Homework Assignment 1

Matthieu Boyer

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1 Exercise 1 - [Edit Distance/Levenshtein Distance]

1.1 Question 1

Proposition 1.1.1 (Complexity and Correction)**.** *If we denote C_f the complexity of f, this algorithm has time complexity $\mathcal{O}C_f\left(\frac{n}{t}\right)^2$. Moreover, it is clear this algorithm is correct as it only just applies*

Algorithme 1 Levenshtein Distance with f

Input S, T, f, t ▷ Two Strings, the function f computing the values and the step t

$\mathbf{D} = \text{zeros}(n+1, n+1)$ ▷ $len(S) = len(T) = n$
for $i \leftarrow 0$to $n+1$ **do**
  $\mathbf{D}[i][0] \leftarrow i$
end for

for $j \leftarrow 0$ to $n+1$ **do**
  $\mathbf{D}[0][j] \leftarrow j$
end for

$\text{up}, \text{left} \leftarrow 0, 0$
while up $< n$ **do**
  left $\leftarrow 0$
  while left $< n$ **do**
    $d \leftarrow \min(n - \text{up}, t)$
    $e \leftarrow \min(n - \text{left}, t)$

    $b \leftarrow \mathbf{D}[\text{up}][\text{left}]$
    $a \leftarrow \mathbf{D}[\text{up} + 1 \rightarrow \text{up} + 1 + d][\text{left}]$
    $c \leftarrow \mathbf{D}[\text{up}][\text{left} + 1 \rightarrow \text{left} + 1 + e]$

    $f(a, b, c, d, e)$ ▷ We can suppose here that f modifies only the last line and column of F in D with side-effect.
    left $\leftarrow$ left $+ e$
    for $i \leftarrow 1$ to $t - 1$ **do**
      $\mathbf{D}[\text{up}+i][\text{left}] \leftarrow \min\left(\mathbf{D}[\text{up} + i][\text{left - 1}] + 1, \mathbf{D}[\text{up} + i - 1][\text{left}] + 1, \mathbf{D}[\text{up} + i - 1][\text{left - 1}] + 1_{\{S[up+i]=T[le}\right.$
    end for
  end while
  up $\leftarrow$ up $+ d$
end while
return $\mathbf{D}[n][n]$