Projet_MODS202_Partie2

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[61]:
```
import pandas as pd
import numpy as np
import matplotlib.pyplot as plt
import scipy.stats as stats
import statsmodels.tsa.stattools as tools
import statsmodels.graphics.tsaplots as tsa
import statsmodels
```

Q1

[120]:
```
df = pd.read_excel('quarterly_projet.xls')

df.dropna()
```

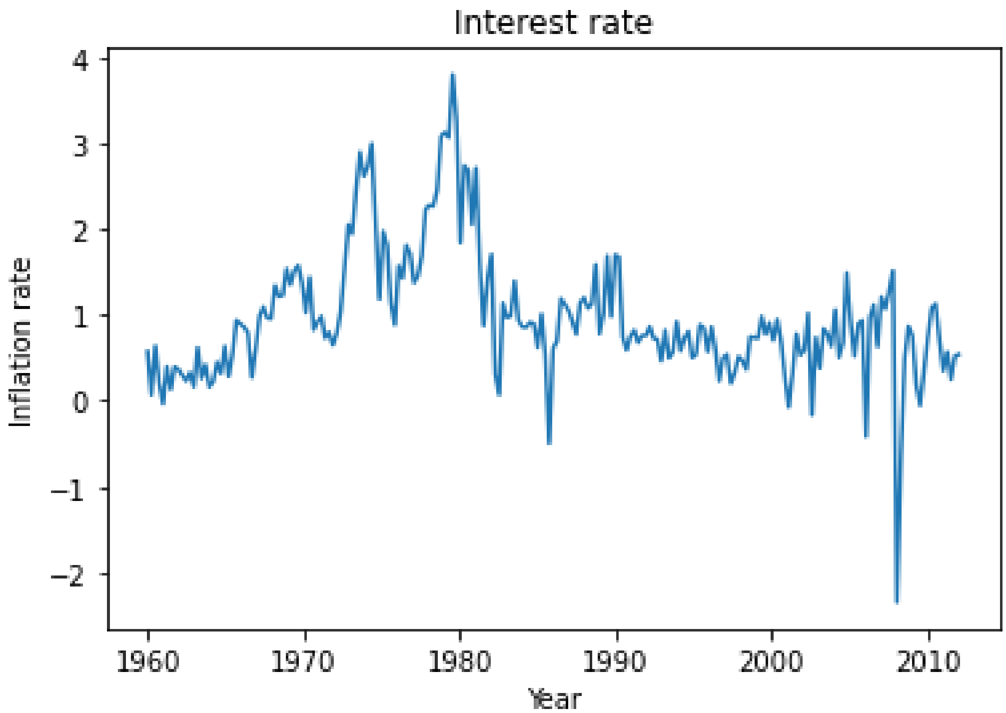
[120]:
```
        DATE   FFR  Tbill  Tb1yr    r5   r10  PPINSA  Finished     CPI  \
0     1960Q1  3.93   3.87   4.57  4.64  4.49   31.67     33.20   29.40
1     1960Q2  3.70   2.99   3.87  4.30  4.26   31.73     33.40   29.57
2     1960Q3  2.94   2.36   3.07  3.67  3.83   31.63     33.43   29.59
3     1960Q4  2.30   2.31   2.99  3.75  3.89   31.70     33.67   29.78
4     1961Q1  2.00   2.35   2.87  3.64  3.79   31.80     33.63   29.84
..       ...   ...    ...    ...   ...   ...     ...       ...     ...
207   2011Q4  0.07   0.01   0.11  0.95  2.05  200.77    192.97  226.97
208   2012Q1  0.10   0.07   0.16  0.90  2.04  202.17    193.73  228.27
209   2012Q2  0.15   0.09   0.19  0.79  1.82  201.80    192.83  228.84
210   2012Q3  0.14   0.10   0.18  0.67  1.64  202.40    195.20  230.03
211   2012Q4  0.16   0.09   0.17  0.69  1.71  202.27    196.20  231.28

     CPICORE    M1NSA     M2SA    M2NSA  Unemp  IndProd     RGDP   Potent  \
0      18.92   140.53    896.1   299.40   5.13    23.93   2845.3   2824.2
1      19.00   138.40    903.3   300.03   5.23    23.41   2832.0   2851.2
2      19.07   139.60    919.4   305.50   5.53    23.02   2836.6   2878.7
3      19.14   142.67    932.8   312.30   6.27    22.47   2800.2   2906.7
4      19.17   142.23    948.9   317.10   6.80    22.13   2816.9   2934.8
..       ...      ...      ...      ...    ...      ...      ...      ...
207   112.50  2165.77  28787.3  9599.47   8.67    95.33  13441.0  14255.9
208   113.12  2213.97  29238.6  9777.03   8.27    96.70  13506.4  14317.4
```

```
209   113.60  2258.30  29611.6   9888.97   8.17    97.27  13548.5  14379.3
210   113.91  2326.47  30251.4  10029.87   8.03    97.39  13652.5  14441.9
211   114.18  2436.73  30938.8  10319.60   7.83    98.01  13665.4  14505.4

     Deflator      Curr
0      18.521    31.830
1      18.579    31.862
2      18.648    32.217
3      18.700    32.624
4      18.743    32.073
..        ...       ...
207   113.987  1055.496
208   114.599  1082.519
209   115.035  1104.500
210   115.810  1119.187
211   116.089  1147.623

[212 rows x 19 columns]
```

Q2

[121]:
```
inf = [100*(df.iloc[i]['CPI']-df.iloc[i-1]['CPI'])/df.iloc[i]['CPI'] for i in␣
 ↪range(1,len(df))]
years = np.linspace(1960,2012,211)

plt.plot(years, inf)
plt.xlabel('Year')
plt.ylabel('Inflation rate')
plt.title("Interest rate")
```

[121]:
```
Text(0.5, 1.0, 'Interest rate')
```



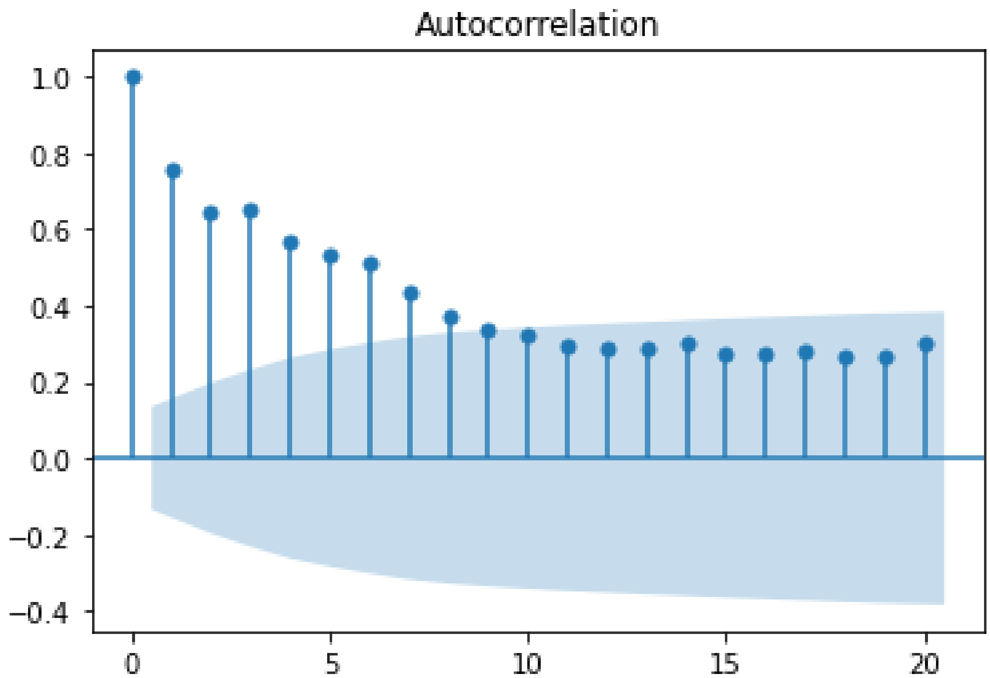
L'inflation augmente globalement jusqu'en 1981, puis elle diminue jusqu'à la crise financière de 2008.

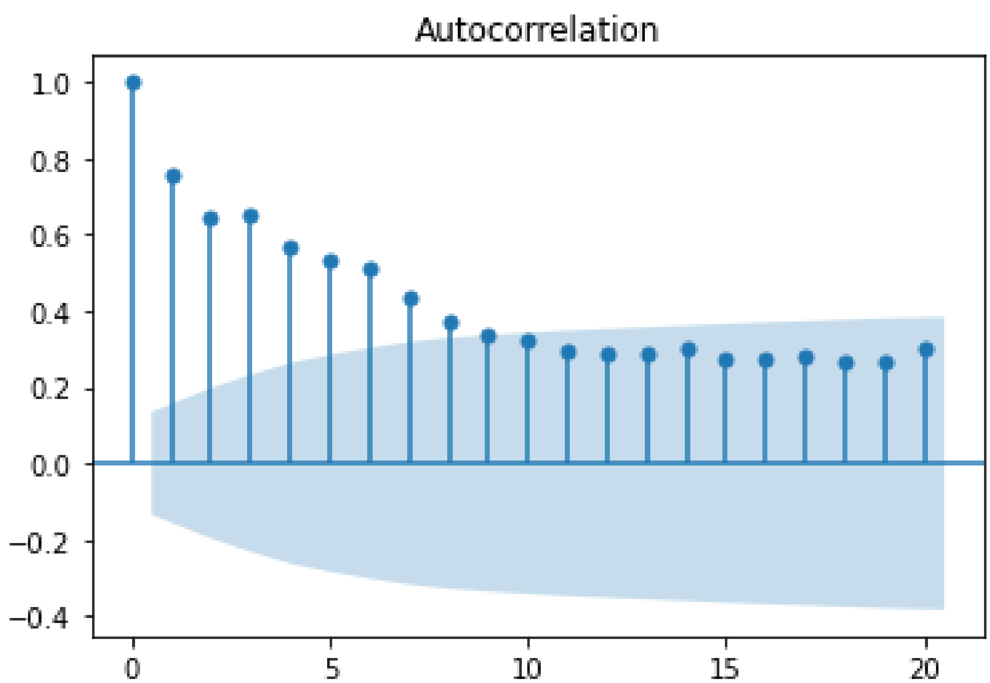
Q3

[122]:
```
tools.acf(inf)
tsa.plot_acf(inf, lags=20)
```

```
/Users/eugeniedulout/opt/anaconda3/lib/python3.8/site-
packages/statsmodels/tsa/stattools.py:657: FutureWarning: The default number of
lags is changing from 40 tomin(int(10 * np.log10(nobs)), nobs - 1) after 0.12is
released. Set the number of lags to an integer to  silence this warning.
  warnings.warn(
/Users/eugeniedulout/opt/anaconda3/lib/python3.8/site-
packages/statsmodels/tsa/stattools.py:667: FutureWarning: fft=True will become
the default after the release of the 0.12 release of statsmodels. To suppress
this warning, explicitly set fft=False.
  warnings.warn(
```

[122]:

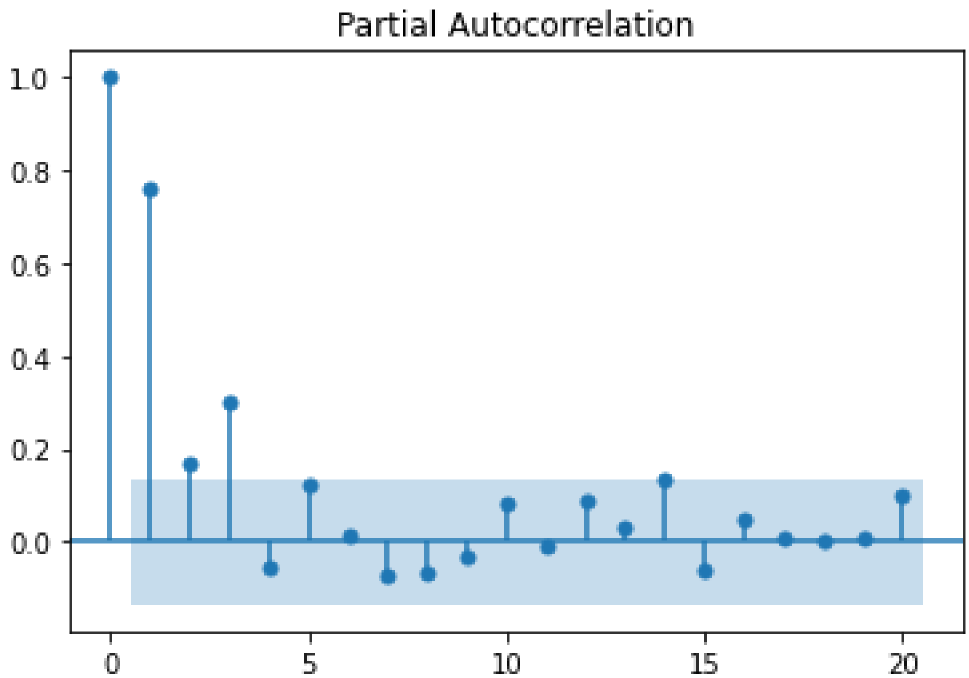


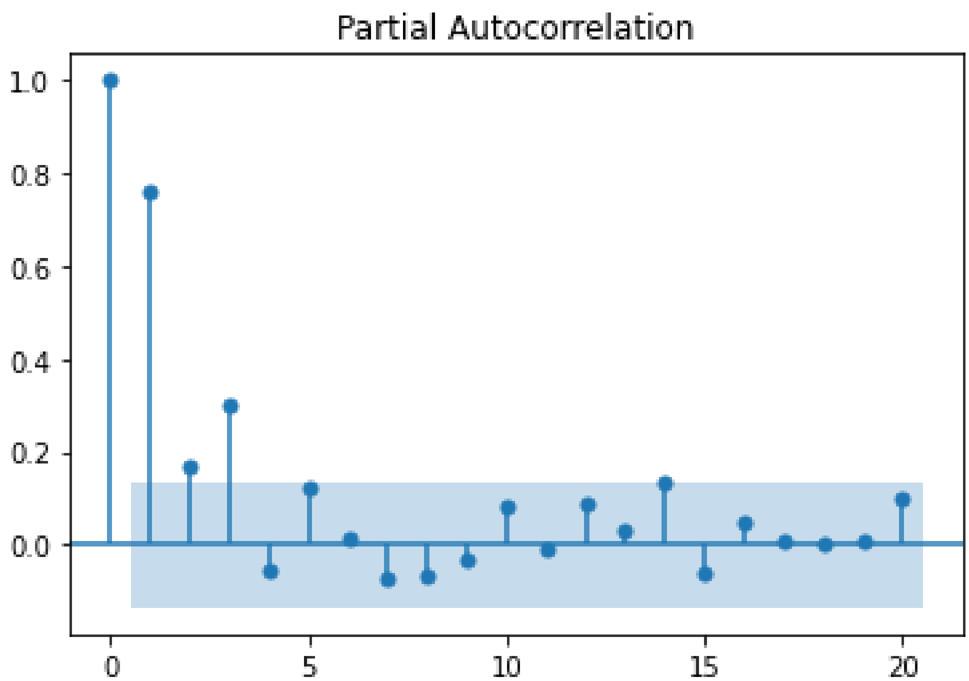


[123]:
```
tools.pacf(inf)
tsa.plot_pacf(inf, lags=20)
```

```
/Users/eugeniedulout/opt/anaconda3/lib/python3.8/site-
packages/statsmodels/tsa/stattools.py:1024: FutureWarning: The default number of
lags is changing from 40 tomin(int(10 * np.log10(nobs)), nobs // 2 - 1) after
0.12is released. Set the number of lags to an integer to  silence this warning.
  warnings.warn(
```

[123]:





L'autocorrelation d'un processus ARMA(p,q) tend exponentiellement vers 0 à partir de l'ordre q+1. Ce comportement est semblable à celui suivi par l'inflation, et on a donc q=3. De plus, l'autocorrélation partielle se trouve dans la zone bleu ou elle oscille autour de 0 à partir d'un certain rang. La série temporelle n'est donc pas stationnaire.

Q4

Stationnarité: Un processus stationnaire est un processus stochastique dans lequel les v.a. suivent toutes la même loi.

Ergodicité:

$Y_T = \frac{1}{T}\int_0^T X(u)du \to E[X(t)]$

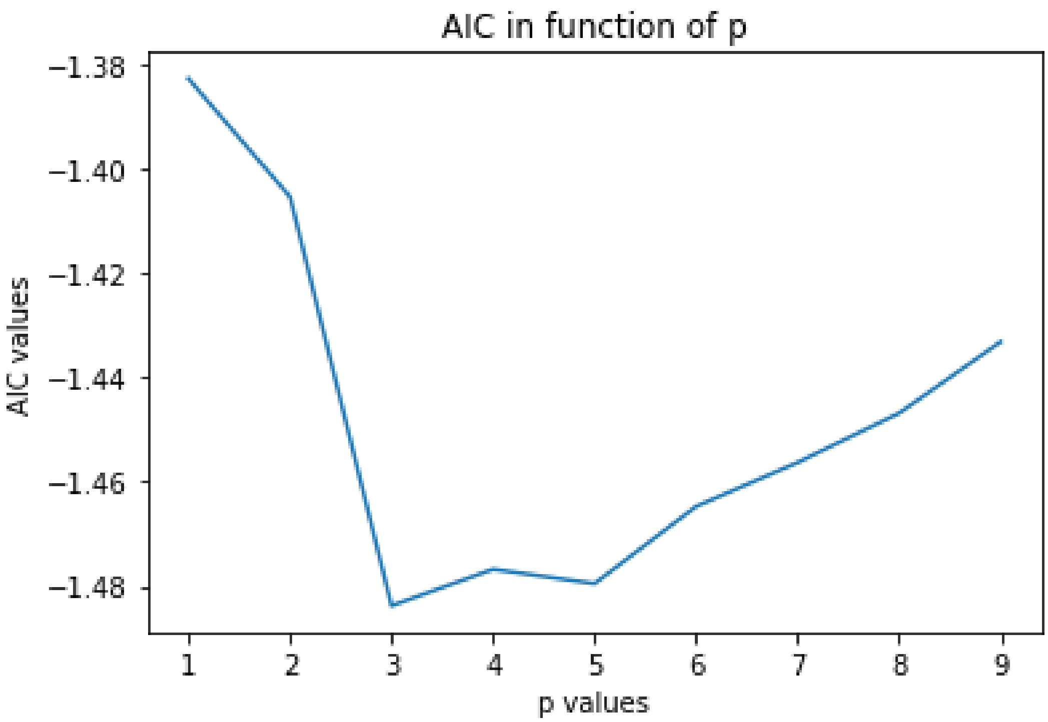
Ces conditions sont nécessaires pour assurer la convergence de la régression linéaire.

Q5

[124]:
```
import statsmodels.tsa.api as smt
l_aic = []
pval = np.arange(1,10)
for p in pval:
    mdl = smt.AR(inf).fit(maxlag=p)
    l_aic.append(mdl.aic)
plt.plot(pval, l_aic)
plt.ylabel("AIC values")
plt.xlabel("p values")
plt.title('AIC in function of p')
plt.show()
print("L'AIC est minimum pour p=3, c'est donc un processus AR(3)")
```

```
/Users/eugeniedulout/opt/anaconda3/lib/python3.8/site-
packages/statsmodels/tsa/ar_model.py:791: FutureWarning:
statsmodels.tsa.AR has been deprecated in favor of statsmodels.tsa.AutoReg and
statsmodels.tsa.SARIMAX.

AutoReg adds the ability to specify exogenous variables, include time trends,
and add seasonal dummies. The AutoReg API differs from AR since the model is
treated as immutable, and so the entire specification including the lag
length must be specified when creating the model. This change is too
substantial to incorporate into the existing AR api. The function
ar_select_order performs lag length selection for AutoReg models.

AutoReg only estimates parameters using conditional MLE (OLS). Use SARIMAX to
estimate ARX and related models using full MLE via the Kalman Filter.
```

```
To silence this warning and continue using AR until it is removed, use:

import warnings
warnings.filterwarnings('ignore', 'statsmodels.tsa.ar_model.AR', FutureWarning)

  warnings.warn(AR_DEPRECATION_WARN, FutureWarning)
```



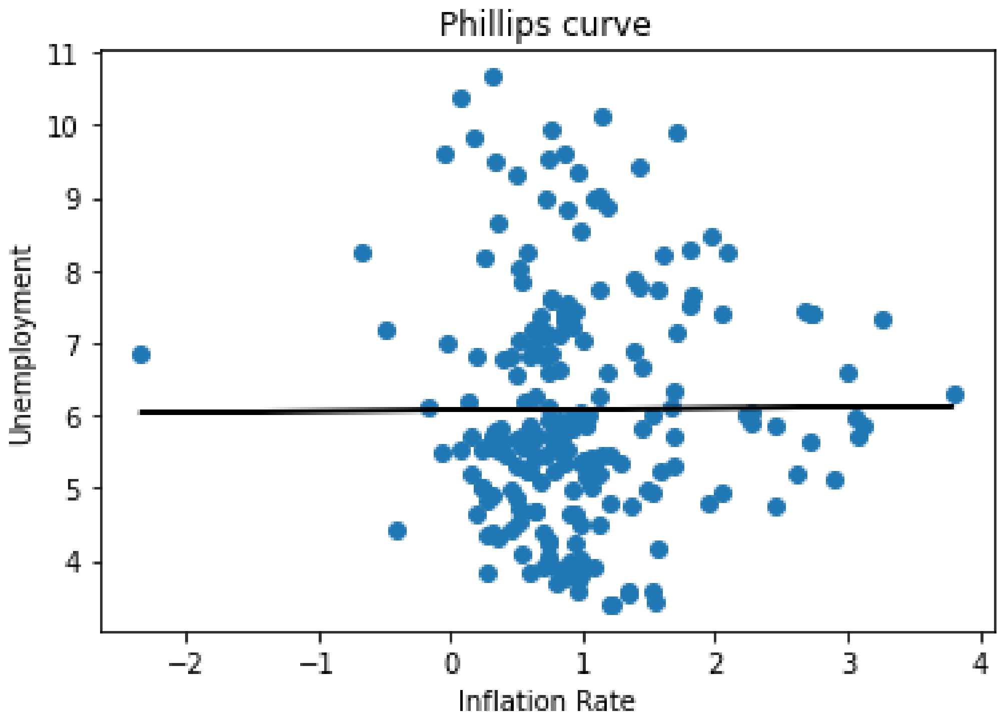

```
L'AIC est minimum pour p=3, c'est donc un processus AR(3)
```

Q6

[125]:
```
Unemp = df['Unemp'].iloc[1:]

unemp_results = statsmodels.api.OLS(Unemp, np.column_stack((np.
 ↪ones(len(inf)),inf))).fit()
print(unemp_results.summary())

list = []
for i in inf:
    list.append(unemp_results.params[0] + unemp_results.params[1] * i)
plt.scatter(inf,Unemp)
plt.plot(inf, list, color='black')
plt.xlabel("Inflation Rate")
plt.ylabel("Unemployment")
```

```python
plt.title("Phillips curve")
plt.show()
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Unemp   R-squared:                       0.000
Model:                            OLS   Adj. R-squared:                 -0.005
Method:                 Least Squares   F-statistic:                  0.008229
Date:                Sun, 21 Nov 2021   Prob (F-statistic):              0.928
Time:                        22:23:57   Log-Likelihood:                -400.28
No. Observations:                 211   AIC:                             804.6
Df Residuals:                     209   BIC:                             811.3
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          6.0735      0.182     33.388      0.000       5.715       6.432
x1             0.0134      0.148      0.091      0.928      -0.279       0.305
==============================================================================
Omnibus:                       13.851   Durbin-Watson:                   0.044
Prob(Omnibus):                  0.001   Jarque-Bera (JB):               15.331
Skew:                           0.659   Prob(JB):                     0.000469
Kurtosis:                       2.935   Cond. No.                         3.00
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

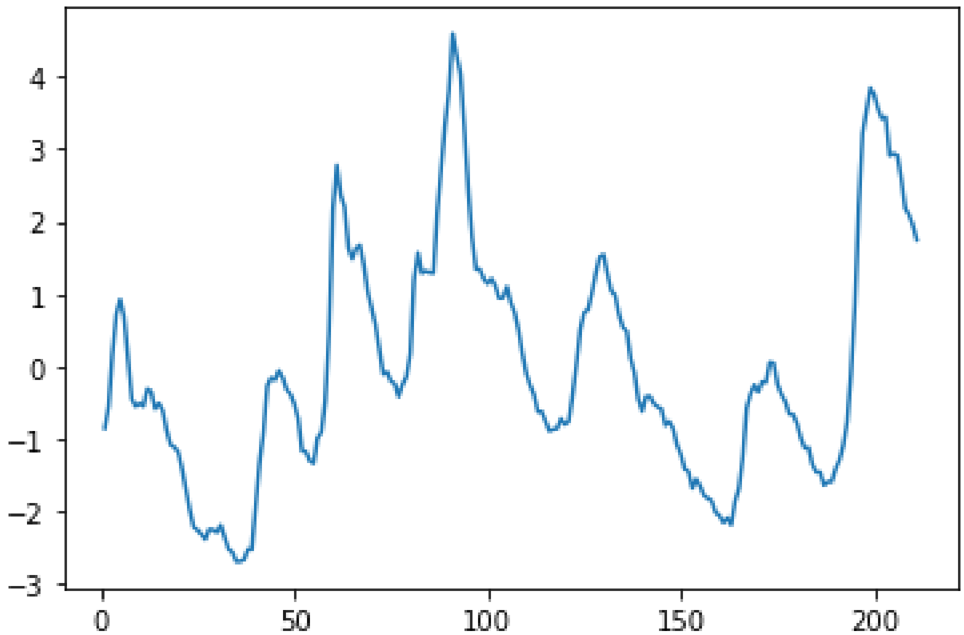


Q7

On plot les résidus du modèle.

[126]:
```
unemp_resid = unemp_results.resid
plt.plot(unemp_resid)
```

[126]: [<matplotlib.lines.Line2D at 0x7fe4568e6ee0>]



On étudie l'autocorrélation des résidus. Pour cela on fait un test d'autocorrélation H0: p=0

[127]:
```
x = np.column_stack((np.ones(len(inf)-1),unemp_resid[0:len(inf)-1]))[:,1]
y = unemp_resid[1:len(inf)]
results = statsmodels.api.OLS(x,y).fit()
print(results.summary())
```

```
                                 OLS Regression Results
================================================================================
=======
Dep. Variable:                      y   R-squared (uncentered):
0.956
Model:                            OLS   Adj. R-squared (uncentered):
0.956
Method:                 Least Squares   F-statistic:
4565.
Date:                Sun, 21 Nov 2021   Prob (F-statistic):
5.77e-144
Time:                        22:23:58   Log-Likelihood:
-69.784
No. Observations:                 210   AIC:
141.6
Df Residuals:                     209   BIC:
144.9
Df Model:                           1
Covariance Type:            nonrobust
==============================================================================
```

```
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
x1             0.9758      0.014     67.566      0.000       0.947       1.004
==============================================================================
Omnibus:                       58.685   Durbin-Watson:                   0.666
Prob(Omnibus):                  0.000   Jarque-Bera (JB):              160.436
Skew:                          -1.196   Prob(JB):                     1.45e-35
Kurtosis:                       6.552   Cond. No.                         1.00
==============================================================================

Notes:
[1] R² is computed without centering (uncentered) since the model does not
contain a constant.
[2] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

La p-valeur est nulle, on rejette donc le test. Les erreurs sont autocorellées, il faut corriger.

Q8

On corrige l'autocorrélation des erreurs.

[128]:
```
Unemp_corrected = [Unemp[i]-results.params[0]*Unemp[i-1] for i in␣
↪range(2,len(Unemp+1))]
x = np.column_stack((np.ones(len(inf)),inf))
x_corrected = x[1:] - results.params[0]*x[0:len(x)-1]
inf_corrected = x_corrected[:,1]
```

Q9

On sépare en deux parties, avant et après la moitié de la période d'observation.

[129]:
```
Unemp_1 = Unemp[0:105]
Unemp_2 = Unemp[106:210]
x1 = x[0:105]
x2 = x[106:210]
inf_1 = inf[0:105]
inf_2 = inf[106:210]
```

Premier SSR:

[130]:
```
res0 = api.OLS(Unemp, x).fit()
u0 = res0.resid
SSR = u0.T@u0
```

Deuxième SSR:

[131]:
```
res1 = api.OLS(Unemp_1, x1).fit()
u1 = res1.resid
SSR1 = u1.T@u1
```

Troisième SSR:

[132]:
```
res2 = api.OLS(Unemp_2, x2).fit()
u2 = res2.resid
SSR2 = u2.T@u2
```

[133]:
```
F = ((SSR-(SSR1+SSR2))/(SSR1+SSR2))*(len(inf)-8)/4
p = stats.f.sf(F,1,len(inf)-2)
print('SSR=',SSR,'SSR1=',SSR1,'SSR2=',SSR2)
print("La p-valeur : ", p)
```

```
SSR= 548.9903905121405 SSR1= 300.53602698671216 SSR2= 234.66519435565817
La p-valeur :  0.25414746341084193
```

La p-valeur est plus importante, le test est stable.

Q10

[134]:
```
#On rajoute les délais d'ordre 1, 2 , 3, 4
n_unemp = len(Unemp)
Unemp_0 = Unemp[0:n_unemp-4]
Unemp_1 = Unemp[1:n_unemp-3]
Unemp_2 = Unemp[2:n_unemp-2]
Unemp_3 = Unemp[3:n_unemp-1]
n_inf = len(inf)
inf0 = inf[0:n_inf-4]
inf1 = inf[1:n_inf-3]
inf2 = inf[2:n_inf-2]
inf3 = inf[3:n_inf-1]

const = np.ones(n_unemp-4)
x = np.column_stack((const,inf0, inf1, inf2, inf3, Unemp_0, Unemp_1, Unemp_2,␣
 ↪Unemp_3))
results = api.OLS(Unemp[4:], x).fit()
print(results.summary())
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Unemp   R-squared:                       0.979
Model:                            OLS   Adj. R-squared:                  0.978
Method:                 Least Squares   F-statistic:                     1143.
Date:                Sun, 21 Nov 2021   Prob (F-statistic):          3.10e-161
Time:                        22:24:01   Log-Likelihood:                 4.5455
No. Observations:                 207   AIC:                             8.909
Df Residuals:                     198   BIC:                             38.90
Df Model:                           8
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          0.1454      0.072      2.007      0.046       0.003       0.288
```

```
x1             0.0174      0.038      0.457      0.648      -0.058       0.092
x2             0.0705      0.040      1.748      0.082      -0.009       0.150
x3            -0.0204      0.041     -0.493      0.623      -0.102       0.061
x4             0.0275      0.038      0.722      0.471      -0.048       0.102
x5            -0.0101      0.070     -0.143      0.886      -0.148       0.128
x6             0.0253      0.135      0.187      0.852      -0.241       0.292
x7            -0.6476      0.134     -4.837      0.000      -0.912      -0.384
x8             1.5931      0.071     22.379      0.000       1.453       1.733
==============================================================================
Omnibus:                       28.701   Durbin-Watson:                   1.997
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               67.462
Skew:                           0.618   Prob(JB):                     2.24e-15
Kurtosis:                       5.509   Cond. No.                         143.
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

Test de Granger:

[135]:
```python
print(tools.grangercausalitytests(np.column_stack((Unemp, inf)),4))
```

```
Granger Causality
number of lags (no zero) 1
ssr based F test:         F=8.3578  , p=0.0042  , df_denom=207, df_num=1
ssr based chi2 test:   chi2=8.4790  , p=0.0036  , df=1
likelihood ratio test: chi2=8.3123  , p=0.0039  , df=1
parameter F test:         F=8.3578  , p=0.0042  , df_denom=207, df_num=1

Granger Causality
number of lags (no zero) 2
ssr based F test:         F=4.1538  , p=0.0171  , df_denom=204, df_num=2
ssr based chi2 test:   chi2=8.5112  , p=0.0142  , df=2
likelihood ratio test: chi2=8.3424  , p=0.0154  , df=2
parameter F test:         F=4.1538  , p=0.0171  , df_denom=204, df_num=2

Granger Causality
number of lags (no zero) 3
ssr based F test:         F=4.9417  , p=0.0025  , df_denom=201, df_num=3
ssr based chi2 test:   chi2=15.3415 , p=0.0015  , df=3
likelihood ratio test: chi2=14.8021 , p=0.0020  , df=3
parameter F test:         F=4.9417  , p=0.0025  , df_denom=201, df_num=3

Granger Causality
number of lags (no zero) 4
ssr based F test:         F=3.7430  , p=0.0058  , df_denom=198, df_num=4
ssr based chi2 test:   chi2=15.6527 , p=0.0035  , df=4
```

```
likelihood ratio test: chi2=15.0892 , p=0.0045  , df=4
parameter F test:         F=3.7430  , p=0.0058  , df_denom=198, df_num=4
{1: ({'ssr_ftest': (8.35783926570071, 0.004249936223841789, 207.0, 1),
'ssr_chi2test': (8.47896737100072, 0.003592759818563595, 1), 'lrtest':
(8.31226620280222, 0.003937822814832033, 1), 'params_ftest': (8.357839265700742,
0.004249936223841704, 207.0, 1.0)},
[<statsmodels.regression.linear_model.RegressionResultsWrapper object at
0x7fe4563ce6a0>, <statsmodels.regression.linear_model.RegressionResultsWrapper
object at 0x7fe4553310d0>, array([[0., 1., 0.]])]), 2: ({'ssr_ftest':
(4.153767777554608, 0.01705317891619206, 204.0, 2), 'ssr_chi2test':
(8.511151622636401, 0.014184920555755756, 2), 'lrtest': (8.342416637257315,
0.015433600146016722, 2), 'params_ftest': (4.153767777554482,
0.01705317891619412, 204.0, 2.0)},
[<statsmodels.regression.linear_model.RegressionResultsWrapper object at
0x7fe45615d4f0>, <statsmodels.regression.linear_model.RegressionResultsWrapper
object at 0x7fe45615d5b0>, array([[0., 0., 1., 0., 0.],
       [0., 0., 0., 1., 0.]])]), 3: ({'ssr_ftest': (4.941739375548519,
0.002477913344302664, 201.0, 3), 'ssr_chi2test': (15.341519255434207,
0.0015468774967208009, 3), 'lrtest': (14.802111319092546, 0.001993810317260139,
3), 'params_ftest': (4.941739375548476, 0.0024779133443027786, 201.0, 3.0)},
[<statsmodels.regression.linear_model.RegressionResultsWrapper object at
0x7fe455316eb0>, <statsmodels.regression.linear_model.RegressionResultsWrapper
object at 0x7fe4553167c0>, array([[0., 0., 0., 1., 0., 0., 0.],
       [0., 0., 0., 0., 1., 0., 0.],
       [0., 0., 0., 0., 0., 1., 0.]])]), 4: ({'ssr_ftest': (3.743043058749731,
0.005845116029327742, 198.0, 4), 'ssr_chi2test': (15.652725518407966,
0.003522375186621331, 4), 'lrtest': (15.089157297033239, 0.004519826626348605,
4), 'params_ftest': (3.743043058749574, 0.005845116029329253, 198.0, 4.0)},
[<statsmodels.regression.linear_model.RegressionResultsWrapper object at
0x7fe455316e80>, <statsmodels.regression.linear_model.RegressionResultsWrapper
object at 0x7fe455316d90>, array([[0., 0., 0., 0., 1., 0., 0., 0., 0.],
       [0., 0., 0., 0., 0., 1., 0., 0., 0.],
       [0., 0., 0., 0., 0., 0., 1., 0., 0.],
       [0., 0., 0., 0., 0., 0., 0., 1., 0.]])])}
```

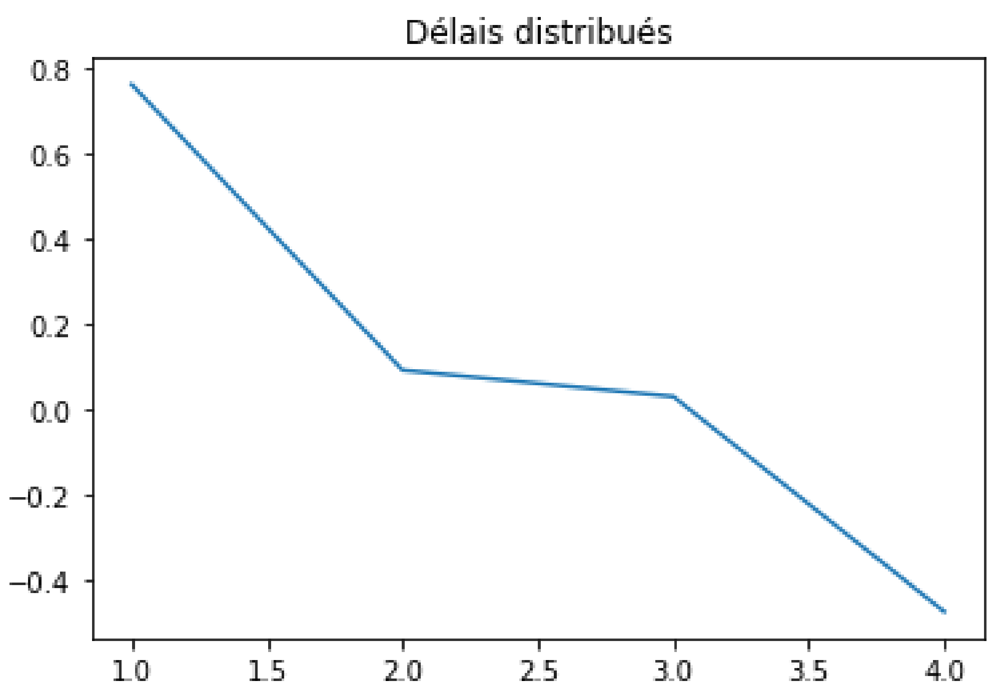
Q11

[136]:
```python
x = np.column_stack((const, inf0, inf1, inf2, inf3))
results = api.OLS(Unemp[4:], x).fit()
print(results.summary())
plt.plot([1,2,3,4], results.params[1:])
plt.title("Délais distribués")
```

```
                            OLS Regression Results
==============================================================================
Dep. Variable:                  Unemp   R-squared:                       0.087
Model:                            OLS   Adj. R-squared:                  0.069
Method:                 Least Squares   F-statistic:                     4.820
Date:                Sun, 21 Nov 2021   Prob (F-statistic):           0.000982
```

```
Time:                        22:24:03   Log-Likelihood:                -384.94
No. Observations:                 207   AIC:                             779.9
Df Residuals:                     202   BIC:                             796.5
Df Model:                           4
Covariance Type:            nonrobust
==============================================================================
                 coef    std err          t      P>|t|      [0.025      0.975]
------------------------------------------------------------------------------
const          5.6859      0.195     29.231      0.000       5.302       6.069
x1             0.7599      0.234      3.248      0.001       0.299       1.221
x2             0.0926      0.261      0.355      0.723      -0.422       0.607
x3             0.0309      0.262      0.118      0.906      -0.485       0.547
x4            -0.4718      0.235     -2.011      0.046      -0.934      -0.009
==============================================================================
Omnibus:                       34.258   Durbin-Watson:                   0.109
Prob(Omnibus):                  0.000   Jarque-Bera (JB):               49.287
Skew:                           0.971   Prob(JB):                     1.98e-11
Kurtosis:                       4.395   Cond. No.                         8.02
==============================================================================

Notes:
[1] Standard Errors assume that the covariance matrix of the errors is correctly
specified.
```

[136]: Text(0.5, 1.0, 'Délais distribués')



[137]:
```
print("L'impact à long terme de l'inflation sur le chômage est de:", results.
↪params[1:].sum()*100, "%")
```

```
L'impact à long terme de l'inflation sur le chômage est de: 41.15788466359076 %
```