

Feuille d'exercice n° 17 : Analyse asymptotique - correction

Exercice 14

1. 0

2. $\frac{1}{6}$

3. $e^{e^{-1}}$

4. e^{-1}

5. $\frac{2}{3}$

6. $\frac{\sqrt{2}}{8x^3}$

7. $\frac{a^3}{b^3}$

8. -1

9. $-\frac{1}{2\sqrt{2}}$

10. $\frac{x^2}{2}$

11. $-\frac{3}{2}\left(x - \frac{\pi}{4}\right)$

12. \sqrt{e}

13. $\frac{1}{\pi}$

14. 1

15. $-x$

Exercice 15 $\lim_{x \rightarrow 1} \frac{x^x - 1}{\ln x} = 1$

$$\lim_{x \rightarrow 0} \left(\frac{x^2}{\ln(\cos x)} + \frac{2}{x^2} \sin^2 x \right) = 0$$

$$\lim_{x \rightarrow \frac{\pi}{4}} \frac{\ln(\sin^2 x)}{\left(\frac{\pi}{2} - x\right)^2} = -\frac{16 \ln 2}{\pi^2}$$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\ln(\sin^2 x)}{\left(\frac{\pi}{2} - x\right)^2} = -1$$

$$\lim_{x \rightarrow +\infty} \sin \frac{1}{x} \tan \left(\frac{2\pi x}{4x+3} \right) = \frac{8}{3\pi}$$

$$\lim_{x \rightarrow 0^+} \ln x \tan(\ln(1+x)) = 0$$

$$\lim_{x \rightarrow e} (\ln x)^{\tan \frac{\pi x}{2e}} = e^{-\frac{2}{\pi}}$$

Exercice 17

1. $\ln(\cos x) = -\frac{1}{2}x^2 - \frac{1}{12}x^4 - \frac{1}{45}x^6 + o(x^6)$
2. $\tan(x) = x + \frac{1}{3}x^3 + \frac{2}{15}x^5 + o(x^5)$
3. $\sin(\tan(x)) = x + \frac{1}{6}x^3 - \frac{1}{40}x^5 + o(x^5)$
4. $(\ln(1+x))^2 = x^2 - x^3 + \frac{11}{12}x^4 + o(x^4)$
5. $\exp(\sin(x)) = 1 + x + \frac{1}{2}x^2 + o(x^3)$
6. $\sin^6(x) = x^6 - x^8 + o(x^9)$

Exercise 18

1. $\sqrt{x+1} = \sqrt{x} + \frac{1}{2\sqrt{x}} - \frac{1}{8x^{3/2}} + o\left(\frac{1}{x^{3/2}}\right)$;
2. $x \ln(x+1) - (x+1) \ln x = 1 - \ln(x) - \frac{1}{2x} + \frac{1}{3x^2} + o\left(\frac{1}{x^2}\right)$;
3. $\left(\frac{x+1}{x}\right)^x = e - \frac{1}{2} \frac{e}{x} + \frac{11}{24} \frac{e}{x^2} + o\left(\frac{1}{x^2}\right)$;
4. $\operatorname{Arctan} x = \frac{\pi}{2} - \frac{1}{x} + \frac{1}{3x^3} + o\left(\frac{1}{x^4}\right)$.

Exercise 19

1. $\frac{\arctan x - x}{\sin x - x} = 2 - \frac{11}{10}x^2 + o(x^2)$;
2. $\ln \sin x = \ln(\sqrt{2}/2) + (x - \pi/4) - (x - \pi/4)^2 + \frac{2}{3}(x - \pi/4)^3 + o((x - \pi/4)^3)$;
3. $(1+x)^{\frac{1}{x}} = e \left(1 - \frac{1}{2}x + \frac{11}{24}x^2 - \frac{7}{16}x^3\right) + o(x^3)$;
4. $x(\sqrt{x^2 + \sqrt{x^4 + 1}} - x\sqrt{2}) = \frac{\sqrt{2}}{8x^2} + o\left(\frac{1}{x^2}\right)$.

Exercise 22

1. $\sqrt[3]{x^3 + x^2} - \sqrt[3]{x^3 - x^2} = x \left(\sqrt{1 + \frac{1}{x}} - \sqrt{1 - \frac{1}{x}}\right) = 1 + \frac{1}{8x^3} + o(1/x^3)$.
2. $\ln(\sqrt{1+x}) = \frac{1}{2} \ln x + \frac{1}{2x} + o(1/x)$.

Exercise 23

1. (a) $\frac{\cos x}{\sqrt{1+x}} = 1 - \frac{x}{2} - \frac{x^2}{8} - \frac{x^3}{16} + \frac{49x^4}{384} + o(x^4)$
- (b) $\frac{\sqrt{1+x}}{\cos x} = 1 + \frac{x}{2} + \frac{3x^2}{8} + \frac{5x^3}{16} + \frac{41x^4}{384} + o(x^4)$
- (c) $\frac{\ln(1+x)}{\cos x} = x - \frac{x^2}{2} + \frac{5x^3}{6} - \frac{x^4}{2} + o(x^4)$

- (d) $\frac{1 + \cos x}{2 + \sin x} = 1 - \frac{x}{2} + \frac{x^3}{12} - \frac{x^4}{16} + o(x^4)$
- (e) $\frac{\sin(x/2)}{e^{2x}} = \frac{x}{2} - x^2 + \frac{47x^3}{48} - \frac{5x^4}{8} + o(x^4)$
- (f) $\frac{\ln(1+x)}{2 - \cos x} = x - \frac{x^2}{2} - \frac{x^3}{6} + o(x^4)$
2. (a) $\frac{\sin(2x - \pi/4)}{\cos x} = 1 + 3(x - \frac{\pi}{4}) + \frac{3}{2}(x - \frac{\pi}{4})^2 + \frac{3}{2}(x - \frac{\pi}{4})^3 + \frac{19}{8}(x - \frac{\pi}{4})^4 + o((x - \frac{\pi}{4})^4)$
- (b) $\frac{\cos(x-1)}{\ln(1+x)} = \frac{1}{\ln 2} - \frac{1}{2(\ln 2)^2}(x-1) + \frac{1}{\ln 2} \left(-\frac{1}{2} + \frac{1}{8 \ln 2} + \frac{1}{4(\ln 2)^2} \right) (x-1)^2 + o((x-1)^2)$
- (c) $\frac{e^{x-1}}{\ln x} = \frac{1}{x-1} + \frac{3}{2} + \frac{11}{12}(x-1) + \frac{3}{8}(x-1)^2 + \frac{71}{720}(x-1)^3 + \frac{41}{1440}(x-1)^4 + o((x-1)^4)$

Exercise 24

1. $e - 1$
2. $-\frac{1}{2}$
3. $-\frac{e}{2}$
4. $\frac{1}{12}$
5. $-\frac{1}{6}$
6. $-\frac{1}{2}$