CoCoMA (2) – Negotiation
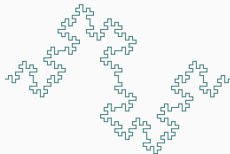
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CoCoMA — Master ANDROIDE

Outline of the Course

- Part I: Bilateral Negotiation
- Part II: Multilateral Negotiation
- Part III: Argument-based Negotiation

General References

Negotiation is a huge topic, it has been studied in many fields for many years.

Raiffa. *The art and science of negotiation*. 1982.

These are some general AI/MAS books, notes, with nice chapters on negotiation:

Wooldridge. *An Introduction to Multiagent Systems*. MIT Press-2004.

Vidal. *Fundamentals of Multiagent Systems*. 2007.

And these are two classic books on the subject:

Rosenschein & Zlotkin. *Rules of Encounter: Designing Conventions for Autamted Negotiation among Agents*. 1994.

Kraus. *Strategic Negotiation in Multiagent Environments*. 2001.

Approaches to Negotiation

It is also common to find the following distinction:

- **Game-theoretic**—use of mathematical tools, as developed in game-theory, to analyze strategical interaction. Provable properties, strong assumptions.

- **Heuristic-based**—design of good strategies in practice, in specific domains of negotiation. More realistic assumptions, more difficult to guarantee properties.

- **Argument-based**—allows the exchange of arguments during negotiation.

The setting

Bilateral Negotiation: The Setting

We first describe the outcome set $\mathcal{O}$. This set may have different characteristics.

Compare the following scenario:

1. we must decide on the next location for the summer school.

$$o_1 = \langle \text{bali} \rangle$$

2. we must divide a chocolate-vanilla cake
   division of a continuous resource.

$$o_1 = \langle 1/3, 2/3 \rangle$$

3. there are 4 candies, we must decide on a complete allocation of resources to children.
   allocation of indivisible resources.

$$o_1 = \langle \{c_1, c_4\}, \{c_2, c_3\} \rangle$$

The Setting: The Outcome Set

The outcome set may be very large, even in the discrete case:

- allocations of indivisible resources
  g goods, so $|\mathcal{O}| = |\mathcal{A}|^g$ outcomes
- choice in a multi-issue domain
  p issues, with D_i the domain of the issue i, so $|\mathcal{O}| = \Pi_i |D_i|$

Example: Choosing the next holiday package:

- $D_d = \{1\text{week}, 2\text{weeks}\}$
- $D_c = \{\text{bali}, \text{lisboa}, \text{moscow}, \text{dakar}\}$
- $D_h = \{\text{pension}, \text{hotel1}, \text{hotel2}, \text{hotel3}, \text{hotel4}\}$
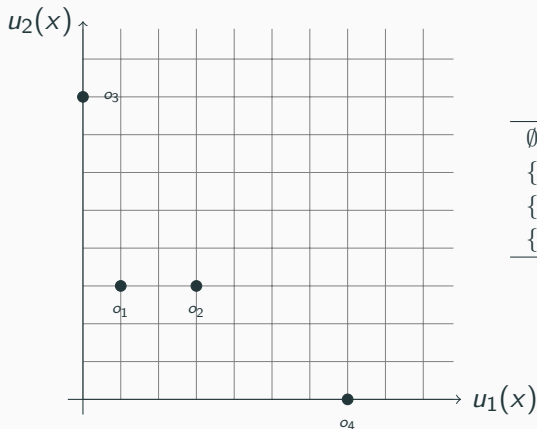- $D_t = \{\text{plane}, \text{bike}, \text{car}\}$

This yields $2 \times 4 \times 5 \times 3 = 120$ outcomes.

Negotiation Domain: Convex Outcome Sets

	u_1	u_2
$\emptyset$	0	0
$\{r_1\}$	1	3
$\{r_2\}$	3	3
$\{r_1, r_2\}$	7	8
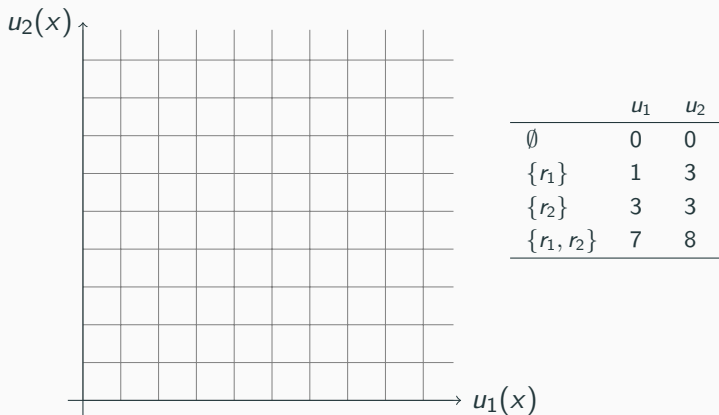
What are the outcomes? Can you place them on this figure?

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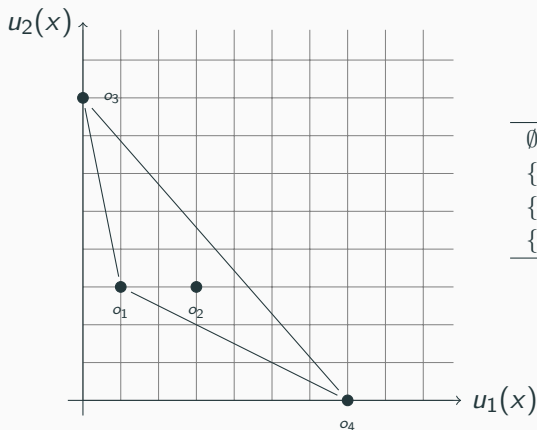
	u_1	u_2
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Randomization between possible outcomes defines a new outcome. For instance, any point on the segment $o_3 - o_4$ is a randomized outcome of the form $\langle p.u_1 + (1 - p).u_1, p.u_2 + (1 - p).u_2 \rangle$. But then the outcome set becomes a convex region.

Some (important) assumptions and remarks

The following remarks are useful:

1. ordinal preferences do not allow interpersonal comparison
2. ordinal preferences cannot represent intensities, cardinal preferences can
3. ordinal preferences can handle incomparabilities, but cardinal preferences cannot
4. explicit representation of cardinal and ordinal preferences require space complexity of $O(|\mathcal{O}|)$ and $O(|\mathcal{O}|^2)$

In the following, we make some assumptions:

1. preferences of agents are common knowledge among all agents (we come back to this later)
2. agents can provide explicit representation of their preferences (more compact way of representing preferences are possible)

The Axiomatic Approach

- We denote by $o^{\downarrow}$ the disagreement (or conflict) point. It indicates the utility that each player gets if the negotiation fails. This needs not be the same for both agents.
- Individual rationality: agents should be better off engaging in the negotiation, that is, for all i, the outcome of the negotiation o must be such that:

$$u_i(o) \geq u_i(o^{\downarrow})$$

	u_1	u_2
$\emptyset$	2	0
$\{r_1\}$	1	3
$\{r_2\}$	3	3
$\{r_1, r_2\}$	7	8

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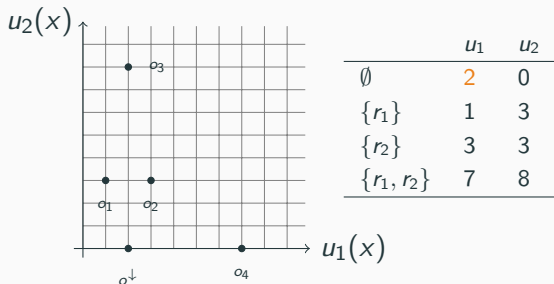
The Negotiation Set

IR plus Pareto brings the negotiation set (Luce and Raiffa, 1957). However, this set contains many possible solutions. Can we restrict further the set of intuitively "fair" outcomes?

Nash (1950) takes an axiomatic approach, and under some assumptions (in particular that the outcome set is convex), shows that the unique solution to a bargaining problem must be the Nash product, provided we accept some "intuitive" axioms.

Nash Bargaining Solution

A bargaining problem is described as a pair $\langle \mathcal{O}, o^{\downarrow} \rangle$.

We write $o^* = NBS(\langle \mathcal{O}, oc \rangle)$ for the outcome selected.

<u>Basic axioms</u>:

- Pareto the solution should be on the Pareto-frontier
- IR the outcome should be individually rational

<u>Additional axioms</u>:

- Symmetry
- Linear Invariance
- Independance of Irrelevant Alternatives

We discuss them in more details now.

Nash Bargaining Solution: Symmetry

Intuitively, symmetry says that agents should be treated the same when their initial situation is equivalent. So,

1. if $u_1(o^\downarrow) = u_2(o^\downarrow)$, and
2. if $\forall o \in \mathcal{O}$: $\exists o' \in \mathcal{O}$ such that $u_1(o) = u_2(o')$ and $u_2(o) = u_1(o')$

then the outcome o^* must be such that $u_1(o^*) = u_2(o^*)$

Nash Bargaining Solution: Linear Invariance

Intuitively, linear invariance says two things:

- independance of scale—the outcome does not depend on the scale used by the agent to represent its utility.
  Suppose agent 1 uses a scale [0,10] to represent its utility, while agent 2 uses a scale [0,100]. The fact that agent 1 enjoys utility 9 and agent 2 utility 50 does not mean that agent 2 is more "happy".

- independance of zero—a translation of the scale of utilities does not affect the outcome.
  Suppose agent 1 uses a scale [0,9], while agent 2 uses a scale [1,10]. The scale of agent 2 can be translated to [0,9] without any consequence on the outcome.

Intuitively, Independence of Irrelevant Alternatives (IIA) says that if the outcome o^* of the negotiation lies in some sub-region of the outcome set, then the negotiation should still select o^* if we restrict the outcome set to this sub-region.

So, removing "irrelevant outcomes" should not affect the result.

More precisely, for any $O \subseteq \mathcal{O}$,
if $NBS(\langle \mathcal{O}, o^\downarrow \rangle) = o^* \in O$ then $NBS(\langle O, o^\downarrow \rangle) = o^*$.

The Nash bargaining solution

The Nash solution is the point o which maximizes

$$(u_1(o) - u_1(o^\downarrow)) \cdot (u_2(o) - u_2(o^\downarrow))$$

This solution verifies the axioms.

Furthermore it is the only solution satisfying exactly these axioms.

Other solutions corresponding to different axioms have been advocated as well (eg. Kalai-Smorodinsky).

The Kalai-Smorodinsky bargaining solution

Replacing IIA by monotony instead characterizes the KS-solution.

Let $b = (b_1, b_2)$ the point of best utilities for 1 and 2. The KS solution is the maximal point o such that:

$$(u_1(o) - u_1(o^\downarrow))/(u_2(o) - u_2(o^\downarrow)) = (b_1(o) - u_1(o^\downarrow))/(b_2(o) - u_2(o^\downarrow))$$

Intuitively: draw a line from $o^\downarrow$ to b.

On this line, the relative gains are equal for both agents.

Pick the outcome which maximizes utilities on this line.

The Kalai-Smorodinsky bargaining solution

Protocols

Properties of Protocols

A protocol specifies the rules of interaction (who can say what?).
Eg, we may allow simultaneous moves, or sequential moves.

A strategy specifies the behavior of the agent (which move to select among all the legal ones?)

We usually require the following properties of protocols+strategies:

- termination—the negotiation will terminate
- guaranteed agreement—the negotiation will end on an agreement (not on the conflict point)
- efficiency—upon termination, the negotiation provides an efficient (eg. Pareto-optimal) outcome
- equilibrium—captures a notion of stability. In particular:
  - symmetric Nash equilibrium: assuming agent 1 uses strategy s, agent 2 cannot be better off using a different strategy than s.
  - subgame perfect equilibrium: in the case of sequential protocol.

Monotonic Concession Protocol

The protocol proceeds in rounds, agents make simultaneous offers. Let o_i^t and o_j^t be the offers made by agent i and j, at round t.

In the initial round, agents make the offer they like, then in the following rounds, each agent must either:

- stick to their previous offer, or
- make a concession (an offer which gives the other more utility)

An agreement is found when, for at least an agent, the offer made by the other agent is at least as good as its own current offer. That is:

$$u_i(o_j^t) \geq u_i(o_i^t) \text{ or } u_j(o_i^t) \geq u_j(o_j^t)$$

(Flip a coin if both agents agree).

A disagreement occurs when both agents stick to their current offer.

Monotonic Concession Protocol: Zeuthen strategy

How should agents play this game? Zeuthen proposes the following:

The willingness to risk conflict (denoted Z_i^t), intuitively captures "how bad" would be a conflict for agent i at round t. It is given by the following formula (assuming $(0, 0)$ for the conflict):

$$Z_i^t = \begin{cases} 1 & \text{if} \quad u_i(o_i^t) = o^\downarrow \\ \frac{u_i(o_i^t) - u_i(o_j^t)}{u_i(o_i^t) - u_i(o^\downarrow)} & \text{otherwise} \end{cases}$$

From this, the Zeuthen strategy is specified as follows, for agent i:

- compute your Z_i^t and that of your partner
- the one with the smallest value should concede
- make the minimal concession making Z_j^t become smaller than Z_i^t
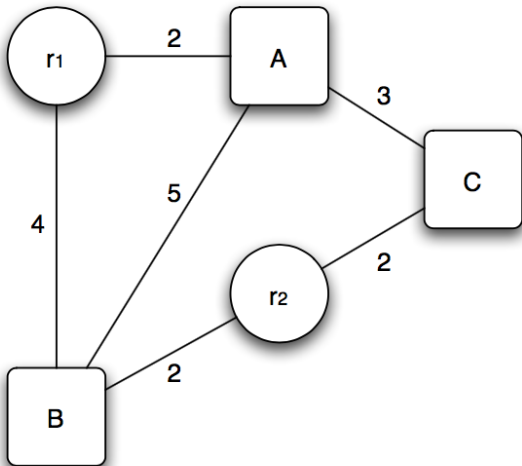
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Monotonic Concession Protocol: Example

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Monotonic Concession Protocol: Example

round	offer a_1	offer a_2	$u_1(o_{a_1}^t), u_1(o_{a_2}^t)$	$u_2(o_{a_1}^t), u_2(o_{a_2}^t)$	Z_1	Z_2
1	$\langle \emptyset, \{a,b,c\} \rangle$	$\langle \{a,b,c\}, \emptyset \rangle$	9,0	0,9	1	1
2	$\langle \{a\}, \{b,c\} \rangle$	$\langle \{a,c\}, \{b\} \rangle$	7,4	3,7	$\frac{3}{7}$	$\frac{4}{7}$
3	$\langle \{a,c\}, \{b\} \rangle$	$\langle \{a,c\}, \{b\} \rangle$	4,4	7,7	stop	stop

Note: at $t = 2$ the offer $a_1 = (b, ac)$ yielding (5,4) is not a correct concession:
while the utility of a_2 grows, it does not ensure that $Z_2 \leq Z_1$ (that the other
one will have to concede in the next round).

Monotonic Concession Protocol: Properties

The properties of the MCP + Zeuthen strategy are as follows:

- termination is guaranteed, as well as agreement upon termination (always at least one agent willing to concede)
- because the offers considered are in the Negotiation Set to start with, Pareto-optimality is obvious

Now a stronger result:

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Explanation: The problem comes from the last step of the protocol. If both agents have the same Z, both are willing to concede, and so one agent can exploit this and deviate to get a better outcome.

Monotonic Concession Protocol

Some final remarks on MCP.

- it is possible to extend the Zeuthen strategy (by allowing a mixed strategy in the last step) to retrieve stability
- a more simple one-step protocol is possible!

The one-step protocol is as follows:

- agents simultaneously make a single offer
- select the one maximizing the product of utilities

What is the best strategy for an agent given this protocol?

Monotonic Concession Protocol

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The one-step protocol is as follows:

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- select the one maximizing the product of utilities

What is the best strategy for an agent given this protocol?
Given this protocol, the strategy for an agent is to select, among the outcomes maximizing, the one giving him the best utility.

Rosenschein & Zlotkin. *Rules of Encounter*. 1993.

Alternating Offers

We now discuss a sequential protocol.

- an agent starts by making an offer. In the next round, the other agent can either accept or make a counter-offer.
- the protocol integrates a discount factor λ_i to capture the fact that negotiation is time constrained. An offer accepted at round t by agent i brings utility $u_i(o^t) \times (\lambda_i)^t$.

The sequential nature of this protocol allows backward induction solving.

Rubinstein. *Perfect equilibrium in a bargaining model*. Econometrica-1982.

Alternating Offers

Set $\lambda = 1$ for agents (they are patient).

- suppose the number of rounds is known in advance. But then the last agent to make an offer gets all the "power". What is his best strategy?

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  Always refuse the offers of the other, then make an offer $\langle 1 - \epsilon, \epsilon \rangle$ in the last round (this last step is actually an ultimatum game: more on this later)

- suppose the number of rounds is not known in advance
  Suppose a_1 uses this strategy: Always propose $(1 - \epsilon, \epsilon)$, and always refuse the offer of the other. What is a_2 best response?

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- suppose the number of rounds is not known in advance
  Suppose a_1 uses this strategy: Always propose $(1 - \epsilon, \epsilon)$, and always refuse the offer of the other. What is a_2 best response? Always refusing yields the conflict outcome. So a_2 must accept at some point, no reason to postpone: accept in the first round. Immediate acceptance of any offer is a Nash equilibrium, given that a_2 knows a_1's strategy.

Alternating Offers: Example

Take $\mathcal{O} = \{o_2, o_3, o_6\}$, with $o_2 = \langle 7, 3 \rangle$, $o_3 = \langle 5, 4 \rangle$, and $o_6 = \langle 4, 7 \rangle$.
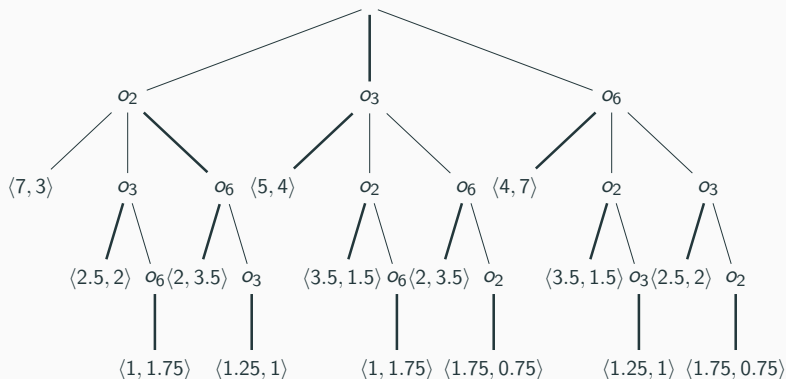

Figure 1: Backward Induction with the alternating-offer protocol

Back to the ultimatum game.

Remember: One agent proposes an offer (say a division of a pie), the offer may either accept or reject. If it accepts the offer is chosen outcome, otherwise the conflict outcome.

What do you a human agent will propose in real life?

A Word of Caution (I)

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What do you a human agent will propose in real life?

- many studies in economics
- usually offers more around a 60/40 division
- importance of social context, reputation, etc.

A Word of Caution (II)

Now consider the following game, known as the centipede game.

There are 100 candies to share, and two agents. The protocol for negotiation is as follows. In each round:

- player i can either take 1 or 2 candies
- if he takes 2 candies, the protocol terminates, and agents keep the candies they have collected so far (the rest is wasted)
- if he takes 1 candy, the protocol continues, by giving the turn to the other agent, and so on.

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Can you analyze this game?

A Word of Caution (II)

Heuristic-Based Negotiation

Often, agents have not an exact knowledge about the preferences of others. It is possible to conceive general profiles of agents, specifying high-level behavior.

- boulware agents—very slow concession until we get close to the deadline, then exponential increase
- conceder agents—prone to concede in the first rounds of negotiation and get close to reserve price, then slow increase

But how can we make sure that a move is indeed a concession in the first place?

Faratin et al.. *Negotiation decision functions for autonomous agents*. Robotics and Autonomous Systems-1998.

Heuristic-Based Negotiation

Consider a multi-issue domain.

Suppose agents' utilities are given as weighted sums. Agents give difference importance to different criteria.

$$u_i(o) = \sum_c w_i^c u_i^c(o)$$

<u>Note</u>: note necessarily zero-sum, nor negatively correlated utilities.

Faratin et al. distinguish:

- response strategies—concession operate on a single issue
- compensation strategies—concede on a single issue but ask more on other issues (trade-offs), so that the new offer has the same utility for itself, but a higher utility for the other one.

Faratin et al.. *Using similarity criteria to make issue trade-offs in automated negotiations*. AIJ-2002.

Heuristic-Based Negotiation

But trying to guess/approximate an agent preference structure based on its negotiation behavior is very challenging!

Idea: seek the offer which is the "closest" from the other agent offer in the preceding move. To do this, compute similarity among offers.

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<u>Idea</u>: seek the offer which is the "closest" from the other agent offer in the preceding move. To do this, compute similarity among offers.

<u>Example:</u>
$\mathcal{D}_c = \{\text{yellow, violet, magenta, green...}\}$
Choose criteria to assess how similar colors are (eg. warmness, perception,...)
$h_w = \{(\text{yellow}, 0.9), (\text{violet}, 0.1), (\text{magenta}, 0.1), (\text{green}, 0.3), ...\}$
$h_p = \{(\text{yellow}, 1), (\text{violet}, 0.5), (\text{magenta}, 0.4), (\text{green}, 0.1), ...\}$

By giving to similarity criteria (eg. 0.7 and 0.3) , we can compute eg.

$$sim(\text{magenta, green}) =$$

$$0.7 \times (1 - |h_w(\text{magenta}) - h_t(\text{green})|) + 0.3 \times (1 - |h_w(\text{magenta}) - h_t(\text{green})|)$$

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Heuristic-Based Negotiation: Example

Opponent modeling

Baarslag et al. find out that opponent modeling essentially tries to model:

- the acceptance strategy
- the deadline
- the preference profile
- the bidding strategy

This can be done thanks to various more or less sophisticated learning techniques (Bayesian learning, non-linear regression, neural networks, etc.)

Baarslag *et al. Learning about the opponent in automated bilateral negotiation: a comprehensive survey of opponent modeling techniques*. JAAMAS-16.

Testing Strategies: Agent Competitions

In recent years, competitions involving negotiating agents have emerged, allowing to test and compare various strategies on different problems.

- ANAC Competition: Automated Negotiating Agents Competition
- TAC: Trading Agent Competition (auctions, etc.)
- Genius platform (negotiation problems, library of agents' strategies) `http://mmi.tudelft.nl/negotiation/index.php/Genius`
- many papers and even books on analysis of the best strategies
- importance of opponent modeling!

Wellmann, Greenwald, & Stone. *Autonomous Bidding Agents: Strategies and Lessons from the TAC competition*. 2007.

Multilateral Negotiation

We will discuss three different settings:

- multilateral negotiation with mediator
- extension of monotonic concession protocol
- negotiation on networks

Single Text Mediated Agent

A first possible approach is to use a mediator.

The protocol is as follows (K is fixed a priori):

```
for t:=1 to K do
begin
  the mediator proposes an offer o ;
  agents votes on o (accept/refuse);
  if all agents accept, then current := o;
end;
```

So the protocol returns the latest unanimously accepted offer.

Starting from an offer, the protocol makes Pareto improvements.

- Is the protocol guaranteed to reach a Pareto-optimal outcome?
- Is the protocol guaranteed to stop when having reached a Pareto optimal outcome?

The outcome is $\langle 10, 2 \rangle$
But the Nash product optimum is $\langle 5, 8 \rangle$
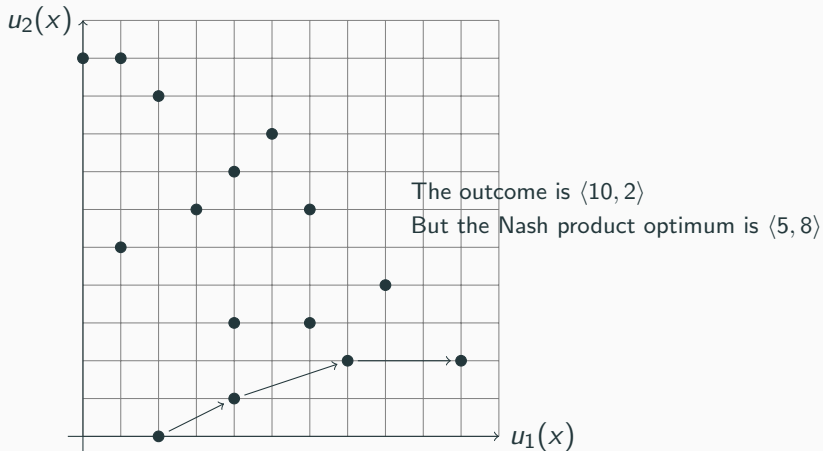
Single Text Mediated Protocol

This protocol may be problematic in some contexts:

- it requires a mediator (not always possible)
- requires many rounds of communication from all agents to the mediator
- it can reach outcomes with very low social welfare

Some extensions have been proposed to try to address some of these limitations:

- use of meta-heuristic techniques to avoid local optima (eg. simulated annealing)
- learning of agents preferences to guide the offer proposal from the mediator

Klein et al.. *Protocols for negotiating complex contracts*. IEEE Intelligent Systems.

Multilateral Negotiation

Is it possible to simply extend the protocols used in the bilateral case? Let us consider the case of the monotonic concession protocols.

Remember:

- An agreement is reached iff one agent makes an offer that is at least as good for each other agent as their own proposal.

But for the notion of concession we run into trouble...

Endriss. *Monotonic Concession Protocols for Multilateral Negotiation*. AAMAS-06.

Multilateral Negotiation

For what it means to concede to many agents?

- **Strong/Weak concession**: make an offer strictly better for all / at least one other agent(s).
- **Pareto concession**: make an offer at least as good for all other agents and strictly better for at least one of them.
- **Egalitarian/Utilitarian/Nash concession**: make an offer such that the min/sum/product of utilities increases
- **Egocentric concession**: make an offer that is worse for yourself.

Termination is guaranteed (except for weak concessions)

Endriss. *Monotonic Concession Protocols for Multilateral Negotiation*. AAMAS-06.

Multilateral Negotiation

- A concession criterion is deadlock-free iff it guarantees that at least one agent can make a concession satisfying the criterion at any stage during negotiation, until an agreement has been reached.

Example: Assume Pareto concessions are used.
Three possible outcomes.

round 1: $o_1 = \langle 3, 2, 1 \rangle$, $o_2 = \langle 1, 3, 2 \rangle$, $o_3 = \langle 2, 1, 3 \rangle$.

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round 2: $o_1 = \langle 1, 3, 2 \rangle$, $o_2 = \langle 2, 1, 3 \rangle$, $o_3 = \langle 3, 2, 1 \rangle$.

No more concessions are possible. An agreement is reached (all agents enjoy utility 1 in their current offer, so any other offer is better).

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Three possible outcomes.

round 1: $o_1 = \langle 2, 1, 3 \rangle$, $o_2 = \langle 3, 2, 1 \rangle$, $o_3 = \langle 1, 3, 2 \rangle$.

Multilateral Negotiation

- A concession criterion is deadlock-free iff it guarantees that at least one agent can make a concession satisfying the criterion at any stage during negotiation, until an agreement has been reached.

Example: Assume Pareto concessions are used.
Three possible outcomes.

round 1: $o_1 = \langle 2, 1, 3 \rangle$, $o_2 = \langle 3, 2, 1 \rangle$, $o_3 = \langle 1, 3, 2 \rangle$.
No more concessions are possible. No agreement is reached!
(all agents enjoy utility 2 in their current offer, so any other makes one of them worse off).

Multilateral Negotiation
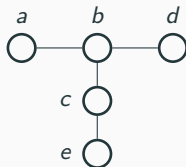
Endriss shows that:

- The weak, the utilitarian, and the egocentric criteria are all deadlock-free.

- The Pareto, the strong, and the egalitarian criteria are not deadlock-free.

- The Nash criterion is deadlock-free iff utilities are required to be positive.

Network Exchange Theory: agents can only negotiate with neighbours.

- agents are now located on a graph G
- each agent can reach an agreement with at most one neighbour
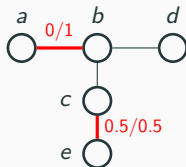- each pair of agents negotiate over the division of 1 euro

Example:

Can you guess how the negotiation will unfold?

Balanced Outcomes

Network Exchange Theory: agents can only negotiate with neighbours.

- agents are now located on a graph G
- each agent can reach an agreement with at most one neighbour
- each pair of agents negotiate over the division of 1 euro

Example:

Intuition: b uses his "power" to have two potential agreements

Balanced Outcomes

More precisely, we can define:

- an outcome is a pair $\langle M, \alpha \rangle$, where M is a matching (which agents agree on a deal), and values α_x for each agent x, with:

  - $\alpha_x + \alpha_y = 1$ when $(x, y) \in M$,
  - $\alpha_x = 0$ when $x \notin M$.

Let β_x be the best alternative for x, that is,
$max\{1 - \alpha_y | (x, y) \in G\}$

- an outcome is stable if $\alpha_x \geq \beta_x$, for all x.

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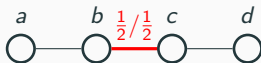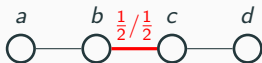

Is this multi-outcome stable?

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Is this multi-outcome stable? Yes!
But contradicted by experiments (b and c have more negotiation

The idea is to strengthen the notion of stability.

- A balanced outcome is an outcome such that, for all $(x, y) \in M$, (α_x, α_y) constitutes a Nash Bargaining Solution considering (β_x, β_y) as the disagreement outcome.

This is not a balanced outcome.

Balanced Outcomes

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- A balanced outcome is an outcome such that, for all $(x, y) \in M$, (α_x, α_y) constitutes a Nash Bargaining Solution considering (β_x, β_y) as the disagreement outcome.

This is not a balanced outcome. Indeed take $(\alpha_a, \alpha_b) = (0.5, 0.5)$.
Given $(\beta_a, \beta_b) = (0, 0.5)$, the surplus $1 - 0.5$ should be evenly divided, yielding $(\alpha_a, \alpha_b) = (0.25, 0.75)$.
But now given $(\beta_c, \beta_d) = (0.25, 0)$, the values of (α_c, α_d) should be modified... $\Rightarrow$ fixed-point definition

- Can you guess the balanced outcome here?

Balanced Outcomes

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- A balanced outcome is an outcome such that, for all $(x, y) \in M$, (α_x, α_y) constitutes a Nash Bargaining Solution considering (β_x, β_y) as the disagreement outcome.

Gives rise to many questions:

- are balanced outcomes guaranteed to exist? (if not, when?)
- are these values rational?
- is it easy to compute these values?
- etc.

Kleinberg & Tardos. *Balanced Outcomes in Social Exchange Networks.* STOC-08.