The Ludwig Wittgenstein Project

Wittgenstein — Notes on Logic

Notes on Logic

Ludwig Wittgenstein

Editor’s Note

Published by the Ludwig Wittgenstein Project.

This digital edition is based on Ludwig Wittgenstein. “Notes on Logic.” Notebooks 1914–1916, edited by G. H. von Wright and G. E. M. Anscombe, Harper & Row, 1969, pp. 93–106. *This original-language text is not in the public domain in its country of origin (the United States of America) but it was released by the copyright holders under Creative Commons Attribution – NonCommercial 4.0 International; it is in the public domain in countries and areas where the copyright term is the author’s life plus 70 years or fewer.*

Notes on Logic

Preliminary

In philosophy there are no deductions; it is purely descriptive. The word “philosophy” ought always to designate something over or under, but not beside, the natural sciences. Philosophy gives no pictures of reality, and can neither confirm nor confute scientific investigations. It consists of logic and metaphysics, the former its basis. Epistemology is the philosophy of psychology. Distrust of grammar is the first requisite for philosophizing. Philosophy is the doctrine of the logical form of scientific propositions (not primitive propositions only). A correct explanation of the logical propositions must give them a unique position as against all other propositions.

I. Bi-polarity of Propositions. Sense and Meaning. Truth and Falsehood

Frege said “propositions are names”; Russell said “propositions correspond to complexes”. Both are false; and especially false is the statement “propositions are names of complexes”. Facts cannot be named. The false assumption that propositions are

names leads us to believe there must be “logical objects”: for the meaning of logical propositions would have to be such things.

What corresponds in reality to a proposition depends upon whether it is true or false. But we must be able to understand a proposition without knowing if it is true or false. What we know when we understand a proposition is this: we know what is the case if it is true and what is the case if it is false. But we do not necessarily know whether it is actually true or false.

Every proposition is essentially true-false. Thus a proposition has two poles (corresponding to case of its truth and case of its falsity). We call this the *sens e* of a proposition. The *meaning* of a proposition is the fact which actually corresponds to it. The chief characteristic of my theory is: *p has the same meaning as not-p* (constituent = particular, component = particular or relation, etc.).

Neither the sense nor the meaning of a proposition is a thing. These words are incomplete symbols. It is clear that we understand propositions without knowing whether they are true or false. But we can only know the meaning of a proposition when we know if it is true or false. What we understand is the sense of the proposition. To understand a proposition p it is not enough to know that p implies “p is true”, but we must also know that ~p implies

“p is false”. This shows the bi-polarity of the proposition. We understand a proposition when we understand its constituents and forms. If we know the meaning of “a” and “b” and if we know what “x R y” means for all x’s and y’s, then we also understand “a R b”. I understand the proposition “a R b” when I know that either the fact that a R b or the fact that not a R b corresponds to it; but this is not to be confused with the false opinion that I understand “a R b” when I know that “a R b or not a R b” is the case.

Strictly speaking, it is incorrect to say we understand the proposition p when we know that “p is true” ≡ p; for this would naturally always be the case if accidentally the propositions to right and left of the symbol ≡ were either both true or both false. We require not only an equivalence but a formal equivalence, which is bound up with the introduction of the form of p. What is wanted is the formal equivalence with respect to the forms of the proposition, i.e. all the general indefinables involved.

There are *positive and negative facts:* if the proposition “This rose is not red” is true, then what it signifies is negative. But the occurrence of the word “not” does not indicate this unless we know that the signification of the proposition “This rose is red” (when it is true) is positive. It is only from both, the negation and the negated proposition, that we can conclude about a characteristic of the signification of the whole proposition. (We are not here speaking

of the negations of *general* propositions, i.e. of such as contain apparent variables. Negative facts only justify the negations of atomic propositions.) Positive and negative facts there are, but not true and false facts.

If we overlook the fact that propositions have a *se nse* which is independent of their truth or falsehood, it easily seems as if true and false were two equally justified relations between the sign and what is signified. (We might then say, e.g., that "q" *signifies* in the true way what "not-q" *signifies* in the false way.) But are not true and false in fact equally justified? Could we not express ourselves by means of false propositions just as well as hitherto with true ones, so long as we know that they are meant falsely? No, for a proposition is true when it is as we assert in the proposition; and accordingly if by "q" we mean "not-q", and it is as we mean to assert, then in the new interpretation "q" is actually true and *not* false. But it is important that we *can* mean the same by "q" as by "not-q", for it shows that neither to the symbol "not" nor to the manner of its combination with "q" does a characteristic of the denotation of "q" correspond.

An analogy for the theory of truth: Consider a black patch on white paper. Then we can describe the form of the patch by mentioning, for each point of the surface, whether it is white or black. To the fact that a point is black corresponds a positive fact;

to the fact that a point is white (not black) corresponds a negative fact. If I designate a point of the surface (one of Frege's "truth-values"), this is as if I set up an assumption to be decided upon. But in order to be able to say of a point that it is black or it is white, I must first know when a point is to be called black and when it is to be called white. In order to be able to say that "p" is true (or false), I must first have determined under what circumstances I call a proposition true, and thereby I determine the *sense* of a proposition. The point in which the analogy fails is this: I can indicate a point of the paper which is white and black, but to a proposition without sense nothing corresponds, for it does not designate a thing (truth-value) whose properties might be called "false" or "true". The verb of a proposition is not "is true" or "is false", as Frege believes, but what is true must already contain the verb.

The comparison of language and reality is like that of a retinal image and visual image: to the blind spot nothing in the visual image seems to correspond, and thereby the boundaries of the blind spot determine the visual image—just as true negations of atomic propositions determine reality.

One is tempted to interpret "not-p" as "everything else, only not p". That from a single fact p an infinity of others, not-not-p, etc., follow is hardly credible. Man possesses an innate capacity for constructing symbols with which *some* sense can be expressed

without having the slightest idea what each word signifies. The best example of this is mathematics, for man has until recently used the symbols for numbers without knowing what they signify or that they signify nothing.

The assertion-sign is logically quite without significance. It only shows, in Frege and in Whitehead and Russell, that these authors hold the propositions so indicated to be true. "⊢", therefore, belongs as little to the proposition as (say) the number of the proposition. A proposition cannot possibly assert of itself that it is true. Assertion is merely psychological. There are only unasserted propositions. Judgment, command and question all stand on the same level; but all have in common the propositional form, and that alone interests us. What interests logic are only the unasserted propositions. When we say A judges that, etc., then we have to mention a whole proposition which A judges. It will not do either to mention only its constituents, or its constituents and form but not in the proper order. This shows that a proposition itself must occur in the statement to the effect that it is judged. For instance, however "not-p" may be explained, the question "What is negated?" must have a meaning. In "A judges (that) p", p cannot be replaced by a proper name. This is apparent if we substitute "A judges that p is true and not-p is false". The proposition "A judges (that) p" consists of the proper name A, the proposition p with its two poles, and A's being relat-

ed to both these poles in a certain way. This is obviously not a relation in the ordinary sense. Every right theory of judgment must make it impossible for me to judge that “this table penholders the book” (Russell’s theory does not satisfy this requirement). The structure of the proposition must be recognized and then the rest is easy. But ordinary language conceals the structure of the proposition: in it relations look like predicates, and predicates like names, etc.

One reason for supposing that not all propositions which have more than one argument are relational propositions is that, if they were, the relations of judgment and inference would have to hold between an arbitrary number of things. The idea that propositions are names for complexes has suggested that whatever is not a proper name is a sign for a relation. Russell, for instance, imagines every fact as a spatial complex, and since spatial complexes consist of things and relations only, therefore he holds all do.

We are very often inclined to explanations of logical functions of propositions which aim at introducing into the function either only the constituents of these propositions, or only their form, etc., and we overlook the fact that ordinary language would not contain the whole propositions if it did not need them.

Names are points, propositions arrows—they have *sense*. The sense of a proposition is determined by the two poles *true* and *false.* The form of a proposition is like a straight line, which divides all points of a plane into right and left. The line does this automatically, the form of the proposition only by convention. It is wrong to conceive every proposition as expressing a relation. A natural attempt at such a solution consists in regarding "not-p" as the opposite of "p", where, then, "opposite" would be the indefinable relation. But it is easy to see that every such attempt to replace functions with sense (ab-functions) by descriptions, must fail.

When we say "A believes p", this sounds, it is true, as if we could here substitute a proper name for "p". But we can see that here a *sense,* not a meaning, is concerned, if we say "A believes that p is true", and in order to make the direction of p even more explicit, we might say "A believes that 'p' is true and 'not-p' is false". Here the bi-polarity of p is expressed, and it seems that we shall only be able to express the proposition "A believes p" correctly by the ab-notation (later explained) by, say, making "A" have a relation to the poles "a" and "b" of a-p-b. The epistemological questions concerning the nature of judgment and belief cannot be solved without a correct apprehension of the form of the proposition.

A proposition is a standard with reference to which facts behave, but with names it is otherwise. Just as one arrow behaves to another arrow by being in the same sense or the opposite, so a fact behaves to a proposition; it is thus bi-polarity and sense come in. In this theory p has the same meaning as not-p but opposite sense. The meaning is the fact. A proper theory of judgment must make it impossible to judge nonsense. The "sense of" an ab function of a proposition is a function of its sense. In not-p, p is exactly the same as if it stands alone (this point is absolutely fundamental). Among the facts which make "p or q" true there are also facts which make "p and q" true; hence, if propositions have only meaning, we ought, in such a case, to say that these two propositions are identical. But in fact their sense is different, and we have introduced sense by talking of all p's and all q's. Consequently the molecular propositions will only be used in cases where their ab-function stands under a generality sign or enters into another function such as "I believe that", etc., because then the sense enters.

II. Analysis of Atomic Propositions, General Indefinables, Predicates, etc.

It may be doubted whether, if we formed all possible atomic propositions, "the world would be completely described if we declared the truth or falsehood of each" (Russell).

If there were a world created in which the principles of logic were true, in that world the whole of mathematics holds. No world can be created in which a proposition is true, unless the constituents of the proposition are created also.

Indefinables are of two sorts: names and forms. Propositions cannot consist of names alone, they cannot be classes of names. A name cannot only occur in two different propositions, but can occur in the same way in both. Propositions, which are symbols having reference to facts, are themselves facts (that this inkpot is on this table may express that I sit in this chair). We must be able to understand propositions we have never heard before. But every proposition is a new symbol. Hence we must have *g eneral* indefinable symbols; these are unavoidable if propositions are not all indefinable. Only the doctrine of general indefinables permits us to understand the nature of functions. Neglect of this doctrine leads us to an impenetrable thicket.

A proposition must be understood when *all* its indefinables are understood. The indefinables in “a R b” are introduced as follows: (1) “a” is indefinable, (2) “b” is indefinable, (3) whatever “x” and “y” may mean, “x R y” says something indefinable about their meaning.

We are not concerned in logic with the relation of any specific name to its meaning and just as little with the relation of a given proposition to reality. We do want to know that our names have meanings and propositions sense, and we thus introduce an indefinable concept "A" by saying "'A' denotes something indefinable", or the form of propositions a R b by saying: "For all meanings of 'x' and 'y', 'x R y' expresses something indefinable about x and y."

The form of a proposition may be symbolized in the following way: Let us consider symbols of the form "x R y", to which correspond primarily pairs of objects of which one has the name "x", the other the name "y". The x's and y's stand in various relations to each other, and among other relations the relation R holds between some but not between others. I now determine the sense of "x R y" by laying down the rule: when the facts behave in regard to "x R y" so that the meaning of "x" stands in the relation R to the meaning of "y", then I say that these facts are "of like sense" (*gleichsinnig*) with the proposition "x R y"; otherwise, "of opposite sense" (*entgegengesetzt*). I correlate the facts to the symbol "x R y" by thus dividing them into those of like sense and those of opposite sense. To this correlation corresponds the correlation of name and meaning. Both are psychological. Thus I understand the form "x R y" when I know that it discriminates the behaviour of x and y according as these stand in the relation R or not. In

this way I extract from all possible relations the relation R, as by a name, I extract its meaning from among all possible things.

There is no *thing* which is the *form* of a proposition, and no name which is the name of a form. Accordingly we can also not say that a relation which in certain cases holds between things holds sometimes between forms and things. This goes against Russell's theory of judgment.

Symbols are not what they seem to be. In "a R b" "R" looks like a substantive but it is not one. What symbolizes in "a R b" is that "R" occurs between "a" and "b". Hence "R" is *not* the indefinable in "a R b". Similarly in "ϕx" "ϕ" looks like a substantive but is not one; in "~p", "~" looks like "ϕ" but is not like it. This is the first thing that indicates there *may* not be logical constants. A reason against them is the generality of logic: logic cannot treat a special set of things.

Russell's "complexes" were to have the useful property of being compounded, and were to combine with this the agreeable property that they could be treated like "simples". But this alone makes them unserviceable as logical types (forms), since there would then have been significance in asserting, of a simple, that it was complex. But a *property* cannot be a logical type.

A false theory of relations makes it easily seem as if the relation of fact and constituent were the same as that of fact and fact-which-follows-from-it. But there is a similarity of the two, expressible thus: $\phi a . \supset_{\phi,\alpha} . a = a$.

Every statement about complexes can be resolved into the logical sum of a statement about the constituents and a statement about the proposition which describes the complex completely. How, in each case, the resolution is to be made, is an important question, but its answer is not unconditionally necessary for the construction of logic. To repeat: every proposition which seems to be about a complex can be analysed into a proposition about its constituents and about the proposition which describes the complex perfectly; i.e. that proposition which is equivalent to saying the complex exists.

III. Analysis of Molecular Propositions: ab-Functions

Whatever corresponds in reality to compound propositions must not be more than what corresponds to their several atomic propositions. Molecular propositions contain nothing beyond what is contained in their atoms; they add no material information above that contained in their atoms. All that is essential about molecular functions is their T-F (true-false) schema (i.e. the statement of the cases where they are true and cases where they are false).

It is *a priori* likely that the introduction of atomic propositions is fundamental for the understanding of all other kinds of propositions. In fact, the understanding of general propositions obviously depends on that of atomic propositions.

One reason for thinking the old notation wrong is that it is very unlikely that from every proposition p, an infinite number of other propositions not-not-p, not-not-not-not-p, etc., should follow. The very possibility of Frege's explanations of "not-p" and "if p then q", from which it follows that "not-not-p" denotes the same as p, makes it probable that there is some method of designation in which "not-not-p" corresponds to the same symbol as "p". But if this method of designation suffices for logic, it must be the right one.

If p = not-not-p, etc., this shows that the traditional method of symbolism is wrong, since it allows a plurality of symbols with the same sense; and thence it follows that in analysing such propositions, we must not be guided by Russell's method of symbolizing.

Naming is like pointing. A function is like a line dividing points of a plane into right and left ones; then "p or not-p" has no meaning because it does not divide the plane. But though a particular proposition, "p or not-p", has no meaning, a general proposition, "For all p's, p or not-p", has a meaning, because this

does not contain the nonsensical function “p or not-p”, but the function “p or not-q”, just as “for all x’s, x R x” contains the function “x R y”.

Logical inferences can, it is true, be made in accordance with Frege’s or Russell’s laws of deduction, but this cannot justify the inference; and therefore they are not primitive propositions of logic. If p follows from q, it can also be inferred from q, and the “manner of deduction” is indifferent.

The reason why “~Socrates” means nothing is that “~x” does not express a property of x. Signs of the forms “p ∨ ~p” are senseless, but not the proposition “(p) p ∨ ~p”. If I know that this rose is either red or not red, I know nothing. The same holds of all ab-functions. The assumption of the existence of logical objects makes it appear remarkable that in the sciences propositions of the form “p ∨ q”, “p ⊃ q”, etc., are only then not provisional when “∨” and “⊃” stand within the scope of a generality-sign (apparent variable). That “or” and “not”, etc., are not relations in the same sense as “right” and “left”, etc., is obvious to the plain man. The possibility of cross-definition in the old logical indefinables shows, of itself, that these are not the right indefinables, and even more conclusively, that they do not denote relations. Logical indefinables cannot be predicates or relations, because propositions, owing to sense, can-

not have predicates or relations. Nor are “not” and “or”, like judgment, *analogous* to predicates and relations, because they do not introduce anything new.

In place of every proposition “p” let us write “${}_{b}{}^{a}p$” . Let every correlation of propositions to each other or of names to propositions be effected by a correlation of their poles “a” and “b”. Let this correlation be transitive. Then accordingly “${}_{b-b}{}^{a-a}p$” is the same symbol as “${}_{b}{}^{a}p$”. Let n propositions be given. I then call a “class of poles” of these propositions every class of n members, of which each is a pole of one of the n propositions, so that one member corresponds to each proposition. I then correlate with each class of poles one of two poles (a and b). The sense of the symbolizing fact thus constructed I cannot define, but I know it.

The sense of an ab-function of p is a function of the sense of p. The ab-functions use the discrimination of facts which their arguments bring forth in order to generate new discriminations. The ab-notation shows the dependence of *or* and *not,* and thereby that they are not to be employed as simultaneous indefinables.

To every molecular function a TF (or ab) scheme corresponds. Therefore we may use the TF scheme itself instead of the function. Now what the TF scheme does is that it correlates the letters T and F

with each proposition. These two letters are the poles of atomic propositions. Then the scheme correlates another T and F to these poles. In this notation all that matters is the correlation of the outside poles to the poles of the atomic propositions. Therefore not-not-p is the same symbol as p. And therefore we shall never get two symbols for the same molecular function. As the ab (TF)-functions of atomic propositions are bi-polar propositions again, we can perform ab operations on them. We shall, by doing so, correlate two new outside poles via the old outside poles to the poles of the atomic propositions.

The symbolizing fact in a-p-b is that *say* a is on the left of p and b on the right of p. [This is quite arbitrary, but if we once have fixed on which order the poles have to stand in, we must of course stick to our convention. If, for instance, "apb" says p, then bpa says *nothing* (it does *not* say ~p). But a-apb-b is the same symbol as apb (here the ab-function vanishes automatically) for here the new poles are related to the same side of p as the old ones. The question is always: how are the new poles correlated to p compared with the way the old poles are correlated to p?] Then, given apb, the correlation of new poles is to be transitive, so that, for instance, if a new pole a in what ever way, i.e. via whatever poles, is correlated to the inside a, the symbol is not changed thereby. It is therefore possible to construct all possible ab-functions by performing one

ab-operation repeatedly, and we can therefore talk of all ab-functions as of all those functions which can be obtained by performing this ab-operation repeatedly (*cf.* Sheffer's work).

Among the facts which make "p or q" true, there are some which make "p and q" true; but the class which makes "p or q" true is different from the class which makes "p and q" true; and only this is what matters. For we introduce this class, as it were, when we introduce ab-functions.

Since the ab-functions of p are again bi-polar propositions, we can form ab-functions of them, and so on. In this way a series of propositions will arise, in which, in general, the *symbolizing* facts will be the same in several members. If now we find an ab-function of such a kind that by repeated applications of it every ab-function can be generated, then we can introduce the totality of ab-functions as the totality of those that are generated by the application of this function. Such a function is $\sim p \vee \sim q$. It is easy to suppose a contradiction in the fact that, on the one hand, every possible complex proposition is a simple ab-function of simple propositions, and that, on the other hand, the repeated application of one ab-function suffices to generate all these propositions. If, e.g., an affirmation can be generated by double negation, is negation in any sense contained in affirmation? Does "p" deny "not-p" or assert "p", or both? And how do matters stand with the defini-

tion of “⊃” by “∨” and “~”, or of “∨” by “~” and “⊃”? And how, e.g., shall we introduce p|q (i.e. ~p ∨ ~q), if not by saying that this expression says something indefinable about all arguments p and q? But the ab-functions must be introduced as follows: The function p|q is merely a mechanical instrument for constructing all possible *symbols* of ab-functions. The symbols arising by repeated application of the symbol “|” do *not* contain the symbol “p|q”. We need a rule according to which we can form all symbols of ab-functions, in order to be able to speak of the class of them; and we now speak of them, e.g., as those symbols of functions which can be generated by repeated application of the operation “|”. And we say now: For all p’s and q’s, “p|q” says something indefinable about the sense of those simple propositions which are contained in p and q.

IV. Analysis of General Propositions

Just as people used to struggle to bring all propositions into the subject-predicate form, so now it is natural to conceive every proposition as expressing a relation, which is just as incorrect. What is justified in this desire is fully satisfied by Russell’s theory of manufactured relations.

If only those signs which contain proper names are complex, then propositions containing nothing but apparent variables would be simple. Then what about their denials? Propositions are always complex, even if they contain no names.

There are no propositions containing real variables. Those symbols which are called propositions in which “variables occur” are in reality not propositions at all, but only schemes of propositions, which do not become propositions unless we replace the variables by constants. There is no proposition which is expressed by “x = x”, for “x” has no signification. But there is a proposition “(x) . x = x”, and propositions such as “Socrates = Socrates”, etc. In books on logic no variables ought to occur, but only general propositions which justify the use of variables. It follows that the so-called definitions in logic are not definitions, but only schemes of definitions, and instead of these we ought to put general propositions. And similarly, the so-called primitive ideas (*Urzeichen*) of logic are not primitive ideas but schemes of them. The mistaken idea that there are *t hings* called facts or complexes and relations easily leads to the opinion that there must be a relation of questioning to the facts, and then the question arises whether a relation can hold between an arbitrary number of things, since a fact can follow from arbi-

trary cases. It is a fact that the proposition which, e.g., expresses that q follows from p and $p \supset q$ is this: $p \,.\, p \supset q \,.\, \supset_{p,q} .\, q$.

Cross-definability in the realm of general propositions leads to quite similar questions to those in the realm of ab-functions. There is the same objection in the case of apparent variables to the usual indefinables as in the case of molecular functions. The application of the ab notation to apparent variable propositions becomes clear if we consider that, for instance, the proposition "for all x, ϕx" is to be true when ϕx is true for all x's, and false when ϕx is false for some x's. We see that *some* and *all* occur simultaneously in the proper apparent variable notation. The notation is

For $(x)\phi x$: a-(x)-.a ϕxb.-($\exists$x)-b and

for $(\exists x)\phi x$: a-($\exists$x)-.a ϕxb.-(x)-b

Old definitions now become tautologous.

A very natural objection to the way in which I have introduced, e.g., propositions of the form x R y is that by it propositions such as ($\exists$ x, y) x R y and similar ones are not explained, which yet obviously have in common with a R b what cRd has in common with a R b. *But* when we introduced propositions of the form x R y we mentioned no one particular proposition of this form; and we only need to introduce $(x,y)\phi(x,y)$ for all ϕ's in any way which

makes the sense of these propositions dependent on the sense of all propositions of the form $\phi(a,b)$, and thereby the justification of our procedure is established.

V. Principles of Symbolism: What Symbolises in a Symbol. Facts for Facts

It is easy to suppose only such symbols are complex as contain names of objects, and that accordingly "$(x,\phi)\phi x$" or "$(\exists x, y)x R y$" must be simple. It is then natural to call the first of these the name of a form, the second the name of a relation. But in that case what *is* the meaning, e.g., of "$\sim(\exists x, y).x R y$"? Can we put "not" before a name? Alternate indefinability shows the indefinables have not yet been reached. The indefinables of logic must be independent of each other. If an indefinable is introduced, it must be introduced in all combinations in which it can occur. We cannot, therefore, introduce it first for one combination, then for another; e.g. if the form $x R y$ has been introduced, it must henceforth be understood in propositions of the form $a R b$ just in the same way as in propositions such as $(\exists x,y)x R y$ and others. We must not introduce it first for one class of cases, then for the other; for it would remain doubtful if its meaning was the same in both cases and there could be no ground for using the same manner of combining symbols in both cases. In short, for the introduction of indefinable symbols

and combinations of symbols the same holds, *mutatis mutandis*, that Frege has said for the introduction of symbols by definitions.

It is impossible to dispense with propositions in which the same argument occurs in different positions. It is obviously useless to replace $\phi(a, a)$ by $\phi(a, b) . a = b$.

It can never express the common characteristic of two objects that we designate them by the same name but otherwise by two different ways of designation, for, since names are arbitrary, we might also choose different names, and where, then, would be the common element in the designations? Nevertheless, one is always tempted, in a difficulty, to take refuge in different ways of designation.

It is to be remembered that names are not things but classes: “A” is the same letter as “A”. This has the most important consequences for every symbolic language.

In regard to notation it is important to observe that not every feature of a symbol symbolizes. In two molecular functions which have the same T-F scheme, what symbolizes must be the same. In “not-not-p”, “not-p” does not occur; for “not-not-p” is the same as “p”, and therefore, if “not-p” occurred in “not-not-p”, it would occur in “p”.

A complex symbol must never be introduced as a single indefinable. Thus, for instance, no proposition is indefinable. For if one of the parts of the complex symbol occurs also in another connection, it must there be reintroduced. And would it then mean the same? The ways in which we introduce our indefinables must permit us to construct all propositions that have sense from these indefinables *alone.* It is easy to introduce “all” and “some” in a way that will make the construction of (say) “(x,y).x R y” possible from “all” and “x R y” *as introduced before.*

One must not say “The complex sign ‘a R b’” says that a stands in the relation R to b; but that “a” stands in a certain relation to “b” says *that* a R b.

Only facts can express sense, a class of names cannot. This is easily shown. In a R b it is not the complex that symbolizes but the fact that the symbol a stands in a certain relation to the symbol b. Thus facts are symbolized by facts, or more correctly: that a certain thing is the case in the symbol says that a certain thing is the case in the world.

VI. Types

No proposition can say anything about itself, because the symbol of the proposition cannot be contained in itself; this must be the basis of the theory of logical types.

It is easy to suppose that “individual”, “particular”, “complex”, etc., are primitive ideas of logic. Russell, e.g., says “individual” and “matrix” are “primitive ideas”. This error is presumably to be explained by the fact that, by employment of variables instead of the generality sign, it comes to seem as if logic dealt with things which have been deprived of all properties except complexity. We forget that the indefinables of symbols (*Urbilder von Zeichen*) only occur under the generality sign, never outside it.

Every proposition which says something indefinable about a thing is a subject-predicate proposition; every proposition which says something indefinable about two things expresses a dual relation between these things, and so on. Thus every proposition which contains only one name and one indefinable form is a subject-predicate proposition, etc. An indefinable symbol can only be a name, and therefore we can know, by the symbol of an atomic proposition, whether it is a subject-predicate proposition.

A proposition cannot occur in itself. This is the fundamental truth of the theory of types. In a proposition convert all indefinables into variables, there then remains a class of propositions which does not include all propositions, but does include an entire type. If we change a constituent a of a proposition $\phi(a)$ into a variable, then there is a class $\hat{p}[(\exists x).\phi x{=}p]$. This class, in general, still depends

upon what, by an *arbitrary convention*, we mean by "ϕx". But if we change into variables all those symbols whose significance was arbitrarily determined, there is still such a class. But this is not now dependent upon any convention, but only upon the nature of the symbol "ϕx". It corresponds to a logical type.

There are two ways in which signs are similar. The names "Socrates" and "Plato" are similar: they are both names. But whatever they have in common must not be introduced before "Socrates" and "Plato" are introduced. The same applies to a subject-predicate form, etc. Therefore, thing, proposition, subject-predicate form, etc., are not indefinables, i.e. types are not indefinables.

Every proposition that says something indefinable about one thing is a subject-predicate proposition, etc. Therefore, we can recognize a subject-predicate proposition, if we know it contains only one name and one form, etc. This gives the construction of types. Hence the type of a proposition can be recognized by its symbol alone.

What is essential in a correct apparent-variable notation is this: (1) it must mention a type of proposition, (2) it must show which components (forms and constituents) of a proposition of this type are constants. Take (ϕ).ϕ!x. Then, if we describe the *kind* of symbols for which ϕ stands, the which, by the above, is enough to determine the type, then auto-

matically “(ϕ).ϕ!x” cannot be fitted by this description, because it *contains* “ϕ!x” and the description is to describe *all* that symbolizes in symbols of the ϕ!x kind. If the description is *thus* completed, vicious circles can just as little occur as can for instance (ϕ).(x)ϕ where (x)ϕ is a subject-predicate proposition.

We can never distinguish one logical type from another by attributing a property to members of the one which we deny to members of the other. Types can never be distinguished from each other by saying (as is currently done) that one has these *but* the other has those properties, for this presupposes that there is a *meaning* in asserting all these properties of both types. And, from this it follows that, at least, these properties may be types, but certainly not the objects of which they are asserted.