equation :

$$\dot{T} = \alpha\, T_{x,x}$$

TF ↓

$$i\omega \tilde{T} = \alpha \tilde{T}_{x,x}$$

solution ↓

$$\tilde{T}(x,\omega) = A\, e^{+(-1)^{\frac{1}{4}} \sqrt{\frac{\omega}{\alpha}} \cdot x} + B\, e^{-(-1)^{\frac{1}{4}} \sqrt{\frac{\omega}{\alpha}} \cdot x}$$

Avec un mur ω :

$$(-1)^{\frac{1}{4}} = \frac{1+i}{\sqrt{2}} \qquad T \xrightarrow[x \to +\infty]{} 0 \Rightarrow A = 0$$

$$\tilde{T}(x,\omega) = \tilde{T}(0,\omega) \cdot e^{-\frac{1+i}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}}\, x}$$

$$\tilde{T}_x(x,\omega) = \tilde{T}(0,\omega)\, e^{-\frac{1+i}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}} \cdot x} \cdot \left(-\frac{1+i}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}}\right)$$

$$\tilde{T}_x(0,\omega) = -\tilde{T}(0,\omega)\, \frac{1+i}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}}$$

cl : $x = 0$

T_{paroi}

$$-k \frac{\partial T}{\partial x} = h(T_i - T_o)$$

$$+\frac{h}{k} \tilde{T}(0,\omega) + \tilde{T}(0,\omega) \frac{(1+i)}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}} = \frac{h}{k} \tilde{T}_i$$

$$\boxed{\tilde{T}(0,\omega)\left[\frac{h}{k} + \frac{(1+i)}{\sqrt{2}} \sqrt{\frac{\omega}{\alpha}}\right] = \frac{h}{k} \tilde{T}_i}$$

Avec un mur d'épaisseur fini L

$$\tilde{T}(x,\omega) = A\, e^{+(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,x} + B\, e^{-(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,x}$$

$$\tilde{T}_x = A(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\, e^{+(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,x} - B(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\, e^{-(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,x}$$

$$\tilde{T}_x(L,\omega) = 0 \rightarrow \quad \frac{A}{B} = e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L}$$

$$\tilde{T}(0,\omega) = A + B = B\left(1 + e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L}\right)$$

$$\tilde{T}_x(0,\omega) = A(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}} - B(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}} = (A-B)(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}$$

$$= B\left(e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L} - 1\right)(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}$$

$$= \tilde{T}(0,\omega)\,\frac{e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L} - 1}{e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L} + 1}\cdot(-i)^{\frac{1}{4}}\cdot\sqrt{\frac{\omega}{\alpha}}$$

CL: $-k\left.\frac{\partial T}{\partial x}\right)_0 = h\left(\tilde{T}_{Air} - \tilde{T}_{x=0}\right)$

$$\tilde{T}(0,\omega)\left[\frac{h}{k} + \underbrace{\frac{1 - e^{-2(-i)^{\frac{1}{4}}\sqrt{\frac{\omega}{\alpha}}\,L}}{1 + e}}_{E}\cdot\underbrace{(-i)^{\frac{1}{4}}}_{\frac{(1+i)}{\sqrt{2}}}\sqrt{\frac{\omega}{\alpha}}\right] = \frac{h}{k}\,\tilde{T}_{AiR}.$$