

Formulaire de trigonométrie

$\tan(x) = \frac{\sin(x)}{\cos(x)}$ définie si $x \neq \frac{\pi}{2} \ (\pi)$	$\cotan(x) = \frac{1}{\tan(x)} = \frac{\cos(x)}{\sin(x)}$ définie si $x \neq 0 \ (\pi)$
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$\cos^2(x) + \sin^2(x) = 1$	$1 + \tan^2(x) = \frac{1}{\cos^2(x)}$ si $x \neq \frac{\pi}{2} \ (\pi)$	$1 + \cotan^2(x) = \frac{1}{\sin^2(x)}$ si $x \neq 0 \ (\pi)$
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$\cos(-a) = \cos(a)$	$\sin(-a) = -\sin(a)$	$\tan(-a) = -\tan(a)$	$\cotan(-a) = -\cotan(a)$
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$\cos(\pi - x) = -\cos(x)$	$\cos\left(\frac{\pi}{2} - x\right) = \sin(x)$	$\cos(\pi + x) = -\cos(x)$	$\cos\left(x + \frac{\pi}{2}\right) = -\sin(x)$
$\sin(\pi - x) = \sin(x)$	$\sin\left(\frac{\pi}{2} - x\right) = \cos(x)$	$\sin(\pi + x) = -\sin(x)$	$\sin\left(x + \frac{\pi}{2}\right) = \cos(x)$
$\tan(\pi - x) = -\tan(x)$	$\tan\left(\frac{\pi}{2} - x\right) = \cotan(x)$	$\tan(\pi + x) = \tan(x)$	$\tan\left(x + \frac{\pi}{2}\right) = -\cotan(x)$

Valeurs remarquables :

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	$\frac{2\pi}{3}$	π
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	-1
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	0
tan	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$		$-\sqrt{3}$	0

Formules d'addition

$\cos(a+b) = \cos(a)\cos(b) - \sin(a)\sin(b)$	$\cos(a-b) = \cos(a)\cos(b) + \sin(a)\sin(b)$
$\sin(a+b) = \sin(a)\cos(b) + \cos(a)\sin(b)$	$\sin(a-b) = \sin(a)\cos(b) - \cos(a)\sin(b)$
$\tan(a+b) = \frac{\tan(a) + \tan(b)}{1 - \tan(a)\tan(b)}$	$\tan(a-b) = \frac{\tan(a) - \tan(b)}{1 + \tan(a)\tan(b)}$

En particulier on a les relations suivantes avec l'angle double :

$\cos(2a) = \cos^2(a) - \sin^2(a) = 2\cos^2(a) - 1 = 1 - 2\sin^2(a)$	$\tan(2a) = \frac{2\tan(a)}{1 - \tan^2(a)}$
$\sin(2a) = 2\sin(a)\cos(a)$	
$\cos^2(a) = \frac{1 + \cos(2a)}{2}$	
$\sin^2(a) = \frac{1 - \cos(2a)}{2}$	

On dispose également de relations avec la tangente de l'angle moitié.

Si $a \neq \pi \ (2\pi)$, on pose $t = \tan\left(\frac{a}{2}\right)$ alors	$\cos(a) = \frac{1 - t^2}{1 + t^2}$	$\sin(a) = \frac{2t}{1 + t^2}$	$\tan(a) = \frac{2t}{1 - t^2}$
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Formules de linéarisation :

$\sin(a) \cos(b) = \frac{1}{2} [\sin(a+b) + \sin(a-b)]$
$\cos(a) \cos(b) = \frac{1}{2} [\cos(a+b) + \cos(a-b)]$
$\sin(a) \sin(b) = -\frac{1}{2} [\cos(a+b) - \cos(a-b)]$

$\sin(p) + \sin(q) = 2 \sin\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$
$\sin(p) - \sin(q) = 2 \cos\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$
$\cos(p) + \cos(q) = 2 \cos\left(\frac{p+q}{2}\right) \cos\left(\frac{p-q}{2}\right)$
$\cos(p) - \cos(q) = -2 \sin\left(\frac{p+q}{2}\right) \sin\left(\frac{p-q}{2}\right)$

Equations trigonométriques

$\cos(a) = \cos(b) \Leftrightarrow \begin{cases} a = b & (2\pi) \\ a = -b & (2\pi) \end{cases}$
$\sin(a) = \sin(b) \Leftrightarrow \begin{cases} a = b & (2\pi) \\ a = \pi - b & (2\pi) \end{cases}$
$\tan(a) = \tan(b) \Leftrightarrow a = b \quad (\pi)$

Lien avec l'exponentielle complexe

$e^{ix} = \cos(x) + i \sin(x)$	
$\cos(x) = \operatorname{Re}(e^{ix}) = \frac{1}{2} (e^{ix} + e^{-ix})$	$\sin(x) = \operatorname{Im}(e^{ix}) = \frac{1}{2i} (e^{ix} - e^{-ix})$
$e^{ia} + e^{ib} = 2 \cos\left(\frac{a-b}{2}\right) e^{i\left(\frac{a+b}{2}\right)}$	$1 + e^{ia} = 2 \cos\left(\frac{a}{2}\right) e^{i\left(\frac{a}{2}\right)}$
$e^{ia} - e^{ib} = 2i \sin\left(\frac{a-b}{2}\right) e^{i\left(\frac{a+b}{2}\right)}$	$1 - e^{ia} = -2i \sin\left(\frac{a}{2}\right) e^{i\left(\frac{a}{2}\right)}$