Fluid dynamics experiments report

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1. Introduction

The viscosity of a fluid is the quantity describing the ability of the fluid to flow, in other words, it is the magnitude of internal friction within the fluid affecting its rate of flow. In this report, we are particularly interested in determining the dynamic viscosity of the golden syrup.
To do so, we conducted two experiments. In the first one, we studied the movement of steel balls bearing through the liquid of different masses. In the second experiment, we studied the spreading of the golden syrup drops on a substrate.
In the first part of this report, we will present the results we got for the first experiment and then use them to estimate the dynamic viscosity of the golden syrup. In the second part, we will present the results of the second experiment and estimate some parameters as including the dynamic viscosity.

2. First experiment

2.1 Theory for the first experiment

2.1.1 Forces

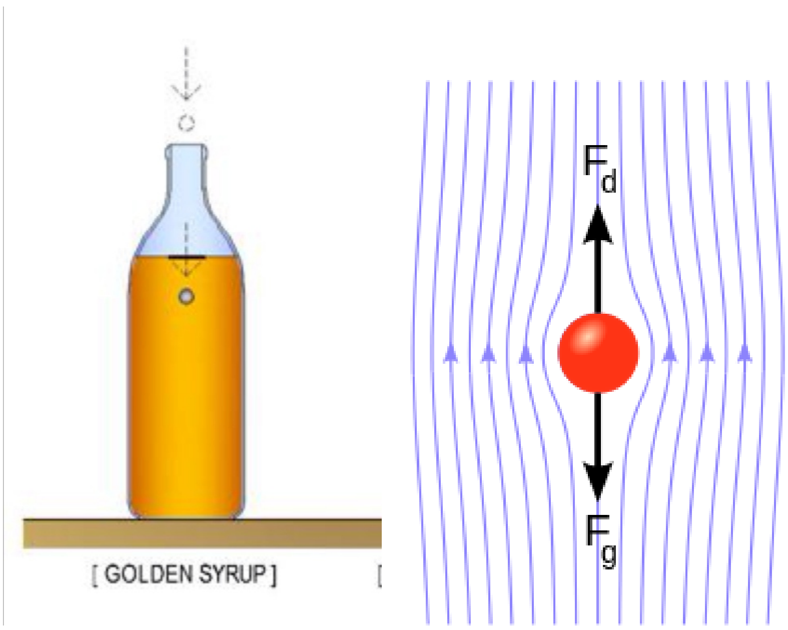

Figure 1: ball bearing through the golden syrup

- gravitational force: $weight = \rho_s V g$, with $\rho_s \sim 8.05 Kg/m^3$ the density of the steel and V the volume of the ball.
- $buoyancy = \rho_f V g$, with ρ_f the density of the fluid

- force due to the viscosity of the fluid: $f = 6\pi a\mu U$, where μ is the viscosity of and a the radius of the ball. The velocity of the balls are subject to the equation:

$$weight - buoyancy - f = \rho_s V \frac{du}{dt} \Leftrightarrow \frac{du}{dt} + \frac{6\pi a\mu}{\rho_s V} u = \frac{\rho_s - \rho_f}{\rho_s} g$$

The solution of that differential equation is

$$u(t) = \frac{\rho_s - \rho_f}{6\pi a\mu} gV \left(1 - \exp\left(\frac{-6\pi a\mu t}{\rho_s V}\right)\right)$$

$V = \frac{4\pi}{3} a^3$. So

$$u(t) = \frac{2a^2 g}{9\mu} (\rho_s - \rho_f) \left(1 - \exp\left(\frac{-9\mu t}{2a^2 \rho_s}\right)\right)$$

We will use this formula and our data to estimate the dynamic viscosity of the fluid.

2.1.2 Boundary conditions

- $u = 0$ at $t = 0$
- $u = 0$ for the liquid far away from the ball.

2.2 Method and material

For this experiment we used the following material:

- Golden syrup
- Six ball bearings of diameter 12.68, 9.52, 7.91, 6.34, 4.75, 3.96
- Stop watch
- Measuring cylinder
- 30 cm Rule

Procedure

- First, we filled the transparent container with the golden syrup
- Second, we released each of the ball as close as possible to the surface of the syrup in the container in order to ensure that the ball starts its movement with zero-velocity.
- Next, we measured the distance travelled at each 5 seconds except the first ball, the biggest one (12.68 mm) for which we did two measurements: 5 and then 10 seconds time interval.
- To determine whether closeness to the wall of the container had any effect on the speed with which the ball falls through the syrup, we repeated the experiment using the 7.91 mm ball, this time releasing it by the side of the container rather than the centre.

2.3 Results and discussion

Data of the experiment:

Time(s)	12.68(mm)	9.52(mm)	7.91(mm)	6.34(mm)	4.75(mm)	3.96 (mm)
0	0	0	0	0	0	0
5	45	30	20	10	8	5
10	45	60	30	30	12	10
15	50	70	50	35	20	13
20	60	80	60	43	25	15
25	75	100	70	50	30	20
30	90	120	80	57	34	25
35	135	150	95	65	42	30
40	145		110	73	50	32
45	150		130	80	56	40
50			140	86	60	42
55				103	65	45
60				115	68	47
65				120	70	51
70				130	73	55
75					77	60
80					80	62
85					85	65
90					90	68
95					93	70
100					97	73
105						75
110						80
115						83
120						85
125						88
130						91
135						93
140						100

This table presents the distance travelled by the different balls.

We plot here the velocity of the balls in each case:

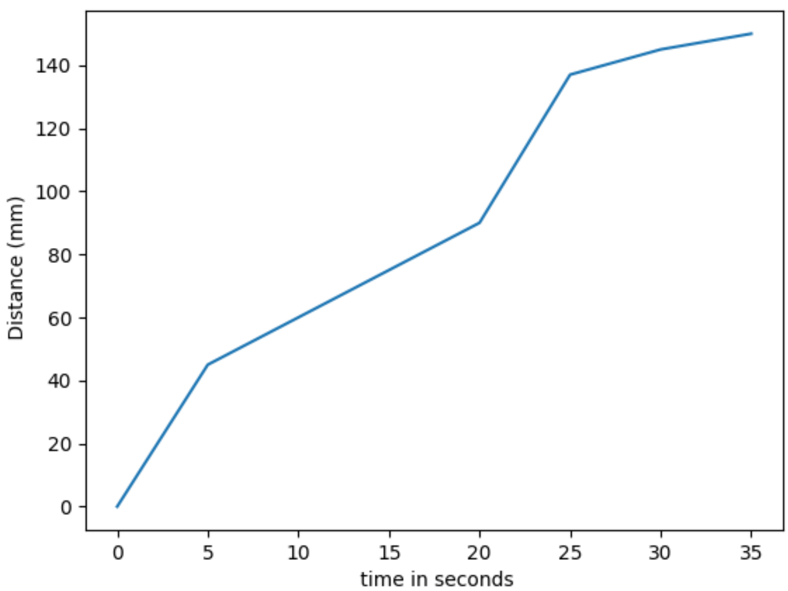

Figure 2: Velocity of the ball of diameter 12.68 mm

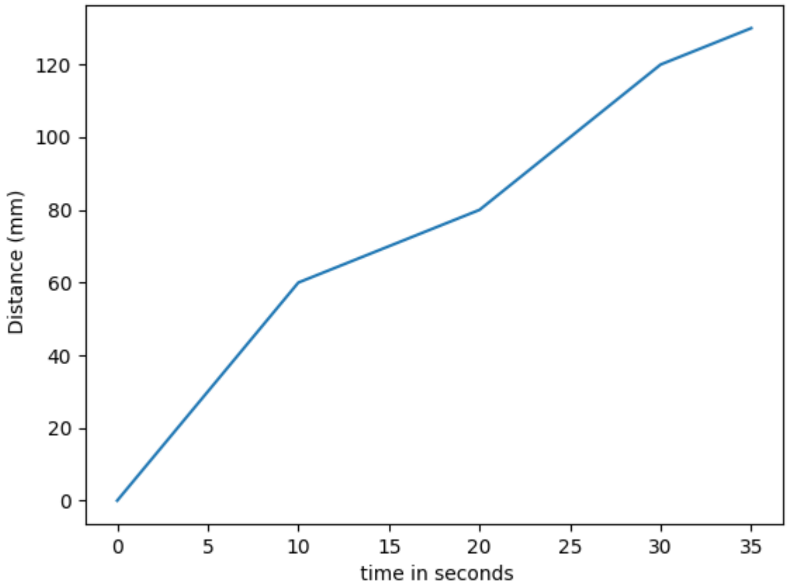

Figure 3: Velocity of the ball of diameter 9.52 mm

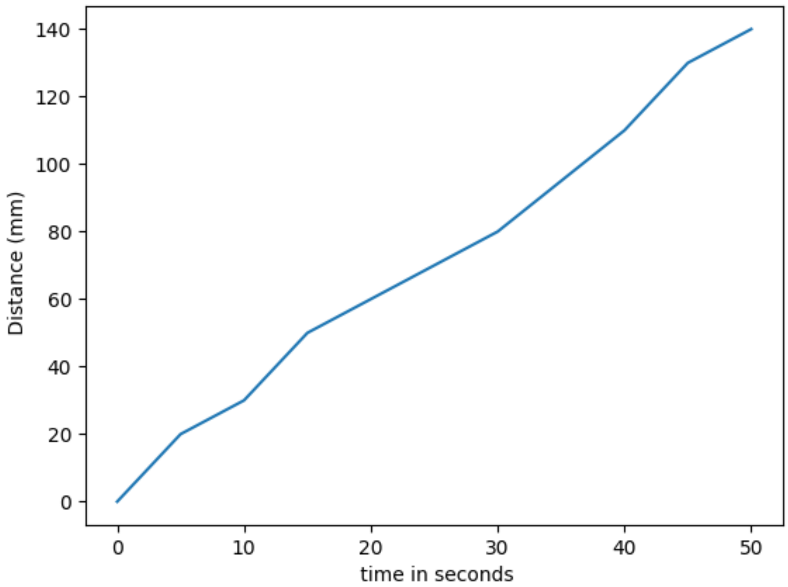

Figure 4: Velocity of the ball of diameter 7.91 mm

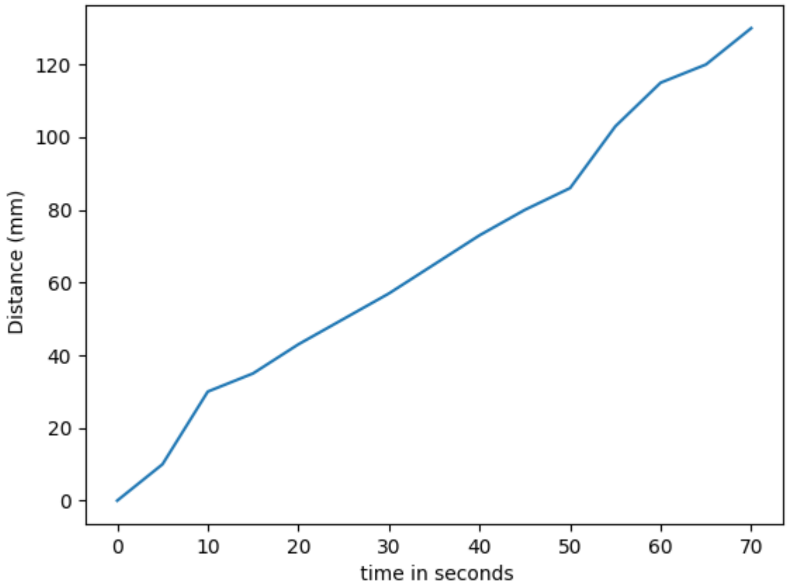

Figure 5: Velocity of the ball of diameter 6.34 mm

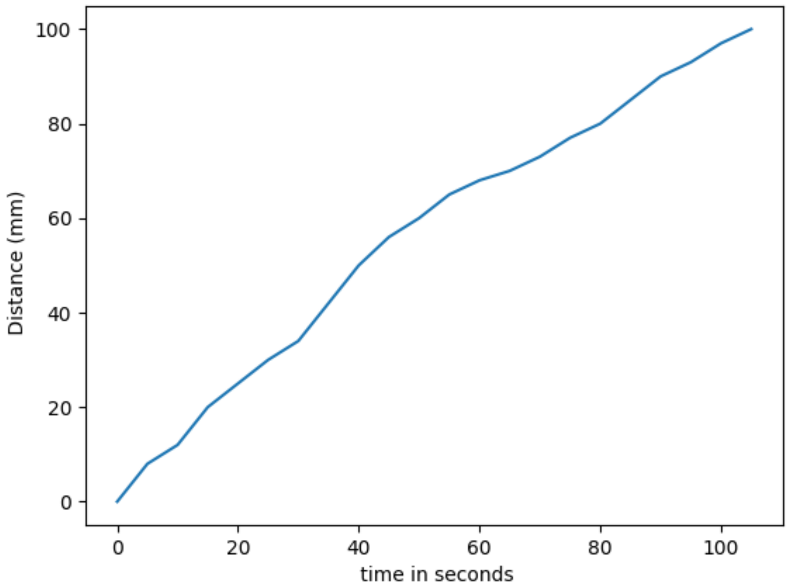

Figure 6: Velocity of the ball of diameter 4.75 mm

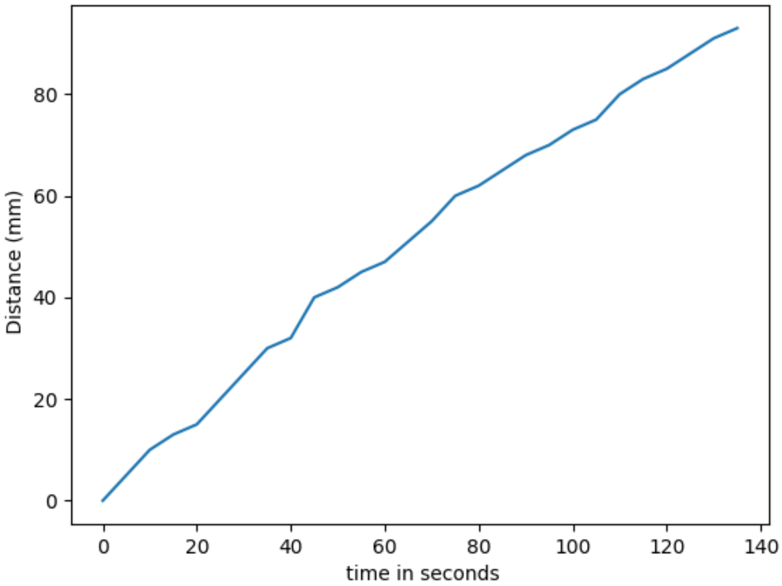

Figure 7: Velocity of the ball of diameter 3.96 mm

- 12.68 mm ball velocity:

The instantaneous velocity at each 5seconds starting at $t = 0\ s$:

$$0.00, 0.009, 0.003, 0.003, 0.003, 0.009, 0.0016 \text{ and } 0.001 \text{ in } ms^{-1}$$

and the average is equal to $0.00375 ms^{-1}$

- 9.52 mm ball velocity:

The instantaneous velocity at each 5 seconds starting at $t = 0\ s$:

$$0.00, 0.006, 0.006, 0.002, 0.002, 0.004, 0.004, 0.002 \text{ in } ms^{-1}$$

and the average is equal to $0.0033 ms^{-1}$

- 7.91 mm ball velocity:

The instantaneous velocity at each 5 seconds starting at $t = 0\ s$:

$$0.00, 0.004, 0.002, 0.004, 0.002, 0.002, 0.002, 0.003, 0.003, 0.004, 0.002 \text{ in } ms^{-1}$$

and the average is equal to $0.0025 ms^{-1}$

- 6.34 mm ball velocity:

The instantaneous velocity at each 5 seconds starting at $t = 0\ s$:

$$0.00, 0.002, 0.004, 0.001, 0.002, 0.001, 0.001, 0.0016, 0.002, 0.001, 0.001, 0.003, 0.002, 0.001, 0.002 \text{ in } ms^{-1}$$

and the average is equal to $0.0017 ms^{-1}$

- 4.75 mm ball velocity:

The instantaneous velocity at each 5 seconds starting at $t = 0\ s$:

$$0.00, 0.016, 0.014, 0.0016, 0.001, 0.001, 0.002, 0.002, 0.002, 0.001, 0.001, 0.001, 0.001, 0.0004, 0.001, 0.001,$$
$$0.001, 0.001, 0.001, 0.001, 0.001, 0.001 \text{ in } ms^{-1}$$

and the average is equal to $0.0009 ms^{-1}$

- 3.96 mm ball velocity:

The instantaneous velocity at each 5 seconds starting at $t = 0\ s$:

$$0.00, 0.001, 0.001, 0.001, 0.0004, 0.001, 0.001, 0.001, 0.0004, 0.0016, 0.0004, 0.001, 0.0004, 0.001, 0.001,$$
$$0.001, 0.0004, 0.001, 0.001, 0.0004, 0.001, 0.0004, 0.001, 0.0006, 0.0004, 0.001, 0.001, 0.0004 \text{ in } ms^{-1}$$

and the average is equal to $0.0007 ms^{-1}$

I wrote a python code to estimate the viscosity of the fluid by minimizing the quantity

$$\sum_{t \in Time} (u(t) - velocity_{measured}(t))^2$$

(least square method) where $Time$ is the time in our data (5s, 10s, 15s, ...)

We found the following estimation in each case:

μ: 12.68 mm ball	μ: 9.52 mm ball	μ: 7.91 mm ball	μ: 6.34 mm ball	μ: 4.75 mm ball	μ: 3.96 mm ball
137.98	89.74	82.18	79.6	87.13	83.72

The average is 93.39 Pa.s
This result is consistent because the theoretical value of the viscosity μ is 100 Pa.s.
We can see that the best estimate comes from the second ball (9.52 mm) with the value 89.74 Pa.s.

3. Second experiment : Gravity Current

3.1 Theory for the second experiment

3.1.1 Forces

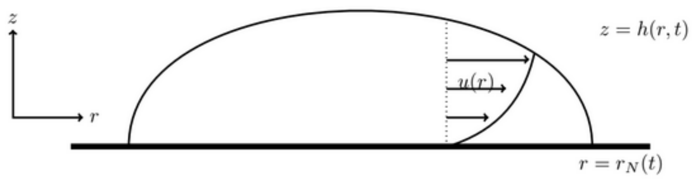

Figure 8:

We have a higher pressure in the middle of the syrup and lower towards the edges. This is due to the position of the centre of mass. The forces controlling the flow of the syrup includes are:

- weight=$\rho_f V g$
- viscous forces which resist to the flow of the syrup
- body forces per unit area

3.1.2 Boundary conditions

- No-slip Condition: The tangential component of the fluid velocity is equal to the tangential component of the velocity of the solid boundary. In our case the solid boundary is rigid and stationary. Therefore, ua = 0 at z = 0.

- No-stress condition: For parallel flow the pressure and the tangential viscous stress are continuous, that is,the top of the current, shear stress and pressure are continuous.

3.2 Method and material

In this experiment, we used:

- a marked horizontal plane
- a stop watch
- a syringe
- a cylinder containing the fluid

The following table contains the results we obtained from observing the spread of the viscous fluid:

	time	Radius 1_1	Radius 1_2	Radius 1_3	20 ml average radius	Radius 2_1	Radius 2_2	Radius 2_3	Radius 2_4	Average radius 15 ml	Radius 3_1	Radius 3_2	Radius 3_3	Radius 3_4	Average radius 10 ml
0	1	3.9	3.8	3.8	3.833333	3.2	3.1	3.10	3.100	3.12500	2.90	3.00	2.900	2.800	2.90000
1	2	4.0	4.2	3.9	4.033333	3.7	3.5	3.50	3.600	3.57500	3.00	3.20	3.000	3.000	3.05000
2	3	4.1	4.5	4.1	4.233333	3.9	3.8	3.70	3.750	3.78750	3.10	3.30	3.100	3.100	3.15000
3	4	4.4	4.7	4.3	4.466667	4.0	3.9	3.80	3.800	3.87500	3.50	3.40	3.150	3.200	3.31250
4	5	4.5	4.8	4.4	4.566667	4.0	4.0	3.90	3.900	3.95000	3.60	3.45	3.180	3.300	3.38250
5	6	4.6	4.9	4.5	4.666667	4.0	4.1	3.95	3.900	3.98750	3.70	3.50	3.190	3.320	3.42750
6	7	4.7	4.9	4.6	4.733333	4.1	4.1	4.00	3.980	4.04500	3.80	3.50	3.190	3.330	3.45500
7	8	4.8	4.9	4.7	4.800000	4.1	4.2	4.10	3.982	4.09550	3.90	3.55	3.195	3.340	3.49625
8	9	4.8	5.0	4.7	4.833333	4.1	4.2	4.10	3.982	4.09550	3.95	3.60	3.195	3.345	3.52250
9	10	4.8	5.0	4.8	4.866667	4.1	4.3	4.10	3.985	4.12125	3.95	3.65	3.196	3.346	3.53550

Figure 9: Table containing the spreading results

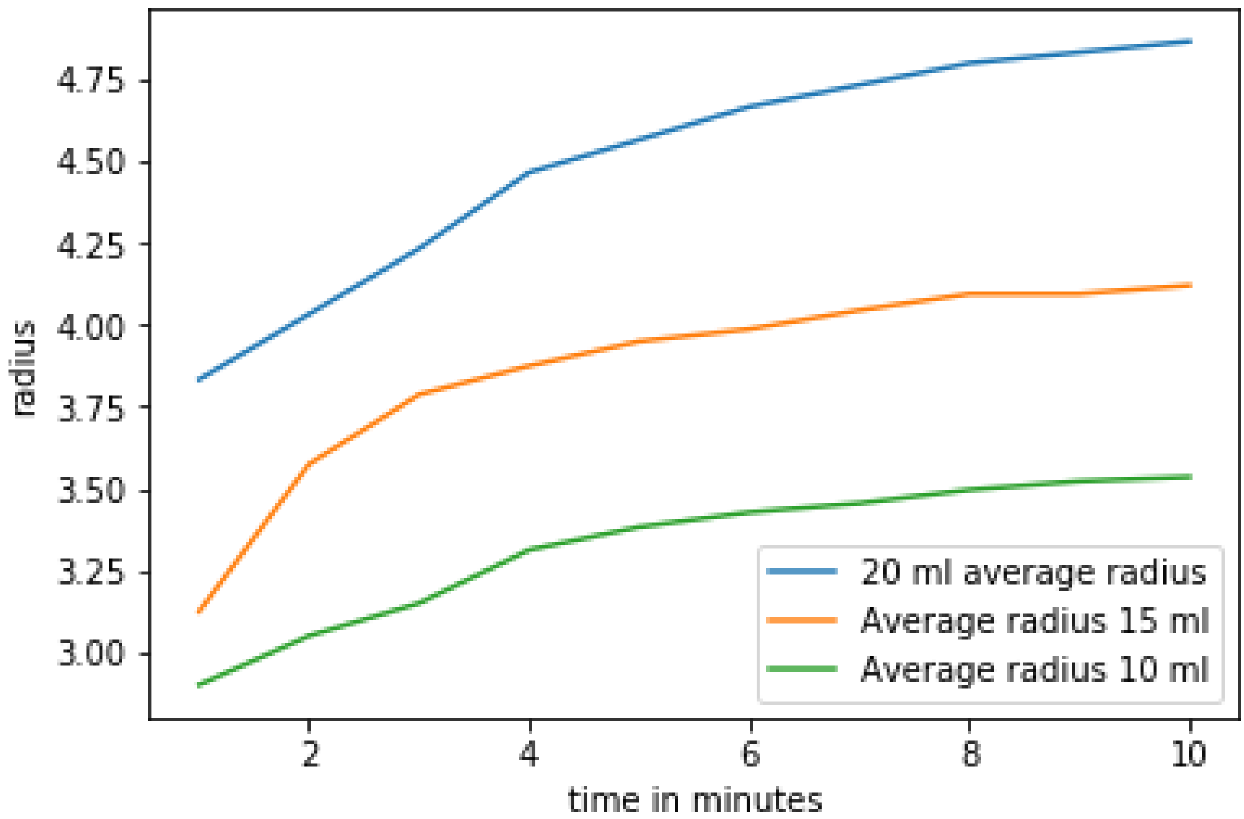

Figure 10: Graph of the radius for each volume

Now we plot the average radius for each volume:

3.3 estimation of b

From the table we were able to estimate the coefficient b if by supposing that radius is $r = t^b$. To do this I also wrote a python code and used the same method as in the first experiment. The ipython code is sent together with this pdf file for the details. We found the following result:

volume	20 ml	15 ml	10 ml
b	0.7773	0.7026	0.6278

The best estimate is $b = 0.6278$

For this value of b, the plot of the function $r = t^b$ is below:

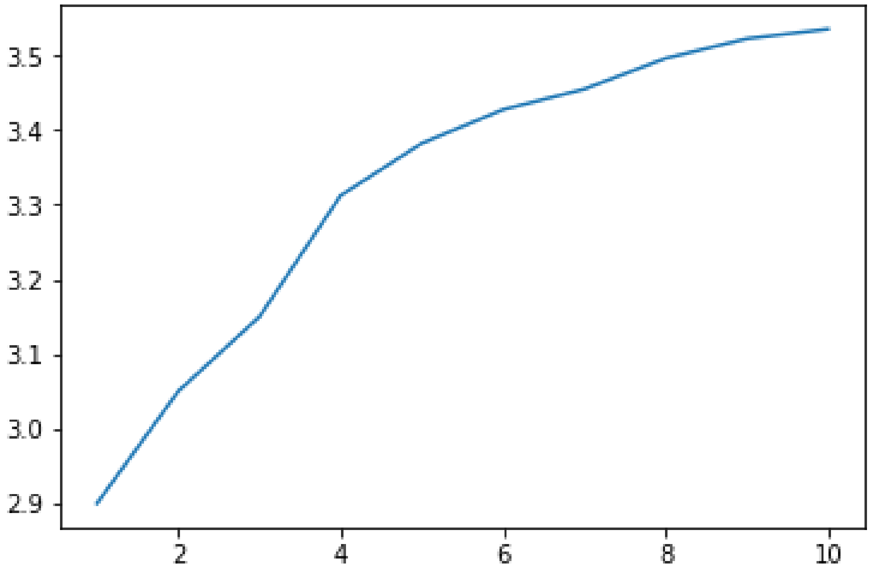

Figure 11: Fit curve

This curve is very similar to the curves above and this confirms an acceptable approximation.

3.3 Viscosity estimation

For the dynamic viscosity, we would use the same method as above.

Conclusion

We were able to measure the distance travelled by ball bearings of different masses each 5 seconds time interval through a marked container of golden syrup in order to try to determine the dynamic viscosity of the syrup. We first solved the differential equation we derived using Newton Law and we used the least square method to estimate the viscosity. It turns out that our results were not bad because we found 89.74 Pa.s and an average of 93.39 Pa.s whereas the theoretical value is 100 Pa.s.

We also studied the spread of the golden syrup when a certain amount is place on a substrate. Assuming that the radius is given by $r = t^b$, we were able to estimate the coefficient b to 0.627.