QOSF Task for admission: Task1

Name: Sunil Kumar Pattnayak

Task:

Given a positive integer "k" and a list of integer numbers, look for the numbers within the list, that are less than k. Consider an appropriate number of qubits and explain why your proposal is valid for all kinds of numbers in case.

Proposed Solution:

Given information:

1. **key**: number to be compared with (k)
2. **list[]**: List of numbers

Output:

1. Numbers from the given list which are **smaller** than key.

Approach:

Programming language: Python
Libraries: Qiskit, Numpy, Math

In this case Grover's algorithm is used to find the required solution.
Grover's algorithm consists of two primary parts.

a. **Oracle:**

This oracle gives f(x) = 1 only when a number given to it as an input is less than **key**. This is incorporated using the **IntegerComparator** from qiskit circuit library.

This is **key-specific,** i.e. the oracle is designed specifically for the key provided in the question and will change of the key is changed.

It compares the number given number with the **key** and returns 1 if the number is less than the key.

b. **Diffusion Matrix:**

This is the standard diffusion matrix used in the Grover's algorithm for inversion around the mean.

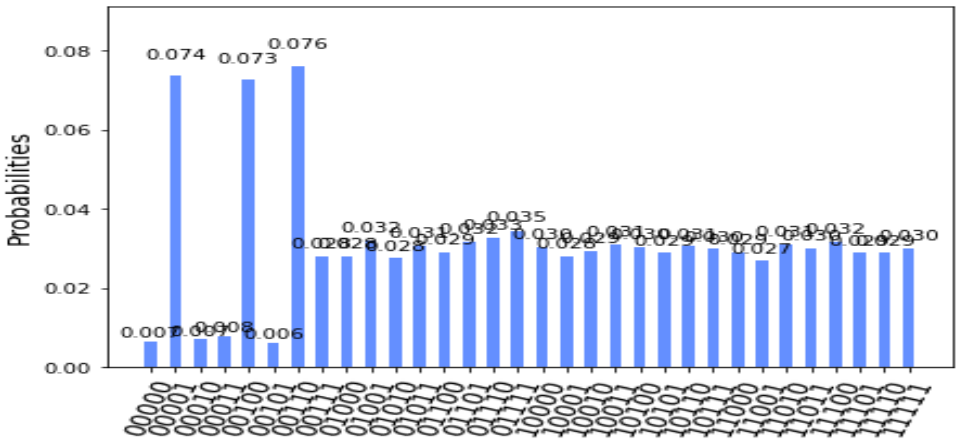
Output:

The solution is achieved using the qasm_simulator available in qiskit and the histogram plot is provided where the answer states are amplified compared to other states.

Example:

>>> less_than_k(7,[4,9,11,14,1,13,6,15])

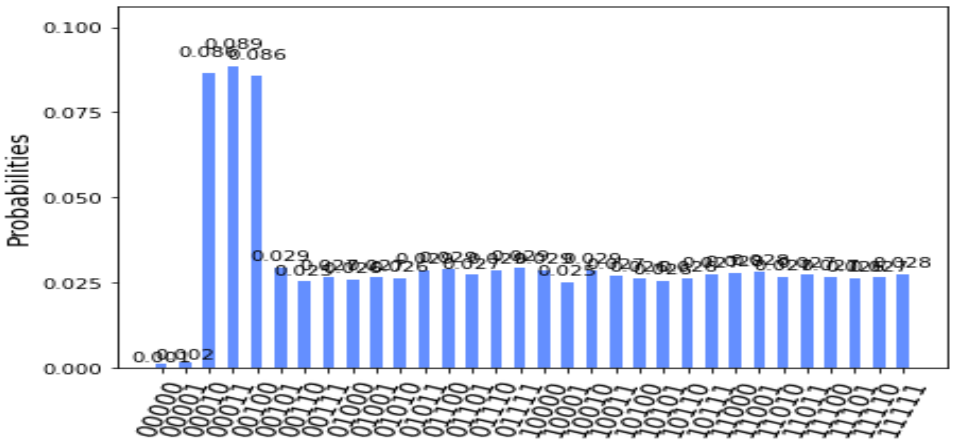
>>>



It is quite clear in the above histogram plot that the numbers, 1('00001'), 4('00100') and 6('00110') are amplified compared to other numbers.

>>> less_than_k(5,[3,7,4,2,11,14,5,15,12])

>>>



It is quite clear in the above histogram plot that the numbers, 2('00010'), 3('00011') and 4('00100') are amplified compared to other numbers.

Remarks:

1. As this solution is based on Grover's search, it is important to note that the solutions will be wrong if the number of iterations will be different than the ideal case. This optimum number in turn depends on the input list size(N) as well as the correct answer list size(k). Here this is kept a value which assumes the sample size of the answer list is much less compared to the input sample size.(k<<N)
   User is advised to change it manually in case the above situation changes by changing the value of iterations by +/- 1.

2. The oracle can be designed from scratch as well for the integer comparison. However, as it was available in qiskit, the same was used.
3. As the input is a superposition state of the numbers from the input list, usual hadamards aren't used. This is incorporated using a custom state defined in the code as per the list entries.
4. The numbers in the list given must be unique.

Circuit for
less_than_k(7,[4,9,11,14,1,13,6,15]) for 2 iterations