Feuille d'exercice n° 16 : **Fractions rationnelles - fiche d'entraînement - correction**

Exercice 1

- $\dfrac{X^7+1}{X^2(X-1)^4} = X+4+\dfrac{4}{X}+\dfrac{1}{X^2}+\dfrac{6}{X-1}+\dfrac{13}{(X-1)^2}+\dfrac{3}{(X-1)^3}+\dfrac{2}{(X-1)^4}$

- $\dfrac{X^2+1}{(X-1)(X-2)} = 1+\dfrac{5}{X-2}-\dfrac{2}{X-1}$

- $\dfrac{X^4-3X+2}{(X-2)^3} = X+6+\dfrac{24}{X-2}+\dfrac{29}{(X-2)^2}+\dfrac{12}{(X-2)^3}$

- $\dfrac{X^2}{(X-1)^2(X+1)^3} = \dfrac{1}{8(X-1)^2}+\dfrac{1}{16(X-1)}+\dfrac{1}{4(X+1)^3}-\dfrac{1}{4(X+1)^2}-\dfrac{1}{16(X+1)}$

- $\dfrac{(X^2+4)^2}{(X^2+1)(X^2-2)^2} = -\dfrac{1}{2}\dfrac{i}{X-i}+\dfrac{1}{2}\dfrac{i}{X+1}-\dfrac{3}{4}\dfrac{\sqrt{2}}{X-\sqrt{2}}+\dfrac{3}{2}\dfrac{1}{(X-\sqrt{2})^2}+\dfrac{3}{4}\dfrac{\sqrt{2}}{X+\sqrt{2}}+\dfrac{3}{2}\dfrac{1}{(X+\sqrt{2})^2}$

- $\dfrac{X^2}{(X^2+X+1)^2} = \dfrac{2}{9}\dfrac{1+2j}{X-j^2}-\dfrac{1}{3}\dfrac{j}{(X-j^2)^2}-\dfrac{2}{9}\dfrac{1+2j}{X-j}+\dfrac{1}{3}\dfrac{1+j}{(X-j)^2}$

Exercice 2   a) $\dfrac{1}{8}\ln\left(\dfrac{(x+3)^6}{|x+5|^5|x+1|}\right)+K)$

b) $\dfrac{1}{x-1}+\ln\left|\dfrac{x-2}{x-1}\right|+K)$

c) $\dfrac{1}{\sqrt{2}}\ln(1+\sqrt{2})\,)$

e) $\ln(4/3))$

f) $\dfrac{1}{8}\text{Arctan}(x)-\dfrac{x(1-x^2)}{8(1+x^2)^2})$

g) $\dfrac{1-\mathrm{e}^{-x}}{2}+\ln(\text{ch}\dfrac{x}{2})+K)$

h) $2/3$

i) $\dfrac{1}{16}(\dfrac{\pi}{4}+\dfrac{1}{3}))$

j) $\dfrac{1}{4}+\dfrac{\pi}{8})$

k) $\dfrac{1}{3}\ln(x+1)-\dfrac{1}{6}\ln(x^2-x+1)+\dfrac{1}{\sqrt{3}}\text{Arctan}\dfrac{2x-1}{\sqrt{3}}+\dfrac{\pi}{6\sqrt{3}})$

l) $\dfrac{\pi}{4}$

m) $-\dfrac{1}{2\sin x}-\dfrac{1}{4}\ln(1-\sin x)+\dfrac{1}{4}\ln(1+\sin x)+K)$