Efficient directed scattering of XUV radiation using high-density spherical clusters

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1 Introduction

Limited size targets interacting with high-intensity coherent radiation is well-studied phenomenon of linear excited surface plasmonic oscillations. Absorption and scattering of incident light in this case good described with Mie theory predicting exist of resonance corresponding to multipole oscillations of part of the target free electrons regarding positive charged ions. In resonance mode efficient exciting of surface plasmons can lead to significant boost internal and external field on fundamental cluster frequency (eigenfrequency). In turn, this can cause enhancement of field scattered on large angles regarding the direction of incident wave.

In micrometer wavelengths photon crystals and lattices can be used for direction or diffraction electromagnetic waves [1], while for x-ray radiation it is possible to use real crystals with regularly placed scattering centers (atoms) with distance of few nanometers [2]. At the same time, large interval between these wavelength orders named XUV (extreme-ultraviolet) is hard to manipulate.

Within the present work we consider the possibility of directed scattering of short wavelength radiation in the XUV range by scattering on suitable spherical clusters. Similar case with cylindrical symmetry (arrays of nanocylinders as scatterers) was researched earlier [3]. Of course, nanocylinders are more suitable regarding the control of size an distance parameters at the target manufacturing stage, but arrays of spherical clusters can make possible to manipulate with light direction in three-dimensional space and give a more optimal spatial configuration.

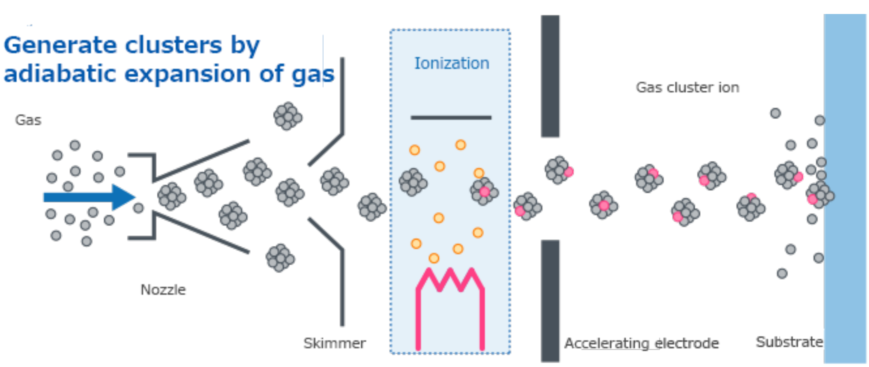

Figure 1: Ionized cluster gas generation process.

It is known that a short intense laser pulse can generate high-order harmonics by interacting with dense solid surfaces. But intensity of high-order harmonics generated in gases is at least 4 orders of magnitude less that is not enough to ionize the target and generate a plasma with fully imaginary refractive index that we need — in our case, spherical clusters are ionized cluster gas (Figure 1). To solve this problem we propose to use intense preceding pulse to pre-ionize the target and reach required plasma generation.

Common interaction scheme is shown in Figure 2. Harmonics in the main pulse have different intensity depending on the angle, that leads to the angle dependence of output radiance shape. The scattering by a single cluster can be completely described in spherical symmetry and the interaction can be easily modeled with the help of particle in cell simulations. We propose to use linear approximation by Mie theory as assessment for further modeling. In general, we concentrate on a theoretical investigation, supported by simulations, and we point out the applicability for experimental realisation.

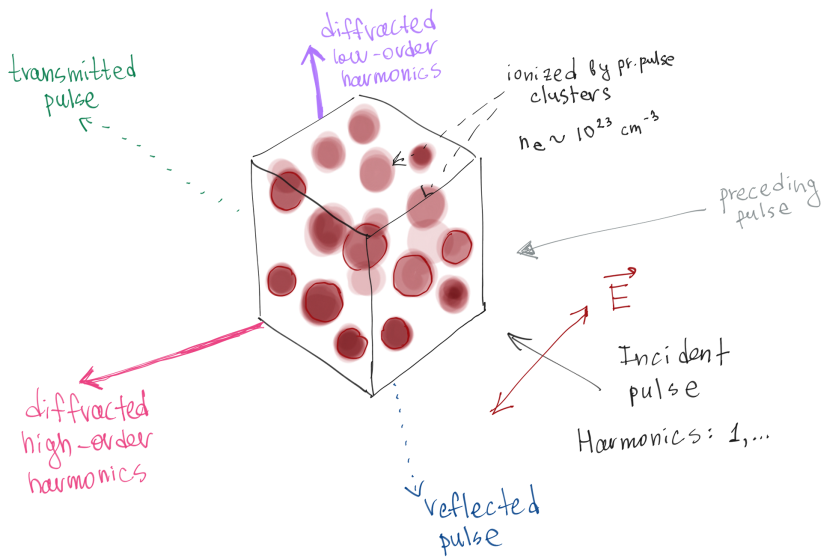

Figure 2: Interaction scheme. The plane of polarization is parallel to one of the faces of cubic region. The dimensions of spherical clusters are about a few nanometers, distance between them is at least wavelength. In general, the distribution of clusters within a cubic region is random, clusters do not intersect the edges of the region and each other.

2 Base model

Let us consoider a single cluster with radius a irradiated by short femtosecond pulse with intensity about $I_h \approx 10^{14}$ W/cm^2. The Drude model yields the dielectric function of the plasma:

$$\varepsilon(\omega) = 1 - \left(\frac{\omega_{pe}}{\omega}\right)^2 \frac{1}{1 + i\beta_e}, \qquad \omega_{pe} = \sqrt{\frac{4\pi e^2 n_e}{m_e}}, \tag{1}$$

where ω — harmonic (angular) frequency under consideration; ω_{pe} — the electron plasma frequency; e, m_e — electron charge and mass; $n_e = Zn_i$ — the electron number density, where Z — average ionization degree, n_i — ion density. $\beta_e = v_e/\omega$ and v_e — electron-ion collision rate in Spitzer approximation. As we are going to consider scattering of harmonic radiation, the cluster should have a density above the critical one for this harmonic: $n_c = \omega^2 m_e / 4\pi e^2$. Thus for example, for 10-th laser harmonic with wavelength $\lambda_L = 830$ nm one obtains condition $n_e > 1.3 \cdot 10^{23}$ cm^{-3}.

The Mie theory can be used for the description of elastic electromagnetic wave scattering by arbitrary sized particles in case of linear interactions and let obtain scattered and internal field. A main step is to solve the scalar Helmholtz Equation in suitable coordinate system andgain the vector solutions. For spherical cluster the solution of corresponding equation can be written in the form of Bessel and Hankel functions of n-th order [4].

Assume an incident plane wave propagating along z axis of cartesian coordinate system and polarized along x axis:

$$\overrightarrow{\mathbf{E}_i} = E_0 e^{i\omega t - ikz}\,\overrightarrow{\mathbf{e}_x}, \tag{2}$$

where $k = \omega/c$ — wavenumber, $\overrightarrow{\mathbf{e}_x}$ — the unit vector of x axis direction and polarization vector:

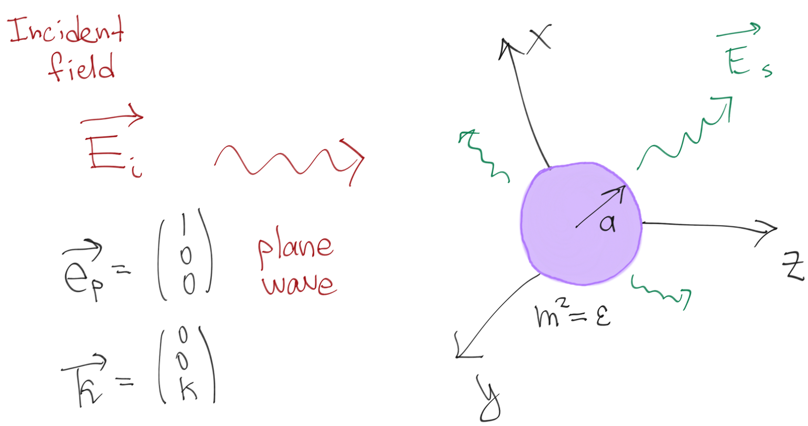

Figure 3: Base model scheme.

Now we can expand the plane wave into series using generalized Fourier expantions. Assuming our media is isotropic we obtain following form of scattered field [4]:

$$\overrightarrow{\mathbf{E}_s} = \sum_{n=1}^{\infty} E_n\Big[ia_n(ka,m)\overrightarrow{\mathbf{N}}^{(3)}_{e1n} - b_n(ka,m)\overrightarrow{\mathbf{M}}^{(3)}_{o1n}\Big], \qquad E_n = i^n E_0 \frac{2n+1}{n(n+1)} \tag{3}$$

n — vector harmonic number after cartesian-spherical coordinate system transformation, $m = \sqrt{\varepsilon(\omega)}$ — refractive index of the target. Vector harmonics coefficients have the following form [4]:

$$a_n(x,m) = \frac{m\psi_n'(x)\psi_n(mx) - \psi_n'(mx)\psi_n(x)}{m\xi_n'(x)\psi_n(mx) - \psi_n'(mx)\xi_n(x)}, \tag{4}$$

$$b_n(x,m) = \frac{\psi_n'(x)\psi_n(mx) - m\psi_n'(mx)\psi_n(x)}{\xi_n'(x)\psi_n(mx) - m\psi_n'(mx)\xi_n(x)}, \tag{5}$$

$\psi_n(z) = zj_n(z)$, $\xi_n(z) = zh_n(z)$ — Riccati-Bessel functions, $h_n = j_n + i\gamma_n$ — spherical Hankel functions of the first kind.

In case of spherical symmetry amplitude of the scattered field is maximum for $m^2 = -(n+1)/n$ when $ka \ll 1$, that gain corresponding set of resonance densities in collision-less case: $n_e = n_c(2n+1)/n$.

$$a_n(x \to 0, m) = \left(1 + 2i\frac{(2n-1)!(2n+1)!}{4^n\, n!(n+1)!}\frac{\left(m^2 + \frac{n+1}{n}\right)}{(m^2-1)}\frac{1}{x^{2n+1}}\right)^{-1}, \qquad b_n(x \to 0, m) = 0 \tag{6}$$

$$a_n(x,m) = \left(1 + i\frac{C_n x^{-1-2n}\left((4(1+n+m^2 n)(-3+4n(1+n)) - 2(m^2-1)(3+n(5+2n+m^2(2n-1)))x^2)\right)}{\pi(m^2-1)(2n+3)(n+1)(4(2n+3)-2(m^2+1)x^2)}\right)^{-1} \tag{7}$$

$$C_n = 2^{1+2n}\Gamma(n-\frac{1}{2})\Gamma(n+\frac{5}{2})$$

References

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