```

```

00225         struct gcd_helper {};
00226
00227         // B = 0, A > 0
00228         template<typename A, typename B>
00229         struct gcd_helper<A, B, std::enable_if_t<
00230             (B::is_zero_t::value) &&
00231             (Ring::template gt_t<A, typename Ring::zero>::value)>> {
00232             using type = A;
00233         };
00234
00235         // B = 0, A < 0
00236         template<typename A, typename B>
00237         struct gcd_helper<A, B, std::enable_if_t<
00238             (B::is_zero_t::value) &&
00239             !(Ring::template gt_t<A, typename Ring::zero>::value)>> {
00240             using type = typename Ring::template sub_t<typename Ring::zero, A>;
00241         };
00242
00243         // B != 0
00244         template<typename A, typename B>
00245         struct gcd_helper<A, B, std::enable_if_t<
00246             (!B::is_zero_t::value)
00247             >> {
00248         private: // NOLINT
00249             // A / B
00250             using k = typename Ring::template div_t<A, B>;
00251             // A - (A/B)*B = A % B
00252             using m = typename Ring::template sub_t<A, typename Ring::template mul_t<k, B>;
00253
00254         public:
00255             using type = typename gcd_helper<B, m>::type;
00256         };
00257
00258         template<typename A, typename B>
00259         using type = typename gcd_helper<A, B>::type;
00260     };
00261 } // namespace internal
00262
00263 // vadd and vmul
00264 namespace internal {
00265     template<typename... vals>
00266     struct vmul {};
00267
00268     template<typename v1, typename... vals>
00269     struct vmul<v1, vals...> {
00270         using type = typename v1::enclosing_type::template mul_t<v1, typename
vmul<vals...>::type>;
00271     };
00272
00273     template<typename v1>
00274     struct vmul<v1> {
00275         using type = v1;
00276     };
00277
00278     template<typename... vals>
00279     struct vadd {};
00280
00281     template<typename v1, typename... vals>
00282     struct vadd<v1, vals...> {
00283         using type = typename v1::enclosing_type::template add_t<v1, typename
vadd<vals...>::type>;
00284     };
00285
00286     template<typename v1>
00287     struct vadd<v1> {
00288         using type = v1;
00289     };
00290 } // namespace internal
00291
00292 template<typename T, typename A, typename B>
00293 using gcd_t = typename internal::gcd<T>::template type<A, B>;
00294
00295 template<typename... vals>
00296 using vadd_t = typename internal::vadd<vals...>::type;
00297
00298 template<typename... vals>
00299 using vmul_t = typename internal::vmul<vals...>::type;
00300
00301 template<typename val>
00302 requires IsEuclideanDomain<typename val::enclosing_type>
00303 using abs_t = std::conditional_t<
00304     val::enclosing_type::template pos_v<val>,
00305     val, typename val::enclosing_type::template sub_t<typename
val::enclosing_type::zero, val>;
00306 } // namespace aerobus
00307
00308 namespace aerobus {

```

```

00324     template<typename Ring, typename X>
00325     requires IsRing<Ring>
00326     struct Quotient {
00329         template <typename V>
00330         struct val {
00331             public:
00332                 using type = abs_t<typename Ring::template mod_t<V, X>>;
00333         };
00334
00336         using zero = val<typename Ring::zero>;
00337
00339         using one = val<typename Ring::one>;
00340
00344         template<typename v1, typename v2>
00345         using add_t = val<typename Ring::template add_t<typename v1::type, typename v2::type>>;
00346
00350         template<typename v1, typename v2>
00351         using mul_t = val<typename Ring::template mul_t<typename v1::type, typename v2::type>>;
00352
00356         template<typename v1, typename v2>
00357         using div_t = val<typename Ring::template div_t<typename v1::type, typename v2::type>>;
00358
00362         template<typename v1, typename v2>
00363         using mod_t = val<typename Ring::template mod_t<typename v1::type, typename v2::type>>;
00364
00368         template<typename v1, typename v2>
00369         using eq_t = typename Ring::template eq_t<typename v1::type, typename v2::type>;
00370
00374         template<typename v1, typename v2>
00375         static constexpr bool eq_v = Ring::template eq_t<typename v1::type, typename v2::type>::value;
00376
00380         template<typename v1>
00381         using pos_t = std::true_type;
00382
00386         template<typename v>
00387         static constexpr bool pos_v = pos_t<v>::value;
00388
00390         static constexpr bool is_euclidean_domain = true;
00391
00397         template<auto x>
00398         using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
00399
00405         template<typename v>
00406         using inject_ring_t = val<v>;
00407     };
00408 } // namespace aerobus
00409
00410 // type_list
00411 namespace aerobus {
00412     template <typename... Ts>
00413     struct type_list;
00414
00416     namespace internal {
00417         template <typename T, typename... Us>
00418         struct pop_front_h {
00419             using tail = type_list<Us...>;
00420             using head = T;
00421         };
00422
00423         template <size_t index, typename L1, typename L2>
00424         struct split_h {
00425             private:
00426                 static_assert(index <= L2::length, "index ouf of bounds");
00427                 using a = typename L2::pop_front::type;
00428                 using b = typename L2::pop_front::tail;
00429                 using c = typename L1::template push_back<a>;
00430
00431             public:
00432                 using head = typename split_h<index - 1, c, b>::head;
00433                 using tail = typename split_h<index - 1, c, b>::tail;
00434         };
00435
00436         template <typename L1, typename L2>
00437         struct split_h<0, L1, L2> {
00438             using head = L1;
00439             using tail = L2;
00440         };
00441
00442         template <size_t index, typename L, typename T>
00443         struct insert_h {
00444             static_assert(index <= L::length, "index ouf of bounds");
00445             using s = typename L::template split<index>;
00446             using left = typename s::head;
00447             using right = typename s::tail;
00448             using ll = typename left::template push_back<T>;
00449             using type = typename ll::template concat<right>;
00450         };

```

```

00451
00452     template <size_t index, typename L>
00453     struct remove_h {
00454         using s = typename L::template split<index>;
00455         using left = typename s::head;
00456         using right = typename s::tail;
00457         using rr = typename right::pop_front::tail;
00458         using type = typename left::template concat<rr>;
00459     };
00460 } // namespace internal
00461
00462 template <typename... Ts>
00463 struct type_list {
00464     private:
00465         template <typename T>
00466         struct concat_h;
00467
00468         template <typename... Us>
00469         struct concat_h<type_list<Us...> {
00470             using type = type_list<Ts..., Us...>;
00471         };
00472
00473     public:
00474         static constexpr size_t length = sizeof...(Ts);
00475
00476         template <typename T>
00477         using push_front = type_list<T, Ts...>;
00478
00479         template <size_t index>
00480         using at = internal::type_at_t<index, Ts...>;
00481
00482         struct pop_front {
00483             using type = typename internal::pop_front_h<Ts...>::head;
00484             using tail = typename internal::pop_front_h<Ts...>::tail;
00485         };
00486
00487         template <typename T>
00488         using push_back = type_list<Ts..., T>;
00489
00490         template <typename U>
00491         using concat = typename concat_h<U>::type;
00492
00493         template <size_t index>
00494         struct split {
00495             private:
00496                 using inner = internal::split_h<index, type_list<>, type_list<Ts...>;
00497
00498             public:
00499                 using head = typename inner::head;
00500                 using tail = typename inner::tail;
00501         };
00502
00503         template <typename T, size_t index>
00504         using insert = typename internal::insert_h<index, type_list<Ts...>, T>::type;
00505
00506         template <size_t index>
00507         using remove = typename internal::remove_h<index, type_list<Ts...>::type;
00508     };
00509
00510 template <>
00511 struct type_list<> {
00512     static constexpr size_t length = 0;
00513
00514     template <typename T>
00515     using push_front = type_list<T>;
00516
00517     template <typename T>
00518     using push_back = type_list<T>;
00519
00520     template <typename U>
00521     using concat = U;
00522
00523     // TODO(jewave): assert index == 0
00524     template <typename T, size_t index>
00525     using insert = type_list<T>;
00526 };
00527 } // namespace aerobus
00528
00529 // i32
00530 namespace aerobus {
00531     struct i32 {
00532         using inner_type = int32_t;
00533         template<int32_t x>
00534         struct val {
00535             using enclosing_type = i32;
00536             static constexpr int32_t v = x;
00537         };
00538     };
00539 }

```



```

00568     template<typename valueType>
00569     static constexpr valueType get() { return static_cast<valueType>(x); }
00570
00572     using is_zero_t = std::bool_constant<x == 0>;
00573
00575     static std::string to_string() {
00576         return std::to_string(x);
00577     }
00578
00581     template<typename valueRing>
00582     static constexpr valueRing eval(const valueRing& v) {
00583         return static_cast<valueRing>(x);
00584     }
00585 };
00586
00588     using zero = val<0>;
00590     using one = val<1>;
00592     static constexpr bool is_field = false;
00594     static constexpr bool is_euclidean_domain = true;
00598     template<auto x>
00599     using inject_constant_t = val<static_cast<int32_t>(x)>;
00600
00601     template<typename v>
00602     using inject_ring_t = v;
00603
00604 private:
00605     template<typename v1, typename v2>
00606     struct add {
00607         using type = val<v1::v + v2::v>;
00608     };
00609
00610     template<typename v1, typename v2>
00611     struct sub {
00612         using type = val<v1::v - v2::v>;
00613     };
00614
00615     template<typename v1, typename v2>
00616     struct mul {
00617         using type = val<v1::v * v2::v>;
00618     };
00619
00620     template<typename v1, typename v2>
00621     struct div {
00622         using type = val<v1::v / v2::v>;
00623     };
00624
00625     template<typename v1, typename v2>
00626     struct remainder {
00627         using type = val<v1::v % v2::v>;
00628     };
00629
00630     template<typename v1, typename v2>
00631     struct gt {
00632         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00633     };
00634
00635     template<typename v1, typename v2>
00636     struct lt {
00637         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00638     };
00639
00640     template<typename v1, typename v2>
00641     struct eq {
00642         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00643     };
00644
00645     template<typename v1>
00646     struct pos {
00647         using type = std::bool_constant<(v1::v > 0)>;
00648     };
00649
00650 public:
00656     template<typename v1, typename v2>
00657     using add_t = typename add<v1, v2>::type;
00658
00664     template<typename v1, typename v2>
00665     using sub_t = typename sub<v1, v2>::type;
00666
00672     template<typename v1, typename v2>
00673     using mul_t = typename mul<v1, v2>::type;
00674
00680     template<typename v1, typename v2>
00681     using div_t = typename div<v1, v2>::type;
00682
00688     template<typename v1, typename v2>
00689     using mod_t = typename remainder<v1, v2>::type;
00690

```

```

00696     template<typename v1, typename v2>
00697     using gt_t = typename gt<v1, v2>::type;
00698
00704     template<typename v1, typename v2>
00705     using lt_t = typename lt<v1, v2>::type;
00706
00712     template<typename v1, typename v2>
00713     using eq_t = typename eq<v1, v2>::type;
00714
00719     template<typename v1, typename v2>
00720     static constexpr bool eq_v = eq_t<v1, v2>::value;
00721
00727     template<typename v1, typename v2>
00728     using gcd_t = gcd_t<i32, v1, v2>;
00729
00734     template<typename v>
00735     using pos_t = typename pos<v>::type;
00736
00741     template<typename v>
00742     static constexpr bool pos_v = pos_t<v>::value;
00743 };
00744 } // namespace aerobus
00745
00746 // i64
00747 namespace aerobus {
00748     struct i64 {
00749         using inner_type = int64_t;
00750         template<int64_t x>
00751         struct val {
00752             using enclosing_type = i64;
00753             static constexpr int64_t v = x;
00754
00755             template<typename valueType>
00756             static constexpr valueType get() { return static_cast<valueType>(x); }
00757
00758             using is_zero_t = std::bool_constant<x == 0>;
00759
00760             static std::string to_string() {
00761                 return std::to_string(x);
00762             }
00763
00764             template<typename valueRing>
00765             static constexpr valueRing eval(const valueRing& v) {
00766                 return static_cast<valueRing>(x);
00767             }
00768         };
00769     };
00770
00771     template<auto x>
00772     using inject_constant_t = val<static_cast<int64_t>(x)>;
00773
00774     template<typename v>
00775     using inject_ring_t = v;
00776
00777     using zero = val<0>;
00778     using one = val<1>;
00779     static constexpr bool is_field = false;
00780     static constexpr bool is_euclidean_domain = true;
00781
00782 private:
00783     template<typename v1, typename v2>
00784     struct add {
00785         using type = val<v1::v + v2::v>;
00786     };
00787
00788     template<typename v1, typename v2>
00789     struct sub {
00790         using type = val<v1::v - v2::v>;
00791     };
00792
00793     template<typename v1, typename v2>
00794     struct mul {
00795         using type = val<v1::v * v2::v>;
00796     };
00797
00798     template<typename v1, typename v2>
00799     struct div {
00800         using type = val<v1::v / v2::v>;
00801     };
00802
00803     template<typename v1, typename v2>
00804     struct remainder {
00805         using type = val<v1::v % v2::v>;
00806     };
00807
00808     template<typename v1, typename v2>
00809     struct gt {
00810         using type = std::conditional_t<(v1::v > v2::v), std::true_type, std::false_type>;
00811     };

```

```

00833     };
00834
00835     template<typename v1, typename v2>
00836     struct lt {
00837         using type = std::conditional_t<(v1::v < v2::v), std::true_type, std::false_type>;
00838     };
00839
00840     template<typename v1, typename v2>
00841     struct eq {
00842         using type = std::conditional_t<(v1::v == v2::v), std::true_type, std::false_type>;
00843     };
00844
00845     template<typename v>
00846     struct pos {
00847         using type = std::bool_constant<(v::v > 0)>;
00848     };
00849
00850     public:
00851     template<typename v1, typename v2>
00852     using add_t = typename add<v1, v2>::type;
00853
00854     template<typename v1, typename v2>
00855     using sub_t = typename sub<v1, v2>::type;
00856
00857     template<typename v1, typename v2>
00858     using mul_t = typename mul<v1, v2>::type;
00859
00860     template<typename v1, typename v2>
00861     using div_t = typename div<v1, v2>::type;
00862
00863     template<typename v1, typename v2>
00864     using mod_t = typename remainder<v1, v2>::type;
00865
00866     template<typename v1, typename v2>
00867     using gt_t = typename gt<v1, v2>::type;
00868
00869     template<typename v1, typename v2>
00870     static constexpr bool gt_v = gt_t<v1, v2>::value;
00871
00872     template<typename v1, typename v2>
00873     using lt_t = typename lt<v1, v2>::type;
00874
00875     template<typename v1, typename v2>
00876     static constexpr bool lt_v = lt_t<v1, v2>::value;
00877
00878     template<typename v1, typename v2>
00879     using eq_t = typename eq<v1, v2>::type;
00880
00881     template<typename v1, typename v2>
00882     static constexpr bool eq_v = eq_t<v1, v2>::value;
00883
00884     template<typename v1, typename v2>
00885     using gcd_t = gcd_t<i64, v1, v2>;
00886
00887     template<typename v>
00888     using pos_t = typename pos<v>::type;
00889
00890     template<typename v>
00891     static constexpr bool pos_v = pos_t<v>::value;
00892     };
00893 } // namespace aerobus
00894
00895 // z/pz
00896 namespace aerobus {
00897     template<int32_t p>
00898     struct zpz {
00899         using inner_type = int32_t;
00900         template<int32_t x>
00901         struct val {
00902             using enclosing_type = zpz<p>;
00903             static constexpr int32_t v = x % p;
00904
00905             template<typename valueType>
00906             static constexpr valueType get() { return static_cast<valueType>(x % p); }
00907
00908             using is_zero_t = std::bool_constant<x % p == 0>;
00909             static std::string to_string() {
00910                 return std::to_string(x % p);
00911             }
00912
00913             template<typename valueRing>
00914             static constexpr valueRing eval(const valueRing& v) {
00915                 return static_cast<valueRing>(x % p);
00916             }
00917         };
00918     };
00919
00920     template<auto x>

```

```

00989     using inject_constant_t = val<static_cast<int32_t>(x)>;
00990
00991     using zero = val<0>;
00992     using one = val<1>;
00993     static constexpr bool is_field = is_prime<p>::value;
00994     static constexpr bool is_euclidean_domain = true;
00995
00996 private:
00997     template<typename v1, typename v2>
00998     struct add {
00999         using type = val<(v1::v + v2::v) % p>;
01000     };
01001
01002     template<typename v1, typename v2>
01003     struct sub {
01004         using type = val<(v1::v - v2::v) % p>;
01005     };
01006
01007     template<typename v1, typename v2>
01008     struct mul {
01009         using type = val<(v1::v * v2::v) % p>;
01010     };
01011
01012     template<typename v1, typename v2>
01013     struct div {
01014         using type = val<(v1::v % p) / (v2::v % p)>;
01015     };
01016
01017     template<typename v1, typename v2>
01018     struct remainder {
01019         using type = val<(v1::v % v2::v) % p>;
01020     };
01021
01022     template<typename v1, typename v2>
01023     struct gt {
01024         using type = std::conditional_t<(v1::v % p > v2::v % p), std::true_type, std::false_type>;
01025     };
01026
01027     template<typename v1, typename v2>
01028     struct lt {
01029         using type = std::conditional_t<(v1::v % p < v2::v % p), std::true_type, std::false_type>;
01030     };
01031
01032     template<typename v1, typename v2>
01033     struct eq {
01034         using type = std::conditional_t<(v1::v % p == v2::v % p), std::true_type, std::false_type>;
01035     };
01036
01037     template<typename v1>
01038     struct pos {
01039         using type = std::bool_constant<(v1::v > 0)>;
01040     };
01041
01042 public:
01043     template<typename v1, typename v2>
01044     using add_t = typename add<v1, v2>::type;
01045
01046     template<typename v1, typename v2>
01047     using sub_t = typename sub<v1, v2>::type;
01048
01049     template<typename v1, typename v2>
01050     using mul_t = typename mul<v1, v2>::type;
01051
01052     template<typename v1, typename v2>
01053     using div_t = typename div<v1, v2>::type;
01054
01055     template<typename v1, typename v2>
01056     using mod_t = typename remainder<v1, v2>::type;
01057
01058     template<typename v1, typename v2>
01059     using gt_t = typename gt<v1, v2>::type;
01060
01061     template<typename v1, typename v2>
01062     static constexpr bool gt_v = gt_t<v1, v2>::value;
01063
01064     template<typename v1, typename v2>
01065     using lt_t = typename lt<v1, v2>::type;
01066
01067     template<typename v1, typename v2>
01068     static constexpr bool lt_v = lt_t<v1, v2>::value;
01069
01070     template<typename v1, typename v2>
01071     using eq_t = typename eq<v1, v2>::type;
01072
01073     template<typename v1, typename v2>
01074     static constexpr bool eq_v = eq_t<v1, v2>::value;
01075
01076
01077
01078
01079
01080
01081
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01084
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01106
01107
01108

```

```

01112     template<typename v1, typename v2>
01113     using gcd_t = gcd_t<i32, v1, v2>;
01114
01117     template<typename v1>
01118     using pos_t = typename pos<v1>::type;
01119
01122     template<typename v>
01123     static constexpr bool pos_v = pos_t<v>::value;
01124 };
01125 } // namespace aerobus
01126
01127 // polynomial
01128 namespace aerobus {
01129     // coeffN x^N + ...
01134     template<typename Ring>
01135     requires IsEuclideanDomain<Ring>
01136     struct polynomial {
01137         static constexpr bool is_field = false;
01138         static constexpr bool is_euclidean_domain = Ring::is_euclidean_domain;
01139
01143         template<typename coeffN, typename... coeffs>
01144         struct val {
01146             using enclosing_type = polynomial<Ring>;
01148             static constexpr size_t degree = sizeof...(coeffs);
01150             using aN = coeffN;
01152             using strip = val<coeffs...>;
01154             using is_zero_t = std::bool_constant<(degree == 0) && (aN::is_zero_t::value)>;
01156             static constexpr bool is_zero_v = is_zero_t::value;
01157
01158         private:
01159             template<size_t index, typename E = void>
01160             struct coeff_at {};
01161
01162             template<size_t index>
01163             struct coeff_at<index, std::enable_if_t<(index >= 0 && index <= sizeof...(coeffs))> {
01164                 using type = internal::type_at_t<sizeof...(coeffs) - index, coeffN, coeffs...>;
01165             };
01166
01167             template<size_t index>
01168             struct coeff_at<index, std::enable_if_t<(index < 0 || index > sizeof...(coeffs))> {
01169                 using type = typename Ring::zero;
01170             };
01171
01172         public:
01173             template<size_t index>
01174             using coeff_at_t = typename coeff_at<index>::type;
01175
01177             static std::string to_string() {
01178                 return string_helper<coeffN, coeffs...>::func();
01179             }
01180
01183             template<typename valueRing>
01184             static constexpr valueRing eval(const valueRing& x) {
01185                 return horner_evaluation<valueRing, val>
01186                     ::template inner<0, degree + 1>
01187                     ::func(static_cast<valueRing>(0), x);
01188             }
01189         };
01190     };
01191
01192     template<typename coeffN>
01193     struct val<coeffN> {
01194         using enclosing_type = polynomial<Ring>;
01195         static constexpr size_t degree = 0;
01196         using aN = coeffN;
01197         using strip = val<coeffN>;
01198         using is_zero_t = std::bool_constant<aN::is_zero_t::value>;
01199
01202         static constexpr bool is_zero_v = is_zero_t::value;
01203
01204         template<size_t index, typename E = void>
01205         struct coeff_at {};
01206
01207         template<size_t index>
01208         struct coeff_at<index, std::enable_if_t<(index == 0)> {
01209             using type = aN;
01210         };
01211
01212         template<size_t index>
01213         struct coeff_at<index, std::enable_if_t<(index < 0 || index > 0)> {
01214             using type = typename Ring::zero;
01215         };
01216
01217         template<size_t index>
01218         using coeff_at_t = typename coeff_at<index>::type;
01219
01220         static std::string to_string() {
01221             return string_helper<coeffN>::func();
01222         }
01223     };

```

```

01228         }
01229
01230         template<typename valueRing>
01231         static constexpr valueRing eval(const valueRing& x) {
01232             return static_cast<valueRing>(aN::template get<valueRing>());
01233         }
01234     };
01235
01237     using zero = val<typename Ring::zero>;
01239     using one = val<typename Ring::one>;
01241     using X = val<typename Ring::one, typename Ring::zero>;
01242
01243 private:
01244     template<typename P, typename E = void>
01245     struct simplify;
01246
01247     template<typename P1, typename P2, typename I>
01248     struct add_low;
01249
01250     template<typename P1, typename P2>
01251     struct add {
01252         using type = typename simplify<typename add_low<
01253             P1,
01254             P2,
01255             internal::make_index_sequence_reverse<
01256                 std::max(P1::degree, P2::degree) + 1
01257             >::type>::type;
01258     };
01259
01260     template<typename P1, typename P2, typename I>
01261     struct sub_low;
01262
01263     template<typename P1, typename P2, typename I>
01264     struct mul_low;
01265
01266     template<typename v1, typename v2>
01267     struct mul {
01268         using type = typename mul_low<
01269             v1,
01270             v2,
01271             internal::make_index_sequence_reverse<
01272                 v1::degree + v2::degree + 1
01273             >::type;
01274     };
01275
01276     template<typename coeff, size_t deg>
01277     struct monomial;
01278
01279     template<typename v, typename E = void>
01280     struct derive_helper {};
01281
01282     template<typename v>
01283     struct derive_helper<v, std::enable_if_t<v::degree == 0> {
01284         using type = zero;
01285     };
01286
01287     template<typename v>
01288     struct derive_helper<v, std::enable_if_t<v::degree != 0> {
01289         using type = typename add<
01290             typename derive_helper<typename simplify<typename v::strip>::type>::type,
01291             typename monomial<
01292                 typename Ring::template mul_t<
01293                     typename v::aN,
01294                     typename Ring::template inject_constant_t<(v::degree)>
01295                 >,
01296                 v::degree - 1
01297             >::type
01298         >::type;
01299     };
01300
01301     template<typename v1, typename v2, typename E = void>
01302     struct eq_helper {};
01303
01304     template<typename v1, typename v2>
01305     struct eq_helper<v1, v2, std::enable_if_t<v1::degree != v2::degree> {
01306         using type = std::false_type;
01307     };
01308
01309     template<typename v1, typename v2>
01310     struct eq_helper<v1, v2, std::enable_if_t<
01311         v1::degree == v2::degree &&
01312         (v1::degree != 0 || v2::degree != 0) &&
01313         std::is_same<
01314             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01315             std::false_type
01316         >::value

```

```

01318     >
01319     > {
01320         using type = std::false_type;
01321     };
01322
01323     template<typename v1, typename v2>
01324     struct eq_helper<v1, v2, std::enable_if_t<
01325         v1::degree == v2::degree &&
01326         (v1::degree != 0 || v2::degree != 0) &&
01327         std::is_same<
01328             typename Ring::template eq_t<typename v1::aN, typename v2::aN>,
01329             std::true_type
01330         >::value
01331     > {
01332         using type = typename eq_helper<typename v1::strip, typename v2::strip>::type;
01333     };
01334
01335     template<typename v1, typename v2>
01336     struct eq_helper<v1, v2, std::enable_if_t<
01337         v1::degree == v2::degree &&
01338         (v1::degree == 0)
01339     > {
01340         using type = typename Ring::template eq_t<typename v1::aN, typename v2::aN>;
01341     };
01342
01343     template<typename v1, typename v2, typename E = void>
01344     struct lt_helper {};
01345
01346     template<typename v1, typename v2>
01347     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01348         using type = std::true_type;
01349     };
01350
01351     template<typename v1, typename v2>
01352     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01353         using type = typename Ring::template lt_t<typename v1::aN, typename v2::aN>;
01354     };
01355
01356     template<typename v1, typename v2>
01357     struct lt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01358         using type = std::false_type;
01359     };
01360
01361     template<typename v1, typename v2, typename E = void>
01362     struct gt_helper {};
01363
01364     template<typename v1, typename v2>
01365     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree > v2::degree)>> {
01366         using type = std::true_type;
01367     };
01368
01369     template<typename v1, typename v2>
01370     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree == v2::degree)>> {
01371         using type = std::false_type;
01372     };
01373
01374     template<typename v1, typename v2>
01375     struct gt_helper<v1, v2, std::enable_if_t<(v1::degree < v2::degree)>> {
01376         using type = std::false_type;
01377     };
01378
01379     // when high power is zero : strip
01380     template<typename P>
01381     struct simplify<P, std::enable_if_t<
01382         std::is_same<
01383             typename Ring::zero,
01384             typename P::aN
01385         >::value && (P::degree > 0)
01386     > {
01387         using type = typename simplify<typename P::strip>::type;
01388     };
01389
01390     // otherwise : do nothing
01391     template<typename P>
01392     struct simplify<P, std::enable_if_t<
01393         !std::is_same<
01394             typename Ring::zero,
01395             typename P::aN
01396         >::value && (P::degree > 0)
01397     > {
01398         using type = P;
01399     };
01400
01401     // do not simplify constants
01402     template<typename P>
01403     struct simplify<P, std::enable_if_t<P::degree == 0>> {
01404         using type = P;

```

```

01405     };
01406
01407     // addition at
01408     template<typename P1, typename P2, size_t index>
01409     struct add_at {
01410         using type =
01411             typename Ring::template add_t<
01412                 typename P1::template coeff_at_t<index>,
01413                 typename P2::template coeff_at_t<index>>;
01414     };
01415
01416     template<typename P1, typename P2, size_t index>
01417     using add_at_t = typename add_at<P1, P2, index>::type;
01418
01419     template<typename P1, typename P2, std::size_t... I>
01420     struct add_low<P1, P2, std::index_sequence<I...> {
01421         using type = val<add_at_t<P1, P2, I>...>;
01422     };
01423
01424     // subtraction at
01425     template<typename P1, typename P2, size_t index>
01426     struct sub_at {
01427         using type =
01428             typename Ring::template sub_t<
01429                 typename P1::template coeff_at_t<index>,
01430                 typename P2::template coeff_at_t<index>>;
01431     };
01432
01433     template<typename P1, typename P2, size_t index>
01434     using sub_at_t = typename sub_at<P1, P2, index>::type;
01435
01436     template<typename P1, typename P2, std::size_t... I>
01437     struct sub_low<P1, P2, std::index_sequence<I...> {
01438         using type = val<sub_at_t<P1, P2, I>...>;
01439     };
01440
01441     template<typename P1, typename P2>
01442     struct sub {
01443         using type = typename simplify<typename sub_low<
01444             P1,
01445             P2,
01446             internal::make_index_sequence_reverse<
01447                 std::max(P1::degree, P2::degree) + 1
01448             >::type>::type;
01449     };
01450
01451     // multiplication at
01452     template<typename v1, typename v2, size_t k, size_t index, size_t stop>
01453     struct mul_at_loop_helper {
01454         using type = typename Ring::template add_t<
01455             typename Ring::template mul_t<
01456                 typename v1::template coeff_at_t<index>,
01457                 typename v2::template coeff_at_t<k - index>
01458             >,
01459             typename mul_at_loop_helper<v1, v2, k, index + 1, stop>::type
01460         >;
01461     };
01462
01463     template<typename v1, typename v2, size_t k, size_t stop>
01464     struct mul_at_loop_helper<v1, v2, k, stop, stop> {
01465         using type = typename Ring::template mul_t<
01466             typename v1::template coeff_at_t<stop>,
01467             typename v2::template coeff_at_t<0>>;
01468     };
01469
01470     template<typename v1, typename v2, size_t k, typename E = void>
01471     struct mul_at {};
01472
01473     template<typename v1, typename v2, size_t k>
01474     struct mul_at<v1, v2, k, std::enable_if_t<(k < 0) || (k > v1::degree + v2::degree)> {
01475         using type = typename Ring::zero;
01476     };
01477
01478     template<typename v1, typename v2, size_t k>
01479     struct mul_at<v1, v2, k, std::enable_if_t<(k >= 0) && (k <= v1::degree + v2::degree)> {
01480         using type = typename mul_at_loop_helper<v1, v2, k, 0, k>::type;
01481     };
01482
01483     template<typename P1, typename P2, size_t index>
01484     using mul_at_t = typename mul_at<P1, P2, index>::type;
01485
01486     template<typename P1, typename P2, std::size_t... I>
01487     struct mul_low<P1, P2, std::index_sequence<I...> {
01488         using type = val<mul_at_t<P1, P2, I>...>;
01489     };
01490
01491     // division helper

```



```

01492     template< typename A, typename B, typename Q, typename R, typename E = void>
01493     struct div_helper {};
01494
01495     template<typename A, typename B, typename Q, typename R>
01496     struct div_helper<A, B, Q, R, std::enable_if_t<
01497         (R::degree < B::degree) ||
01498         (R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01499         using q_type = Q;
01500         using mod_type = R;
01501         using gcd_type = B;
01502     };
01503
01504     template<typename A, typename B, typename Q, typename R>
01505     struct div_helper<A, B, Q, R, std::enable_if_t<
01506         (R::degree >= B::degree) &&
01507         !(R::degree == 0 && std::is_same<typename R::aN, typename Ring::zero>::value)>> {
01508     private: // NOLINT
01509         using rN = typename R::aN;
01510         using bN = typename B::aN;
01511         using pT = typename monomial<typename Ring::template div_t<rN, bN>, R::degree -
01512             B::degree>::type;
01513         using rr = typename sub<R, typename mul<pT, B::type>::type>::type;
01514         using qq = typename add<Q, pT>::type;
01515     public:
01516         using q_type = typename div_helper<A, B, qq, rr>::q_type;
01517         using mod_type = typename div_helper<A, B, qq, rr>::mod_type;
01518         using gcd_type = rr;
01519     };
01520
01521     template<typename A, typename B>
01522     struct div {
01523         static_assert(Ring::is_euclidean_domain, "cannot divide in that type of Ring");
01524         using q_type = typename div_helper<A, B, zero, A::q_type>;
01525         using m_type = typename div_helper<A, B, zero, A::mod_type>;
01526     };
01527
01528     template<typename P>
01529     struct make_unit {
01530         using type = typename div<P, val<typename P::aN>::q_type>;
01531     };
01532
01533     template<typename coeff, size_t deg>
01534     struct monomial {
01535         using type = typename mul<X, typename monomial<coeff, deg - 1>::type>::type;
01536     };
01537
01538     template<typename coeff>
01539     struct monomial<coeff, 0> {
01540         using type = val<coeff>;
01541     };
01542
01543     template<typename valueRing, typename P>
01544     struct horner_evaluation {
01545         template<size_t index, size_t stop>
01546         struct inner {
01547             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01548                 constexpr valueRing coeff =
01549                     static_cast<valueRing>(P::template coeff_at_t<P::degree - index>::template
01550             get<valueRing>());
01551                 return horner_evaluation<valueRing, P>::template inner<index + 1, stop>::func(x *
01552             accum + coeff, x);
01553             };
01554         template<size_t stop>
01555         struct inner<stop, stop> {
01556             static constexpr valueRing func(const valueRing& accum, const valueRing& x) {
01557                 return accum;
01558             }
01559         };
01560     };
01561
01562     template<typename coeff, typename... coeffs>
01563     struct string_helper {
01564         static std::string func() {
01565             std::string tail = string_helper<coeffs...>::func();
01566             std::string result = "";
01567             if (Ring::template eq_t<coeff, typename Ring::zero>::value) {
01568                 return tail;
01569             } else if (Ring::template eq_t<coeff, typename Ring::one>::value) {
01570                 if (sizeof...(coeffs) == 1) {
01571                     result += "x";
01572                 } else {
01573                     result += "x^" + std::to_string(sizeof...(coeffs));
01574                 }
01575             } else {

```

```

01576         if (sizeof...(coeffs) == 1) {
01577             result += coeff::to_string() + " x";
01578         } else {
01579             result += coeff::to_string()
01580                 + " x^" + std::to_string(sizeof...(coeffs));
01581         }
01582     }
01583
01584     if (!tail.empty()) {
01585         result += " + " + tail;
01586     }
01587
01588     return result;
01589 }
01590 };
01591
01592 template<typename coeff>
01593 struct string_helper<coeff> {
01594     static std::string func() {
01595         if (!std::is_same<coeff, typename Ring::zero>::value) {
01596             return coeff::to_string();
01597         } else {
01598             return "";
01599         }
01600     }
01601 };
01602
01603 public:
01604     template<typename P>
01605     using simplify_t = typename simplify<P>::type;
01606
01607     template<typename v1, typename v2>
01608     using add_t = typename add<v1, v2>::type;
01609
01610     template<typename v1, typename v2>
01611     using sub_t = typename sub<v1, v2>::type;
01612
01613     template<typename v1, typename v2>
01614     using mul_t = typename mul<v1, v2>::type;
01615
01616     template<typename v1, typename v2>
01617     using eq_t = typename eq_helper<v1, v2>::type;
01618
01619     template<typename v1, typename v2>
01620     using lt_t = typename lt_helper<v1, v2>::type;
01621
01622     template<typename v1, typename v2>
01623     using gt_t = typename gt_helper<v1, v2>::type;
01624
01625     template<typename v1, typename v2>
01626     using div_t = typename div<v1, v2>::q_type;
01627
01628     template<typename v1, typename v2>
01629     using mod_t = typename div_helper<v1, v2, zero, v1>::mod_type;
01630
01631     template<typename coeff, size_t deg>
01632     using monomial_t = typename monomial<coeff, deg>::type;
01633
01634     template<typename v>
01635     using derive_t = typename derive_helper<v>::type;
01636
01637     template<typename v>
01638     using pos_t = typename Ring::template pos_t<typename v::aN>;
01639
01640     template<typename v>
01641     static constexpr bool pos_v = pos_t<v>::value;
01642
01643     template<typename v1, typename v2>
01644     using gcd_t = std::conditional_t<
01645         Ring::is_euclidean_domain,
01646         typename make_unit<gcd_t<polynomial<Ring>, v1, v2>::type,
01647         void>;
01648
01649     template<auto x>
01650     using inject_constant_t = val<typename Ring::template inject_constant_t<x>>;
01651
01652     template<typename v>
01653     using inject_ring_t = val<v>;
01654 };
01655 } // namespace aerobus
01656
01657 // fraction field
01658 namespace aerobus {
01659     namespace internal {
01660         template<typename Ring, typename E = void>
01661         requires IsEuclideanDomain<Ring>
01662         struct _FractionField {};
01663     }
01664 }

```

```

01707
01708     template<typename Ring>
01709     requires IsEuclideanDomain<Ring>
01710     struct _FractionField<Ring, std::enable_if_t<Ring::is_euclidean_domain> {
01711         static constexpr bool is_field = true;
01712         static constexpr bool is_euclidean_domain = true;
01713
01714     private:
01715         template<typename val1, typename val2, typename E = void>
01716         struct to_string_helper {};
01717
01718         template<typename val1, typename val2>
01719         struct to_string_helper <val1, val2,
01720             std::enable_if_t<
01721                 Ring::template eq_t<
01722                     val2, typename Ring::one
01723                 >::value
01724             >
01725         > {
01726             static std::string func() {
01727                 return val1::to_string();
01728             }
01729         };
01730
01731         template<typename val1, typename val2>
01732         struct to_string_helper<val1, val2,
01733             std::enable_if_t<
01734                 !Ring::template eq_t<
01735                     val2,
01736                     typename Ring::one
01737                 >::value
01738             >
01739         > {
01740             static std::string func() {
01741                 return "(" + val1::to_string() + " ) / ( " + val2::to_string() + " )";
01742             }
01743         };
01744     };
01745
01746     public:
01747         template<typename val1, typename val2>
01748         struct val {
01749             using x = val1;
01750             using y = val2;
01751             using is_zero_t = typename val1::is_zero_t;
01752             static constexpr bool is_zero_v = val1::is_zero_t::value;
01753
01754             using ring_type = Ring;
01755             using enclosing_type = _FractionField<Ring>;
01756
01757             static constexpr bool is_integer = std::is_same_v<val2, typename Ring::one>;
01758
01759             template<typename valueType>
01760             static constexpr valueType get() { return static_cast<valueType>(x::v) /
01761 static_cast<valueType>(y::v); }
01762
01763             static std::string to_string() {
01764                 return to_string_helper<val1, val2>::func();
01765             }
01766
01767             template<typename valueRing>
01768             static constexpr valueRing eval(const valueRing& v) {
01769                 return x::eval(v) / y::eval(v);
01770             }
01771         };
01772
01773         using zero = val<typename Ring::zero, typename Ring::one>;
01774         using one = val<typename Ring::one, typename Ring::one>;
01775
01776         template<typename v>
01777         using inject_t = val<v, typename Ring::one>;
01778
01779         template<auto x>
01780         using inject_constant_t = val<typename Ring::template inject_constant_t<x>, typename
01781 Ring::one>;
01782
01783         template<typename v>
01784         using inject_ring_t = val<typename Ring::template inject_ring_t<v>, typename Ring::one>;
01785
01786         using ring_type = Ring;
01787
01788     private:
01789         template<typename v, typename E = void>
01790         struct simplify {};
01791
01792         // x = 0
01793         template<typename v>
01794         struct simplify<v, std::enable_if_t<v::x::is_zero_t::value> {

```

```

01821         using type = typename _FractionField<Ring>::zero;
01822     };
01823
01824     // x != 0
01825     template<typename v>
01826     struct simplify<v, std::enable_if_t<!v::x::is_zero_t::value> {
01827     private:
01828         using _gcd = typename Ring::template gcd_t<typename v::x, typename v::y>;
01829         using newx = typename Ring::template div_t<typename v::x, _gcd>;
01830         using newy = typename Ring::template div_t<typename v::y, _gcd>;
01831
01832         using posx = std::conditional_t<
01833             !Ring::template pos_v<newx>,
01834             typename Ring::template sub_t<typename Ring::zero, newx>,
01835             newx>;
01836         using posy = std::conditional_t<
01837             !Ring::template pos_v<newy>,
01838             typename Ring::template sub_t<typename Ring::zero, newy>,
01839             newy>;
01840     public:
01841         using type = typename _FractionField<Ring>::template val<posx, posy>;
01842     };
01843
01844     public:
01845     template<typename v>
01846     using simplify_t = typename simplify<v>::type;
01847
01848     private:
01849     template<typename v1, typename v2>
01850     struct add {
01851     private:
01852         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01853         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01854         using dividend = typename Ring::template add_t<a, b>;
01855         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01856         using g = typename Ring::template gcd_t<dividend, diviser>;
01857
01858     public:
01859         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01860             diviser>;
01861     };
01862
01863     template<typename v>
01864     struct pos {
01865     private:
01866         using type = std::conditional_t<
01867             (Ring::template pos_v<typename v::x> && Ring::template pos_v<typename v::y>) ||
01868             (!Ring::template pos_v<typename v::x> && !Ring::template pos_v<typename v::y>),
01869             std::true_type,
01870             std::false_type>;
01871     };
01872
01873     template<typename v1, typename v2>
01874     struct sub {
01875     private:
01876         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01877         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01878         using dividend = typename Ring::template sub_t<a, b>;
01879         using diviser = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01880         using g = typename Ring::template gcd_t<dividend, diviser>;
01881
01882     public:
01883         using type = typename _FractionField<Ring>::template simplify_t<val<dividend,
01884             diviser>;
01885     };
01886
01887     template<typename v1, typename v2>
01888     struct mul {
01889     private:
01890         using a = typename Ring::template mul_t<typename v1::x, typename v2::x>;
01891         using b = typename Ring::template mul_t<typename v1::y, typename v2::y>;
01892
01893     public:
01894         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;
01895     };
01896
01897     template<typename v1, typename v2, typename E = void>
01898     struct div {};
01899
01900     template<typename v1, typename v2>
01901     struct div<v1, v2, std::enable_if_t<!std::is_same<v2, typename
01902         _FractionField<Ring>::zero>::value> {
01903     private:
01904         using a = typename Ring::template mul_t<typename v1::x, typename v2::y>;
01905         using b = typename Ring::template mul_t<typename v1::y, typename v2::x>;
01906
01907     public:
01908         using type = typename _FractionField<Ring>::template simplify_t<val<a, b>;

```

```

01907         };
01908
01909     template<typename v1, typename v2>
01910     struct div<v1, v2, std::enable_if_t<
01911         std::is_same<zero, v1>::value && std::is_same<v2, zero>::value> {
01912         using type = one;
01913     };
01914
01915     template<typename v1, typename v2>
01916     struct eq {
01917         using type = std::conditional_t<
01918             std::is_same<typename simplify_t<v1>::x, typename simplify_t<v2>::x>::value &&
01919             std::is_same<typename simplify_t<v1>::y, typename simplify_t<v2>::y>::value,
01920             std::true_type,
01921             std::false_type>;
01922     };
01923
01924     template<typename v1, typename v2, typename E = void>
01925     struct gt;
01926
01927     template<typename v1, typename v2>
01928     struct gt<v1, v2, std::enable_if_t<
01929         (eq<v1, v2>::type::value)
01930         >> {
01931         using type = std::false_type;
01932     };
01933
01934     template<typename v1, typename v2>
01935     struct gt<v1, v2, std::enable_if_t<
01936         (!eq<v1, v2>::type::value) &&
01937         (!pos<v1>::type::value) && (!pos<v2>::type::value)
01938         >> {
01939         using type = typename gt<
01940             typename sub<zero, v1>::type, typename sub<zero, v2>::type
01941             >::type;
01942     };
01943
01944     template<typename v1, typename v2>
01945     struct gt<v1, v2, std::enable_if_t<
01946         (!eq<v1, v2>::type::value) &&
01947         (pos<v1>::type::value) && (!pos<v2>::type::value)
01948         >> {
01949         using type = std::true_type;
01950     };
01951
01952     template<typename v1, typename v2>
01953     struct gt<v1, v2, std::enable_if_t<
01954         (!eq<v1, v2>::type::value) &&
01955         (!pos<v1>::type::value) && (pos<v2>::type::value)
01956         >> {
01957         using type = std::false_type;
01958     };
01959
01960     template<typename v1, typename v2>
01961     struct gt<v1, v2, std::enable_if_t<
01962         (!eq<v1, v2>::type::value) &&
01963         (pos<v1>::type::value) && (pos<v2>::type::value)
01964         >> {
01965         using type = typename Ring::template gt_t<
01966             typename Ring::template mul_t<v1::x, v2::y>,
01967             typename Ring::template mul_t<v2::y, v2::x>
01968         >;
01969     };
01970
01971     public:
01972     template<typename v1, typename v2>
01973     using add_t = typename add<v1, v2>::type;
01974
01975     template<typename v1, typename v2>
01976     using mod_t = zero;
01977
01978     template<typename v1, typename v2>
01979     using gcd_t = v1;
01980
01981     template<typename v1, typename v2>
01982     using sub_t = typename sub<v1, v2>::type;
01983
01984     template<typename v1, typename v2>
01985     using mul_t = typename mul<v1, v2>::type;
01986
01987     template<typename v1, typename v2>
01988     using div_t = typename div<v1, v2>::type;
01989
01990     template<typename v1, typename v2>
01991     using eq_t = typename eq<v1, v2>::type;
01992
01993     template<typename v1, typename v2>

```

```

02021         static constexpr bool eq_v = eq<v1, v2>::type::value;
02022
02026         template<typename v1, typename v2>
02027         using gt_t = typename gt<v1, v2>::type;
02028
02032         template<typename v1, typename v2>
02033         static constexpr bool gt_v = gt<v1, v2>::type::value;
02034
02037         template<typename v1>
02038         using pos_t = typename pos<v1>::type;
02039
02042         template<typename v>
02043         static constexpr bool pos_v = pos_t<v>::value;
02044     };
02045
02046     template<typename Ring, typename E = void>
02047     requires IsEuclideanDomain<Ring>
02048     struct FractionFieldImpl {};
02049
02050     // fraction field of a field is the field itself
02051     template<typename Field>
02052     requires IsEuclideanDomain<Field>
02053     struct FractionFieldImpl<Field, std::enable_if_t<Field::is_field> {
02054         using type = Field;
02055         template<typename v>
02056         using inject_t = v;
02057     };
02058
02059     // fraction field of a ring is the actual fraction field
02060     template<typename Ring>
02061     requires IsEuclideanDomain<Ring>
02062     struct FractionFieldImpl<Ring, std::enable_if_t<!Ring::is_field> {
02063         using type = _FractionField<Ring>;
02064     };
02065 } // namespace internal
02066
02070     template<typename Ring>
02071     requires IsEuclideanDomain<Ring>
02072     using FractionField = typename internal::FractionFieldImpl<Ring>::type;
02073 } // namespace aerobus
02074
02075 // short names for common types
02076 namespace aerobus {
02077     using q32 = FractionField<i32>;
02082     using fpq32 = FractionField<polynomial<q32>>;
02085     using q64 = FractionField<i64>;
02087     using pi64 = polynomial<i64>;
02089     using pq64 = polynomial<q64>;
02091     using fpq64 = FractionField<polynomial<q64>>;
02096     template<typename Ring, typename v1, typename v2>
02097     using makefraction_t = typename FractionField<Ring>::template val<v1, v2>;
02098
02102     template<int64_t p, int64_t q>
02103     using make_q64_t = typename q64::template simplify_t<
02104         typename q64::val<i64::inject_constant_t<p>, i64::inject_constant_t<q>>;
02105
02109     template<int32_t p, int32_t q>
02110     using make_q32_t = typename q32::template simplify_t<
02111         typename q32::val<i32::inject_constant_t<p>, i32::inject_constant_t<q>>;
02112
02117     template<typename Ring, typename v1, typename v2>
02118     using addfractions_t = typename FractionField<Ring>::template add_t<v1, v2>;
02123     template<typename Ring, typename v1, typename v2>
02124     using mulfractions_t = typename FractionField<Ring>::template mul_t<v1, v2>;
02125 } // namespace aerobus
02126
02127 // taylor series and common integers (factorial, bernoulli...) appearing in taylor coefficients
02128 namespace aerobus {
02129     namespace internal {
02130         template<typename T, size_t x, typename E = void>
02131         struct factorial {};
02132
02133         template<typename T, size_t x>
02134         struct factorial<T, x, std::enable_if_t<(x > 0)>> {
02135             private:
02136                 template<typename, size_t, typename>
02137                 friend struct factorial;
02138             public:
02139                 using type = typename T::template mul_t<typename T::template val<x>, typename factorial<T,
x - 1>::type>;
02140                 static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02141         };
02142
02143         template<typename T>
02144         struct factorial<T, 0> {
02145             public:

```

```

02146         using type = typename T::one;
02147         static constexpr typename T::inner_type value = type::template get<typename
T::inner_type>();
02148     };
02149 } // namespace internal
02150
02151 template<typename T, size_t i>
02152 using factorial_t = typename internal::factorial<T, i>::type;
02153
02154 template<typename T, size_t i>
02155 inline constexpr typename T::inner_type factorial_v = internal::factorial<T, i>::value;
02156
02157 namespace internal {
02158     template<typename T, size_t k, size_t n, typename E = void>
02159     struct combination_helper {};
02160
02161     template<typename T, size_t k, size_t n>
02162     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k <= (n / 2) && k > 0)> {
02163         using type = typename FractionField<T>::template mul_t<
02164             typename combination_helper<T, k - 1, n - 1>::type,
02165             makefraction_t<T, typename T::template val<n>, typename T::template val<k>>>;
02166     };
02167
02168     template<typename T, size_t k, size_t n>
02169     struct combination_helper<T, k, n, std::enable_if_t<(n >= 0 && k > (n / 2) && k > 0)> {
02170         using type = typename combination_helper<T, n - k, n>::type;
02171     };
02172
02173     template<typename T, size_t n>
02174     struct combination_helper<T, 0, n> {
02175         using type = typename FractionField<T>::one;
02176     };
02177
02178     template<typename T, size_t k, size_t n>
02179     struct combination {
02180         using type = typename internal::combination_helper<T, k, n>::type::x;
02181         static constexpr typename T::inner_type value =
02182             internal::combination_helper<T, k, n>::type::template get<typename
T::inner_type>();
02183     };
02184 } // namespace internal
02185
02186 template<typename T, size_t k, size_t n>
02187 using combination_t = typename internal::combination<T, k, n>::type;
02188
02189 template<typename T, size_t k, size_t n>
02190 inline constexpr typename T::inner_type combination_v = internal::combination<T, k, n>::value;
02191
02192 namespace internal {
02193     template<typename T, size_t m>
02194     struct bernoulli;
02195
02196     template<typename T, typename accum, size_t k, size_t m>
02197     struct bernoulli_helper {
02198         using type = typename bernoulli_helper<
02199             T,
02200             addfractions_t<T,
02201                 accum,
02202                 mulfractions_t<T,
02203                     makefraction_t<T,
02204                         combination_t<T, k, m + 1>,
02205                         typename T::one>,
02206                         typename bernoulli<T, k>::type
02207                     >,
02208                     k + 1,
02209                     m>::type;
02210     };
02211
02212     template<typename T, typename accum, size_t m>
02213     struct bernoulli_helper<T, accum, m, m> {
02214         using type = accum;
02215     };
02216
02217     template<typename T, size_t m>
02218     struct bernoulli {
02219         using type = typename FractionField<T>::template mul_t<
02220             typename internal::bernoulli_helper<T, typename FractionField<T>::zero, 0, m>::type,
02221             makefraction_t<T,
02222                 typename T::template val<static_cast<typename T::inner_type>(-1)>,
02223                 typename T::template val<static_cast<typename T::inner_type>(m + 1)>
02224             >
02225         >;
02226     };
02227
02228     template<typename floatType>

```

```

02243         static constexpr floatType value = type::template get<floatType>();
02244     };
02245
02246     template<typename T>
02247     struct bernoulli<T, 0> {
02248         using type = typename FractionField<T>::one;
02249
02250         template<typename floatType>
02251         static constexpr floatType value = type::template get<floatType>();
02252     };
02253 } // namespace internal
02254
02255 template<typename T, size_t n>
02256 using bernoulli_t = typename internal::bernoulli<T, n>::type;
02257
02258 template<typename FloatType, typename T, size_t n>
02259 inline constexpr FloatType bernoulli_v = internal::bernoulli<T, n>::template value<FloatType>;
02260
02261 namespace internal {
02262     template<typename T, int k, typename E = void>
02263     struct alternate {};
02264
02265     template<typename T, int k>
02266     struct alternate<T, k, std::enable_if_t<k % 2 == 0> {
02267         using type = typename T::one;
02268         static constexpr typename T::inner_type value = type::template get<typename
02269 T::inner_type>();
02270     };
02271
02272     template<typename T, int k>
02273     struct alternate<T, k, std::enable_if_t<k % 2 != 0> {
02274         using type = typename T::template sub_t<typename T::zero, typename T::one>;
02275         static constexpr typename T::inner_type value = type::template get<typename
02276 T::inner_type>();
02277     };
02278 } // namespace internal
02279
02280 template<typename T, int k>
02281 using alternate_t = typename internal::alternate<T, k>::type;
02282
02283 namespace internal {
02284     template<typename T, int n, int k, typename E = void>
02285     struct stirling_helper {};
02286
02287     template<typename T>
02288     struct stirling_helper<T, 0, 0> {
02289         using type = typename T::one;
02290     };
02291
02292     template<typename T, int n>
02293     struct stirling_helper<T, n, 0, std::enable_if_t<(n > 0)> {
02294         using type = typename T::zero;
02295     };
02296
02297     template<typename T, int n>
02298     struct stirling_helper<T, 0, n, std::enable_if_t<(n > 0)> {
02299         using type = typename T::zero;
02300     };
02301
02302     template<typename T, int n, int k>
02303     struct stirling_helper<T, n, k, std::enable_if_t<(k > 0) && (n > 0)> {
02304         using type = typename T::template sub_t<
02305             typename stirling_helper<T, n-1, k-1>::type,
02306             typename T::template mul_t<
02307                 typename T::template inject_constant_t<n-1>,
02308                 typename stirling_helper<T, n-1, k>::type
02309             >;
02310     };
02311 } // namespace internal
02312
02313 template<typename T, int n, int k>
02314 using stirling_signed_t = typename internal::stirling_helper<T, n, k>::type;
02315
02316 template<typename T, int n, int k>
02317 using stirling_unsigned_t = abs_t<typename internal::stirling_helper<T, n, k>::type>;
02318
02319 template<typename T, int n, int k>
02320 static constexpr typename T::inner_type stirling_signed_v = stirling_signed_t<T, n, k>::v;
02321
02322 template<typename T, int n, int k>
02323 static constexpr typename T::inner_type stirling_unsigned_v = stirling_unsigned_t<T, n, k>::v;
02324
02325 template<typename T, size_t k>
02326 inline constexpr typename T::inner_type alternate_v = internal::alternate<T, k>::value;
02327
02328 namespace internal {

```



```

02355     template<typename T, auto p, auto n, typename E = void>
02356     struct pow {};
02357
02358     template<typename T, auto p, auto n>
02359     struct pow<T, p, n, std::enable_if_t<(n > 0 && n % 2 == 0)> {
02360         using type = typename T::template mul_t<
02361             typename pow<T, p, n/2>::type,
02362             typename pow<T, p, n/2>::type
02363         >;
02364     };
02365
02366     template<typename T, auto p, auto n>
02367     struct pow<T, p, n, std::enable_if_t<(n % 2 == 1)> {
02368         using type = typename T::template mul_t<
02369             typename T::template inject_constant_t<p>,
02370             typename T::template mul_t<
02371                 typename pow<T, p, n/2>::type,
02372                 typename pow<T, p, n/2>::type
02373             >
02374         >;
02375     };
02376
02377     template<typename T, auto n, auto p>
02378     struct pow<T, n, p, std::enable_if_t<p == 0> { using type = typename T::one; };
02379 } // namespace internal
02380
02385     template<typename T, auto p, auto n>
02386     using pow_t = typename internal::pow<T, p, n>::type;
02387
02392     template<typename T, auto p, auto n>
02393     static constexpr typename T::inner_type pow_v = internal::pow<T, p, n>::type::v;
02394
02395     namespace internal {
02396         template<typename, template<typename, size_t> typename, class>
02397         struct make_taylor_impl;
02398
02399         template<typename T, template<typename, size_t> typename coeff_at, size_t... Is>
02400         struct make_taylor_impl<T, coeff_at, std::integer_sequence<size_t, Is...> {
02401             using type = typename polynomial<FractionField<T>>::template val<typename coeff_at<T,
Is::type...>;
02402         };
02403     }
02404
02409     template<typename T, template<typename, size_t index> typename coeff_at, size_t deg>
02410     using taylor = typename internal::make_taylor_impl<
02411         T,
02412         coeff_at,
02413         internal::make_index_sequence_reverse<deg + 1>::type;
02414
02415     namespace internal {
02416         template<typename T, size_t i>
02417         struct exp_coeff {
02418             using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02419         };
02420
02421         template<typename T, size_t i, typename E = void>
02422         struct sin_coeff_helper {};
02423
02424         template<typename T, size_t i>
02425         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02426             using type = typename FractionField<T>::zero;
02427         };
02428
02429         template<typename T, size_t i>
02430         struct sin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02431             using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02432         };
02433
02434         template<typename T, size_t i>
02435         struct sin_coeff {
02436             using type = typename sin_coeff_helper<T, i>::type;
02437         };
02438
02439         template<typename T, size_t i, typename E = void>
02440         struct sh_coeff_helper {};
02441
02442         template<typename T, size_t i>
02443         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02444             using type = typename FractionField<T>::zero;
02445         };
02446
02447         template<typename T, size_t i>
02448         struct sh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02449             using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02450         };
02451
02452         template<typename T, size_t i>

```

```

02453     struct sh_coeff {
02454         using type = typename sh_coeff_helper<T, i>::type;
02455     };
02456
02457     template<typename T, size_t i, typename E = void>
02458     struct cos_coeff_helper {};
02459
02460     template<typename T, size_t i>
02461     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02462         using type = typename FractionField<T>::zero;
02463     };
02464
02465     template<typename T, size_t i>
02466     struct cos_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02467         using type = makefraction_t<T, alternate_t<T, i / 2>, factorial_t<T, i>>;
02468     };
02469
02470     template<typename T, size_t i>
02471     struct cos_coeff {
02472         using type = typename cos_coeff_helper<T, i>::type;
02473     };
02474
02475     template<typename T, size_t i, typename E = void>
02476     struct cosh_coeff_helper {};
02477
02478     template<typename T, size_t i>
02479     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02480         using type = typename FractionField<T>::zero;
02481     };
02482
02483     template<typename T, size_t i>
02484     struct cosh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02485         using type = makefraction_t<T, typename T::one, factorial_t<T, i>>;
02486     };
02487
02488     template<typename T, size_t i>
02489     struct cosh_coeff {
02490         using type = typename cosh_coeff_helper<T, i>::type;
02491     };
02492
02493     template<typename T, size_t i>
02494     struct geom_coeff { using type = typename FractionField<T>::one; };
02495
02496
02497     template<typename T, size_t i, typename E = void>
02498     struct atan_coeff_helper;
02499
02500     template<typename T, size_t i>
02501     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02502         using type = makefraction_t<T, alternate_t<T, i / 2>, typename T::template val<i>;
02503     };
02504
02505     template<typename T, size_t i>
02506     struct atan_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02507         using type = typename FractionField<T>::zero;
02508     };
02509
02510     template<typename T, size_t i>
02511     struct atan_coeff { using type = typename atan_coeff_helper<T, i>::type; };
02512
02513     template<typename T, size_t i, typename E = void>
02514     struct asin_coeff_helper;
02515
02516     template<typename T, size_t i>
02517     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02518         using type = makefraction_t<T,
02519             factorial_t<T, i - 1>,
02520             typename T::template mul_t<
02521                 typename T::template val<i>,
02522                 T::template mul_t<
02523                     pow_t<T, 4, i / 2>,
02524                     pow<T, factorial<T, i / 2>::value, 2
02525                 >
02526             >
02527         >>;
02528     };
02529
02530     template<typename T, size_t i>
02531     struct asin_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02532         using type = typename FractionField<T>::zero;
02533     };
02534
02535     template<typename T, size_t i>
02536     struct asin_coeff {
02537         using type = typename asin_coeff_helper<T, i>::type;
02538     };
02539

```

```

02540     template<typename T, size_t i>
02541     struct lnpl_coeff {
02542         using type = makefraction_t<T,
02543             alternate_t<T, i + 1>,
02544             typename T::template val<i>;
02545     };
02546
02547     template<typename T>
02548     struct lnpl_coeff<T, 0> { using type = typename FractionField<T>::zero; };
02549
02550     template<typename T, size_t i, typename E = void>
02551     struct asinh_coeff_helper;
02552
02553     template<typename T, size_t i>
02554     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02555         using type = makefraction_t<T,
02556             typename T::template mul_t<
02557                 alternate_t<T, i / 2>,
02558                 factorial_t<T, i - 1>
02559             >,
02560             typename T::template mul_t<
02561                 typename T::template mul_t<
02562                     typename T::template val<i>,
02563                     pow_t<T, (factorial<T, i / 2>::value), 2>
02564                 >,
02565                 pow_t<T, 4, i / 2>
02566             >
02567         >;
02568     };
02569
02570     template<typename T, size_t i>
02571     struct asinh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02572         using type = typename FractionField<T>::zero;
02573     };
02574
02575     template<typename T, size_t i>
02576     struct asinh_coeff {
02577         using type = typename asinh_coeff_helper<T, i>::type;
02578     };
02579
02580     template<typename T, size_t i, typename E = void>
02581     struct atanh_coeff_helper;
02582
02583     template<typename T, size_t i>
02584     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 1> {
02585         // 1/i
02586         using type = typename FractionField<T>::template val<
02587             typename T::one,
02588             typename T::template inject_constant_t<i>;
02589     };
02590
02591     template<typename T, size_t i>
02592     struct atanh_coeff_helper<T, i, std::enable_if_t<(i & 1) == 0> {
02593         using type = typename FractionField<T>::zero;
02594     };
02595
02596     template<typename T, size_t i>
02597     struct atanh_coeff {
02598         using type = typename atanh_coeff_helper<T, i>::type;
02599     };
02600
02601     template<typename T, size_t i, typename E = void>
02602     struct tan_coeff_helper;
02603
02604     template<typename T, size_t i>
02605     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02606         using type = typename FractionField<T>::zero;
02607     };
02608
02609     template<typename T, size_t i>
02610     struct tan_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02611     private:
02612         // 4^((i+1)/2)
02613         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02614         // 4^((i+1)/2) - 1
02615         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02616         // (-1)^((i-1)/2)
02617         using altp = typename FractionField<T>::template inject_t<alternate_t<T, (i - 1) / 2>;
02618         using dividend = typename FractionField<T>::template mul_t<
02619             altp,
02620             FractionField<T>::template mul_t<
02621                 _4p,
02622                 FractionField<T>::template mul_t<
02623                     _4pml,
02624                     bernoulli_t<T, (i + 1)>
02625                 >

```

```

02626         >
02627         >;
02628     public:
02629         using type = typename FractionField<T>::template div_t<dividend,
02630             typename FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
02631     };
02632
02633     template<typename T, size_t i>
02634     struct tan_coeff {
02635         using type = typename tan_coeff_helper<T, i>::type;
02636     };
02637
02638     template<typename T, size_t i, typename E = void>
02639     struct tanh_coeff_helper;
02640
02641     template<typename T, size_t i>
02642     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) == 0> {
02643         using type = typename FractionField<T>::zero;
02644     };
02645
02646     template<typename T, size_t i>
02647     struct tanh_coeff_helper<T, i, std::enable_if_t<(i % 2) != 0> {
02648     private:
02649         using _4p = typename FractionField<T>::template inject_t<pow_t<T, 4, (i + 1) / 2>;
02650         using _4pml = typename FractionField<T>::template sub_t<_4p, typename
FractionField<T>::one>;
02651         using dividend =
02652             typename FractionField<T>::template mul_t<
02653                 _4p,
02654                 typename FractionField<T>::template mul_t<
02655                     _4pml,
02656                     bernoulli_t<T, (i + 1)>>::type;
02657     public:
02658         using type = typename FractionField<T>::template div_t<dividend,
02659             FractionField<T>::template inject_t<factorial_t<T, i + 1>>>;
02660     };
02661
02662     template<typename T, size_t i>
02663     struct tanh_coeff {
02664         using type = typename tanh_coeff_helper<T, i>::type;
02665     };
02666 } // namespace internal
02667
02668 template<typename Integers, size_t deg>
02669 using exp = taylor<Integers, internal::exp_coeff, deg>;
02670
02671 template<typename Integers, size_t deg>
02672 using expml = typename polynomial<FractionField<Integers>>::template sub_t<
02673     exp<Integers, deg>,
02674     typename polynomial<FractionField<Integers>>::one>;
02675
02676 template<typename Integers, size_t deg>
02677 using lnpl = taylor<Integers, internal::lnpl_coeff, deg>;
02678
02679 template<typename Integers, size_t deg>
02680 using atan = taylor<Integers, internal::atan_coeff, deg>;
02681
02682 template<typename Integers, size_t deg>
02683 using sin = taylor<Integers, internal::sin_coeff, deg>;
02684
02685 template<typename Integers, size_t deg>
02686 using sinh = taylor<Integers, internal::sh_coeff, deg>;
02687
02688 template<typename Integers, size_t deg>
02689 using cosh = taylor<Integers, internal::cosh_coeff, deg>;
02690
02691 template<typename Integers, size_t deg>
02692 using cos = taylor<Integers, internal::cos_coeff, deg>;
02693
02694 template<typename Integers, size_t deg>
02695 using geometric_sum = taylor<Integers, internal::geom_coeff, deg>;
02696
02697 template<typename Integers, size_t deg>
02698 using asin = taylor<Integers, internal::asin_coeff, deg>;
02699
02700 template<typename Integers, size_t deg>
02701 using asinh = taylor<Integers, internal::asinh_coeff, deg>;
02702
02703 template<typename Integers, size_t deg>
02704 using atanh = taylor<Integers, internal::atanh_coeff, deg>;
02705
02706 template<typename Integers, size_t deg>
02707 using tan = taylor<Integers, internal::tan_coeff, deg>;
02708
02709 template<typename Integers, size_t deg>
02710 using tanh = taylor<Integers, internal::tanh_coeff, deg>;
02711 } // namespace aerobus

```

```

02762
02763 // continued fractions
02764 namespace aerobus {
02773     template<int64_t... values>
02774     struct ContinuedFraction {};
02775
02776     template<int64_t a0>
02777     struct ContinuedFraction<a0> {
02781         using type = typename q64::template inject_constant_t<a0>;
02783         static constexpr double val = static_cast<double>(a0);
02784     };
02785
02786     template<int64_t a0, int64_t... rest>
02787     struct ContinuedFraction<a0, rest...> {
02792         using type = q64::template add_t<
02793             typename q64::template inject_constant_t<a0>,
02794             typename q64::template div_t<
02795                 typename q64::one,
02796                 typename ContinuedFraction<rest...>::type
02797             >;
02799         static constexpr double val = type::template get<double>();
02800     };
02801
02802     using PI_fraction =
02803     ContinuedFraction<3, 7, 15, 1, 292, 1, 1, 1, 2, 1, 3, 1, 14, 2, 1, 1, 2, 2, 2, 2, 1>;
02804     using E_fraction =
02805     ContinuedFraction<2, 1, 2, 1, 1, 4, 1, 1, 6, 1, 1, 8, 1, 1, 10, 1, 1, 12, 1, 1, 14, 1, 1>;
02806     using SQRT2_fraction =
02807     ContinuedFraction<1, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2, 2>;
02808     using SQRT3_fraction =
02809     ContinuedFraction<1, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2, 1, 2>;
02810 // NOLINT
02811 } // namespace aerobus
02812
02813 // known polynomials
02814 namespace aerobus {
02815     // CChebyshev
02816     namespace internal {
02817         template<int kind, size_t deg>
02818         struct chebyshev_helper {
02819             using type = typename pi64::template sub_t<
02820                 typename pi64::template mul_t<
02821                     typename pi64::template mul_t<
02822                         pi64::inject_constant_t<2>,
02823                         typename pi64::X>,
02824                     typename chebyshev_helper<kind, deg - 1>::type
02825                 >,
02826                 typename chebyshev_helper<kind, deg - 2>::type
02827             >;
02828         };
02829
02830         template<>
02831         struct chebyshev_helper<1, 0> {
02832             using type = typename pi64::one;
02833         };
02834
02835         template<>
02836         struct chebyshev_helper<1, 1> {
02837             using type = typename pi64::X;
02838         };
02839
02840         template<>
02841         struct chebyshev_helper<2, 0> {
02842             using type = typename pi64::one;
02843         };
02844
02845         template<>
02846         struct chebyshev_helper<2, 1> {
02847             using type = typename pi64::template mul_t<
02848                 typename pi64::inject_constant_t<2>,
02849                 typename pi64::X>;
02850         };
02851     } // namespace internal
02852
02853     // Laguerre
02854     namespace internal {
02855         template<size_t deg>
02856         struct laguerre_helper {
02857             private:
02858                 // Lk = (1 / k) * ((2 * k - 1 - x) * Lkm1 - (k - 2) Lkm2)
02859                 using lnm2 = typename laguerre_helper<deg - 2>::type;
02860                 using lnm1 = typename laguerre_helper<deg - 1>::type;
02861                 // -x + 2k-1
02862                 using p = typename pq64::template val<
02863                     typename q64::template inject_constant_t<-1>,
02864                     typename q64::template inject_constant_t<2 * deg - 1>;
02865                 // 1/n

```

```

02869         using factor = typename pq64::template inject_ring_t<
02870             q64::val<typename i64::one, typename i64::template inject_constant_t<deg>>>;
02871
02872     public:
02873         using type = typename pq64::template mul_t <
02874             factor,
02875             typename pq64::template sub_t<
02876                 typename pq64::template mul_t<
02877                     p,
02878                     lnm1
02879                 >,
02880             typename pq64::template mul_t<
02881                 typename pq64::template inject_constant_t<deg-1>,
02882                 lnm2
02883             >
02884         >
02885     >;
02886 };
02887
02888 template<>
02889 struct laguerre_helper<0> {
02890     using type = typename pq64::one;
02891 };
02892
02893 template<>
02894 struct laguerre_helper<1> {
02895     using type = typename pq64::template sub_t<typename pq64::one, typename pq64::X>;
02896 };
02897 } // namespace internal
02898
02899 // Bernstein
02900 namespace internal {
02901     template<size_t i, size_t m, typename E = void>
02902     struct bernstein_helper {};
02903
02904     template<>
02905     struct bernstein_helper<0, 0> {
02906         using type = typename pi64::one;
02907     };
02908
02909     template<size_t i, size_t m>
02910     struct bernstein_helper<i, m, std::enable_if_t<
02911         (m > 0) && (i == 0)>> {
02912         using type = typename pi64::mul_t<
02913             typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02914             typename bernstein_helper<i, m-1>::type>;
02915     };
02916
02917     template<size_t i, size_t m>
02918     struct bernstein_helper<i, m, std::enable_if_t<
02919         (m > 0) && (i == m)>> {
02920         using type = typename pi64::template mul_t<
02921             typename pi64::X,
02922             typename bernstein_helper<i-1, m-1>::type>;
02923     };
02924
02925     template<size_t i, size_t m>
02926     struct bernstein_helper<i, m, std::enable_if_t<
02927         (m > 0) && (i > 0) && (i < m)>> {
02928         using type = typename pi64::add_t<
02929             typename pi64::mul_t<
02930                 typename pi64::sub_t<typename pi64::one, typename pi64::X>,
02931                 typename bernstein_helper<i, m-1>::type>,
02932             typename pi64::mul_t<
02933                 typename pi64::X,
02934                 typename bernstein_helper<i-1, m-1>::type>;
02935     };
02936 } // namespace internal
02937
02938 namespace known_polynomials {
02939     enum hermite_kind {
02940         probabilist,
02941         physicist
02942     };
02943 }
02944
02945 // hermite
02946 namespace internal {
02947     template<size_t deg, known_polynomials::hermite_kind kind>
02948     struct hermite_helper {};
02949
02950     template<size_t deg>
02951     struct hermite_helper<deg, known_polynomials::hermite_kind::probabilist> {
02952     private:
02953         using hnm1 = typename hermite_helper<deg - 1,
02954             known_polynomials::hermite_kind::probabilist>::type;
02955         using hnm2 = typename hermite_helper<deg - 2,

```

```

known_polynomials::hermite_kind::probabilist>::type;
02958
02959     public:
02960         using type = typename pi64::template sub_t<
02961             typename pi64::template mul_t<typename pi64::X, hnm1>,
02962             typename pi64::template mul_t<
02963                 typename pi64::template inject_constant_t<deg - 1>,
02964                 hnm2
02965             >
02966         >;
02967     };
02968
02969     template<size_t deg>
02970     struct hermite_helper<deg, known_polynomials::hermite_kind::physicist> {
02971     private:
02972         using hnm1 = typename hermite_helper<deg - 1,
02973 known_polynomials::hermite_kind::physicist>::type;
02973         using hnm2 = typename hermite_helper<deg - 2,
02974 known_polynomials::hermite_kind::physicist>::type;
02974
02975     public:
02976         using type = typename pi64::template sub_t<
02977             // 2X Hn-1
02978             typename pi64::template mul_t<
02979                 typename pi64::val<typename i64::template inject_constant_t<2>,
02980                 typename i64::zero>, hnm1>,
02981
02982             typename pi64::template mul_t<
02983                 typename pi64::template inject_constant_t<2*(deg - 1)>,
02984                 hnm2
02985             >
02986         >;
02987     };
02988
02989     template<>
02990     struct hermite_helper<0, known_polynomials::hermite_kind::probabilist> {
02991     public:
02992         using type = typename pi64::one;
02993     };
02994
02995     template<>
02996     struct hermite_helper<1, known_polynomials::hermite_kind::probabilist> {
02997     public:
02998         using type = typename pi64::X;
02999     };
03000
03001     template<>
03002     struct hermite_helper<0, known_polynomials::hermite_kind::physicist> {
03003     public:
03004         using type = typename pi64::one;
03005     };
03006
03007     template<>
03008     struct hermite_helper<1, known_polynomials::hermite_kind::physicist> {
03009     public:
03010         // 2X
03011         using type = typename pi64::template val<typename i64::template inject_constant_t<2>,
03012         typename i64::zero>;
03013     };
03014     } // namespace internal
03015
03016     // legendre
03017     namespace internal {
03018     template<size_t n>
03019     struct legendre_helper {
03020     private:
03021         // 1/n constant
03022         // (2n-1)/n X
03023         using fact_left = typename pq64::monomial_t<make_q64_t<2*n-1, n>, 1>;
03024         // (n-1) / n
03025         using fact_right = typename pq64::val<make_q64_t<n-1, n>;
03026     public:
03027         using type = pq64::template sub_t<
03028             typename pq64::template mul_t<
03029                 fact_left,
03030                 typename legendre_helper<n-1>::type
03031             >,
03032             typename pq64::template mul_t<
03033                 fact_right,
03034                 typename legendre_helper<n-2>::type
03035             >
03036         >;
03037     };
03038
03039     template<>
03040     struct legendre_helper<0> {
03041     public:
03042         using type = typename pq64::one;
03043     };
03044
03045     template<>
03046     struct legendre_helper<1> {

```

```

03041         using type = typename pq64::X;
03042     };
03043 } // namespace internal
03044
03045 // bernoulli polynomials
03046 namespace internal {
03047     template<size_t n>
03048     struct bernoulli_coeff {
03049         template<typename T, size_t i>
03050         struct inner {
03051             private:
03052                 using F = FractionField<T>;
03053             public:
03054                 using type = typename F::template mul_t<
03055                     typename F::template inject_ring_t<combination_t<T, i, n>,
03056                     bernoulli_t<T, n-i>
03057                 >;
03058         };
03059     };
03060 } // namespace internal
03061
03062 namespace known_polynomials {
03070     template<size_t deg>
03071     using chebyshev_T = typename internal::chebyshev_helper<1, deg>::type;
03072
03073     template<size_t deg>
03074     using chebyshev_U = typename internal::chebyshev_helper<2, deg>::type;
03075
03076     template<size_t deg>
03077     using laguerre = typename internal::laguerre_helper<deg>::type;
03078
03079     template<size_t deg>
03080     using hermite_prob = typename internal::hermite_helper<deg, hermite_kind::probabilist>::type;
03081
03082     template<size_t deg>
03083     using hermite_phys = typename internal::hermite_helper<deg, hermite_kind::physicist>::type;
03084
03085     template<size_t i, size_t m>
03086     using bernstein = typename internal::bernstein_helper<i, m>::type;
03087
03088     template<size_t deg>
03089     using legendre = typename internal::legendre_helper<deg>::type;
03090
03091     template<size_t deg>
03092     using bernoulli = typename internal::bernoulli_coeff<deg>::template inner, deg>;
03093 } // namespace known_polynomials
03094 } // namespace aerobus
03095
03096 #ifdef AEROBUS_CONWAY_IMPORTS
03097 // conway polynomials
03098 namespace aerobus {
03099     template<int p, int n>
03100     struct ConwayPolynomial {};
03101
03102 #ifndef DO_NOT_DOCUMENT
03103     #define ZPV ZPZ::template val
03104     #define POLYV aerobus::polynomial<ZPV>::template val
03105     template<> struct ConwayPolynomial<2, 1> { using ZPV = aerobus::zpv<2>; using type =
03106         POLYV<ZPV<1>, ZPV<1>; }; // NOLINT
03107     template<> struct ConwayPolynomial<2, 2> { using ZPV = aerobus::zpv<2>; using type =
03108         POLYV<ZPV<1>, ZPV<1>, ZPV<1>; }; // NOLINT
03109     template<> struct ConwayPolynomial<2, 3> { using ZPV = aerobus::zpv<2>; using type =
03110         POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03111     template<> struct ConwayPolynomial<2, 4> { using ZPV = aerobus::zpv<2>; using type =
03112         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03113     template<> struct ConwayPolynomial<2, 5> { using ZPV = aerobus::zpv<2>; using type =
03114         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03115     template<> struct ConwayPolynomial<2, 6> { using ZPV = aerobus::zpv<2>; using type =
03116         POLYV<ZPV<1>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<0>, ZPV<1>; }; // NOLINT
03117     template<> struct ConwayPolynomial<2, 7> { using ZPV = aerobus::zpv<2>; using type =
03118         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>; }; // NOLINT
03119     template<> struct ConwayPolynomial<2, 8> { using ZPV = aerobus::zpv<2>; using type =
03120         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>; }; // NOLINT
03121     template<> struct ConwayPolynomial<2, 9> { using ZPV = aerobus::zpv<2>; using type =
03122         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>; }; //
03123         NOLINT
03124     template<> struct ConwayPolynomial<2, 10> { using ZPV = aerobus::zpv<2>; using type =
03125         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>,
03126         ZPV<1>; }; // NOLINT
03127     template<> struct ConwayPolynomial<2, 11> { using ZPV = aerobus::zpv<2>; using type =
03128         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>,
03129         ZPV<1>; }; // NOLINT
03130     template<> struct ConwayPolynomial<2, 12> { using ZPV = aerobus::zpv<2>; using type =
03131         POLYV<ZPV<1>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<0>, ZPV<1>, ZPV<1>, ZPV<1>, ZPV<0>, ZPV<1>,
03132         ZPV<0>, ZPV<1>; }; // NOLINT

```



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```

05069     template<> struct ConwayPolynomial<983, 3> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<1>, ZPZV<978>; }; // NOLINT
05070     template<> struct ConwayPolynomial<983, 4> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<5>, ZPZV<567>, ZPZV<5>; }; // NOLINT
05071     template<> struct ConwayPolynomial<983, 5> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<8>, ZPZV<978>; }; // NOLINT
05072     template<> struct ConwayPolynomial<983, 6> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<849>, ZPZV<296>, ZPZV<228>, ZPZV<5>; }; // NOLINT
05073     template<> struct ConwayPolynomial<983, 7> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<978>; }; // NOLINT
05074     template<> struct ConwayPolynomial<983, 8> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<738>, ZPZV<276>, ZPZV<530>, ZPZV<5>; }; //
NOLINT
05075     template<> struct ConwayPolynomial<983, 9> { using ZPZ = aerobus::zpz<983>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<858>, ZPZV<87>, ZPZV<978>;
}; // NOLINT
05076     template<> struct ConwayPolynomial<991, 1> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<985>; }; // NOLINT
05077     template<> struct ConwayPolynomial<991, 2> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<989>, ZPZV<6>; }; // NOLINT
05078     template<> struct ConwayPolynomial<991, 3> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<985>; }; // NOLINT
05079     template<> struct ConwayPolynomial<991, 4> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<10>, ZPZV<794>, ZPZV<6>; }; // NOLINT
05080     template<> struct ConwayPolynomial<991, 5> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<3>, ZPZV<985>; }; // NOLINT
05081     template<> struct ConwayPolynomial<991, 6> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<637>, ZPZV<855>, ZPZV<278>, ZPZV<6>; }; // NOLINT
05082     template<> struct ConwayPolynomial<991, 7> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<7>, ZPZV<985>; }; // NOLINT
05083     template<> struct ConwayPolynomial<991, 8> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<15>, ZPZV<941>, ZPZV<786>, ZPZV<234>, ZPZV<6>; }; //
NOLINT
05084     template<> struct ConwayPolynomial<991, 9> { using ZPZ = aerobus::zpz<991>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<9>, ZPZV<466>, ZPZV<222>, ZPZV<985>;
}; // NOLINT
05085     template<> struct ConwayPolynomial<997, 1> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<990>; }; // NOLINT
05086     template<> struct ConwayPolynomial<997, 2> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<995>, ZPZV<7>; }; // NOLINT
05087     template<> struct ConwayPolynomial<997, 3> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<2>, ZPZV<990>; }; // NOLINT
05088     template<> struct ConwayPolynomial<997, 4> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<4>, ZPZV<622>, ZPZV<7>; }; // NOLINT
05089     template<> struct ConwayPolynomial<997, 5> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<10>, ZPZV<990>; }; // NOLINT
05090     template<> struct ConwayPolynomial<997, 6> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<981>, ZPZV<58>, ZPZV<260>, ZPZV<7>; }; // NOLINT
05091     template<> struct ConwayPolynomial<997, 7> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<1>, ZPZV<990>; }; // NOLINT
05092     template<> struct ConwayPolynomial<997, 8> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<934>, ZPZV<473>, ZPZV<241>, ZPZV<7>; }; //
NOLINT
05093     template<> struct ConwayPolynomial<997, 9> { using ZPZ = aerobus::zpz<997>; using type =
POLYV<ZPZV<1>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<0>, ZPZV<39>, ZPZV<732>, ZPZV<616>, ZPZV<990>;
}; // NOLINT
05094 #endif // DO_NOT_DOCUMENT
05095 } // namespace aerobus
05096 #endif // AEROBUS_CONWAY_IMPORTS
05097
05098 #endif // __INC_AEROBUS__ // NOLINT

```

## Chapter 10

# Examples

### 10.1 QuotientRing

inject a 'constant' in quotient ring

inject a 'constant' in quotient ring<i32, i32::val<2>>::inject\_constant\_t<1>>

Template Parameters

x	a 'constant' from Ring point of view
---	--------------------------------------

### 10.2 type\_list

A list of types <int, double, float>

A list of types <int, double, float>

Template Parameters

...Ts	types to store and manipulate at compile time
-------	---

### 10.3 i32::template

inject a native constant

inject a native constant

Template Parameters

x	inject_constant_2<2> -> i32::template val<2>
---	--

## 10.4 i32::add\_t

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

addition operator yields  $v1 + v2$   $\langle i32::val\langle 2 \rangle, i32::val\langle 3 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.5 i32::sub\_t

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

subtraction operator yields  $v1 - v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.6 i32::mul\_t

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

multiplication operator yields  $v1 * v2$   $\langle i32::val\langle 3 \rangle, i32::val\langle 2 \rangle \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.7 i32::div\_t

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

division operator yields  $v1 / v2$   $\langle i32::val\langle 7 \rangle, i32::val\langle 2 \rangle \rangle \rightarrow i32::val\langle 3 \rangle$

Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32



## 10.8 i32::gt\_t

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

strictly greater operator ( $v1 > v2$ ) yields  $v1 > v2$  <i32::val<7>, i32::val<2>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.9 i32::eq\_t

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

equality operator (type) yields  $v1 == v2$  as `std::integral_constant<bool>` <i32::val<2>, i32::val<2>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.10 i32::eq\_v

equality operator (boolean value)

equality operator (boolean value)

### Template Parameters

<i>v1</i>	
<i>v2</i>	<i32::val<1>, i32::val<1>>

## 10.11 i32::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

greatest common divisor yields  $GCD(v1, v2)$  <i32::val<6>, i32::val<15>>

### Template Parameters

<i>v1</i>	a value in i32
<i>v2</i>	a value in i32

## 10.12 i32::pos\_t

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

positivity operator yields  $v > 0$  as `std::true_type` or `std::false_type` `<i32::val<1`

Template Parameters

$v$	a value in i32
-----	----------------

## 10.13 i32::pos\_v

positivity (boolean value) yields  $v > 0$  as boolean value

positivity (boolean value) yields  $v > 0$  as boolean value

Template Parameters

$v$	a value in i32 <code>&lt;i32::val&lt;1&gt;&gt;</code>
-----	---

## 10.14 i64::template

injects constant as an i64 value

injects constant as an i64 value

Template Parameters

$x$	<code>inject_constant_t&lt;2&gt;</code>
-----	---

## 10.15 i64::add\_t

addition operator

addition operator

Template Parameters

$v1$	: an element of <code>aerobus::i64::val</code>
$v2$	: an element of <code>aerobus::i64::val</code> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.16 i64::sub\_t

subtraction operator

subtraction operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.17 i64::mul\_t

multiplication operator

multiplication operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.18 i64::div\_t

division operator integer division

division operator integer division

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>, i64::val<2>>

## 10.19 i64::mod\_t

modulus operator

modulus operator

Template Parameters

<i>v1</i>	: an element of <a href="#">aerobus::i64::val</a>
<i>v2</i>	: an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>>

## 10.20 i64::gt\_t

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

strictly greater operator yields  $v1 > v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;1&gt;&gt;</code>

## 10.21 i64::lt\_t

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

strict less operator yields  $v1 < v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.22 i64::lt\_v

strictly smaller operator yields  $v1 < v2$  as boolean value

strictly smaller operator yields  $v1 < v2$  as boolean value

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;1&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.23 i64::eq\_t

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

equality operator yields  $v1 == v2$  as `std::true_type` or `std::false_type`

### Template Parameters

<code>v1</code>	: an element of <a href="#">aerobus::i64::val</a>
<code>v2</code>	: an element of <a href="#">aerobus::i64::val</a> <code>&lt;i64::val&lt;2&gt;, i64::val&lt;2&gt;&gt;</code>

## 10.24 i64::eq\_v

equality operator yields  $v1 == v2$  as boolean value

equality operator yields  $v1 == v2$  as boolean value

Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<2>, i64::val<2>>

## 10.25 i64::gcd\_t

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of i64::val

greatest common divisor yields  $GCD(v1, v2)$  as instantiation of i64::val

Template Parameters

$v1$	: an element of <a href="#">aerobus::i64::val</a>
$v2$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<6>, i64::val<15>>

## 10.26 i64::pos\_t

is v positive yields  $v > 0$  as std::true\_type or std::false\_type

is v positive yields  $v > 0$  as std::true\_type or std::false\_type

Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>>
-----	---

## 10.27 i64::pos\_v

positivity yields  $v > 0$  as boolean value

positivity yields  $v > 0$  as boolean value

Template Parameters

$v$	: an element of <a href="#">aerobus::i64::val</a> <i64::val<1>>
-----	---

## 10.28 polynomial

makes the constant (native type) polynomial `a_0`

makes the constant (native type) polynomial `a_0`

Template Parameters

<code>x</code>	<code>&lt;i32&gt;::template inject_constant_t&lt;2&gt;</code>
----------------	---

## 10.29 q32::add\_t

addition operator

addition operator

Template Parameters

<code>v1</code>	a value
<code>v2</code>	a value <code>&lt;q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;2&gt;&gt;, q32::val&lt;i32::val&lt;1&gt;, i32::val&lt;3&gt;&gt;&gt;</code>

## 10.30 FractionField

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$ .

Fraction field of an euclidean domain, such as  $\mathbb{Q}$  for  $\mathbb{Z}$

Template Parameters

<code>Ring</code>	<code>&lt;i64&gt;</code> is q64 (rationals with 64 bits numerator and denominator)
-------------------	--

## 10.31 aerobus::ContinuedFraction

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$

represents a continued fraction  $a_0 + \frac{1}{a_1 + \frac{1}{a_2 + \dots}}$  [ [https://en.wikipedia.org/wiki/Continued\\_fraction](https://en.wikipedia.org/wiki/Continued_fraction)](See in Wikipedia)

Template Parameters

<code>...values</code>	are <code>int64_t</code>
------------------------	-----------------------------

$\langle 1, 1, 1 \rangle$  represents  $1 + \frac{1}{1}$

## 10.32 PI\_fraction::val

representation of  $\pi$  as a continued fraction -> 3.14...

## 10.33 E\_fraction::val

approximation of  $e$  -> 2.718...

approximation of  $e$  -> 2.718...





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