1 Question 1

■ **Notation 1.1** For $F \subseteq E$ and $v \in A \cup B$, let us define $F(v) = \{e \in F \mid \exists v', e = (v, v') \vee e = (v', v)\}$.

We then define the matroids $\mathbb{A} = (E, \mathcal{A})$, $\mathbb{B} = (E, \mathcal{B})$ where :

$$\mathcal{A} = \{I \subseteq E \mid |I(a)| \leq 1 \forall a \in A\}$$
$$\mathcal{B} = \{I \subseteq E \mid I(h) \in \mathcal{M}_b \forall b \in B\}$$

We then see that $M \subseteq E$ is a matching if and only if $|M| = |A|$ and M is an independent set of $\mathcal{A}$ and $\mathcal{B}$.
Then, since $|A| = \max_{I \in \mathcal{A}} |I|$, we just need to show that $\min_{X \subseteq E} r_A(X) + r_B(E \setminus X) \geq |X|$.

2 Question 2

Let $F = 2^I$ and let us denote by $g : 2^{\mathcal{F}} \to \mathbb{R}^+$ the function that to a family of sets gives their combined profit. Clearly, g is submodular. Furthermore we denote by X_0 the emptyset, and by X_i the set of items taken after i knapsacks were filled by our algorithm. Since we apply the FPTAS k times, and since g is submodular, we have :

$$g(X_i) - g(X_{i-1}) \geq \frac{OPT - g(X_{i-1})}{k} \tag{1}$$

for each i, where OPT is the weight of an optimal solution. Then, we have :

$$g(X_1) - g(X_0) = g(X_1) \geq \frac{OPT}{k} = OPT(1 - \left(1 - \frac{1}{k}\right)) \tag{2}$$

and then :

$$g(X_2) \geq OPT(1 - \left(1 - \frac{1}{k}\right)^2)$$

By induction :

$$g(X_i i) \geq OPT(1 - \left(1 - \frac{1}{k}\right)^i)$$

And thus :

$$g(X_k) \geq OPT(1 - \left(1 - \frac{1}{k}\right)^k) \geq OPT(1 - \frac{1}{e})$$

3 Question 3