Énoncé : Les membres d'une société internationale sont originaires de six pays différents. La liste des membres contient 1978 noms numérotés de 1 à 1978. Montrer qu'il y a un membre dont le numéro vaut la somme des numéros de deux autres membres venant du même pays ou le double du numéro d'un compatriote.

Solution (in english please) : The trick is to use differences.

6.329 = 1974, so at least 330 members come from the same country, call it C_1. Let their numbers be $a_1 < a_2 < \cdots < a_{330}$. Now take the 329 differences $a_2 - a_1, a_3 - a_1, \cdots, a_{330} - a_1$. If any of them are in C_1, then we are home, so suppose they are all in the other five countries.

At least 66 must come from the same country, call it C_2. Write the 66 as $b_1 < b_2 < \cdots < b_{66}$. Now form the 65 differences $b_2 - b_1, b_3 - b_1, \cdots, b_{66} - b_1$. If any of them are in C_2, then we are home. But each difference equals the difference of two of the original a_i s, so if it is in C_1 we are also home.

So suppose they are all in the other four countries. At least 17 must come from the same country, call it C_3. Write the 17 as $c_1 < c_2 < \cdots < c_{17}$. Now form the 16 differences $c_2 - c_1, c_3 - c_1, \cdots, c_{17} - c_1$. If any of them are in C_3, we are home. Each difference equals the difference of two b_i s, so if any of them are in C_2 we are home. [For example, consider $c_i - c_1$. Suppose $c_i = b_n - b_1$ and $c_1 = b_m - b_1$, then $c_i - c_1 = b_n - b_m$, as claimed.]. Each difference also equals the difference of two a_i s, so if any of them are in C_1, we are also home. [For example, consider $c_i - c_1$, as before. Suppose $b_n = a_j - a_1$, $b_m = a_k - a_1$, then $c_i - c_1 = b_n - b_m = a_j - a_k$, as claimed.]

So suppose they are all in the other three countries. At least 6 must come from the same country, call it C_4. We look at the 5 differences and conclude in the same way that at least 3 must come from C_5. Now the 2 differences must both be in C_6 and their difference must be in one of the $C_1, \cdots, C_6$ giving us the required sum.