# **ANNEXE** : *Formulaire de trigonométrie*

**Angles particuliers**

| $x$ | $0$ | $\frac{\pi}{6}$ | $\frac{\pi}{4}$ | $\frac{\pi}{3}$ | $\frac{\pi}{2}$ | $\pi$ |
|---|---|---|---|---|---|---|
| $\cos x$ | $1$ | $\frac{\sqrt{3}}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{1}{2}$ | $0$ | $-1$ |
| $\sin x$ | $0$ | $\frac{1}{2}$ | $\frac{\sqrt{2}}{2}$ | $\frac{\sqrt{3}}{2}$ | $1$ | $0$ |
| $\tan x$ | $0$ | $\frac{\sqrt{3}}{3}$ | $1$ | $\sqrt{3}$ | $\Delta$ | $0$ |

**Dérivées**

| | | |
|---|---|---|
| $\cos' x$ | $=$ | $-\sin x$ |
| $\sin' x$ | $=$ | $\cos x$ |
| $\tan' x$ | $=$ | $1+\tan^2 x = \frac{1}{\cos^2 x}$ |
| $\cot' x$ | $=$ | $-1-\cot^2 x = -\frac{1}{\sin^2 x}$ |

**Premières formules**

| | | | | |
|---|---|---|---|---|
| $\tan a = \frac{\sin a}{\cos a}$ | $\cot a = \frac{1}{\tan a} = \frac{\cos a}{\sin a}$ | $\cos^2 a + \sin^2 a = 1$ | $1+\tan^2 a = \frac{1}{\cos^2 a}$ | $1+\cot^2 a = \frac{1}{\sin^2 a}$ |

**Parité et symétries**

| | | |
|---|---|---|
| $\cos(-a) = \cos a$ | $\sin(-a) = -\sin a$ | $\tan(-a) = -\tan a$ |
| $\cos(\pi - a) = -\cos a$ | $\sin(\pi - a) = \sin a$ | $\tan(\pi - a) = -\tan a$ |
| $\cos(\pi + a) = -\cos a$ | $\sin(\pi + a) = -\sin a$ | $\tan(\pi + a) = \tan a$ |
| $\cos\left(\frac{\pi}{2} - a\right) = \sin a$ | $\sin\left(\frac{\pi}{2} - a\right) = \cos a$ | $\tan\left(\frac{\pi}{2} - a\right) = \cot a$ |
| $\cos\left(\frac{\pi}{2} + a\right) = -\sin a$ | $\sin\left(\frac{\pi}{2} + a\right) = \cos a$ | $\tan\left(\frac{\pi}{2} + a\right) = -\cot a$ |

**Formules de développement**

| | |
|---|---|
| $\cos(a+b) = \cos a\cos b - \sin a \sin b$ | $\cos(a-b) = \cos a\cos b + \sin a\sin b$ |
| $\sin(a+b) = \sin a\cos b + \sin b\cos a$ | $\sin(a-b) = \sin a\cos b - \sin b\cos a$ |
| $\tan(a+b) = \frac{\tan a + \tan b}{1-\tan a\tan b}$ | $\tan(a-b) = \frac{\tan a - \tan b}{1+\tan a\tan b}$ |

En particulier : **Formules de duplication** :

| | | |
|---|---|---|
| $\cos(2a) = \cos^2 a - \sin^2 a = 1 - 2\sin^2 a = 2\cos^2 a - 1$ | $\sin(2a) = 2\sin a\cos a$ | $\tan(2a) = \left(\frac{2\tan a}{1-\tan^2 a}\right)$ |

**Transformation de produits en sommes** :

$$\cos a\cos b = \frac{1}{2}\left(\cos(a+b)+\cos(a-b)\right)$$
$$\sin a\sin b = \frac{1}{2}\left(\cos(a-b)-\cos(a+b)\right)$$
$$\sin a\cos b = \frac{1}{2}\left(\sin(a+b)+\sin(a-b)\right)$$

En particulier : **Linéarisation** :

$$\cos^2 a = \frac{1+\cos 2a}{2}$$
$$\sin^2 a = \frac{1-\cos 2a}{2}$$

**Transformation de sommes en produits.**

$$\cos p + \cos q = 2\cos\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\cos p - \cos q = -2\sin\frac{p+q}{2}\sin\frac{p-q}{2}$$
$$\sin p + \sin q = 2\sin\frac{p+q}{2}\cos\frac{p-q}{2}$$
$$\sin p - \sin q = 2\sin\frac{p-q}{2}\cos\frac{p+q}{2}$$

**Arc moitié.** Posons $t = \tan\frac{a}{2}$.

$$\cos a = \frac{1-t^2}{1+t^2} \qquad \sin a = \frac{2t}{1+t^2} \qquad \tan a = \frac{2t}{1-t^2}$$

**Factorisation d'une combinaison linéaire de** $\cos x$ **et** $\sin x$ : $a, b \neq 0$ :

$$\boxed{a\cos x + b\sin x = \sqrt{a^2+b^2}\cos(x-\theta)} \quad \text{où} \quad \begin{cases} \cos\theta = \frac{a}{\sqrt{a^2+b^2}} \\ \sin\theta = \frac{b}{\sqrt{a^2+b^2}} \end{cases}$$

**Equations trigonométriques**

$$\sin x = \sin a \iff x \equiv a\ [2\pi] \text{ ou } x \equiv \pi - a\ [2\pi]$$
$$\cos x = \cos a \iff x \equiv a\ [2\pi] \text{ ou } x \equiv -a\ [2\pi]$$
$$\tan x = \tan a \iff x \equiv a\ [\pi]$$